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Summer School on Novel Quantum Phases and Non-Equilibrium Phenomena in Cold Atomic Gases

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Quantum noise and other probes of strong correlations in ultracold atom systems

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Quantum noise and strong correlations in ultracold atom systems

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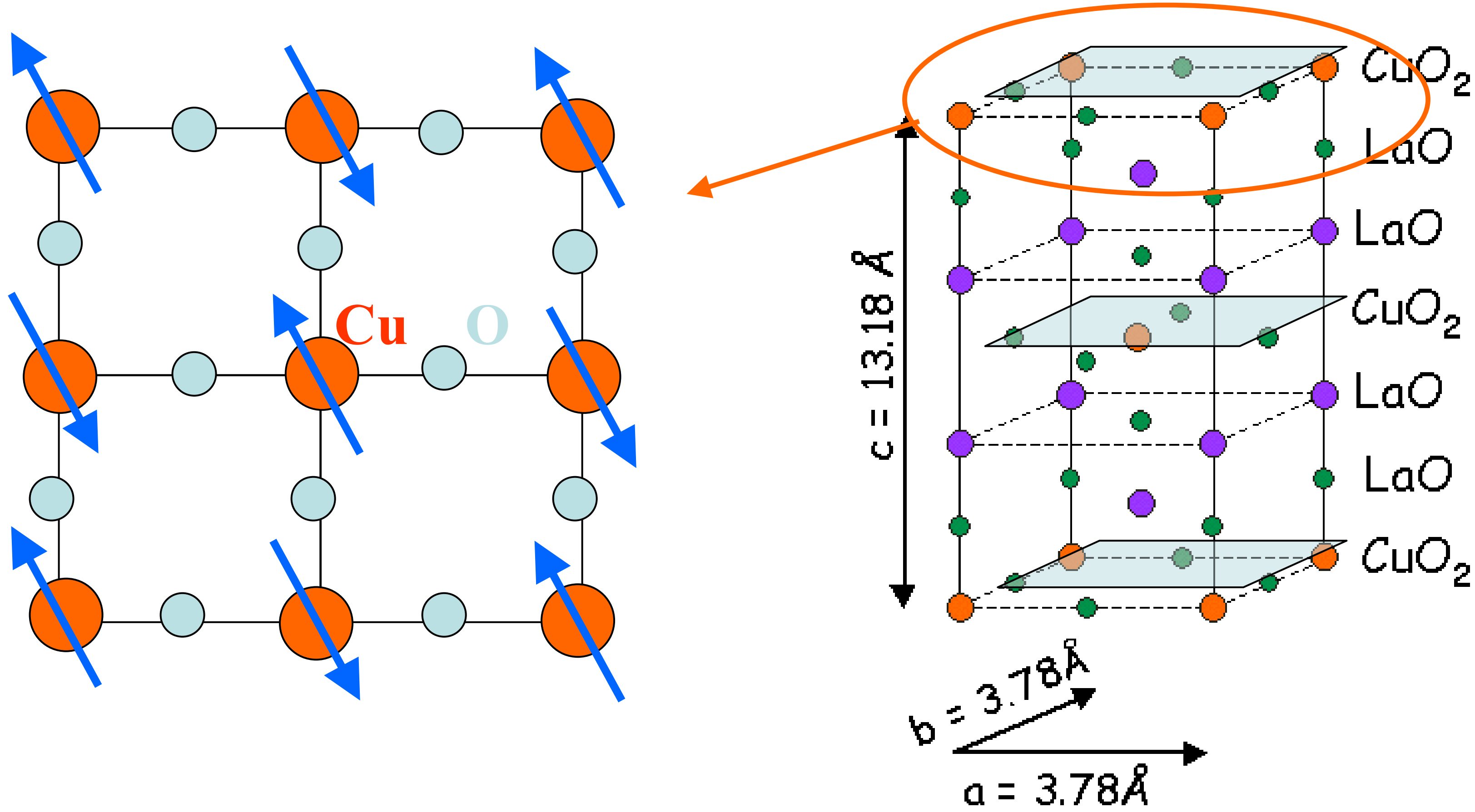
Challenges in strongly correlated systems

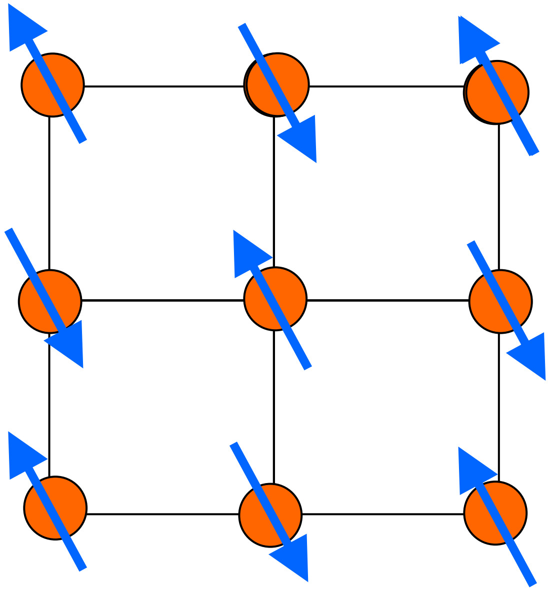
A case study: High T_C cuprates



$$H = -t \sum_{\langle ij \rangle \sigma} c^{+}_{i\sigma} c_{j\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow} + \ldots ?$$

Parent compounds – ½ filling

$$H = -t \sum_{\langle ij \rangle} \left(c^{\dagger}_{i\sigma} c_{j\sigma} + H.c. \right) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$



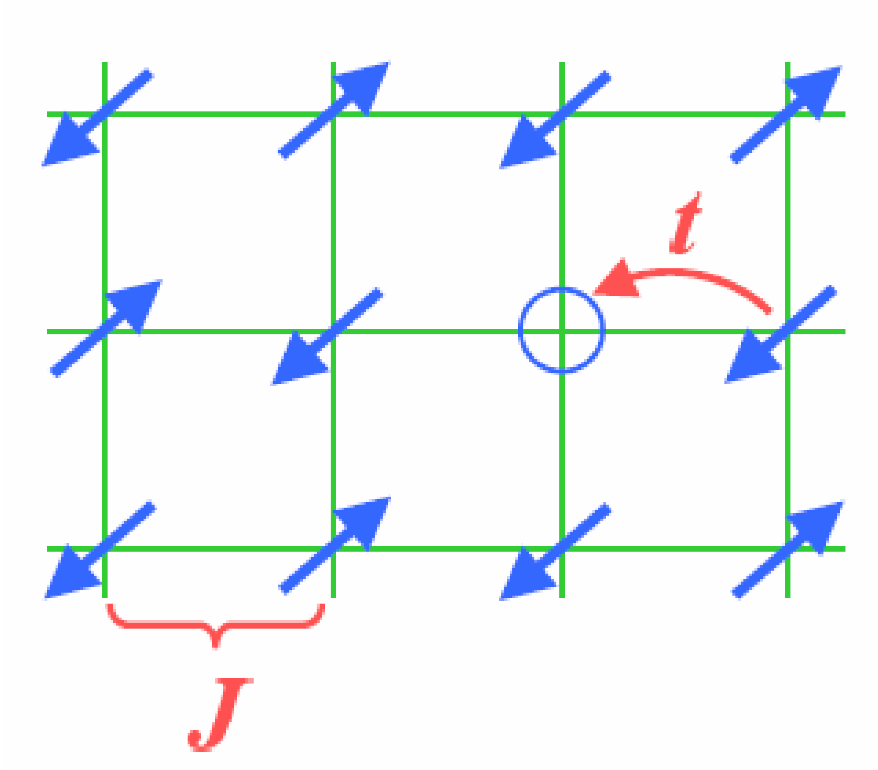
Electron localized because of large local repulsion

Spin degeneracy removed by 2nd order perturbation theory

$$H_{eff} = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j \qquad J \sim \frac{t^2}{U}$$

Antiferromagnetic Mott insulator

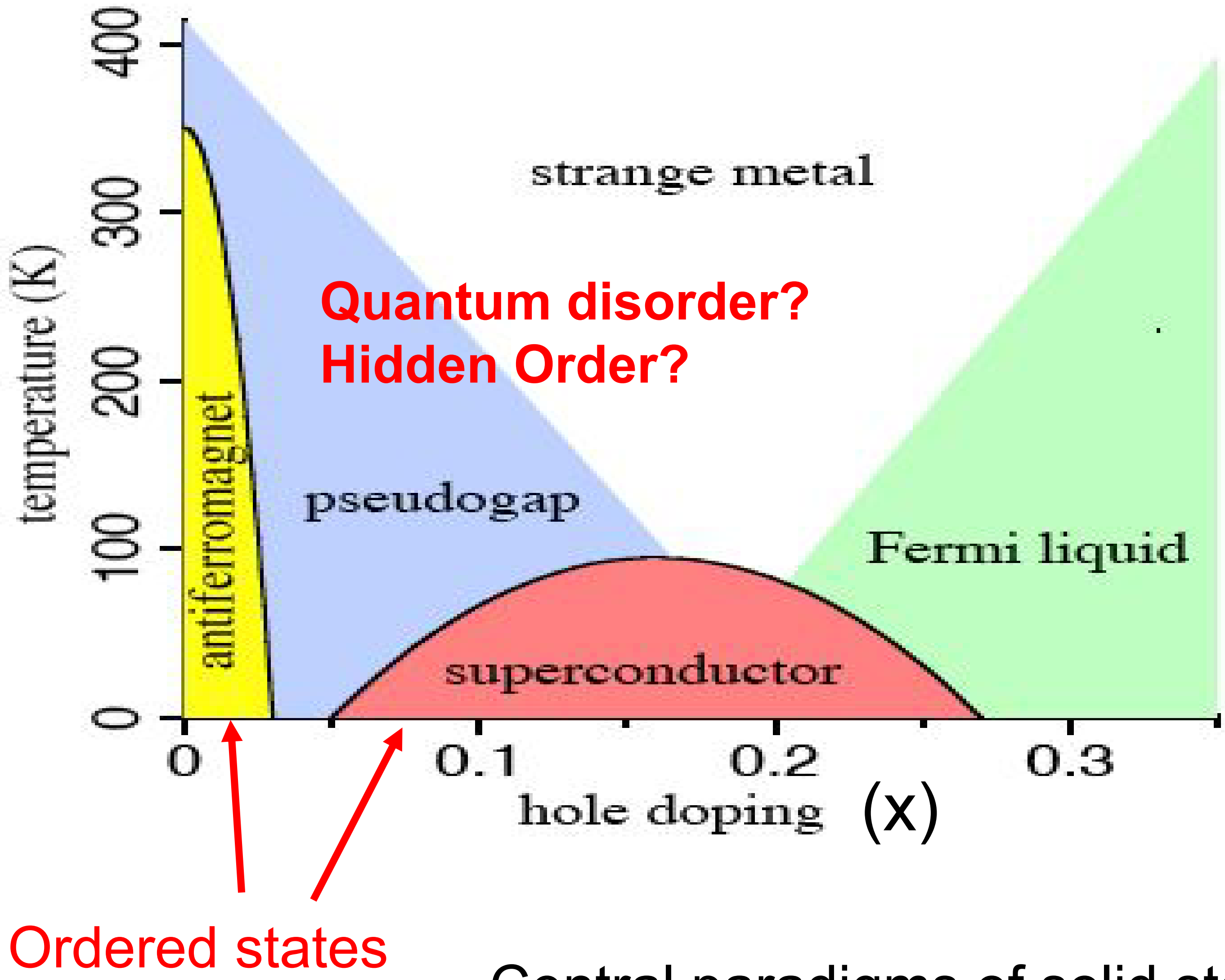
Chemical hole doping



Fate of the doped Mott antiferromagnet ?

- Highly correlated spin and hole dynamics
- No small parameter

Phase diagram



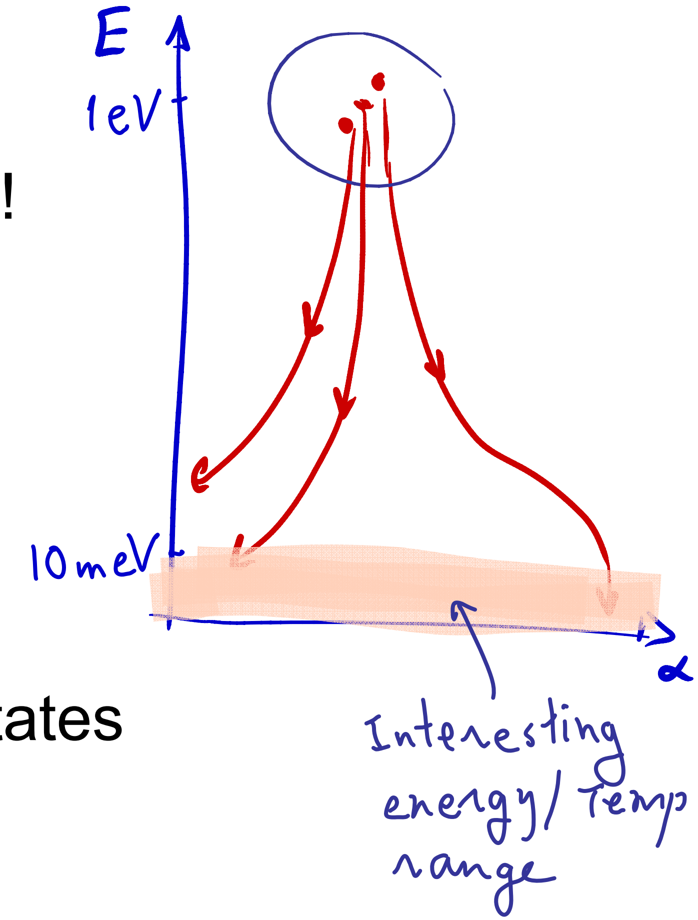
Central paradigms of solid state physics fail:
Fermi liquid, Landau theory, BCS

Challenges in strongly correlated systems

- Essentially impossible to predict phase diagram from the microscopics!

- Many competing orders
  - Are there hidden order parameters?
  - New emergent collective modes?

- Characterizing quantum disordered states
  - “Order” without broken symmetry?
  - Topological order?



How can ultracold atoms help?

- Serve as "Quantum simulators".

  e.g. realization of the repulsive Hubbard model

  See Hofstetter et. al. PRL (2002)

- Explore strongly correlated systems in new regimes

  Non equilibrium quantum dynamics

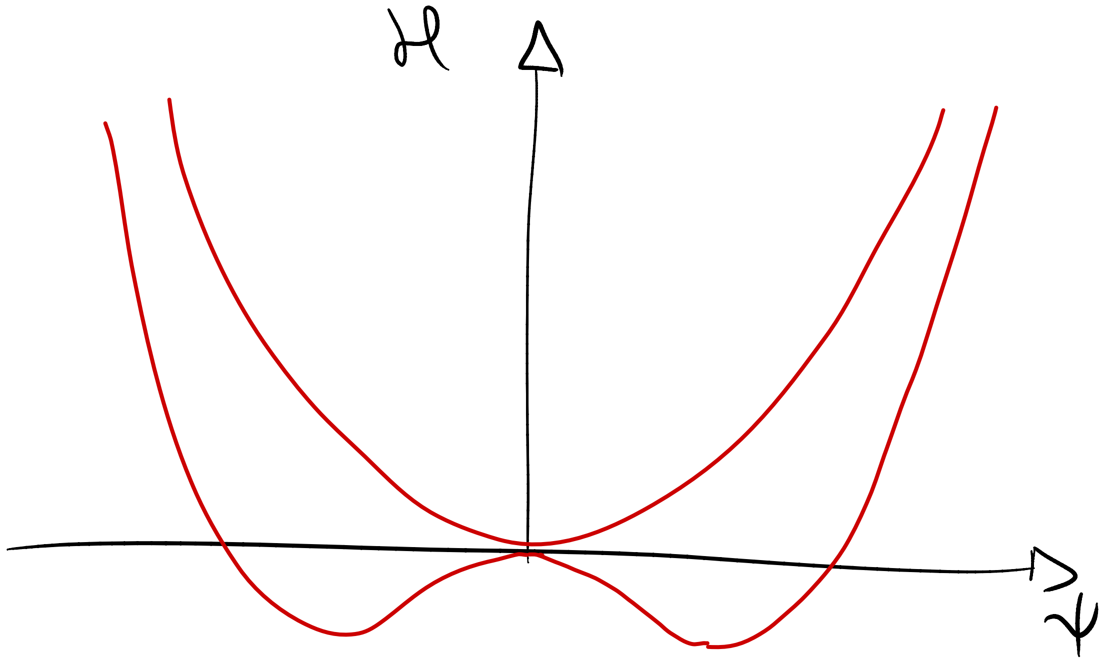
  Dipolar interactions

- New measurement techniques are needed!

  Quantum noise interferometry

  Interference of independent condensates

Broken symmetry: Landau theory



$$H_{quantum} \rightarrow \mathcal{H}_{cl}[\psi(x)]$$

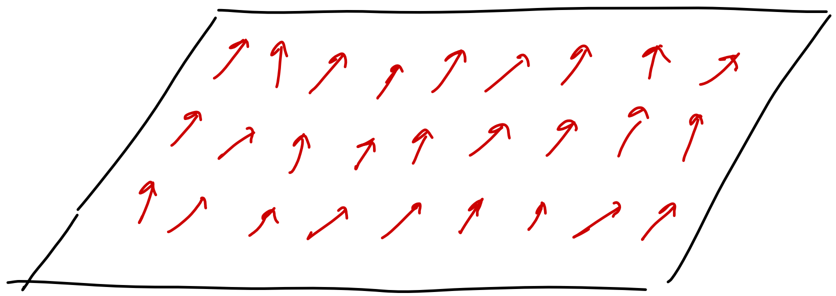
$$-i\frac{\partial \hat{\psi}}{\partial t} = [\hat{H}, \hat{\psi}] \longrightarrow$$ Effective classical dynamics of local order parameters

Landau Theory Example 1

Gross-Pitaevskii description of bose superfluids

$$\langle b^{\dagger}(x)\, b(0)\rangle \to \text{const}$$

$$\langle b(x)\rangle = \psi(x) = \sqrt{\rho(x)}\, e^{i\varphi(x)}$$



$$v_s(x) = \frac{\hbar}{m}\nabla\varphi(x)$$

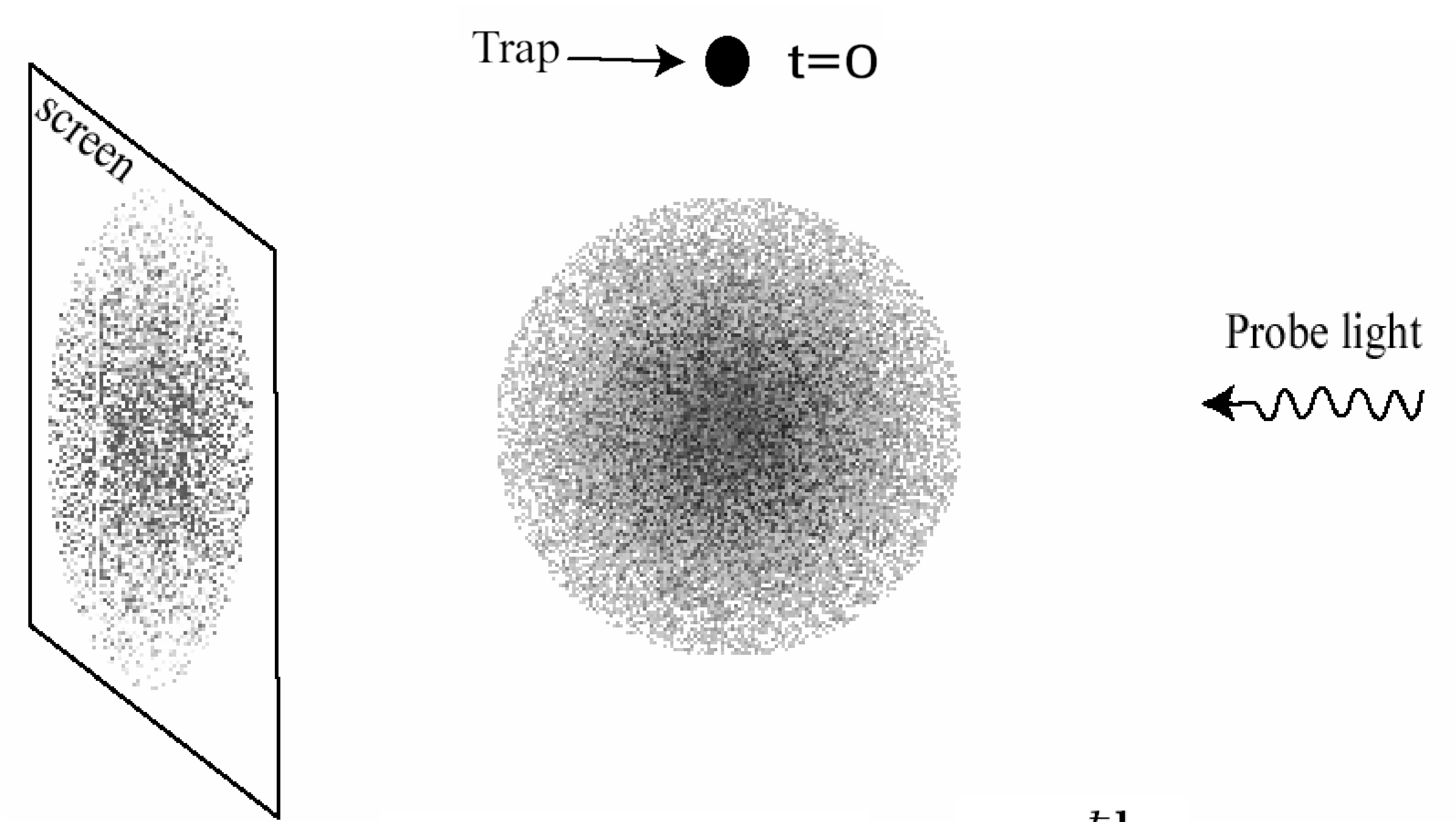
Broken U(1) symmetry

<u>Classical</u> field equation for the condensate wave function

$$-i\frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + \left[V(x) + u|\psi|^2\right]\psi$$

Enormous success in describing ultra cold dilute atomic gasses !

Detection of superfluid order: time of flight imaging

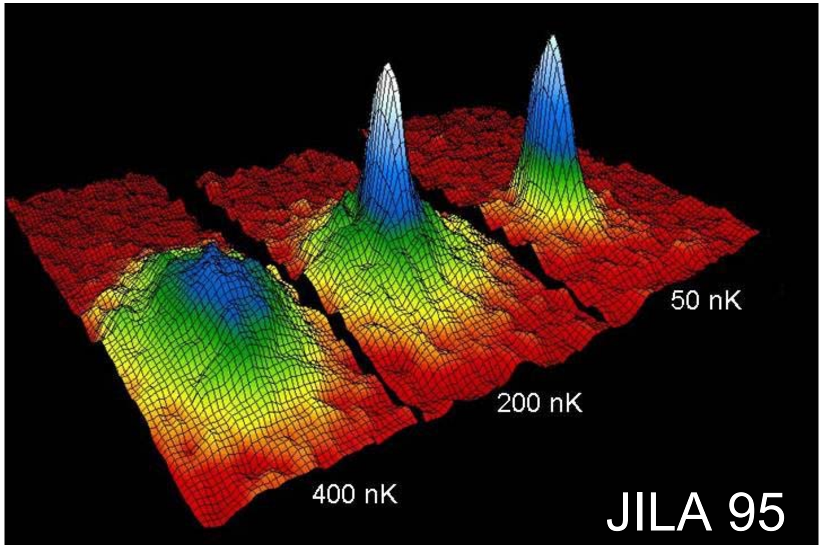


$$\langle n(\mathbf{r})\rangle_t \sim \langle n_{\mathbf{k}}\rangle_{t=0} \qquad \mathbf{r} = \frac{\hbar \mathbf{k}}{m} t$$

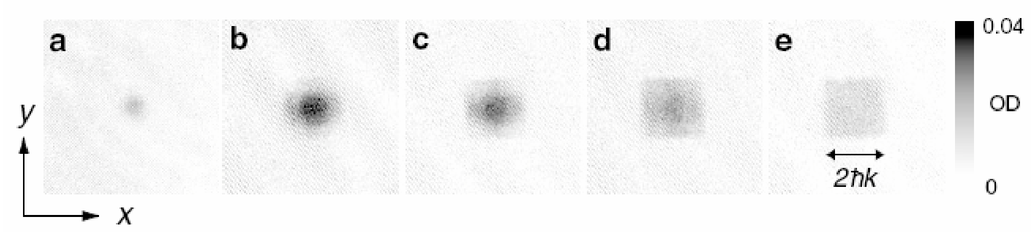
Detection of superfluid order: time of flight imaging

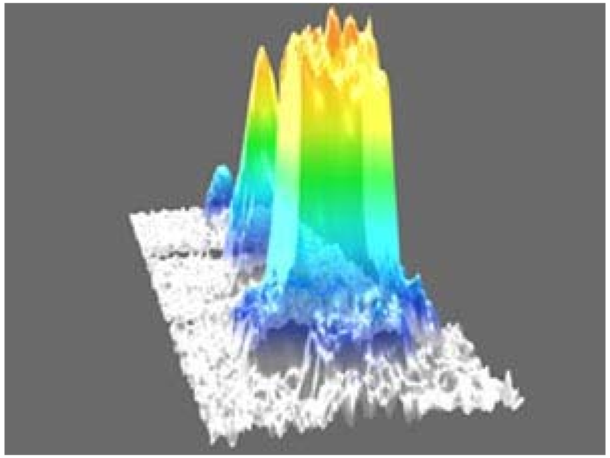
$$\langle b(x_i) \rangle = \Psi$$

$$\langle n_k \rangle = \frac{1}{V} \sum_{ij} e^{i \bar{k} \cdot (\bar{x}_i - \bar{x}_j)} \langle b^+(x_i) b(x_j) \rangle \sim |\Psi|^2 \delta(\bar{k})$$



Fermi systems





Lattice fermions:
M. Kohl et. Al. (ETH), PRL 2005

Fermions cannot condense but allow other types of order:
Density wave (DW), Spin-density wave/Antiferromagnet (SDW), Superconductivity (SC)

We shall see that all of these admit a unified description in terms of a condensate of fermion pairs.

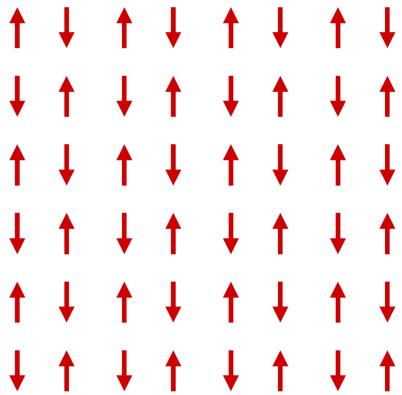
Antiferromagnetic order

$$H = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

$$e^{i\vec{\pi}\cdot\vec{x}_{ij}} \langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle \longrightarrow M_{stag}^2$$

$$\vec{M}_i = \langle \mathbf{S}_i \rangle \neq 0$$

Broken Su(2) symmetry



SDW order parameter ($\mathbf{q} \sim 2\mathbf{k}_{\mathrm{f}}$): $\langle \vec{m}_{\mathbf{q}} \rangle \neq 0$

$$\vec{m}_{\mathbf{q}} = \frac{1}{V} \sum_j e^{i\mathbf{q}\cdot\mathbf{x}_j} \mathbf{S}_j = \frac{1}{2V} \sum_j e^{i\mathbf{q}\cdot\mathbf{x}_j} \psi_{js}^\dagger \vec{\sigma}_{ss'} \psi_{js'} = \frac{1}{2} \sum_{\mathbf{k}} \psi_{\mathbf{k}+\mathbf{q}}^\dagger \vec{\sigma} \psi_{\mathbf{k}}$$

$m_q^\alpha |FS\rangle$ Is a spin-1 bosonic excitation of the Fermi sea

$\langle m_q^\alpha \rangle \neq 0$ $\rightsquigarrow$ Condensation of a spin-1 boson

Unified mean field description of ordered states

$$|\,\Psi_{SC}\,\rangle = \prod_{\mathbf{k}} (u_{\mathbf{k}} + v_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\uparrow} c^{\dagger}_{-\mathbf{k}\downarrow})\,|\,0\,\rangle \quad \Rightarrow \quad \psi = \sum_{\mathbf{k}} u_{\mathbf{k}} v_{\mathbf{k}}$$

particle-particle condensate

$$|\,\Psi_{SDW}\,\rangle = \prod_{\mathbf{k}} [u_{\mathbf{k}} + v_{\mathbf{k}} (c^{\dagger}_{\mathbf{k}+\mathbf{q}} \vec{\sigma} c_{\mathbf{k}}) \cdot \mathbf{n}]\,|\,FS\,\rangle \quad \Rightarrow \quad \langle \vec{m}_{\mathbf{q}} \rangle = \mathbf{n} \sum_{\mathbf{k}} u_{\mathbf{k}} v_{\mathbf{k}}$$

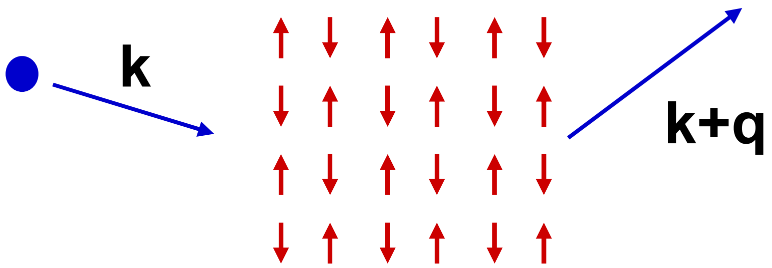
particle-hole condensates

$$|\,\Psi_{CDW}\,\rangle = \prod_{\mathbf{k}} (u_{\mathbf{k}} + v_{\mathbf{k}} c^{\dagger}_{\mathbf{k}+\mathbf{q}} c_{\mathbf{k}})\,|\,FS\,\rangle \quad \Rightarrow \quad \langle \rho_{\mathbf{q}} \rangle = u_{\mathbf{k}} v_{\mathbf{k}}$$

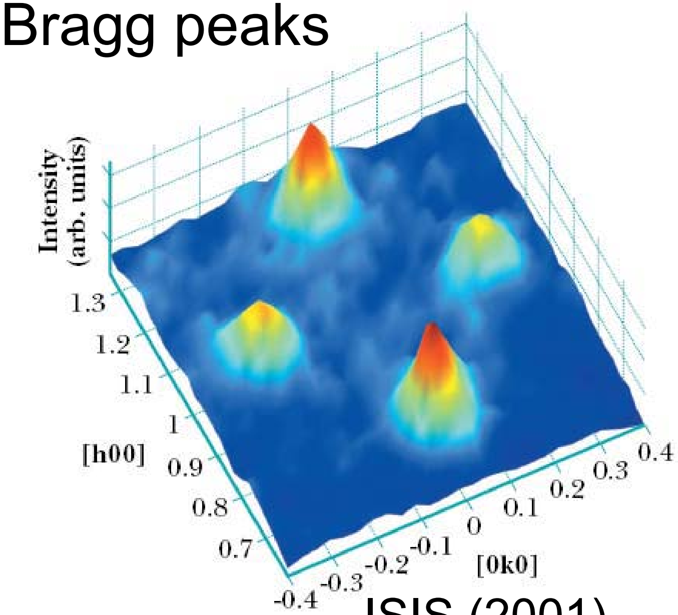
Is there a unified detection scheme?

Standard probe of order in solids:

Elastic neutron scattering:



Magnetic Bragg peaks



ISIS (2001)

Measures:

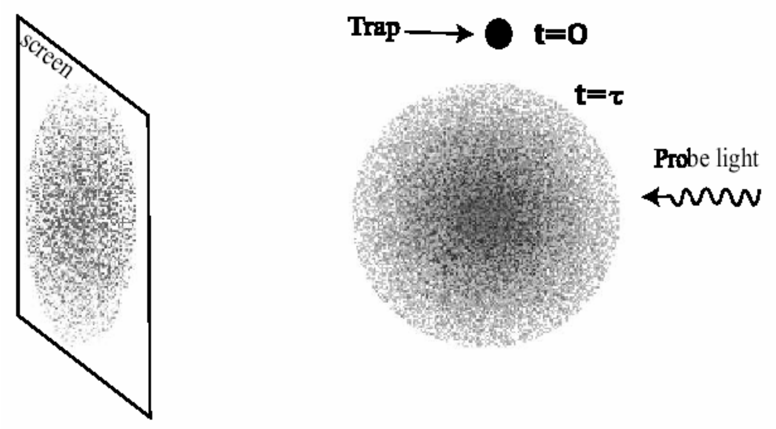
$$\langle \vec{m}_{-q} \cdot \vec{m}_{q} \rangle \quad , \quad \langle \rho_{-q}\, \rho_{q} \rangle$$

Neutrons (like any external probe) couple only to local densities (spin or charge). Cannot detect pairing correlations!

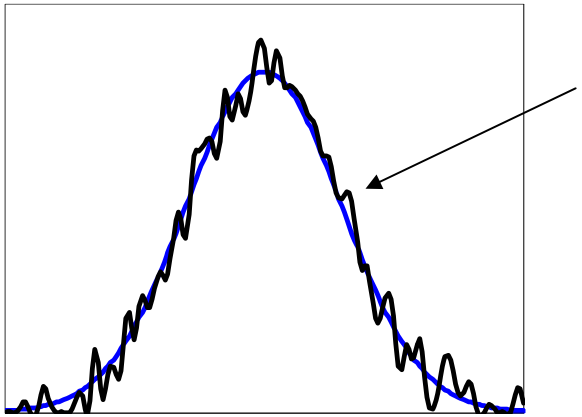
Can we develop a more general scheme for ultracold atoms?

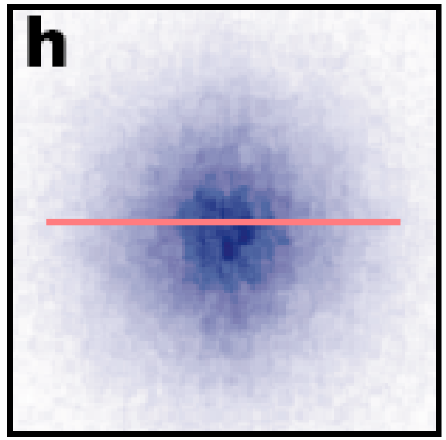
Quantum noise interferometry

Altman , Demler and Lukin, PRA 2004



$$\langle n(\mathbf{r})\rangle_t \sim \langle n_{\mathbf{k}}\rangle_{t=0}$$





Correlations: $\langle n_{\mathbf{k}} n_{\mathbf{k}'}\rangle_{con} = \langle n_{\mathbf{k}} n_{\mathbf{k}'}\rangle_0 - \langle n_{\mathbf{k}}\rangle_0 \langle n_{\mathbf{k}'}\rangle_0$

number correlations in momentum space

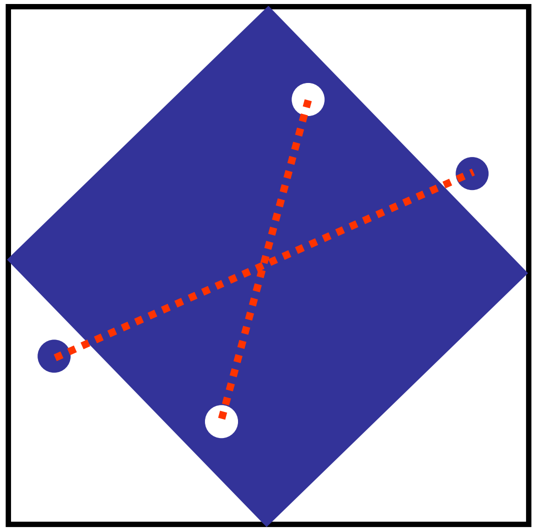
Noise correlations in ordered states

Superconductivity (p-p):

$$\prod_{\mathbf{k}} (u_{\mathbf{k}} + v_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\uparrow} c^{\dagger}_{-\mathbf{k}\downarrow}) \,|\, 0 \,\rangle$$

$$\langle n_{\mathbf{k}} n_{\mathbf{k}'} \rangle_{con} = \sum_{\mathbf{G}} F_{\mathbf{G}} |u^{\star}_{\mathbf{k}} v_{\mathbf{k}}|^2 \delta(\mathbf{k} + \mathbf{k}' + \mathbf{G})$$

(correlation → Peaks)

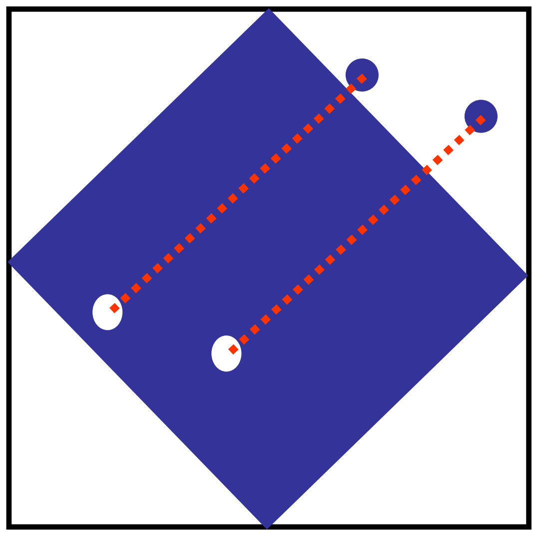


Spin/charge density wave (p-h):

$$\prod_{\mathbf{k}} (u_{\mathbf{k}} + v_{\mathbf{k}} c^{\dagger}_{\mathbf{k}+\mathbf{q}\sigma} c_{\mathbf{k}\sigma'}) \,|\, FS \,\rangle$$

$$\langle n_{\mathbf{k}} n_{\mathbf{k}'} \rangle_{con} = -\sum_{\mathbf{G}} F_{\mathbf{G}} |u^{\star}_{\mathbf{k}} v_{\mathbf{k}}|^2 \delta(\mathbf{k} - \mathbf{k}' - 2\mathbf{k}_f + \mathbf{G})$$

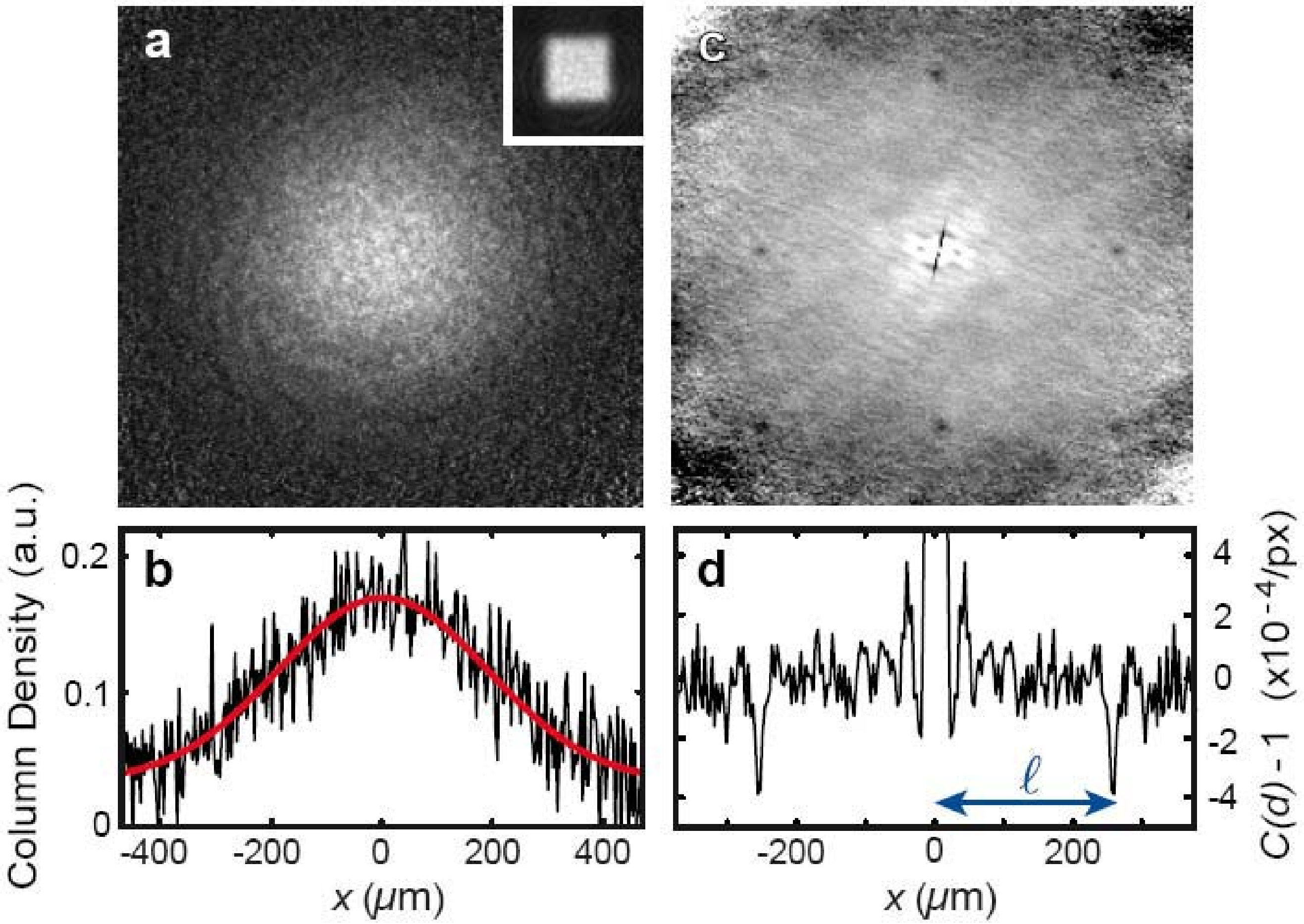
(anticorrelation → Peaks)



Note: δ-functions accquires width ~1/L for finite clouds of spatial size L

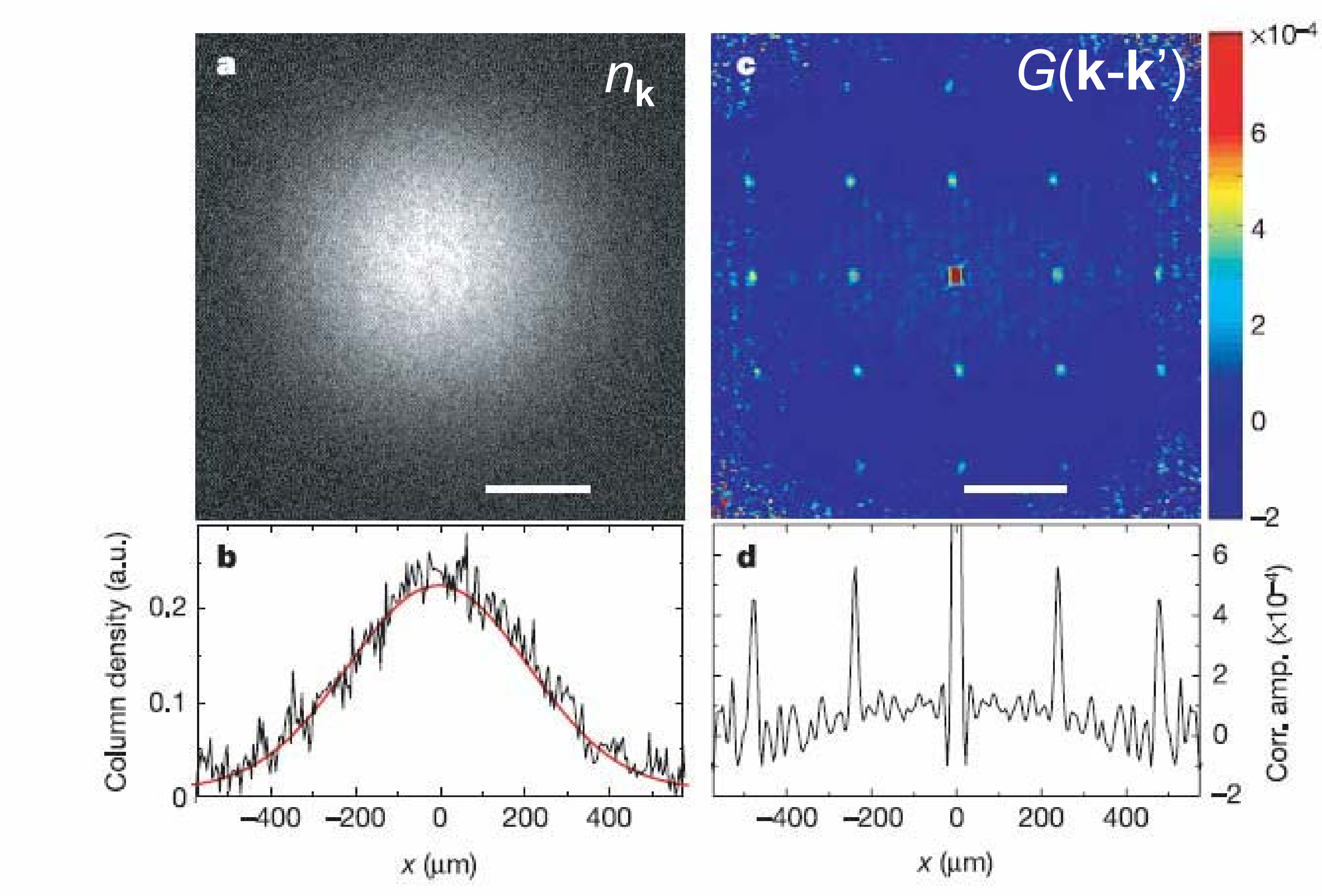
Noise correlations in a fermion band insulator (lattice induced CDW)

Mainz experiment: T. Rom et al. Nature (2006)

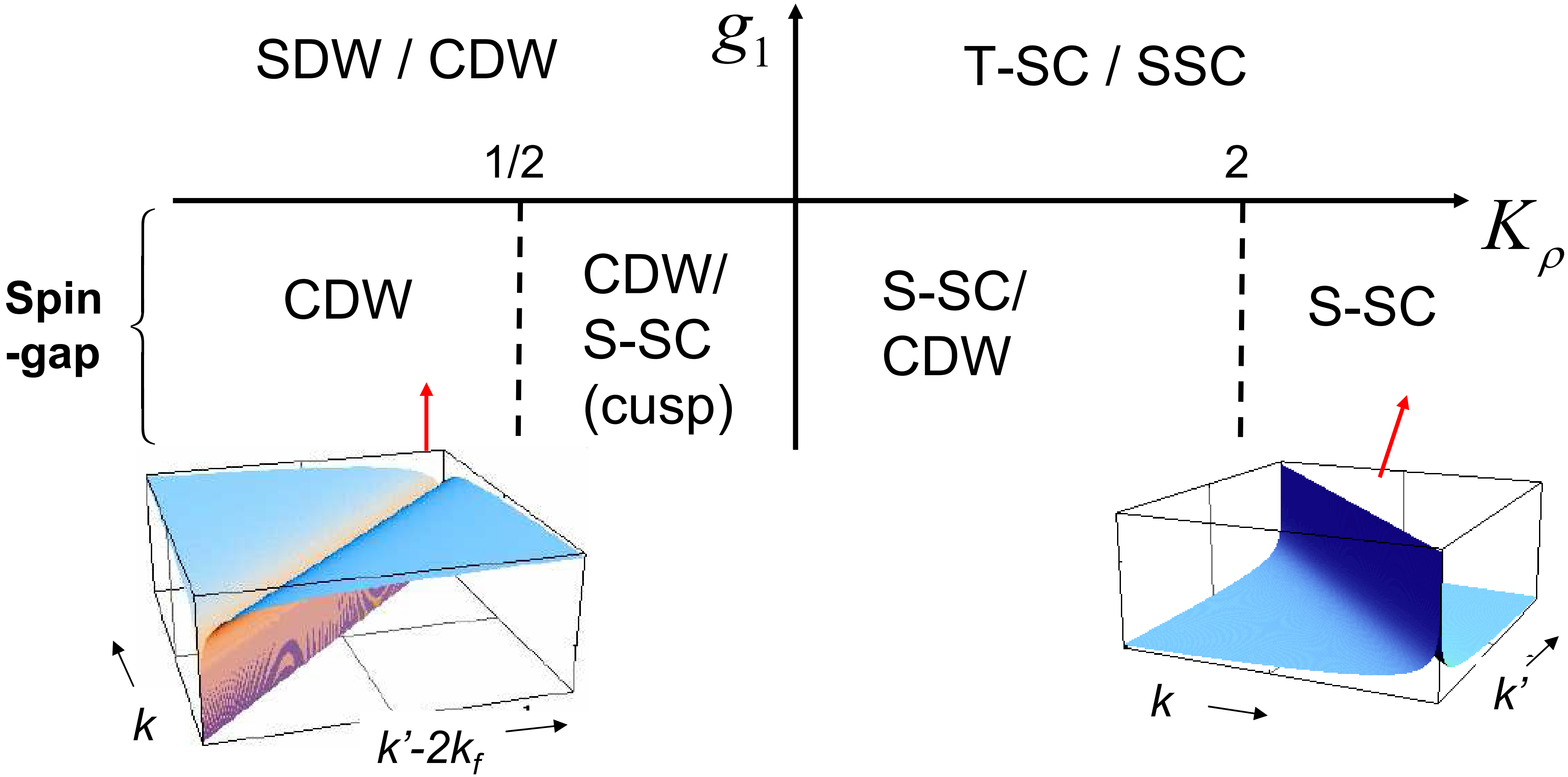


Noise correlations in a boson Mott insulator. (Lattice induced CDW)

Mainz experiment: Foelling et al., Nature (2005)



Beyond mean field: Itinerant fermions in 1d



$$\langle n_{\sigma\mathbf{k}} n_{\sigma\mathbf{k}'} \rangle_{con} \sim -|k - k' - 2k_f|^{K_\rho - 1}$$

$$\langle n_{\uparrow\mathbf{k}} n_{\downarrow\mathbf{k}'} \rangle_{con} \sim |k + k'|^{-1+1/K_\rho}$$

L. Mathey, EA, A. Vishwanath cond-mat/0507108

Beyond mean field: localized spin-½ particles on a lattice

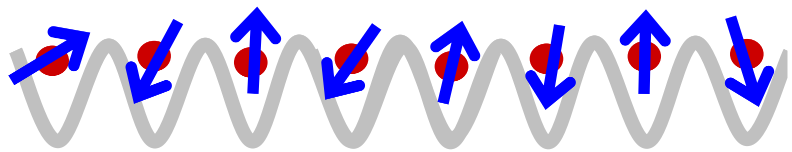
For localized particles we can show quite generally:

$$\langle n_{\mathbf{k}} n_{\mathbf{k}'} \rangle_{con} \sim \eta \sum_{\mathbf{G}} \delta(\mathbf{k} - \mathbf{k}' + \mathbf{G}) + \eta \sum_{ij} e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{R}_{ij}} \langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle$$

$\eta = \pm 1 \quad \text{bosons/fermions}$

Altman , Demler and Lukin, PRA 2004

Example: 1d Heisenberg chain

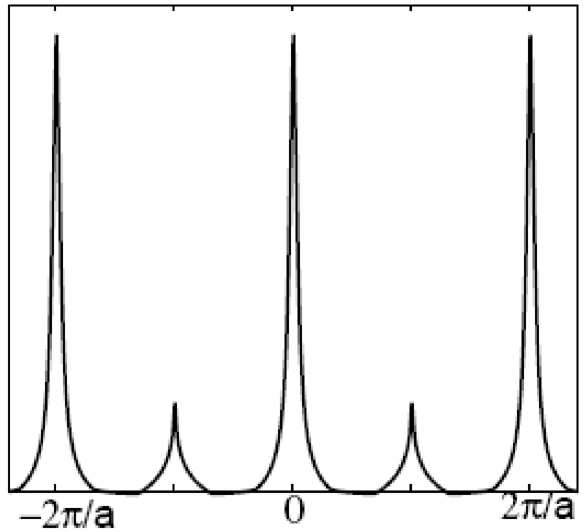


Spin correlations

$$\langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle \sim \frac{(-1)^{i-j}}{|x_i - x_j|}$$

Noise correlations

$$\langle n_k n_{k'} \rangle_{con} \sim \eta \sum_{n} \delta(k - k' + 2\pi n/a) - \ln(k - k' - \pi + 2\pi n/a)$$



Quantum disordered states

Example: Heisenberg spin chains

Spin ½ -- Exactly solvable by Bethe Ansatz

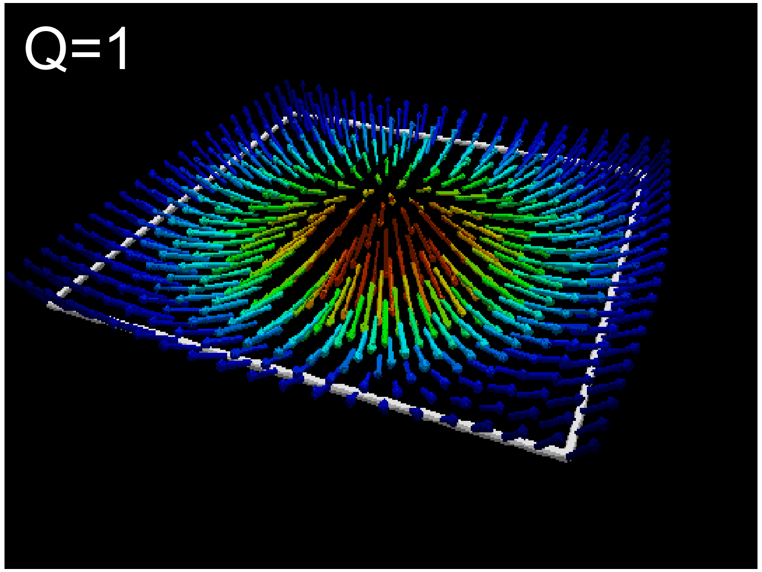
⇨ Gapless excitations, Power-law correlations

Long believed to apply for all spins S until Haldane (83) pointed to a fascinating difference between integer and half-integer spin chains

Large S -- Mapping to O(3) NLσM (Haldane 83)

$$Z \approx \int \mathcal{D}\hat{\mathbf{n}} \exp\left[i2\pi S Q[\hat{\mathbf{n}}] - \frac{1}{g}\int dx_0 dx_1 \partial_\mu \hat{\mathbf{n}} \cdot \partial_\mu \hat{\mathbf{n}}\right]$$

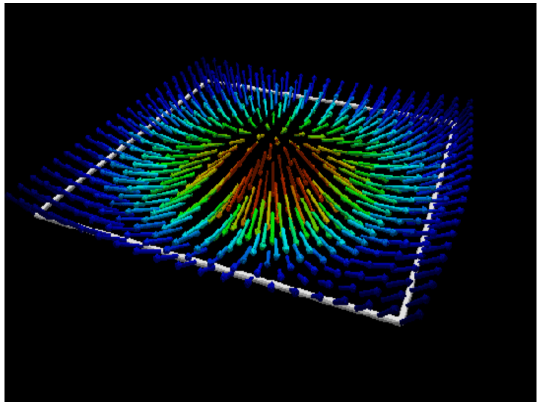
Topological Berry phase term



Q = Skyrmion number.

Integer number depending on the topology of the field configuration

Example: Heisenberg spin chains



<u>Integer spin:</u> $e^{i2\pi SQ} = 1$

$$Z \approx \int \mathcal{D}\hat{\mathbf{n}} \exp\left[-\frac{1}{g}\int dx_0 dx_1 \partial_\mu \hat{\mathbf{n}} \cdot \partial_\mu \hat{\mathbf{n}}\right]$$

Only real weights → "Classical" NLσM

Spin liquid ground state $\langle \mathbf{S}(x) \cdot \mathbf{S}(0) \rangle = (-1)^x e^{-x/\xi}$

Gapped (spin-1) excitations: $\Delta = c/\xi$

More careful analysis reveals subtle non local order:

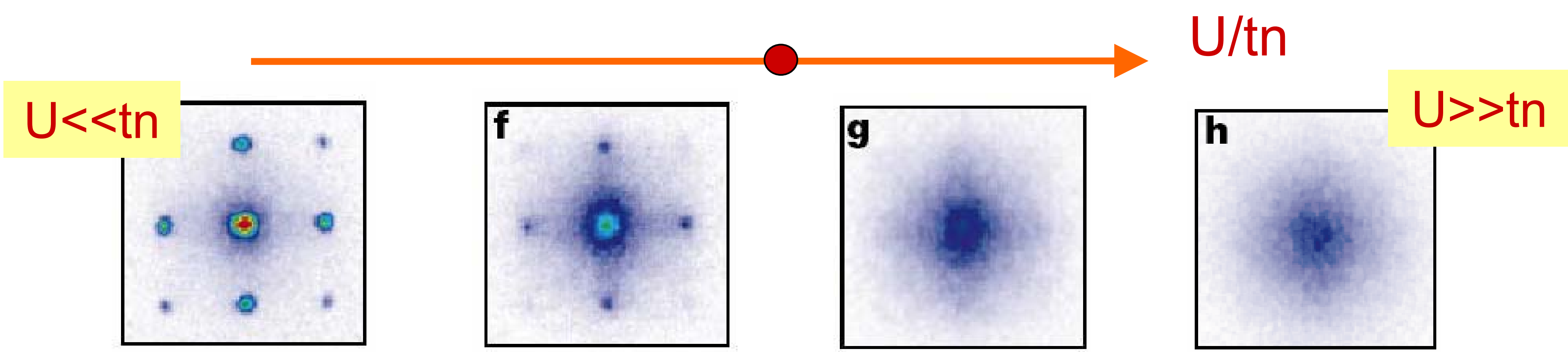
$$\langle S_i^\alpha \exp\left(i\pi \sum_{i<l<j} S_l^\alpha\right) \cdot S_j^\alpha \rangle \rightarrow \text{const}$$

Den Nijs & Rommelse (89)

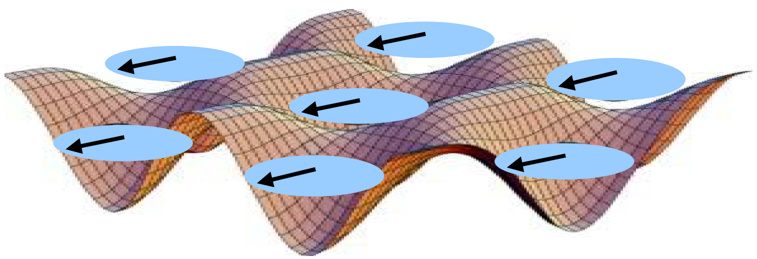
Superfluid to Mott-insulator transition of bosons

Theory: Fisher et al. PRB (89); Exp: Greiner et al. Nature (01)

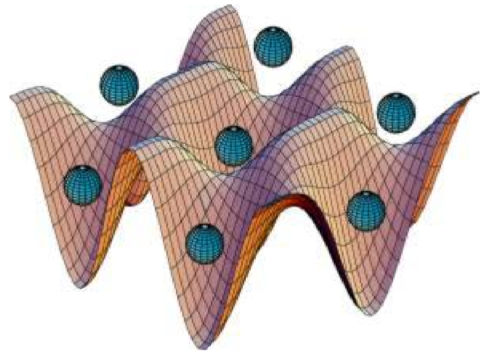
$$H_{eff} = \frac{U}{2}\sum_i (n_i - \bar{n})^2 - t\bar{n}\sum_{\langle ij \rangle}\cos(\varphi_i - \varphi_j) \qquad [\varphi_j, n_i] = i\delta_{ij}$$



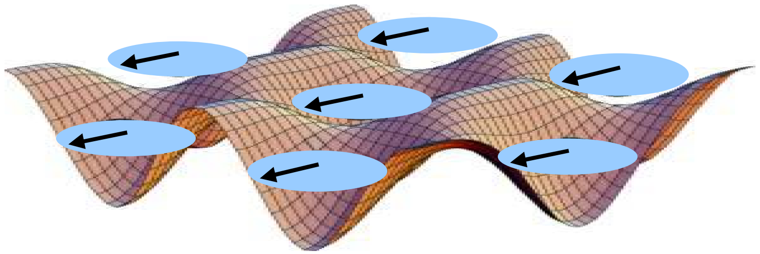
Superfluid

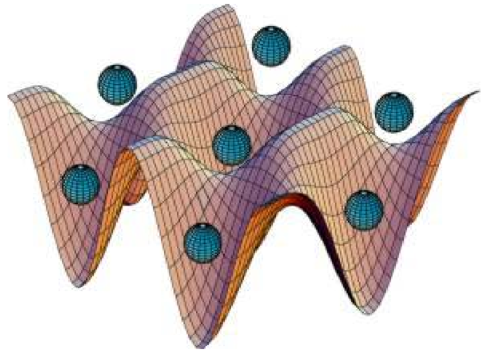


Mott-insulator



Both phases admit a simple classical (local) description!





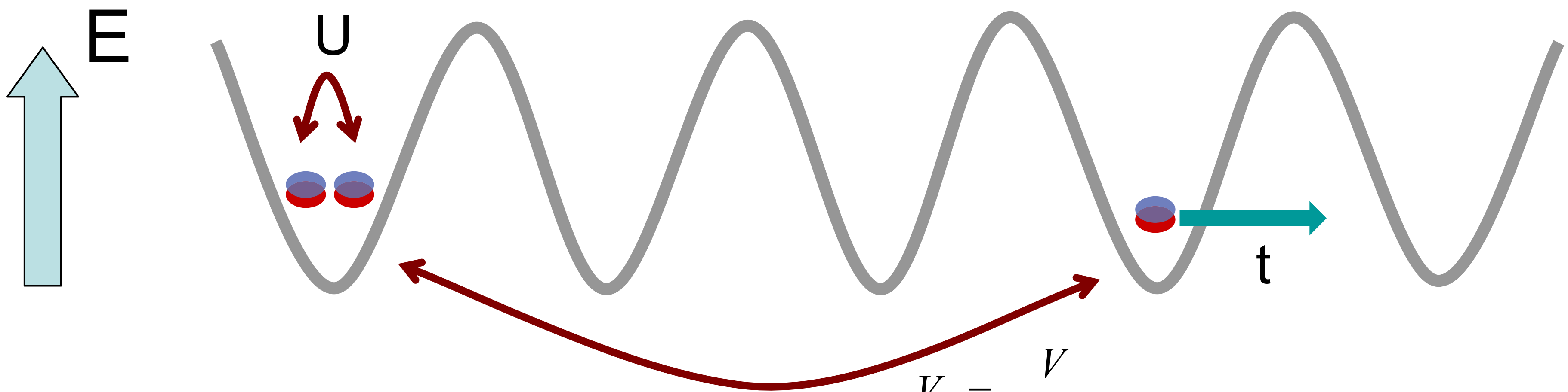
Can we get something more interesting
from plain vanilla bosons (spinless)?

Let’s see …

Polar molecules or atoms in a 1d optical lattice

Atoms with large magnetic dipole moment (Cr @ Stuttgart):
Stuhler et. al. PRL **95**, 150406 (2005).

Polar Molecules (underway)
Doyle (Harvard), Demille (Yale), Grim (Innsbruck) ….



$$V_{ij} = \frac{V}{|i-j|^3}$$

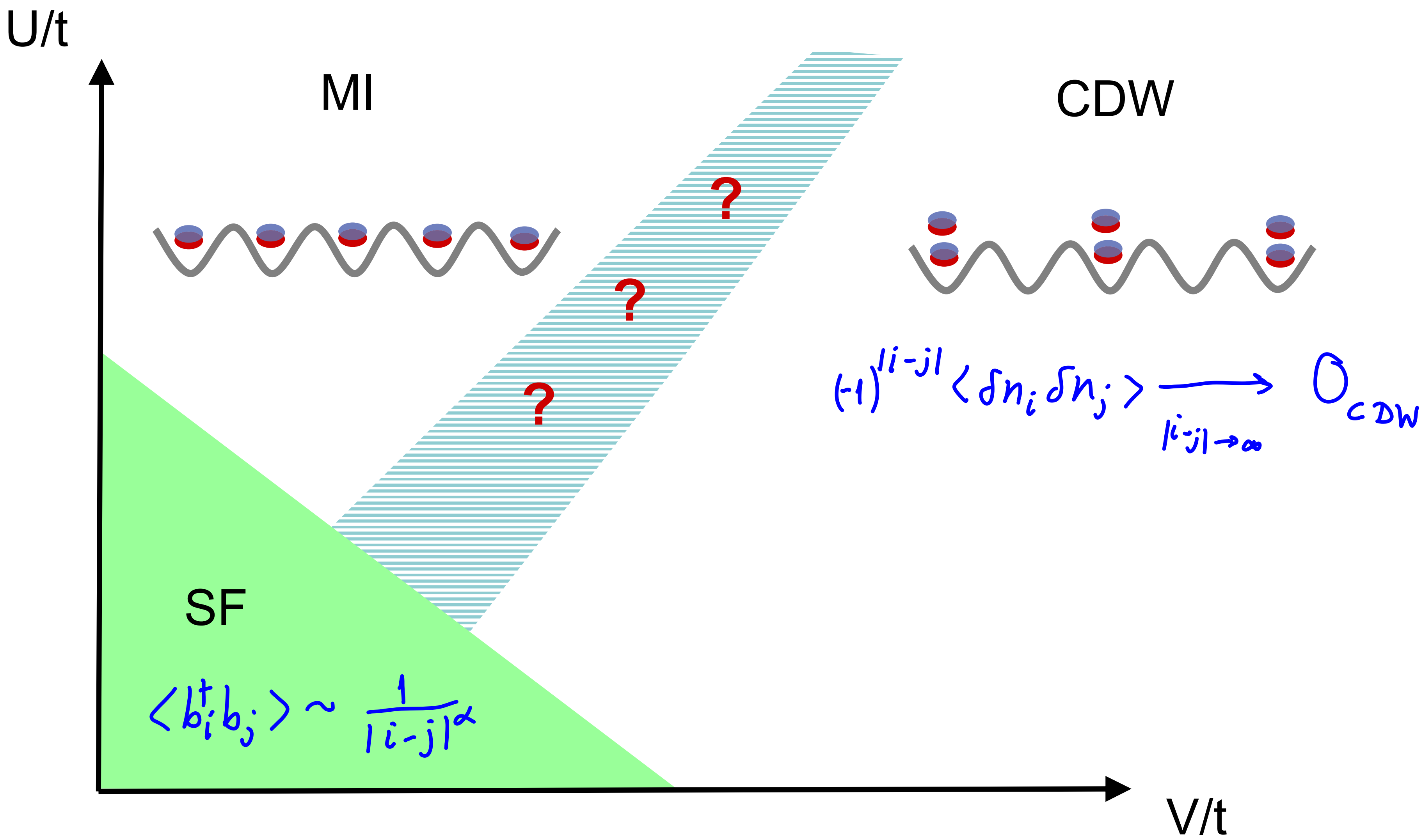
$$H = -t\sum_i (b_i^\dagger b_{i+1} + H.c.) + \frac{U}{2}\sum_i n_i(n_i - 1) + \sum_{i,r} \frac{V}{r^3} n_i n_{i+r}$$

Plane vanilla (spinless)bosons – but with long range interaction

We will focus on integer filling (say one atom per site)

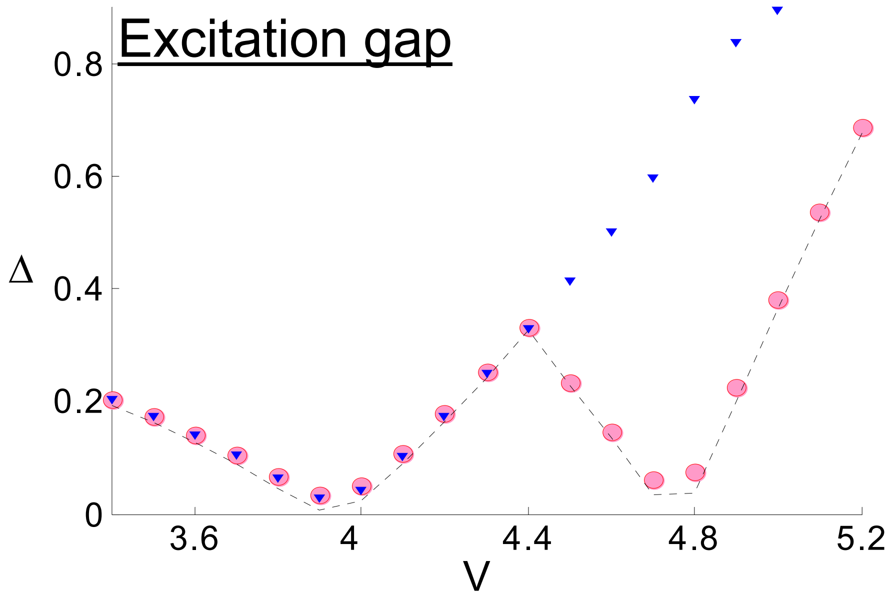
Conventional phases

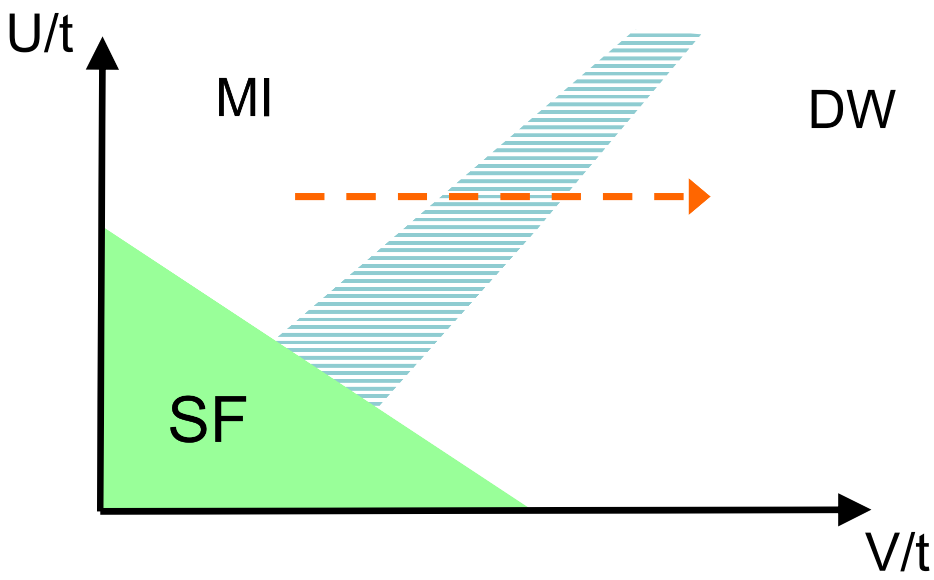
(integer filling)



Numerical investigation with DMRG

Filling: $\bar{n} = 1$ Length: L=256



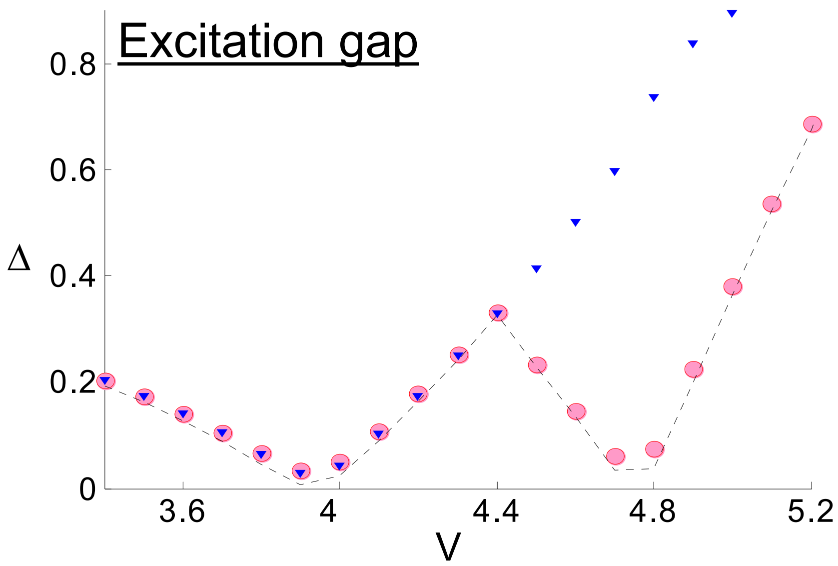


New insulating phase between the two quantum phase transitions!

How is it characterized?

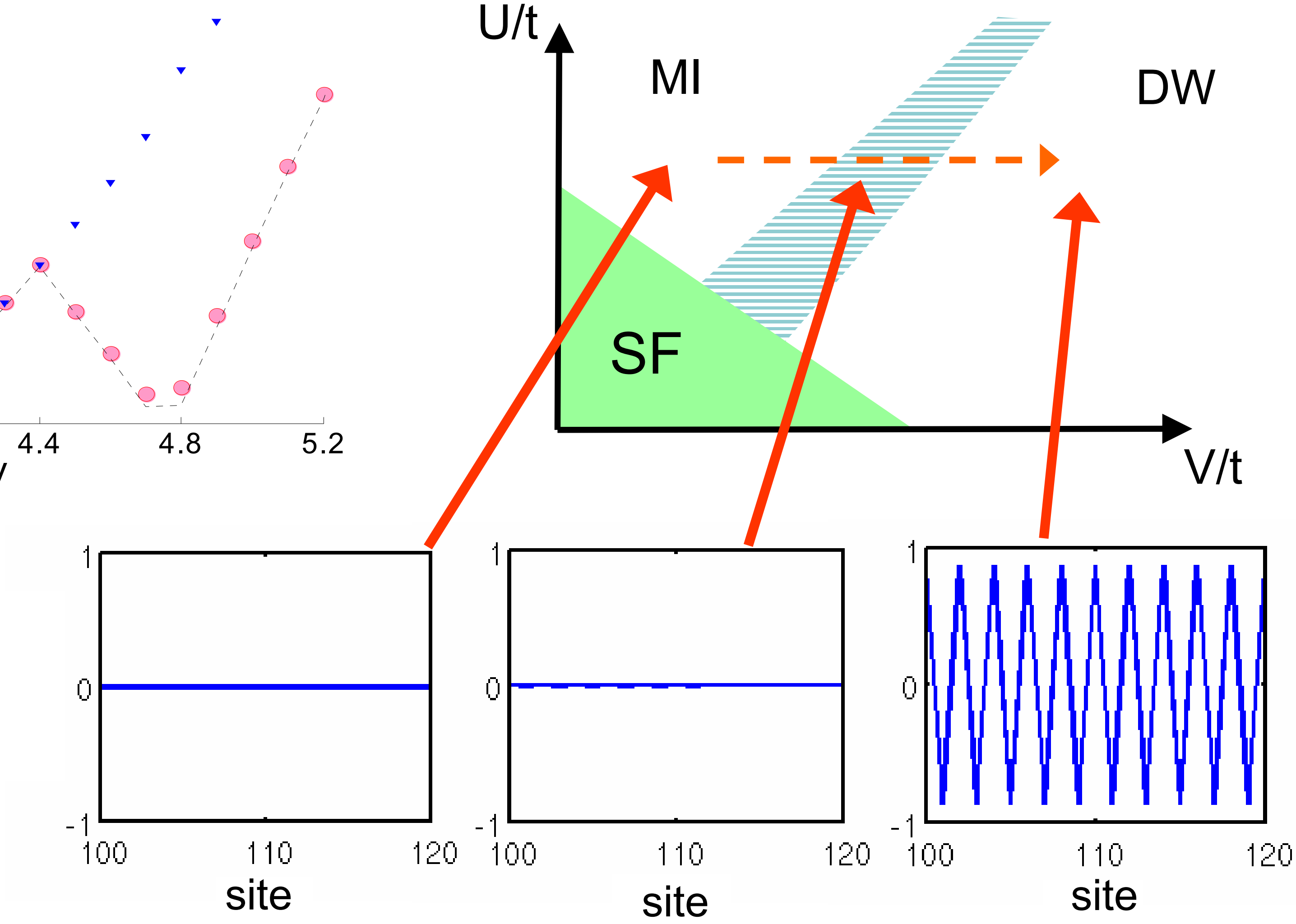
Numerical investigation with DMRG

Filling: $\overline{n} = 1$ Length: L=256



Local density:

$$\langle n_i \rangle - \bar{n}$$



Indistinguishable from Mott by local density. Is there hidden order?

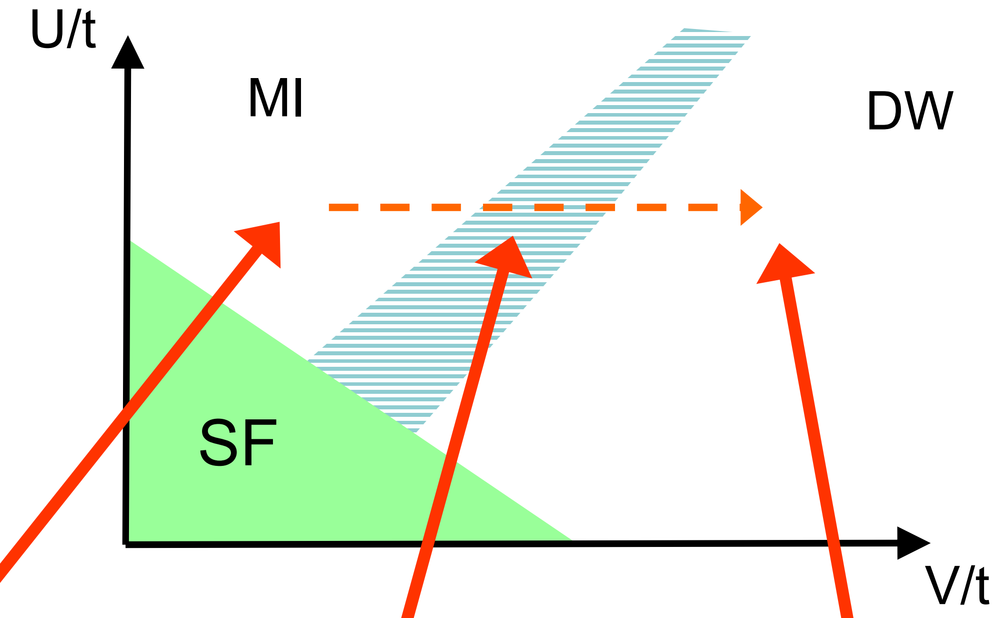
Yes! Highly non local string correlations

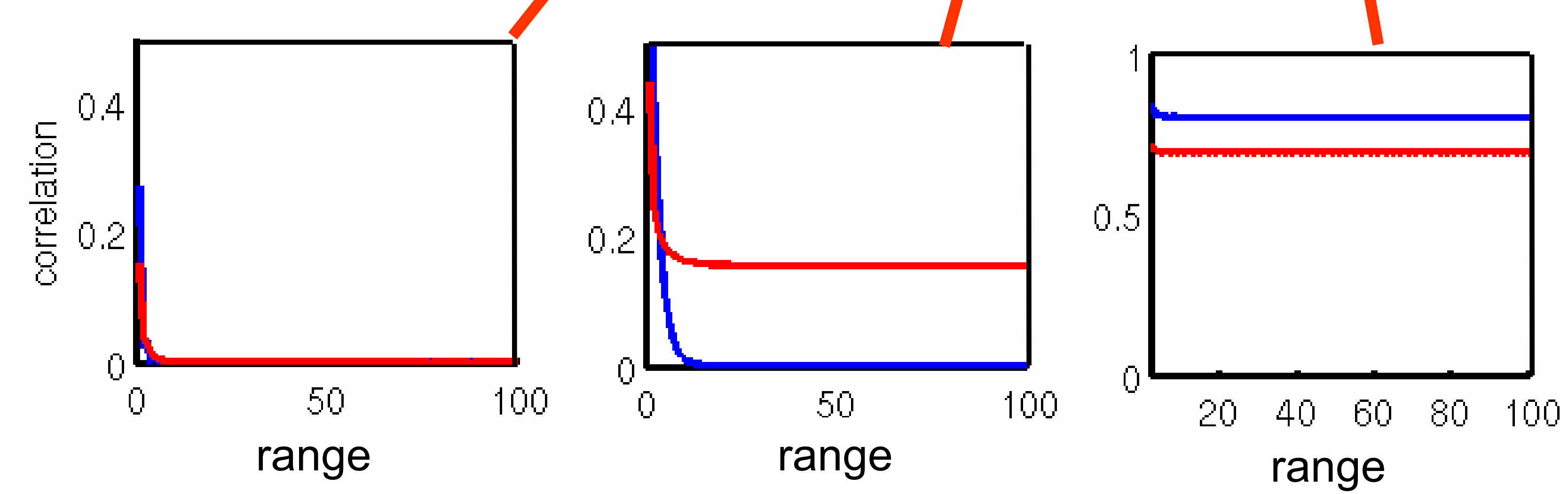
Density:

$$(-1)^{|i-j|}\langle \delta n_i \delta n_j \rangle$$

String:

$$\left\langle \delta n_i e^{i\pi \sum_{k=i}^{j} \delta n_k} \delta n_j \right\rangle$$

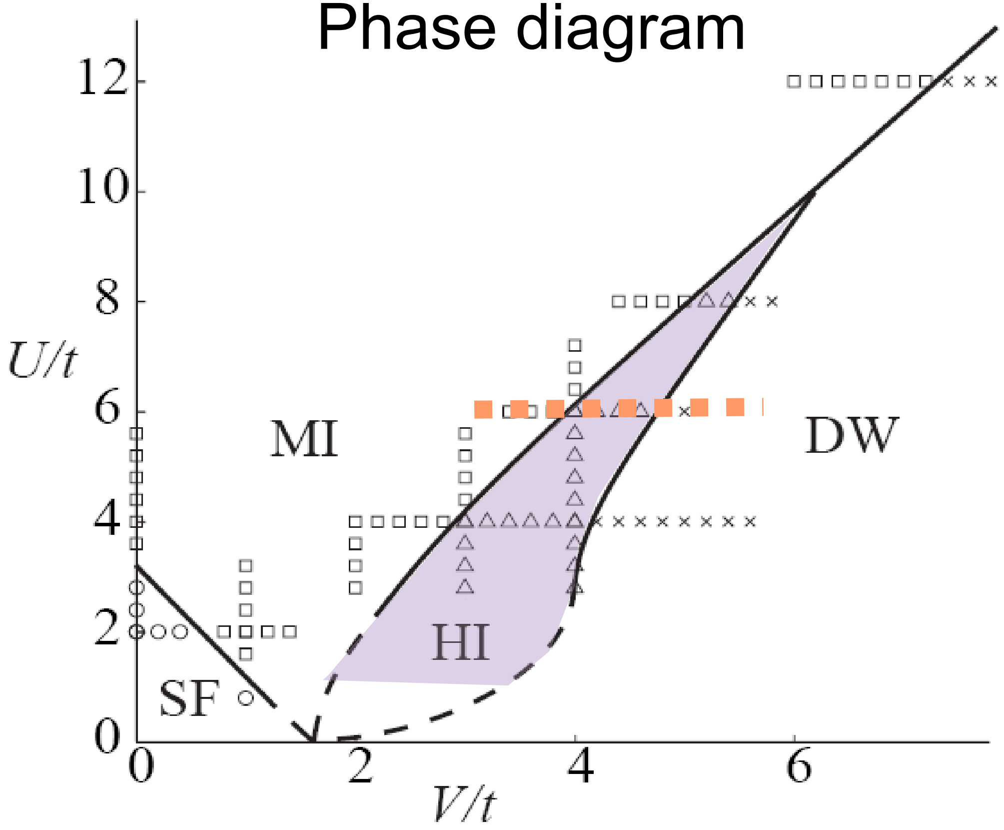




New insulating phase of bosons characterized by a highly non local order parameter !

$$O_{string} = \langle \prod_{i<j} e^{i\pi \delta n_i} \delta n_j \rangle =$$

Phase diagram

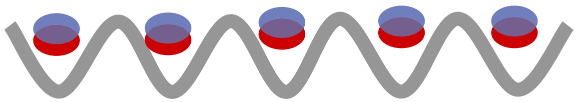


Caricature ground state by analogy to spin-1

Truncate Hilbert space to 3 occupation states: $S_i^z = n_i - \overline{n}$

$$H_{eff} = J \sum_i (S_i^+ S_{i+1}^- + \text{H.c.}) + \sum_i V S_i^z S_j^z + \frac{U}{2}(S_i^z)^2 \quad + \text{ p-h sym. breaking}$$

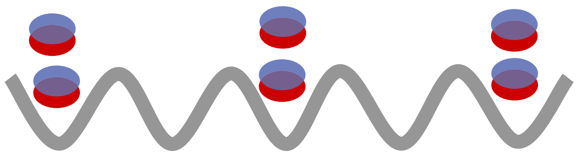
Mott insulator:



$\Rightarrow$ $|0\ 0\ 0\ 0\ 0\ 0\ 0\ 0 \ldots\rangle$

Density wave:

particle = (+) hole = (−)



$\Rightarrow$ $|+-+-+-+-\ldots\rangle$

String:

$$\left\langle S_i^z e^{i\pi \sum_{k=i}^{j} S_k^z} S_j^z \right\rangle$$

$|+00-0000+-0000+00-0\ldots\rangle$

$+\ |00+0000-000000+00-0\ldots\rangle + \ldots$

- Huge quantum superposition of configurations
- Alternate ordering of particles and holes

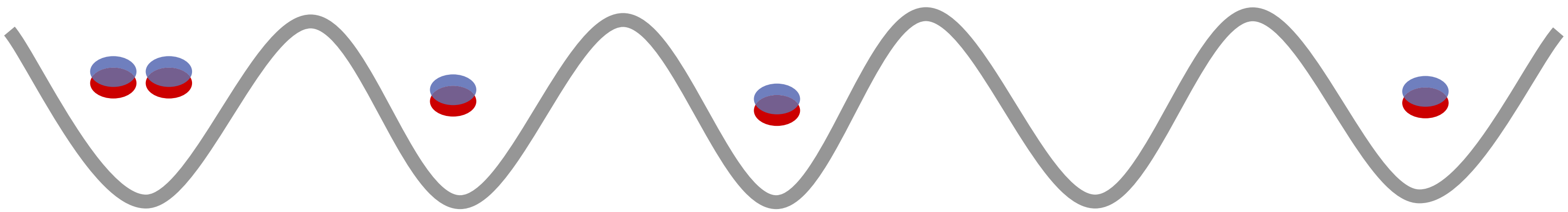
How to detect the hidden order ?

<u>Challenge:</u>
Experimental probes couple to ***local*** observables which cannot distinguish the Mott from the string-ordered ground state.

<u>Solution:</u>
We will show that the elementary excitations of those two phases are dramatically different

Measuring excitation spectra

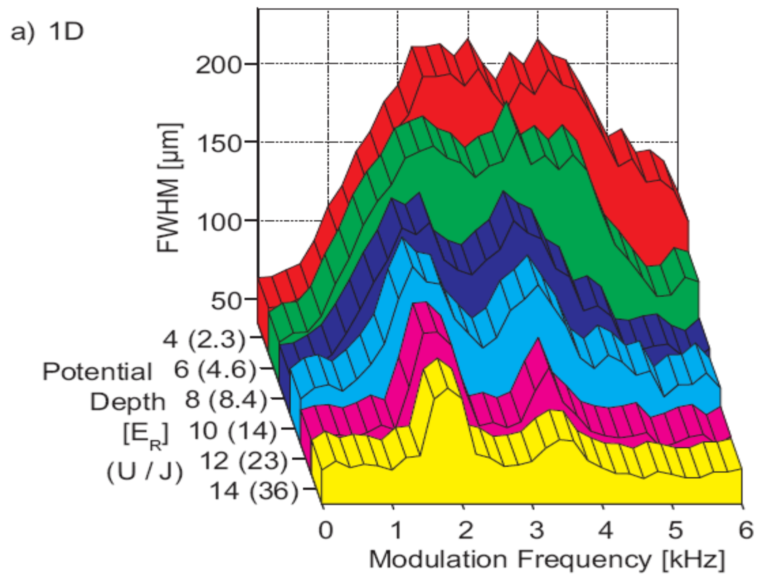
1. Periodic modulation of the lattice intensity:



$$H = H_0 + h\cos(\omega t)\sum_i (b_i^\dagger b_{i+1} + \mathrm{H.c})$$

Absorption spectrum in linear response:

$$I(\omega) \sim \sum_\alpha \left| \langle \psi_\alpha | \hat{T} | \psi_0 \rangle \right|^2 \delta(\omega_{\alpha 0} - \omega)$$

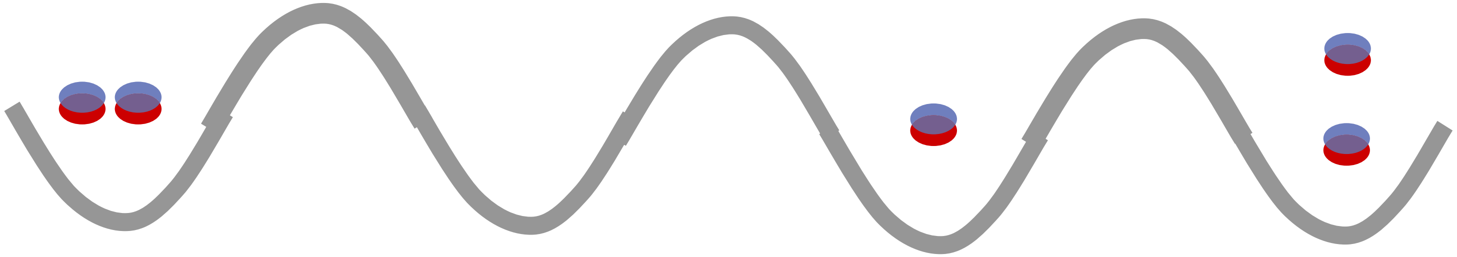


Used to probe excitations in SF and MI

Stoferle *et. al*., PRL 04 (ETH)

Measuring excitation spectra

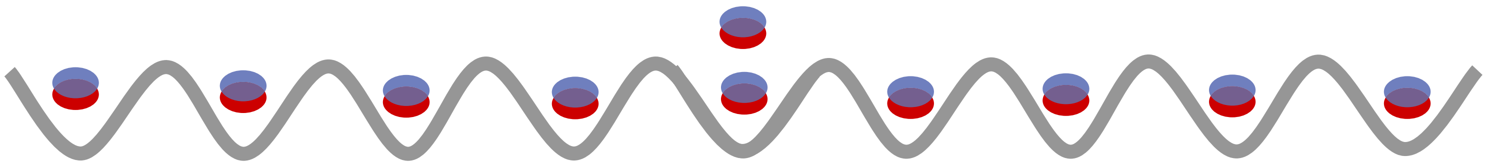
2. Bragg spectroscopy



$$S(\mathbf{q},\omega) = \sum_n \left|\langle n|\delta\rho_{-\mathbf{q}}|0\rangle\right|^2 \delta(\hbar\omega - \hbar\omega_{n0})$$

Excitations of the Mott insulator

Particle:



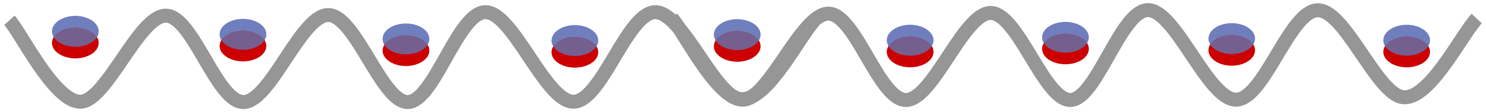
Excitations of the Mott insulator

Hole:

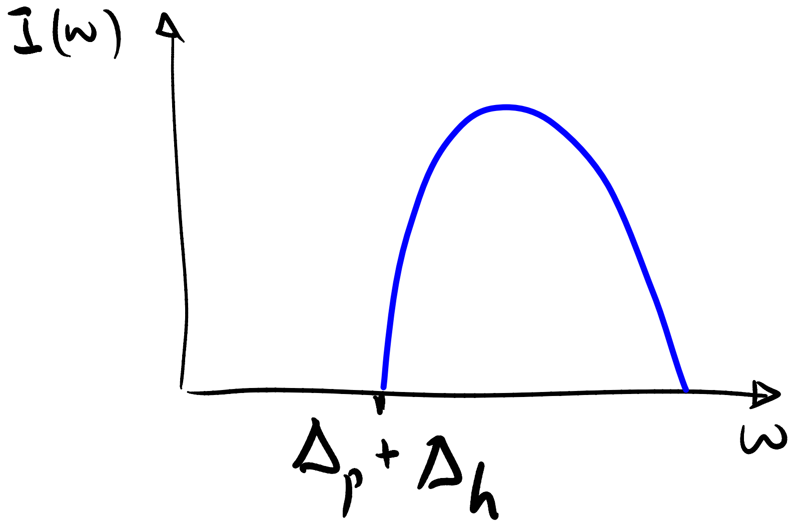
Excitations of the Mott insulator

Iucci et. al PRA 73, 041608 (06), Kollath et. al. PRL 97, 050402 (06)
Huber et. al. PRB 75, 085106 (07)

Lattice modulations or Bragg spectroscopy
only excite particle-hole pairs



Observe only particle-hole
continuum (No single mode peak)

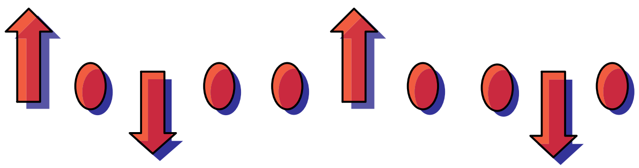


Excitations of the Haldane insulator -Theory

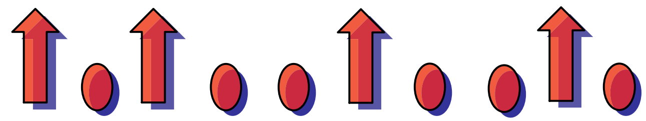
To enable a local description apply a non-local unitary transformation:

$$U: \quad \left\langle S_i^z e^{i\pi \sum_{k=i}^{j} S_k^z} S_j^z \right\rangle \rightarrow \left\langle S_i^z S_j^z \right\rangle$$

Kennedy & Tasaki, PRL (91)
Oshikawa, Phys.Scr.T (92)



$$U = \prod_{j<k} \exp(i\pi S_j^z S_k^x)$$



String order

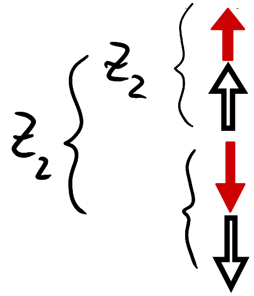
Ferromagnetic order

Transformed Hamiltonian:

$$\tilde{H} = UHU = \sum_j -S_j^x S_{j+1}^x + S_j^y \exp(i\pi S_j^z + i\pi S_{j+1}^x) S_{j+1}^y - V \sum_j S_j^z S_{j+1}^z + \frac{U}{2} \sum_i (S_j^z)^2$$

explicit Z_2 X Z_2 symmetry!

Single kink excitations



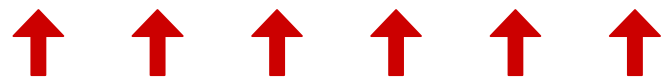
$= \alpha|S_z = 0\rangle + \beta|S_z = 1\rangle$

$= \alpha|S_z = 0\rangle - \beta|S_z = 1\rangle$

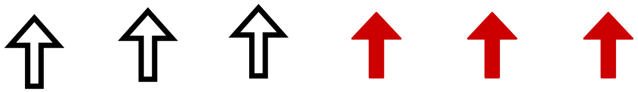
$= \alpha|S_z = 0\rangle + \beta|S_z = -1\rangle$

$= \alpha|S_z = 0\rangle - \beta|S_z = -1\rangle$

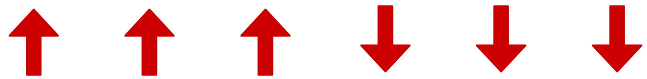
Variational ground state (one of 4):

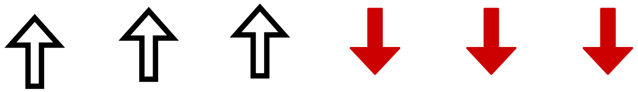
$|\Psi_0\rangle =$ 

Excitations:

$S_i^z|\Psi_0\rangle =$  $\rightarrow \sum_i e^{ikx_i} S_i^z |\Psi_0\rangle$

Neutral excitation

$S_i^x|\Psi_0\rangle =$  $\rightarrow \sum_i e^{ikx_i} S_i^x |\Psi_0\rangle$

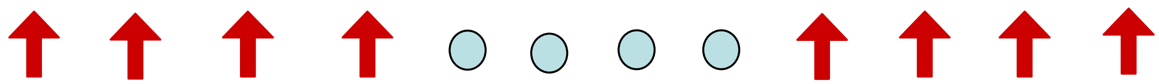
$S_i^y|\Psi_0\rangle =$  $\rightarrow \sum_i e^{ikx_i} S_i^y |\Psi_0\rangle$

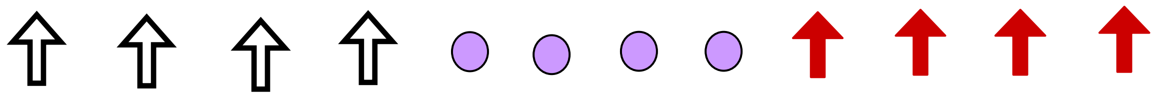
See also Arovas,Auerbach,Haldane (88) Fath and Solyom (93)

"Two particle" excitations

That don't change total particle number

Two branches classified by a topological number (N_{kinks}):

$S_i^\alpha S_j^\alpha \,|\, \Psi_0 \,\rangle \; =$ 

$S_i^x S_j^y \,|\, \Psi_0 \,\rangle \; =$ 

Insert Particle-hole: $S_i^+ S_j^- = (S_i^x S_j^x + S_i^y S_j^y) - i(S_i^x S_j^y - S_i^y S_j^x)$

Breaks up into pairs of kinks from the two different branches !

“Two particle” excitations

That don’t change total particle number

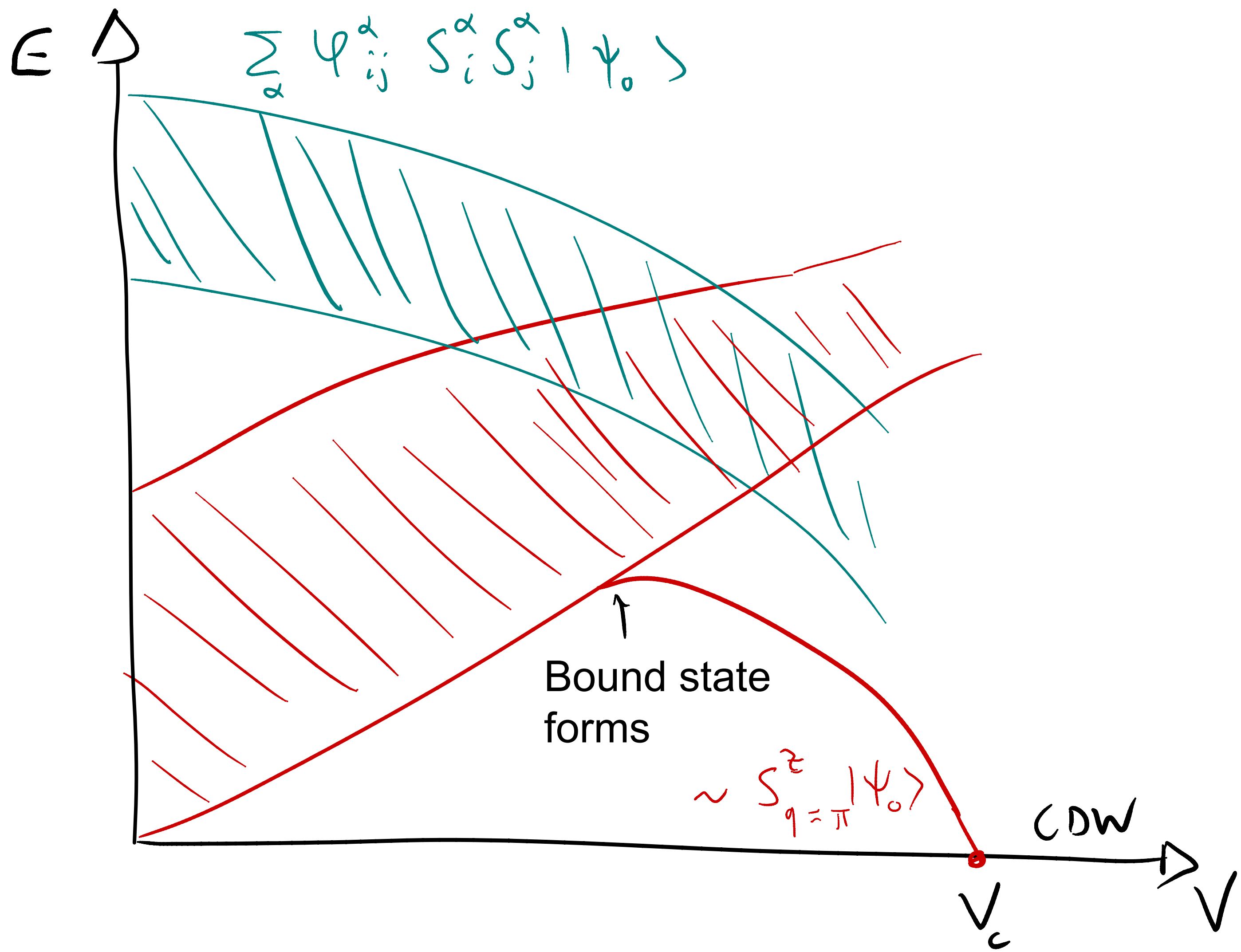
Couples to lattice modulation

$(S_i^+ S_j^- + S_i^- S_j^+)$

$\eta_{ij} S_i^x S_j^y |\psi_0\rangle$

Couples to Bragg

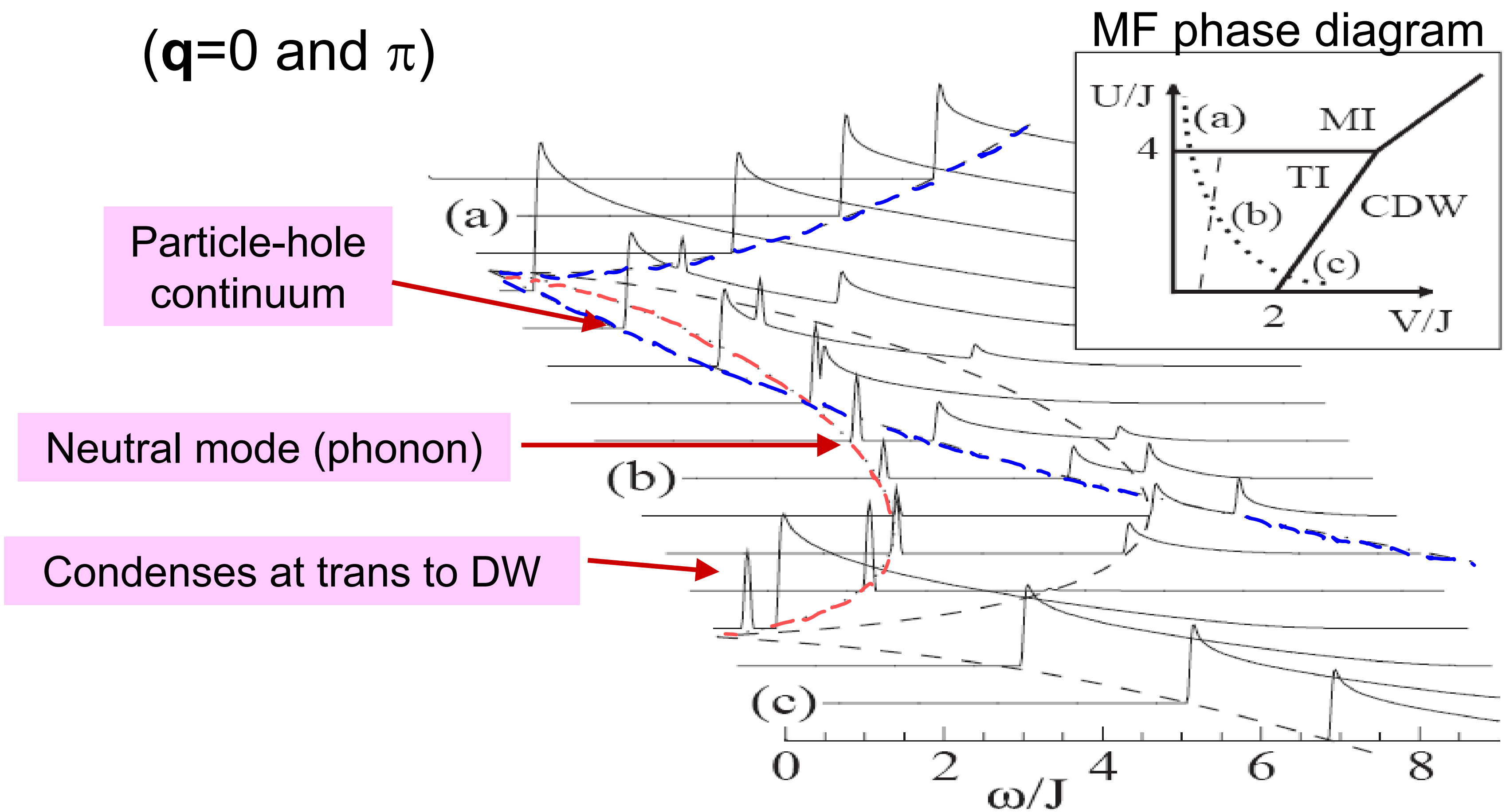
(S_q^z)



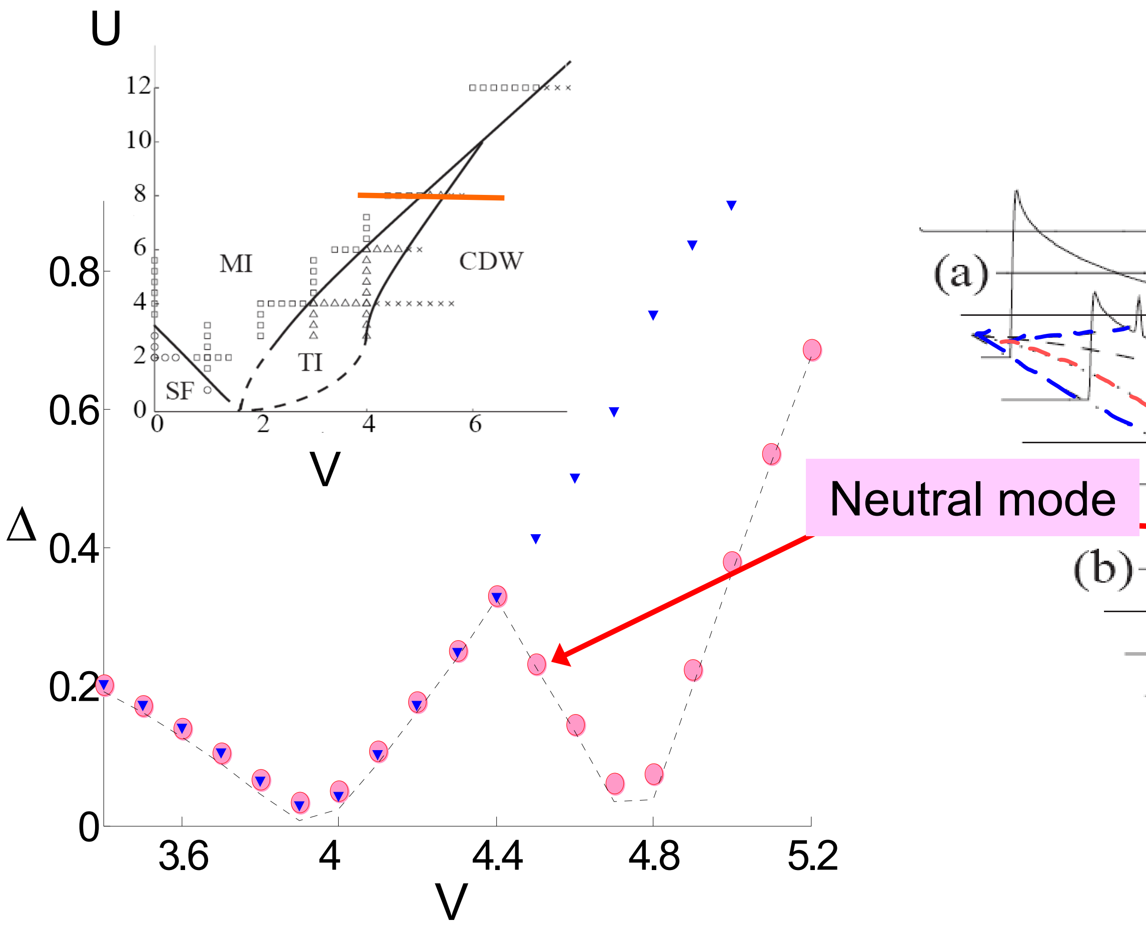
New resonance in the absorption spectrum

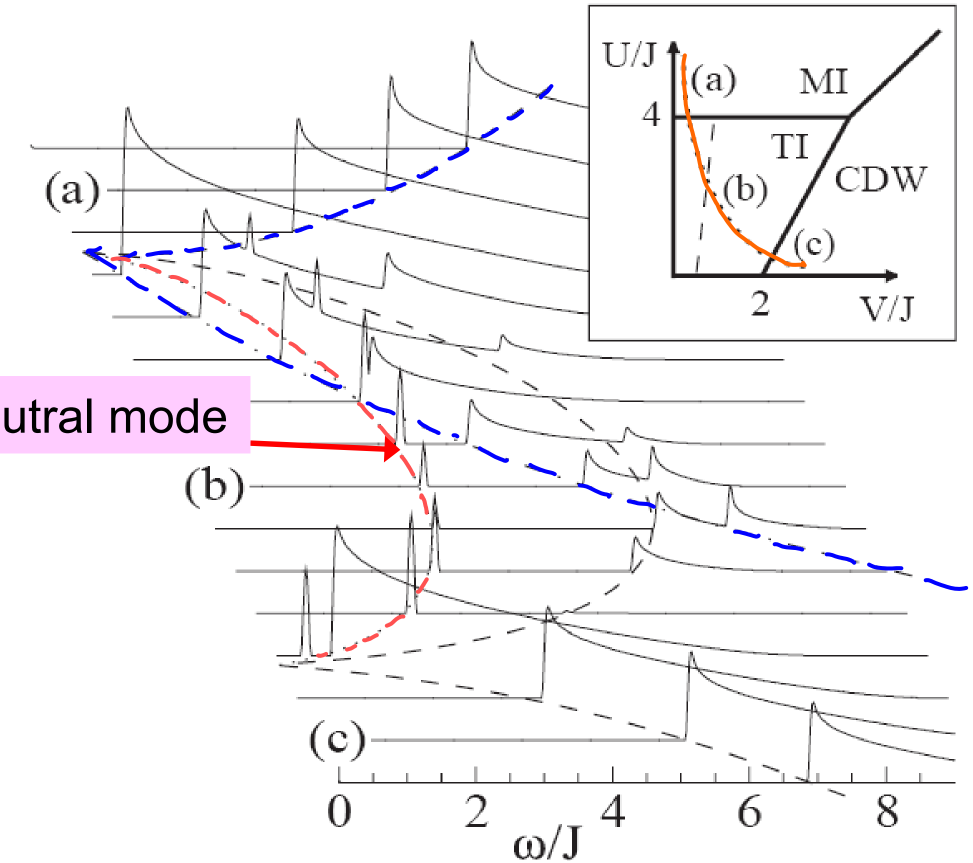
Explicitly compute energies and matrix elements in the variational excitations

(**q**=0 and π)



Compare with numerics





Extensions

- Phase diagram by field theoretic analysis (Bosonization)
- Analysis of coupled chains (Bosonization, DMRG)

Realistic experiments:

- Full control of tunneling between chains
  (via transverse lattice potential)

- Limited control over interchain dipolar interaction
  (via angle of polarizing field)

Conclusions

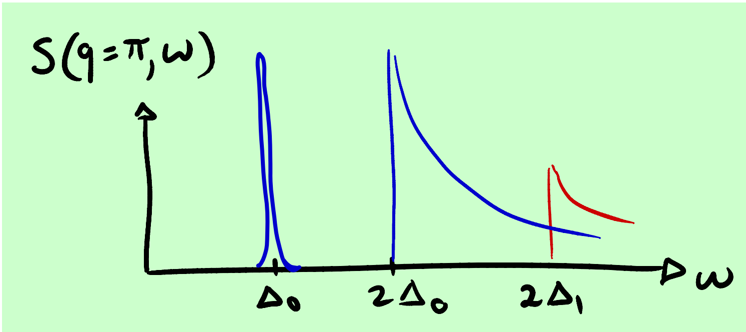
- Systems of ultracold atoms are geared to address open questions of strongly correlated quantum systems

- Quantum noise interferometery:
  More to time of flight imaging than $\langle n_{\mathbf{k}} \rangle$.
  Probes many-body correlations.

  Altman, Demler, Lukin, PRA 70, 013603 (2004)

- New insulating phase of bosons with dipolar interaction.
  Hidden order.
  New collective mode.

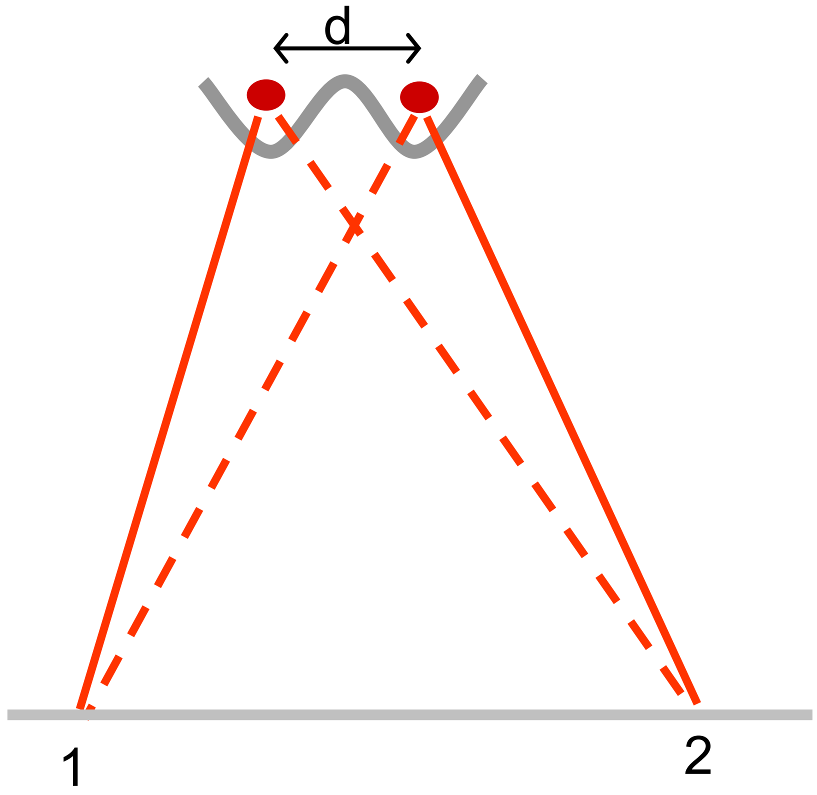
  Dalla Torre, Berg, and Altman
  PRL 97, 260401 (2006)



Appendices

Noise correlations as a manifestation of the Hanbury-Brown Twiss effect

Two sources:



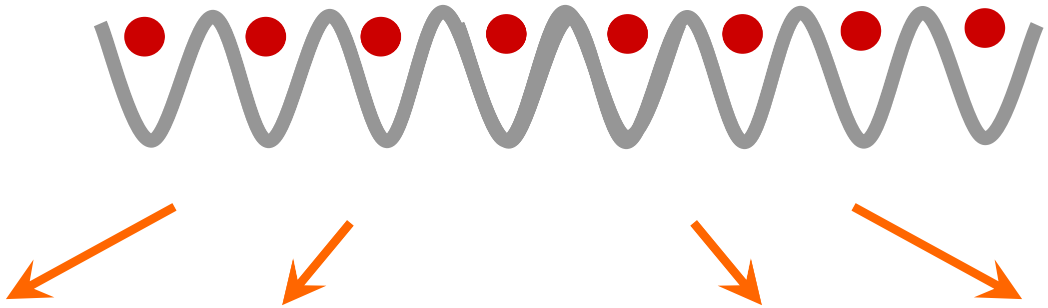
$$k_{1,2} = \frac{m}{\hbar t} r_{1,2}$$

Quantum interference between two possible two-particle paths

$$\langle \rho_1 \rho_2 \rangle - \langle \rho_1 \rangle \langle \rho_2 \rangle = \pm \cos[d(k_1 - k_2)]$$

(−) for fermions

Lattice:

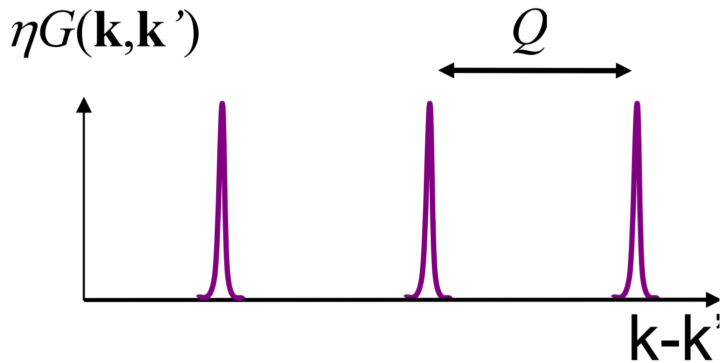


Bosons at **lattice-momentum** q expand as plane waves with wave-vectors $k = q\,,\, q+Q\,,\, q+2Q\,,\, q+3Q\,,\,\ldots$

$$k_{1,2} = \frac{m}{\hbar t} r_{1,2}$$

If $k_1 - k_2 = nQ$ then the two particles originate from the same lattice-momentum q :

- Enhanced correlation (bunching - bosons)
- antibunching – fermions

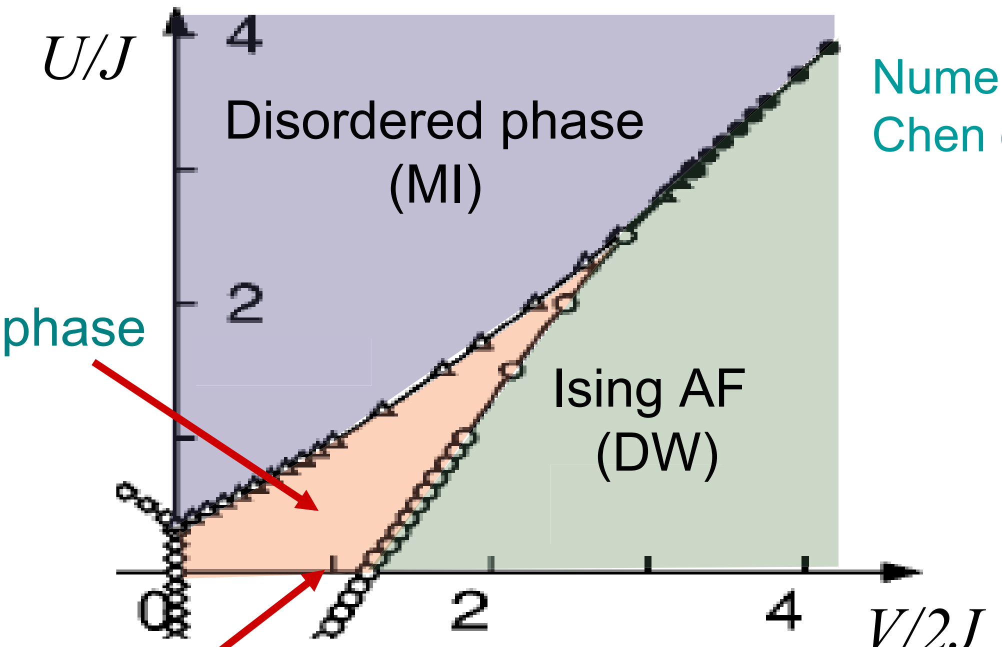


Haldane gap in the anisotropic spin-1 chain

(excluding terms that break particle-hole symmetry)

$$H_{eff} = J\sum_i (S_i^+ S_{i+1}^- + \mathrm{H.}c.) + \sum_i V S_i^z S_j^z + \frac{U}{2}(S_i^z)^2$$

Haldane Gapped phase
Haldane (83)



Numerical phase diagram
Chen et. al. PRB (2003)

Heisenberg point

2

- String order in the Haldane gapped phase of spin-1 chains
- Breaking of hidden Z_2 X Z_2 symmetry   Den Nijs & Rommelse (89)