The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA International Atomic Energy Agency

1856-33

2007 Summer College on Plasma Physics

30 July - 24 August, 2007

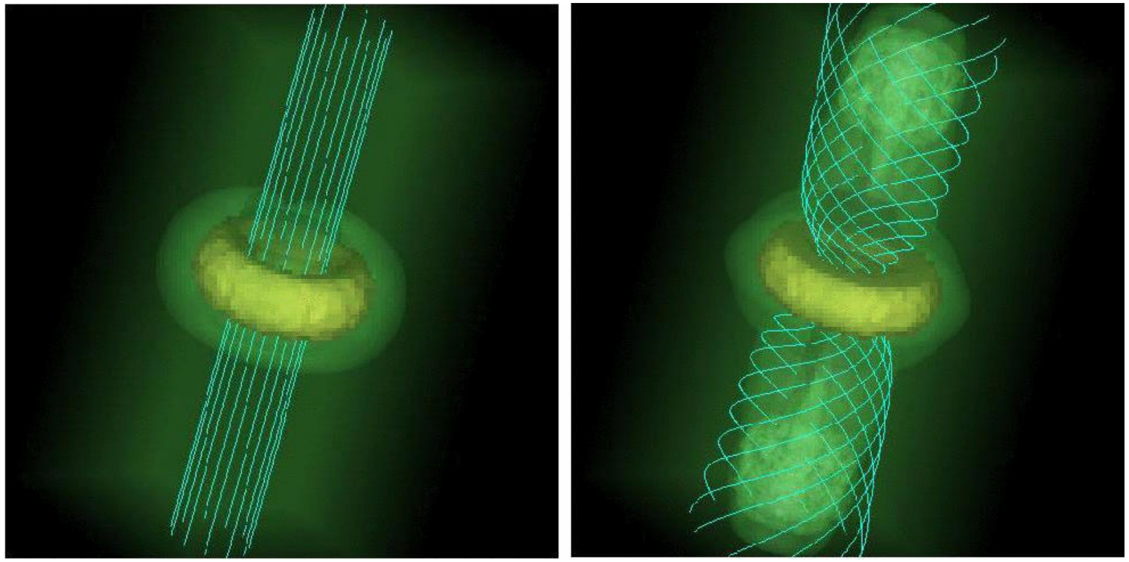
Numerical Laboratory for Plasma Astrophysics

R. Matsumoto

R. Matsumoto
Chiba University
Chiba Shi, Japan

2007 Summer College on Plasma Physics

Numerical Laboratory for Plasma Astrophysics



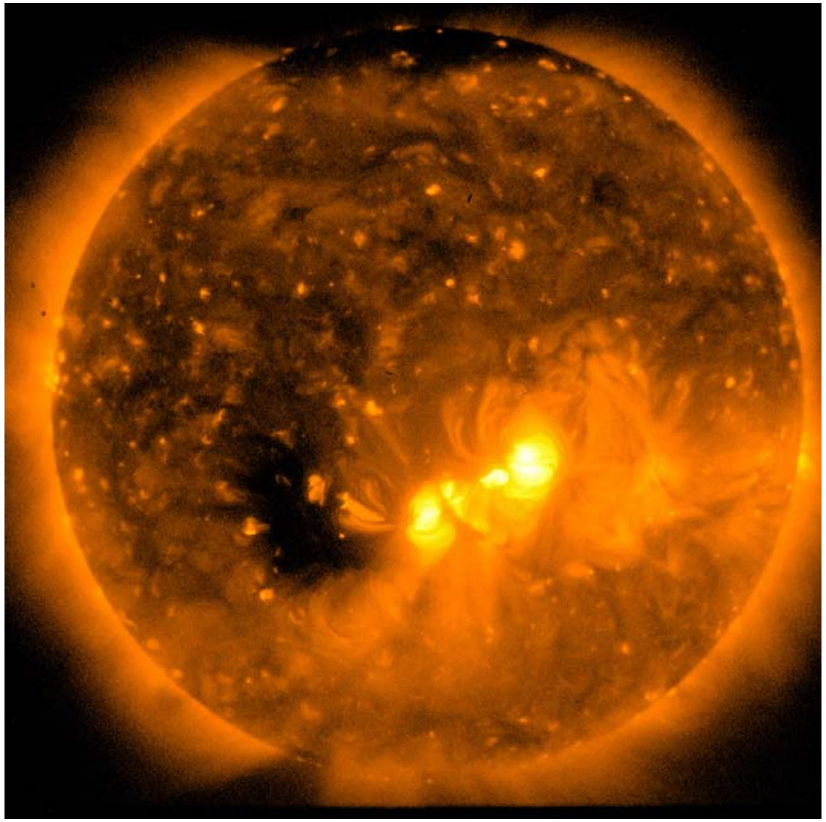
Ryoji Matsumoto
(Chiba University)

Contents

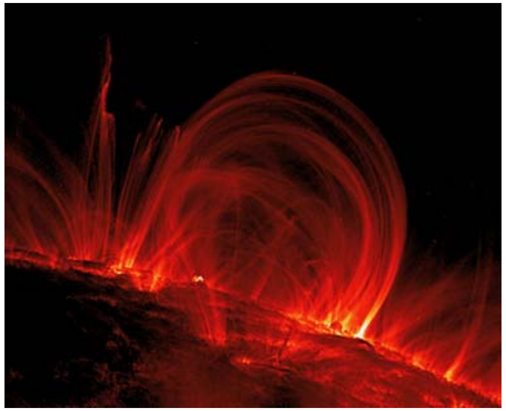
- Introduction
- Numerical MHD Laboratory for Astrophysics
- Simulation Engines
- Examples of Astrophysical MHD Simulations
- Summary and Future

1. Introduction

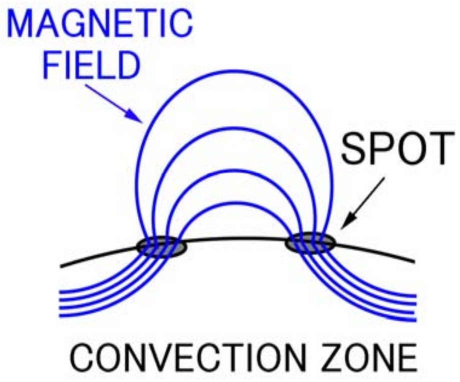
- Magnetic fields often play essential roles in astrophysical phenomena

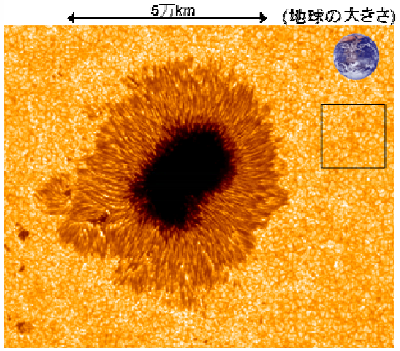


X-ray image of the Sun by HINODE satellite



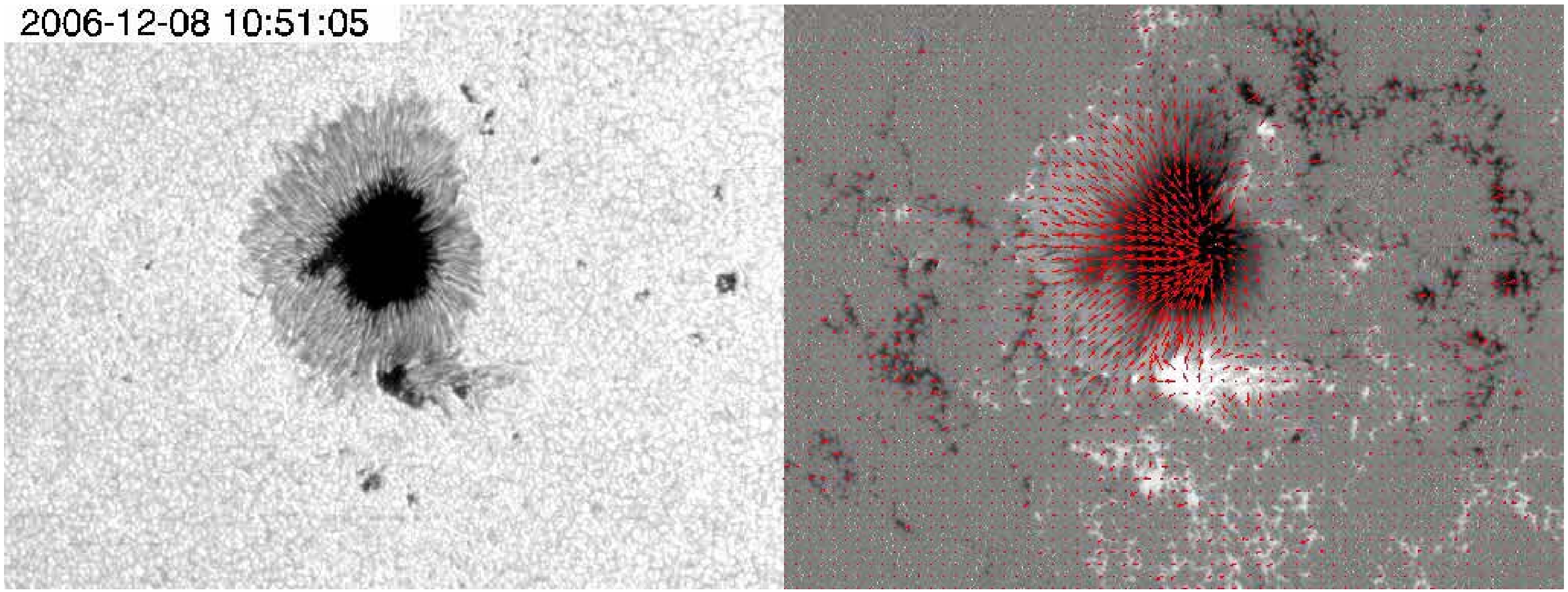
Magnetic Loops Observed by TRACE



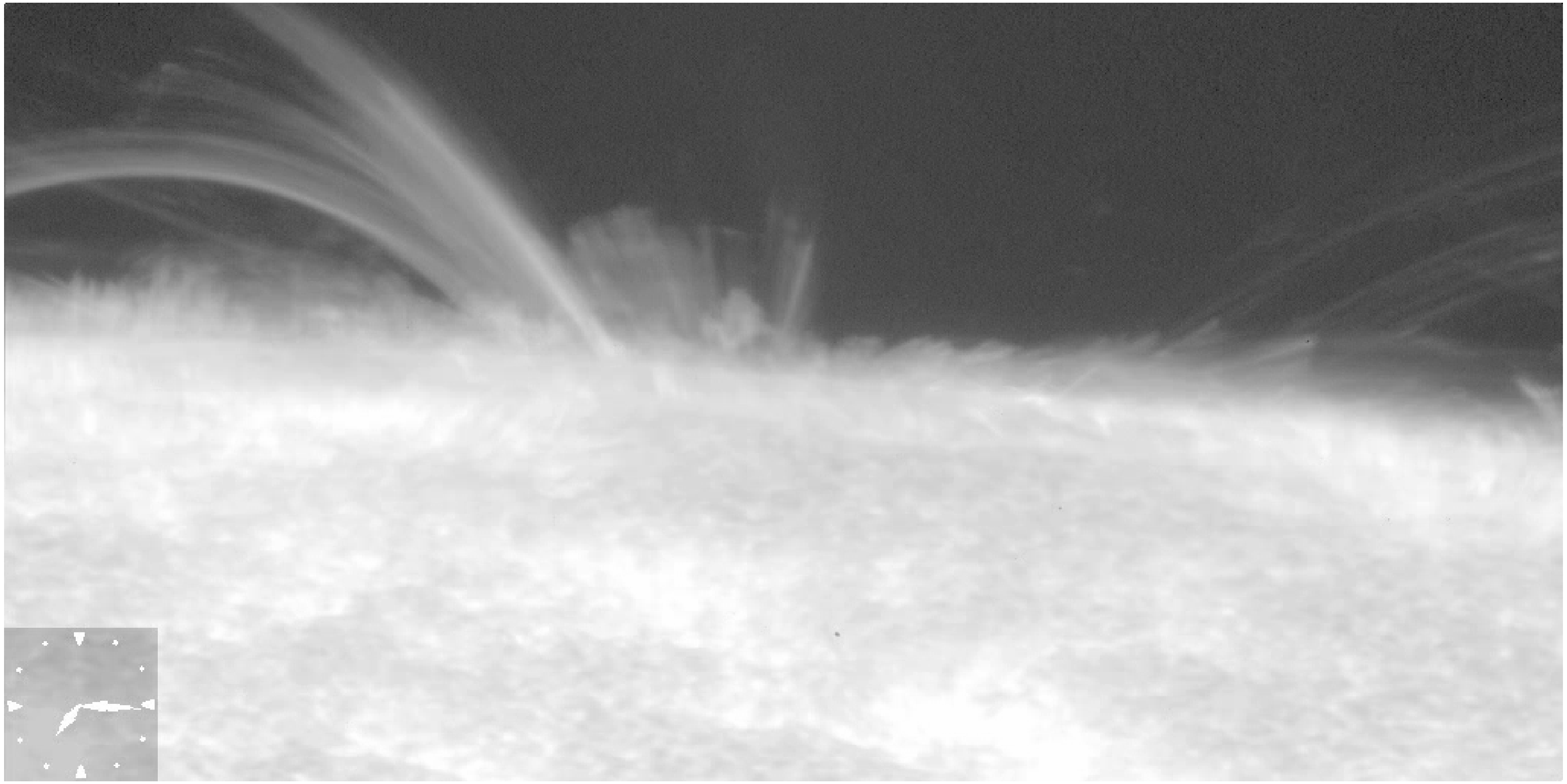


Sunspot

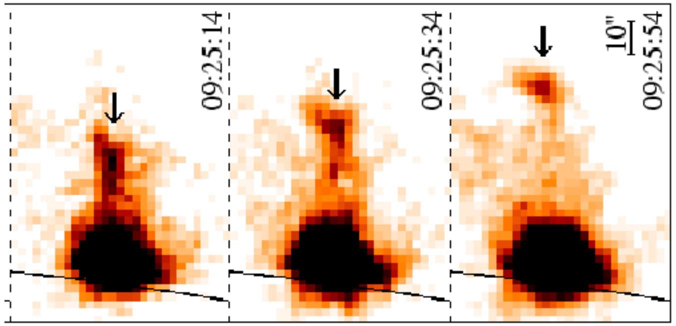
HINODE Observation of The Time Evolution of Sunspots

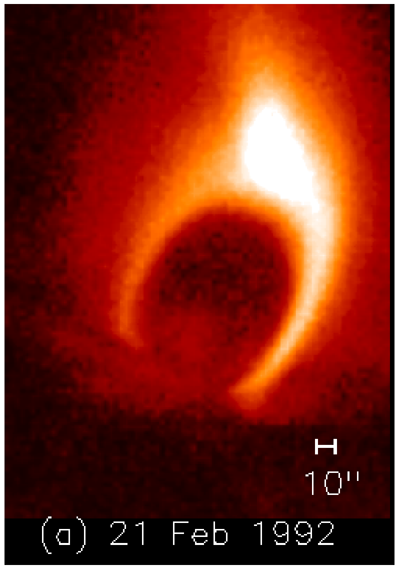


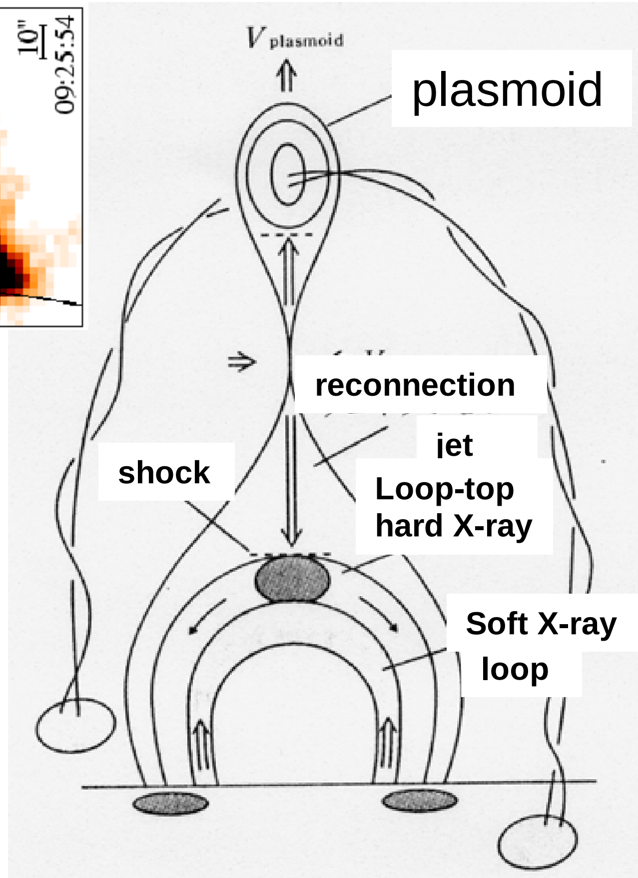
Plasma Motion Observed by the Optical Telescope of HINODE Satellite



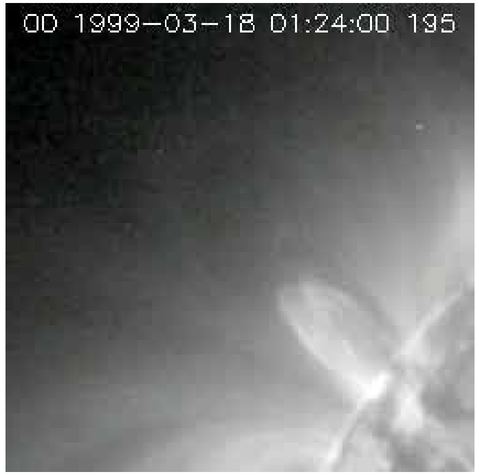
Magnetic Energy Release in the Solar Corona: Solar Flares

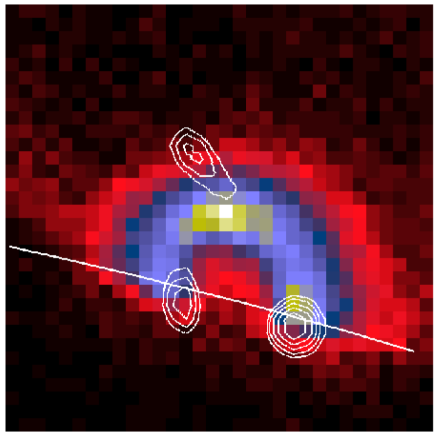




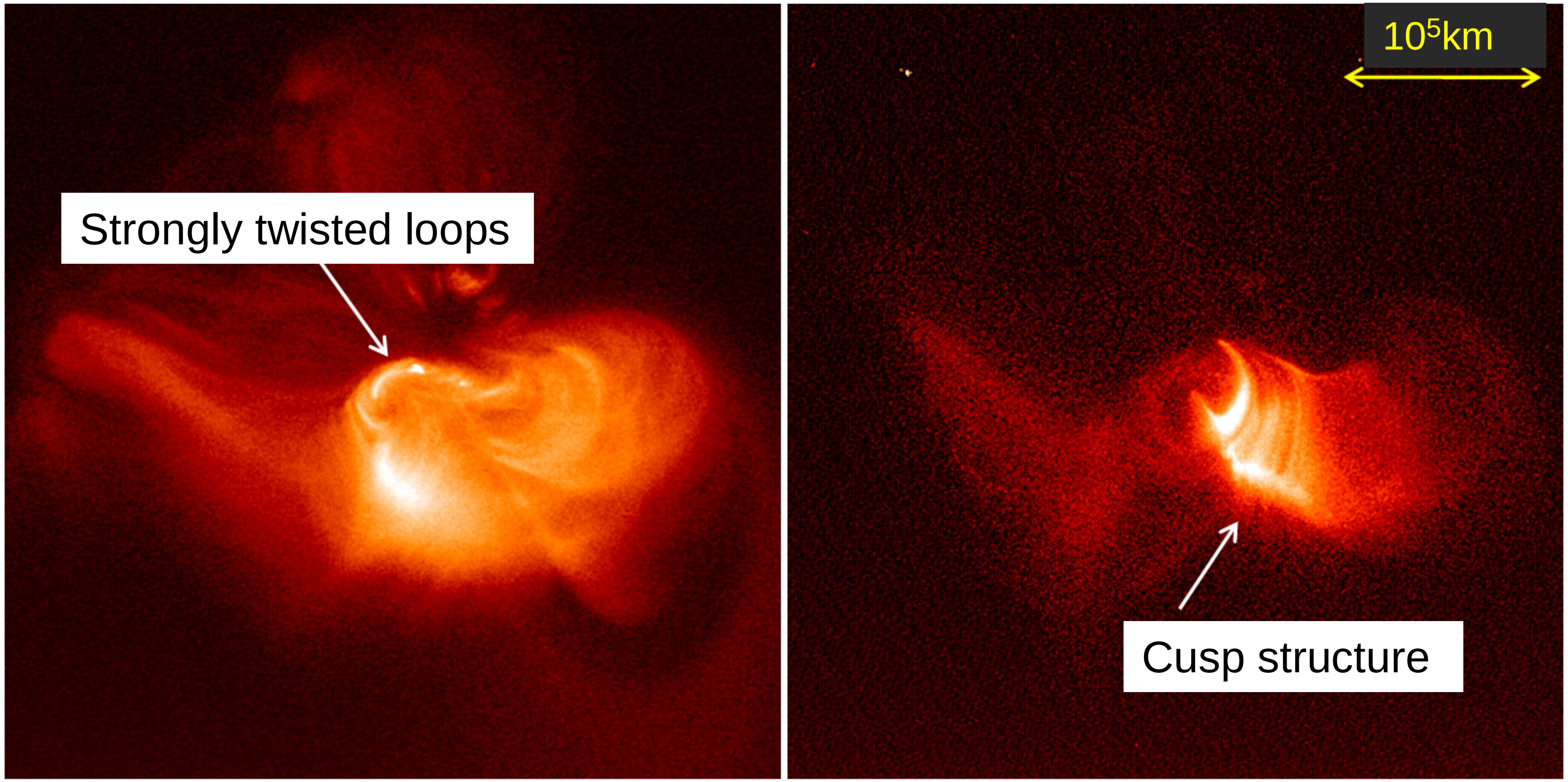


Shibata and Yokoyama 1995





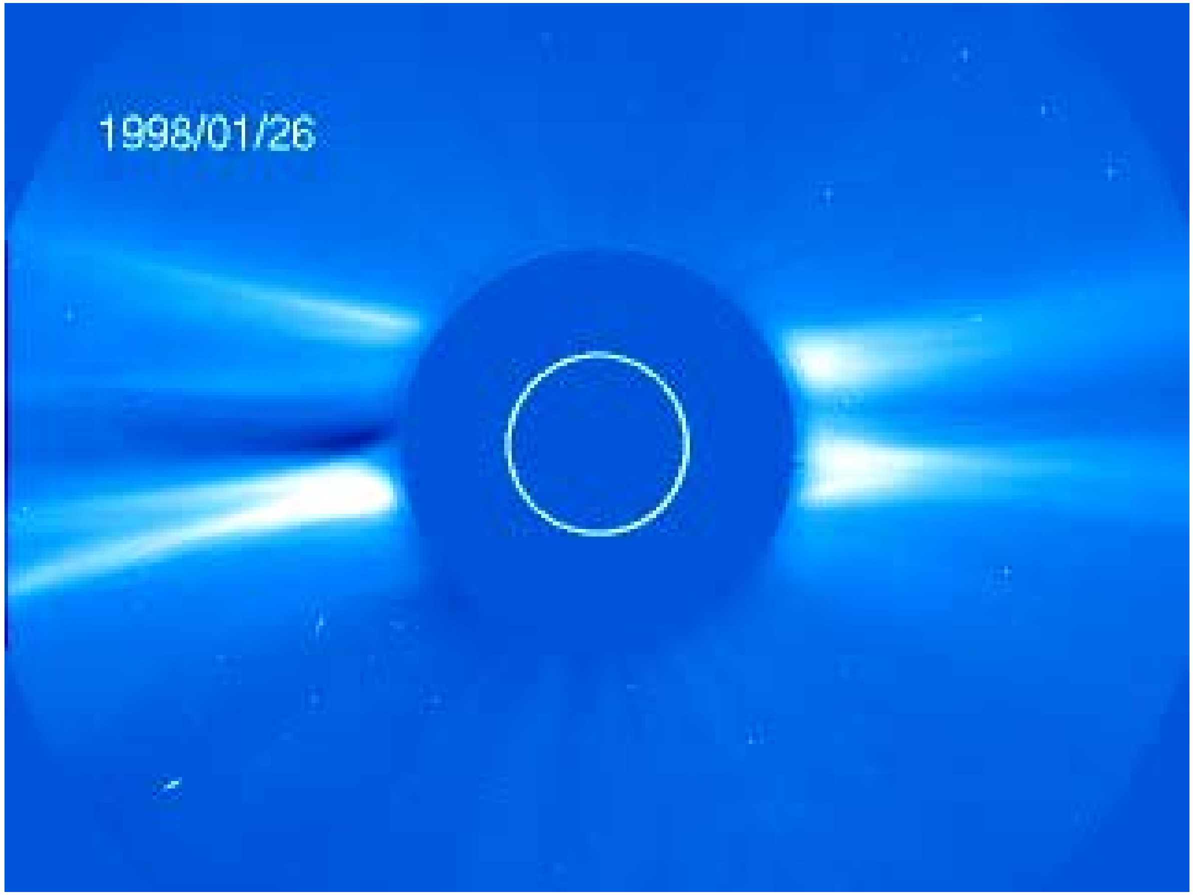
Hinode Observation of X-ray Loops Before and After a Flare



Before

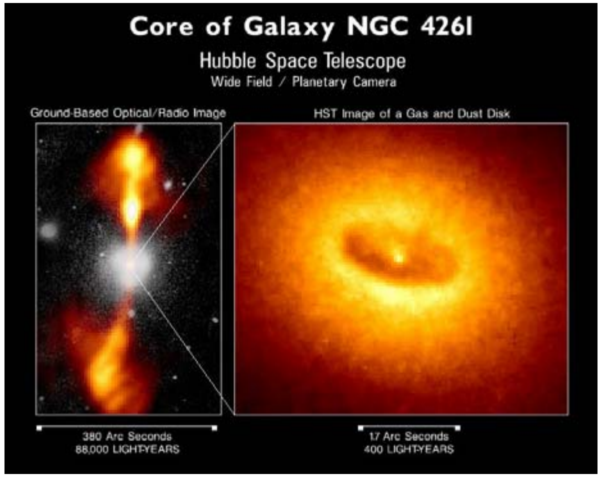
After

Outflows from the Sun

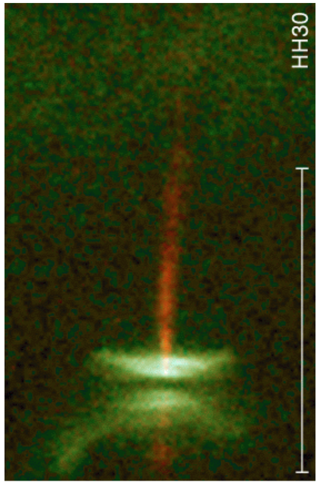


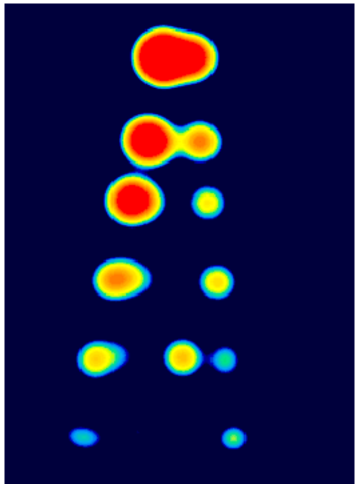
Solar Wind Observed by the SOHO Satellite

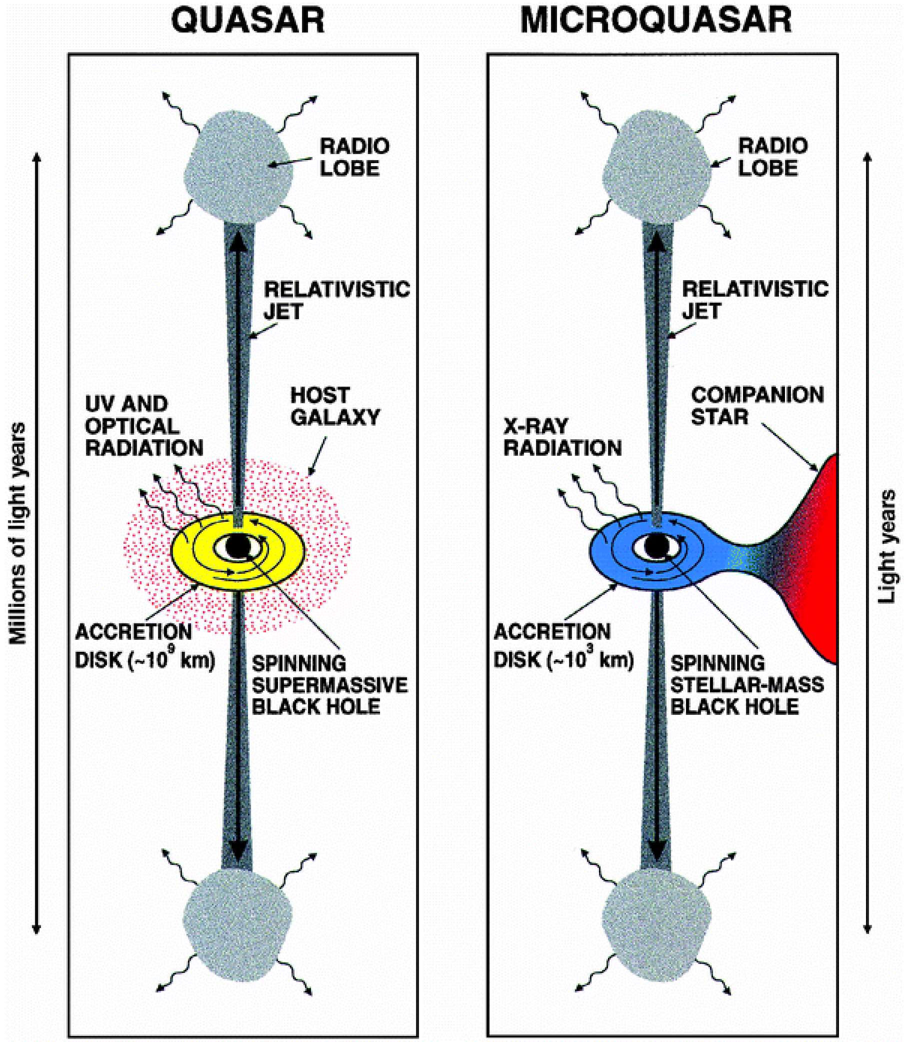
Astrophysical Jets and Disks



AGNs





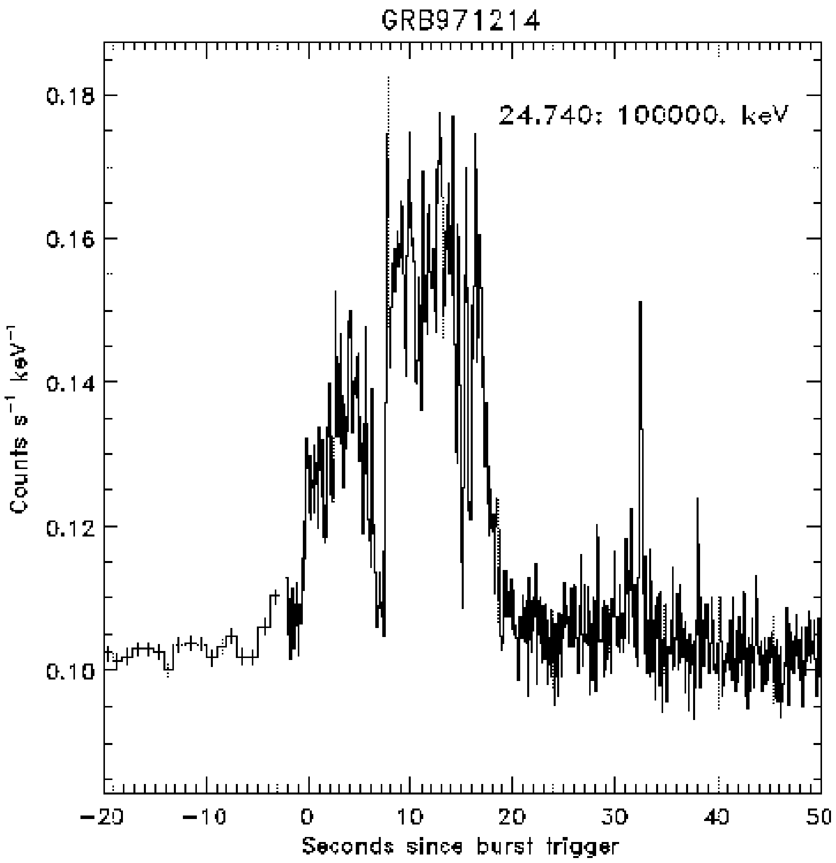


Protostars
(Burrows 1995)

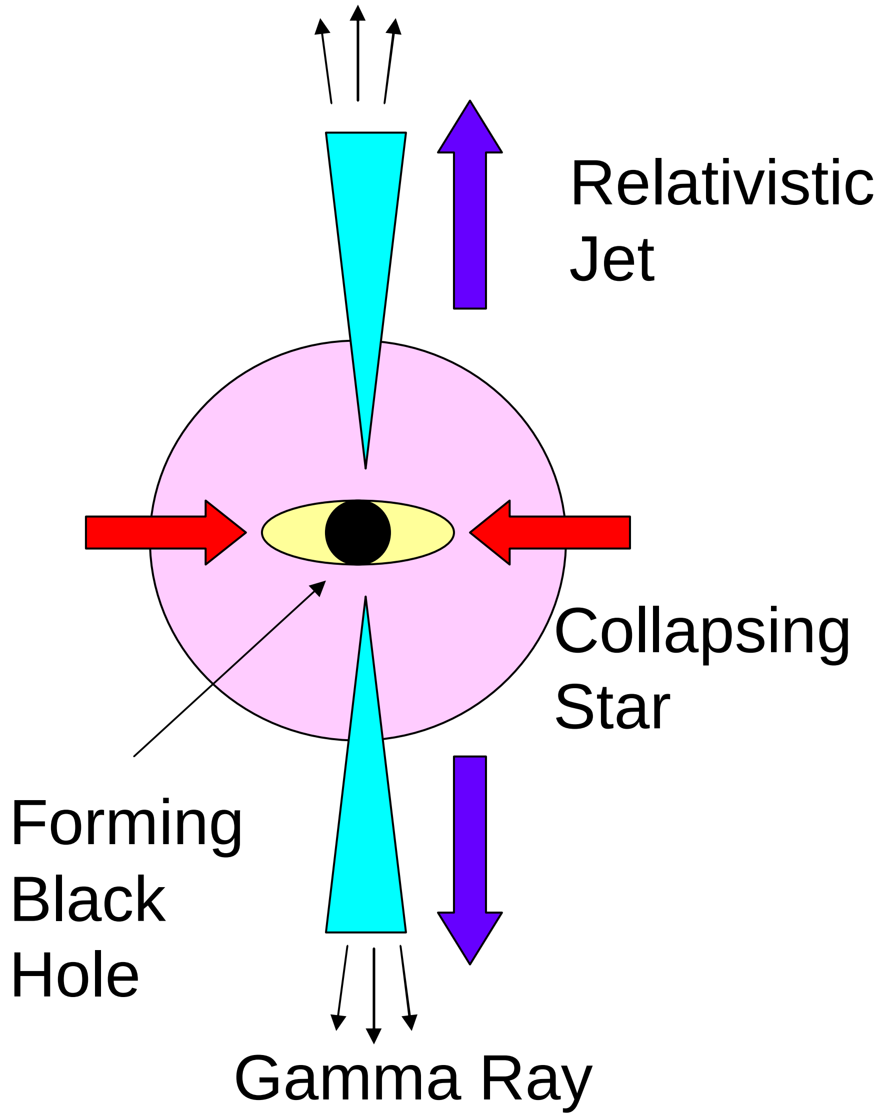
Microquasar
(Mirabel et al. 1994)

Mirabel, Rodriguez 1998

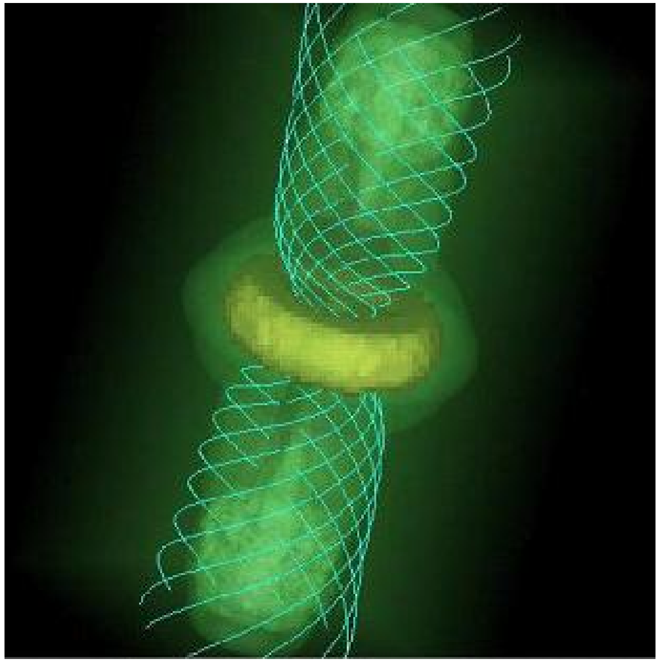
Gamma Ray Bursts



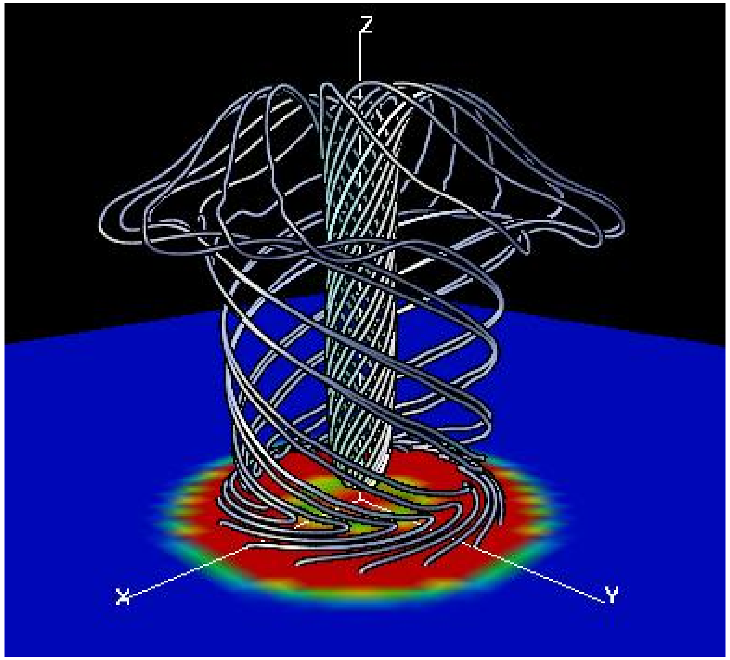
time (second)



MHD Simulation of Astrophysical Jets



Magnetocentrifugally driven jet ejected from accretion disks



Magnetic Tower Jet : Light jet dominated by Poynting flux

MHD Equations

$$\frac{\partial \rho}{\partial t} + \nabla(\rho \mathbf{v}) = 0$$

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \rho(\mathbf{v} \bullet \nabla)\mathbf{v} = -\nabla P + \frac{(\nabla \times \mathbf{B}) \times \mathbf{B}}{4\pi} + \rho \mathbf{g}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$$

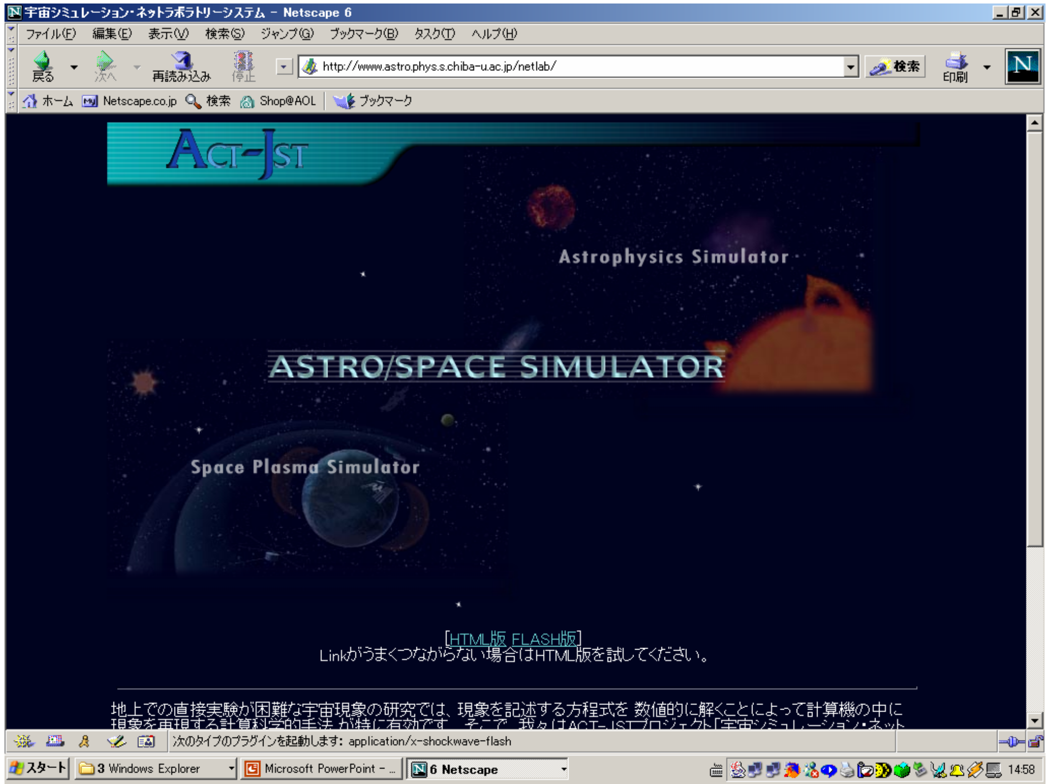
$$\frac{\partial \rho \varepsilon}{\partial t} + \nabla(\rho \varepsilon \mathbf{v}) + P \nabla \mathbf{v} = Q_J + Q_{vis} - Q_{rad}$$

MHD Simulations in Astrophysics

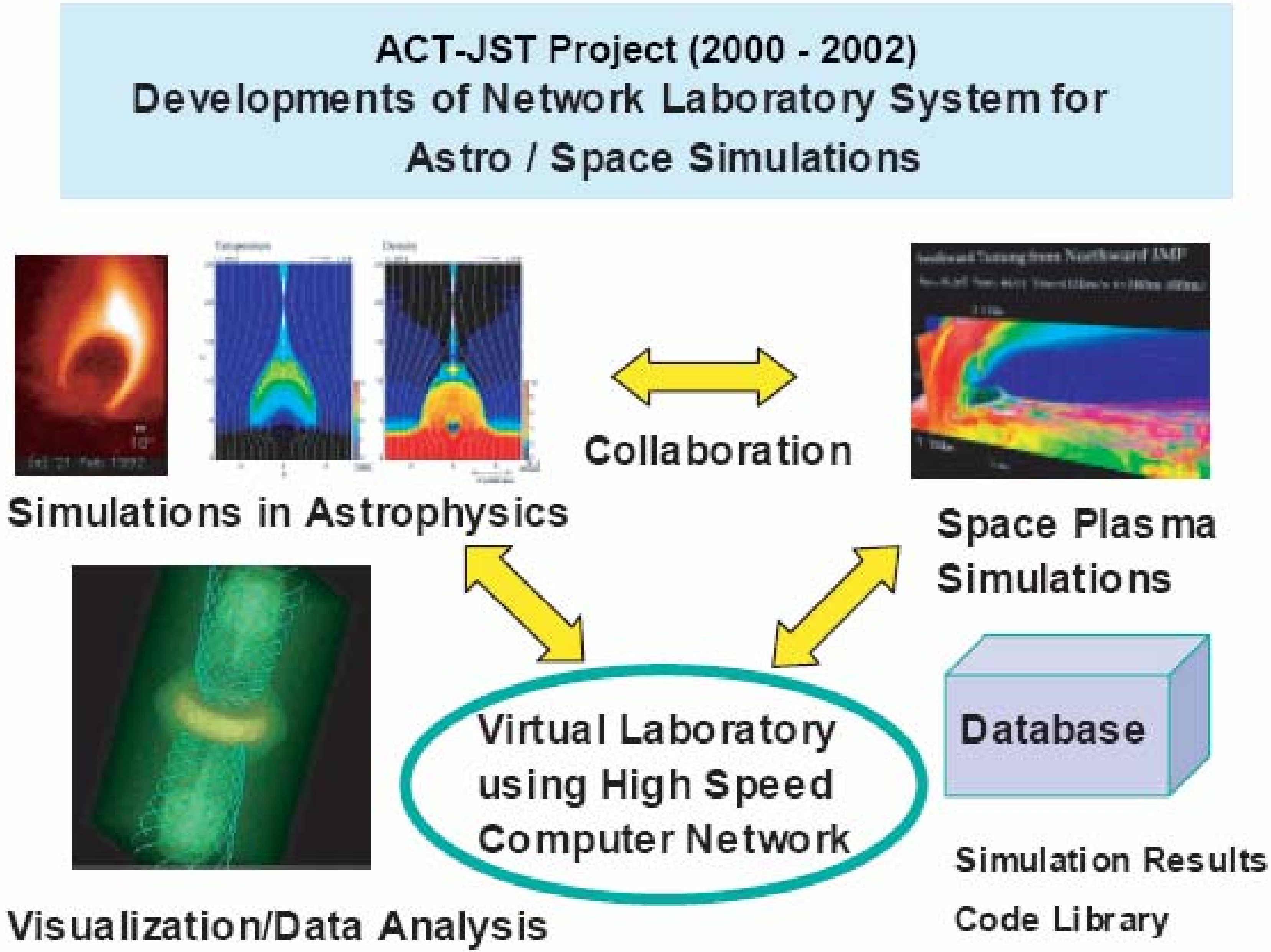
- Gravity is often important
- The backgound is not uniform
- We need to implement open boundary conditions
- Nonlinear growth of instability leads to astrophysically interesting phenomena such as flares and outflows
- We have to deal with shock waves

2. Numerical MHD Laboratory for Astrophysics

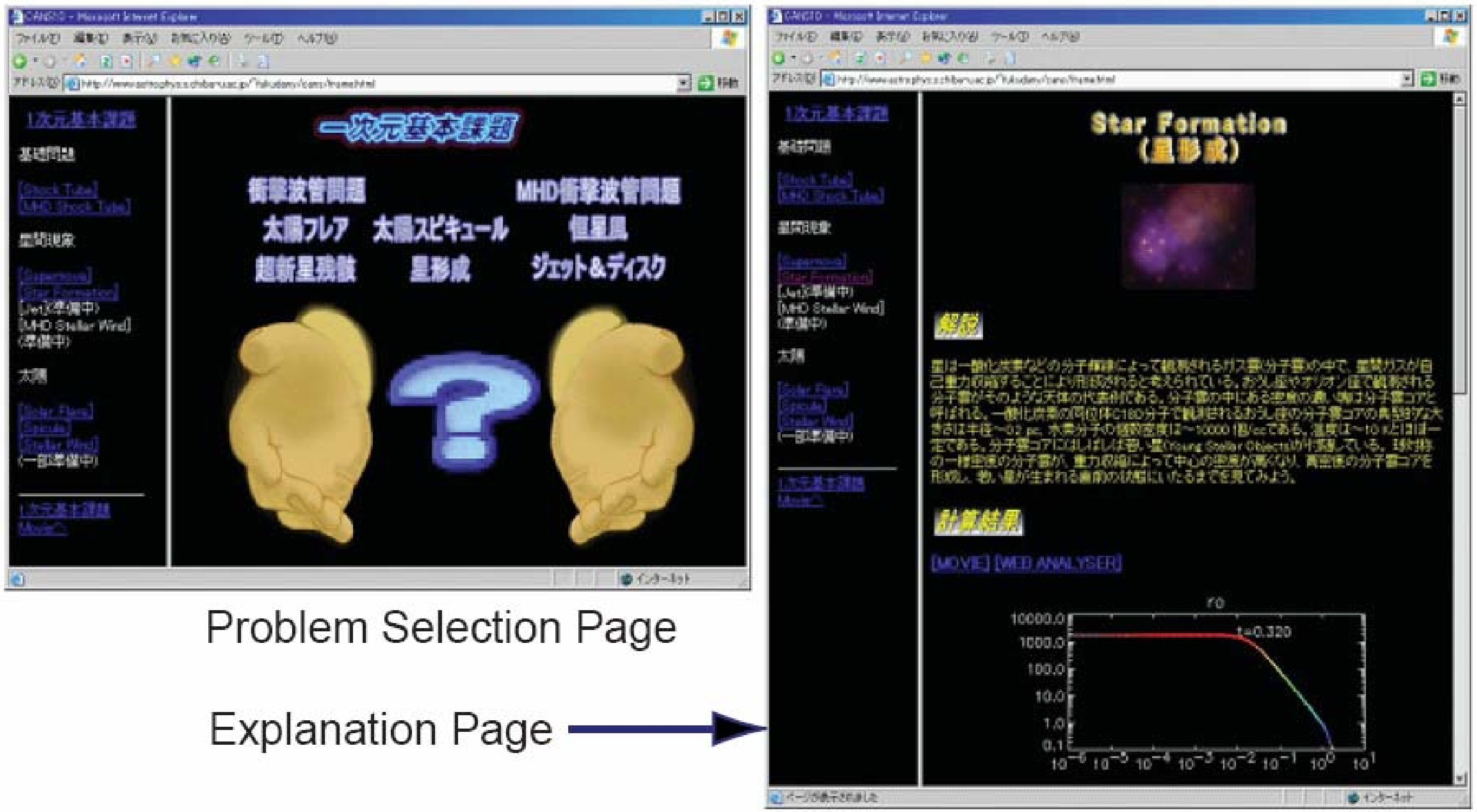
- We use computers as telescopes to explore the extreme universe
- Numerical Simulations extend our perspective

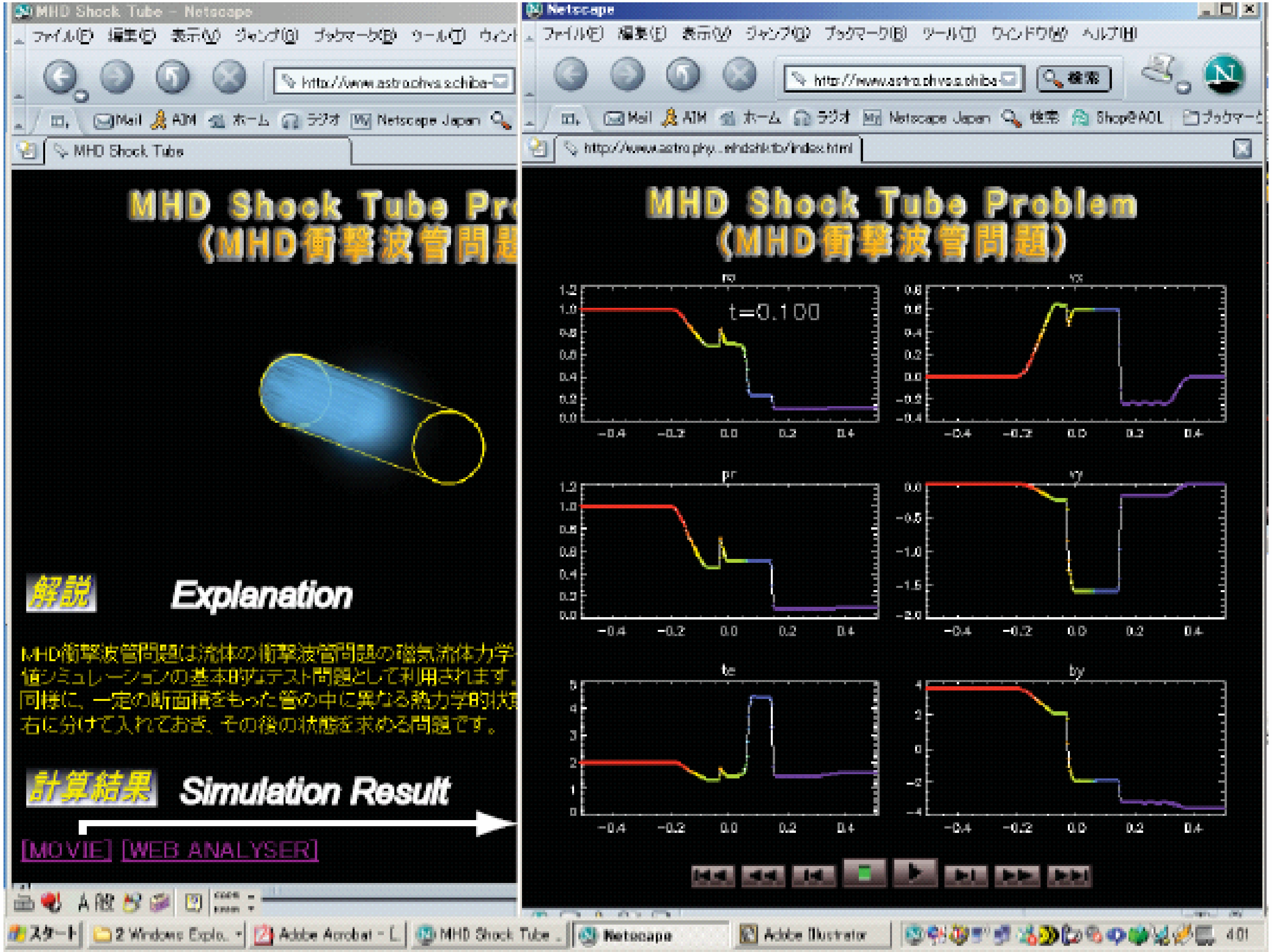


http://www.astro.phys.s.chiba-u.ac.jp/netlab/

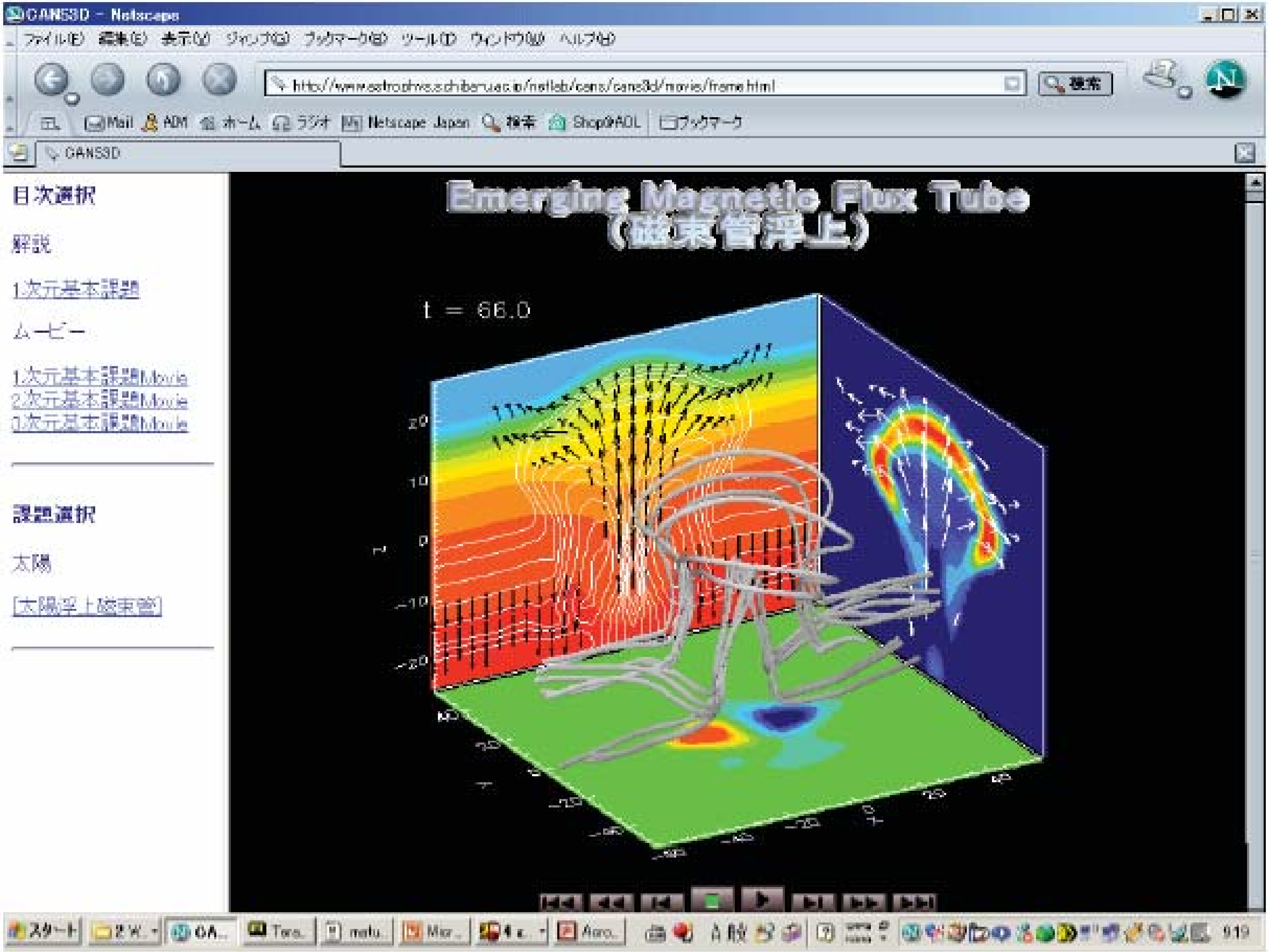

ACT-JST Project (2000 - 2002)
Developments of Network Laboratory System for
Astro / Space Simulations
Collaboration
Simulations in Astrophysics
Space Plasma
Simulations
Virtual Laboratory
using High Speed
Computer Network
Database
Simulation Results
Code Library
Visualization/Data Analysis

Contents of the AstroSimulator Page



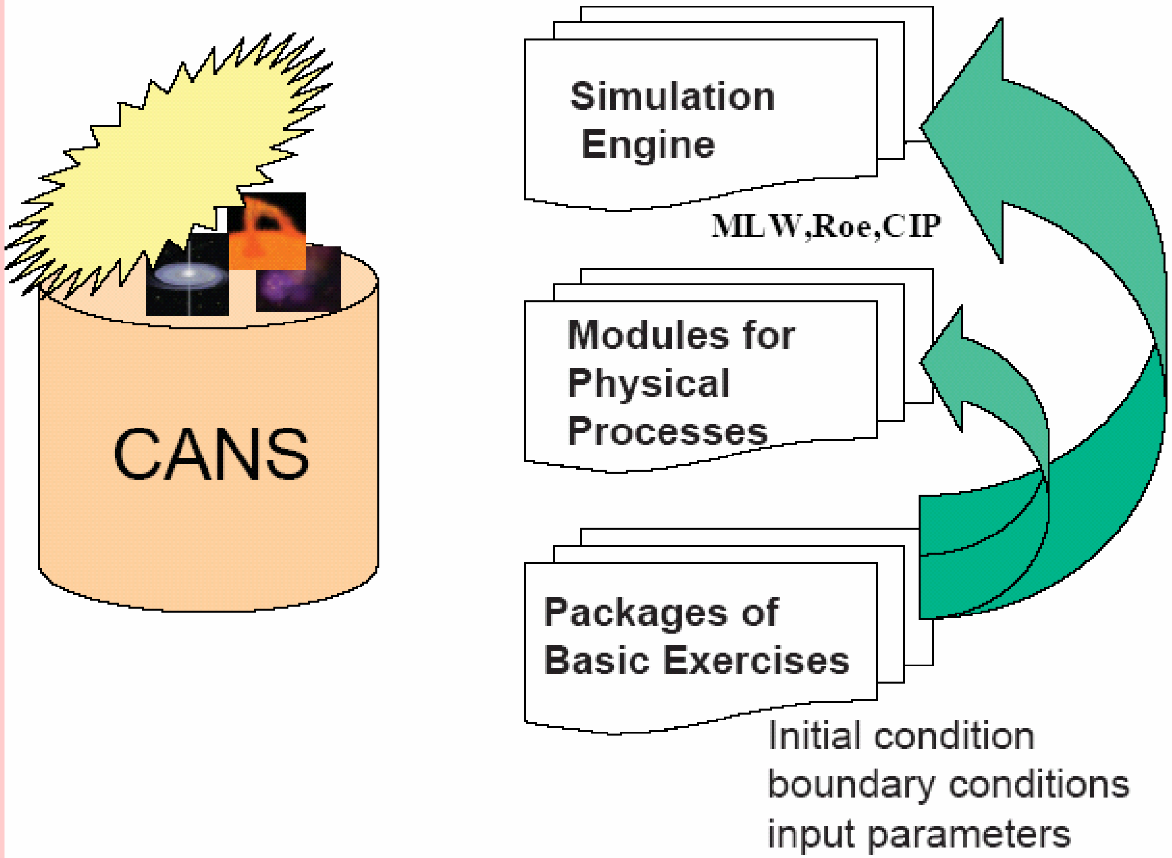

MHD Shock Tube - Netscape
MHD Shock Tube
(MHD衝撃波管問題
解説
Explanation
MHD衝撃波管問題は流体の衝撃波管問題の磁気流体力学
値シミュレーションの基本的なテスト問題として利用されます。
同様に、一定の断面積をもった管の中に異なる熱力学的状
右に分けて入れておき、その後の状態を求める問題です。
計算結果
Simulation Result
[MOVIE] [WEB ANALYSER]
Netscape
MHD Shock Tube Problem
(MHD衝撃波管問題)
ro
vx
t=0.100
pr
vy
te
by

Movie Page of 2D Basic Simulation Exercises


CANS3D - Netscape
ファイル(F) 編集(E) 表示(V) ジャンプ(G) ブックマーク(B) ツール(T) ウィンドウ(W) ヘルプ(H)
Mail AIM ホーム ラジオ Netscape Japan 検索 Shop@AOL ブックマーク
CANS3D
目次選択
解説
1次元基本課題
ムービー
1次元基本課題Movie
2次元基本課題Movie
3次元基本課題Movie
課題選択
太陽
[太陽浮上磁束管]
Emerging Magnetic Flux Tube
（磁束管浮上）
t = 66.0
スタート

Coordinated Astronomical Numerical Software (CANS)

- Main Content of the Astro-Simulation Laboratory
- Integrated Simulation code for Astrophysical MHD Simulations
- Library of Basic Simulation exercises + pluggable modules
- Users can carry out new simulations by slightly modifying the package closest to their problem.



CANS: Simulation Modules

```
>cd cans
>ls
Develop.txt Models.pdf README     cans1d/ cans3d/ htdocs/
Makefile    NonLTE     Readme.pdf cans2d/ cansnc/ idl
```

```
>cd cans2d
>ls
bc/          cndbicg/        cndsor/       cndsormpi     common/
commonmpi/   hdmlw/          htcl/         md_advect/    md_awdecay/
md_cloud/    md_cme/         md_cndsp/     md_cndtb/     md_corjet/
md_efr/      md_itmhdshktb/  md_itshktb/   md_jetprop/   md_kh/
md_mhd3dkh/  md_mhd3dshktb/  md_mhdcloud/  md_mhdcondtb/ md_mhdgwave/
md_mhdkh/    md_mhdshktb/    md_mhdsn/     md_mhdwave/   md_mri/
md_parker/   md_reccnd/      md_recon/     md_recon3/    md_rt/
md_sedov/    md_shkref/      md_shktb/     md_sndwave/   md_thinst/
mdp_awdecay/ mdp_cme/        mdp_cndsp/    mdp_cndtb/    mdp_corjet/
mdp_efr/     mdp_itmhdshktb/ mdp_itshktb/  mdp_jetprop/  mdp_kh/
mdp_mhd3kh/  mdp_mhd3shktb/  mdp_mhdcndtb/ mdp_mhdkh/    mdp_mhdshktb/
mdp_mhdsn/   mdp_mhdwave/    mdp_mri/      mdp_recon/    mdp_recon3/
mdp_rt/      mdp_sedov/      mdp_shkref/   mdp_shktb/    mdp_thinst/
```

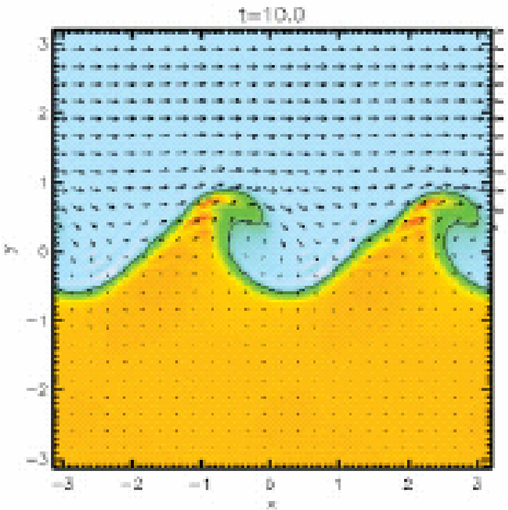
An Example of Basic Simulation Exercises : MHD supernova

```
>cd cans2d/md_mhdsn
>ls
Makefile Makefile-nc Makefile-pgnc anime.pro bnd.f
main.f   model.f     pldt.pro      rddt.pro  rdnc.pro
```

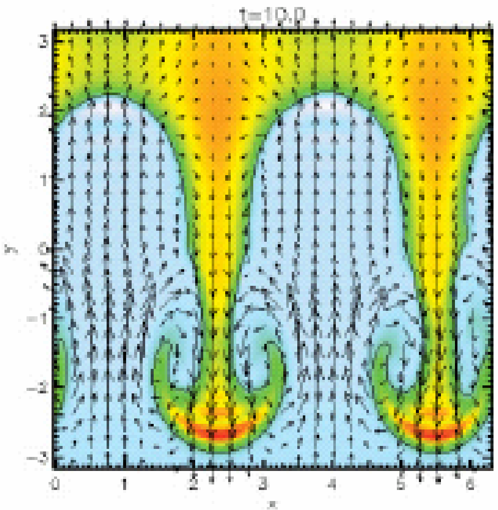
Compile and Run

```
>make 'FC=f77' ←並 'FC=mpif77' to use parallelized modules
>ls
Makefile Makefile-nc Makefile-pgnc a.out    anime.pro
ay.dac   bnd.f       bnd.o         bx.dac   bz.dac
main.f   main.o      model.f       model.o  params.txt
pldt.pro pr.dac      rddt.pro      rdnc.pro ro.dac
t.dac    vx.dac      vz.dac        x.dac    z.dac
```

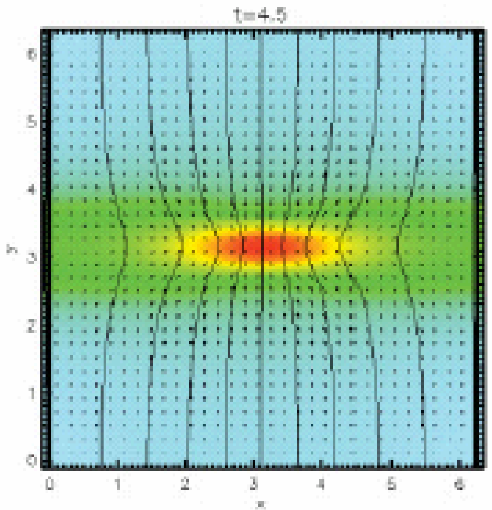
Simulation results : *.dac



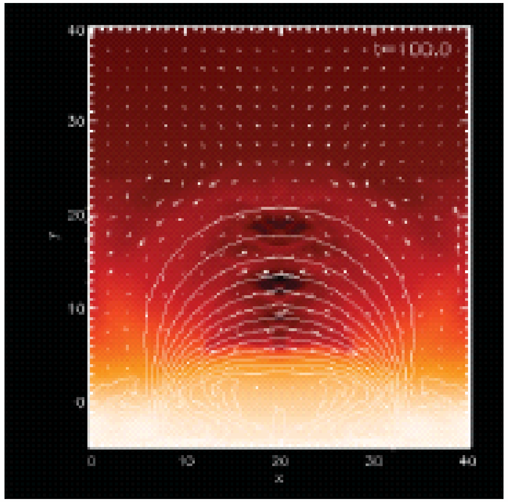
Kelvin Helmholtz Instability



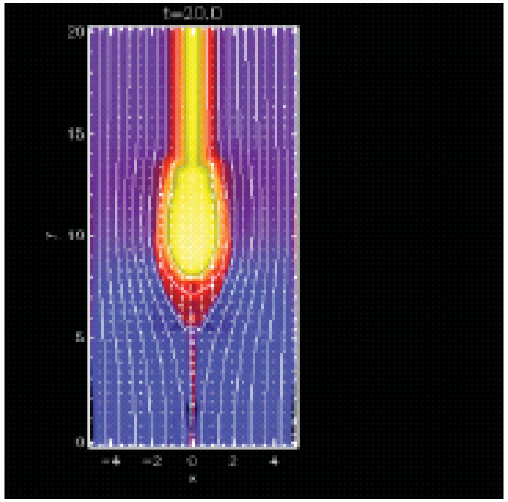
Rayleigh Taylor Instability



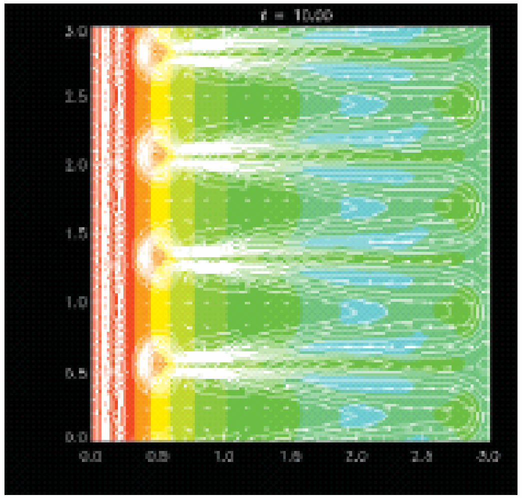
Collapse of Magnetized Cloud



Emergence of Magnetic Flux



Magnetic Reconnection



Magnetorotational Instability

CANS: Available Platforms

- You need a Fortran Compiler
- Implemented Machines
  - Linux PC, Sun、SGI WS, cygwin, VPP5000, Earth Simulator, …
  - Optimized for Vector-parallel processors
  - We prepared modules parallelized by MPI
  - Parallel Performance is more than 99.9% on Earth Simulator
- DATA Visualization
  - Use IDL
- DATA I/O ：
  - netCDF
  - Other portable format

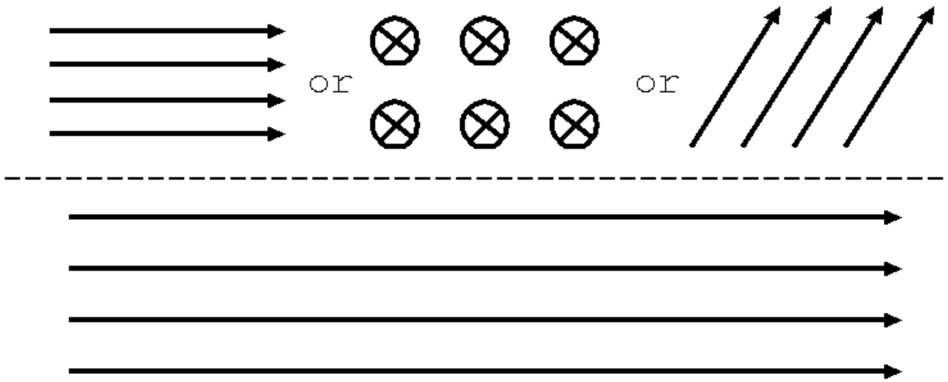
Simulation School

Snapshots of Simulation School

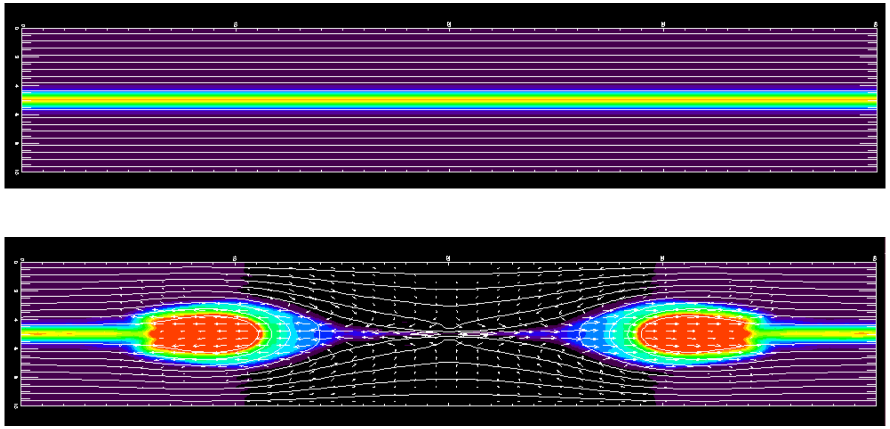


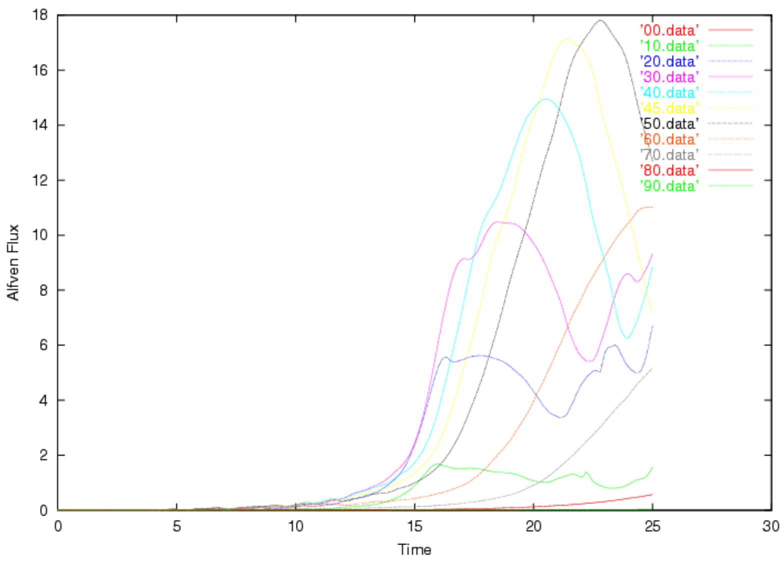


Examples of Group Projects in the Simulation School

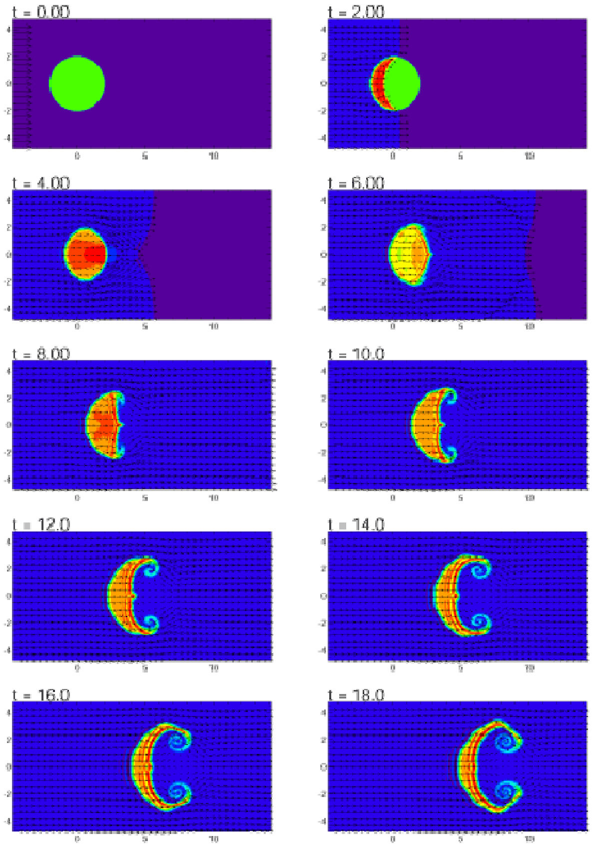


Study the dependence of the Alfven wave flux on twist angle of magnetic reconnection

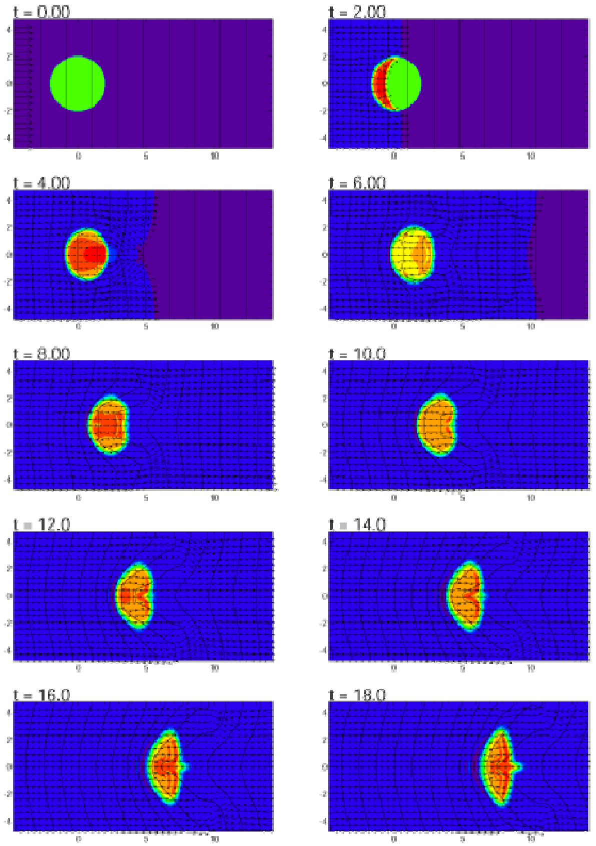




Stabilization of KH Instability by Magnetic Field

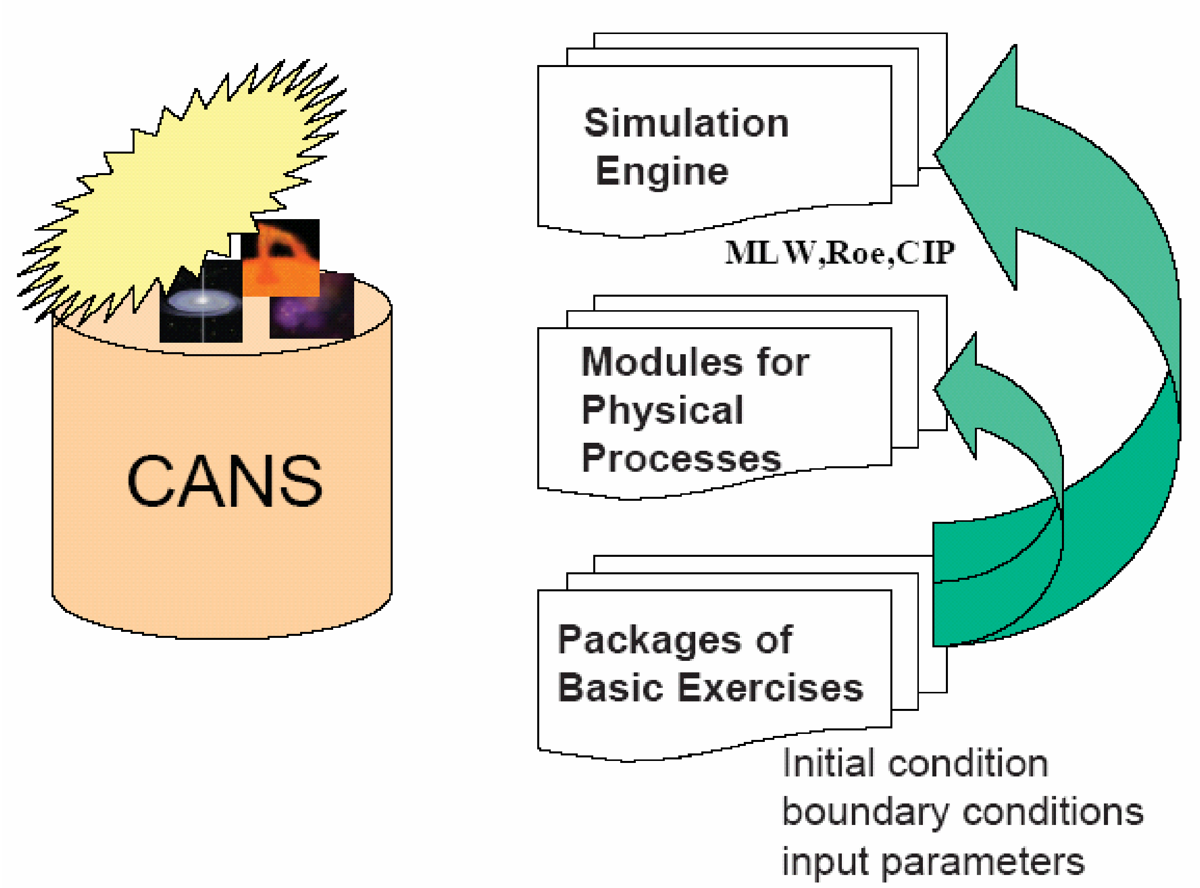


Without Magnetic Field



With Magnetic Field

3. Simulation Engines



Finite Difference Solutions for System Equations

- Basic Equation in Conservation Form

$$\frac{\partial \boldsymbol{u}}{\partial t} + \frac{\partial \boldsymbol{f}}{\partial x} = 0$$

In ideal hydrodynamics,

$$\boldsymbol{u} = \begin{pmatrix} \rho \\ \rho v \\ \dfrac{\rho v^2}{2} + \dfrac{P}{\gamma - 1} \end{pmatrix} \quad \boldsymbol{f} = \begin{pmatrix} \rho v \\ \rho v^2 + P \\ \dfrac{\rho v^3}{2} + \dfrac{\gamma P v}{\gamma - 1} \end{pmatrix}$$

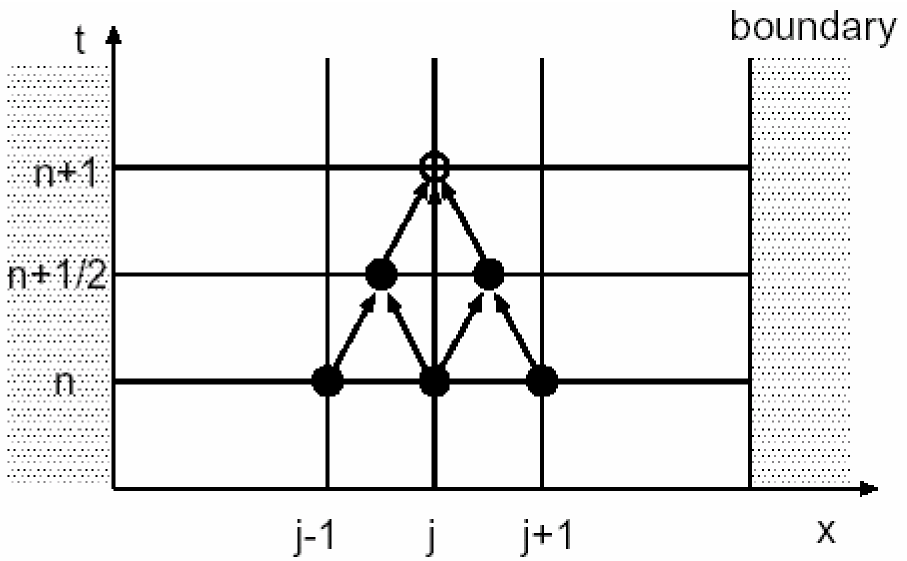
Basic Equations in Ideal MHD

$$\boldsymbol{u} = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ B_y \\ B_z \\ \rho E \end{pmatrix} \quad \boldsymbol{f} = \begin{pmatrix} \rho u \\ \rho u^2 + P + \dfrac{{B_y}^2 + {B_z}^2 - {B_x}^2}{8\pi} \\ \rho u v - \dfrac{B_x B_y}{4\pi} \\ \rho u w - \dfrac{B_x B_z}{4\pi} \\ B_y u - v B_x \\ B_z u - w B_x \\ \rho H u - \dfrac{B_x (B_x u + B_y v + B_z w)}{4\pi} \end{pmatrix}$$

$$E = \frac{u^2 + v^2 + w^2}{2} + \frac{P}{(\gamma - 1)\rho} + \frac{{B_x}^2 + {B_y}^2 + {B_z}^2}{8\pi\rho}$$

$$H = \frac{u^2 + v^2 + w^2}{2} + \frac{\gamma P}{(\gamma - 1)\rho} + \frac{{B_x}^2 + {B_y}^2 + {B_z}^2}{4\pi\rho}$$

Two-step Lax-Wendroff Scheme



$$U_{j+1/2}^{n+1/2} = \frac{U_{j+1}^{n} + U_{j}^{n}}{2} - \frac{1}{2}\frac{\Delta t}{\Delta x}(F_{j+1}^{n} - F_{j}^{n})$$

$$U_{j}^{n+1} = U_{j}^{n} - \frac{\Delta t}{\Delta x}(F_{j+1/2}^{n+1/2} - F_{j-1/2}^{n+1/2})$$

Approximate Riemann Solver

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{A}\frac{\partial \boldsymbol{u}}{\partial x} = 0$$ Where $$\boldsymbol{A} = \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{u}}$$

$$|\boldsymbol{A} - \lambda \boldsymbol{I}| = 0$$ → eigenvalues

We can diagonalize the Matrix A by using Right eigenvectors and left eigenvectors of A

$$\boldsymbol{\Lambda} = \boldsymbol{LAR}$$

$$= \begin{pmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \lambda_3 \end{pmatrix}$$

By defining

$$d\mathbf{w} = \mathbf{L}d\mathbf{u}$$ we obtain $$\frac{\partial w_i}{\partial t} + \lambda_i \frac{\partial w_i}{\partial x} = 0$$

Numerical Flux

- We can apply upwind scheme for each wave

$$\frac{\partial w_i}{\partial t} + \lambda_i \frac{\partial w_i}{\partial x} = 0$$

- Numerical Flux

$$\begin{aligned}\tilde{\boldsymbol{f}}_{u,j+1/2} &= \boldsymbol{R}\,\tilde{\boldsymbol{f}}_{w,j+1/2} \\ &= \frac{1}{2}\left[\boldsymbol{f}_{u,j+1} + \boldsymbol{f}_{u,j} - \boldsymbol{R}\,|\boldsymbol{\Lambda}|\,\boldsymbol{L}\,(\boldsymbol{u}_{j+1} - \boldsymbol{u}_j)\right]\end{aligned}$$

$$\boldsymbol{u}_j(t + \Delta t) = \boldsymbol{u}_j(t) - \frac{\Delta t}{\Delta x}\left(\tilde{\boldsymbol{f}}_{u,j+1/2} - \tilde{\boldsymbol{f}}_{u,j-1/2}\right)$$

Roe Average

- Numerical Flux is computed by using the following average (Roe Average)

$$
\begin{aligned}
\bar{\rho} &= \sqrt{\rho_{j+1}\rho_j} \\
\bar{v} &= \frac{\sqrt{\rho_{j+1}}v_{j+1} + \sqrt{\rho_j}v_j}{\sqrt{\rho_{j+1}} + \sqrt{\rho_j}} \\
\bar{H} &= \frac{\sqrt{\rho_{j+1}}H_{j+1} + \sqrt{\rho_j}H_j}{\sqrt{\rho_{j+1}} + \sqrt{\rho_j}} \\
\bar{c}_{\mathrm{s}}^2 &= (\gamma - 1)\left(\bar{H} - \frac{\bar{v}^2}{2}\right)
\end{aligned}
$$

Property U

- The velocity matrix A computed by Roe Average satisfies
  - For any u_j and u_{j+1}

$$\mathbf{f}_{j+1} - \mathbf{f}_j = \mathbf{A}(\mathbf{u}_{j+1}, \mathbf{u}_j)(\mathbf{u}_{j+1} - \mathbf{u}_j)$$

  - All eigenvectors are real
  - When $u_{j+1} = u_j$

$$\boldsymbol{A} = \partial \boldsymbol{f} / \partial \boldsymbol{u}$$

- These properties are called “Property U”

CIP-MOCCT Scheme

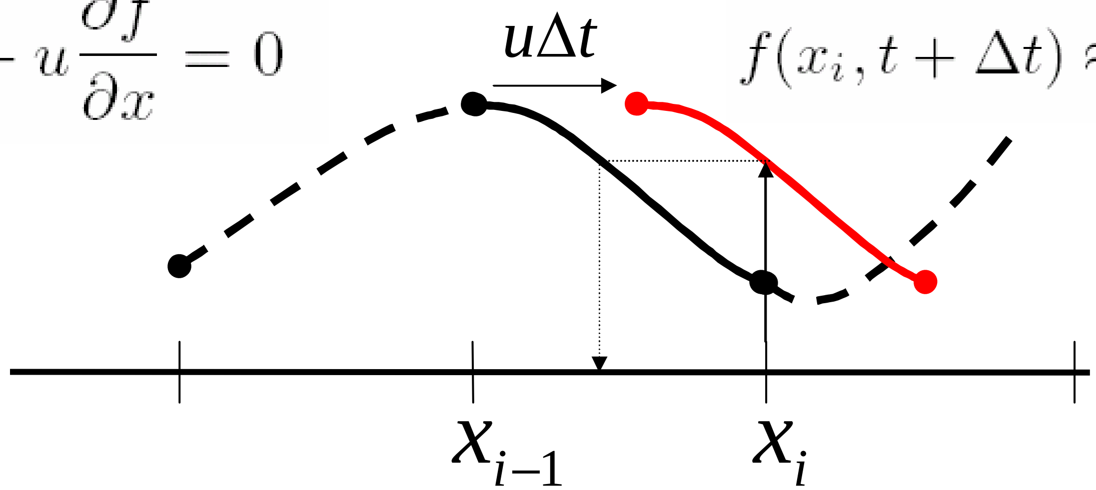
- CIP scheme has been developed by Yabe (1991)

Cubic Interpolated Propagation

Constrained Interpolation Profile

$$\frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} = 0$$

$$f(x_i, t+\Delta t) \approx f(x_i - u_i\Delta t, t)$$

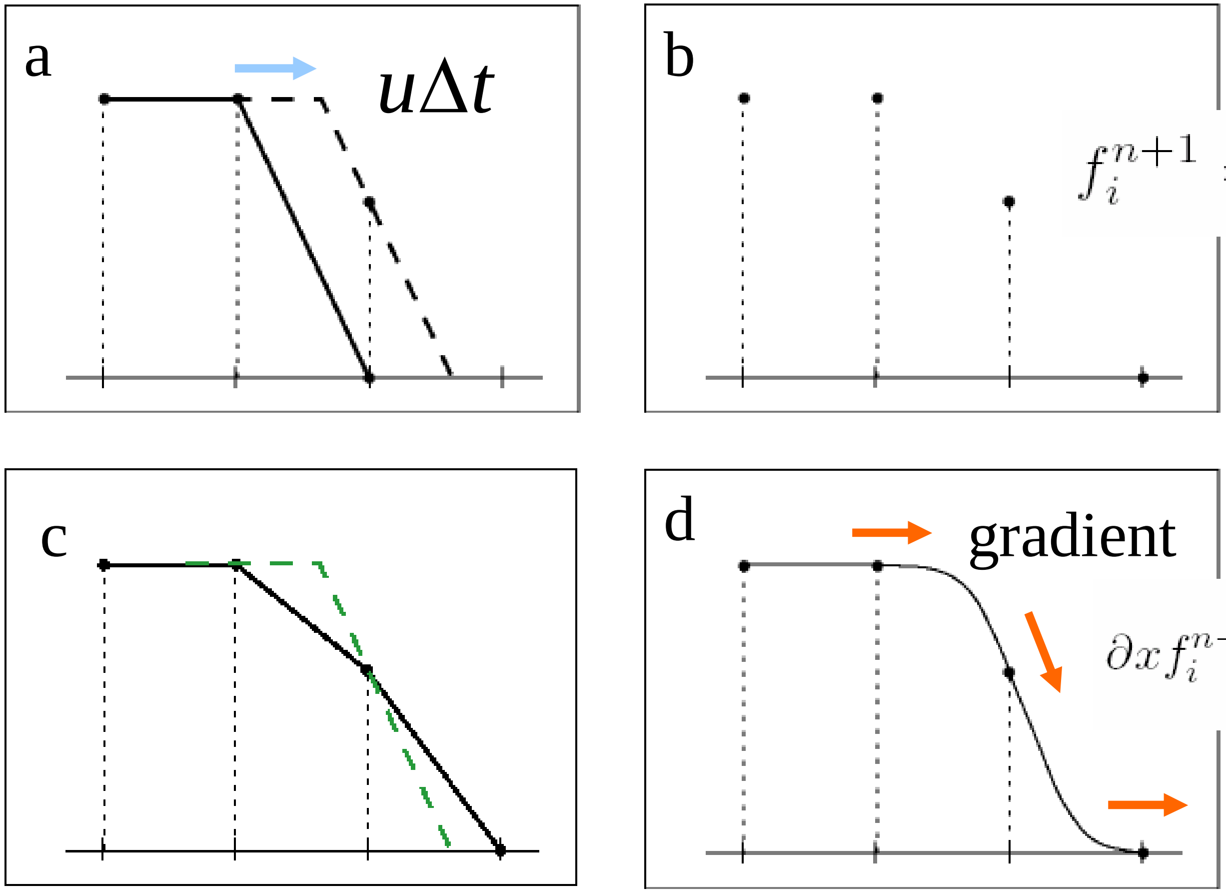


Coefficients a,b,c,d are determined by $F(x_{i-1})=F_{i-1}$, $F(x_i)=F_i$, $G(x_{i-1})=G_{i-1}$ and $G(x_i)=G_i$

$$F_i(x) = a_i(x-x_i)^3 + b_i(x-x_i)^2 + c_i(x-x_i) + d_i$$

$$G_i(x) = 3a_i(x-x_i)^2 + 2b_i(x-x_i) + c_i \quad [\; x_{i-1} \le x \le x_i \;]$$

CIP Transports both Physical Quantities and their Gradient



$$f_i^{n+1} = F(x_i - u\Delta t)$$

$$\partial x f_i^{n+1} = \frac{dF(x_i - u\Delta t)}{dx}$$

$= G(x_i - u\Delta t)$

CIP Scheme for MHD Equations

Transport

Source Term

$$\frac{\partial \rho}{\partial t} + (\mathbf{u} \cdot \nabla)\rho = -\rho \nabla \cdot \mathbf{u}$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{1}{\rho}\nabla(p + \frac{B^2}{8\pi}) + \frac{1}{4\pi\rho}(\mathbf{B} \cdot \nabla)\mathbf{B} + \mathbf{Q_f}$$

$$\frac{\partial p}{\partial t} + (\mathbf{u} \cdot \nabla)p = -\gamma p \nabla \cdot \mathbf{u} + \mathbf{Q_p}$$

$$\frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{u} \times \mathbf{B}) = 0$$

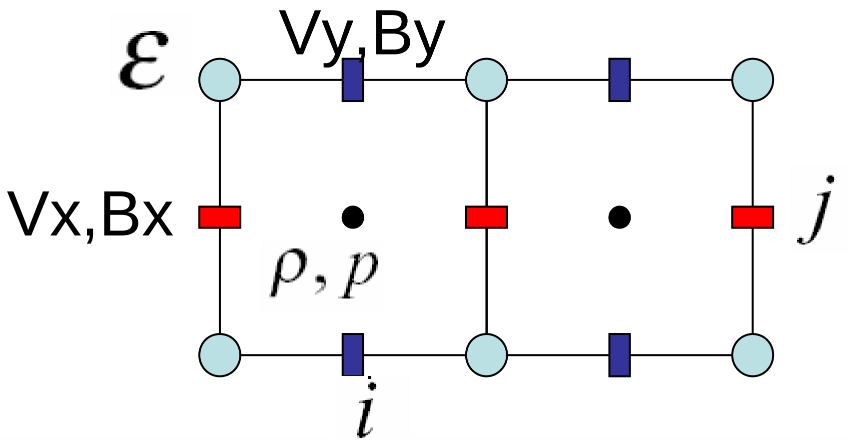
$$\nabla \cdot \mathbf{B} = 0$$

Special Care should be taken for induction equation and div B=0

CT (Constrained Transport) scheme

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}) \longrightarrow \frac{\partial B_x}{\partial t} = -\frac{\partial \varepsilon}{\partial y}, \quad \frac{\partial B_y}{\partial t} = \frac{\partial \varepsilon}{\partial x}$$

$$\varepsilon = -(\mathrm{V}_x B_y - \mathrm{V}_y B_x)$$



Staggered mesh

div B=0 is satisfied

$$\frac{B^{n+1}_{x(i+1/2,j)} - B^{n}_{x(i+1/2,j)}}{\Delta t} = -\frac{\varepsilon^{n+1/2}_{(i+1/2,j+1/2)} - \varepsilon^{n+1/2}_{(i+1/2,j-1/2)}}{\Delta y}$$

$$\frac{B^{n+1}_{y(i,j+1/2)} - B^{n}_{y(i,j+1/2)}}{\Delta t} = \frac{\varepsilon^{n+1/2}_{(i+1/2,j+1/2)} - \varepsilon^{n+1/2}_{(i-1/2,j+1/2)}}{\Delta x}$$

MOC scheme (Stone and Norman 1992)

$$\frac{\partial \mathrm{V}_y}{\partial t} = \frac{B_y}{4\pi\rho}\frac{\partial B_y}{\partial x} - \frac{\partial}{\partial x}\left(\mathrm{V}_x \mathrm{V}_y\right)$$

$$\frac{\partial B_y}{\partial t} = B_x \frac{\partial \mathrm{V}_y}{\partial x} - \frac{\partial}{\partial x}\left(\mathrm{V}_x B_y\right)$$

$$\varepsilon = -\left(\mathrm{V}_x^* B_y^* - \mathrm{V}_y^* B_x^*\right)$$

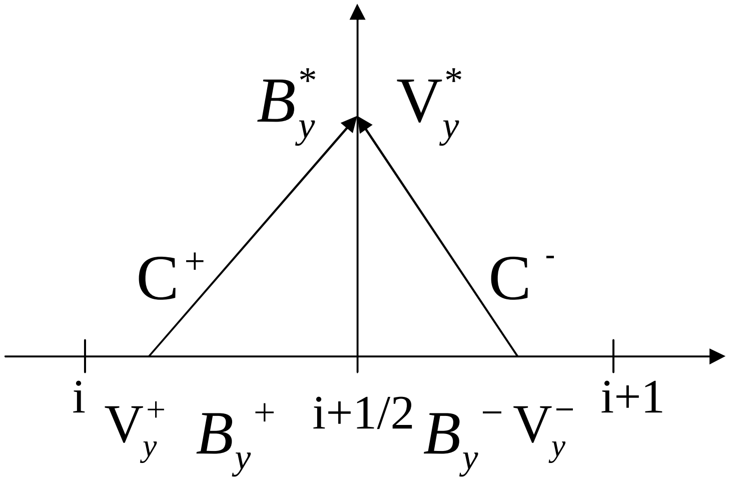
Compute vx*,By* etc by Method of Characteristics

$$\frac{D\mathrm{V}_y}{Dt} \mp \frac{1}{\sqrt{4\pi\rho}}\frac{DB_y}{Dt} = 0, \qquad \frac{D}{Dt} = \frac{\partial}{\partial t} + \left(\mathrm{V}_x \pm \frac{B_x}{\sqrt{4\pi\rho}}\right)\frac{\partial}{\partial x}$$

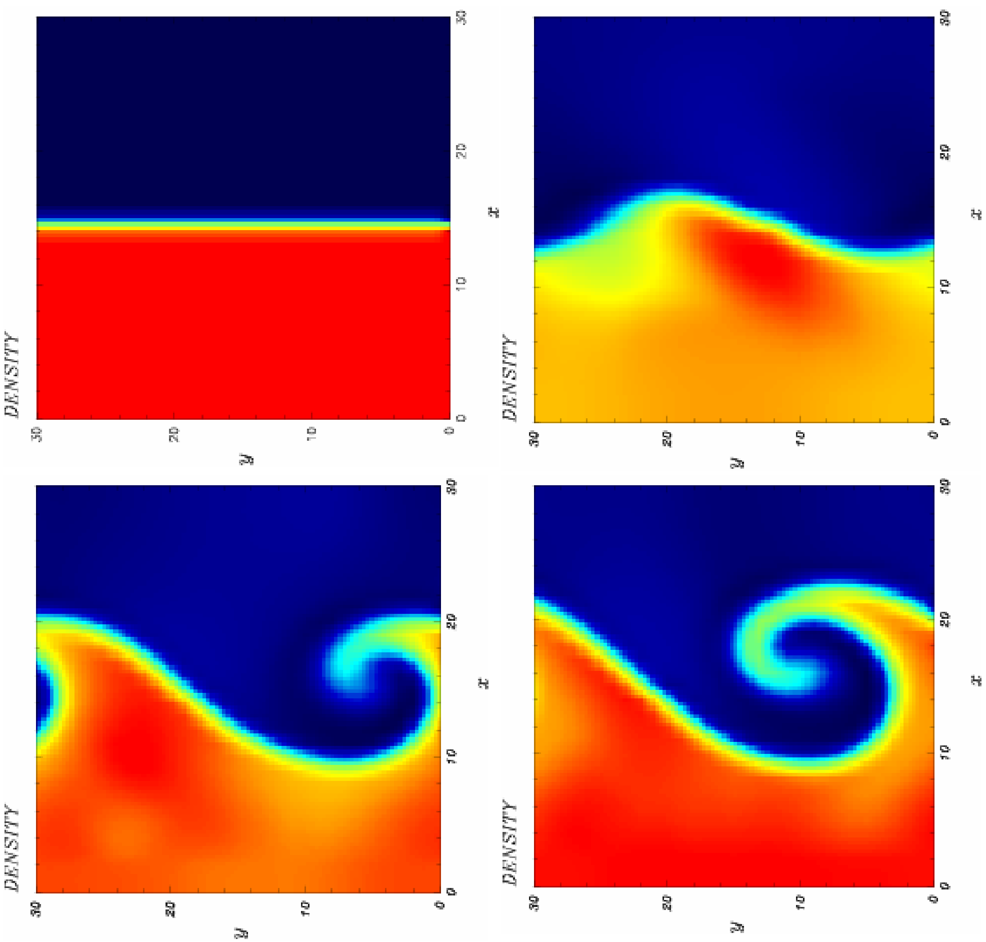
When ρ is constant,

$$\left(\mathrm{V}_y^* - \mathrm{V}_y^+\right) - \frac{1}{\sqrt{4\pi\rho^+}}\left(B_y^* - B_y^+\right) = 0$$

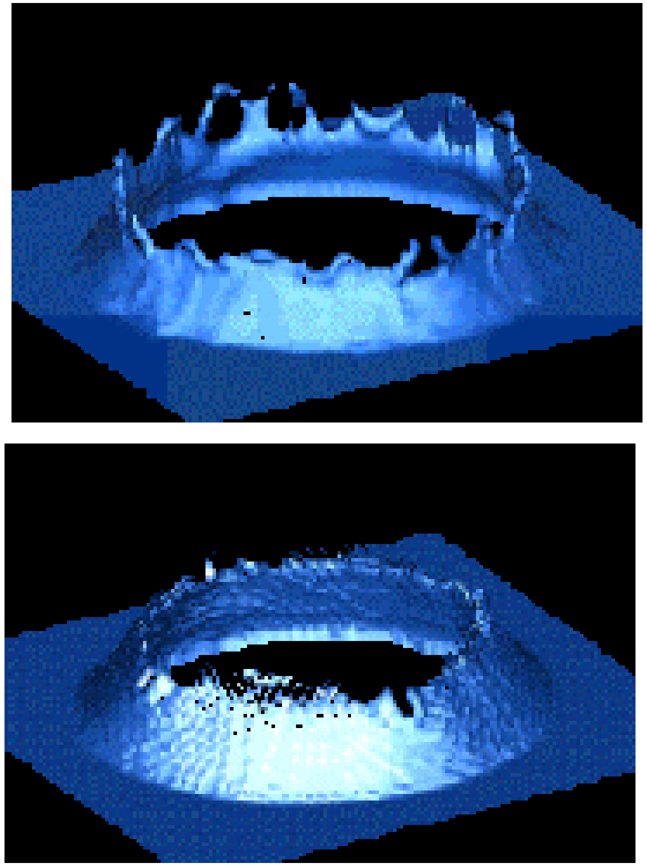
$$\left(\mathrm{V}_y^* - \mathrm{V}_y^-\right) + \frac{1}{\sqrt{4\pi\rho^-}}\left(B_y^* - B_y^-\right) = 0$$



Example of Simulation using CIP



Grid number: 31×31

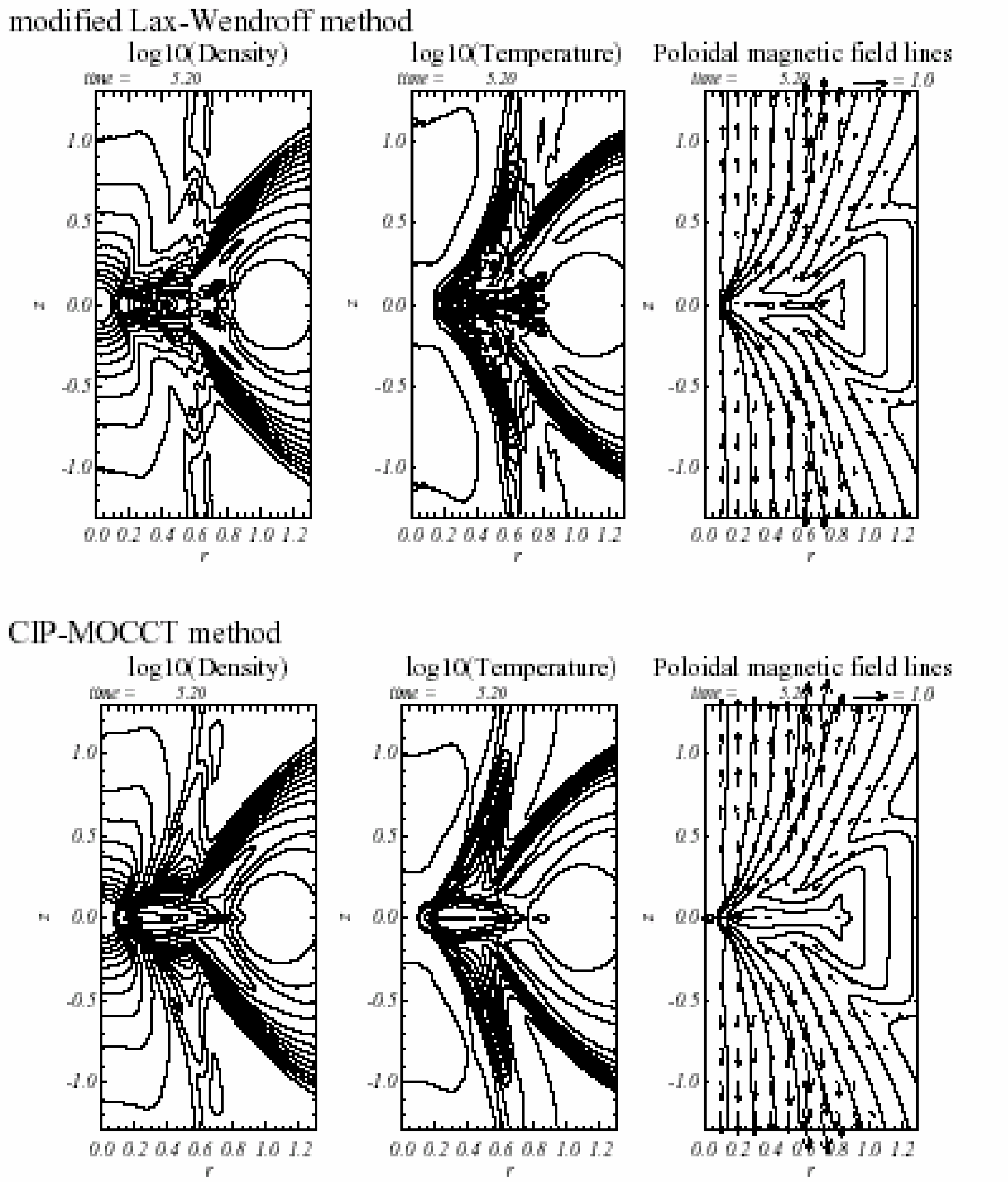


Simulation of Milk Crown

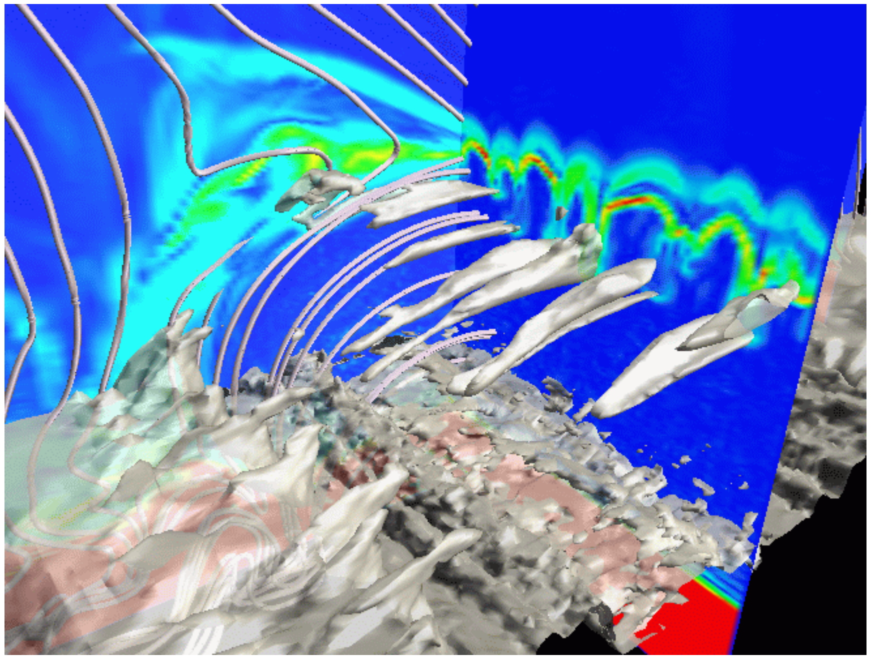
by using CIP

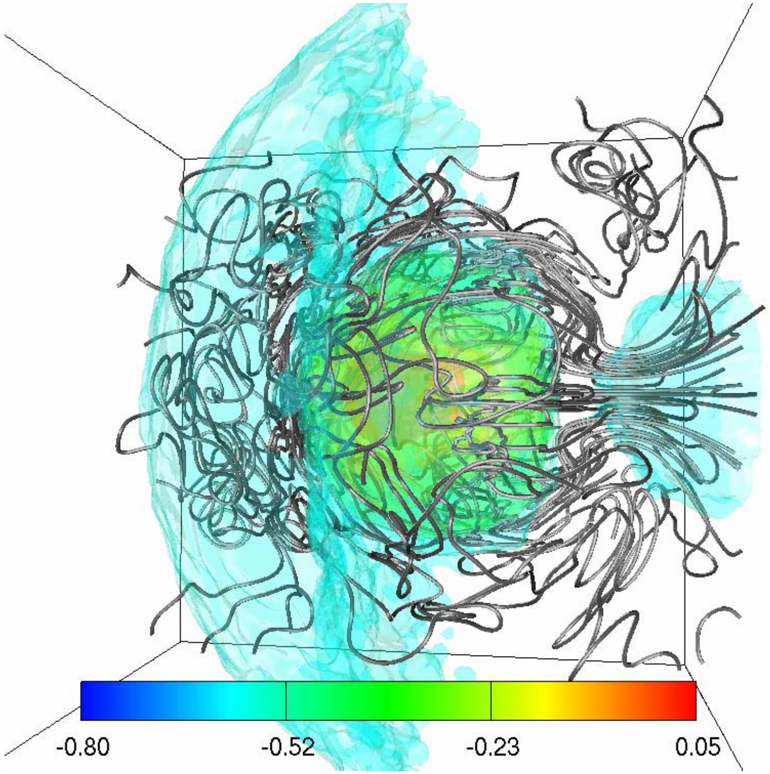
Comparison between MLW scheme and CIP-MOCCT scheme

(Kudoh et al. 1998)



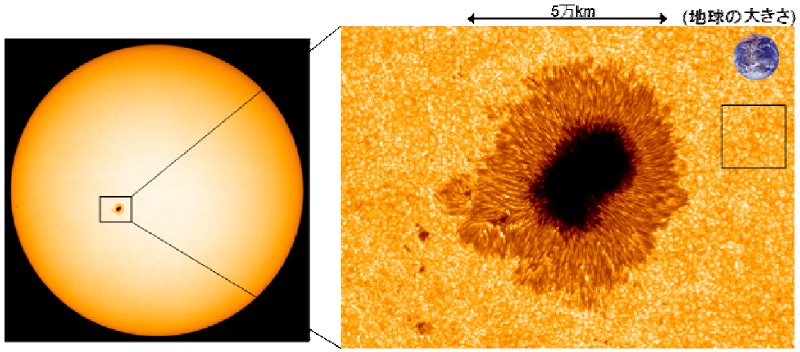
4. Examples of MHD Simulations using CANS



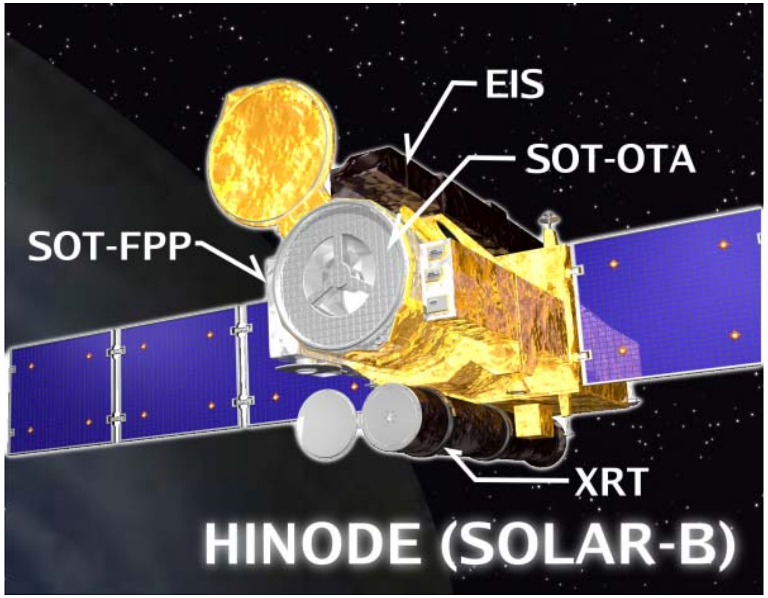


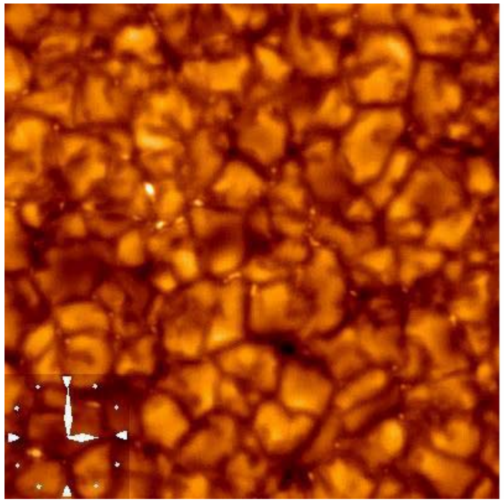
Solar Convection Observed by HINODE

Launch of HINODE
２００６．９．２３

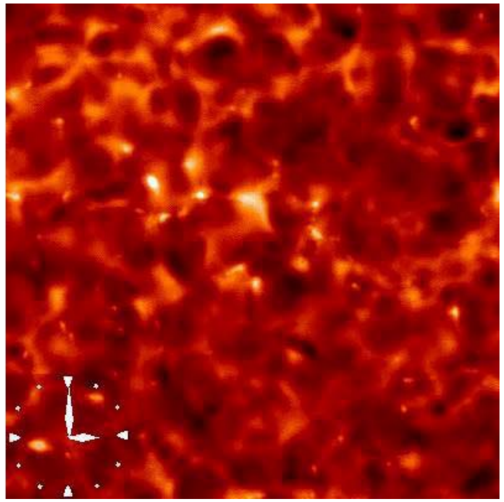


Optical Telescope



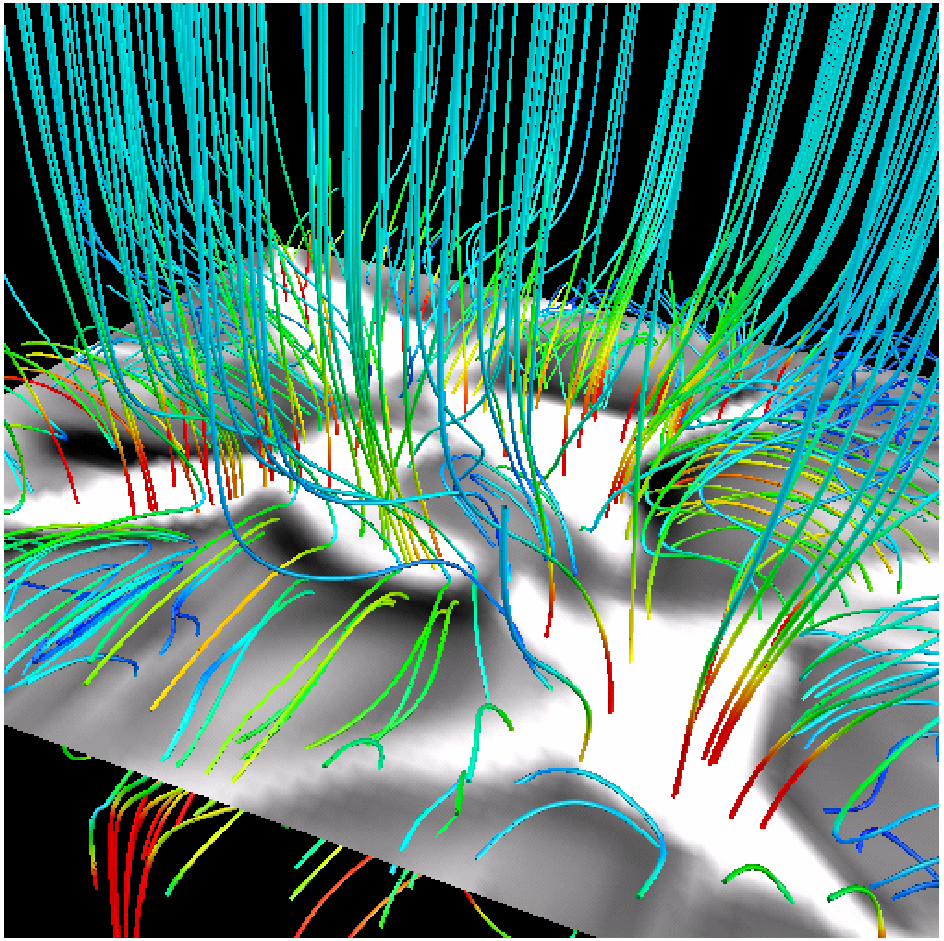


G band
(Photosphere)

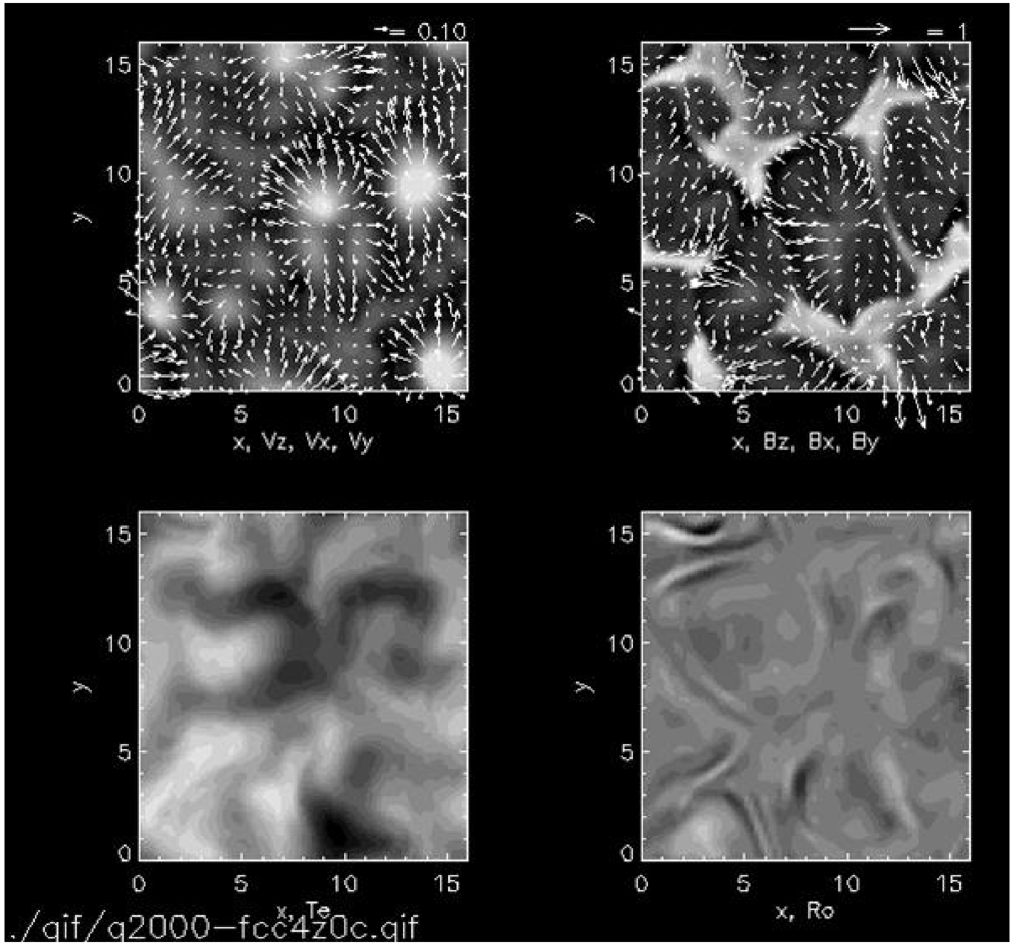


Ca H line
(Chromosphere)

MHD Simulation of Magneto-Convection

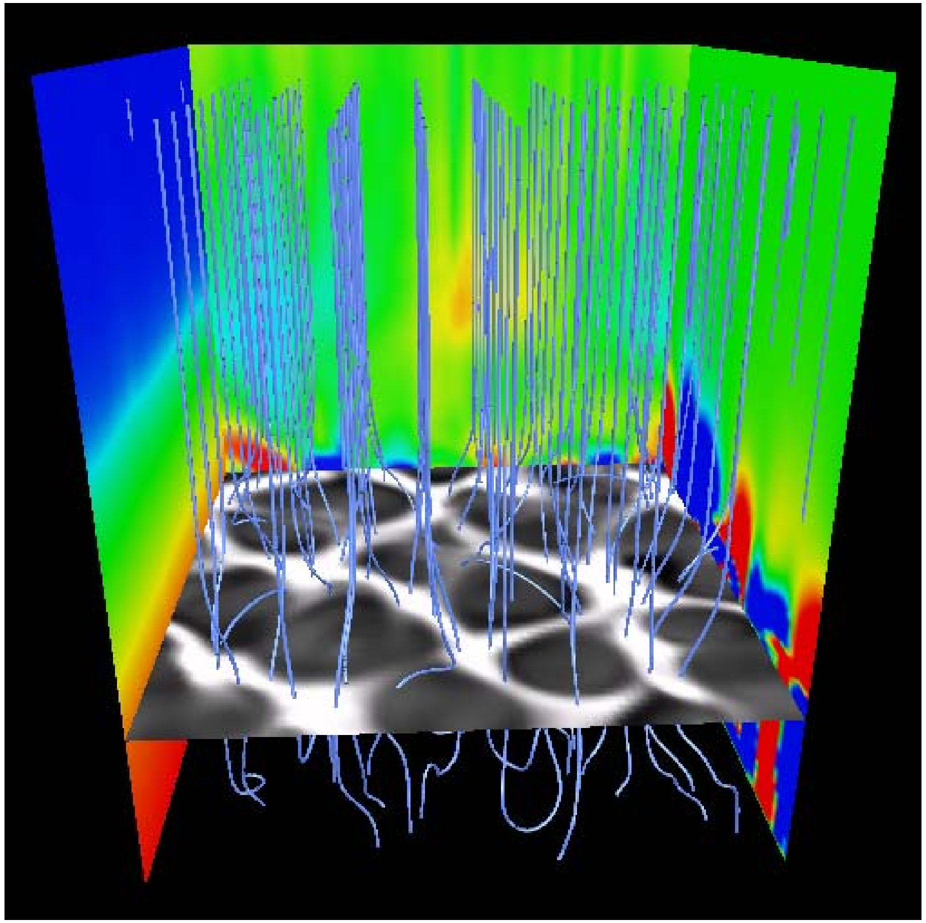


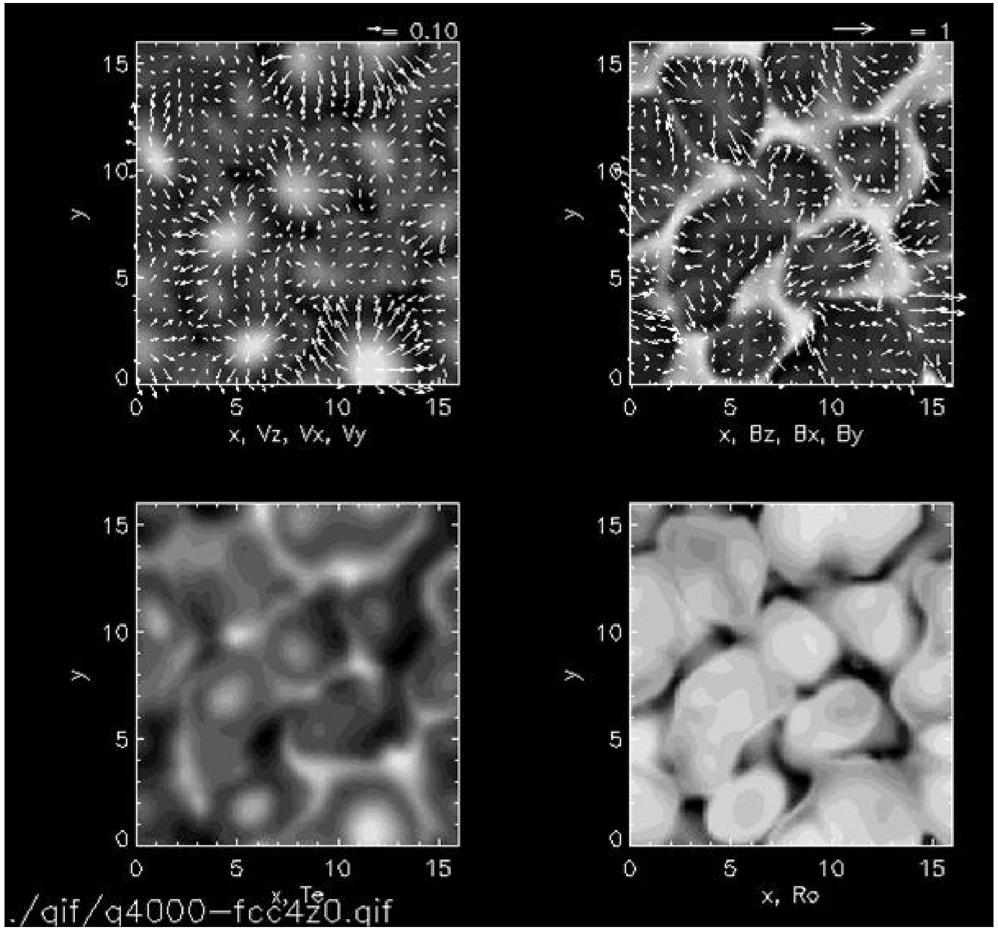
16H×16H×30H   β=300 at z=0



Isobe et al. 2007

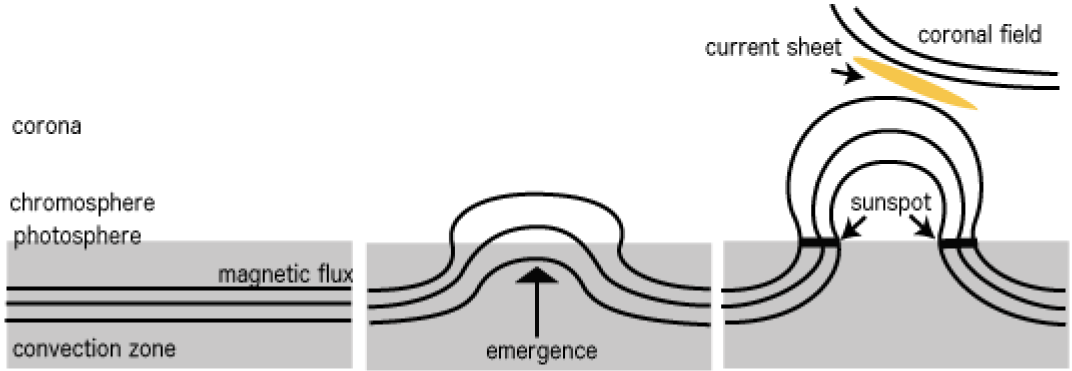
Simulation of Magneto-Convection when the Magnetic Field is Strong

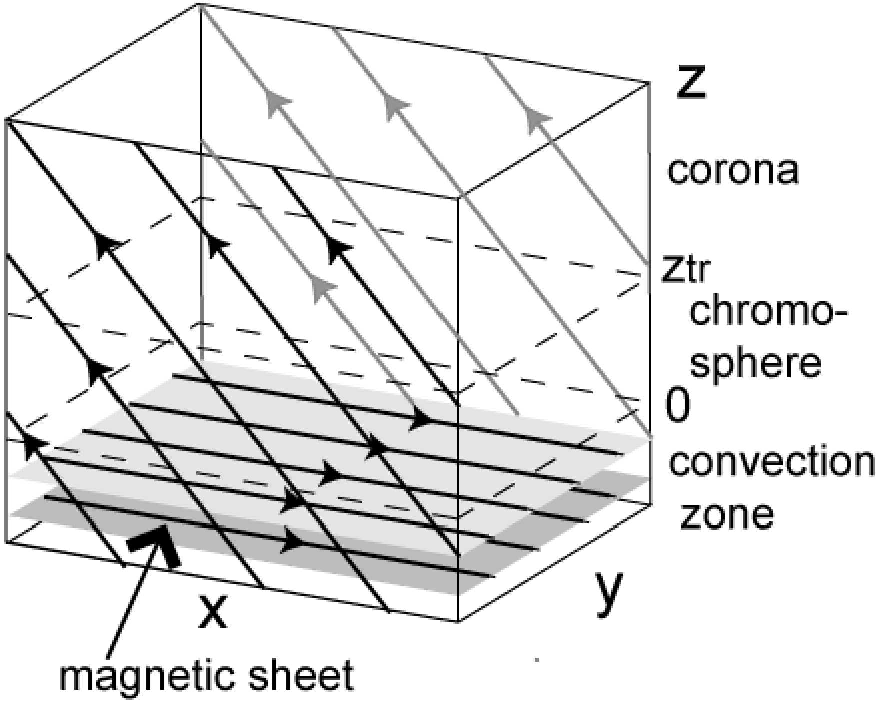


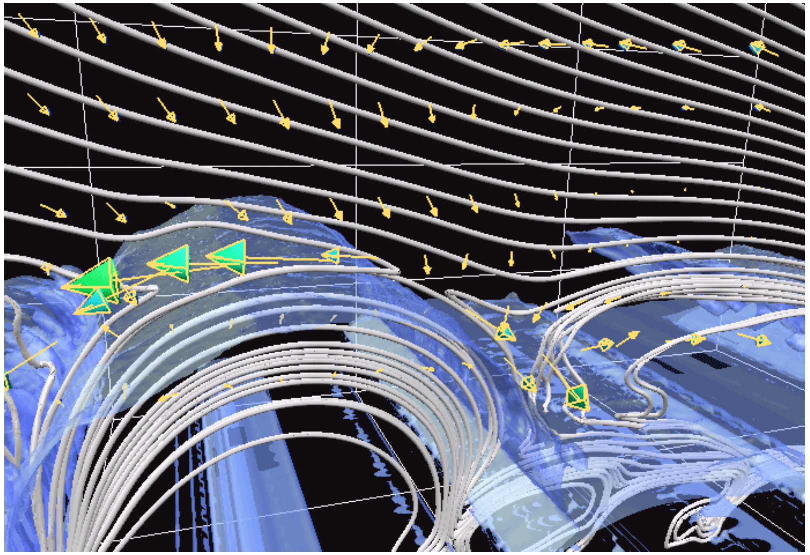


$\beta = 150$ at z=0

3D MHD Simulation of Solar Emerging Magnetic Flux

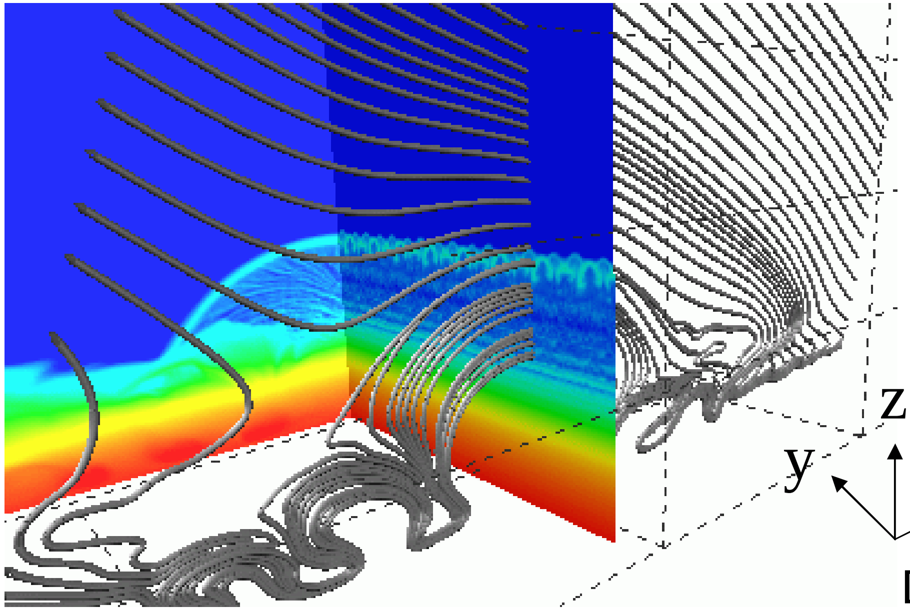


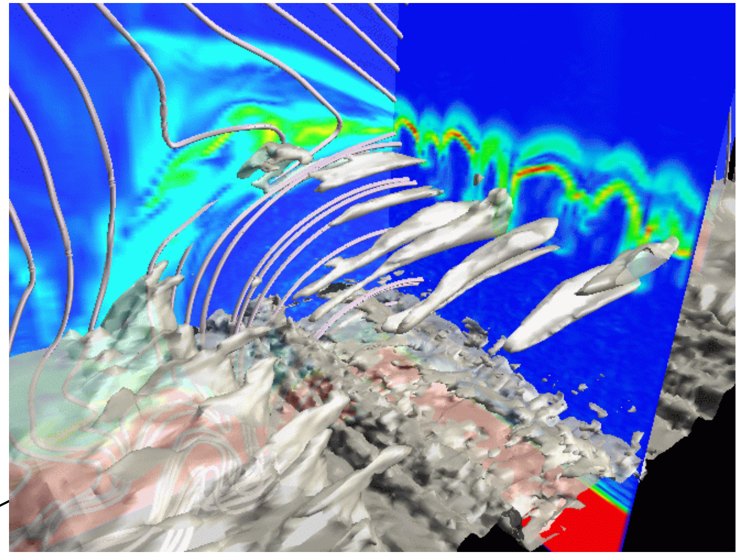




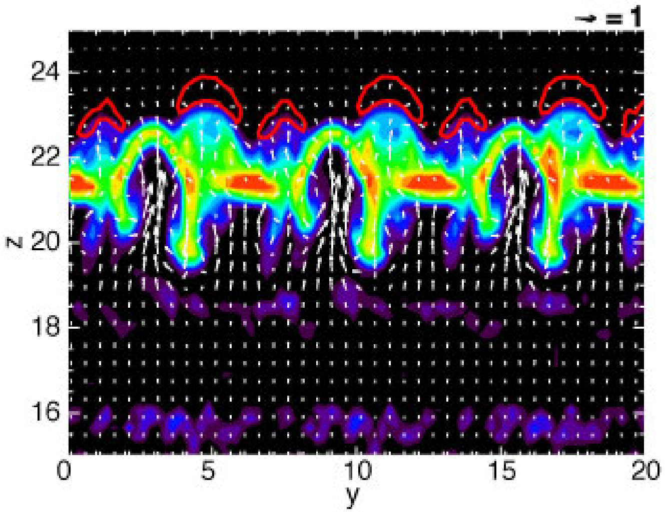
Isobe et al. 2006

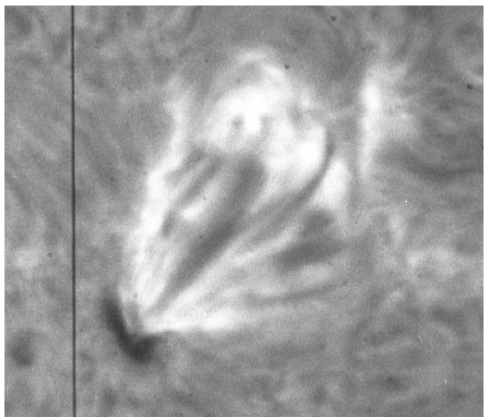
Structure Formation by Rayleigh Taylor Instability





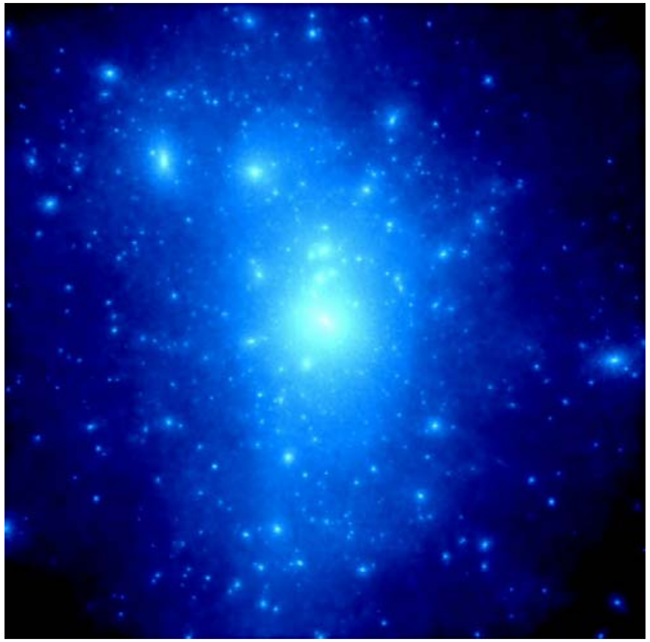
Density isosurface and Current Density



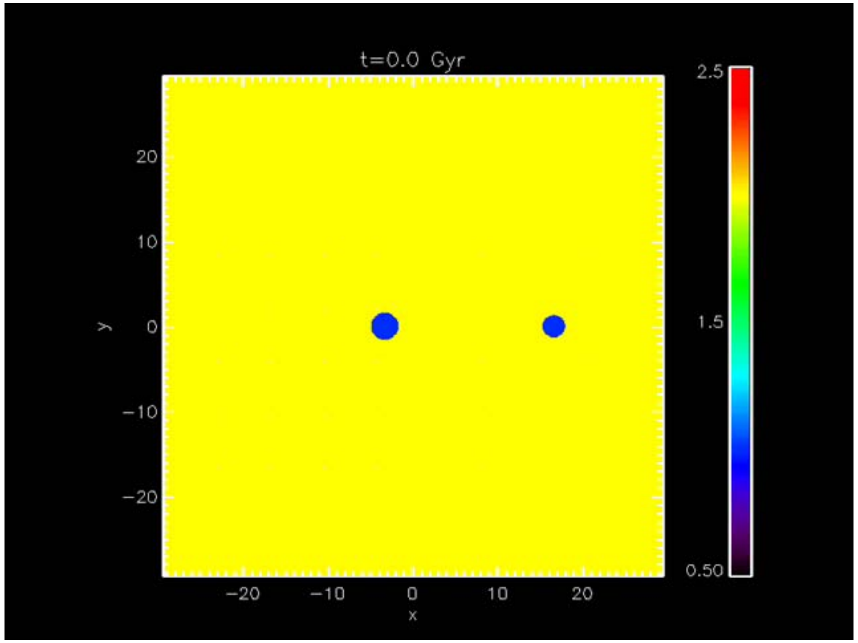


Hα Arch Filament (Hida Observatory, Kyoto Univ)

Numerical Simulations of X-ray Emitting Plasma in Cluster of Galaxies

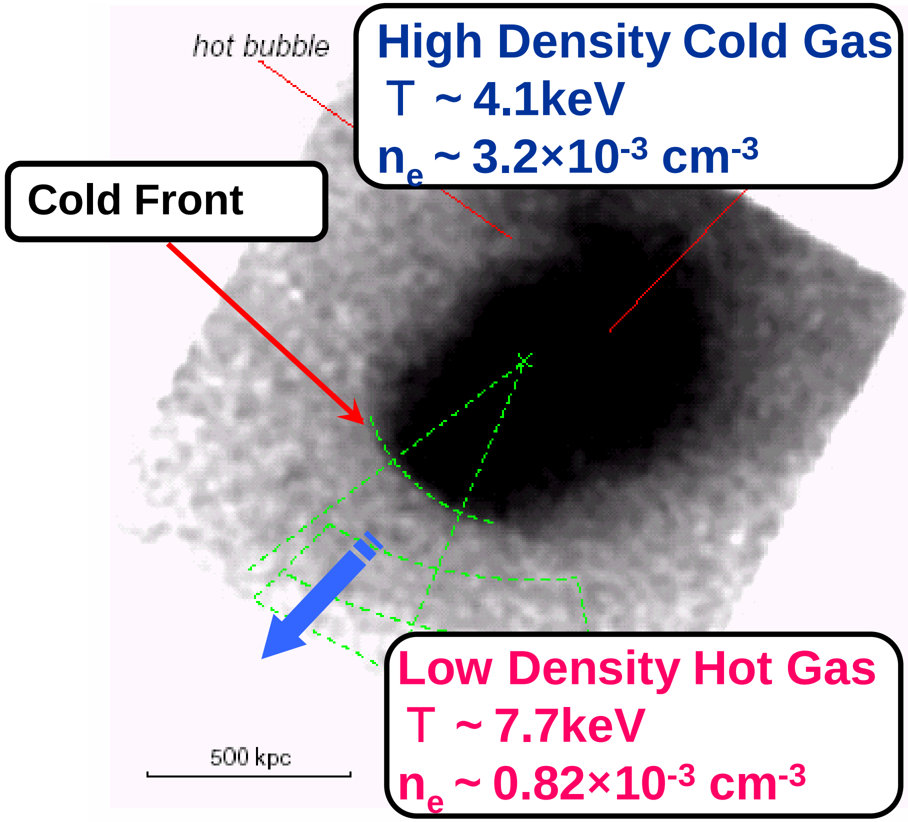


Distribution of dark matter obtained by N-body simulation by Yahagi (2002)

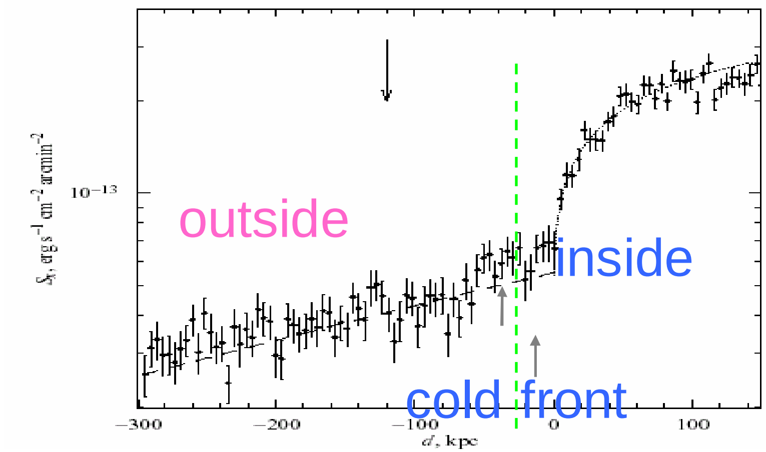


Hydrodynamical simulation of a moving subclump in cluster of galaxies by Asai (2005)

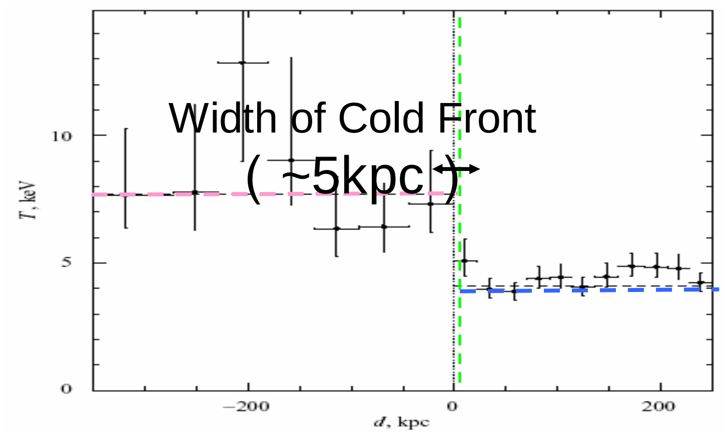
Cold Fronts in Subclumps Moving in Cluster of Galaxies



A3667 (Vikhlinin et al. 2001)

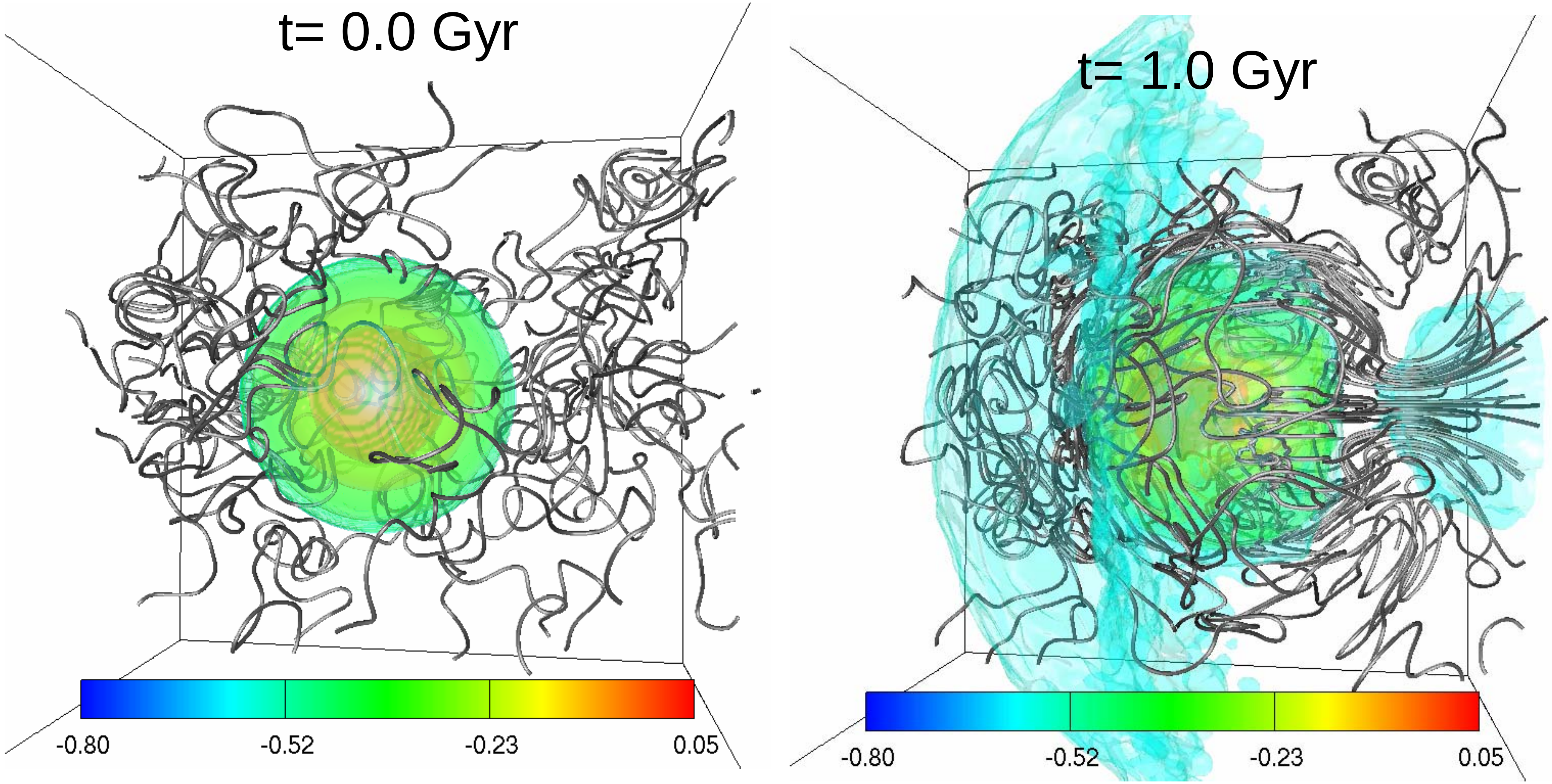


X-ray Intensity



Temperature

3D MHD Simulation of Subclumps Moving in Cluster of Galaxies



Asai, N., Fukuda, N., and Matsumoto, R. 2007

Basic Equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\rho \left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right] = -\nabla p + \frac{(\nabla \times \mathbf{B}) \times \mathbf{B}}{4\pi} - \rho \nabla \psi$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B})$$

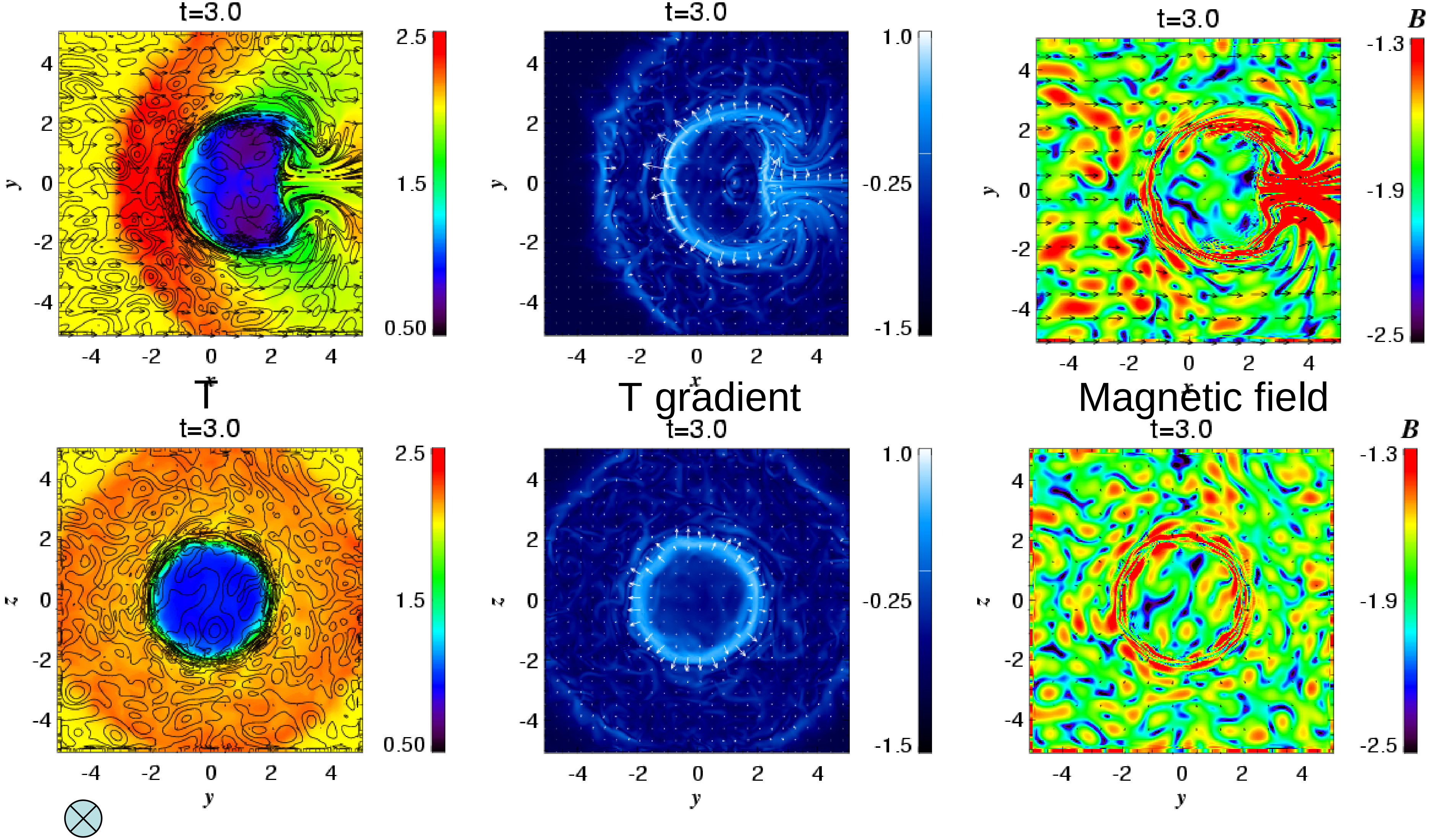
$$\frac{\partial}{\partial t}\left[\frac{1}{2}\rho v^2 + \frac{B^2}{8\pi} + \frac{p}{\gamma - 1} \right] + \nabla \cdot \left[\left(\frac{1}{2}\rho v^2 + \frac{\gamma p}{\gamma - 1} \right) \mathbf{v} + \frac{-(v \times \mathbf{B}) \times \mathbf{B}}{4\pi} - \kappa \nabla T \right] = -\rho \mathbf{v} \cdot \nabla \psi$$

Thermal Conductivity

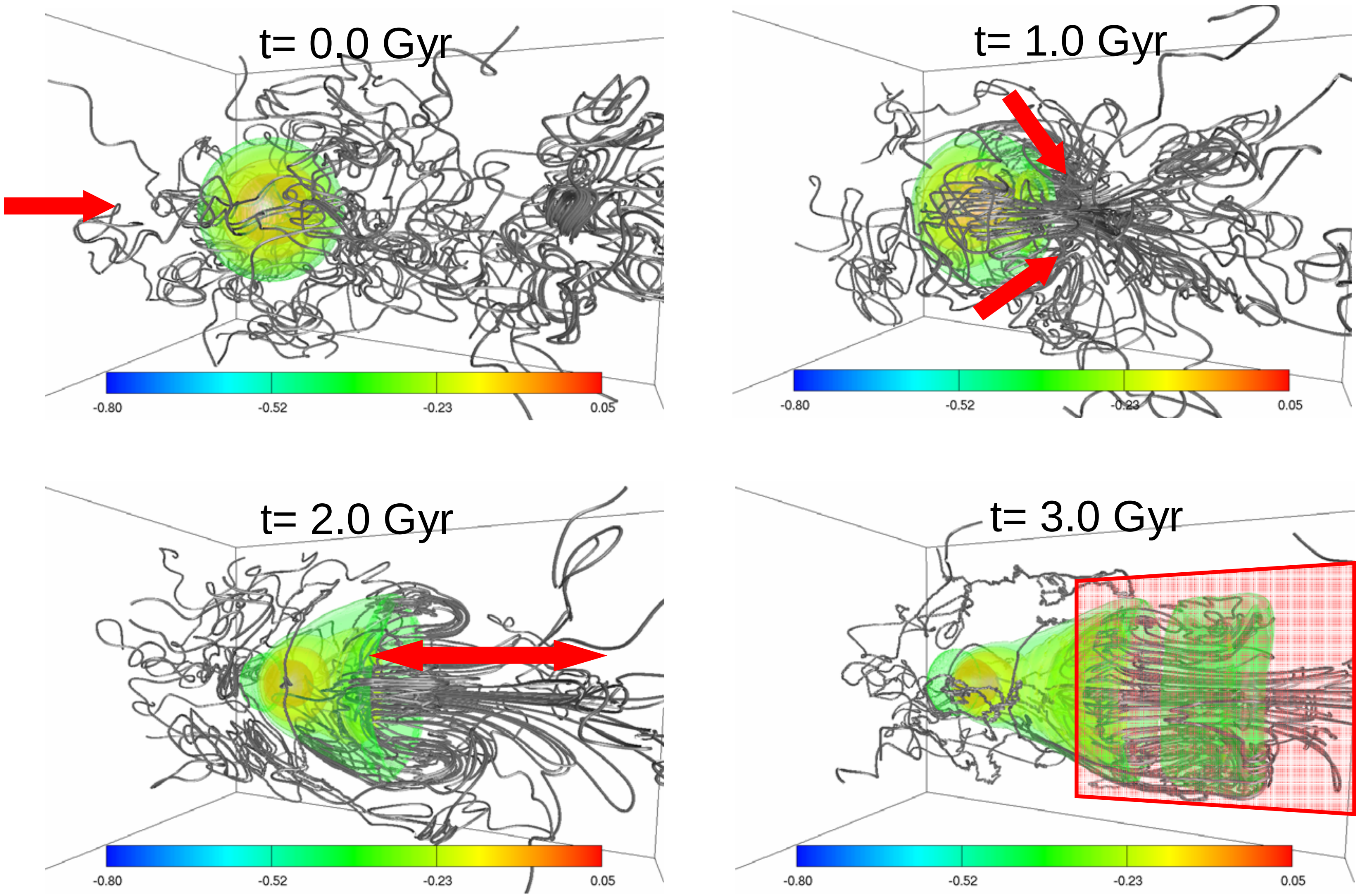
$\kappa_{//} = \kappa_0 T^{5/2}$ $\kappa_{\perp} = 0$

κ_0 : Spitzer Conductivity

Suppression of Thermal Conduction



Magnetic Field Amplification



Summary and Future

- MHD simulations are powerful in studying nonlinear phenomena in astrophysical plasmas
- We have developed a virtual laboratory system for astrophysical MHD simulations
- We are going to implement Relativistic MHD modules, Radiation MHD modules, and nested grid modules
- Micro-Macro coupling such as the connection between MHD simulations and particle simulations is a future work
- Please visit our web-site
  - http://www.astro.phys.s.chiba-u.ac.jp/netlab/

END