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Laser-based ion acceleration:theoretical models

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Laser-based ion acceleration:
theoretical models

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Outline of the 2nd lecture

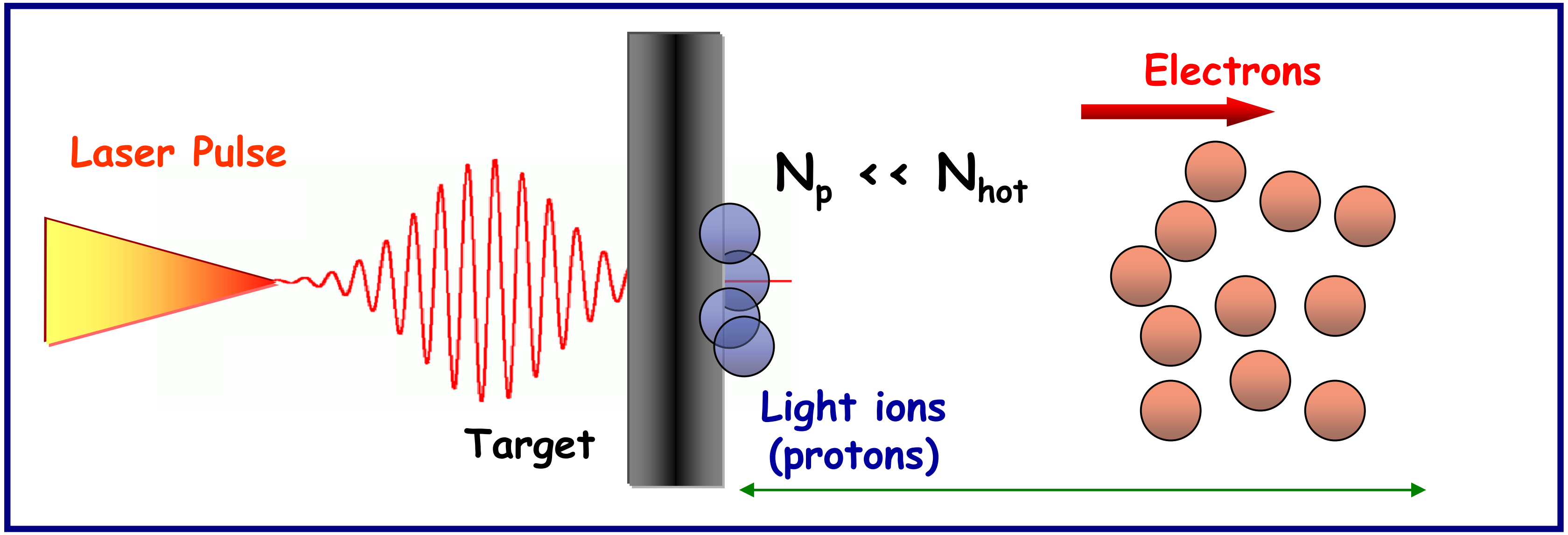
- Effects of non quasi-neutrality for sub-nanosecond expansion
    - order of magnitude and scaling of the accelerating field
    - early investigations of non-neutral plasma expansion
    - Mora's numerical model: maximum ion energy and spectrum

- focusing on the very early ion acceleration, when most energetic ions are produced

- quasi-static isothermal models: time-scales ≤ 100 fs (1 fs = 10^{-15} s)
    - the "truncation" of the domain of the Poisson equation
    - a self-consistent model
    - accelerating E field in the presence of two electron populations with different temperatures

- conclusions, future perspectives, general summary

Ion acceleration mechanism - 1

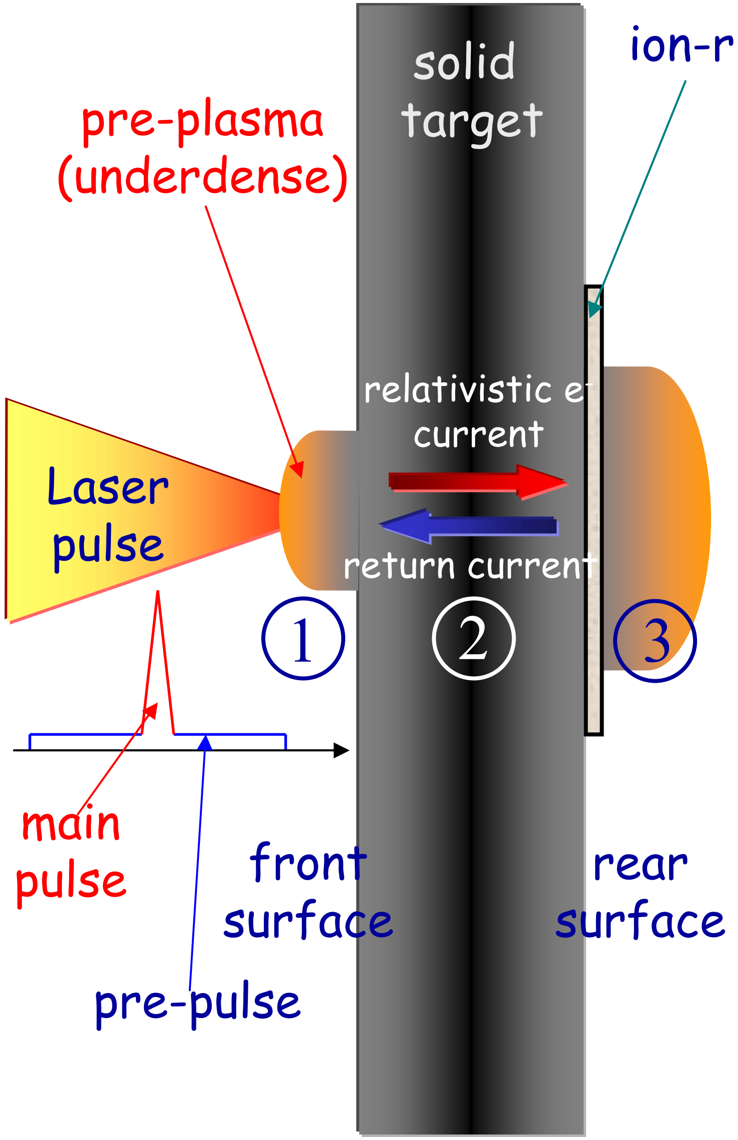


There is common agreement on the fact that the protons are accelerated by the electrostatic field set up by fast electrons propagating into or leaving the target.

Target Normal Sheath Acceleration mechanism (TNSA)

[S.C. Wilks, *et al.*, *Phys. Plasmas* **8**, 542 (2001)]

Ion acceleration mechanism - 2



1: laser pulse-front surface interaction;
generation of relativistic el. current

2: hot electron propagation in the target
possible if a return current is generated

3: expansion in vacuum;
charge separation at the rear surface;
generation of intense electric fields

Protons: bulk (CH) and/or contamination layers (oils, water vapour…)
and/or coated layers

Heavy ions: bulk and/or coated layers

Ion acceleration mechanism - 3

Let us simply estimate the field at the rear surface:

order of magnitude of electron energy ~ T_e

order of magnitude of el. cloud extention ~ λ_{de}

$$\longrightarrow \boxed{E \sim \frac{T_e}{e\lambda_{de}}}$$

[S.C. Wilks, *et al.*, *Phys. Plasmas* **8**, 542 (2001)]

From our Q.N. analysis, remember that $\boxed{E_{QN} \sim \dfrac{T_e}{eL_n}}$

Estimate of T_e: ponderomotive potential is the main mechanism of electron heating…

$$T_e = mc^2\left[\left(1 + \frac{I\lambda^2(\mu m)}{\alpha 1.4x10^{18}}\right)^{1/2} - 1\right]$$

[S.C. Wilks, W.L. Kruer, *IEEE Trans. Quantum Electr.* **33**, 1954 (1997)]

$\alpha \sim 1$ or 2 dep. polarization

Ion acceleration mechanism - 4

For ultraintense ultrashort laser pulses
($I > 10^{18}$ W/cm^2, $\lambda \sim 1$ μm, $n_e \sim n_c \sim 10^{21}$ cm^{-3}) we have

$T_e \sim$ MeV , $\lambda_{de} \sim$ μm ($\ll L_n$)

$$\boxed{E \sim \frac{\mathrm{MV}}{\mu\mathrm{m}} \ggg E_{QN}}$$

such fields can accelerate ion up to several MeV on a micrometer scale length!!

...more on plasma expansion in vacuum

Effects of non-neutrality in plasma expansion (still 1D)

$$\frac{\partial N_i}{\partial t} + \frac{\partial}{\partial x}(N_i v_i) = 0 \qquad M\left(\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x}\right) = -e\frac{\partial \phi}{\partial x}$$

charge separation

$$\frac{\partial^2 \phi}{\partial x^2} = -4\pi e(N_i - N_e)$$

electrons in equilibrium with the e.s. potential

$$N_e = N_0 \exp\left(\frac{e\phi}{T_e}\right)$$

$$\begin{cases} \partial^2_{xx}\phi = 4\pi e\left(N_0 e^{e\phi/T_e} - ZN_{0i}\right) & \text{... for } x < 0 \\ \partial^2_{xx}\phi = 4\pi e N_0 e^{e\phi/T_e} & \text{... for } x > 0 \end{cases}$$

Equations valid at $t = 0$

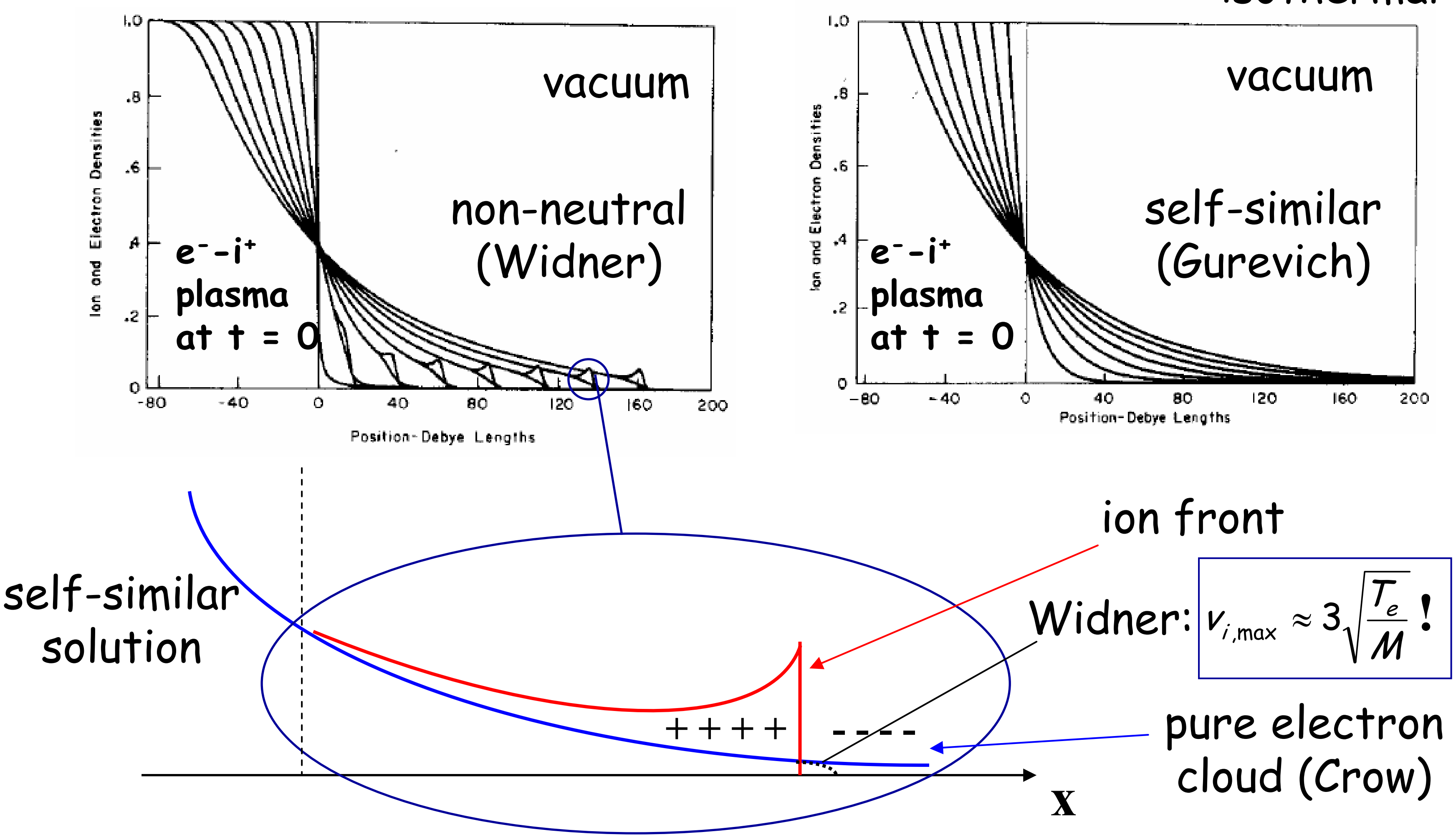
$$\boxed{\frac{e\phi(x)}{T_e} = -2\ln\left(1 + \frac{1}{\sqrt{2e}}\frac{x}{\lambda_D}\right) - 1}$$

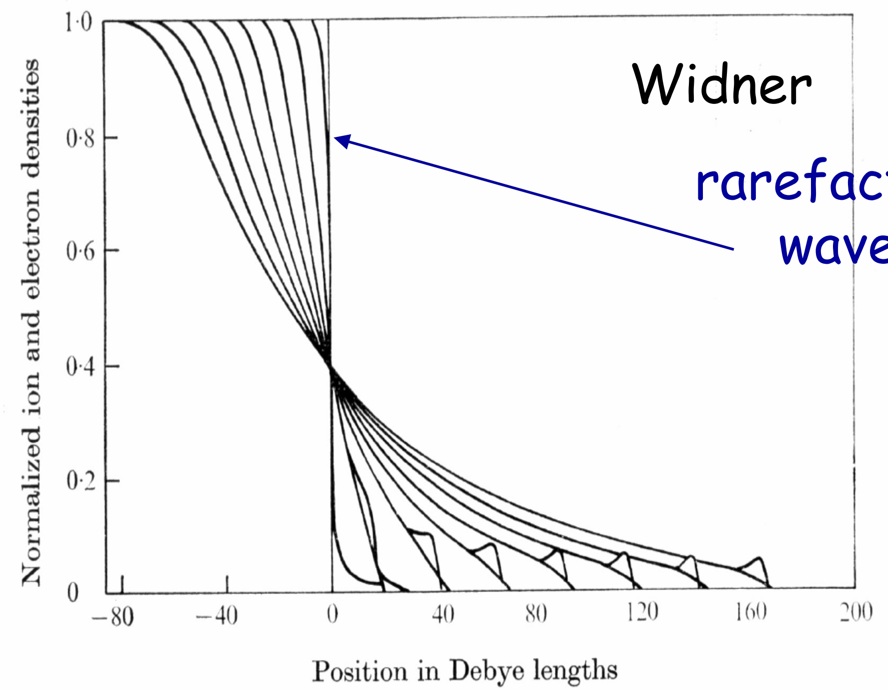
"*Plasma expansion into a vacuum*"
M. Widner, *et al.*, *Phys. Fluids* **14**, 795 (1971)
"*The expansion of a plasma into a vacuum*"
J.E. Crow, *et al.*, *J. Plasma Phys.* **14**, 65 (1975)

...more on plasma expansion in vacuum

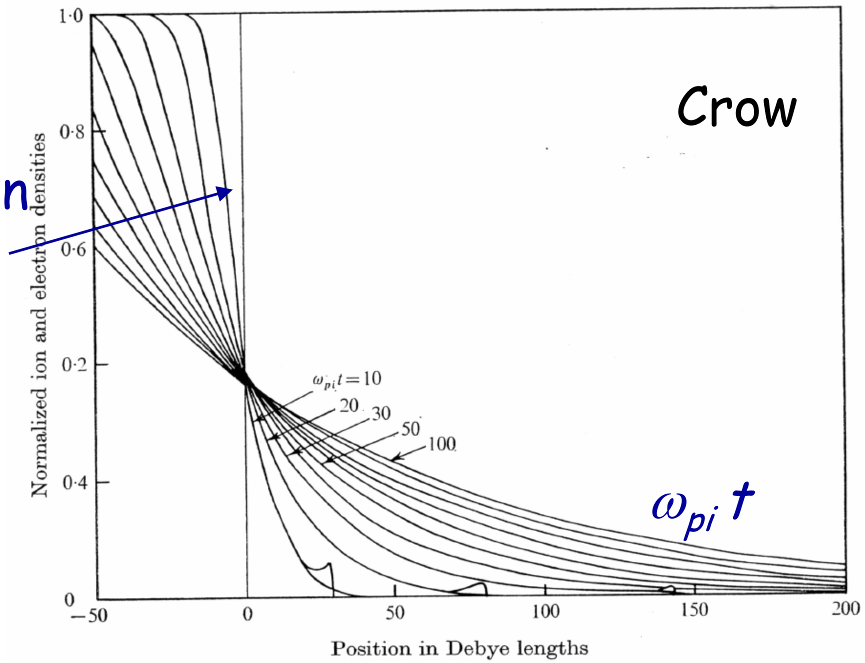
numerical integration of fluid eqs. with $T_i = 0$ and $\boxed{T_e = \text{const.}}$

isothermal model

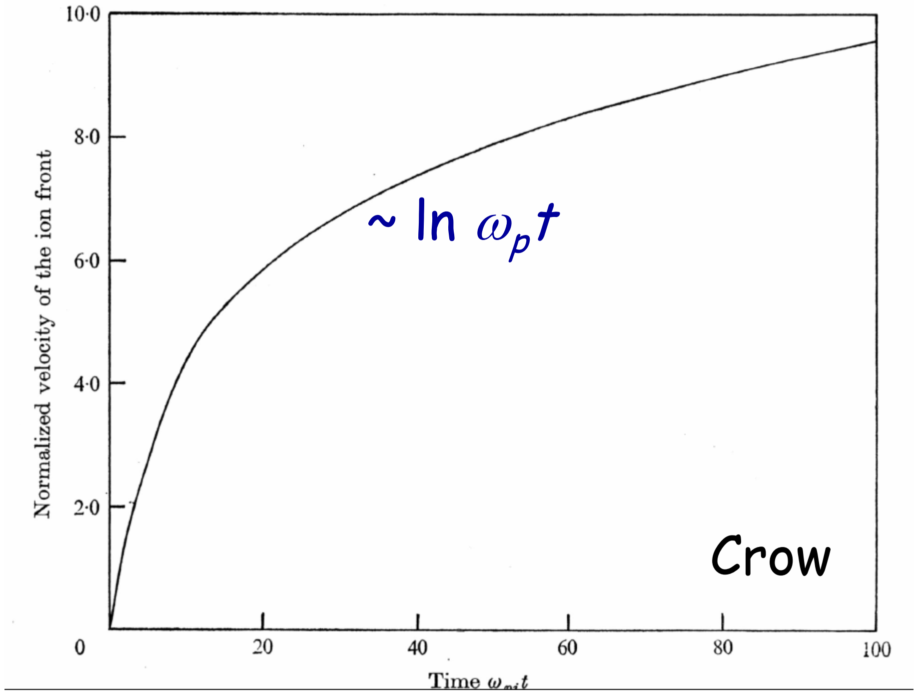




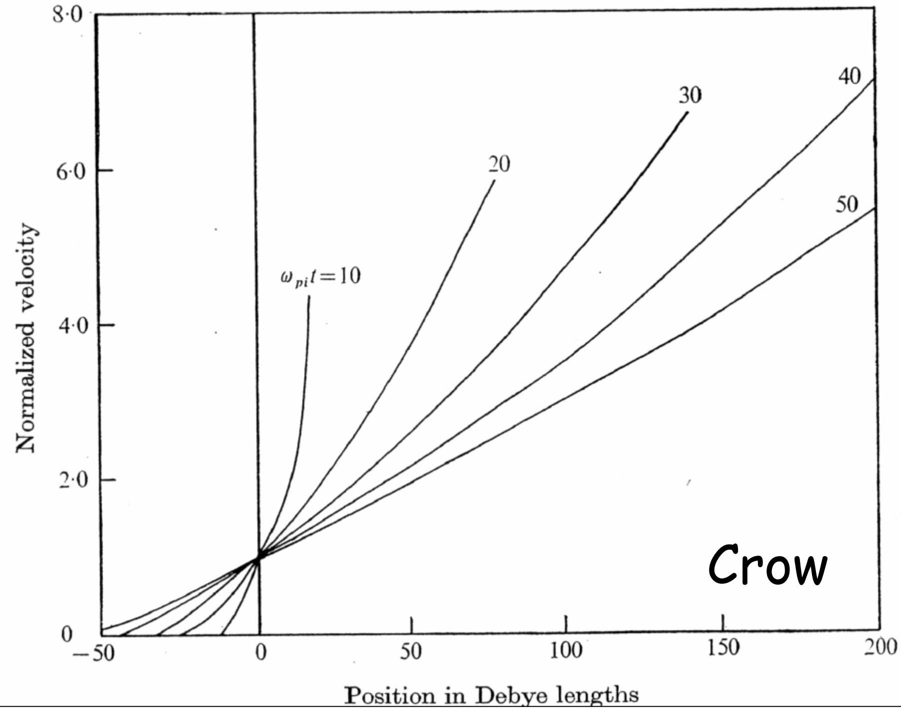
Widner
rarefaction
wave
Normalized ion and electron densities
Position in Debye lengths
1·0
0·8
0·6
0·4
0·2
0
−80
−40
0
40
80
120
160
200



Crow
Normalized ion and electron densities
Position in Debye lengths
$\omega_{pi}t=10$
20
30
50
100
$\omega_{pi}\ t$
1·0
0·8
0·6
0·2
0·4
0
−50
0
50
100
150
200



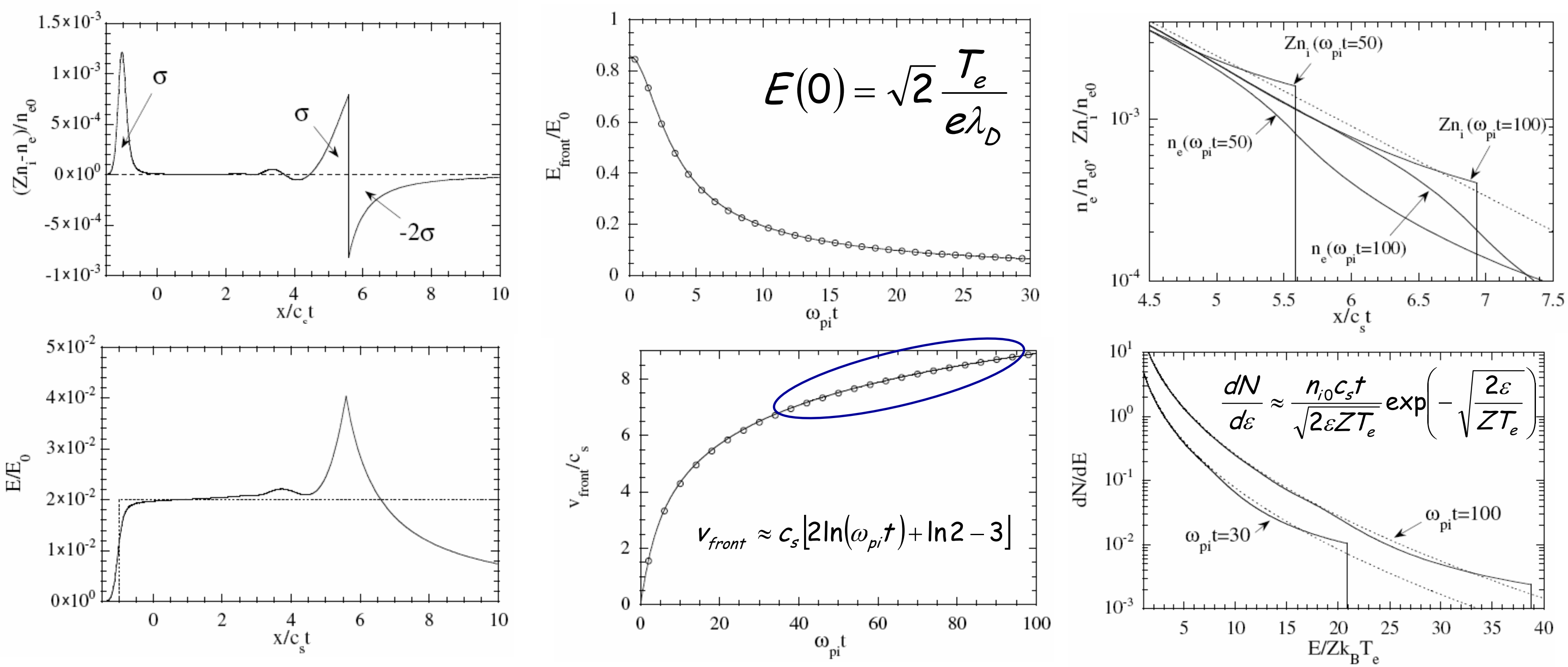
~ ln $\omega_p t$
Crow
Normalized velocity of the ion front
Time $\omega_{pi}t$
10·0
8·0
6·0
4·0
2·0
0
20
40
60
80
100



Crow
Normalized velocity
Position in Debye lengths
$\omega_{pi}t=10$
20
30
40
50
8·0
6·0
4·0
2·0
0
−50
0
50
100
150
200

Mora's hydrodynamic code - 1

"*Plasma expansion into a vacuum*"
P. Mora, *Phys. Rev. Lett.* **90**, 185002 (2003)
- ion fluid equations (T_i = 0) + Boltzmann electron distribution + Poisson equation (…like before!)
- Lagrangian code: Poisson is integrated between $x = x_{front}$ and $x = \infty$



Mora's hydrodynamic code - 2

Interpolating the numerical result, the following approximate expressions for the time dependence of the physical quantities at the ion front can be obtained:

$$E_{front} \approx \sqrt{\frac{2}{e}} \frac{E(0)}{\sqrt{1+\tau^2}} \qquad v_{front} \approx 2c_s \ln\left(\tau + \sqrt{\tau^2+1}\right) \qquad \tau = \frac{\omega_{pi} t}{\sqrt{2e}}$$

$$\varepsilon_{front} \approx 2ZT_e\left[\ln\left(\tau + \sqrt{\tau^2+1}\right)\right]^2 \qquad \frac{dN}{d\varepsilon} \approx \frac{n_{i0} c_s \tau}{\omega_{pi}\sqrt{ZT_e}} exp\left(-\sqrt{\frac{2\varepsilon}{ZT_e}}\right)$$

maximum kinetic energy at the ion front

[P. Mora, *Phys. Rev. Lett.* **90**, 185002 (2003)]

note that $\varepsilon_{front} \rightarrow \infty$ as $\tau \rightarrow \infty$

(consequence of Boltzmann, see below!!)

a maximum acceleration time must be introduced to describe the energy cut-off

Mora's hydrodynamic code - 3

[J. Fuchs, *et al.*, *Nature Phys.* **2**, 48 (2006)]

in order to fit several experimental data,

$t \approx 1.3\tau_{pulse}$

should be assumed.

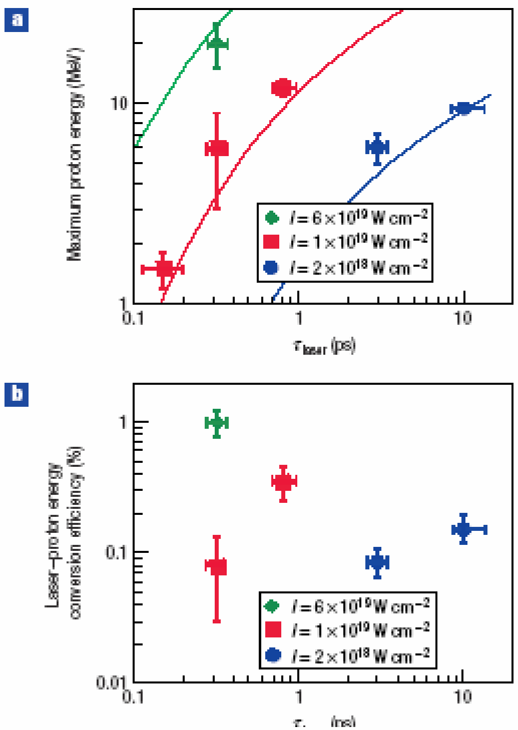


Figure 3 Longer pulses improve the laser-accelerated proton maximum as well as the energy conversion efficiency. a, Maximum energy of the proton beam and **b**, laser–proton energy conversion efficiency (for protons with energy >4 MeV) as a function of the laser pulse duration for three different laser intensities; the laser energy is increased with the laser pulse duration to keep the laser intensity constant for each group of points. The lines are calculations for each intensity using the fluid model. Error bars on the laser pulse duration represent the shot-to-shot fluctuation combined with the estimated error linked to assuming different pulse shapes for the pulse-duration retrieval. Vertical error bars are estimated similarly to Fig. 1.

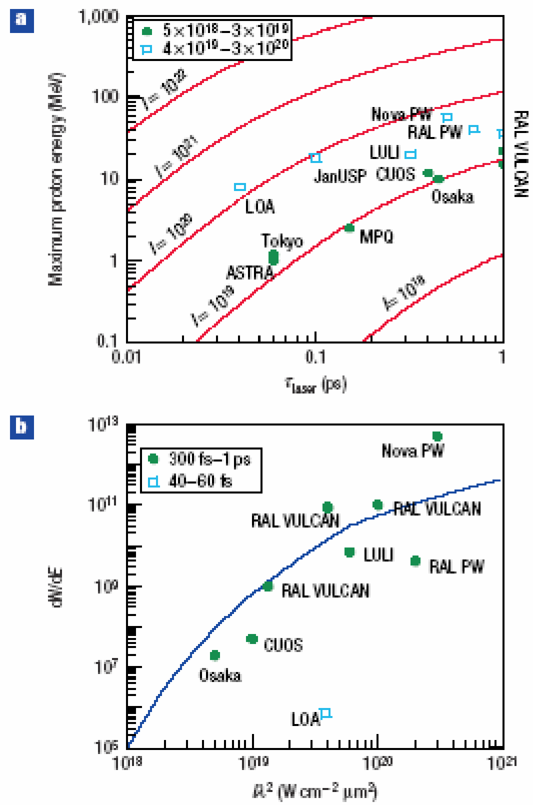


Figure 4 Comparison between fluid-model predictions and previously published data. a, Maximum proton energy as a function of laser pulse duration. Circles and squares are experimental data for the two intensity ranges; the intensities are in units of W cm^{-2}. Lines represent calculations for various laser intensities, as indicated in units of W cm^{-2}, using the fluid model assuming 20-µm-thick targets and a 10 µm FWHM laser spot size. **b**, Number of protons in a 1 MeV bin around 10 MeV as a function of laser intensity multiplied by the laser wavelength squared. The last parameter is chosen as it governs the hot electron temperature T_p. Circles and squares are experimental data for the two laser-pulse-duration ranges shown. The line is given by the fluid model assuming 20-µm-thick targets, a 10 µm FWHM laser spot size and a 0.5 ps laser pulse

…it is necessary to change this scaling for very short pulses (< 100 fs)

[P. Antici, PhD thesis (2007)]

Mora's hydrodynamic code - 4

This model has been found very useful and it is widely used to interpret many experimental data.

Limits of this kind of description:

- the accelerated protons/ions, are a thin layer of positive charge rather than a semi-infinite expanding plasma

- the empirical acceleration time can be unphysical in several situations, i.e. not corresponding to the actual ion acceleration time (in principle not directly related to τ_{pulse});
  - too short for very short pulses,
  - too long for the most energetic part of the spectrum and long pulses

- divergent maximum ion energy (see below!!)

Quasi-static theoretical models for ion acceleration - 1

The following physical picture can be assumed to properly describe the most energetic accelerated ions:

- at the rear surface, hot electrons create a non-neutral charge sheet which is the source of an electric field

- light ions (mainly protons) form a thin layer at the rear surface, while the main target is made of heavier ions

- during the characteristic acceleration time of the light ions hot electrons are almost isothermal (cooling mechanisms become important at longer times) while heavier ions are almost immobile:

- until the total number of accelerated light ions remains significantly lower than the total number of hot electrons, the field is not heavly affected by their motion

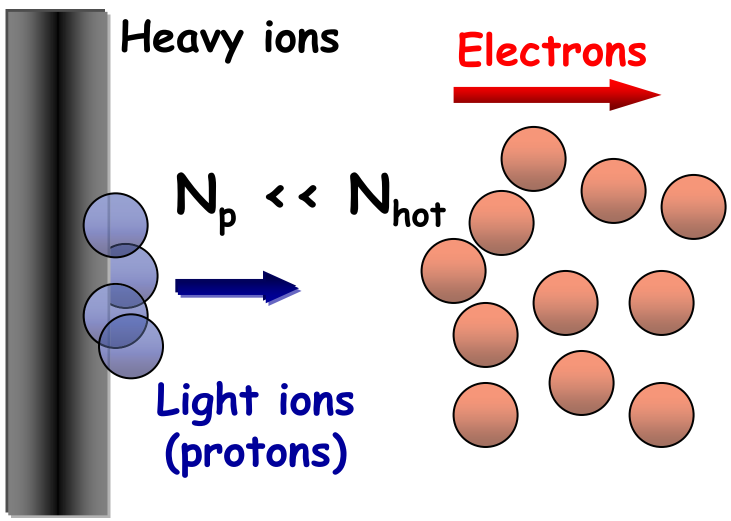
→ the accelerating field can be assumed as quasy-static!

Quasi-static theoretical models for ion acceleration - 2

Therefore, it is possible to build a model
for the laser-based ion acceleration in which:

- focus is on the accelerating electric field:
  - Hot electrons isothermal (Boltzmann)
  - Heavy ions immobile

- light ions accelerated
  in this field and treated as
  test particles



we do not speak any more of "*plasma expansion*" but of "*particle acceleration*"

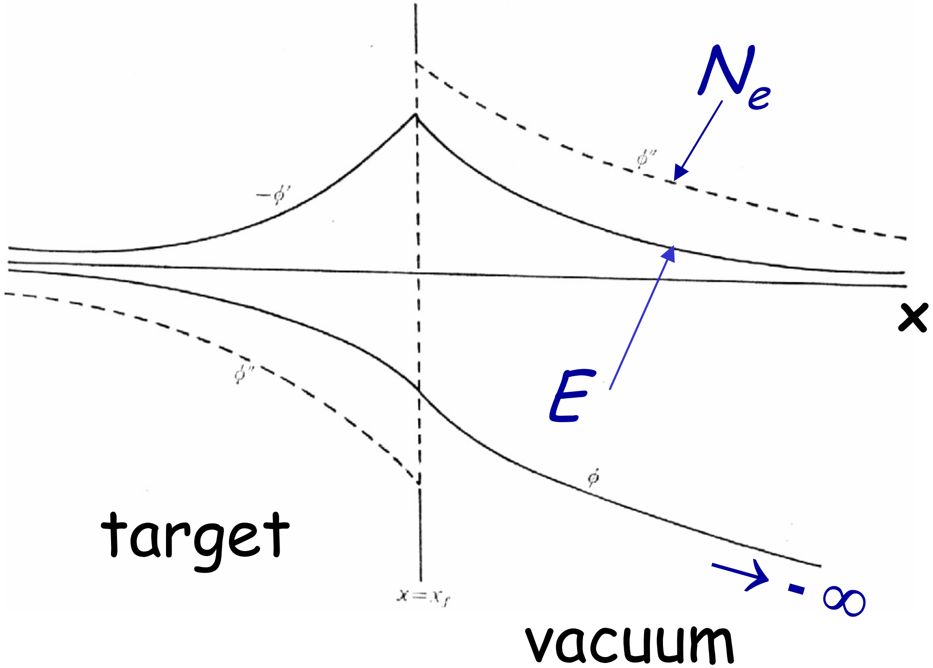
This description can be the most suitable to describe the most energetic part of the ion spectrum

Boltzmann distribution & infinite space

Now, if we assume: [J.E. Crow, *et al.*, *J. Plasma Phys.* **14**, 65 (1975)]

- Boltzmann density distribution of isothermal electrons
- Poisson equation extending over a semi-infinite domain

$$\begin{cases} \partial^2_{xx}\phi = 4\pi e\left(N_0 e^{e\phi/T_e} - ZN_{0i}\right) & \text{... for } x < 0 \\ \partial^2_{xx}\phi = 4\pi e N_0 e^{e\phi/T_e} & \text{... for } x > 0 \end{cases}$$



Exact solution for x > 0

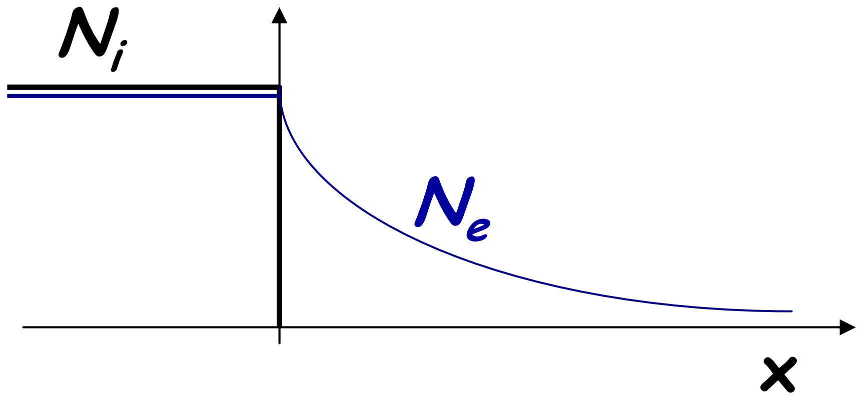
$$\frac{e\phi(x)}{T_e} = -2\ln\left(1 + \frac{1}{\sqrt{2e}}\frac{x}{\lambda_D}\right) - 1$$

isothermal electrons inevitably give
$\phi(x \to \infty) \to -\infty$!!!
in order to have $N_e(x \to \infty)$
and $E_x(x \to \infty) \to 0$

an ion initially at rest in x = 0,
will get an infinite energy
in an infinite time! (in 3D as well !)

Boltzmann distribution & infinite space

…i.e.: on the problem of maximum ion energy



$$N_e = N_0 e^{e\phi/T_e}$$

$$n_e \to 0 \;\; as \;\; x \to +\infty \quad \Longrightarrow \quad \phi \to -\infty \;\; as \;\; x \to +\infty$$

- regardless the dimensionality, final ion energy diverges
- remove the isothermal assumption

V.F. Kovalev, *et al.*, *JETP,* **95**, 226 (2002)
P. Mora, *Phys. Rev. E* **72**, 056401 (2005); *Phys. Pl.* **12**, 112102 (2005)
S. Betti, *et al.*, *Pl. Phys. Contr. Fus.* **47**, 521 (2005)

- isothermal models: introduce "truncation mechanisms"

Y. Kishimoto, *et al.*, *Phys. Fluids* **26**, 2308 (1983)
M.Passoni, M.Lontano, *Laser Part. Beams* **22**, 171 (2004)
M. Lontano, M. Passoni, *Phys.Plasmas*, **13**,042102 (2006)

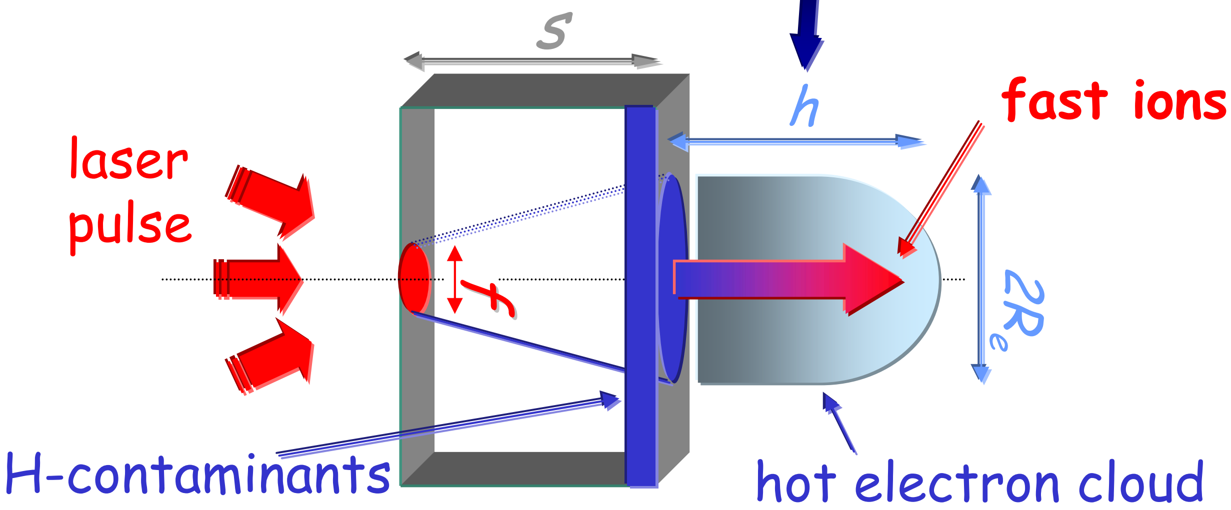
1T model in a finite interval - 1

[M.Passoni, M.Lontano, *Laser Part. Beams* **22**, 163 (2004)]

- 1-dim, 1-T equilibrium hot electron population created by the laser pulse and generating the electric field

1-dim Poisson-Boltzmann equation

- choose a finite spatial extension, h, of the hot electron cloud



1T model in a finite interval - 2

$$\begin{cases} \dfrac{d^2\phi(x)}{dx^2} = 4\pi N_{eh} e^{e\phi(x)/T_e} & \text{... 1D } \textit{Poisson-Boltzmann} \text{ equation} \\ x = [0,h] & \text{...domain of integration} \\ \phi(h) = 0\,,\quad \phi'(h) = 0 & \text{...boundary conditions} \end{cases}$$

$h \longrightarrow$ ... estimated *on physical grounds*,
by the *energy conservation of a hot electron...*

"*Interaction of a beam of fast electrons with solids*"
V.I.Tikhonchuk, *Phys.Plasmas*, **9**,1416 (2002)

1T model in a finite interval - 3

Electrost. potential $\longrightarrow$ $$\phi(x) = \frac{T_e}{e} \ln\left[1 + \tan^2\left(\frac{h - x}{\sqrt{2}\lambda_{Dh}}\right)\right]$$

Electric field $\longrightarrow$ $$E(x) = \sqrt{2}\frac{T_e}{\lambda_{Dh} e}\tan\left(\frac{h - x}{\sqrt{2}\lambda_{Dh}}\right)$$

Hot electron density $\longrightarrow$ $$N(x) = N_{eh}\left[1 + \tan^2\left(\frac{h - x}{\sqrt{2}\lambda_{Dh}}\right)\right]$$

T_e hot electron temperature,

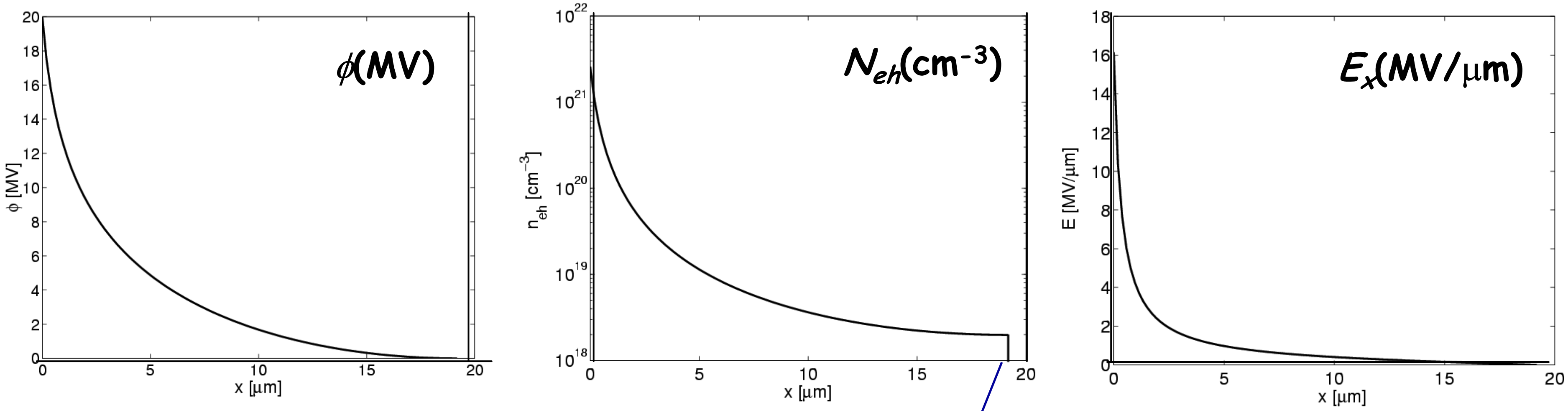
$N_{eh} = N_e(h)$ and $\lambda_{Dh} = \left(\dfrac{T_e}{4\pi N_{eh} e^2}\right)^{1/2}$

$$T_e = T_e(I_{pulse})$$
$$h = h(T_e, s_{target})$$
$$N_{eh} = N_{eh}(E_{pulse}, h)$$

1T model in a finite interval - 4

From this results, we can calculate the physical quantities for various experimental conditions…for example:

E.L. Clark, *et al.*, *Phys. Rev. Lett.* **84**, 670 (2000)
(ε_L = 50 J, I = 5×10^{19} W/cm^2, s_{target} = 125 μm, η_e = 0.2, θ = 25°)



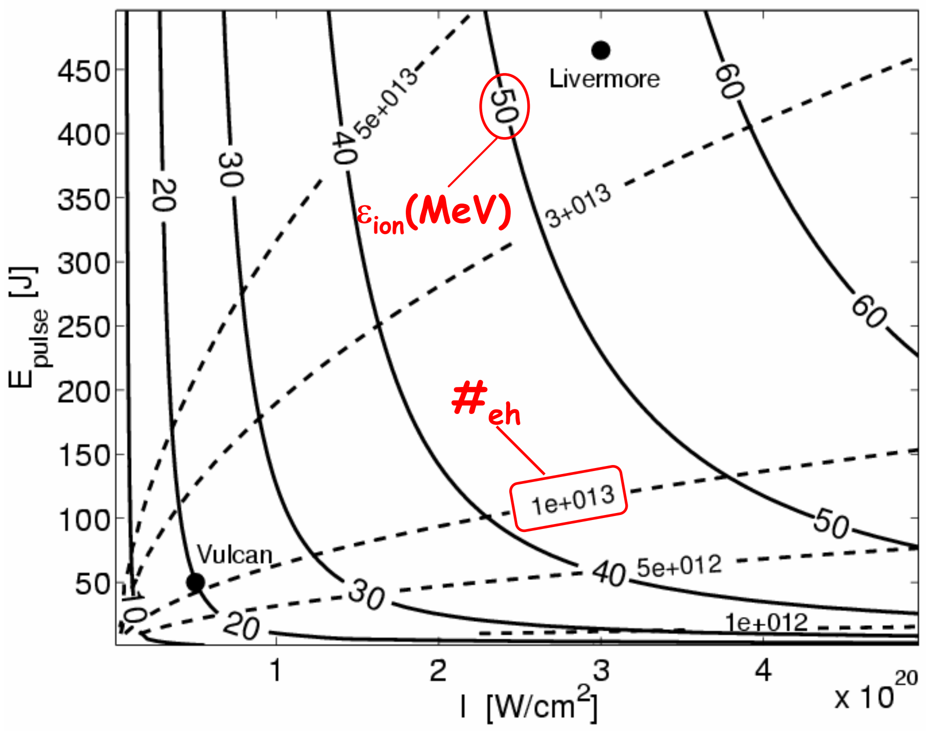
$T_e \approx$ 2.6 MeV $\Rightarrow \lambda_{Dh} \approx$ 0.4 - 8 μm $h \approx$ 20 μm !

1T model in a finite interval - 5

[M.Passoni, M.Lontano, *Laser Part. Beams* **22**, 163 (2004)]

Maximum ion energy: $\varepsilon_{ion}^{max} = Ze\phi(0) = ZT_e \ln\left[1 + \tan^2\left(\frac{h}{\sqrt{2}\lambda_{Dh}}\right)\right]$

to be compared with the experiments…



LIVERMORE [Snavely *et al.*, *PRL* **85**, 2945 (2000)]
E_{pulse} ~ 450 *J*, *I* ~ 3x10^{20} *W/cm*^{2}, L ~ 100-125 μm
max proton energy ~ 58 *MeV*

MOD. 1T
average el. field ~ 4 MV/ μm
max electric field ~ 50 MV/μm
max proton energy ~ 55-60 MeV

VULCAN [Clark *et al.*, *PRL* **84**, 670 (2000)]
E_{pulse} ~ 50 *J*, *I* ~ 5x10^{19} *W/cm*^{2}, L ~ 125 μm
max proton energy ~ 18 *MeV*

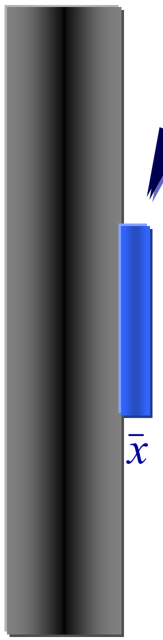
MOD. 1T
average el. field ~ 1 MV/ μm
max electric field ~ 16 *MV/μm*
max proton energy ~ 20 *MeV*

1T model in a finite interval - 6

Proton energy spectrum: from conservation in phase space $N_i(x)dx = N_i(\varepsilon)d\varepsilon$

proton coating on the rear surface:

Proton coating: so-called "double layer"

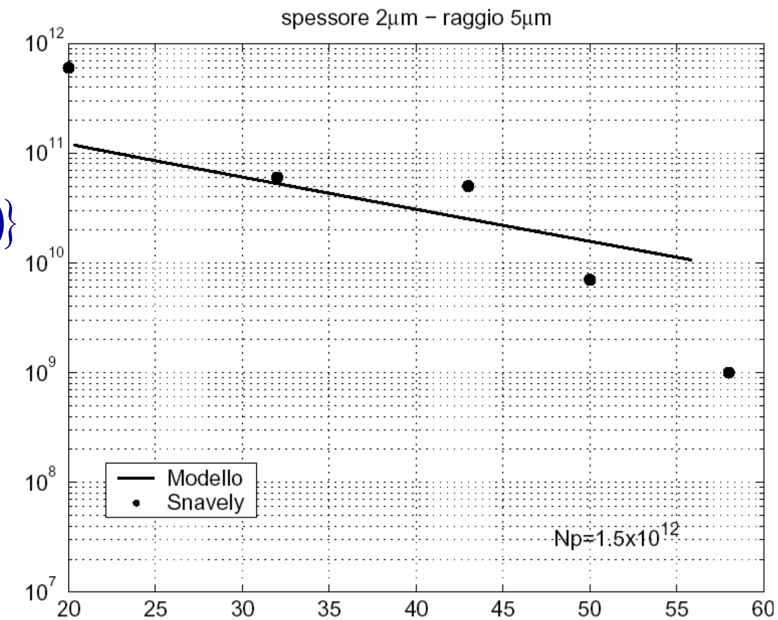


$$\frac{dN_i}{d\varepsilon} = \frac{N_i \lambda_{Dh}}{\sqrt{2T_e \bar{x}}\left(e^{\varepsilon/T_e} - 1\right)^{1/2}} \left\{H(\varepsilon - \varepsilon_0) - H(\varepsilon - \varepsilon_{\bar{x}})\right\}$$

$$\varepsilon_0 = Ze\phi(0) = ZT_e \ln\left[1 + \tan^2\left(\frac{h}{\sqrt{2}\lambda_{Dh}}\right)\right]$$

$$\varepsilon_{\bar{x}} = Ze\phi(\bar{x}) = ZT_e \ln\left[1 + \tan^2\left(\frac{h - \bar{x}}{\sqrt{2}\lambda_{Dh}}\right)\right]$$

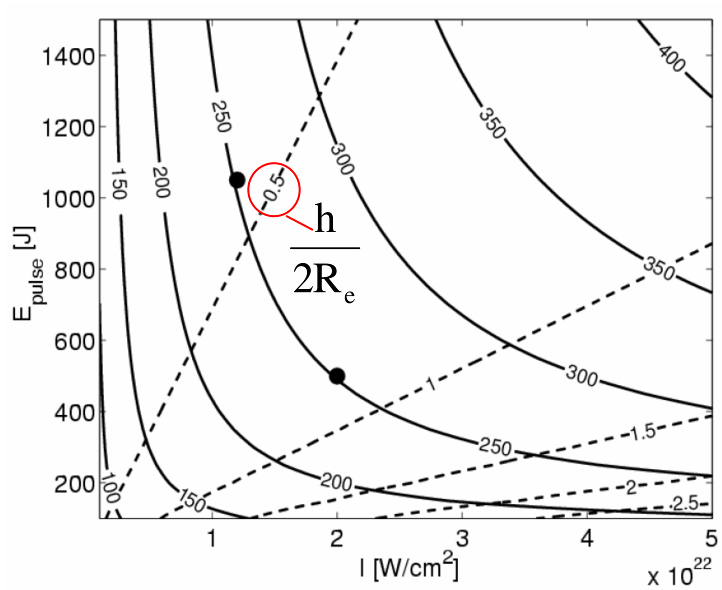
Contaminants: comparison with exps.

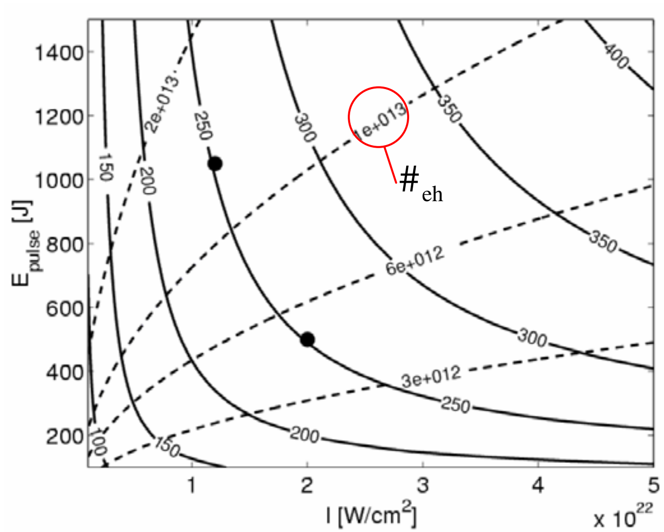


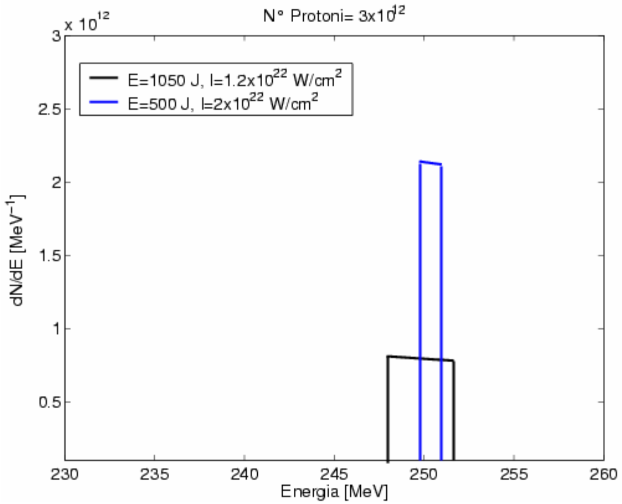
Dots: Snavely *et al.*, *PRL* **85**,2945 (2000)

1T model in a finite interval - 7

Application to hadrontherapy ($E_{req} \approx 250$ MeV)







s_{target} = 100 μm
η = 30 %
θ = 25°

planar 1D model applies if $h/2R_e \leq 1$

Role of "bound" electrons - 1

How to build a more self-consistent description??

Kinetic approach

- consider the Maxwell-Boltzmann (M-B) distribution function

$$f_e(\mathbf{r},v^2) = \tilde{N}\left(\frac{m}{2\pi T_e}\right)^{3/2} \exp\left\{-\frac{\varepsilon(\mathbf{r},v^2)}{T_e}\right\}$$

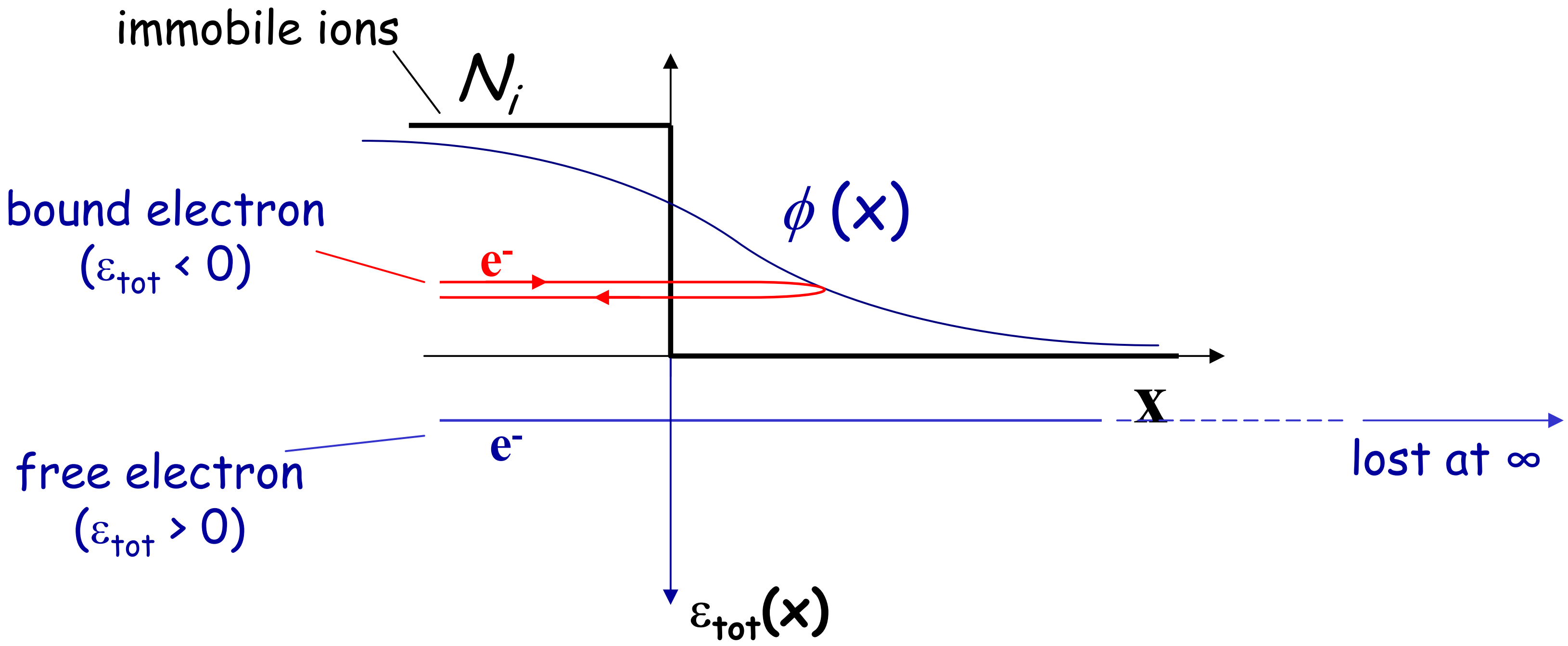
where the total (non relativistic, NR) electron energy is

$$\varepsilon(\mathbf{r},v^2) = \frac{1}{2}mv^2 - e\phi(\mathbf{r})$$

Y. Kishimoto, *et al.*, *Phys. Fluids* **26**, 2308 (1983)

- at the rear surface of the target, only "trapped" electrons are bound to the positively charged ion target ⇒ $\varepsilon(\mathbf{r},v^2) < 0$; "passing" electrons, with $\varepsilon(\mathbf{r},v^2) > 0$, leave the system forever

Role of "bound" electrons - 2



therefore only the density of "trapped" electrons enters Poisson eq.; that is, integrating M-B over the energies in the range

$$-e\phi(\mathbf{r}) < \varepsilon < 0$$

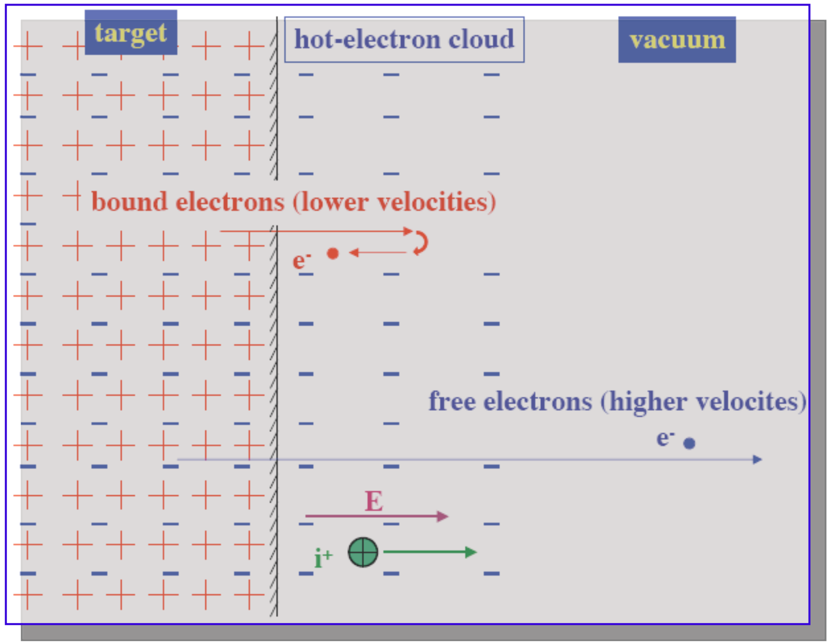
we get the bound (trapped) electron density $N_{tr}(\mathbf{r})$

1T trapped electron model - 1

$$f_e(x,p) = \frac{\tilde{n}}{2mcK_1\left(\frac{mc^2}{T}\right)} exp\left(-\frac{\varepsilon(x,p)}{T_e}\right)$$

$$n_{tr}(\mathbf{r}) = \iiint_{\varepsilon(\mathbf{r},\mathbf{p})\leq 0} d^3 p f_e(\mathbf{r},\mathbf{p})$$

$$\varepsilon(\mathbf{r},\mathbf{p}) \leq 0 \;\Rightarrow\; |\mathbf{p}| \leq p_{max}(\mathbf{r})$$



$$\varepsilon(x,p) = mc^2(\gamma - 1) - e\phi(x) \leq 0$$

$$\Rightarrow \gamma \leq \gamma_{max}(x) \equiv 1 + \frac{e}{mc^2}\phi(x)$$

$$p^2 \leq p_{max}^2 \equiv m^2c^2\left[\left(\frac{e\phi}{mc^2}\right)^2 + \frac{2e\phi}{mc^2}\right]$$

$$\nabla^2\varphi = N_{tr}(\varphi) \qquad \varphi = \frac{e\phi}{T_e},\; N = \frac{n}{\tilde{n}}$$

"*E.S. field distribution at the sharp interface between high density matter and vacuum*"
M. Lontano, M. Passoni, *Phys.Plasmas*, **13**, 042102 (2006)

1T trapped electron model - 2

- First NR limit. Integrating over v, for $v^2 < \dfrac{2e\phi(x)}{m}$ Poisson eq. becomes

$$\frac{d^2\varphi}{dx^2} = N_{tr}(\xi) = \Phi\left(\sqrt{\varphi}\right)e^{\varphi}$$ (out of the target, 1D, dimensionless units)

- integrating twice $(\varphi = 0$ where $\varphi'_\xi = 0)$, we get the implicit form

$$\int_{\varphi_0}^{\varphi(\xi)} \frac{d\varphi'}{\left[\Phi\left(\sqrt{\varphi'}\right)e^{\varphi} - \frac{2}{\sqrt{\pi}}\sqrt{\varphi'}\right]^{1/2}} = -\sqrt{2}\xi$$ $\xi = x/\lambda_D$ (λ_D from $\tilde{n}$)

- in the small amplitude limit, $|\varphi| \ll 1$

$$\varphi(\xi) \approx \left[\varphi_0^{1/4} - \left(\frac{1}{6\sqrt{\pi}}\right)^{1/2}\xi\right]^4$$

$$\varphi_0 = \varphi(\xi = 0)$$

- $\varphi(\xi)$ = 0, with its three derivatives (i.e. **electric field and electron density**!)

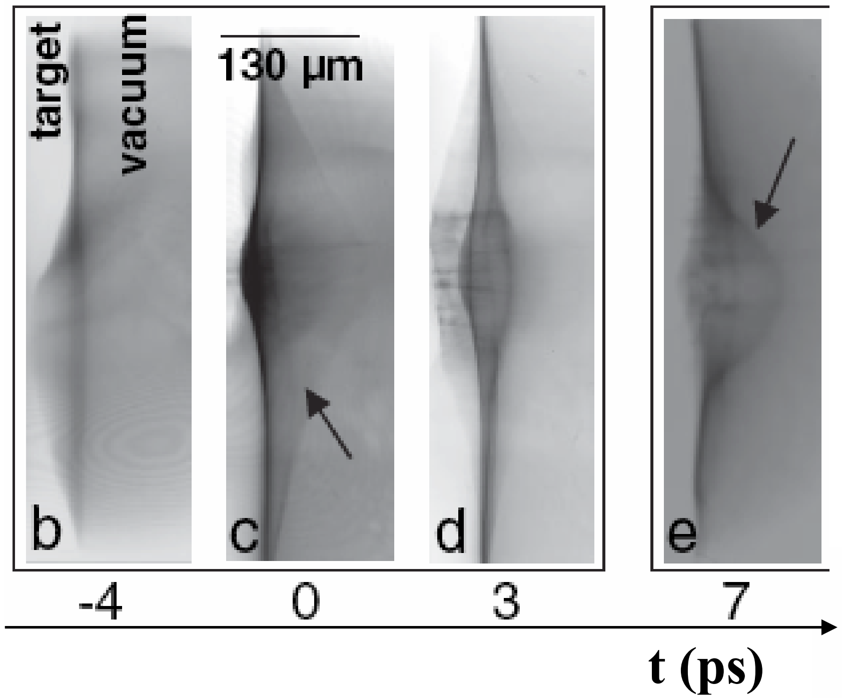
at a finite $\xi = \xi_f = \sqrt{6\pi^{1/2}\varphi_0^{1/4}}$

weakly depending on φ_0 (that is on plasma parameters ...)

LULI

[L. Romagnani, *et al.*, *P.R.L.* **95**, 195001 (2005)
M. Borghesi, *et al.*, *Fus. Sc. & Techn.* **49**, 412 (2005)]

proton imaging of rear field



experimental data could be best reproduced by PIC simulations by assuming a field which becomes zero at a finite distance $h \approx 20\ \mu m$ from the rear surface

interaction CPA_1
$I \approx 3.5 \times 10^{18}\ W/cm^2$
$\tau \approx 1.5$ ps
1 - 40 μm, Al, Au bent foils

$T_e \approx$ 500 keV
$\varepsilon_{int} \approx$ 6-7 MeV
$E \approx 3 \times 10^{10}$ V/m

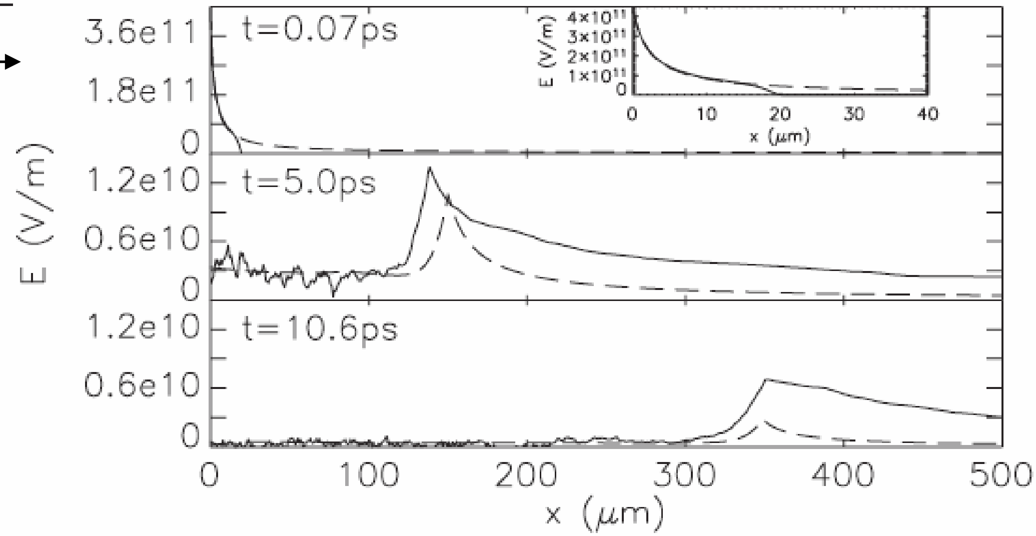


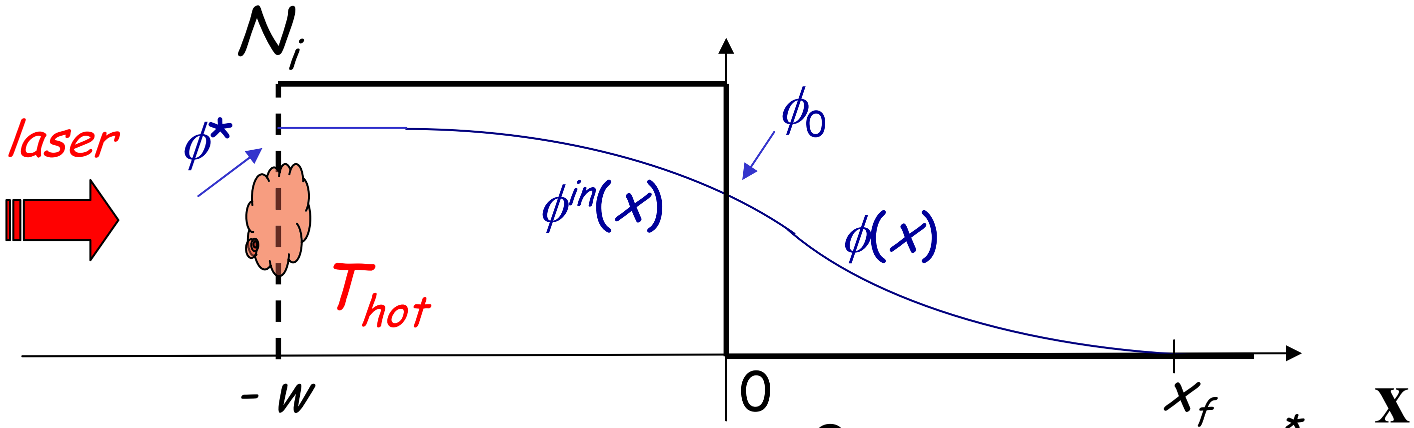
FIG. 3. Field profiles from PIC (solid line) and fluid (dashed line) simulations at three different times and for $T_{e0} = 500$ keV and $n_{e0} = 3 \times 10^{19}\ cm^{-3}$.

1T trapped electron model - 3

- determination of ξ_f from the knowledge of φ_0

- φ_0 is related to the hot electron parameters inside the target as far as the front side ($-\xi_w = -w/\lambda_D < \xi < 0$)

- here the ions and cold electrons forming the solid target provide the positively charged background density $ZN_i - N_{cold} = N_L$



- for bound electrons $\varepsilon = \varepsilon_k - \varphi < 0 \Rightarrow \varepsilon_{k,max} = \varphi^*$

max value of electron energy

max value of electrostatic potential inside the target

1T trapped electron model - 4

The problem can be solved analytically in the ultra-relativistic limit (which is appropriate near and inside the target for typical parameters)

$$|\mathbf{p}|/mc >> 1 \qquad f_e^{UR}(x,p) = \tilde{n}\frac{c}{2T_e} exp\left(-\frac{c|\mathbf{p}| - e\phi(x)}{T_e}\right) \qquad p^2 \le p_{max}^2 \equiv \left(\frac{e\phi}{c^2}\right)^2$$

We obtain $\varphi_0 = \dfrac{(2\varphi^* - 1)e^{\varphi^*} - \varphi^* + 1}{2\left(e^{\varphi^*} - 1\right)} \qquad \varphi_0 = \varphi_0\left(\varphi^*\right)$

From this we get the maximum ion energy $\varepsilon_{max}^i = Z\varphi_0 T_{hot}$

and ion energy spectrum $$n_i(\varepsilon_i) = \frac{H(\varepsilon_i - \varphi_0) - H(\varepsilon_i - \varphi_0 - \Delta\varphi)}{\sqrt{2}Z\left[exp\left(\frac{\varepsilon}{Z}\right) - \frac{\varepsilon}{Z} - 1\right]^{1/2}}$$

1T trapped electron model - 5

The maximum electron energy $\varepsilon_{e,max}$ = φ* as a scaling law

How to obtain $\varepsilon_{e,max}$ = φ* ?
Difficult both theoretically and experimentally…

However, from the analysis of several published results (i.e. starting with observed proton energies and using the model to infer φ*)
we get the fitting

$$\boxed{\varepsilon_{e,max} = \frac{K_{e,max}}{T_e} = A + B\ln\left[E_L(J)\right] = \varphi^*}$$

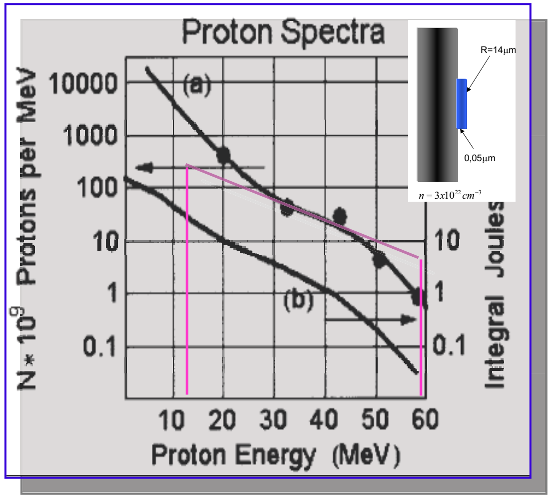
$A = 3.8$, $B = 0.8$ where E_L is the laser energy

[P. Tacconi, *Master Thesis in Nucl. Eng.*, Polytechnic of Milan (2006)]

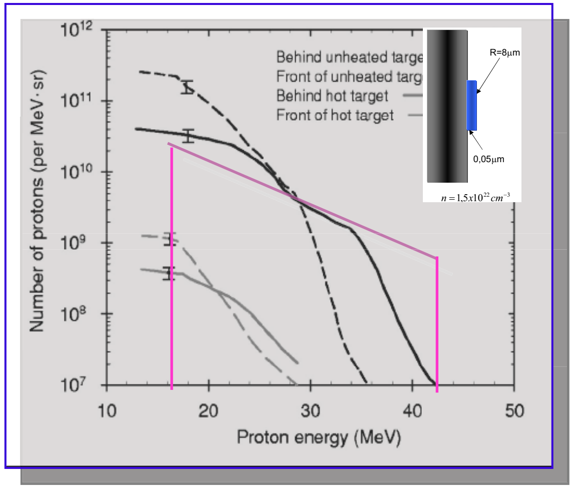
1T trapped electron model - 6

Comparison with experimental data

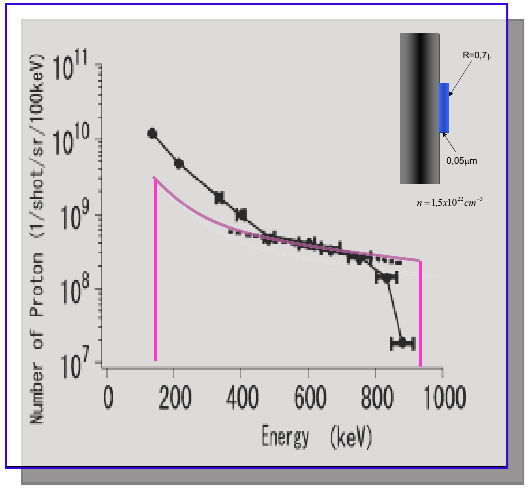
$D = 2R = f_L + w_t \tan \vartheta$ - model



[R.A. Snavely, *et al.*,
Phys. Rev. Lett., **85**, 2945 (2000)]



[P. McKenna, *et al.*,
Phys. Rev. E, **70**, 036405 (2004)]



[M. Nishiuchi, *et al.*,
Phys. Lett. A, **357**, 339 (2006)]

$E_L = 500\ J$

$I_L = 3\times10^{20}\ W/cm^2$

$w_t = 100\ \mu m$

$\tau_L = 0.5\ ps$

$n_{e,L}^{est} = 4.4\times10^{21}\ cm^{-3}$

$T_e = 7\ MeV$

$K_{e,\max} = 61\ MeV$

$E_{i,\max}^{mea} = 58\ MeV$

$E_{i,\max}^{mod} = 58.4\ MeV$

$E_L = 400\ J$

$I_L = 2\times10^{20}\ W/cm^2$

$w_t = 100\ \mu m$

$\tau_L = 0.7\ ps$

$n_{e,L}^{est} = 3.6\times10^{21}\ cm^{-3}$

$T_e = 5.7\ MeV$

$K_{e,\max} = 46.7\ MeV$

$E_{i,\max}^{mea} = 44\ MeV$

$E_{i,\max}^{mod} = 45.9\ MeV$

$E_L = 0.25\ J$

$I_L = 3\times10^{18}\ W/cm^2$

$w_t = 3\ \mu m$

$\tau_L = 70\ fs$

$T_e = 0.26\ MeV$

$K_{e,\max} = 1\ MeV$

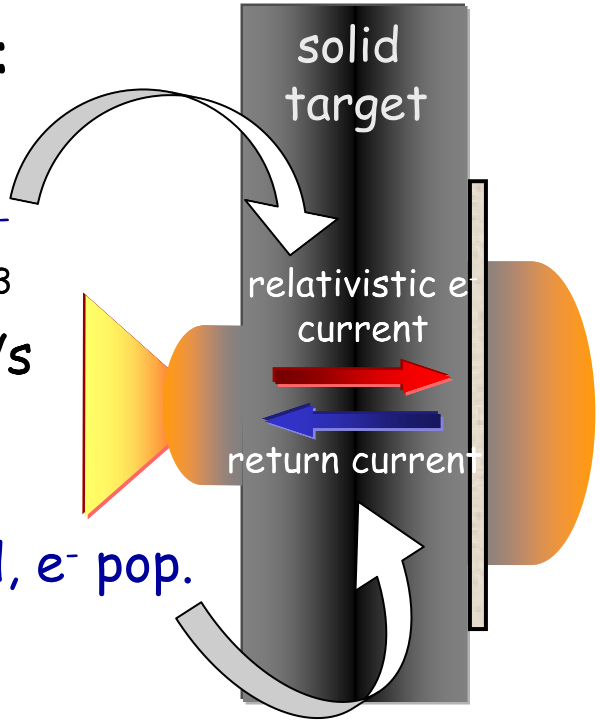
$E_{i,\max}^{mea} = 0.88\ MeV$

$E_{i,\max}^{mod} = 0.92\ MeV$

Limits of 1T models

- at least two electron population exist:

i) "hot" (fast, *h*), low density, laser-generated, e^-
density ~ critical density ~ 10^{20}-10^{21} cm^{-3}
T_h ~ ponderomotive potential ~ few MeVs

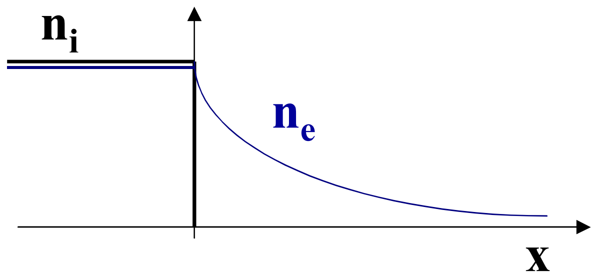


ii) "cold" (bulk, *c*), high density, ohmically heated, e^- pop.
density ~ solid density ~ 10^{23}-10^{24} cm^{-3}
T_c= ? ...> T_{room}

affects the properties of the self-consistent electric field induced on the rear surface
→ effects on ion acceleration

2T model: the electric field - 1

Poisson – Boltzmann eq.



$$\partial^2_{xx}\phi = 4\pi e\left(N_{0h}e^{e\phi/T_h} + N_{0c}e^{e\phi/T_c} - ZN_{0i}\right)$$ inside the target

$$\partial^2_{xx}\phi = 4\pi e\left(N_{0h}e^{e\phi/T_h} + N_{0c}e^{e\phi/T_c}\right)$$ outside the target

with the conditions

$ZN_{0i} = N_{0h} + N_{0c}$ and

$$E(-\infty) \to 0 \Rightarrow \phi'(-\infty) \to 0$$
$$E(+\infty) \to 0 \Rightarrow \phi'(+\infty) \to 0$$
$$n_{eh}(+\infty), n_{ec}(+\infty) \to 0 \Rightarrow \phi(+\infty) \to -\infty$$

(!)

2T model: the electric field - 2

introducing $\boxed{a \equiv \frac{N_{0c}}{N_{0h}}, \quad b \equiv \frac{T_c}{T_h}}$ implicit solutions can be found

inside the target

$$\int_{\varphi(0)}^{\varphi(x)} \frac{d\varphi}{\left[\exp\varphi + ab\exp(\varphi/b) - (1+ab) - (1+a)\varphi\right]^{1/2}} = -\sqrt{2}\,\frac{x}{\lambda_{Dh}}$$

outside the target

$$\int_{\varphi(0)}^{\varphi(x)} \frac{d\varphi}{\left[\exp\varphi + ab\exp(\varphi/b)\right]^{1/2}} = -\sqrt{2}\,\frac{x}{\lambda_{Dh}}$$

where $\varphi \equiv \frac{e\phi}{T_h}$ and $\lambda_{Dh} = \sqrt{\frac{T_h}{4\pi N_{0h} e^2}}$

"*Charge separation effects in solid targets and ion acceleration with a two-temperature electron distribution*", M. Passoni, *et al.*, *Phys. Rev. E* **69**, 026411 (2004)

2T model: the electric field - 3

continuity of ϕ and E in $x = 0$ gives

$$\phi(0) = -\frac{T_h}{e}\frac{1+ab}{1+a} = -\frac{\langle T_e \rangle}{e}$$

electrostatic potential at $x = 0$

$$E(0) = \sqrt{2}\,\frac{T_h}{e\lambda_{Dh}}\left\{\exp\left[-b\left(\frac{1+ab}{b+ab}\right)\right] + ab\exp\left(\frac{1+ab}{b+ab}\right)\right\}^{1/2}$$

maximum value of the electric field

2T model: the electric field -results

- max. electric field never below

$$\sqrt{2}\,\frac{T_h}{e\lambda_{Dh}}$$

$$\left(\text{1-T sol. } E_{max} = \sqrt{\frac{2}{e}}\,\frac{T_h}{e\lambda_{Dh}}\right)$$

- key parameter $\boxed{ab = p_{0c}/p_{0h}}$

$$ab << 1 \quad \Rightarrow \quad E_{max} \approx \underline{\sqrt{2}}\,\frac{T_h}{e\lambda_{Dh}} \qquad ab >> 1 \quad \Rightarrow \quad E_{max} \approx \underline{\sqrt{\frac{2}{e}ab}}\,\frac{T_h}{e\lambda_{Dh}}$$

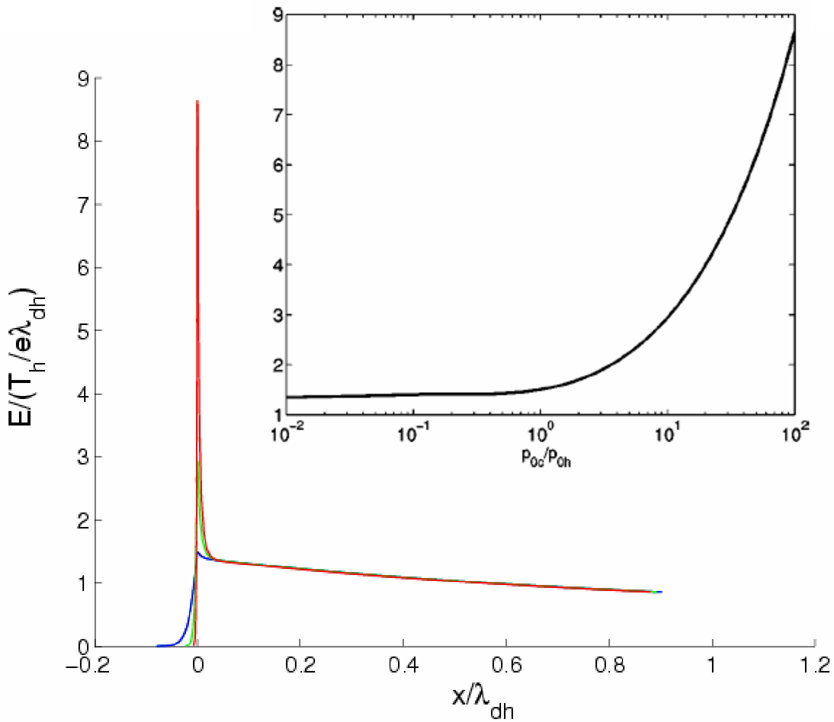
- the penetration of the electric field inside the target is determined by the cold el. population; E penetrates over a few cold Debye lengths λ_{Dc}

$$\boxed{E \approx \exp\left(x/\lambda_{Dc}\right)}$$

- if p_{0c} is high enough, cold electrons influence the field outside the target, over a few λ_{Dc}

$$\boxed{\lambda_{Dc}\,(\sim \text{nm}) \ll \lambda_{Dh}\,(\sim \mu\text{m})}$$

2T model: the electric field -results

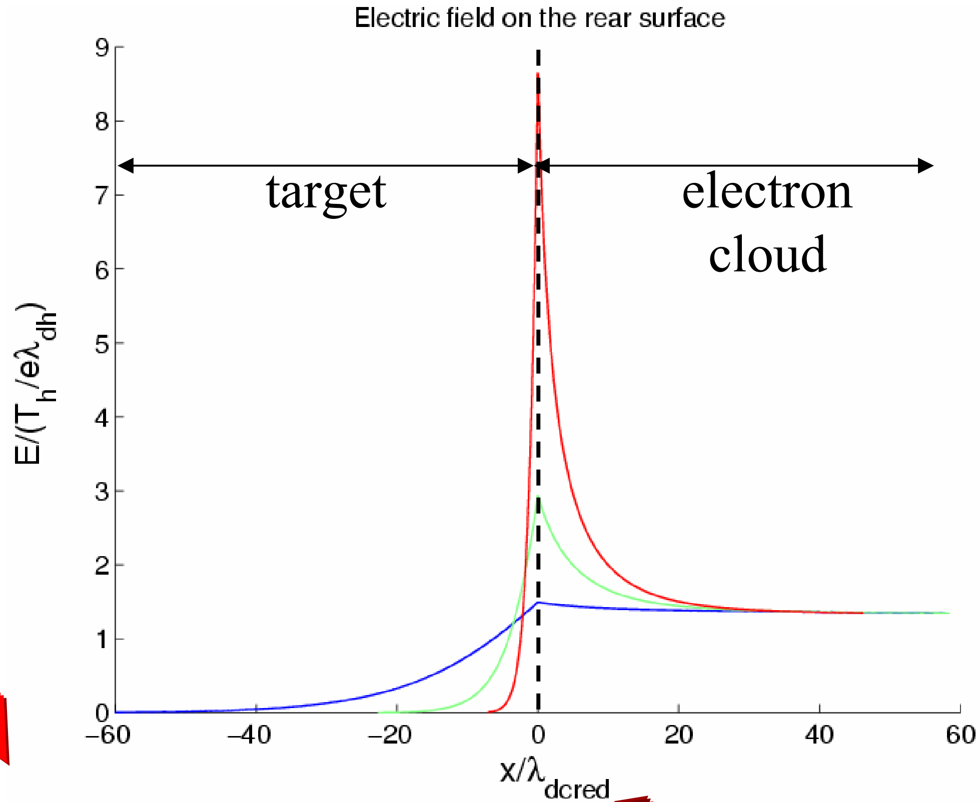


Blue: p_{0c} / p_{0h} =1

Green: p_{0c} / p_{0h} =10

Red: p_{0c} / p_{0h} =100

b =0.01



different
scale!

2T model: cold electrons heating - 1

background electrons heated by the return current

$$p_{heat} = \eta j^2$$

neglecting thermal conduction, $T_e(t)$ given by Fourier eq.

$$C(T_e)\frac{\partial T_e}{\partial t} = \eta(T_e, T_i) j^2$$

assume a power-law dependence on T_e
for the heat capacity C and the electrical resistivity η:

$$\eta = \eta_k \left(\frac{T_e}{T_k}\right)^{\alpha} \qquad C(T_e) = C_k \left(\frac{T_e}{T_k}\right)^{\beta}$$

2T model: cold electrons heating - 2

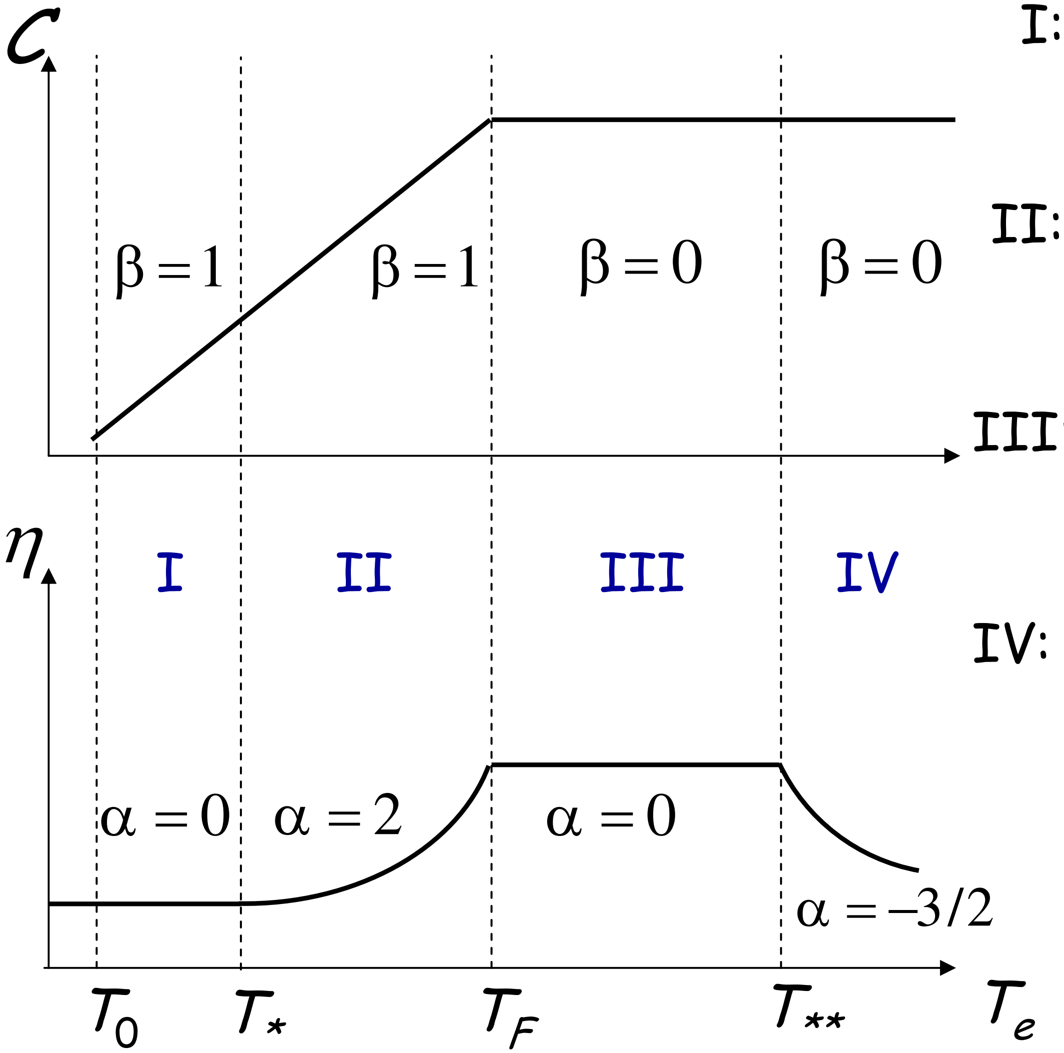
analytical solutions can be found

$$T_e(t) = T_k \left\{ 1 + [1 - (\alpha - \beta)] \frac{\eta_k j^2}{C_k T_k} (t - t_k) \right\}^{\frac{1}{1-(\alpha-\beta)}} \qquad \alpha - \beta < 1$$

$$T_e(t) = T_k \exp\left[\frac{\eta_k j^2}{C_k T_k} (t - t_k) \right] \qquad \alpha - \beta = 1$$

M. Passoni, *et al.*, *Phys. Rev. E* **69**, 026411 (2004)

2T model: cold electrons heating - 3



I: degenerate el. gas heat capac.
room resistivity (e-phonon)

II: degen. el. gas heat capacity
resistivity $\propto T_e^2$ (e-e coll.)

III: ideal el. gas heat capacity
const. resistivity (grey region!)

IV: ideal el. gas heat capacity
hot plasma resistivity (Spitzer)

[Ashcroft & Mermin, *Solid State Physics*]

[Landau & Lishfitz, *Statistical Physics II* (vol. IX) *Physical Kinetics* (vol. X)]

2T model: cold electrons heating - 5

- Heating duration time? Pulse duration time, lower limit:
    - "short" pulses ~ 50 fs
    - "long" pulses ~ 0.5 – 1 ps

- Estimates for Al (n_e=1.8x10^{23} cm^{-3} , T_F=11.6 eV):
    - T_*~ 1eV ; T_{**}~ 100 eV
    - for "short" pulses: T_c ~ 100 eV
    - for "long" pulses : T_c ~ 1 keV

2T model: ion detachment - 1

Without coating layer, "bulk" protons only (CH targets):

- Electric field inside the target:
    - ~ MV/μm >> 100 MV/cm needed to detach an at. layer (binding energy ~ eV, interlayer distance d~10^{-1} nm)
    - it penetrates over a few λ_{Dc}

For Al: with T_c=1 keV, λ_{Dc} ~ d = 2×10^{-1} nm

10^{12} ions detached with area π (50 μm)^2

- During pulse time, peak moves inwards at speed ~ c_s

c_s~ 10^7 cm/s = 100 nm/ps

"short" pulses: $c_s\tau$ ~ 5 nm

"long" pulses: $c_s\tau$ ~ 50 nm

2T model: ion detachment - 2

With coating layer (or thin contaminants layer):

[S.V. Bulanov, *et al.*, *Pl. Phys. Rep.* **28**, 453 (2002); **28**, 975 (2002)
T.Zh. Esirkepov, *et al. Phys. Rev. Lett.* **89**, 175003 (2002)]

- usually low density $\longrightarrow$ does not alter *E(x)* distribution

- instantaneous ionization

- tot. # of accelerated protons depends on layer thickness, experimentally not established...

ex.: assume N_{coat}~10^{22} cm^{-3} $\longrightarrow$ 10^{12} protons within 10 nm

This value has to be compared with the total number of hot electrons (~ several times 10^{13})...

2T model: ion acceleration

- max value of el. field close to target higher than in 1-T
  → it can affect acceleration

- the peak in E influences the proton trajectory
  → application as an"injector"

- influence on the so-called "double-layer" acc. scheme
  → E peak is close to the proton layer

- penetration of the electric field inside the target is determined by the cold el. population;

the effect of the background e^- population is important in the description of the rear acceleration mechanism

Quasi-static models: several arguments for discussion

- quasi-static models give simple expressions for the maximum ion energy and for the energy spectrum

- they are effective to determine the properties of the most energetic ions

- the experimental results are in quite good agreement with the expectations of q.-s. models

- however, they hold for a very short time (in this sense, they are complementary to non-neutral fluid models)

Other quasi-static models…

" *Theory of Laser Acceleration of Light-Ion Beams from Interaction of Ultrahigh-Intensity Lasers with Layered Targets* "
B.J. Albright, *et al.*, *Phys. Rev. Lett.* **97**, 115002 (2006)
- extention of the 2T model to describe layered targets

" *Analytical Model for Ion Acceleration by High-Intensity Laser Pulses* "
J. Schreiber, *et al. Phys. Rev. Lett.* **97**, 045005 (2006)
- surface charge model exploiting radial symmetry for the electric field

" *The laser proton acceleration in the strong charge separation regime* "
M. Nishiuchi, *et al.*, *Phys. Lett. A* **357**, 339 (2006)
- approach analogous to the 1T model to interpret experiments

" *Effect of Target Composition on Proton Energy Spectra in Ultraintense Laser-Solid Interactions* "
A.P. Robinson, *et al.*, *Phys. Rev. Lett.* **96**, 035005 (2006)
- study of the effects of a non negligible proton density in the target

Further theoretical references…

…not exaustive list…

"*Ion acceleration in expanding multi-species plasmas*"
V. Yu. Bychenkov *et al.*, *Phys. Plasmas,* 11, 3242 (2004)

"*Ion acceleration in short-laser-pulse interaction with solid foils* "
V. T. Tikhonchuk, *et al.*, *Plasma Phys. Controlled Fusion* 47 , B869 2005.

"*Collisionless expansion of a Gaussian plasma into a vacuum*"
P. Mora, *Phys. Plasmas* 12 , 112102 (2005)

" *Thin-foil expansion into a vacuum*"
P. Mora, *Phys. Rev. E* 72 , 056401 (2005)

"Test ion acceleration in a dynamic planar electron sheath "
M.M. Basko, *Eur. Phys. J. D,* **41** , 641 (2007)

References on PIC simulations…

Particle In Cell simulation is a numerical approach to explore the physics of UU laser-matter interaction close to the experimental conditions.
About laser-ion acceleration:

- Y. Sentoku, *et al.*, *Phys. Rev. E* 62, 7271 (2000)
- A. Pukhov *Phys. Rev. Lett.* 86 3562 (2001)
- S.C. Wilks, *et al.*, *Phys. Plasmas* 8, 542 (2001)
- T. Zh. Esirkepov, *Phys. Rev. Lett.* 89 175003 (2002)
- S.V. Bulanov, *et* al., *Plasma Phys. Rep.* **28**, 975 (2002)
- T. Nakamura, S. Kawata, *Phys. Rev. E* 67, 026403 (2003)
- T.Zh. Esirkepov, *et al.*, *Phys. Rev. Lett.* 92, 175003 (2004)
- T.Zh. Esirkepov, *et al.*, *Phys. Rev. Lett.* 96, 105001 (2006)

Hot research issues…

The field of laser-based ion acceleration is extraordinary active…some examples:

- elimination of the pre-pulse to allow:
    - efficient TNSA front acceleration
    - more efficient electron heating
    - use of ultrathin targets (promising to increase ion properties)

- control of the beam properties
    - Achievement of monoenergetic ion beams

- investigation of new accelation schemes

- construction of satisfactory theoretical descriptions of these issues

- development of the possible applications

[Laser and Plasma Accelerator Workshop 2007, Azores, 9-13 July]

General summary

- laser-driven ion acceleration is a promising charged particle acceleration method: however several issues are still to be solved

- ultra-short pulses and thin targets are preferable

- laser: rep.rate / contrast (pre-plasma effects)

- quasi-monoenergetic ion beams and control are huge challenges

- modelling: non-local dynamics / main acceleration mechanism or a combination of several / PIC simulations are approaching actual experimental condition

- analytical models: mainly limited to 1-dim dynamics, which nevertheless is ok as soon as $h < \varnothing$ / isothermal / quasi-static; however good comparison with experiments