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Electro-Acoustic Solitary And Shock Waves In Dusty Plasmas

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ELECTRO-ACOUSTIC SOLITARY AND SHOCK WAVES IN DUSTY PLASMAS

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OUTLINE

- ❑ Introduction
- ❑ Static Dust: DIA Waves
  - ❖ DIA Solitary Waves
  - ❖ DIA Shock Waves
- ❑ Mobile Dust: DA Waves
  - ❖ DA Solitary Waves
  - ❖ DA Shock Waves
- ❑ Summary

❑ Introduction

- **How are new waves generated in DPs?**

  What are DIA and DA waves?

❖ **We consider a water wave in a pond:**

- **A very small fish (even moving very fast) cannot significantly modify it.**
- **But, if in front of the wave an elephant just stands or slightly moves up and down. The wave can be significantly modified or a new wave can be generated associated with the slight movement of this elephant.**

1. **The inequality $n_{i0}/n_{e0} >> 1$ (caused by the static negatively charged dust) introduces a new type of ion-acoustic waves (compression and rarefaction of ion density) known as DIA waves (Shukla & Silin 1992).**

2. **The mobile charged dust introduce a very low frequency and long wavelength waves (compression and rarefaction of dust density) known as DA waves (Shukla 1989).**

1. Static Dust: DIA Waves: $\frac{\omega}{k} = \sqrt{\left(\frac{n_{i0}}{n_{e0}}\right)\left(\frac{T_e}{m_i}\right)}$

- The compression and rarefaction of ion density in presence of static negatively charged dust which cause $n_{i0}/n_{e0} >> 1$.

- The restoring force comes from the electron thermal pressure and the inertia is provided by the ion mass.

- The phase speed of the DIA waves can be 10 --- 100 times higher than the ion-acoustic speed $C_i=(T_e/m_i)^{1/2}$ because of the factor n_{i0}/n_{e0} $(=1+Z_d n_{d0}/n_{e0})$ which can be 10^2 --- 10^4 for many space and laboratory dusty plasma situations.

- DIA waves has been theoretically predicted by **Shukla & Silin (1992)** and experimentally observed by **D'Angelo et al (1996)**.

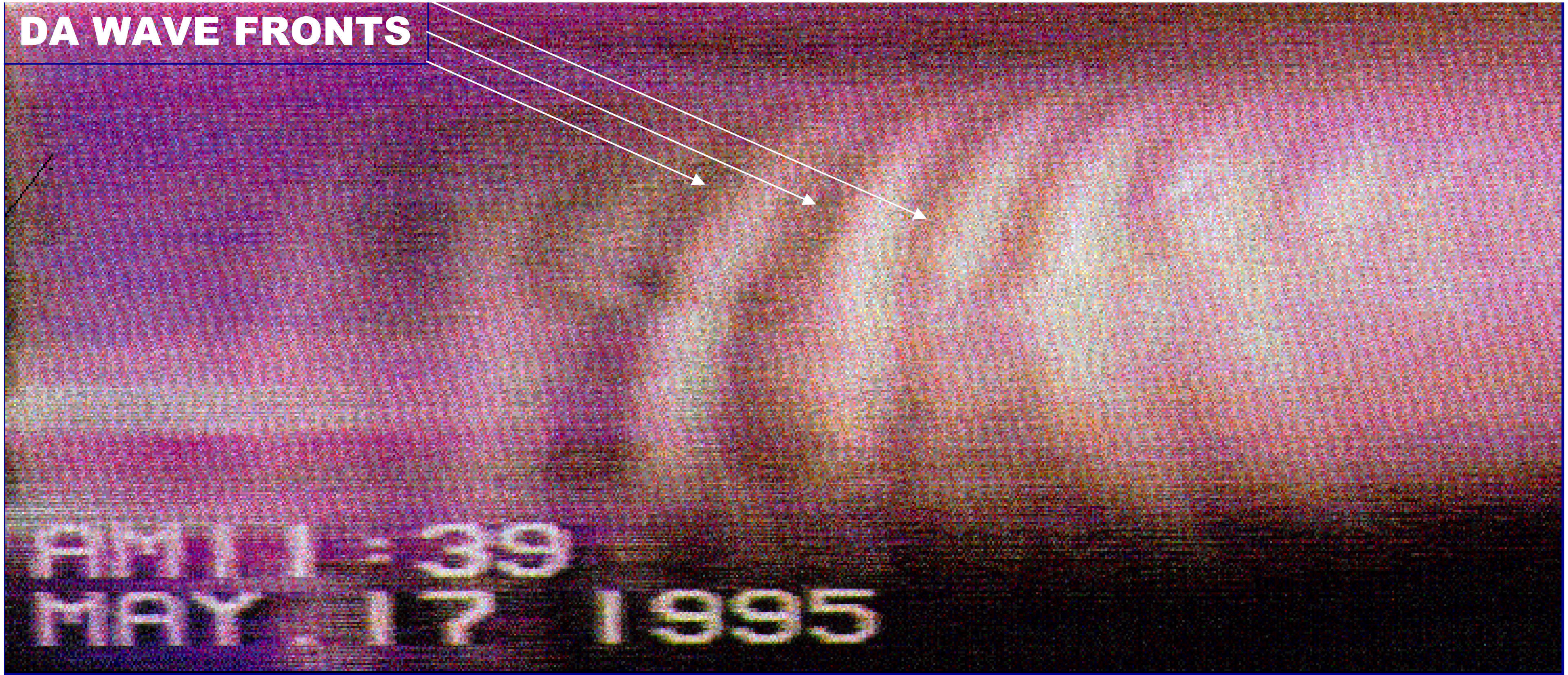
2. Mobile Dust: DA Waves: $\frac{\omega}{k} \approx \sqrt{(\frac{Z_d n_{d0}}{n_{i0}})(\frac{Z_d T_i}{m_d})}$

- The waves associated with the dynamics of dust, i.e., associated with the compression and rarefaction of dust density.

- The restoring force comes from electron and ion thermal pressures and the inertia is provided by dust mass. However, T_e and $k\lambda_D$ are not appeared in above dispersion relation, where $k\lambda_D \ll 1$ and $n_{e0}T_i/n_{i0}T_e \ll 1$ are assumed.

- A very low frequency and long wavelength wave mode, where f=10 - 100 Hz and λ=0.5 - 5.0cm.

- The DA waves are visible, even with naked eyes.



- The idea of DA waves was first presented by **Shukla** at Capri Meeting in 1989. Later, DA waves were theoretically predicted by **Rao, Yu & Shukla (1990)** and experimentally observed by **Barkan, Merlino & D`Angelo (1995).**

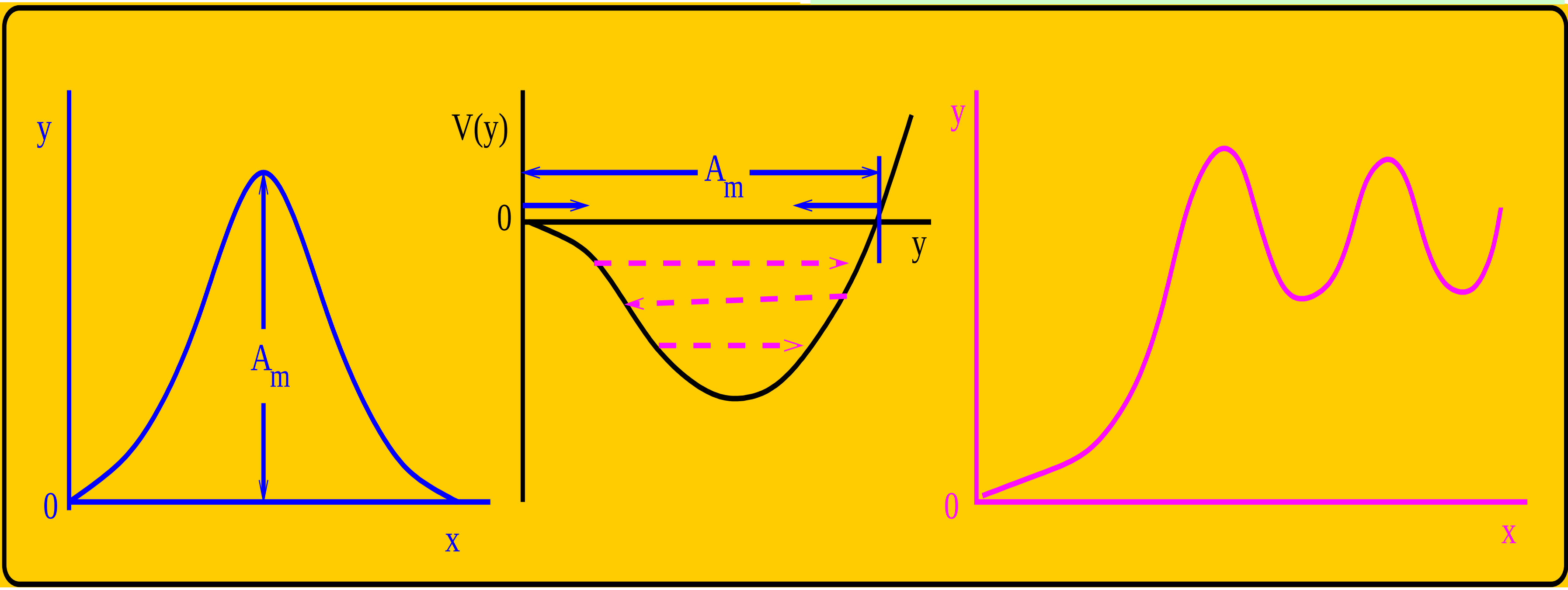
The aim of the present lecture is present the underlying physics of the basic features of solitary and shock structures associated with such DIA and DA waves.

➢ What are solitary waves? How do they form? How do they convert to shock-like structures?

- **Solitary waves** are hump/dip shaped nonlinear waves of permanent profile due to balance between nonlinearity and dispersion (without dissipation). However, when dissipation is important, **shock waves** are formed.

The formation of arbitrary amplitude stationary solitary and shock waves

$$\frac{1}{2}\left(\frac{dy}{dx}\right)^2 + V(y) = 0$$



- **Evolution equation for small amplitude solitary/shock waves: K-dV-Burgers equation (Karpman 1975):**

$$\frac{\partial y}{\partial t} + Ay\frac{\partial y}{\partial x} + B\frac{\partial^3 y}{\partial x^3} = C\frac{\partial^2 y}{\partial x^2}$$

The term containing B (C) is the dispersive (dissipative) term

When the dispersive term is much more important than the dissipative term: K-dV equation:

$$\frac{\partial y}{\partial t} + Ay\frac{\partial y}{\partial x} + B\frac{\partial^3 y}{\partial x^3} = 0$$

$$y = \left(\frac{3v_0}{A}\right) \operatorname{sech}^2\left[(x - v_0 t)\sqrt{\frac{v_0}{4B}}\right]$$

When the dissipative term is much more important than the dispersive term we have: Burgers equn:

$$\frac{\partial y}{\partial t} + Ay\frac{\partial y}{\partial x} = C\frac{\partial^2 y}{\partial x^2}$$

$$y = \left(\frac{v_0}{A}\right)\left\{1 - \tanh\left[(x - v_0 t)\frac{v_0}{2C}\right]\right\}$$

❑ Static Dust: DIA Waves

▪ Model:

We consider an unmagnetized DP with static dust [Shukla & Silin 1992]. The dynamics of 1D DIA waves:

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial z}(n_i u_i) = 0 \qquad (1)$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial z} = -\frac{\partial \phi}{\partial z} \qquad (2)$$

$$\frac{\partial^2 \phi}{\partial z^2} = \mu \exp(\phi) - n_i + (1-\mu) \qquad (3)$$

where n_i, u_i, , t, z are normalized by n_{i0}, C_i, T_e/e, $1/\omega_{pi}$, $\mu^{1/2}\lambda_{de}$, respectively, and $\mu = n_{e0}/n_{i0} = 1 - Z_d n_d / n_{i0}$.

❖ DIA SWs:

- **Small Amplitude:**

RPM [Washimi & Taniuti 1966] reduces (1)- (3) to a K-dV Eq:

$$\frac{\partial\phi}{\partial\tau} + a_s\phi\frac{\partial\phi}{\partial\zeta} + b_s\frac{\partial^3\phi}{\partial\zeta^3} = 0$$

where $\zeta=\varepsilon^{1/2}(z - v_0t)$, $\tau= \varepsilon^{3/2}t$, $a_s=(3-1/\mu)\ \mu^{1/2}/2$, and $b_s=(2\mu)^{-3/2}$.

The stationary SW solution of this K-dV Eq:

$$\phi = \phi_m \mathrm{sech}^2[(\zeta - u_0\tau)/\Delta_s]$$

where $_m=3u_0/a_s$ and $\Delta_s=(4b_s/u_0)^{1/2}$.

Since a_s is positive (negative) when $\mu>(<)1/3$, small amplitude DIA SWs with $>(<)0$ exist when $\mu>(<)1/3$.

- **Arbitrary Amplitude:**

SPA [Sagdeev 1966] reduces (1) - (3) to an energy integral:

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2 + V(\phi) = 0$$

where ξ=z - Mt and V( ) reads [Bharuthram & Shukla 1992]:

$$V(\phi) = \mu[1-\exp(\phi)]-(1-\mu)\phi+M^2\left(1-\sqrt{1-2\phi/M^2}\right)$$

It is clear: V( ) = dV( )/d = 0 at =0. So SW solution of this energy integral exists if $(d^2V/d\ ^2)_{=0}$< 0 and $(d^3V/d\ ^3\ _{=0}$>(<) 0 for >(<)0. The vanishing of quadratic term: M_c=$\mu^{-1/2}$. At M=M_c the cubic term of V( ) becomes (3-1/μ) $\mu^{1/2}$/2= a_s.

This means that arbitrary amplitude DIA SWs with >(<)0 exist when μ>(<)1/3.

• Effects of Non-Planar Geometry

The dynamics of DIA waves in planar (v=0; r=z) or non-planar [cylindrical (v=1) or spherical (v=2)] geometry:

$$\frac{\partial n_i}{\partial t} + \frac{1}{r^\nu}\frac{\partial}{\partial r}(r^\nu n_i u_i) = 0$$

$$\frac{\partial u_i}{\partial t} + u_i\frac{\partial u_i}{\partial r} = -\frac{\partial \phi}{\partial r}$$

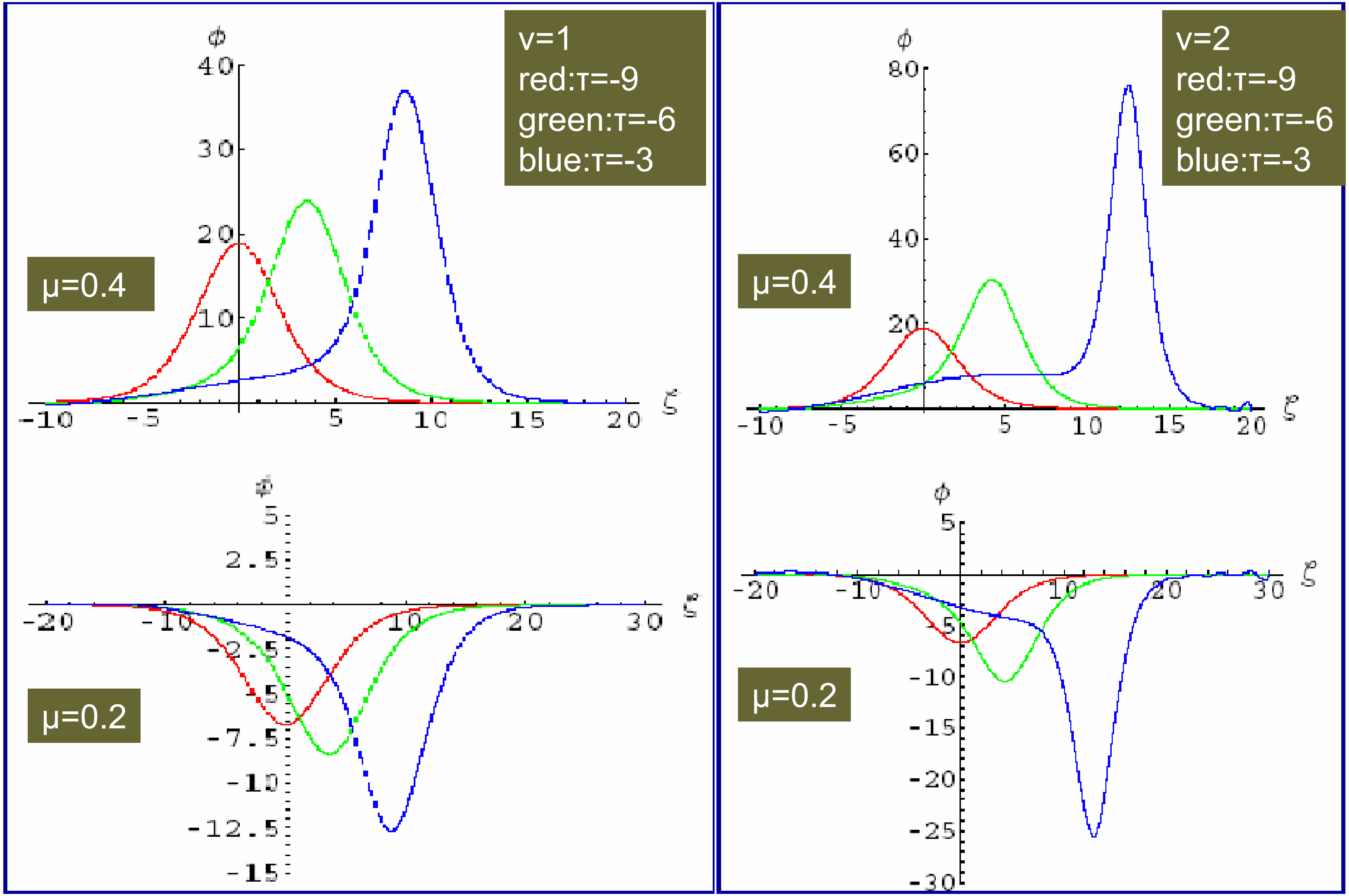
$$\frac{1}{r^\nu}\frac{\partial}{\partial r}\left(r^\nu \frac{\partial \phi}{\partial r}\right) = \mu\exp(\phi) - n_i + (1-\mu)$$

RPM [Maxon & Vieceli 1974] reduces these Eqs. to a mK-dV Eq:

$$\frac{\partial \phi}{\partial \tau} + \frac{\nu}{2\tau}\phi + a_s\phi\frac{\partial \phi}{\partial \zeta} + b_s\frac{\partial^3 \phi}{\partial \zeta^3} = 0$$

where $\zeta=-\varepsilon^{1/2}(r + v_0 t)$, $\tau=\varepsilon^{3/2}t$. The 2nd term arises due to the effects of non-planar geometry. We have numerically solved, and studied the effects of cylindrical (ν=1) and spherical (ν=2) geometries on the

geometries on time-dependent DIA SWs. The numerical results are as follows [Mamun & Shukla 2002]:



DIA Shock Waves

Theoretical Model:

We consider an unmagnetized, dissipative DP [Nakamura et al 1999; Shukla 2000] which can be described by (1), (3) and

$$\frac{\partial u_i}{\partial t} + u_i\frac{\partial u_i}{\partial z} = -\frac{\partial \phi}{\partial z} - \eta_i\frac{\partial^2 u_i}{\partial z^2} \quad (6)$$

RPM and $\eta_i=\varepsilon^{1/2}\eta_0$ reduce (1), (3) and (6) to a K-dV-Bergers Eq:

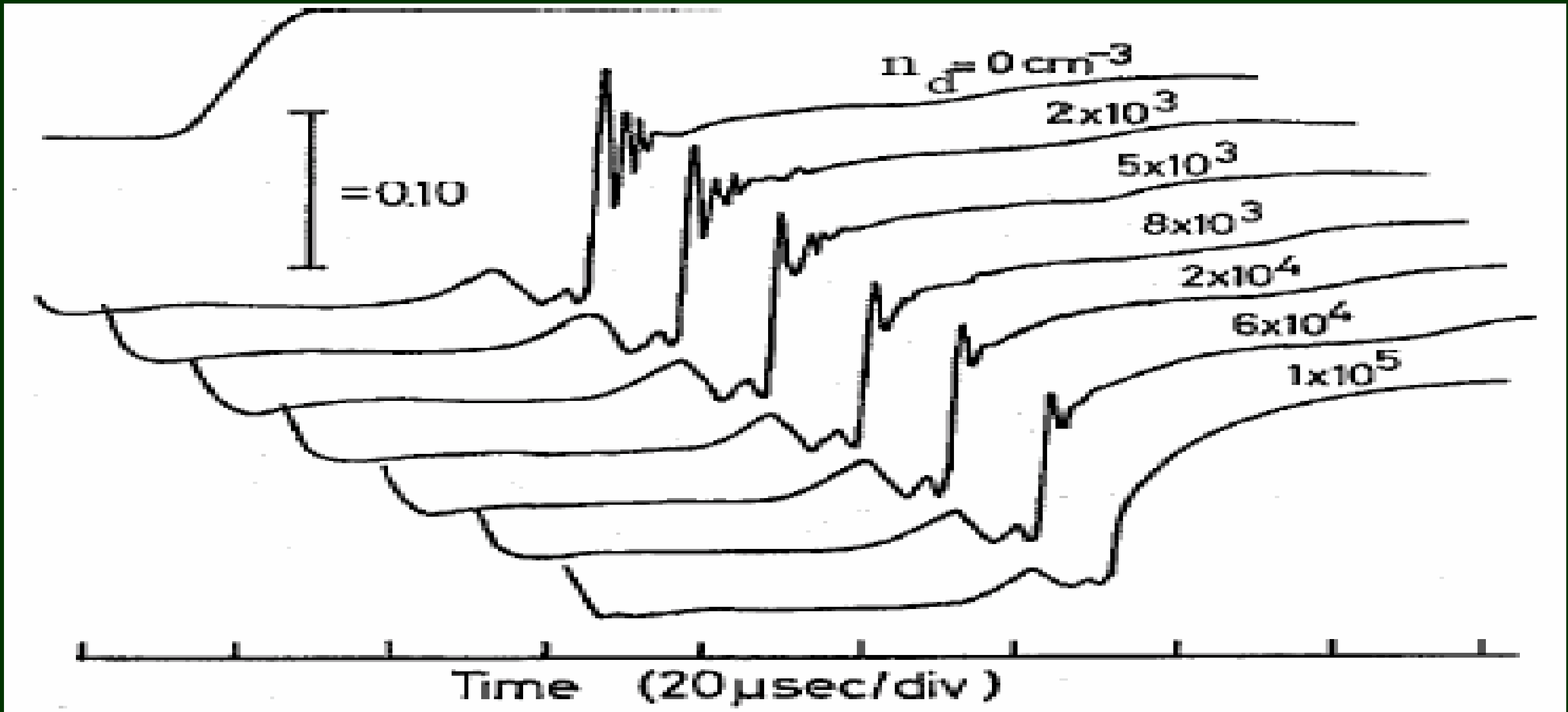
$$\frac{\partial \phi}{\partial \tau} + a_s\phi\frac{\partial \phi}{\partial \zeta} + b_s\frac{\partial^3 \phi}{\partial \zeta^3} = \eta\frac{\partial^2 \phi}{\partial \zeta^2} \quad (7)$$

where $\eta=\eta_0/2V_0$.

Eq. (7) has monotonic shock solution for $\eta^2>4U_0b_s$ (where U_0=shock speed) and oscillatory shock solution for $\eta^2<4U_0b_s$ [Karpman 1975; Shukla & Mamun 2001]. For very small η, i.e. $\eta^2<<4U_0b_s$, shock waves will have oscillatory profiles in which first few oscillations will be close to solitons moving with U_0.

▪ Experimental Observation

- DIA shocks were experimentally observed by Nakamura et al (1999). The plasma parameters used for this experiment: $n_e \approx 10^8$ cm-3, $T_e \approx 10^4$ K, $T_i \approx 0.1 T_e$, $Z_d \approx 10^5$ for $n_d < 10^3$ cm^{-3} and $Z_d \approx 10^2$ for $n_d < 10^5$ cm^{-3}.



- The oscillatory shock waves were excited in a plasma first without dust and then with dust: as n_d is increased, oscillatory wave structures behind the shock decreases and completely disappears at $n_d = 10^3$ cm^{-3}.

A source of Dissipation: Dust Charge Fluctuation

To find an alternate mechanism for the formation of DIA shock waves, we consider charge fluctuating dust grains. Then the dynamics of DIA waves can be described by (1), (2) and

$$\frac{\partial^2 \phi}{\partial z^2} = \mu \exp(\phi) - n_i + (1-\mu) Z_d \quad (8)$$

Z_d (normalized by Z_{d0}) is not constant, but varies according to

$$\eta \frac{\partial Z_d}{\partial t} = \mu\beta \exp(\phi - \alpha Z_d) - \beta_i n_i u_i \left(1 + \frac{2\alpha Z_d}{u_i^2}\right) \quad (9)$$

where $\eta=[\alpha m_e(1-\mu)/2m_i]^{1/2}$, $\beta=(r_d/a_d)1/2$, $\beta_i=\beta(\pi m_e/m_i)^{1/2}$, $a_d=n_{d0}^{-1/3}$ and $\alpha=Z_{d0}e^2/T_e r_d$. At equilibrium we have

$$\mu\beta \exp(-\alpha) = \beta_i u_0 (1 + 2\alpha/u_0^2)$$

where u_0 (normalized by C_i) is the streaming speed of ions.

- **RPM, $\eta_c=\varepsilon^{1/2}\eta_{c0}$ reduce (1), (2), (8), (9) to K-dV-Bergers Eq.**

$$\frac{\partial\phi}{\partial\tau}+A\phi\frac{\partial\phi}{\partial\zeta}+B\frac{\partial^3\phi}{\partial\zeta^3}=C\frac{\partial^2\phi}{\partial\zeta^2} \qquad (10)$$

where [Mamun & Shukla 2001]

$$A=\frac{3c+bw_0-\beta_2 w_0^4}{w_0(2c+bw_0)},\quad B=\frac{w_0^3}{2c+bw_0},\quad C=\frac{Bv_0\eta_0\beta_0\mu_1}{\alpha(\beta_e+2\beta_i/u_0)},$$

$$b=\frac{\mu_1\beta_i}{\alpha u_\beta},\quad c=1+\frac{\mu_1 u_2 u_0\beta_i}{\alpha u_\beta},\quad \beta_0=\frac{1}{\alpha u_\beta}\left[\beta_e-\frac{\beta_i w_2}{w_0}+\frac{2\alpha\beta_i}{w_0 u_0}\left(1-\frac{1}{w_0}\right)\right],$$

$$\beta_1=\beta_e[1+(\alpha\beta_0-2)\alpha\beta_0]-\frac{2\beta_i}{w_0^3}\left[1+\frac{w_\alpha}{u_0^3}\right],\quad \beta_2=\mu+\frac{\beta_1\mu_1}{\alpha u_\beta},\quad \mu_1=1-\mu,$$

$$w_0=v_0-u_0,\ u_1=1-2\alpha/u_0,\ u_2=1+2\alpha/u_0^2,\ u_\beta=\beta_e+2\beta_i/u_0,\ \beta_e=\mu\beta\exp(-\alpha),$$

$$w_1=1-u_0/w_0,\ w_2=1+u_0/w_0,\ \text{and}\ w_\alpha=2\alpha w_0 w_1(1-\beta_0 w_0 u_0).$$

- **Two situations for analytic solution of Eq. (10):**

- **$C^2>4U_0B$: This corresponds to monotonic shock solution of Eq. (10). For $C^2>>4U_0B$ its monotonic shock solution [Karpman 1975] is**

$$\phi = \frac{U_0}{A}\left[1 - \tanh\left(\frac{U_0}{2C}(\zeta - U_0\tau)\right)\right]$$

- **$C^2<4U_0B$: This corresponds to oscillatory shock solution of Eq. (10). For $C^2<<4U_0B$ its oscillatory shock solution [Karpman 1975] is**

$$\phi = \phi_0 + K\exp\left[\frac{C}{2B}(\zeta - U_0\tau)\right]\cos\left[\sqrt{\frac{U_0}{B}}(\zeta - U_0\tau)\right]$$

where ${}_0$= (ζ=0) and K is a constant. This solution means that for very small C, shock waves will have oscillatory profiles in which first few oscillations will be close to solitons moving with U_0.

- **Therefore, dust charge fluctuation can act as a source of dissipation which may responsible for the formation of monotonic or oscillatory DIA shock profiles depending on the plasma parameters.**

❑ Mobile Dust: DA Waves

▪ Model:

We consider an unmagnetized DP with mobile dust [Rao et al 1990]. **The dynamics of 1D DA waves:**

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial z}(n_d u_d) = 0 \quad (11)$$

$$\frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial z} = \frac{\partial \phi}{\partial z} \quad (12)$$

$$\frac{\partial^2 \phi}{\partial z^2} = n_d + \mu_e \exp(\sigma_i \phi) - \mu_i \exp(-\phi) \quad (13)$$

where n_d, u_d, , t, z are normalized by n_{d0}, C_d, T_i/e, $1/\omega_{pd}$, C_d/ω_{pd}, respectively, $\mu_e=1/(\delta-1)$, $\mu_i=\delta/(\delta-1)$, $\delta=n_{i0}/n_{e0}$, $\sigma_i=T_i/T_e$.

❖ DA SWs:

- **Small Amplitude:**

RPM [Washimi & Taniuti 1966] reduces (11)- (13) to a K-dV Eq.

$$\frac{\partial\phi}{\partial\tau} + a_s\phi\frac{\partial\phi}{\partial\zeta} + b_s\frac{\partial^3\phi}{\partial\zeta^3} = 0$$

$$a_s = -\frac{v_0^3}{(\delta-1)^2}[\delta^2 + (3\delta+\sigma_i)\sigma_i + \frac{1}{2}\delta(1+\sigma_i^2)]$$

and $b_s = v_0^3/2$ **. The stationary SW solution of this K-dV Eq.**

$$\phi = \phi_m\mathrm{sech}^2[(\zeta - u_0\tau)/\Delta_s]$$

where $_m = 3u_0/a_s$ and $\Delta_s = (4b_s/u_0)^{1/2}$.

Since $\delta > 1$ and $\sigma_i > 0$, i.e. a_s is always negative, small amplitude DA SWs with <0 can only exist [Mamun 1999].

- ## Arbitrary Amplitude:

SPA [Sagdeev 1966] reduces (11) - (13) to an energy integral:

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2 + V(\phi) = 0$$

where V( ) reads [Mamun 1999]:

$$V(\phi) = \mu_i[1 - \exp(-\phi)] + \frac{\mu_e}{\sigma_i}[1 - \exp(\sigma_i\phi)] + M^2\left[1 - \left(1 + \frac{2\phi}{M^2}\right)^{1/2}\right]$$

V( ) = dV( )/d = 0 at =0. So SW solution of this energy integral exists if $(d^2V/d\ ^2)_{=0}$< 0 and $(d^3V/d\ ^3{}_{=0}$>(<) 0 for >(<)0. The vanishing of quadratic term: $M_c=[(\delta-1)/(\delta+\sigma_i)]^{1/2}$. At $M=M_c$ the cubic term of V( ):

$$-\frac{1}{3(\delta-1)^2}\left[\delta^2 + (3\delta+\sigma_i)\sigma_i + \frac{1}{2}\delta(1+\sigma_i^2)\right]$$

This means that arbitrary amplitude DA SWs with <0 can only exist.

Effects of Dust of Opposite Polarity

We consider an UDP with dust of opposite polarity [Mamun & Shukla 2002; Mamun 2007]. The dynamics of 1D DA waves:

$$\frac{\partial N_1}{\partial t} + \frac{\partial}{\partial x}(N_1 U_1) = 0, \quad (1)$$

$$\frac{\partial U_1}{\partial t} + U_1 \frac{\partial U_1}{\partial x} = \frac{\partial \Psi}{\partial x}, \quad (2)$$

$$\frac{\partial N_2}{\partial t} + \frac{\partial}{\partial x}(N_2 U_2) = 0, \quad (3)$$

$$\frac{\partial U_2}{\partial t} + U_2 \frac{\partial U_2}{\partial x} = -\alpha \frac{\partial \Psi}{\partial x}, \quad (4)$$

$$\frac{\partial^2 \Psi}{\partial x^2} = N_1 - \mu_2 N_2 + \mu_e e^{\sigma \Psi} - \mu_i e^{-\Psi}, \quad (5)$$

subscript 1 (2) corresponds to - (+) dust, N_1 (N_2) are normalized by n_{10} (n_{02}), U_1 and U_2 by $C_1=(Z_1 k_B T_i/m_1)^{1/2}$, ψ by $k_B T_i/e$, $\alpha = Z_2 m_1/Z_1 m_2$, $\mu_e = n_{e0}/Z_1 n_{10}$, $\mu_i = n_{i0}/Z_1 n_{10}$, $\sigma = T_i/T_e$ and $\mu_2 = 1 + \mu_e - \mu_i$.

- **Small Amplitude:**

RPM [Washimi & Taniuti 1966] reduces (1) - (5) to a K-dV Eq:

$$\frac{\partial \psi^{(1)}}{\partial \tau} + A\psi^{(1)}\frac{\partial \psi^{(1)}}{\partial \zeta} + B\frac{\partial^3 \psi^{(1)}}{\partial \zeta^3} = 0, \qquad (6)$$

where $\zeta = \epsilon^{1/2}(x - V_0 t)$, $T = \epsilon^{2/3} t$. The coefficients A & B are given by [Sayed & Mamun 2007]

$$A = \frac{1}{2V_0(1 + z\beta\mu_2)}[3z^2\beta^2\mu_2 - 3 - V_0^4(\mu_e\sigma^2 - \mu_i)],$$
$$B = \frac{V_0^3}{2(1 + z\beta\mu_2)}, \qquad (7)$$

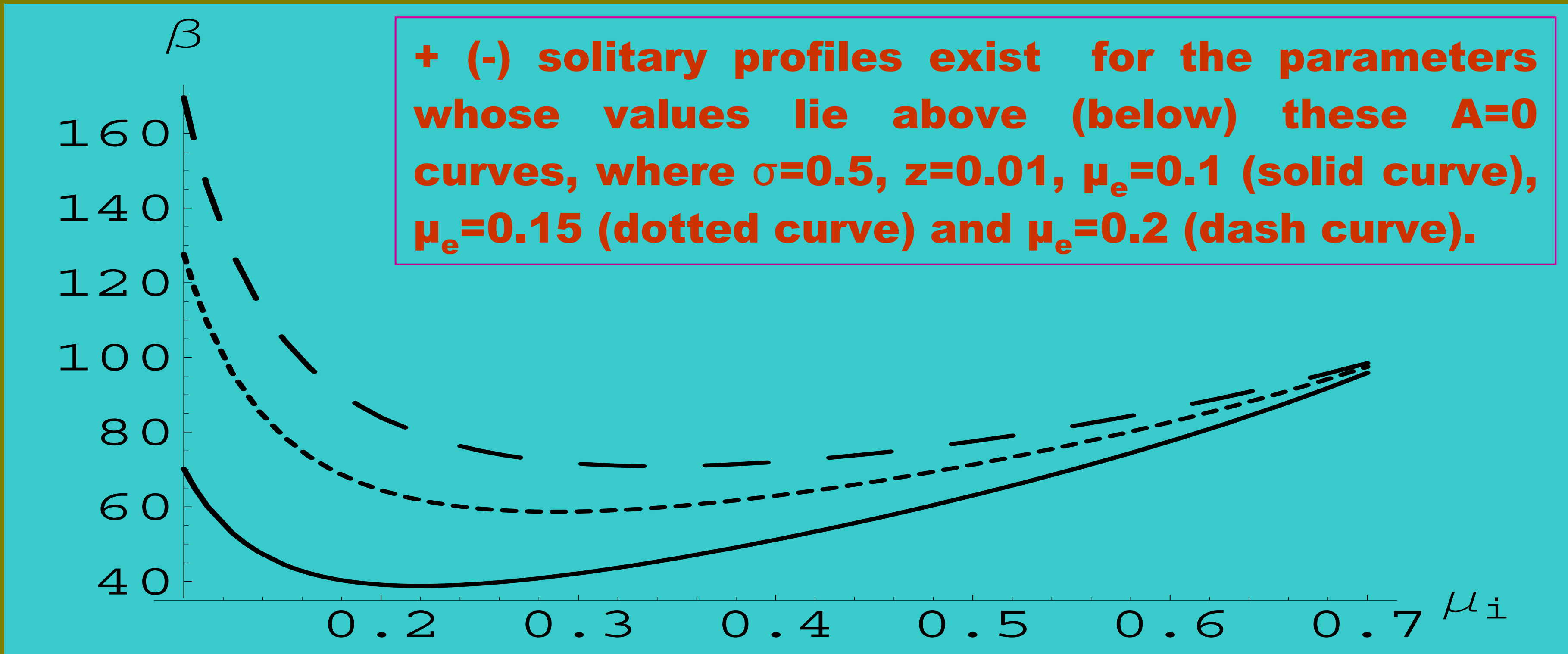
where $z = Z_2/Z_1$ and $\beta = m_1/m_2$. The stationary solution of (6):

$$\psi^{(1)} = \psi_m \mathrm{sech}^2[(\zeta - U_0\tau)/\Delta], \qquad (8)$$

where the amplitude ψ_m and the width Δ are given by

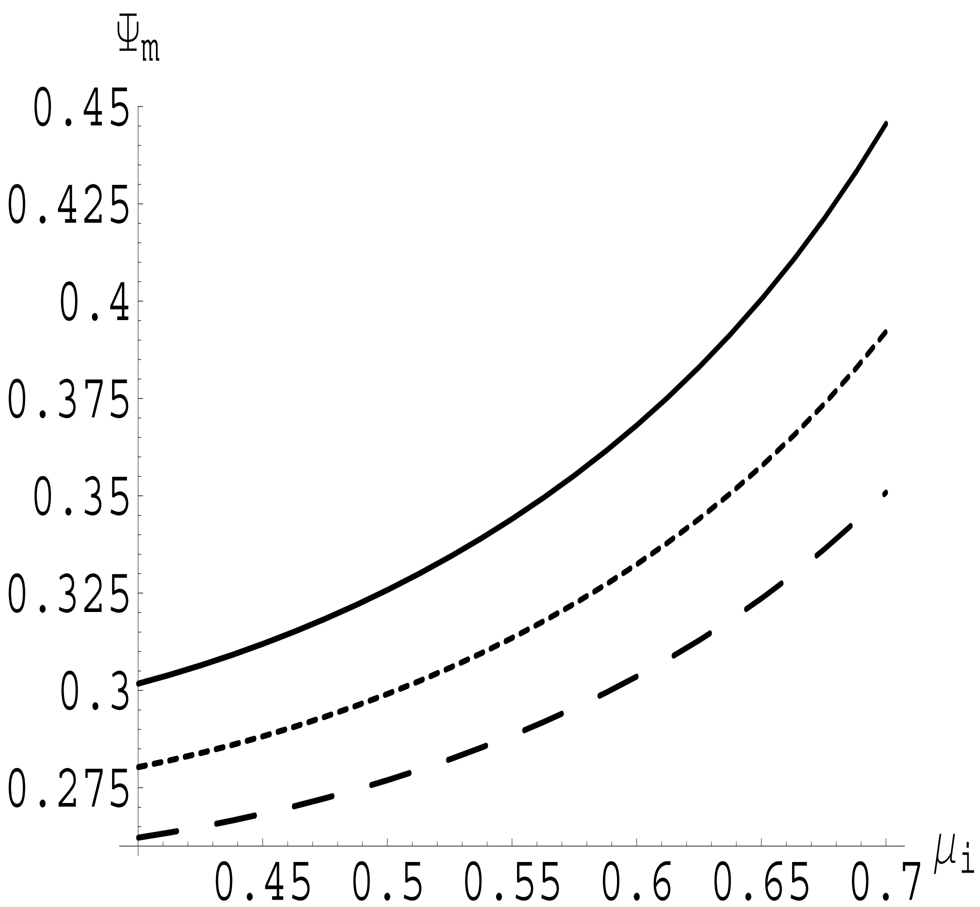
$$\psi_m = \frac{3U_0}{A}, \qquad (9)$$
$$\Delta = \sqrt{4B/U_0}.$$

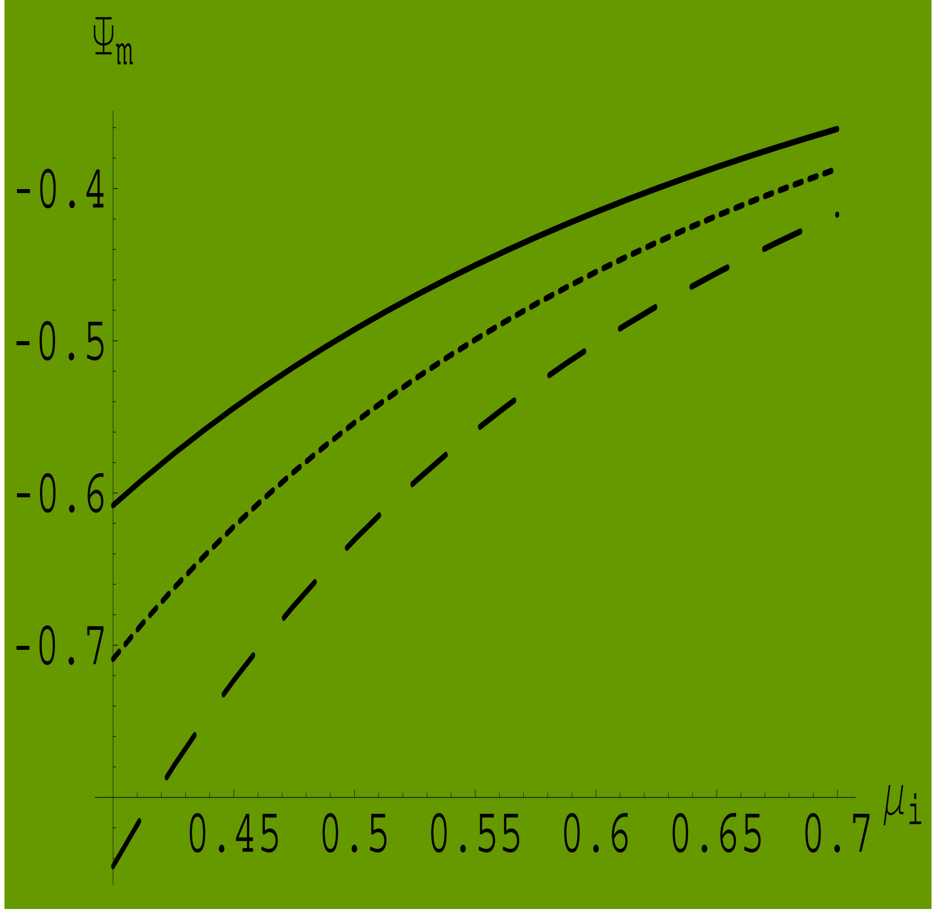
Eqs. (7) – (9) implies that the solitary potential profile is + (-) if A> (<) 0. So, we have numerically analyzed A and obtain A=0 curves displayed in figure below:



We have graphically shown how amplitude of + (corresponding to parameters whose values lie above A=0 curves) and – (corresponding to the parameters whose values lie below A=0 curves) solitary potential profiles vary with μ_i.

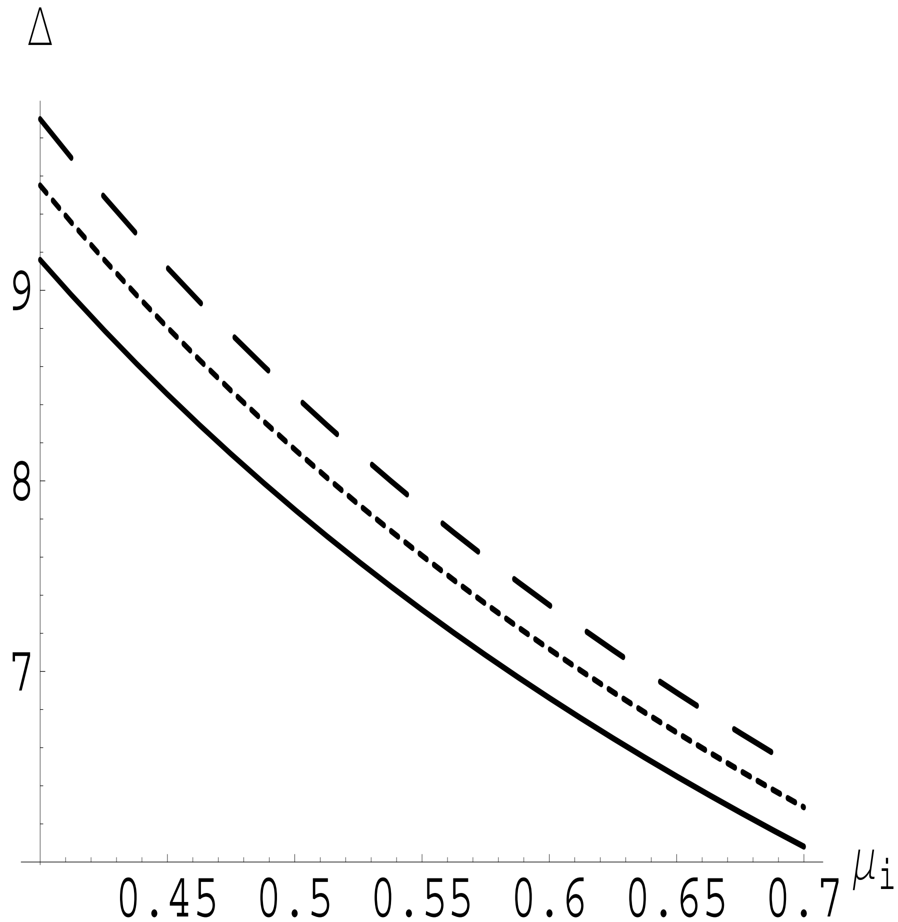
$z=0.01$, $\sigma=0.5$, $\mu_e=0.2$, $\beta=150$ (solid curve), $\beta=160$ (dotted curve), and $\beta=170$ (dash curve).	**$z=0.01$, $\sigma=0.5$, $\mu_e=0.2$, $\beta=5$ (solid curve), $\beta=10$ (dotted curve), and $\beta=15$ (dash curve).**

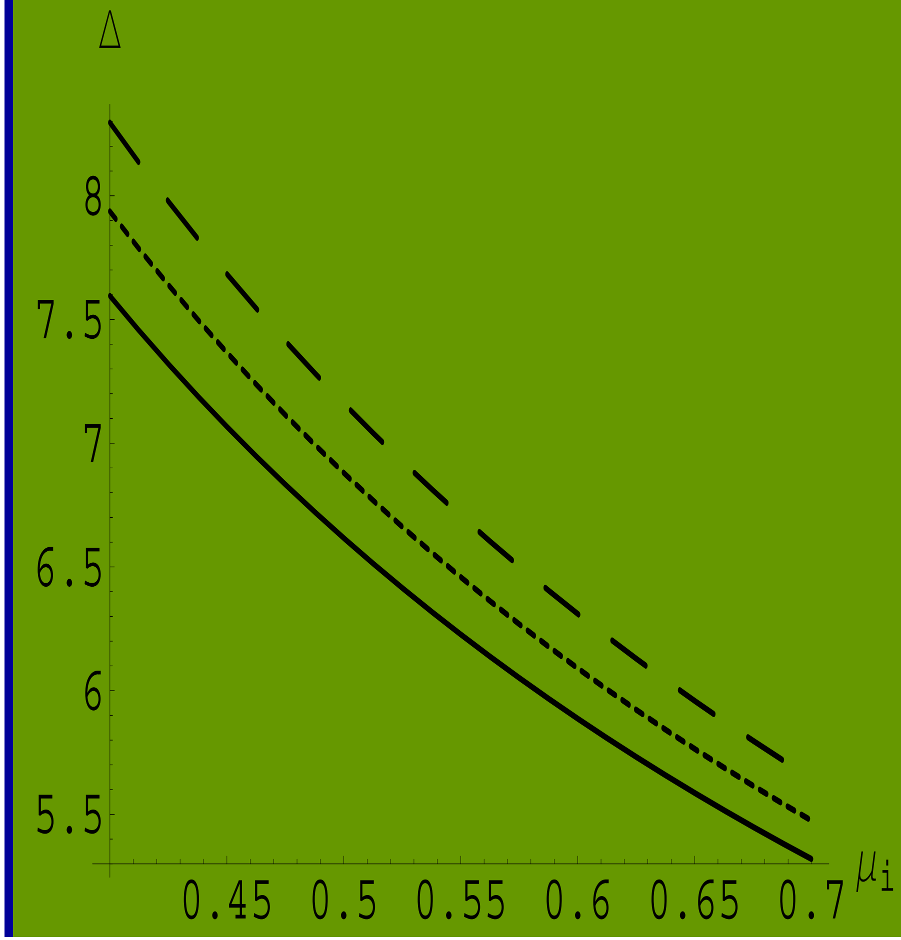




We have graphically shown how width of + (corresponding to parameters whose values lie above A=0 curves) and – (corresponding to the parameters whose values lie below A=0 curves) solitary potential profiles vary with μ_i.

z=0.01, σ=0.5, μ_e=0.2, β=150 (solid curve), β=160 (dotted curve), and β=170 (dash curve).	z=0.01, σ=0.5, μ_e=0.2, β=5 (solid curve), β=10 (dotted curve), and β=15 (dash curve).





- **Arbitrary Amplitude:**

SPA [Sagdeev 1966] reduces (1) - (5) to an energy integral:

$$\frac{1}{2}\left(\frac{d\Psi}{d\xi}\right)^2 + V(\Psi) = 0, \qquad (6)$$

where ξ=x - Mt and V(Ψ) reads [Mamun 2007]:

$$\begin{aligned} V(\Psi) &= M^2\left(1+\frac{\mu_2}{\alpha}\right)+\frac{\mu_e}{\sigma}+\mu_i \\ &- M^2\left(1+\frac{2\Psi}{M^2}\right)^{\frac{1}{2}} - \frac{\mu_2 M^2}{\alpha}\left(1-\alpha\frac{2\Psi}{M^2}\right)^{\frac{1}{2}} \\ &- \frac{\mu_e}{\sigma}e^{\sigma\Psi} - \mu_i e^{-\Psi}. \end{aligned}$$

Clearly, V(Ψ) = dVdΨ= 0 at Ψ=0. So SW solution of (6) exists if $(d^2V/d\Psi^2)_{\Psi=0} < 0$ and $(d^3V/d\Psi^3{}_{\Psi=0} > (<)\ 0$ for Ψ>(<)0.

The expansion of $V(\Psi)$ around $\Psi = 0$ is

$$V(\Psi) = C_2\Psi^2 + C_3\Psi^3 + \cdots,$$

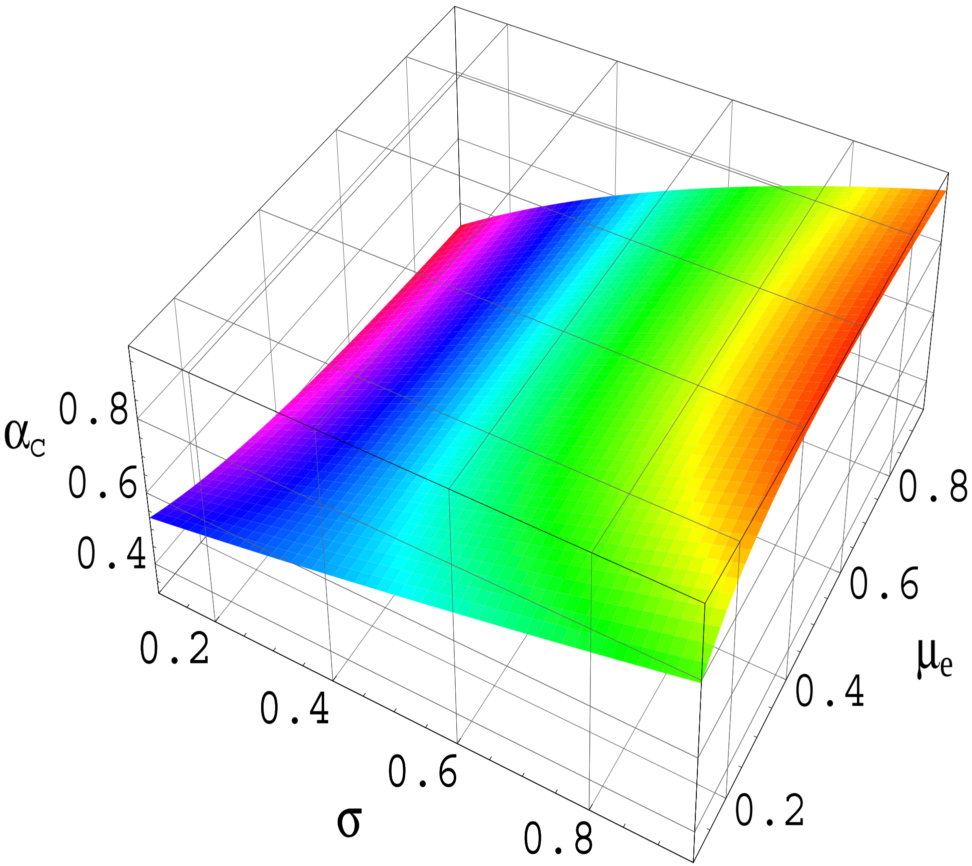
where

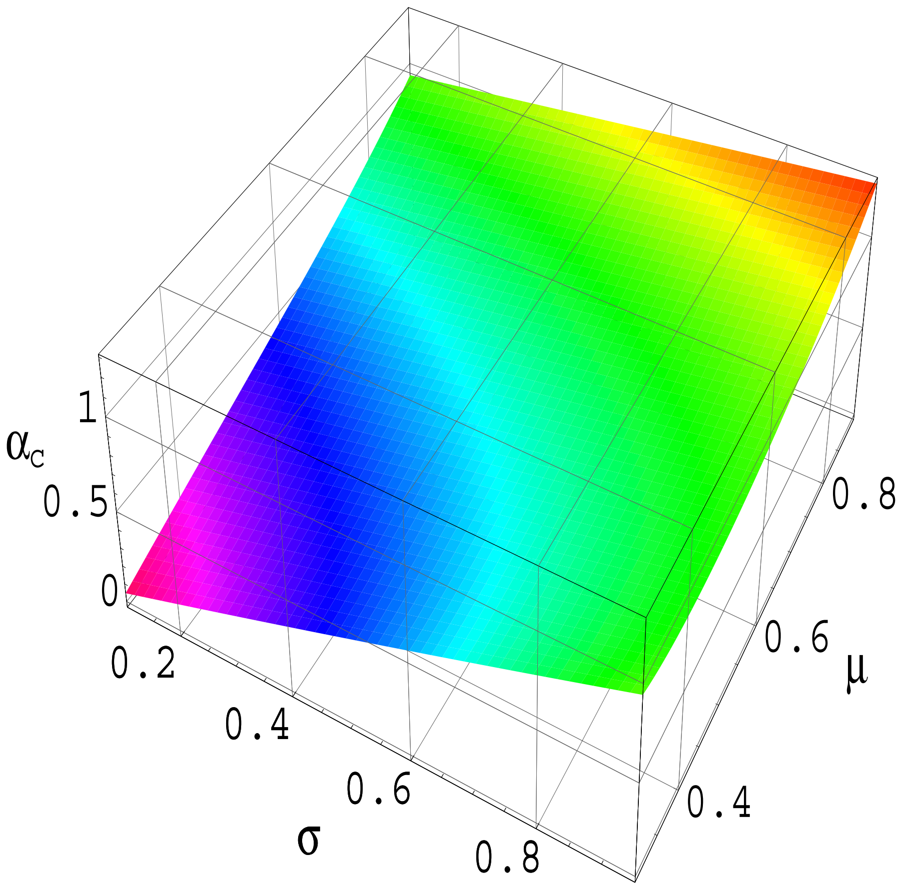
$$C_2 = \frac{1}{2M^2}(1+\alpha\mu_2) - \frac{1}{2}(\mu_i + \sigma\mu_e),$$
$$C_3 = \frac{1}{2M^4}(-1+\alpha^2\mu_2) - \frac{1}{6}(\mu_i - \sigma^2\mu_e).$$

- **So, C_2=0 gives the critical Mach number: $M_c=[(1+\alpha\mu_2)/(\mu_i+\sigma\mu_e)]^{1/2}$ and $C_3(M=M_c)=0$ gives α_c [a value of α below which –ve solitary potential exist and above which –ve and +ve solitary potentials coexist.**

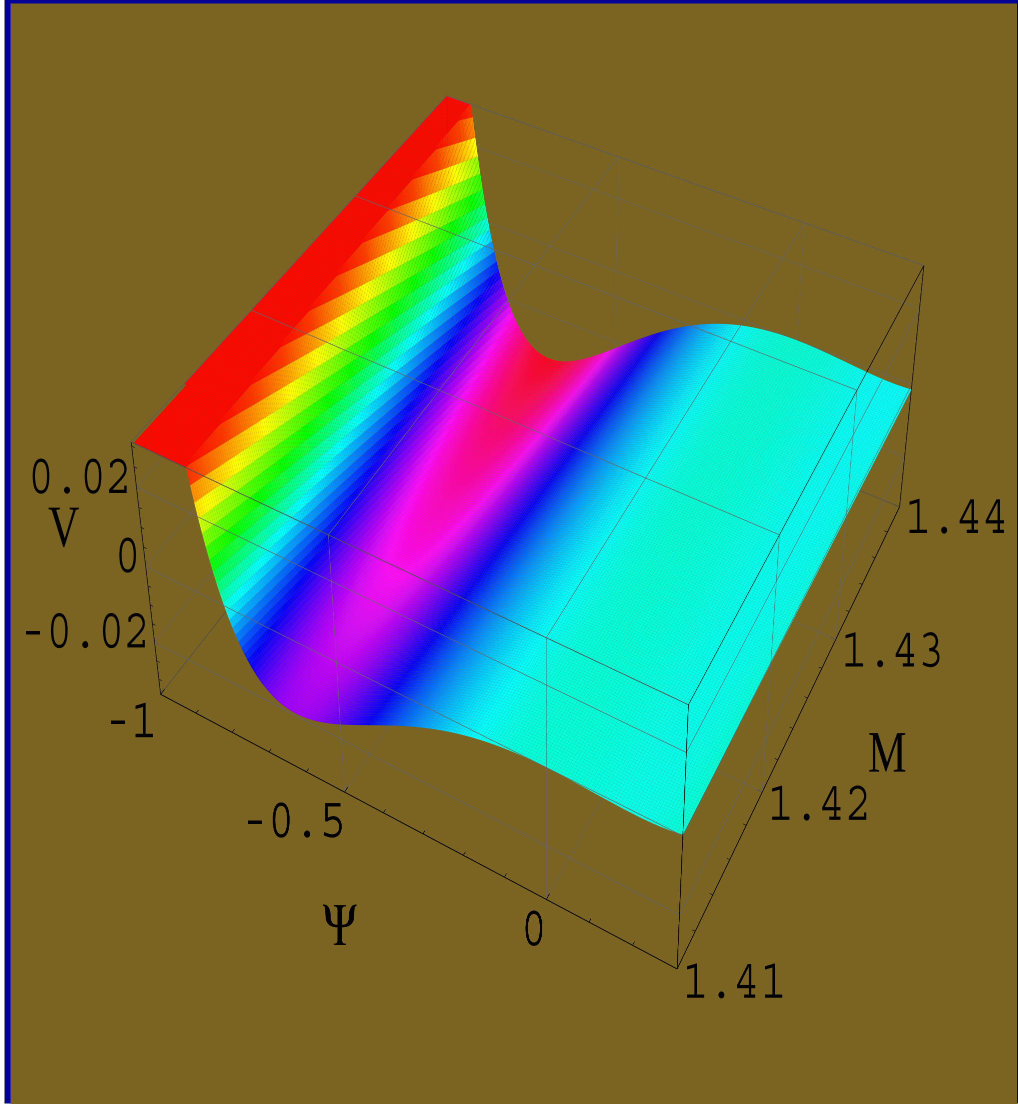
- **We have drawn $C_3(M=M_c)=0$ plot and have found the parametric regimes for existence of –ve solitary potential and the coexistence of –ve and +ve solitary potentials. Then we have chosen an appropriate set of parameters from this plot, and shown the coexistence of –ve and +ve solitary potentials by analyzing V(Ψ). The results are displayed in following figures:**

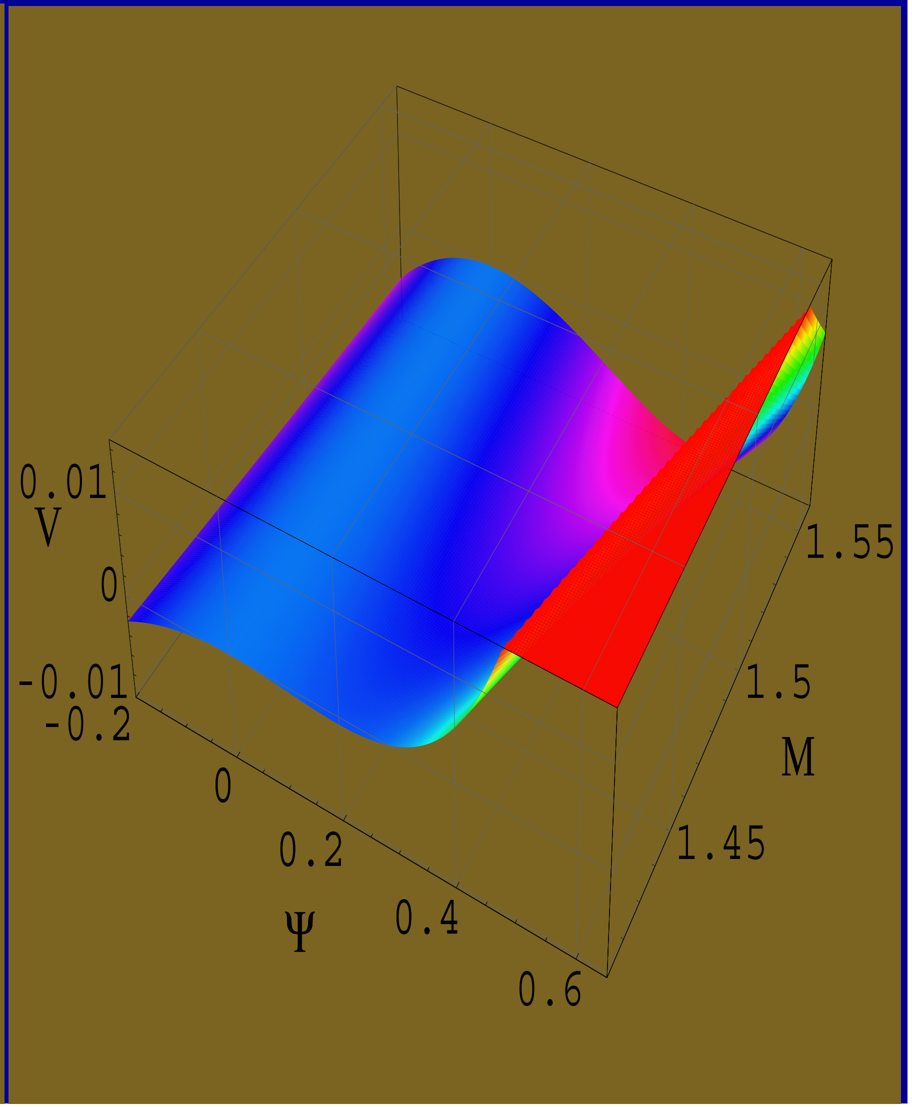
Showing the parametric regimes (values of $\alpha=Z_1m_2/Z_2m_1$, $\mu_e=n_{e0}/n_{10}$, $\mu_i=n_{i0}/n_{10}$ & $\sigma=T_i/T_e$) for the coexistence of –ve & +ve solitary potentials (values of α, μ_e μ_i & σ lying above the surface), and the existence of –ve solitary potential (values of α, μ_e, μ_i & σ lying below the surface).





Showing the coexistence of –ve & +ve solitary potentials: Potential wells are formed in both –ve & +ve Ψ-axis for the same set of dusty plasma parameters: σ=0.5, μe=0.2, μi =0.8, α=1.5 and M=1.41.





❖ DA Shock Waves

We consider an unmagnetized strongly coupled DP described by GH equations [Kaw & Sen 1998]: Eqs. (11), (13) and

$$(1+\tau_m D_t)\left[n_d\left(D_t u_d + \nu_{dn} u_d - \frac{\partial \phi}{\partial z}\right)\right] = \eta_d \frac{\partial^2 u_d}{\partial z^2} \quad (16)$$

where

$$D_t = \frac{\partial}{\partial t} + u_d \frac{\partial}{\partial z},\ \eta_d = (\tau_d / m_d n_{d0} \lambda_{Dm}^2)[\eta_b + (4/3)\zeta_b]$$

is the normalized longitudinal viscosity coefficient, τ_m is the normalized visco-elastic relaxation time, η_b and ζ_b are shear and bulk viscosity coefficients, ν_{dn} is the normalized dust-neutral collision frequency. All transport coefficients are well defined by Kaw & Sen (1999), Mamun et al (2000), Shukla & Mamun (2001), etc.

- **Using RPM, $\eta_d=\varepsilon^{1/2}\eta_0$ and Eqs. (11), (13) & (16) we have:**

$$\frac{\partial\phi}{\partial\tau}+A\phi\frac{\partial\phi}{\partial\zeta}+B\frac{\partial^3\phi}{\partial\zeta^3}=C\frac{\partial^2\phi}{\partial\zeta^2} \qquad (17)$$

where

$$A=\frac{1}{2v_0}(\nu_{dn}\tau_m-a_{\delta\sigma})\left(1+\frac{1}{2}\nu_{dn}\tau_m\right)^{-1}$$

$$B=\frac{1}{2}v_0^3\left(1+\frac{1}{2}\nu_{dn}\tau_m/2\right)^{-1}$$

$$C=\frac{1}{2}\eta_0\left(1+\frac{1}{2}\nu_{dn}\tau_m\right)^{-1}$$

$$a_{\delta\sigma}=\frac{2v_0^4}{(\delta-1)^2}\left[\delta^2+(3\delta+\sigma_i)\sigma_i+\frac{1}{2}\delta(1+\sigma_i^2)\right]$$

It is obvious that for a collision-less limit (v_{dn}=0) A<0, B>0, and C>0, but for a highly collision limit ($v_n\tau_m$>2, since $a_{\delta\sigma}\approx 2$ for δ=10 and σ_i=1) A>0, B>0 and C>0.

- **There are two situations for analytic solutions of Eq. (17):**

- **$C^2>4U_0B$: This corresponds to monotonic shock solution of Eq. (17). For $C^2>>4U_0B$ its monotonic shock solution [Karpman 1975] is**

$$\phi = \frac{U_0}{A}\left[1 - \tanh\left(\frac{U_0}{2C}(\zeta - U_0\tau)\right)\right]$$

- **$C^2<4U_0B$: This corresponds to oscillatory shock solution of Eq. (17). For $C^2<<4U_0B$ its oscillatory shock solution [Karpman 1975] is**

$$\phi = \phi_0 + K\exp\left[\frac{C}{2B}(\zeta - U_0\tau)\right]\cos\left[\sqrt{\frac{U_0}{B}}(\zeta - U_0\tau)\right]$$

where $_0$= (ζ=0) and K is a constant. This solution means that for very small η_d shock waves will have oscillatory profiles in which first few oscillations will be close to solitons moving with U_0.

- **Therefore, strong correlation of dust can act as a source of dissipation which may responsible for the formation of monotonic or oscillatory DA shock profiles depending on the plasma parameters.**

❑ Summary

- The dust particle does not only modify the existing plasma waves, but also introduces a number of new eigen modes, e.g. DIA, DA, DL, etc., which in nonlinear regime form different types of interesting coherent structures.
- The basic features of DIA & DA SWs [Bharuthram & Shukla 1992; Mamun et al 1996] in comparison with IA SWs:

IA SWs ($V_{Te}>V_P>V_{Ti}$)	DIA SWs ($V_{Te}>V_P>V_{Ti}$)	DA SWs ($V_{Ti}>V_P>V_{Td}$)
>0 (only)	>0 when $\mu>1/3$ <0 when $\mu<1/3$	<0 (only)
$n_e>0$ $n_i>0$	$n_e>(<)0$ when $\mu>(<)1/3$ $n_i>(<)$ 0 when $\mu>(<)1/3$	$n_e<0$; $n_i>0$ $n_d<0$
$V>C_i$ $C_i=(T_e/m_i)^{1/2}$	$V>C_i\,\mu^{-1/2}$ $\mu=n_{e0}/n_{i0}$	$V>C_d$ $C_d=(Z_dT_i/m_d)^{1/2}$

- **Effects of ion (dust) fluid temperature DIA and DA SWs**: as ion (dust) fluid temperature increases, amplitude of DIA (DA) SWs decreases, but their width increases [Mamun 1997; Sayed & Mamun 2007].

- **Effects of a non-planer geometry on DIA and DA SWs**: it reduces to a modified K-dV equation containing an extra-term (ν/2T) with ν=1 (2) is for cylindrical (spherical) geometry [Mamun & Shukla 2001, 2002].

- **Effects of dust charge fluctuation on DIA SWs**: The dust grain charge fluctuations do not only change the amplitude and width of DIA SWs, but also provide a source of dissipation, and may be responsible for the formation of DIA shock waves [Mamun & Shukla 2002].

- **Effects of fast ions on DA SWs**: The presence of fast ions may allow compressive and rarefactive SWs to coexist: electrostatic SWs observed by Freja and Viking spacecrafts [Mamun et al 1996].

- **Effects of trapped ion distribution on DA SWs**: it gives rise to a modified K-dV equation exhibiting stronger nonlinearity: smaller width & larger propagation speed [Mamun et al 1996; Mamun 1997].

- <u>Effects of dust of opposite polarity on DA SWs</u>: The dynamics of positive dust (in addition to negative ones) may allow negative and positive SWs to coexist [Mamun & Shukla 2002, Mamun 2007].

- <u>Effect of strong dust correlation on DA SWs</u>: The strong dust correlation provides a source of dissipation, and is responsible forthe formation of DA shock waves [Shukla & Mamun 2001]. The combined effects of strong dust correlation and trapped ion distribution reduce to a modified K-dV-Burgers equation with some new features [Mamun et al 2004].

- Because of time limit, I confined my talk to an unmagnetized DP. However, a number of investigations [Mamun 1998; Kotsarenko et al 1998a] on DA SWs in a magnetized DP have been made: (i) external magnetic field makes ES-SWs more spiky and (ii) ES-SWs becomes unstable: multi-dimensional instability [Mamun 1998b, 1998c].

- The physics of nonlinear waves that we have discussed must play a significant role in understanding the properties of localized ES structures in space & laboratory dusty plasmas.

THANK YOU ALL