The Abdus Salam
International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

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International Atomic Energy Agency

1858-18

School on Physics, Technology and Applications of Accelerator Driven Systems (ADS)

19 - 30 November 2007

ADS Dynamics
"Accelerator-Driven System Dynamics.
Part II"

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Accelerator-driven and advanced system dynamics

Part II

Politecnico di Torino
Dipartimento di Energetica

Outline

- New challenges in the simulation of the neutron dynamics of ADS
- Models and methods
- Time-dependent transport models
- An advanced application: dynamics of fluid fuel systems
- Simulation of source experiments

The role of delayed neutrons

- Time-dependent analysis of nuclear systems can be done only taking account of delayed emissions from fission
- On the basis of elementary physics considerations, a multiplying system evolution is regulated by the exponential law $\exp((\delta k/\Lambda)t)$, where ....

The role of delayed neutrons

Λ is a "characteristic" time
- No delayed neutrons: 10^{-4} - 10^{-6} s
- With delayed neutrons: $\Lambda + \beta/\lambda \sim 10^{-1}$ s ($\lambda \sim 10^{-1}$ s^{-1})

Evolution is dominated by delayed neutrons (for sub-prompt-critical systems)

Note: β is an important dynamic parameter (the *physical* fraction β is: for U235: 0.0065, for Pu239: 0.0022)

Time-scales in the dynamics of nuclear reactors

- Prompt neutron (very fast) scale, connected to the lifetime of prompt neutrons (10^{-4} - 10^{-6} s)
- Delayed emission scale, connected to evolution of delayed neutron precursors (10^{-1} - 10^{1} s)
- Thermal-hydraulic scale (feedback), connected to the evolution of temperatures and hydraulic parameters (10^{-1} - 10^{2} s)
- Control scale, connected to the movement of masses in the system (control rods, poisons)
- Nuclide transmutation scale, connected to neutron transmutation phenomena ($>10^{2}$ s)

Time-scales in the dynamics of nuclear reactors

Very different time-scales

the physico-mathematical problem
is **stiff**

Time-scales in the dynamics of nuclear reactors

- We now focus our interest on the dynamics of nuclear systems during operational and accidental transients
  - Nuclide transmutation can be neglected, but still
    - Delayed emissions
    - Thermal feedback

    are to be considered.

Basic equations for neutron dynamics (1)

Boltzmann transport equation in presence of delayed emissions:

$$\begin{cases} \dfrac{\partial n(\mathbf{r},E,\Omega,t)}{\partial t} = \hat{\mathrm{B}}(t)n(\mathbf{r},E,\Omega,t) + \\ \qquad \displaystyle\sum_{i=1}^{6} \lambda_i \frac{\chi_i(E)}{4\pi} C_i(\mathbf{r},t) + S(\mathbf{r},E,\Omega,t) \\ \dfrac{\partial(\chi_i(E)C_i(\mathbf{r},t)/4\pi)}{\partial t} = \hat{\mathrm{M}}_i(t)n(\mathbf{r},E,\Omega,t) - \\ \qquad \lambda_i \dfrac{\chi_i(E)}{4\pi} C_i(\mathbf{r},t) \end{cases}$$

Basic equations for neutron dynamics (2)

$$\frac{\partial n(\mathbf{r},E,\Omega,t)}{\partial t} = \hat{\mathrm{B}}(t)n(\mathbf{r},E,\Omega,t) + \sum_{i=1}^{6}\lambda_i\frac{\chi_i(E)}{4\pi}C_i(\mathbf{r},t) + S(\mathbf{r},E,\Omega,t)$$

$$\hat{\mathrm{L}}(t) = -\Omega\cdot\nabla v(E) - \Sigma(r,E,t)v(E) + \int dE' \oint d\Omega' v(E')\Sigma_s(r,E'\to E,\Omega'\cdot\Omega,t)$$

$$\hat{\mathrm{M}}_p(t) = \sum_j \frac{\chi_p^j(E)}{4\pi}\int dE' \oint d\Omega' v(E')(1-\beta^j)\times \nu^j(E')\Sigma_f^j(\mathbf{r},E',t)$$

Leakage

Balance operator

Prompt multiplication

$$\hat{\mathrm{B}}(t) = \hat{\mathrm{L}}(t) + \hat{\mathrm{M}}_p(t)$$

Basic equations for neutron dynamics (3)

$$\frac{\partial(\chi_i(E)C_i(\mathbf{r},t)/4\pi)}{\partial t} = \hat{\mathrm{M}}_i(t) n(\mathbf{r},E,\Omega,t) - \lambda_i \frac{\chi_i(E)}{4\pi} C_i(\mathbf{r},t)$$

Delayed multiplication

$$\hat{\mathrm{M}}_i(t) = \sum_j \frac{\chi_i(E)}{4\pi} \int dE' \oint d\Omega' v(E') \beta_i^j \times \nu^j(E') \Sigma_f^j(r,E',t)$$

Operators can be time-dependent because of:

- effects independent of neutron flux (perturbations)
- non-linear effects (feedback)

Challenges in the simulation of neutron dynamics

- The Boltzmann equation is a very challenging problem

Example: 3D calculation of a nuclear reactor

- Space: ~ $(10^2)^3 = 10^6$ meshes
- Angle: ~ 10^2 directions (S_8 in 3D)
- Energy: ~ 10^1 - 10^2 groups

→ ~ 10^9 - 10^{10} unknowns for a steady-state calculation

- Time: Δt ~ 10^{-6} s

→ ~ 10^6 pseudo-stationary calculation per second in time-dependent evaluation

It yields too much physical detail

In real systems only integral quantities can be observed

Challenges in the simulation of neutron dynamics

- ***Need to construct simplified models (multigroup, diffusion...) based on physical assumptions***

- ***Need of numerical algorithms (discretizations, expansions)***

Development of approximate models and algorithms

important: establish adequateness of approximations for the problem considered (benchmarks)

Models and methods for neutron dynamics

- Point kinetics
  - Derivation of the model and physical interpretation
- Quasi-static method
  - Improved quasi-statics
  - Predictor-Corrector quasi-statics
- Multipoint kinetics
  - Features of MPK approach

Point kinetics

the neutron distribution is factorized in an amplitude (time-dependent) and a shape (time independent)

$$n(r, E, \Omega, t) = P(t)\varphi(r, E, \Omega; \cancel{t})$$

<u>Critical systems</u>

Shape: fundamental eigenfunction of the model

$$\left(\hat{\mathbf{L}}_0 + \frac{1}{k}\hat{\mathbf{M}}_0\right)\varphi = 0$$

<u>Subritical systems</u>

Shape: steady-state solution, dominated by the source

$$\left(\hat{\mathbf{L}}_0 + \hat{\mathbf{M}}_0\right)\varphi + S_0 = 0$$

Point kinetics

The factorized form is introduced into the balance equations

$$\begin{cases} P\cancel{\dfrac{\partial\varphi}{\partial t}} + \varphi\dfrac{dP}{dt} = P\hat{\mathrm{B}}\varphi + \displaystyle\sum_{i=1}^{6} \lambda_i \left(\frac{\chi_i}{4\pi} C_i\right) + S \\ \dfrac{\partial(\chi_i C_i/4\pi)}{\partial t} = P\hat{\mathrm{M}}_i\varphi - \lambda_i \left(\dfrac{\chi_i}{4\pi} C_i\right) \end{cases}$$

and is projected on a weighting function w:

?

$$\begin{cases} \langle w \mid \varphi \rangle \dfrac{dP}{dt} = \left\langle w \mid \hat{\mathbf{B}}\varphi \right\rangle P + \displaystyle\sum_{i=1}^{6} \lambda_i \left\langle w \mid \left(\frac{\chi_i}{4\pi} C_i\right)\right\rangle + \langle w | S \rangle \\ \left\langle w \mid \dfrac{\partial(\chi_i C_i/4\pi)}{\partial t} \right\rangle = \left\langle w \mid \hat{\mathbf{M}}_i\varphi \right\rangle P - \lambda_i \left\langle w \mid \left(\dfrac{\chi_i}{4\pi} C_i\right)\right\rangle \end{cases}$$

Point kinetics

Weight w → solution of the adjoint steady-state problem

<u>Critical systems</u>

$$\left(\hat{\mathbf{L}}_0^\dagger + \frac{1}{k}\hat{\mathbf{M}}_0^\dagger\right) N_0^\dagger = 0$$

<u>Subritical systems</u>

$$\left(\hat{\mathbf{L}}_0^\dagger + \hat{\mathbf{M}}_0^\dagger\right) N_0^\dagger + S_0^\dagger = 0$$

The procedure is standard for critical reactors, while for subcritical source-driven systems the question on the adjoint source arises

definition can be given on the basis of

physical consideration and *variational principles*

Point kinetics

Integral quantities are evaluated and the differential equations for the amplitudes are derived:

$$
\left\{
\begin{array}{l}
\left\langle N_0^\dagger \right| \\
\left\langle N_0^\dagger \right|
\end{array}
\right.
\left\{
\begin{array}{l}
\dfrac{dP(t)}{dt} = \dfrac{\rho(t) - \tilde{\beta}}{\Lambda} P(t) + \displaystyle\sum_{i=1}^{6} \lambda_i \tilde{C}_i(t) + \tilde{S} \rangle + \left\langle N_0^\dagger \, |S\right\rangle \\
\dfrac{dC_i(t)}{dt} = \dfrac{\tilde{\beta}}{\Lambda} P(t) - \lambda_i \tilde{C}_i(t) \qquad C_i \big) \Big\rangle
\end{array}
\right.
$$

having introduced the definition of the kinetic parameters

$$
\rho(t) = \frac{\left\langle N_0^+ \right| \delta \hat{\mathrm{K}} \varphi \Big\rangle}{\left\langle N_0^+ \right| \hat{\mathrm{M}} \varphi \Big\rangle} \qquad
\tilde{\beta}_i = \frac{\left\langle N_0^+ \right| \hat{\mathrm{M}}_i \varphi \Big\rangle}{\left\langle N_0^+ \right| \hat{\mathrm{M}} \varphi \Big\rangle} \qquad
\Lambda = \frac{\left\langle N_0^+ \right| \varphi \rangle}{\left\langle N_0^+ \right| \hat{\mathrm{M}} \varphi \Big\rangle}
$$

Point kinetics

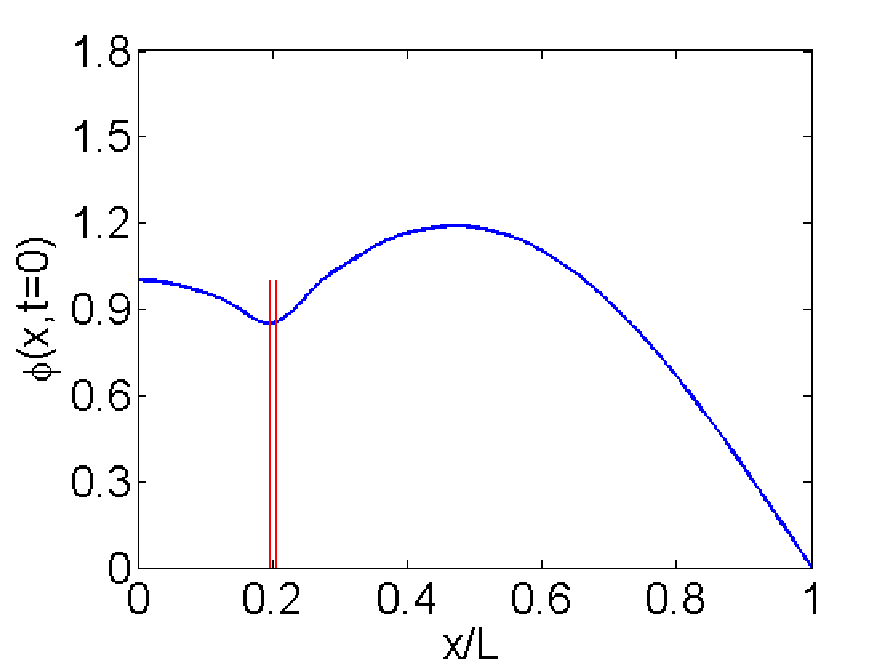
- Characteristics of the **point kinetic** approximation:
  - no space distortion during the transient
  - the evolution is space-time separable
  - any point is representative of the whole system

The approximation is poor when localized phenomena (e.g. control rod insertion) are concerned

Point kinetics - results

- Transient following extraction of a control device in a critical system

  - Simplified 1D system
  - Exact solution vs point kinetic results



Initial shape

Point kinetics - results

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- Simplified 1D system
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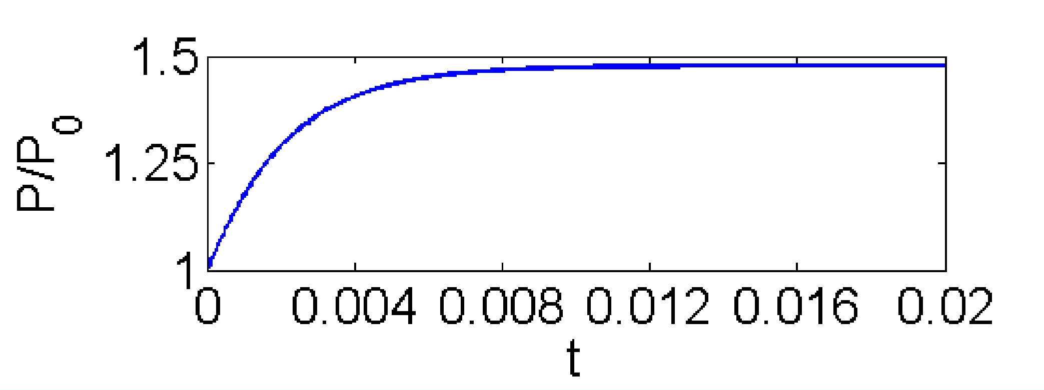
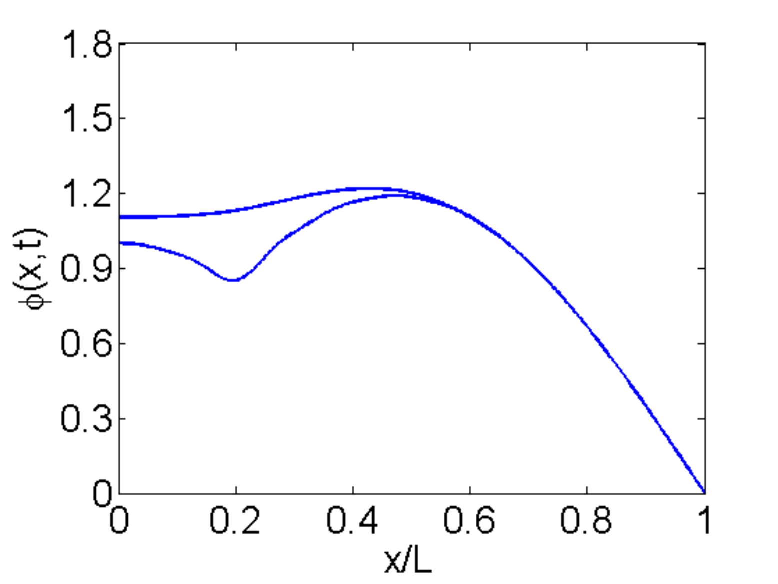
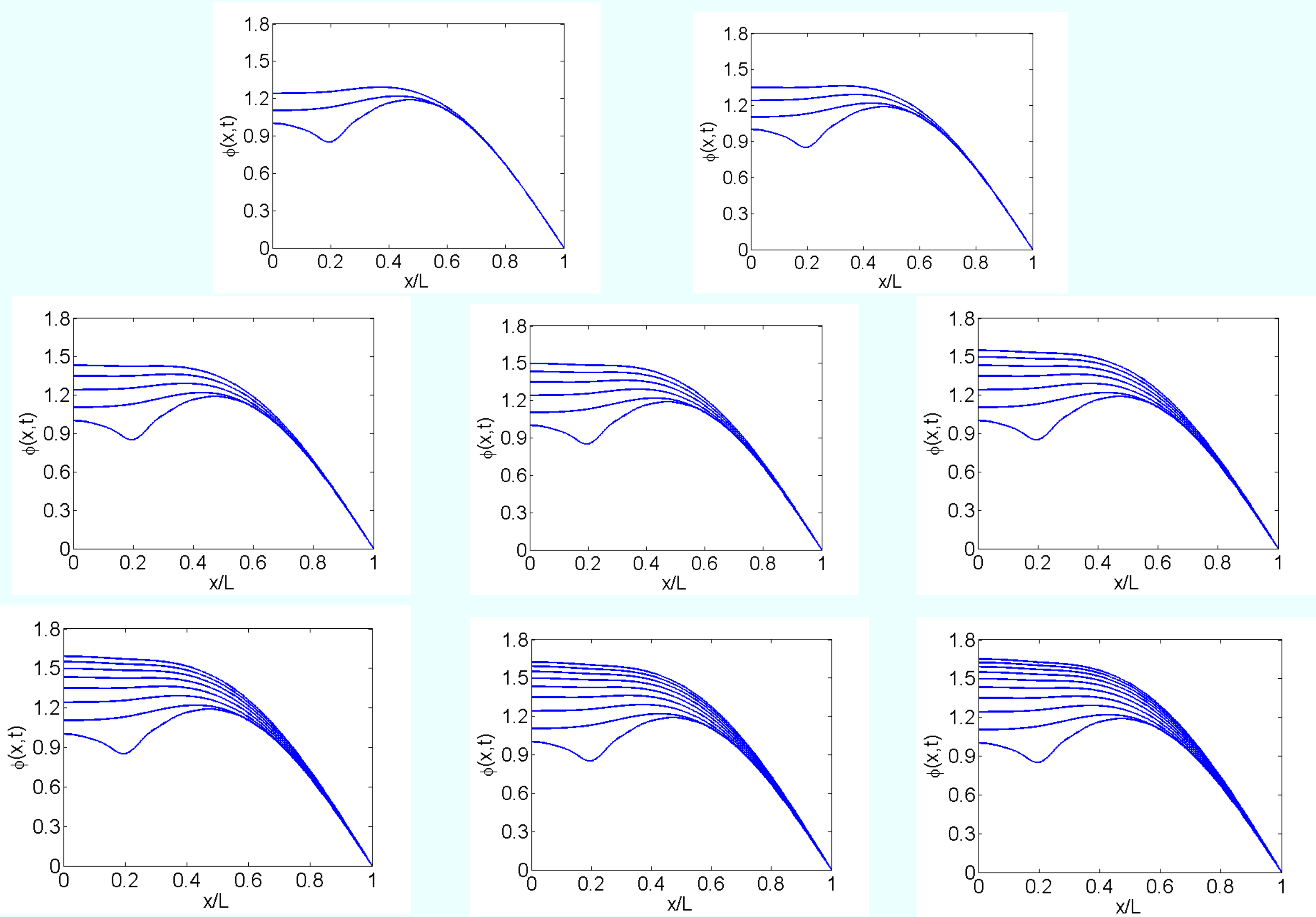




Point kinetics - results



Point kinetics - results

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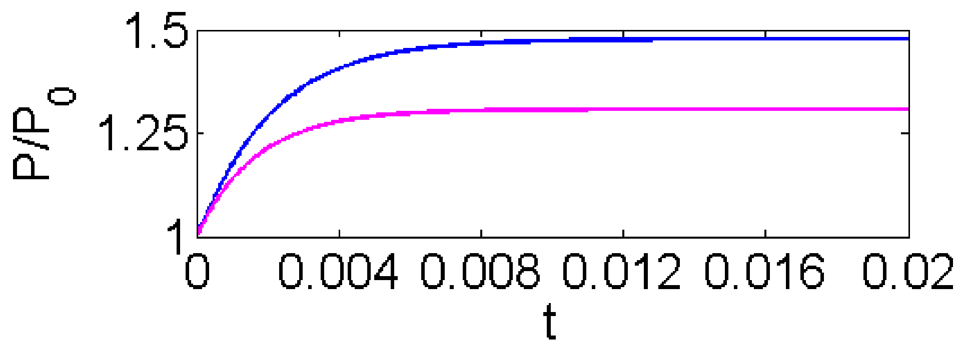
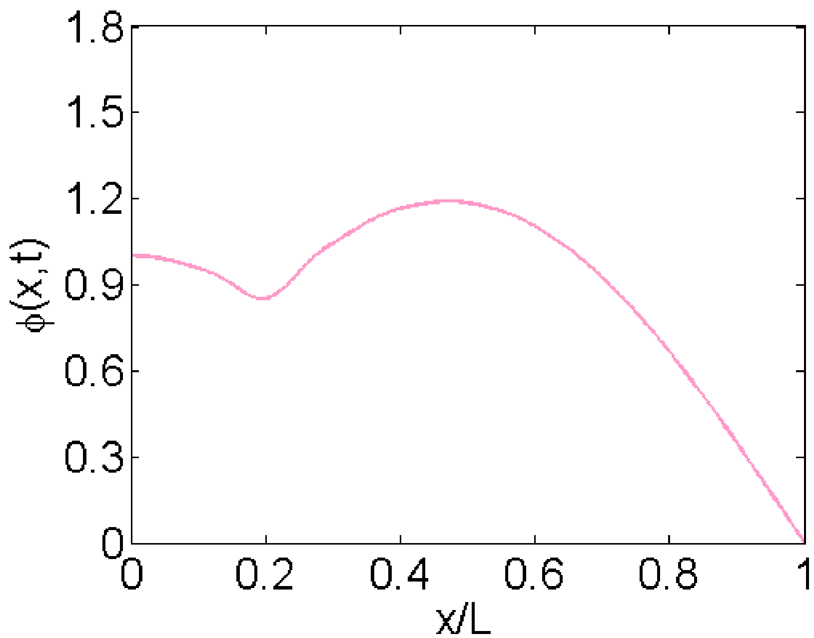




Point kinetics

- Results produced with point kinetics underestimate real power evolution

↓

not reliable for safety assessment

- Spatial/spectral effects are neglected

- Need for a more sophisticated method, able to take into account these effects…

Quasi-static method

Quasi-statics

- The factorization procedure is generalized as:

$$n(r,E,\Omega,t) = P(t)\varphi(r,E,\Omega;t)$$

No approximation introduced

Amplitude: fast evolving phenomena

Shape: slowing evolving phenomena

inserted into the t-d model and projected on a weight

$$\begin{cases} \left\langle w \,\middle|\, \dfrac{\partial \varphi}{\partial t} \right\rangle P + \langle w \mid \varphi \rangle \dfrac{dP}{dt} = \left\langle w \,\middle|\, \hat{\mathbf{B}}\varphi \right\rangle P + \displaystyle\sum_{i=1}^{6} \lambda_i \left\langle w \,\middle|\, \left(\dfrac{\chi_i}{4\pi} C_i \right) \right\rangle + \langle w \mid S \rangle \\ \left\langle w \,\middle|\, \dfrac{\partial \left(\chi_i C_i / 4\pi\right)}{\partial t} \right\rangle = \left\langle w \,\middle|\, \hat{\mathbf{M}}_i \varphi \right\rangle P - \lambda_i \left\langle w \,\middle|\, \left(\dfrac{\chi_i}{4\pi} C_i \right) \right\rangle \end{cases}$$

Quasi-statics

- Again, the weight is the solution of the adjoint model:

$$\begin{cases} \cancel{\dfrac{d}{dt}\left\langle N_0^\dagger \mid \varphi \right\rangle} P + \left\langle N_0^\dagger \mid \varphi \right\rangle \dfrac{dP}{dt} = \left\langle N_0^\dagger \mid \hat{\mathbf{B}}\varphi \right\rangle P + \displaystyle\sum_{i=1}^{6} \lambda_i \left\langle N_0^\dagger \mid \left(\frac{\chi_i}{4\pi} C_i \right) \right\rangle + \left\langle N_0^\dagger \mid S \right\rangle \\ \left\langle N_0^\dagger \mid \dfrac{\partial \left(\chi_i C_i / 4\pi \right)}{\partial t} \right\rangle = \left\langle N_0^\dagger \mid \hat{\mathbf{M}}_i \varphi \right\rangle P - \lambda_i \left\langle N_0^\dagger \mid \left(\frac{\chi_i}{4\pi} C_i \right) \right\rangle \end{cases}$$

and a normalization condition is introduced to make the factorization unique

$$\frac{d}{dt}\left\langle N_0^\dagger \mid \varphi \right\rangle = 0$$

Quasi-statics

- The final form of the equation for the apmlitude is the well-known point model:

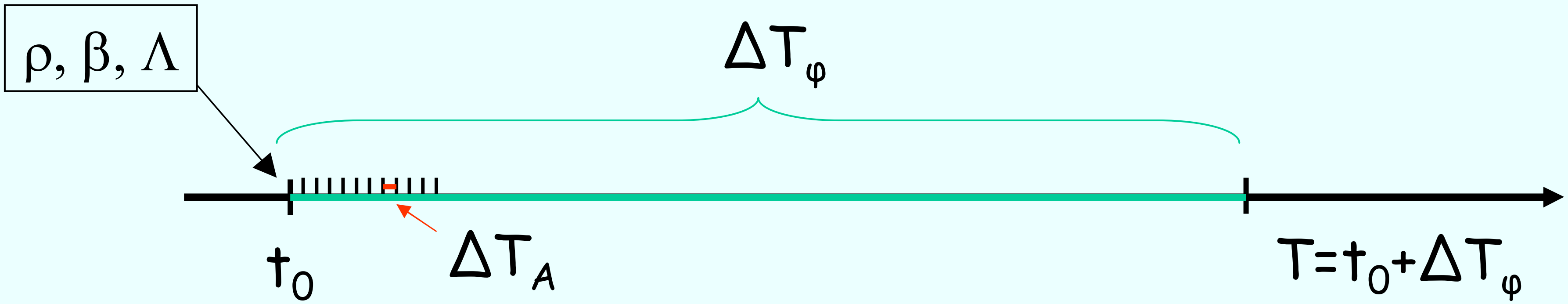
$$\begin{cases} \dfrac{dP(t)}{dt} = \dfrac{\rho(t) - \tilde{\beta}}{\Lambda} P(t) + \displaystyle\sum_{i=1}^{6} \lambda_i \tilde{C}_i(t) + \tilde{S} \\ \dfrac{dC_i(t)}{dt} = \dfrac{\tilde{\beta}}{\Lambda} P(t) - \lambda_i \tilde{C}_i(t) \end{cases}$$

but the kinetic parameters depend on the shape function, which is the other unknown of the problem

⇒ **Two time-scales solution**

Improved quasi-statics

- The solution is obtained on a two-scale frame:
  - Evaluation of the kinetic parameters with the shape at time t_0 (if t_0=0, the initial shape is used)
  - Solution of the point model on time interval $[t_0,T]$ with a fine time mesh ΔT_A
  - Solution of the shape model (computationally expensive) on $\Delta T_\varphi = T - t_0$ to update shape function



Improved quasi-statics

- Characteristics of the algorithm:
  - The model is non linear

$$
\begin{cases}
\left\langle N_0^\dagger \mid \varphi \right\rangle \dfrac{dP}{dt} = \left\langle N_0^\dagger \mid \hat{\mathbf{B}}\boxed{\varphi} \right\rangle \boxed{P} + \displaystyle\sum_{i=1}^{6} \lambda_i \left\langle N_0^\dagger \mid \left(\frac{\chi_i}{4\pi} C_i \right) \right\rangle + \left\langle N_0^\dagger \mid S \right\rangle \\
\left\langle N_0^\dagger \mid \dfrac{\partial \left(\chi_i C_i / 4\pi \right)}{\partial t} \right\rangle = \left\langle N_0^\dagger \mid \hat{\mathbf{M}}_i \varphi \right\rangle P - \lambda_i \left\langle N_0^\dagger \mid \left(\dfrac{\chi_i}{4\pi} C_i \right) \right\rangle
\end{cases}
$$

  - The normalization condition needs to be fulfilled

Iterations on the solution of the shape model are performed

Improved quasi-statics

- Iterative procedure for the shape update (1)
  - Solution of the shape model with known P and dP/dt:

$$\frac{\varphi^{(n+1)} - \varphi^{(n)}}{\Delta T_\varphi} P(T) + \varphi^{(n+1)} \left.\frac{dP}{dt}\right|_T = \hat{\mathbf{B}}\varphi^{(n+1)} P(T) + \sum_{i=1}^{6} \lambda_i \left(\frac{\chi_i}{4\pi} C_i^{(n+1)}\right) + S^{(n+1)}$$

$$\frac{\left(\chi_i C_i^{(n+1)}/4\pi\right) - \left(\chi_i C_i^{(n)}/4\pi\right)}{\Delta T_\varphi} = \hat{\mathbf{M}}_i \varphi^{(n+1)} P(T) - \lambda_i \left(\frac{\chi_i}{4\pi} C_i^{(n+1)}\right)$$

  - Renormalization of the shape

$$\tilde{\varphi}^{(n+1)} = \frac{\left\langle N_0^\dagger \mid \varphi_0 \right\rangle}{\left\langle N_0^\dagger \mid \varphi^{(n+1)} \right\rangle} \varphi^{(n+1)}$$

Check on error on shape

Improved quasi-statics

- Iterative procedure for the shape update (2)
  - Computation of kinetic parameters with $\tilde{\varphi}^{(n+1)}$
  - Modification of P (continuity of total power)

$$\left\langle \hat{\mathbf{M}}^{(n+1)} \tilde{\varphi}^{(n+1)} \right\rangle P^{(n+1)} = \left\langle \hat{\mathbf{M}}^{(n)} \varphi^{(n)} \right\rangle P^{(n)}$$

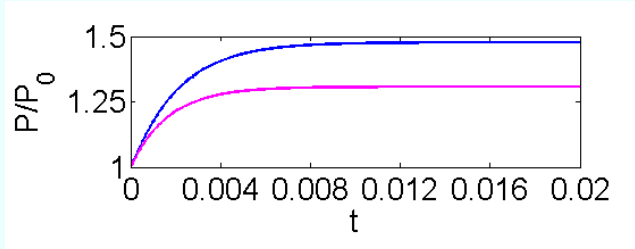
  - Modification of dP/dt (fulfillment of point model with updated kinetic parameters)

$$\begin{cases} \left.\dfrac{dP}{dt}\right|^{(n+1)} = \dfrac{\rho^{(n+1)} - \tilde{\beta}^{(n+1)}}{\Lambda} P^{(n+1)} + \sum_{i=1}^{6} \lambda_i \tilde{C}_i + \tilde{S} \\ \left.\dfrac{dC_i}{dt}\right|^{(n+1)} = \dfrac{\tilde{\beta}^{(n+1)}}{\Lambda} P^{(n+1)} - \lambda_i \tilde{C}_i \end{cases}$$

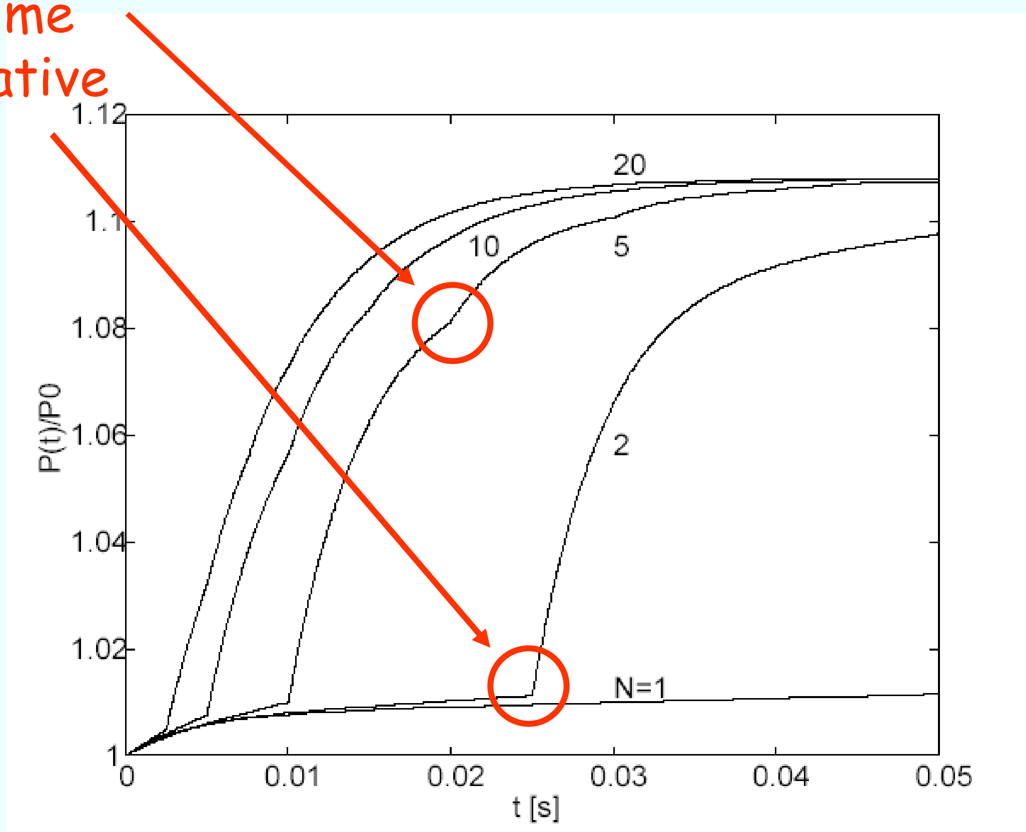
  - Substitution into the shape model and…

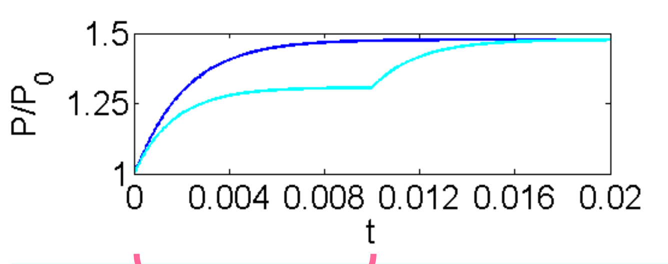
Improved quasi-statics - Results

- Improvement of the dynamic simulation of the transient



discontinuities of time derivative





ΔT_φ

Improved quasi-statics

- Characteristics of the method:
  - Spatial and spectral effects can be taken into account
  - Solution converges to reference when ΔT_φ is reduced
  - The method can allow to obtain high quality results with reduced computational time

BUT

  - The definition of the interval ΔT_φ largely influences the quality of the results (need of adaptive procedure)
  - The convergence of the shape is not always ensured
  - The iterative procedure of the shape update can be time consuming when large modifications of the shape are involved
  - The procedure can become too expensive computationally

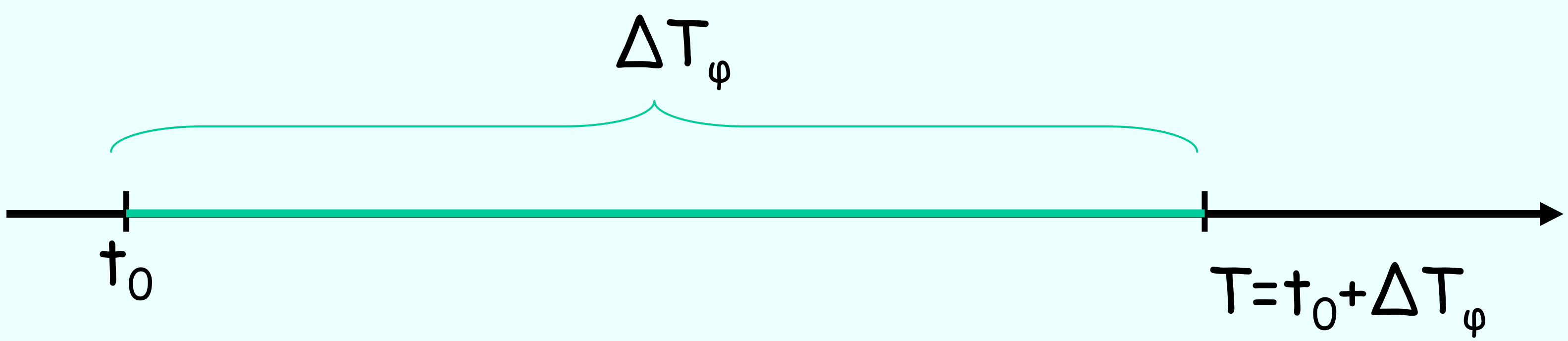
Needs for alternative numerical schemes
to avoid the non-linearity of the problem

Predictor-corrector quasi-statics

Predictor-Corrector quasi-statics

- Scheme for the solution of the quasi-static equation, avoiding the non linearity of the model
  - Solution of the balance model on the time mesh ΔTφ

$$\begin{cases} \dfrac{\partial n}{\partial t} = \hat{\mathbf{B}} n + \displaystyle\sum_{i=1}^{6} \lambda_i \frac{\chi_i}{4\pi} C_i + S \\ \dfrac{\partial \left(\chi_i C_i / 4\pi \right)}{\partial t} = \hat{\mathbf{M}}_i n - \lambda_i \dfrac{\chi_i}{4\pi} C_i \end{cases}$$

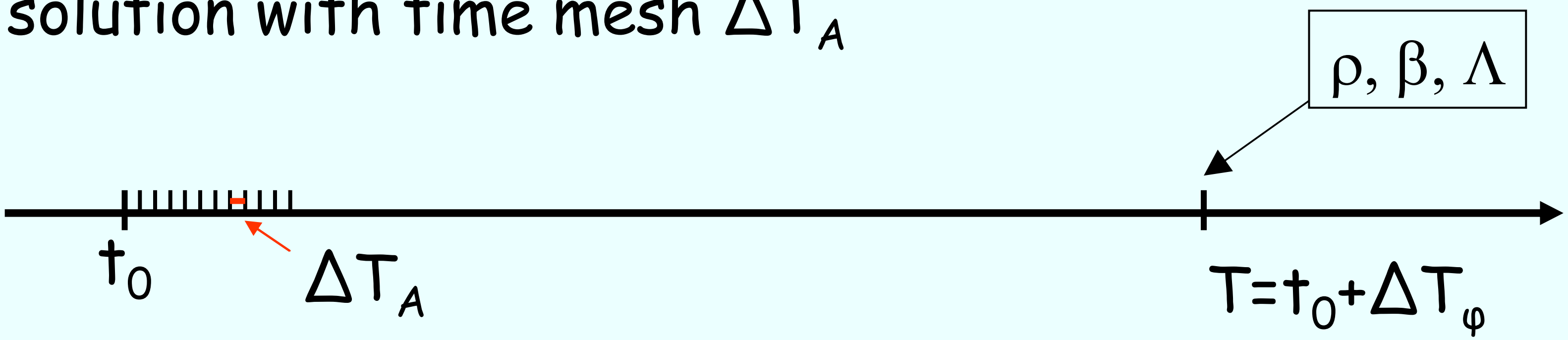


Predictor-Corrector quasi-statics

- Scheme for the solution of the quasi-static equation, avoiding the non linearity of the model (2)
  - Renormalization if the flux in order to obtain a proper shape function

$$\varphi^{(n+1)} = \frac{\left\langle N_0^\dagger \mid \varphi_0 \right\rangle}{\left\langle N_0^\dagger \mid n^{(n+1)} \right\rangle} n^{(n+1)}$$

  - Evaluation of kinetic parameters and point kinetic solution with time mesh ΔT_A

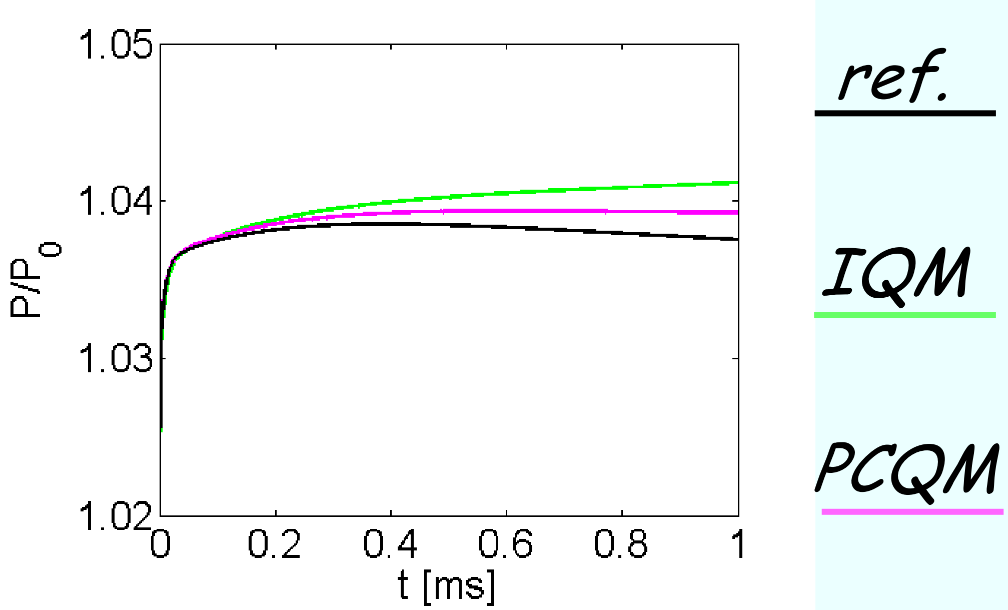


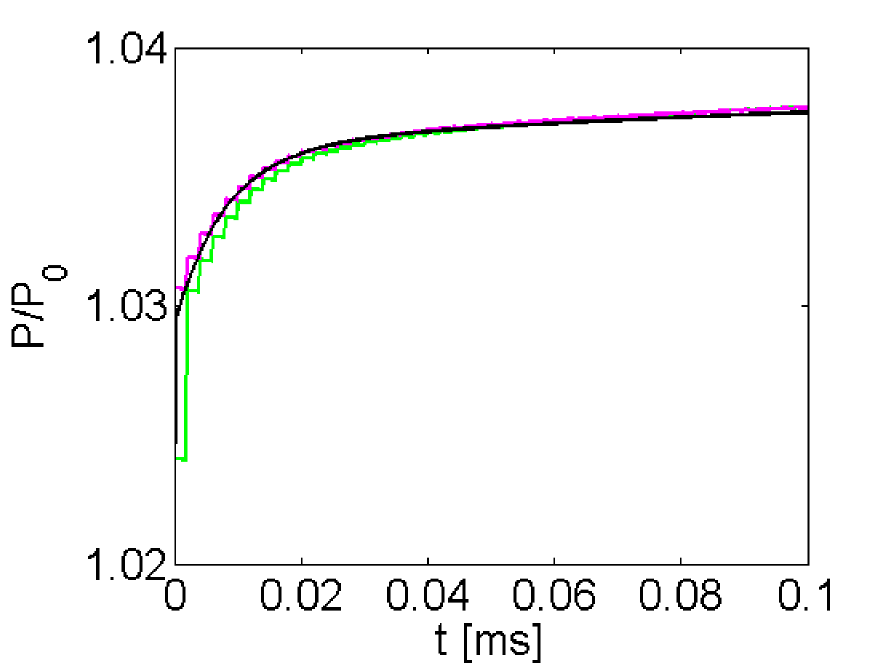
Predictor-Corrector quasi-statics

- Characteristics of PC quasi-statics
  - No iterations to fulfil normalization are required
  - Kinetic parameters used for point-kinetic calculations are more suitable to describe the transient during $\Delta T\varphi$ and can provide more accurate results
  - The computational effort can be effectively reduced with respect to IQM
  - When transients with large power effects and small shape modifications are involved, PCQM can fail in reducing computational time (point-like transients)

P-C quasi-statics - Results

Convergence of IQM and PCQM to the reference solution





Different performance of kinetic parameters generated with IQM and PCQM

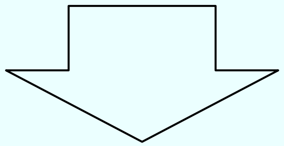
Further improvements

- The factorization procedure can be improved, subdividing the domain in several regions of the phase space
- This approach can be very effective when loosely coupled systems are concerned

Multipoint method

The multipoint method

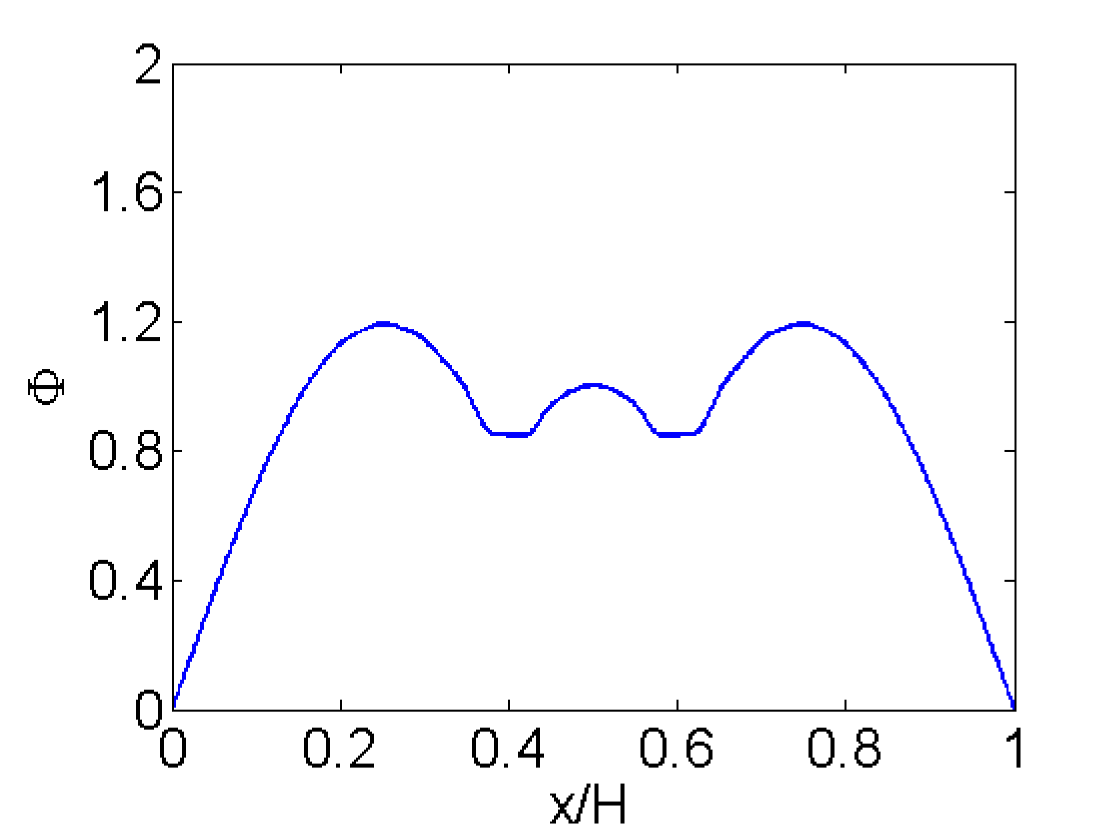
- The method can be viewed as an extension of the point kinetic model
- The domain considered in the phase space is subdivided in K (reasonably small) regions (points)
- The neutron density in each region is factorized in a product of amplitude and shape



- K point-like systems for the amplitudes P_K are obtained, all coupled by integral coefficients obtained by the projection technique

The multipoint method

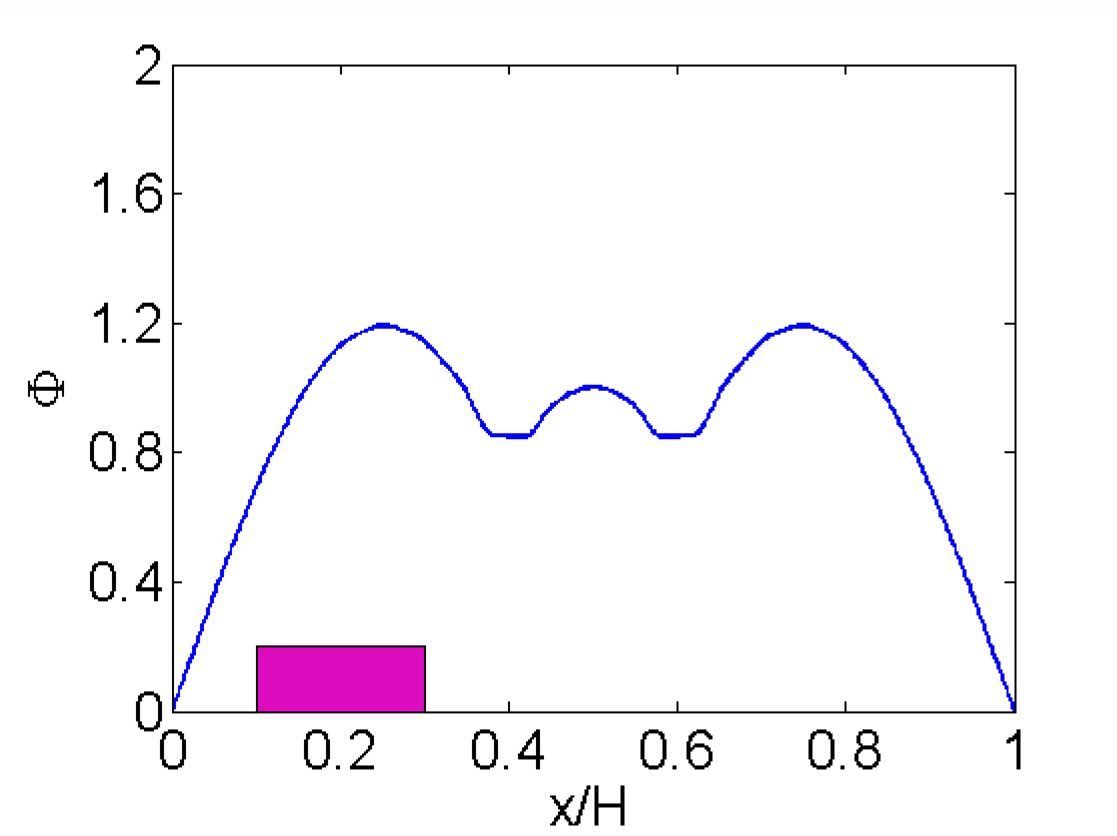
- Example of the multipoint phylosophy



Initial neutron density

The multipoint method

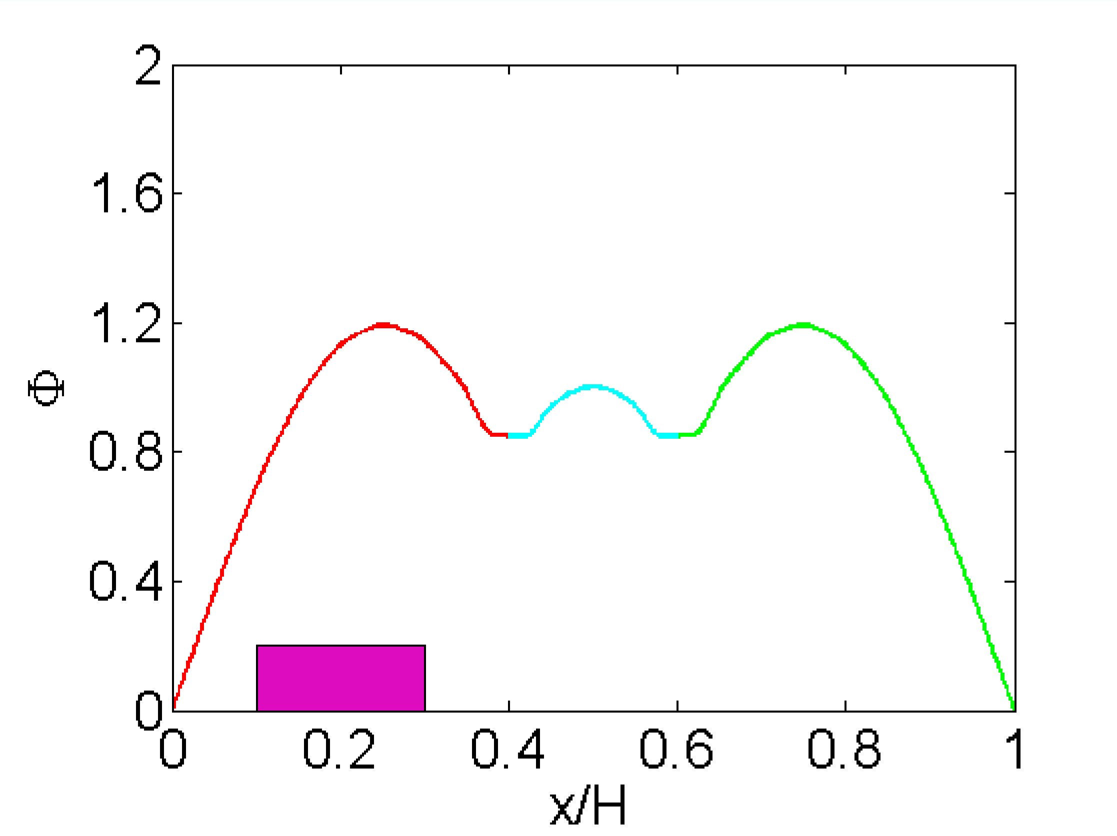
- Example of the multipoint phylosophy



Localized perturbation

The multipoint method

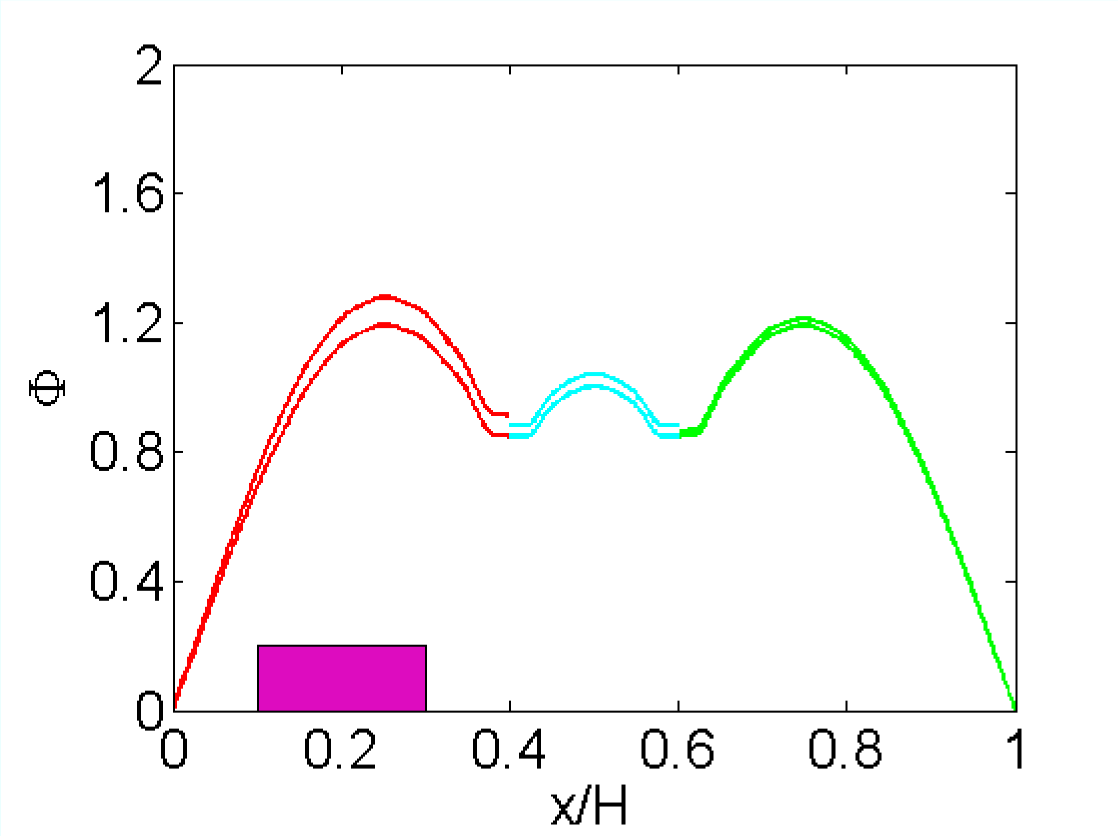
- Example of the multipoint phylosophy



Different regions are affected differently by the perturbation…

The multipoint method

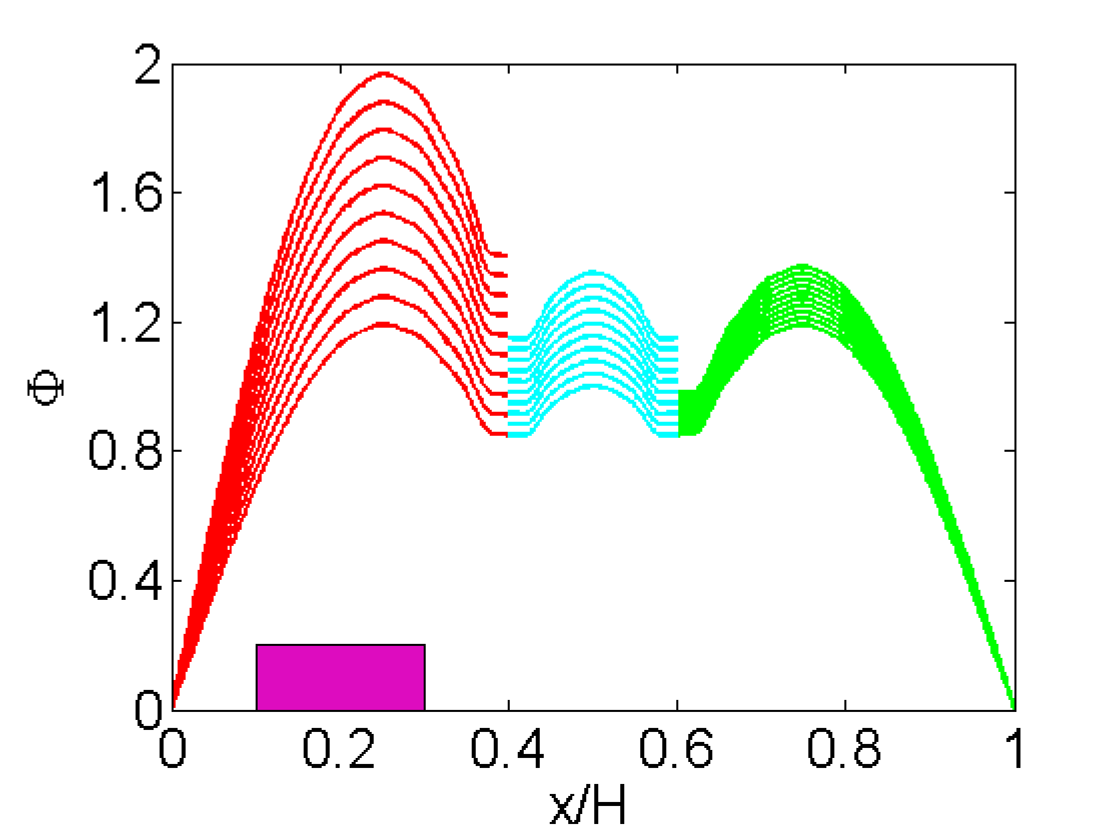
- Example of the multipoint phylosophy



… and are simulated with different amplitude functions

The multipoint method

- Example of the multipoint phylosophy



… and are simulated with different amplitude functions

The multipoint method

Balance equations in discretized form and phase-space subdivision:

$$\begin{cases} \dfrac{1}{v_m}\dfrac{d\phi_{nm}}{dt} = \sum_{n'}\sum_{m'} k_{nm,n'm'}\phi_{n'm'} + \\ \qquad \sum_{i=1}^{6} \lambda_i \chi_{i,m} C_{i,n} + S_{nm} \\ \dfrac{dC_{i,n}}{dt} = \beta_i \sum_{m'} f_{nm'}\phi_{nm'} - \lambda_i C_{i,n} \qquad i = 1, 2, ..., 6 \end{cases}$$

$$\phi_{nm}(t) = \phi(\mathbf{r}_n, V_m, t) \qquad C_{i,n}(t) = C_i(\mathbf{r}_n, t)$$

$$\phi_{nm}(t) = A_{NM}(t)\varphi_{nm}(t) \qquad \mathbf{r}_n, V_m \in \Gamma_{NM}$$

The multipoint method

Regionwise inner products $\langle w|\, g\rangle = \left[\sum_n \sum_m\right]_{NM} w_{nm} g_{nm}$

Introduce factorization (shape equations – known amplitudes):

$$\begin{cases} \dfrac{1}{v_m}\varphi_{nm}\dfrac{dA_{NM}}{dt} + \dfrac{1}{v_m}A_{NM}\dfrac{d\varphi_{nm}}{dt} = \\ \displaystyle\sum_{N'}\sum_{M'}\left[\sum_{n'}\sum_{m'}\right]_{N'M'} k_{nm,n'm'}\varphi_{n'm'}A_{N'M'} + \\ \qquad \displaystyle\sum_{i=1}^{6}\lambda_i\chi_{i,m}C_{i,n} + S_{nm} \\ \\ \dfrac{dC_{i,n}}{dt} = \beta_i \displaystyle\sum_{M'}\left[\sum_{m'}\right]_{M'} f_{nm'}\varphi_{nm'}A_{NM'} - \lambda_i C_{i,n} \\ \qquad i = 1, 2, ..., 6, \quad \mathbf{r}_n, V_m \in \Gamma_{NM} \end{cases}$$

The multipoint method

Project on weight (amplitude equation – known shape):

$$
\begin{cases}
\dfrac{dA_{NM}}{dt} = \sum_{N'} \sum_{M'} K_{NM,N'M'} A_{N'M'} + \\
\qquad \sum_{i=1}^{6} \lambda_i C_{i,NM} + S_{NM} \\
\dfrac{dC_{i,NM}}{dt} = \beta_i \sum_{M'} F_{i,NM,M'} A_{NM'} - \lambda_i C_{i,NM} \\
\quad i = 1, 2, ..., 6,
\end{cases}
$$

The multipoint method

Normalization condition (its application may require iteration):

$$\frac{d}{dt}\left[\sum_n \sum_m\right]_{NM} w_{nm}\frac{1}{v_m}\varphi_{nm}(t) = \frac{d}{dt}\gamma_{NM} = 0$$

Kinetic effective parameters and source are introduced

Multipoint effective terms

$$K_{NM,N'M'} = \frac{1}{\gamma_{NM}} \left[\sum_n \sum_m \right]_{NM} \left(w_{nm} \left[\sum_{n'} \sum_{m'} \right]_{N'M'} k_{nm,n'm'} \varphi_{n'm'} \right)$$

coupling terms

effective source

$$S_{NM} = \frac{1}{\gamma_{NM}} \left[\sum_n \sum_m \right]_{NM} w_{nm} S_{nm}$$

$$C_{i,NM} = \frac{1}{\gamma_{NM}} \left[\sum_n \sum_m \right]_{NM} w_{nm} \chi_{i,m} C_{i,n}$$

effective delayed concentration

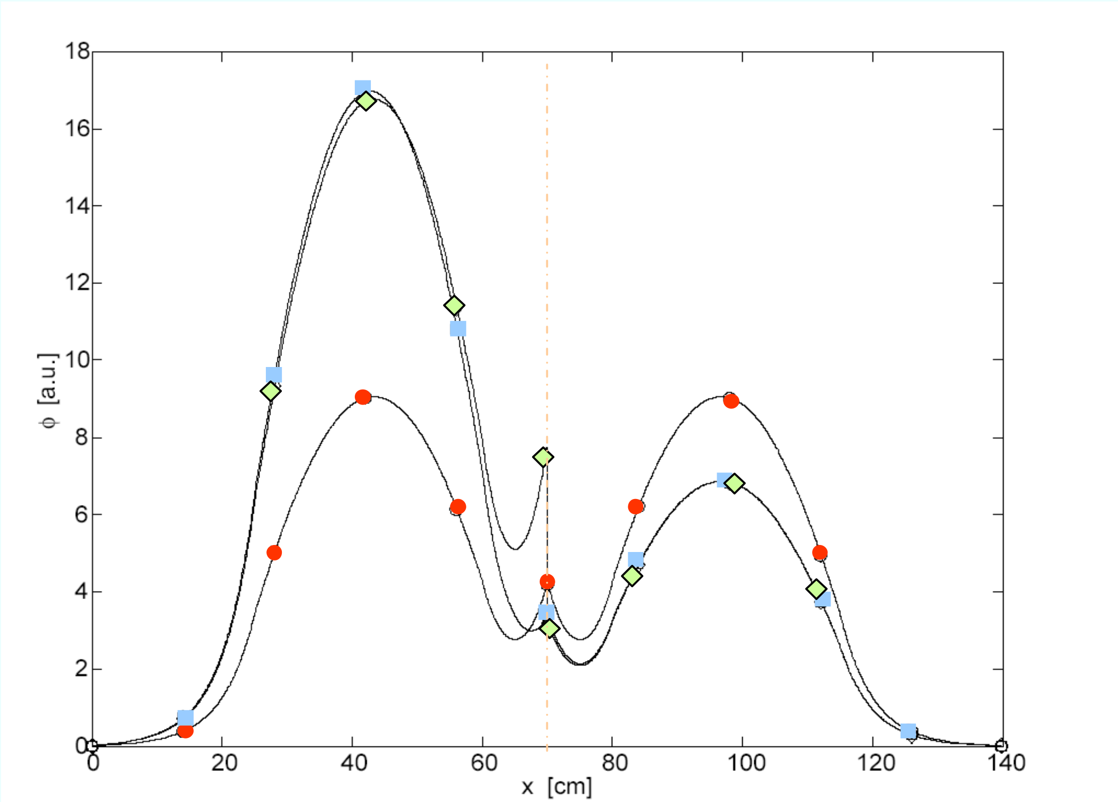
delayed fission term

$$F_{i,NM,M'} = \frac{1}{\gamma_{NM}} \left[\sum_n \sum_m \right]_{NM} w_{nm} \chi_{i,m} \beta_i \left[\sum_{m'} \right]_{M'} f_{nm'} \varphi_{nm'}$$

Multipoint features

Multipoint can be used in quasi-statics
Graph to show features of multipoint:

Circle (●) : PK
Square (■): exact
Diamond (◇): 2-point



Effect of choice of points

Different subdivision of the phase space has influence on the accuracy of the results

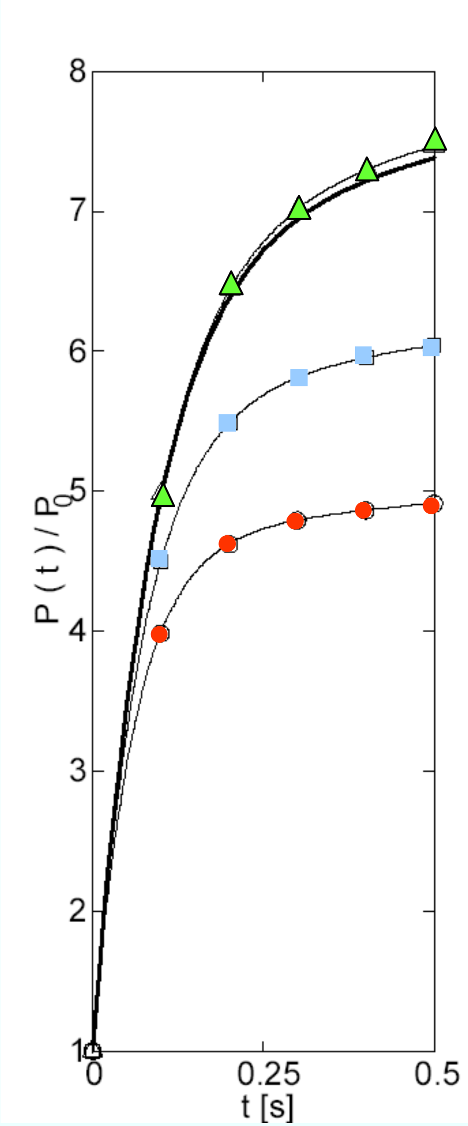
Bold: exact

Circle (●) : PK

Square (■): 2-point

Triangle (▲): 2-point

■ and ▲ are characterized by different subdivisions of the spatial domain



Effect of choice of points

- The different subdivision of the spatial domain are evidenced

  Bold: exact

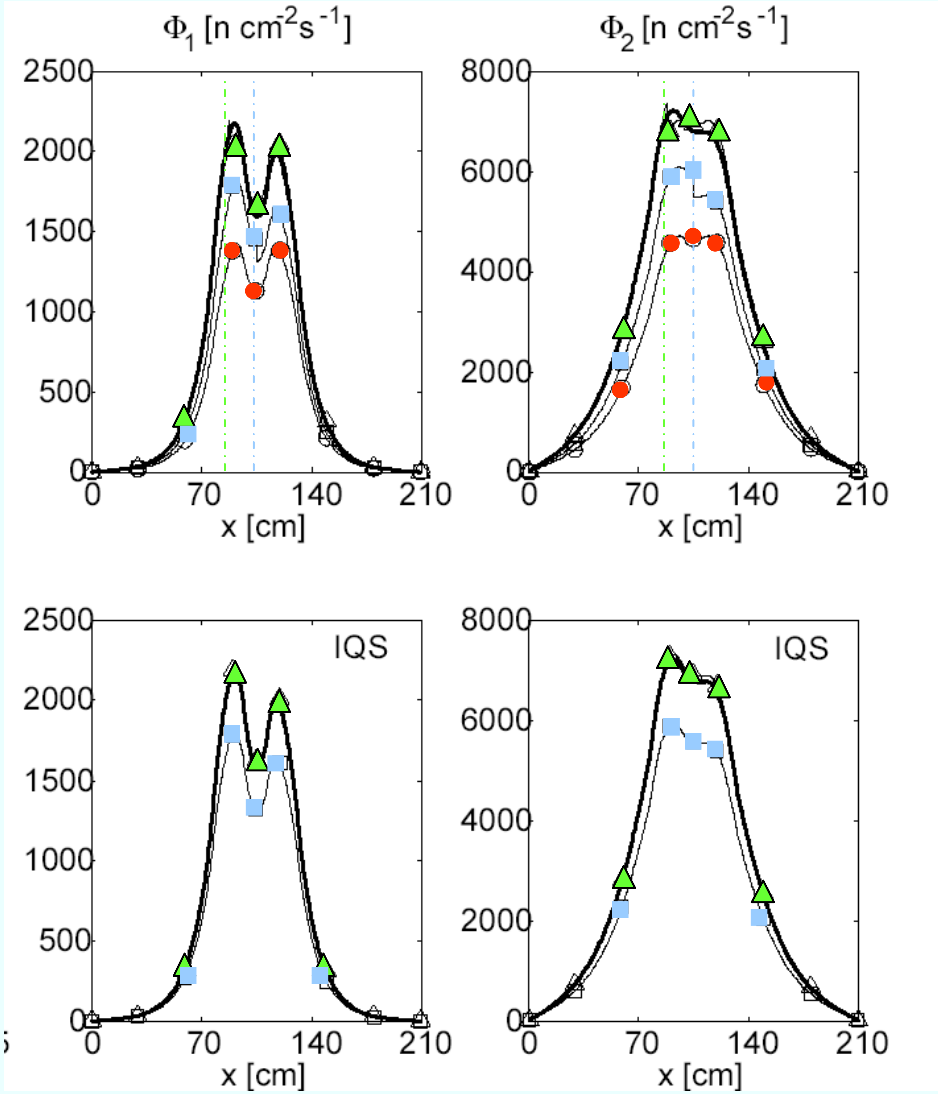
  Circle (•) : PK

  Square (▪): 2-point

  Triangle (▲): 2-point

- Update of shape functions through quasi-static procedure

  Continuity of fluxes



Time-dependent transport models

- <u>Transport effects in source-driven experiments</u>
  - study of basic phenomena
  - evaluation of limits of diffusion theory referring to reference configurations
  - study of propagation phenomena in pulsed experiments:
    - Analysis of sharp space-energy wave-front appearance
    - Identification of limits of low-order transport models
    - Separate analysis of model- and numerically-induced effects
    - Analytical validation tools for numerical schemes in the solution of time-dependent transport problems
    - Application to subcritical source-driven systems -> neutron pulsed-source

Analytical approach to kinetic models

Propagation phenomena can be properly accounted for by solving transport models

Signal transmission at finite velocity

(while diffusion models propagate signals at infinite velocity)

The analytical approach allows to produce benchmark solutions, extremely useful when dealing with innovative systems (e.g. Accelerator-Driven Systems)

Objective of the work

- Study the propagation of a neutron pulse adopting different transport models
  - analytical approach for the time integration of the problem
  - analytical treatment of space and angle dependences (when possible!)
- Define a "reference" solution (exact), useful to evaluate the approximations intoduced by different transport models
- Perform the analysis of the effect of space and angle discretization without any time-discretization effect

Objective of the work

- Separate physical and numerically-induced effects:
  - Physical effects
    - Propagation at finite speed
    - High frequency effects
  - Model effects
    - Wave-like behavior of P_N models (telegrapher's equation)

      Time-dependent ray-effects
  - Numerically-induced effects
    - Ray-effects in space (multi-D S_N)
    - Oscillations due to spatial discretization schemes

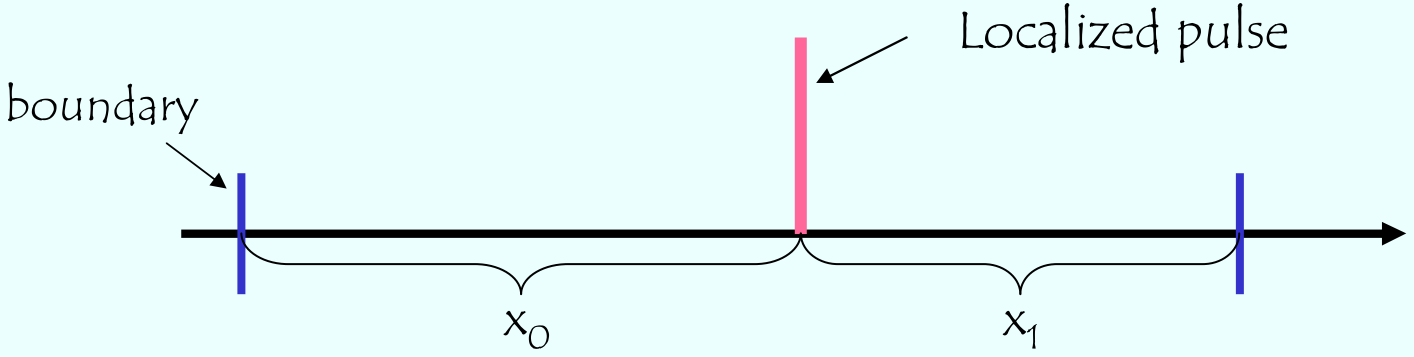
A reference solution: the space asymptotic method

- Approach to transport equation in the frequency domain by superposition of spatial waves
  - series involving Helmholtz eigenfunctions vanishing at the physical boundary of the system (complete set!)
- unable to correctly describe the exact boundary behavior of the neutron flux, but...

A reference solution: the space asymptotic method

... can describe exactly the propagation of a localized pulse for all times shorter than the time taken by neutrons to reach the boundary

$$t<x_1/v$$



A reference solution: the space asymptotic method

Consider the transport equation in 1D:

$$\frac{1}{v}\frac{\partial\varphi(x,\mu,t)}{\partial t}+\mu\frac{\partial\varphi(x,\mu,t)}{\partial x}+\sigma\varphi(x,\mu,t)=\frac{c\sigma}{2}\int_{-1}^{1}d\mu'\varphi(x,\mu',t)+\frac{1}{2}S(x,t)$$

perform the Laplace transform with respect to the time variable

$$\frac{s}{v}\varphi(x,\mu,s)+\mu\frac{\partial\varphi(x,\mu,s)}{\partial x}+\sigma\varphi(x,\mu,s)=\frac{c\sigma}{2}\int_{-1}^{1}d\mu'\varphi(x,\mu',s)+\frac{1}{2}S(x,s)$$

and the Fourier transform for the space variable (infinite medium)

$$\varphi(B,\mu,s)\left[\left(\sigma+\frac{s}{v}\right)-iB\mu\right]=\frac{c\sigma}{2}\int_{-1}^{1}d\mu'\varphi(B,\mu',s)+\frac{1}{2}S(B,s)$$

A reference solution: the space asymptotic method

Starting from this formulation

$$\varphi(B,\mu,s)\left[\left(\sigma+\frac{s}{v}\right)-iB\mu\right]=\frac{c\sigma}{2}\int_{-1}^{1}d\mu'\varphi(B,\mu',s)+\frac{1}{2}S(B,s)$$

different approaches are possible:

- Expansion of angular variable → spherical harmonics (approximated to order L)
- Integration of the equation over μ , after recasting (no further approximation introduced)

$$\int_{-1}^{1}d\mu\,\varphi(B,\mu,s)\quad \Phi(B,s)=\frac{A_{00}(B,s)}{1-c\sigma A_{00}(B,s)}S(B,s)\quad \ldots\,\mu'\varphi(B,\mu',s)+\int_{-1}^{1}d\mu\,S(B,s)\Big]$$

$A_{00}(B,s)$

A reference solution: the space asymptotic method

- Considering a 1D symmetric system, the source term can be expanded in terms of Fourier-transformed Helmholtz eigenfunctions (vanishing on the physical boundary of the system)

$$S(B,s)=\sum_{n=1}^{\infty} s_n(s)\sqrt{\frac{2}{a}}\frac{1}{2}\left[\delta(B-B_n)+\delta(B+B_n)\right]$$

and perform the inverse transforms:

Fourier

$$\Phi(x,s)=\sum_{n=1}^{\infty}\frac{A_{00}(B_n,s)}{1-c\sigma A_{00}(B_n,s)}s_n(s)\sqrt{\frac{2}{a}}\cos(B_n x)$$

Laplace

$$\Phi(x,t)=\sum_{n=1}^{\infty}\left\{\int dt'\left[\frac{1}{2\pi i}\int_{c_n-i\infty}^{c_n+i\infty} ds\frac{A_{00}(B_n,s)}{1-c\sigma A_{00}(B_n,s)}e^{s(t-t')}\right]s_n(t')\right\}\sqrt{\frac{2}{a}}\cos(B_n x)$$

A reference solution: the space asymptotic method

- Focusing on the Laplace transform, the function:

$$\Gamma(B_n, s) = \frac{A_{00}(B_n, s)}{1 - c\sigma A_{00}(B_n, s)}$$

where

$$A_{00}(B, s) = \frac{1}{2}\int_{-1}^{1} d\mu \frac{1}{\left(\sigma + \frac{s}{v}\right) - iB\mu} = \frac{1}{B}\arctan\frac{B}{\sigma + \frac{s}{v}} = \frac{1}{2i}\log\left[\frac{\sigma + s/v + iB_n}{\sigma + s/v - iB_n}\right]$$

needs to be studied, to determine its singularities:

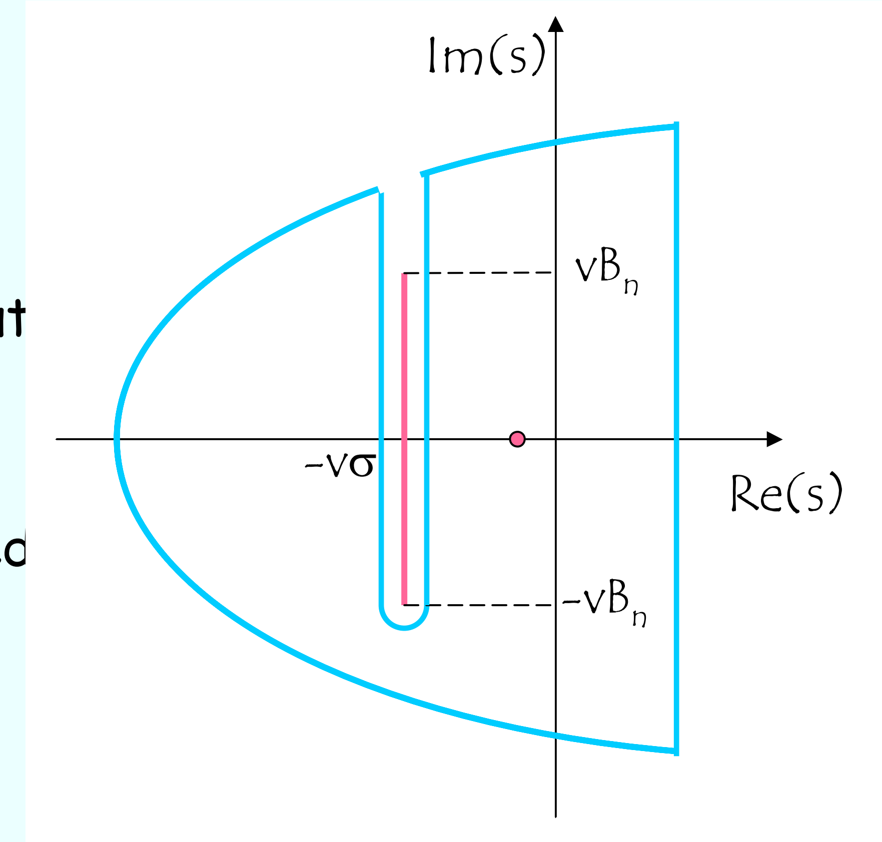
- Location of poles (discrete spectrum) $1 - c\frac{\sigma}{B_n}\arctan\frac{B_n}{\sigma + \frac{s}{v}} = 0$
- Presence of continuum spectrum

$$\mathrm{Im}\left[\frac{\sigma + s/v + iB_n}{\sigma + s/v - iB_n}\right] = 0 \qquad \mathrm{Re}\left[\frac{\sigma + s/v + iB_n}{\sigma + s/v - iB_n}\right] < 0$$

A reference solution: the space asymptotic method

Integral in the complex plane corresponding to each B_n:

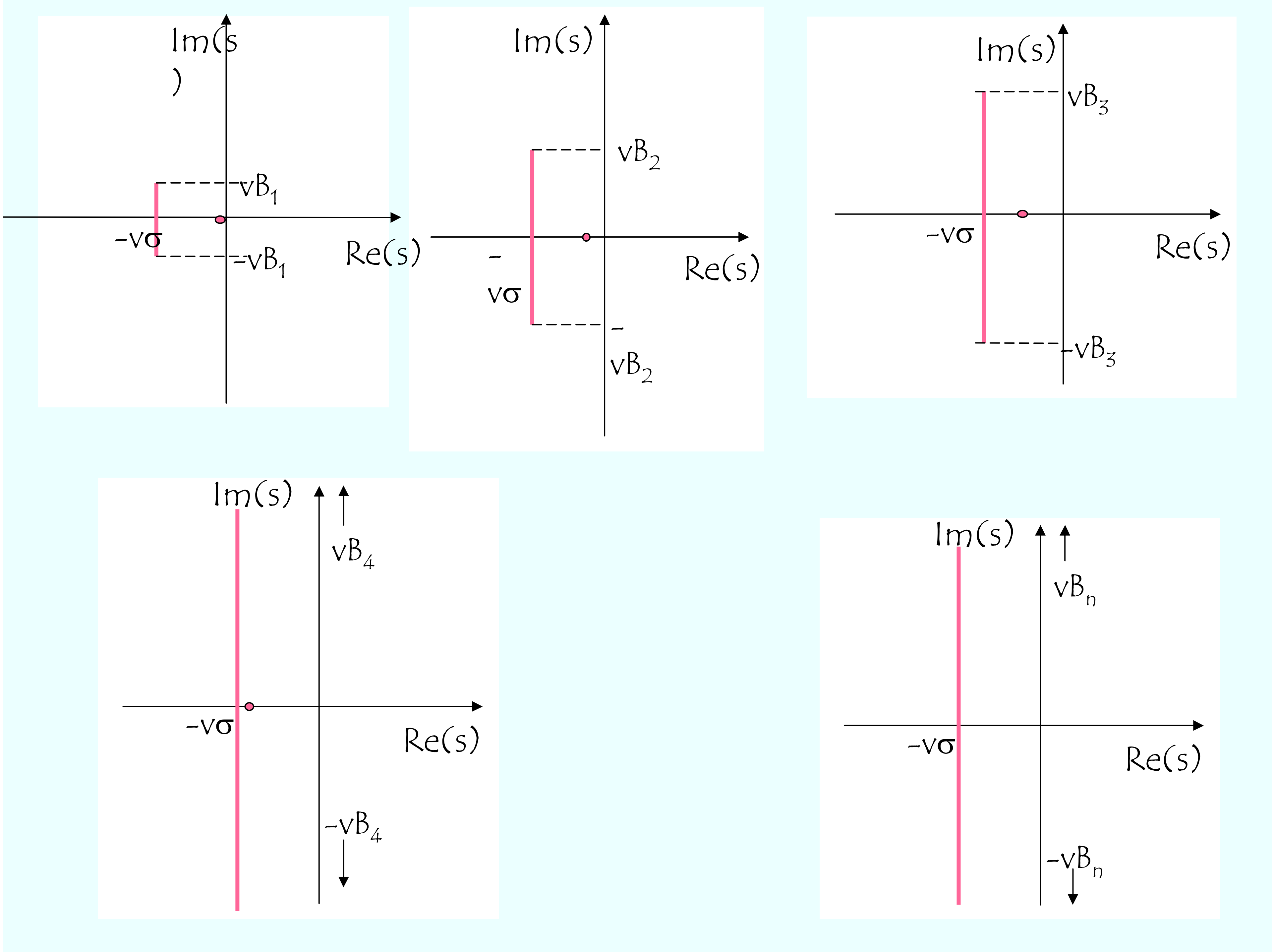
- Real pole s_n;
- Cut from $(-v\sigma-vB_n)$ to $(-v\sigma+vB_n)$



NOTE 1: for increasing n
the singularity moves towards the cut and disappears

NOTE 2: critical condition is obtained when the pole corresponding to the first harmonics is equal to zero:

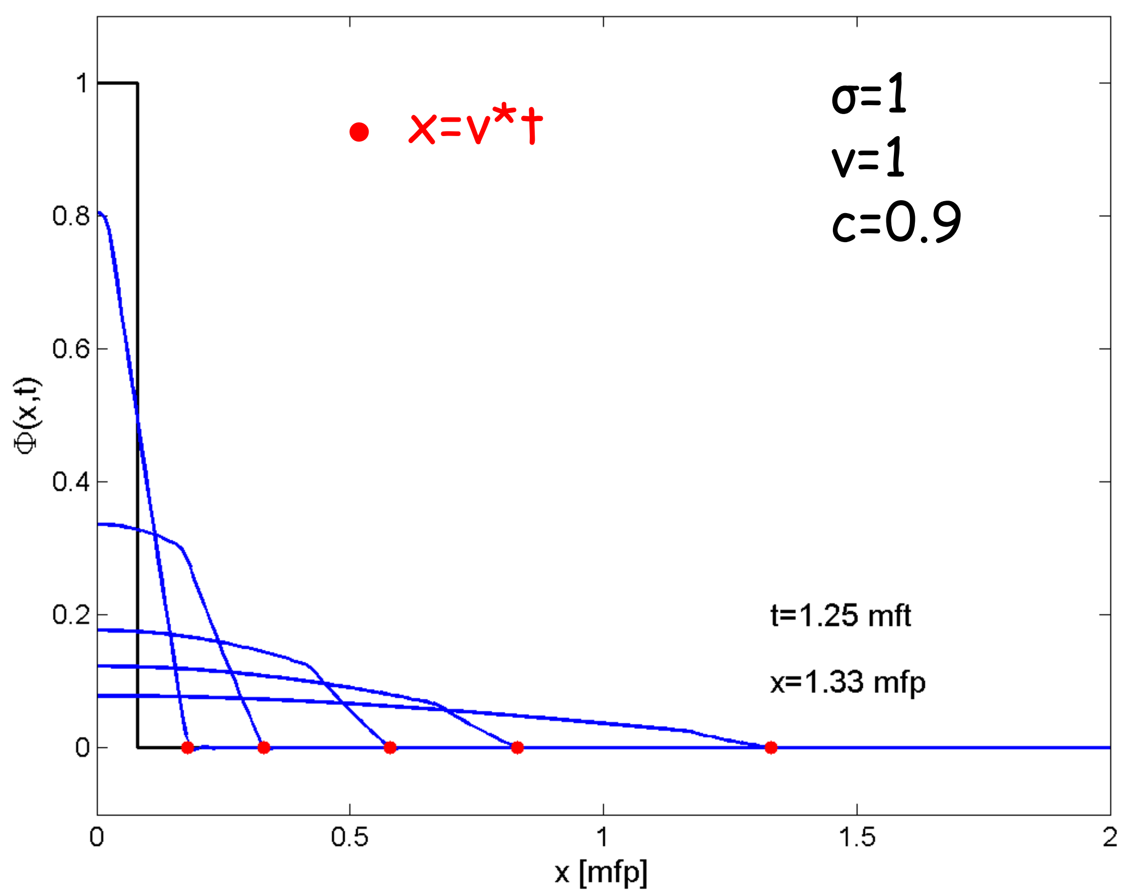
$$1 - c\sigma A_{00}(B_1, 0) = 1 - c\sigma \frac{1}{B_1} \arctan \frac{B_1}{\sigma} = 0$$


Im(s)
vB1
-vσ
-vB1
Re(s)
Im(s)
vB2
-vσ
Re(s)
-vB2
Im(s)
vB3
-vσ
Re(s)
-vB3
Im(s)
vB4
-vσ
Re(s)
-vB4
Im(s)
vBn
-vσ
Re(s)
-vBn

Space asymptotic method: propagation of a source pulse

- Slab geometry
- Isotropic space-localized pulsed source (initial condition for the flux)
- Isotropic scattering
- The solution obtained represents the "reference" case, useful to compare to different transport approximations and diffusion model

Space asymptotic method: propagation of a source pulse



Solution of P_N and S_N models

- S_N and P_{N-1} are proved to be equivalent in 1D configurations (with special care to the choice of the boundary conditions for the P_N model)

Differences are introduced by the discretization/solution schemes adopted

➲ The solution of the P_N model can be approached in the asymptotic framework starting from the telegrapher's equation

➲ The S_N model is solved by spatial discretization of the unknown and analytical time integration

Solution of S_N model in 1-D configurations

- S_N model

$$
\begin{cases}
\dfrac{1}{v}\dfrac{\partial \varphi_{j+}}{\partial t} + \mu_j \dfrac{\partial \varphi_{j+}}{\partial x} + \sigma \varphi_{j+} = \dfrac{c\sigma}{2} \displaystyle\sum_{j=1}^{N/2} w_j \left(\varphi_{j+} + \varphi_{j-}\right) + \dfrac{S}{2} \\
\dfrac{1}{v}\dfrac{\partial \varphi_{j-}}{\partial t} - \mu_j \dfrac{\partial \varphi_{j-}}{\partial x} + \sigma \varphi_{j-} = \dfrac{c\sigma}{2} \displaystyle\sum_{j=1}^{N/2} w_j \left(\varphi_{j+} + \varphi_{j-}\right) + \dfrac{S}{2} \\
\qquad\qquad j = 1, 2, ..., N/2,
\end{cases}
$$

With initial and boudary conditions (vacuum):

$$
\varphi_{j+}(-H/2, t) = 0, \qquad \varphi_{j-}(H/2, t) = 0, \qquad j = 1, 2, ..., N/2
$$

Solution of P_N model in 1-D configurations

- P_1 model

$$\begin{cases} \frac{1}{v}\frac{\partial \phi_0(x,t)}{\partial t} + \sigma_a \phi_0(x,t) + \frac{\partial \phi_1(x,t)}{\partial x} = S(x,t) \\ \frac{3D}{v}\frac{\partial \phi_1(x,t)}{\partial t} + D\frac{\partial \phi_0(x,t)}{\partial x} + \phi_1(x,t) = 0 \end{cases}$$

- Boundary conditions:
  - Marshak (zero incoming current)

$$\phi_0(\pm\frac{a}{2},t) \mp 2\phi_1(\pm\frac{a}{2},t) = 0$$

$$\int_0^1 \left(\frac{1}{2}\phi_0(-\frac{a}{2},t) + \frac{3}{2}\mu\phi_1(-\frac{a}{2},t)\right)\mu d\mu = \int_{-1} \left(\frac{1}{2}\phi_0(\frac{a}{2},t) + \frac{3}{2}\mu\phi_1(\frac{a}{2},t)\right)\mu d\mu = 0$$

  - Mark (zero incoming flux)

$$\phi_0(\pm\frac{a}{2},t) \mp \sqrt{3}\phi_1(\pm\frac{a}{2},t) = 0$$

Consistent with BC in S_2 model

$$\frac{1}{2}\phi_0(-\frac{a}{2},t) + \frac{\sqrt{3}}{2}\phi_1(-\frac{a}{2},t) = \frac{1}{2}\phi_0(\frac{a}{2},t) - \frac{\sqrt{3}}{2}\phi_1(\frac{a}{2},t) = 0$$

Solution of S_N model in 1-D configurations

- Spatial discretization of the model:

1. Diamond difference

$$\begin{cases} \dfrac{1}{v}\dfrac{d\varphi_{j+}^{(i)}}{dt} + \dfrac{2\mu_j}{\Delta_i}\left(\varphi_{j+}^{(i)} - \varphi_{j+}^{(i-1/2)}\right) + \sigma\varphi_{j+}^{(i)} = \dfrac{c\sigma}{2}\displaystyle\sum_{j=1}^{N/2} w_j\left(\varphi_{j+}^{(i)} + \varphi_{j-}^{(i)}\right) + \dfrac{S^{(i)}}{2} \\ \dfrac{1}{v}\dfrac{d\varphi_{j-}^{(i)}}{dt} - \dfrac{2\mu_j}{\Delta_i}\left(\varphi_{j-}^{(i+1/2)} - \varphi_{j-}^{(i)}\right) + \sigma\varphi_{j-}^{(i)} = \dfrac{c\sigma}{2}\displaystyle\sum_{j=1}^{N/2} w_j\left(\varphi_{j+}^{(i)} + \varphi_{j-}^{(i)}\right) + \dfrac{S^{(i)}}{2} \\ \qquad j = 1, 2, ..., N/2, \qquad i = 1, 2, ..., I, \end{cases}$$

with transmission relation:

$$\varphi_{j\pm}^{(i)} = \frac{1}{2}\left(\varphi_{j\pm}^{(i-1/2)} + \varphi_{j\pm}^{(i+1/2)}\right)$$

Solution of S_N model in 1-D configurations

- Spatial discretization of the model:

 2. Linear discontinuous

$$\frac{1}{v}\frac{d}{dt}\frac{\varphi_{j+}^{i+1/2,L}+\varphi_{j+}^{i-1/2,R}}{2}+\mu_j\frac{\varphi_{j+}^{i+1/2,L}-\varphi_{j+}^{i-1/2,L}}{\Delta_i}+\sigma_i\frac{\varphi_{j+}^{i+1/2,L}+\varphi_{j+}^{i-1/2,R}}{2}$$

$$=\frac{c\sigma}{2}\sum_{j=1}^{N/2}w_j\left(\frac{\varphi_{j+}^{i+1/2,L}+\varphi_{j+}^{i-1/2,R}}{2}+\frac{\varphi_{j-}^{i+1/2,L}+\varphi_{j-}^{i-1/2,R}}{2}\right)+\frac{S^{i+1/2,L}+S^{i-1/2,R}}{4},$$

NOTE: time derivative is not discretized

→ system of first-order differential equations

→ solution of the problem in matrix form

Solution of S_N model in 1-D configurations

- Spatial discretization of the model:

$$\frac{1}{v}\frac{d}{dt}\frac{\varphi_{j-}^{i+1/2,L}+\varphi_{j-}^{i-1/2,R}}{2}-\mu_j\frac{\varphi_{j-}^{i+1/2,R}-\varphi_{j-}^{i-1/2,R}}{\Delta_i}+\sigma_i\frac{\varphi_{j-}^{i+1/2,L}+\varphi_{j-}^{i-1/2,R}}{2}$$
$$=\frac{c\sigma}{2}\sum_{j=1}^{N/2}w_j\left(\frac{\varphi_{j+}^{i+1/2,L}+\varphi_{j+}^{i-1/2,R}}{2}+\frac{\varphi_{j-}^{i+1/2,L}+\varphi_{j-}^{i-1/2,R}}{2}\right)+\frac{S^{i+1/2,L}+S^{i-1/2,R}}{4}$$

$$\frac{1}{v}\frac{d}{dt}\frac{\varphi_{j-}^{i+1/2,L}+2\varphi_{j-}^{i-1/2,R}}{3}-\mu_j\frac{\varphi_{j-}^{i+1/2,L}-\varphi_{j-}^{i-1/2,R}}{\Delta_i}+\sigma_i\frac{\varphi_{j-}^{i+1/2,L}+2\varphi_{j-}^{i-1/2,R}}{3}$$
$$=\frac{c\sigma}{2}\sum_{j=1}^{N/2}w_j\left(\frac{\varphi_{j+}^{i+1/2,L}+2\varphi_{j+}^{i-1/2,R}}{3}+\frac{\varphi_{j-}^{i+1/2,L}+2\varphi_{j-}^{i-1/2,R}}{3}\right)+\frac{S^{i+1/2,L}+2S^{i-1/2,R}}{6}.$$

NOTE: time derivative is not discretized
→ system of first-order differential equations
→ solution of the problem in matrix form

Solution of S_N model in 1-D configurations

Matrix form of the problem:

$$\frac{d\mathbf{X}}{dt} + \hat{A}\mathbf{X} = \mathbf{S}$$

X is the unknown flux vector

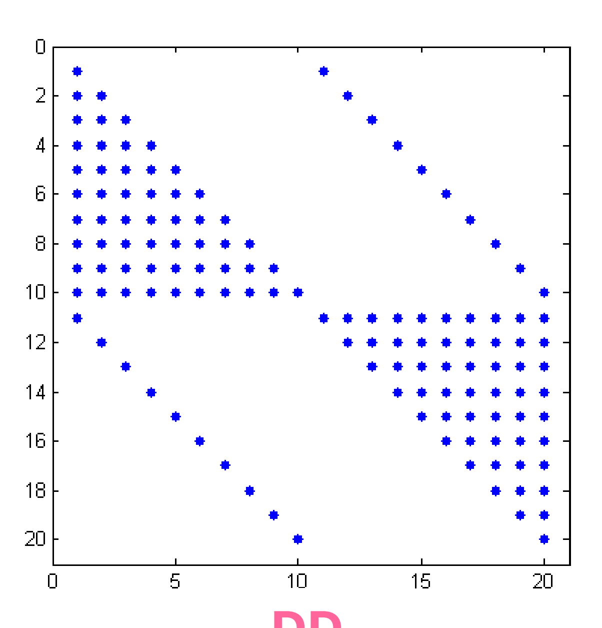
- DD: **X**=$\{\varphi^{(i)}{}_{j\pm}\}$, dimension I × N
- LD: **X**=$\{\varphi^{(i\pm 1/2)}{}_{j\pm}\}$, dimension 2I × N

- In DD the interface fluxes are eliminated through the transmission relation

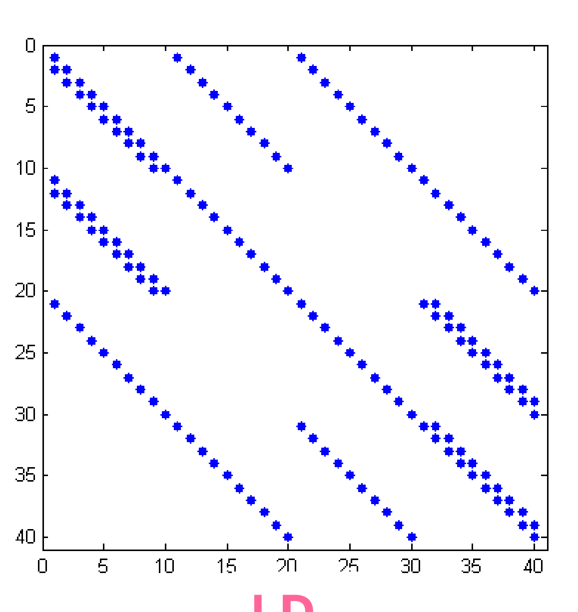
The solution for an initial condition is obtained analytically in terms of the eigenvectors of matrix Â:

$$\mathbf{X}(t) = \sum_{k=1}^{I\times N} c_k(t)\mathbf{u}_k = \sum_{k=1}^{I\times N} \left(c_k(0)\exp(-\omega_k t)\right)\mathbf{u}_k$$

$$= \sum_{k=1}^{I\times N} \left(\mathbf{u}_k^+ \cdot \mathbf{X}(0)\exp(-\omega_k t)\right)\mathbf{u}_k.$$



DD



LD

Solution of P_1 model in 1-D configurations

- P_1 model

$$\begin{cases} \frac{1}{v}\frac{\partial \phi_0(x,t)}{\partial t} + \sigma_a \phi_0(x,t) + \frac{\partial \phi_1(x,t)}{\partial x} = S(x,t) \\ \frac{3D}{v}\frac{\partial \phi_1(x,t)}{\partial t} + D\frac{\partial \phi_0(x,t)}{\partial x} + \phi_1(x,t) = 0 \end{cases}$$

with Mark boudary conditions:

$$\phi_0(\pm\frac{a}{2},t) \mp \sqrt{3}\phi_1(\pm\frac{a}{2},t) = 0$$

This problem can be given a different formulation:

telegrapher's equation

The telegrapher's equation

Consider the second equation of the P_1 model:

$$\frac{3D}{v}\frac{\partial \phi_1(x,t)}{\partial t} + D\frac{\partial \phi_0(x,t)}{\partial x} + \phi_1(x,t) = 0$$

- Steady state → Fick's law → diffusion model
- Time-dependent situation → integrate with respect to time:

$$\phi_1(x,t) = \cancel{\phi_1(x,0)e^{-\frac{v}{3D}t}} - D\int_0^t \frac{\partial \phi_0(x,t')}{\partial x}\frac{v}{3D}e^{-\frac{v}{3D}(t-t')}dt'$$

- If v→∞ we obtain the original Fick's law (infinite speed signal transmission)
- The current at time t depends on the gradient of the flux at preceding times, through the kernel v/3D exp(-(v/3D)t)

The telegrapher's equation

Substituting the integral in the first equation of the P_1 model we obtain the integro-differential form of the telegrapher's equation:

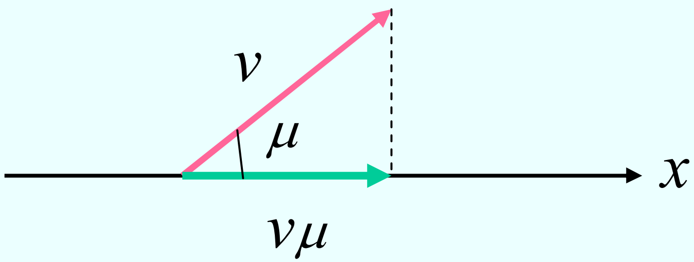
$$\frac{1}{v}\frac{\partial \phi_0(x,t)}{\partial t} + \sigma_a \phi_0(x,t) - \frac{v}{3D}\int_0^t D\frac{\partial^2 \phi_0(x,t')}{\partial x^2} e^{-\frac{v}{3D}(t-t')} dt' = S(x,t)$$

Alternatively, the telegrapher's equation can be derived in second-order formulation:

$$\frac{1}{v}\frac{\partial}{\partial t}\left(1 + \frac{3D}{v}\frac{\partial}{\partial t}\right)\Phi(x,t) = D\frac{\partial^2 \Phi(x,t)}{\partial x^2} + \left(1 + \frac{3D}{v}\frac{\partial}{\partial t}\right)\left[S(x,t) - \sigma_a \Phi(x,t)\right]$$

The telegrapher's equation

- Second-order time derivative → wave propagation at speed $v/\sqrt{3}$ (projection of the velocity on the x-axis)



  - The solution of the problem in asymptotic theory is equivalent to directly project the Laplace-transformed flux in the P_1 model on the Helmholtz eigenfunctions;

$$\Phi(x,s) = \sum_{n=0}^{\infty} A_n(s)\varphi_n(x)$$

  - The solution for the projection coefficients for the pulse propagation comes out as:

$$A_n(s) = \frac{\dfrac{3D}{v^2}p + \dfrac{1}{v}}{\dfrac{3D}{v^2}p^2 + \dfrac{1+3D\sigma_a}{v}p + (\sigma_a + DB_n^2)} A_n(0)$$

this expression can be derived alternatively….

The telegrapher's equation

… from the exact kernel of the transport equation and introducing P_1 hypotesis!!!!

$$\Gamma(B_n, s) = \frac{A_{00}(B_n, s)}{1 - c\sigma A_{00}(B_n, s)}$$

Number of directions of P_{M-1}-S_M

$$A_{00}(B_n, s) = \frac{1}{2}\int_{-1}^{1} d\mu \frac{1}{\left(\sigma + \frac{s}{v}\right) - iB_n\mu} \cong \sum_{m=1}^{M/2} \frac{\left(\sigma + \frac{s}{v}\right) w_m}{\left(\sigma + \frac{s}{v}\right)^2 + (B_n\mu_m)^2}$$

Then, for M=2, introducing the definitions of D and σ_a the kernel becomes:

$$\Gamma(B_n, s) = \frac{\left(\frac{3D}{v}s + 1\right)}{\frac{3D}{v^2}s^2 + (1 + 3D\sigma_a)\frac{s}{v} + (\sigma_a + DB_n^2)}$$

2 discrete poles in $\mathbb{C}$ for each n

No continuum spectrum

The diffusion model

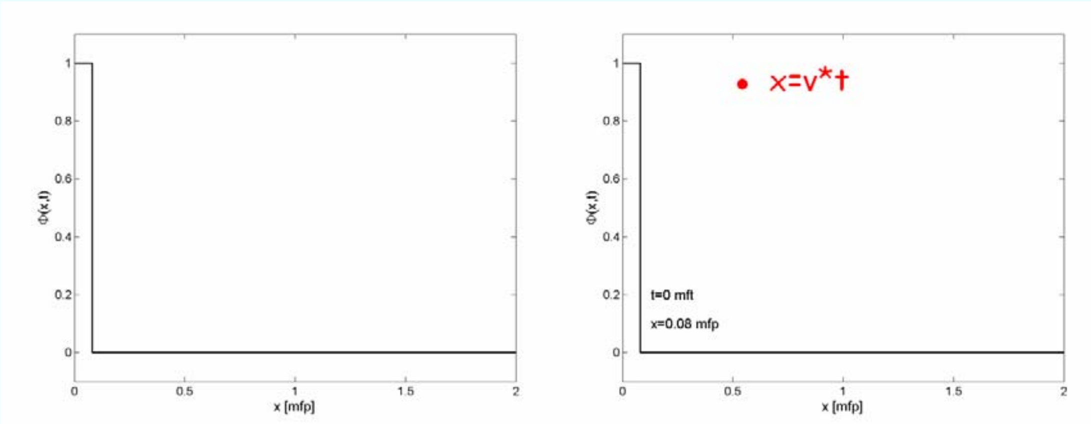
- Starting from the P1 model, the diffusion case can be derived, by adding some further hypotheses:
  - Infinite-velocity v→∞
  - Collision-dominated medium c→1

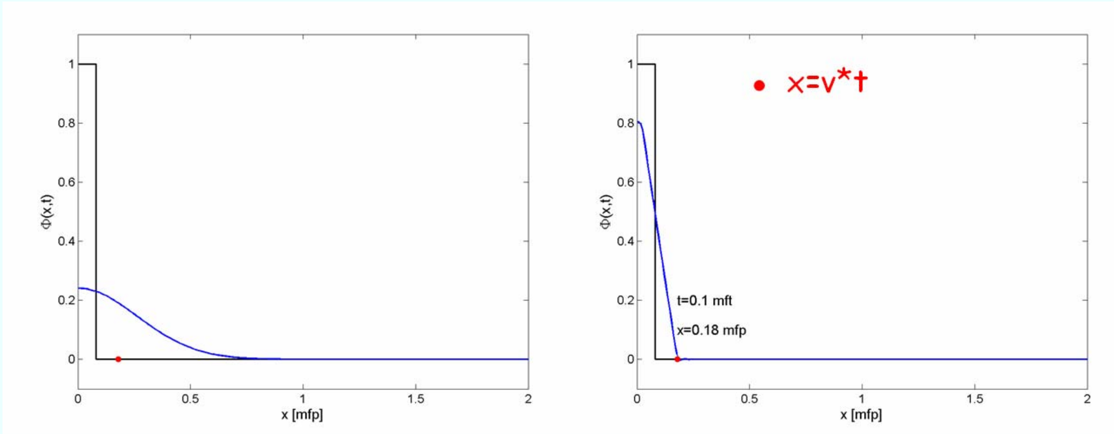
$$\Gamma(B_n, s) = \frac{1}{\frac{s}{v} + (\sigma_a + DB_n^2)}$$

1 discrete pole in R for each n

We now compare the "reference" result with P_1, S_2 (spatially discretized) and diffusion results

Propagation of a source pulse: asymptotic method vs. diffusion

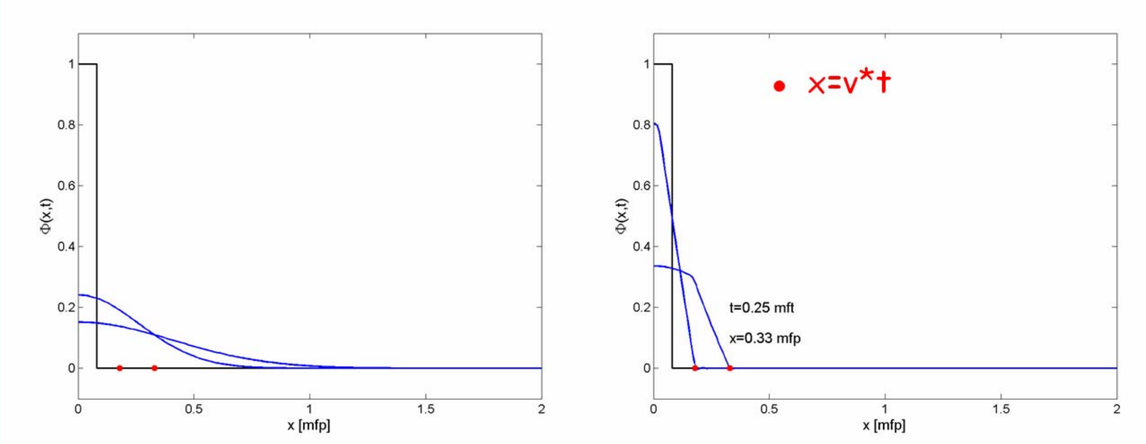


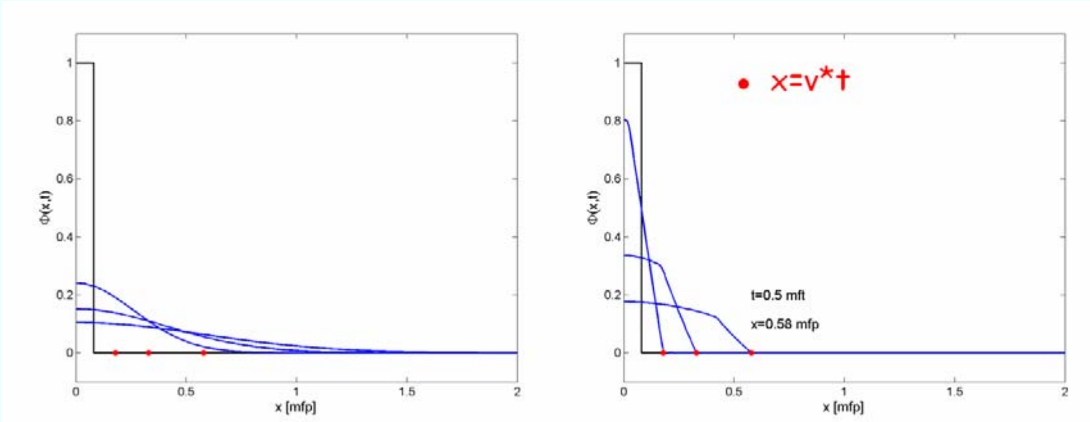


Diffusion is totally inadequate to describe this phenomenon

Superposition of real exponential ⇔ Propagation at infinite speed

Propagation of a source pulse: asymptotic method vs. diffusion





Diffusion is totally inadequate to describe this phenomenon

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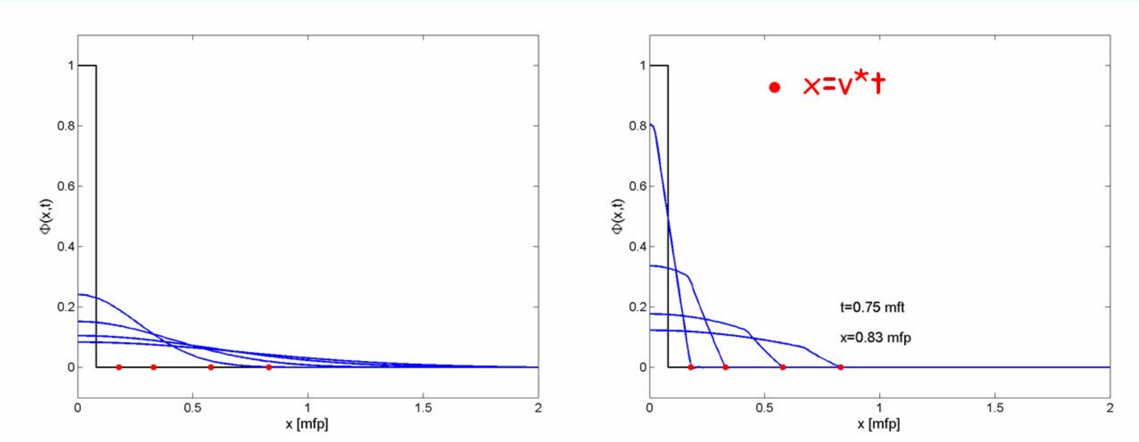

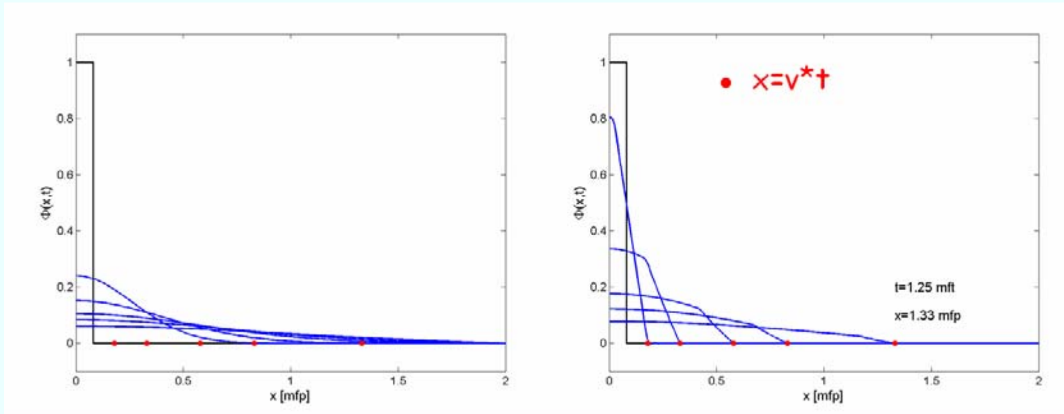

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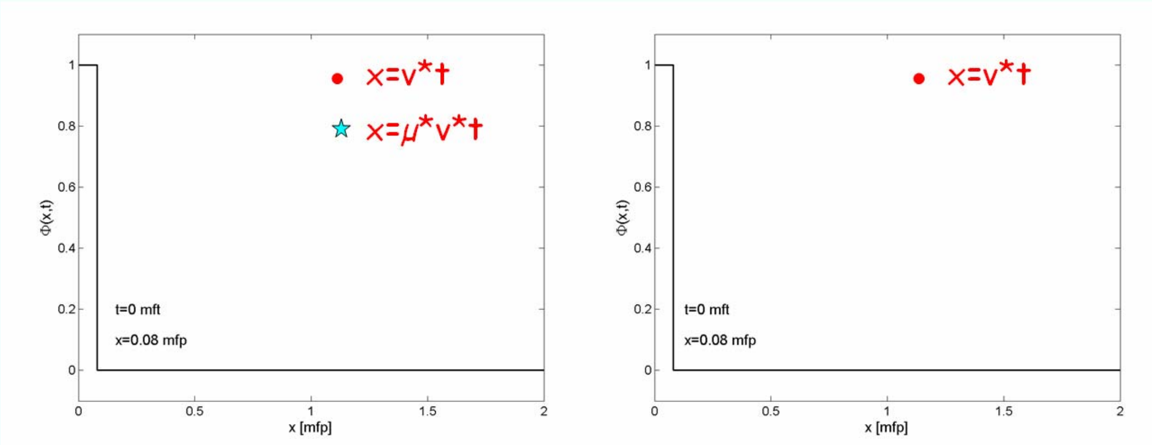
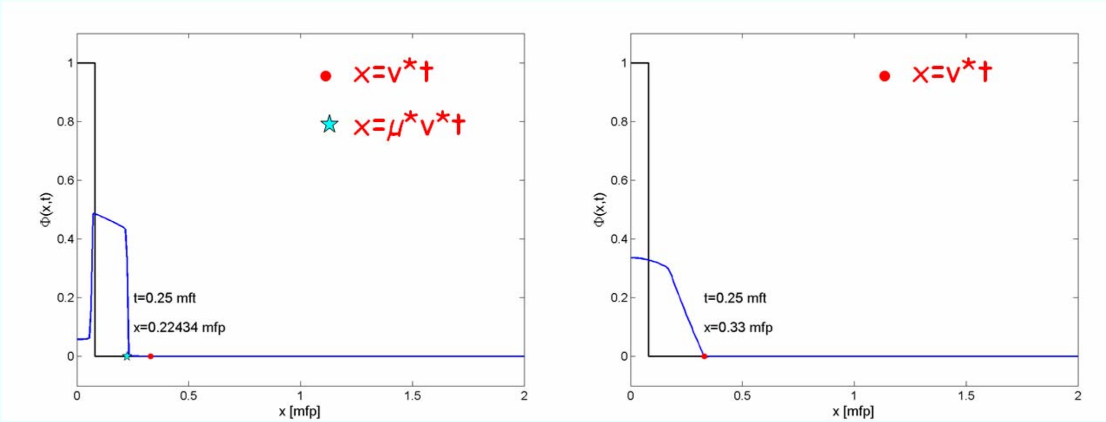
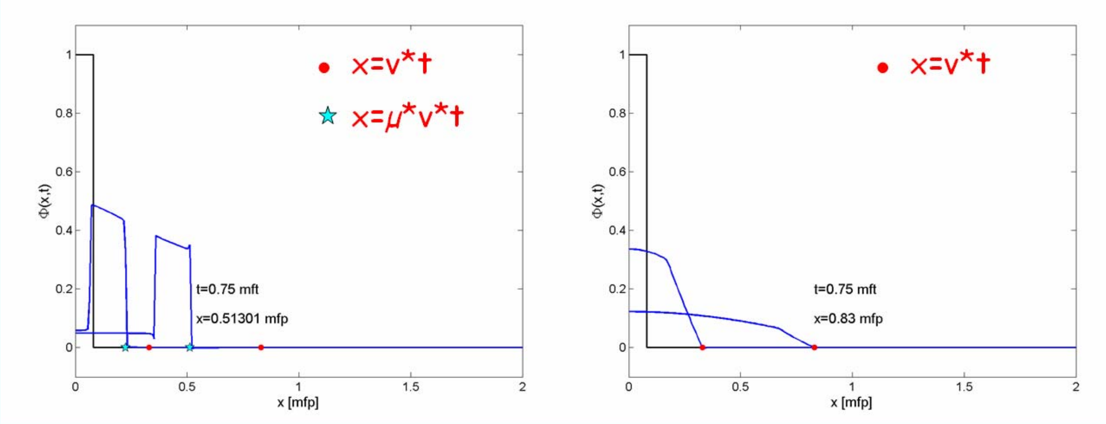
Propagation of a source pulse: asymptotic method vs. P_1

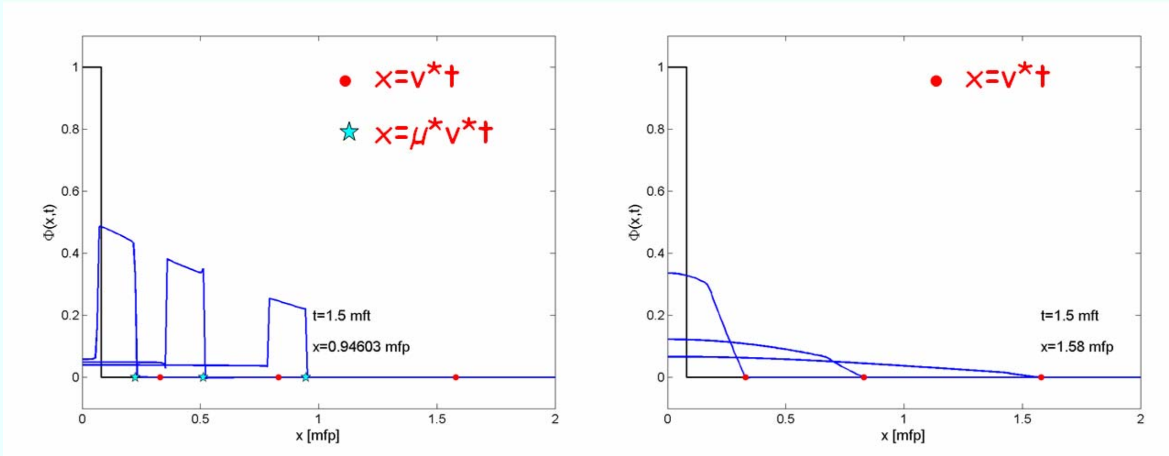




P_1 presents a wave-like propagation of the uncollided part + forward collided part of the pulse

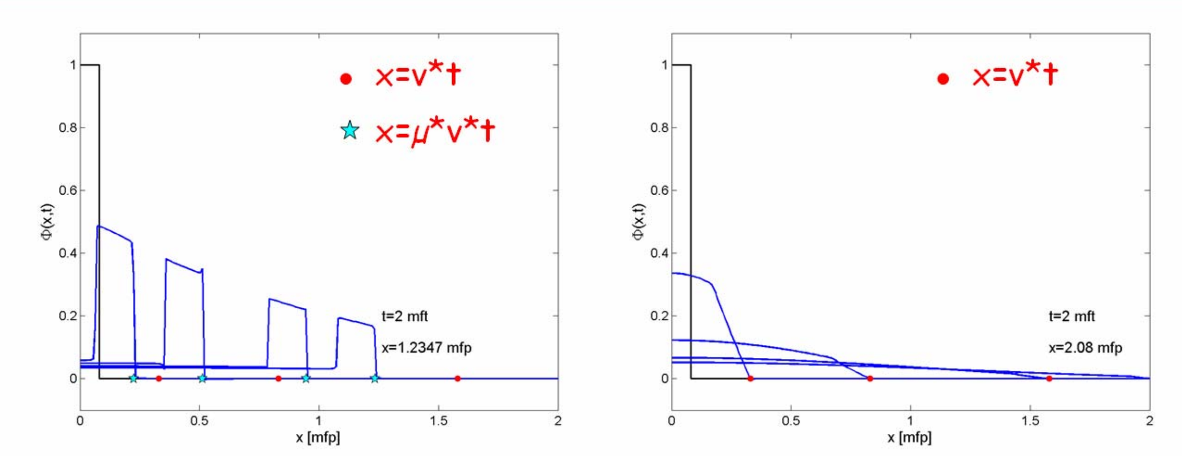
Propagation of a source pulse: asymptotic method vs. P_1

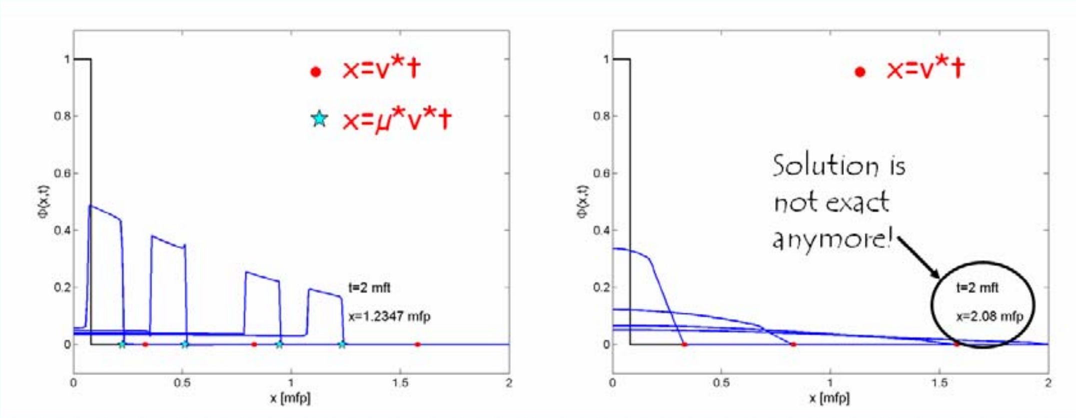




P_1 presents a wave-like propagation of the uncollided part + forward collided part of the pulse

Propagation of a source pulse: asymptotic method vs. P_1

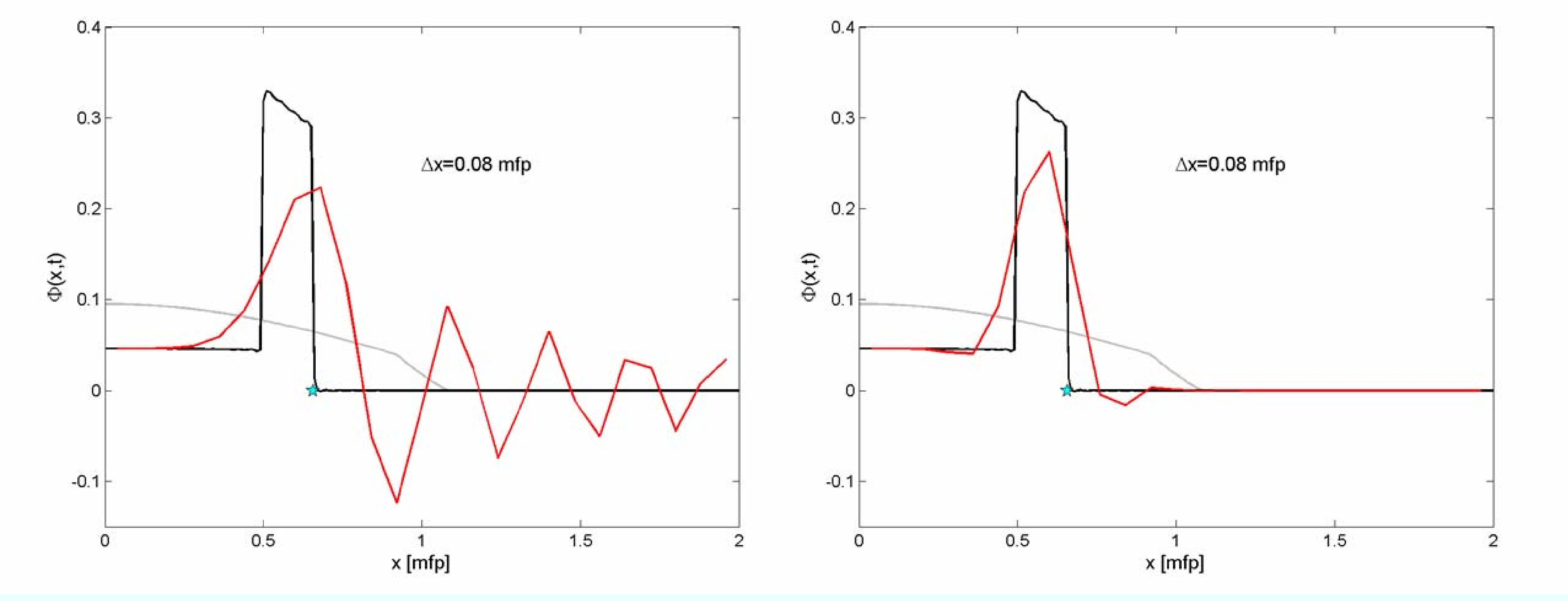




P_1 presents a wave-like propagation of the uncollided part + forward collided part of the pulse

Propagation of a source pulse: P_1 vs. S_2

Since P_1 and S_2 in 1D are equivalent, comparisons are made to evidence the effect of spatial discretization of S_2



☆ $x=\mu*v*t$

Propagation of a source pulse: P_1 vs. S_2

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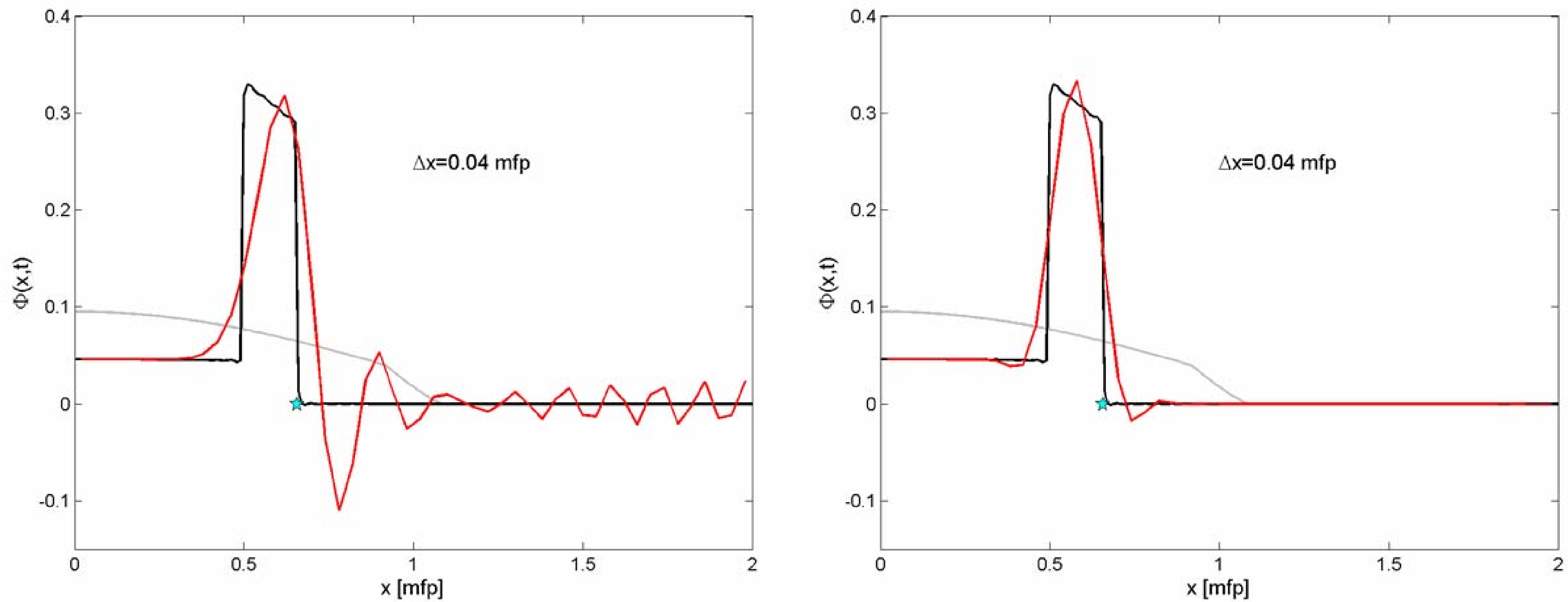

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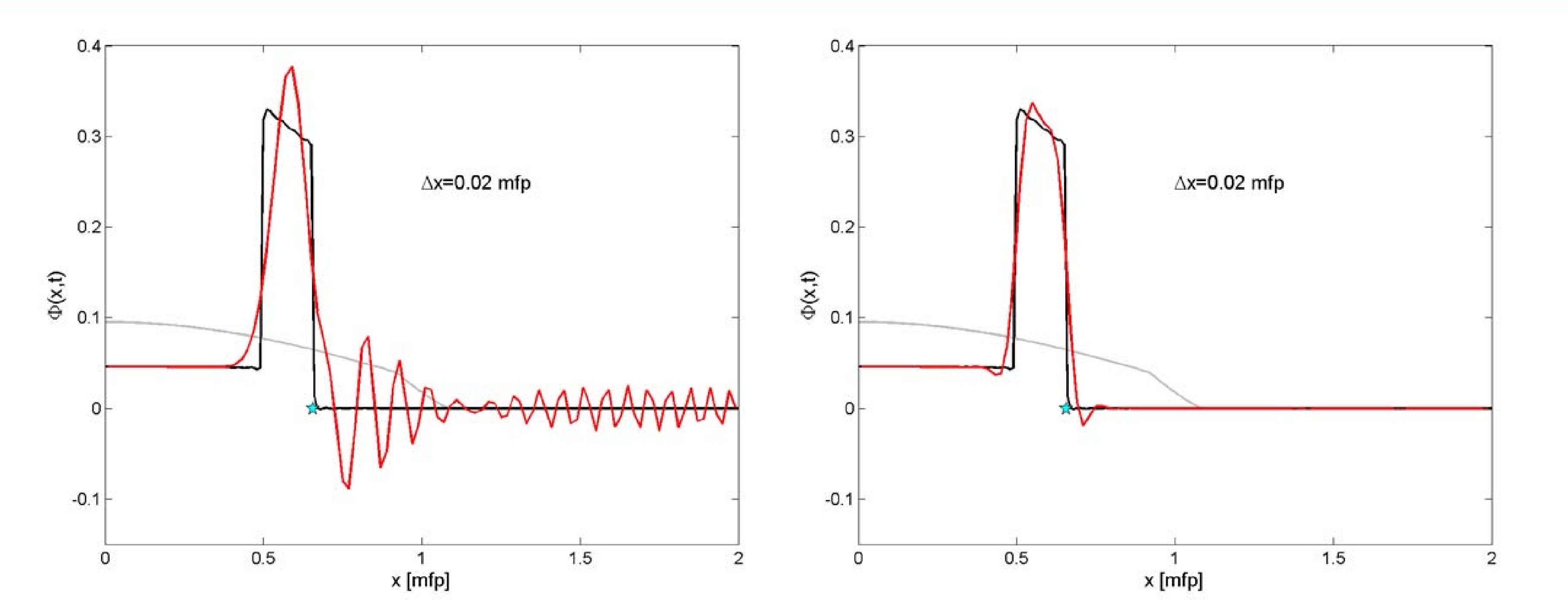

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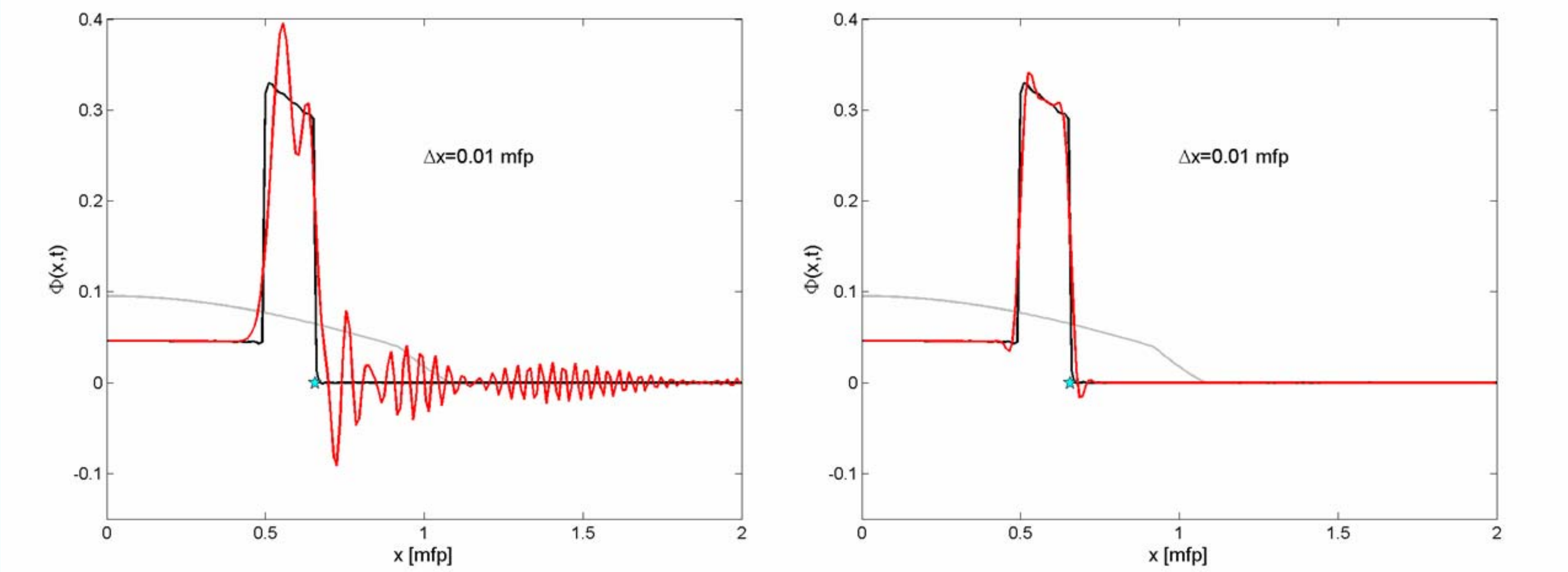
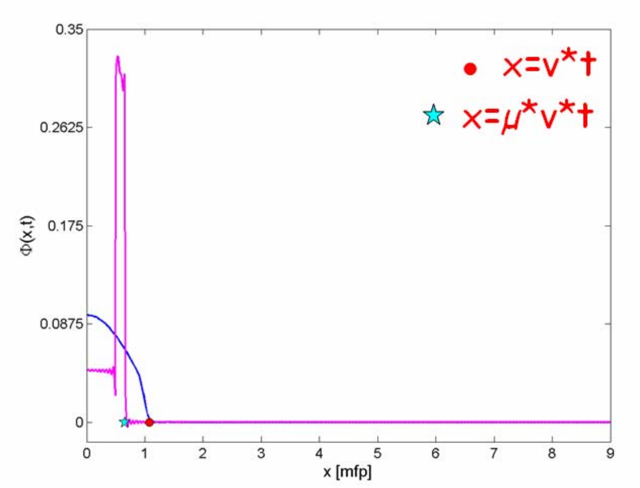
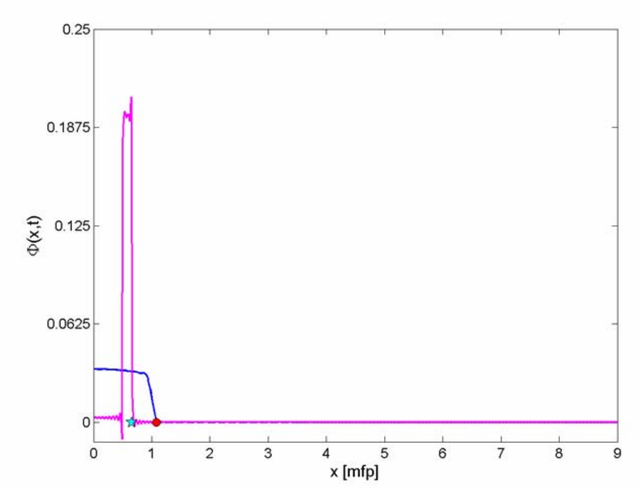


☆ $x=\mu^{*}v^{*}t$

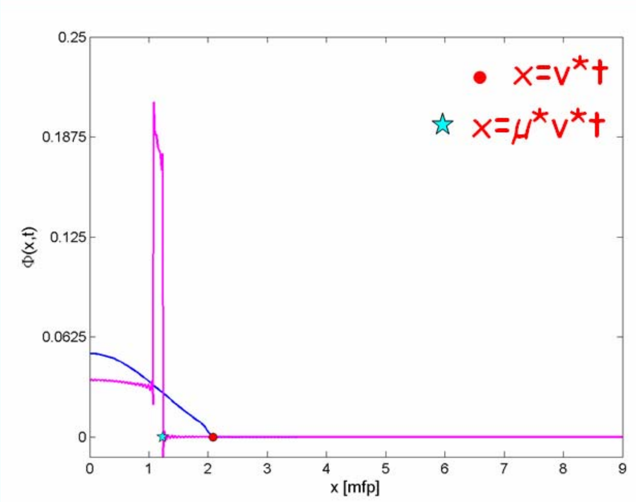
Propagation of a source pulse: asymptotic method vs. P_1 (influence of c)



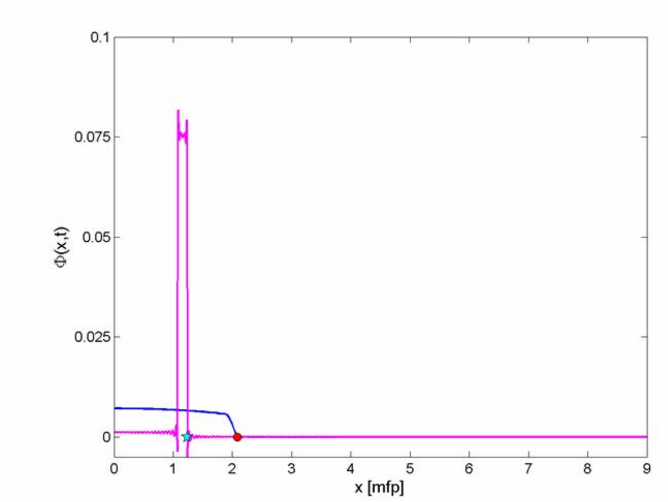
c=0.9



c=0.1

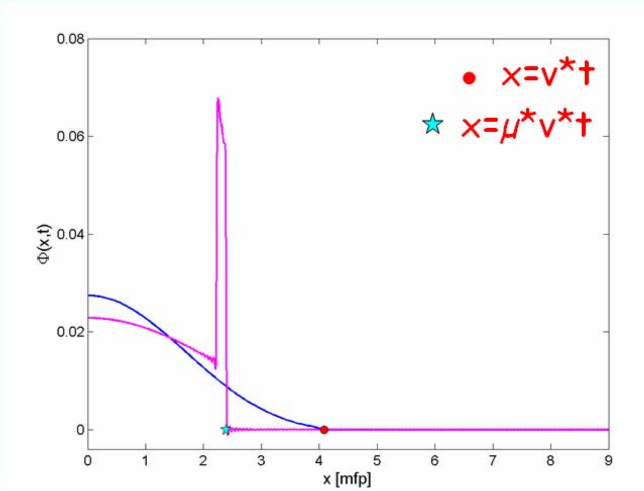


c=0.9



c=0.1

Propagation of a source pulse: asymptotic method vs. P_1 (influence of c)



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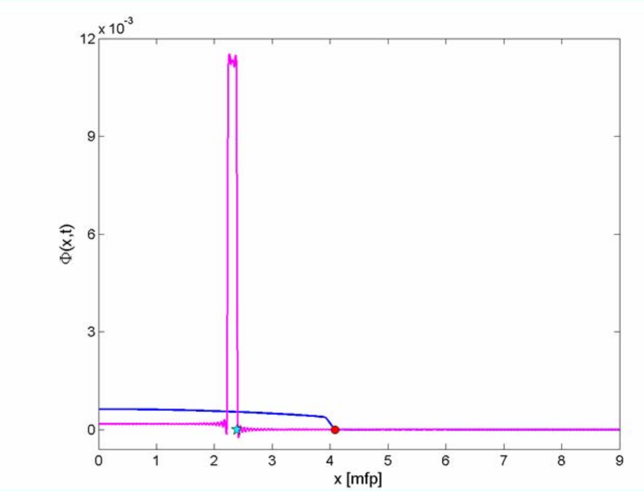
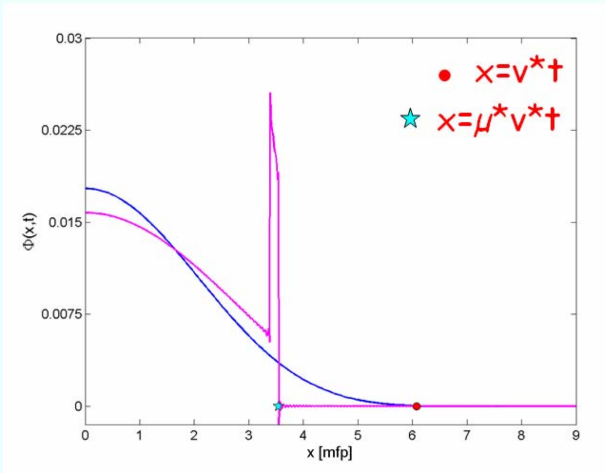
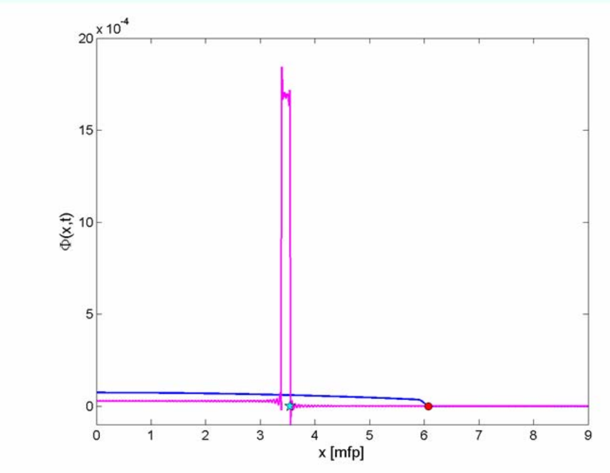


c=0.1

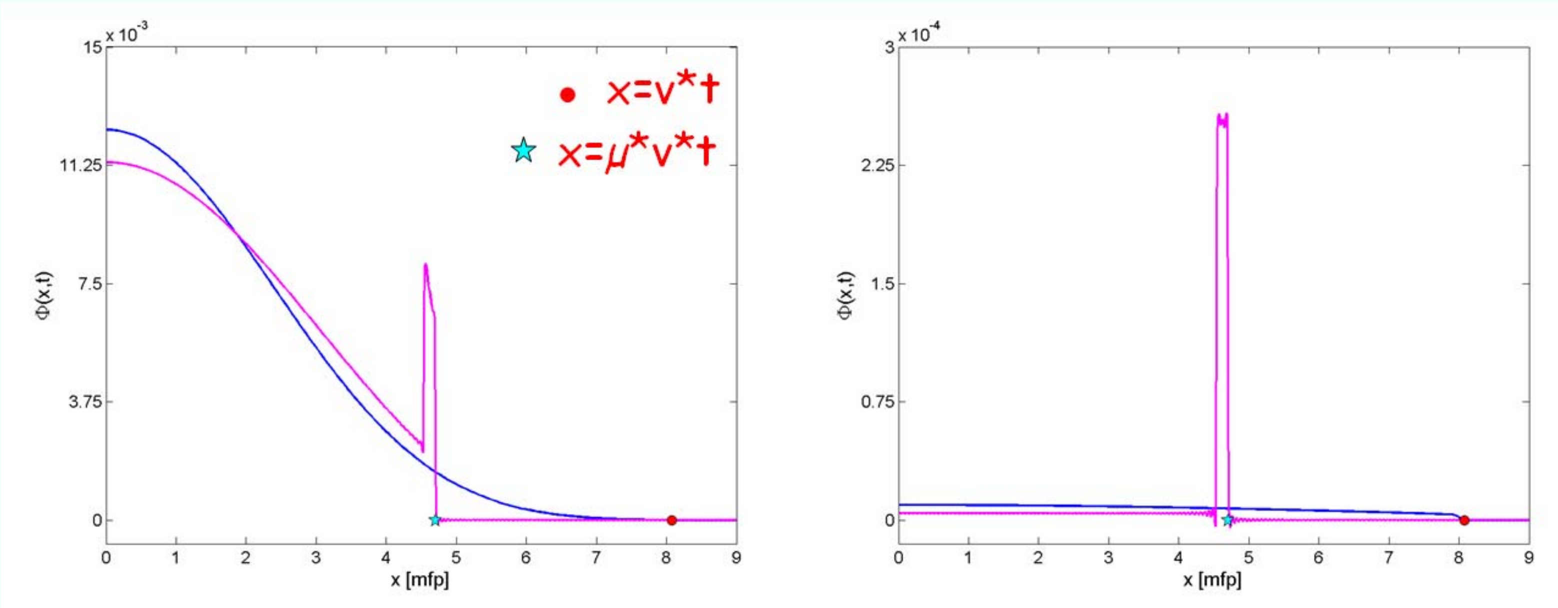


c=0.9



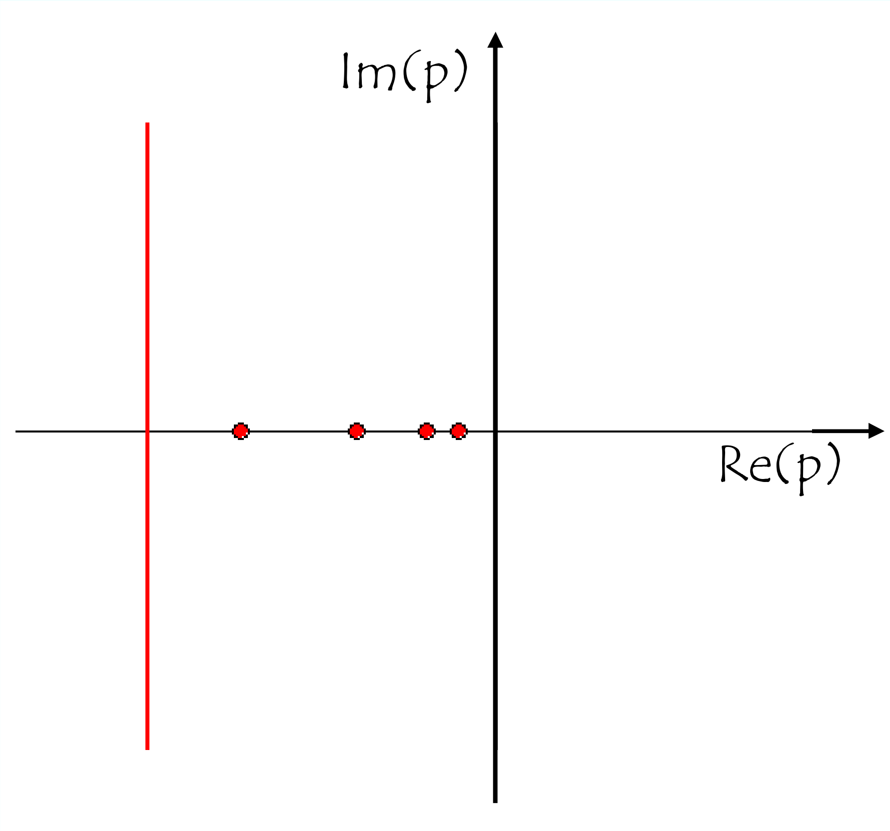
c=0.1

Propagation of a source pulse: asymptotic method vs. P_1 (influence of c)

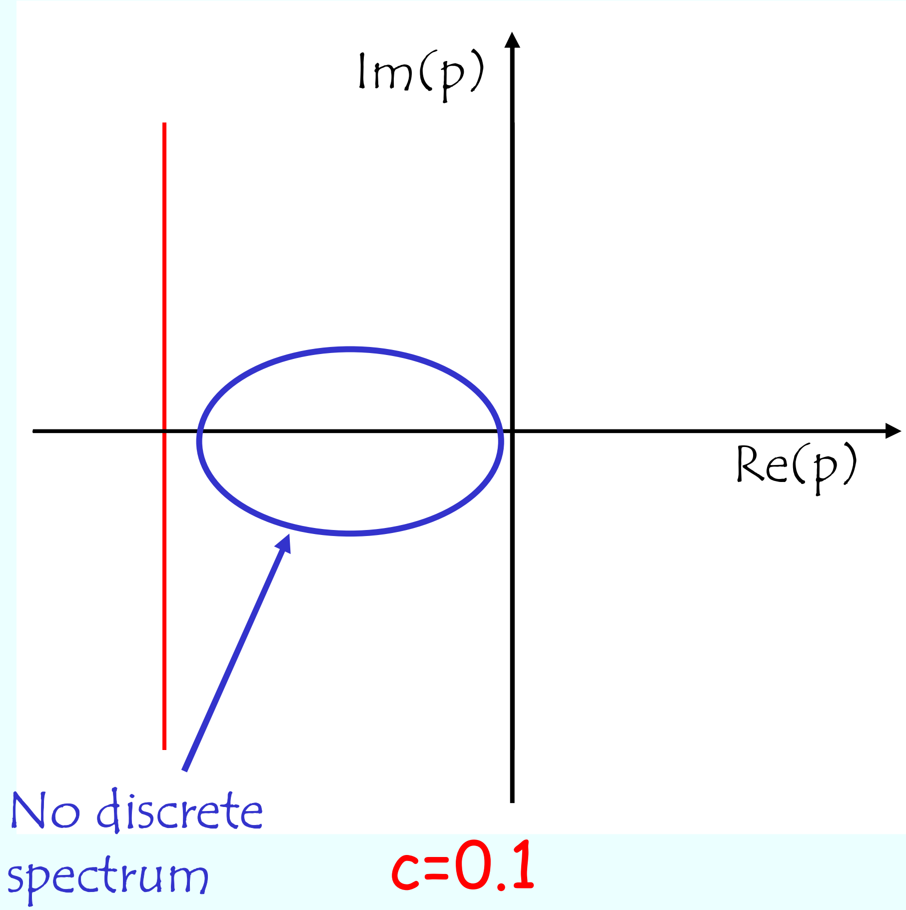


c=0.9 c=0.1

Propagation of a source pulse: spectrum of the asymptotic method
(influence of c)



c=0.9



c=0.1

Solution of the P_3-S_4 problem

- Higher order P_N-S_N methods are analyzed with a similar procedure;
  - P_3: space asymptotic method
  - S_4: spatial discretization with DD and LD and analytical time integration

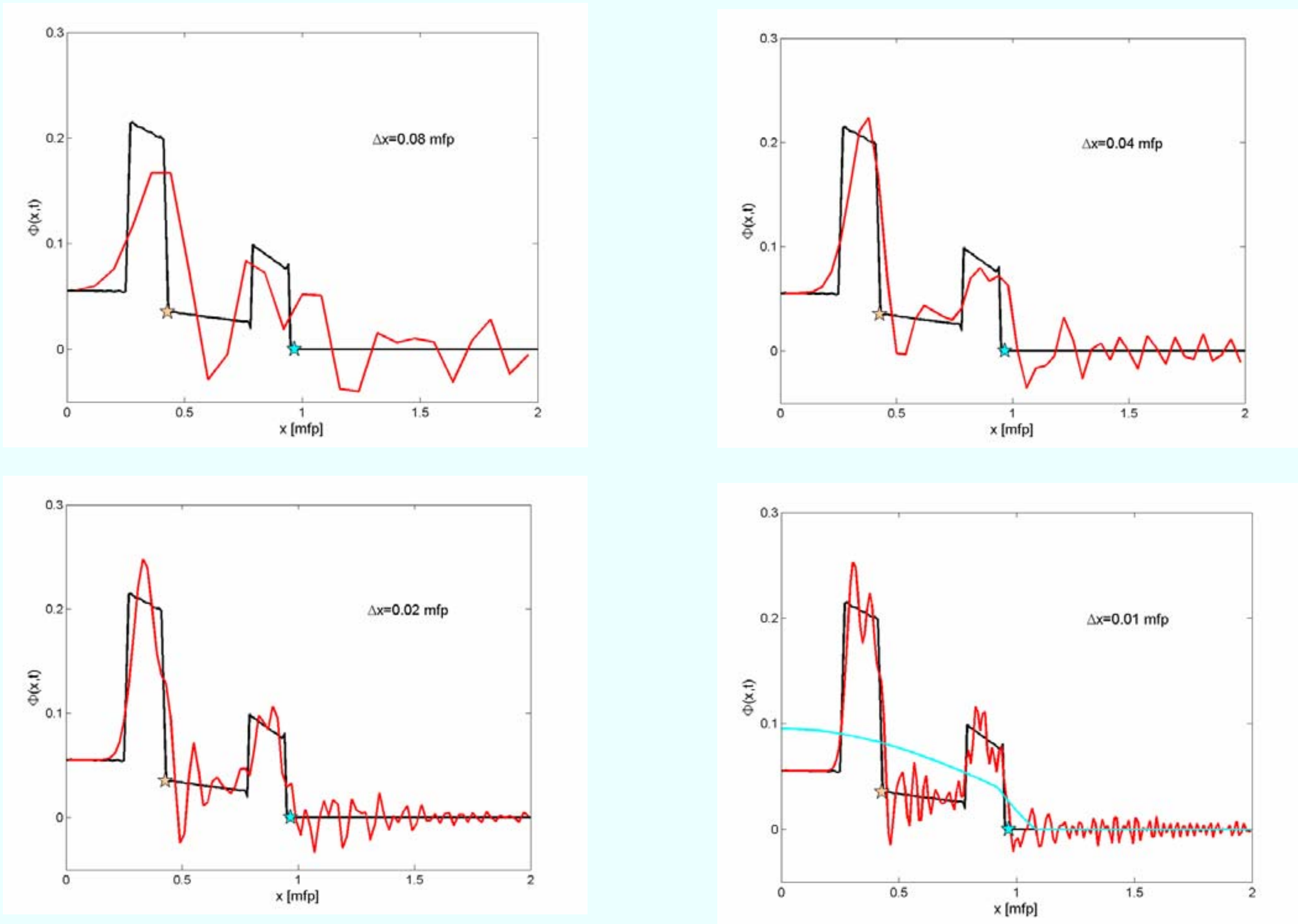
- The propagation of pulses at different velocities (corresponding to the directions μ_1 and μ_2) is observed

Solution of the S_4 problem - DD

Influence of space discretization → numerically-induced oscillation

☆ $x=\mu_1 * v * t$

☆ $x=\mu_2 * v * t$

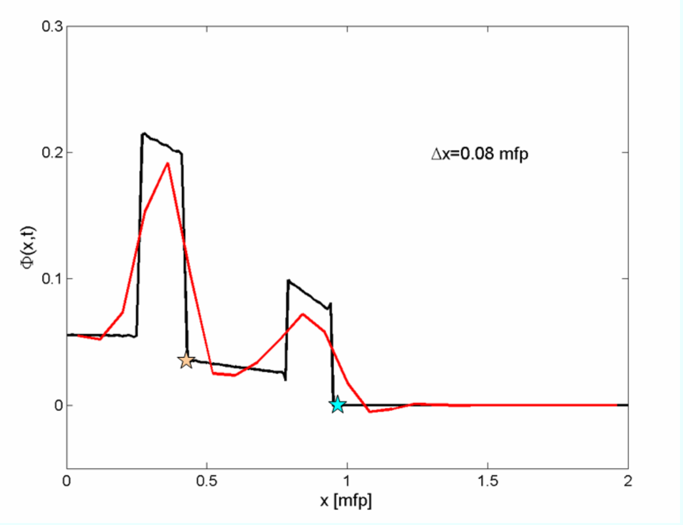


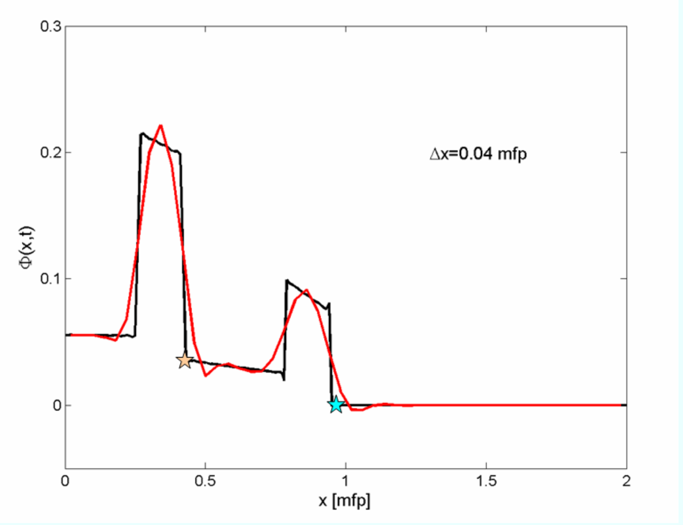
Solution of the S_4 problem - LD

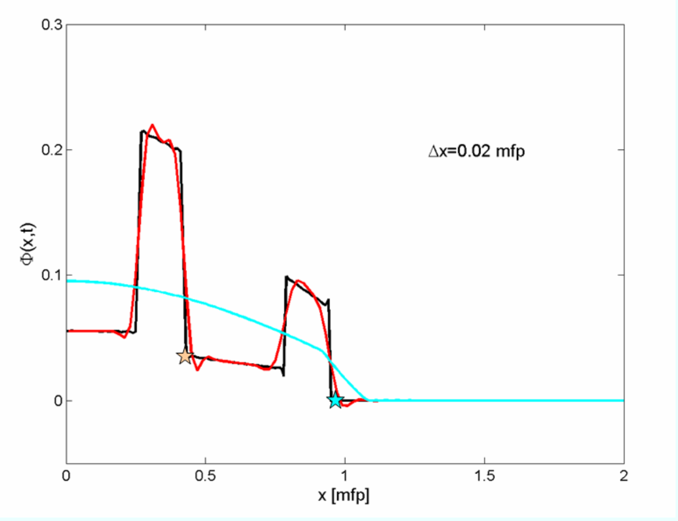
Influence of space discretization → smoothening of oscillation with linear discontinuous

☆ $x=\mu_1 * v * t$

☆ $x=\mu_2 * v * t$





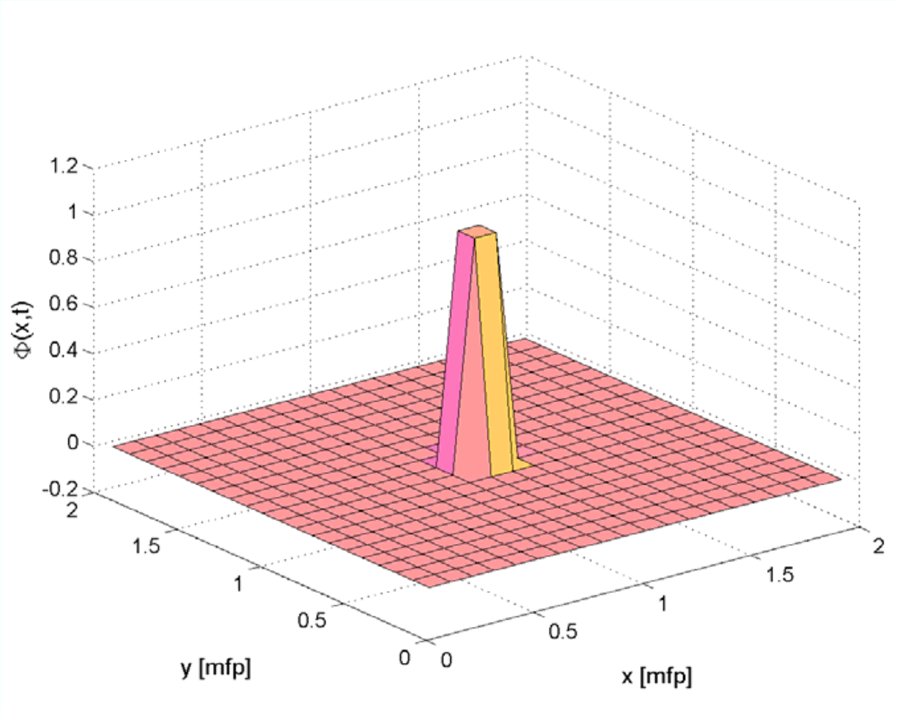


Solution of P_1-S_2 models in 2-D configurations

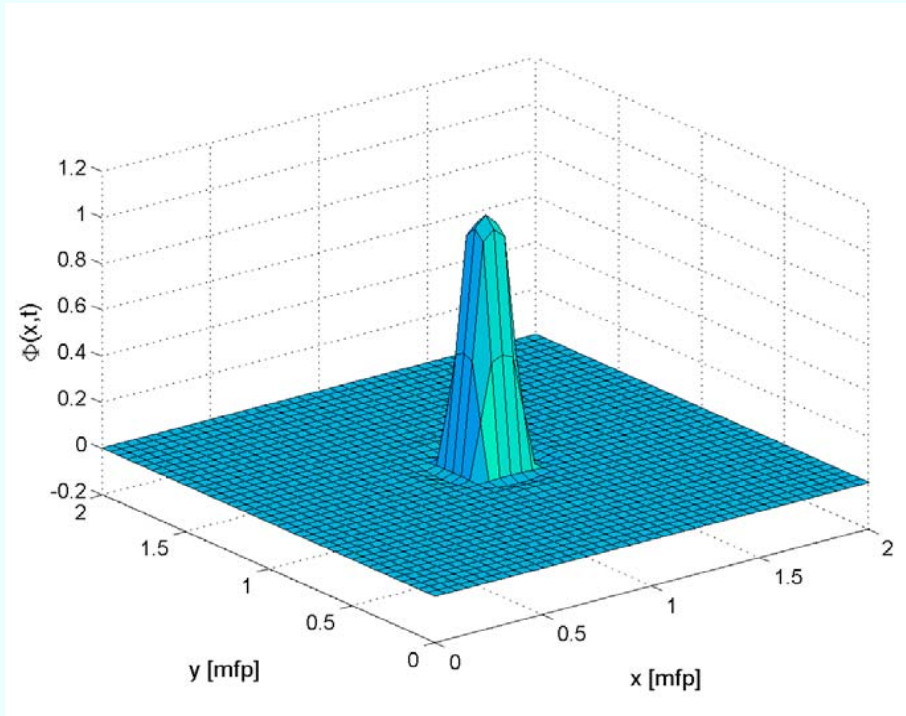
- P_1 and S_2 model are no longer equivalent:
  - P_1: time ray effects (model effects)
  - S_2: space and time ray effects (effects due to the model itself and the angular discretization)
- Solution methods:
  - P_1: Laplace transform
  - S_2: space discretization and analytical time integration

Solution of P_1-S_2 models in 2-D configurations

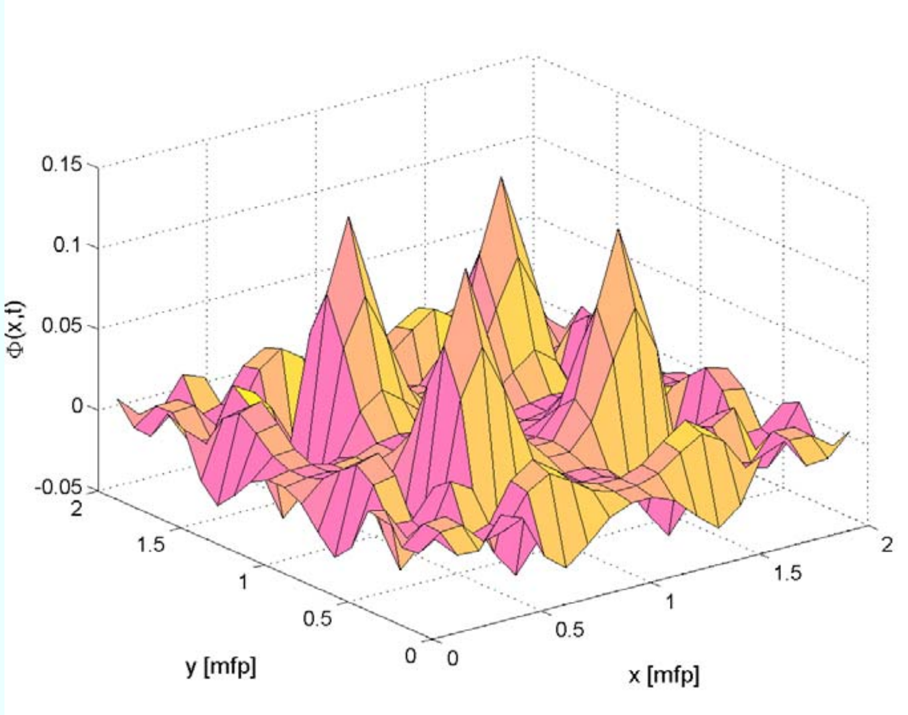
P_1



S_2

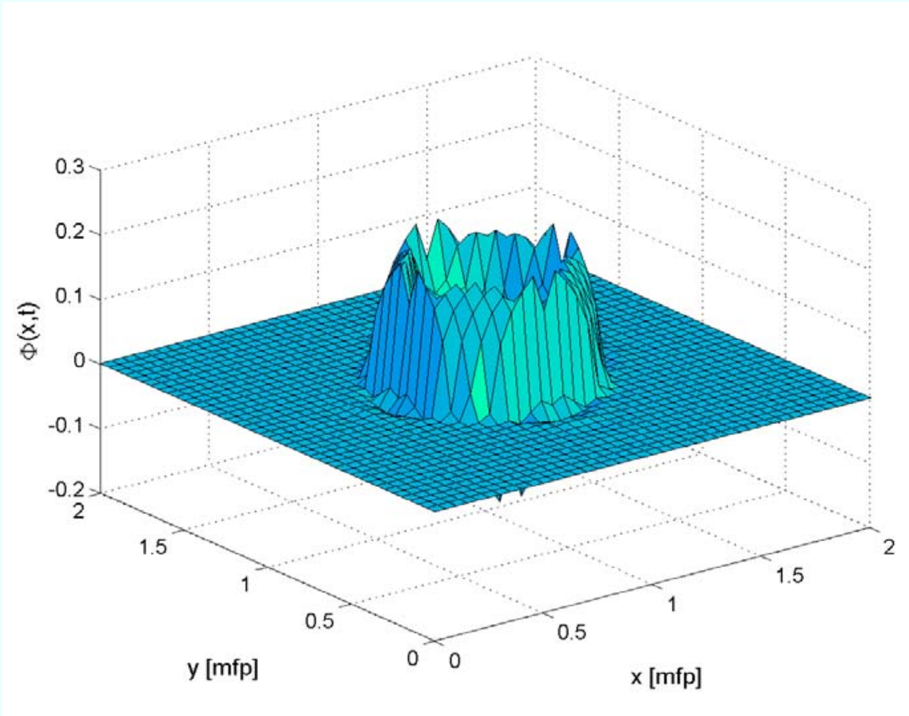


Solution of P_1-S_2 models in 2-D configurations



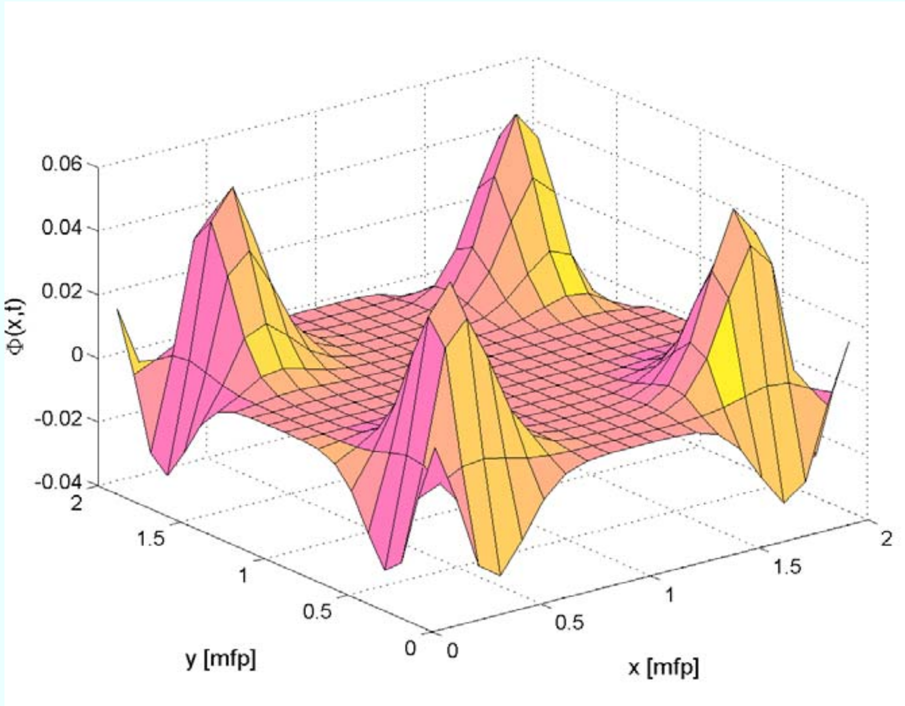
S_2

P_1

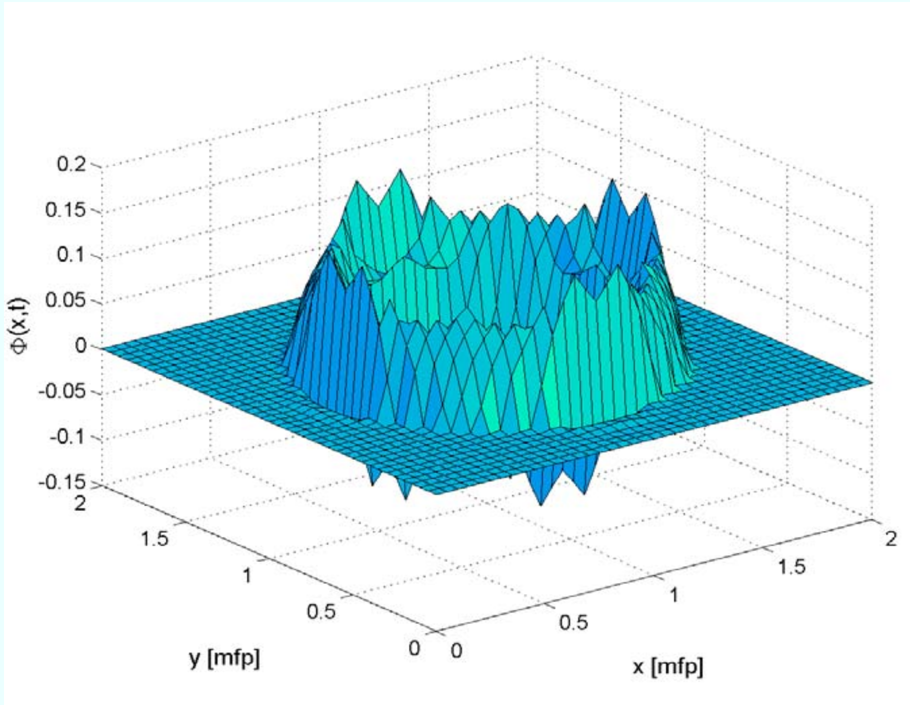


Solution of P_1-S_2 models in 2-D configurations

P_1



S_2



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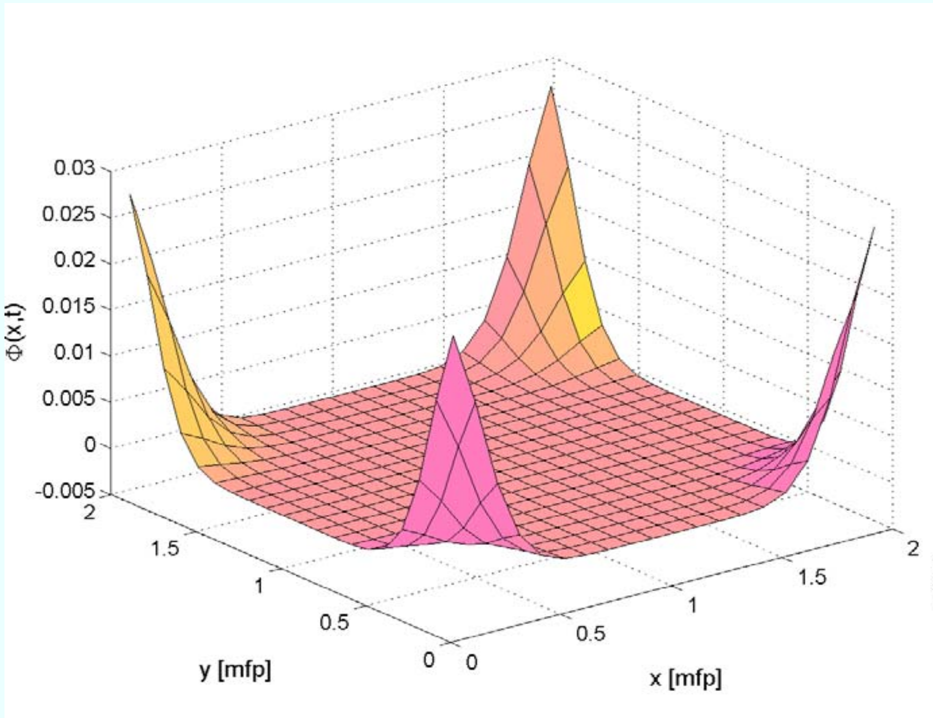
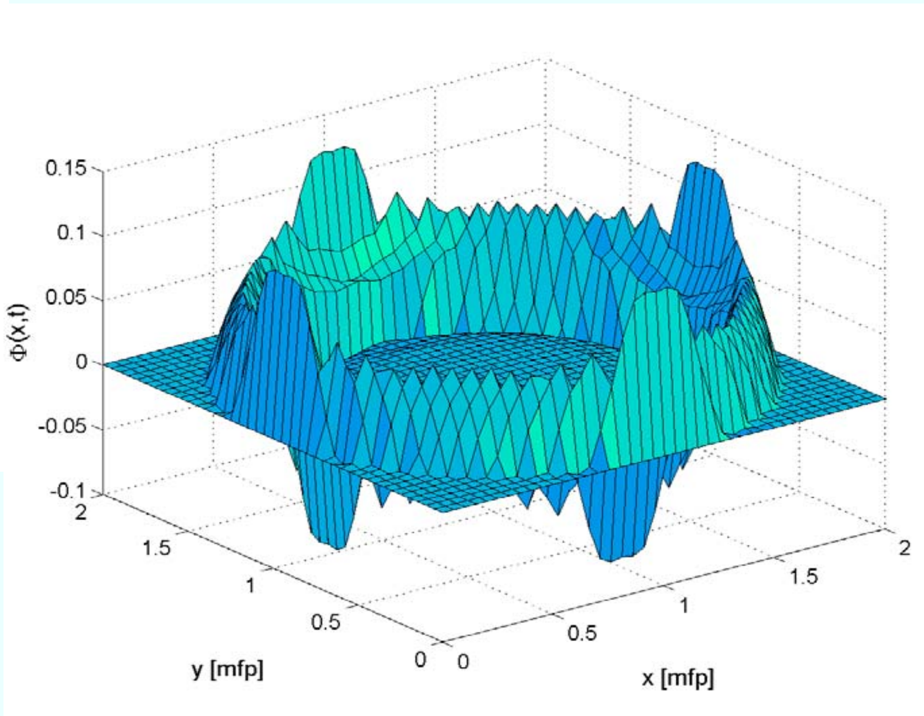


S_2

P_1



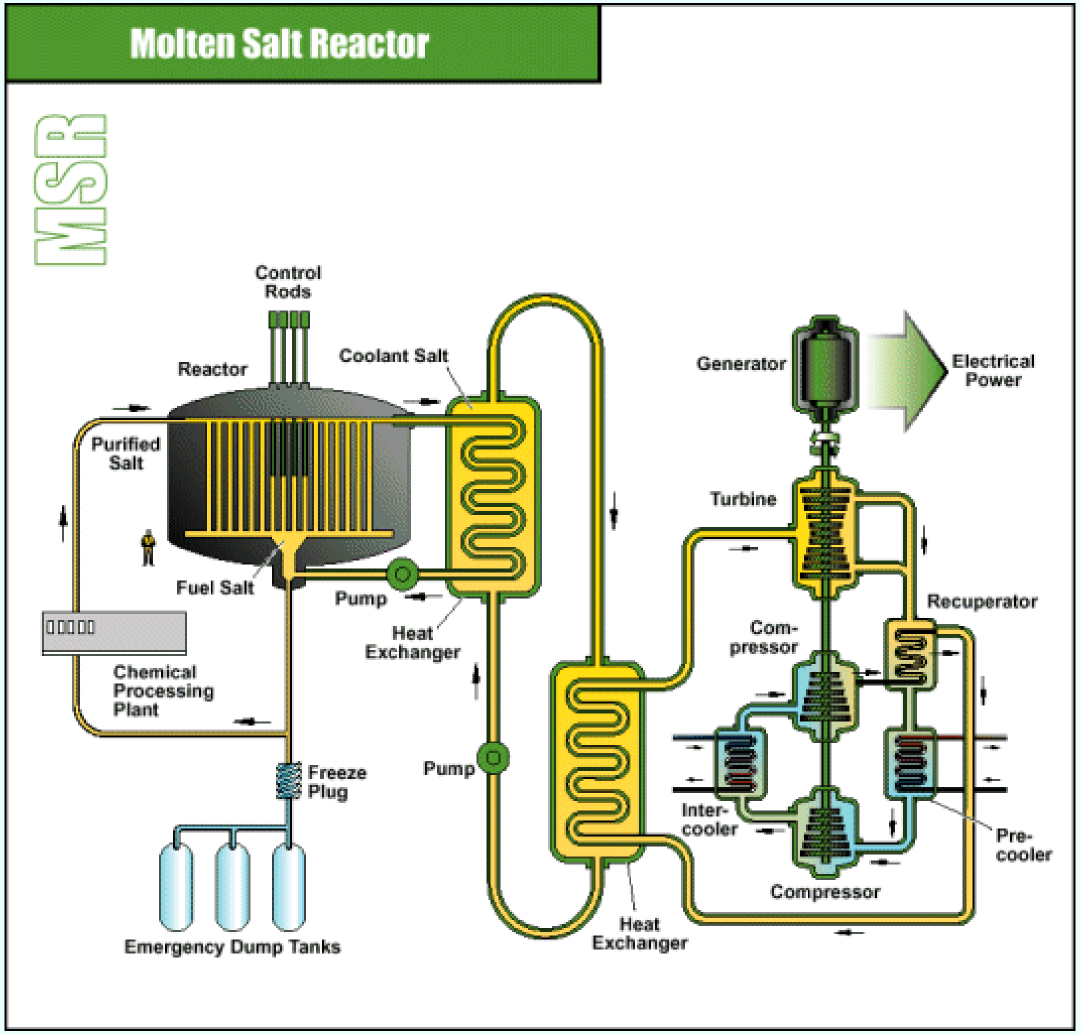
An application: dynamics of fluid fuel systems

- The multiplying medium is in a fluid phase, flowing through the reactor core

the fissile salt acts both as nuclear fuel and system coolant

- The presence of a velocity field in the multiplying medium affects its neutronic behavior → need to develop suitable models and tools for reactor physics analysis

An application: dynamics of fluid fuel systems



Neutronic model for MSR (1)

- Balance equations for neutrons in presence of delayed emissions:

$$\begin{cases} \dfrac{\partial n(\mathbf{r}, E, \mathbf{\Omega}, t)}{\partial t} = \left[\hat{\mathrm{L}}(t) + \hat{\mathrm{M}}_p(t)\right] n(\mathbf{r}, E, \mathbf{\Omega}, t) + \displaystyle\sum_{i=1}^{R} \mathcal{E}_i(\mathbf{r}, E, t) + S(\mathbf{r}, E, \mathbf{\Omega}, t) \\ \dfrac{1}{\lambda_i} \dfrac{\partial \mathcal{E}_i(\mathbf{r}, E, t)}{\partial t} + \dfrac{1}{\lambda_i} \nabla \cdot (\mathbf{u}(\mathbf{r}, t) \mathcal{E}_i(\mathbf{r}, E, t)) = \hat{\mathrm{M}}_i(t) n(\mathbf{r}, E, \mathbf{\Omega}, t) - \mathcal{E}_i(\mathbf{r}, E, t) \\ \qquad i = 1, 2, \ldots, R \end{cases}$$

where the delayed emissions are defined as

$$\mathcal{E}_i(\mathbf{r}, E, t) = \lambda_i C_i(\mathbf{r}, t) \frac{\chi_i(E)}{4\pi}$$

Neutronic model for MSR (2)

- Equation for neutron density is unchanged

$$\frac{\partial n(\mathbf{r}, E, \mathbf{\Omega}, t)}{\partial t} = \left[\hat{\mathrm{L}}(t) + \hat{\mathrm{M}}_p(t)\right] n(\mathbf{r}, E, \mathbf{\Omega}, t) + \sum_{i=1}^{R} \mathcal{E}_i(\mathbf{r}, E, t) + S(\mathbf{r}, E, \mathbf{\Omega}, t)$$

the time scale of prompt neutron production is much faster than fluid-dynamic phenomena

- A streaming term appears in the precursor equations

$$\frac{1}{\lambda_i} \frac{\partial \mathcal{E}_i(\mathbf{r}, E, t)}{\partial t} + \frac{1}{\lambda_i} \nabla \cdot (\mathbf{u}(\mathbf{r}, t) \mathcal{E}_i(\mathbf{r}, E, t)) = \hat{\mathrm{M}}_i(t) n(\mathbf{r}, E, \mathbf{\Omega}, t) - \mathcal{E}_i(\mathbf{r}, E, t)$$

the mathematical nature of the equation is changed

Neutronic model for MSR (3)

Appropriate boundary conditions must be introduced for the precursors:

$$\mathcal{E}_i(\mathbf{r},E,t)\mathbf{u}(\mathbf{r},t)\cdot(-\mathbf{n}(\mathbf{r})) = \int_{\mathcal{A}_{out}} \mathcal{E}_i(\mathbf{r}',E,t-\tau(\mathbf{r}'\rightarrow\mathbf{r}))e^{-\lambda_i\tau(\mathbf{r}'\rightarrow\mathbf{r})}\times$$
$$\mathbf{u}(\mathbf{r}',t-\tau(\mathbf{r}'\rightarrow\mathbf{r}))\cdot\mathbf{n}(\mathbf{r}')\,\mathfrak{F}(\mathbf{r}'\rightarrow\mathbf{r})d\mathcal{A}' \qquad \mathbf{r}\in\mathcal{A}_{in}$$

ingoing flow of precursors

Flow redistribution function

outgoing flow of precursors

The flow function is a *probability density function*, normalized as:

$$\int_{\mathcal{A}_{in}} \mathfrak{F}(\mathbf{r}'\rightarrow\mathbf{r})\,d\mathcal{A}=1, \qquad \forall\mathbf{r}'\in\mathcal{A}_{out}.$$

Effects of fuel motion

- STEADY STATE

- Dependence of the multiplication eigenvalue on the velocity field and delayed neutron characteristics

reactivity loss

- Spatial redistribution of the delayed neutron precursors importance and reduction of their importance

- TIME-DEPENDENT

- Reduced effective delayed neutron fraction

new definition of β_{eff}

- Prompter dynamic response

- Peculiar effects connected to the undecayed fraction of precursors re-entering in the system

Extension of quasi-statics to fluid fuel systems

In order to properly study the physical phenomena typical of fluid fuel systems, the standard tools for dynamic analysis need to be consistently extended and adapted

- The point kinetic model is reformulated consistently for fluid-fuel systems, applying Henry procedure of factorization and projection of the neutron density and delayed emissions

- The quasi-static scheme is applied, taking particular care of the normalization conditions to be fullfilled

Point Kinetics

Point models are based on the factorization of the unknowns and the projection of the factorized problem on a weighting function

a reference configuration is considered:

Subcritical systems

$$\begin{cases} \left[\hat{\mathrm{L}}_0 + \hat{\mathrm{M}}_{p,0}\right] N_0(\mathbf{r}, E, \mathbf{\Omega}) + \sum_{i=1}^{R} \mathcal{E}_{i,0}(\mathbf{r}, E) + S_0(\mathbf{r}, E, \mathbf{\Omega}) = 0 \\ \frac{1}{\lambda_i} \nabla \cdot (\mathbf{u}_0 \mathcal{E}_{i,0}(\mathbf{r}, E)) = \hat{\mathrm{M}}_{i,0} N_0(\mathbf{r}, E, \mathbf{\Omega}) - \mathcal{E}_{i,0}(\mathbf{r}, E), \qquad i = 1, 2, ..., R \end{cases}$$

Still differential!

Point Kinetics

A proper inner product definition is introduced:

$$\mathbf{w} = (n, \mathcal{E}_1, ..., \mathcal{E}_R)^t \qquad \mathbf{w}^\dagger = (n^\dagger, \mathcal{E}_1^\dagger, ..., \mathcal{E}_R^\dagger)$$

$$(\mathbf{w}^\dagger, \mathbf{w}) = \sum_{n=1}^{R+1} \langle w_n^\dagger \mid w_n \rangle = \sum_{n=1}^{R+1} \int dV \int dE \oint d\Omega w_n^\dagger w_n$$

And the *solution of the* consistently derived steady-state *adjoint problem* is used as weighting function

Point Kinetics

Direct
matrix operator

$$\begin{bmatrix} \hat{\mathrm{L}}_0 + \hat{\mathrm{M}}_{p,0} & \frac{1}{4\pi}\oint d\boldsymbol{\Omega} & \dots & \frac{1}{4\pi}\oint d\boldsymbol{\Omega} \\ \hat{\mathrm{M}}_{1,0} & -1 - \frac{1}{\lambda_1}\boldsymbol{\nabla}\cdot(\mathbf{u}_0 & & \\ \dots & & \dots & \\ \hat{\mathrm{M}}_{R,0} & & & -1 - \frac{1}{\lambda_R}\boldsymbol{\nabla}\cdot(\mathbf{u}_0 \end{bmatrix}$$

Adjoint
matrix operator
(transpose with adjoints)

$$\begin{bmatrix} \hat{\mathrm{L}}_0^\dagger + \hat{\mathrm{M}}_{p,0}^\dagger & \hat{\mathrm{M}}_{1,0}^\dagger & \dots & \hat{\mathrm{M}}_{R,0}^\dagger \\ \frac{1}{4\pi}\oint d\boldsymbol{\Omega} & -1 + \frac{1}{\lambda_1}\boldsymbol{\nabla}\cdot(\mathbf{u}_0 & & \\ \dots & & \dots & \\ \frac{1}{4\pi}\oint d\boldsymbol{\Omega} & & & -1 + \frac{1}{\lambda_R}\boldsymbol{\nabla}\cdot(\mathbf{u}_0 \end{bmatrix}$$

Point Kinetics

The adjoint equations take the form:

$$\begin{cases} \left[\hat{\mathrm{L}}_0^\dagger + \hat{\mathrm{M}}_{p,0}^\dagger\right] N_0^\dagger(\mathbf{r}, E, \mathbf{\Omega}) + \sum_{i=1}^{R} \hat{\mathrm{M}}_{i,0}^\dagger \mathcal{E}_{i,0}^\dagger(\mathbf{r}, E) + S_0^\dagger(\mathbf{r}, E, \mathbf{\Omega}) = 0 \\ \frac{1}{4\pi} \oint d\Omega N_0^\dagger(\mathbf{r}, E, \mathbf{\Omega}) + \frac{1}{\lambda_i} \mathrm{u}_0 \cdot \nabla \left(\mathcal{E}_{i,0}^\dagger(\mathbf{r}, E)\right) - \mathcal{E}_{i,0}^\dagger(\mathbf{r}, E) = 0, \qquad i = 1, 2, ..., R \end{cases}$$

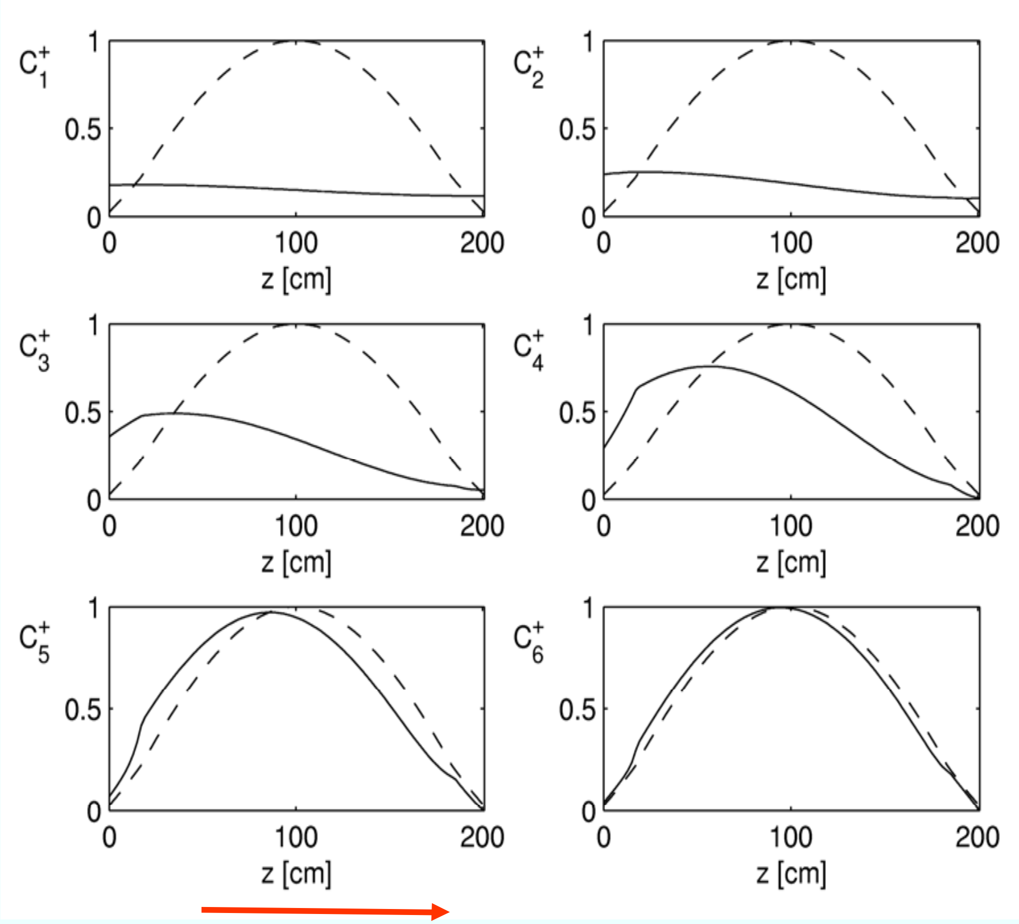
with associate boundary conditions for adjoint delayed emissions:

$$\mathcal{E}_i^\dagger(\mathbf{r}, E) = \int_{\mathcal{A}_{in}} \mathcal{E}_i^\dagger(\mathbf{r}', E) e^{-\lambda_i \tau(\mathbf{r} \to \mathbf{r}')} \mathfrak{F}(\mathbf{r} \to \mathbf{r}')\, d\mathcal{A}', \qquad \mathbf{r} \in \mathcal{A}_{out}$$

Point Kinetics

Physical interpretation of the adjoint functions $\mathcal{E}_i^\dagger(\mathbf{r},E)$ in terms of importance:

Importance of a delayed neutron emitted isotropically by a precursor of the i-th family



$u(z)\mathbf{e}_z$

Point Kinetics

Neutron density and delayed emissions are factorized as follows:

$$n(\mathbf{r}, E, \mathbf{\Omega}, t) = A(t)\phi(\mathbf{r}, E, \mathbf{\Omega}; t)$$

$$\mathcal{E}_i(\mathbf{r}, E, t) = G_i(t)e_i(\mathbf{r}, E; t) \qquad i = 1, 2, ..., R$$

These expressions are then introduced into the balance equations:

shape equations
$$\begin{cases} A(t)\dfrac{\partial \phi}{\partial t} + \phi\dot{A} = \left[\hat{\mathrm{L}} + \hat{\mathrm{M}}_p\right]\phi A + \displaystyle\sum_{i=1}^{R} G_i e_i + S \\ \dfrac{\partial e_i}{\partial t} G_i + e_i \dot{G}_i + \mathbf{\nabla} \cdot (\mathbf{u} e_i)\, G_i = \lambda_i \hat{\mathrm{M}}_i \phi A - \lambda_i e_i G_i \\ \qquad\qquad i = 1, 2, ..., R, \end{cases}$$

Point Kinetics

The system is projected on the adjoint functions

$$\begin{cases} \cancel{\frac{d}{dt}\left\langle N_0^\dagger \middle| \phi \right\rangle} A + \left\langle N_0^\dagger \middle| \phi \right\rangle \dot{A} = \left\langle N_0^\dagger \middle| \left[\hat{\mathrm{L}} + \hat{\mathrm{M}}_p\right] \phi \right\rangle A + \sum_{i=1}^{R} \left\langle N_0^\dagger \middle| e_i \right\rangle G_i + \left\langle N_0^\dagger \middle| S \right\rangle \\ \cancel{\frac{d}{dt}\left\langle \mathcal{E}_{i,0}^\dagger \middle| e_i \right\rangle} G_i + \left\langle \mathcal{E}_{i,0}^\dagger \middle| e_i \right\rangle \dot{G}_i + \left\langle \mathcal{E}_{i,0}^\dagger \middle| \nabla \cdot (\mathrm{u} e_i) \right\rangle G_i = \\ \qquad = \left\langle \mathcal{E}_{i,0}^\dagger \middle| \lambda_i \hat{\mathrm{M}}_i \phi \right\rangle A - \left\langle \mathcal{E}_{i,0}^\dagger \middle| \lambda_i e_i \right\rangle G_i \\ \qquad i = 1, 2, ..., R \end{cases}$$

amplitude equations

and normalization conditions are imposed, to make the factorization unique:

$$\frac{d}{dt}\left\langle N_0^\dagger \middle| \phi \right\rangle = \frac{d\gamma_0}{dt} = 0,$$

$$\frac{d}{dt}\left\langle \mathcal{E}_{i,0}^\dagger \middle| e_i \right\rangle = \frac{d\eta_{i,0}}{dt} = 0, \qquad i = 1, 2, ..., R$$

Point Kinetics

A point model is obtained:

New definition of β_{eff}!

$$\begin{cases} \dfrac{dA}{dt} = \alpha A + \sum_{i=1}^{R} \mu_i G_i + \tilde{S}, \\ \dfrac{dG_i}{dt} = \vartheta_i A - \Lambda_i G_i, \qquad i = 1, \ldots, R \end{cases}$$

with a different definition of the kinetic parameters with respect to a solid fuel reactor.

Note: the kinetic parameters are functions of the neutron and precursor shapes!!!

Reformulated kinetic parameters

New definition of the effective delayed neutron fraction:

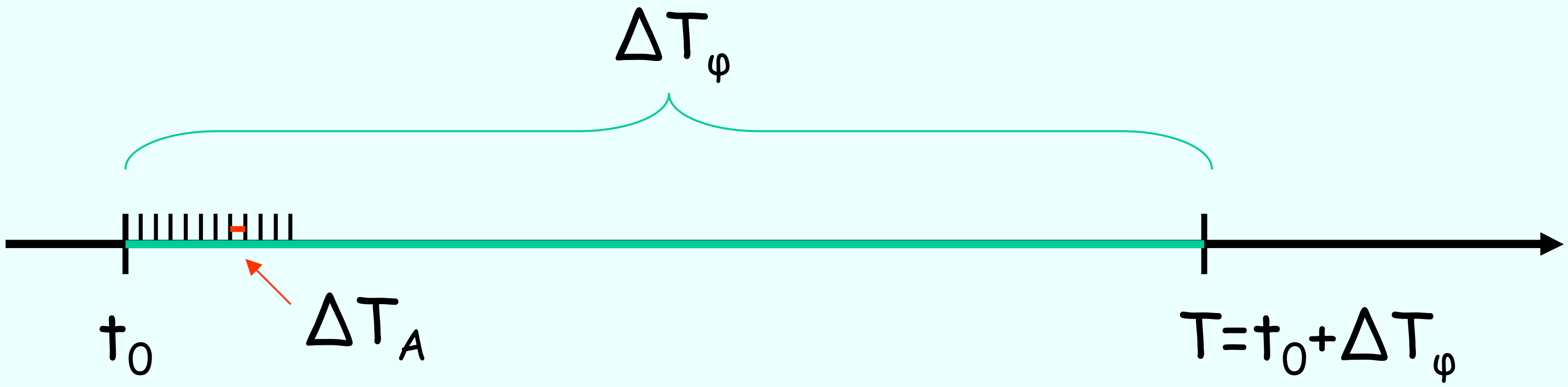
$$\tilde{\beta}_i = \frac{\left\langle \mathcal{E}^{\dagger}_{i,0} \middle| \hat{\mathrm{M}}_i \phi \right\rangle}{\mathcal{F}}$$

- weighted on the adjoint delayed emissions
- takes into account the modification introduced by the fuel motion
- in absence of motion reduces to the standard definition

	1	2	3	4	5	6	**tot**
$\lambda_i\ [s^{-1}]$	0.01	0.03	0.12	0.31	1.4	3.9	
$\beta_i\ [pcm]$	24	123	117	263	108	45	**680**
$\tilde{\beta}_i\ [pcm]$	7	38	52	182	104	45	**428**
$\Delta\beta_i/\beta_i\%$	71	69	56	31	4	0	**37**

Quasi Statics

- To perform quasi-static calculations, the point model solution is calculated over a time interval ΔT_φ, using a fine time discretization ΔT_A;



Quasi Statics

- To perform quasi-static calculations, the point model solution is calculated over a time interval ΔT_φ, using a fine time discretization ΔT_A;

- At time T the shape functions are updated, solving the full space-time problem on the time interval t_0-T:

$$\begin{cases} A(t)\dfrac{\partial \phi}{\partial t} + \phi \dot{A} = \left[\hat{\mathrm{L}} + \hat{\mathrm{M}}_p\right] \phi A + \displaystyle\sum_{i=1}^{R} G_i e_i + S \\ \dfrac{\partial e_i}{\partial t} G_i + e_i \dot{G}_i + \boldsymbol{\nabla} \cdot (\mathbf{u} e_i)\, G_i = \lambda_i \hat{\mathrm{M}}_i \phi A - \lambda_i e_i G_i \\ \qquad\qquad\qquad\qquad i = 1, 2, ..., R, \end{cases}$$

Quasi Statics

- To perform quasi-static calculations, the point model solution is calculated over a time interval ΔT_{φ}, using a fine time discretization ΔT_A;

- At time T the shape functions are updated, solving the full space-time problem on the time interval t_0-T:

  Discretized with an implicit Euler scheme

$$\begin{cases} \dfrac{\phi_j^{n+1} - \phi^n}{\Delta T_\phi} + \phi_j^{n+1} \left.\dfrac{\dot{A}}{A}\right|_{T,j} = \left[\hat{\mathrm{L}} + \hat{\mathrm{M}}_p\right] \phi_j^{n+1} + \displaystyle\sum_{i=1}^{R} e_{i,j}^{n+1} \left.\dfrac{G_i}{A}\right|_{T,j} + \left.\dfrac{S}{A}\right|_{T,j}, \\ \dfrac{e_{i,j}^{n+1} - e_i^n}{\Delta T_\phi} + e_{i,j}^{n+1} \left.\dfrac{\dot{G}_i}{G_i}\right|_{T,j} + \nabla \cdot \left[(\mathbf{u} e_{i,j})^{n+1}\right] = \lambda_i \hat{\mathrm{M}}_i \phi_j^{n+1} \left.\dfrac{A}{G_i}\right|_{T,j} - \lambda_i e_{i,j}^{n+1} \\ \qquad i = 1, \ldots, R, \end{cases}$$

Normalization of the solution

- The solutions obtained for the shape functions must be rescaled to fulfill normalization conditions:

$$\phi_{j+1/2}^{n+1} = \frac{\gamma_0}{\left\langle N_0^\dagger \middle| \phi_j^{n+1} \right\rangle} \phi_j^{n+1}$$

$$e_{i,j+1/2}^{n+1} = \frac{\eta_{i,0}}{\left\langle \mathcal{E}_{i,0}^\dagger \middle| e_{i,j}^{n+1} \right\rangle} e_{i,j}^{n+1} \qquad i = 1, \ldots, R$$

and are used for the recalculation of the kinetic parameters.

⇓

This non linear procedure implies the need to perform iterations on the solution obtained

Iterations on quasi static solution

- Neutron amplitude A (imposed continuity of system power during the transient):

discontinuous in t=T

$$\left\langle \Sigma_f \left| \phi_{j+1/2}^{n+1} \right\rangle A|_{T,j+1} = \langle \Sigma_f \left| \phi_j^{n+1} \right\rangle A|_{T,j}$$

- Delayed amplitude G_i (imposed fulfillment of boundary conditions):

$$\begin{aligned} G_i|_{T,j+1}\, e_{i,j+1/2}^{n+1} \mathbf{u} \cdot (-\mathbf{n}) \quad = \quad & \int_{\mathcal{A}_{out}} G_i(T - \tau(\mathbf{r}' \to \mathbf{r}))e_i(\mathbf{r}', E; T - \tau(\mathbf{r}' \to \mathbf{r}))e^{-\lambda_i \tau(\mathbf{r}' \to \mathbf{r})} \times \\ & \mathbf{u}\,(\mathbf{r}', T - \tau(\mathbf{r}' \to \mathbf{r})) \cdot \mathbf{n}\,(\mathbf{r}')\, \mathfrak{F}\,(\mathbf{r}' \to \mathbf{r})\, d\mathcal{A}'. \end{aligned}$$

- Time derivatives of A and G_i (update of kinetic parameters):

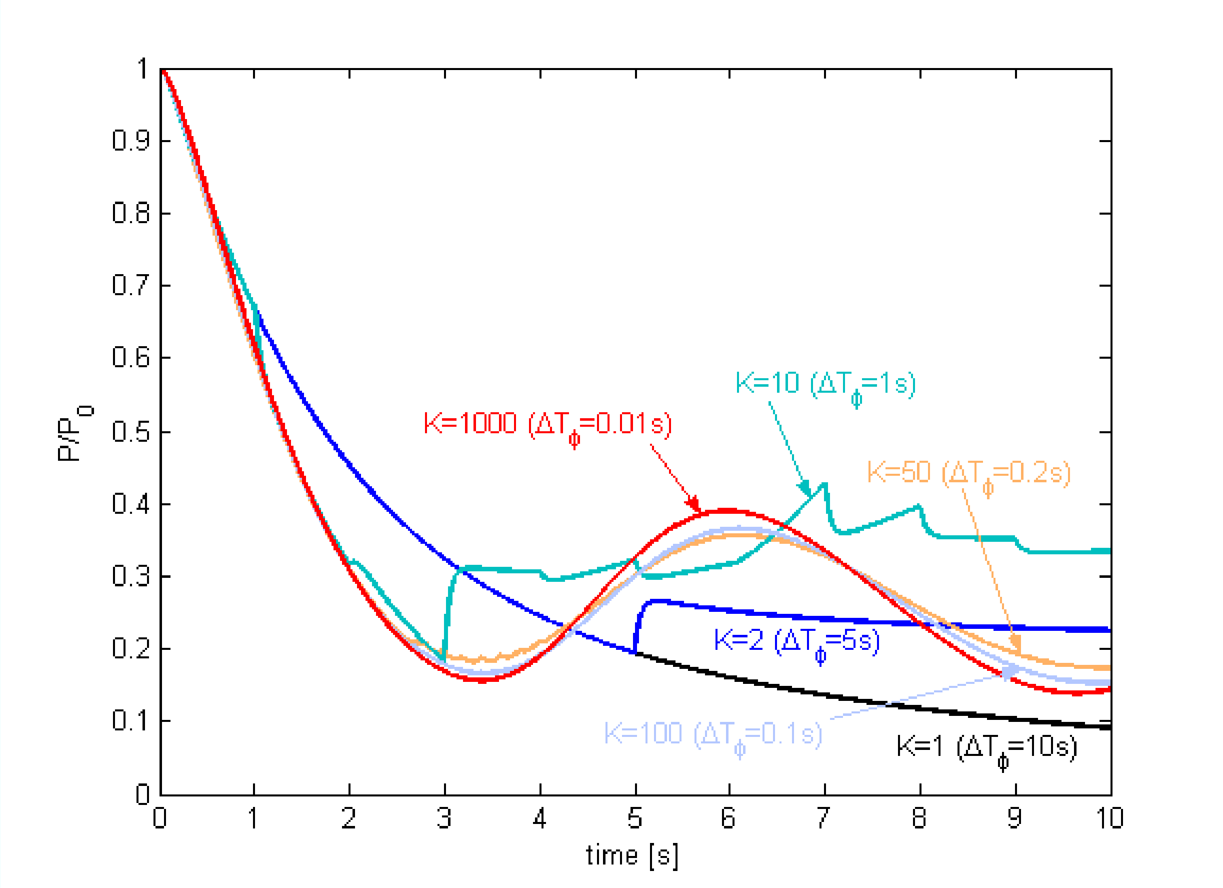
$$\begin{cases} \dot{A}\Big|_{T,j+1} = \alpha^j \; A|_T + \sum_{i=1}^{R} \mu_i^j \; G_i|_{T,j+1} + \tilde{S}_j, \\ \dot{G}_i\Big|_{T,j+1} = \vartheta_i^j \; A|_T - \Lambda_i^j \; G_i|_{T,j+1}, \qquad i = 1, ..., R \end{cases}$$

Numerical results

Non compensated fuel velocity transient in a critical system

($\Delta\rho$=-131pcm)

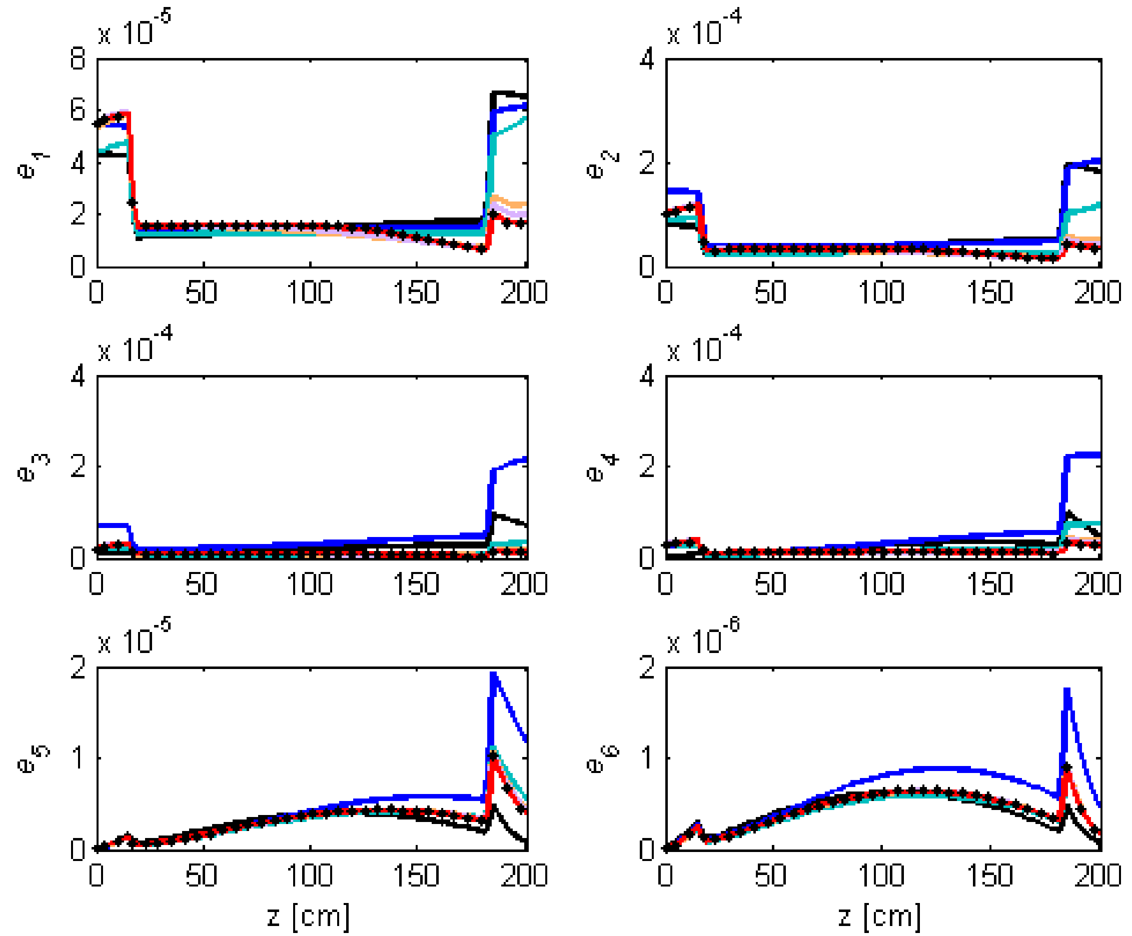
Power evolution



Numerical results

Non compensated fuel velocity transient in a critical system

Modification of shape functions



Shape functions used for kinetic parameter calculation at time t=5 s

Numerical results

Non compensated fuel velocity transient in a critical system

Computational effort

recalculations	1	2	10	50	100	1000	reference
Rel. time	0.001	0.043	0.158	0.337	0.481	0.515	1.000

Simulation of source experiments

- Experiments are under way to study the physics of source-driven systems
- Reactivity reconstruction from local flux measurements is an important aspect
- Pulsed experiments are considered
- Flux interpretation needs to account for spectral and spatial effects

Simulation of source experiments

- Inverse methods are easily implemented from point kinetics models
- It has been observed that satisfactory results are obtained if the global system power is used in inverse methods associated to *global* kinetic parameters

However

- Only signals from local flux detectors are available

Simulation of source experiments

- Signals from local detectors have to be (importance) averaged
- Strong importance effects can be observed in source-driven problems
- More suitable and problem-oriented weighting procedures must be used in inverse frameworks to construct ad-hoc kinetic parameters

Scope of the analysis

- *Study importance weighting procedures to generate point models that are adequate to accurately simulate local flux signal evolutions in a pulsed experiment*
- *Re-define time-dependence of source to better represent the physical response*

Methodology

Integral parameters are derived by taking the factorized balance equations

$$\boxed{n(\vec{x},t) = A(t)\varphi(\vec{x};t)}$$

$$\begin{cases} \dfrac{\partial(A\varphi)}{\partial t} = \hat{L}\varphi A + \hat{F}_p\varphi A + \lambda C + S \\ \dfrac{\partial C}{\partial t} = -\lambda C + \hat{F}_d\varphi A \end{cases}$$

project them on a weight

$$\begin{cases} \langle\psi|\varphi\rangle \dfrac{dA}{dt} = \langle\psi|\hat{L}\varphi\rangle A + \left\langle\psi\middle|\hat{F}_p\varphi\right\rangle A + \lambda\langle\psi|C\rangle + \langle\psi|S\rangle \\ \dfrac{d\langle\psi|C\rangle}{dt} = -\lambda\langle\psi|C\rangle + \left\langle\psi\middle|\hat{F}_d\varphi\right\rangle A \end{cases}$$

Methodology

obtaining a system of equations in time for amplitude functions with a point-like structure

$$
\begin{cases}
\dfrac{dA}{dt} = \dfrac{\langle \psi | \hat{L}\varphi \rangle}{\langle \psi | \varphi \rangle} A + \dfrac{\langle \psi | \hat{F}_p \varphi \rangle}{\langle \psi | \varphi \rangle} A + \lambda \tilde{C} + \dfrac{\langle \psi | S \rangle}{\langle \psi | \varphi \rangle} \\
\dfrac{d\tilde{C}}{dt} = -\lambda \tilde{C} + \dfrac{\langle \psi | \hat{F}_d \varphi \rangle}{\langle \psi | \varphi \rangle} A
\end{cases}
$$

Questions:

- Choice of the shape function φ ? **Initial reference configuration**
- Choice of the weighting function ? → definition of **various point kinetic models** with different objectives

Global and Local Point Kinetics (*gpk*) and (*lpk*)

weighting function → adjoint problem

Source-driven adjoint critical adjoint (eigenvalue)

- the adjoint source can be assumed as the local detector where the flux measurement is taken → *lpk*
- global weighting, i.e. assuming as adjoint source the fission productivity $\nu\Sigma_f$, suitable to predict power evolution→ *gpk*

Pulsed-source experiments

- Localized source in space and time
- Analysis of the evolution of the flux after the source pulse
- Comparison of different options for the construction of the point model adopted in the interpretation of the results

Pulsed-source experiments

- Analytical and semi-analytical study of a source pulse in a 1D system:
  - Comparison of exact results with point kinetic models;
  - Influence of the distance of the detector from the source:
    - Time-delay of the response of the system
    - Modification of the time behavior of the source

Pulsed-source experiments

- Point kinetic models:
  - Adjoint functions:
    - Local: $S^{\dagger} = \delta(x - x_0)$ → lpk
    - Global: $S^{\dagger} = \nu\Sigma_f$ → gpk
    - Critical problem → cpk

- Shape functions (initial ref. configuration is φ=0)
    - Equilibrium configuration with external source → φ_S
    - Critical configuration → φ_{cr}

Pulsed-source experiments

- Results for a simplified configuration, adopting analytical approach:
  - Similar performances of all options in reproducing the power evolution
  - Results with poor accuracy in flux prediction when the detector is placed far from the source
  - Strong higher harmonic effects for the flux for detectors near the source
  - lpk shows better performance in preserving areas (important when area methods are used in the interpretation of measurements)

Pulsed-source experiments

Flux at detector position

- Flux response at spatial points far from the source is mainly influenced by the time delay of the neutron signal detected

It is necessary to modify the time behavior of the source in order to simulate the real signal

  - Evaluation of time of flight to detector position
  - Evaluation of mean travel time (which distribution?)
  - Convolution of external source with system response (use of Green function)

Treatment of source delay

- Time of flight → $t_{x_0} = {}^{x_0}\!/_{v}$

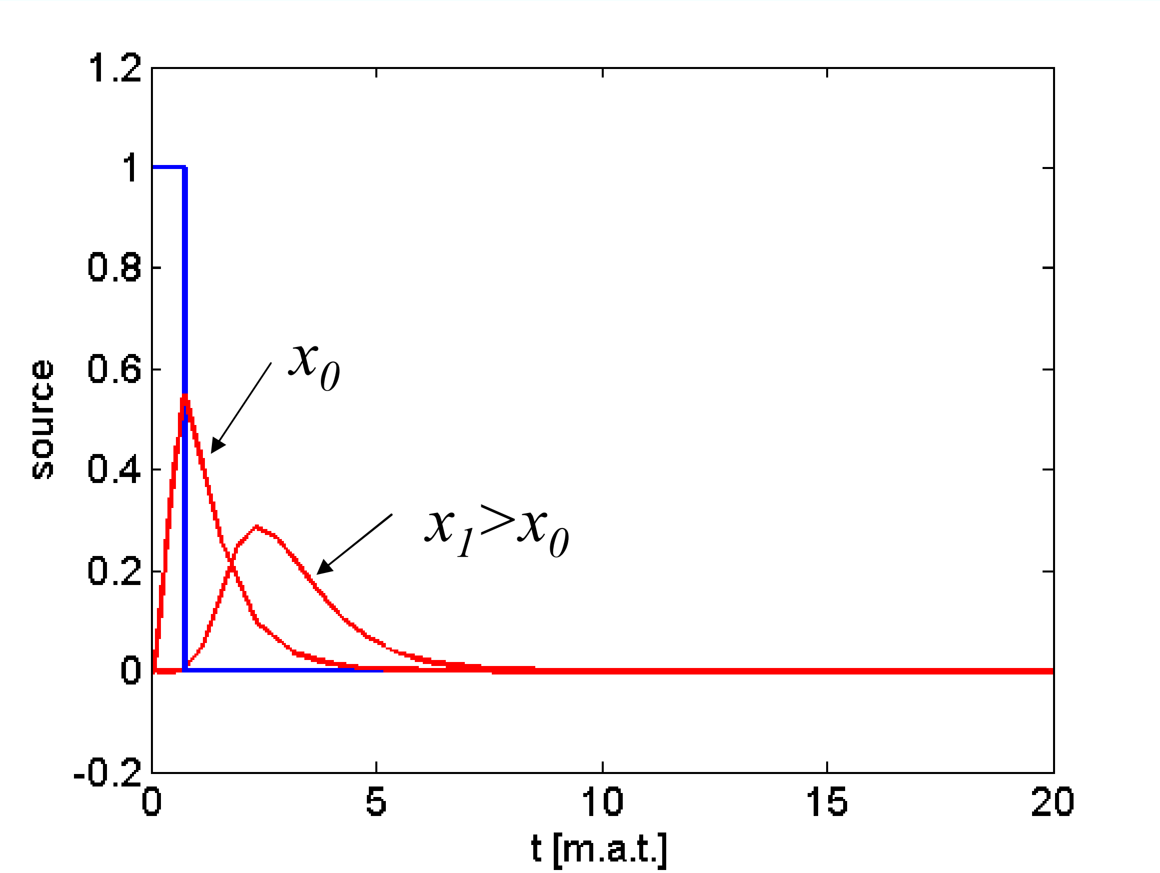
- Mean travelling time → $t_{x_0} = \int t' \cdot f(x_0, t')dt'$

$$S(x,t) = \delta(x)\vartheta(t) \longrightarrow \tilde{S}(x,t) = \delta(x)\vartheta(t - t_{x_0})$$

- Source convolution:

$$\tilde{\vartheta}(t) = \int \vartheta(t')G(x_0, t' \rightarrow t)dt'$$

Treatment of source delay



Treatment of source delay

$$x_0 = 4.4L$$

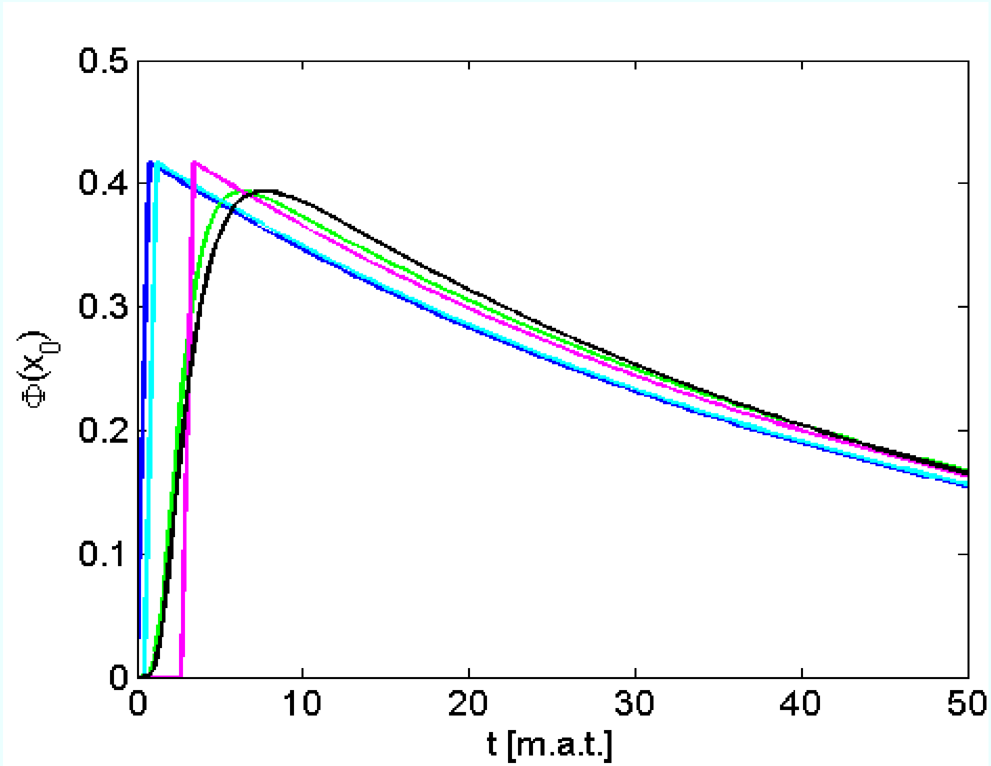
Exact solution

Point kinetics

$t_{x_0} = {}^{x_0}\!/_{v} = 6.15\mu s = 0.46$ m.a.t.

$t_{x_0} = \int t' \cdot f(x_0, t') dt' = 36 \mu s = 2.7 m.a.t$

Convolution with Green function



Treatment of source delay

$$x_0 = 0.9L$$

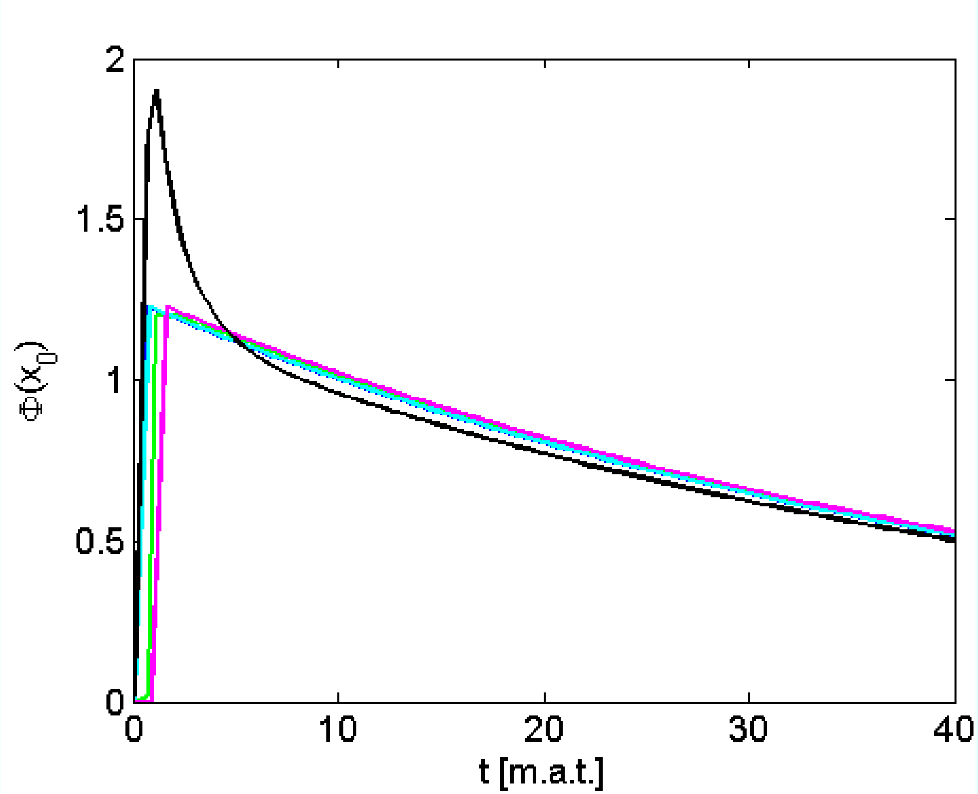
Exact solution

Point kinetics

$$t_{x_0} = {x_0}/{v} = 1.16\mu\text{s}=0.087 \text{ m.a.t.}$$

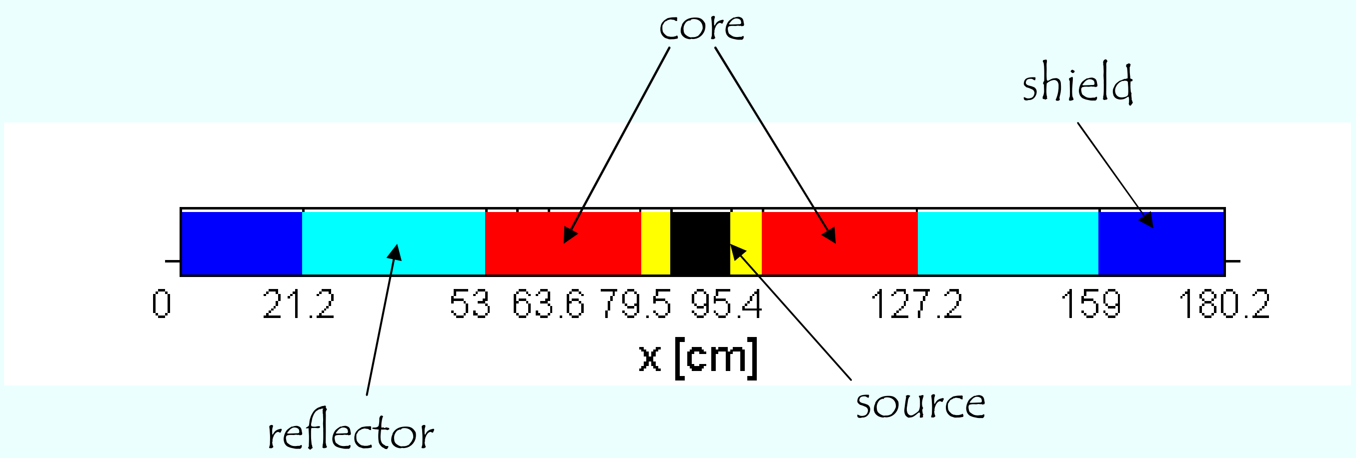
$$t_{x_0} = \int t' \cdot f(x_0, t')dt' = 12.3\mu s = 0.92 m.a.t.$$

Convolution with Green function

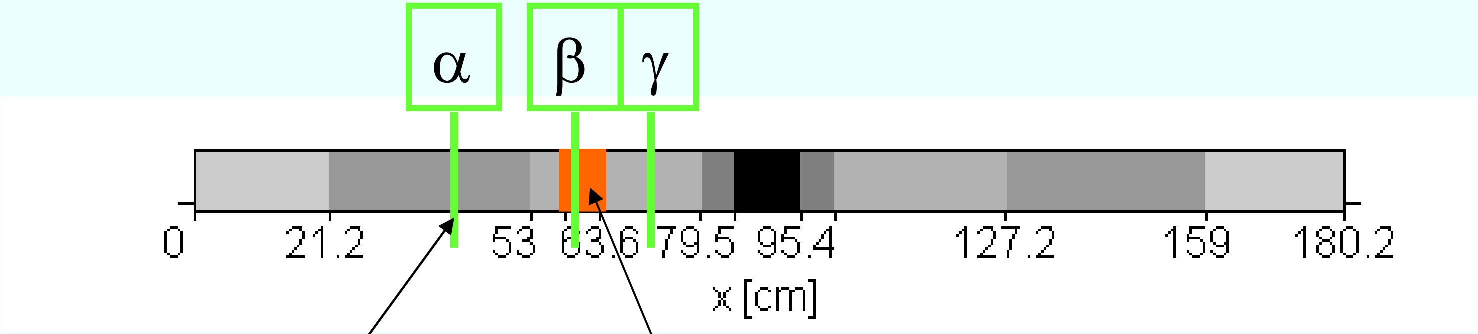


Use of lpk in transient evaluation

- 3-group solution in a MUSE-like configuration (1D)
- Localized perturbations of cross sections
- Evaluation of power and local flux using gpk and lpk



Use of lpk in transient evaluation



Monochromatic detectors $d_{\alpha,g}$

$\Delta\Sigma_{a,g} < 0$

Improved performances with lpk

<u>Transient 1</u>: $\Delta\Sigma_{a,1} < 0$

$\Delta\rho$=729pcm

- P/P_0= 1.286

$\Delta P_{gpk}= (P_{gpk}-P)/P[\%]=-1.78$

- $\Phi(x_{\alpha,1})/\ \Phi_0(x_{\alpha,1})=1.377$

$\Delta\ \Phi\ _{gpk}[\%]=-8.33$

$\Delta\ \Phi\ _{lpk}[\%]=-0.46$

- $\Phi(x_{\alpha,2})/\ \Phi_0(x_{\alpha,2})=1.316$

$\Delta\ \Phi_{gpk}[\%]=-4.05$

$\Delta\ \Phi_{lpk}[\%]=-1.46$

- $\Phi(x_{\alpha,3})/\ \Phi_0(x_{\alpha,3})=1.309$

$\Delta\ \Phi_{gpk}[\%]=-3.56$

$\Delta\ \Phi_{lpk}[\%]=-1.56$

- $\Phi(x_{\beta,1})/\ \Phi_0(x_{\beta,1})=1.316$

$\Delta\ \Phi_{gpk}[\%]=-4.05$

$\Delta\ \Phi_{lpk}[\%]=-1.45$

- $\Phi(x_{\beta,2})/\ \Phi_0(x_{\beta,2})=1.300$

$\Delta\ \Phi_{gpk}[\%]=-2.92$

$\Delta\ \Phi_{lpk}[\%]=-1.63$

- $\Phi(x_{\beta,3})/\ \Phi_0(x_{\beta,3})=1.297$

$\Delta\ \Phi_{gpk}[\%]=-2.68$

$\Delta\ \Phi_{lpk}[\%]=-1.69$

- $\Phi(x_{\gamma,1})/\ \Phi_0(x_{\gamma,1})=1.368$

$\Delta\ \Phi_{gpk}[\%]=-7.72$

$\Delta\ \Phi_{lpk}[\%]=-0.65$

- $\Phi(x_{\gamma,2})/\ \Phi_0(x_{\gamma,2})=1.326$

$\Delta\ \Phi_{gpk}[\%]=-4.78$

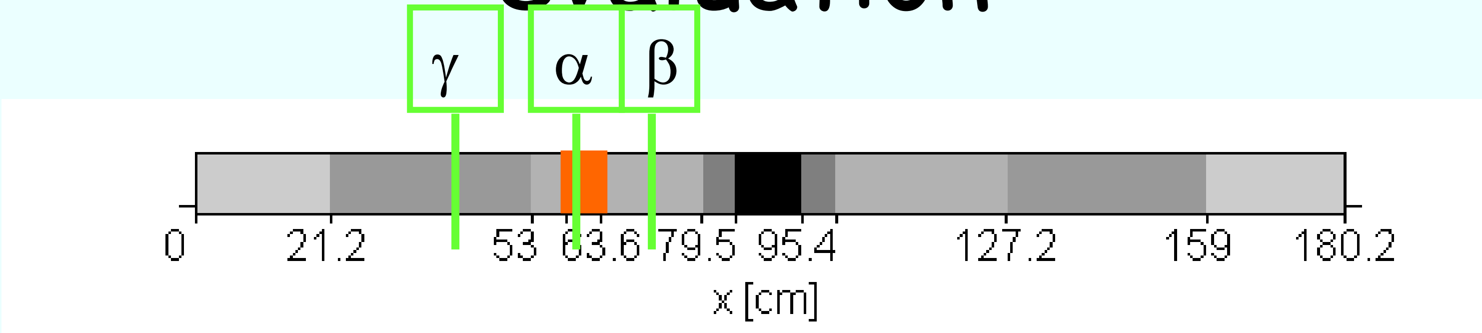
$\Delta\ \Phi_{lpk}[\%]=-1.34$

- $\Phi(x_{\gamma,3})/\ \Phi_0(x_{\gamma,3})=1.320$

$\Delta\ \Phi_{gpk}[\%]=-4.36$

$\Delta\ \Phi_{lpk}[\%]=-1.45$

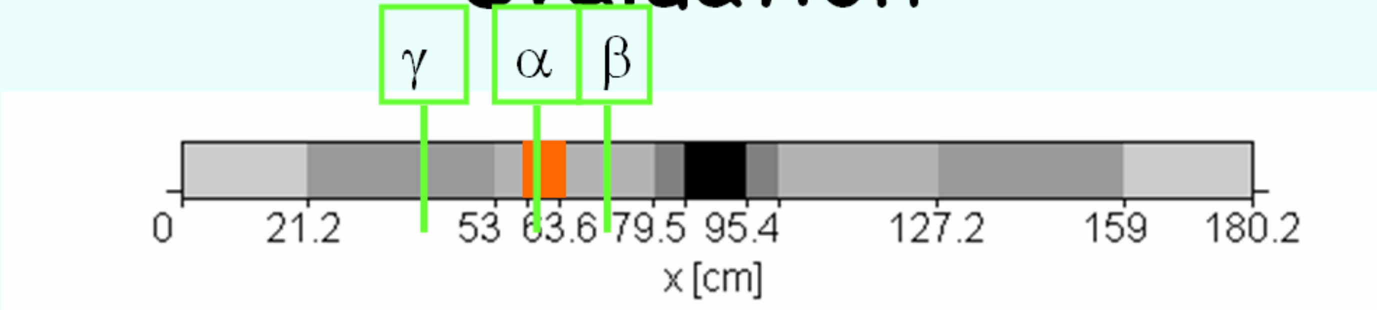
Use of Ipk in transient evaluation



Transient 1: $\Delta\Sigma_{a,1} < 0$ $\Delta\rho$=729pcm

	α	β	γ
g= 1	**-0.46**	**-1.45**	**-0.65**
g= 2	-1.46	-1.63	-1.34
g= 3	-1.56	-1.69	-1.45

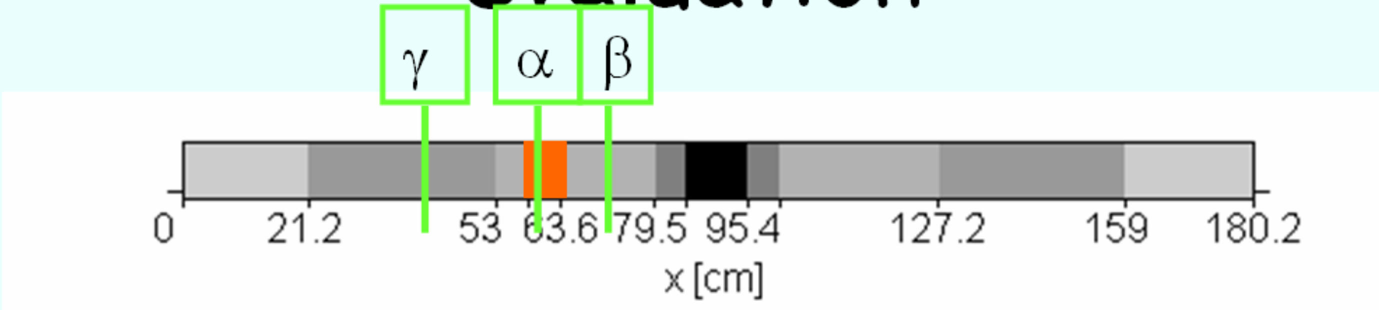
Use of Ipk in transient evaluation



Transient 2: $\Delta\Sigma_{a,2} < 0$ $\Delta\rho$=1157pcm

	α	β	γ
g= 1	-1.55	-2.31	-1.35
g= 2	**-0.43**	**-1.74**	**-0.55**
g= 3	-1.32	-1.98	-0.92

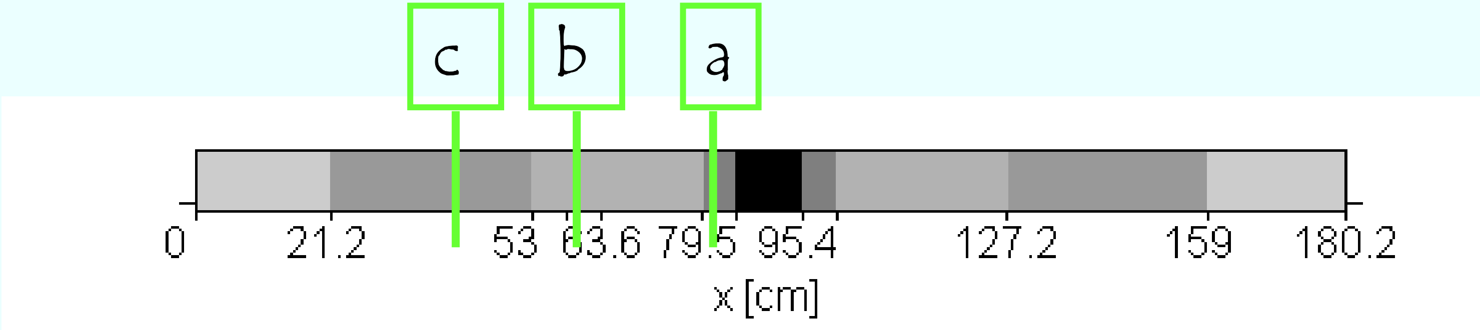
Use of Ipk in transient evaluation



<u>Transient 3</u>: $\Delta\Sigma_{a,3} < 0$ $\Delta\rho = 499$pcm

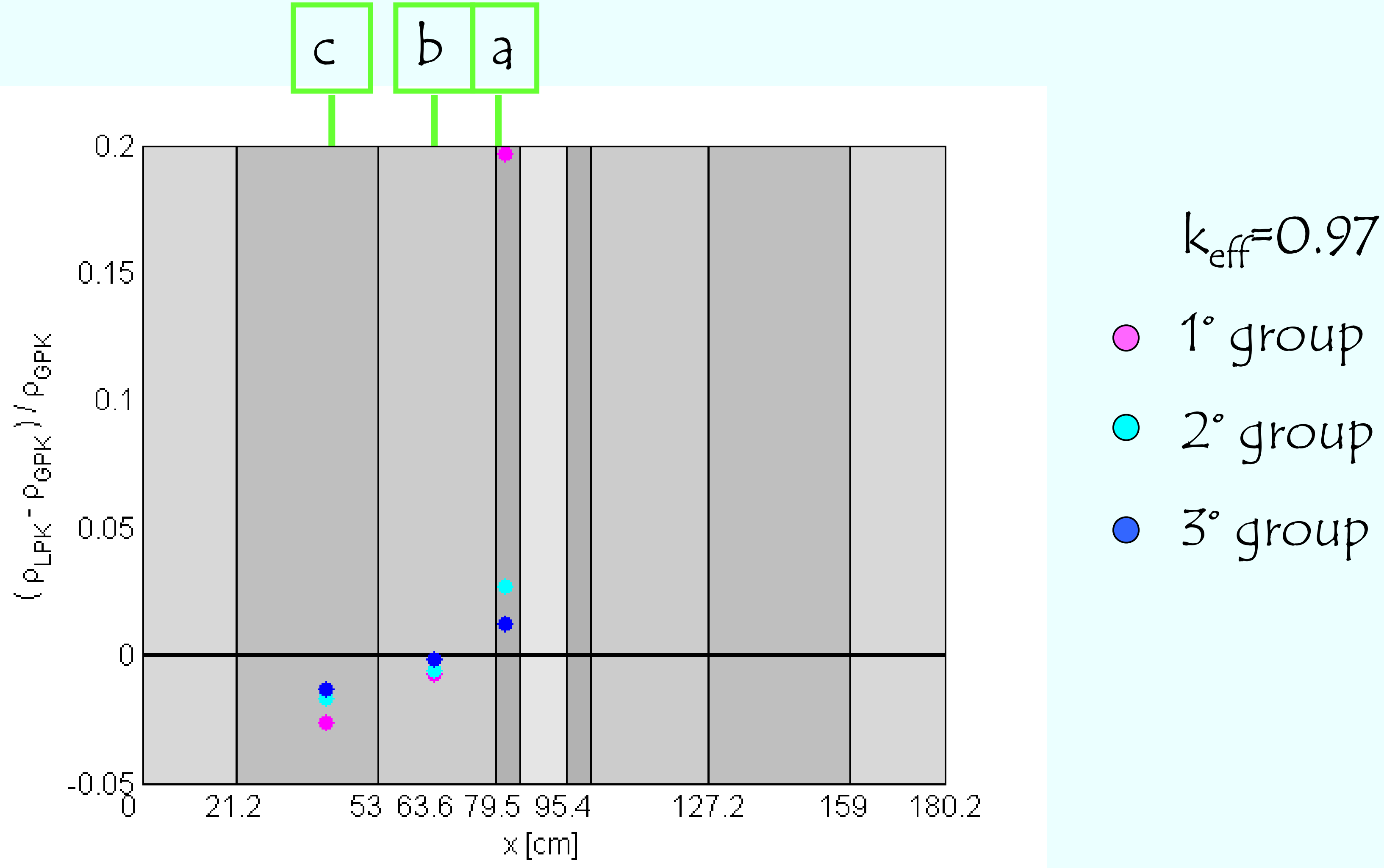
	α	β	γ
g= 1	-0.84	-0.90	-0.82
g= 2	-0.87	-0.91	-0.86
g= 3	<u>**-0.38**</u>	<u>**-0.79**</u>	<u>**-0.75**</u>

Reactivity evaluation in pulsed-source experiments

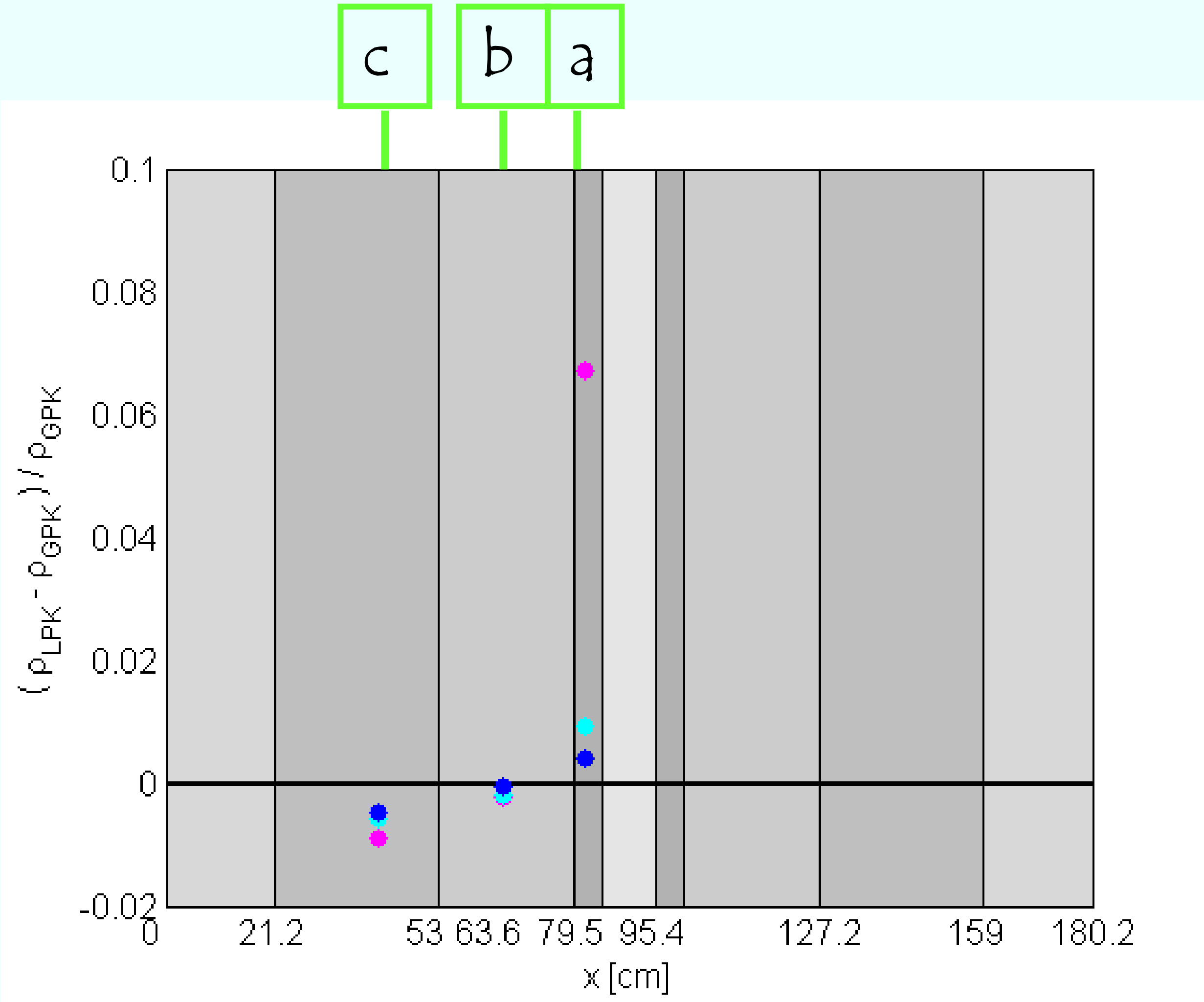


- 3-group evaluation of a subcritical system
- Evaluation of reactivity from Ipk for different detectors (at 3 spatial positions a-b-c and within each of 3 energy groups)

Reactivity evaluation in pulsed-source experiments



Reactivity evaluation in pulsed-source experiments



k_{eff}=0.99

1° group

2° group

3° group