The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA International Atomic Energy Agency

1859-7

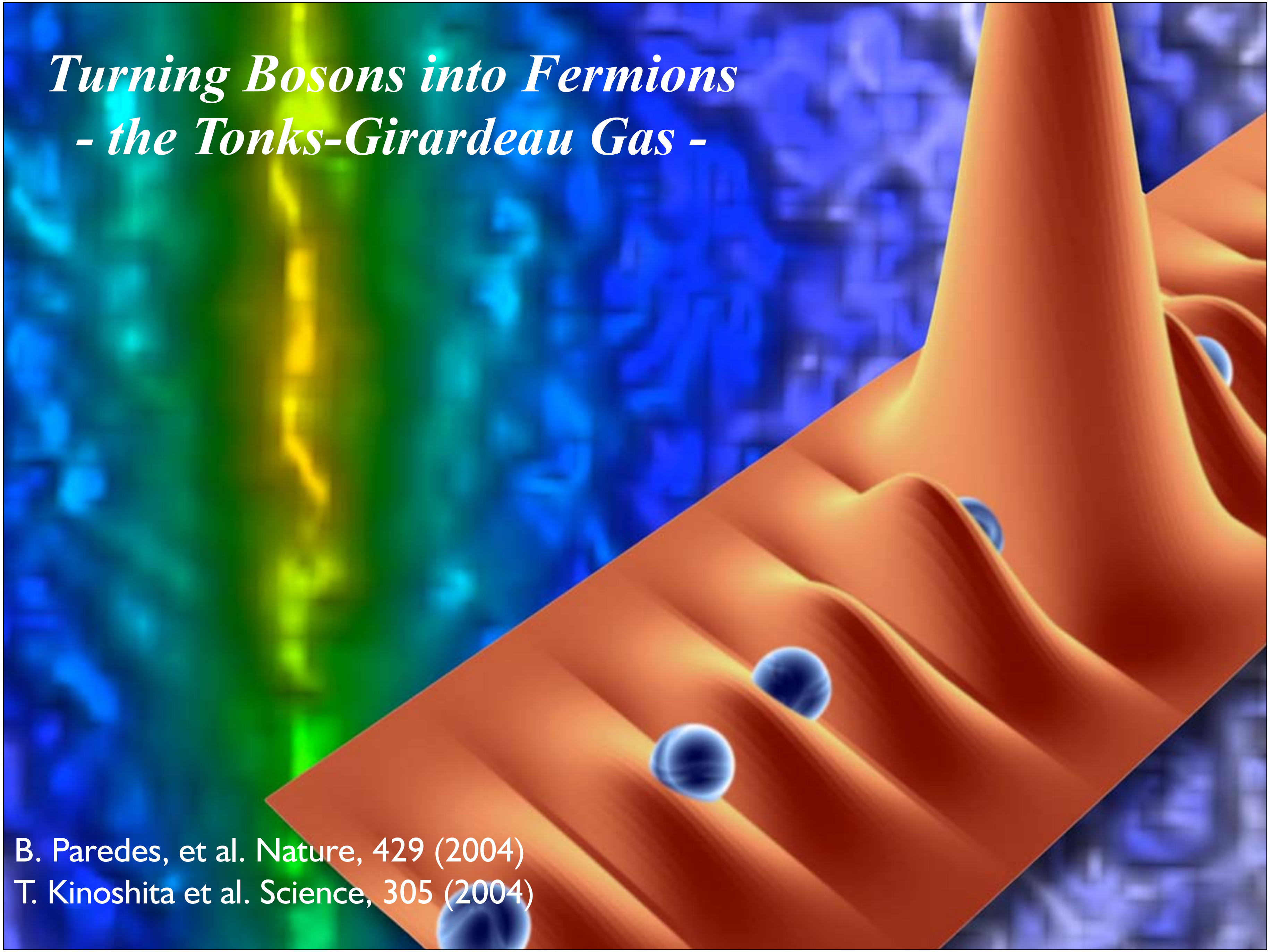
Summer School on Novel Quantum Phases and Non-Equilibrium Phenomena in Cold Atomic Gases

27 August - 7 September, 2007

Introduction to experiments in optical lattices - Part II & III

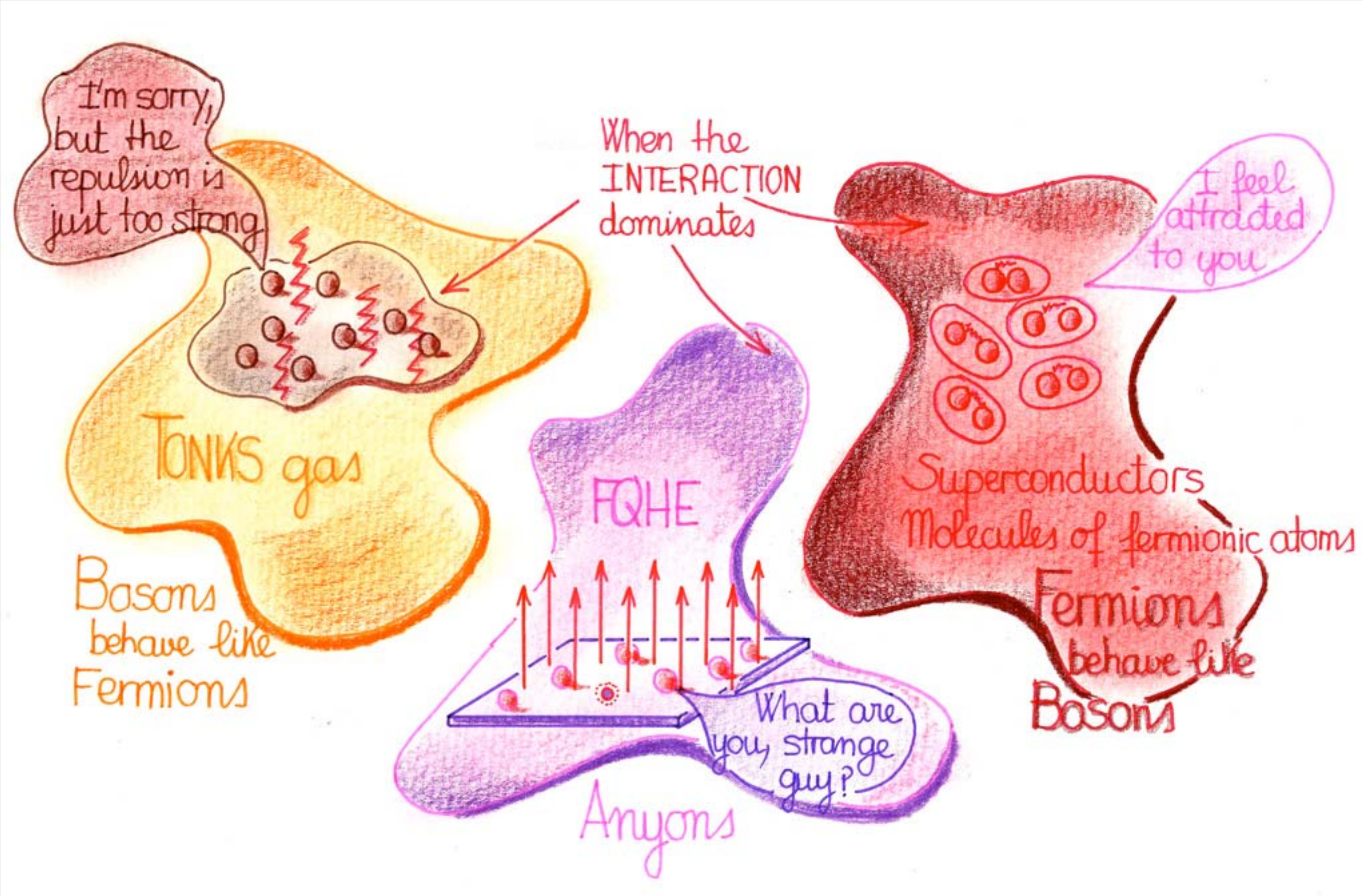
Immanuel Bloch
University of Mainz

Turning Bosons into Fermions
- the Tonks-Girardeau Gas -

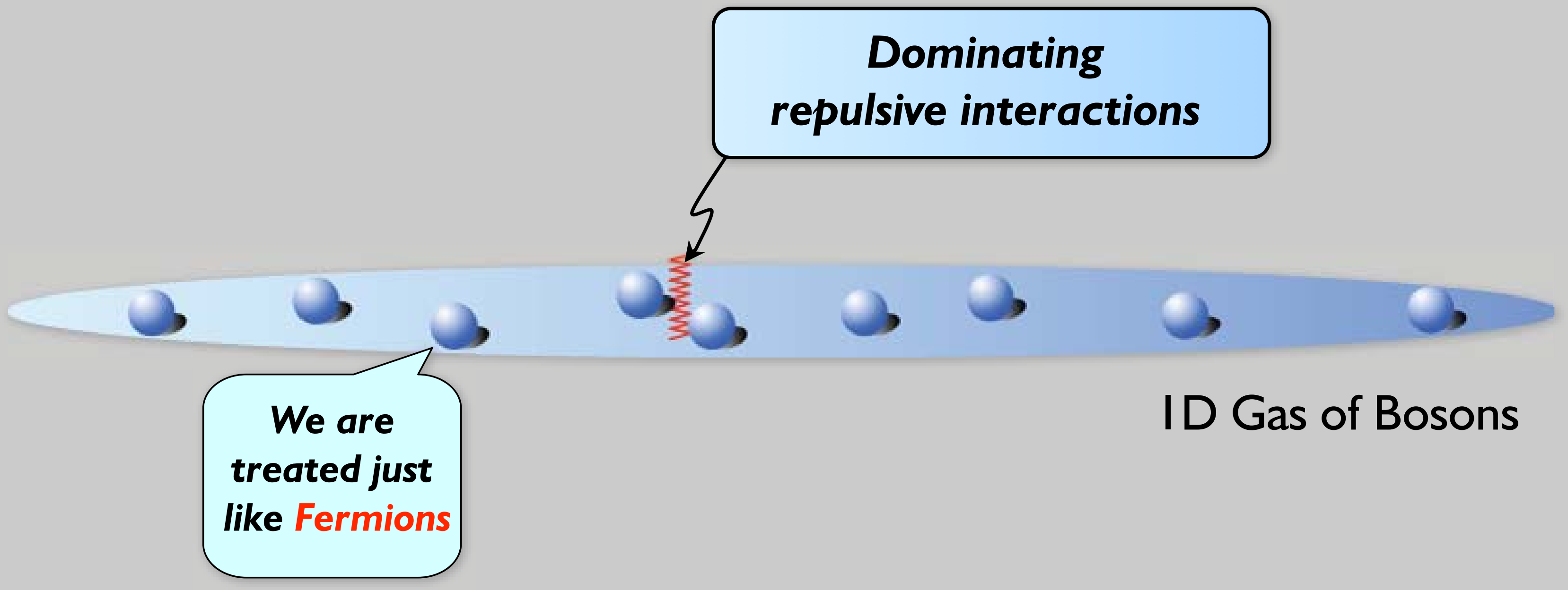


B. Paredes, et al. Nature, 429 (2004)
T. Kinoshita et al. Science, 305 (2004)

Strong interactions lead to novel phenomena…



What is a Tonks-Girardeau Gas?



A Gas of FERMIONIZED Bosons

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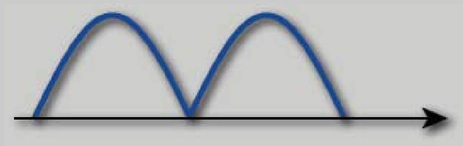
Tonks-Girardeau Gas - Fermionization

Equivalence between

Hard-Core Bosons

dominating interactions

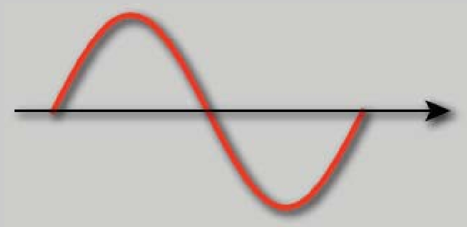
$$\Psi_B = \left|\Psi_F\right|$$



Free Fermions

Pauli exclusion principle

$$\Psi_F = \begin{vmatrix} \phi_1(x_1) & \cdots & \phi_1(x_N) \\ \vdots & \ddots & \vdots \\ \phi_N(x_1) & \cdots & \phi_N(x_N) \end{vmatrix}$$



(M.D. Girardeau, J. Math. Phys. 1960)

Bosons behave like Fermions – Not Quite

Density distribution: $|\Psi_B(x)|^2 = |\Psi_F(x)|^2$

identical to the one of free fermions!
(absolute value of det does not matter)

Entropy: identical to the one of free fermions!

Energy spectrum: identical to the one of free fermions!

Correlation function: $g^{(1)}(x) = \langle \psi_B^\dagger(0)\psi_B(x)\rangle \neq \langle \psi_F^\dagger(0)\psi_F(x)\rangle$

different to the one of free fermions!
(absolute value of det matters)

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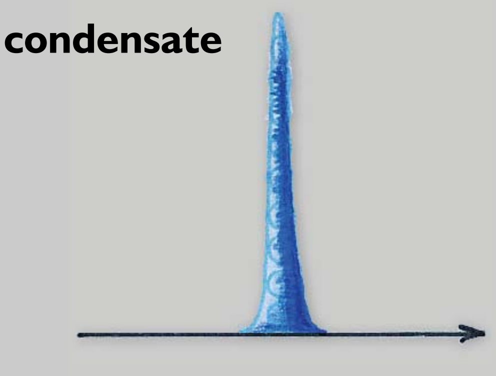
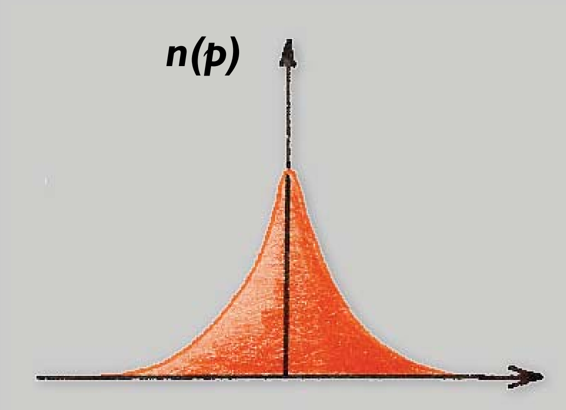
Bosons behave like Fermions – Not Quite

Momentum Distribution: $n(p) \propto \int e^{-ipx} g^{(1)}(x)\,dx$

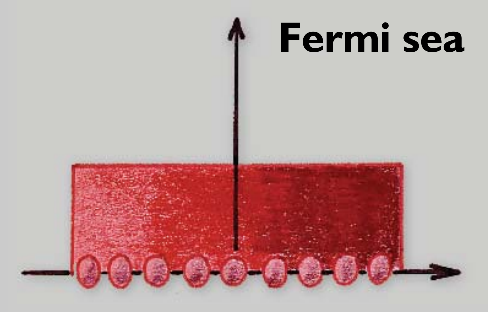
different to the one of free fermions!
(FT of correlation function)



Bosons



FERMIONIZED Bosons



Fermions

Momentum distribution is characteristic for a Tonks-Girardeau gas!

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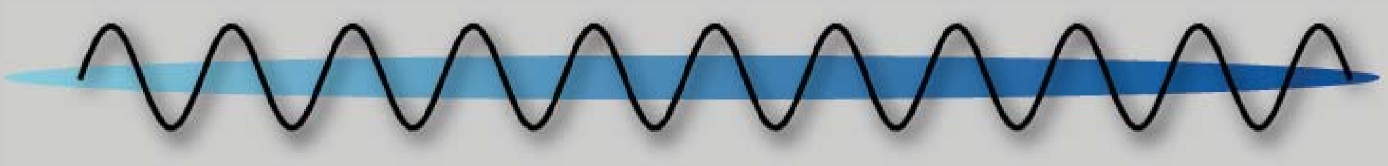
Realizing the Tonks Regime

We need:

$$\gamma = \frac{\text{Interaction Energy}}{\text{Kinetic Energy}} >> 1$$

Use shallow optical lattice to increase effective mass m*!



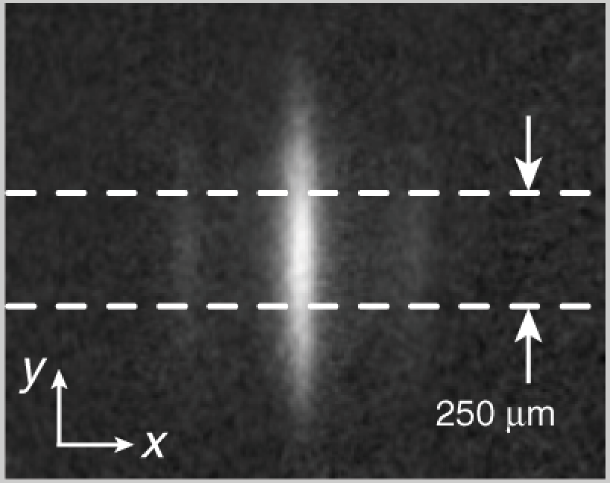
effectively

low filling factor important!

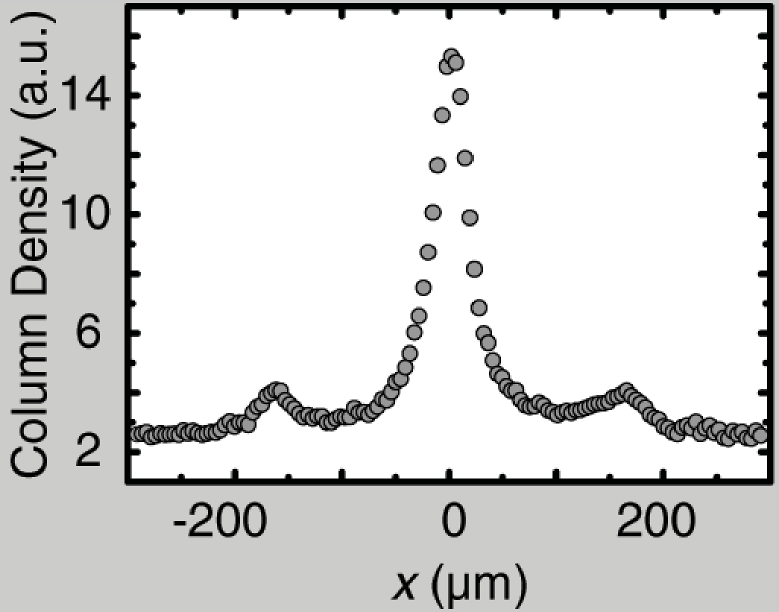
(exp. with high filling fraction T. Stöferle et al., Phys. Rev. Lett. (2004)

Typical Absorption Images After Time Of Flight

Observe fast expansion in radial direction



Due to the low atom number we average horizontal profiles within the white dashed lines.



Challenge:
Fully explain momentum distributions!

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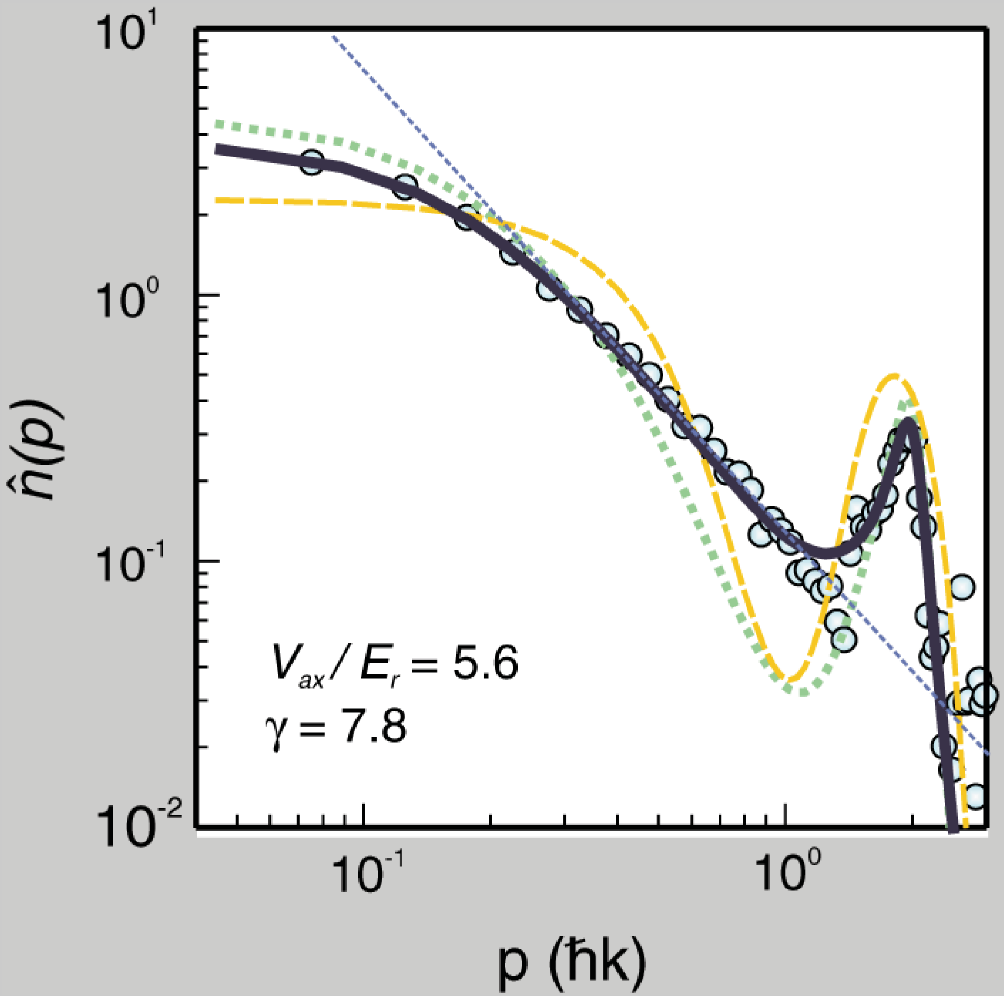
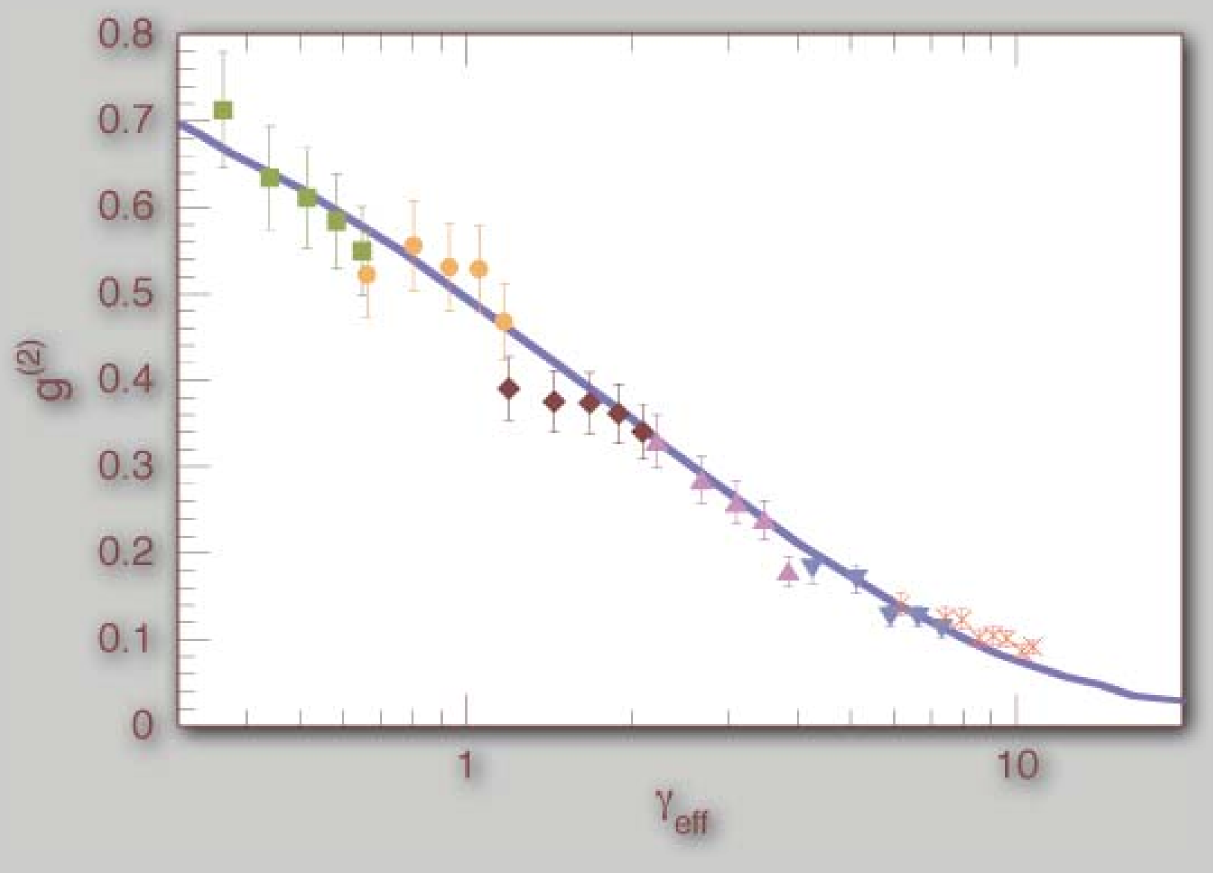
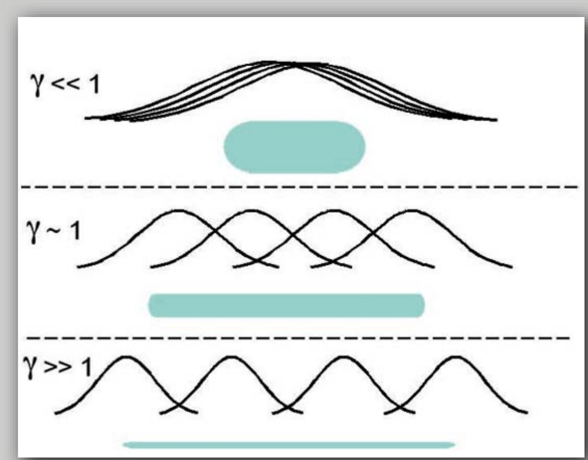
Example of Momentum Profile



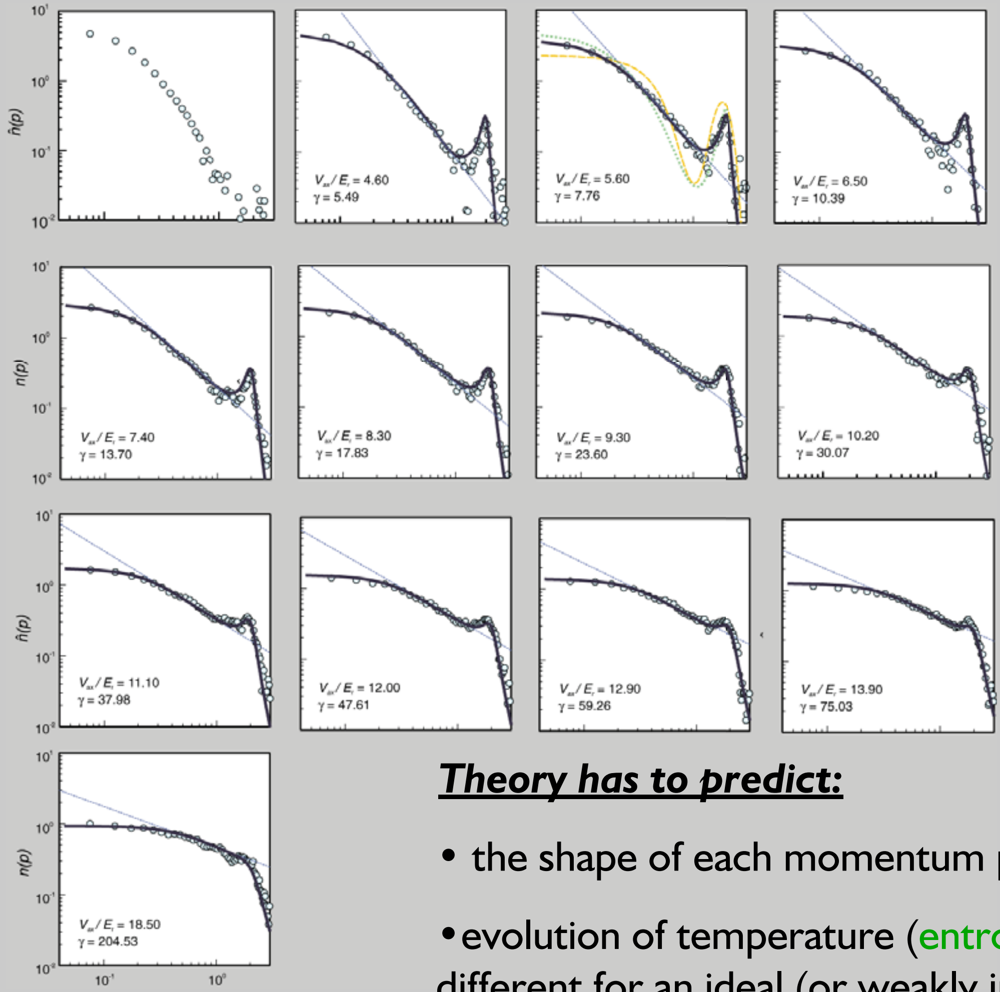
Measuring $g^{(2)}$ of a Tonks Gas





T. Kinoshita et al., Science **305**, 1125 (2004)
T. Kinoshita et al. PRL **95**, 190406 (2005)

Comparison Theory-Experiment (All Series)



Only two fit parameters for whole series!

$$k_B T_0 / J \approx 0.5$$

$$N_{0,0} = 18$$

Theory has to predict:

- the shape of each momentum profile
- evolution of temperature (entropy) which is very different for an ideal (or weakly interacting) Bose and Fermi gas

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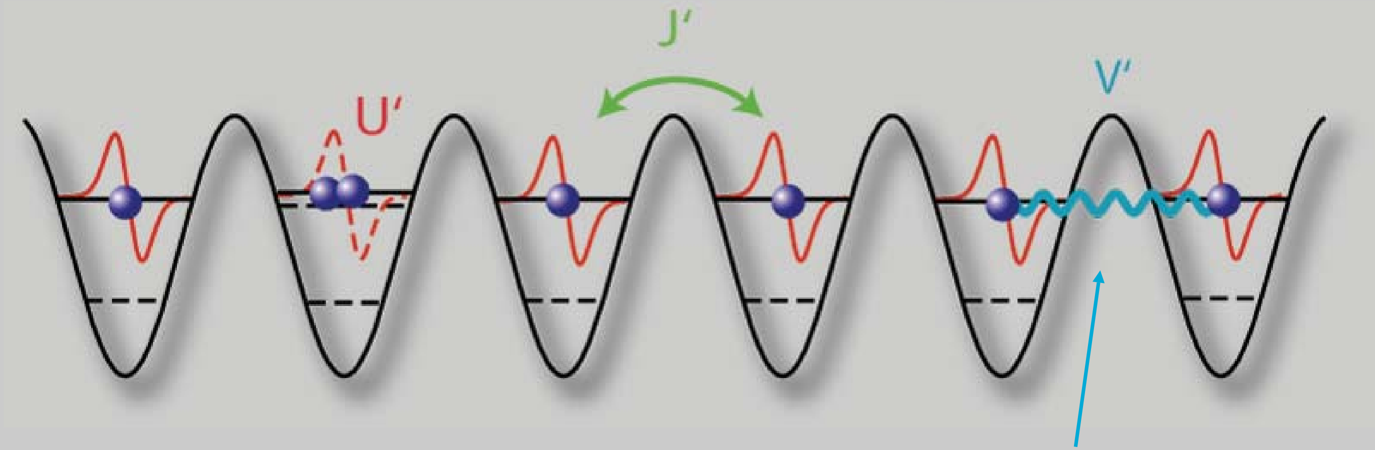
Anisotropic Multiorbital MI Physics

In this talk: **We create an atomic system in the 2nd band**



Inter-site interaction
A. Isacsson and S.M. Girvin, PRA 72, 053604 (2005)

Motivation:

New phases:

- **Supersolid phase**
- **Density wave**

V. W. Scarola and S. Das Sarma , PRL 95, 033003 (2005)
P. Sengupta et al., PRL 94, 207202 (2005)

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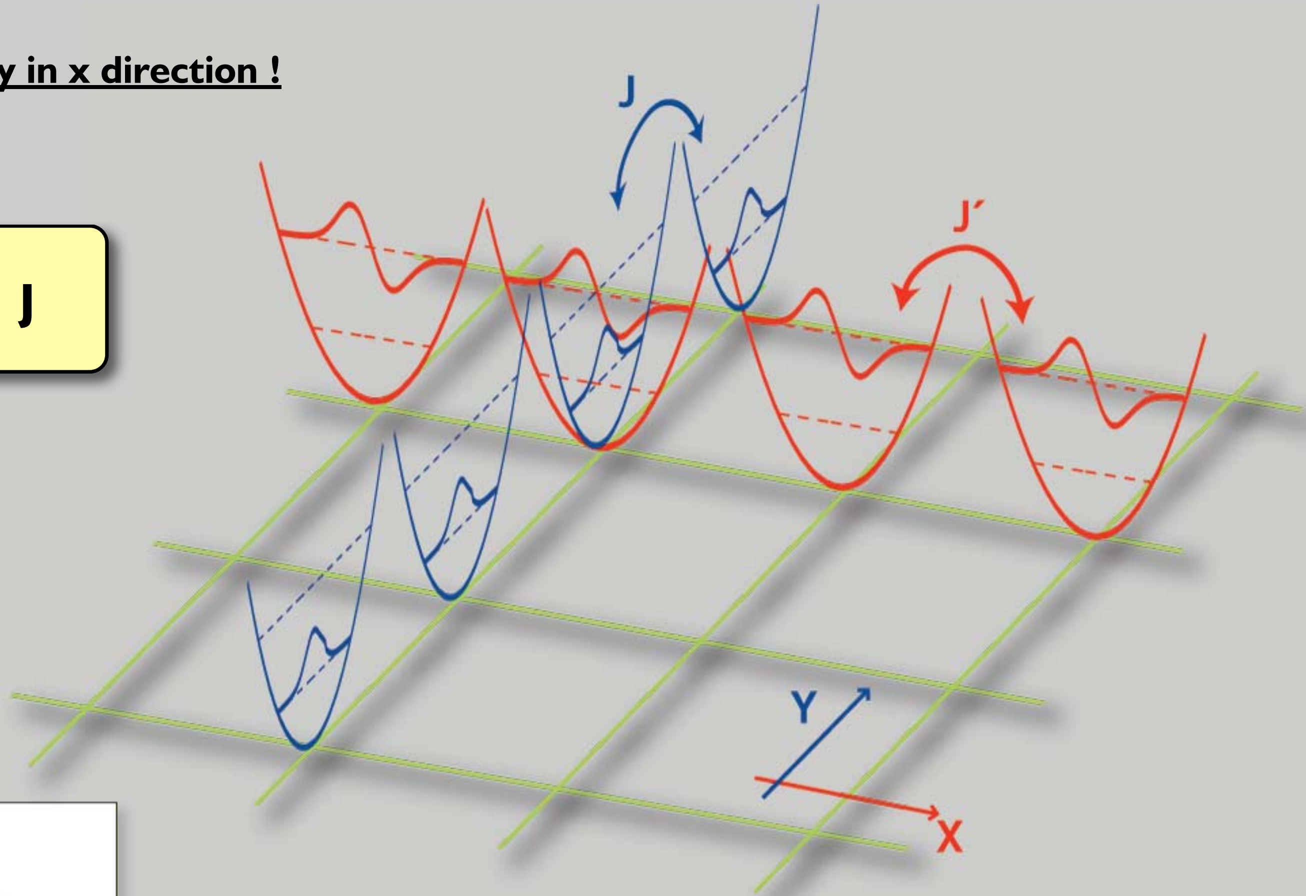
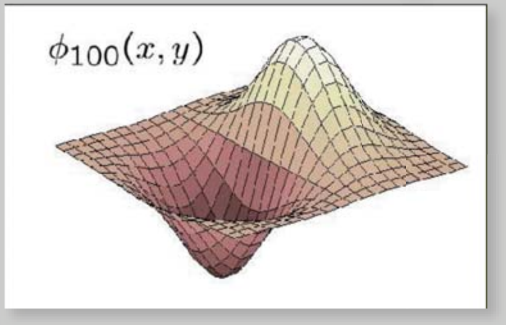
Atoms to the 2nd band in X

<u>Excitation only in x direction !</u>

J´ >> J





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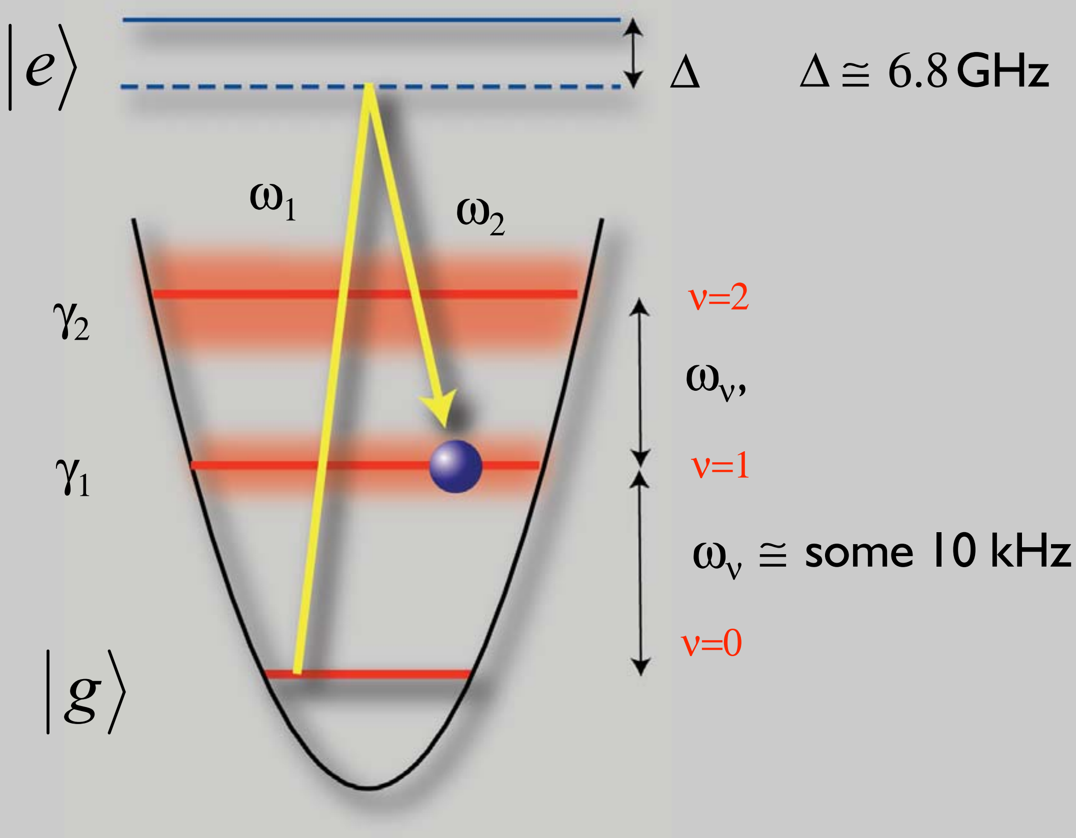
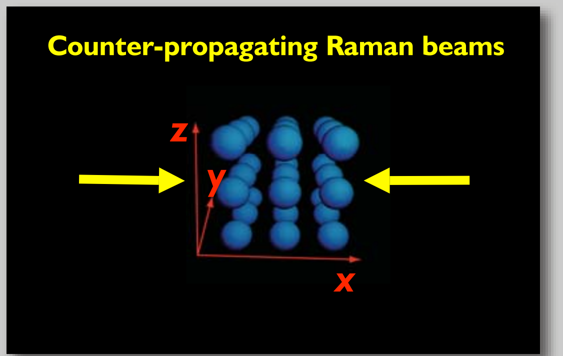
Populating the first vibrational band in X direction

Raman Process

$$\omega_v = \omega_1 - \omega_2$$





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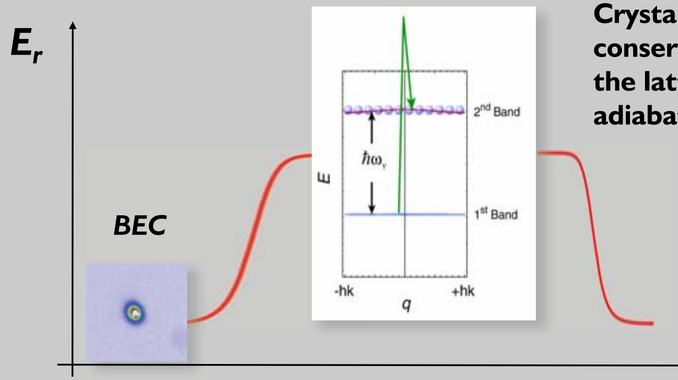
Imaging of Excitation to 2nd Band:

- ***Ramping down the lattice adiabatically:***

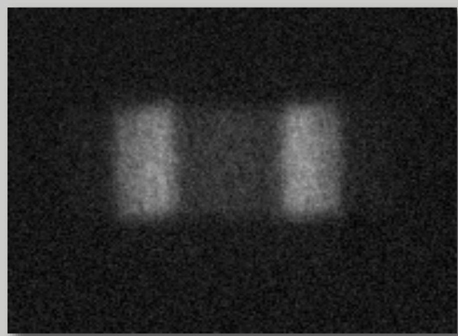


Crystal momentum is conserved while lowering the lattice depth adiabatically !

TOF

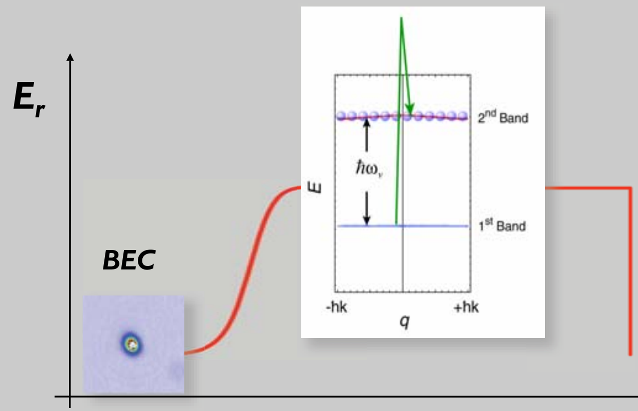
Population of nth band is mapped onto nth Brillouin zone !

Brillouin zone mapping



Time

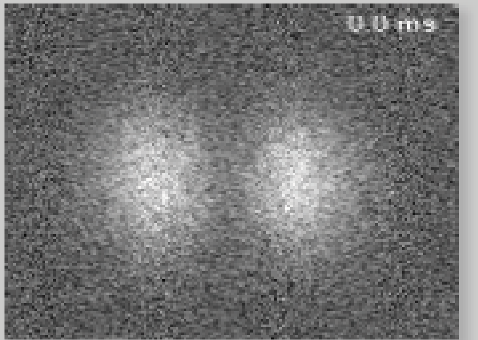
- ***Switching off the lattice fast:***



TOF

Incoherent sum over localized wave functions

Wave function mapping



Time

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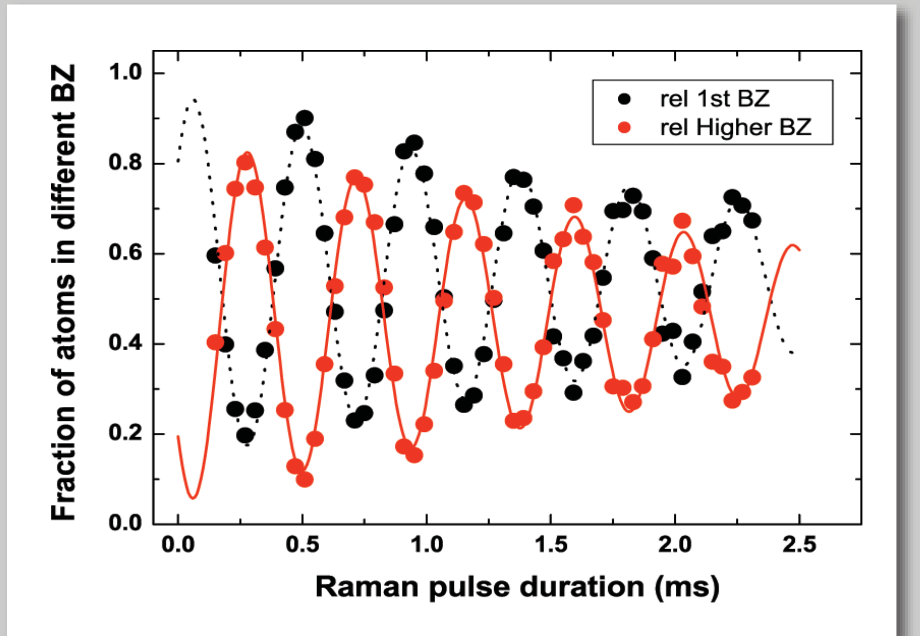
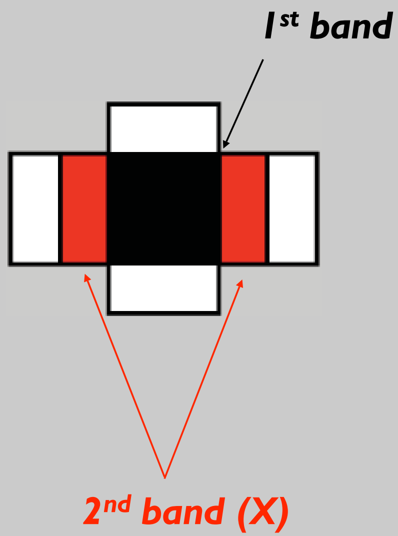
Coherent Population Transfer to 2nd Band

Rabi oscillation

X-lattice: 40Er ; Y-& Z-lattice: 55Er (Mott Insulator)

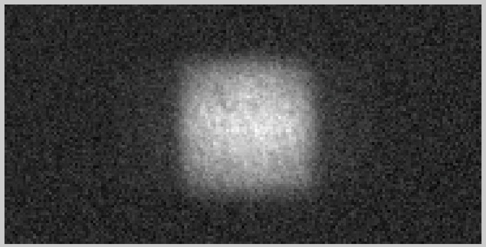




Fit parameter:

ω_v = ***(37,11 ± 0,05)kHz***

width = (1,8 ± 0,1)kHz



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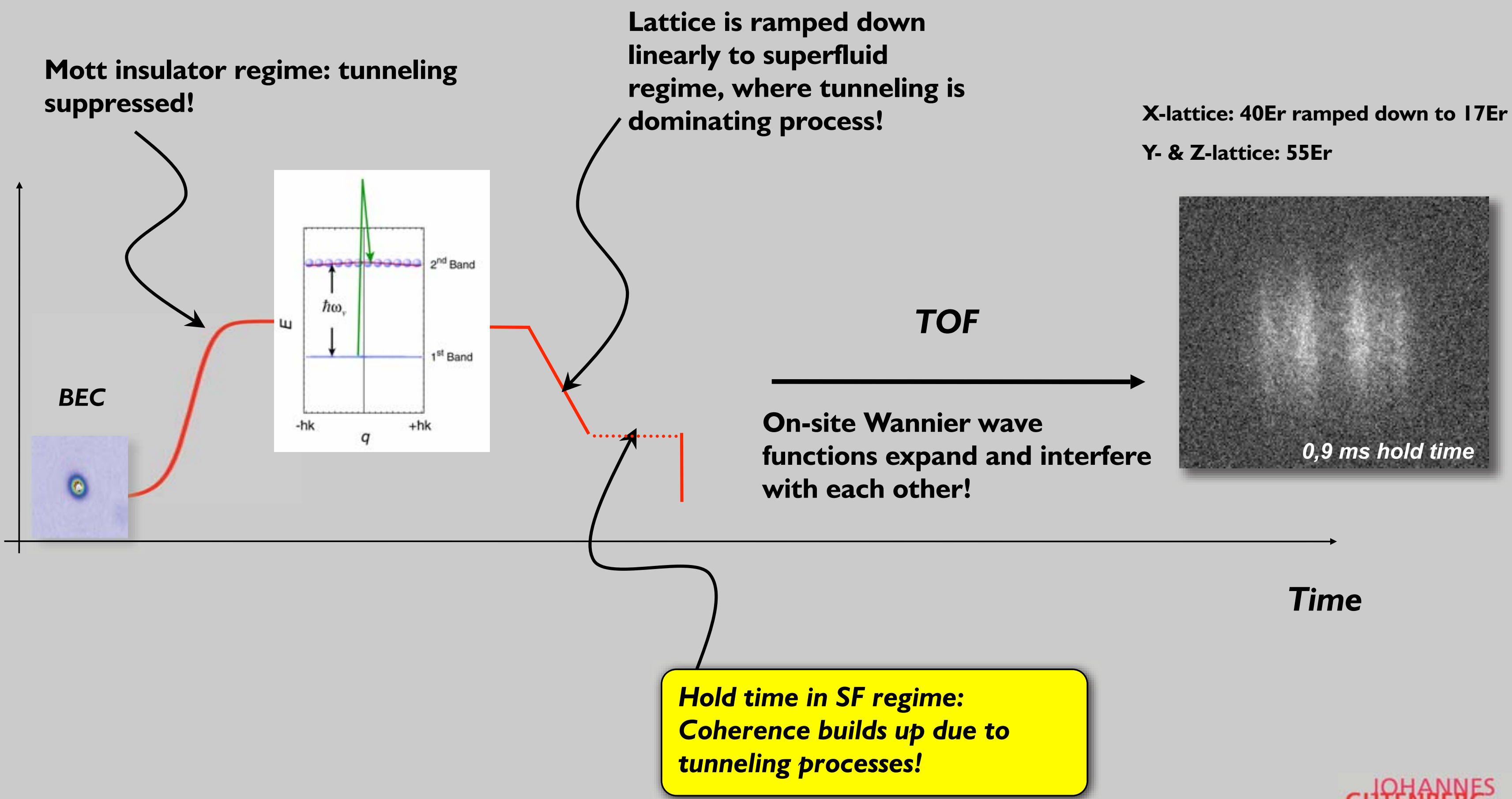
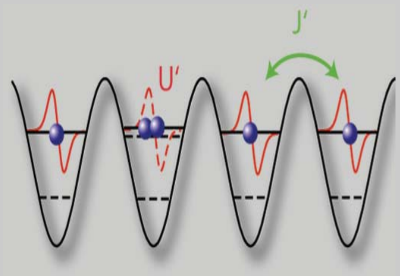
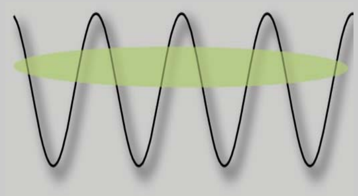
Insulating - coherent transition in 1^{st} excited Bloch band



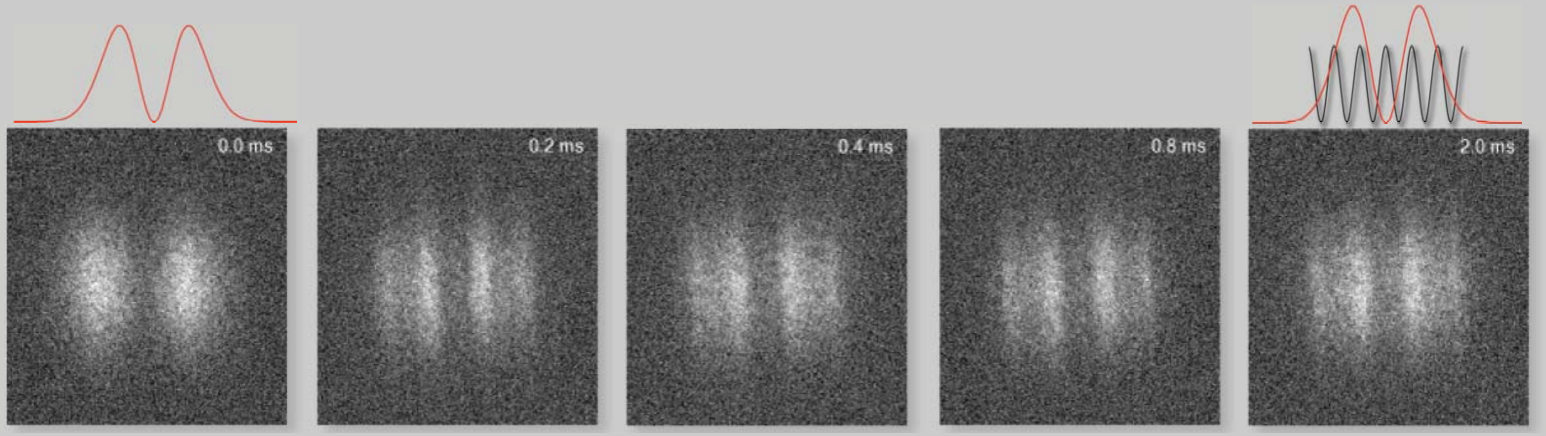
1D Insulating – delocalized Transition in 1st excited Band

Insulating: U´ » J´



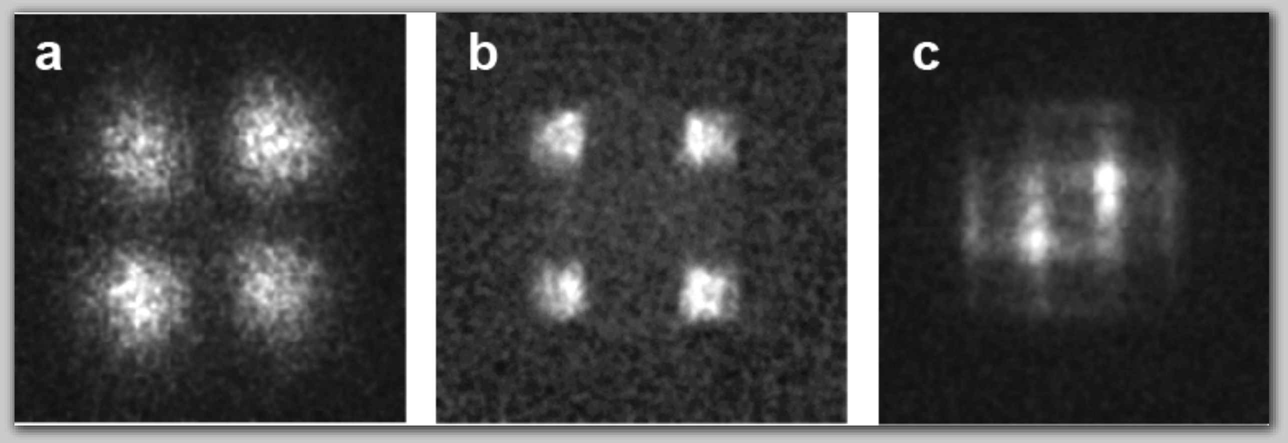


SF: U´ » J´



X-lattice: 40Er ramped down to 17Er ; Y- & Z-lattice: 55Er

2D Excitations



sudden switch off

adiabatic switch off

emergence of coherence

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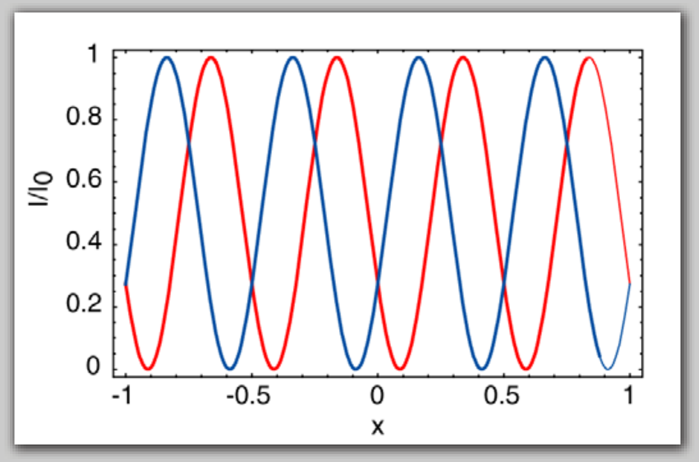
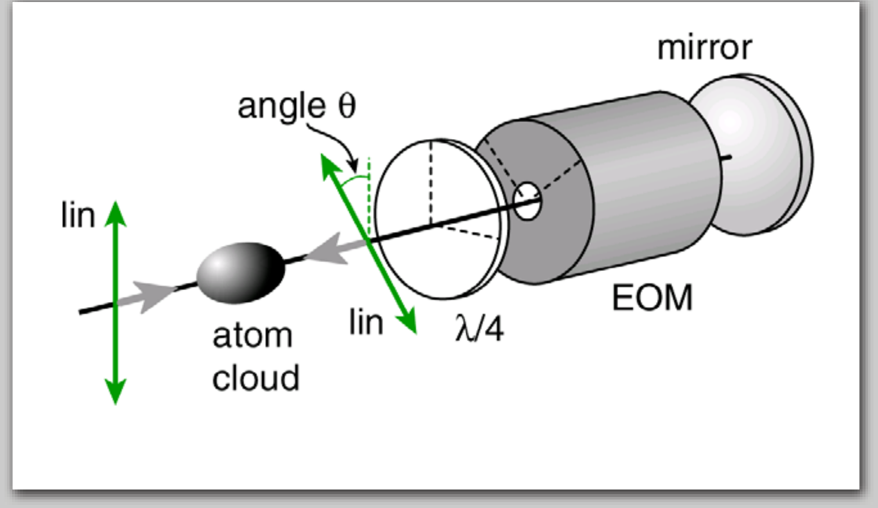
Entangling Neutral Atoms

Moving the Lattice Potentials

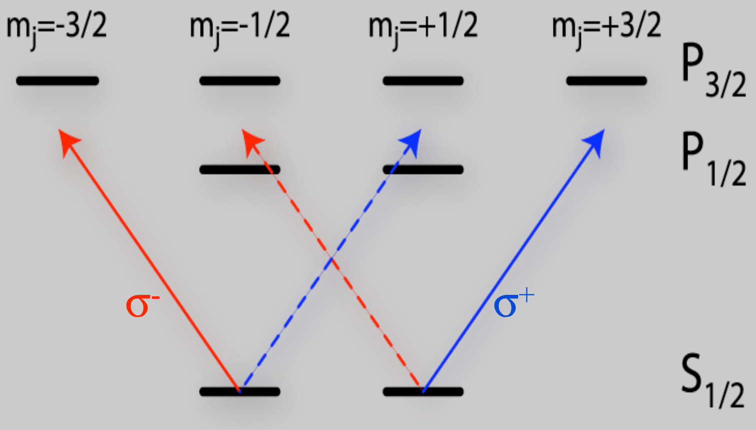
$$I_- = I_0 \sin^2(kx - \theta/2)$$
$$I_+ = I_0 \sin^2(kx + \theta/2)$$



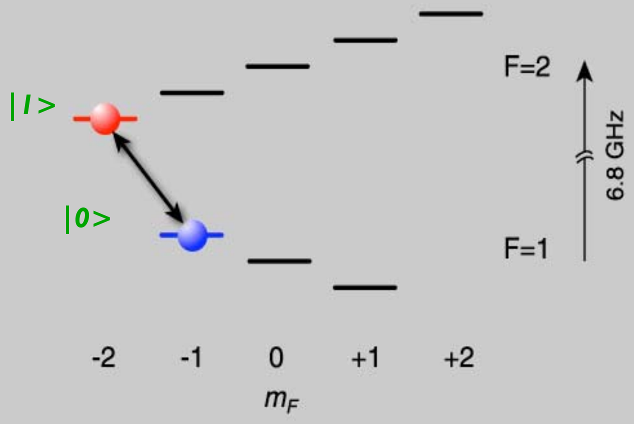


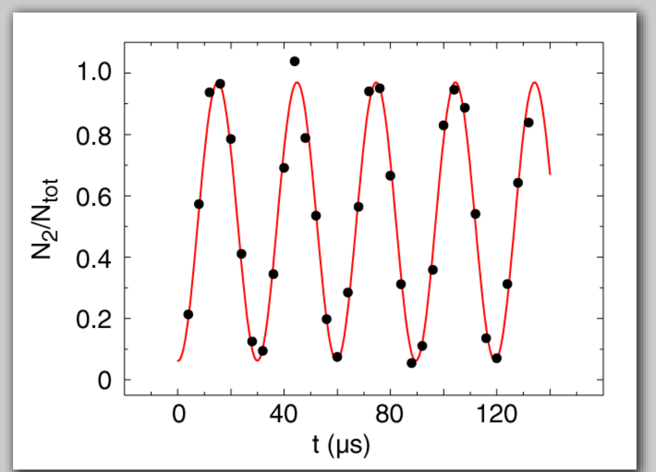
State Selective Lattice Potentials

^{87}Rb Fine- structure



Hyperfine structure





$$V_1(x,\theta) = V_-(x,\theta)$$

$$V_0(x,\theta) = \frac{1}{4}V_-(x,\theta) + \frac{3}{4}V_+(x,\theta)$$

Delocalization "by Hand"

TOF

↑

π/2 microwave pulse

↑

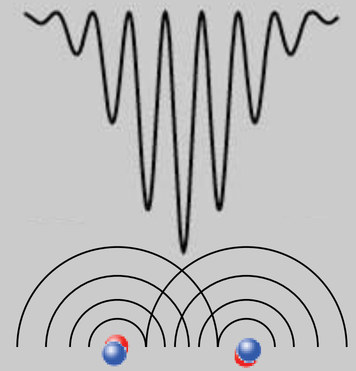
Shift

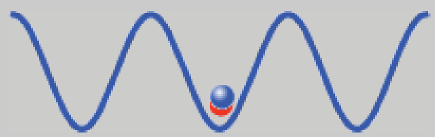
↑

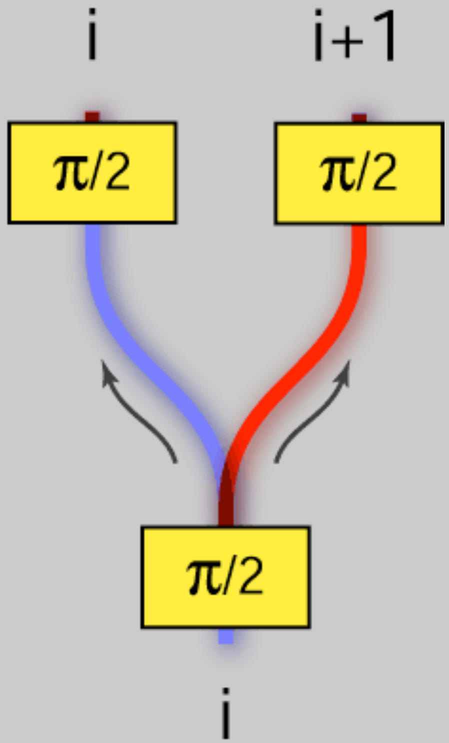
π/2 microwave pulse

↑

Initial state |0>

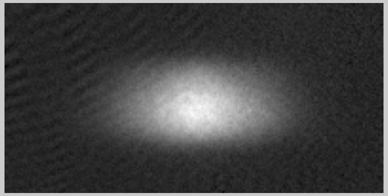






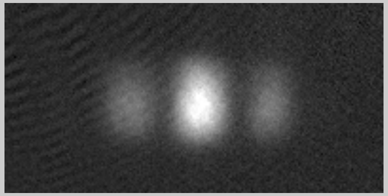
Shifting is Coherent !

Localized

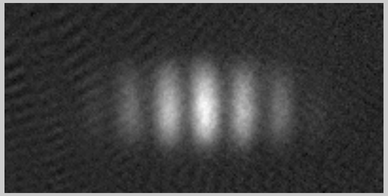


Delocalized

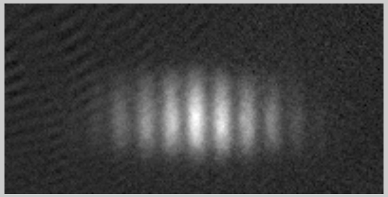
2 sites

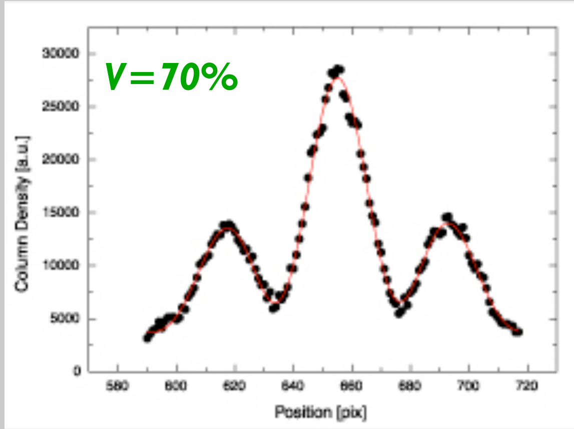


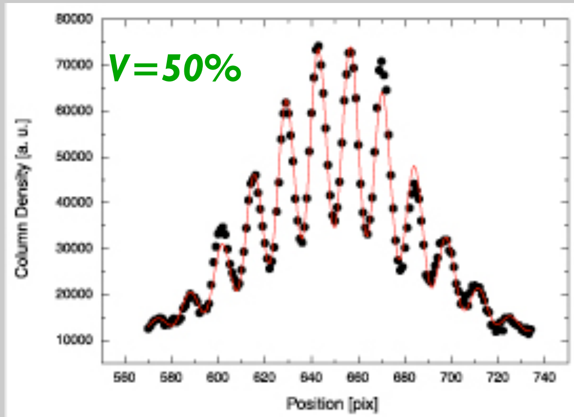
3 sites



4 sites







Able to delocalize atoms over up to 7 lattice sites !

O. Mandel et al., Phys. Rev. Lett. **91**, 010407 (2003)

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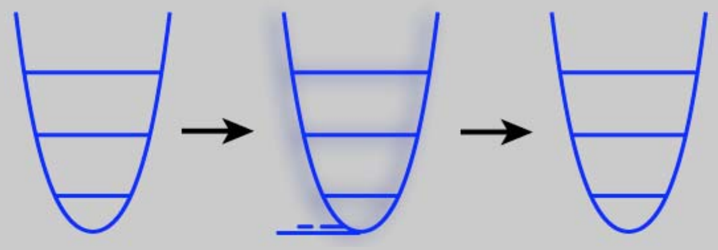
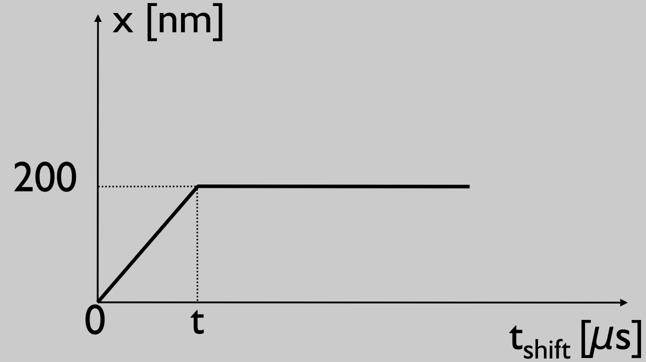
Moving Atoms in Harmonic Potentials

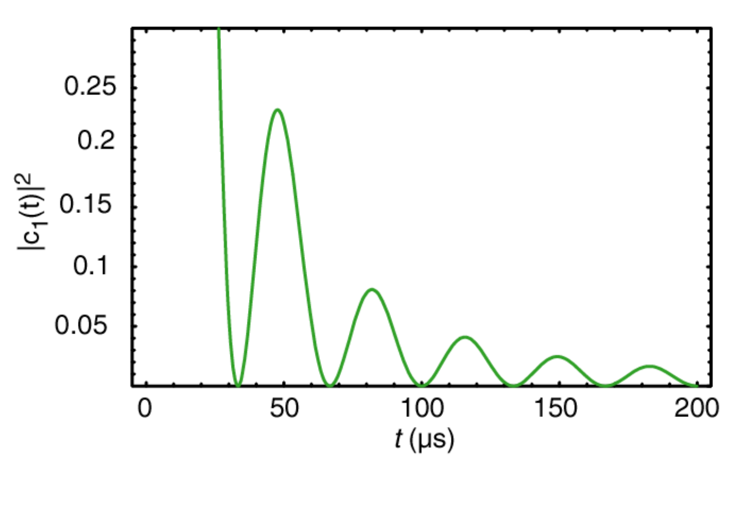




How fast can we move, in order to avoid excitations ?

For a constant shift velocity v, first order perturbation theory yields:

$$|c_1(t)|^2 = \frac{2v^2}{(a_0\omega)^2}\sin^2(\omega t/2)$$



ω=2π30 kHz, Δx = 200 nm

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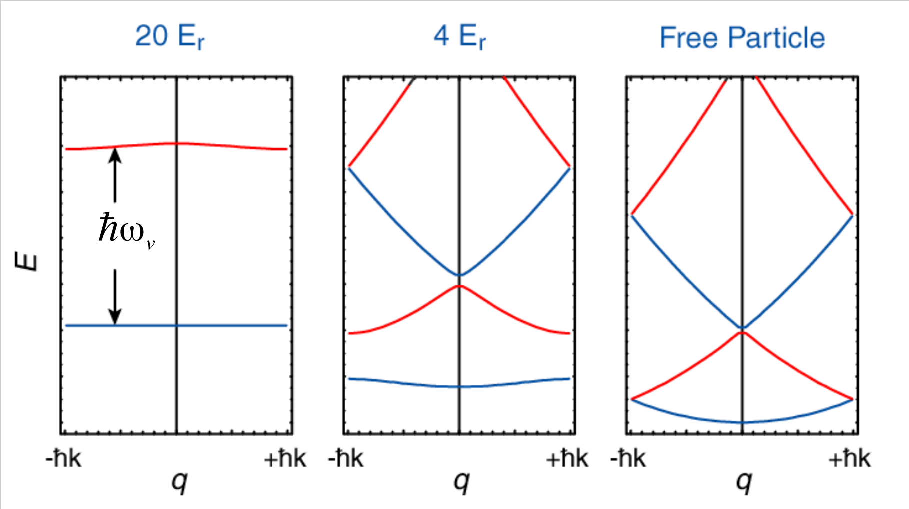
Mapping the Population of the Energy Bands onto the Brillouin Zones

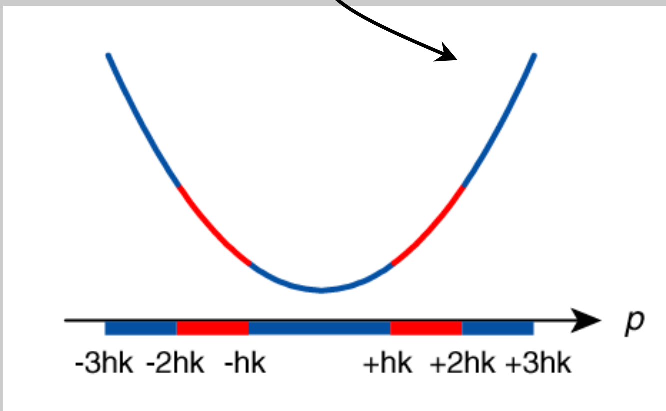


Crystal momentum

Crystal momentum is conserved while lowering the lattice depth adiabatically !

Population of nth band is mapped onto nth Brillouin zone !

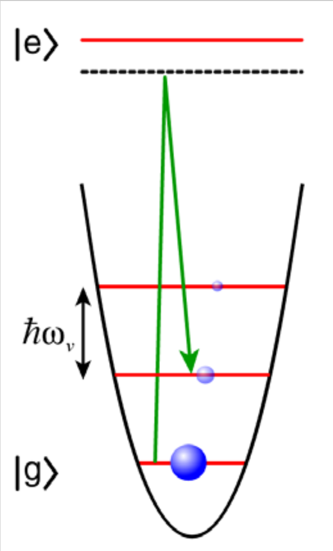
A. Kastberg et al. PRL 74, 1542 (1995)
M. Greiner et al. PRL 87, 160405 (2001)



Free particle momentum

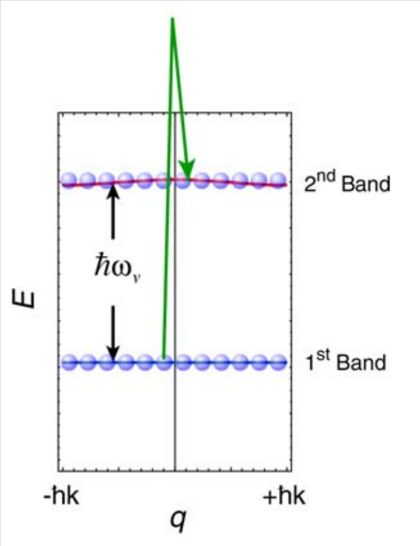
Populating Higher Energy Bands

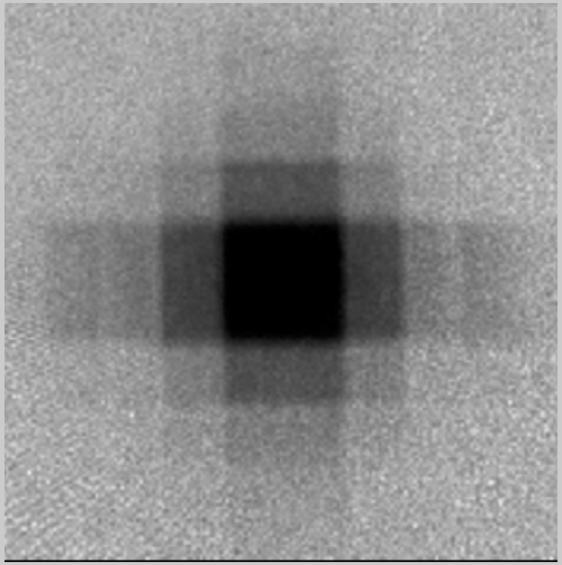
Single lattice site



Stimulated Raman transitions between vibrational levels are used to populate higher energy bands.

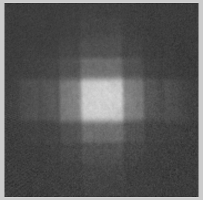
Energy bands





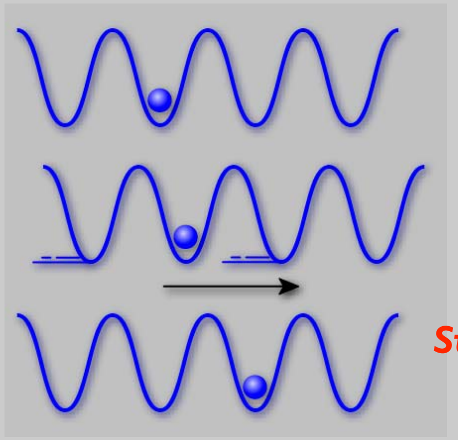
Measured Momentum Distribution !

Measuring the Excitation Probability vs. Shift Velocity



Population of higher vibrational states (energy bands) can be mapped onto the corresponding Brillouin zones by adiabatically decreasing the lattice potential !

A. Kastberg et al. PRL (1995)
M. Greiner et al. PRL (2001)



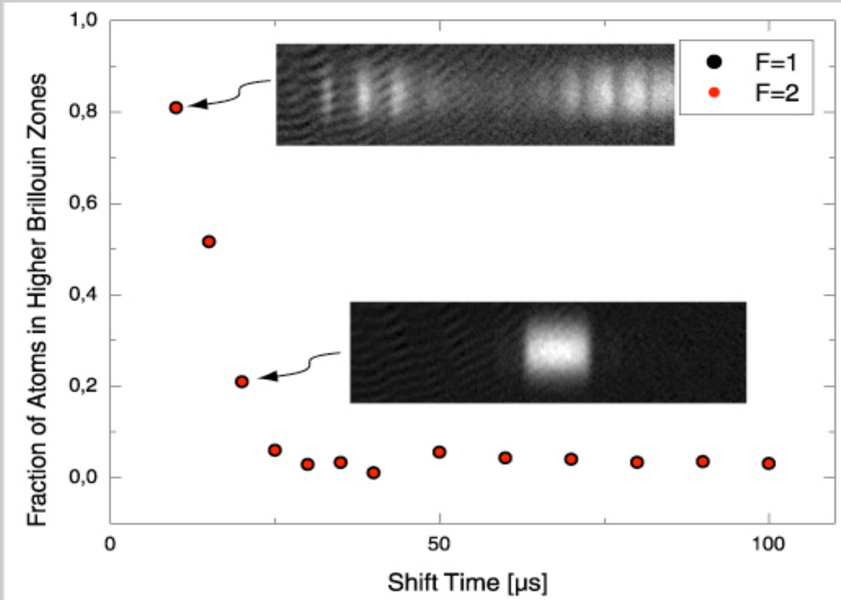
***Start** with ground state atoms*

↓

Constant Velocity

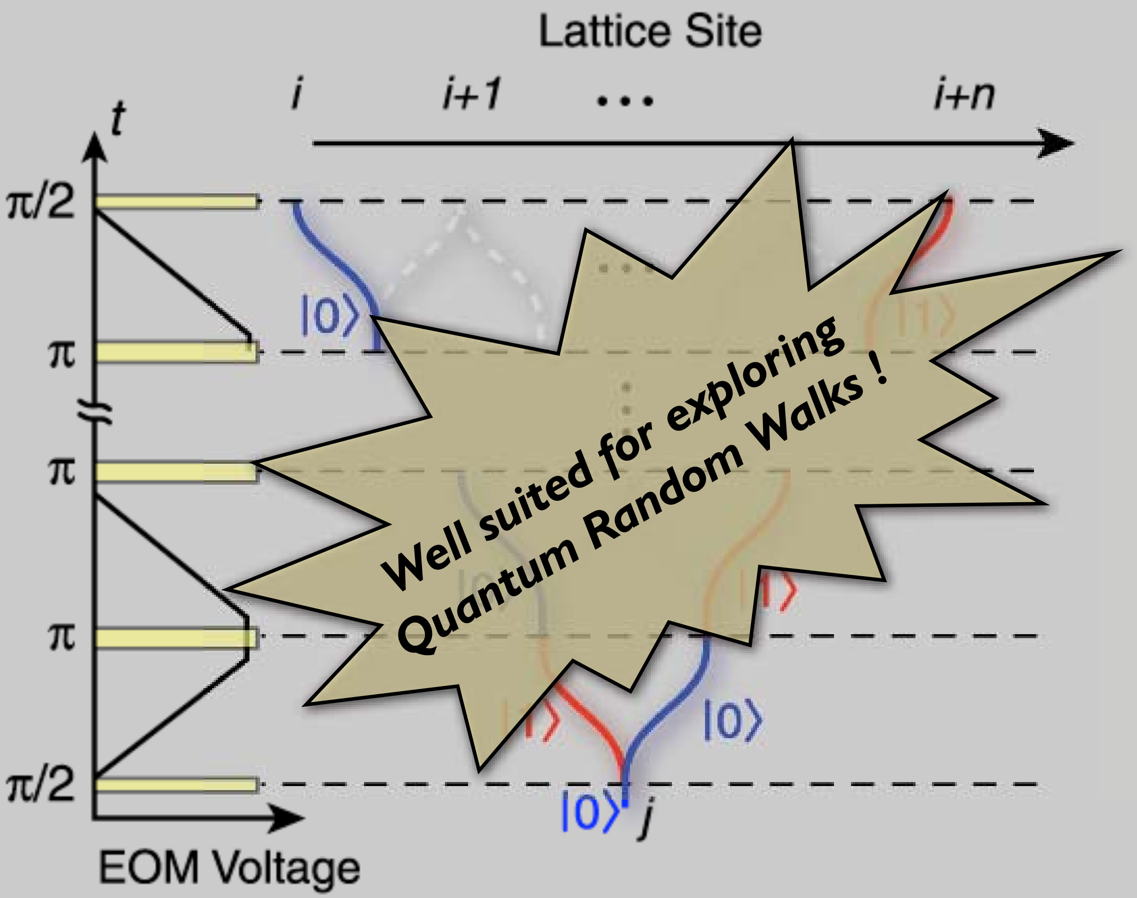
↓

***Stop**; measure remaining atoms in ground state*



Atoms moved over a distance of approx. 200 nm

Complete Sequence used in the Experiment



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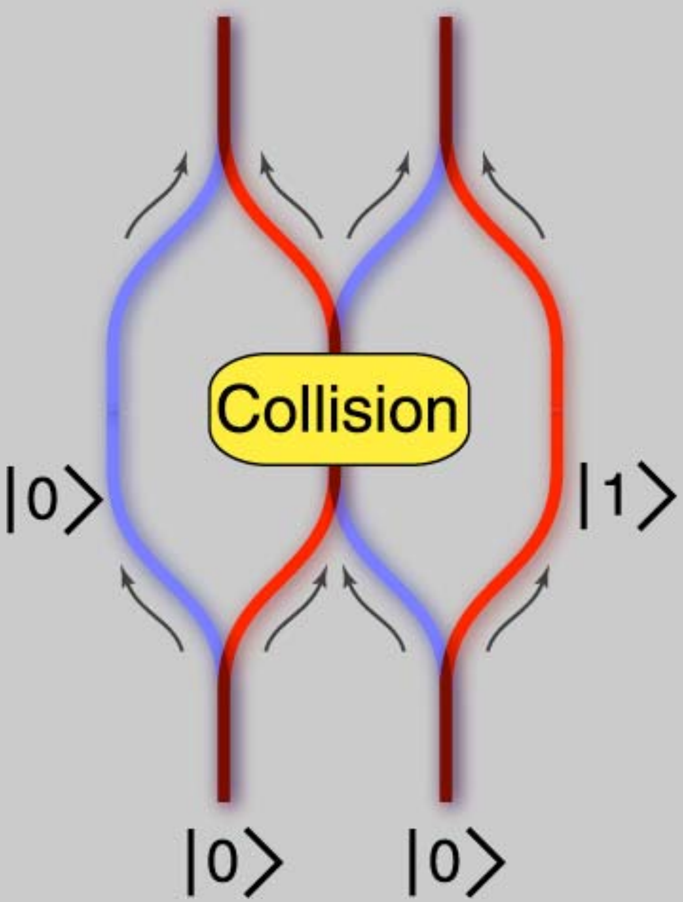
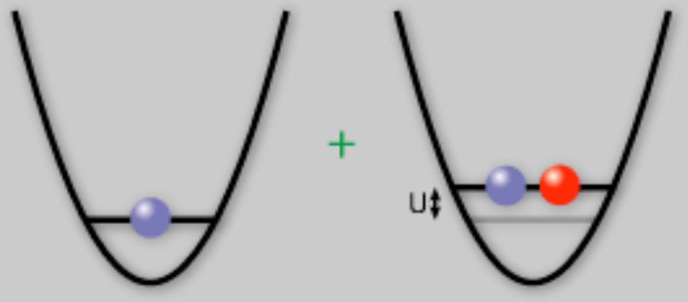
Controlled Collisions





$$U = \frac{4\pi\hbar^2 a}{m} \int |w_0(x)|^2 |w_1(x)^2| d^3x$$

$$E = U\, n_0 n_1$$

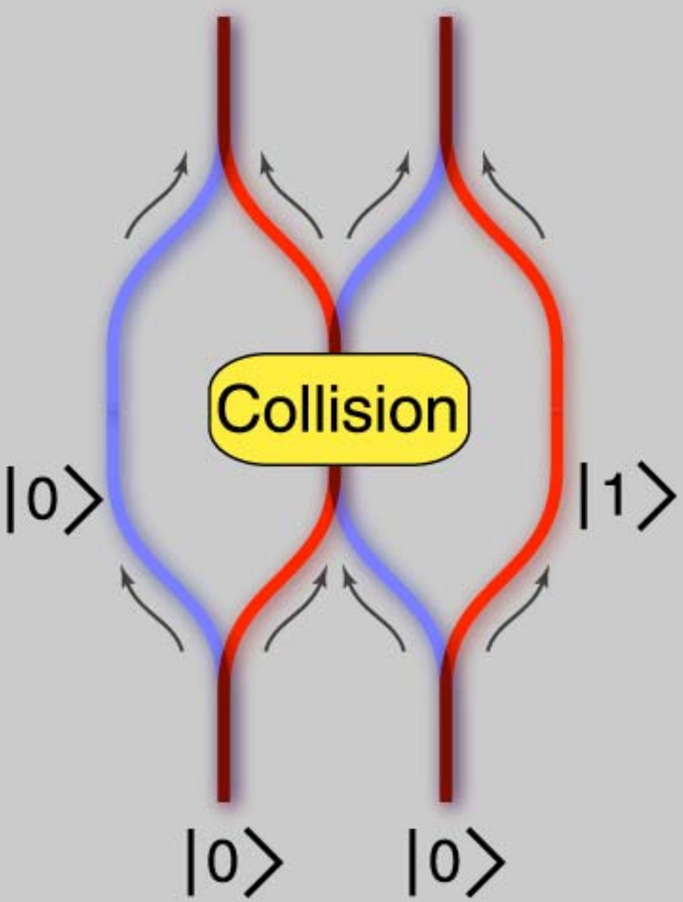
In time t_{Hold} a phase factor of

$$e^{i\phi} = e^{iEt_{hold}/\hbar}$$

is acquired.

D. Jaksch et al., PRL 82,1975 (1999)
O. Mandel et al, PRL 91, 010407 2003 , O. Mandel et al., Nature, 425, 937, 2003

Building a Quantum Gate



Input state	Final state
$\|0\rangle\|0\rangle$	$\|0\rangle\|0\rangle$
$\|0\rangle\|1\rangle$	$\|0\rangle\|1\rangle$
$\|1\rangle\|0\rangle$	$\|1\rangle\|0\rangle \times e^{i\phi}$
$\|1\rangle\|1\rangle$	$\|1\rangle\|1\rangle$

Fundamental quantum gate for neutral atoms

D. Jaksch et al., PRL 82,1975 (1999)

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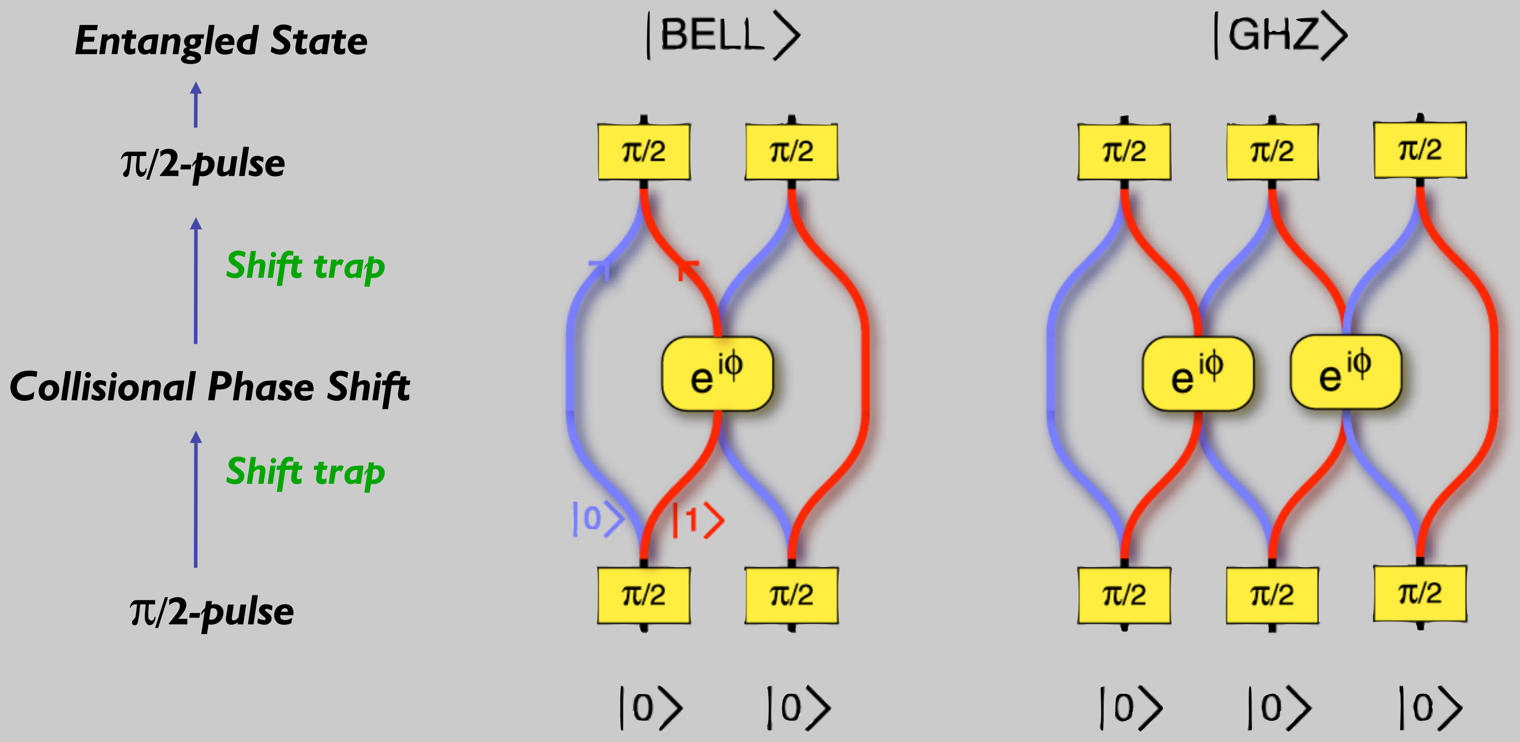
Controlled Collisions Entanglement



With N atoms one obtains maximally entangled cluster states from such a sequence !

$$U(\phi) = \exp\left(-i\phi \sum_j \frac{1+\sigma_z^{(j)}}{2}\,\frac{1-\sigma_z^{(j+1)}}{2}\right)$$

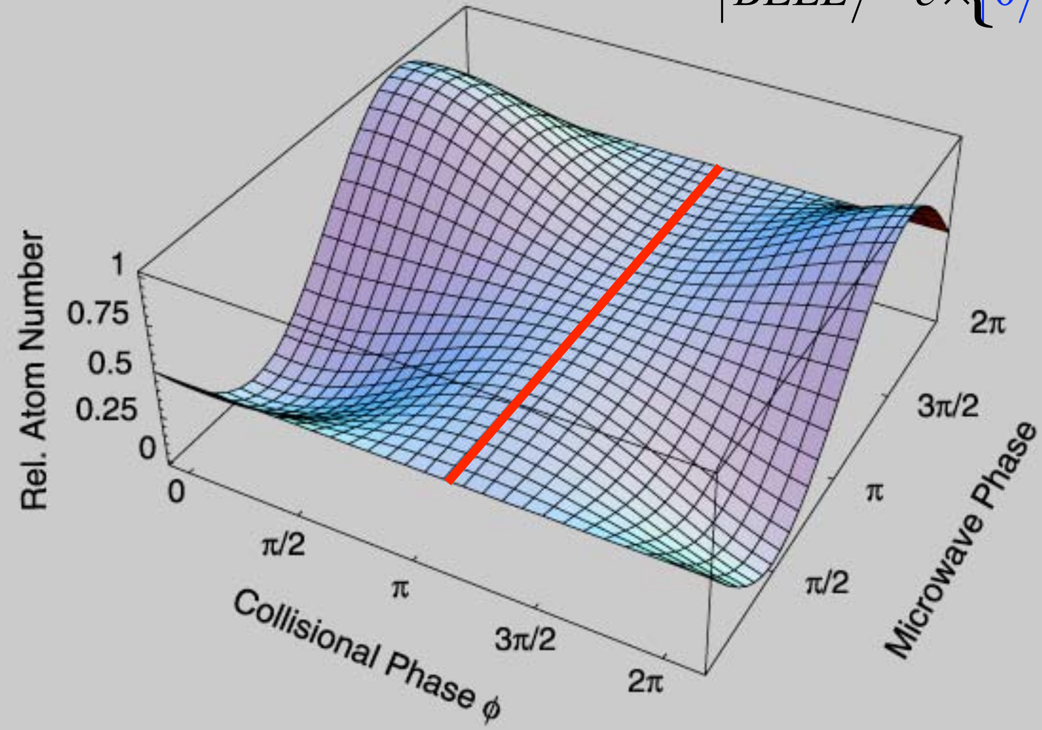
D. Jaksch et al., PRL 82,1975 (1999), A: Sorensen & K. Molmer, PRL 83, 2274 (1999)
H.-J. Briegel & R. Raussendorf PRL 86, 910 (2001) & PRL 86, 5188 (2001),
Experiment: O. Mandel et al., Nature, 425, 937, 2003

Collapse and Revival of the Ramsey fringe

$$\phi = \pi \Rightarrow \psi = |\mathrm{BELL}\rangle$$

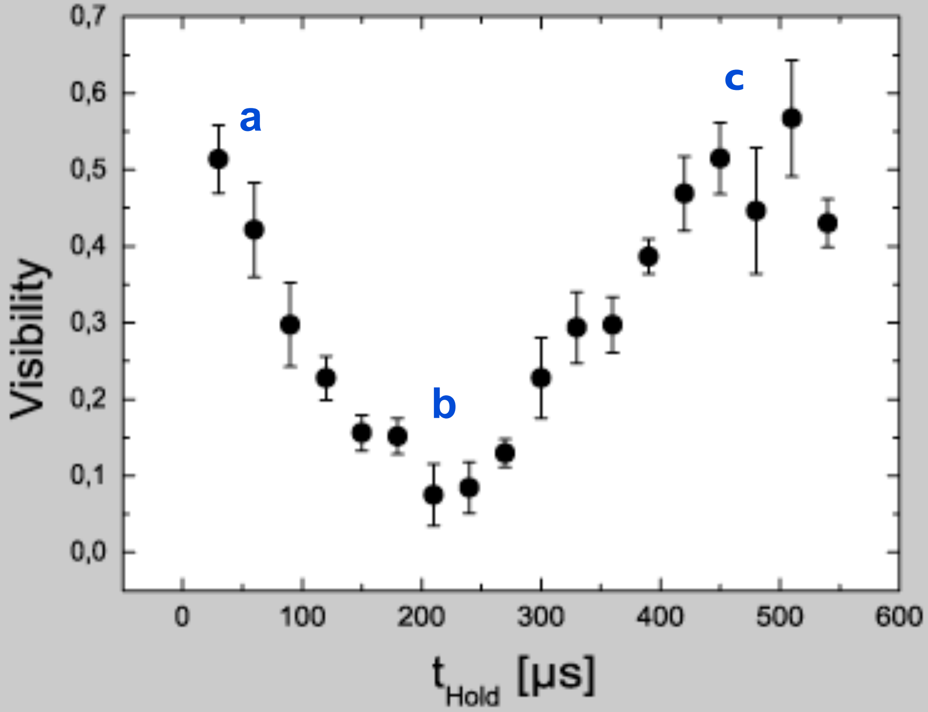
$$|BELL\rangle = c \times \{|0\rangle \otimes (|0\rangle - |1\rangle) + |1\rangle \otimes (|0\rangle + |1\rangle)\}$$

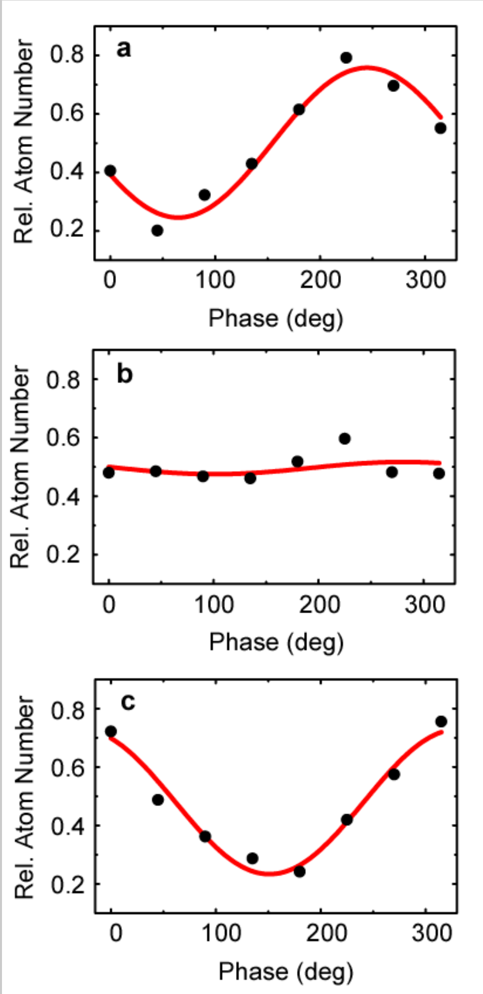
One atom per site



D. Jaksch, PhD-Thesis, Innsbruck

Entanglement Dynamics I





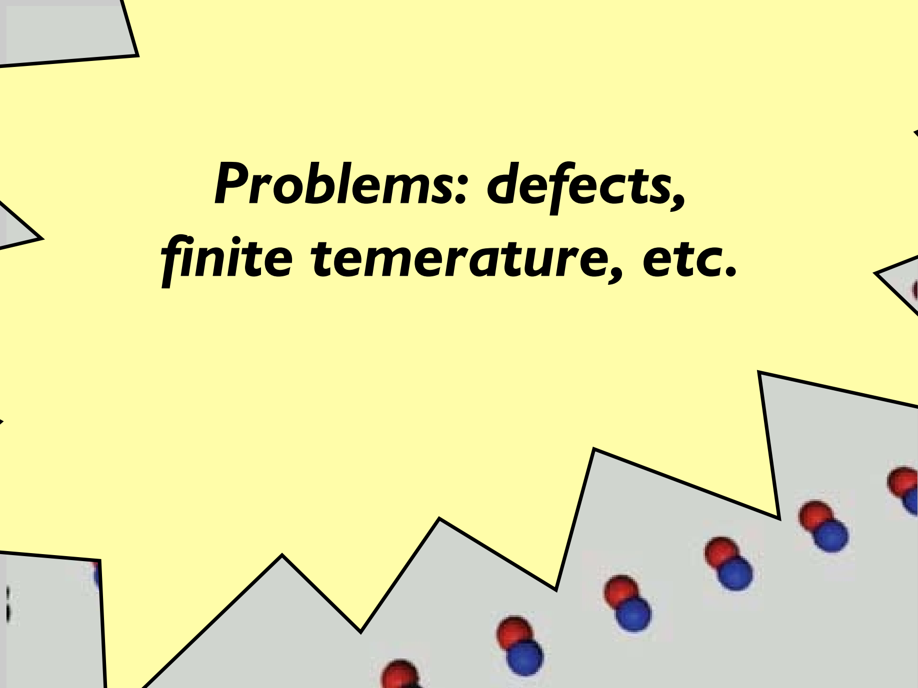
The gate operation does not cause a loss of visibility!

1D/2D Controlled Collisions

Experiments here: *3600 copies of one dim onal arrays of 60 ato*

Straightforward extension: *2D o tan r s with o 216.000*

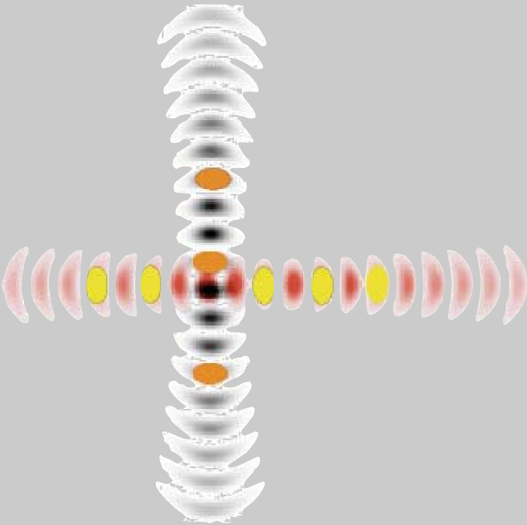
trapped atoms !!!

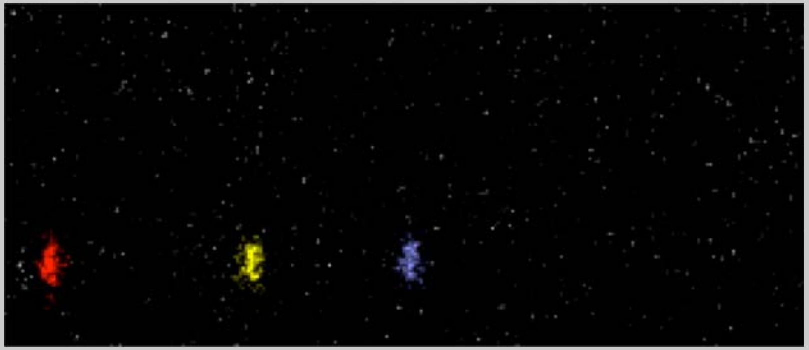


Problems: defects,
finite temerature, etc.

H.-J. Briegel & R. Raussendorf PRL 86, 910 (2001) & PRL 86, 5188 (2001).

Positioning Neutral Atoms



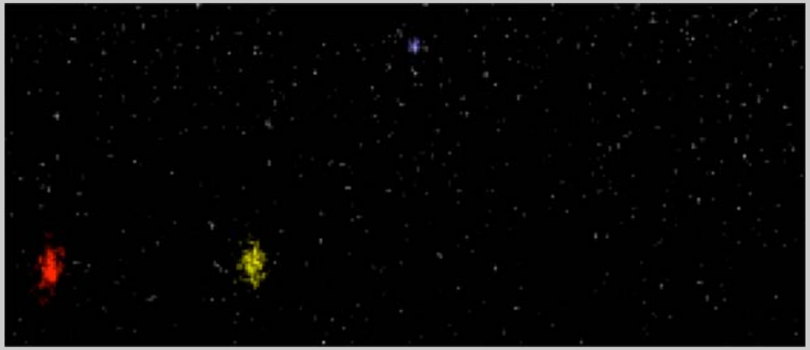


Positioning and Transport
with Submicron Precision

Phys. Rev. Lett. **95**, 033002 (2005)

D. Meschede & A. Rauschenbeutel
(University of Bonn)

Positioning Neutral Atoms



Positioning and Transport
with Submicron Precision
Phys. Rev. Lett. **95**, 033002 (2005)

D. Meschede & A. Rauschenbeutel

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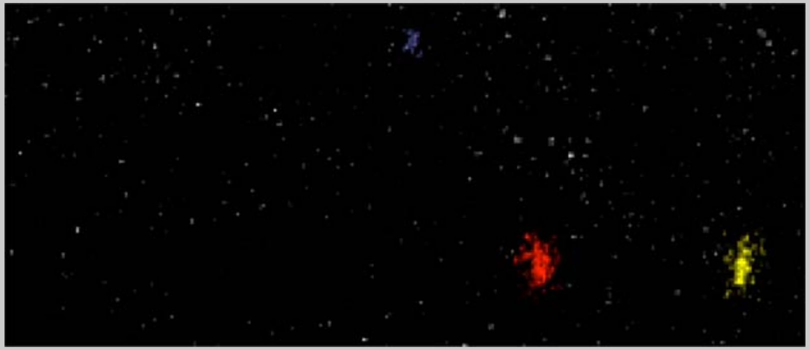
Positioning Neutral Atoms



Positioning and Transport
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Phys. Rev. Lett. **95**, 033002 (2005)

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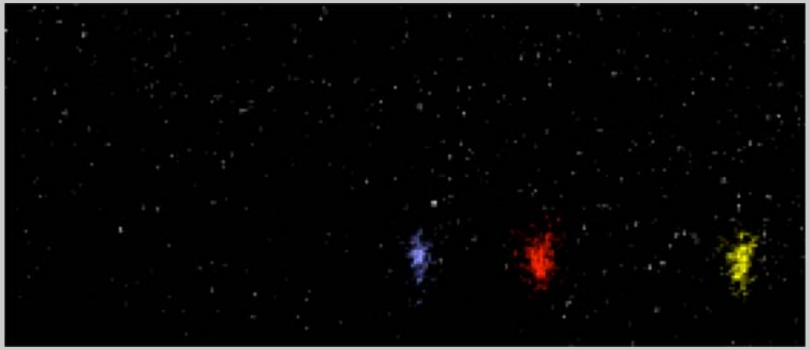
Positioning Neutral Atoms



Positioning and Transport
with Submicron Precision

Phys. Rev. Lett. **95**, 033002 (2005)

D. Meschede & A. Rauschenbeutel

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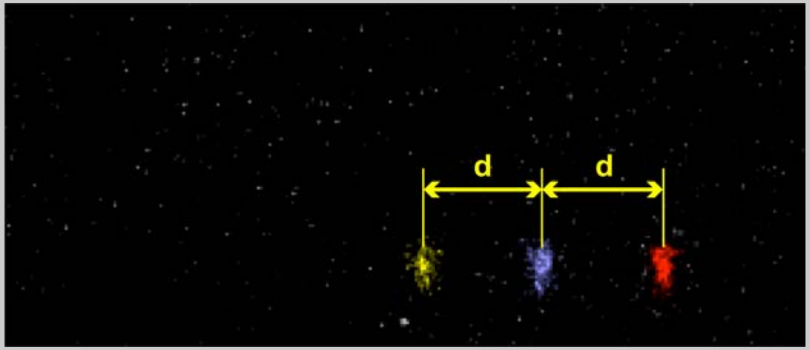
Positioning Neutral Atoms

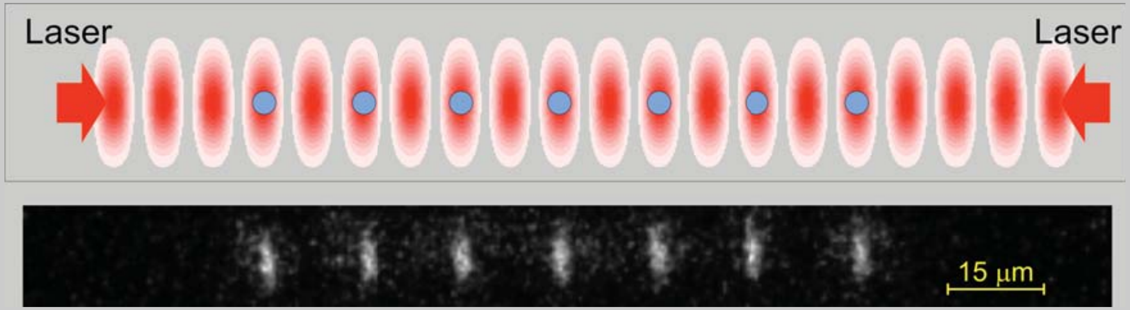


Positioning and Transport
with Submicron Precision

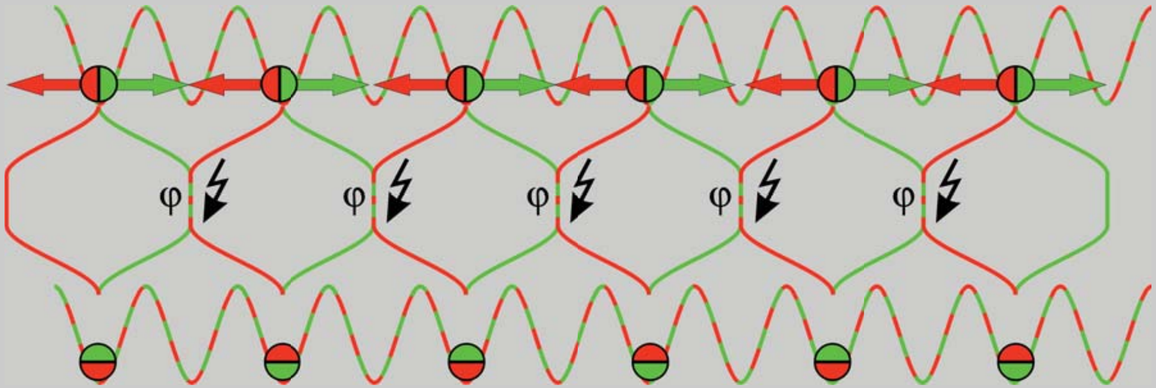
Phys. Rev. Lett. **95**, 033002 (2005)

D. Meschede & A. Rauschenbeutel

Adressable Cluster States



1. ***Sort***
2. ***Sideband Cooling***
3. ***Controlled Collisions***



From D. Meschede & A. Rauschenbeutel
(University of Bonn)

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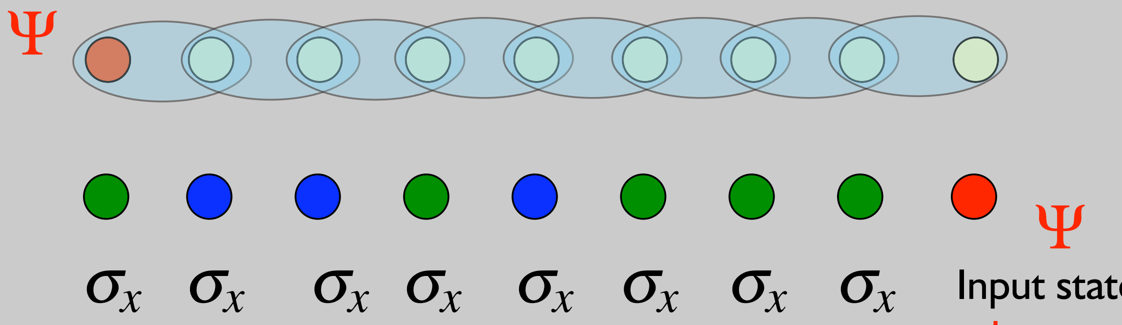
Properties of Cluster States

Cluster states are ressource states for quantum computing!

Generation e.g. by: $U(\phi) = \exp\left(-i\phi \sum_j \frac{1+\sigma_z^{(j)}}{2} \frac{1-\sigma_z^{(j+1)}}{2}\right)$ $\phi = \pi, 3\pi, 5\pi, \ldots$

$$|\phi_N\rangle = \frac{1}{2^{N/2}} \bigotimes_{j=1}^{N} \left(|0\rangle_j \sigma_z^{(j+1)} + |1\rangle_j\right)$$

Moving Information through a Cluster state:



Input state has been teleported through the cluster!

R. Raussendorf & H. Briegel PRL **86**, 5188 (2001)

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One-Way Quantum Computing

- ***Quantum Computing with Universal Resource States***
- ***Single Site Adressability Crucial***
- ***Search for Novel Universal Resource States***
- ***Lifetime of Large Entangled Multi Particle States***
- ***Decoherence***

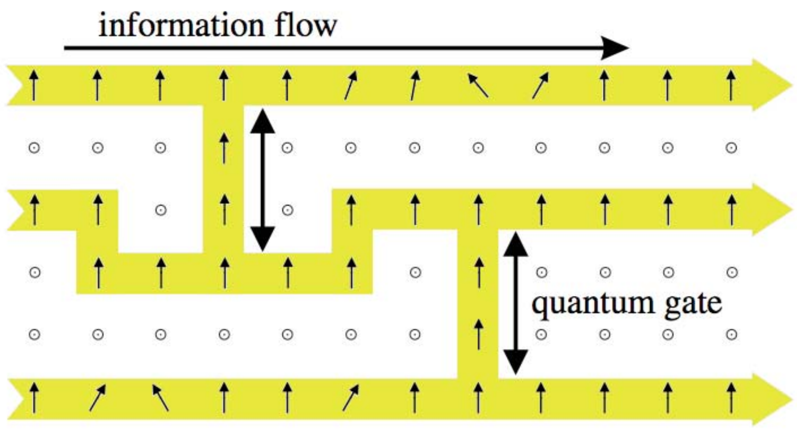
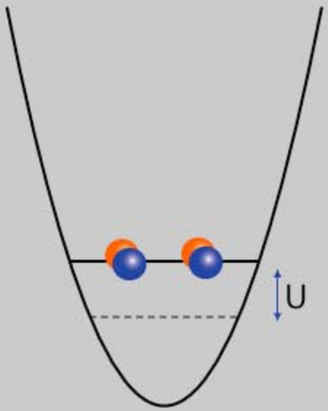


FIG. 1. Quantum computation by measuring two-state particles on a lattice. Before the measurements the qubits are in the cluster state $|\Phi\rangle_C$ of (1). Circles $\odot$ symbolize measurements of σ_z, vertical arrows are measurements of σ_x, while tilted arrows refer to measurements in the x-y plane.

R. Raussendorf & H.J. Briegel, PRL **86**, 5188 (2001)
Experiments with Photons, see e.g.:
P. Walther et al., Nature **434**, 169 (2005)

Controlling the effective interaction

Two atoms in a coherent superposition of the internal states:



$$\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) \otimes \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

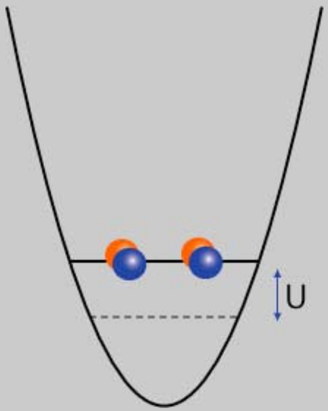
starting point

$$\frac{1}{2}\left(e^{i\phi_{00}}|00\rangle + e^{i\phi_{01}}|01\rangle + e^{i\phi_{10}}|10\rangle + e^{i\phi_{11}}|11\rangle\right)$$

$$\phi_{i,j} = t \times \frac{4\pi \times \hbar^2 \times a_{i,j}}{m} \int d^3x \, |w_i(x)|^2 \, |w_j(x)|^2$$

Controlling the effective interaction

Two atoms in a coherent superposition of the internal states:



$$\frac{1}{\sqrt{2}}\left(e^{i\phi_{00}}\left|00\right\rangle + e^{i\phi_{01}}\left|01\right\rangle + e^{i\phi_{10}}\left|10\right\rangle + e^{i\phi_{11}}\left|11\right\rangle\right)$$

$$\phi_{i,j} = t \times \frac{4\pi \times \hbar^2 \times a_{i,j}}{m} \int d^3x \left|w_i(x)\right|^2 \left|w_j(x)\right|^2$$

Entanglement evolution for

$$\chi \cdot t = \phi_{00} + \phi_{11} - 2\phi_{01} \neq 0$$

A. Sørensen et al., Nature 409, 63 (2001),
A. Micheli et al. PRA 67, 013607 (2003)

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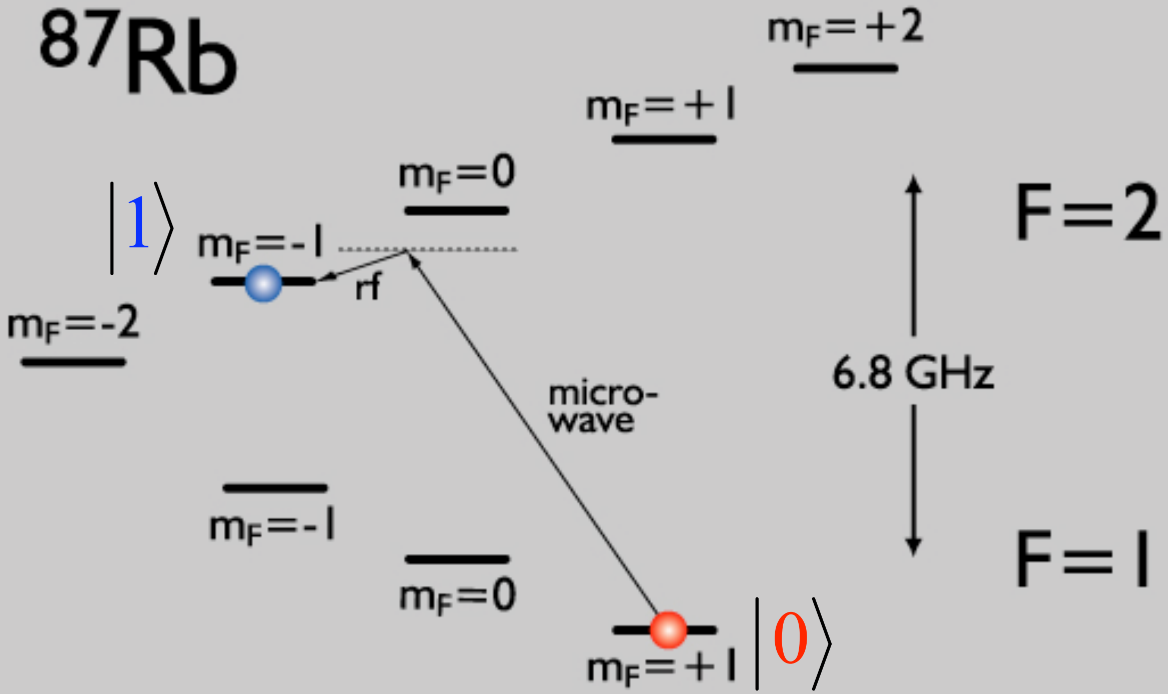
State preparation



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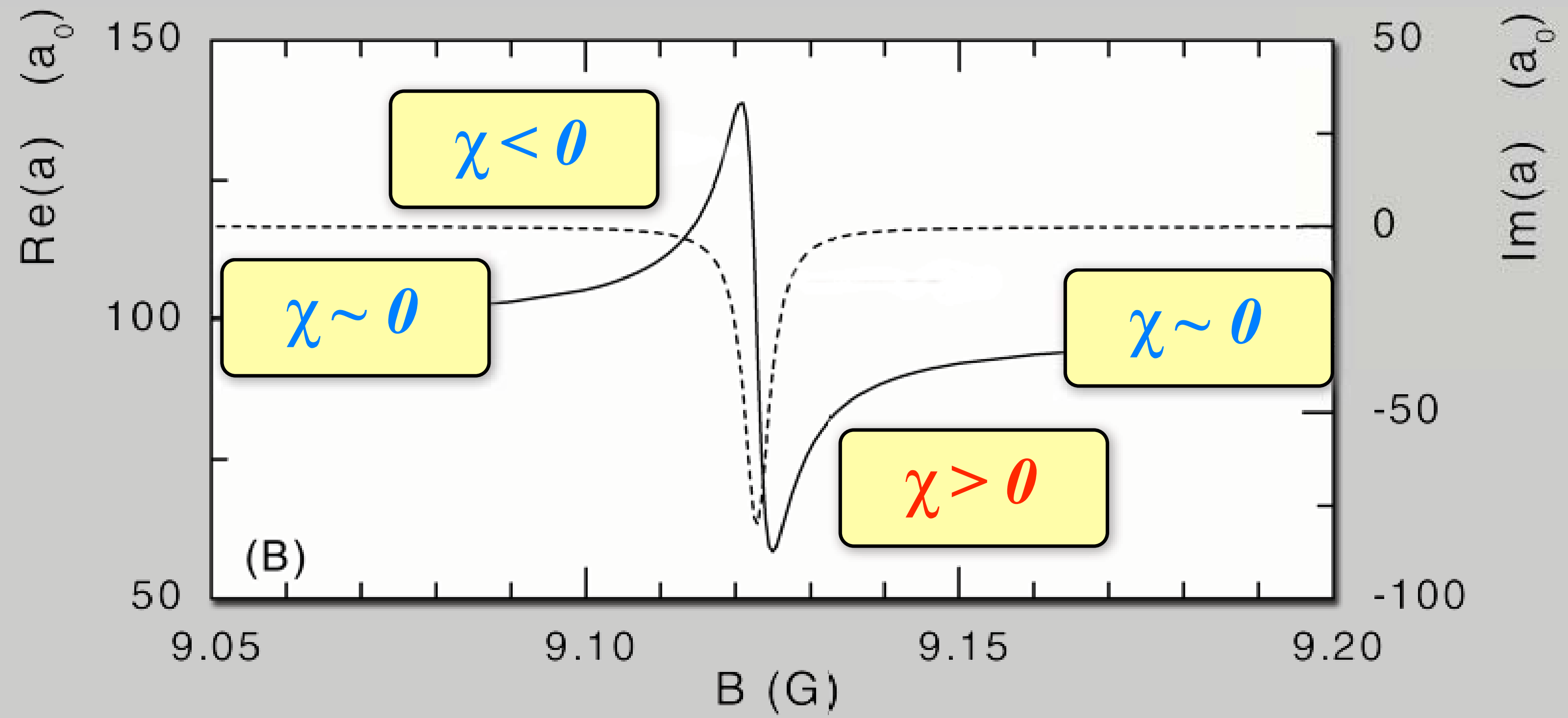
Hyperfine Feshbach resonance for ^{87}Rb

$$|F=1, m_F=+1\rangle + |F=2, m_F=-1\rangle:$$



Extracted from: E.G.M. van Kempen et. al, PRL 88 , 093201 (2002)

see also M. Erhard et al., PRA 69, 032705 (2004)

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Two atoms per site

$$|0\rangle \otimes |0\rangle$$

Two atoms per site

$$|0\rangle \otimes |0\rangle \xrightarrow{\pi/2} \frac{1}{2}\left(|0\rangle + |1\rangle\right) \otimes \left(|0\rangle + |1\rangle\right)$$

Two atoms per site

$$|0\rangle \otimes |0\rangle \xrightarrow{\pi/2} \frac{1}{2}\left(|0\rangle + |1\rangle\right) \otimes \left(|0\rangle + |1\rangle\right)$$

$$\frac{1}{2}\left(e^{i\phi_{00}}|0\rangle|0\rangle + e^{i\phi_{01}}|0\rangle|1\rangle + e^{i\phi_{10}}|1\rangle|0\rangle + e^{i\phi_{11}}|1\rangle|1\rangle\right)$$

$$\phi_{ij} \equiv \frac{4\pi\,\hbar\,a_{ij}}{m_{Rb}} \cdot t_{WW} \cdot \int |\psi_i|^2 |\psi_j|^2\, d^3x$$

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Two atoms per site

$$|0\rangle \otimes |0\rangle \xrightarrow{\pi/2} \frac{1}{2}\left(|0\rangle + |1\rangle\right) \otimes \left(|0\rangle + |1\rangle\right)$$

$$\frac{1}{2}\left(e^{i\phi_{00}}|0\rangle|0\rangle + e^{i\phi_{01}}|0\rangle|1\rangle + e^{i\phi_{10}}|1\rangle|0\rangle + e^{i\phi_{11}}|1\rangle|1\rangle\right)$$

$$\begin{aligned} \phi_{01} &= \phi_{10} \\ \phi_{00} &\approx \phi_{11} \\ \Delta\phi &= \phi_{00} - \phi_{10} \end{aligned}$$

Global phase

$$\Rightarrow \frac{1}{2} e^{i\phi_{00}} \left(|0\rangle|0\rangle + e^{i\Delta\phi}|0\rangle|1\rangle + e^{i\Delta\phi}|1\rangle|0\rangle + |1\rangle|1\rangle\right)$$

Two atoms per site

$|0\rangle \otimes |0\rangle$ → $\pi/2$ → $\frac{1}{2}\left(|0\rangle + |1\rangle\right) \otimes \left(|0\rangle + |1\rangle\right)$

$$\frac{1}{2}\left(e^{i\phi_{00}}|0\rangle|0\rangle + e^{i\phi_{01}}|0\rangle|1\rangle + e^{i\phi_{10}}|1\rangle|0\rangle + e^{i\phi_{11}}|1\rangle|1\rangle\right)$$

$$\frac{1}{2}\left(|0\rangle|0\rangle + e^{i\Delta\phi}|0\rangle|1\rangle + e^{i\Delta\phi}|1\rangle|0\rangle + |1\rangle|1\rangle\right)$$

$\pi/2$ (Phase α) →

$$\frac{1}{2}\left[|0\rangle|0\rangle \cdot \left(1 + e^{i\Delta\phi}\right) + |1\rangle|1\rangle \cdot \left(1 - e^{i\Delta\phi}\right)\right]$$

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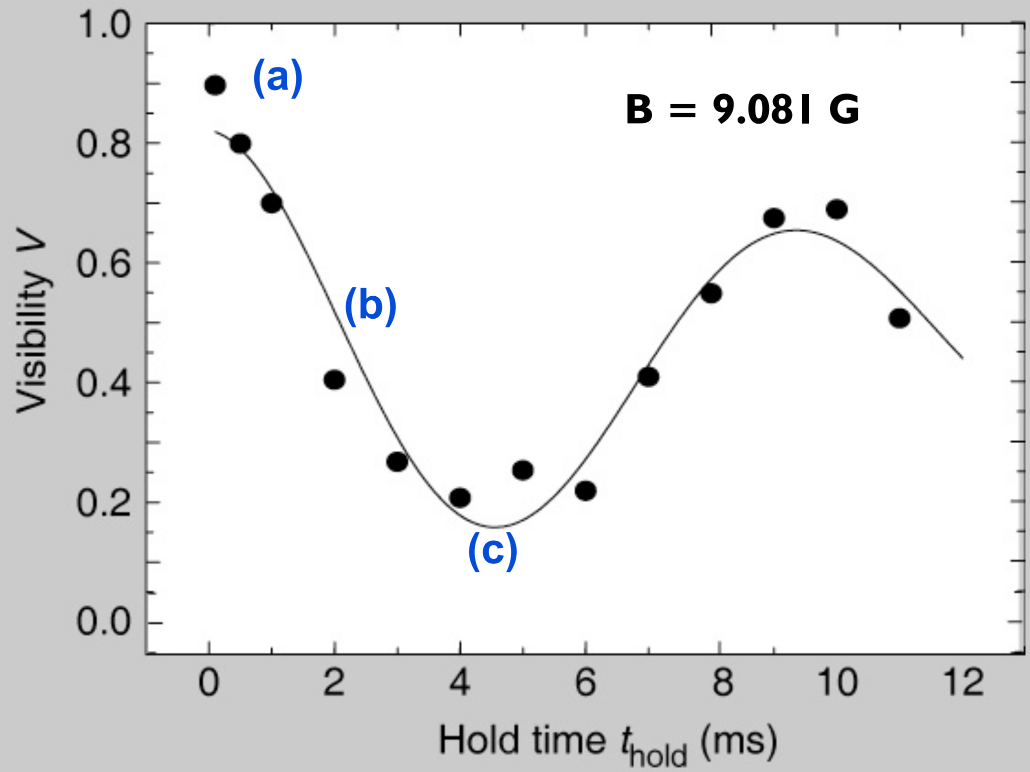
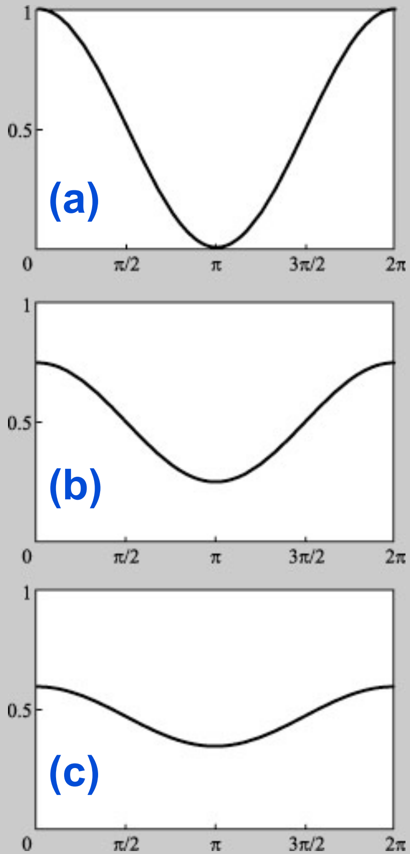
Experimental results

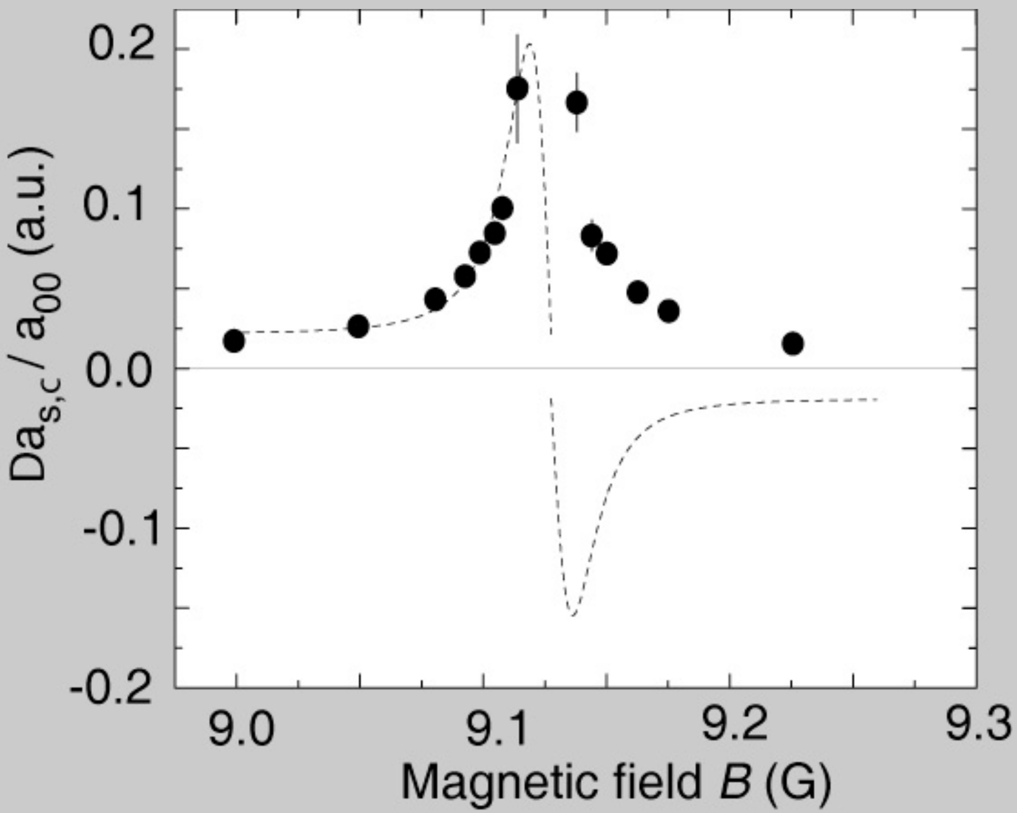




System shows entanglement oscillations!

A. Widera et al., PRL **92**, 160406 (2004)

Measurement of elastic channel

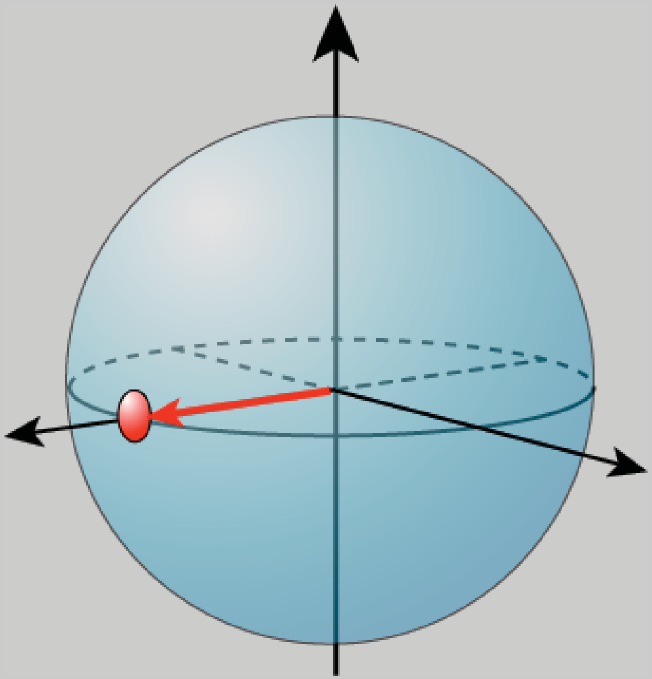


$$\Delta a_{s,\chi} = -\frac{1}{2}\left(a_{00} + a_{11} - 2a_{01}\right)$$

A. Widera et al., PRL **92**, 160406 (2004)

Nonlinear Quantum Spin Dynamics in Bose-Einstein Condensates

From Spin Squeezing to Schrödinger Cats
- Nonlinear Quantum Spin Dynamics -



What happens if you tune interactions in larger ensembles?

$$\left(\hat{a}^{\dagger}+\hat{b}^{\dagger}\right)^{\otimes N}|0\rangle$$

$$\hat{H}=\chi\hat{S}_z^2$$

$$\chi = a_{aa}+a_{bb}-2a_{ab}$$

A. Sørensen et al., Nature 409, 63 (2001),
L. You, PRA (2002)
A. Micheli et al. PRA 67, 013607 (2003)

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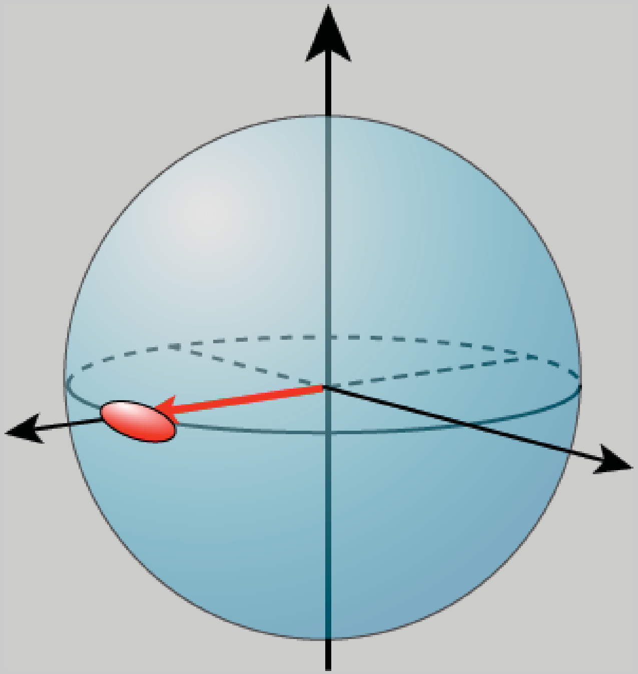
Short timescale evolution



Spin squeezing! (see exp. E. Polzik)

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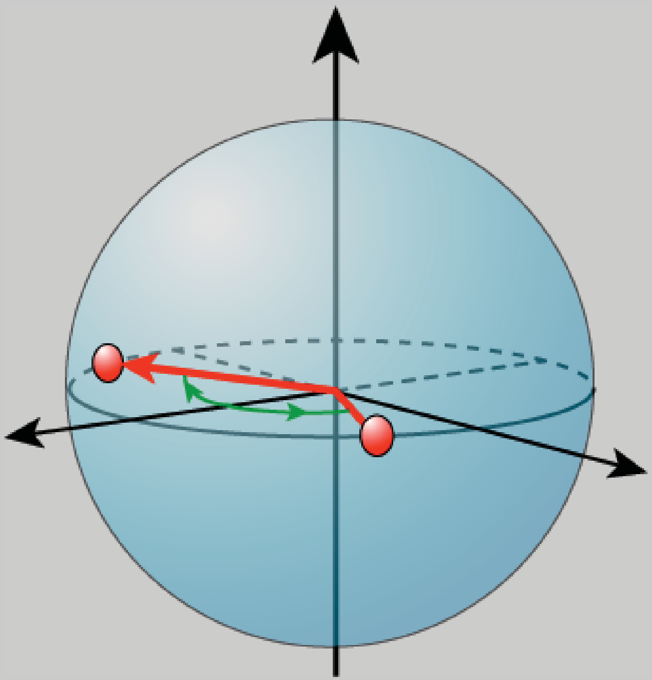
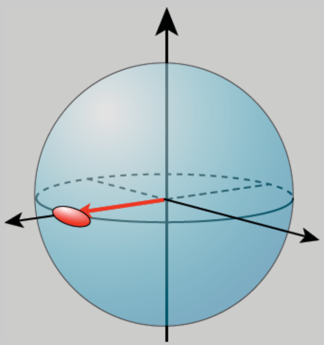
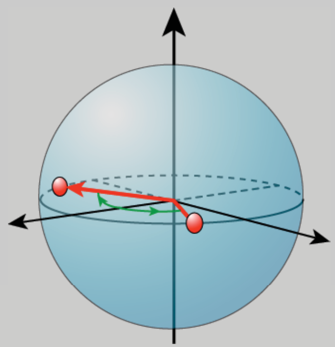
On longer timescales a Schrödinger cat state forms

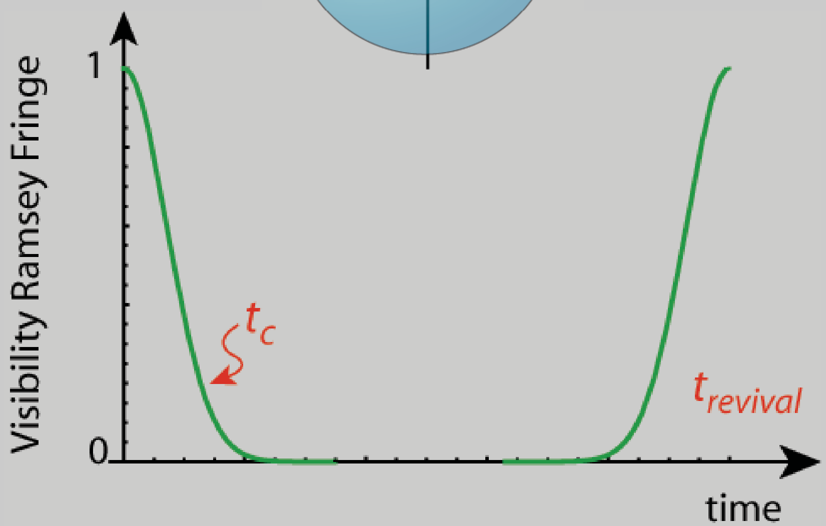


Ramsey Fringe Visibility Evolution

π/2 → ***Tune interactions and wait*** → π/2

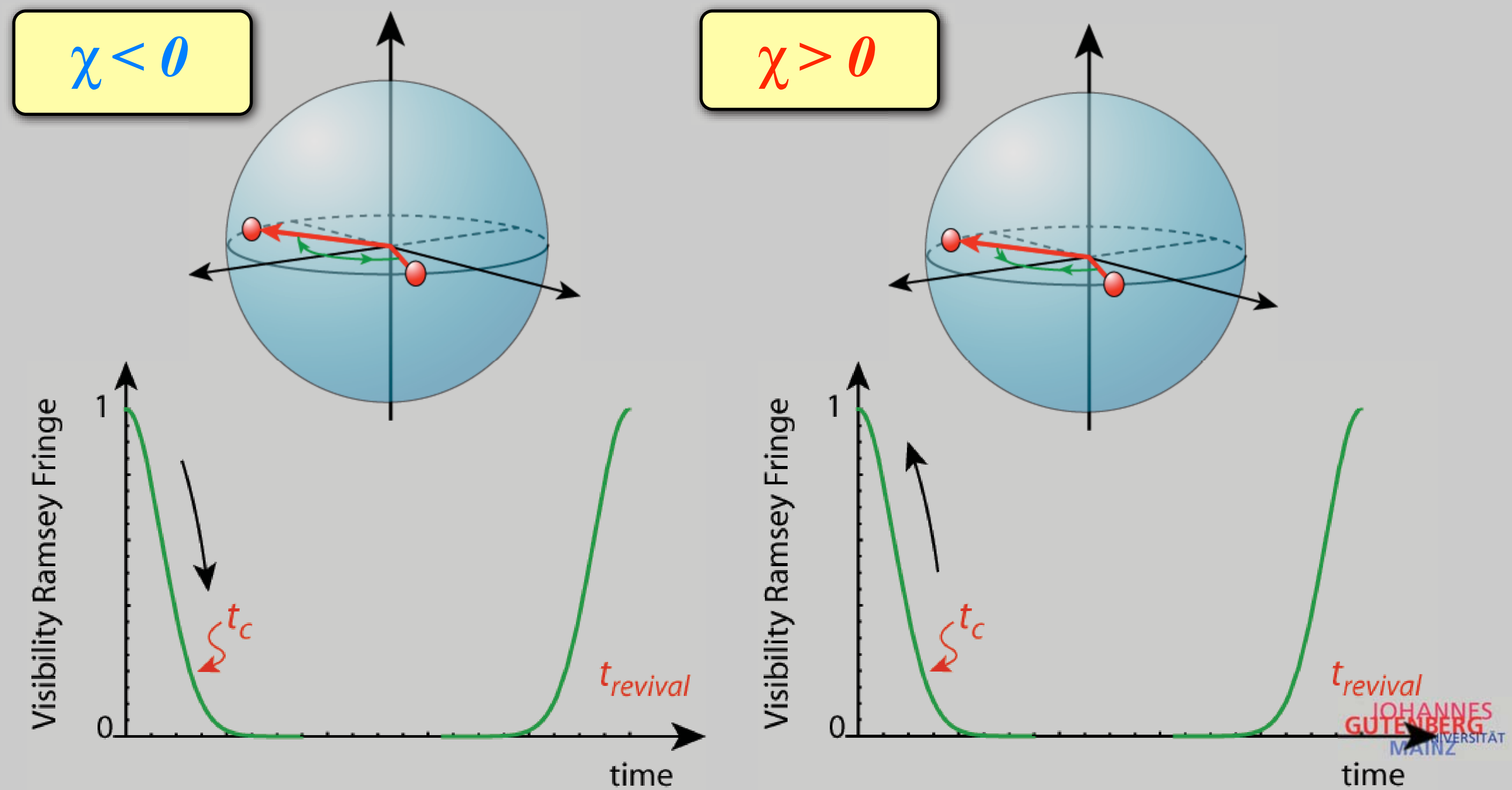






Decoherence Problems

Full revival can so far not be reached due to decoherence, but time evolution can be reversed!



Proposal:

E. Altman, E. Demler & M. Lukin PRA (2004)

A. Polkovnikov et al., PNAS (2006)

Experiment:

Fölling et al., Nature (2005),

Greiner et al., PRL (2005)

Rom et al., Nature (2006)

related work:

Bach & Rzazewski, PRA (2004)

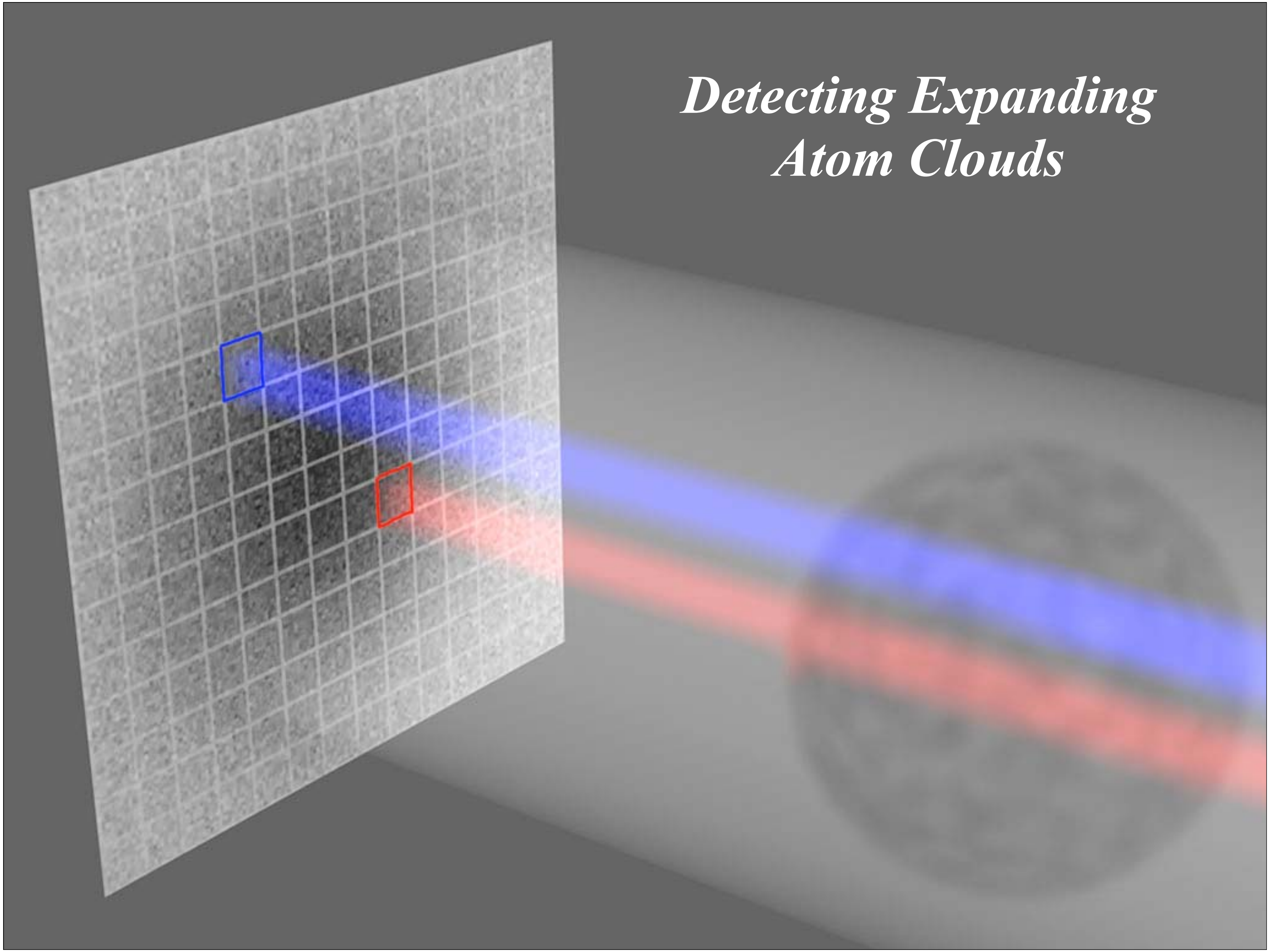
Z. Hadzibabic et al. PRL (2004),

Yasuda & Shimizu, PRL (1996),

Schellekens et al., Science (2005),

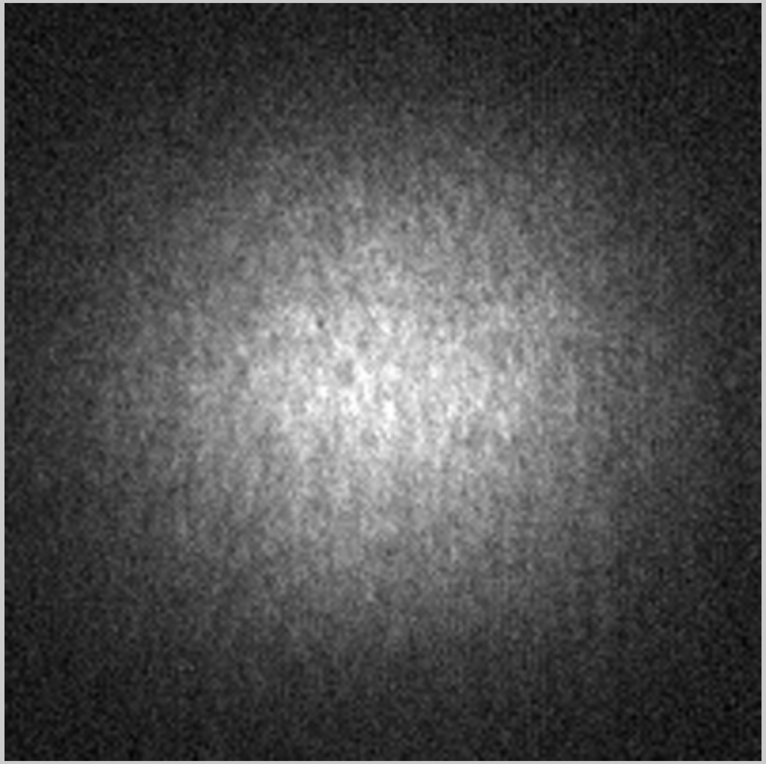
Öttl et al., PRL (2005),

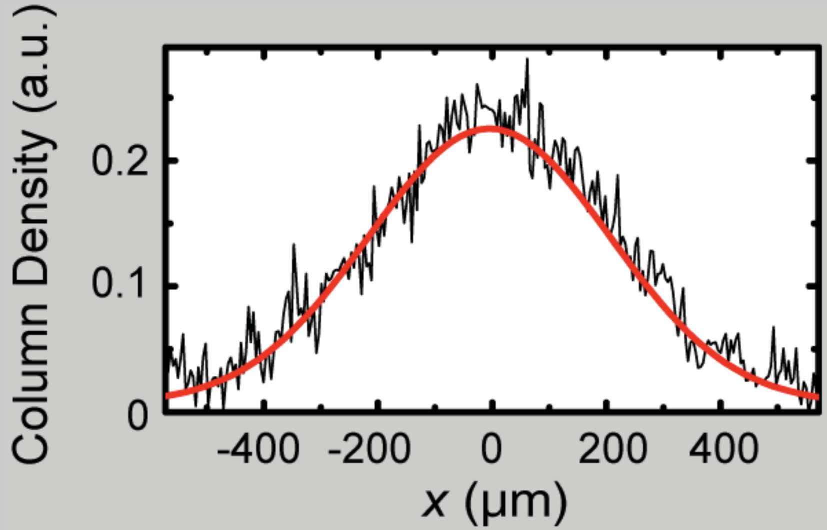
Estève et al., PRL (2006)


Detecting Expanding
Atom Clouds

Typically Noise in Images of a Mott Insulator

Single Image

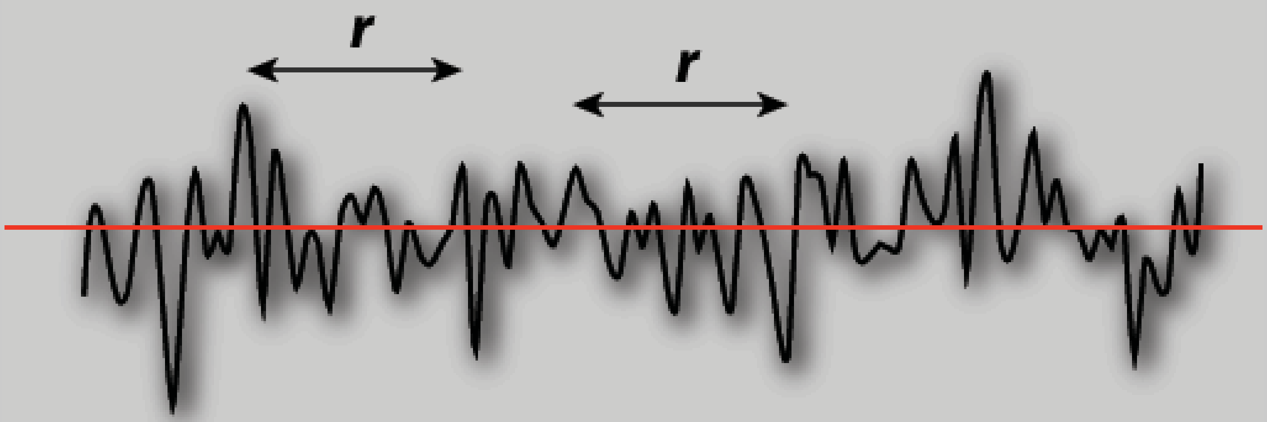




Fluctuations due to Atomic Shot Noise

$$\sigma \sim \sqrt{N_{bin}}$$

Corrleations in Noise?



Hanbury-Brown Twiss effect correlates fluctuations at special distances r!

Quantitatively

$g^{(2)}(r)-1>0$	*Noise correlated (Bosons)*
$g^{(2)}(r)-1=0$	*Noise uncorrelated*
$g^{(2)}(r)-1<0$	*Noise anti-correlated (Fermions)*

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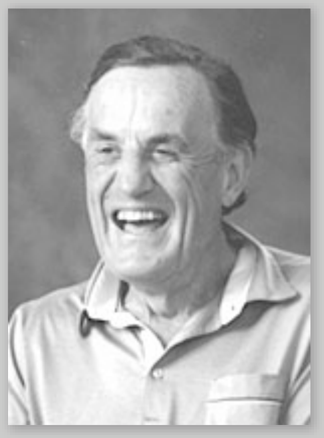
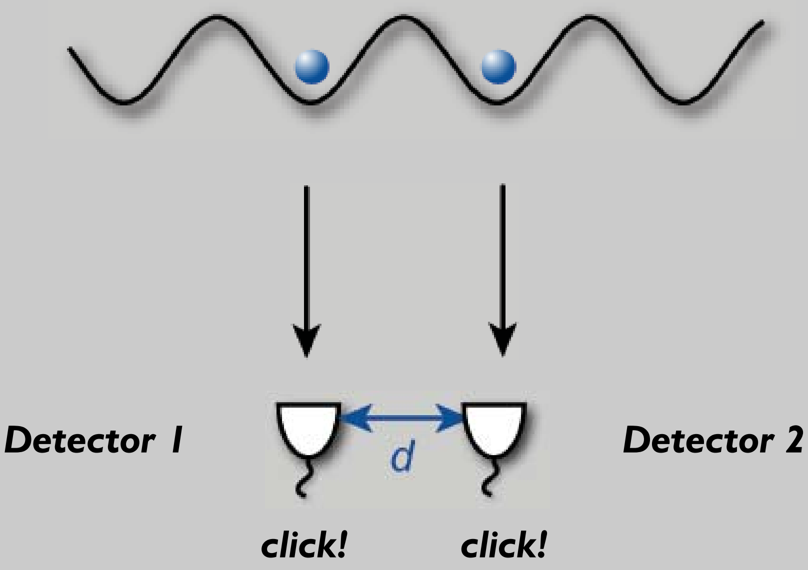
- Hanbury Brown-Twiss Effect for Atoms (1) -

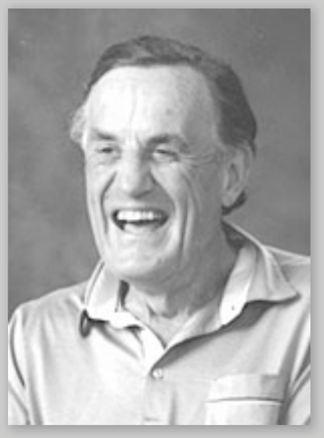


Hanbury Brown
1916-2002

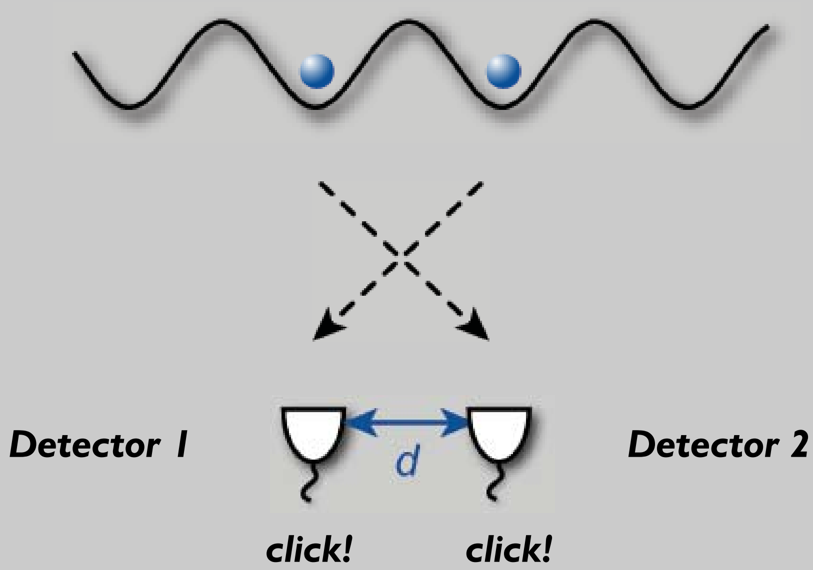


- Hanbury Brown-Twiss Effect for Atoms (2) -

There's another ways....



Hanbury Brown
1916-2002



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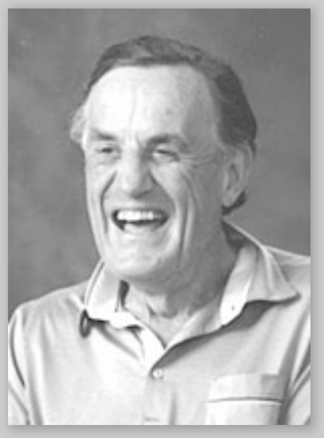
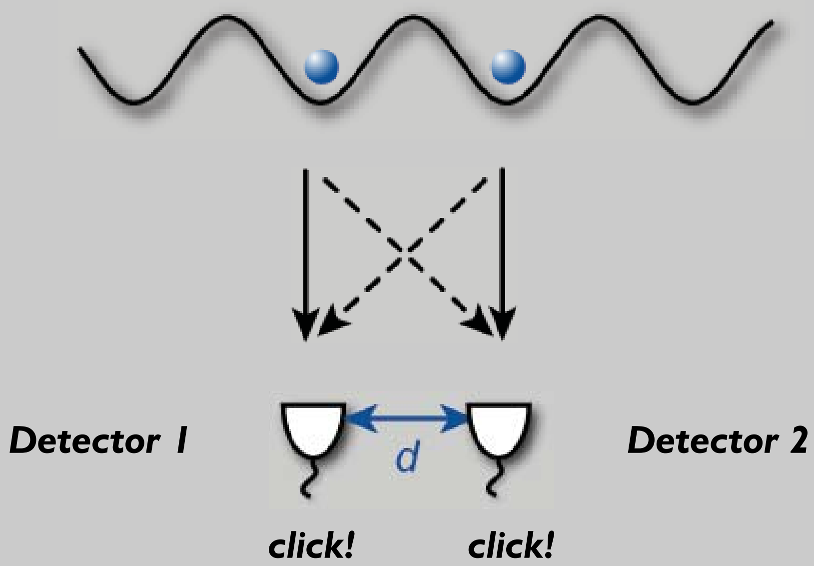
- Hanbury Brown-Twiss Effect for Atoms (3) -

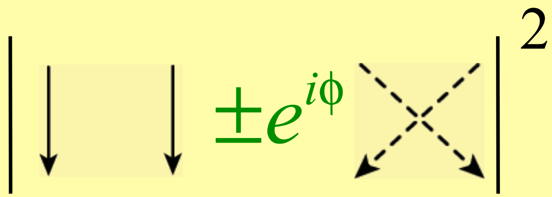
Cannot fundamentally distinguish between both paths...



Hanbury Brown
1916-2002

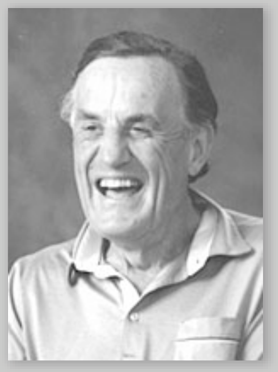


Two Particle Detection probability

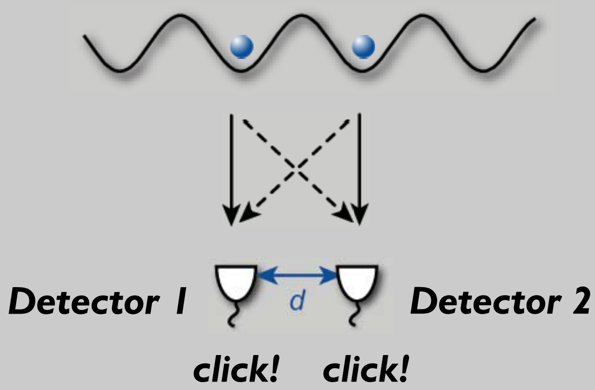


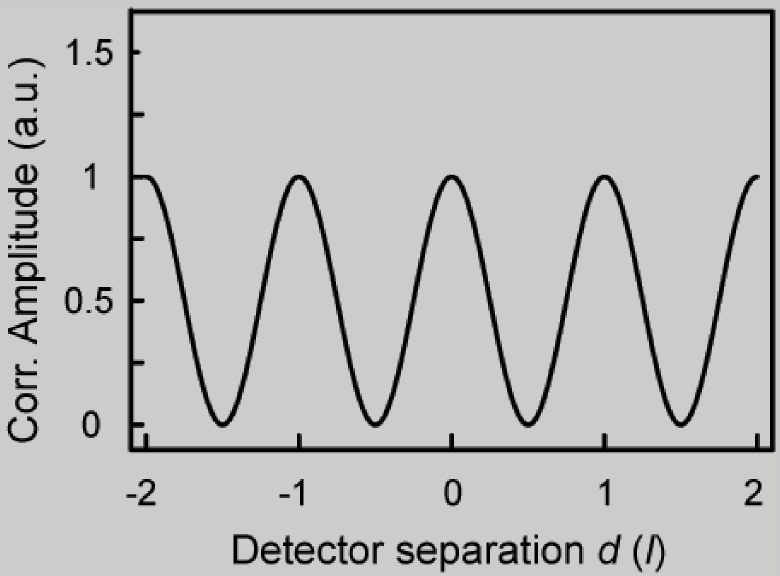
- Hanbury Brown-Twiss Effect for Atoms (4) -

Interference in Two-Particle Detection Probability!



Hanbury Brown
1916-2002

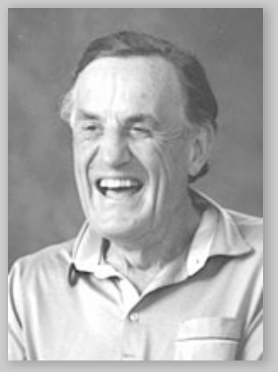




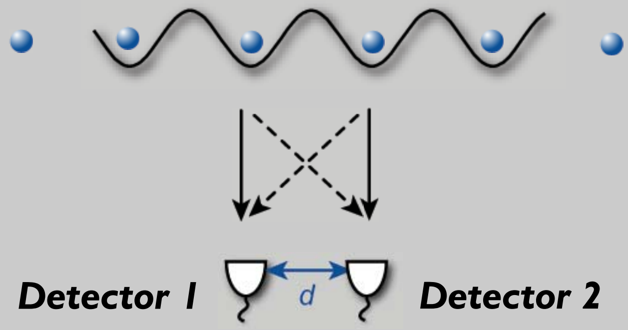
$$l = \frac{h}{m \times a_{lat}} t$$

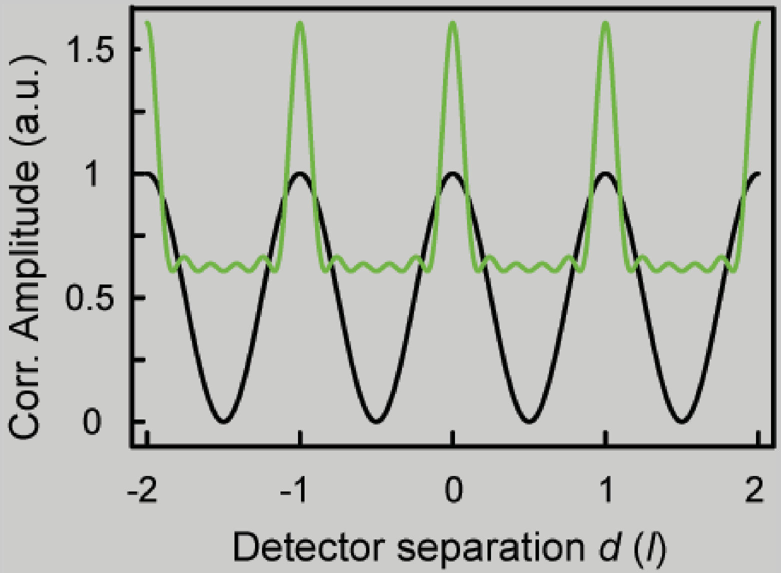
- Multiple Wave Hanbury Brown-Twiss Effect (4) -

Interference in Two-Particle Detection Probability!



Hanbury Brown
1916-2002





$$l = \frac{h}{m a_{lat} t}$$

Deriving the Noise Correlation Signal (1)

In Time of Flight we measure:

$$\begin{aligned}\langle \hat{n}_{3D}(\mathbf{x})\rangle_{\text{tof}} &= \langle \hat{a}^\dagger_{tof}(\mathbf{x})\hat{a}_{\text{tof}}(\mathbf{x})\rangle_{\text{tof}} \\ &\approx \langle \hat{a}^\dagger(\mathbf{k})\hat{a}(\mathbf{k})\rangle_{\text{trap}} = \langle \hat{n}_{3D}(\mathbf{k})\rangle_{\text{trap}}\end{aligned}$$

where

$$\mathbf{k} = M\mathbf{x}/\hbar t$$

In Noise Correlations we measure:

$$\begin{aligned}&\langle \hat{n}_{3D}(\mathbf{x})\hat{n}_{3D}(\mathbf{x}')\rangle_{\text{tof}} \approx \langle \hat{a}^\dagger(\mathbf{k})\hat{a}(\mathbf{k})\hat{a}^\dagger(\mathbf{k}')\hat{a}(\mathbf{k}')\rangle_{\text{trap}} = \\ &\langle \hat{a}^\dagger(\mathbf{k})\hat{a}^\dagger(\mathbf{k}')\hat{a}(\mathbf{k}')\hat{a}(\mathbf{k})\rangle_{\text{trap}} + \delta_{\mathbf{k}\mathbf{k}'}\langle \hat{a}^\dagger(\mathbf{k})\hat{a}(\mathbf{k})\rangle_{\text{trap}} \quad .\end{aligned}$$

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Deriving the Noise Correlation Signal (2)

$$\hat{a}(\mathbf{k}) = \int e^{-i\mathbf{k}\mathbf{r}}\hat{\psi}(\mathbf{r})d^3r$$ with $$\hat{\psi}(\mathbf{r}) = \sum_{\mathbf{R}} \hat{a}_{\mathbf{R}} w(\mathbf{r}-\mathbf{R})$$

⇒ $$\hat{a}(\mathbf{k}) = \tilde{w}(\mathbf{k}) \sum_{\mathbf{R}} e^{-i\mathbf{k}\mathbf{R}} \hat{a}_{\mathbf{R}}$$ Plug this into four operator correlator

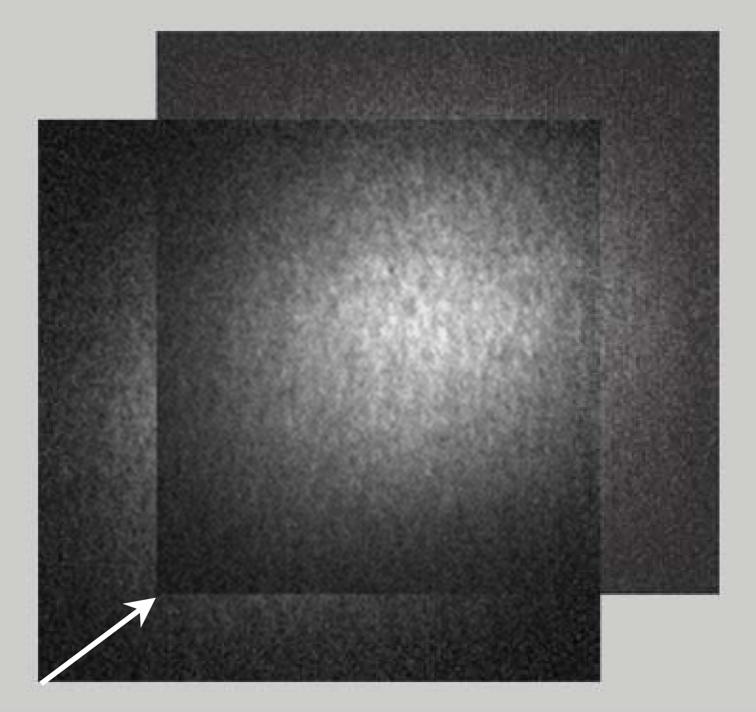
For Mott state or Fermi gas, one has $\langle \hat{a}^\dagger_{\mathbf{R}} \hat{a}_{\mathbf{R}'} \rangle = n_{\mathbf{R}}\, \delta_{\mathbf{R},\mathbf{R}'}$

which yields:

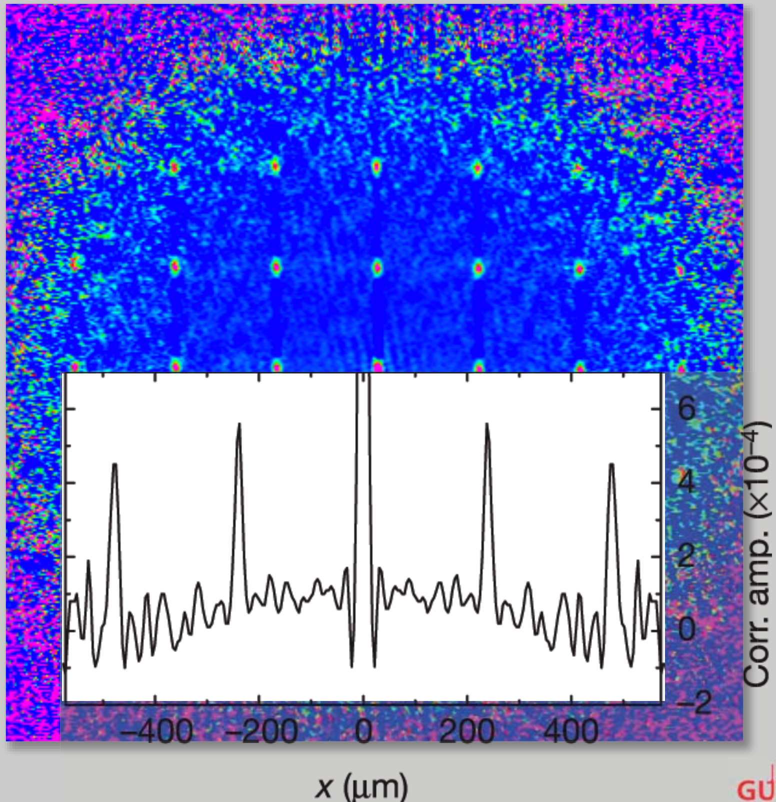
$$\langle \hat{n}_{3D}(\mathbf{x}) \hat{n}_{3D}(\mathbf{x}') \rangle = |\tilde{w}(M\mathbf{x}/\hbar t)|^2 |\tilde{w}(M\mathbf{x}'/\hbar t)|^2 N^2 \times \left[1 \pm \frac{1}{N^2} \left| \sum_{\mathbf{R}} e^{i(\mathbf{x}-\mathbf{x}')\cdot\mathbf{R}(M/\hbar t)} n_{\mathbf{R}} \right|^2 \right]$$

Information in the Noise – Correlations become visible!

$$g^{(2)}_{\exp}(\mathbf{b}) = \frac{\int \langle n(\mathbf{x}+\mathbf{b}/2)\cdot n(\mathbf{x}-\mathbf{b}/2)\rangle\, d^2\mathbf{x}}{\int \langle n(\mathbf{x}+\mathbf{b}/2)\rangle\langle n(\mathbf{x}-\mathbf{b}/2)\rangle\, d^2\mathbf{x}}$$



Fölling et al. Nature, 434, p. 481 (2005)



Correlation function!

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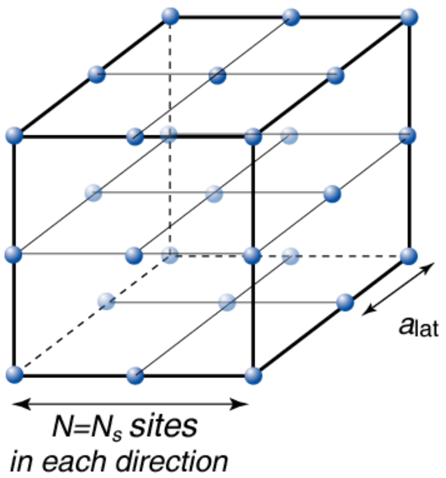
How large are the correlations ?



Coherence length :
also ideal peak width

$$L_{coh} \sim \frac{\hbar t}{m N_s a_{lat}} = \frac{l}{N_s}$$

Great spatial resolution :
fringe spacing l >> L_{coh} >> **res.**

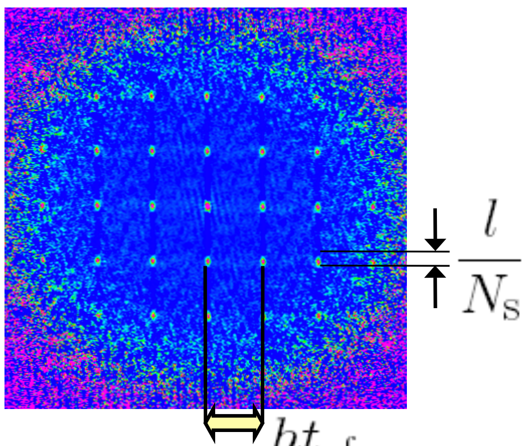
$$\mathcal{C}_{max} \approx 1 + 1$$

Poor spatial resolution :
res. >> fringe spacing l >> L

$$\mathcal{C}_{max} \approx 1 + \frac{1}{N_s^3}$$

Intermediate spatial resolution :
fringe spacing l >> **res.** >> L_{coh}

$$\mathcal{C}_{max} \approx 1 + \left(\frac{L_{coh}}{\mathrm{res}}\right)^3$$

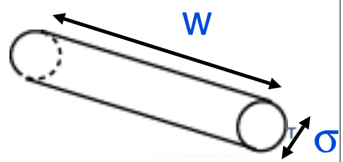


$$l = \frac{h t_{of}}{m a_{lat}}$$

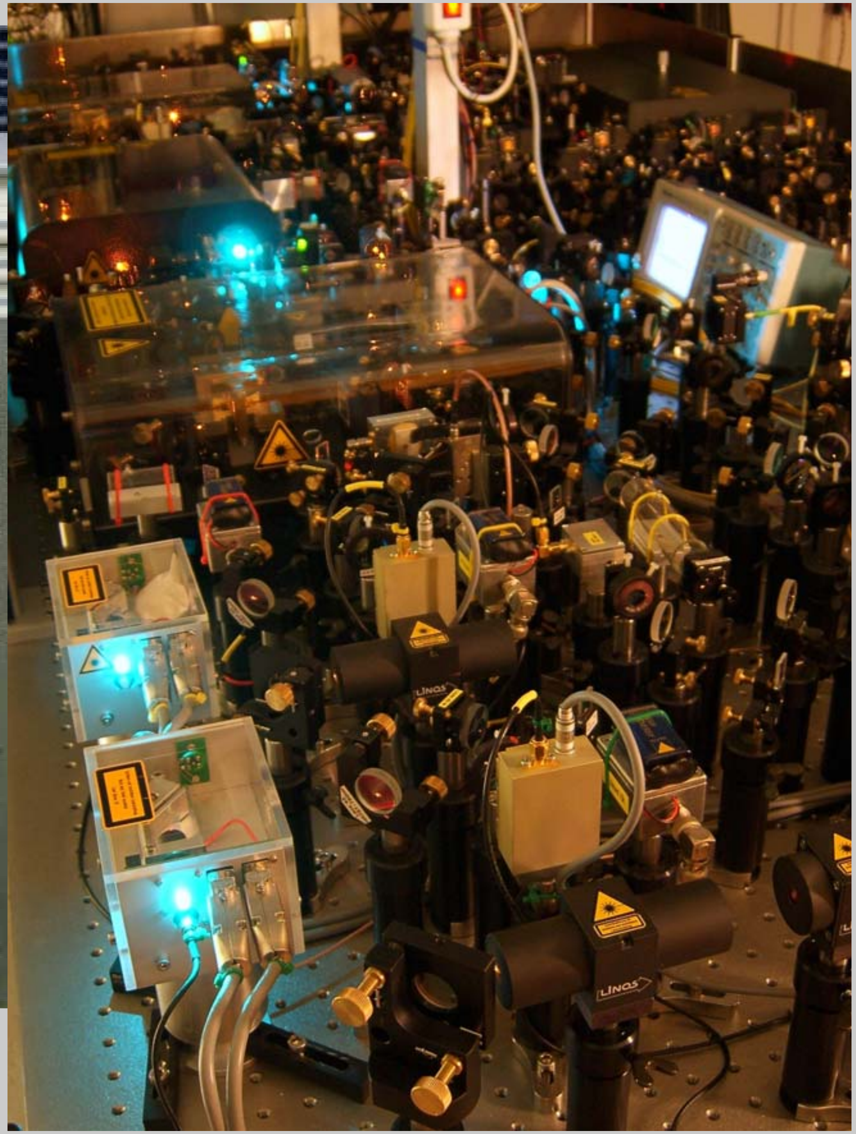
$$\mathcal{C}_{max} \approx 1 + \frac{1}{N_s} \cdot \frac{1}{N_s^2}\left(\frac{l}{\sigma}\right)^2$$

Imaging plane:
l >> σ > L_{coh}

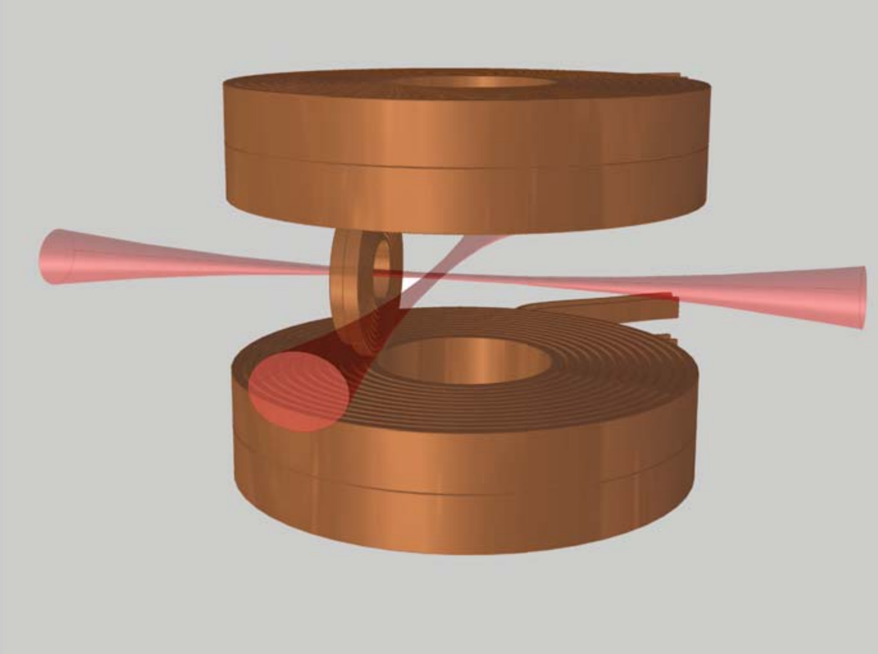
Probe direction : w >> l



Let’s change the sign…



Sympathetic Cooling of ^{40}K-^{87}Rb in Crossed Dipole Trap:



After final cooling in optical dipole trap

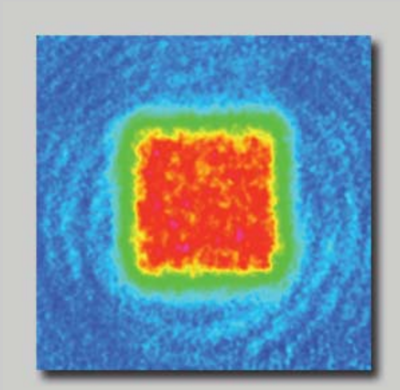
2×10^5 ^{87}Rb (almost pure condensate)

2.5×10^5 ^{40}K

After removal of ^{87}Rb

2×10^5 ^{40}K @ $T/T_F=0.2$

Then load into 3D optical lattice and create a fermionic band insulator!



Adiabatic mapping:
theory: A. Kastberg et al. PRL (1995)
exp: M. Greiner et al., PRL (2001), M. Köhl et al. PRL (2005)

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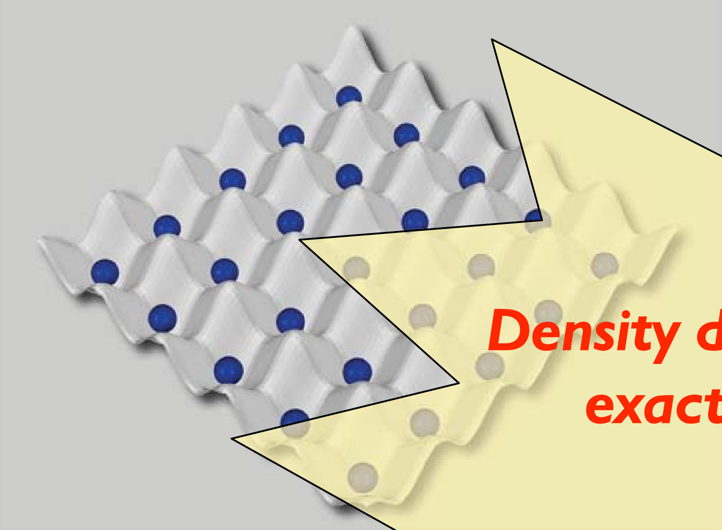
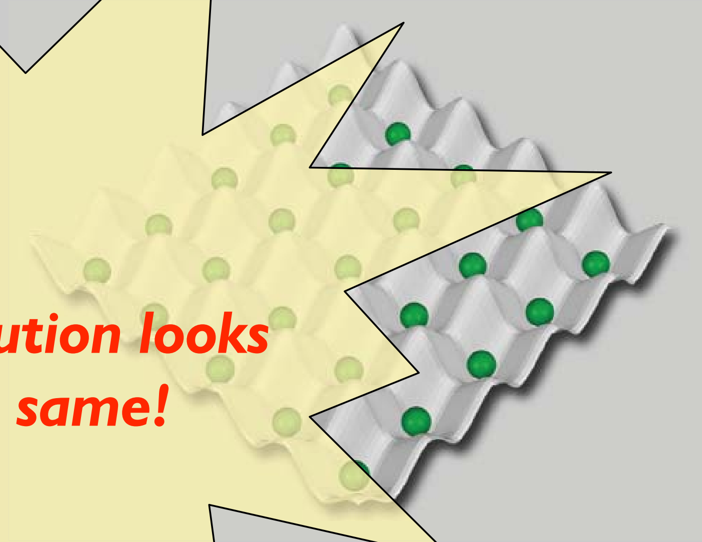
Mott insulator – Fermionic Band Insulator



Mott insulator

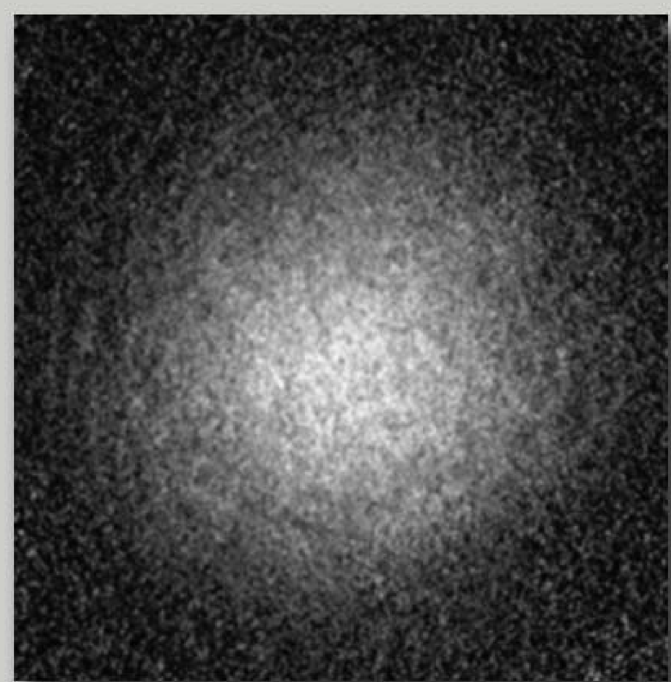


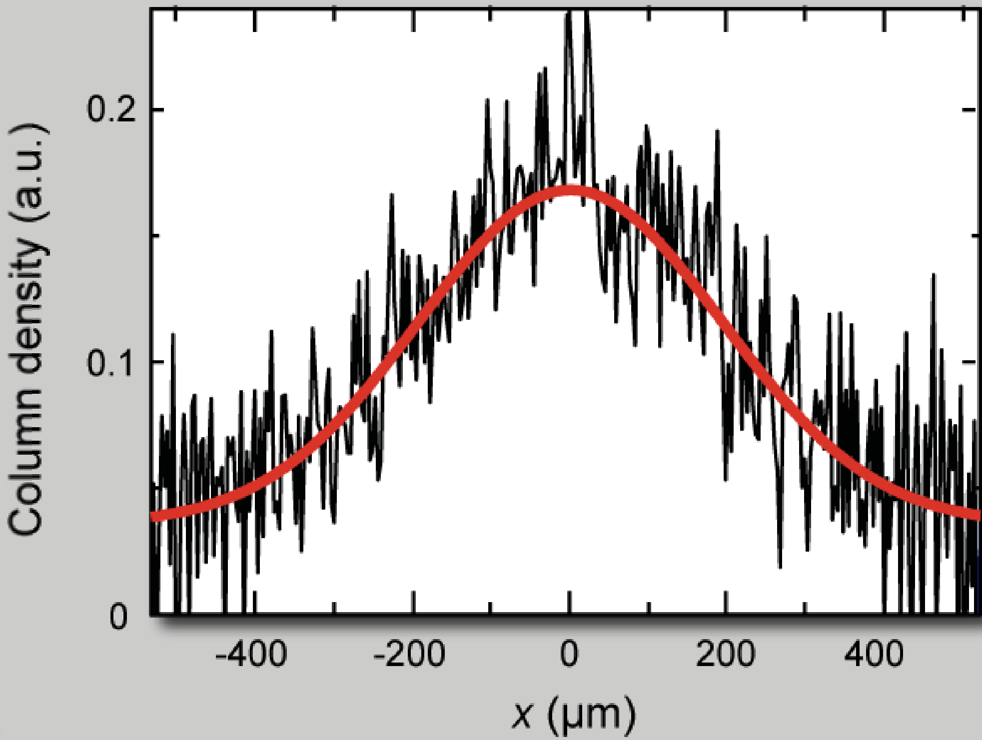
Fermionic band insulator

Density distribution looks exactly the same!

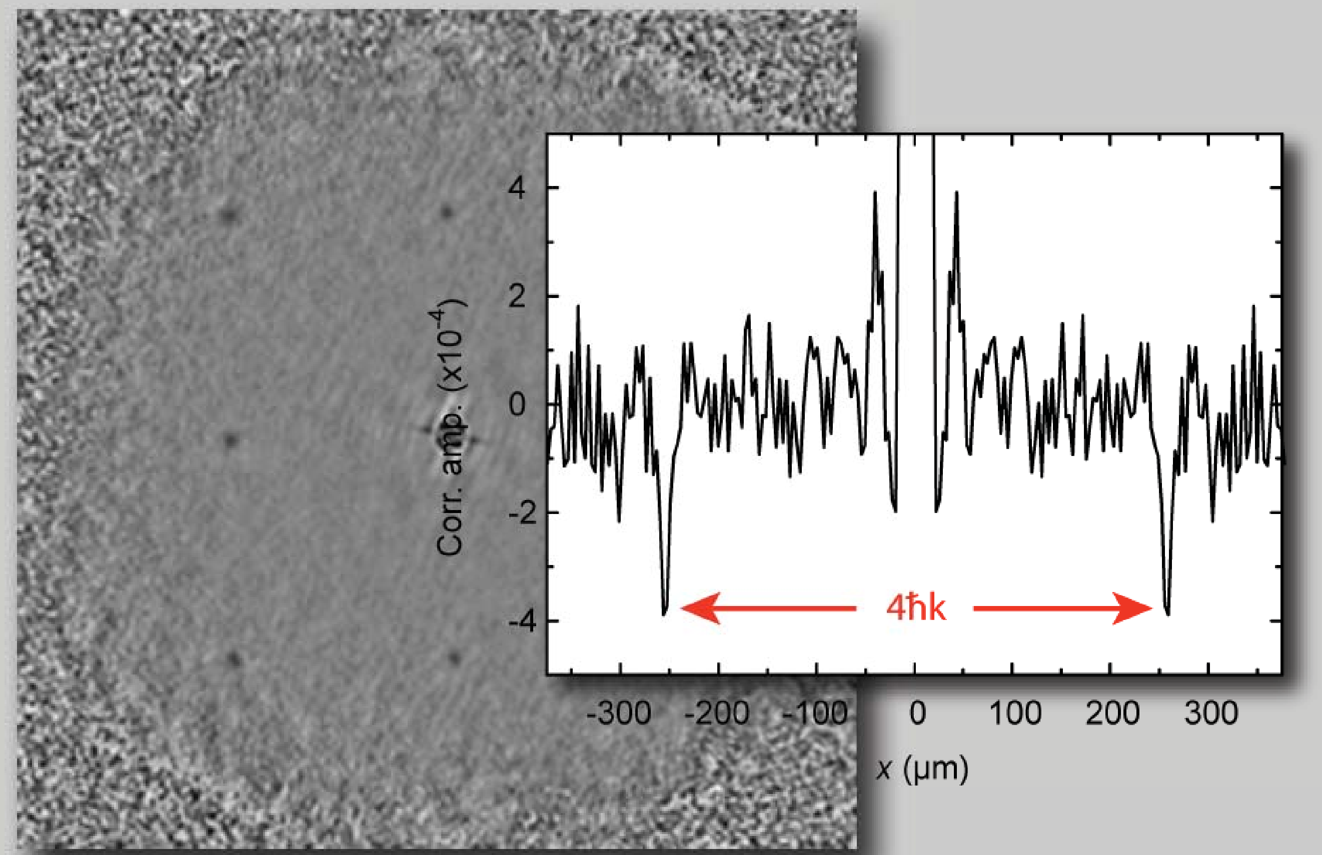
Dominating repulsive interactions mimic Pauli principle!

Releasing the Fermi Gas





Noise Correlations of a Degenerate Fermi Gas



Rom et al.
Nature **444**, 733 (2006)

First observation of fermionic antibunching for neutral atoms (maybe neutral particles)! (see also Jeltes et al., Nature **445**, 402 (2007))

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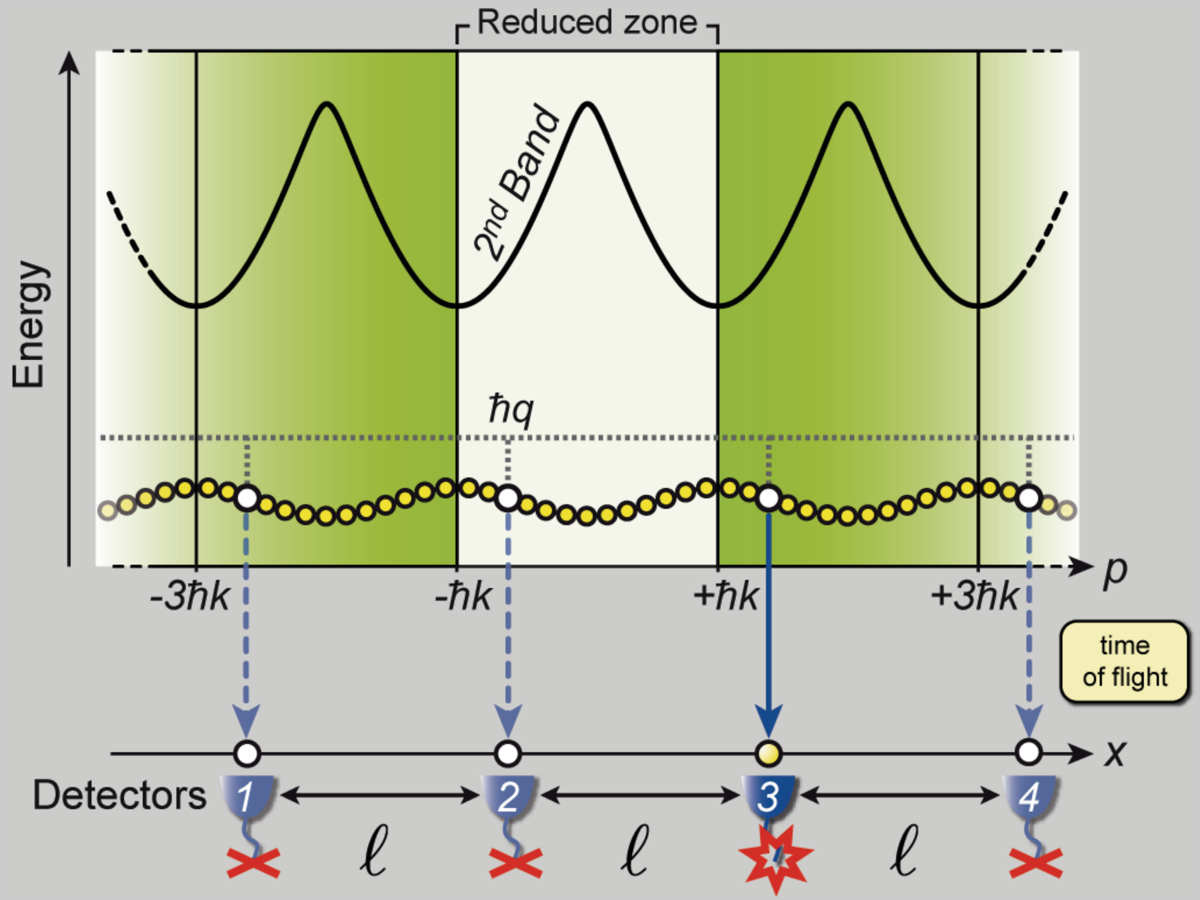
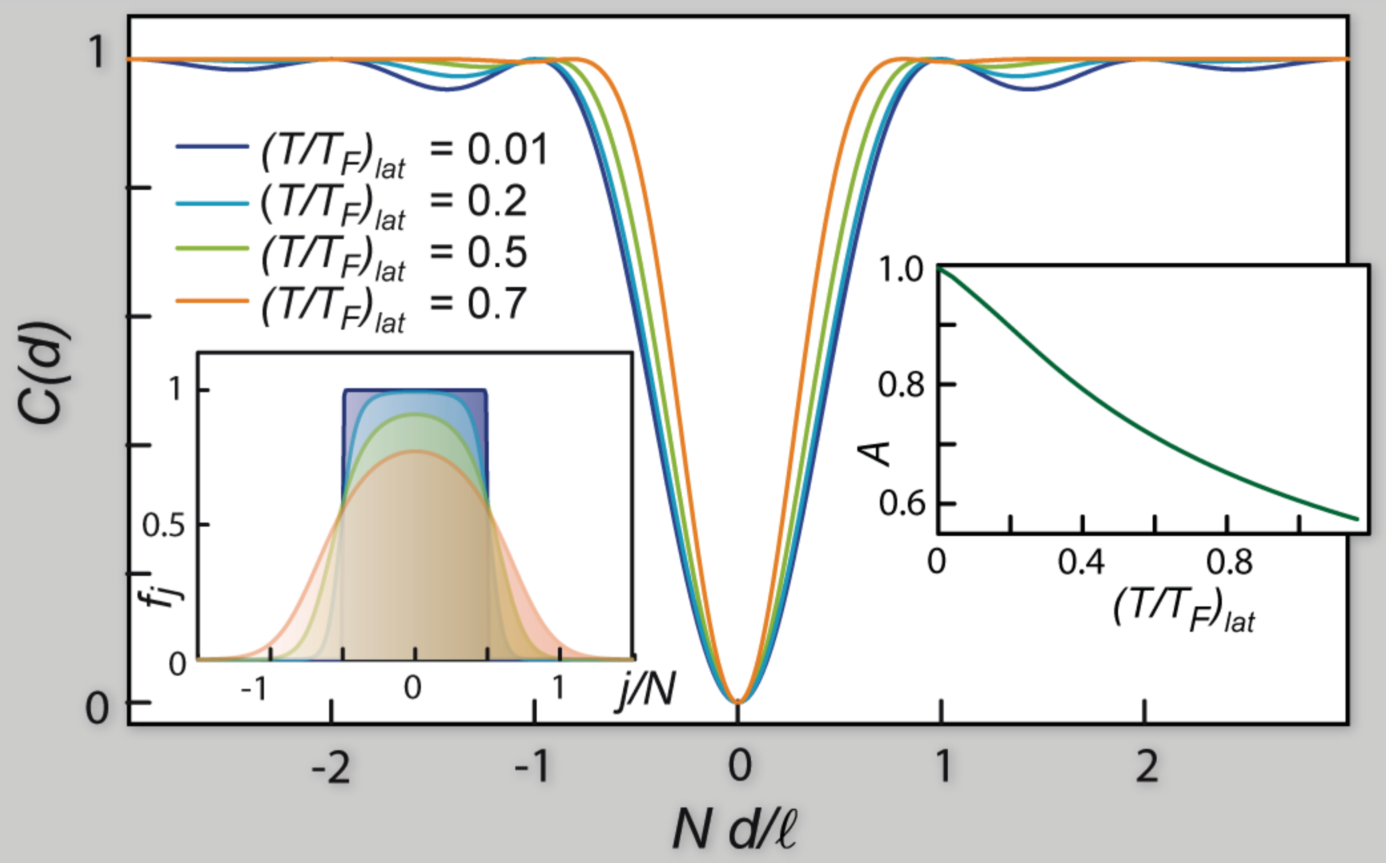
An Alternative Description

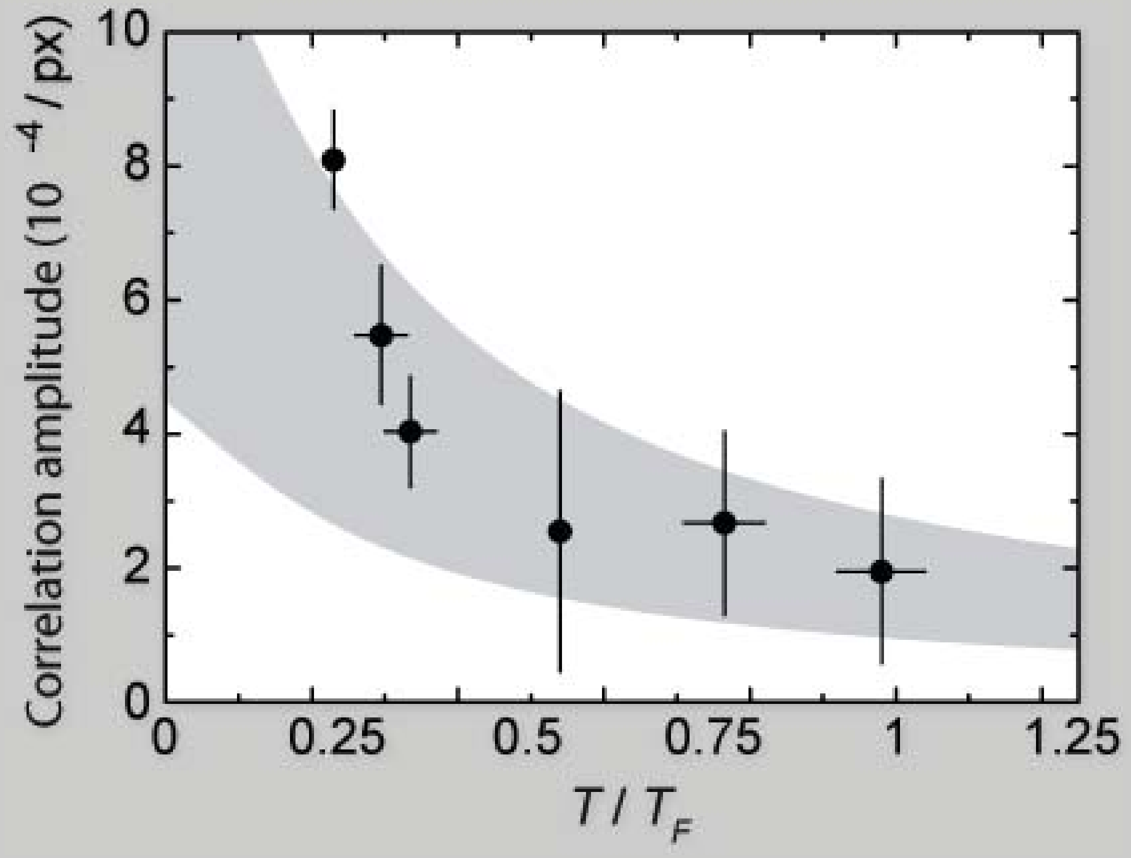


Finite Temperature Effects

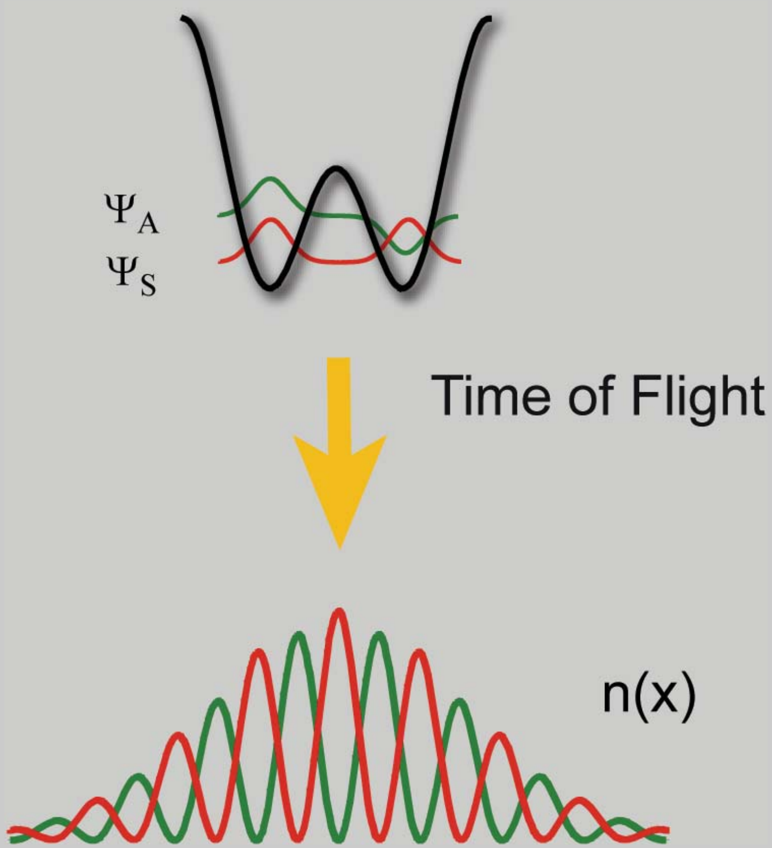


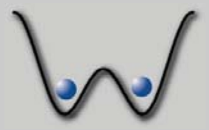
$$\left\langle \hat{n}(x_1)\hat{n}(x_2)\right\rangle_{TOF} = \left\langle \hat{n}(k_1)\hat{n}(k_2)\right\rangle_{in\ trap}$$

Finite Temperature - Experiment



Why Bosons and Fermions are Different in their Correlations





Bosons

$$\Psi_L \otimes \Psi_R + \Psi_L \otimes \Psi_R$$

$$\Updownarrow$$

$$\Psi_S \otimes \Psi_S + \Psi_A \otimes \Psi_A$$

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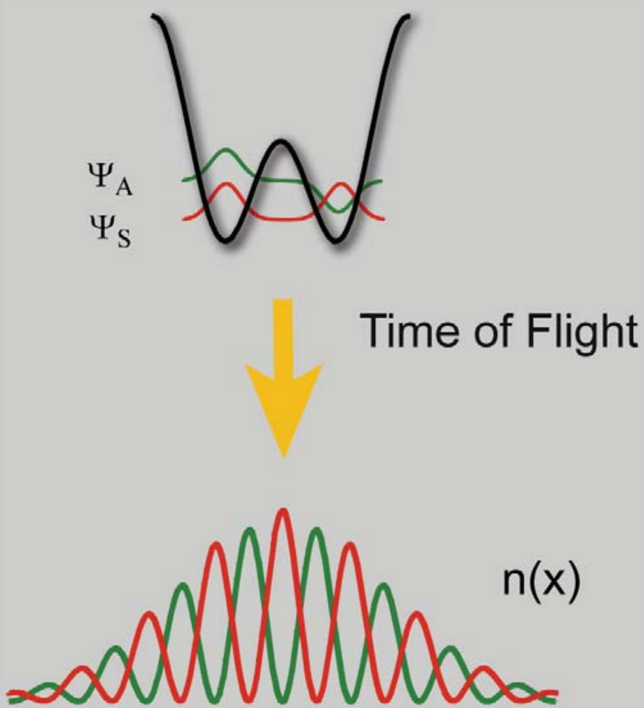
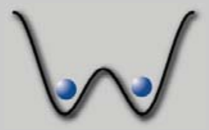
Why Bosons and Fermions are Different in their Correlations





Bosons

$$\Psi_L \otimes \Psi_R + \Psi_L \otimes \Psi_R$$

$$\Updownarrow$$

$$\Psi_S \otimes \Psi_S + \Psi_A \otimes \Psi_A$$

Fermions

$$\Psi_L \otimes \Psi_R - \Psi_L \otimes \Psi_R$$

$$\Updownarrow$$

$$\Psi_S \otimes \Psi_A - \Psi_A \otimes \Psi_S$$

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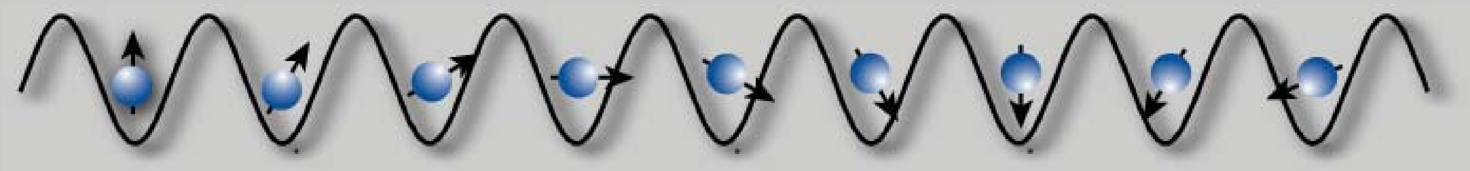
Now detection of many strongly correlated quantum states becomes possible!

Antiferromagnet

Spin wave



Charge density wave

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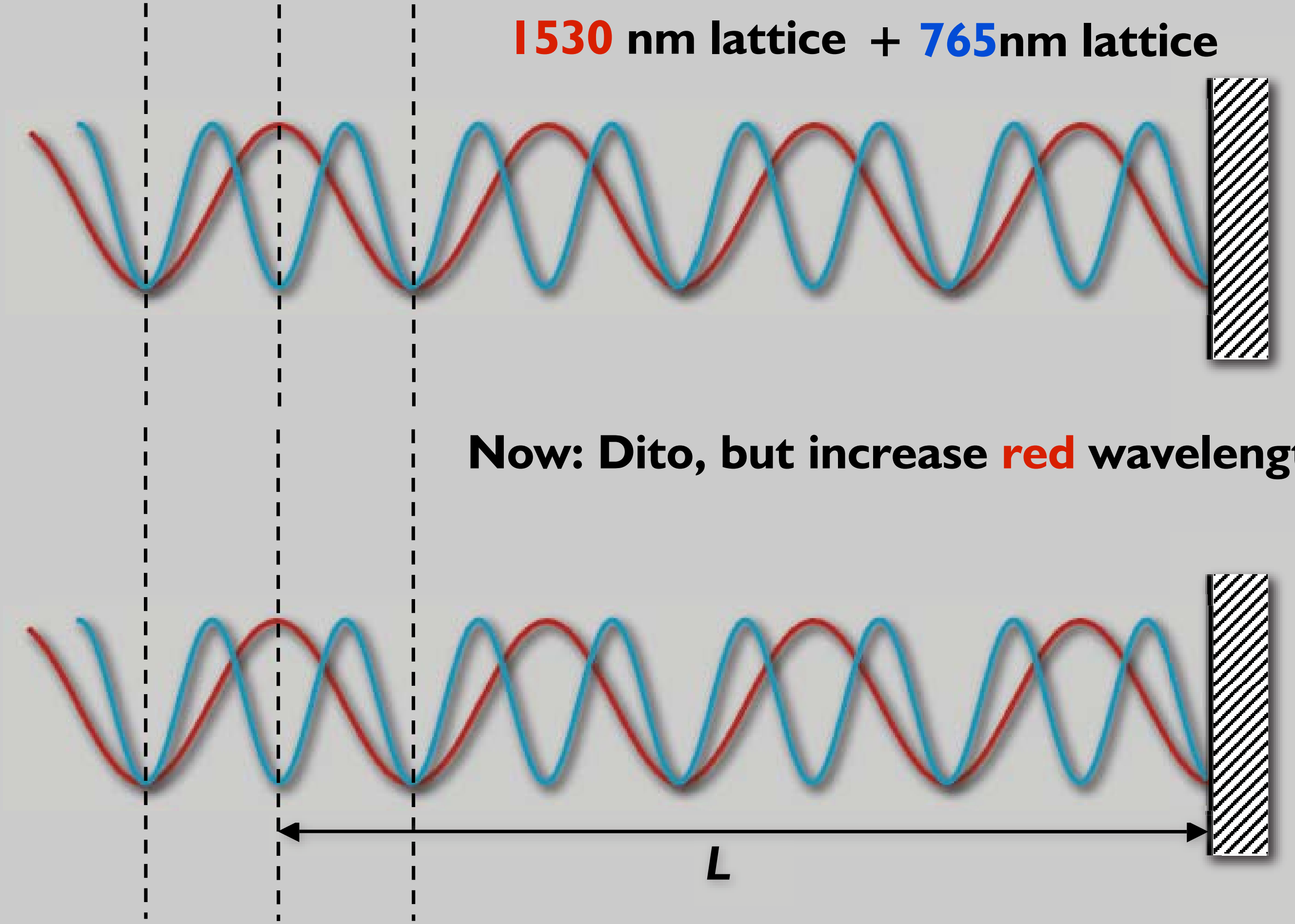
Optical Superlattices

See also Related Experiments at NIST (T. Porto & W. D. Phillips)

How to make a Superlattice

1530 nm lattice + 765nm lattice



Now: Dito, but increase red wavelength

In reality *L*=0.25m: 500 MHz freq change

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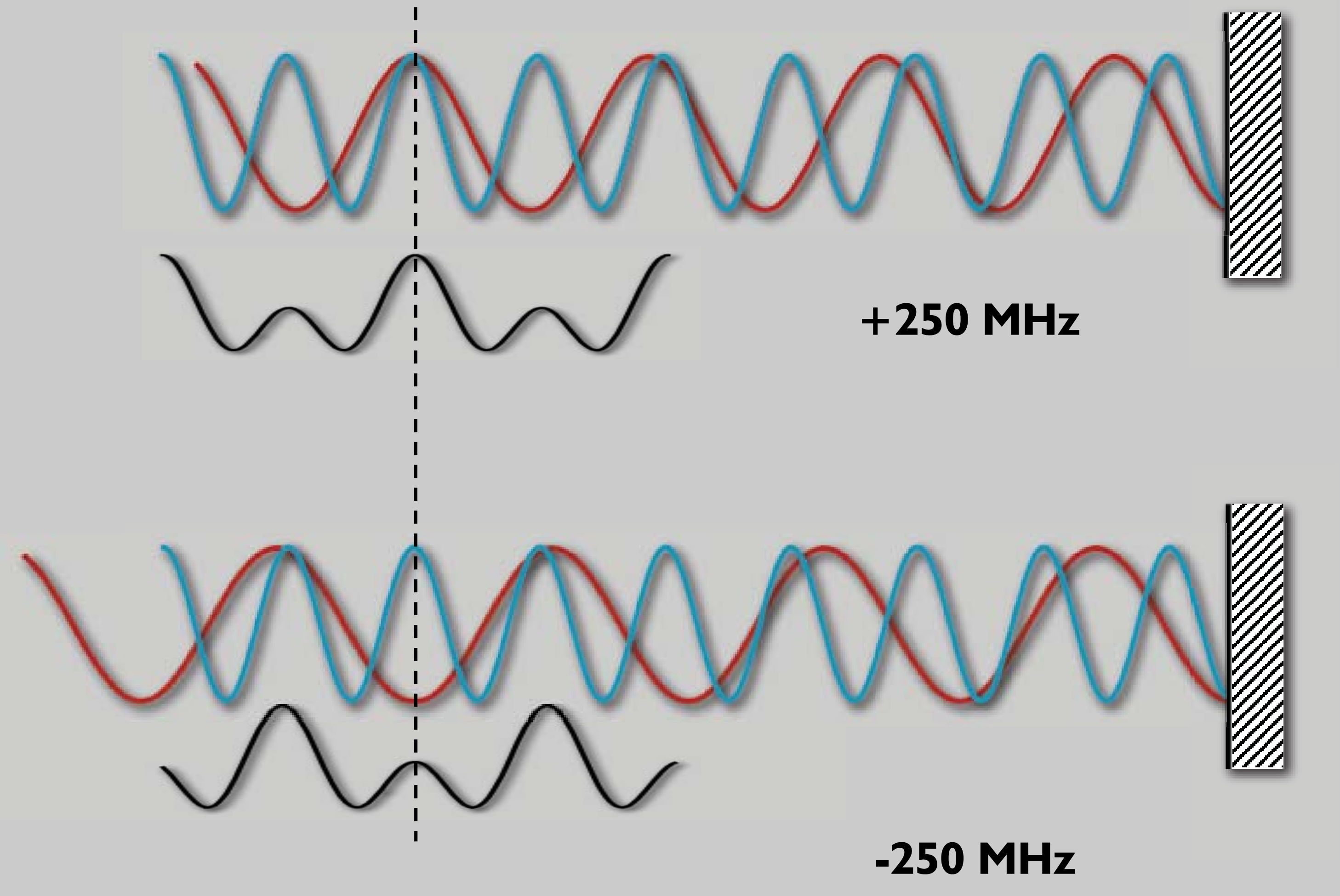
How to make a Superlattice II



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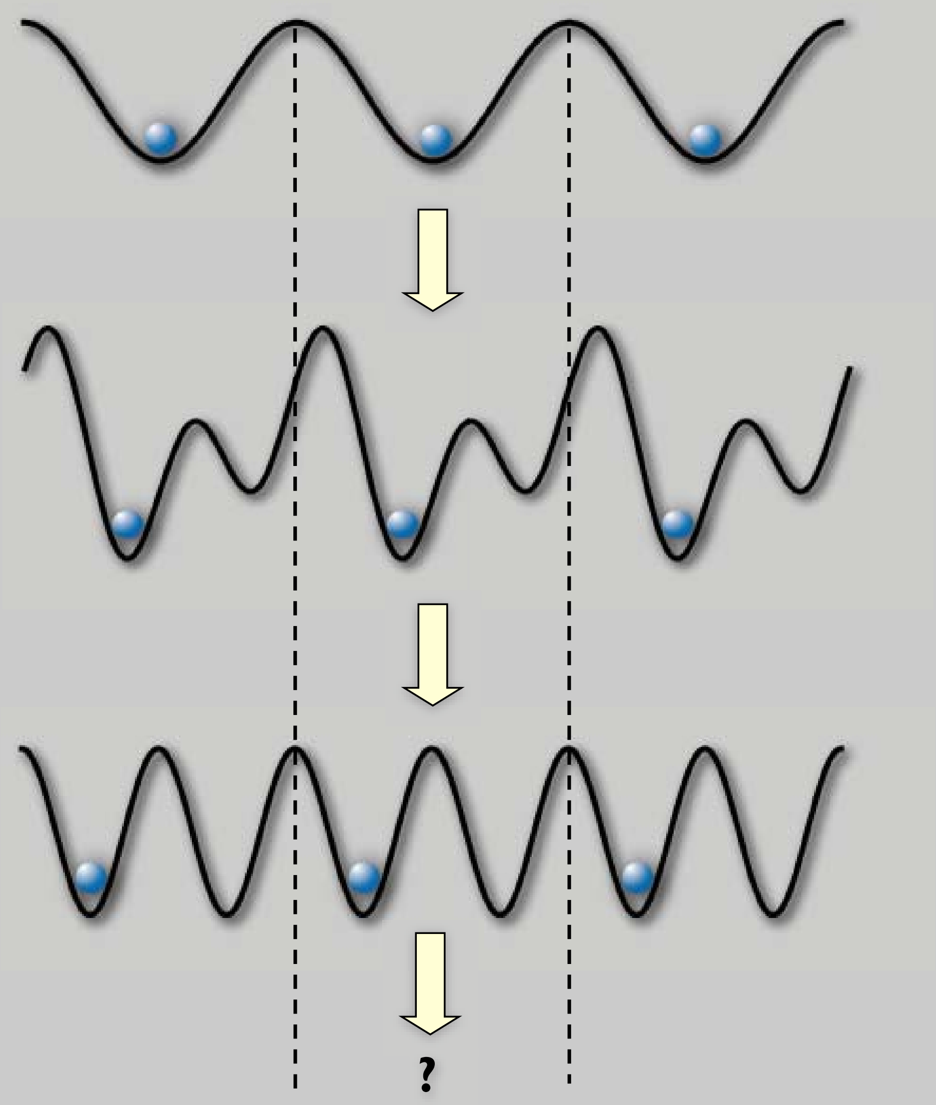
Patterned loading of the short lattice



Mott State in long lattice

Increase short lattice

Switch off long lattice

Release

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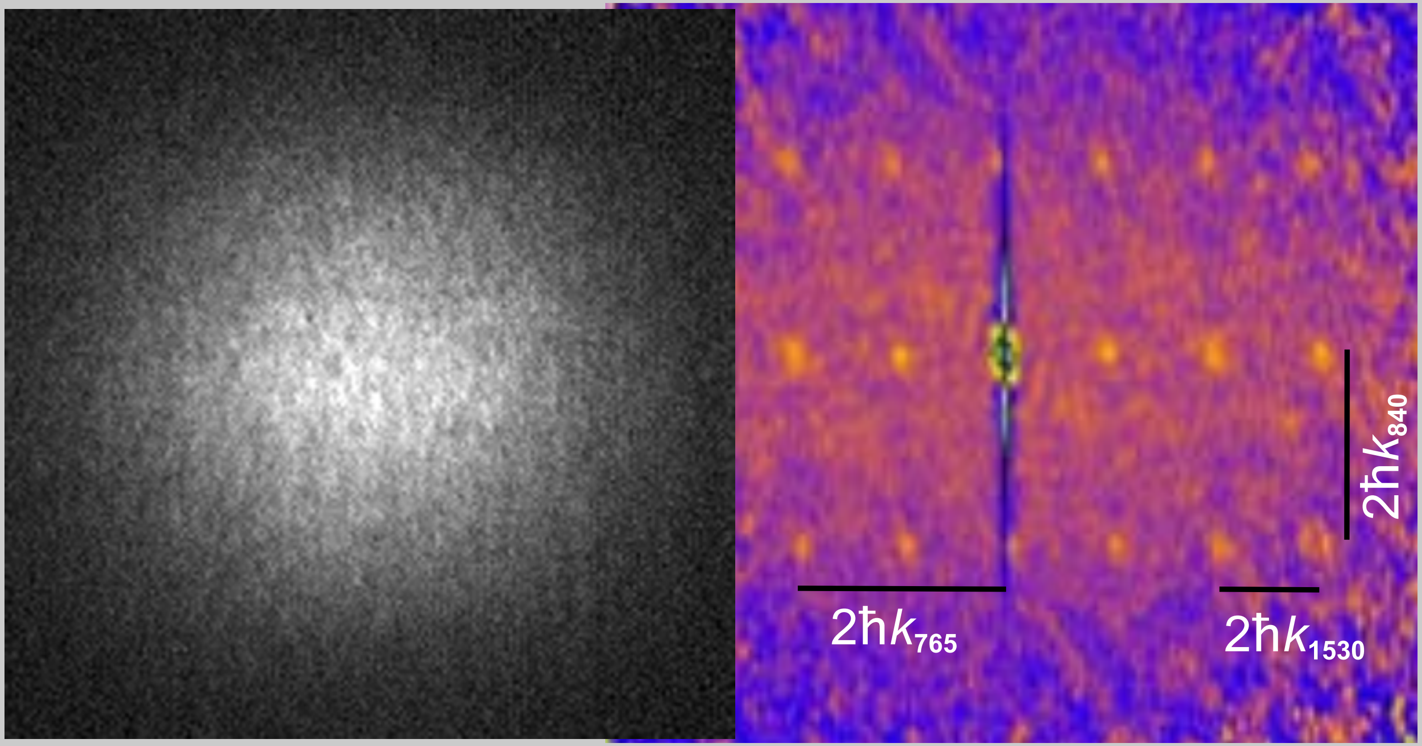
Patterned correlations



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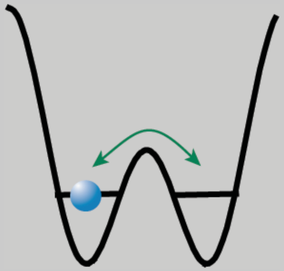
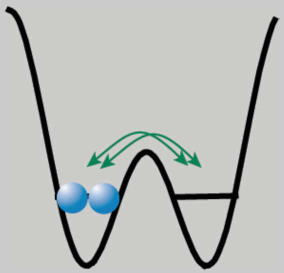
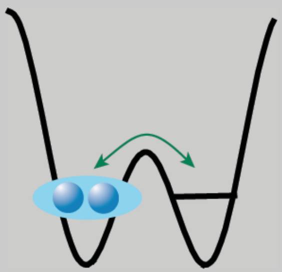
Tunneling of one or two atoms



1) Resonant tunneling between the two wells with frequency $2J$



2) Two atoms, no interaction: tunneling is independent



3) Cooperative tunneling of attractively bound objects (Cooper pairs, molecules)

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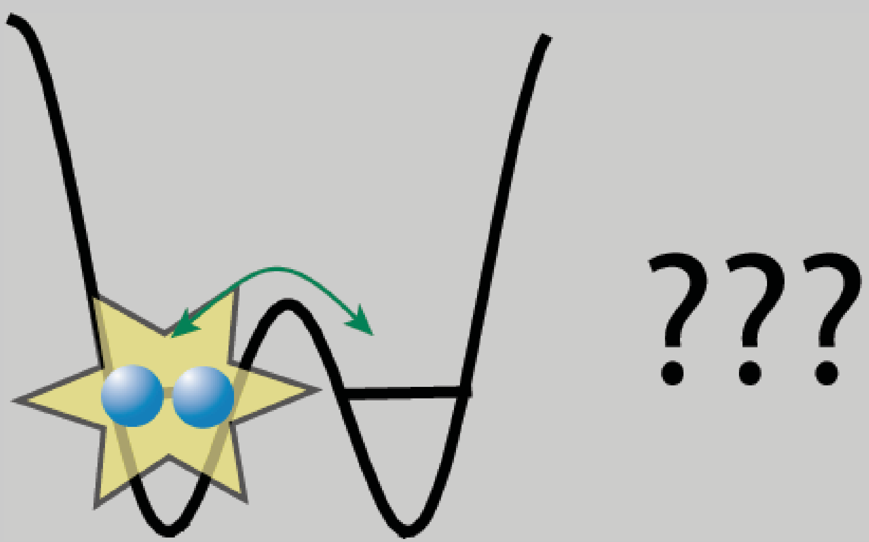
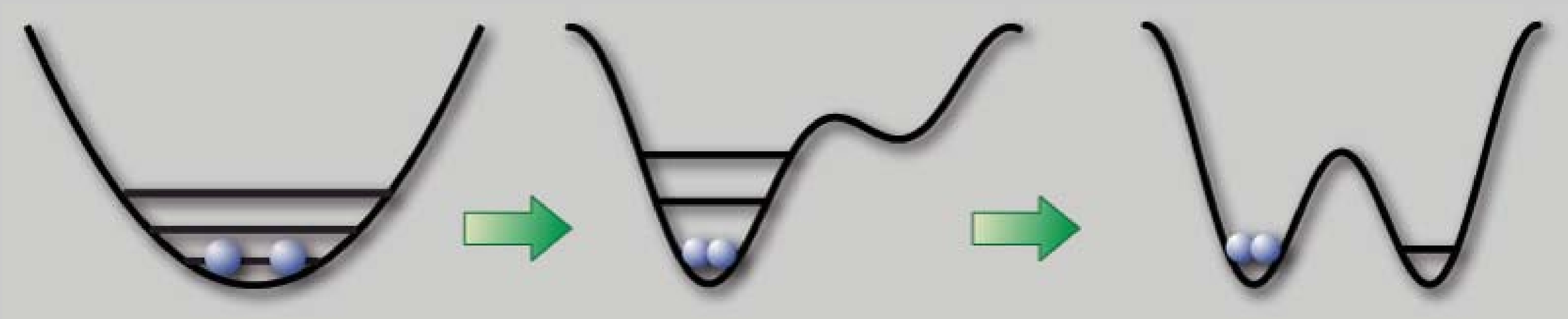
What about interacting atoms?



S. Fölling et al., Nature in press
stability of pairs, see: K. Winkler et al, Nature 441, (2006)

State Preparation

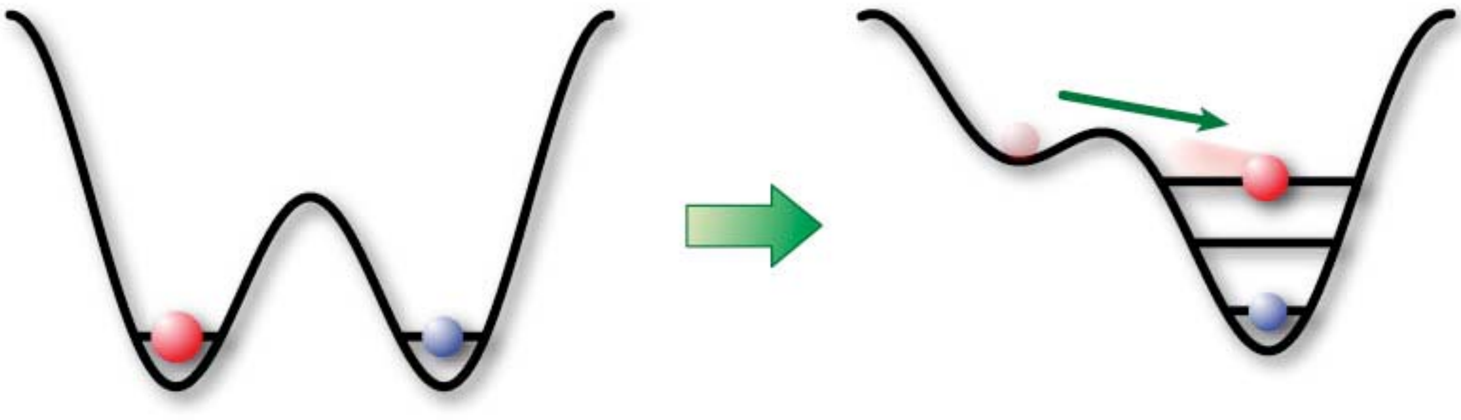


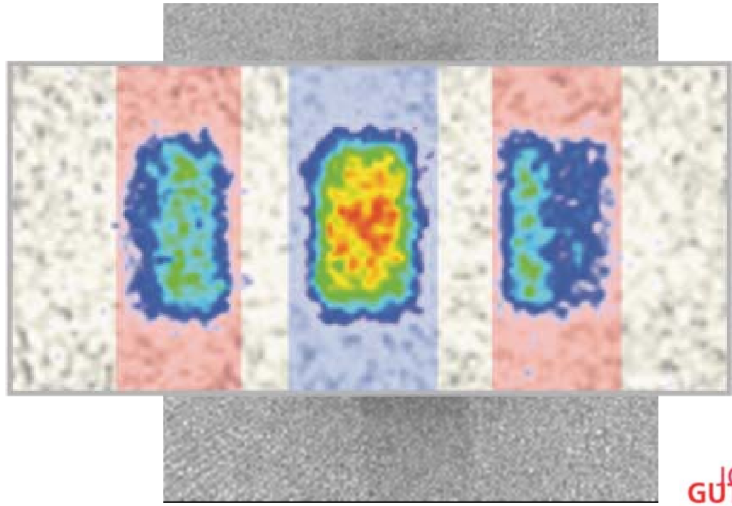
Two Atoms in „Big Well“

Tilted Double Well

Balanced Double Well

Population Imbalance Measurement

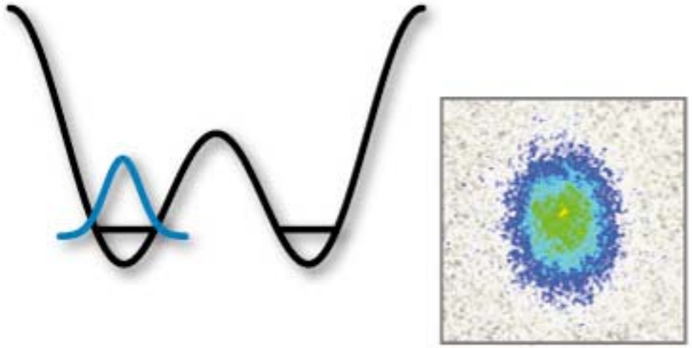


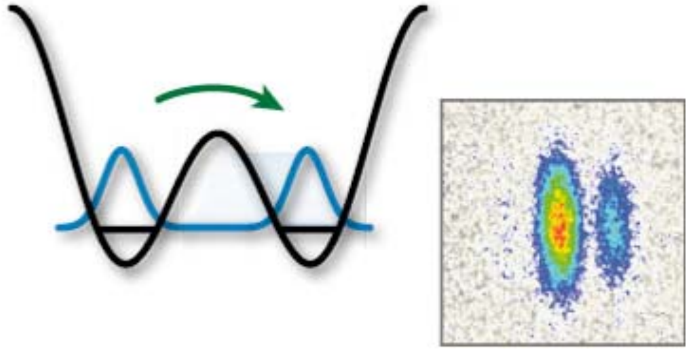


Map Left-Right Populations onto Band Populations

see also: Sebby-Strabley et al., quant-ph/0701110

Phase Measurement

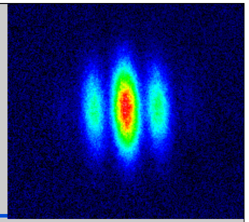


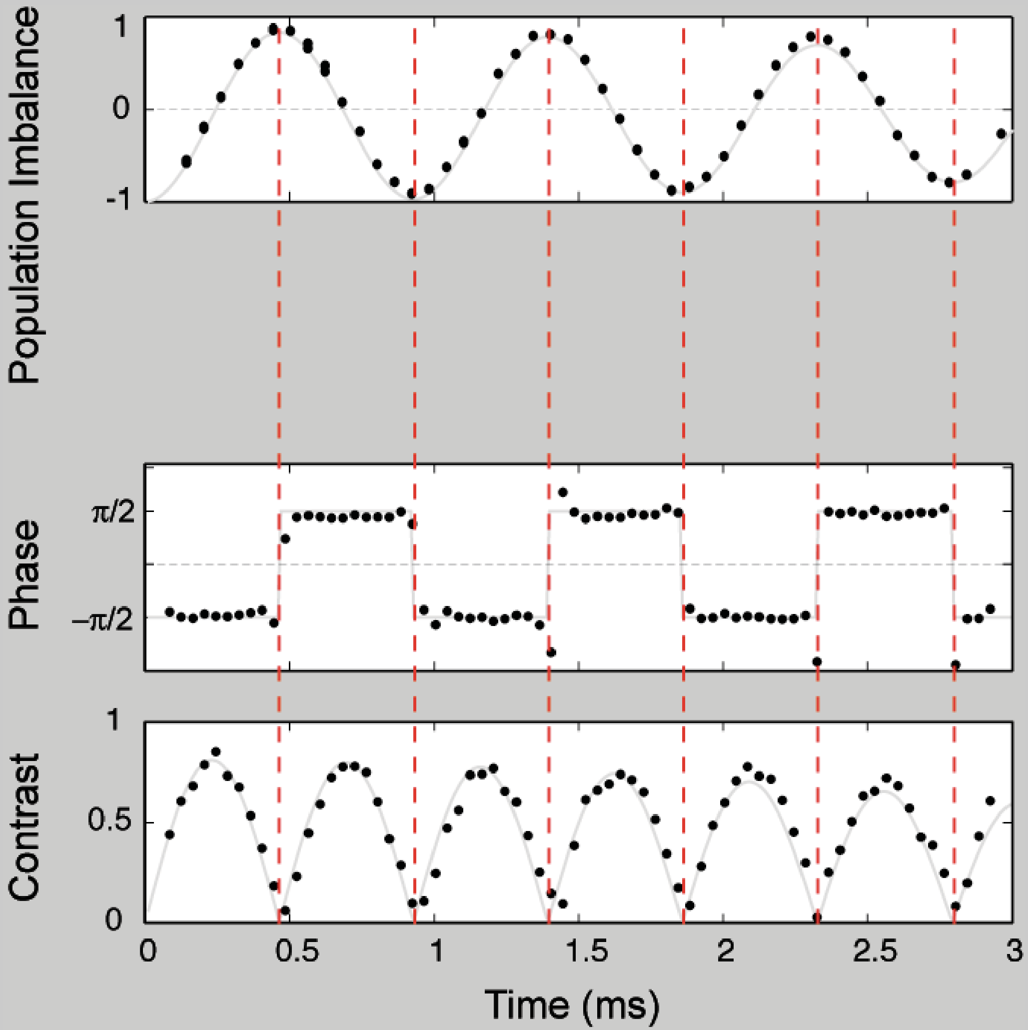


Localized Particle yields no interference pattern

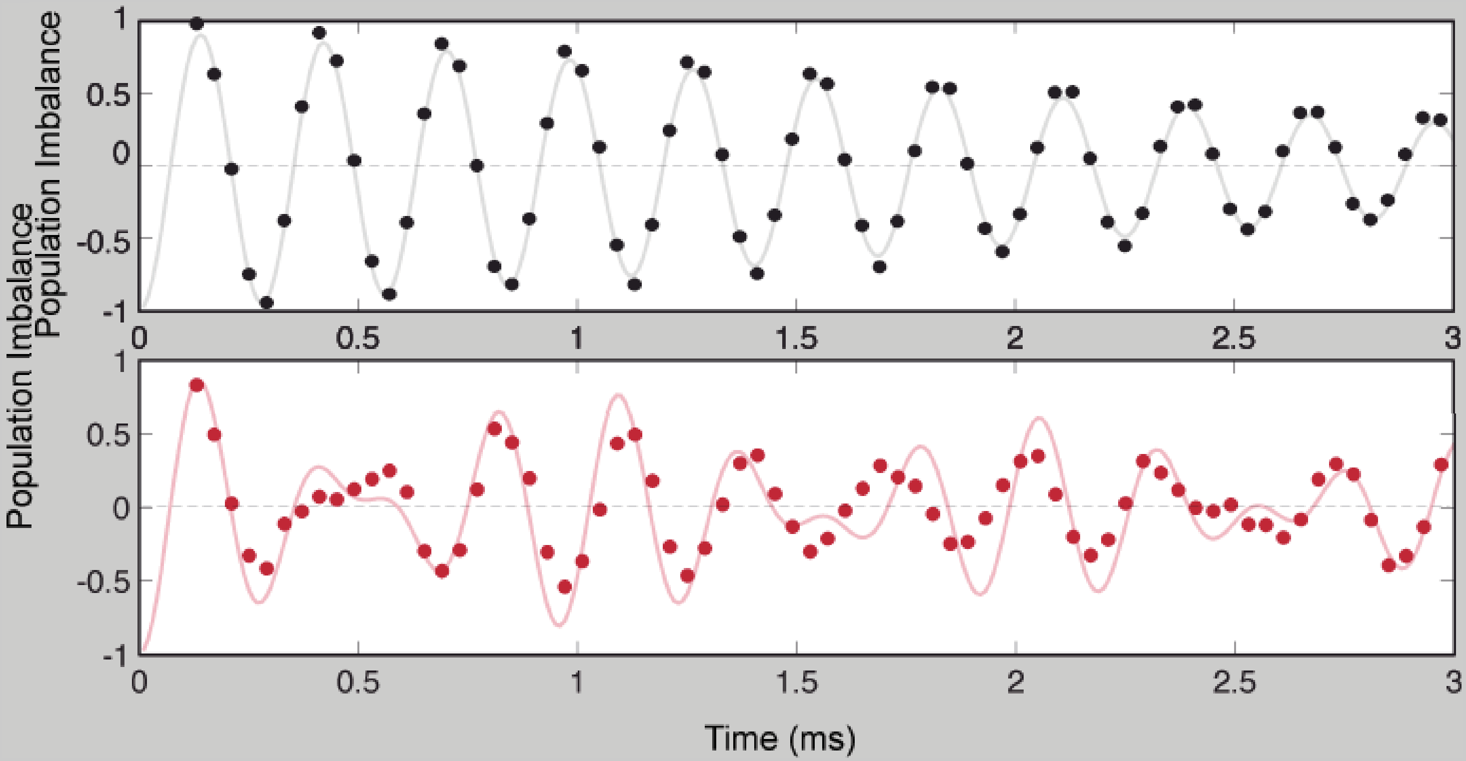
Phase of superposition state can be read out through phase of interference pattern.

Single particle tunneling

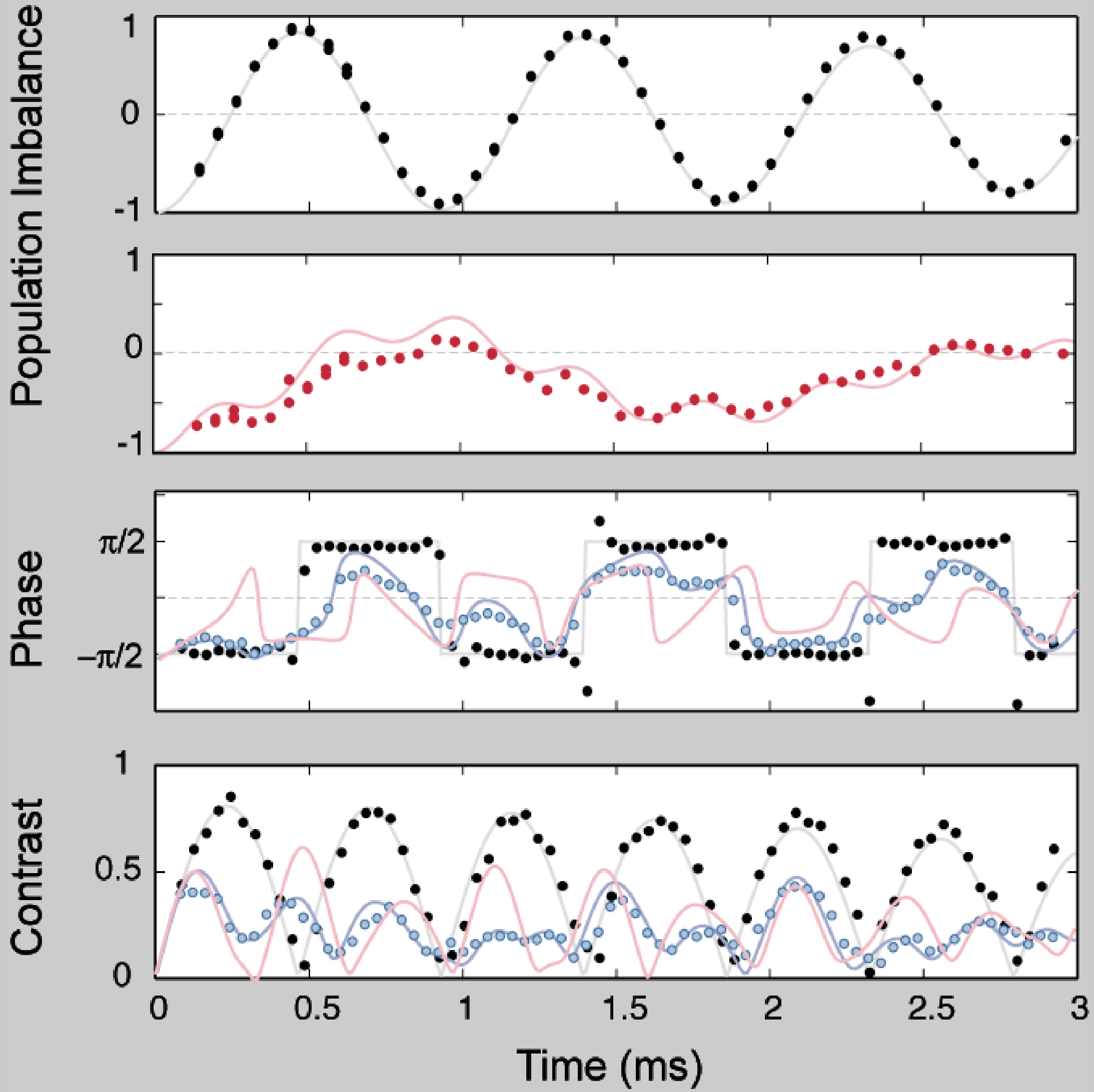




Correlated Pair Tunneling J/U=1.5



Correlated Particle Tunneling J/U=0.2



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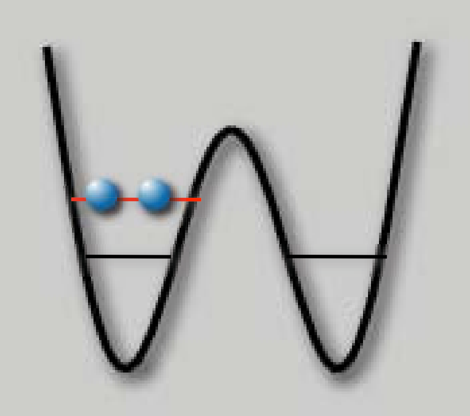
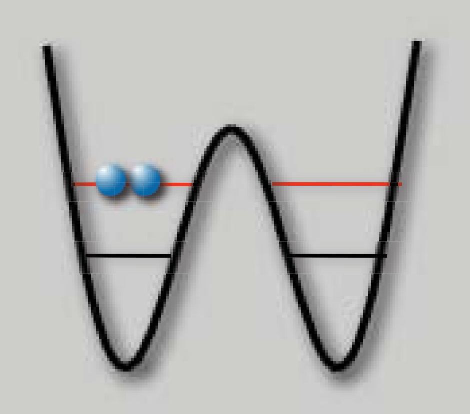
Tunneling under Repulsive Interactions



Single atom tunneling
Transition is detuned by *U*
Off-resonant tunneling between the two wells with frequency

$$2\sqrt{4J^2+U^2}$$



Simultaneous tunneling is resonant – with tunneling rate – co-tunnelling

$$J^2/U$$

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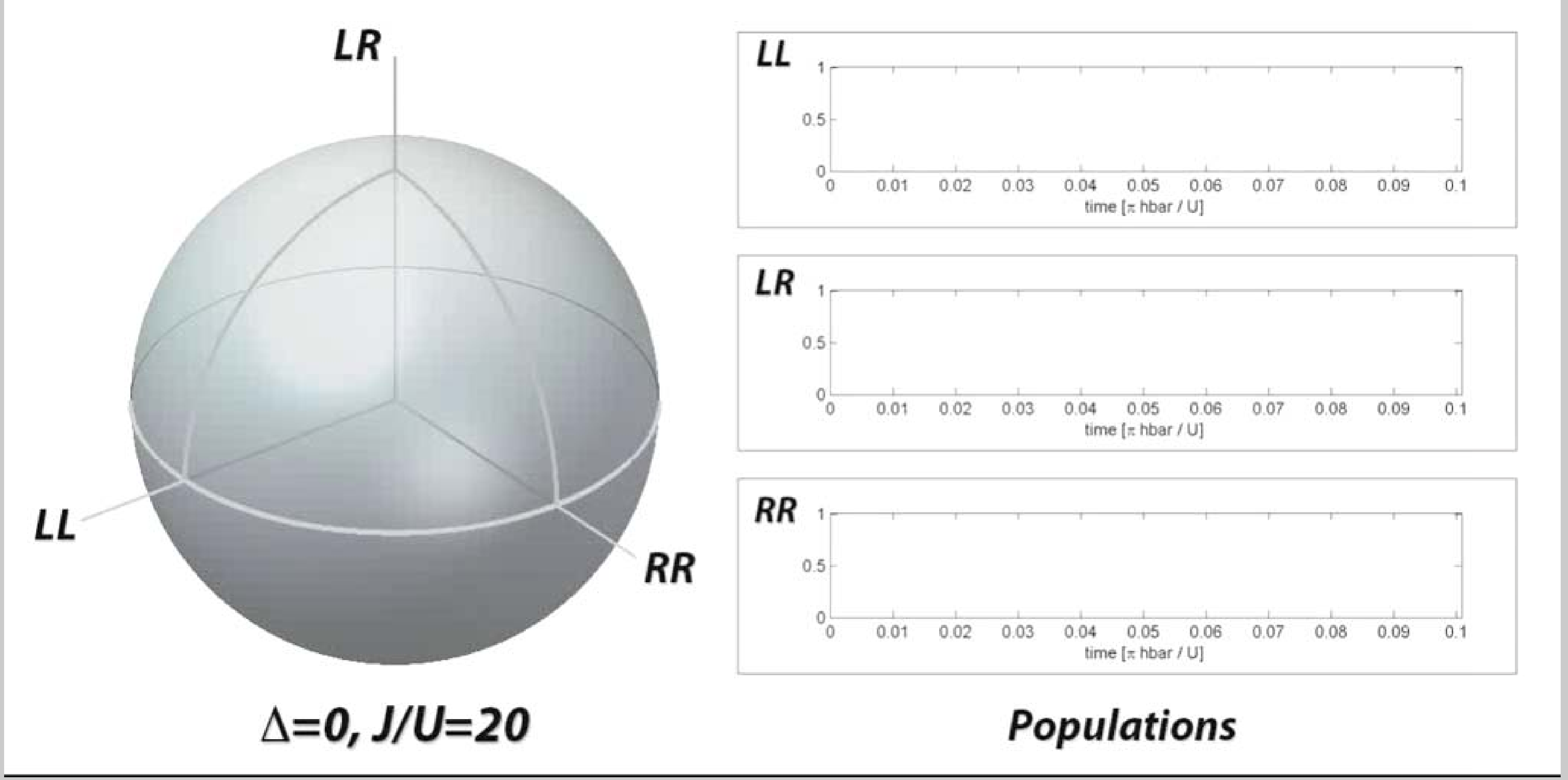
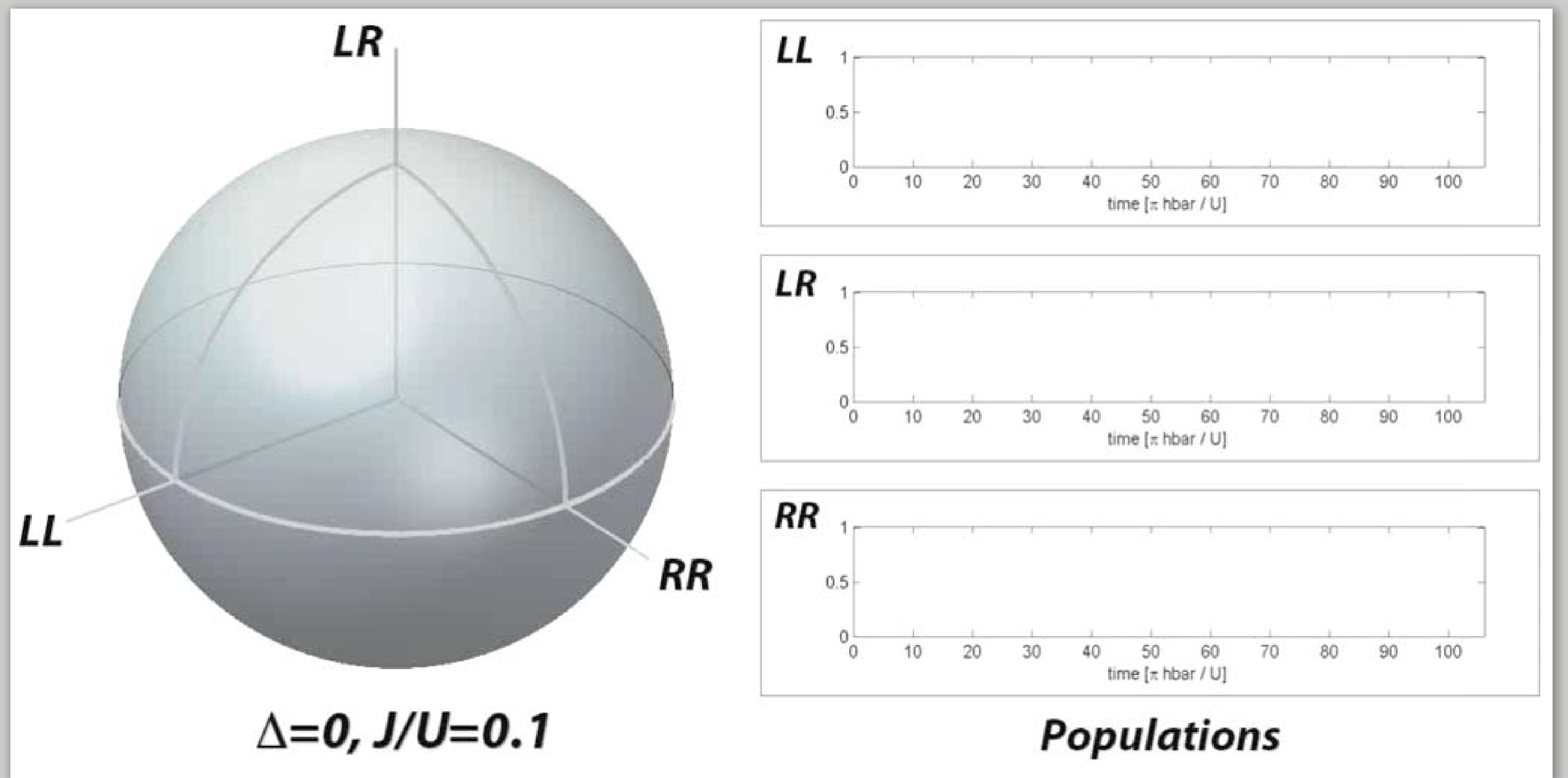
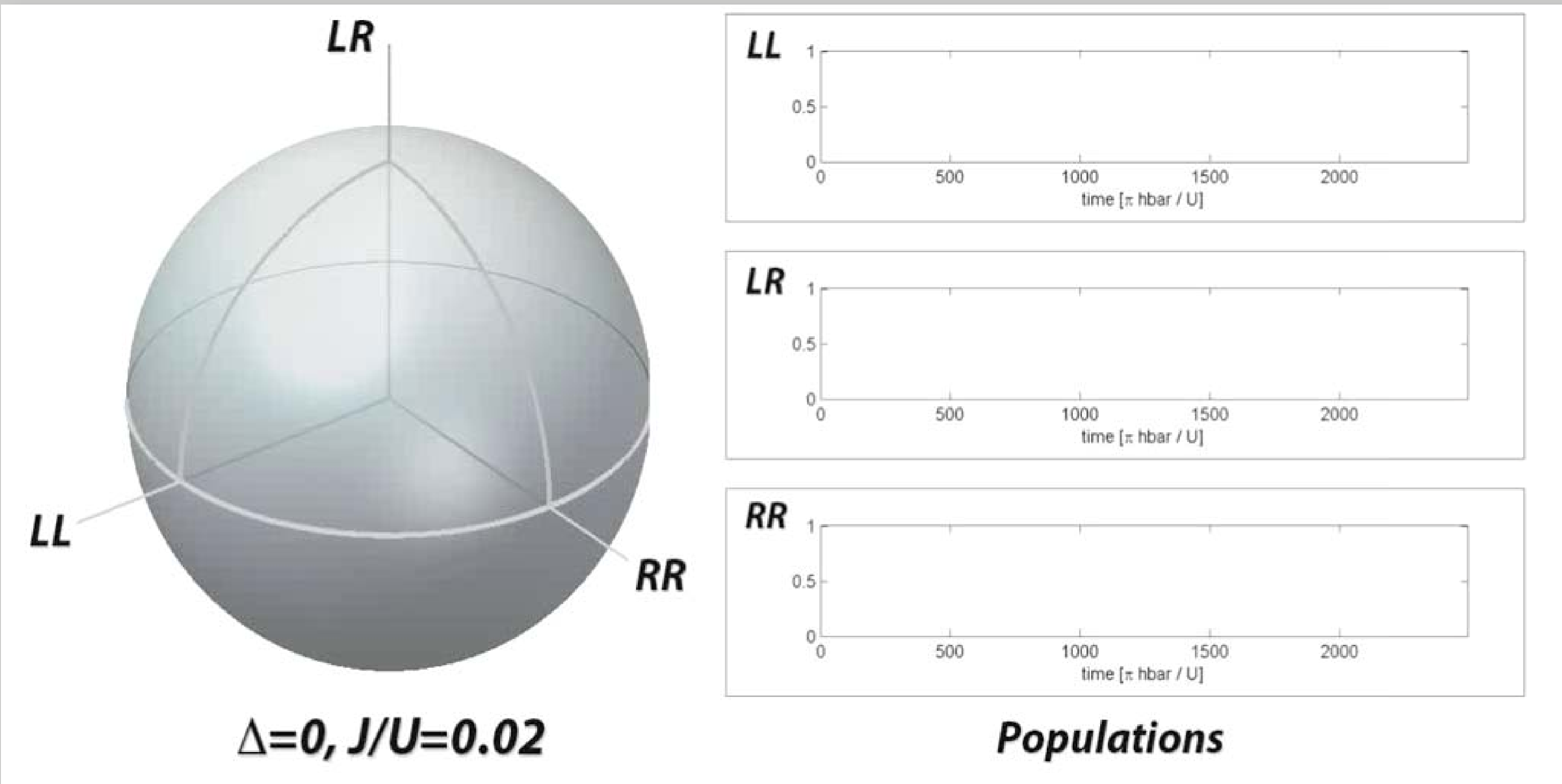
Atom Pair Tunneling J/U=20



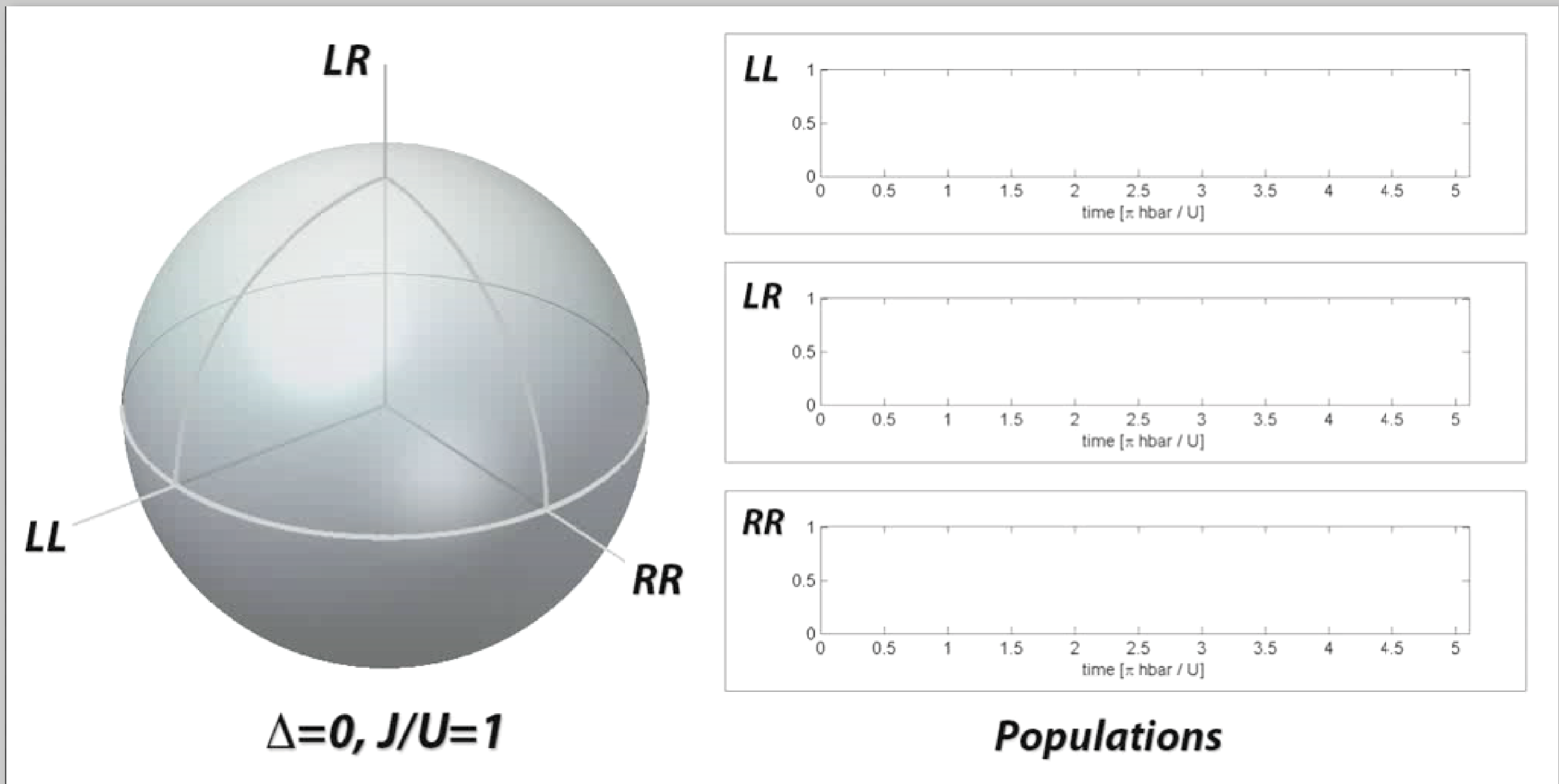
Atom Pair Tunneling J/U=0.1



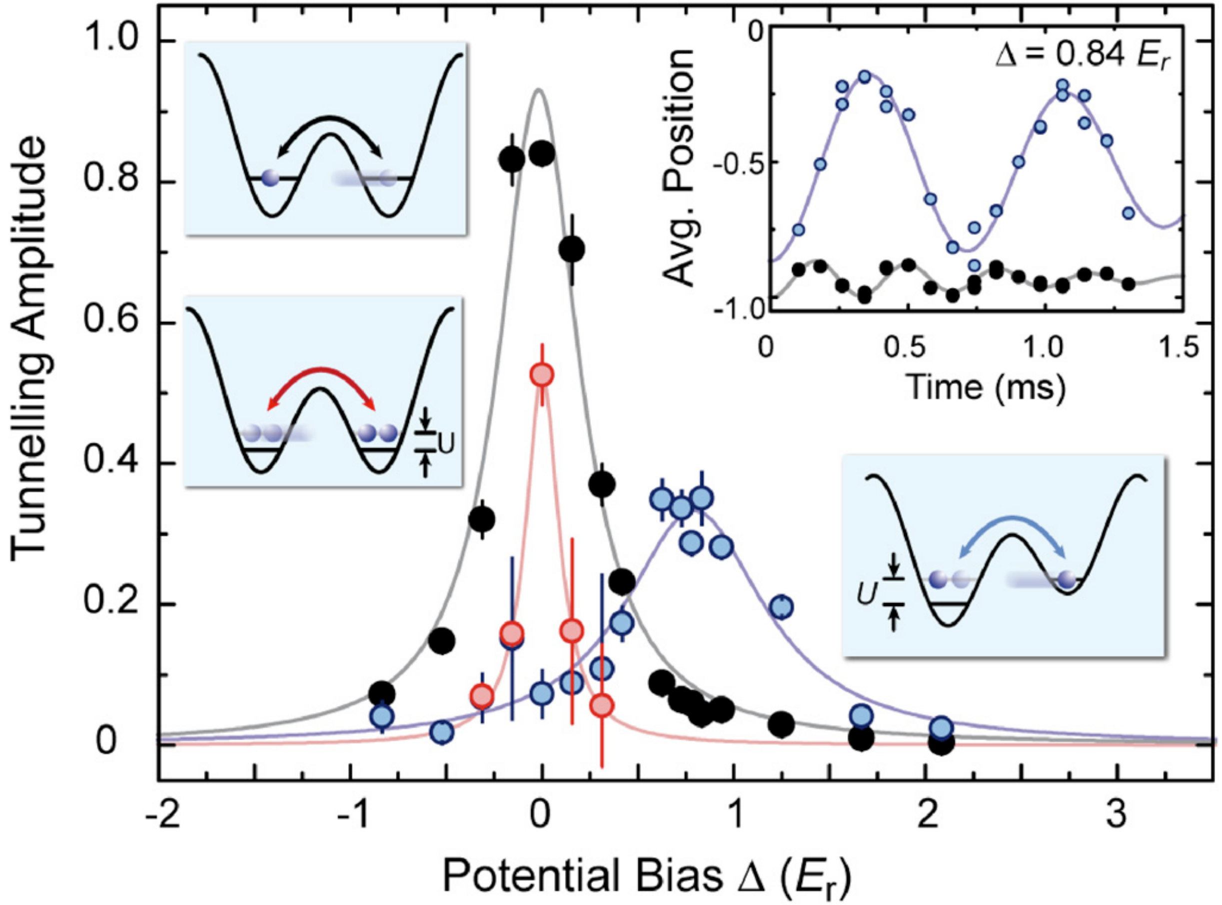
Atom Pair Tunneling J/U=0.02



Atom Pair Tunneling J/U=1



Correlated Tunneling (2) - Conditional Tunneling



Second atom acts as switch!

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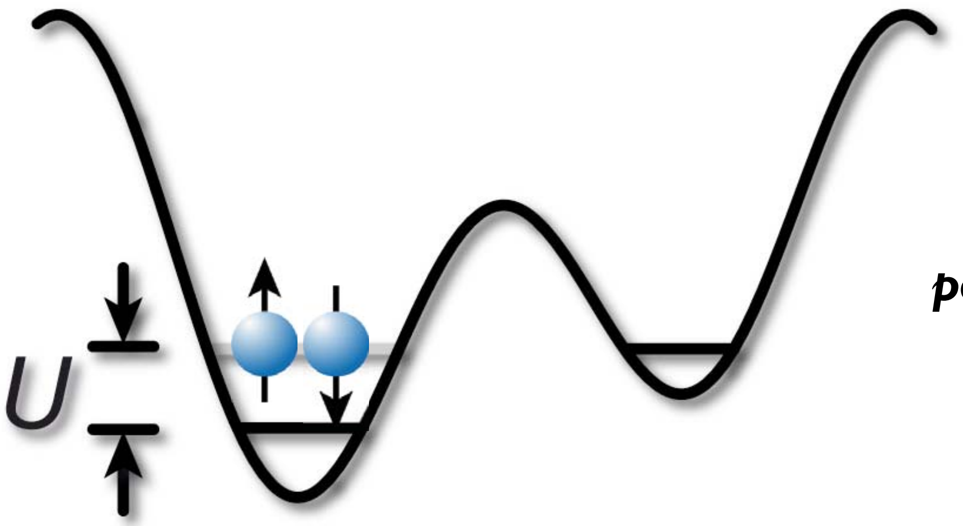
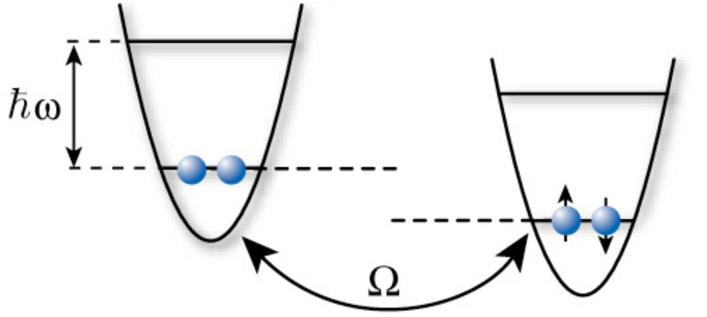
Entangling Atoms via Resonance Tunneling



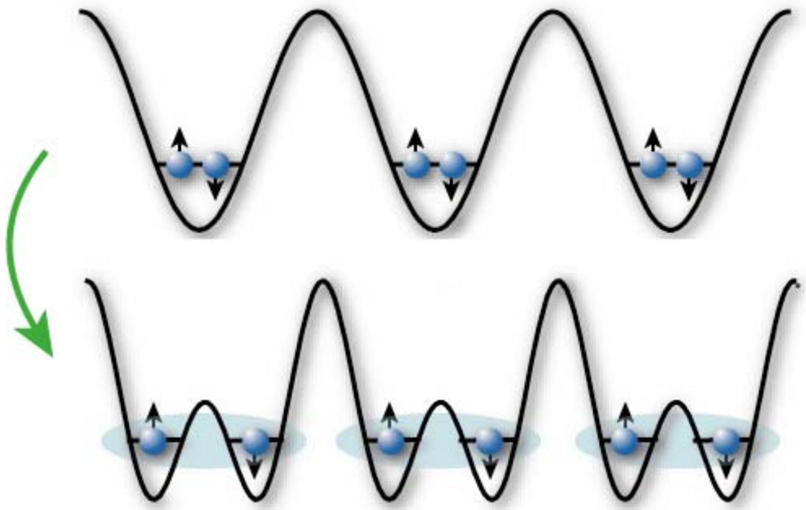
For spin independent potential, we cannot tell which spin state tunneled.

$$\frac{1}{\sqrt{2}}\left(\left|\uparrow\right\rangle_L \left|\downarrow\right\rangle_R + \left|\downarrow\right\rangle_L \left|\uparrow\right\rangle_R\right)$$

Robust multi-particle entanglement via spin changing collisions



A. Widera et al.,
Phys. Rev. Lett., 95,190405, (2005)



$$\left(|\uparrow,\downarrow\rangle + |\downarrow,\uparrow\rangle\right)\otimes|0,0\rangle$$

Spin Triplet

$$|\uparrow\rangle_L|\downarrow\rangle_R + |\downarrow\rangle_L|\uparrow\rangle_R$$

Entangled Bell state

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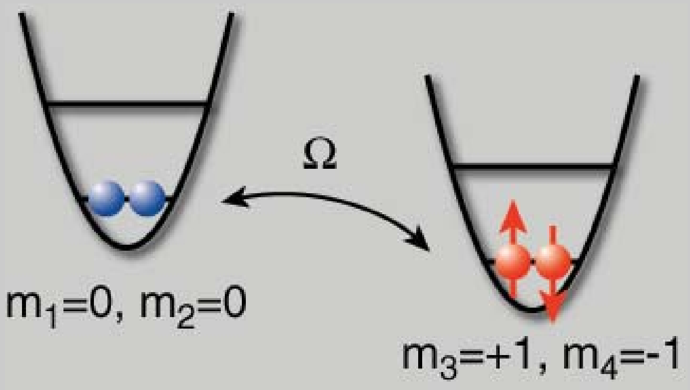
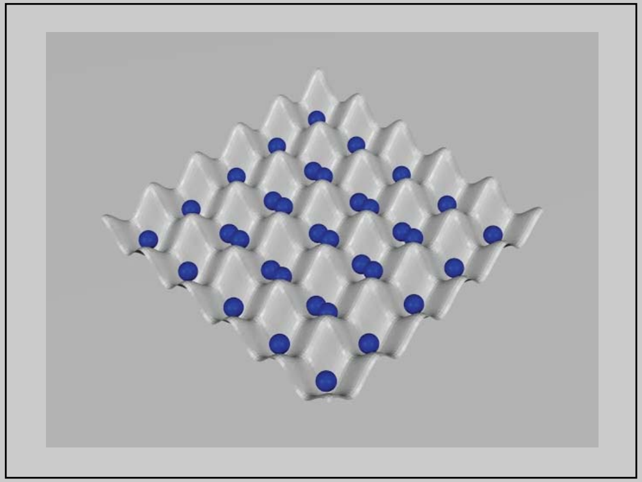
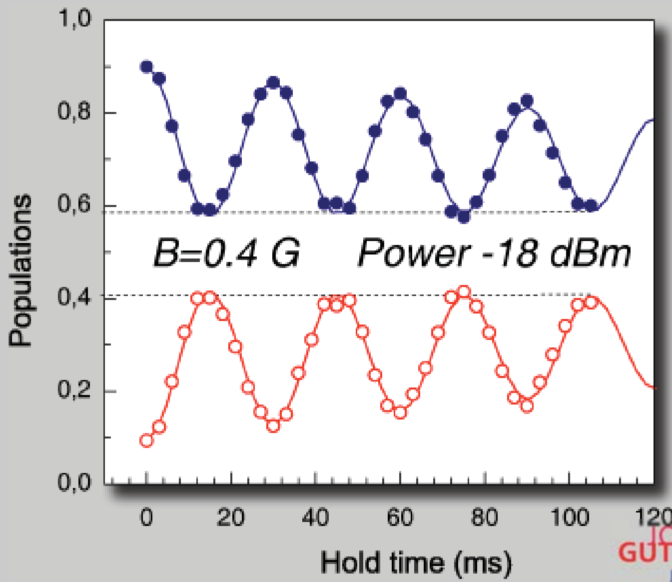
Spin Changing Collisions in an Optical Lattice





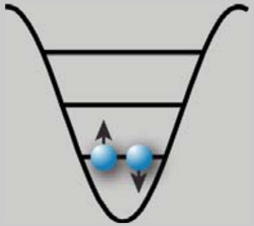


Collisionally induced
„Rabi-Type" Oscillations

$$|0,0\rangle \leftrightarrow \left(|\uparrow,\downarrow\rangle + |\downarrow,\uparrow\rangle\right)/\sqrt{2}$$

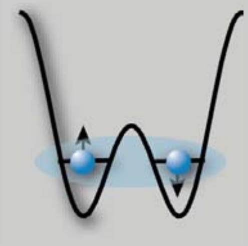
A. Widera et al., PRL 95, 190405 (2005)
Spinor dynamics without lattices:
Hamburg, GeorgiaTech, Berkley

How can we detect the Bell pairs? (1)



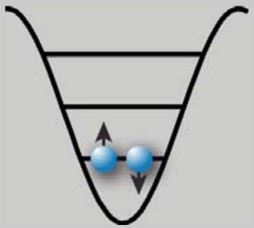
$$\left(|\uparrow,\downarrow\rangle+|\downarrow,\uparrow\rangle\right)\otimes|0,0\rangle$$

Split



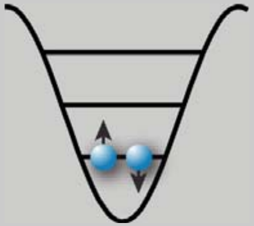
$$|\uparrow\rangle_L|\downarrow\rangle_R+|\downarrow\rangle_L|\uparrow\rangle_R$$

Unite



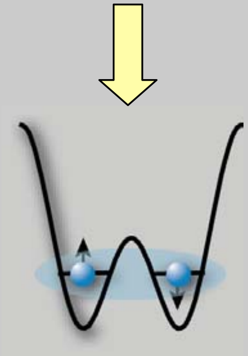
$$\left(|\uparrow,\downarrow\rangle+|\downarrow,\uparrow\rangle\right)\otimes|0,0\rangle$$

How can we detect the Bell pairs? (2)



$$\left(|\uparrow,\downarrow\rangle + |\downarrow,\uparrow\rangle\right)\otimes|0,0\rangle$$

Split



$$|\uparrow\rangle_L |\downarrow\rangle_R + |\downarrow\rangle_L |\uparrow\rangle_R$$

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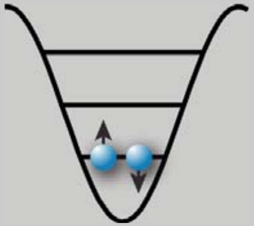
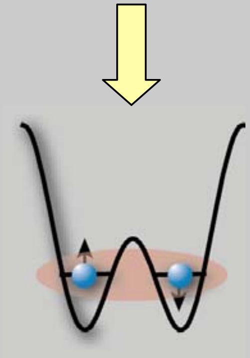
How can we detect the Bell pairs? (2)



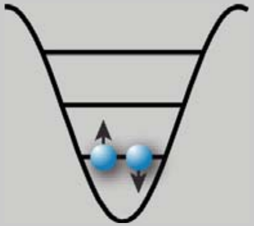
$$\left(\left|\uparrow,\downarrow\right\rangle+\left|\downarrow,\uparrow\right\rangle\right)\otimes\left|0,0\right\rangle$$

Split



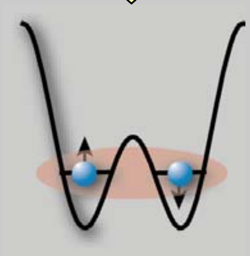
$$\left|\uparrow\right\rangle_L\left|\downarrow\right\rangle_R-\left|\downarrow\right\rangle_L\left|\uparrow\right\rangle_R$$

How can we detect the Bell pairs? (2)



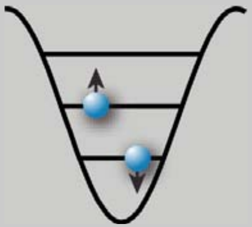
$$\left(|\uparrow,\downarrow\rangle + |\downarrow,\uparrow\rangle\right)\otimes|0,0\rangle$$

Split



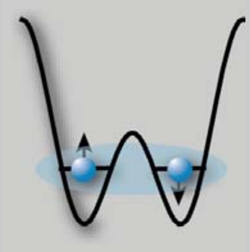
$$|\uparrow\rangle_L|\downarrow\rangle_R - |\downarrow\rangle_L|\uparrow\rangle_R$$

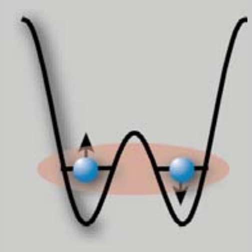
Unite



$$\left(|\uparrow,\downarrow\rangle - |\downarrow,\uparrow\rangle\right)\otimes\left(|0,1\rangle - |1,0\rangle\right)$$

Magnetic Gradient Fields Induce Singlet-Triplet Oscillations



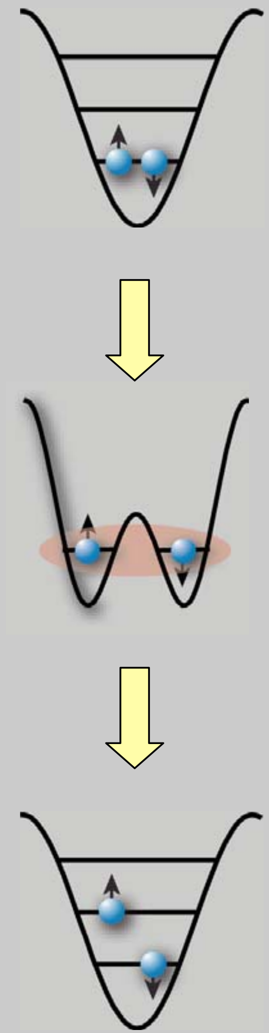


$\left|\uparrow\right\rangle_L \left|\downarrow\right\rangle_R + \left|\downarrow\right\rangle_L \left|\uparrow\right\rangle_R$ ⟺ $\left|\uparrow\right\rangle_L \left|\downarrow\right\rangle_R - \left|\downarrow\right\rangle_L \left|\uparrow\right\rangle_R$

$$\left|\uparrow\right\rangle_L \left|\downarrow\right\rangle_R + e^{2i\phi(t)} \left|\downarrow\right\rangle_L \left|\uparrow\right\rangle_R$$

$$\phi(t) = \mu_B B' d_{DW}$$

How can we detect the Bell pairs? (2)



$$\left(|\uparrow,\downarrow\rangle+|\downarrow,\uparrow\rangle\right)\otimes|0,0\rangle$$

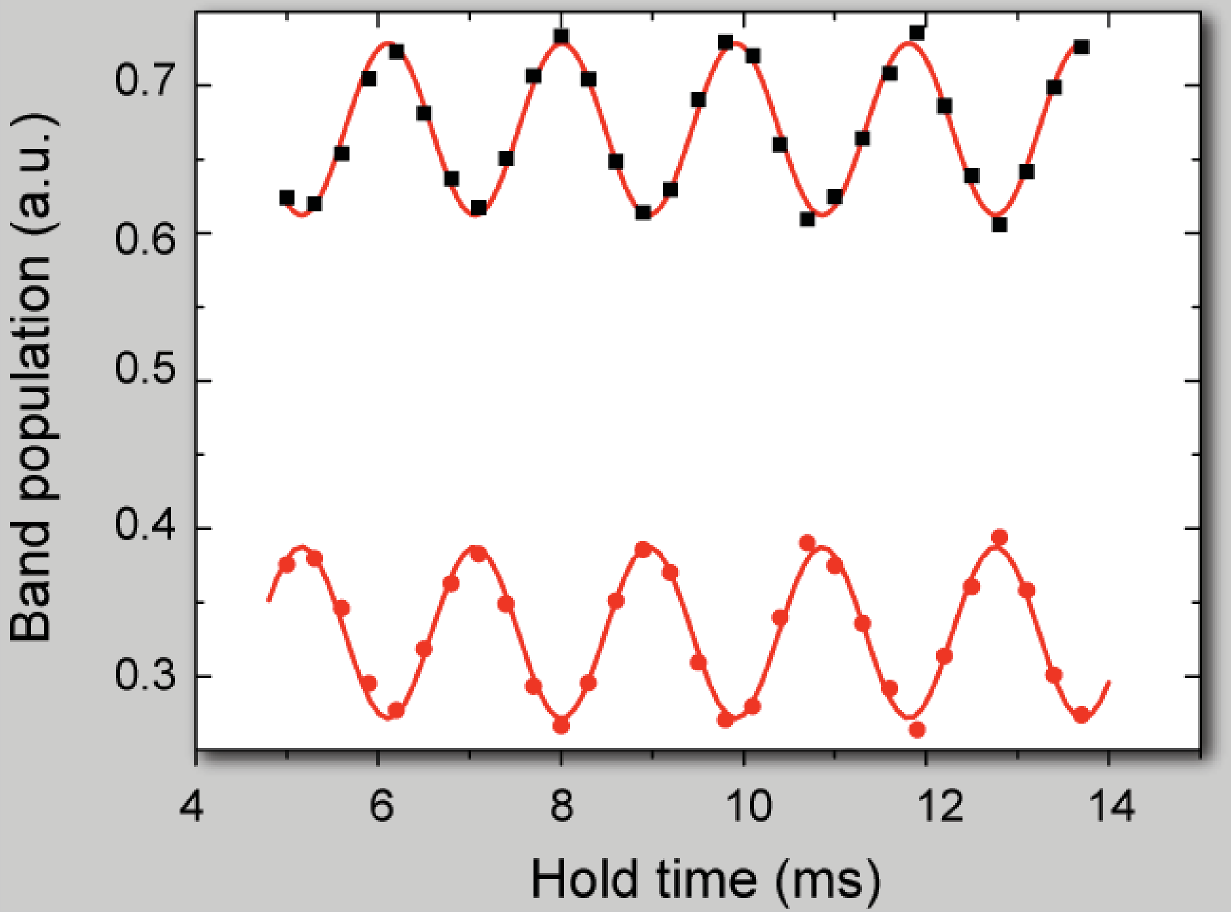
Split

When uniting bosonic spin singlet states, one particle has to occupy the excited band!

Unite

$$\left(|\uparrow,\downarrow\rangle+|\downarrow,\uparrow\rangle\right)\otimes\left(|0,1\rangle+|1,0\rangle\right)$$

Singlet-Triplet Spin Oscillations



See ion trap exps: C.F. Roos et al., PRL **92**, 220402 (2004),
C. Langer et al., PRL **95**, 060502 (2005)

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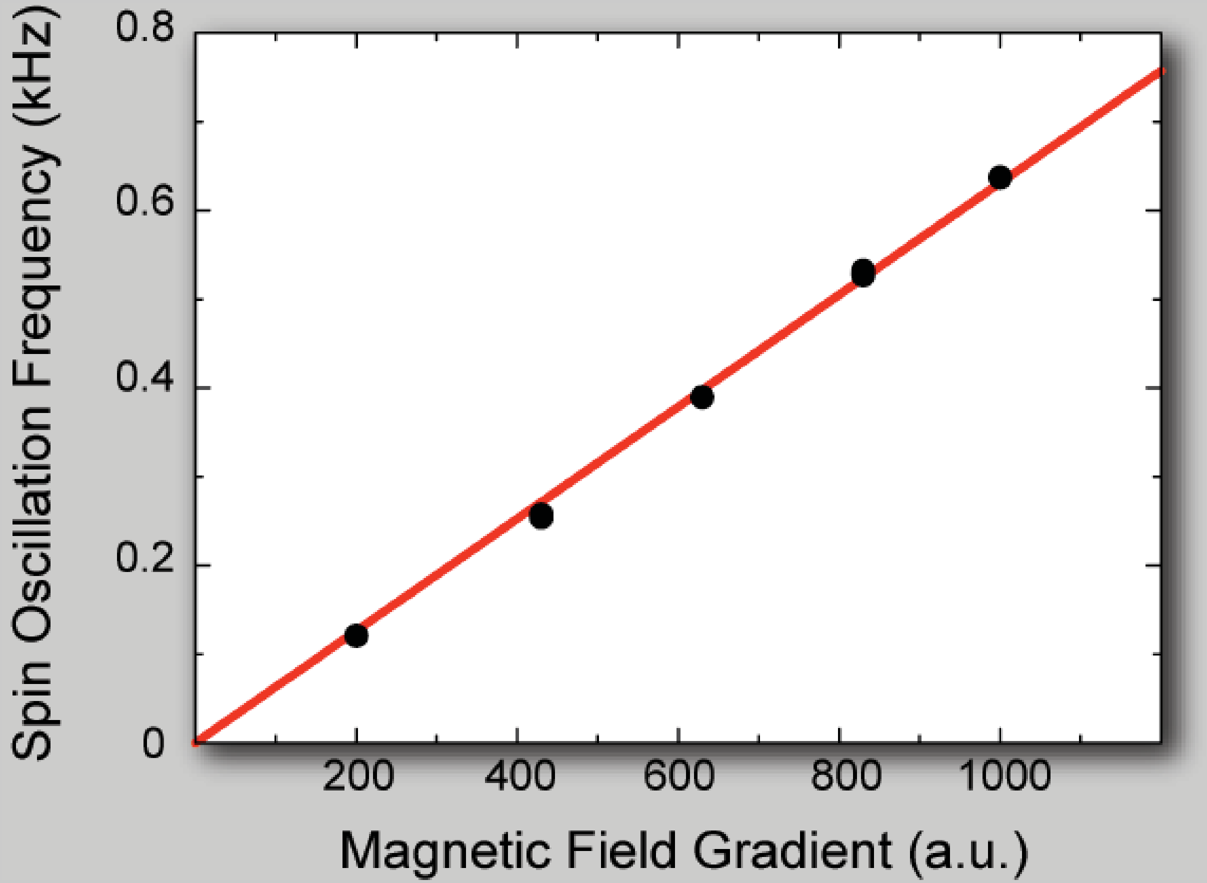
Oscillation Frequency vs. B-Field Gradient



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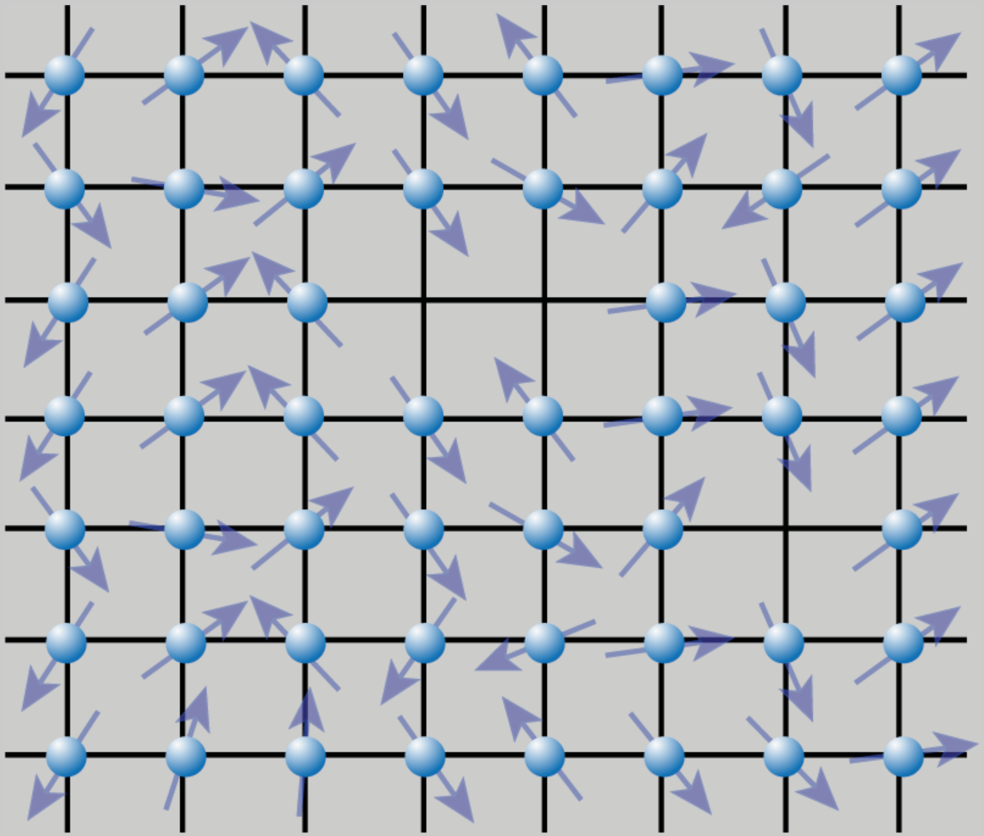
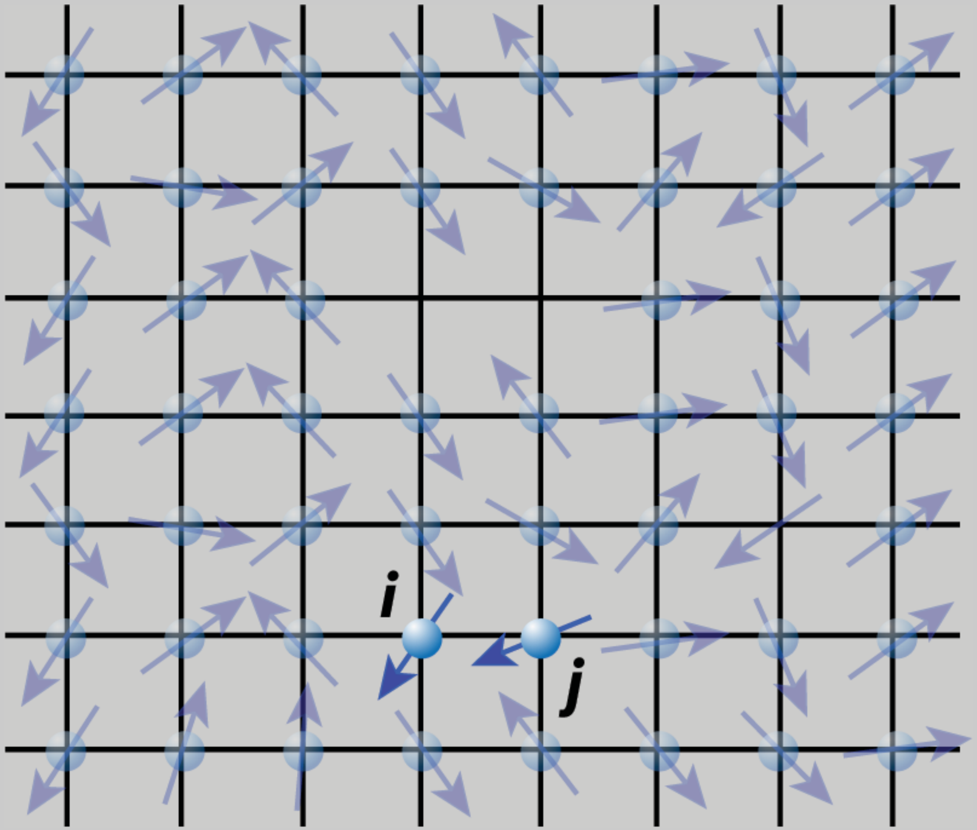
Quantum Spin Systems in Optical Lattices



***Double occupancy suppressed* in strongly interacting regime of Mott insulator.**

Quantum Spin Systems in Optical Lattices



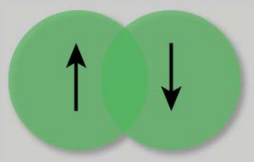
In strongly correlated electron system spin-spin interactions exist.

$$-J_{ex}\vec{S}_i \cdot \vec{S}_j$$

Origin of Spin-Spin Interactions – Exchange Interactions

$$-J_{ex}\vec{S}_1 \cdot \vec{S}_2$$
$$J_{ex} > 0$$

In Atoms
(e.g. excited state Helium)



$$-J_{ex}\vec{S}_i \cdot \vec{S}_j$$
$$J_{ex} < 0$$

In Molecules
(e.g. In molecule)

***Direct overlap of electronic wave functions* determines**
strength of exchange interactions
(typically very short ranged)

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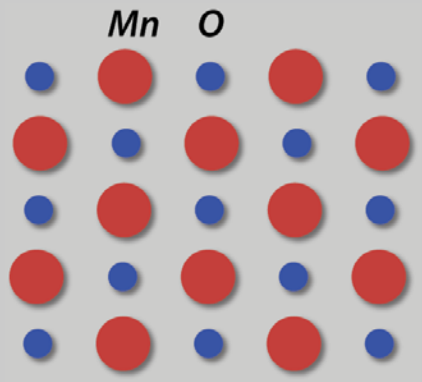
Origin of Spin-Spin Interactions – Exchange Interactions



?

Important ionic solids with no direct exchange between magnetic ions show magnetic ordering (MnO, CuO)!

„Super"-exchange interactions must be at work!

P.W. Anderson, Phys. Rev. **79**, 350 (1950)

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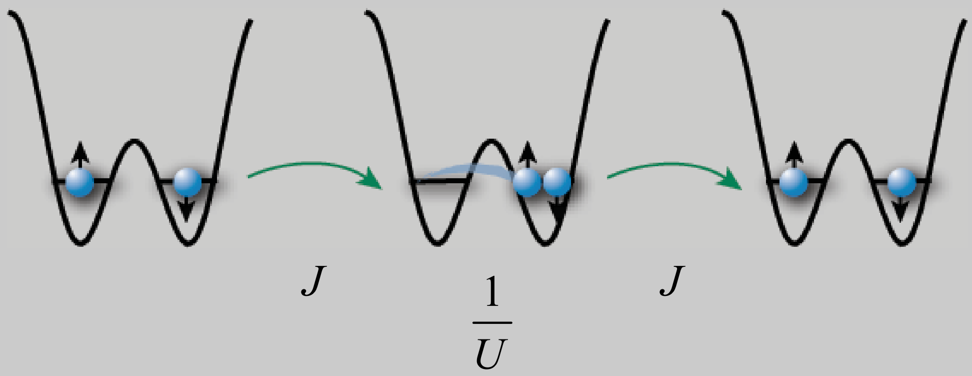
Quantum Magnetism

Second order hopping processes form the basis of superexchange interactions! (see e.g. A. Auerbach, Interacting Electrons and Quantum Magnetism)



$$-J\sum_{\langle i,j\rangle}\hat{a}_i^\dagger\hat{a}_j+\frac{1}{2}U\sum_i\hat{n}_i(\hat{n}_i-1)$$

$$H=-J_{ex}\sum_{\langle i,j\rangle}\mathbf{S}_i\cdot\mathbf{S}_j$$

$$J_{ex}\propto\frac{J^2}{U}$$

Ultracold atoms allow tuning of Spin-Hamiltonians

$$H=\sum_{\langle i,j\rangle}\left[\lambda_{\mu z}\hat{\sigma}_i^z\hat{\sigma}_j^z\pm\lambda_{\mu\perp}\left(\hat{\sigma}_i^x\hat{\sigma}_j^x+\hat{\sigma}_i^y\hat{\sigma}_j^y\right)\right]$$

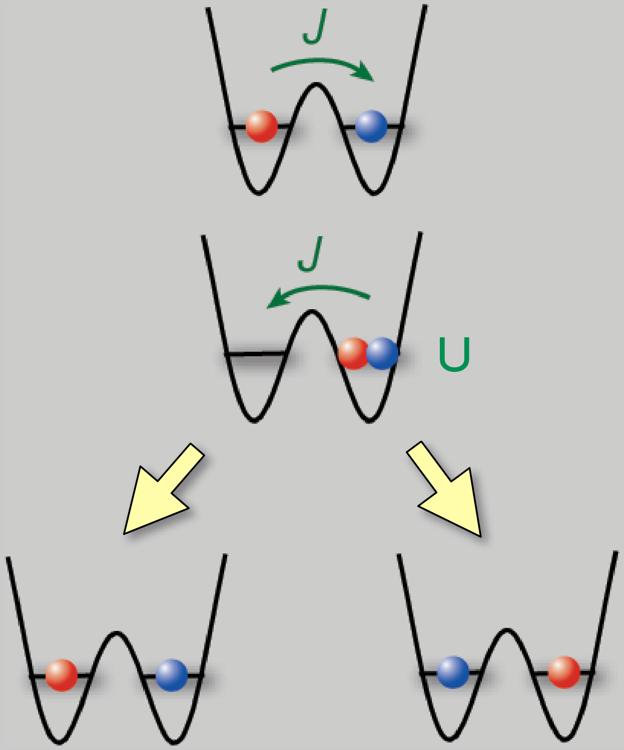
$$\lambda_{\mu z}=\frac{t_{\mu\uparrow}^2+t_{\mu\downarrow}^2}{2U_{\uparrow\downarrow}}-\frac{t_{\mu\uparrow}^2}{U_\uparrow}-\frac{t_{\mu\downarrow}^2}{U_\downarrow}$$

$$\lambda_{\mu\perp}=\frac{t_{\mu\uparrow}t_{\mu\downarrow}}{U_{\uparrow\downarrow}}$$

L.M. Duan et al., PRL **91**, 090402 (2003),
E. Altman et al., NJP **5**, 113 (2003), A.B. Kuklov et al. PRL **90**, 100401 (2003)

Deriving the Effective Spin Hamiltonian (1)

How do we get from $-J\sum_{\langle i,j\rangle}\hat{a}_i^\dagger\hat{a}_j+\frac{1}{2}U\sum_i\hat{n}_i(\hat{n}_i-1)$ to $H=-J_{ex}\sum_{\langle i,j\rangle}\mathbf{S}_i\cdot\mathbf{S}_j$?



$\hat{1} \quad + \quad \hat{X}_{LR}$

Deriving the Effective Spin Hamiltonian (2)

Second order hopping can be written as

$$H = -2\frac{J^2}{U}\left(1 + \hat{X}_{LR}\right)$$

$$\hat{X}_{LR}\left[\frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}}\right] = -\left[\frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}}\right]$$

$$\hat{X}_{LR}\left[\frac{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle}{\sqrt{2}}\right] = +\left[\frac{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle}{\sqrt{2}}\right]$$

$$H = -J_{ex}\,\hat{P}_{\text{triplet}}$$

0 Singlet

-J Triplet

Deriving the Effective Spin Hamiltonian (3)

$$\hat{P}_{\text{triplet}} = \hat{P}_{S=1}$$

$$\begin{aligned}
\mathbf{S}_L \cdot \mathbf{S}_R &= \frac{(\mathbf{S}_L + \mathbf{S}_R)^2}{2} - \frac{3}{4} \\
&= \frac{S(S+1)}{2} - \frac{3}{4} \\
&= \hat{P}_{S=1} - \frac{3}{4}
\end{aligned}$$

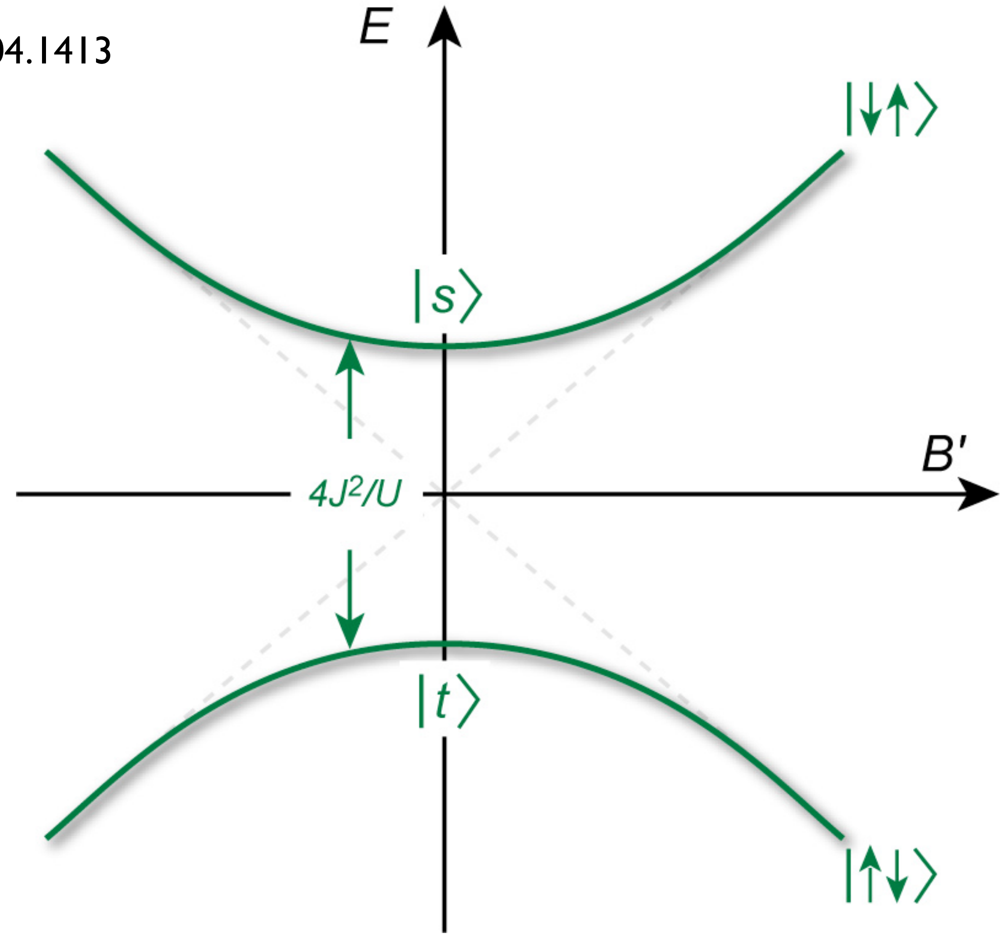
$$H = -J_{ex}\left(\mathbf{S}_L \cdot \mathbf{S}_R + \frac{3}{4}\right)$$

Direct Detection of Superexchange Interactions

A. M. Rey et al., cond-mat/0704.1413

$|s\rangle = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

$|t\rangle = |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$



$$H_{eff} = -J_{ex}\,\vec{S}_L \cdot \vec{S}_R - \mu_B B'\left(S_{z,L} - S_{z,R}\right)$$

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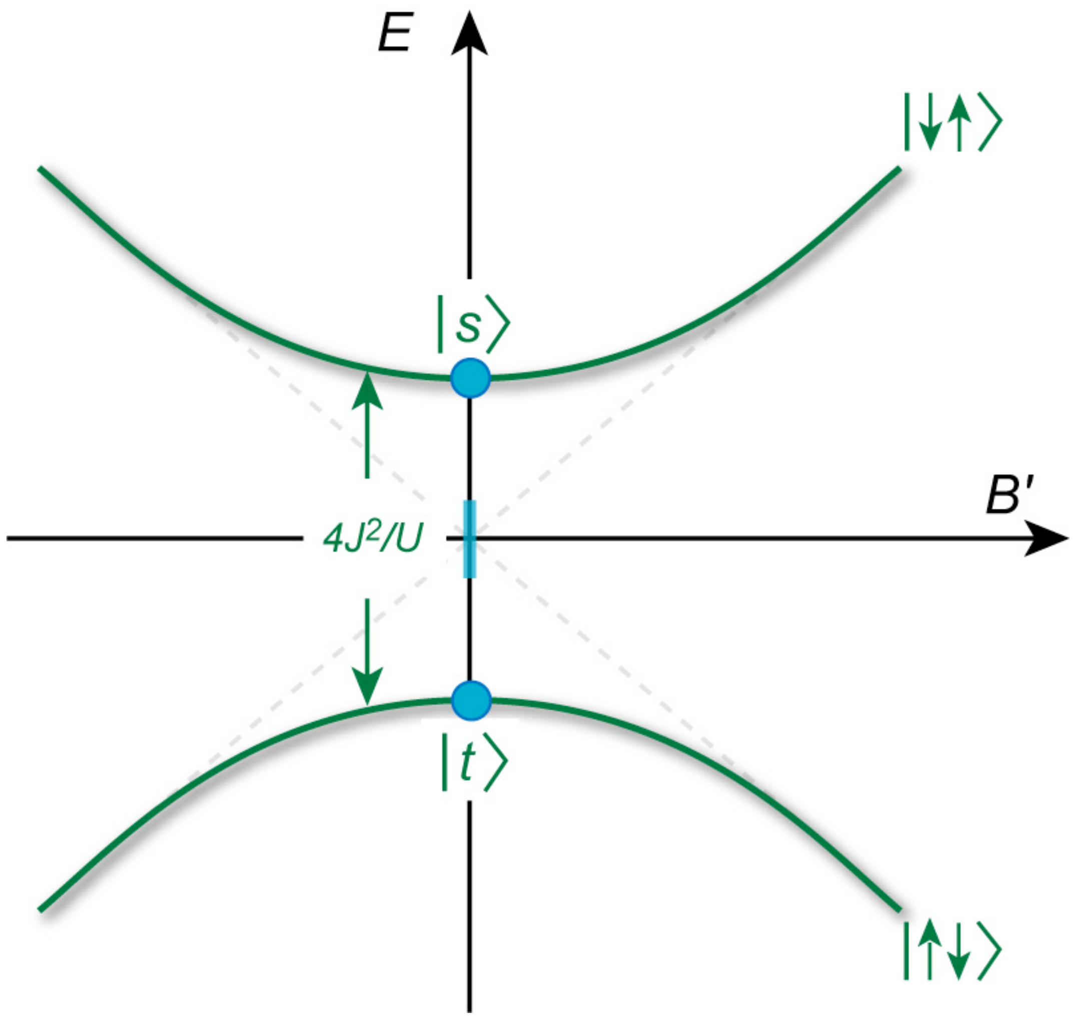
Direct Detection of Superexchange Interactions (2)



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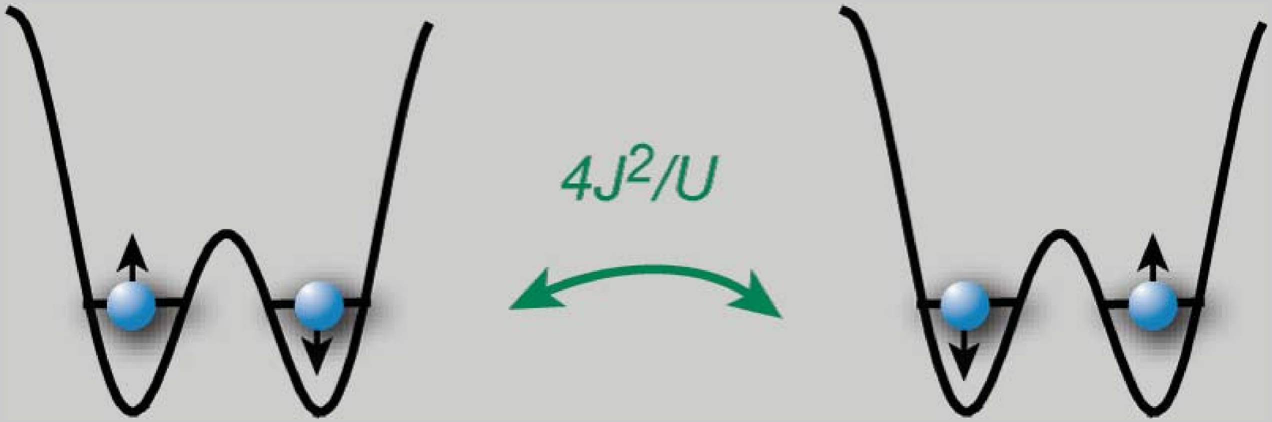
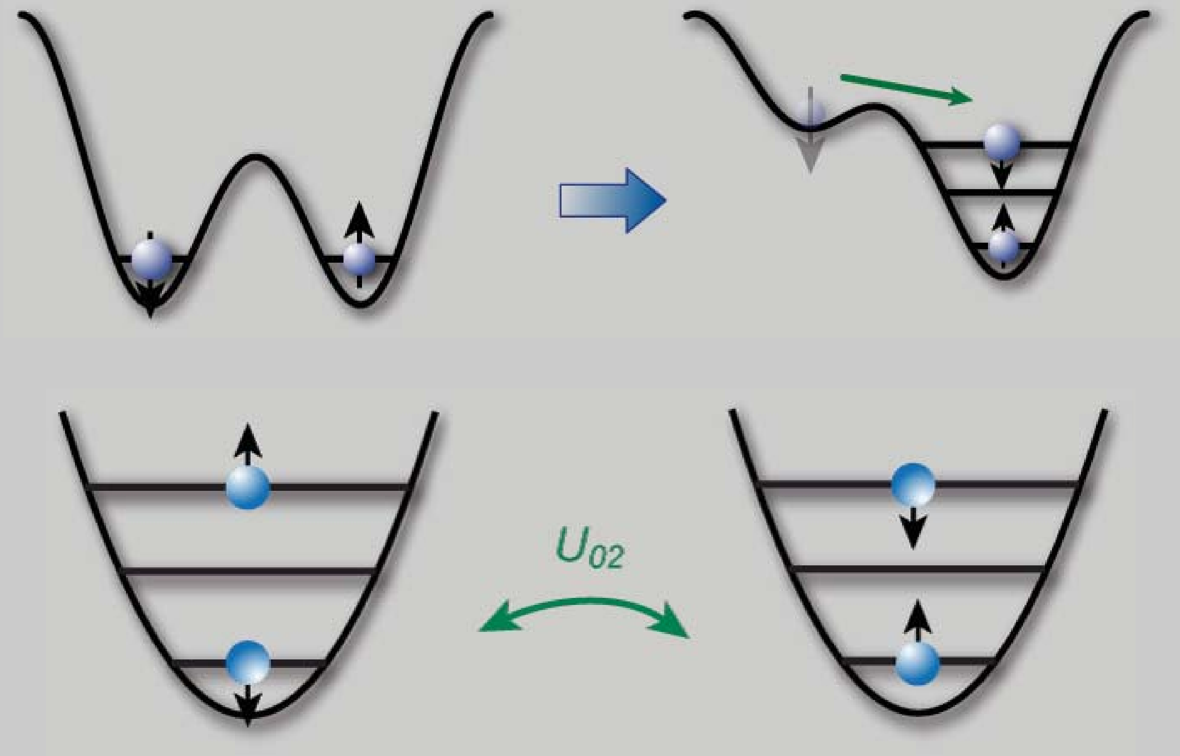
Superexchange induced flopping



$$H_{eff} = -J_{ex}\vec{S}_i \cdot \vec{S}_j$$
$$= -\frac{J_{ex}}{2}\left(\hat{S}_i^+\hat{S}_j^- + \hat{S}_i^-\hat{S}_j^+\right) - J_{ex}\hat{S}_i^z\hat{S}_j^z$$

One Other Feature – Onsite Exchange Interactions



Onsite exchange interaxtions can modify population

see M. Anderlini et al., Nature **448**, 452 (2007)

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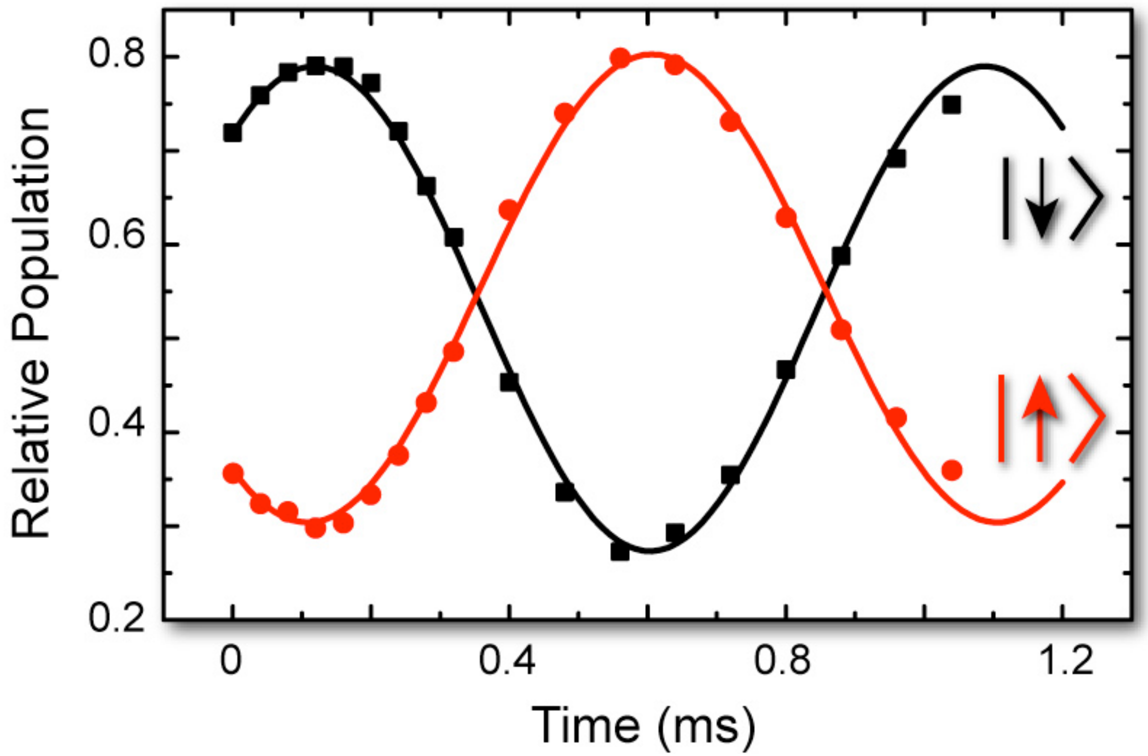
Onsite Exchange Interaction - Data



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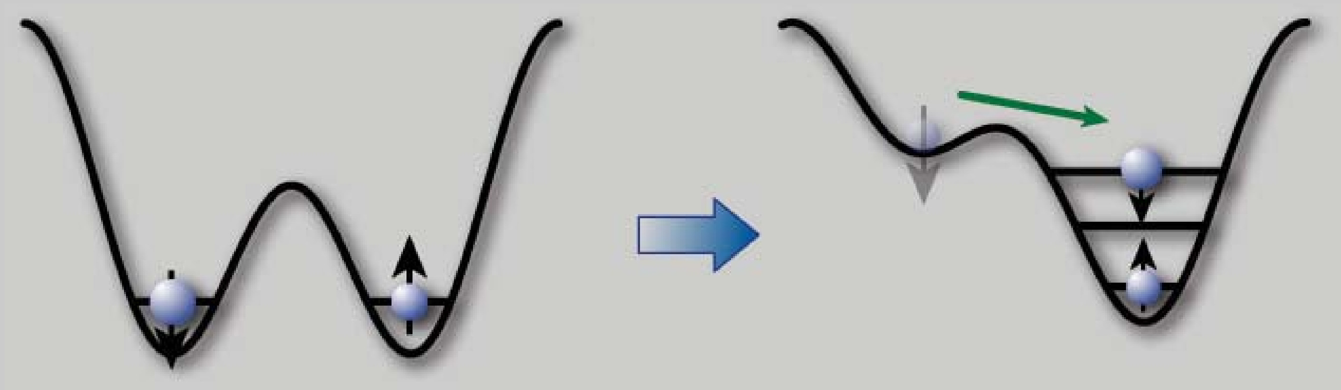
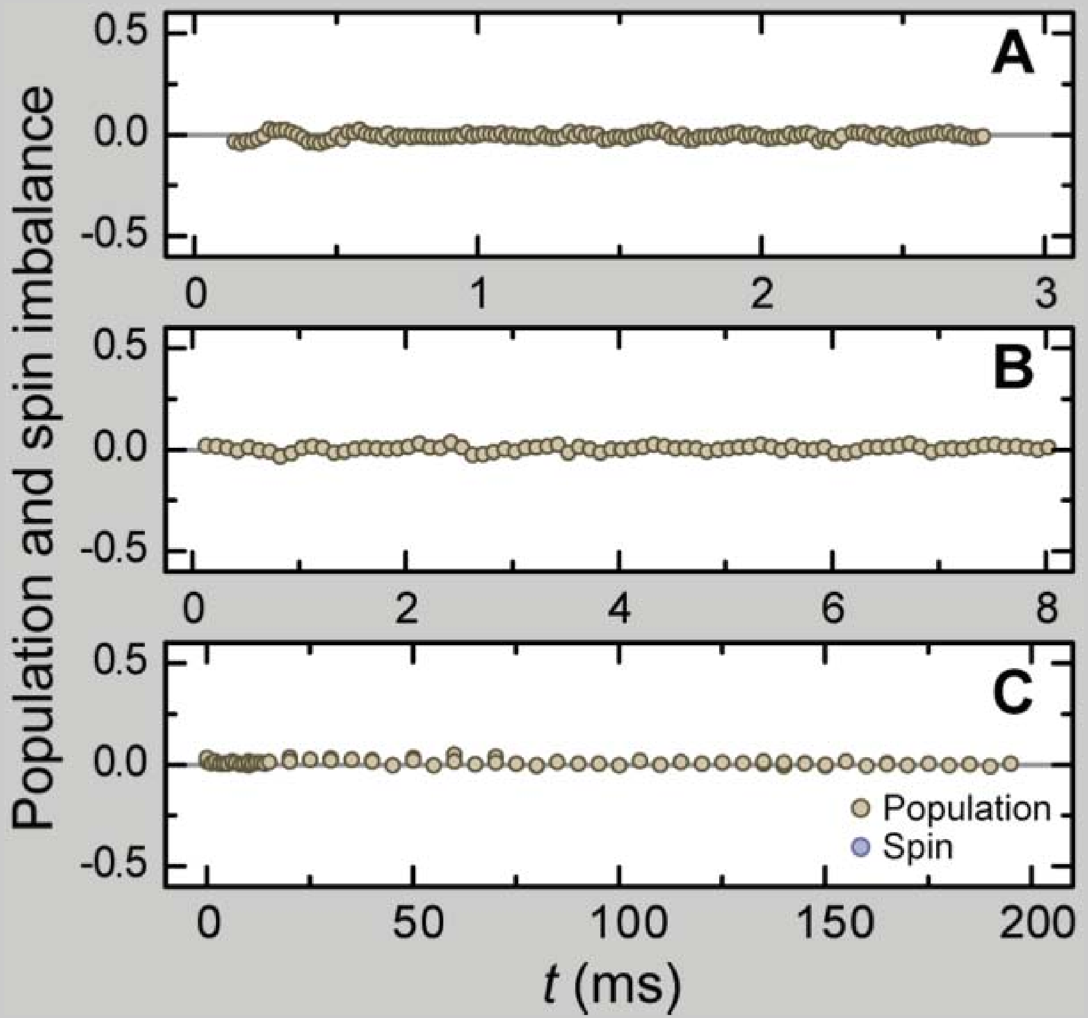
Mapping the Spins



Initial AF order verified in the experiment!

Superexchange induced flopping



$J/U = 1.25$
$V_{short} = 6\ E_r$

$J/U = 0.26$
$V_{short} = 11\ E_r$

$J/U = 0.05$
$V_{short} = 17\ E_r$

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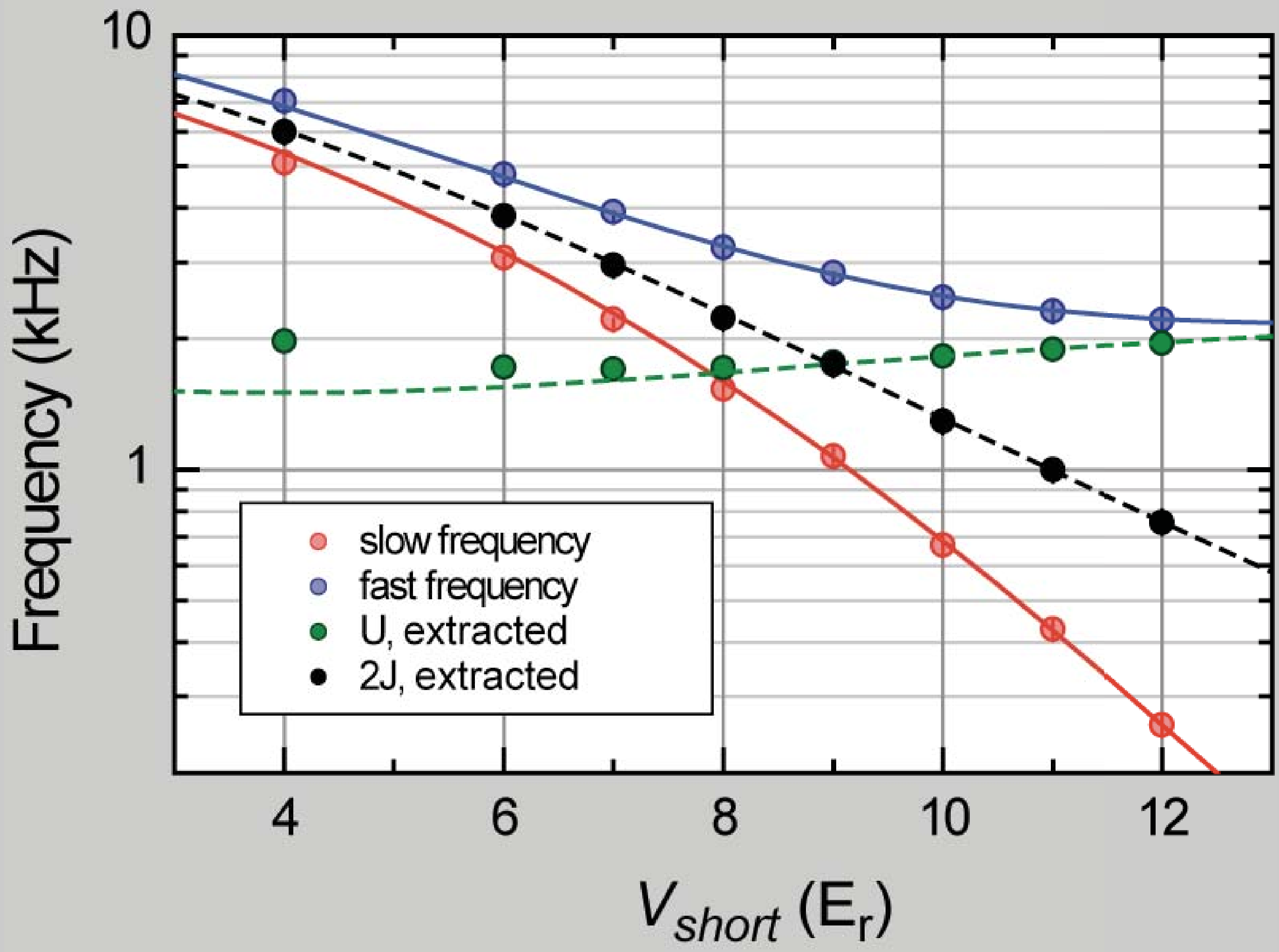
Oscillation Frequencies (1)



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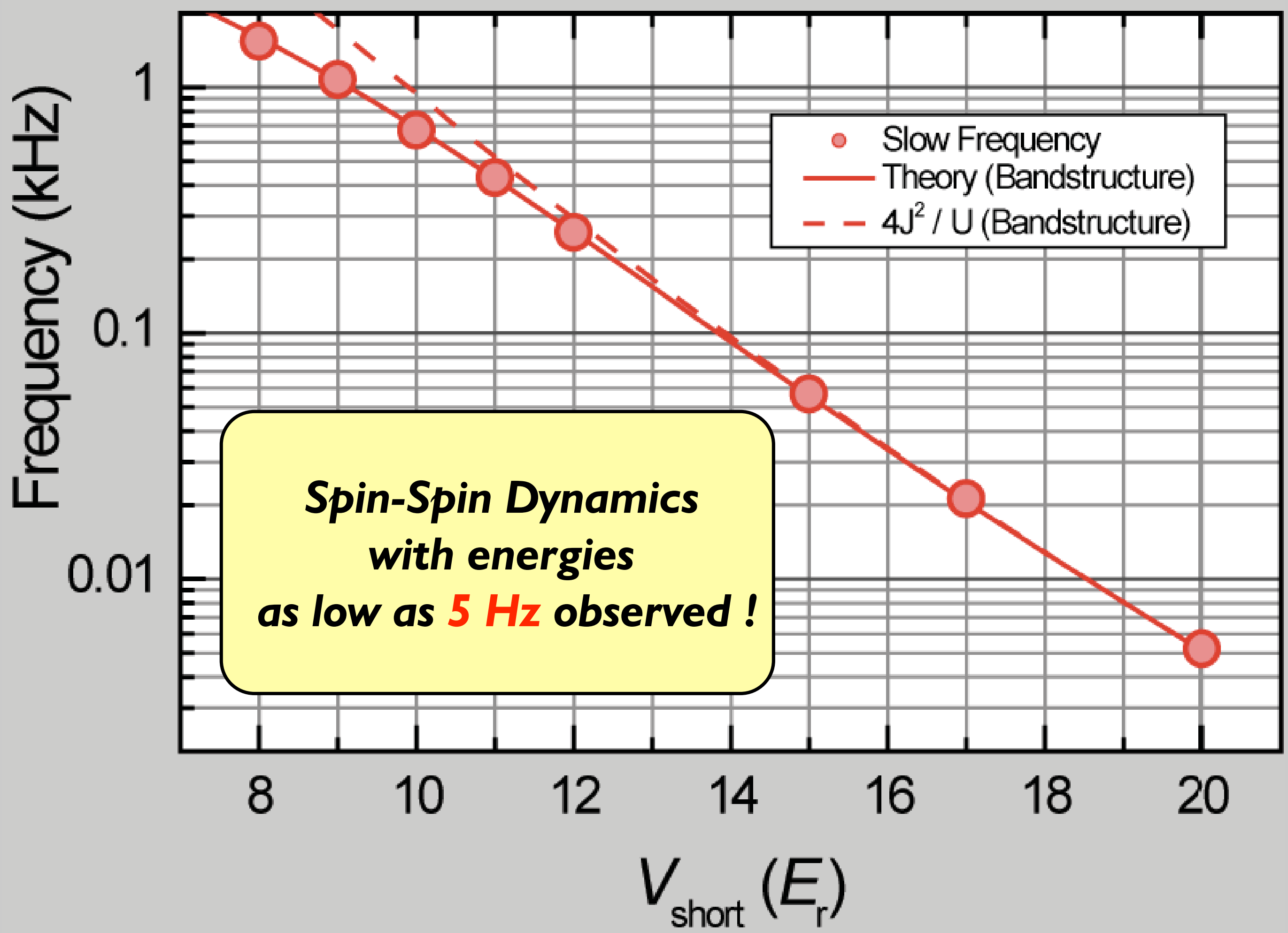
Oscillation Frequencies (2)



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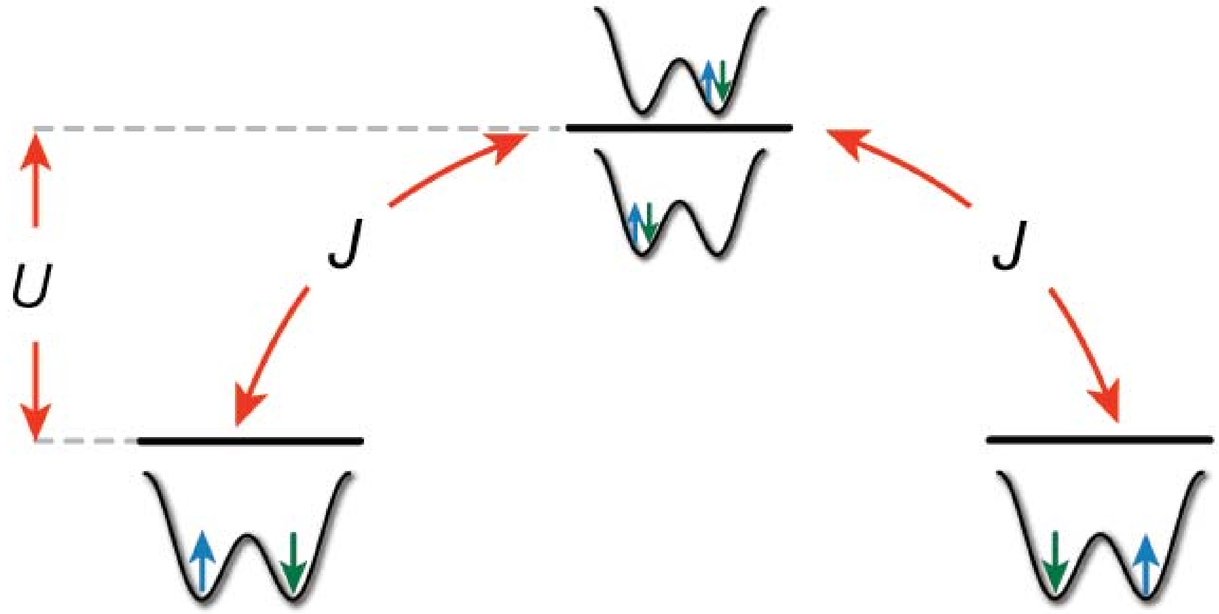
Controlling Superexchange Interactions

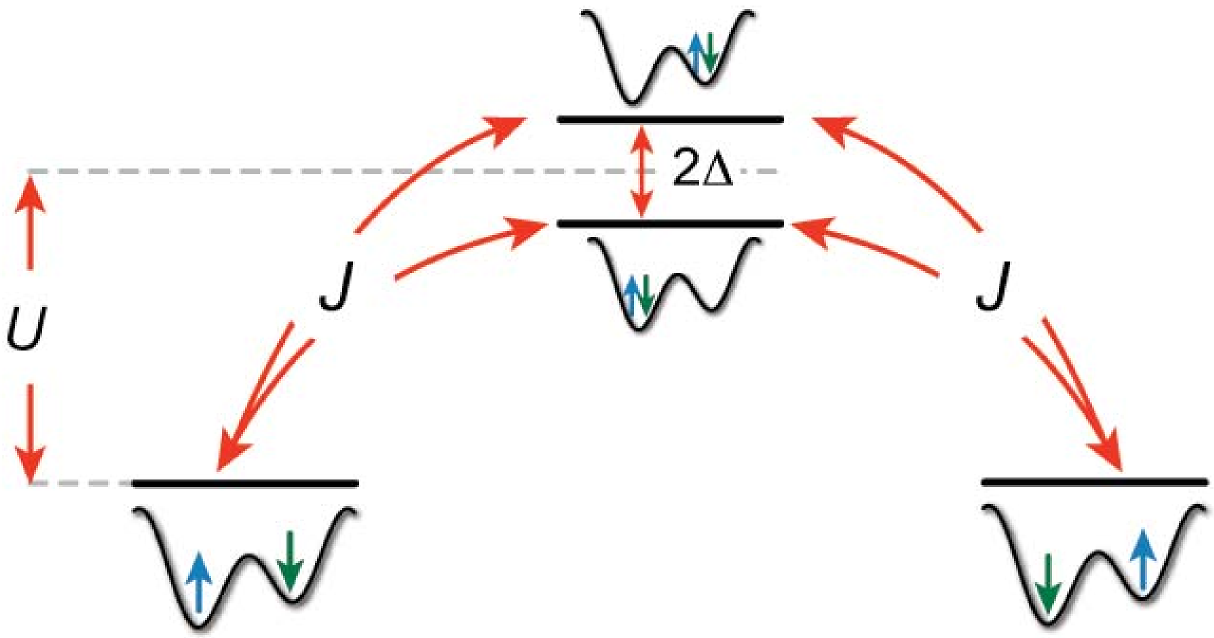
$$H_{eff} = -J_{ex}\vec{S}_i \cdot \vec{S}_j$$



$$J_{ex} \propto \frac{J^2}{U} + \frac{J^2}{U} = \frac{2J^2}{U}$$

Controlling Superexchange Interactions

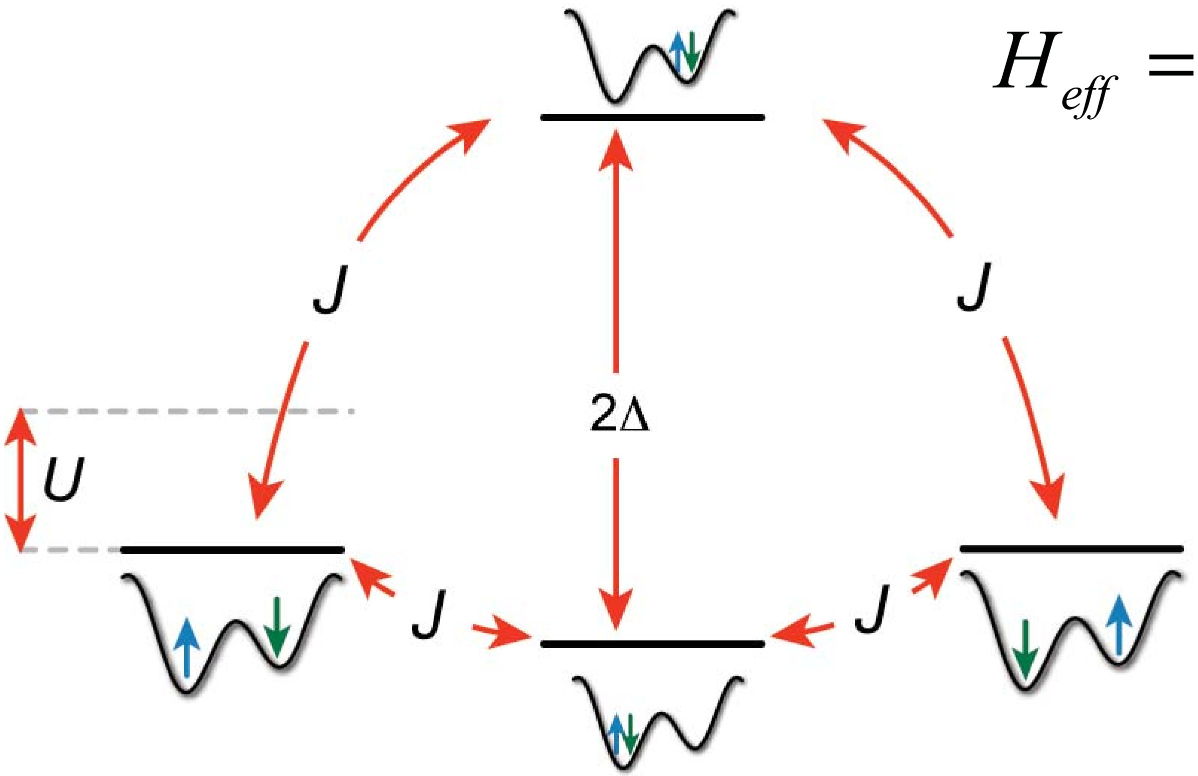
$$H_{eff} = -J_{ex}\vec{S}_i \cdot \vec{S}_j$$



$$J_{ex} \propto \frac{J^2}{U+\Delta} + \frac{J^2}{U-\Delta}$$

Controlling Superexchange Interactions

$$H_{eff} = -J_{ex}\vec{S}_i \cdot \vec{S}_j$$

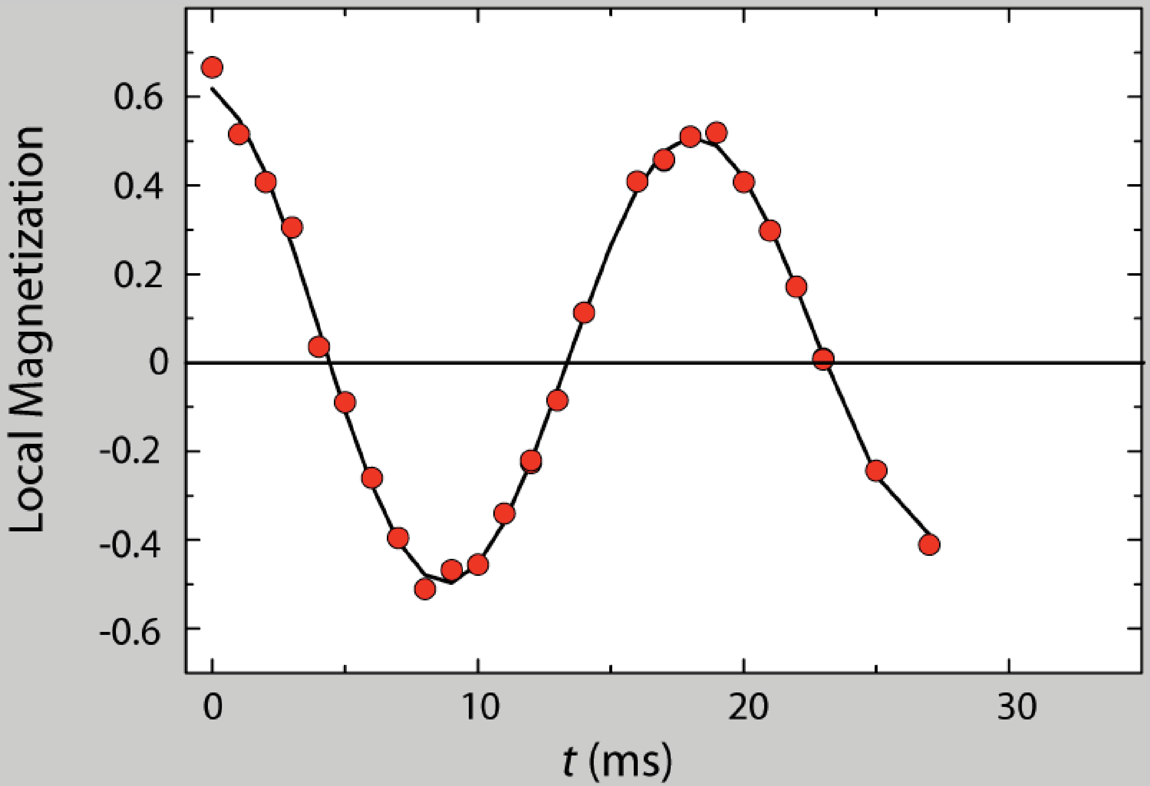


$$J_{ex} = \frac{J^2}{U+\Delta} + \frac{J^2}{U-\Delta}$$

Controlling Superexchange Interactions

Time evolution under action of ferromagnetic superexchange

$$H_{eff} = -J_{ex}\vec{S}_i \cdot \vec{S}_j$$



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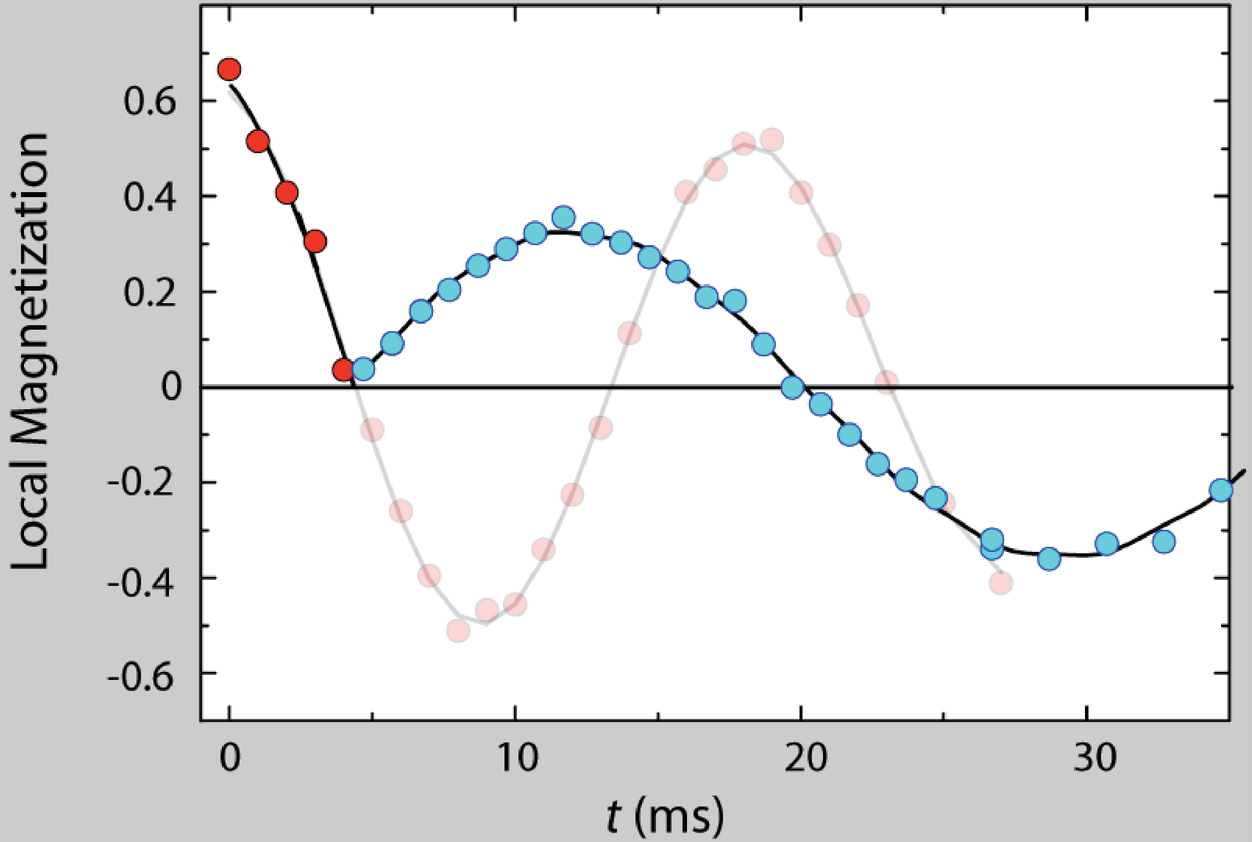
Controlling Superexchange Interactions

From ferromagnetic to antiferromagnetic superexchange interactions

$$H_{eff} = -J_{ex}\vec{S}_i \cdot \vec{S}_j \quad \Rightarrow \quad H_{eff} = +J'_{ex}\vec{S}_i \times \vec{S}_j$$



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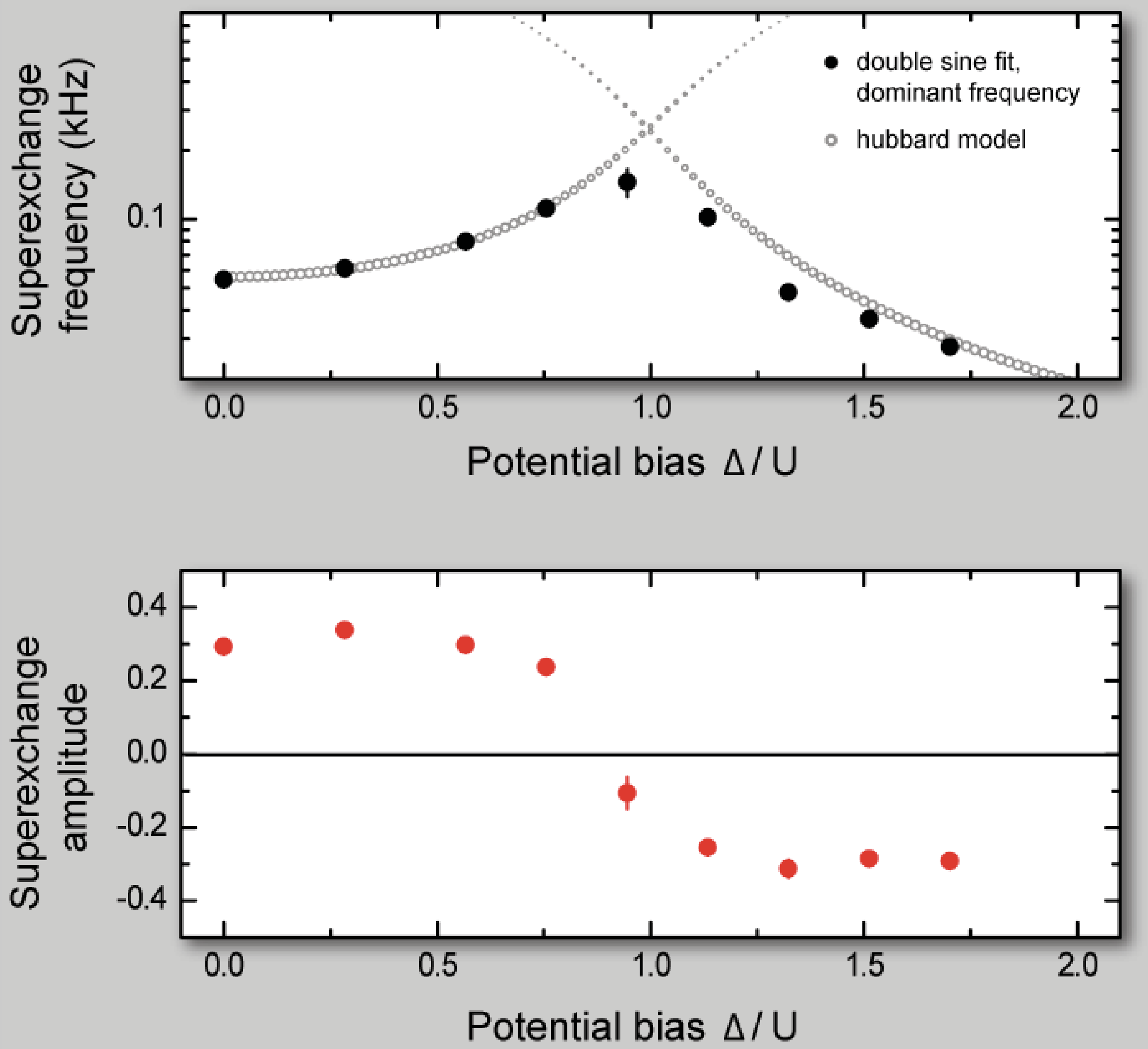
Controlling Superexchange Interactions



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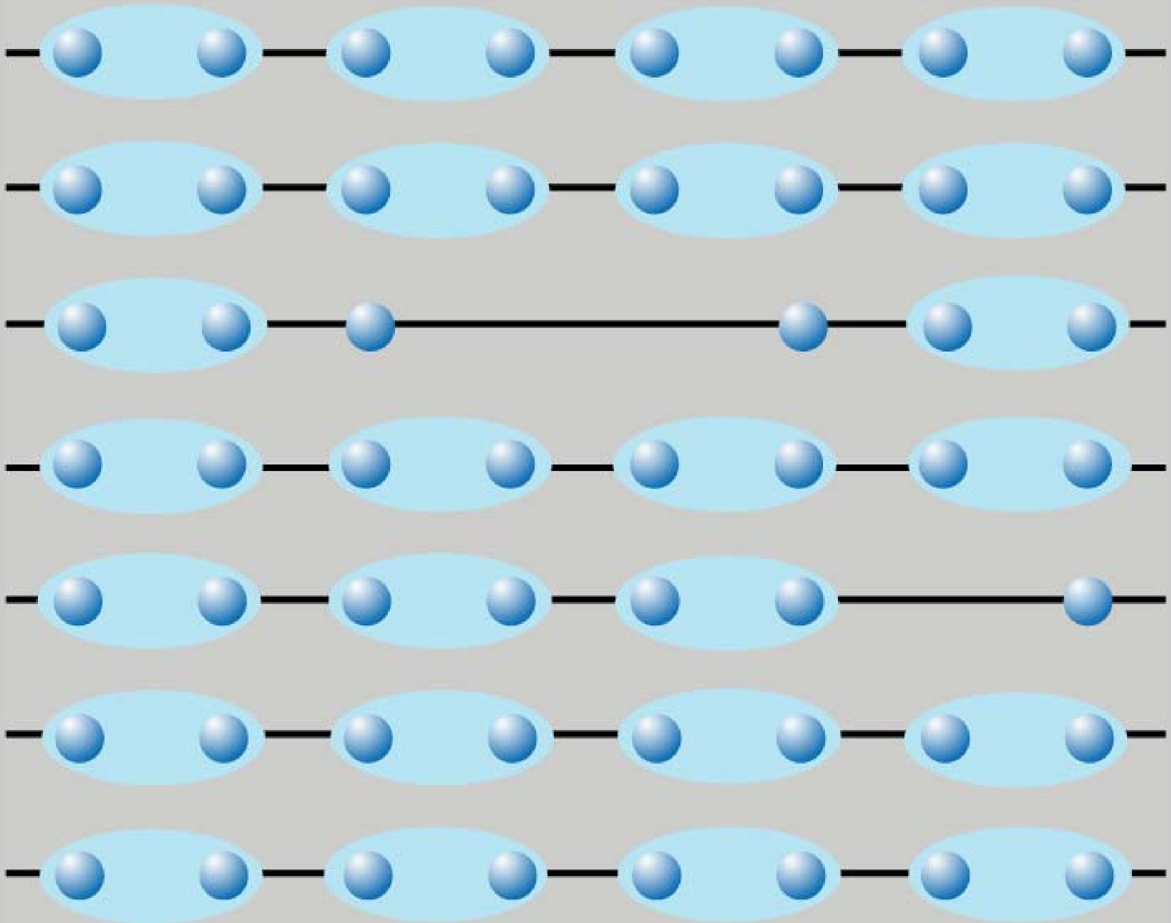
Dynamics of Valence Solid-Type States?



A. M. Rey et al., cond-mat/0704.1413

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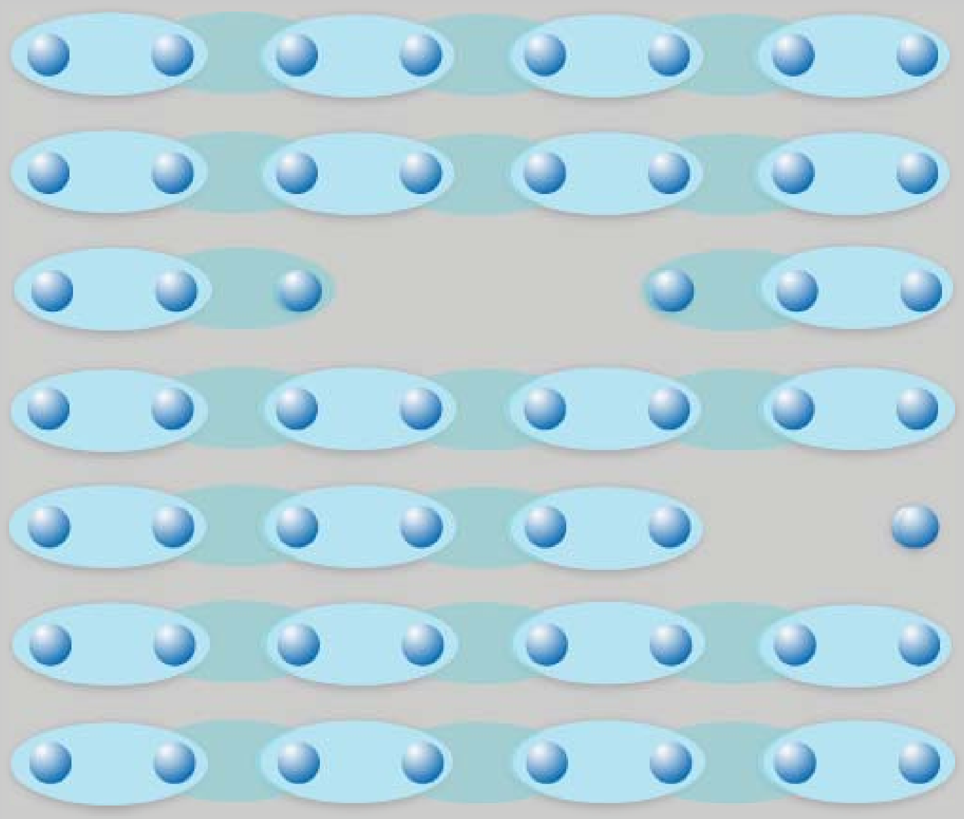
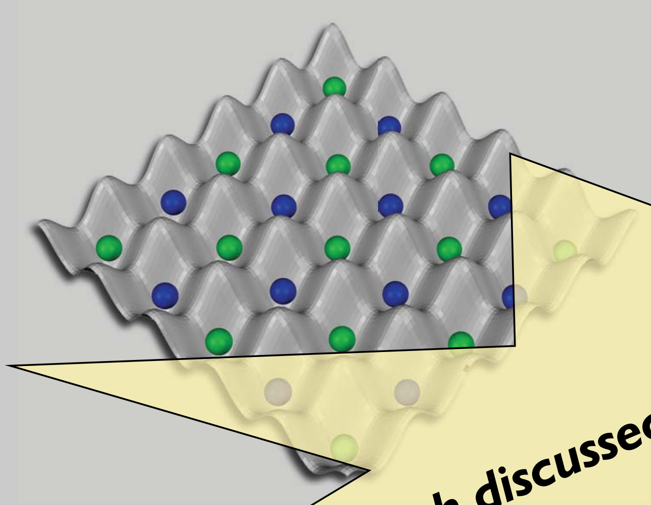
Large Entangled States



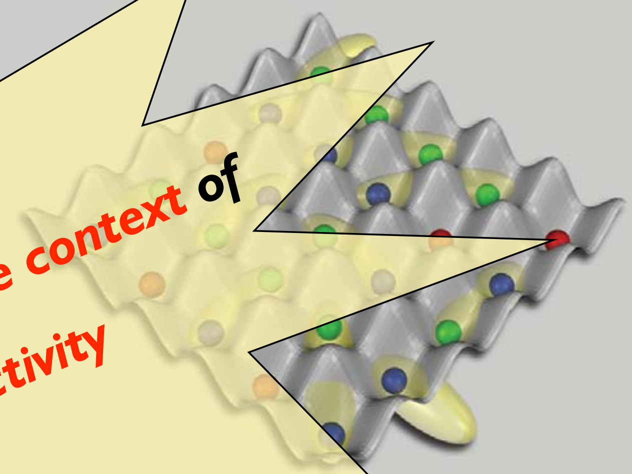
Pushbutton *simultaneous creation* of *thousands of entangled Bell pairs* in a single experiment.

***Connection of entanglement* possible via e.g. superexchange interactions.**

Outlook – Controlled Disorder & Interactions



Antiferromagnetic Mott Insulator



Spin liquid

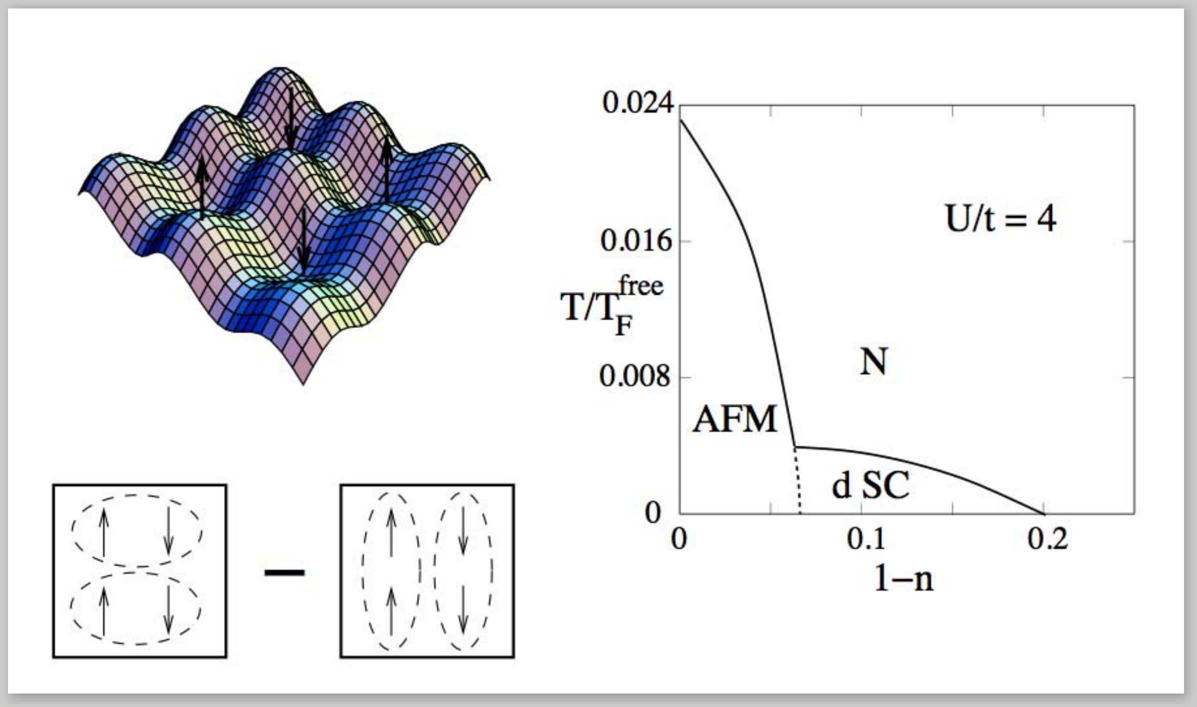
Much discussed scenario in the context of High-Tc superconductivity

Doping destroys AF order and possibly creates a spin liquid with superconducting phase!

P.A. Lee, N. Nagaosa, X.-G. Wen, Rev. Mod. Phys., 78 (2006)

Hubbard Model and High-Tc

Can we help to identify the phase diagram of the Hubbard model?



W. Hofstetter, J.I Cirac, P. Zoller, E. Demler, M. D. Lukin, PRL **89**, 220407 2002.

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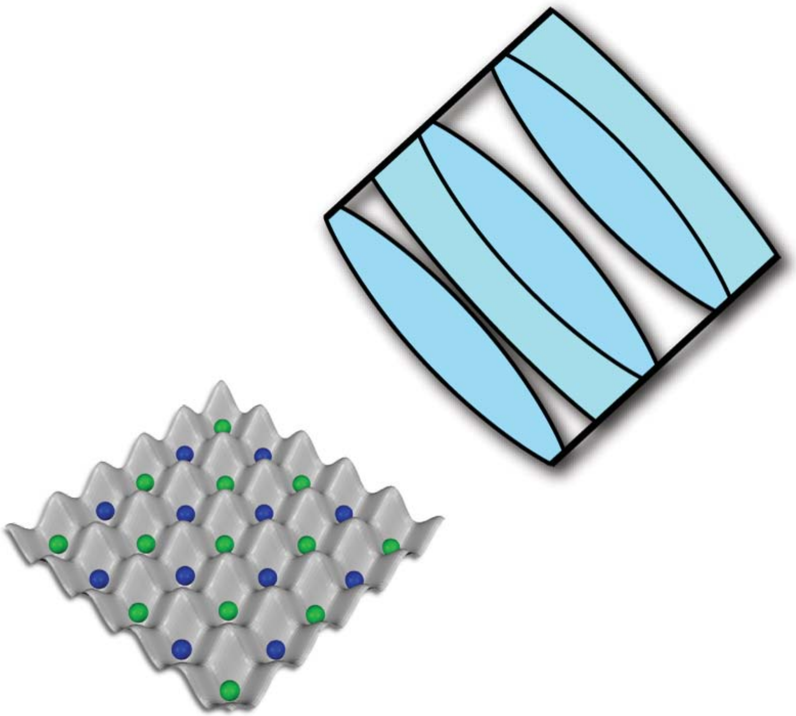
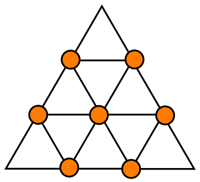
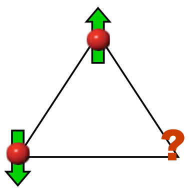
Towards Single Site Imaging



- *Manipulate and detect single atoms in parallel, spatially resolved*
- *See 100x100 Atoms, spin state resolved in the lattice*
- *Reveal Dynamics*
- *Measure Spin-Spin, Density-Density Correlation functions*
- *Different Lattice Geomteries Triangular/Hexagonal*
- *Frustration Effects*





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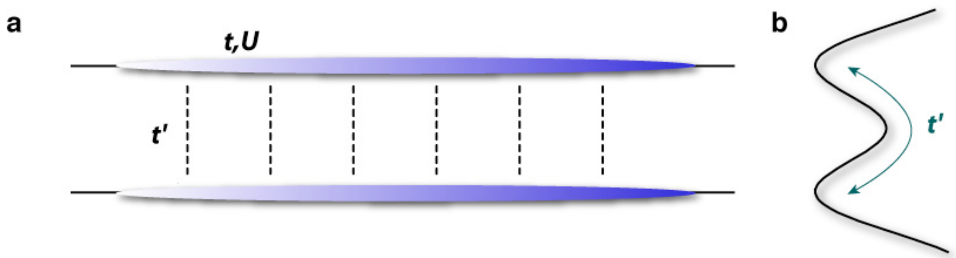
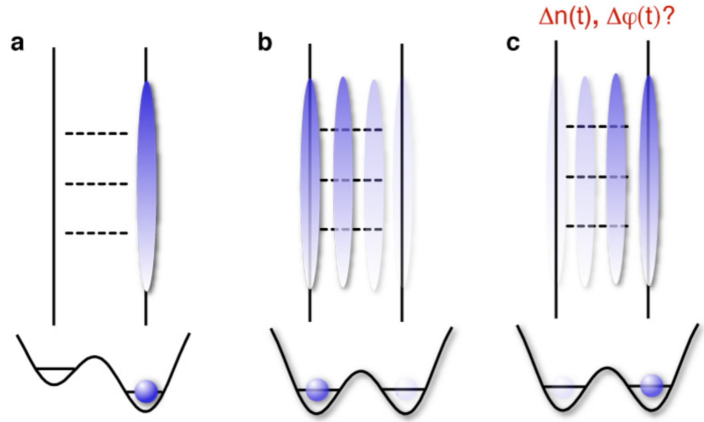
Nonequilibrium Dynamics in Many-Body Systems

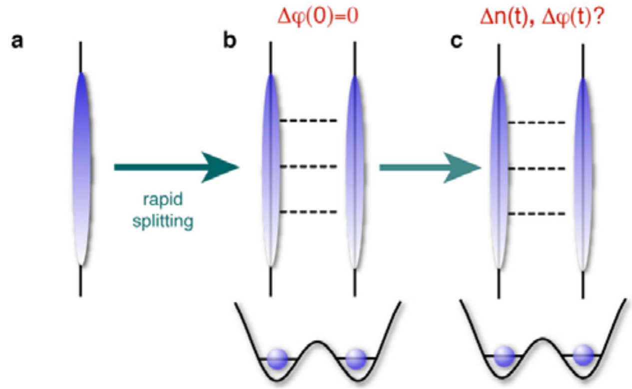


Realization of coupled one dimensional quantum systems – tuneable from weakly interacting to strongly interacting



Out of equilibrium dynamical evolution

See also Th. Giamarchi,
Quantum Physics in One Dimension
Numerical Methods: DMRG, MPS (see Cirac)



Generation of nonclassical correlations and mesoscopic Schrödinger cat states.

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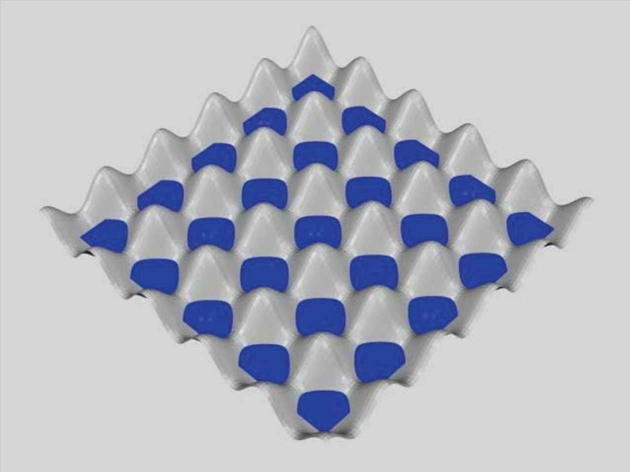
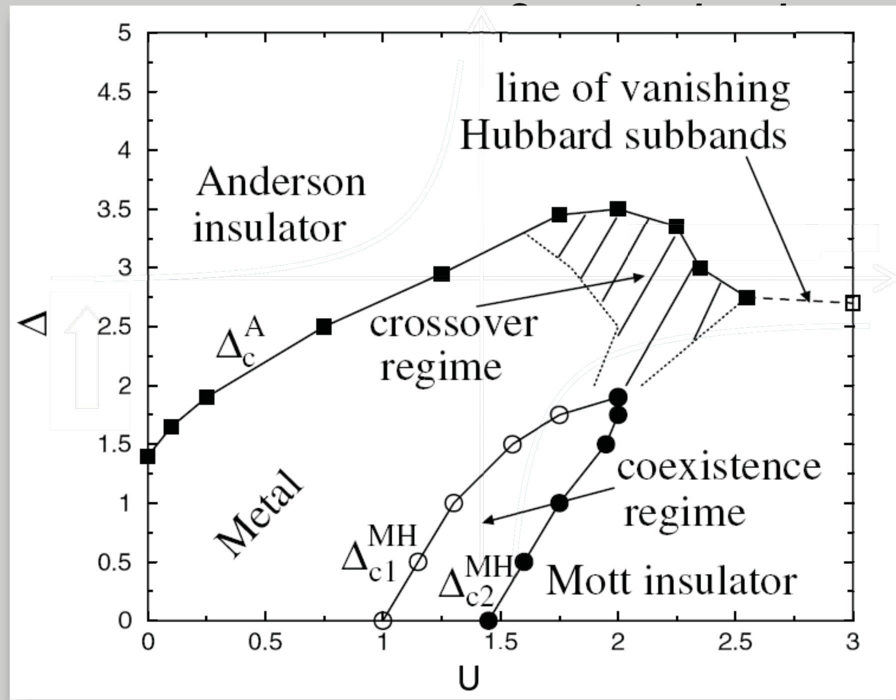
Outlook – Controlled Disorder & Interactions



K. Günter et al., PRL **96**, 180402 (2006),
C. Ospelkaus et al., PRL **96**, 180403 (2006)

L. Fallani et al., PRL **98**, 130404 (2006)

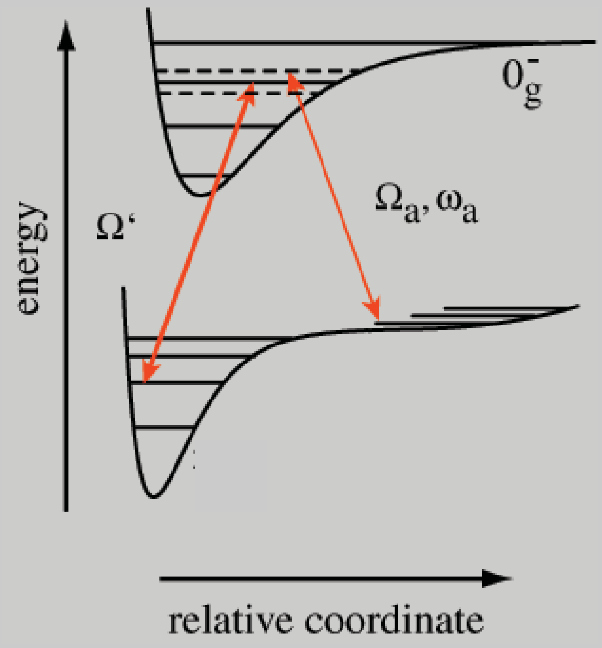
D. Clément et al., PRL **95**, 170409 (2005), C. Fort et al., PRL **95**, 170410 (2005), T. Schulte et al., PRL **95**, 170411 (2005)



K. Byczuk, W. Hofstetter & D. Vollhardt, PRL 94, 056404 (2005)

Controlled disorder via second atomic species & Feshbach resonance

Generation of Polar Ground State Molecules

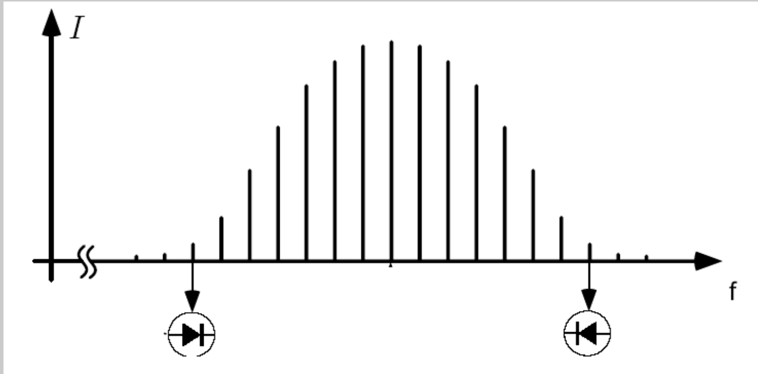


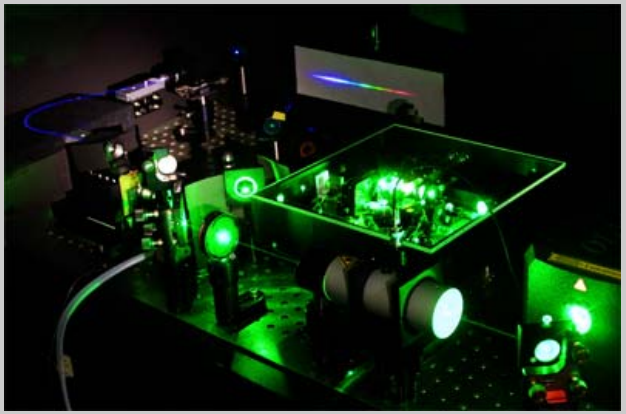
Goal: create polar molecules in any specified rovibrational states

⇨ ***Strong spin-spin interactions***

See A. Micheli, G.K. Brennen & P. Zoller, Nat. Phys. **2**, 341 2006.

Femtosecond Laser is a „Ruler" in Frequency Space





Th. W. Hänsch et al. (MPQ Garching)

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Many-Body Physics with Ultracold Gases

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(Dated: March 2007)

This article reviews recent experimental and theoretical progress on many-body phenomena in dilute, ultracold gases. Its focus are effects beyond standard weak-coupling descriptions, like the Mott-Hubbard-transition in optical lattices, strongly interacting gases in one dimension or quasi two-dimensional gases in fast rotation. Strong correlations in fermionic gases are discussed in optical lattices or near Feshbach resonances in the BCS-BEC crossover.

Contents

I. INTRODUCTION

The achievement of Bose-Einstein-Condensation (BEC) (Anderson *et al.*, 1995; Bradley *et al.*, 1995; Davis *et al.*, 1995) and of Fermi degeneracy (DeMarco and Jin, 1999; Truscott *et al.*, 2001) in ultracold, dilute gases has opened a new chapter in atomic and molecular physics in which the particle statistics and their interactions, rather than the study of single atoms or photons, are at center stage. For a number of years, a main focus in this field has been to explore the wealth of phenomena associated with the existence of coherent matter waves. Major examples include the observation of interference of two overlapping condensates (Andrews *et al.*, 1997), of long range phase coherence (Bloch *et al.*, 2000) or of quantized vortices and vortex lattices

arXiv:0704.3011

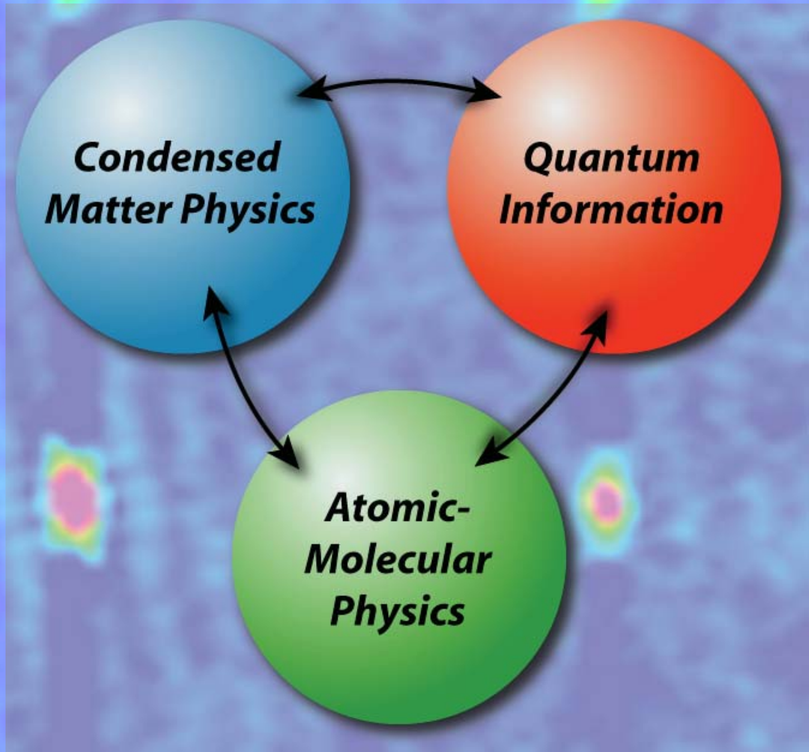
Fermions in Lattices
(Hubbard Model, Superconductivity)

Bose-Fermi mixtures

Disordered Systems

Quantum Magnets
(in spin mixtures, Ising, XY model, Heisenberg model)

Quantum Ladders
static and dynamic properties



Towards a Quantum Computer

Create **large scale entanglement**

Robust qubits

fast quantum gates

Decoherence?

single site addressing ?

Spin squeezing

High precision spectroscopy, Search for **EDM**
Controlled Molecule Formation in arbitrary quantum states
Formation of **heteronuclear molecules** with dipole moments
Control interaction properties
(mag. & opt. Feshbach resonances)