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Theory of cosmological seed magnetic fields.

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Theory of cosmological seed magnetic fields

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Abstract

A theory for the generation of seed magnetic field and plasma flow on cosmological scales driven by externally given baro-clinic vectors is presented. The Beltrami-like plasma fields can grow from zero values at initial time $t = 0$ from a non-equilibrium state. An exact analytical solution of the set of two fluid equations is obtained which is valid for both small and large plasma β-values. Weaknesses of previous models for seed magnetic field generation are also pointed out. The estimate of the magnitude of the galactic seed magnetic field turns out to be $10^{-14}G$ and may vary depending upon the scale lengths of the density and temperature gradients. The seed magnetic field may be amplified later by $\alpha\omega$-dynamo (or by some other mechanism) to the present observed values of $\sim (2-10)\mu G$. The theory has been applied to laser-induced plasmas as well and the estimate of the magnetic field's magnitude is in agreement with the experimentally observed values.

I. Introduction

In spite of a great deal of research work in this direction there is still not a convincing theory for the generation of seed magnetic field on cosmological and laboratory scales [1]. Extensive literature has appeared on galactic and intergalactic magnetic fields [2, 3, 4, 5]. The simplest justification can be that the generation of magnetic fields was a feature of initial conditions of the universe. However a more appealing hypothesis is that they are created by the physical mechanism operating after the Big Bang. The magnetic fields have been observed in almost all astronomical environments like the galaxies, intergalactic space, active galactic nuclei (AGN), galaxy clusters etc.

Most of the studies of magnetic field generation are based on single fluid magnetohydrodynamics (MHD). This model is very useful for the study of large scale magnetic phenomena. Therefore the MHD-based theoretical models and numerical simulations are very helpful to study the astrophysical magnetic fields [6, 7, 8].

In principal, the dynamo paradigm is incomplete because it is unable to explain the creation of initial magnetic field, the seed [1]. It is possible that the magnetic field is highly amplified by the $\alpha\omega$-dynamo effect later, but there must be some seed field already present for this action.

The magnetic fields of magnitudes of the order of 2-10 μG have been observed in many galaxies [9, 10]. The galactic magnetic fields have components parallel to the galactic disk planes as well as along the vertical directions [5, 11, 12]. Biermann battery effect [13] is the most widely studied mechanism for the generation of seed magnetic fields. Several modifications to the basic Biermann model and computer simulations have also been presented [14, 15, 16, 17]. More than a decade ago, it was shown that the seed magnetic field on galactic scale can be produced by the electron Biermann-type diffusion processes ([18]. Unlike the original Biermann process these mechanisms do not require rotation of the system.

The strong magnetic fields produced in laser-induced plasmas [19, 20] strengthen the argument that plasma dynamics can create large magnetic fields. Most of the theoretical models to explain the creation of magnetic fields in laser plasma systems are also based on electron baroclinic term ($\nabla T_e \times \nabla n_e$) (where n_e and T_e are the electron density and temperature, respectively). In these works, the ions are assumed to be stationary and electrons are treated to be inertia-less. The equations of the single fluid electron magnetohydrodynamics (EMHD) are used in these models [21, 22]. The weaknesses and contradictions of EMHD theory have also been discussed in a few research papers [23] with reference to magnetic field generation.Therefore we note that the seed magnetic field generation has not been explained in a true sense by MHD and EMHD models. The ideal MHD equations conserve the magnetic flux as "circulation", so that they cannot explain the generation of magnetic field. A theory for the self-excitation of transverse electromagnetic waves due to anisotropy of electron velocity distribution has also been presented many decades ago [24].

The magnetic fields in galaxies have coherent structures of the scales of tens of kilo parsec (kpc) winding around the galaxy which is larger than the seed from any given star or stellar binary system. These regular structures have superimposed on them shorter scale irregular structures as well [26]. These fields are believed to have been growing with the evolution of the universe during times of the order of billions of years$\sim 10^9 yrs$.

On the other hand, magnetic fields have been observed in laser-induced plasma experiments of the order of mega Gauss. In early experiments [19] the growth time of the field was $\tau \sim 10^{-9}S$ and the spatial scale size was of the order of fuel pallet diameter ($\sim \mu m$). In later experiments with intense lasers of short pulse duration $\tau \sim 10^{-12}S$, the growth times were correspondingly much shorter than a nanosecond. The recent experiments use very intense laser beams and produced plasmas are relativistic and degenerate. Such systems are not under consideration here. The classical laser-plasma dynamics are also discussed because they have a similarity with the galactic magnetic field problem, in our opinion. The previous theoretical models based on EMHD assume ions to be static and electrons to be inertial-less. These assumptions need to be analyzed very carefully. First we note that to assume ions to be static and electrons to be inertia-less (to ignore displacement current in Maxwell's equation) one needs the limits (i) $\omega_{pi} \ll |\partial_t| \ll \omega_{pe}$ (where $\omega_{pj} = \left(\frac{4\pi n_0 e^2}{m_j}\right)^{\frac{1}{2}}$ are the plasma oscillation frequency of the jth species, and $j = e, i$) and (ii) $|\partial_t| \ll c|\nabla|$. The assumption of inertia-less electrons also helps in using the steady state equation of motion for electrons. Note that for hydrogen plasma $\frac{\omega_{pi}}{\omega_{pe}} \sim \frac{1}{43}$. If we define a smallness parameter $\epsilon \sim O\left(\frac{1}{6.5}\right)$ or $\epsilon \sim \frac{1}{7}$, only then the limit (i) can be applicable and we can work on a time scale where ions are static and electrons are inertia-less. But even for this narrow window, the length of time say τ, for the generation of field may not satisfy the condition (i) easily. Secondly as soon as the magnetic field is created, the nonlinear Lorentz force term $\mathbf{v}_e \times \mathbf{B}$ comes into play and it's role must be taken into account.

In laser induced plasmas, it is assumed that the magnetic field is produced in time τ such that $\omega_{pe}^{-1} << \tau << \omega_{pi}^{-1}$ and hence $m_e \to 0$ and $m_i \to \infty$. The electron equation of motion is written as

$$0 = -en_e\mathbf{E} - \nabla p_e$$

The electron flow is ignored and $B = 0$ at $t = 0$ is assumed. The back reaction of the field on electron motion at times $0 < t$ is assumed to be negligible (which is not justified). The curl of the above equation gives,

$$\nabla \times \mathbf{E} = -\frac{1}{e}\nabla \times \left(\frac{\nabla p_e}{n_e}\right)$$

Then using Faraday's law one obtains,

$$\partial_t \mathbf{B} = \frac{c}{e}\left(\nabla T_e \times \nabla \ln n_e\right)$$

If the vectors ∇T_e and ∇n_e are assumed to be in xy-plane with $\frac{\nabla T_e}{T_e} = \kappa_T \mathbf{y} =$

constant and $\frac{\nabla n_e}{n_e} = \kappa_n \mathbf{x} =$ constant, then $\mathbf{B}$ is along z-axis as

$$\partial_t \mathbf{B} = \left(\frac{c}{e} \frac{T_e}{L_n L_T} \right) \mathbf{z}$$

where $L_n = \kappa_n^{-1}$ and $L_T = \kappa_T^{-1}$ are density and temperature gradient scale lengths, respectively. Integrating the above equation from 0 to τ, one gets,

$$B = \frac{c}{e} \frac{T_e}{L_n L_T} \tau$$

Let $c_s = \left(\frac{T_e}{m_i} \right)^{\frac{1}{2}}$ be the ion sound speed and assume $\tau = \frac{L_n}{c_s}$. Then for laser plasmas with $T_e \sim 1keV$, $n_0 \sim 10^{20} cm^{-3}$, $c_s \sim 3 \times 10^7 cms^{-1}$ and $L_T \sim 0.005cm$, one obtains $|B| \sim \frac{cT_e}{ec_s L_T} = 0.64 \times 10^6 G$ [20].

There are many contradictions in this model. For example $\mathbf{B}$ grows with time and hence it's back reaction on the electron fluid after initial times $0 < t$ does not remain negligible. It is also important to note that the time $\tau = \frac{L_n}{c_s}$ is the ion time scale, therefore to consider ions to be static is not reasonable. Moreover in laser-induced dense plasmas $\omega_{pi} \sim 10^{12} - 10^{13} radS^{-1}$ and pulse duration (specially in the early experiments) was of the order $\sim 10^{-9} S$. Even in later experiments $\tau \leq 10^{-12} S$ was the case while $\omega_{pi}^{-1} \leq 10^{-12} S^{-1}$.

Therefore ions should not be treated as static [23]. On galactic scales, the time is so long ($\sim 10^9$ years) and density gradient length is so large ($\sim$ tens of pc) that electrons seem to be in equilibrium and their motions on λ_e scale is also irrelevant. In this situation only the electron current can not be responsible for magnetic field generation. If $\partial_t \mathbf{B} \neq 0$ due to $(\nabla T_e \times \nabla n_e \neq 0)$, then ion dynamics due to $(\nabla T_i \times \nabla n_i)$-term should also be taken into account. Even if $n_e \sim n_i = n$ is assumed, the condition $T_i \neq T_e$ should hold otherwise plasma is in thermal equilibrium and significant currents can not exist. Moreover, the magnetic field can not be time-dependent if plasma is in equilibrium. Hence in equilibrium $\partial_t \mathbf{B} = 0$ as well as $\nabla T_i \times \nabla n = 0, (j = e, i)$ should hold.

The above discussion shows that Biermann battery formulism is very restrictive and can be valid only on a very limited time scale which may not be useful for finding a general mechanism for the creation of seed magnetic fields.

A few years ago, a theory for the generation of magnetic field and plasma flow based on two-fluid equations has been presented [25]. In this work it is assumed that at time $t = 0$, the plasma has $\nabla n \times \nabla T_j \neq 0$ (where $n_e \sim n_i \simeq n$ and T_j denote the temperature of jth species). The plasma evolves on a slow time scale due to given form of baro-clinic vectors $(\nabla \psi \times \nabla T_j)$ where $\psi = \ln n$. A particular solution of the two-fluid equations is obtained and it is shown that Beltrami-like field and flow can be generated by the external gradients when the plasma is in a non-equilibrium state with $T_e \neq T_i$. The terms $(\nabla \psi \times \nabla T_j)$ are considered to be the functions of only (x, y) coordinates and all the plasma fields are assumed to have the similar form as that of the source terms. The self-consistent fields are separated into growing and ambient (static) parts, and each part has different geometric character- the toroidal magnetic field and poloidal flow grow simultaneously, while the poloidal magnetic field and toroidal flow are static. Then the instabilities of some ambient inhomogeneous plasma (the stationary fields in the model) may create magnetic fields and flows with specific forms due to the baroclinic terms. A particular form of the baroclinic vectors is chosen such that all the nonlinear terms vanish and we succeed to obtain two linear equations which have an exact solution.

However, in this theoretical model there is a weakness that poloidal components of magnetic field and toroidal component of plasma flow are assumed to be time-independent. Therefore some static non-zero magnetic field and flow field have to prevail in the system.

In the present investigation [27], we modify the previous theory [25] to explain the creation of all the components of seed magnetic field and plasma flow from $t = 0$ due to externally given forms of baroclinic vectors ($\nabla n \times \nabla T_j \neq 0$). This theoretical model is applicable to very large plasma β-values as well. Furthermore the spatial and temporal scales of two fluid and one fluid plasma models will be discussed briefly in the context of magnetic field generation and the problems associated with these scales will be pointed out. It seems necessary to mention here that the inclusion of electron pressure term in the ideal MHD set of equations changes the scenario.

It is important to note that the present theory is different from Biermann battery concept and dynamo mechanism in the sense that it takes into account the dynamics of both ions and electrons. This work also points out that the Biermann battery is not necessary for the creation of magnetic field. Rather the field and plasma flow both are simultaneously created by the pressure gradients. However, the pioneering idea of Biermann is used to generate magnetic field and flow. That is the non-collinear density and temperature gradients are assumed to be the source of plasma evolution. The term $\nabla \times \left(\frac{\nabla p_e}{n_e}\right) \neq 0$ is one of the source terms for the generation of magnetic field and it modifies the scope of MHD and Hall magnetohydrodynamics (HMHD).

II. Mathematical model

We show that the set of nonlinear two-fluid equations (with inertia-less electrons) along with Maxwell's equations can have an analytical solution when the pressure gradients ∇p_e and ∇p_i have particular spatial structures. It is interesting to find out an analytical solution of a complicated system of equations to understand the basic physics of plasma evolution along with the creation of magnetic field and flow on the lines of Ref. [25].

The flows $\mathbf{v}_j$ and magnetic field $\mathbf{B}$ vectors will be defined in terms of a few scalar fields (ϕ, u, χ, h) which will be assumed to be correlated. These relationships will cause cancellations of all nonlinear terms in the set of equations. The density will be defined as $\ln n = \psi$, and it will become independent of time. All scalar fields (ϕ, u, χ, h) will be assumed to be functions of ψ. We shall choose particular spatial dependences of ∇n, ∇T_e and ∇T_i. Then all fields $\mathbf{v}_i$ and $\mathbf{B}$ will automatically become functions of externally given baroclinic vectors. We shall see that for such a choice of the form of gradients, the nonlinear equations for electrons and ions reduce to two linear equations and all the assumptions made on the way are satisfied. The flow $\mathbf{v}_i$ and mangnetic field $\mathbf{B}$ will appear to grow from $t = 0$ due to $(\nabla\psi \times \nabla T_j)$ vectors from a non-equilibrium state with $T_e \neq T_i$ and $n_e \simeq n_i = n$.

The equation of motion for inertial-less electrons is,

$$0 \simeq -en_e \left(\mathbf{E} + \frac{1}{c}\mathbf{v}_e \times \mathbf{B}\right) - \nabla p_e \tag{1}$$

The ion momentum conservation gives,

$$m_i n_i \left(\partial_t + \mathbf{v}_i . \nabla\right) \mathbf{v}_i = en_i \left(\mathbf{E} + \frac{1}{c}\mathbf{v}_i \times \mathbf{B}\right) - \nabla p_i \tag{2}$$

The continuity equation for jth species (j = e,i) can be written as,

$$\partial_t n_j + \nabla . \left(n_j \mathbf{v}_j\right) = 0 \tag{3}$$

Ampere's law becomes,

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J} \tag{4}$$

under the approximation $|\partial_t| \ll \omega_{pe}, |c\nabla|$.

We need Faraday's law as well i.e.

$$\nabla \times \mathbf{E} = -\frac{1}{c}\partial_t \mathbf{B} \tag{5}$$

Instead of Poisson equation, the quasi-neutrality $(n_e \sim n_i \sim n)$ is used which defines the current as $\mathbf{J} = en(\mathbf{v}_i - \mathbf{v}_e)$ and it yields

$$\mathbf{v}_e = \mathbf{v}_i - \frac{\mathbf{J}}{en} \tag{6}$$

The equations of state are defined as,

$$p_j = n_j T_j \tag{7}$$

Let both electron and ion fluids be incompressible $(\nabla . \mathbf{v}_j = 0)$ and densities be time-independent $(\partial_t n_j = 0)$. Then equation(3) demands,

$$\nabla\psi . \mathbf{v}_j = 0 \tag{8}$$

Let us assume $\mathbf{E} = -\nabla\Phi - \frac{1}{c}\partial_t \mathbf{A}$. After taking curl of equation (1), we use Eqs. (5) and (6) to obtain,

$$\partial_t \mathbf{B} + \nabla \times \left[\mathbf{B} \times (\mathbf{v}_i - \frac{c}{4\pi e}\frac{\nabla \times \mathbf{B}}{n})\right] = \frac{c}{e}(\nabla T_e \times \nabla\psi) \tag{9}$$

Then using $\mathbf{v} . \nabla\mathbf{v} = \frac{1}{2}\nabla v^2 - \mathbf{v} \times (\nabla \times \mathbf{v})$ and taking curl of Eq.(2), we obtain,

$$\partial_t(a\mathbf{B}+\nabla\times\mathbf{v}_i)-\nabla\times[\mathbf{v}_i\times(a\mathbf{B}+\nabla\times\mathbf{v}_i)]=-\frac{1}{m_i}(\nabla T_i\times\nabla\psi) \tag{10}$$

where $a=\frac{e}{m_i c}$.

Now we define the fluid fields as,

$$\mathbf{v}_i=(\nabla\phi\times\mathbf{z}+u\mathbf{z})f(t)=(\partial_y\phi,-\partial_x\phi,u)f \tag{11}$$

$$\mathbf{B}=(\nabla\chi\times\mathbf{z}+h\mathbf{z})f(t)=(\partial_y\chi,-\partial_x\chi,h)f \tag{12}$$

where the scalar fields $\phi, u, \chi$and h are functions of x, y and t only. Note that ϕ is a scalar field different from electrostatic potential Φ . Equation (8) demands,

$$\{\phi,\psi\}=0 \tag{13}$$

where $\{\phi,\psi\}=\partial_y\phi\partial_x\chi-\partial_x\phi\partial_y\chi$.

Using definitions (11) and (12) one finds,

$$\nabla\times[\mathbf{v}_i\times(a\mathbf{B}+\nabla\times\mathbf{v}_i)]$$

$$=f^2\left[-\partial_y\{\phi,(a\chi+u)\},\partial_x\{\phi,(a\chi+u)\},\{(ah-\nabla_\perp^2\phi),\phi\}+\{(a\chi+u),u\}\right] \tag{14}$$

If we assume

$$\{\phi,\chi\}=0,\{\phi,u\}=0,\{h,\phi\}=0,\{\nabla^2\phi,\phi\}=0 \tag{15}$$

then the nonlinear terms of Eq. (10) vanish, and it reduces to

$$\partial_t(a\mathbf{B}+\nabla\times\mathbf{v}_i)=-\frac{1}{m_i}(\nabla T_i\times\nabla\psi) \tag{16}$$

The condition $\{\nabla^2\phi,\phi\}=0$ may be satisfied if,

$$\nabla_\perp^2\phi=-\lambda\phi \tag{17}$$

where λ is an arbitrary constant.

Note that

$$\nabla\times(\mathbf{v}_e+\mathbf{B})=-\nabla\times\left[\mathbf{B}\times\left(\mathbf{v}_i-b\frac{\nabla\times\mathbf{B}}{n}\right)\right]$$

where $b=\frac{c}{4\pi e}$ and this term also disappears because due to (15) the following conditions also hold,

$$\left\{\nabla^2\chi,\chi\right\}=0;\{\psi,h\}=0;\{\psi,\chi\}=0 \tag{18}$$

Let us assume

$$\mathbf{B}=g\left(\nabla\times\mathbf{v}_i\right) \tag{19}$$

where g is an arbitrary constant. Equation (19) along with (11) and (12) gives,

$$\chi = gu \tag{20}$$

and

$$h = -g\nabla^2\phi \tag{21}$$

Then the equations (9) and (10), respectively, yield

$$\partial_t \mathbf{B} = \frac{c}{e}\left(\nabla T_e \times \nabla\psi\right) \tag{22}$$

and

$$\left(a + g^{-1}\right)\partial_t \mathbf{B} = -\frac{1}{m_i}\left(\nabla T_i \times \nabla\psi\right) \tag{23}$$

where $a + g^{-1} \neq 0$ must hold.

Equations (22) and (23) are similar to Eqs. (17) and (19) of [25] if these are normalized in the same way and if we assume $\partial_t\chi = 0, \partial_t u = 0$. In this case the x and y components of the baroclinic vectors $(\nabla T_j \times \nabla\psi)$ must vanish.

If the Hall term and pressure term both are ignored, then curl of Eq.(1) yields the simplest form of Ohm's law as,

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{v}_i \times \mathbf{B}) \tag{24}$$

The nonlinear term on rhs vanishes due to the forms of $\mathbf{v}_i$ and $\mathbf{B}$ assumed in Eqs. (11) and (12), respectively. This means that the magnetic field generation mechanism is killed when ∇p_e-term is ignored in Ohm's law. If Hall-term is ignored, then ideal MHD equations are modified by the inclusion of ∇p_e-term in Ohm's law. Hence a source of magnetic field generation is added to the MHD equaitons. Since the nonlinear terms vanish, therefore the present theory is applicable to both MHD and HMHD scales. The HMHD spatial scale is the ion skin depth $\lambda_i = \frac{c}{\omega_{pi}}$ and MHD can be used even for larger scales. It will be shown later that the present model is applicable in the limit of infinitely large plasma β-values as well. The collisions have been ignored here just for simplicity.

It is important to note that we must have $\partial_t(\varphi, \chi, u, h) \neq 0$ so that all the components of magnetic field $\mathbf{B}$ and flow $\mathbf{v}_i$ evolve from $t = 0$ [26]. If the nonlinear terms of Eqs.(9) and (10) do not vanish, then an analytical solution of these equations is not easy to find. A numerical simulation is required in this case.

There is another important point to be noticed here. In equations (22) and (23), the scales of $\mathbf{B}$ (and correspondingly $\mathbf{v}_i$ through (19)) should be the same as that of $(\nabla T_j \times \nabla\psi)$ which are the driving terms for plasma evolution. The short scale nonlinear phenomena have disappeared automatically under the conditions (15) and (18). Then $\mathbf{B}$ is directly proportion to $(\nabla T_j \times \nabla\psi)$.

A few comments on the scales of applicability of the present model seem to be necessary at this stage. In the previous work [25], the set of Eq.(1-7) were normalized following the procedure of Ref.[28] where the double curl Beltrami steady state was investigated. A few years ago [29], the analysis of two interesting scales of magneto-plasmas has been presented. In this work the Hall-term is treated as a singular perturbation in the two-fluid equations. A smallness parameter $\epsilon_i = \frac{\lambda_i}{L_0}$ is introduced where $\lambda_i = \frac{c}{\omega_{pi}}$ is the ion skin depth and L_0 is a characteristic length and may be equal to the size of the system. The HMHD has both the macroscopic and microscopic scales superimposed on each other. In the limit $\epsilon_i \to 0$, the system degenerates into standard MHD with a single relevant (the macroscopic) scale. In this work, variables have been normalized as follows,

$\mathbf{x} = L_0\tilde{\mathbf{x}}, \mathbf{B} = B_0\tilde{\mathbf{B}}, n = n_0\tilde{n}, t = \left(\frac{L_0}{v_a}\right)\tilde{t},$

$p_j = \left(\frac{B_0^2}{4\pi}\right)\tilde{p}_j, \mathbf{v}_j = v_A\tilde{\mathbf{v}}, \mathbf{E} = \left(\frac{v_A B_0}{c}\right)\tilde{\mathbf{E}}$, where $v_A = \frac{B_0}{\sqrt{4\pi n_0 m_i}}$ is the Alfven speed.

Under this normalization, the Eqs.(9) and (10) become, respectively, as

$$\partial_{\tilde{t}}\tilde{\mathbf{B}} + \tilde{\nabla} \times \left[\left(\tilde{\mathbf{B}} \times \tilde{\mathbf{v}}\right) - \epsilon_i \frac{1}{\tilde{n}}\tilde{\mathbf{B}} \times \left(\tilde{\nabla} \times \tilde{\mathbf{B}}\right)\right] = -\epsilon_i \left(\tilde{\nabla}\tilde{\psi} \times \tilde{\nabla}\tilde{T}_e\right) \tag{25}$$

and

$$\partial_{\tilde{t}}\left(\tilde{\mathbf{B}} + \epsilon_i\tilde{\nabla} \times \tilde{\mathbf{v}}_i\right) - \nabla \times \left[\tilde{\mathbf{v}} \times \left(\tilde{\mathbf{B}} + \epsilon_i \nabla \times \tilde{\mathbf{v}}\right)\right] = \epsilon_i \left(\tilde{\nabla}\tilde{\psi} \times \tilde{\nabla}\tilde{T}_i\right) \tag{26}$$

The superscript tilde ($\sim$) denotes the normalized quantities and operators. Note that if $L_0 = \lambda_i$ is assumed, then Eq.(25) and (26) are the same as Eq.(6) and (7) of Ref.[25] because $\frac{v_A}{L_0} = \frac{v_A}{\lambda_i} = \Omega_i$ which normalizes time in this case.

We have shown that if all plasma fields are assumed to be driven by baroclinic vectors and $\nabla\psi$ and ∇T_j have suitable spatial structures, then the Eqs. (25) and (26) will reduce, respectively, to

$$\partial_{\tilde{t}}\tilde{\mathbf{B}} = -\epsilon_i \left(\tilde{\nabla}\tilde{\psi} \times \tilde{\nabla}\tilde{T}_e\right) \tag{27}$$

and

$$\left(1 + \epsilon_i g^{-1}\right)\partial_{\tilde{t}}\tilde{\mathbf{B}} = \epsilon_i \left(\tilde{\nabla}\tilde{\psi} \times \tilde{\nabla}\tilde{T}_i\right) \tag{28}$$

where $\tilde{\psi} = \ln\tilde{n}$ and

$$\tilde{\mathbf{B}} = \tilde{g}\left(\tilde{\nabla} \times \tilde{\mathbf{V}}\right) \tag{29}$$

has been assumed with $\tilde{g}$ to be an arbitrary constant.

The theoretical studies of Refs. [28, 29] do not discuss any limit on plasma $\beta = c_s/v_A$. Since they deal with steady state, therefore we think that they are more relevant for the case $\beta < 1$ otherwise plasma will be expanding. The expansion rate may be assumed to be constant. However, we do not discuss the steady state problem here.

For an interesting comparison with the case of evolving plasma with $1 < \beta$, we analyze the two fluid equations using another normalization scheme 2 (say) and let the previous one be called as scheme 1.

Since we are interested in the seed magmatic field generation by plasma dynamics and assume $\mathbf{B} = 0$ at $t = 0$, therefore it is preferable to normalize the pressures with some arbitrary pressure $p_0 = n_0 T_0$. Let us use the following normalization,

$\mathbf{x} = L_0 \tilde{\mathbf{x}}, \mathbf{B} = B_0 \tilde{\mathbf{B}}, n = n_0 \tilde{n}, t = \frac{L_0}{c_s} \tilde{t},$

$p_j = \frac{p_j^2}{n_0 T_0}, \mathbf{v}_j = c_s \tilde{\mathbf{v}}_j, \mathbf{E} = \frac{c_s B_0}{c} \tilde{\mathbf{E}}$

Then Eqs.(25) and (26) can be written, respectively, as

$$\partial_{\tilde{t}} \tilde{\mathbf{B}} + \tilde{\nabla} \times \left[\left(\tilde{\mathbf{B}} \times \tilde{\mathbf{v}} \right) - \frac{1}{\tilde{n}} \frac{\rho_s}{L_0} \frac{1}{\beta} \tilde{\mathbf{B}} \times \left(\tilde{\nabla} \times \tilde{\mathbf{B}} \right) \right] = -\frac{\rho_s}{L_0} \left(\tilde{\nabla} \tilde{\psi} \times \tilde{\nabla} \tilde{T}_e \right) \tag{30}$$

and

$$\partial_{\tilde{t}} \left(\tilde{\mathbf{B}} + \frac{\rho_s}{L_0} \tilde{\nabla} \times \tilde{\mathbf{v}} \right) - \tilde{\nabla} \times \left[\left(\tilde{\mathbf{v}} \times \tilde{\mathbf{B}} \right) + \frac{\rho_s}{L_0} \tilde{\mathbf{v}} \times \left(\tilde{\nabla} \times \tilde{\mathbf{v}} \right) \right] = \frac{\rho_s}{L_0} \left(\tilde{\nabla} \tilde{\psi} \times \tilde{\nabla} \tilde{T}_i \right) \tag{31}$$

where$\rho_s = \frac{c_s}{\Omega_i}$ and $\rho_s \to \infty$ as $t \to 0$ with $\mathbf{B} = 0$.

Note that $\rho_s^2 / \lambda_i^2 = \frac{c_s^2}{v_A^2} = \beta$ and hence for $\beta \lesssim 1$, the scheme 1 is suitable and for $1 < \beta$, the scheme 2 is useful, in our opinion.

If $1 < \beta$ is assumed, then we may consider two cases,a) $1 < \beta \lesssim m_i/m_e$ and b) $\frac{m_i}{m_e} < \beta$.

Let $\rho_s \sim L_0$, and $\epsilon = \frac{1}{\beta}$ in case a, then the Hall-term again becomes a singular perturbation defining a microscopic scale. In the limit $1 \ll \beta$, the Hall term becomes negligibly small and the system reduces to MHD case having only the macroscopic scale.

Let us use ISM parameters $T_e \sim 1kev$ and $n_0 \sim 1cm^{-3}$ with $\lambda_i \sim 1.87 \times 10^4 cm$ assuming Hydrogen plasma. We shall present an estimate of the seed field to be $10^{-14}G$ in Sec. IV. Previous studies have predicted the galactic seed field generated in $\tau = 10^9 yrs$ as $[B] \sim \left(10^{-15} - 10^{-17}\right) Gauss$ [18] and these values are still flexible. So, we choose $[B] \sim 10^{-14}G$ which predicts $\rho_s \sim 10^{-2}pc$ (parsec) and it is not negligibly small compared to the characteristic length $\left(L_0 \sim 1pc \simeq 10^{18}cm\right)$ in this case while $\epsilon_i = \frac{\lambda_i}{L_0} \sim 10^{-14}$, $\beta = 0.2 \times 10^{18}$ and $\Omega_i \sim 10^{-11} radS^{-1}$ at the peak value of seed magnetic field generated in time $\tau \sim 10^9 yrs$ after the Big Bang. It may be mentioned here that we need an estimate for ∇T_e to evaluate $|B|$. Here a particular value $T_e \sim 1KeV$ has been used just for an analysis of the scales. Since the gradient scale lengths of density and temperature are of the order of $L_0 \sim 1pc$ (may vary from 1 to $10^3 pc$, for example), therefore the normalization scheme 2 is relevant. We shall choose density gradient scale length $\mu_1^{-1} \sim L_n \sim 10^3 pc$ in Sec. IV, then the time is normalized to the factor $\frac{L_n}{c_s} \sim \frac{1}{30} \times 10^9 yrs$ which is reasonable because the time of evolution is assumed to be $\tau \sim 10^9 yrs$.

Now we look at the laser-induced plasmas. Let us choose Eq. (30) to look into the scales. When non-linear terms vanish it becomes,

$$\partial_{\tilde{t}} \tilde{\mathbf{B}} = -\frac{\rho_s}{L_0} \left(\tilde{\nabla} \tilde{\psi} \times \tilde{\nabla} \tilde{T}_e \right) \tag{32}$$

Let us assume density gradient along x-axis and temperature gradient along y-axis. If the scale lengths of density and temperature gradients are L_n and L_{T} respectively and $\frac{T_e}{T_0} \sim 1$, then integrating Eq. (32) for $t = 0 \rightarrow \tau$, we can write it as,

$$\mathbf{B} = -\left\{ \frac{c}{e} \left(\frac{T_e}{L_n L_T} \right) \tau \right\} \hat{\mathbf{z}} \tag{33}$$

Note that Eq. (33) is the same as Eq. (5-79) of Ref. [20]. For the laser plasma with $T_e \sim 1KeV, n_0 \sim 10^{20} cm^{-3}, c_s \sim 3 \times 10^7 cmS^{-1}$ and $L_n \sim L_T \sim 0.005cm$, one obtains $|\mathbf{B}| \simeq 0.6 \times 10^6 G$ [20] where $\tau \simeq \frac{L_n}{c_s}$ has been used. We have already explained that Eq. (33) alone must not be used to determine $|\mathbf{B}|$ .But here we use this equation just to understand the scales of applicability. For this plasma we find $\lambda_i \sim 2.25 \times 10^{-4} cm, \epsilon_i \sim 5.6 \times 10^{-2}$ and $\tau = \frac{L_n}{c_s} \sim 1.29 \times 10^{-10} S$ which is smaller than $10^{-9} S$ (the laser pulse duration time in the initial experiments). Since $\frac{m_e}{m_i} < \epsilon_i < 1$, therefore in this case the Hall-Term is a singular perturbation. Our model is applicable to HMHD scale and we may use normalization scheme 1. Note that $1 < \beta \sim 62$ for this plasma.

Important point to note is that in the case $\frac{1}{\beta} \rightarrow 0$ the Hall-term is vanishingly small and the model is applicable to MHD scales. But for $L_0 \sim \lambda_i$, our model is applicable to HMHD scale. It may also be mentioned here that we have used the peak value of seed field to estimate ρ_s in case of ISM. But it can be even larger than $10^{-2} pc$ corresponding to smaller values of B.

Our main aim is to predict the generation of 'seed' magnetic field. Therefore, it seems better to use the equations in physical units without normalization to keep the option open for application to different systems.

III. Seed magnetic field and flow

Let us assume

$$\psi = \psi_0 e^{\mu_1 x} \cos \mu_2 y \tag{34}$$

$$T_e = T_{00e} + T_{0e}^{'} (y - z) f(t) \tag{35}$$

and

$$T_i = T_{00i} + T_{0i}^{'} (y - z) f(t) \tag{36}$$

Here $\mu_1, \mu_2, T_{00j}, T_{0j}^{'}$ are constants whereas T_{0j} are in units of eV and $T_{0j}^{'}$ denote the temperature gradients. Note that Eq. (17) is satisfied with $\lambda = (\mu_2^2 - \mu_1^2)$.

Then

$$\nabla T_e \times \nabla \psi = f(t) T_{0e}^{'} \psi_0 e^{\mu_1 x} (-\mu_2 \sin \mu_2 y, -\mu_1 \cos \mu_2 y, -\mu_1 \cos \mu_2 y) \tag{37}$$

and

$$\nabla T_i \times \nabla \psi = f(t) T_{0i}^{'} \psi_0 e^{\mu_1 x} (-\mu_2 \sin \mu_2 y, -\mu_1 \cos \mu_2 y, -\mu_1 \cos \mu_2 y) \tag{38}$$

Now we shall see that all the fields ϕ, ψ, χ and h have the forms similar to ψ.

If

$$f(t) = e^{\gamma t} \tag{39}$$

is assumed (where γ is a constant), then equation(22) gives,

$$\chi = \frac{c}{e} \frac{T_{0e}^{'}}{\gamma} \psi \tag{40}$$

and

$$h = -\frac{c}{e} \frac{\mu_1 T_{0e}^{'}}{\gamma} \psi \tag{41}$$

where $\gamma \neq 0$. Similarly Eq.(23) yields,

$$\chi = -\frac{T_{0i}^{'}}{m_i \gamma (a + g^{-1})} \psi \tag{42}$$

and

$$h = \frac{T_{0i}^{'} \mu_1}{m_i \gamma (a + g^{-1})} \psi \tag{43}$$

The relations (40-43) require,

$$T_{0e}^{'} = -\frac{T_{0i}^{'}}{(a + g^{-1})} \tag{44}$$

The fields u and ϕ are related with χ and h through Eqs.(20) and (21). Therefore all the components of $\mathbf{B}$ and $\mathbf{v}_i$ become functions of ϕ which has a linear relationship with ψ. Therefore the conditions (15) and (18) are satisfied along with equation(13). The components of the fields $\mathbf{B}$ and $\mathbf{v}_j$ have the structural forms either of type $F_1 = e^{\mu_1 x} \cos \mu_2 y$ or $F_2 = e^{\mu_1 x} \sin \mu_2 y$. The forms of these functions in xy-plane are shown in Fig. 1 and Fig. 2, respectively, for $\mu_1 x = -1 \rightarrow 0$ and $\mu_2 y = 0 \rightarrow 2\pi$.

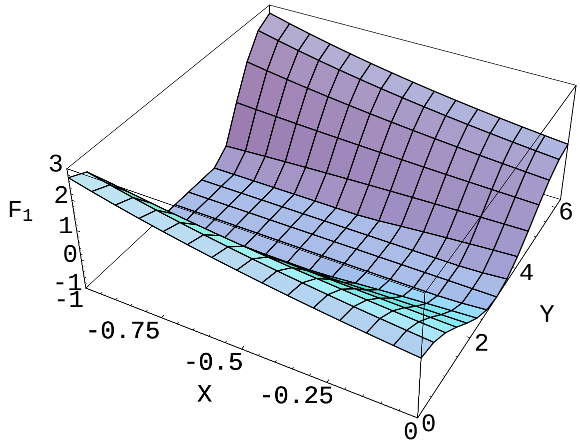

Fig.1. The function $F_1(x, y)$ is plotted for $\mu_1 x = 0 \to -1$ and $0 \leqslant \mu_2 y \leqslant 2\pi$.

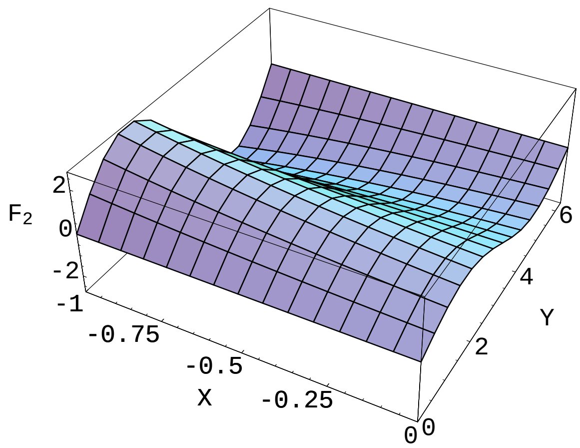

Fig. 2. The function $F_2(x, y)$ is plotted for $\mu_1 x = 0 \to -1$ and $0 \leqslant \mu_2 y \leqslant 2\pi$.

IV. Applications

We apply the theory to both cosmological and laser-induced plasmas. We may consider any one of the equations (22) and (23) to estimate $|B|$. Let us choose Eq.(22) which for $f(t) = 1$ becomes,

$$\partial_t \mathbf{B} = \frac{cT_{0e}^{'}}{e} \psi_0 e^{\mu_1(x)} \left(-\mu_2 \sin \mu_2 y, -\mu_1 \cos \mu_2 y, -\mu_1 \cos \mu_2 y\right) \tag{45}$$

Integrating this equation from $t = 0$ to $t = \tau$, we obtain,

$$\mathbf{B} = \frac{cT_{0e}^{'}}{e} \psi_0 e^{\mu_1(x)} \left(-\mu_2 \sin \mu_2 y, -\mu_1 \cos \mu_2 y, -\mu_1 \cos \mu_2 y\right) \tau \tag{46}$$

which gives

$$|B| \simeq \frac{1}{\sqrt{2}} \frac{cT_{0e}^{'}}{e} \psi_0 e^{\mu_1 x} \left(\mu_2^2 + 2\mu_1^2\right)^{\frac{1}{2}} \tau \tag{47}$$

Let us try to find out the origin of galactic magnetic fields. We assume that the seed magnetic field is very weak and it is amplified later on by $\alpha\omega$ dynamo (or by some other mechanism). Here our aim is to generate the magnetic field and flow by the given baroclinic vector.

Let (y_1, z_1) be the point inside ISM but the centre of galaxy is far away point (0,0) on yz plane and it is ignored assuming that the conditions are very different there. Let (y_2, z_2) be the edge point region under consideration and $y_1 \neq z_1, (y - z) \neq 0$ but $y_2 - y_1 = z_2 - z_1 = d$ so that the temperature gradient along y and z directions is equal in magnitude. The density is assumed to decrease exponentially away from x_1 along x-axis, therefore we have $\mu_1 < 0$. Let $x_2 - x_1 = x_0$ such that $x_0\mu_1 = -1$ and $\psi_0 = 3$ so that $\psi_0 e^{-1}$ at the edge of the region is not less than 1 and hence density remains positive. Then $T_{0e}^{'} = -\frac{k_\beta 10^6}{100pc}$ is assumed.

Let $x_0 = 10d = 10^3 pc$ and $\mu_2 d = 2\pi$. Then $(\mu_2^2 + 2\mu_1^2)^{\frac{1}{2}} = \frac{0.062}{(pc)}$ and hence, we have $|B|$ in Guass as,

$$|B| \sim \left[(1.5)\, 10^{-23}\right] \tau \tag{48}$$

In Eq.(48), τ is in years. If $\tau = 10^9$ yrs is assumed, we obtain the seed magnetic field to be

$$|B| = 10^{-14} G \tag{49}$$

which is 10^3 times larger than the previously estimated field [18].

This increase in the estimated magnitude is due to steeper temperature gradients assumed in the calculation which is necessary to fulfill the condition $0 < \lambda$.

This theoretical model can be applied to laser-induced plasma experiments as well. We may define,

$$\kappa_T = \left| \frac{1}{T_e} \frac{dT_e}{dy} \right| = \left| \frac{1}{T_e} \frac{dT_e}{dz} \right|$$

and hence $T_{0e}^{'} = T_{0e}\kappa_T$. Let $\mu_1 \simeq \mu_2 = \mu$ and $(\mu_2^2 + 2\mu_1^2)^{\frac{1}{2}} \simeq \sqrt{3}\mu$ where $\mu = \kappa_n = \left| \frac{1}{n_0)} \frac{dn}{dx} \right|$ and $\mu_1 x_0 = -1$. Then we can express equation (47) as,

$$|B| \sim \left\{ \sqrt{\frac{3}{2}} \psi_0 \left(\frac{cT_e}{e} \right) \kappa_n \kappa_T e^{-1} \right\} \tau \tag{50}$$

Let $L_n = \frac{1}{\kappa_n}$ and $L_T = \frac{1}{\kappa_T}$ be the density and temperature scale lengths along x-axis and along y and z axes, respectively. Then the above expression can be written as,

$$|B| \sim b_0 \left(\frac{cT_{0e}}{e} \frac{1}{L_n L_T} \right) \tau \tag{51}$$

where $b_0 = \sqrt{\frac{3}{2}} \psi_0 e^{-1}$. Note that equation (51) is the same as equation (5.79) of Ref. [20] and we take the same example of laser-induced plasma as chosen in this reference i.e. $T_{e0} \sim 10^3 eV, c_s \sim 3 \times 10^7 cmS^{-1}, \tau = \frac{L_n}{c_s}$. We assume $\psi_0 = 3$ (to have density positive at the edge region as well) and find $b_0 \simeq 1.36$. Therefore

$$|B_0| \sim (8.7) \times 10^5 G \tag{52}$$

Note that if $b_0 = 1$, we have exactly the same value of $|B| \sim 0.64 \times 10^6 G$ as estimated in Ref. [20].

But the important point to note is that, the term $\mathbf{v}_e \times \mathbf{B}$ has not been dropped and ion dynamics has also been taken into account because $\tau = \frac{L_n}{c_s} \gg \omega_{pi}^{-1}$.

V. Discussion

A theory for the generation of seed magnetic fields on cosmological scales has been presented which can be applicable to laser-induced plasmas as well. The present theoretical model is actually a modified form of the previous work [25]. In Ref. [25], one has to assume poloidal components of the magnetic field and toroidal components of the flow to be static. But in the present theoretical model[27], all plasma fields can grow from their zero values at $t = 0$ due to the source terms $(\nabla\psi \times \nabla T_j)$. An exact analytical solution of a set of highly nonlinear two-fluid partial differential equations of a hot inhomogeneous plasma has been obtained. The plasma is assumed to be produced with electron and ion pressure gradients in a state of non-equilibrium $(T_i \neq T_e)$. The baroclinic vectors $(\nabla\psi \times \nabla T_e)$ and $(\nabla\psi \times \nabla T_i)$ then become the source for plasma evolution creating it's flow $\mathbf{v}_i$ and magnetic field $\mathbf{B}$. These vectors are expressed in terms of scalar fields (ϕ, u, χ, h) in Eqs. (11) and (12). A relationship between $\mathbf{v}_i$ and $\mathbf{B}$ is assumed to be given by Eq. (19). The plasma fields are assumed to satisfy the conditions of Eqs. (15) and (18). Then the forms of $\nabla\psi$ and ∇T_j are chosen in Eqs. (34), (35) and (36) in such a way that $(\nabla\psi \times \nabla T_j)$ vectors become proportional to ψ. Then all other fields can also be expressed in terms of ψ in Eqs. (40-43) and hence all the assumed conditions in Eqs. (13), (15) and (18) are satisfied.

This is a particular solution of the plasma equations. But it shows how the plasma can evolve form $t = 0$ with $B = 0$ generating it's magnetic field and flow. It has been shown that the present theoretical model is applicable to both HMHD and MHD scales.

Since $(\nabla\psi \times \nabla T_j)$ are the generating forces therefore the scales of $\mathbf{v}_i$ and $\mathbf{B}$ should be of the same order of magnitude as that of $\nabla\psi$ and ∇T_j as is clear from equations (22) and (23). The short scale phenomena superimposed upon such fields do not participate in the creation of magnetic field and flow on very large scales. Several effects like dissipation and compressibility have been ignored.

It has not been discussed here that how the particular spatial structures of density and temperature gradients are produced. But these simplifications and assumptions have been made to present an exact solution of the complex partial differential equations. The estimates of the magnetic fields at both cosmological and laboratory scales are very close to observations. It is necessary to explain why our estimate of the 'seed' magnetic field is 10^3-times larger than that made in Ref. [18]. The reason for this larger estimate of seed field is that we assume the density gradient scale length (μ_1) about 10 -times larger and temperature gradient scale length (μ_2) about 10^2-times smaller than that used in Ref. [18]. These parameters can be varied because the interstellar cloud can have regions of different magnitudes of the scales of gradients, in our opinion.

It must be pointed out here that our assumption of equal magnitudes of temperature gradients along +ve y-axis and (-ve) z-axis forces us to choose $y_0 = |y_2 - y_1| = |z_2 - z_1| = z_0$. Then the requirement $y_0\mu_2 = 2\pi$ along with $|\mu_1| < |\mu_2|$ (to have $\lambda = \mu_2^2 - \mu_1^2 > 0$) compels us to assume steeper temperature gradients along both the axes compared to Ref. [18]. However, the future observations about the gradients can be useful to make the estimate for B-field magnitude more exact.

It is important to point out that the seed magnetic field investigated in Ref. [18] has only non-zero component along z-axis while $B_x = B_y = 0$ is assumed because $(\nabla T_e \times \nabla n_e)$ is along z-axis. In our theoretical model, the baro-clinic vectors $(\nabla T_j \times \nabla \psi)$ have all the three components to be non-zero and hence the galactic seed magnetic field has three components which is in agreement with the observations [11, 12].

The different values of the magnitudes of the density and temperature gradients can also be used for the generation of the magnetic and flow fields with $y_0 \neq z_0$ and $\mu_1 < \mu_2$. But in these cases, analytical solution of the set of nonlinear partial differential equations may not be found.

The present theory can have wider applications to many astrophysical situations. However, the numerical simulations will be required, if plasma is considered to be non ideal and if the forms of ∇n and ∇T_j are chosen to be different from the ones used in the present analytical calculations.

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