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**Plasma Accelerators** 

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## **Trieste Summer School, 2007**

# **Plasma Accelerators**

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## Introduction

- General Principles
  - Laser-plasma accelerators
  - Electron-beam-plasma accelerators
  - Theory Simulations
  - Experiments
  - Future









## CERN – LEP schematic











## **SLAC Schematic**



## Plasma Accelerator Progress and the Livingston Curve



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### Particle Accelerators – compact to country size Rich Physics and Applications



## <u>Compact</u>

- Verified Standard Model of Elem particles
- W, Z bosons
- Quarks, gluons and quarkgluon plasmas
- Asymmetry of matter and anti-matter
- In pursuit of the Higgs Boson (cause of mass)

- Medicine
  - Cancer therapy, imaging
- Industry and Gov't
  - Killing anthrax
  - lithography
- Light Sources (synchrotrons)
  - Bio imaging
  - Condensed matter science

### NGC6251 – Astronomical Accelerator!





## **Highest Energy Cosmic Rays**





Examples of Energetic Particle Acceleration

- Cosmic Rays
  - $e^{-}$ , p<sup>+</sup>, He<sup>+</sup>, heavy ions  $\Rightarrow 10^{7} 3x10^{20} \text{ eV}$
- Solar flares

 $- e^{-}, p^{+} \Rightarrow 10^{5} - 10^{9} eV$ 

- Aurora in planetary magnetospheres  $- e^{-}, p^{+} \Rightarrow 10^{4} - 10^{6} eV$
- Runaways in tokamaks

 $- e^{-} \Rightarrow 10^{5} - 10^{6} \text{ eV}$ 

• Laser fusion

 $- e^{-} \Rightarrow 10^{5} - 10^{7} eV$ 

Plasma accelerators

 $-e^{-} \Rightarrow 1 \text{ GeV} - 100 \text{GeV}, \text{ ions 10's MeV}$ 



## **Plasma Acceleration**

- Electric fields generated in a plasma can accelerate electrons, protons, ions to high energies
- Definition

$$\Delta T = q v_0(t) \cdot E(r_0(t), t) dt$$

- $\Delta T$  is change in particle kinetic energy
- *E* is the electric field
- q is the particle charge
- $v_0$  is the particle velocity
- Deterministic System
  - E(r,t) does not change significantly during the acceleration phase.
- Stochastic
  - E(r,t) changes in a random fashion.

### **Proposed Acceleration Mechanisms**

## Coherent $t_{acc} > t_{change}$

## **E-Fields**

- Generated by reconnection inductively driven field
- Double layers conservative field cannot energise particle, can only locally accelerate them (KE at expanse of PE)
- Coherent waves

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- Acceleration mechanisms, Beat-Wave, wake field need large amplitudes or large distances of uniform plasma
- Coulombic Explosions



## **Stochastic Acceleration**

- In laboratory, space and astrophysical plasmas stochastic acceleration by turbulent wave fields is important.
- In astrophysics, the best known stochastic acceleration is Fermi acceleration.
- Fermi acceleration -- cosmic ray particles accelerated by colliding with moving magnetized structures i.e. galaxies.
- The collision is effective via the magnetic mirror and the particle either gains or losses energy proportional to the velocity of the structure.



## The acceleration takes place via many small steps with multiple collisions between particles and magnetic clouds.

**Fermi Acceleration** 

- Acceleration is non-isotropic energy being put into the parallel direction.
- Energy distribution among a large number of particles is redistributed among a few very high energy particles.

Note: Fermi acceleration requires pre-acceleration.



Simplest is Cerenkov resonance  $\omega = kv$ 

- corresponds to law of momentum and energy conservation particle emits wave quantum  $\hbar\omega, \hbar k$ 

Wave-Particle Resonance

$$\Delta E = \hbar \omega = \Delta \underline{p}.\underline{v}$$
 where  $\Delta \underline{p} = \hbar \underline{k}$  &  $\omega = \underline{k}.\underline{v}$ 

### In a Magnetic Field

- Resonant particle moves along a spiral on Larmor orbit  $\omega_c = \frac{1}{2}$
- Particle may be treated as an oscillator with energy  $n\hbar\omega_c = (n = 0, 1, 2, ...)$  moving along *B*
- Both energy of translational motion and oscillator energy may change

$$\begin{split} \Delta E &= \hbar \omega = \Delta p_z v_z + n \hbar \omega_c \qquad \Delta p_z = \hbar k_z \\ \omega &= k_z v_z + n \omega_k \qquad \qquad n = 0, \pm 1, \pm 2, \dots \end{split}$$



Plasma Acceleration Technologies

## **Electron Accelerators**

- 1) Electron beam-induced Plasma Wakefield Accelerator (PWA)
- 2) Laser-induced Plasma Wakefield Accelerators
  - a) Plasma Beat-Wave Accelerator (PBWA)
  - b) Laser Plasma Wakefield Accelerator (LPWA)
  - c) Self-Modulated Laser Wakefield Accelerator (SMLWA)
- 3) Inverse Cherenkov Laser Accelerator
- 4) Surfatron Accelerators



### Plasma Accelerators – Why Plasmas?

## **Conventional Accelerators**

- Limited by peak power and breakdown
- 20-100 MeV/m
- Large Hadron Collider (LHC)
   -- 27km, 2010
- Plans for "Next" Linear Collider (NLC) -- 100km ?

### <u>Plasma</u>

- No breakdown limit
- 10-100 GeV/m

### <u>Laser</u>

• >1 GeV in several cm

### <u>e-beam</u>

• ~100 GeV in 1m

### Particle Accelerators – Requirements for High Energy Physics



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- High Energy
- High Luminosity (event rate)
  - L=fN<sup>2</sup>/ $4\pi\sigma_x\sigma_y$
- High Beam Quality
  - Energy spread  $\delta\gamma/\gamma \sim .1 10\%$
  - Low emittance:  $\epsilon_n \sim \gamma \sigma_y \theta_y < 1$  mm-mrad
- Low Cost (one-tenth of \$6B/TeV)
  - Gradients > 100 MeV/m
  - Efficiency > few %



**Compact Proton/Heavy Ion Accelerators** 

- 1) Laser-induced Coulomb Explosions
- 2) Laser-induced Plasma Accelerator
- 3) Laser Shock Wave Accelerator
- **4)** Laser  $\underline{\mathbf{v}}_p \times \underline{B}$  Accelerator
- 5) Electron Ring Accelerator

## **Synchrotron Sources**

Laser Undulator - radiation generation system



- Lasers Terawatt, Petawatt Compact Lasers 10<sup>12</sup>–10<sup>15</sup> Watts already exist.
  - -Some with high rep. Rates *i.e.* **10 Hz**.
  - -Capable of 10<sup>19</sup>-10<sup>21</sup> Watts/cm<sup>2</sup> on target.
  - -Future ~10<sup>23</sup> Watts/cm<sup>2</sup> using OPCPA.
- Electrons Beams Shaped electron beams such as the Stanford/USC/UCLA experiment generate 100*GV/m* accelerating gradient using the 30 – 50*GeV* beam in a 1 meter long Lithium Plasma.







### **Radiation Pressure**

#### Keyhole Nebula



Hubble Heritage

NASA and The Hubble Heritage Team (STScI) · Hubble Space Telescope WFPC2 · STScI-PRC00-06

 At 10<sup>21</sup> Wcm<sup>2</sup>, the light pressure will exceed 10<sup>9</sup> atmospheres, compressing material to densities found inside stars.

Eta Carinae PRC96-23a · ST Scl OPO · June 10, 1996 J. Morse (U. CO), K. Davidson, (U. MN), NASA HST · WFPC2



- The electric field of a laser in vacuum is given by  $E_{\perp} = 30\sqrt{I}$  V/cm
- For short pulse intense lasers,

$$P = 10 \text{ TW}, \ \lambda_0 = 1 \ \mu \text{m}, \ I = 1.6 \text{x} 10^{18} \text{ W/cm}^2$$
  
 $E_\perp = 40 \text{ GV/cm}$ 

- Unfortunately, this field is perpendicular to the direction of propagation and no significant acceleration takes place.
- The longitudinal electric field associated with electron plasma waves can be extremely large and can accelerate charged particles.





~1 GeV/cm

- Conventional accelerators limited by electrical breakdown of accelerating structures
- Plasmas are already broken down.
  - The accelerating fields limited only by plasma density.
- Plasmas can support longitudinal accelerating fields moving close to the speed of light; Relativistic electron plasma waves.
- Lasers easily couple to plasmas and can generate relativistic electron plasma waves.

### Laser Plasma Accelerators

- Large accelerating fields
- No electrical breakdown limit
- But also, Low repetition rates
- ... and can be expensive!



A high energy electron bunch



Laser Wake Field Accelerator(LWFA, SMLWFA, PBWA) λ A single short-pulse of photons

Drive beam λ Trailing beam λ

Accelerators\*

### \*Godfather of the field: Prof. John Dawson

In the Plasma Beat Wave Accelerator (PBWA) a relativistic plasma wave is resonantly excited by the "ponderomotive" force of two lasers separated by the plasma frequency  $\omega_{p}$ .

The two laser beams beat together forming a modulated beat pattern in the plasma.



Plasma Beat Wave Accelerator (PBWA)

• For relativistic plasma wave the accelerating field E<sub>||</sub> is given by  $E_{\parallel} = \varepsilon \sqrt{n_0}$  V/cm

 $\varepsilon$  is the fractional electron density bunching, n<sub>0</sub> is the plasma density. For  $n_0 = 10^{18} \text{ cm}^{-3}$ ,  $\varepsilon = 10\% \implies E_{\parallel} = 10^8 \text{ V/cm}$ 



## **Plasma Beat Wave**

Relativistic plasma wave driven by beating 2 lasers in a<br/>  $\begin{array}{l} plasma \\ \omega_1 - \omega_2 \cong \omega_p \\ k_1 - k_2 \cong k_p \end{array} \quad \text{energy} \\ \hline k_1 - k_2 \cong k_p \\ \hline momentum \\ \hline momentum$ 

Then 
$$k_1 - k_2 \sim \Delta k$$
  
 $\omega_1 - \omega_2 \sim \Delta \omega$   $\left\{ \begin{array}{c} \Delta \omega \\ \Delta k \end{array} = v_g \end{array} \right\}$ 

 $v_{g} \text{ is the group velocity of the laser beat pattern.}$ But  $k_{1} - k_{2} \sim k_{p}$ ;  $\omega_{1} - \omega_{2} \sim \omega_{p}$  $\Rightarrow \frac{\omega_{p}}{k_{p}} = v_{ph} \equiv v_{g} \qquad v_{g} = c \left(1 - \frac{\omega_{p}^{2}}{\omega_{1,2}}\right)^{1/2}$ 

For  $\omega_1, \omega_2 \rangle \rangle \omega_p \Rightarrow \mathbf{v}_g \approx \mathbf{c} \Rightarrow$  "Hence relativistic"



## **Poisson's Equation**

From Poisson's equation we can estimate how large these longitudinal • electron plasma waves can be  $\nabla \cdot E = 4\pi e \delta n_a$ 

 $\delta n_{\rm e}$  is perturbed electron density of the plasma ions immobile on short time scale.

Largest field exists for  $\delta n_e = n_0$  *i.e.* background density.

• Electron plasma waves oscillate with frequency  $\omega_n = (4\pi n_0 e^2 / m_a)^{1/2}$ cgs, or  $(n_0 e^2 / m_e \varepsilon_0)^{1/2}$  MKS.

Relativistic plasma waves have phase velocities close to c

*i.e.* 
$$\frac{\omega_p}{k_p} \cong c.$$

With Poisson's equation we get •

Poisson's equation we get 
$$eE_{MAX} \approx \frac{4\pi n_0 e^2}{(\omega_p/c)} \approx 0.97\sqrt{n} \text{ eV/cm}$$
  
*i.e.*  $eE_{MAX} \approx \sqrt{n} \text{ eV/cm}$ 

2

## PBWA: Evolution of Laser Intensity and Accelerating Field









- The Lorentz force due to the interaction of the lasers produces a longitudinal force or "ponderomotive" force of the beat pattern proportional to the gradient of
- Each beat (pulse) adds to the plasma waves amplitude. ٠
  - The growth in time

$$eE_{\parallel} = \frac{m_e c\omega_p}{4} \int_0^{t_0} \alpha_1 \alpha_2 dt$$

- $\alpha_{1,2} = eE_{\perp 1,2} / m_e c \omega_{1,2}$  is the quiver velocity of an electron in the laser field, ٠ normalized to the speed of light c
- From Gauss's Law the accelerating field  $E_{\parallel}$  can be estimated ٠

$$E_{\parallel} = \varepsilon \frac{m_e c \omega_p}{e}$$
$$E_{\parallel} = \varepsilon \sqrt{n_0} \quad \text{V/cm}$$

111 00

or

$$E_{\parallel} = \varepsilon \sqrt{n_0} \quad \text{V/em}$$

 $\epsilon$  is plasma wave amplitude (fractional electron density bunching  $n_1/n_0$ , ۰  $n_0$  is ambient density).





- For  $n_0 = 10^{18}$  cm<sup>-3</sup>,  $\varepsilon = n_1 / n_0 = 10\%$
- Gain in energy of electron  $\Delta W$
- $\gamma = \omega_1 / \omega_p$  is the Lorentz factor
- For a neodymium laser,  $\omega_1/\omega_p \sim 30$  , and  $n_0 \sim 10^{18} \, {\rm cm}^{-3}$

$$E_{p} = \frac{e}{k_{p}} \delta n = \frac{ec}{\omega_{p}} \delta n$$

$$l = \frac{\lambda_{p}}{2} \frac{\omega^{2}}{\omega_{p}^{2}} = \frac{\lambda_{p}}{2} \gamma^{2}$$

$$l = \frac{\lambda_{p}}{2} \chi \implies \chi = \frac{c}{c - v_{ph}} \implies v_{ph} \approx c \left(1 - \frac{\omega_{p}^{2}}{\omega^{2}}\right)^{\frac{1}{2}}$$

$$eEl = 2\varepsilon \gamma^{2} m_{e} c^{2}$$

• Maximum energy gain  $\Delta W \approx eE_p I$   $\Delta W = 2 \varepsilon \gamma^2 m_e c^2$ 

 $\Delta W \approx 100 \; MeV$ 

#### The Relativistic Plasma Wave is described by



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$$\frac{\partial^2}{\partial t^2} + \omega_p^2 \bigg) \delta n = \frac{3}{8} \omega_p^2 \frac{\delta n^2}{n_0^2} \delta n - \frac{n_0}{2} \omega_p^2 \alpha_1 \alpha_2 e^{-i\delta}$$
$$\alpha_j = \frac{eE}{m_e \omega_j c} \quad ; \ \delta = \omega_1 - \omega_2$$

For  $\alpha_1 = \alpha_2 =$ constant

$$\frac{\delta n}{n_0} = \frac{\delta n_0(0)}{n_0} + \frac{1}{4}\alpha_1 \alpha_2 \omega_p t$$

<u>Linear growth:</u> However due to 1<sup>st</sup> term on RHS i.e. cubic nonlinearity wave saturates before reaching wave breaking limit  $\delta n/n$ ~ 1; acts as nonlinear frequency shift.

$$\frac{\partial n}{n} \max = \left(\frac{16}{3}\alpha_1\alpha_2\right)^{7/3} = \varepsilon$$

**Relativistic mass increase of electrons reduces natural frequency** 

$$\omega_p^1 = \omega_p \left( 1 - \frac{3}{8} \frac{v_{osc}^2}{c^2} \right)^{\frac{1}{2}} = \omega_p \left( 1 - \frac{3}{8} \frac{\delta n^2}{n^2} \right)^{\frac{1}{2}}$$

This results in  $\omega_1 - \omega_2 \neq \omega_p \Rightarrow$  non-resonant interaction  $\Rightarrow$  saturation.

## **Beat Wave Growth**



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## **Computer Simulations**



PLASMA WAVE (*left*) from a computer simulation of the beatwave method generates an electric field (*right*). If a group of charged particles was injected into a plasma and moved in phase with the electric field, the group would accelerate.



#### **Beat Wave Generated Plasma Wave**



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 PBWA – Plasma Beat Wave Accelerator



[S. Wilks *et al*. PPG. 1262]







## Surfing The Waves!

At 10<sup>21</sup> W.cm<sup>-2</sup>, electrons will be accelerated to beyond 100 MeV, generating gamma rays, proton beams and exotic isotopes.



#### Laser Wakefield Experiments (LWFA)



Confirmation of wakefield generation - J.R. Marques *et al.* PRL <u>76</u> 3566 (1996) C.W. Siders *et al.* PRL <u>76</u> 3570 (1996)

Acceleration - F. Amiranoff et al. PRL 81 995 (1998)





## **Plasma Wakes**







Plasma wake is a relativistic electron plasma wave

$$v_{ph} \leq c$$

Plasma Wakes (2)

• Capable of growing to large values

$$E_W = \varepsilon \sqrt{n_e} \qquad \varepsilon < 1$$

• For  $n_e \sim 10^{14} \text{ cm}^{-3}$  and  $\varepsilon \sim 10\%$ 

 $E_W \approx 10^8 V/m$  or 1 GeV in 10m

• For  $n_e \sim 10^{20} \text{ cm}^{-3}$  $E_W \approx 10^{11} \text{ V/m}$  or 1 TeV in 10 m

#### Linear Plasma Wakefield Theory

$$(\partial_t^2 + \omega_p^2) \frac{n_1}{n_o} = -\omega_p^2 (\frac{n_b}{n_o} + k_p^2 \nabla^2 \sqrt{1 + a_o^2})$$

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Large wake for a beam density  $n_b \sim n_o$  or laser amplitude  $a_o = eE_o/m\omega_o c \sim 1$ for  $\tau_{pulse}$  of order  $\omega_p^{-1} \sim 100$  fs  $(10^{16}/n_o)^{1/2}$  and speed  $\sim c = \omega_p/k_p$ 

$$\nabla \bullet E = -4\pi e n_1 \Rightarrow eE = \frac{n_1}{n_o} \sqrt{\frac{n_o}{10^{16} cm^{-3}}} 10 \, GeV/m \cos\omega_p (t - z/c)$$

#### But interesting wakes are very nonlinear => PIC simulations



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SUPERIOR TÉCNICO science & Technology Facilities Council Rutherford Appleton Laboratory Laser & Electron Wakes

#### Nonlinear wakes are similar with laser or particle beam drivers: 3-D PIC OSIRIS Simulation (self-ionized gas)





# Plasma waves driven by electrons, photons, and neutrinos



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#### **Electron beam**

$$\left(\partial_t^2 + \omega_{pe0}^2\right)\delta n_e = -\omega_{pe0}^2 n_{e-beam}$$

 $\delta n_e$  ... Perturbed electron plasma density

#### **Photons**

$$\left(\partial_t^2 + \omega_{pe0}^2\right)\delta n_e = \frac{\omega_{pe0}^2}{2m_e}\nabla^2 \int \frac{d\mathbf{k}}{(2\pi)^3} \mathbf{h} \frac{N_{\gamma}}{\omega_{\mathbf{k}}}$$

**Neutrinos** 

$$(\partial_t^2 + \omega_{pe0}^2)\delta n_e = \frac{\sqrt{2}n_{e0}G_F}{m_e}\nabla^2 n_v$$

Ponderomotive force physics/9807049 physics/9807050

Kinetic/fluid equations for electron beam, photons, neutrinos coupled with electron density perturbations due to PW

Self-consistent picture of collective e,y,v-plasma interactions



via Forward Raman Scattering in 1-D limit or via an Envelope-Self-Modulation instability in 2-D limit.



GW

<u>Pulse Power</u> > relativistic self-focusing  $P > P_c = 17 \frac{\omega^2}{\omega_n^2}$ 



#### Stimulated Raman & Modulational Instability



Physical mechanism for stimulated Raman scattering driven by Ponderomotive force.

## **Self-Modulated Laser Pulse**



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## **Surfatron Acceleration**

- As a laser plasma accelerator with unlimited acceleration, main limitation is finite transverse dimension
  - T. Katsouleas, J.M. Dawson PRL (1983)
- Can overcome the main weakness of diffusive shock acceleration
   the injection problem.
  - Shock surfing M. Lee, V. Shapiro, R. Sagdeev JGR (1996)
- Simulations of Raman backscatter in magnetized plasma ⇒ Upper hybrid mode → accelerates electrons

– J.M. Dawson *et al.* PRL (1983)

- Solar proton acceleration to 10 GeV within seconds
- Supernovæ remnant acceleration of both electrons and ions
- Acceleration in radio jets
- Produces ring velocities distributions possible source of
   Oveletree Meser Instability



## **Surfatron Concept**

• Similar to process of acceleration in  $\bot$  shocks

Refs (1+4) deal with plasma waves moving  $\perp$  to  $\underline{B}_0$  field

- assumed to be generated in  $\perp$  shocks -

Refs (2+3) deal with  $\perp$  shocks [shock drift acceleration]

<u>Surfatron</u> a) phase locking particles in plasma wave
 b) indefinite energy gain

<u>Assumptions</u> particle trapped in wave moving  $\perp$  to <u>B</u><sub>0</sub>

- The  $-ev_{ph} \times B$  force deflects particle across wave front (c.f. surfer)
- Particle<sup>c</sup> acquires v<sub>y</sub> velocity<u>,ev<sub>y</sub>×B</u> force; presses particle against wave <u>counteracts</u> (eE<sub>p</sub>sin kx) accelerating wave ⇒ stable fixed point.

Ideal to have wave moving at  $v_x \le c$  to be significant.

**Bibliography** 

- 1) Katsouleas and Dawson, PRL, <u>51</u>, 392 (1983)
- 2) Cairns, PRL, JPP (1973)
- 3) Sagdeev and Shapiro, JETP LETT, <u>17</u>, 279 (1973)
- 4) Gribov, Sagdeev, Shapiro, Shev., JETP LETT, <u>42</u>, 63 (1985)

relativistic non-relativistic non-relativistic relativistic



## Surfatron Acceleration

- Waves such as Bernstein, upper-hybrid, lower-hybrid, • magnetosonic etc., have a common damping mechanism resulting in energization perpendicular to the magnetic field STOCHASTIC (random heating)
  - Energization

**NON-STOCHASTIC** (regular acceleration, linear or non-linear)

Non-stochastic acceleration occurs above and below the stochastic threshold  $\frac{E}{B_0} = \frac{1}{4} \left(\frac{\omega_c}{\omega}\right)^{1/3} V_{ph}$ (Karney, C.F.F. 1978)

E - electric field amplitude  $\omega_{c}$  - cyclotron frequency v<sub>ph</sub> - wave phase velocity

 $B_0$  - ambient magnetic field  $\omega$  - wave frequency



## **Surfatron Model**







## **Surfatron Acceleration**

In the non-linear regime, trapped particles are accelerated across the • wavefronts in a direction perpendicular to the wave propagation by the v<sub>ph</sub>xB electric field.

Test particle mod Model  $\mathbf{B}_{0}, \hat{\mathbf{z}}$ ,  $\mathbf{E}$ 

 $m\gamma$ 

Relativistic equa motion for an ele

Test particle model  
Model 
$$\mathbf{B}_{0}, \hat{\mathbf{z}}, \mathbf{E}, \hat{\mathbf{y}}$$
,  
Relativistic equations of  
motion for an electron are  
 $\mathbf{B}_{0}, \hat{\mathbf{z}}$   
 $\mathbf{B}_{0}, \hat{\mathbf{z}}$   
 $\mathbf{B}_{0}, \hat{\mathbf{z}}$   
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In the wave frame, a trapped particle comes to an equilibrium position (x') against the side of the potential well where the <u>x</u> forces balance:

$$E_0 \sin k' x_1' \approx \gamma_{ph} V_y' B / c \rightarrow \gamma_{ph} B / c$$



## Katsouleas, 1984





Laser Plasma Experiments

Experiments are being conducted world-wide using the Plasma Beat Wave Scheme in the UK (RAL), USA (UCLA), France (CNRS) and Japan (KEK).

#### RAL EXPERIMENTS

- 2 variations a) Plasma Beat Wave Accelerator b) Self-Modulated Laser Wakefield Accelerator and
- b) The Self-Modulated Laser Wakefield Accelerator uses a short pulse intense laser (Vulcan)

  - P = 25 TW (25x10<sup>12</sup> Watts)
    τ = 500-800 fsec (500-800x10<sup>-15</sup> seconds)
  - Maximum Intensity I = 6x10<sup>18</sup> Watts/cm<sup>2</sup>
  - Plasma Density n<sub>PLASMA</sub> = 1.5x10<sup>19</sup> cm<sup>-3</sup>
- Particles accelerated from background to 44 MeV in a distance of 0.035 cm. This corresponds to an accelerating field of

$$E_{\parallel} \approx 1 \text{ GV/cm}$$

 $\Rightarrow$  Particle energy ~ 100 MeV L ~ 1-2 mm; N ~ 10<sup>8</sup>.



λ Plasma Beat Wave Accelerator(PBWA)
 Two-frequencies, i.e., a train of pulses



λ Self Modulated Laser Wake Field Accelerator(SMLWFA)
 Raman forward scattering instability







#### UCLA Plasma Beat Wave Accelerator

CO<sub>2</sub> Laser, 2 wavelengths  $\lambda_1 = 10.29 \ \mu m$ ,  $\lambda_2 = 10.59 \ \mu m$ ,  $\lambda_p = 360 \ \mu m$ ,  $n_{PLASMA} = 10^{16} \ cm^{-3}$ ,  $\alpha_1 = 0.17$ ,  $\alpha_2 = 0.07$ ,  $\tau = 150$  psec.

Electrons injected at 2.1 MeV are accelerated to 30 MeV in a plasma length of 1 cm. This corresponds to an accelerating field  $E_{||} = 30 \text{ MeV/cm}$ 

Amplitude of relativistic plasma wave  $\varepsilon = 30\%$   $n_0 = \delta n/n_0$ 1stCsuckepssful demonstrationett., 70, 37 (1994) M. Everett et al. Nature, 368, 527 (1994)



#### **Overview of Beatwave Expt.**

Experiment consists of five major components ...



## **Experiment – Schematic**

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### **UCLA Experimental Results**

Electrons injected from 0.3 GHz rf LINAC ⇒ train of pulses, <10ps duration



- 1% or 10<sup>5</sup> electrons are accelerated in the diffraction length of ~1cm.
- 2→30 MeV Gradient of 3 GeV/m



The interplay leads to highly nonlinear selfmodulation

How can we understand this interplay and how does it effect selftrapped acceleration? <u>ct = 0.00 mm</u>

Evolution of a 5x10<sup>18</sup> W/cm<sup>2</sup> laser propagating through a 1.4x10<sup>18</sup> cm<sup>-3</sup> plasma











#### Evolution of Laser Intensity and Accelerating Electric Field





#### Santala et al., PRL, 2001.



FIG. 3. A typical electron spectrum (unfolded) measured by an on-axis electron spectrometer. Ponderomotive scalings (Ref. [12]) at  $10^{19}$  and  $10^{20}$  W/cm<sup>2</sup> are also shown.



# Laser Acceleration Results:

V. Malka *et al.*, LOA France, *Science*, 2002.

> •30TW, 35 fs laser •200 MeV energy gain in a 1mm gas jet (> 200 GeV/m) •Highly collimated beam:  $\epsilon$   $n \sim 1$  mm-mrad Fritzler et al., PRL92(2004)





#### Laser Wakefield Acceleration



Electron spectra for  $n_e = 2.5 \times 10^{19}$  cm<sup>-3</sup> (blue) and for  $n_e = 6 \times 10^{19}$  cm<sup>-3</sup> (red).

FWHM of the angular distribution of the electron beam.

Malka et al., 2002.





## The Bubble Regime

30 September 2004 International weekly journal of science

## Dream beam

The dawn of compact particle accelerators

Offshore tune ranches A threat to US waters?

The Earth's hum Sounds of air and sea

technology feature RNA interference

Protein folding Escape from the ribosome

Human ancestry One from all and all from one


#### Electron Acceleration by Intense Ultrashort Laser Pulse

0.06 n\_



Transmitted laser intensity as a function of wavelength in vacuum (blue) and in the plasma (red).

170 200 547 230 517 577 15 (mu) Y 0 B D  $E_x$  (m<sub>0</sub> c $\omega_p$ /e) -2 200 547 170 230 517 577 X (µm) X (µm)

Electron density cuts along the z=0plane and RPW electric field along the laser axis after propagation of 210 µm (A, B) and 560 µm (C, D).

Malka et al., 2002.



Courtesy J. Faure, LOA

Divergence FWHM = 6 mrad



### Results on e-beam : Energy distribution improvements



N.B. : color tables are different J. Faure, LOA



#### Mono-energetic spectra can be observed at higher power (∆E/E = 6 %)



E ~ 500 mJ, pulse duration ~ 40 fsec Focal spot ~ 25  $\mu$ m Density ~ 2 x 10<sup>19</sup> cm<sup>-3</sup>

Significant shot-to-shot fluctuations in a) energy spread b) peak energy

Careful control of laser and plasma conditions is necessary

Courtesy: K. Krushelnick, IC

# 85 MeV e-beam with %-level energy spread observed from laser accelerator



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#### Beam profile





(~40 Rayleigh lengths)

20

2 [µm]

256 cells

136 µm









# Beam loading of first bunch contributes to the generation of a second bunch



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### Laser Wakefield



Figure 1 Wakefield acceleration. a, In a plasma excited by a laser pulse, the wake potential rises until it steepens and breaks. Electrons from the plasma are caught in the 'whitewater' and surf the wave. b, The load of the electrons deforms the wake, stopping further trapping of electrons from the plasma. c, As the electrons surf to the bottom of the wake potential, they each arrive bearing a similar amount of energy.



#### **Production of a Monoenergetic Beam**

- 1. Excitation of wake (e.g., self-modulation of laser)
- 2. Onset of self-trapping (e.g., wavebreaking)
- 3. Termination of trapping (e.g., beam loading)
- 4. Acceleration
  - If > or < dephasing length: large energy spread
  - If ~ dephasing length: monoenergetic



Optimal choice of the plasma density: the smallest possible density For conditions 1 -4 to be fulfilled.



### **Proton Acceleration**



At present, experiments achieve 10s of MeV per nucleon. Future experiments aim to reach 100s of MeV to GeV.

#### Heavy Ion/Proton Generation by Ultraintense Lasers



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Clark *et al.*, 2000, PRL, <u>85</u>, 1654.

FIG. 4. Maximum ion energy as a function of  $I\lambda^2$ . Data from Refs. [7] and [10] are indicated by circles. Squares denote data from experiments discussed here.



2-D PIC Simulations of the Underdense Laser-plasma Interaction Show That Energies Higher Than U<sub>pond</sub> Can Be Produced.





Typical ponderomotive scaling predicts T<sub>hot</sub> ~ 2 MeV.

# However, slope temperature measured from simulations

gives  $T_{hot} \sim 7$  MeV for an intensity of  $10^{20}$  W/cm<sup>2</sup>, agreeing with experimental electron spectrometer

#### Laser-acceleration of *ions* from solid targets



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if target is heated \ efficient acceleration of heavy ions

[M. Hegelich et al., Phys. Rev. Lett. 89, 085002 (2002).]

### **Proton Energy Scaling**

FOCUS



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#### Recent Highlight: Record beam quality en < .004 mm-mrad!

#### 10x lower $\varepsilon$ than conventional ion injectors



Intense Relativistic Beams in Plasmas: PW/m<sup>2</sup> New Plasma Physics

- Wake generation/ particle acceleration
- Focusing
- Hosing
- "Collective Refraction"
- Radiation generation
- Ionization effects

- Compact accelerators
- Plasma lens/astro jets
- E-cloud instability/LHC
- Fast kickers
- Tunable light sources
- Beam prop. physics/X-ray lasers

#### Beam-Induced Plasma Wakefield Accelerator (PWA)

#### • P. Chen, J.M. Dawson et al.



# density fluctuations of a plasma (plasma wave)

Experiments: E157 Stanford Linear Accelerator SLAC/UCLA
⇒ 30 GeV driver, 10<sup>14</sup> cm<sup>-3</sup> plasma
Argonne National Lab (ANL/UCLA)
⇒ aiming for 1 GeV/m, 10<sup>12</sup> particles per
pulse, 100 Hz repetition rate,
emittance of 1mm.mrad



- Front of beam generates plasma wakefield
- Tail of beam particles interacts with wakefield
   Accelerated and decelerated beam.



**Cerenkov radiation** 



**Physical Principles of the Plasma Wakefield** Science & Technology Facilities Council Rutherford Appleton Laboratory

- Plasma is used to create a longitudinal accelerating gradient
- Space charge of drive beam displaces plasma electrons



Accelerator

- Plasma ions exert restoring force => Space charge oscillations
- Wake Phase Velocity = Beam Velocity (like wake on a boat)

• Wake amplitude  $\propto N_b/\sigma_z^2$  (for  $2\sigma_z \approx \lambda_p \propto \frac{1}{\sqrt{n_o}}$ )



### **Plasma Wakefield**

• The maximum plasma wake amplitude of an electron bunch scales as current over pulse length  $(\tau=2\sigma_z/c)$ 

$$eE_z \cong 1GeV/m\frac{I}{kAmp}\cdot\frac{4\,ps}{\tau}$$

• So, a 1 kAmp pulse, of duration 4 picoseconds is needed to generate a 1 GeV/m wake.



#### Electron Beam Generated Plasma Wave

 Similar to laser wakefield generation.



[S. Wilks *et al*. PPG. 1262]



#### PIC Simulation of Beam and Plasma Wakes





#### Bi-Gaussian shape $\sigma_z = 1.2 \text{ c/}\omega_p, \ n_b/n_p = 26$





Wedge shape w/ beam load beam length =  $6 c/\omega_{p}$ ,  $n_b/n_p = 8.4$ ,  $N_{drive} = 3x10^{10}$ ,  $N_{trailing} = 0.5x10^{10}$ 



#### PWFA Experiments @SLAC Share Common Apparatus

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#### Layout of SLAC-FFTB Plasma Wakefield Experiment

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#### E157 Experimental Layout – Plasma "Turbo-



<u>GOALS</u> 1 GeV/m accelerating gradient 10<sup>12</sup> particles/pulse 100 Hz Repetition rate Emittance < 1mm.mrad

<u>To date</u> positron beam accel. E ~ 50MV/m e-beam gains 350 MeV

Reference: R. Assmann *et al.*, Phys.Plasma (2000)



## Different spot sizes lead to different focusing forces and betatron





450

OTR images  $\approx 1$  m from plasma exit ( $\varepsilon_x \neq \varepsilon_v$ ) *n<sub>e</sub>*=0 2mm e<sup>-</sup> 200 250

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200 250 300 350 400



 Ideal Plasma Lens in Blow-Out Regime



 Plasma Lens with Aberrations



e+: halo formation from non uniform focusing

M.J. Hogan *et al.*, PRL, 2002; Also J. Ng et al., 2001 (**F/r~GG/cm**)



### **Betatron X-rays**

Joshi *et al.*, Physics of Plasmas, <u>9</u>, 1845, 2002.





S. Wang et al. Phys. Rev. Lett. Vol. 88 Num. 13

FIG. 8. (Color) (a) The estimated (triangles) and the measured (dots) x-ray energy in the 5–30 keV range as a function of plasma density. The solid line is a quadratic fit to the data. (b) Processed image produced on a fluorescent screen as recorded by a CCD camera showing the betatron x-rays produced by the plasma  $n_p = 2 \times 10^{13}$  cm<sup>-3</sup> (circle at the top) and a vertical stripe of remnant synchrotron radiation produced by a dipole bend magnet.



### Wakefield Afterburner

### High Energy Plasma Accelerator Energy Doubler for a Linear Collider



- Accelerating Gradients ~ 8 GeV/m
- SLAC 50 GeV beams boosted in a Plasma Wakefield Accelerator (PWFA) to ~ 100 GeV
- Possible to reach energies suitable for Higgs Boson detection



### Laser Wakefield Acceleration

#### LASER WAKEFIELD ACCELERATOR

A tabletop plasma accelerator consists of: High intensity laser beam; supersonic jet of helium gas (below). The beam produces a plasma in the gas jet, and the wakefield accelerates some of the dislodged electrons.

Electron beams (right). (a) Previous work demonstrated some electrons were accelerated to 100 MeV. However, all energies down to 0 MeV were also present and the beam was very wide. (b) By contrast, a newly discovered "bubble"regime produces a narrow beam of monoenergetic180 MeV electrons which are of much greaterpractical use.

Electron energy spectra





### **Plasma Afterburner**

#### PLASMA AFTERBURNER

Plasma wakefield acceleration was recently demonstrated in an experiment using a beam of the Stanford Linear Collider (SLC). The accelerator added 4 GeV of energy to an electron beam in just 10 centimeters—an energy gain that would require a 200-meter section of a conventional microwave accelerator.

In the experiment, an oven vaporized lithium pellets. An intense electron pulse (*red*) ionized the vapor to produce a plasma. The pulse blew out the plasma electrons (*blue*), which then set up a wakefield, or a charge disturbance, behind the pulse. Electrons located in that wakefield experienced powerful acceleration (*orange arrows*).



In the absence of the lithium (a), SLC's 30-GeV beam was quite monoenergetic (energy is plotted vertically). After passing through 10 centimeters of lithium plasma (b), most of the beam particles lost energy in generating the plasma wakefield (red tail). The wakefield accelerated a small number of electrons that happened to be at the back of the pulse to higher energy (blue region at top).





#### BOOSTING A CONVENTIONAL ACCELERATOR



**Boosting a Conventional Accelerator** 

A Plasma Afterburner (Energy Doubler) of Relevance to Future Colliders Could be Demonstrated at SLAC



S. Lee et al., Phys. Rev. STAB, 2001



#### High energy Plasma Science with an Ultrarelativistic electron beam

• Joshi *et al.*, Physics of Plasmas, <u>9</u>, 1845, 2002.



FIG. 1. The experimental set-up for studying beamplasma interaction effects with a 28.5 GeV electron or positron beam that was used in most of the experiments described in the text. The detection set-up for x-rays in the forward direction is not shown.

# Acceleration Of Electrons & Positrons: E-162

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Electrons



Some electrons gained 280 MeV (200 MeV/m)
Now going for 2 GeV at a rate of 10,000 MeV/m this month at SLAC



• Loss  $\approx$  50 MeV • Gain  $\approx$  75 MeV

B. Blue et al., Phys. Rev. Lett. 2003

R. Bingham, Nature, News and Views, 2003


#### **PWA Experiments at SLAC**



# E-164X Breaks GeV Barrier



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L≈10 cm,  $n_e \approx 2.55 \times 10^{17} \text{ cm}^{-3} \text{ N}_b \approx 1.8 \times 10^{10}$ 



Energy gain exceeds  $\approx$  4 GeV in 10 cm



# SLAC Plasma Wakefield Expt.



a,b) Density of electron pulse (brown) and plasma electrons (blue) at two different points in the plasma (12.3 and 81.9 cm). Scalloping features are the result of increasing focusing force.

c) Maximum energy reached after 85 cm. Saturation occurs due to the beam head spreading to the point that it can no longer ionize the lithium vapour.

- a) Energy spectrum of electrons in the 30-100 GeV range. Electrons reach 85 GeV ( $3x10^6$  e/GeV).
- b) Experimental (blue) and simulation (red)

Reference: Blumenfeld et al. Nature (2007)

# Plasma Density in Wakefield



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# Plasma Density in Wakefield



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#### Acceleration Of Electrons & Positrons: E-162

Electrons





• Loss  $\approx$  50 MeV • Gain  $\approx$  75 MeV

B. Blue et al., Phys. Rev. Lett. 2003

R. Bingham, Nature, News and Views, 2003

# Focusing of e-/e+ Beam

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OTR images  $\approx 1$  m from plasma exit ( $\varepsilon_x \neq \varepsilon_v$ )



*n*<sub>*a*</sub>≈10<sup>13</sup> cm<sup>-3</sup>



 Ideal Plasma Lens in Blow-Out Regime





 Plasma Lens with Aberrations



• e<sup>+</sup>: halo formation from non uniform focusing M.J. Hogan *et al.*, PRL, 2002; Also J. Ng et al., 2001





<u>Key Issue</u>	Experiment	Theory/Simulation
Acceler. Length $mm \rightarrow cm+$	Channel Formation	1-to-1 models parallel
	Plasma Sources	3-D hybrid
Beam Quality	Injectors	Beam Dynamics
$\Delta\gamma$	50 fs bunch	matching $\beta$
ε	50 <i>µm</i> spot	injection phase
Ν	Blowout regime	
Efficiency		Drive beam evolution

(new) load Shaped driver and

**Transformer Ratio** 

$$L_{dif} \simeq \pi L_R = \pi^2 w_0^2 / \lambda$$



3 Limits to Energy gain DW=eE<sub>z</sub>L<sub>acc</sub> (laser

driver)

order mm! (but overcome w/ channels or relativistic self-focusing)

Dephasing:

• Diffraction:



•Depletion:

$$\Delta W_{ch}[MeV] \sim 60 \left(\lambda_p / w_0\right)^2 P[TW]$$



# Plasma channel: structure for guiding and acceleration



- Hydro-dynamically formed plasma channel
  - On-axis axicon (C.G. Durfee and H. Milchberg, PRL 71 (1993) )
  - Ignitor-Heater (P. Volfbeyn et al., Phys. Plasmas 6 (1999))
  - Discharge assisted (E. Gaul et al., Appl. Phys. Lett. 77 (2000))
  - Cluster jets (Kim et al., PRL 90 (2003))
- Discharge ablated capillary discharges (Y. Ehrlich et al., PRL 77 (1996))
- Z-pinch discharge (T. Hosokai et al., Opt. Lett. 25 (2000))
- Hydrogen filled capillary discharge (D. Spence and S. M. Hooker, JOSA B (2000))
- Glass capillaries (B. Cross et al., IEEE Trans. PS 28(2000), Y. Kitagawa PRL (2004))



- Relativistic Plasma Wave Acceleration
- The problem is to generate large amplitude plasma wave travelling with a velocity close to the speed of light *c*
- 4 Approaches
  - 1. Plasma Beat Wave
  - 2. Laser Plasma Wakefield
  - 3. Self-Modulated Laser Wakefield (RFS)
  - 4. Electron Beam Plasma Wakefield

### We are on the path to the energy frontier



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## Requirements for High Energy Experiments

- Use Collider Parameters
- Luminosity =  $10^{31}$  cm<sup>-2</sup> sec<sup>-1</sup>
- Beam Energy ~ 1 TeV
- No. of particles per pulse ~ 10<sup>11</sup>
- Total Laser Energy (assuming 5% transfer efficiency) = 320 kJ/pulse
- Multiple staging required
- For a 100 stage accelerator requires 100 x 3 kJ lasers
- Power requirements :

 $P_{TOTAL} = 320 \text{ kJ x } f$  (pulse rate second<sup>-1</sup>)  $P_{TOTAL} = 1 \text{ GW Power}$ 



## Laser plasma accelerators and fast ignition both rely on ultraintense laser pulses (CPA)

#### $I \ge 10^{18} \text{ W/cm}^2$

• Lorentz factor for oscillating electron

 $\gamma \sim 2-10$ 

- For fast ignition a channel is formed guiding an intense laser pulse through the long scale length plasma to the compression core.
- Can the laser pulse propagate through the plasma and hit the core?
- Can instabilities like RFS absorb the energy?

⇒ relativistic electrons

 $\Rightarrow$  pions





- Laser Plasma Accelerators > 1 GeV
- 85 GeV achieved by SLAC e-beam Wakefield Experiment in 85cm.
- Numerous applications for 100 MeV-10 GeV beams – Medicine, Light Sources, Industry
- Ultra-High energies can be achieved by using a plasma afterburner on existing facilities – energies can be boosted up to 100 GeV



# Pulsars – The Crab Nebula

## **HST** optical picture



VIS X-RAY

The Crab Nebula is the remnant of a supernova explosion seen by the ancient Chinese in AD 1054. The Crab pulsar is almost unique in being detectable from radio right down to gamma-ray wavelengths.









#### **Solar Energetic Proton Spectrum**



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# **Plasma Acceleration**

In space plasmas most of the energy is stored in the magnetic fields.

- *e.g.* magnetic loops in the Sun, pile-up of magnetic flux at the bow shock, the front of comets and the tail of magnetospheres.
- Through a violent disruption effect like reconnection of magnetic fields energy is transferred to the ions

#### $\Rightarrow$ lon streams

Observed in the magnetotail and the Sun?

- The solar wind is also a source of free energy which can be used *i.e.* Interaction of the solar wind with the bow shock and with comets.
- Supernovæ remnants the shock catches up with the circumstellar material to create very hot plasma.
- Relativistic jets particles accelerated to relativistic energies. Also highly collimated.



#### Feedback mechanism for stimulated Raman scattering.



An EM wave ( $\omega_0$ ,  $\underline{k}_0$ ) scatters into two co-propagating sidebands ( $\omega_p \pm \omega_0$ ,  $\underline{k}_p \pm \underline{k}_0$ ) and a plasma wave ( $\omega_p$ ,  $\underline{k}_p$ )

Plasma wave

$$V_{ph} = \frac{\omega_p}{k_p} \approx C$$

The spatiotemporal growth rate is described as a gain

ľ

$$G = \left(\frac{1}{2\pi g}\right)^{\frac{1}{2}} e^g$$

$$g = \frac{a_0}{\sqrt{2}} \left( 1 + \frac{a_0^2}{2} \right) \left( \frac{\omega_p}{\omega_0} \right)^2 \frac{\omega_0}{c} \sqrt{x\varphi}$$

(Mori et al.1994)



- Fermi or stochastic acceleration of particles in turbulent fields is possible.
  - e.g. Pitch angle scattering from Alfvén waves

$$\lambda_A \cong \rho_i$$

Mechanism needs pre-accelerator

$$P_{initial} \cong m_i V_A$$

$$\omega = k_{||} V_A = k_{||} v_{||} + \mathbf{n} \Omega^*$$

-  $\Omega^*$  relativistic gyro frequency

#### Magneto acoustic turbulence + LH Turbulence

Ideal for protons etc., surfatron mechanism (similar to shock acceleration) Possible to produce relativistic electrons (Lembege+Dawson *Phys.Fluids*, **B**1, 1001 [1989])

#### Protons

$$E_{x} \cong \frac{m_{i}}{e} V_{A}^{2} \left(\frac{c}{\omega_{pe}}\right)^{-1} (M-1)^{3/2}$$
$$v_{\max} \cong \left(\frac{m_{i}}{m_{e}}\right)^{\frac{1}{2}} V_{A} (M-1)^{3/2}$$
$$E_{proton} \cong 1 \ GeV$$



**Steady State Solution** 

- The average gain is  $\Delta T = \pm v T/c$ , where T is the kinetic energy and v is the speed of the magnetic cloud.
- Positive gain for particle and cloud moving towards each other, negative gain for moving away.
- Statistically more head on collisions particles gain energy on average.

$$\frac{dT}{dt} = n \left(\frac{v}{c}\right)^2 T$$

*n* is number of collisions per unit time

• If particles are lost at a rate  $I/\tau$ , then obtain power law distribution

f number of particles in range dT then  $\partial f = \partial f$ 

$$\frac{\partial f}{\partial t} = -\frac{\partial}{\partial t} (\alpha T f) - \frac{f}{\alpha}$$
$$\alpha = \left(\frac{v}{c}\right)^2 n$$

• Steady state solution  $\implies$   $f = const. \times T^{-(1+1/\alpha \tau)}$