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Laser speckle: a wonderful tool for quantum gases in disordered potential (Part I & II)

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ICTP Summer School on Novel Quantum Phases and Non-equilibrium Phenomena in Cold Atomic Gases, Trieste, August 28, 2007

From classical to quantum localization: laser speckle wonderful

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QUDEDIS ESF/PESC Programme

From classical to Anderson localization: laser speckle wonderful

1. Localization with cold atoms in laser speckle: why, how?

   A well controlled system

2. Suppression of the expansion of a 1D BEC released in a speckle

   Classical localization

3. Towards Anderson localization?

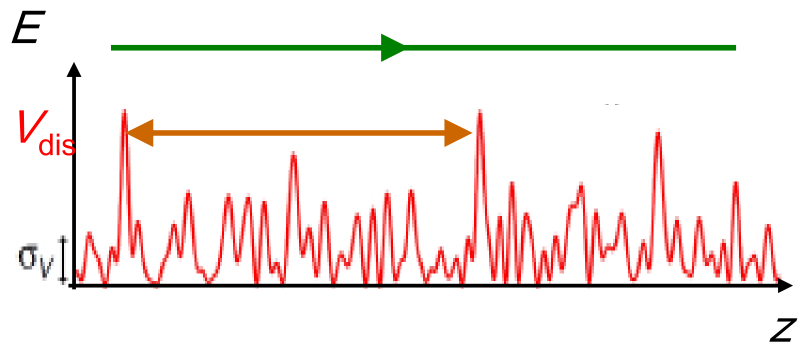
   An appealing scenario

Localization: a naive view (1)

Localization of particles in a disordered potential: a scenario to model insulators (electrons localization due to impurities)

Classical localization

energy smaller than the two highest potential barriers: confined classical motion



- $E \leq\sim 3\bar{V}$ localization
- $E \geq\sim 4\bar{V}$: no barrier higher than E; unbounded motion

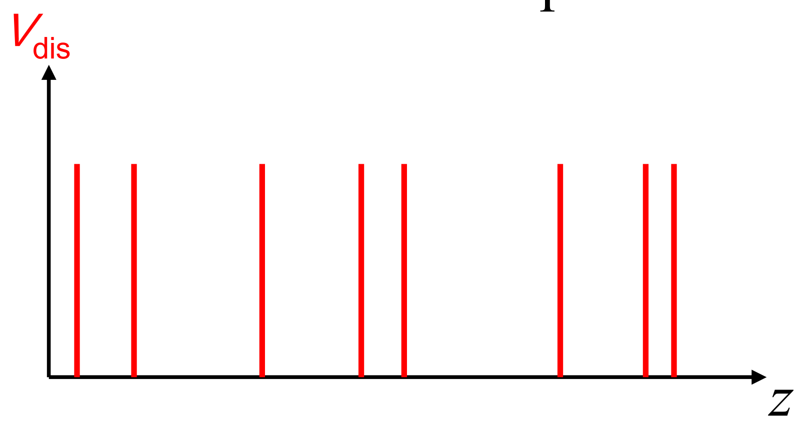
Can we have localization for $E >>> V$? Anderson localization

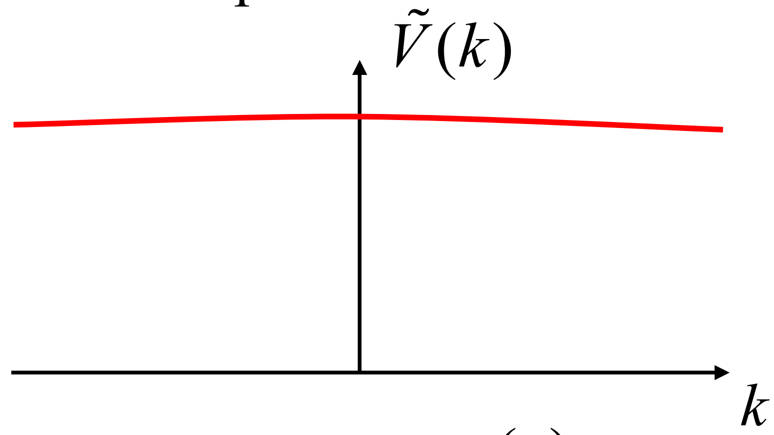
Localization: a naive view (2)

Localization of particles in a disordered potential: a scenario to model insulators with impurities

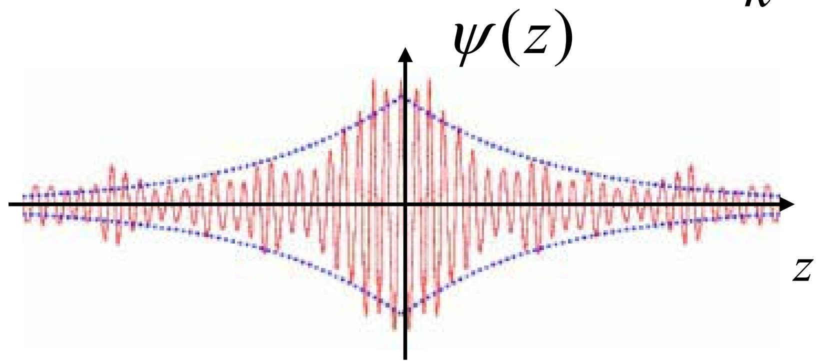
Quantum (Anderson like) localization 1D

Disordered potential with a broad k spectrum





Any de Broglie wave with momentum p is Bragg reflected on Fourier component $k = p / \hbar$: exponential envelope.



The experimental quest of Anderson localization

Electromagnetic waves: problem of absorption

Microwaves (cm) on dielectric spheres: discrimination between absorption and localization by study of statistical fluctuations of transmission
Chabonov et al., Nature 404, 850 (2000)

Light on dielectric microparticles (TiO_2):

- Exponential transmission observed, but possible role of absorption
  Wiersma et al., Nature 390, 671 (1997)
- Discrimination between absorption and localization (time resolved transmission) close to the localization threshold: Störzer et al., Phys Rev Lett 96, 063904 (2006)

Difficult to obtain $\ell < \lambda / 2\pi$ (ℓ = mean free path)

Ioffe-Regel criterion: mobility edge $k < 1 / \ell$

No direct observation of the exponential profile of the localized function

Most of these limitations do not apply to the 2D localization of light observed in disordered two-dimensional photonic lattices: T. Schwartz et al. (M. Segev), Nature 446, 52 (2007).

Ultra cold atoms (matter waves)
Good candidate to observe AL

Good features

- Controllable dimensionality, geometry (size >> λ_{dB})
- Wavelength λ_{dB} "easily" controllable over many orders of magnitude (1 nm to 10 μm)
- Pure potentials (no absorption), with "easily" controllable amplitude and statistical properties
- Many observation tools: light scattering or absorption, Bragg spectroscopy, …

A new feature: interactions between atoms

- A hindrance to observe AL (pure wave effect)
- New interesting problems

Disordered potentials for cold atoms

Magnetic potential on an atom chip

- Sub micron structures (near field), designed at will (e-beam lithography), hard to modify in real time; extra randomness a problem, in spite of progress in understanding and controlling it (J. Estève et al. PRA 2004, JB Trebbia et al. PRL 2007)

RF dressed potentials (Villetaneuse)

- Small enough structures still to be demonstrated

Impurity atoms in a lattice (Castin, exp in Hamburg)

- Not easy to pin the impurities

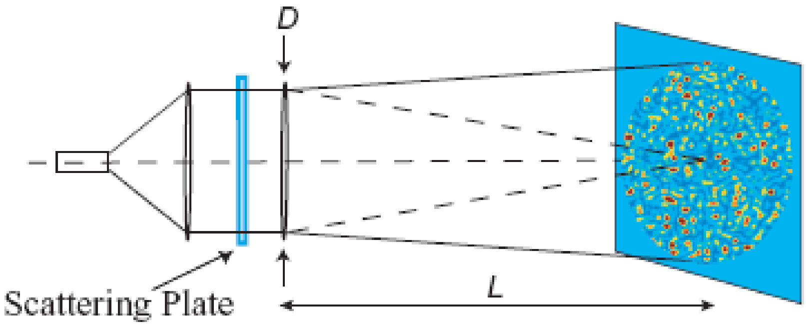
Optical dipole potential (R. Roth and K. Burnett, B. Damski et al., PRL 2003)

- laser beam transmitted through a random mask (Hanover): hard to make small structures (optical resolution)

- Optical dipole potential: created by a laser speckle = our favorite

Optical speckle random potential

Blue detuned light creates a repulsive potential $V \propto \frac{I}{\delta}$



Laser speckle: very well controlled random intensity pattern (Gaussian random process, central limit theorem)

Spatial distribution controlled by aperture. Autocorrelation function rms width (speckle grain size)

$$\sigma_R \simeq \frac{\lambda L}{\pi D}$$

Calibrated for $\sigma_R > 1\mu\text{m}$

Exponential intensity distribution

$$P(I) = \frac{1}{\bar{I}} \exp\{-\frac{I}{\bar{I}}\}$$

Calibrated by RF spectroscopy (light shifts distribution)

D. Clément *et al.*, New J. Phys. 8, 165 (2006)

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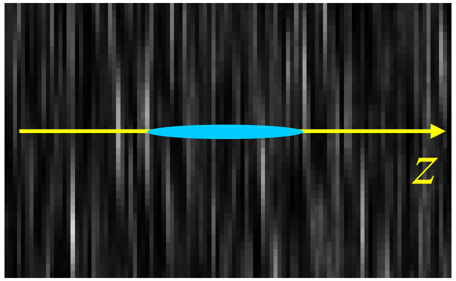
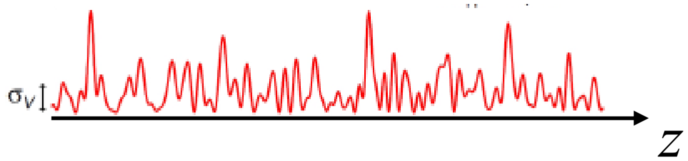
A 1D random potential for an elongated BEC

Cylindrical lens = anisotropic speckle, elongated along x and y





$$\sigma_x, \sigma_y \approx 50\,\mu\text{m}$$

$$\sigma_z \approx 1\ \mu\text{m}$$

BEC elongated along z : $2R_z^{\text{TF}} \approx 300\,\mu\text{m}$; $2R_\perp^{\text{TF}} \approx 3\,\mu\text{m}$

Released along z, confinement kept transversely during expansion

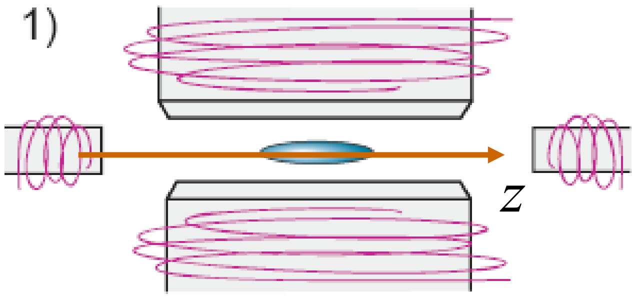
1 D situation for the expanding BEC.

Many speckle grains covered (self averaging system = ergodic)

Potential amplitude $\bar{V}$ tunable between 0 and 4 kHz (direct calibration) :
smaller than trapped BEC chemical potential $\mu_{\text{TF}} = 5$ kHz

1D expansion of an elongated BEC

1)

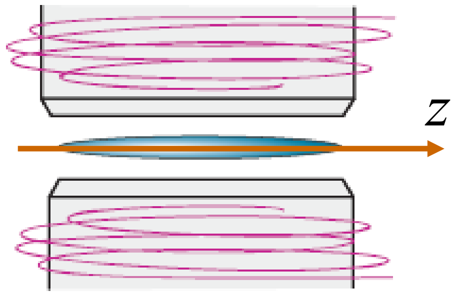


- Elongated BEC trapped

  $2R_z^{\mathrm{TF}} \approx 300\,\mu\mathrm{m}$; $2R_\perp^{\mathrm{TF}} \approx 3\,\mu\mathrm{m}$

  $\omega_\mathrm{z} / 2\pi = 6\,\mathrm{Hz}$; $\omega_\perp / 2\pi = 600\,\mathrm{Hz}$

2)

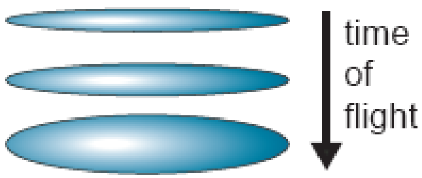


- Axial trapping released

  $\omega_\mathrm{z} / 2\pi \sim 1\,\mathrm{Hz}$

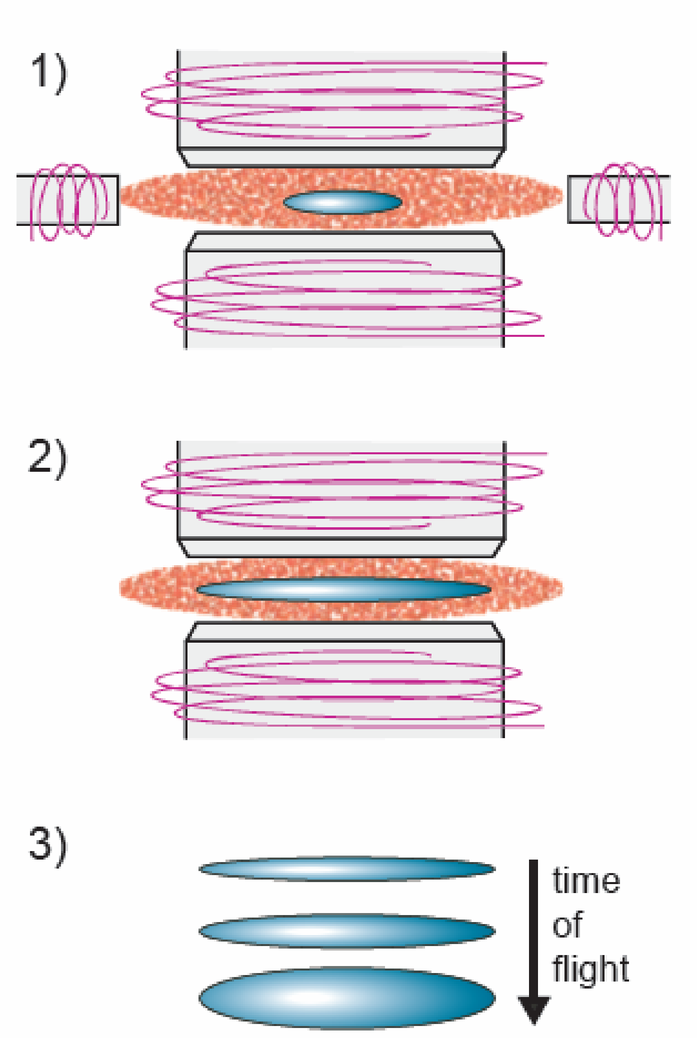
- BEC expands along z, still transversely confined

3)



- At time τ : switch off transverse trapping, free fall
- Absorption image allows to measure length at τ

1D BEC expansion in a random potential



- Elongated BEC trapped
- Apply random potential
- Hold for 200 ms

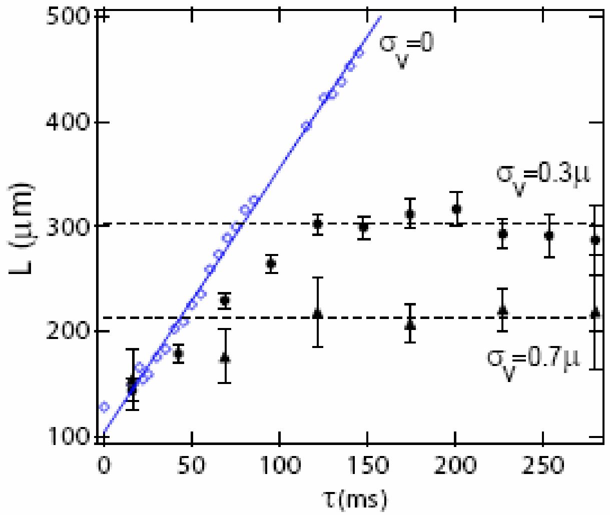
- Axial trapping released

$$\omega_z / 2\pi \sim 1\text{Hz}$$

- BEC expands along z, still transversely confined

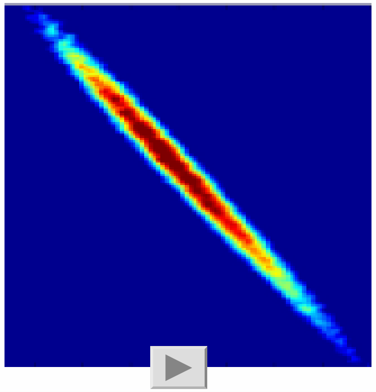
- At time τ: switch off transverse trapping, free fall
- Absorption image allows to measure length at τ

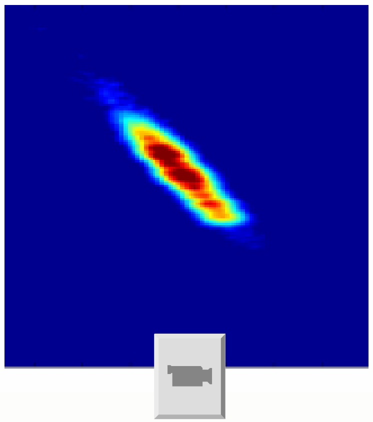
Expansion stopping in the random potential

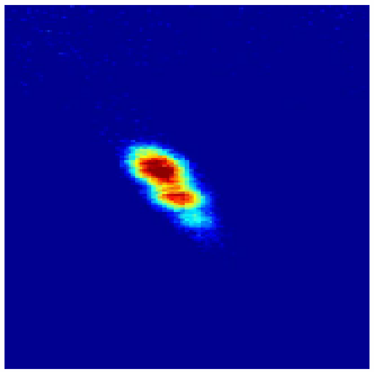


- ◇ Free axial expansion $\sigma_V = 0$
- ● With random potential $\sigma_V = 0.3\mu_{TF}$
- ▲ With random potential $\sigma_V = 0.6\mu_{TF}$

D. Clément, A. Varòn., M. Hugbart, J. Retter et al., Phys. Rev. Lett., 95,170409 *(2005)*







See also *C. Fort et al.*,PRL,95,170410(2005) in Florence and *T. Schulte et al.*,PRL,95,170411(2005) in Hannover

How to understand the expansion stopping?

Anderson localization?

What are the relevant parameters?

Energies

- Chemical potential μ_{in}
- Random potential V_{dis}
- Kinetic energy E_k

Initially $\mu_{in} > V_{dis} \gg E_{k\,in}$

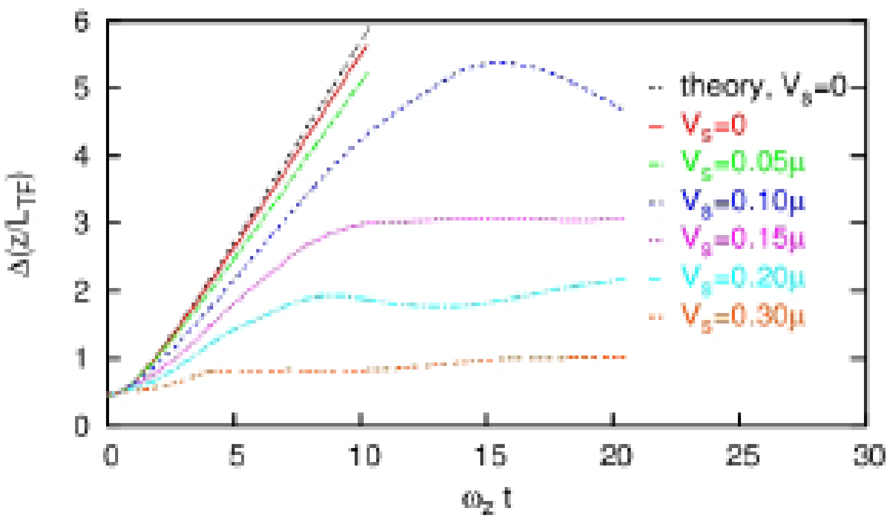
Disordered potential correlation length σ_R compared to healing length

Initially $\xi_{in} < \sigma_R \Leftrightarrow \mu_{in} > \dfrac{\hbar^2}{2M\sigma_R^{\ 2}}$

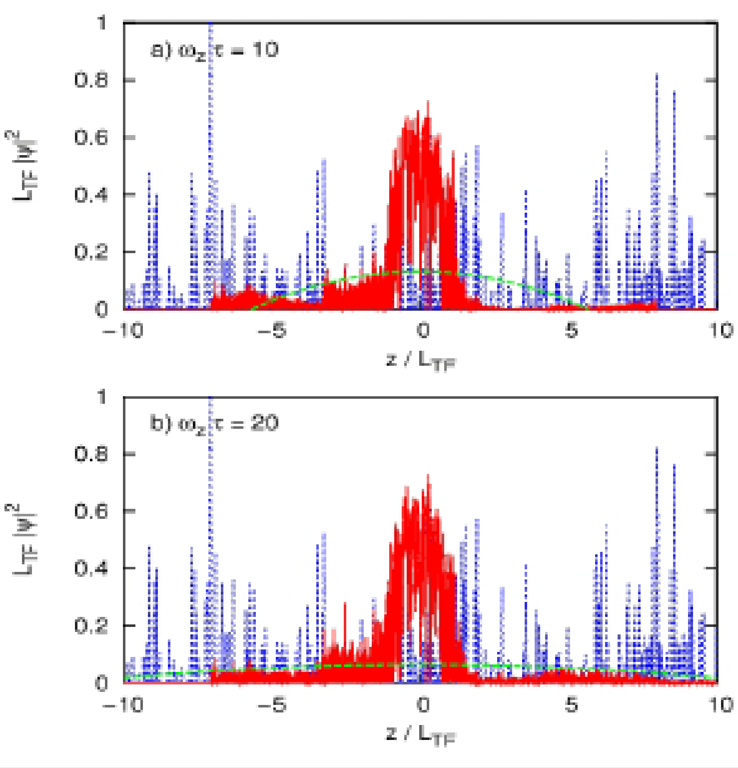
Numerical study

Numerical resolution of the GPE
(with optical speckle potential)

Reproduces quite well the
experimental behavior



Plot of the density profile
vs time suggests a scenario



Scenario for the stopping of the core expansion

Kinetic energy negligible: Thomas Fermi regime with $\mu_{in} > V_{dis}$:

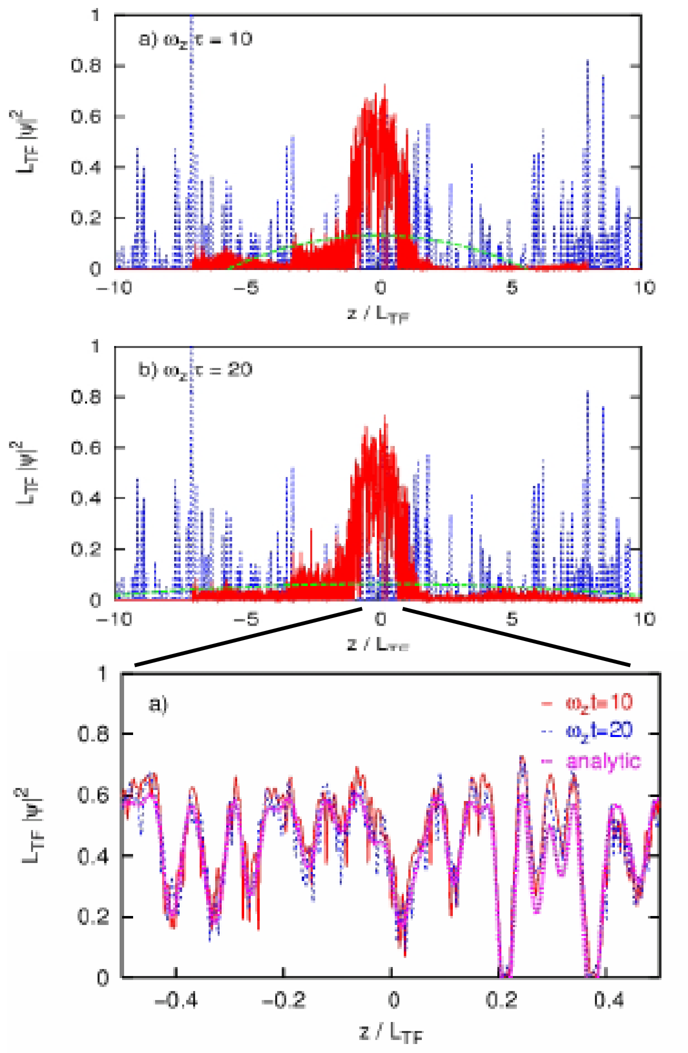
$\Rightarrow$ density $n(z)$ locally follows the disordered potential, with a parabolic envelope (initial Thomas Fermi profile)

$\Rightarrow$ expansion : $n(z)$ decreases until local chemical potential becomes equal to a local maximum of $V_{dis}(z)$

$\Rightarrow$ $n(z) = 0$ at peaks of $V_{dis}(z)$: fragmentation and expansion stopping

Competition between the interaction energy and the disordered potential

Averaged density reminiscent of the scaling solution of the expansion

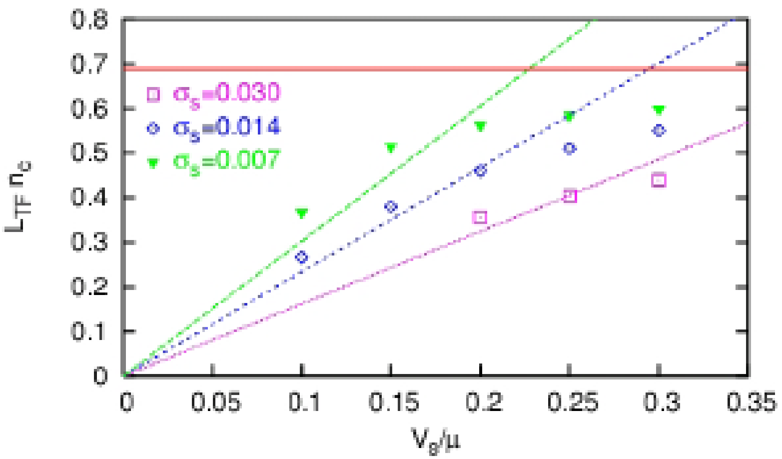


Density of the core after blocking

Simple estimate of n_0 based on the fragmentation scenario and on the speckle statistics:

probability to have two close peaks with heights $V_{\mathrm{peak}} > g_{1D}\, n_0$

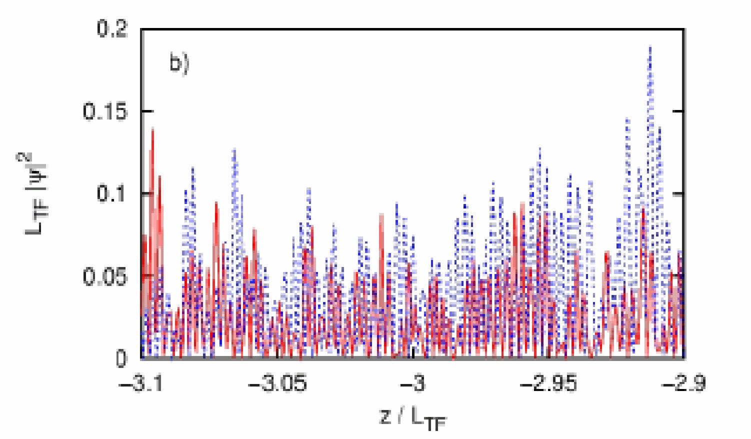
$$\Longrightarrow \quad n_0 \simeq 1.25 \left(\frac{V_R}{g_{1D}}\right) \ln\left(\frac{0.47 L_{TF}}{\Delta z}\right)$$



Reasonable agreement with results from numerics

Scenario for the expansion stopping in the wings

Observation (numerical calculations): average density constant, but detailed profile never stops evolving.



Regime of negligible interaction energy (n small) and large kinetic energy (a fraction of μ_{in})

Competition between kinetic energy and the disordered potential: many reflections back and forth.

Anderson localization?

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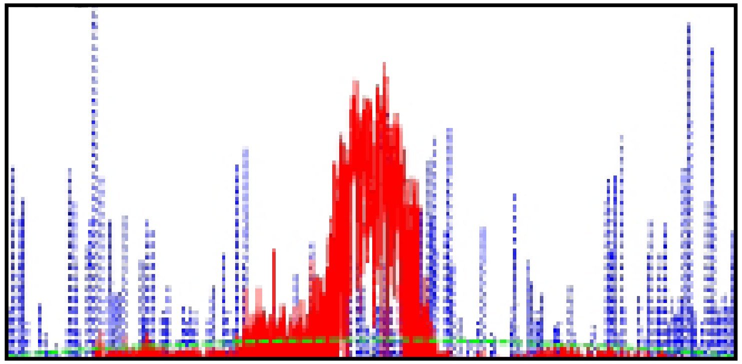
Observation (numerical calculations): average density constant, but detailed profile never stops evolving.



Regime of negligible interaction energy (*n* small) and large kinetic energy (a fraction of μ_{in})

Competition between kinetic energy and the disordered potential: many reflections back and forth.

Anderson localization?

No! Classical reflections on peaks higher than kinetic energy ☹

What prevented us from observing AL?

Anderson localization demands many (quantum) reflections
with $R \ll 1$

In the core

Kinetic energy negligible. Competition between interaction energy and disordered potential, leading to fragmentation. Kinetic energy too small to have quantum reflections on peaks.

In the wings

Competition between kinetic energy and disordered potential: classical reflection on large peaks, not quantum reflection.

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Weak disorder: a regime for AL?

Anderson localization demands many (quantum) reflections with $R \ll 1$

A good candidate: expanding BEC in a weak disorder $V_{dis} \ll \mu_{in}$

Almost free expansion. In the wings:

- Density small enough that interaction energy is negligible
- Initial energy transformed into kinetic energy large compared to V_{dis}: almost free de Broglie waves with $k_{max} \sim 1 / \xi_{in}$

If disordered potential has a broad enough spectrum of spatial frequencies, hope for AL

$\Rightarrow$ speckle grain size σ_R must be small compared to $1 / k_{max} \sim \xi_{in}$

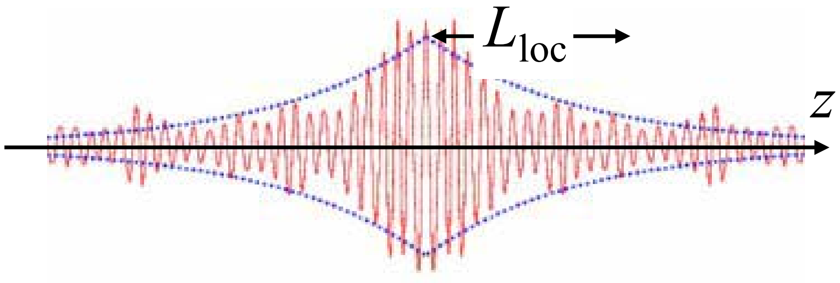
In previous experiments: $\sigma_R > \xi_{in}$ ☹

Localization in weak disorder for a given energy

Energy $E = \frac{\hbar^2 k^2}{2M} >> V_{dis}$

Interaction energy negligible

Example of numerical solution



Localized wave function

L_{loc} can be calculated using the phase formalism approach

$$\phi_k(z) = r(z)\sin[\theta(z)]$$
$$\partial_z \phi_k(z) = kr(z)\cos[\theta(z)]$$

- Solve Schrödinger equ. with these variables
- Calculate $$\frac{1}{L_{loc}(k)} = \gamma(k) = -\lim_{|z|\to\infty}\left\langle \frac{\log\{r(z)\}}{|z|} \right\rangle$$

Lyapunov coefficient

Localization length for a given *k*

Perturbative solution of the Schrödinger equation, in the presence of a weak disorder

$$\rightarrow \gamma(k) \simeq \frac{\sqrt{2\pi}}{8\sigma_R}\left(\frac{V_R}{E}\right)^2 (k\sigma_R)^2 \hat{c}(2k\sigma_R)$$

provided that $V_R\sigma_R \ll \hbar^2 k/2m\left(k\sigma_\mathrm{R}\right)^{1/2}$

$\sigma_V = \bar{V}$

width of the autocorr. funct. of the disordered potential

FT of the autocorrelation funct. of the disordered potential

Localization of wave *k* if $\hat{c}(2k\sigma_\mathrm{R}) \neq 0$

i.e. if the disorder contains a component able to Bragg reflect the plane wave *k*.

Localization length for a given *k*

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$\sigma_V = \bar{V}$

width of the autocorr. funct. of the disordered potential

FT of the autocorrelation funct. of the disordered potential

Localization if $\hat{c}(2k\sigma_R) \neq 0$

Corrections based on a diagrammatic approach (Gogolin):

$$\langle|\phi_k(z)|^2\rangle = \frac{\pi^2\gamma(k)}{2}\int_0^\infty du\, u\, \sinh(\pi u)\left(\frac{1+u^2}{1+\cosh(\pi u)}\right)^2 e^{-2(1+u^2)\gamma(k)|z|}$$

Localization length for a given *k*

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$\sigma_V = \bar{V}$

width of the autocorr. funct. of the disordered potential

Localization if $\hat{c}(2k\sigma_R) \neq 0$

Corrections based on a diagrammatic approach (Gogolin):

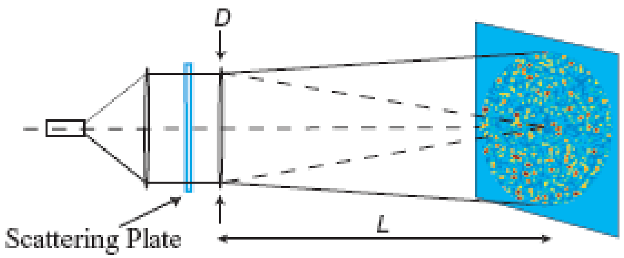
$$\langle|\phi_k(z)|^2\rangle = \left(\frac{\pi^{7/2}}{64\sqrt{2\gamma(k)}}\right)\frac{e^{-2\gamma(k)|z|}}{|z|^{3/2}}$$

Same localization length

Conclusion unchanged

Case of a speckle potential: existence of an effective mobility edge

Case of a speckle potential



In 1D,

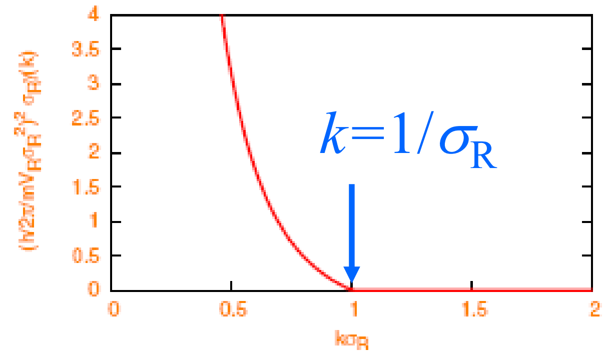
$$C(\Delta z) = V_R^2 \, sinc^2(\Delta z/\sigma_R)$$

$$\hat{c}(\kappa) = \sqrt{\pi/2}(1-\kappa/2)\Theta(1-\kappa/2)$$

$$\hat{c}(2k\sigma_R) = 0$$

$$\gamma(k) = \frac{1}{L_{\text{loc}}(k)} = 0$$

for $k > 1/\sigma_R$



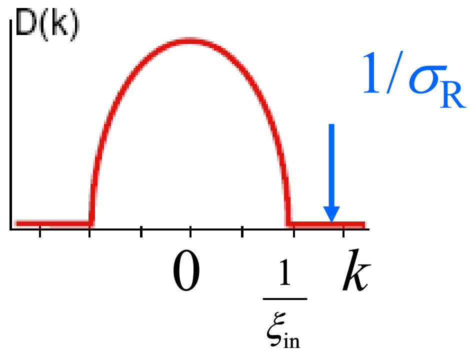
Effective mobility edge

Only waves with $k < 1/\sigma_R$ localize

Expansion of a BEC in a weak speckle potential: below the "mobility edge"

Assume an initial free expansion of the BEC, and take its *k* spectrum in the asymptotic regime (interaction energy negligible)

$$\mathcal{D}(k) \simeq \frac{3N\xi_{in}}{4}(1 - k^2\xi_{in}^2)\Theta(1 - k\xi_{in})$$



If $\frac{1}{\xi_{in}}$ < mobility edge $1/\sigma_R$

all *k* components localize. Addition of densities: density profile.

$$n_0(z) \propto \frac{\exp\{-2\gamma(1/\xi_{in})\,|z|\}}{|z|^{7/2}}, \quad |z| \to \infty$$

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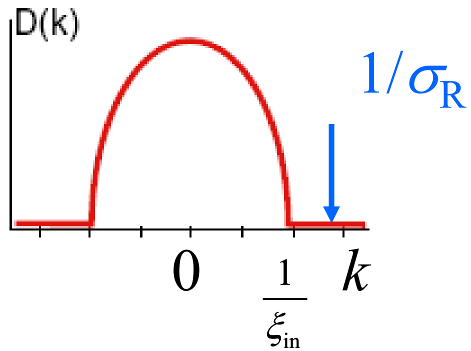

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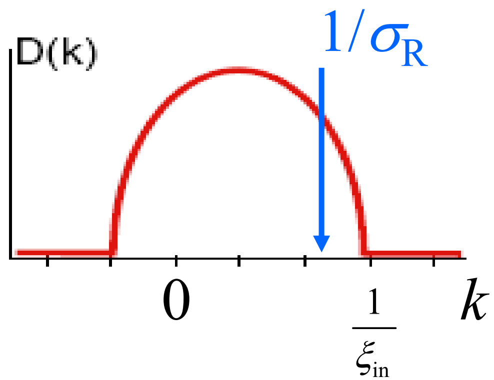
$$n_0(z) \propto \frac{\exp\{-2\gamma(1/\xi_{in})\,|z|\}}{|z|^{7/2}}, \quad |z| \to \infty$$

Looks like AL!

Expansion of a BEC in a weak speckle potential: above the "mobility edge"

Assume an initial free expansion of the BEC, and take its *k* spectrum in the asymptotic regime (interaction energy negligible)

$$\mathcal{D}(k) \simeq \frac{3N\xi_{in}}{4}(1-k^2\xi_{in}^2)\Theta(1-k\xi_{in})$$



If $\frac{1}{\xi_{in}}$ > mobility edge $1/\sigma_R$

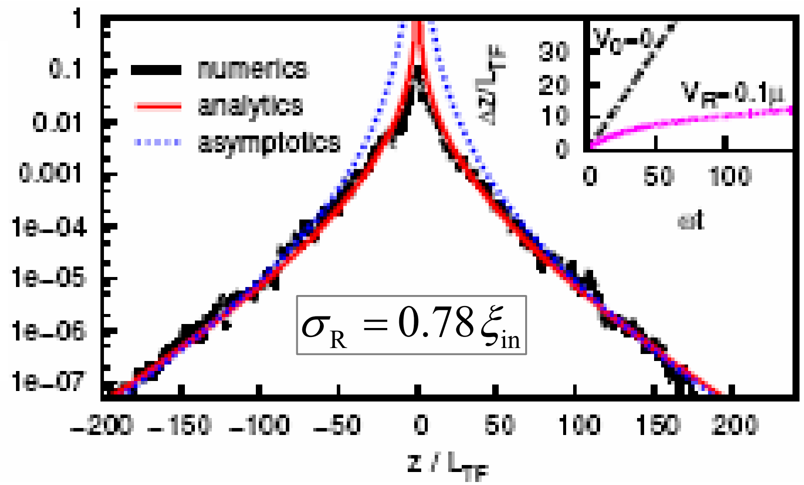
k components with $k > 1/\sigma_R$ do not localize.

Algebraic localization! No AL (exponential).

Previous experiments in that regime: $\sigma_R > \xi_{in}$ ☹

Expansion of a BEC in a weak speckle potential: numerical calculation (case of $\sigma_R < \sigma_{in}$)

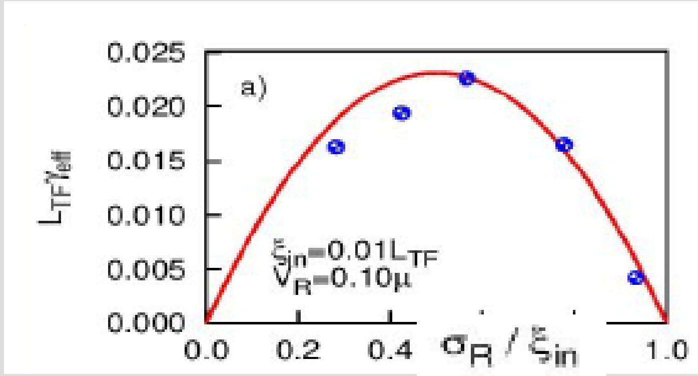
- expansion of a 1D BEC (GP eq.)
- disorder present from the begining
- interactions maintained during the whole expansion



Exponential localization in the wings

Comparison to the analytic result (asymptotic):

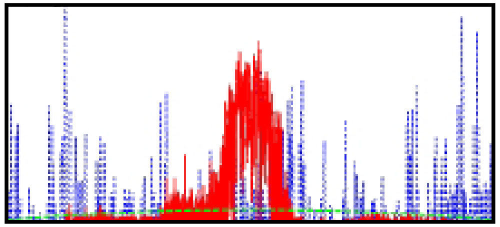
$$n_0(z) \propto \frac{\exp\{-2\gamma(1/\xi_{in})\,|z|\}}{|z|^{7/2}}$$



From classical to Anderson localization in a speckle potential: a (provisional) conclusion

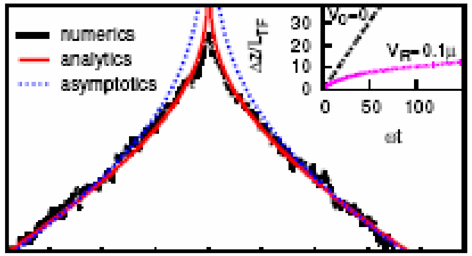
Strong disorder ($V_{\mathrm{dis}} \sim \mu_{\mathrm{in}}$)

Classical localization due to trapping between large peaks of the disordered potential, in the core and in the wings

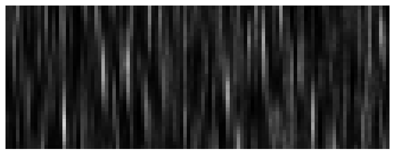


Weak disorder ($V_{\mathrm{dis}} << \mu_{\mathrm{in}}$)

Anderson localization predicted in the wings. Numbers encouraging enough that the experiment is underway ($\sigma_R = 0.6\ \xi_{\mathrm{in}}$). Hope to observe the density profile of localized wave function



Speckle: a wonderful tool for both theorists and experimentalists: well controlled random process.



References to our work on quantum gases in (speckle) random potential

Localization (coll. G. Shlyapnikov)

- D. Clément et al., PRL 95, 170409 (2005); NJP 8, 165 (2006)
- L. Sanchez-Palencia et al., PRL 98, 210401 (2007)

States of repulsive Bose Gases (coll. M. Lewenstein)

- L. Sanchez-Palencia, PRA 74, 053625 (2006)
- P. Lugan et al., PRL 98, 170403 (2007)

Localization of quasiparticles of an interacting BEC

- P. Lugan et al., cond mat 0707.2918, to appear in PRL

See poster tomorrow

Groupe d'Optique Atomique du Laboratoire Charles Fabry de l'Institut d'Optique

Graduate students and post docs welcome

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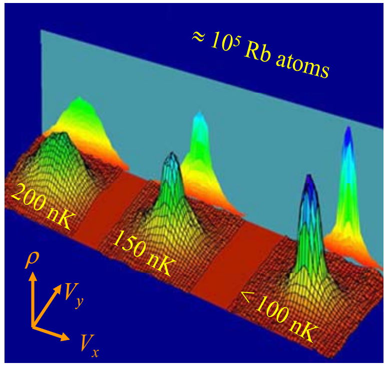
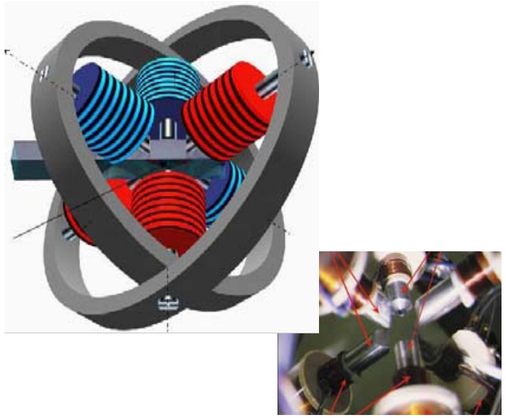
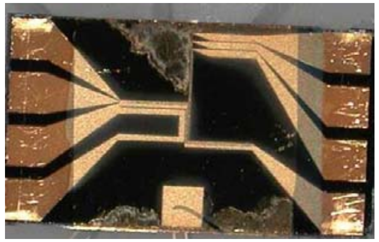
Bose Einstein Condensates

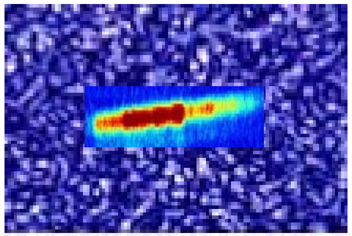




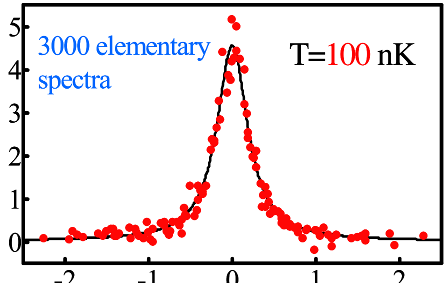
In a ferromagnetic yoke



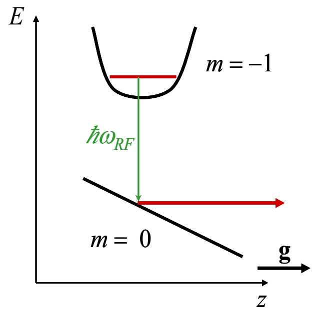
On an atom chip (applications)



In a random potential (localisation)



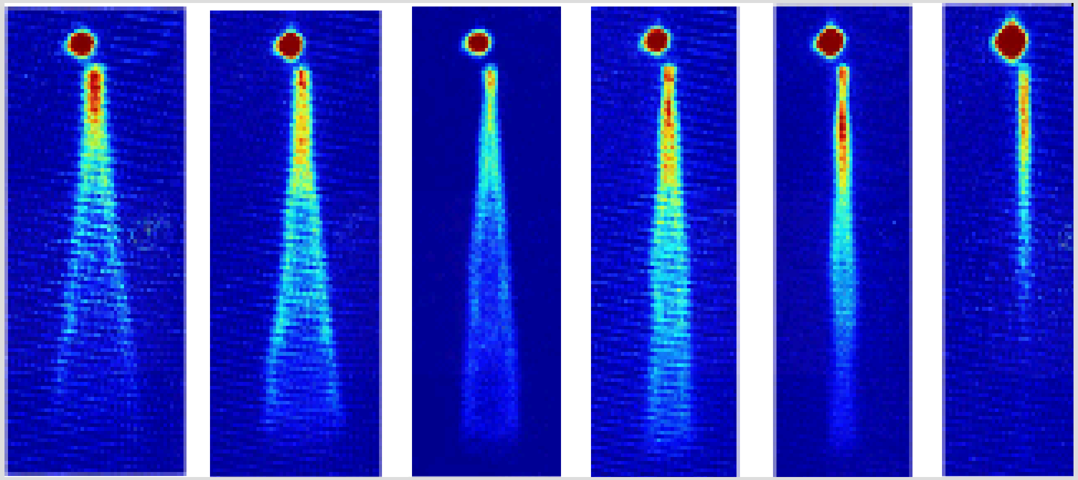
Coherence studies



Atom lasers

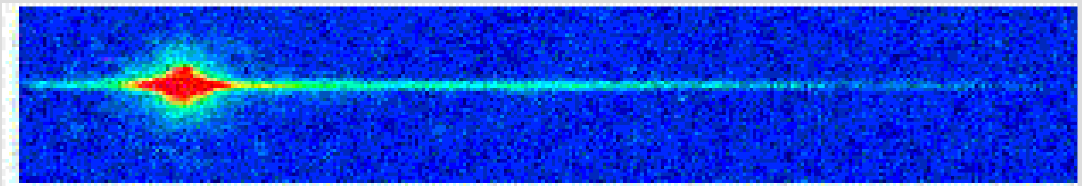
Atom lasers

Transverse structure: the M^2 factor



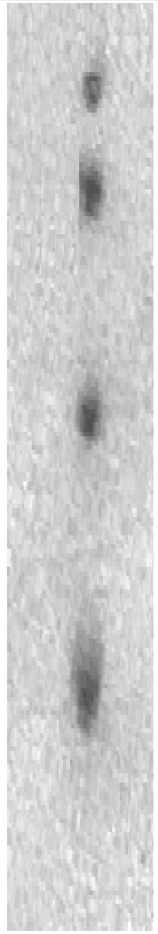
J.-F. Riou et al., 2005

A guided atom

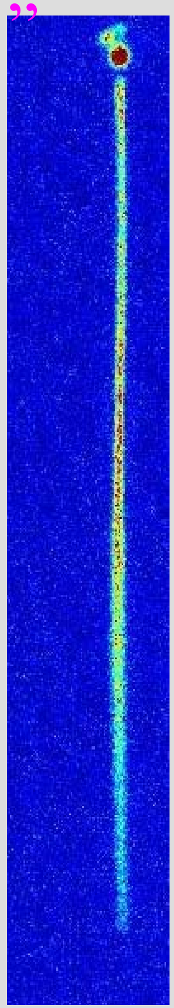


W. Guérin et al., 2006

Mode locked

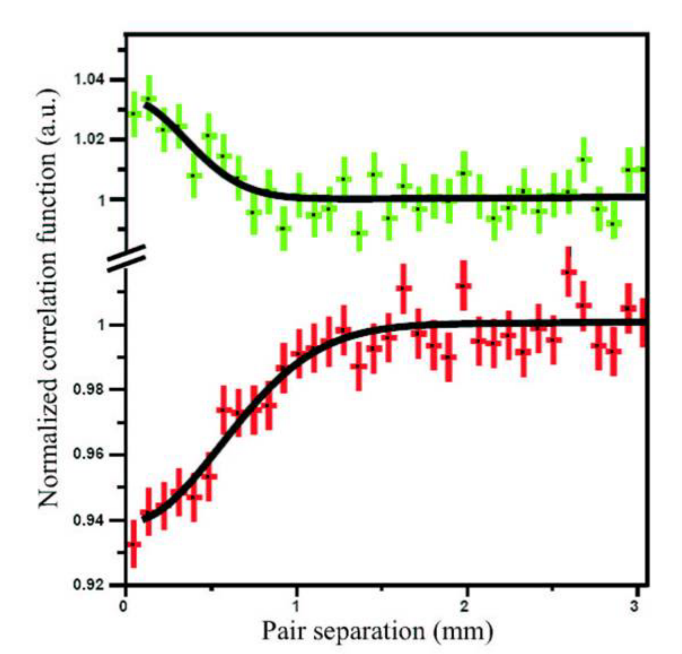


"CW"



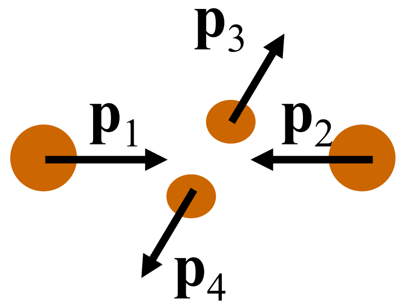
Quantum atom optics with He*

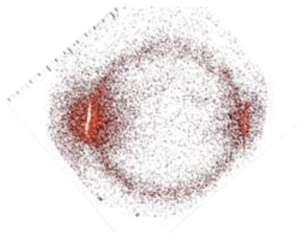
Atomic Hanbury Brown and Twiss effect for fermions and bosons

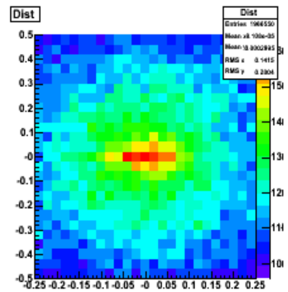


A fully quantum effect

Pairs of correlated atoms







Entanglement?