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Current driven acoustic perturbations in partially ionized collisional plasmas

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Plan

1. Fluid modeling of electron flow driven IA mode in plasma with magnetized electrons

- model and effects of electron inertia
- fluid description of Landau damping
- angle dependent instability threshold
- inhomogeneous electron flow
- plasma with two ion species

2. Gas acoustic and ion acoustic modes

- ion-neutral and ion-electron collisions
- the dynamics of neutrals
- electromagnetic effects

Based on the following publications:

- J. Vranjes and S. Poedts, Unstable ion sound in plasmas with drifting electrons, Eur. Phys. J. D **40**, 257-262 (2006).
- J. Vranjes and S. Poedts, Properties of acoustic mode in partially ionized and dusty plasma, Phys. Plasmas **13**, 052103 (2006).
- J. Vranjes, S. Poedts, and M. Y. Tanaka, Fluid modeling of electron flow driven ion acoustic mode in collisional plasma with magnetized electrons, Phys. Plasmas **13**, 122103 (2006).
- J. Vranjes, B. P. Pandey, and S. Poedts, Gas acoustic and ion acoustic waves in a partially ionized plasma with magnetized electrons, Phys. Plasmas **14**, 032106 (2007).

Works inspired by:

- P. Kaw, Phys. Lett. A **44**, 427 (1973).
- N. D'Angelo, G. Joyce, and M. E. Pesses, Astrophys. J. **229**, 1138 (1979).
- H. K. Andersen, N. D'Angelo, V. O. Jensen, P. Michelsen, and P. Nielsen, Phys. Fluids **11**, 1177 (1968).

1. Fluid modeling of electron flow driven IA mode

- In a weakly ionized plasma immersed in a magnetic field, the ion-neutral collision frequency may exceed the ion gyro-frequency so that the ions are un-magnetized. On the other hand, the electron-neutral collisions may still occur less frequent than the electron gyro-frequency. In such a situation, acoustic-type oscillations of the ion gas may propagate at an angle with respect to the magnetic field, and the oscillations may have a sound-type character as long as the magnetized electrons are able to follow by moving only along the magnetic field lines. Collisions determine the mode behavior in such a partially ionized plasma. The plasma-neutrals collisions, roughly speaking, dominate the collisions between charged particles as long as the ionization ratio n_{i0}/n_{n0} remains below the value $3 \cdot 10^{10}/T_e^2$, where T_e is in units of K.

- Ion sound wave in a hot ion plasma is Landau damped; waves with wavelengths far exceeding the electron Debye radius may propagate if

$$f(\tau) \equiv (z_i \tau)^{3/2} \text{exp}(-z_i \tau/2) \ll 1, \quad \tau = \frac{T_e}{T_i}.$$

z - the ion charge number. For an electron-proton plasma $f(\tau)$ negligible for $\tau \geq 15$. For higher ion charges, e.g. $z_i = 5$, $f(\tau)$ becomes negligible already at $\tau \geq 3 \Rightarrow$ highly charged and hot ions practically do not contribute to the Landau damping as long as $\tau \geq 3$.

- In the presence of an electron stream v_0 the icrement/decrement of the ion sound in an electron-proton plasma

$$\omega_i = \left(\frac{\pi}{8}\right)^{1/2} k c_s \left[\left(\frac{m_e}{m_i}\right)^{1/2}\left(\frac{v_0}{c_s} - 1\right) - \tau^{3/2} \mathrm{exp}\left(-\frac{\tau}{2}\right)\right],$$

- the mode unstable provided that

$$v_0 > c_s \left[1 + (m_i/m_e)^{1/2} f(\tau)\right]. \tag{1.1}$$

For $f(1) = 0.6$ the instability develops if $v_0 > 27c_s$, while for $f(10) = 0.2$ the threshold is still high, viz. $v_0 > 10c_s$.

- Hence, electron-current-driven instability does exist within the collision-less kinetic description where it appears due to the positive slope in the electron distribution function in the domain around the wave phase speed. It is necessary that the electron macroscopic speed exceeds the sound speed $c_s = (\kappa T_e/m_i)^{1/2}$. However, the actual instability threshold is usually much higher.

- In the presence of an electron drift/flow, perpendicular or parallel to the magnetic lines, the collisional ion sound becomes unstable even within the fluid theory.
- In the fluid case of a collisional plasma, and for an obliquely propagating sound mode, the instability threshold is angle dependent and it may be lower compared to the collision-less kinetic case.

Model and effects of electron inertia

- The dominant electron collisions are those with the neutrals

$$m_e n_e \left[\frac{\partial \vec{v}_e}{\partial t} + (\vec{v}_e \cdot \nabla)\vec{v}_e\right] = e n_e \nabla\phi - e n_e \vec{v}_e \times \vec{B} - \kappa T_e \nabla n_e$$
$$- m_e n_e \nu_{en}(\vec{v}_e - \vec{v}_n). \tag{1.2}$$

- Perturbations propagating obliquely with respect to the magnetic field $\vec{B}_0 = B_0 \vec{e}_z$, and with the background of a neutral gas.

- The perturbed electron velocity

$$v_{ez1} = -\frac{ek_z}{m_e\omega_e}\phi_1 + \frac{v_{Te}^2 k_z}{\omega_e}\frac{n_{e1}}{n_{e0}}. \tag{1.3}$$

$$v_{e\perp} = \frac{1}{B_z}\vec{e}_z \times \nabla_\perp\phi + \frac{\nu_e}{\Omega_e}\frac{\nabla_\perp\phi}{B_z} - \frac{v_{Te}^2\nu_e}{\Omega_e^2}\frac{\nabla_\perp n_e}{n_e} - \frac{v_{Te}^2}{\Omega_e}\vec{e}_z \times \frac{\nabla_\perp n_e}{n_e}$$
$$-\frac{1}{\Omega_e}\left(\frac{\partial}{\partial t} + \vec{v}_e \cdot \nabla\right)\vec{e}_z \times \vec{v}_{e\perp} - \frac{\nu_e}{\Omega_e^2}\left(\frac{\partial}{\partial t} + \vec{v}_e \cdot \nabla\right)\vec{v}_{e\perp}. \tag{1.4}$$

$\omega_e = \omega_0 + i\nu_e$, $\omega_0 = \omega - k_z v_0$, $v_{Te}^2 = \kappa T_e/m_e$, $\vec{v}_0 = v_0\vec{e}_z$. The ω_0 term appears in ω_e from the left-hand side of the electron momentum equation as the finite electron mass effect.

- Conditions used

$$\omega \ll \nu_e, \quad \nu_e \ll \Omega_e. \tag{1.5}$$

The first inequality is to be satisfied in order to use the fluid equations for the electrons, implying also negligible electron kinetic effects, i.e., the electron mean free path is much shorter than the parallel wave-length; the second part implies magnetized electrons.

- <u>Comment 1</u>: the flow v_0 may be driven by external conditions, e.g., by an electric field E_0 yielding in the general case

$$v_{i0} = \frac{eE_0}{m_i \nu_i}\left(1 - \frac{m_i}{m_e}\frac{\nu_{ie}}{\nu_e}\right)\left(1 - \frac{\nu_{ie}\nu_{ei}}{\nu_i \nu_e}\right)^{-1}, \qquad \nu_i = \nu_{ie} + \nu_{in}, \tag{1.6}$$

$$v_{e0} = \frac{eE_0}{m_e \nu_e}\left[\frac{m_e}{m_i}\frac{\nu_{ei}}{\nu_i}\left(1 - \frac{m_i}{m_e}\frac{\nu_{ie}}{\nu_e}\right)\frac{1}{1 - \frac{\nu_{ie}\nu_{ei}}{\nu_i \nu_e}} - 1\right]. \tag{1.7}$$

  In a weakly ionized plasma $v_{i0} \approx eE_0/(m_i \nu_{in})$ and $v_{e0} \approx -eE_0/(m_e \nu_{en})$. The ion velocity usually much smaller and can be omitted, or the equations could be conveniently written in the ion reference frame.

- <u>Comment 2</u>: on the ambient electric field in purely ionized plasmas. The collisional cross section for Coulomb-type interactions proportional to $1/v_{T\alpha}^2$, yielding the mean free path λ proportional to $v_{T\alpha}^4$. For λ of the order of the plasma scale, a runaway effect takes place. The effect important because of the acceleration of particles that are already faster (they feel less collisions) $\Rightarrow$ the number of fast particles increases. The intensity of the applied electric field E_0 must exceed the Dreicer's critical value

$E_D = e/(4\pi\varepsilon_0 r_D^2)$, where r_D is the plasma Debye radius. However, we are dealing with a weakly ionized plasma where the effect in general should have no importance. To have the effect, the collision cross section for plasma-neutral collisions should be proportional to the inverse thermal velocity (with some exponent) which usually is not so.

- The electron continuity

$$\frac{n_{e1}}{n_{e0}} = -\frac{ek_z^2}{m_e\omega_0\omega_e\alpha_e}\left(1 - \frac{\omega_e^2 k_\perp^2}{\Omega_e^2 k_z^2}\right)\phi_1. \tag{1.8}$$

Here,

$$\alpha_e = 1 - \frac{k_z^2 v_{Te}^2}{\omega_0\omega_e} + \rho_e^2 k_\perp^2\left(1 + i\frac{\nu_e}{\omega_0}\right), \quad \rho_e = v_{Te}/\Omega_e. \tag{1.9}$$

In the absence of collisions and for inertia-less electrons Eq. (1.8) yields the standard Boltzmann's distribution.

- <u>Collisional ions</u>: in the proper fluid limit of negligible trapping effect (wave frequency lower than the ion collision frequency) and small Landau damping (the ion mean free path lower than the wave-length), and in the case when they are not magnetized,

$$\frac{\Omega_i}{\nu_i} \ll 1, \tag{1.10}$$

$\Rightarrow$ the ions may be treated within the fluid theory, and the terms parallel and perpendicular to the magnetic field are irrelevant $\Rightarrow$ the ion perturbations given by $\sim$ exp$(-i\omega t + i\vec{k}\cdot\vec{r})$; $\vec{r}$ denotes an arbitrary direction with respect to $\vec{B}_0 = B_0\vec{e}_z$.

- The ion momentum and continuity yield

$$\frac{n_{i1}}{n_{i0}} = \frac{ek^2}{m_i}\frac{\phi_1}{\omega^2 + i\nu_i\omega - k^2 v_{Ti}^2}. \tag{1.11}$$

- Typically, in the case of an ordinary ion sound mode, the electron inertia is neglected, resulting in a Boltzmann distribution for the electrons, which is always a good approximation because $(\omega/k)^2/v_{Te}^2 \simeq m_e/m_i \ll 1$. In the present case of an obliquely propagating sound wave, and for electrons participating in the perturbations by moving only along the magnetic field lines, the above small factor appears multiplied by $k^2/k_z^2 > 1$, and consequently the electron inertia may not always be negligible.

- The derivations are considerably simplified when the electron mass correction in the perpendicular dynamics is neglected. From the real and the imaginary terms with the mass corrections in Eq. (1.8) if

$$\xi \equiv \frac{k_\perp^2}{k_z^2}\frac{\nu_e^2}{\Omega_e^2} < 1, \tag{1.12}$$

the perpendicular part can be omitted, and only the parallel electron dynamics becomes modified due to the electron mass corrections through the term ω_0 comprised in ω_e. In view of $\omega \ll \nu_e$, $\nu_e \ll \Omega_e$ the condition (1.12) is usually satisfied, otherwise electrons would not be able to follow by moving only in the parallel direction. The conditions (1.5), (1.10), and (1.12) will be checked for every set of parameters used later in the text.

- Dispersion equation:

$$\omega^2 = k^2 c_s^2\left(1+\frac{1}{\tau}\right) - \frac{m_e}{m_i}\frac{k^2}{k_z^2}(\omega - k_z v_0)^2 - i\left[\nu_i\omega + \frac{m_e}{m_i}\frac{k^2}{k_z^2}\nu_e(\omega - k_z v_0)\right]. \tag{1.13}$$

- $(\omega - k_z v_0)^2$ - the effect of the left-hand side of the electron momentum eq. In the absence of collisions with neutrals and for perturbations along the magnetic lines, this term yields an electron two stream instability of the sound mode, which sets in provided that

$$v_0^2 > v_{Te}^2 \left(1 + \frac{1}{\tau}\right) . \tag{1.14}$$

The threshold of this reactive instability is very high.

- Another kind of instability, with a much lower threshold, is obtained in the presence of collisions. From Eq. (1.13)

$$\omega_i = -\frac{\nu_i}{2(1-\chi)}\left(1 - \frac{\nu_e}{\nu_i}\chi\right), \quad \chi = \frac{m_e}{m_i}\frac{k^2}{k_z^2}\left(\frac{k_z v_0}{\omega_r} - 1\right). \tag{1.15}$$

  The χ in denominator appears as the electron mass effect.

- The collision frequency of plasma species with neutrals $\nu_{e,i} = \sigma_{(e,i)n} n_n v_{T(e,i)}$; $\sigma_{in}(m_i, m_n, T_i)$ generally larger than $\sigma_{en}(m_n, T_e)$; both cross sections strongly dependent on the temperature of the colliding plasma particle.

$$\frac{\nu_i}{\nu_e} = \left(\frac{m_e}{m_i \tau}\right)^{1/2} \frac{\sigma_{in}(m_i, m_n, T_i)}{\sigma_{en}(m_n, T_i)} = \beta \left(\frac{m_e}{m_i \tau}\right)^{1/2}, \quad \beta > 1.$$

- For e–H collisions the cross section at 1 eV around $2.5 \cdot 10^{-19}$ m^2. For H$^+$–H momentum transfer at 1 eV it is 10^{-18} m^2 $\Rightarrow \beta = \sigma_{in}/\sigma_{en} \approx 4$, and $\nu_i/\nu_e \approx 0.09/\tau^{1/2}$, or $\nu_e/\nu_i = 10.7\tau^{1/2}$. This ratio does not change much, even for lower temperatures that are of particular interest for the partially ionized plasma studied here. Consequently, in most practical situations when $T_e \geq T_i$ or T_i is not very much larger than T_e we have $\nu_e > \nu_i$ and the sign of ω_i changes primarily due to the term in the numerator of Eq. (1.15). We are interested in values of τ close to unity, firstly in order to investigate the hot ion case, and secondly to have the conditions (1.5), (1.10), (1.12) well satisfied.

- The instability condition

$$\frac{v_0}{v_s} > \frac{k}{k_z}\left(1 + \frac{m_i}{m_e}\frac{\nu_i}{\nu_e}\frac{k_z^2}{k^2}\right) = \frac{1}{\cos\varphi}\left(1 + \frac{m_i}{m_e}\frac{\nu_i}{\nu_e}\cos^2\varphi\right). \qquad (1.16)$$

- Similar expression obtained in J. Vranjes and S. Poedts, Phys. Plasmas **13**, 052103 (2006) by using a kinetic description for the ions. Angle dependent instability threshold for an ion sound mode driven by the constant electron flow directed along the magnetic field lines. Fluid instability in the presence of electron collisions.

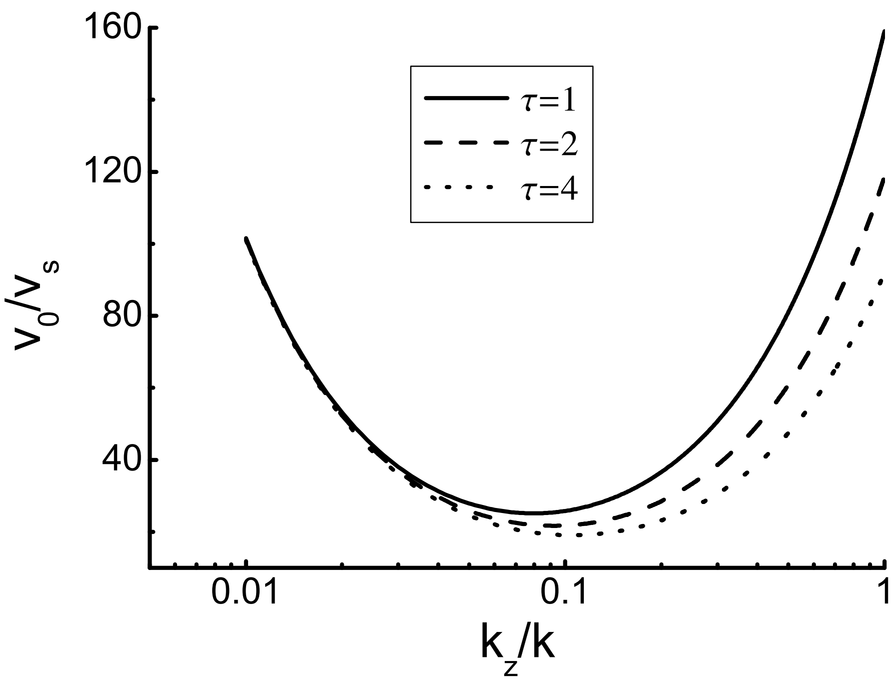


Fig. 1: The angle dependent instability threshold for electron-H$^+$ plasma in H-gas.

- The threshold presented in Fig. 1 in terms of k_z/k for hydrogen and for the electron energy of 1 eV for which $\sigma_{en} = 2.5 \cdot 10^{-19}$ m^2, and for three values of $\tau = 1,\ 2,\ 4$. The corresponding H$^+$–H collision cross sections for the momentum transfer (in 10^{-19} m^2) are $\sigma_{in} = 9.24$, 9.8, and 10.64, respectively. The collision frequency ratio ν_i/ν_e is 0.086, 0.064, and 0.049, respectively. The mode most easily excited at given large angles (around $k_z/k = 0.1$) with respect to the driving electron flow which is in the z-direction. The minimum instability threshold v_0/v_s for the three lines: 25.1, 21.7, 19. Compare this with the threshold for the mode propagating along the magnetic lines which for the given values of τ is 159, 118, and 92. Note that taking Ω_e/ν_e, in the interval $20-80$ we have the condition (1.10) reasonably satisfied as it takes values from the interval $0.12\tau^{1/2} - 0.48\tau^{1/2}$, while the left-hand side of the condition (1.12) takes values from the interval $0.25 - 0.015$, respectively. So the parameters used above satisfy the model, particularly for τ close to unity.

- The reason for the instability seen by observing the phase difference between the electron and ion perturbations. From the electron equation when the electron inertia is neglected, the density perturbations become related to the potential by

$$\frac{n_{e1}}{n_{e0}} = \frac{e\phi_1}{\kappa T_e}\frac{1}{1+\gamma^2}(1+i\gamma), \quad \gamma = -\frac{\nu_{en}(v_0 - \omega/k_z)}{k_z v_{Te}^2}. \tag{1.17}$$

From the ion part, omitting the ion temperature, the ion density perturbation

$$\frac{n_{i1}}{n_{i0}} = \frac{e\phi_1}{\kappa T_e}\frac{k^2 c_s^2}{\omega^2 + \nu_i^2}\left(1 - i\frac{\nu_i}{\omega}\right). \tag{1.18}$$

When the phase of the electron density oscillations advances the phase of the ion density oscillations, the electrons drive the ion perturbations and the instability grows. Since the ion phase is negative, the instability sets in if $Arg[n_{e1}/n_{e0}] < Arg[n_{i1}/n_{i0}]$. This yields

$$-\frac{\nu_{en}(v_0 - \omega/k_z)}{k_z v_{Te}^2} < -\frac{\nu_i}{\omega},$$

which is in fact the same as the instability condition (1.16), provided $\omega \simeq kc_s$.

- The dependence on the angle (or on k_z/k) due to the interplay of the two k_z terms in Eq. (1.16), i.e., $(1/k_z)(1 + ak_z^2)$; the second one comprises the ion collisions. Two limits: $+\infty$ for $k_z = 0$, and $1 + m_i\nu_i/(m_e\nu_e)$ for $k_z/k = 1$. This creates the dip in the threshold profile (cf. Fig. 1), with the minimum which shifts towards smaller values of k_z when the ion mass is increased (see below).

- Note: for some gasses and ions the sign in Eq. (1.15) can change by the term in the denominator! Table 1 - for several types of ions in gases of particular interest for laboratory investigations (helium, neon and argon). Here, $\nu_i/\nu_e > 1$ for lithium and potassium in the neon and argon gasses and for equal electron and ion temperatures. This is due to much larger cross sections for collisions between these large target atoms and large colliding ions, compared to the electron collisions.

- In this case the instability threshold considerably higher

$$\frac{v_0}{v_s} > \frac{k}{k_z}\left(1 + \frac{m_i}{m_e}\frac{k_z^2}{k^2}\right). \tag{1.19}$$

Tabel 1.1: *The collision cross sections and collision frequencies for electrons, and several ion species, in helium, neon and argon gasses, in units of* 10^{-20} *m*^{2} *and at electron and ion temperatures of 0.1 eV, i.e.,* $\tau = 1$*. The values in brackets are for electrons at 1 eV, or* $\tau = 10$*.*

helium gas					neon gas			argon gas		
σ_{eHe}	σ_{H^+He}	σ_{He^+He}	σ_{Li^+He}	σ_{K^+He}	σ_{eNe}	σ_{Li^+Ne}	σ_{K^+Ne}	σ_{eAr}	σ_{Li^+Ar}	σ_{K^+Ar}
5.86 (6.85)	28	50	106	165	0.7 (1.62)	120	259	0.45 (1.05)	303	580
$\nu_{H^+}/\nu_e = 0.11(0.03)$ $\nu_{He^+}/\nu_e = 0.097$ $\nu_{Li^+}/\nu_e = 0.16(0.04)$ $\nu_{K^+}/\nu_e = 0.1(0.028)$					$\nu_{Li^+}/\nu_e = 1.5(0.2)$ $\nu_{K^+}/\nu_e = 1.38(0.19)$			$\nu_{Li^+}/\nu_e = 5.9(0.27)$ $\nu_{K^+}/\nu_e = 4.8(0.65)$		

Omnia in numero et mensura!

- The essential difference between the electron flow driven instability (1.16) and the one obtained from the collision-less kinetic theory. The latter is the electron inertia effect, in the limit $T_e \gg T_i$ obtained from

$$\omega_i = kv_s(\pi/8)^{1/2}\left[(v_0/c_s-1)(m_e/m_i)^{1/2}-\tau^{3/2}\text{exp}(-\tau/2)\right]. \qquad (1.20)$$

It is a purely collision-less plasma instability. In a strongly collisional plasma not expected to play a significant role. On the other hand, in our present case the driving current term associated with the electron collisions and by its nature it is a fluid effect.

Useful references:

- M. Mitchner and C. H. Kruger, *Partially Ionized Gasses* (John Willey and Sons, New York, 1973) p. 102.
- Y. P. Raizer, *Gas discharge physics* (Springer-Verlag, Berlin Heidelberg, 1991) p. 25.
- V. E. Fortov and I. T. Iakubov, *The physics of non-ideal plasma* (World Scientific, Singapore, 2000) p. 380.
- A. Zecca, G. P. Karwasz, and R. S. Brusa, Riv. Nuovo Cim. **19**, 128 (1996).
- P. S. Krstic and D. R. Schultz, J. Phys. B: At. Mol. Opt. Phys. **32**, 3485 (1999).

Fluid description of Landau damping

- In a collisional plasma the ion Landau damping is not expected to play an important role[1] as long as the ion mean free path is much shorter that the wave-length. This verified experimentally[2] even for $\tau \approx 1$: the strong-weak damping transition observed at $\omega \sim \nu_i$.

- The inclusion of the ion Landau damping effects implies a proper kinetic domain in which the collisions are neither too strong (too short ion mean free path) nor too weak (implying that ion trapping effects must be included). In this case, the ion equation Eq. (1.11) replaced by its kinetic counterpart [J. Vranjes and S. Poedts, Eur. Phys. J. D **40**, 257 (2006); J. Vranjes and S. Poedts, Phys. Plasmas **13**, 052103 (2006)]

$$n_{i1}/n_{i0} = -e\phi_1[1 - J_i(\omega/(kv_{Ti}))]/(\kappa T_i),$$

  $J_i(\psi)$ denotes the plasma dispersion function.

- In 1979 a fluid model was introduced in N. D'Angelo, G. Joyce, and M. E. Pesses, Astrophys. J. **229**, 1138 (1979) in order to describe the Landau damping effects on

[1] T. H. Stix, Waves in plasmas (AIP, New York, 1992), p. 184

[2] N. D'Angelo, Astrophys. J. **154**, 401 (1968) and references cited therein

the solar wind fast streams with a spatially varying ratio $\tau(= T_e/T_i)$. Within the distances 0.8 - 1 AU from the Sun, where the ratio τ is of the order of unity, the Landau damping on ions is significant and it counteracts the steepening of sound perturbations. Further away, for τ above 4 or 5 it becomes less important and the steepening takes place again. To describe this effect within a collision-less fluid theory, an effective 'viscous' term is introduced in the ion momentum equation $\mu_L \nabla^2 \vec{v}_i$, where μ_L is chosen in such a way to mimic the known properties of the Landau effect. These include the fact that the ratio δ/λ, between the attenuation length δ and the wavelength, is (i) independent of the wavelength, (ii) independent of the plasma density n, and (iii) dependent in a prescribed way on τ. These requirements are fulfilled by

$$\mu_L = \frac{m_i n_{i0} v_s \lambda}{2\pi^2 \delta/\lambda}. \tag{1.21}$$

Here, δ/λ in terms of τ satisfies an experimental curve which is such that the attenuation is strong at $\tau \approx 1$ and weak for higher values of τ.

- Demonstration:

$$\frac{\partial v_{i1}}{\partial t} = -\frac{e}{m_i}\frac{\partial \phi_1}{\partial r} - \frac{\kappa T_i}{m_i n_0}\frac{\partial n_{i1}}{\partial r} + \frac{\mu_L}{m_i n_0}\frac{\partial^2 v_{i1}}{\partial r^2}.$$

  This combined with the ion continuity and the Boltzmann distribution for electrons

$$\omega^2 + i\mu_0\omega k^2 - k^2(c_s^2 + v_{Ti}^2) = 0, \quad \mu_0 = \mu_L/(m_i n_{i0}).$$

  Setting $\omega = \omega_r + i\omega_{if}$ we have

$$\omega_{if} = -\mu_0 k^2/2 = -v_s/(\lambda d), \quad \omega_r^2 = k^2 v_s^2 - \mu_0^2 k^4/4, \quad d = \delta/\lambda. \quad (1.22)$$

- From the standard kinetic theory in the limit $T_e \gg T_i$ the Landau damping of the IA wave for singly charged ions given by the approximate formula

$$\omega_i = k v_s (\pi/8)^{1/2}\left[(m_e/m_i)^{1/2} + \tau(3+\tau)^{1/2}\mathrm{exp}[-(3+\tau)/2]\right]. \quad (1.23)$$

- In comparison to the exact solution[3] the damping rate (1.23) in terms of τ has a somewhat different behavior: at small values of τ it has a local maximum. The exact solution for the decrement is a monotonic, decreasing function of τ in the interval $\tau > 1$.

[3] F. F. Chen, Introduction to plasma physics and controlled fusion (Plenum Press, New York, 1984) p. 272

- Using the data and graph from D'Angelo et al[4]. we find that the 'fluid' attenuation length d introduced above can be expressed by the following approximate fitting formula to give the same decrement as the exact kinetic expression:

$$d \equiv \delta/\lambda \approx 0.2751 + 0.0421\,\tau + 0.089\,\tau^2 - 0.011785\,\tau^3 + 0.0012186\,\tau^4. \tag{1.24}$$

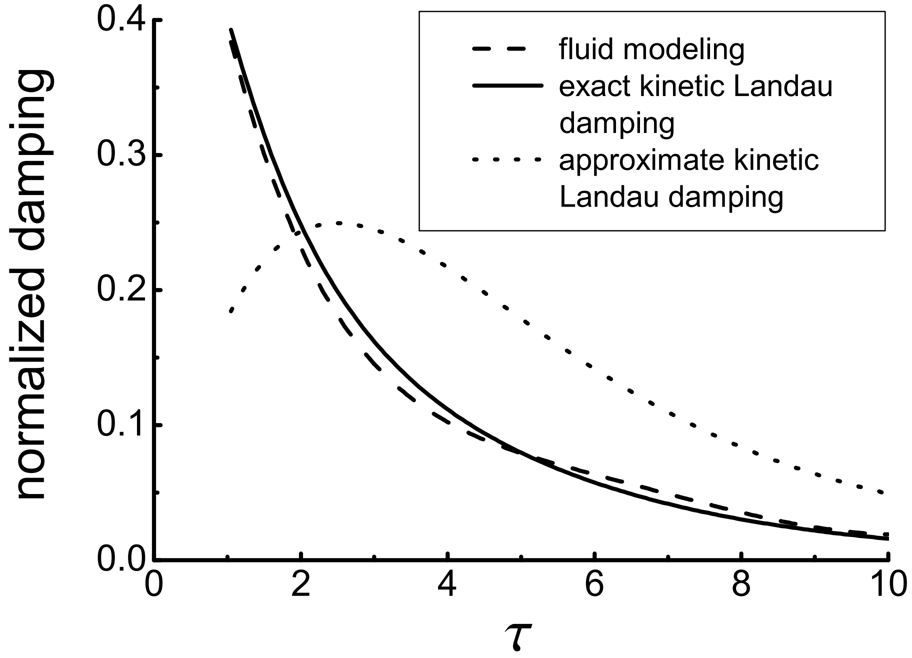


Fig. 2: Comparison of the fluid model decrement ω_{if} and the kinetic decrement ω_{ik} (normalized to ω_r) of the ion acoustic mode in a plasma with hot ions.

[4] Astrophys. J. **229**, 1138 (1979)

- Hence, the fluid 'viscosity' term (1.21) and the corresponding attenuation length (1.24) can be successfully used as a first approximation in the practical fluid description of the Landau damping.

IA mode with modeled Landau damping

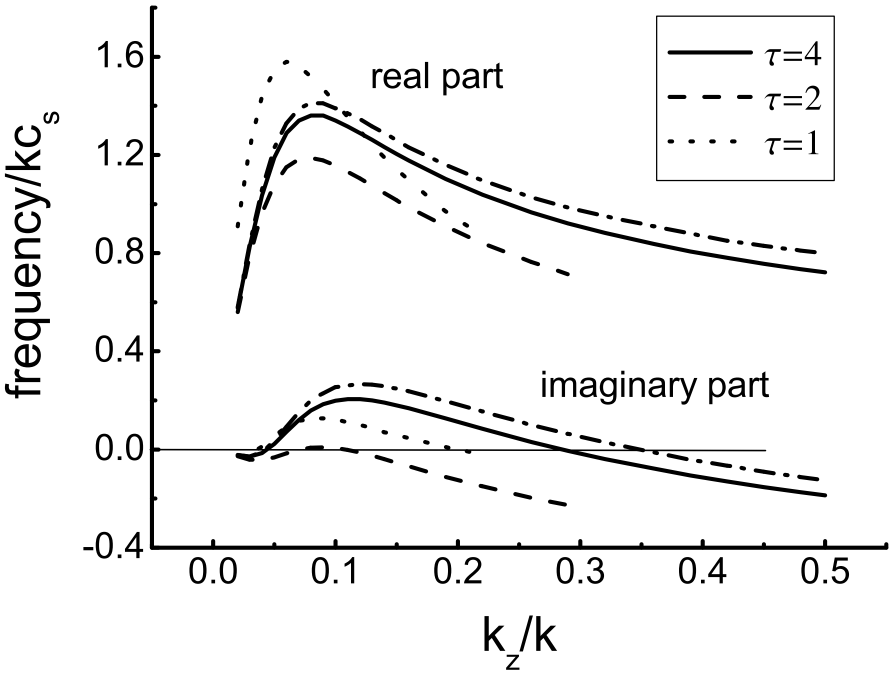


Fig.3: The angle dependent frequency of the ion acoustic mode for hot ions and Landau damping modeled by (1.21) and (1.24), for $v_0 = 30c_s$ and $\nu_e = 30kc_s$. The dotted lines are for $\tau = 1$ and $v_0 = 50c_s$. For comparison with full line, the dash-dot line is for $\tau = 4$ without the Landau damping.

Inhomogeneous electron flow

- In the case when the electron flow along the magnetic field has a small gradient in the x direction the perturbations may be taken in the form $\widehat{f}(x)$exp$(-i\omega t+ik_y y+ik_z z)$, where $|\partial/\partial x| \ll k_y,\ k_z$.

- Electrons described by[5]:

$$\frac{n_{e1}}{n_{e0}}\left[1-\frac{\omega_e k_y^2 v_{Te}^2}{\omega_0(\omega_e^2-\Omega_e^2)}-\frac{k_z^2 v_{Te}^2}{\omega_0\omega_e}\left(1-\frac{k_y v_0'\Omega_e}{k_z(\omega_e^2-\Omega_e^2)}\right)\right]$$

$$=-\left[\frac{\omega_e k_y^2}{\omega_0(\omega_e^2-\Omega_e^2)}+\frac{k_z^2}{\omega_0\omega_e}\left(1-\frac{k_y v_0'\Omega_e}{k_z(\omega_e^2-\Omega_e^2)}\right)\right]\frac{e\phi_1}{m_e}. \tag{1.25}$$

Here $v_0' = dv_0/dx$ and we shall further assume $|\omega_e^2| \ll \Omega_e^2$.

[5] The same as B. Eliasson, P. K. Shukla, and J. O. Hall, Phys. Plasmas **13**, 024502 (2006)

- Dispersion equation:

$$\omega^2 - k^2 v_{Ti}^2 + i\delta_i\omega - k^2 c_s^2 \frac{1 + \dfrac{k_y\Gamma}{k_z}}{1 + \dfrac{k_y\Gamma}{k_z} - \dfrac{k_y^2}{k_z^2}\dfrac{\omega_{e1}^2}{\Omega_e^2}}$$

$$+\frac{m_e}{m_i}\frac{k^2}{k_z^2}\left(\omega_0^2 + i\nu_e\omega_0\right)\frac{1 + \rho_e^2 k_y^2\left(1 + i\dfrac{\nu_e}{\omega_0}\right)}{1 + \dfrac{k_y\Gamma}{k_z} - \dfrac{k_y^2}{k_z^2}\dfrac{\omega_e^2}{\Omega_e^2}} = 0. \tag{1.26}$$

$$\Gamma = (dv_0/dx)/\Omega_e, \quad k^2 = k_y^2 + k_z^2, \quad \delta_i = \nu_i + \mu_0 k^2.$$

- The real part, i.e., without any dissipation yields the shear flow instability.

- When $1 > |k_y\Gamma/k_z| > |k_y^2\omega_e^2/(k_z^2\Omega_e^2)|$, setting $\omega = \omega_r + i\omega_i$, from the imaginary part of (1.26):

$$\omega_i \approx -\frac{\delta_i}{2}\left(1-\frac{\nu_e}{\delta_i}\frac{\chi}{1+\dfrac{k_y\Gamma}{k_z}}\right)\left(1-\frac{\chi}{1+\dfrac{k_y\Gamma}{k_z}}\right)^{-1}. \tag{1.27}$$

- If $|\nu_e| > |\delta_i|$ the instability sets in provided that

$$\begin{aligned}\frac{v_0}{v_s} &> \frac{k}{k_z}\left[1+\frac{\delta_i}{\nu_e}\frac{m_i}{m_e}\left(1+\Gamma\frac{(1-k_z^2/k^2)^{1/2}}{k_z/k}\right)\frac{k_z^2}{k^2}\right]\\ &= \frac{1}{\cos\varphi}\left[1+\frac{\delta_i}{\nu_e}\frac{m_i}{m_e}(1+\Gamma\tan\varphi)\cos^2\varphi\right].\end{aligned} \tag{1.28}$$

- Clearly, except for the mode propagating parallel to the magnetic field, a negative shear flow gradient can considerably reduce the instability threshold.

Plasma with two ion species

- Examples of multi-ion plasmas:

  - in the upper atmosphere of the Sun, a mixture of hydrogen and helium, while in the solar photosphere it is believed that the main constituents of the plasma are hydrogen and metal ions (with a typical mass of about 35 proton mass).

  - in the terrestrial ionosphere, the most abundant ions in the day-time E-region and lower F-region (i.e. for an altitude h between 120 and 160 km) are molecular ions NO^+ and O_2^+.

  - in a narrow region around $h = 160$ km the third (atomic-ion) component O^+ with the same number density is present. Above this layer mainly a two component electron-O^+ plasma.

  - in the laboratory, classical experiments involve the argon-helium[6], argon-neon[7], or xenon-helium plasmas[8].

  - each ion specie, in principle, includes an additional branch of acoustic oscillations

[6] B. D. Fried, R. B. White, and T. K. Samec, Phys. Fluids **14**, 2388 (1971)
[7] Y. Nakamura, M. Nakamura, and T. Itoh, Phys. Rev. Lett. **37**, 209 (1976).
[8] I. Alexeff, W. D. Jones, and D. Montgomery, Phys. Rev. Lett. **19**, 422 (1967)

and various new physical effects emerge, both in the fluid and kinetic domain.

- in a highly ionized plasma, there is an additional friction between the two ion species. The damping of the heavy ion mode is strongly increased in the presence of even small concentration of light ions.

- the addition of light ions effectively modifies τ. This means that the Landau damping can be controlled, turned on and off [details in B. D. Fried, R. B. White, and T. K. Samec, Phys. Fluids **14**, 2388 (1971); Y. Nakamura, M. Nakamura, and T. Itoh, Phys. Rev. Lett. **37**, 209 (1976)].

- similar effect when an additional hot electron component is added [J. Vranjes and S. Poedts, Eur. Phys. J. D **40**, 257 (2006); F. F. Chen, *Introduction to plasma physics and controlled fusion*, p. 409].

- Instability for ions a and b separately:

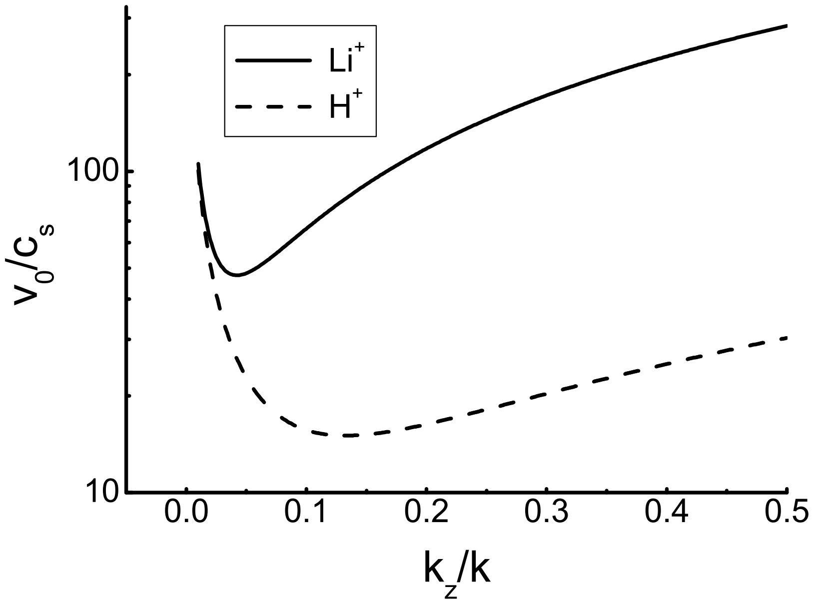


Fig. 4: The instability threshold for two separate cases: (i) H$^+$ sound in an e-H$^+$-He plasma (dashed line), and (ii) Li$^+$ sound in an e-Li$^+$-He plasma (full line), for $\nu_e = 50kc_s$ and $T_{H^+} = T_{Li^+} = T_e/10$.

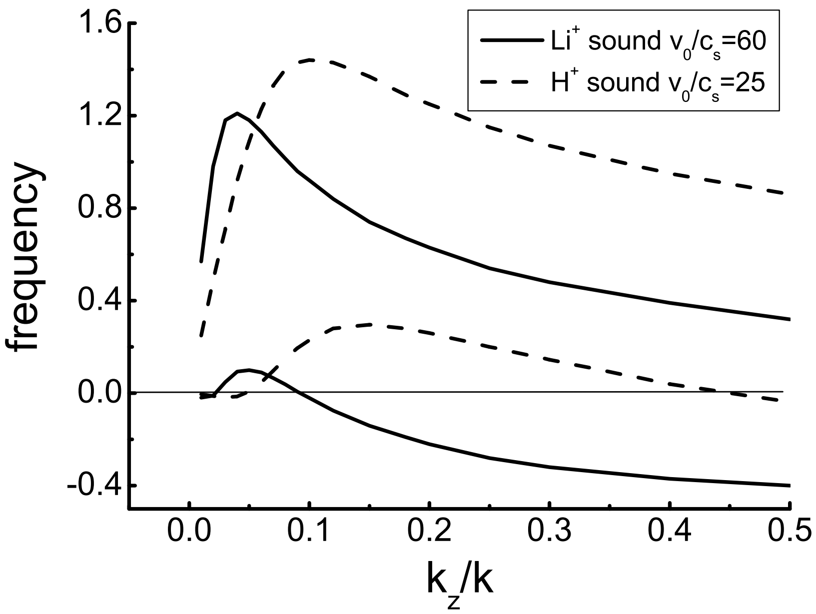


Fig. 5: Frequency (two upper lines) and increment/decrement (two lower lines) normalized to kc_s, for the two separate cases from the upper figure.

- Normalized dispersion equation for $e - a - b$ plasma

$$\frac{\zeta_1}{\omega^2 + i\delta_a\omega - 1/\tau_a} + \frac{\zeta_2}{\omega^2 + i\delta_b\omega - m_a/(m_b\tau_b)}$$
$$+\frac{1}{(\omega - k_z v_0/k)[\omega - k_z v_0/k + i\nu_e) - m_a k_z^2/(m_e k^2)]} = 0. \qquad (1.29)$$

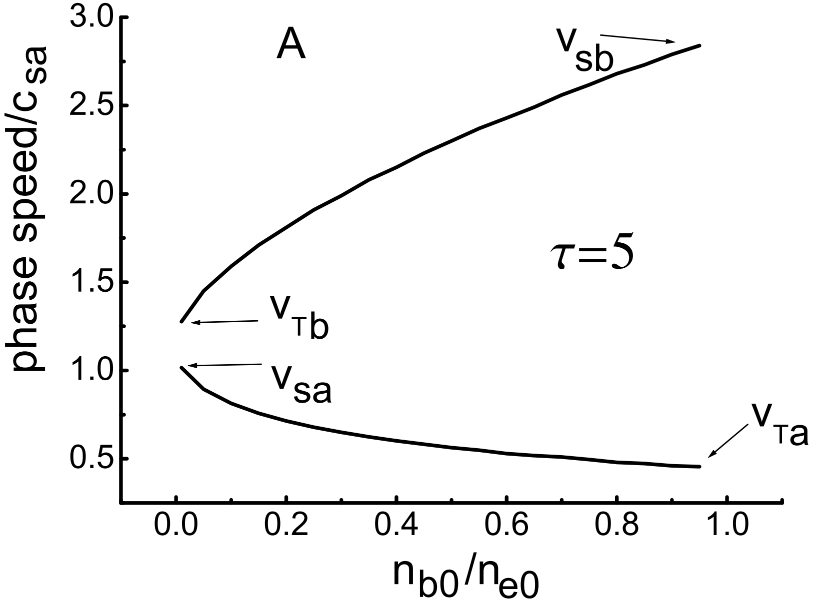
$\omega,\ \nu_{a,b},\ \delta_{a,b}$ all normalized to kc_{sa}, and v_0 is normalized to $c_{sa} = (\kappa T_e/m_a)^{1/2}$

$$\delta_a = \nu_a + \frac{1}{\pi d_a}\left(1 + \frac{1}{\tau_a}\right)^{1/2}, \quad \delta_b = \nu_b + \frac{1}{\pi d_b}\left[\frac{m_a}{m_b}\left(1 + \frac{1}{\tau_b}\right)\right]^{1/2},$$

$$\zeta_1 = \frac{m_e(1 - z_b\eta)z_a k^2}{m_a k_z^2}, \quad \zeta_2 = \frac{m_e \eta z_b^2 k^2}{m_b k_z^2}, \quad \nu_a = \beta_a\left(\frac{m_e}{m_a\tau_a}\right)^{1/2}\frac{\nu_e}{kc_{sa}},$$

$$\nu_b = \beta_b\left(\frac{m_e}{m_b\tau_b}\right)^{1/2}\frac{\nu_e}{kc_{sa}}, \quad \eta = n_{b0}/n_{e0}.$$

For $d_{a,b}$ we use the polynomials (1.24) with the corresponding $\tau_{a,b} = T_e/T_{a,b}$.



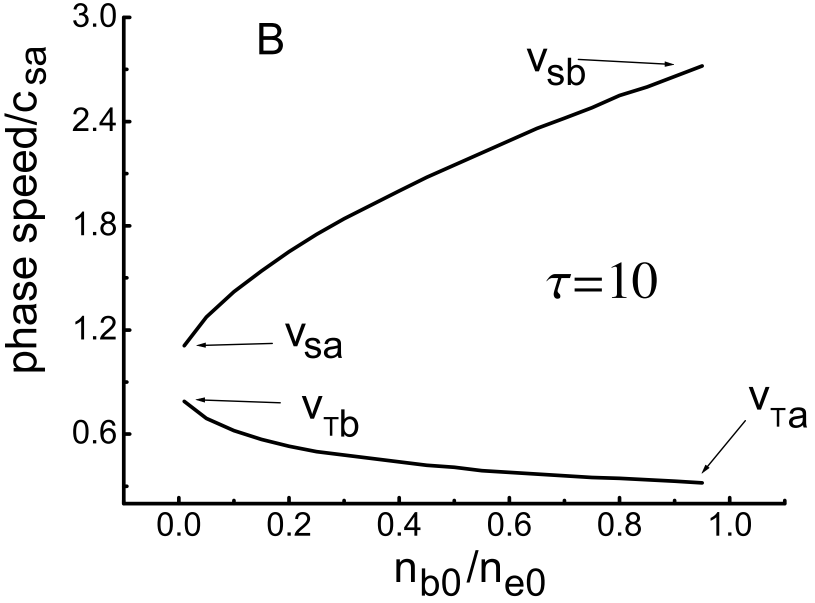


Fig. 6: Phase speed of sound modes in collision-less ideal e-Li$^+$-H$^+$-He plasma without flows and Landau damping, for $k_z = k$.

A: the case $v_{sa} < v_{Tb}$, $\tau = 5$.

B: the case $v_{sa} > v_{Tb}$, $\tau = 10$. *All velocities are normalized to* c_{sa}.

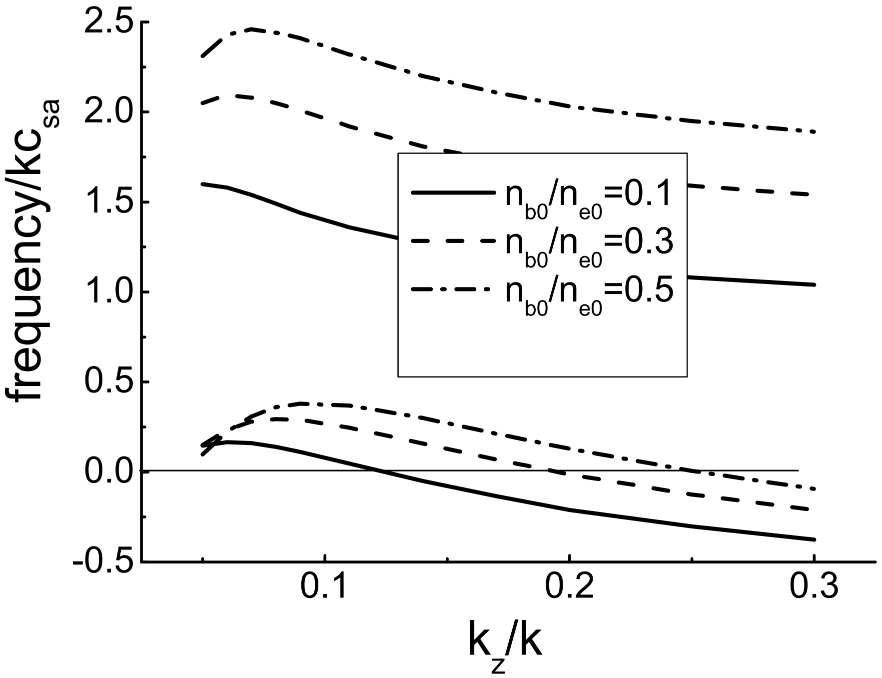


Fig. 6: Real (three upper lines) and imaginary (three lower lines) parts of the frequency (normalized to kc_{sa}) of Li$^+$ sound wave dependent on the angle of propagation and the number density of H$^+$ ions. Here $v_0 = 60c_{sa}$, $\nu_e = 50kc_{sa}$, and $\tau = 10$.

Short summary:

Fluid analysis is presented of the ion sound mode in a weakly ionized collisional plasma. The ion-neutral collision frequency exceeds the ion gyro-frequency while the electrons remain magnetized. Under these conditions, an ion sound wave can propagate at arbitrary angles with respect to the direction of the magnetic field. In the presence of an electron flow along the magnetic lines the sound mode can grow. Due to the electron collisions the mode is unstable while ion collisions cause an angle dependent instability threshold which is such that the mode is most easily excited at very large angles. Hot ion effects are included in the study by means of an effective viscosity which effectively describes the ion Landau damping effect. In the presence of an additional light ion specie, the mode frequency and increment in a certain parameter range are increased.

2. Gas acoustic and ion acoustic modes

The effect of ion-electron collisions on the angle dependent instability threshold

- Plasma with additional significant collisions between charged species
- Instability provided that:

$$V \equiv \frac{v_0}{c_s} > \frac{k_z}{k}\left(1+\frac{T_i}{T_e}\right)^{1/2}\left[\frac{\nu_{ei}}{\nu_{ei}+\nu_{en}}\left(\frac{k^2}{k_z^2}-1\right)+\right.$$
$$\left.\frac{\nu_{en}}{\nu_{ei}+\nu_{en}}\frac{k^2}{k_z^2}+\frac{m_i}{m_e}\frac{\nu_{in}+\mu_0 k^2}{\nu_{ei}+\nu_{en}}\right]. \tag{1.30}$$

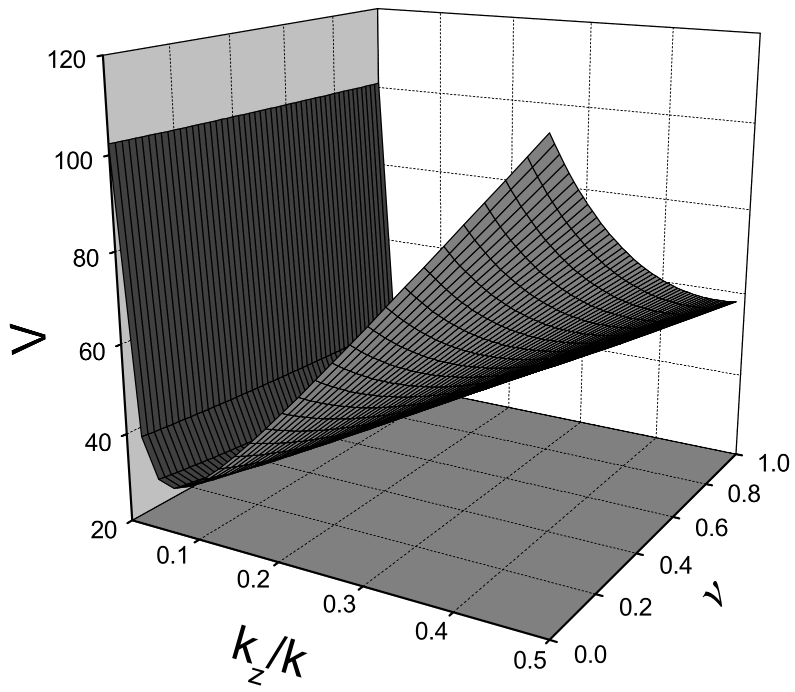


Fig. 7: The normalized threshold velocity $V \equiv v_0/c_s$ for the instability in terms of k_z/k and $\nu \equiv \nu_{ei}/\nu_{en}$. The unstable values are located above the surface.

- In Fig. 7: hydrogen plasma in a neutral hydrogen gas; $\tau = 1$, and $\sigma_{en} = 2.5 \cdot 10^{-19}$ m^2, $\sigma_{in} = 9.24 \cdot 10^{-19}$ m^2 at the temperature of 1 eV, and we have chosen $\widehat{\nu}_{en} = 30$. For these parameters $d \simeq 0.4$, and $\widehat{\nu}_{in} = 2.6$. The additional electron-ion collisions drastically reduce the velocity threshold at small angle of propagation (i.e., for k_z/k close to 1).

- The parameter ν can be conveniently expressed as

$$\nu \equiv \frac{\nu_{ei}}{\nu_{en}} \simeq 2 \cdot 10^3 X T_{6000}^{-2}, \tag{1.31}$$

where $X \equiv n_{i0}/n_{n0}$ is the fractional ionization, and T_{6000} is written in the units of 6000 K. In the lower solar chromosphere at the altitude of about 1000 km, the neutral hydrogen and electron number densities are[9] respectively, $n_{n0} = 3.15 \cdot 10^{19}$ m^{-3} and $n_{e0} = 10^{17}$ m^{-3}, and thus, $X \sim 10^{-3}$. Thus, the instability threshold corresponding to the lower chromosphere will be near $\nu = 1$. Note that at the same time, for a magnetic field of 0.1 T, the protons still remain un-magnetized $\Omega_i/\nu_{it} \simeq 0.2$, while the electrons are magnetized $\Omega_e/\nu_{et} \simeq 16$.

- In the star forming molecular cloud regions $T = 10 - 100$ K, and the above ratio (1.31) will become $\nu \simeq 6 \cdot 10^6 X T_{10}^{-2}$. The ionization fraction is very low in the dense region of the molecular clouds, typically $X \sim 10^{-8} - 10^{-10}$. Note that this is the region where ions are also un-magnetized ($\Omega_i/\nu_{it} \ll 1$). This raises the possibility of applying this instability to the star forming, dense molecular cloud regions in the interstellar medium. However, the assumption about the stationary

[9] J. E. Vernazza, E. H. Avrett, and R. Loeser, Astrophys. J. Suppl. **45**, 635 (1981).

neutral background is not always applicable and thus in the following sections we shall relax this constraint.

The dynamics of neutrals and the angle dependent frequencies

- The neutral momentum and continuity equations

$$v_{n1} = \left(\frac{i\omega\nu_{ni}}{\omega\omega_n - k^2 v_{Tn}^2} \right) v_{i1}. \tag{1.32}$$

  Eq. (1.32) describes acoustic waves in the neutral gas which is coupled to the ions due to collisions; small longitudinal perturbations of the form $\sim$ exp$(-i\omega t + i\vec{k}\cdot\vec{r})$, propagating in an arbitrary direction $\vec{r}$ which makes an angle ψ with the magnetic field lines $\vec{B}_0 = B_0\vec{e}_z$; $\omega_n \equiv \omega + i\nu_{ni}$, $v_{Tn}^2 = \kappa T_n/m_n$, and v_{i1} is the perturbed ion velocity in the same $\vec{r}$ direction.

- The ion momentum equation includes $-m_i n_{i0}\nu_{in}(v_{i1} - v_{n1})$; similar extra term in the electron momentum as well.

- The dispersion equation:

$$\left[\omega^2 - k^2 c_s^2\left(1+\frac{1}{\tau}\right)\right](\omega\omega_n - k^2 v_{Tn}^2) = -\nu_{ni}\omega^2\left(\nu_{in} + \nu_{en}\frac{m_e}{m_i}\right)$$
$$-i(\omega\omega_n - k^2 v_{Tn}^2)\left\{\omega(\nu_{in} + \mu_0 k^2) + \frac{m_e}{m_i}\left[\omega\left(\nu_{ei}\left(\frac{k^2}{k_z^2} - 1\right) + \nu_{en}\frac{k^2}{k_z^2}\right)\right.\right.$$
$$\left.\left. - k_z v_0(\nu_{ei} + \nu_{en})\frac{k^2}{k_z^2}\right]\right\}. \qquad (1.33)$$

- When the neutral gas is perturbed or when the perturbations in the ionized component induce (due to the friction) perturbations of the neutral background, the dispersion equation is solved for the parameters $\tau = 4$, $\tau_n = 4$, $\mu = m_i/m_e = 1838$, $\mu_n = m_i/m_n = 1$, $\widehat{\nu}_{en} = 30$, and $V = 30$. We have taken $n_{e0} = n_{i0} = 6 \cdot 10^{16}$ m^{-3} and $n_{n0} = 10^{19}$ m^{-3}, which yields $X \equiv n_{i0}/n_{n0} = 0.006$ and $\nu \equiv \nu_{ei}/\nu_{en} = 0.916$. The results are presented in Fig. 8, with the remarkable angle dependent behavior of the IA mode. The neutral acoustic mode has nearly a constant frequency $\omega_n \simeq 0.5$ and a very small decrement $\simeq -0.005$. The real and imaginary parts of the ion acoustic mode frequency change in the presence of electron-ion collisions ν although the ionization is relatively small. Note that the assumed value of $\widehat{\nu}_{en} = 30$ in principle fixes the wavelength of fluctuations. For example, assuming $T = 5000$ K, one has $c_s = 6.4$ km/s and $\nu_{en}/(kc_s) = 30$ implies a wavelength of 0.7 m.

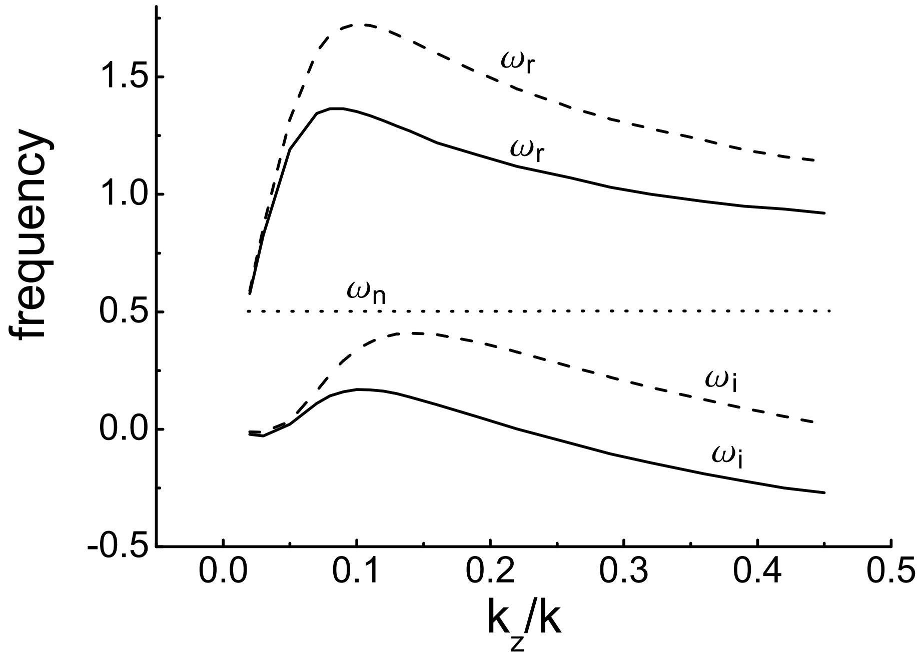


Fig. 8: Normalized real ω_r and imaginary ω_i parts of the angle dependent ion acoustic frequency for $\nu \equiv \nu_{ei}/\nu_{en} = 0$ (full lines) and $\nu = 0.916$ (dashed lines), in terms of k_z/k. The dotted line ω_n describes the neutral acoustic mode.

Wave spectrum in terms of v_0: one example

- The dispersion equation solved in terms of the driving velocity v_0 for a fixed $k = 0.002$ m^{-1} ($\lambda = 3.14$ km), and for the following plasma parameters that may be taken as typical for the lower solar atmosphere: $T_e = T_i = T_n \simeq 5000$ K, $n_{e0} = n_{i0} = 8 \cdot 10^{16}$ m^{-3}, $n_{n0} = 3 \cdot 10^{20}$ m^{-3}, thus $X = 2.6 \cdot 10^{-4}$. This further yields $\nu_{in} = 3.5 \cdot 10^6$ Hz, $\nu_{en} = 3 \cdot 10^7$ Hz, and $\nu_{ei} = 3.8 \cdot 10^6$ Hz. For a magnetic field $B_0 = 0.01$ T this yields un-magnetized ions and magnetized electrons, viz. $\Omega_i/\nu_{it} = 0.27$ while $\Omega_e/\nu_{et} = 53.4$. For these parameters, the plasma β is $1.3 \cdot 10^{-4} < m_e/m_i = 5.4 \cdot 10^{-4}$, implying we are in the proper electrostatic limit. Here, the nominal values of the IA and GA frequencies are 18.2 Hz, and 12.9 Hz, respectively.

- In Fig. 9, the ion acoustic (IA) frequency and increment/decrement are presented in terms of v_0 for two angles of propagation, viz. $k_z = 0.25\,k$ and $k_z = 0.05\,k$. When $k_z = 0.25\,k$, the IA mode becomes unstable already for $v_0 \simeq 4\,c_s$ (see the corresponding dashed line). In the other case, when $k_z = 0.05\,k$, both the frequency and the increment/decrement are lower, and the instability takes place for $v_0 > 20\,c_s$.

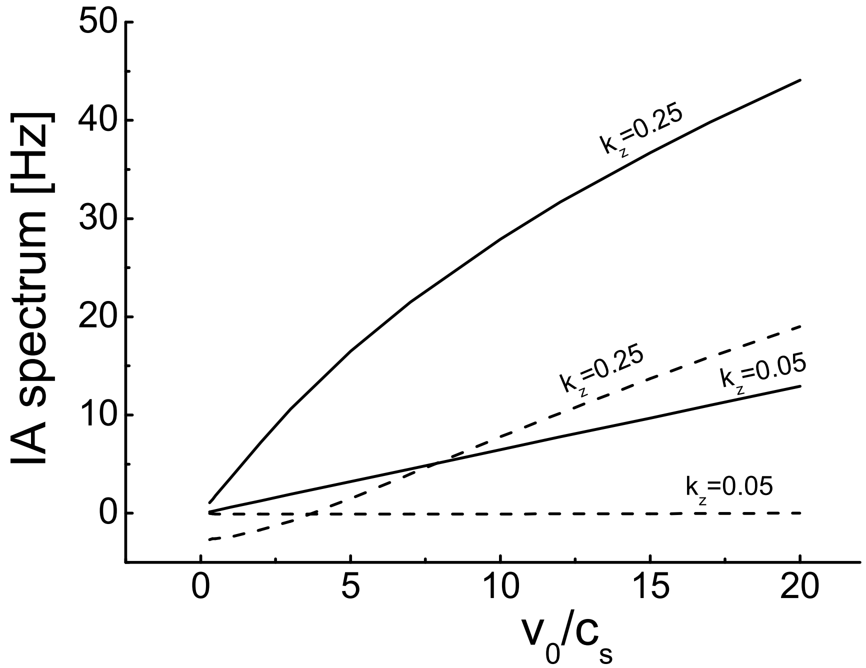


Fig. 9: Unstable IA mode for two angles of propagation $k_z/k = 0.25,\ 0.05$, for wavelengths $\lambda = 3.14$ km. Full lines indicate real parts of frequencies and dashed lines indicate increments/decrements.

- The response of the neutral gas is negligible because (e.g., for $k_z = 0.25\,k$) for the given range of v_0 the GA frequency changes between 0.006 Hz and 0.06 Hz, while the corresponding decrement is in the range -0.18 Hz and -0.02 Hz. For the given wavelengths the collisions with the ions completely destroy the GA mode. This is checked by reducing the collision frequency by hand, yielding in the end the above given nominal value of 12.9 Hz.

Electromagnetic perturbations

- Assume now that the magnetic lines are perturbed. Typically this is due to the difference in the parallel motion of electrons and ions, which implies a perturbed parallel current and a perturbed perpendicular magnetic component according to the Ampère law. In the case of magnetized ions that are tied to the magnetic field lines, such perturbations propagate along the field lines at the Alfvén speed. However, this is not the situation in the present model. For the electromagnetic (EM) perturbations, the dynamics of neutrals is unchanged. The small perturbations of the magnetic field do not change the ion magnetization and thus the Lorentz force in the ion momentum equation can still be omitted. For not so small plasma β, assuming only perpendicular bending of the magnetic field lines, we express the perturbations of the EM field in terms of potentials $\vec{E}_1 = -\nabla\phi_1 - \partial\vec{A}_{z1}/\partial t$ and $\vec{B}_1 = \nabla\times\vec{A}_{z1} = -\vec{e}_z\times\nabla_\perp A_{z1}$. The ion momentum equation now includes the new term $-e\, n_{i0}\,\partial_t A_{z1}\,\vec{e}_z$. Consequently, the ion dynamics in $\vec{k}$-direction comprises the new term $A_k = k_z\,A_{z1}/k$, so that

$$v_{i1} = \frac{ek}{m_i\omega_2}\left(1+\frac{k^2v_{Ti}^2}{\omega\omega_2 - k^2v_{Ti}^2}\right)\left(\phi_1 - \frac{\omega}{k}\frac{k_z}{k}A_{z1}\right). \tag{1.34}$$

- This results in the modified ion equation

$$\frac{n_{i1}}{n_{i0}} = \frac{ek^2}{m_i(\omega\omega_2 - k^2 v_{Ti}^2)} \left(\phi_1 - \frac{\omega}{k}\frac{k_z}{k} A_{z1} \right). \tag{1.35}$$

- The electron perpendicular velocity

$$\vec{v}_{e\perp 1} = \frac{1}{1 + \nu_e^2/\Omega_e^2} \left[\frac{1}{B_0} \vec{e}_z \times \nabla_\perp \phi_1 + \frac{\nu_e}{\Omega_e} \frac{\nabla_\perp \phi_1}{B_0} - \frac{v_{Te}^2}{\Omega_e} \frac{\nu_e}{\Omega_e} \frac{\nabla_\perp n_{e1}}{n_0} \right.$$

$$- \frac{v_{Te}^2}{\Omega_e} \vec{e}_z \times \frac{\nabla_\perp n_{e1}}{n_0} - v_0 \vec{e}_z \times \nabla_\perp A_{z1} + \frac{\nu_{ei}}{\Omega_e} \vec{e}_z \times \vec{v}_{i\perp 1} + \frac{\nu_{ei}\nu_e}{\Omega_e^2} \vec{v}_{i\perp 1}$$

$$\left. + \frac{\nu_{en}}{\Omega_e} \vec{e}_z \times \vec{v}_{n\perp 1} + \frac{\nu_{en}\nu_e}{\Omega_e^2} \vec{v}_{n\perp 1} \right].$$

- This is to be used in the $\nabla_\perp \cdot \vec{v}_{e\perp 1}$ term in the electron continuity equation. Clearly, all non-vanishing terms are multiplied by the small ratio of the collision and gyro-frequencies, so that, like the previous case, the electron perpendicular dynamics can be neglected.

- The electron parallel dynamics now includes the vector potential, and as a result there appears a new term in the electron continuity

$$-\frac{ie\omega k_z A_{z1}}{m_e \nu_e \omega_0} \tag{1.36}$$

- From the Ampère law

$$\frac{k^2\lambda_i^2\omega_2}{k_z}\left(\frac{k_z^2}{k^2}-1\right)A_{z1} = \frac{im_i\omega\omega_2}{m_e(\nu_e\omega_0 + ik_z^2 v_{Te}^2)}\left(\phi_1 - \frac{\omega}{k_z}A_{z1}\right)$$
$$+\left(1+\frac{k^2 v_{Ti}^2}{\omega\omega_2 - k^2 v_{Ti}^2}\right)\left[-1+\frac{\omega}{\nu_e\omega_0 + ik_z^2 v_{Te}^2}\left(\nu_{ei}+\frac{i\nu_{en}\nu_{ei}\omega}{\omega\omega_n - k^2 v_{Tn}^2}\right)\right]\times$$
$$\times\left(\phi_1 - \frac{\omega}{k_z}\frac{k_z}{k}A_{z1}\right). \tag{1.37}$$

Here, $\lambda_i = c/\omega_{pi}$, where c is the speed of light and ω_{pi} is the ion plasma frequency.

- Continuity equations:

$$\frac{k^2}{m_i}\left\{\frac{1}{\omega\omega_2 - k^2 v_{Ti}^2} - \frac{k_z^2}{k^2}\frac{\nu_{ei}}{\omega_2(\nu_e\omega_0 + ik_z^2 v_{Te}^2)}\left(1 + \frac{k^2 v_{Ti}^2}{\omega\omega_2 - k^2 v_{Ti}^2}\right)\left[1 + \frac{i\omega\nu_{ni}\nu_{en}}{\nu_{ei}(\omega\omega_n - k^2 v_{Tn}^2)}\right]\right\} \times \left(\phi_1 - \frac{\omega}{k_z}\frac{k_z}{k}A_{z1}\right)$$

$$= \frac{ik_z^2}{m_e(\nu_e\omega_0 + ik_z^2 v_{Te}^2)}\left(\phi_1 - \frac{\omega}{k_z}A_{z1}\right). \tag{1.38}$$

- In the electromagnetic collision-less case without the electron flow and Landau damping, Eqs. (1.37), (1.38) yield

$$(\omega^2 - k^2 v_{Tn}^2)\left[k^4 k_z^2 v_s^2 \lambda_i^2 - \left(k^2(1 + k_z^2\lambda_i^2) - k_z^2\right)\omega^2\right] = 0.$$

Here, we have the GA mode uncoupled

- The IA mode which is modified due to the electromagnetic effects yielding

$$\omega^2 = k^2 c_s^2 \frac{k_z^2 \lambda_i^2}{1 + k_z^2 \lambda_i^2 - \dfrac{k_z^2}{k^2}}. \tag{1.39}$$

  Hence, in the absence of Alfvén waves (un-magnetized ions), the parallel propagation ($k_z = k$) yields an ordinary ion sound mode. For any other angle of propagation (except for $k_z \to 0$) the ion sound mode is electromagnetically modified and becomes dispersive.

- The collisions couple the two modes and the full dispersion equation is given by

$$k^4\lambda_i^2 v_{Tn}^2\left(k_z^2 v_s^2 - i\nu_e\omega_0\frac{m_e}{m_i}\right) + \omega_n\omega^3\left(1-\frac{k_z^2}{k^2}\right)$$
$$+k_z^2\omega^2\left[\lambda_i^2\frac{m_e}{m_i}\left(\nu_{en}\nu_{ni} - i\nu_{ei}\omega_n\right) + v_{Tn}^2 + \omega_n\omega_2\lambda_i^2\right]$$
$$-k^2\omega\left\{\omega v_{Tn}^2 - i\lambda_i^2\nu_e\omega_0\omega_n\frac{m_e}{m_i} + k_z^2\lambda_i^2\left[v_{Tn}^2\left(\omega_2 - i\nu_{ei}\frac{m_e}{m_i}\right)+\omega_n v_s^2\right]\right\} = 0. \tag{1.40}$$

$$\omega_0 = \omega - k_z v_0, \quad \omega_n = \omega + i\nu_{ni}, \quad \omega_2 = \omega + i(\nu_{in} + \mu_0 k^2) + \frac{\nu_{in}\nu_{ni}\omega}{\omega\omega_n - k^2 v_{Tn}^2}.$$

In order to compare the ES and EM cases, we use the same set of parameters as in the previous cases. This is only for the sake of comparison because, in principle, the EM effects imply a higher plasma β. The most visible difference is in the graph of the IA mode increment, which is now much higher for the velocity v_0 above certain critical value. Physically, here we have the bending of the magnetic lines representing an additional obstacle for the electron motion in the z-direction, and as a result the mode is more unstable.