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Luttiger liquid state in the 1D dipolar gas

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Luttinger liquid state in the 1D dipolar gas

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Repulsive bosons in one dimension

Lieb-Liniger model (2nd quantization):

$$H = -\frac{\hbar^2}{2m}\int dx \psi_B^\dagger \frac{\partial^2}{\partial x^2}\psi_B + g\int dx \psi_B^\dagger(x)\psi_B^\dagger(x)\psi_B(x)\psi_B(x),$$

- $g > 0$: repulsive interactions
- Bethe Ansatz integrable model: thermodynamics "easy", correlation functions much harder.
- $\Rightarrow$ search an easier limit.

Fermionization

$g \to \infty \Rightarrow$ the wavefunction vanishes for $x_i = x_j \Rightarrow$ mapping to free spinless fermions (Girardeau 1960).
With second quantization:

$$\psi_B(x) = \psi_F(x) \exp\left[i\pi \int^x dx' \psi_F^\dagger \psi_F(x')\right]$$

and:

$$H = -\frac{\hbar^2}{2m} \int dx \psi_F^\dagger \frac{\partial^2 \psi_F}{\partial x^2}$$

Ground state properties

Fill the Fermi sea : $k_F = \pi \rho_0$ $(\rho_0 = N/L)$
Fermi velocity: $v_F = \pi \hbar \rho_0 / m$
Energy per unit length: $e(\rho_0) = \frac{\pi^2 \hbar^2}{6} \rho_0^3$.
Low energy description: only excitations near the Fermi points.
$\psi_F(x) = e^{i\pi\rho_0 x}\psi_{F,+}(x) + e^{-i\pi\rho_0 x}\psi_{F,-}(x)$.
effective Hamiltonian:

$$H = -i\hbar v_F \int dx (\psi_{F,+}\partial_x \psi^\dagger_{F,+} - \psi_{F,-}\partial_x \psi^\dagger_{F,-})$$

$$\Rightarrow \rho(x) = \psi^\dagger_F(x)\psi_F(x) = \psi^\dagger_{F,+}(x)\psi_{F,+}(x) + \psi^\dagger_{F,-}(x)\psi_{F,-}(x) + e^{2i\pi\rho_0 x}\psi^\dagger_{F,-}(x)\psi_{F,+}(x) + e^{-2i\pi\rho_0 x}\psi^\dagger_{F,+}(x)\psi_{F,-}(x)$$

Bosonization description

See T. Giamarchi's lectures.

$$H = \hbar v_F \int \frac{dx}{2\pi} \left[(\pi \Pi)^2 + (\partial_x \phi)^2 \right],$$

$[\phi(x), \Pi(x')] = i\delta(x - x')$, $\theta(x) = \pi \int^x \Pi(x')dx'$

$\psi_{F,\pm}(x) = \frac{e^{i(\theta \mp \phi)(x)}}{\sqrt{2\pi\alpha}}$, $\rho(x) = \rho_0 - \frac{1}{\pi}\partial_x \phi(x) + \frac{\cos(2\pi\rho_0 x - 2\phi)}{\pi\alpha}$

$\psi_B(x) = \frac{e^{i\theta(x)}}{\sqrt{2\pi\alpha}} [1 + \cos(2\phi(x) - 2\pi\rho_0 x)]$

Off Diagonal Long Range Order ?

$\langle\psi_B^\dagger(x)\psi_B(x')\rangle = \frac{1}{2\pi a}\langle e^{i\theta(x)}e^{-i\theta(x')}\rangle + \text{osc.}$ We have:

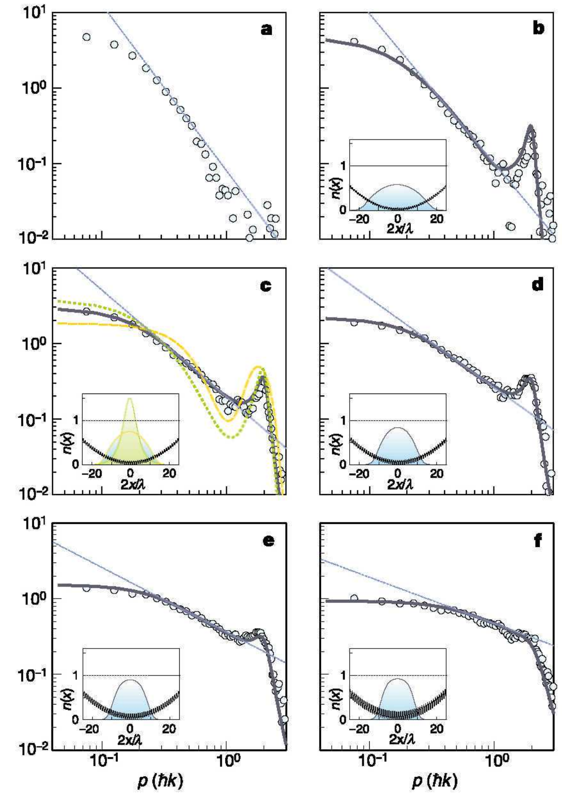
$$\langle e^{i\theta(x)}e^{-i\theta(x')}\rangle \sim \left(\frac{\alpha}{|x-x'|}\right)^{1/2}$$

Therefore:

$$\lim_{|x-x'|\to\infty}\langle\psi_B^\dagger(x)\psi_B(x')\rangle = 0,$$

No Off Diagonal Long range order, but quasi long range order (algebraic decay instead of exponential decay).

Experimental realization of a Tonks-Girardeau gas



From B. Paredes, A. Widera, V. Murg, O. Mandel, S. Fölling, I. Cirac, G. V. Shlyapnikov, T. W. Hänsch and I. Bloch, Nature **429**, 277 (2004).

Luttinger liquid physics

No BEC, no crystalline order, only quasi-long range order
$\rightarrow$ fluctuating "supersolid"
gapless excitations with linear spectrum
FDM Haldane 1980.

$$
\begin{aligned}
H &= \hbar \int \frac{dx}{2\pi} \left[uK(\pi\Pi)^2 + \frac{u}{K}(\partial_x \phi)^2 \right] \\
\psi_B(x) &= e^{i\theta} \left[\sum_{m=0} B_m \cos(2m\phi - 2m\pi\rho_0 x) \right] \\
\rho_B(x) &= \rho_0 - \frac{\partial_x \phi}{\pi} + \sum_{m=1}^{\infty} A_m \cos(2m\phi(x) - 2m\pi\rho_0 x)
\end{aligned}
$$

Correlation functions

$$\langle e^{in\theta(x)} e^{-in\theta(x')} \rangle = \left(\frac{\alpha}{|x-x'|}\right)^{\frac{n^2}{2K}}$$

$$\langle e^{i2m\phi(x)} e^{-i2m\phi(x')} \rangle = \left(\frac{\alpha}{|x-x'|}\right)^{2m^2 K}$$

In finite size L, $x - x' \to \frac{L}{\pi} \sin \frac{\pi(x-x')}{L}$.

- Dominant ODLRO for $K \to \infty$
- Dominant crystalline order for $K \to 0$.
- $K = \infty \to$ Free Bose gas.
- $K \searrow$ as repulsion $\nearrow$.
- $K = 1 \to$ Tonks-Girardeau gas.

Luttinger parameters (I)

Galilean boost: $\psi_B(x) \to e^{imvx/\hbar}\psi_B(x)$.
$\Leftrightarrow \theta(x) \to \theta(x) + mvx/\hbar$
$\Rightarrow \pi\Pi(x) \to \pi\Pi(x) + mv/\hbar$

$$\Delta E = L\frac{uKm^2v^2}{2\pi\hbar} = \frac{1}{2}Nmv^2$$

if Galilean invariant.

$$\Rightarrow uK = \frac{\hbar\pi\rho_0}{m},$$

$\Rightarrow uK$ is unrenormalized by interactions in a Galilean invariant model.

Luttinger parameters (II)

$\rho_0 \to \rho_0 + \delta\rho_0$
$\Leftrightarrow \partial_x \phi \to \partial_x \phi - \pi \delta\rho_0.$

$$\Delta E = \frac{\hbar u \pi (\delta\rho_0)^2}{2K} = \frac{1}{2}\frac{\partial^2 e}{\partial \rho^2}(\delta\rho_0)^2$$

$$\Rightarrow \frac{\hbar \pi u}{K} = \frac{\partial^2 e}{\partial \rho^2}$$

In the Lieb-Liniger gas

$K \geq 1$ for all interaction.
$\rightarrow$ Always odd-diagonal quasi-long range order
$\rightarrow$ No sign of crystallization
$\Rightarrow$ Can one construct a model with $K \leq 1$?

One dimensional bosonic dipolar gas

$$H = \sum_i \frac{p_i^2}{2m} + \frac{1}{4\pi\epsilon_0} \sum_{i<j} \frac{\vec{d}_i \cdot \vec{d}_j - 3 d_i^x d_j^x}{|x_i - x_j|^3},$$

$\vec{d}_i$: vector dipole moment.
Fully polarized case $\Rightarrow \vec{d}_i = d\hat{e}_z$
Typical kinetic energy: $\mathcal{K} = \hbar^2 \rho_0^2/m$
Typical potential energy: $\mathcal{U} = d^2 \rho_0^3 / 4\pi\epsilon_0$.
$\mathcal{K} \gg \mathcal{U} \Leftrightarrow \rho_0 r_0 \ll 1$
$\mathcal{K} \ll \mathcal{U} \Leftrightarrow \rho_0 r_0 \gg 1$

$$r_0 = \frac{md^2}{2\pi\epsilon_0\hbar^2}$$

Low density limit $\rho_0 r_0 \ll 1$

Two-body problem in the center of mass frame:

$$\frac{\hbar^2}{m}\left(-\frac{d^2}{dx^2}+\frac{r_0}{2x^3}\right)\psi = E\psi$$

WKB approximation in the classically forbidden region:

$$\psi(x) = \exp[-C(r_0/|x|)^{1/2}]$$

$\rightarrow$ Fermionization expected and $K \simeq 1$.

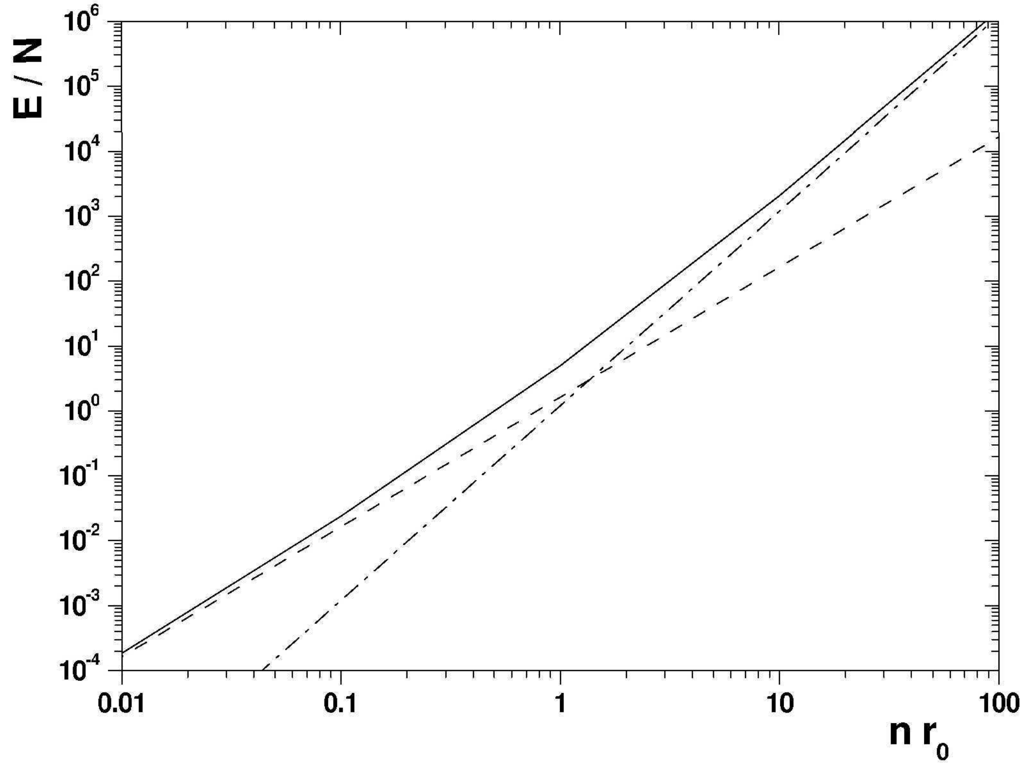
High density limit $\rho_0 r_0 \gg 1$

Potential energy dominates → Crystal with lattice spacing $1/\rho_0 = a$.
Energy per unit length $= \frac{d^2}{4\pi\epsilon_0}\zeta(3)\rho_0^4$
This crystal is melt by the quantum fluctuations.

$$K \sim \frac{1}{\sqrt{6\zeta(3) r_0 \rho_0}}$$

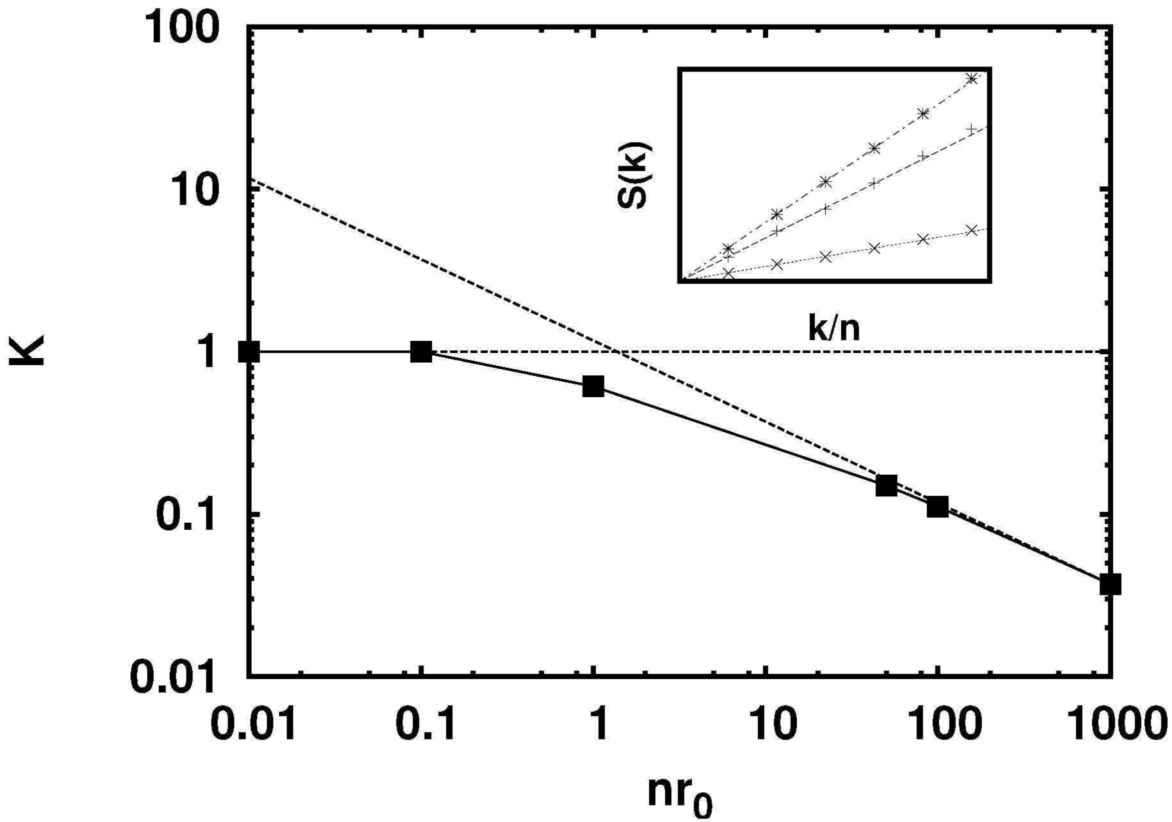
⇒ Crossover from Tonks-Girardeau at low density to quasi-solid ("super-Tonks") at high density.

Energy vs Density by DMC calculation



From A.S.Arkhipov, G.E.Astrakharchik, A.V.Belikov and Yu. E. Lozovik, JETP Lett. **82**, 39 (2005)

Behavior of Luttinger exponent as a function of density by RQMC



From R. Citro, EO, S. De Palo and M. L. Chiofalo Phys. Rev. A **75**, 051602(R) (2007).

Structure factor

$$S(q) = \int_0^L dx e^{iqx} \left[\langle \rho(x)\rho(0)\rangle - \langle \rho_0^2 \rangle\right]$$

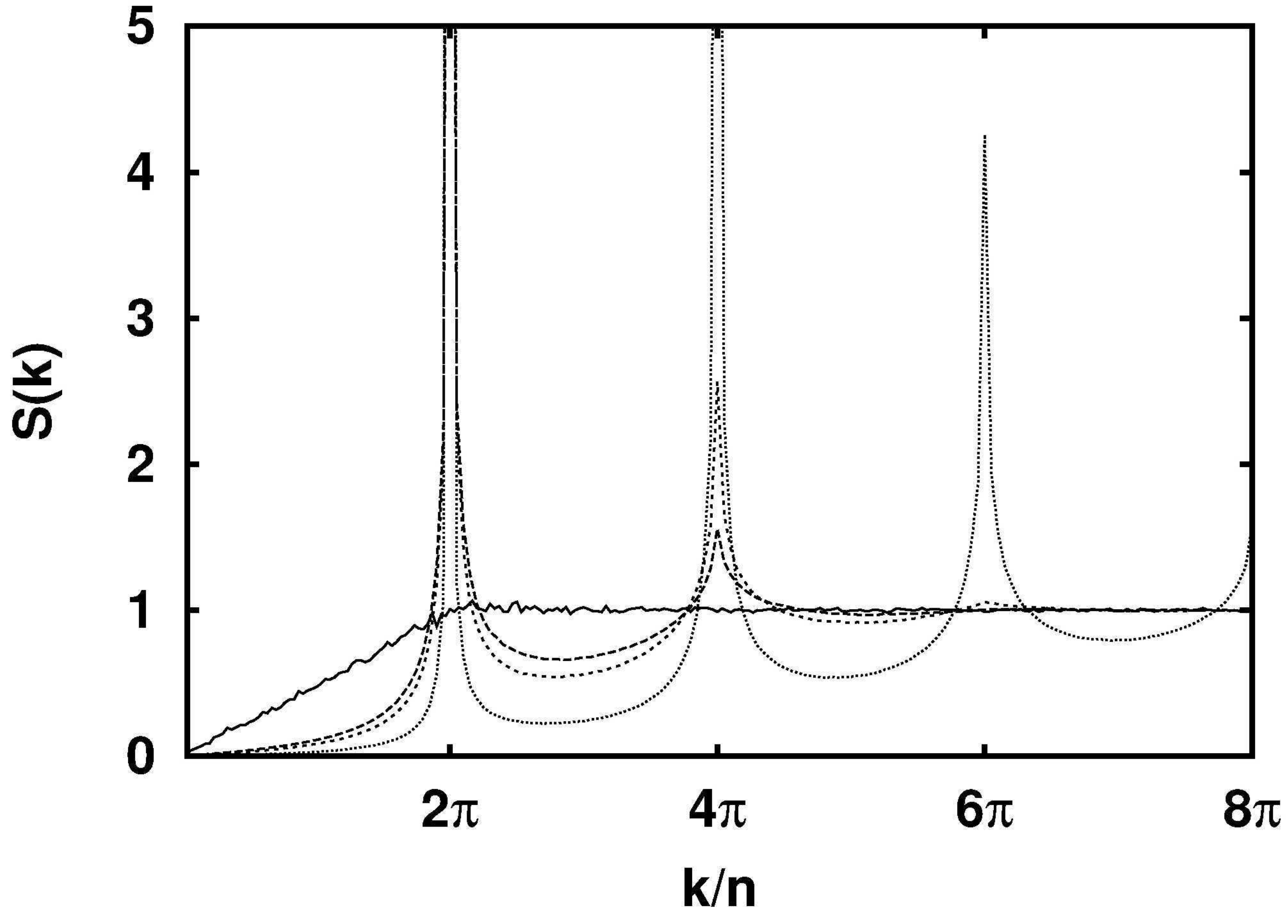
$$S(q) = \frac{K|q|}{2\pi} (q \ll \pi\rho_0)$$

$$S(q) \sim L^{1-2m^2K} \frac{\Gamma(1-2m^2K)\Gamma(m^2K + kL/(2\pi))}{\Gamma(1-m^2K+kL/(2\pi))} \sin(\pi m^2 K)$$

$q = k + 2m\pi\rho_0, 2m^2K < 1.$

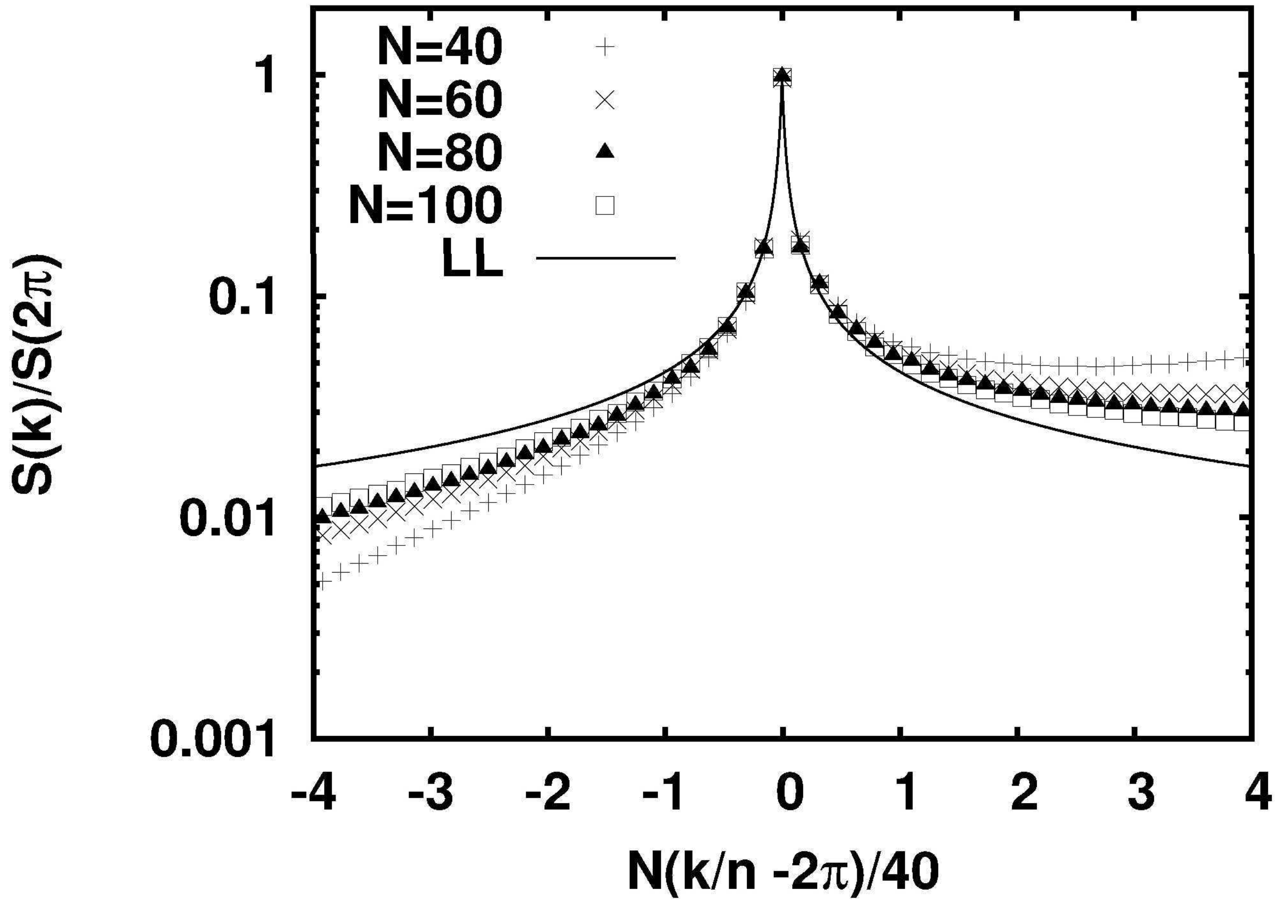
$\Rightarrow$ Development of quasi-Bragg peaks as $K \to 0$.

Development of strong fluctuations towards the crystal state



From R. Citro, EO, S. De Palo and M. L. Chiofalo Phys. Rev. A **75**, 051602(R) (2007).

Luttinger liquid scaling of the peaks for fixed density



From R. Citro, EO, S. De Palo and M. L. Chiofalo Phys. Rev. A **75**, 051602(R) (2007).

Experimental systems ?

- magnetic dipoles: ^{52}Cr atoms ($6\mu_B$) Stuhler et al. Phys. Rev. Lett. **95**, 150406 (2005).
- electric dipoles: polar molecules (SrO) Büchler et al. Phys. Rev. Lett., **98**, 060404 (2007).

Condition for one dimensional regime: $\rho_0 \ell_\perp \ll 1$ where $\ell_\perp = (r_0/(4a_\perp))^{1/5} a_\perp$.

Magnetic dipoles (Chromium)

$\ell_\perp = 31$ nm, $r_0 = 4.8$ nm $\Rightarrow \rho_0 r_0 \lesssim 0.1$

Only the Tonks-Girdardeau regime should be observable !

Electric dipoles (SrO)

$\ell_\perp = 0.2\ \mu\text{m},\ r_0 = 240\ \mu\text{m} \Rightarrow \rho_0 r_0 \lesssim 10^3$

The "super-Tonks" regime could be observed !

But:

- Trapped molecules are difficult to cool [Eur. Phys. J. D **31**, 149 (2004).]
- Forming molecules by photoassociation or via Feshbach resonances [Rev. Mod. Phys. **78** 001311 (2006)] does not yield molecules in their ground state.

Open questions

- Influence of a shallow longitudinal trapping potential (in progress)
- more realistic interaction potential (taking into account transverse confinement, and confinement induced resonances)
- Case with more than one transverse channel occupied (analogies with coupled spin chain problems: Haldane gap ?)
- Array of 1D dipolar gases: possibility of crystallization ?

References

- R. Citro, EO, S. De Palo and M. L. Chiofalo Phys. Rev. A **75**, 051602(R) (2007).
- P. Pedri, S. De Palo, EO, R. Citro, M. L. Chiofalo `arXiv:0708.2789`