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Summer School on Novel Quantum Phases and Non-Equilibrium Phenomena in Cold Atomic Gases

27 August - 7 September, 2007

Strongly correlated systems: from electronic materials to cold atoms to photons (Parts I & II)

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Strongly correlated many-body systems: from electronic materials to cold atoms to photons

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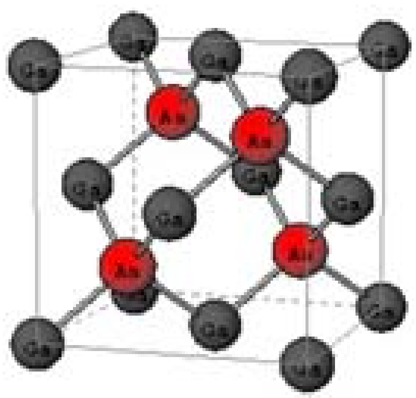
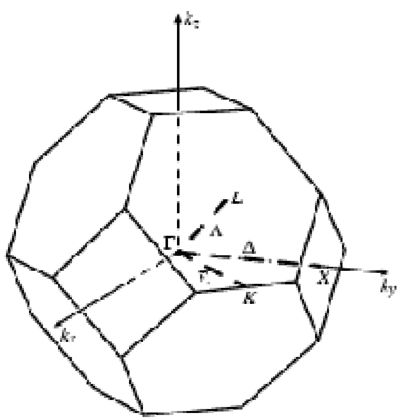
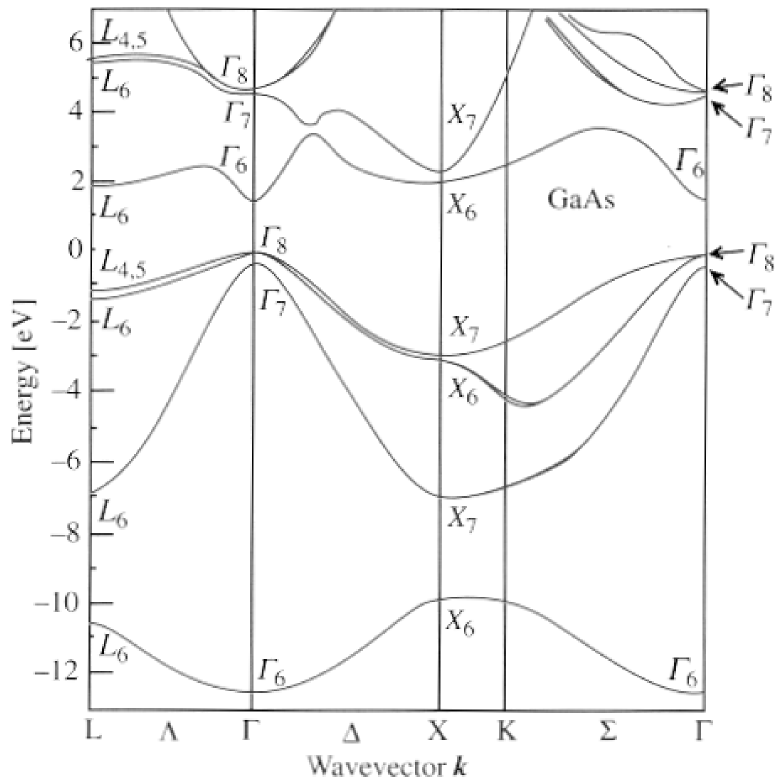
Strongly correlated electron systems

“Conventional” solid state materials

Bloch theorem for non-interacting
electrons in a periodic potential

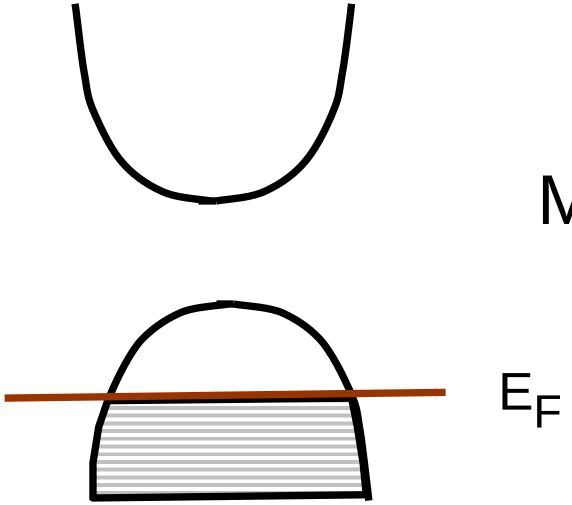




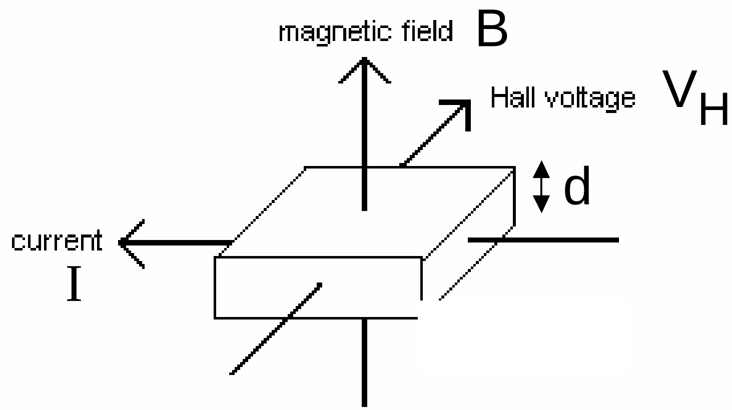


Electronic band structure of GaAs

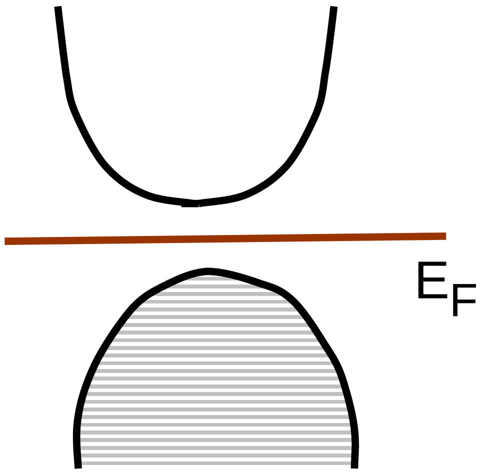
Consequences of the Bloch theorem



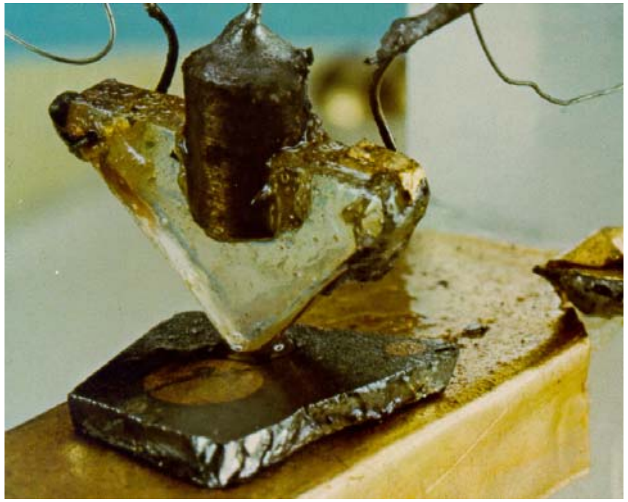
Metals



$$V_H = \frac{I\,B}{n\,e^*\,d}$$



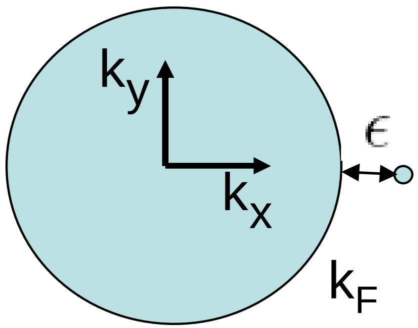
Insulators
and
Semiconductors



First semiconductor transistor

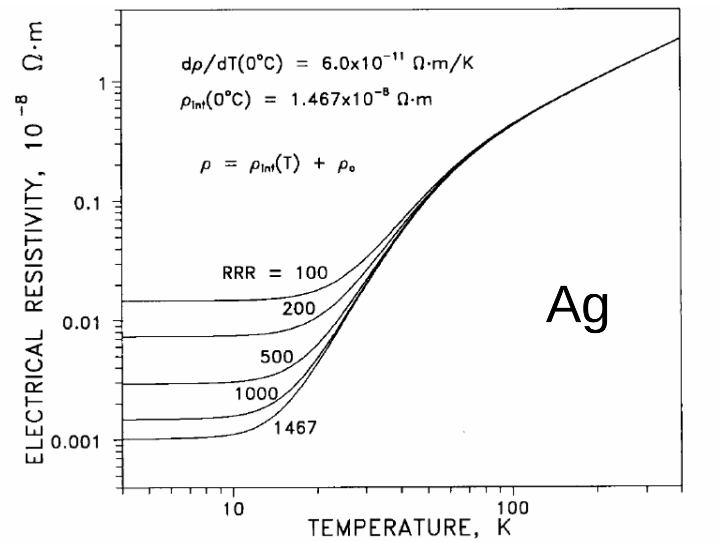
“Conventional” solid state materials

Electron-phonon and electron-electron interactions
are irrelevant at low temperatures

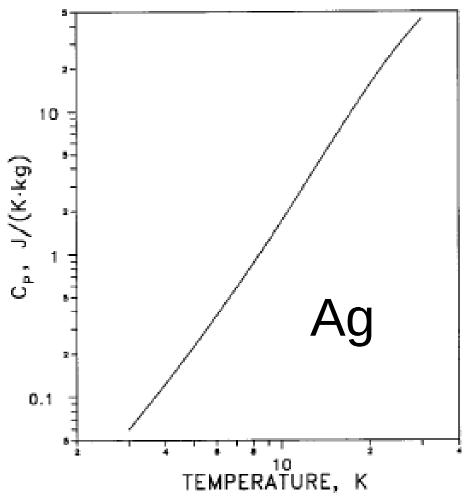


$$\frac{1}{\tau_{e-e}} \sim \epsilon^2 \qquad \frac{1}{\tau_{e-ph}} \sim \epsilon^3$$

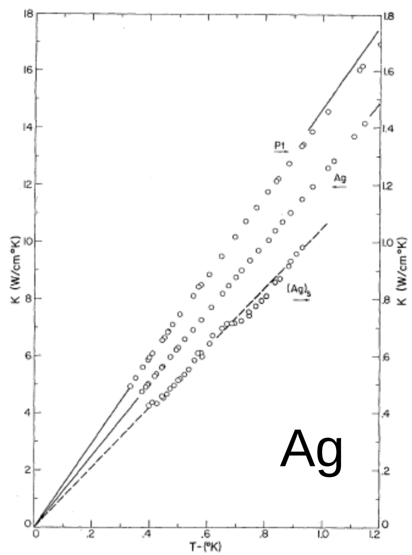
Landau Fermi liquid theory: when frequency and temperature are smaller than E_F electron systems are equivalent to systems of non-interacting fermions



$$\rho = \rho_0 + aT^2$$

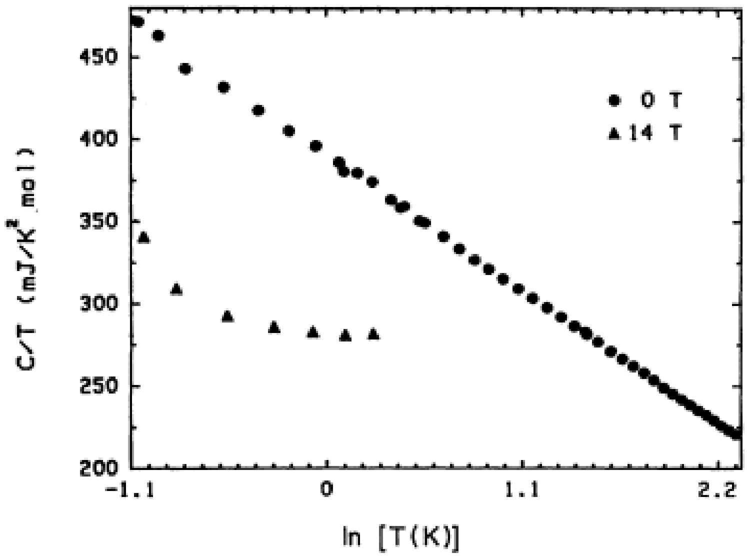


$$c/T = \text{const}$$



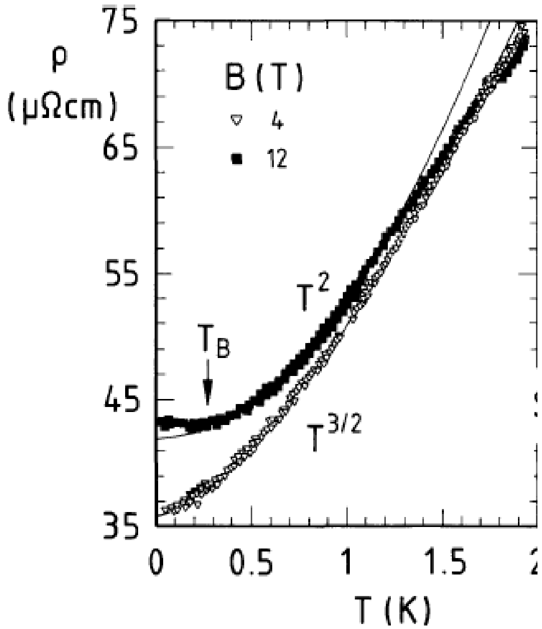
$$\kappa/T = \text{const}$$

Non Fermi liquid behavior in novel quantum materials

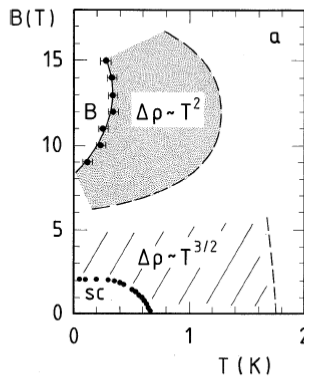


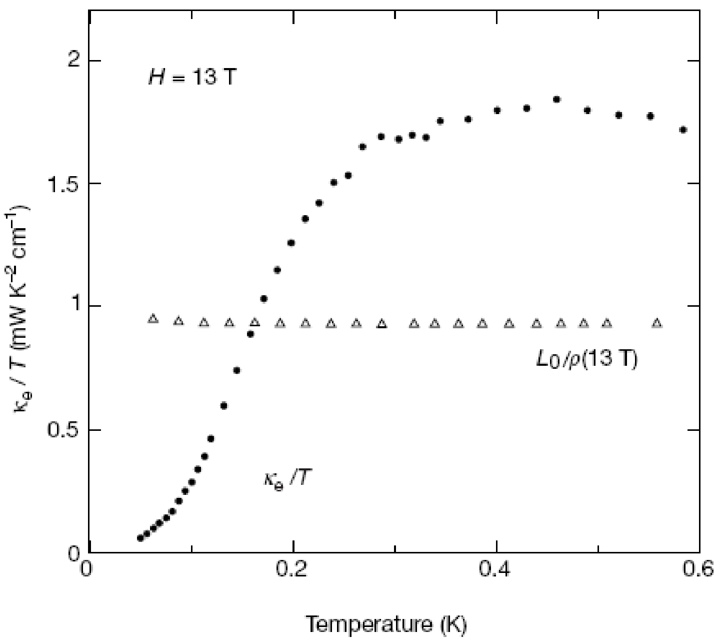
$UCu_{3.5}Pd_{1.5}$
Andraka, Stewart,
PRB 47:3208 (93)

Violation of the
Wiedemann-Franz law
in high Tc superconductors
Hill et al., Nature 414:711 (2001)



$CeCu_2Si_2$ Steglich et al.,
Z. Phys. B 103:235 (1997)



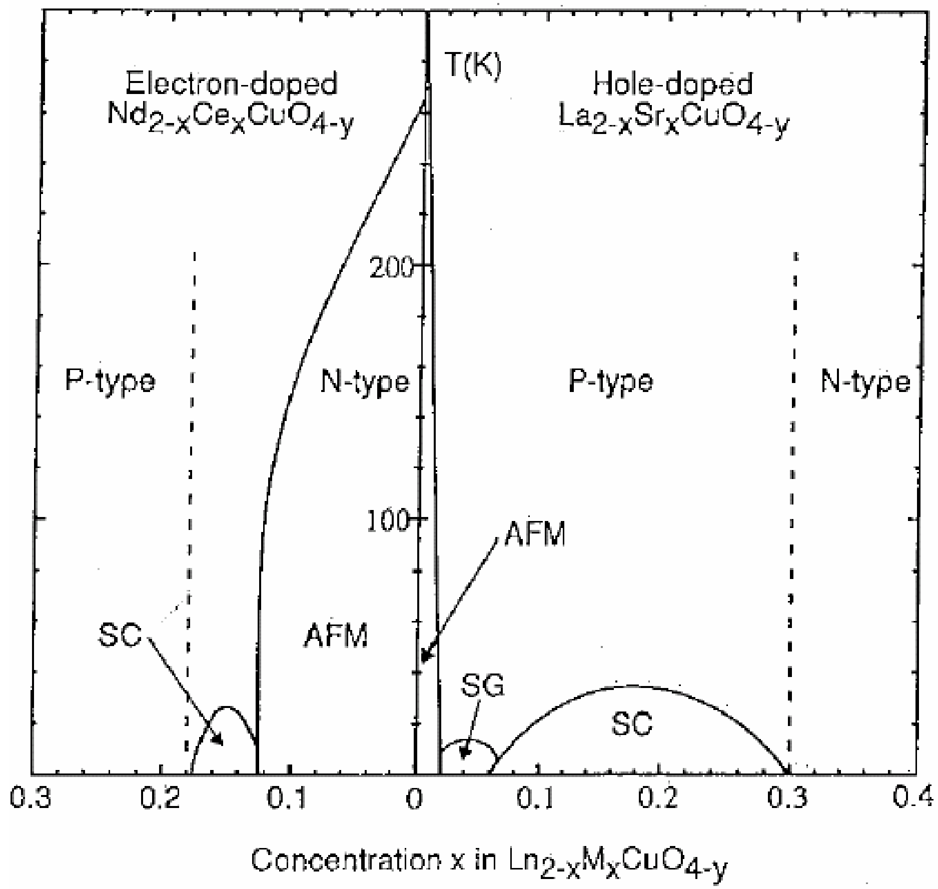


Puzzles of high temperature superconductors

Unusual “normal” state

Maple, JMMM 177:18 (1998)

Resistivity, opical conductivity,
Lack of sharply defined quasiparticles,
Nernst effect



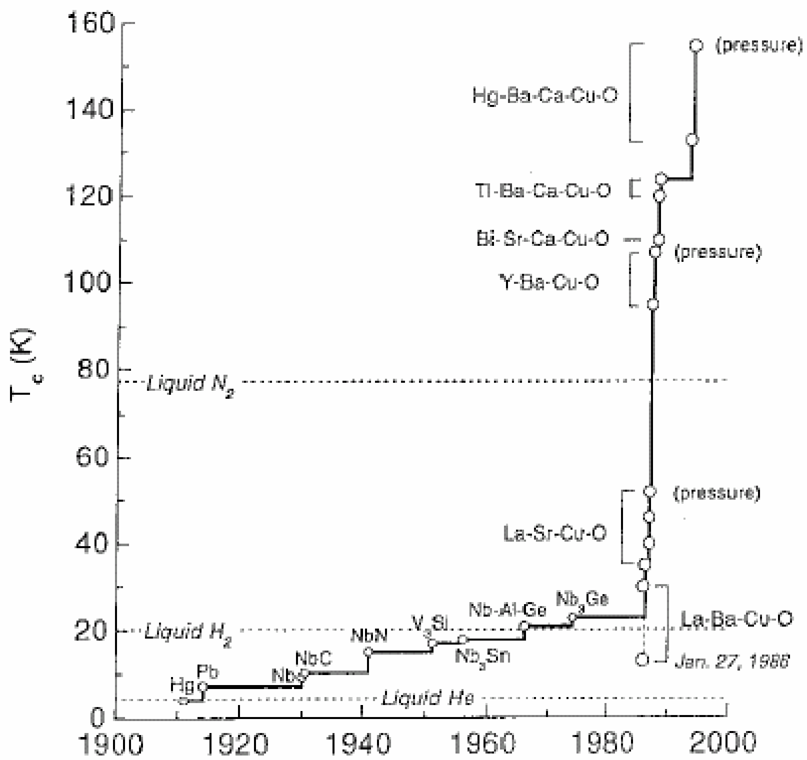
Mechanism of Superconductivity

High transition temperature,
retardation effect, isotope effect,
role of elecron-electron
and electron-phonon interactions

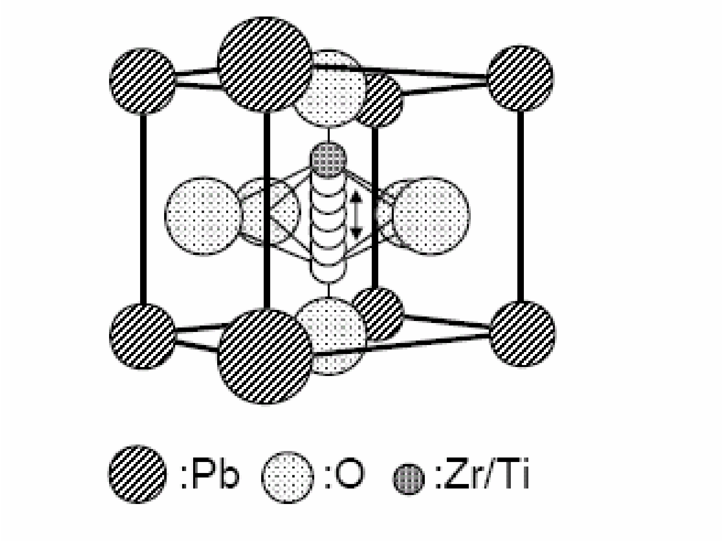
Competing orders

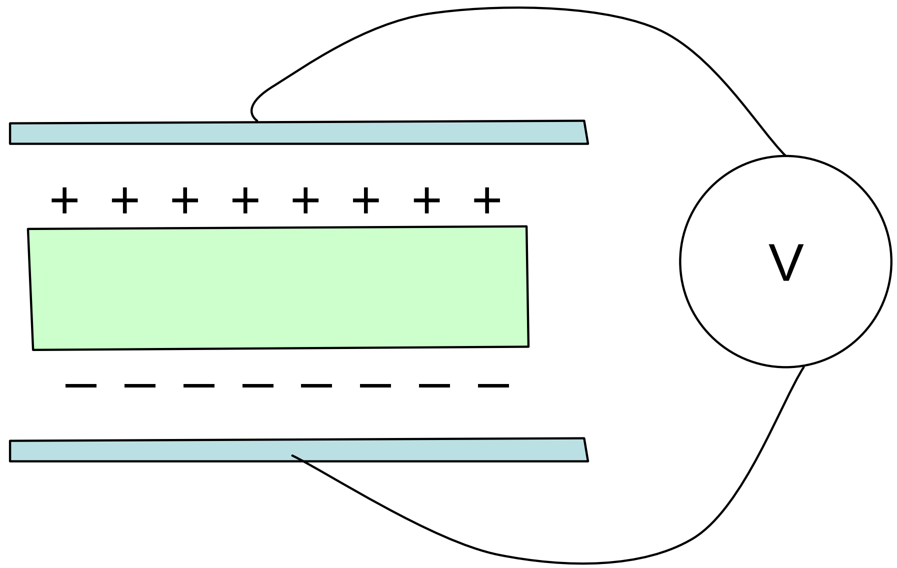
Role of magnetsim, stripes,
possible fractionalization

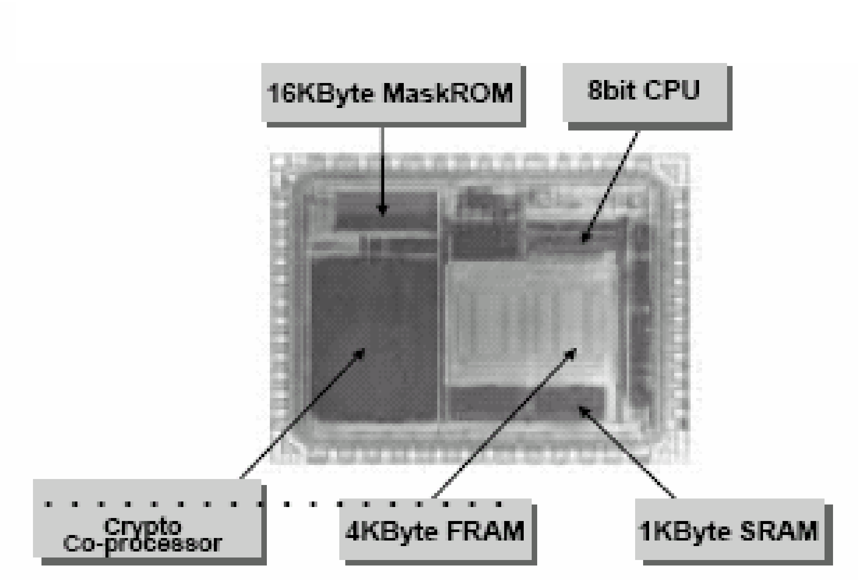
Applications of quantum materials: High Tc superconductors



Applications of quantum materials: Ferroelectric RAM







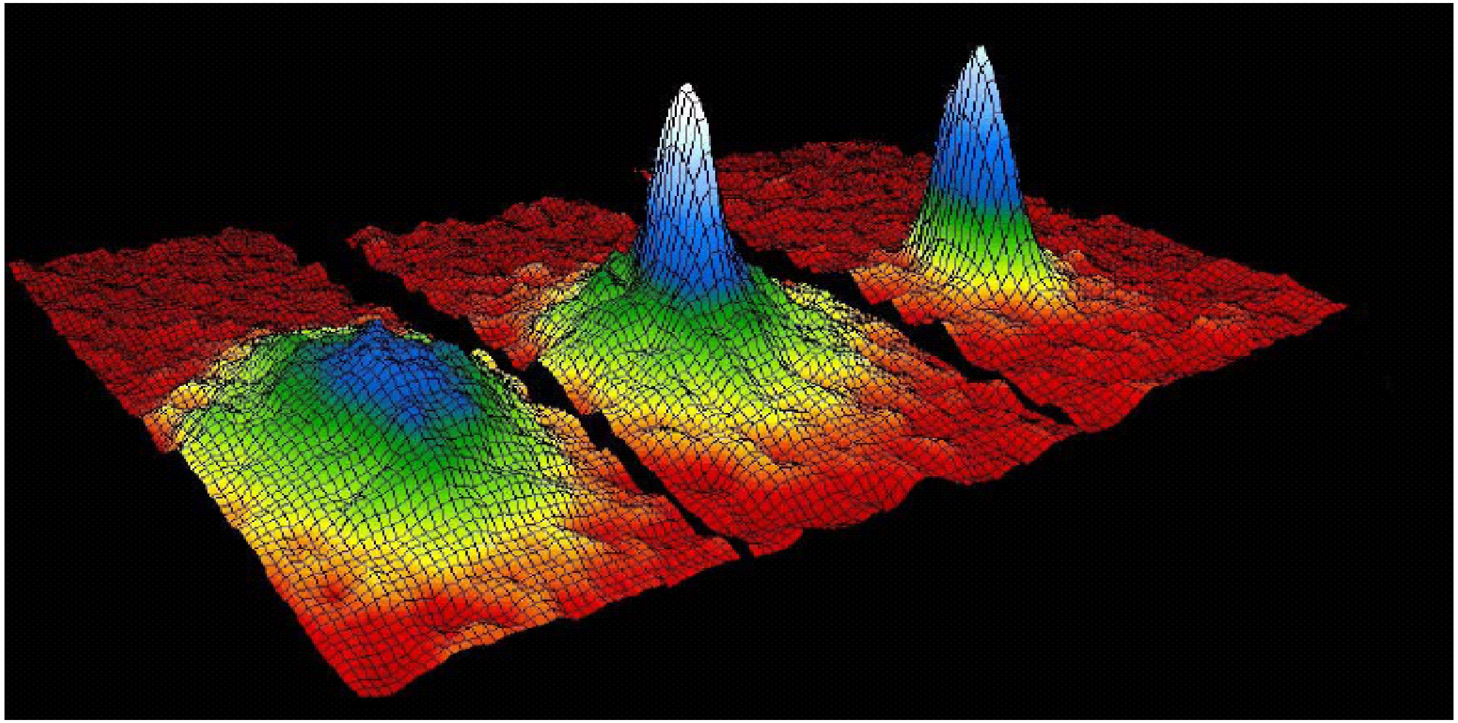
FeRAM in Smart Cards

Non-Volatile Memory

High Speed Processing

Modeling strongly correlated systems using cold atoms

Bose-Einstein condensation



Cornell et al., Science 269, 198 (1995)

$$n \sim 10^{14}\mathrm{cm}^3 \quad T_{\mathrm{BEC}} \sim 1\mu\mathrm{K}$$

Ultralow density condensed matter system

Interactions are weak and can be described theoretically from first principles

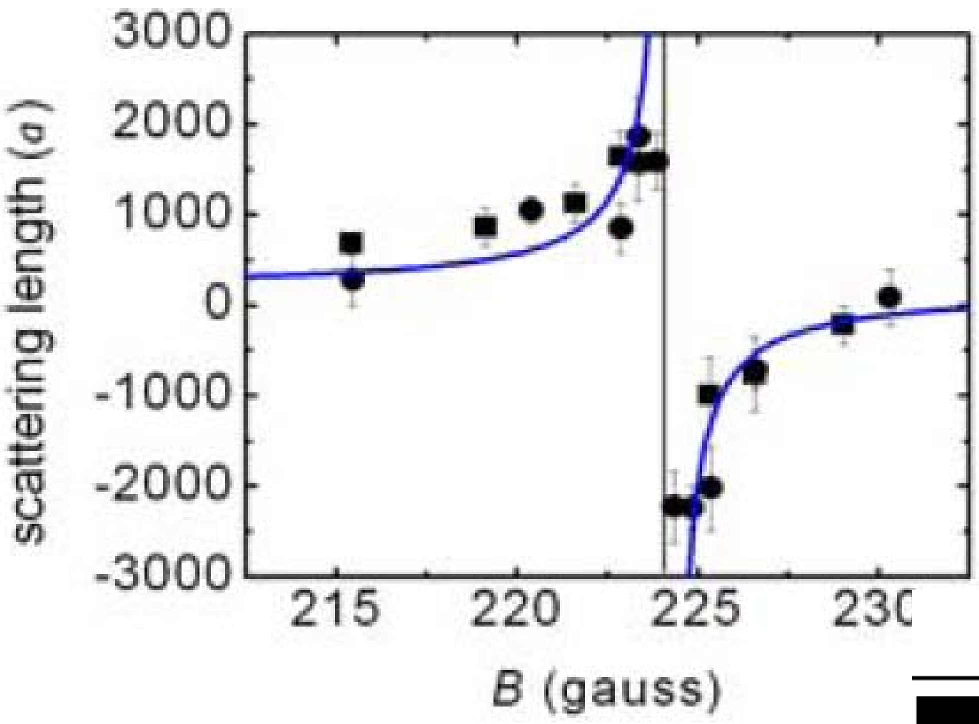
New Era in Cold Atoms Research

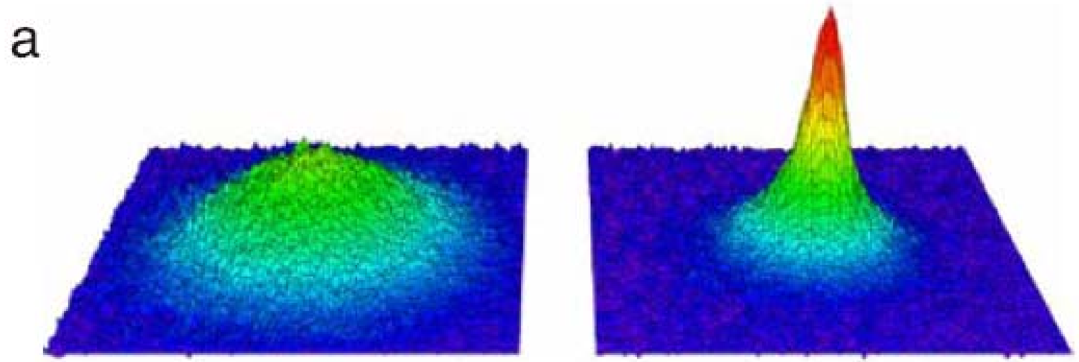
Focus on Systems with Strong Interactions

- Feshbach resonances
- Rotating systems
- Low dimensional systems
- Atoms in optical lattices
- Systems with long range dipolar interactions

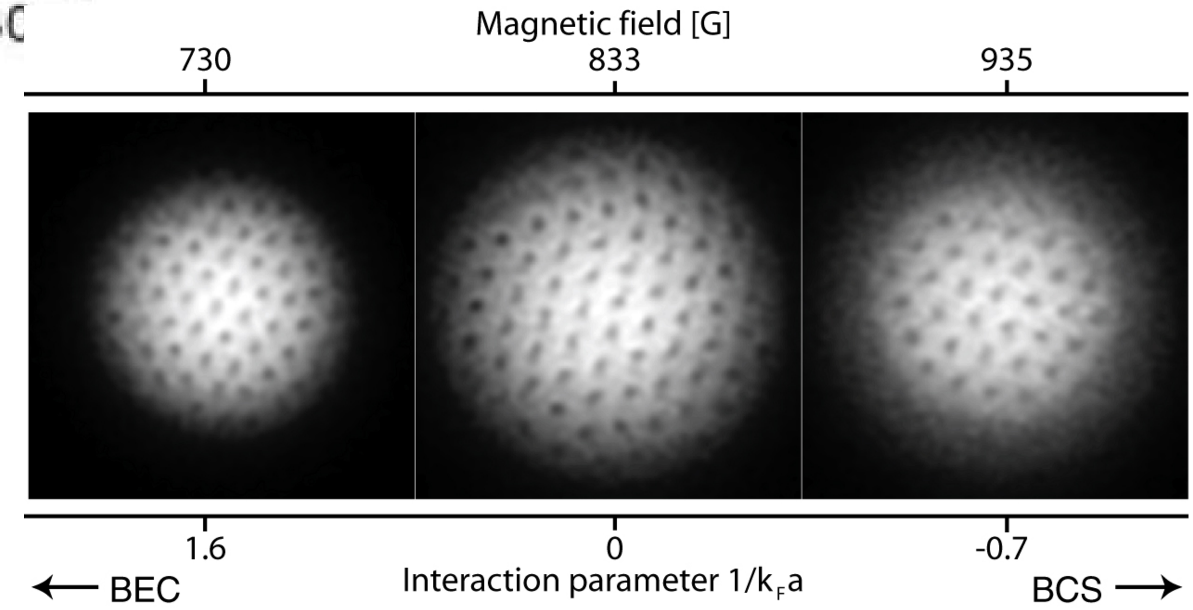
Feshbach resonance and fermionic condensates

Greiner et al., Nature 426:537 (2003); Ketterle et al., PRL 91:250401 (2003)

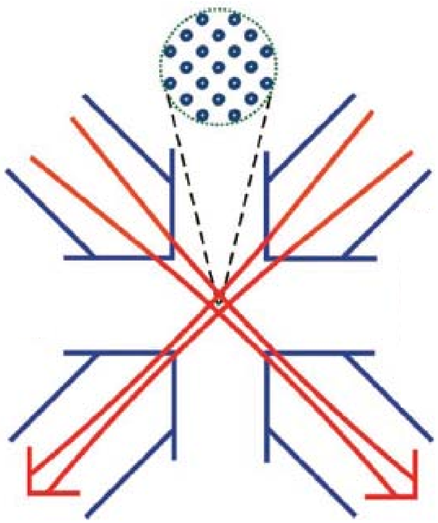




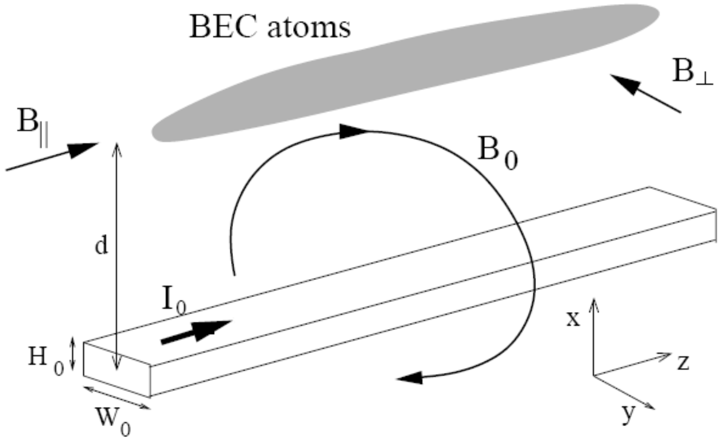
Ketterle et al.,
Nature 435, 1047-1051 (2005)



One dimensional systems



1D confinement in optical potential
Weiss et al., Science (05);
Bloch et al.,
Esslinger et al.,



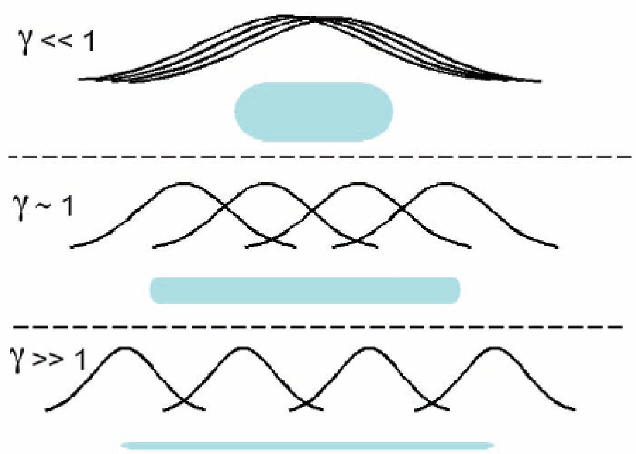
One dimensional systems in microtraps.
Thywissen et al., Eur. J. Phys. D. (99);
Hansel et al., Nature (01);
Folman et al., Adv. At. Mol. Opt. Phys. (02)

$$E_{kin} \sim \frac{\hbar^2}{m\, d^2} \sim \frac{\hbar^2\, n^2}{m}$$

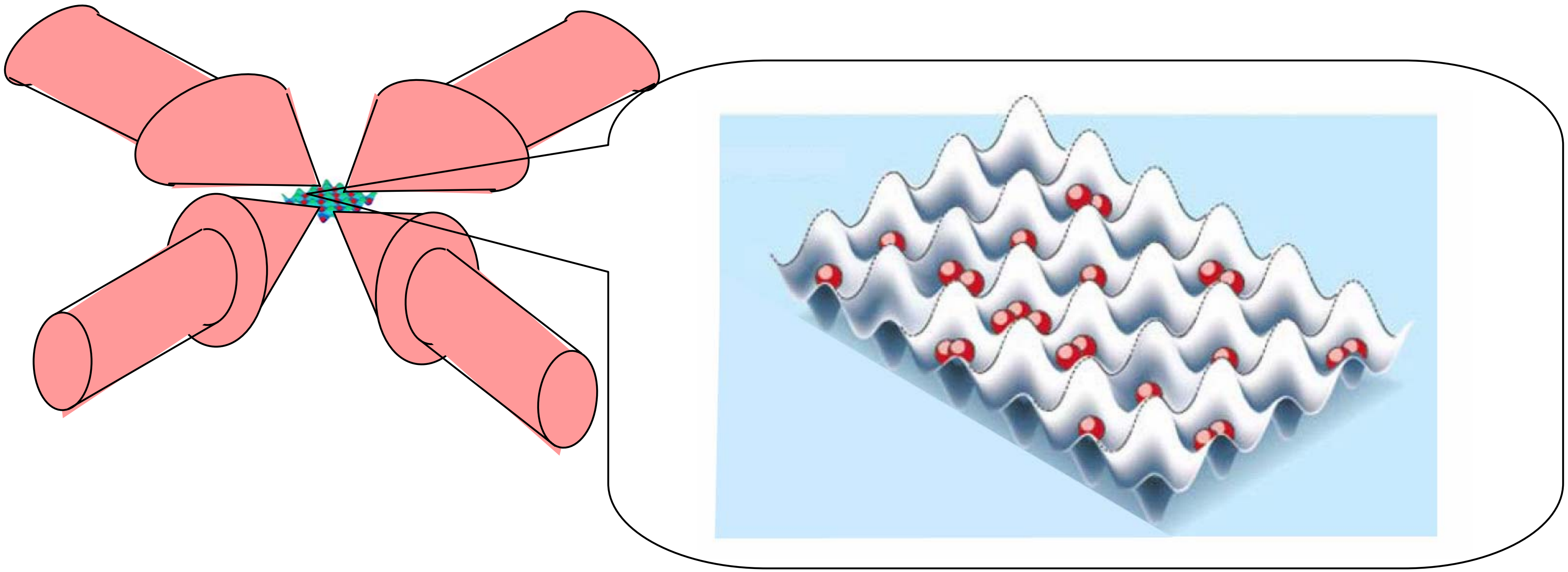
$$E_{int} \sim g\, n$$

$$\gamma = \frac{E_{int}}{E_{kin}} \sim \frac{g\, m}{\hbar^2\, n}$$

Strongly interacting regime can be reached for low densities



Atoms in optical lattices



Theory: Jaksch et al. PRL (1998)

Experiment: Kasevich et al., Science (2001);
Greiner et al., Nature (2001);
Phillips et al., J. Physics B (2002)
Esslinger et al., PRL (2004);
and many more …

Strongly correlated systems

Electrons in Solids

$$E_{\rm int} \sim 1 \div 4 \text{ eV} \sim 10^4 \text{ K}$$

$$E_{\rm kin} \sim 1 \div 10 \text{ eV} \sim 10^5 \text{ K}$$

Atoms in optical lattices

$$E_{\rm int} \sim E_{\rm kin} \sim 10 \text{ kHz} \sim 10^{-6} \text{ K}$$

Simple metals $E_{\rm int} < E_{\rm kin}$

Perturbation theory in Coulomb interaction applies.
Band structure methods wotk

Strongly Correlated Electron Systems $E_{\rm int} \geq E_{\rm kin}$

Band structure methods fail.

Novel phenomena in strongly correlated electron systems:

Quantum magnetism, phase separation, unconventional superconductivity, high temperature superconductivity, fractionalization of electrons …

Strongly correlated systems of photons

Strongly interacting photons

- Usual approach: atoms inside resonant optical microcavities (cavity QED)
  - Enhancement due to many round-trip interactions between photon and atom
  - Narrow bandwidth
- Phase transitions in coupled networks of cavities?

Strongly Correlated Photons in a Two-Dimensional Array of Photonic Crystal Microcavities

Y.C. Neil Na,[1] Shoko Utsunomiya,[2] Lin Tian,[1] and Yoshihisa Yamamoto[1,2]
[1] E. L. Ginzton Laboratory, Stanford University, Stanford, CA 94305, USA
[2] National Institute of Informatics, Hitotsubashi, Chiyoda-ku, Tokyo 101-8430, Japan

Strongly interacting polaritons in coupled arrays of cavities

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Quantum phase transitions of light

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[1] Centre for Quantum Computer Technology, School of Physics, The University of Melbourne, Victoria 3010, Australia
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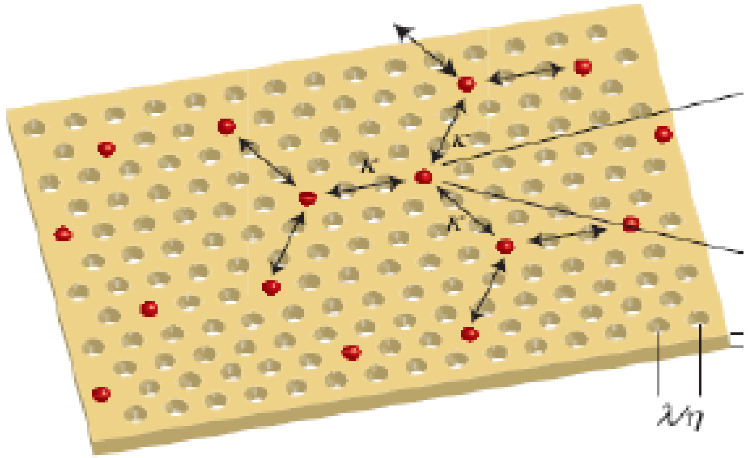
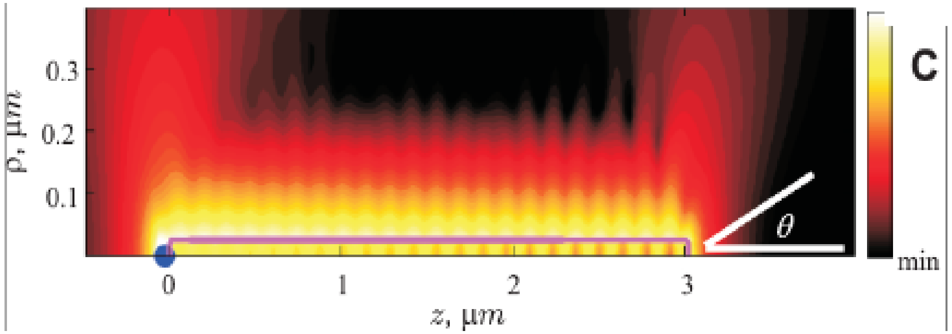


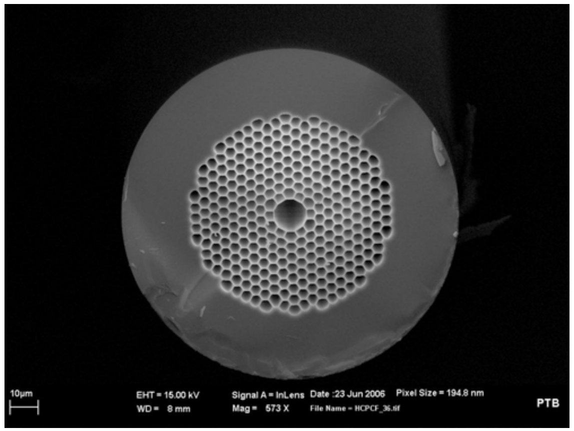
Novel approach to strong coupling

- Strong coupling of atom to photons in single-mode, 1-D waveguides
  - Large interaction with tightly-confined modes, $g \sim 1/\sqrt{A_{eff}}$
  - Broadband

Nanoscale surface plasmons



Atoms in a hollow core photonic crystal fiber



particles: $$i\,\partial_t \hat{\psi} = -\frac{1}{2\mathrm{m}}\partial_z^2 \hat{\psi} + g\,\hat{\psi}^\dagger \hat{\psi}\,\hat{\psi}$$

photons: $$i\,\partial_t \hat{E} = -\alpha\,\partial_z^2 \hat{E} + \chi^{(3)} \hat{E}^\dagger \hat{E}\,\hat{E}$$

α – group velocity dispersion

$\chi(3)$ – nonlinear susceptibility

Strongly interacting photons in 1-D optical waveguides

BEFORE: two level systems and insufficient mode confinement

Interaction corresponds to attraction. Physics of solitons (e.g. Drummond)

Weak non-linearity due to insufficient mode confining

Limit on non-linearity due to photon decay

NOW: EIT and tight mode confinement

Sign of the interaction can be tuned

Tight confinement of the electromagnetic mode enhances nonlinearity

Strong non-linearity without losses can be achieved using EIT

Fermionized photons are possible (D. Chang et al.)

Why are we interested in making strongly correlated systems of cold atoms (and photons) ?

New Era in Cold Atoms Research

Focus on Systems with Strong Interactions

Goals

- Resolve long standing questions in condensed matter physics (e.g. origin of high temperature superconductivity)

- Resolve matter of principle questions (e.g. existence of spin liquids in two and three dimensions)

- Study new phenomena in strongly correlated systems (e.g. coherent far from equilibrium dynamics)

Outline

- Introduction
- Basics of cold atoms in optical lattices

  Bose Hubbard model. Superfluid to Mott transition.

  Dynamical instability.
- Two component Bose mixtures

  Quantum magnetism
- Fermions in optical lattices

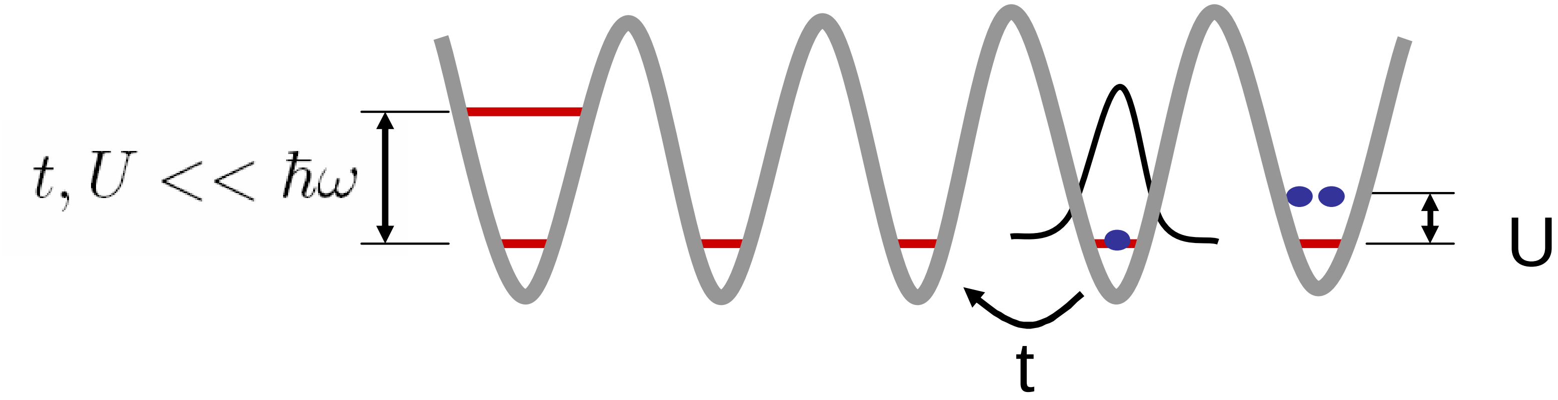
  Pairing in systems with repulsive interactions. High Tc mechanism
- Low-dimensional Bose systems in and out of equilibrium

  Analysis of correlations beyond mean-field

Emphasis: detection and characterzation of many-body states

Atoms in optical lattices. Bose Hubbard model

Bose Hubbard model

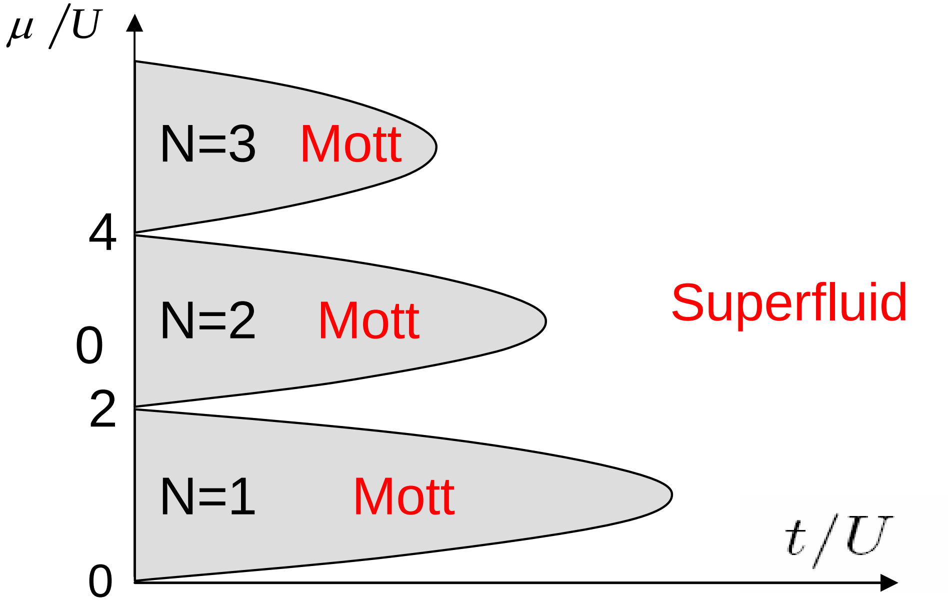


$$\mathcal{H} = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + U \sum_i n_i (n_i - 1) - \mu \sum_i n_i$$

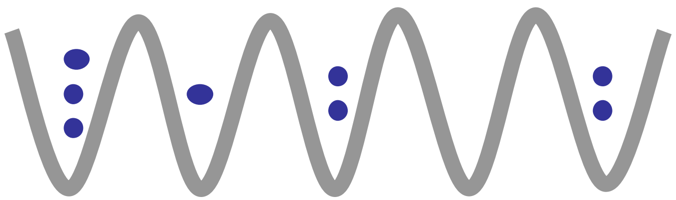
t – tunneling of atoms between neighboring wells

U – repulsion of atoms sitting in the same well

Bose Hubbard model. Mean-field phase diagram

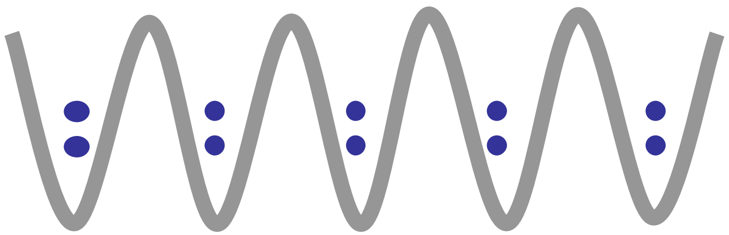


M.P.A. Fisher et al.,
PRB40:546 (1989)



$$U << Nt$$

Superfluid phase
Weak interactions



$$U >> Nt$$

Mott insulator phase
Strong interactions

Bose Hubbard model

$$\mathcal{H} = \cancel{-t \sum_{\langle ij \rangle} b_i^\dagger b_j} + U \sum_i n_i (n_i - 1) - \mu \sum_i n_i$$

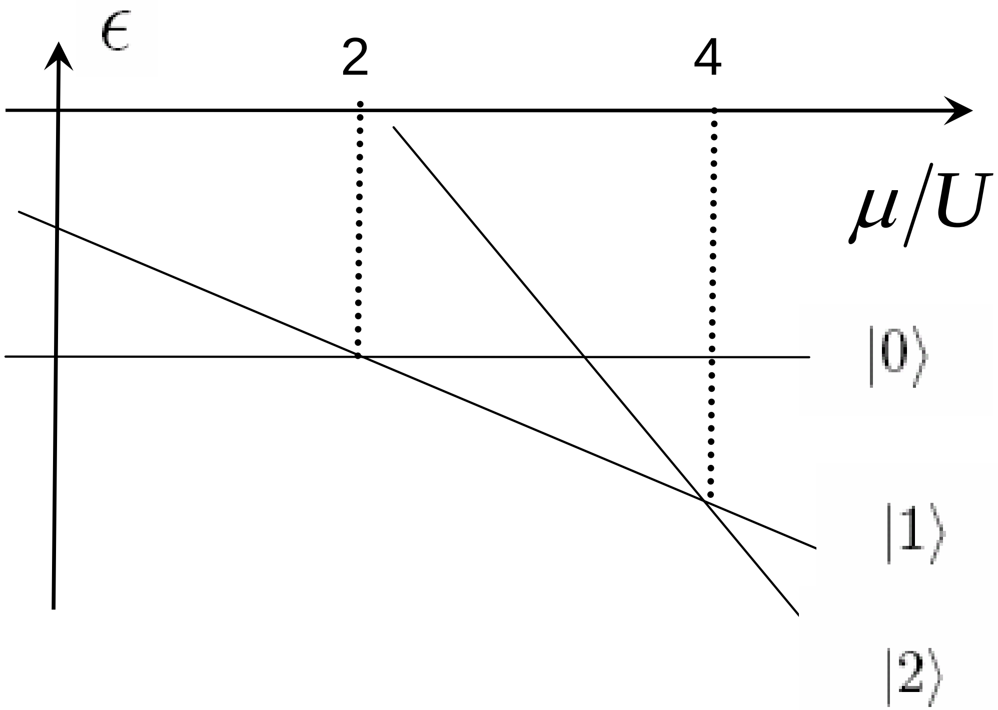
Set $t = 0$ Hamiltonian eigenstates are Fock states $|n\rangle = \frac{1}{\sqrt{n!}} \left(b_i^\dagger\right)^n |0\rangle$

$|0\rangle$ $\epsilon = 0$

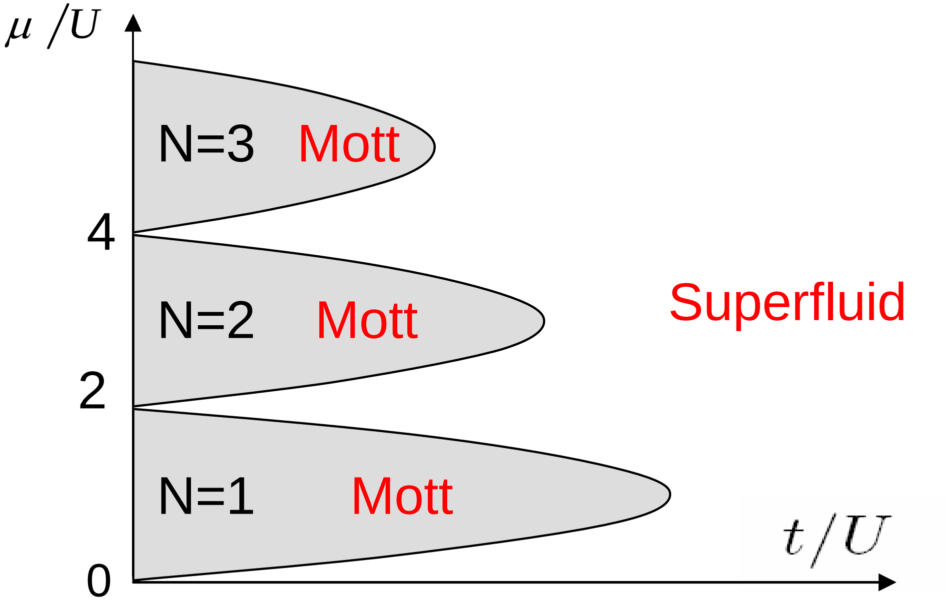
$|1\rangle$ $\epsilon = -\mu$

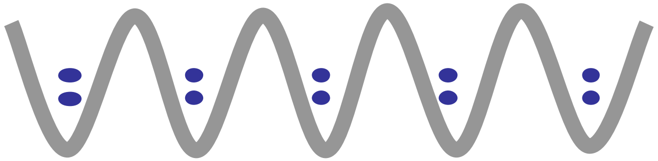
$|2\rangle$ $\epsilon = 2U - 2\mu$

$|3\rangle$ $\epsilon = 6U - 3\mu$

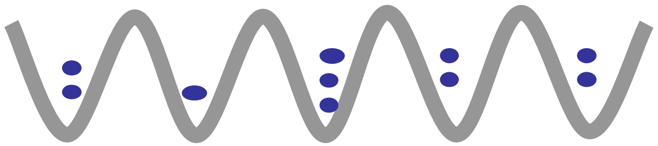


Bose Hubbard Model. Mean-field phase diagram





Mott insulator phase



Particle-hole excitation $\Delta E \sim U - N\,t$

Tips of the Mott lobes $U \sim N\,t$

Gutzwiller variational wavefunction

$$|\Psi\rangle = \prod_i (f_0 \; |0\rangle + f_1 \; |1\rangle + f_2 \; |2\rangle + \dots)_i$$

$$= \prod_i (f_0 + f_1 \; b_i^\dagger + \frac{f_2}{\sqrt{2}} (b_i^\dagger)^2 + \dots) \; |0\rangle_i$$

Normalization $\quad |f_0|^2 + |f_1|^2 + |f_2|^2 + \cdots = 1$

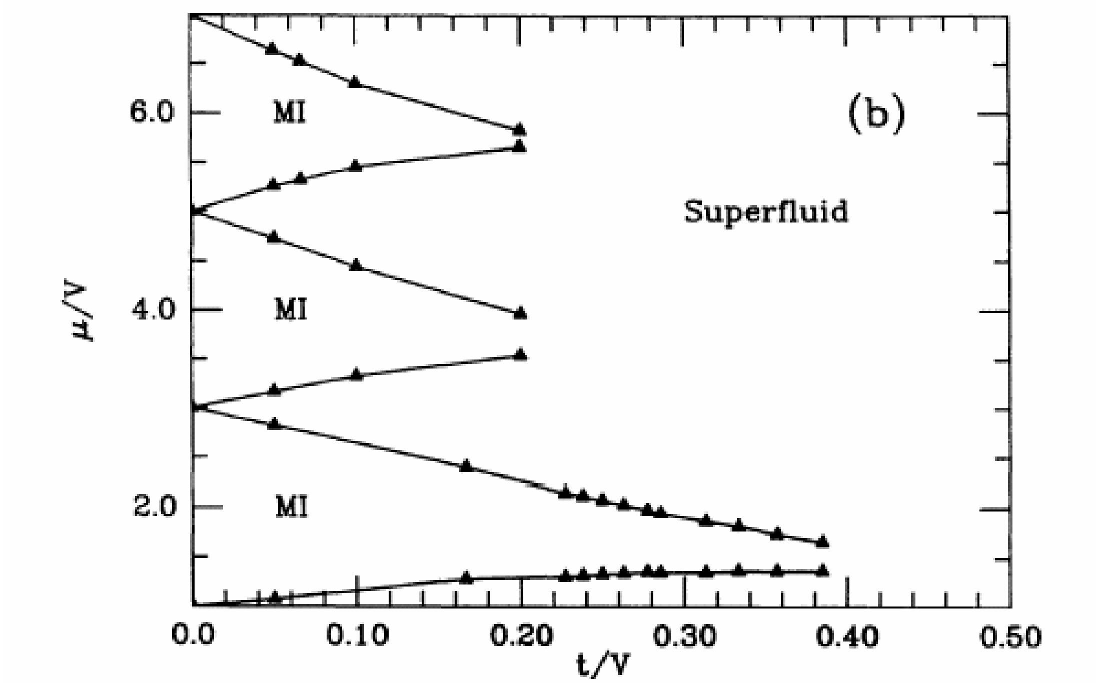
Interaction energy $\quad \epsilon_{\mathrm{U}} = 2 \; U \; |f_2|^2 + 6 \; U \; |f_3|^2 + \dots$

Kinetic energy $\quad \epsilon_{\mathrm{t}} = -zt \left| \; f_0^* f_1 + \sqrt{2} \; f_1^* f_2 + \sqrt{3} \; f_2^* f_3 + \dots \; \right|^2$

z – number of nearest neighbors

Phase diagram of the 1D Bose Hubbard model. Quantum Monte-Carlo study

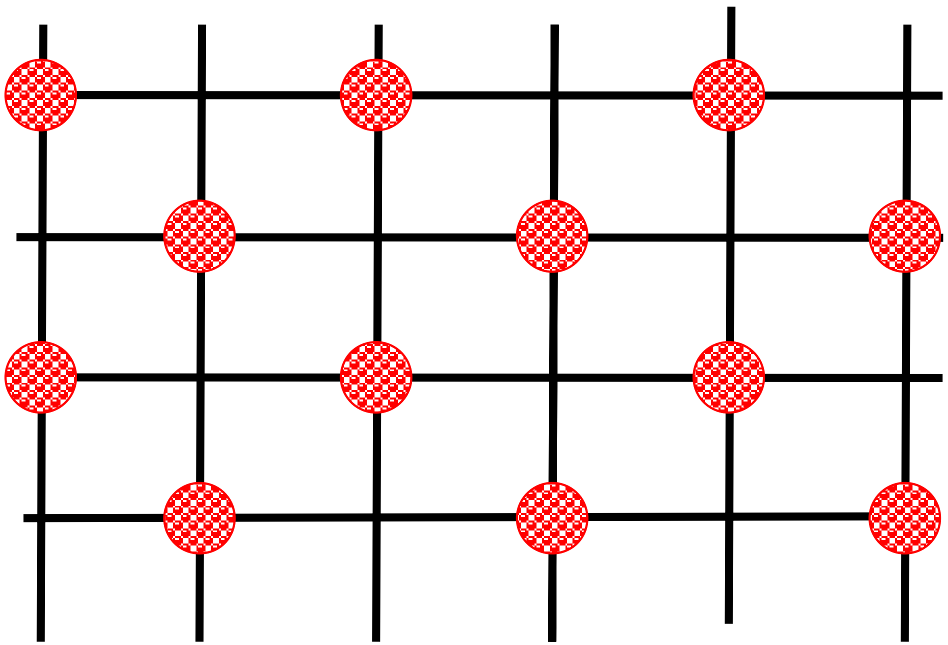
Batrouni and Scaletter, PRB 46:9051 (1992)



Extended Hubbard Model

$$\mathcal{H} = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + \sum_{ij} n_i U_{ij} n_j - \mu \sum_i n_i$$

U_0 - on site repulsion $\qquad$ U_1 - nearest neighbor repulsion

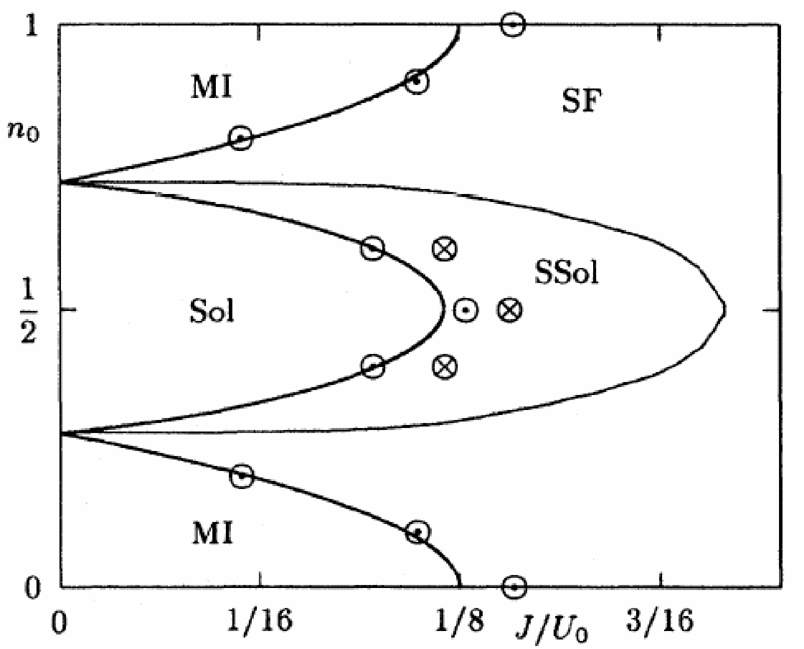


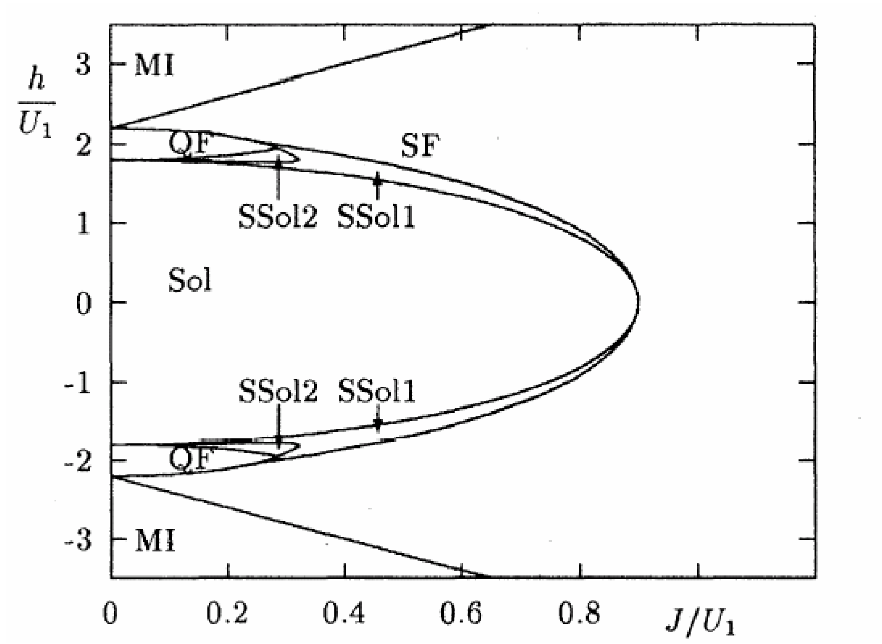
Checkerboard phase:

Crystal phase of bosons.
Breaks translational symmetry

Extended Hubbard model. Mean field phase diagram

van Otterlo et al., PRB 52:16176 (1995)



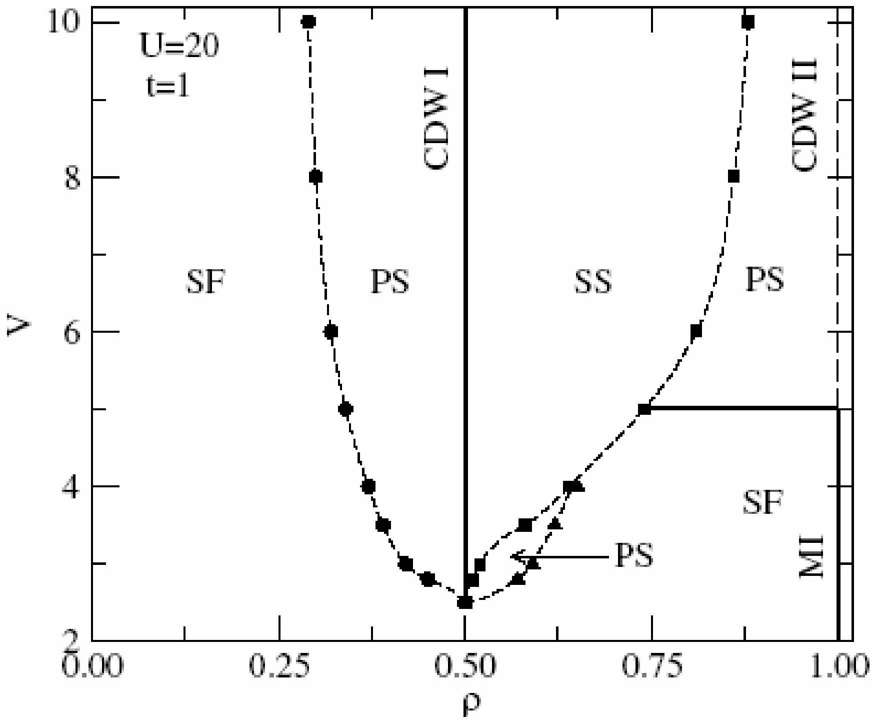


$$\frac{U_1}{U_0} = \frac{1}{5}$$

Hard core bosons. $\frac{U_2}{U_1} = \frac{1}{10}$

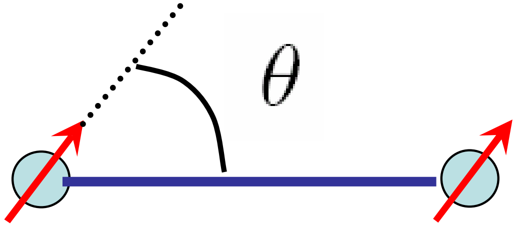
Supersolid – superfluid phase with broken translational symmetry

Extended Hubbard model.
Quantum Monte Carlo study

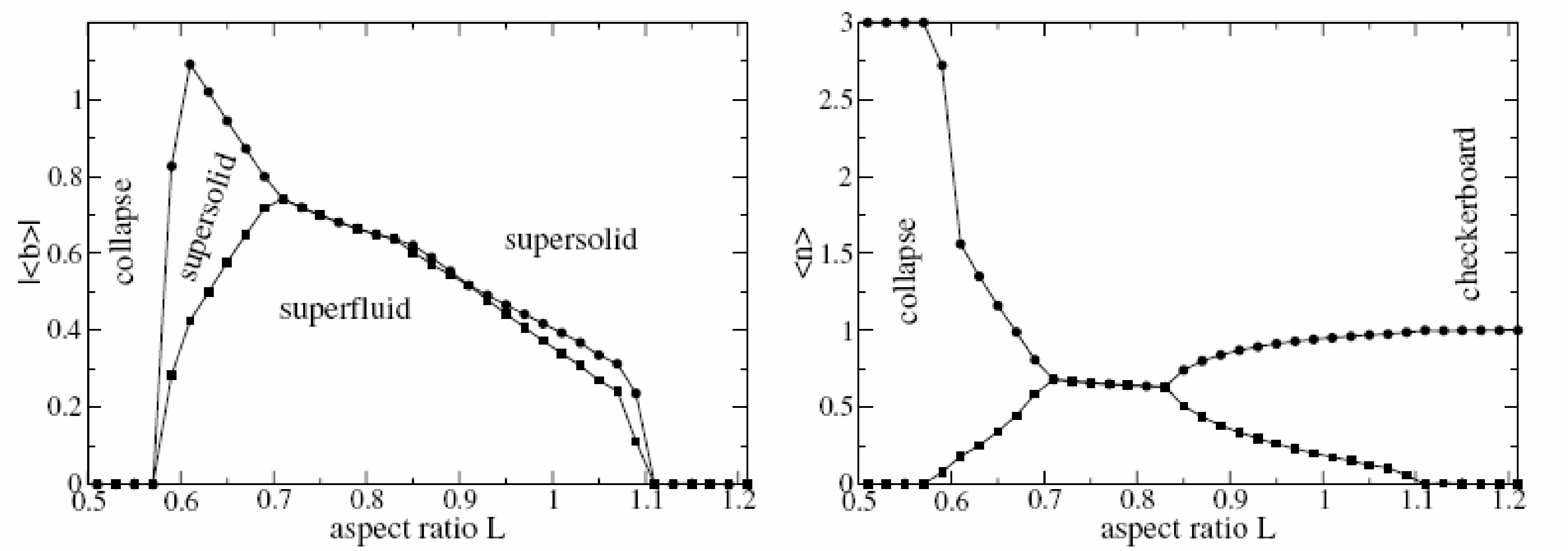


Sengupta et al., PRL 94:207202 (2005)

Dipolar bosons in optical lattices



$$V_{\mathrm{int}} = d^2 \frac{1 - 3\cos^2\theta}{|\mathbf{r} - \mathbf{r}'|^3} + \frac{4\pi\hbar^2 a}{m}\,\delta(\mathbf{r} - \mathbf{r}')$$

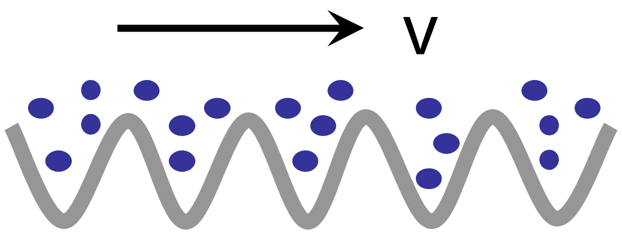


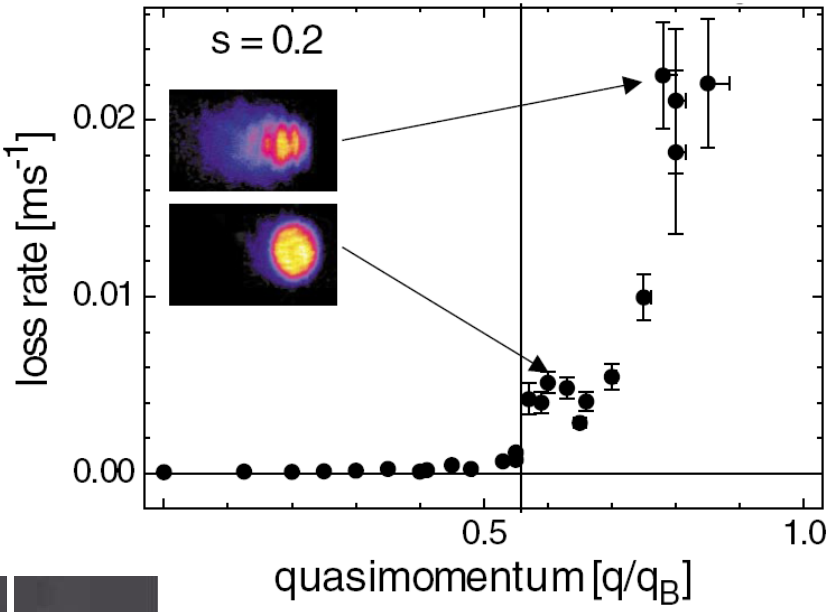
Goral et al., PRL88:170406 (2002)

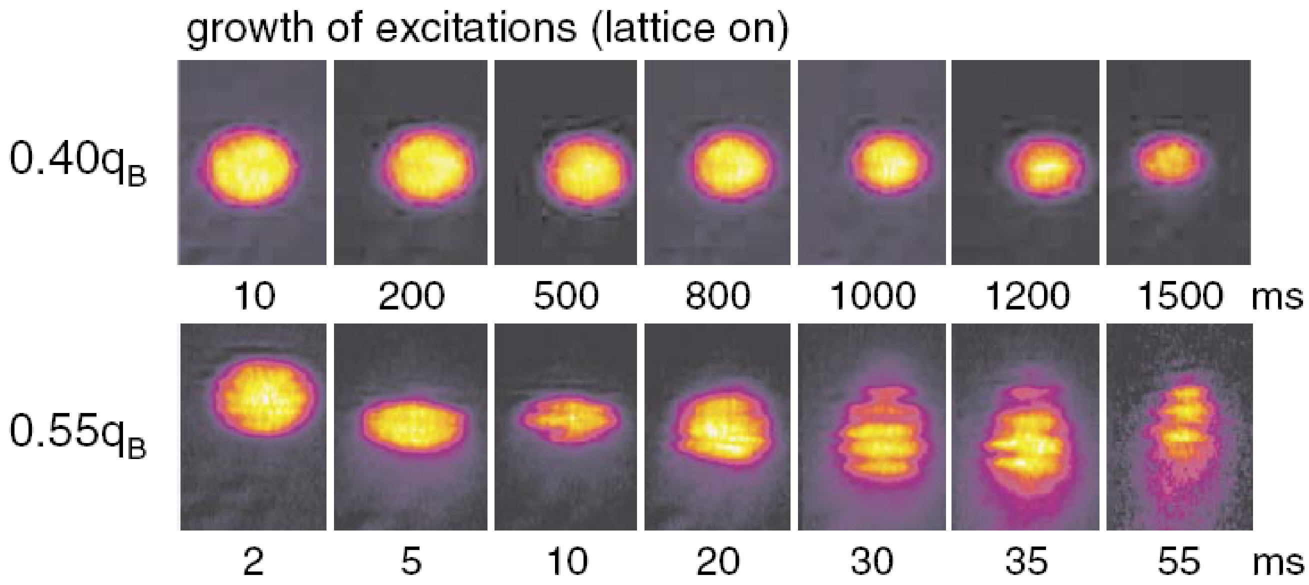
Bose Hubbard model away from equilibrium. Dynamical Instability of strongly interacting bosons in optical lattices

Moving condensate in an optical lattice. Dynamical instability

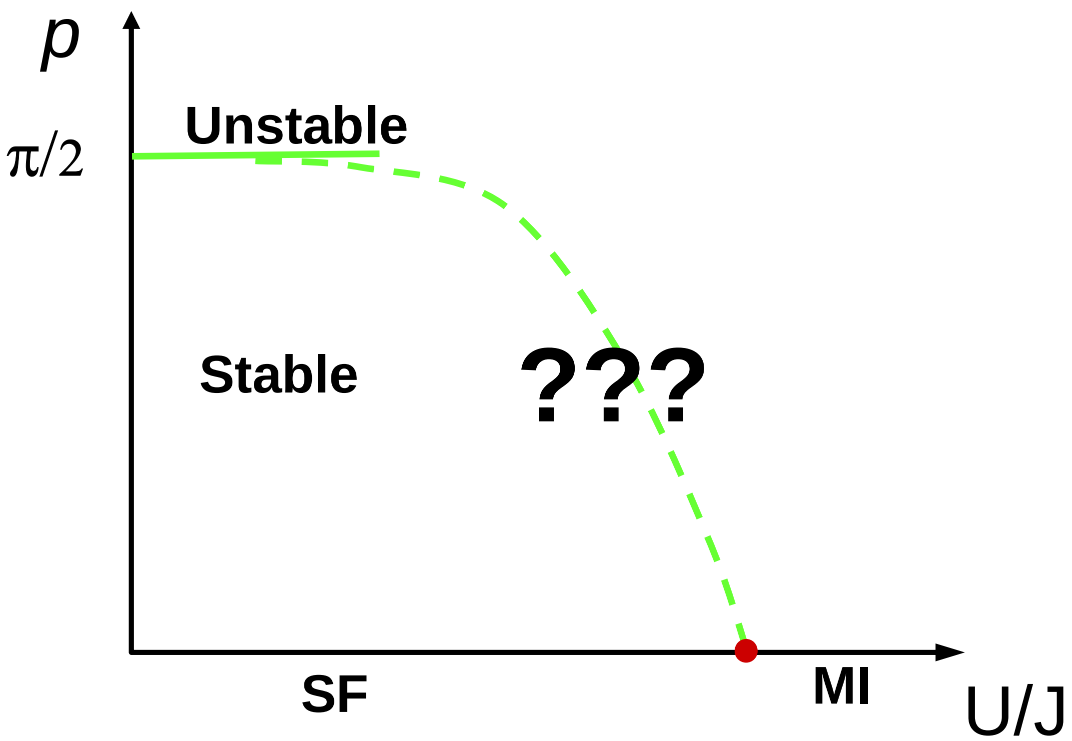
Theory: Niu et al. PRA (01), Smerzi et al. PRL (02)
Experiment: Fallani et al. PRL (04)



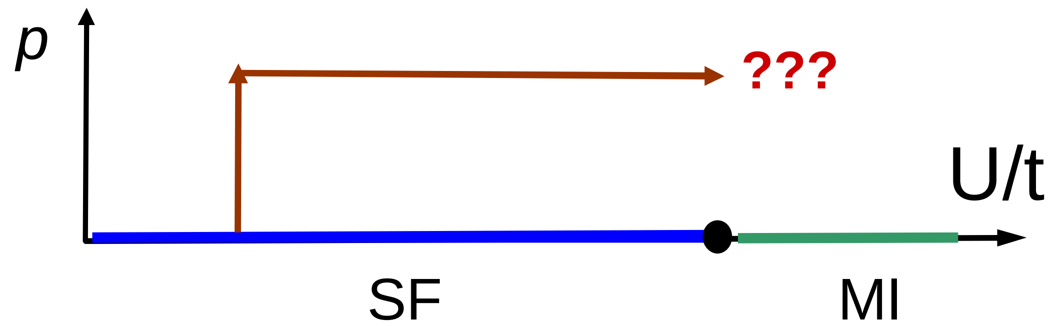




Question: How to connect
the dynamical instability (irreversible, classical)
to the superfluid to Mott transition (equilibrium, quantum)



Possible experimental sequence:



Dynamical instability

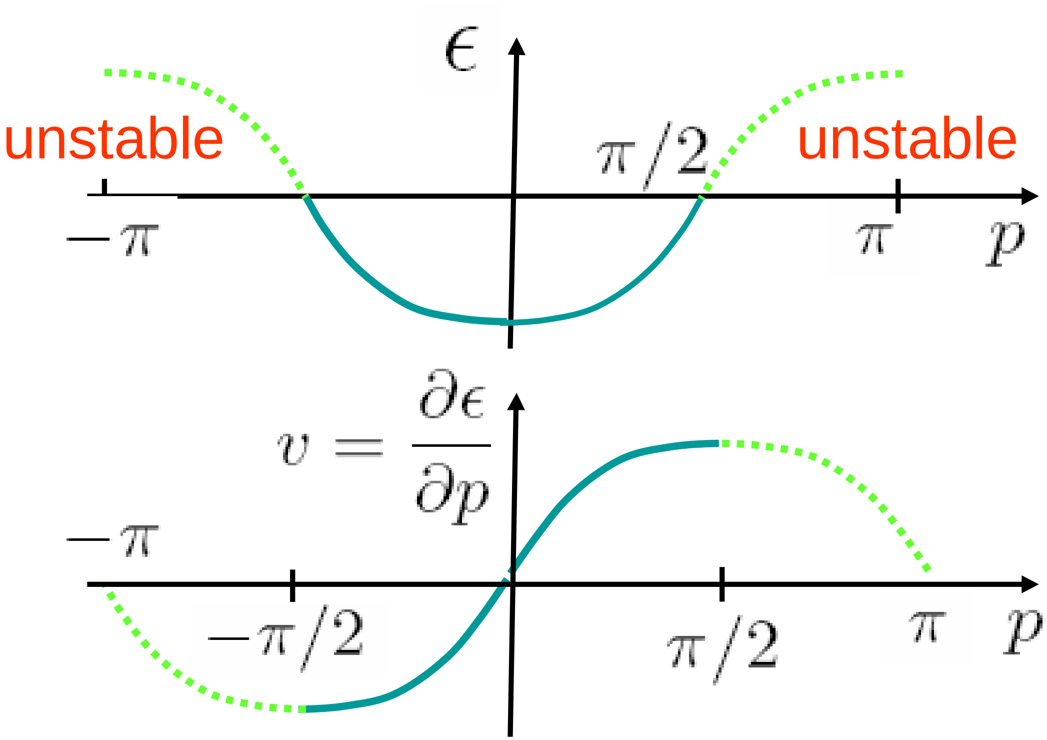
Classical limit of the Hubbard model. $N\,t >> U$

Discreet Gross-Pitaevskii equation

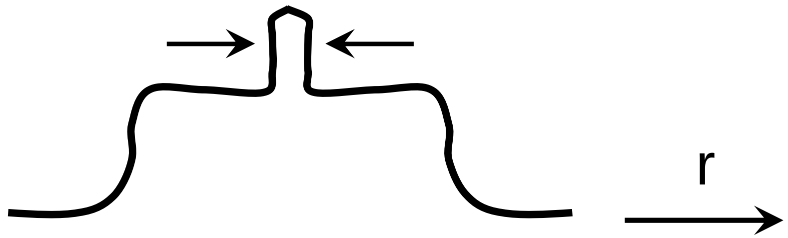
$$i\,\frac{d\,\Psi_j}{d\,t} = -t \sum_{\langle k \rangle} \Psi_k + U\,|\Psi_j|^2\,\Psi_j$$

Current carrying states $\Psi_j \sim e^{ipx_j}$

Linear stability analysis: States with p> are unstable



Amplification of density fluctuations



Dynamical instability for integer filling

Order parameter for a current carrying state $\Psi_j(p) = A(p)\ e^{ipx_j}$

Current $J(p) = |A(p)|^2 \sin(p)$

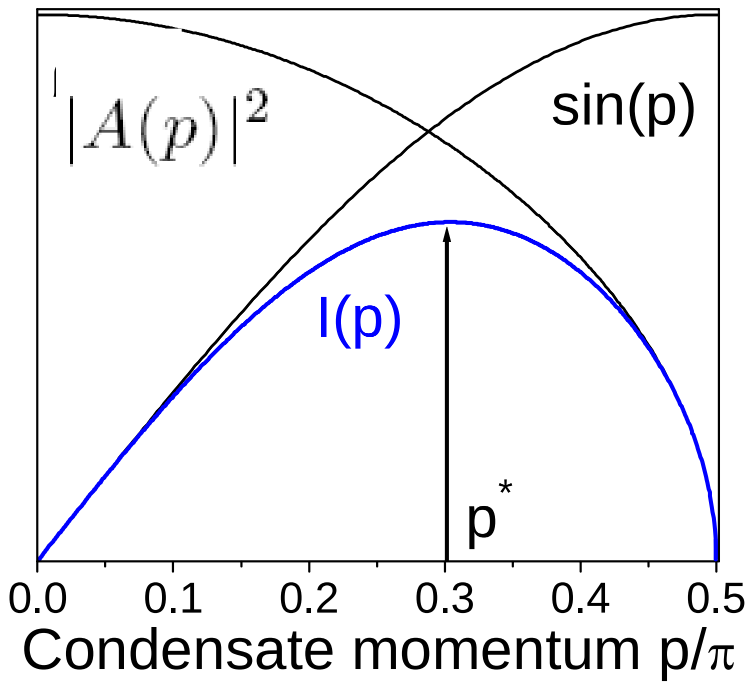
GP regime $A(p) = \sqrt{N}$. Maximum of the current for $p = \pi/2$

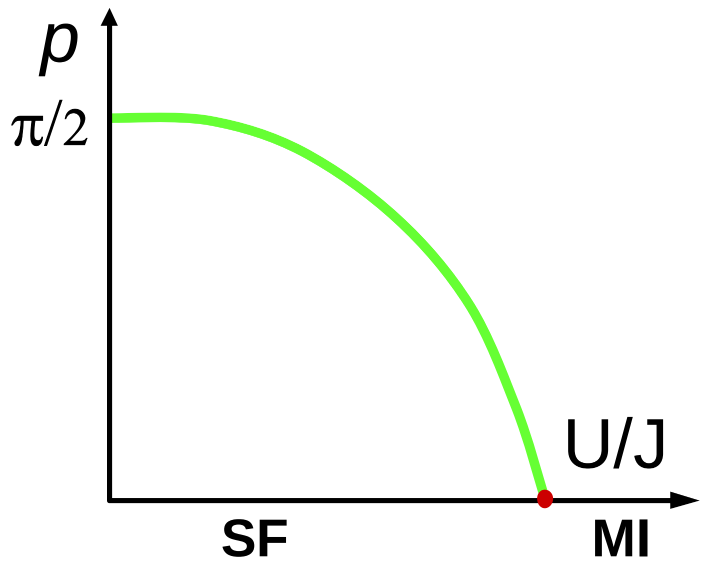
When we include quantum fluctuations, the amplitude of the order parameter is suppressed

$$\frac{A(p=0)}{\sqrt{N}} \approx 1 - \left(\frac{U}{Nt}\right)^{1/2}$$

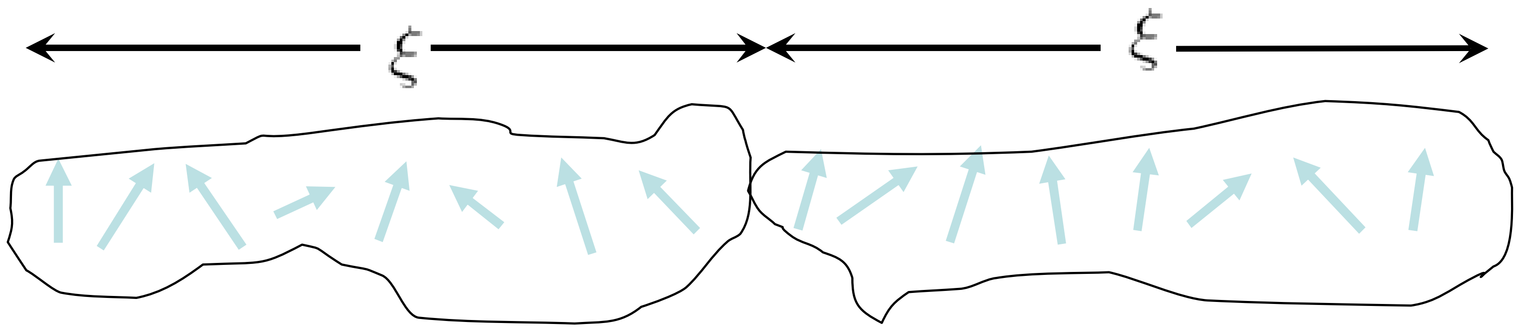
$A(p)$ decreases with increasing phase gradient p

Dynamical instability for integer filling





Vicinity of the SF-I quantum phase transition.
Classical description applies for $L > \xi \sim (U_c - U)^{-1/2}$



Dynamical instability occurs for $p\xi \approx \pi/2$

Dynamical instability. Gutzwiller approximation

Wavefunction

$$|\Psi(t)\rangle = \prod_j \left[\sum_{n=0}^{\infty} f_{jn}(t) \, | \, n \rangle_j \right]$$

Time evolution

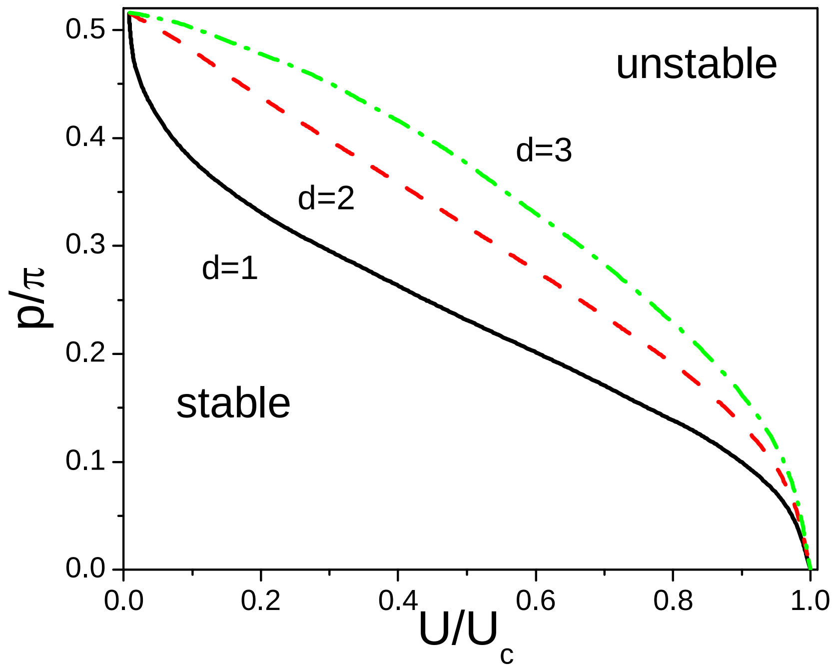
$$-i \frac{df_{jn}}{dt} = -t \, (\, f_{jn-1} \, \phi_j \, + \, f_{jn+1} \, \phi_j^* \,) \, + \, \frac{U}{2} \, n \, (n-1) \, f_{jn}$$

$$\phi_j(t) = \sum_{\langle i \rangle} \langle \, \Psi(t) \, | \, a_i \, | \, \Psi(t) \, \rangle$$

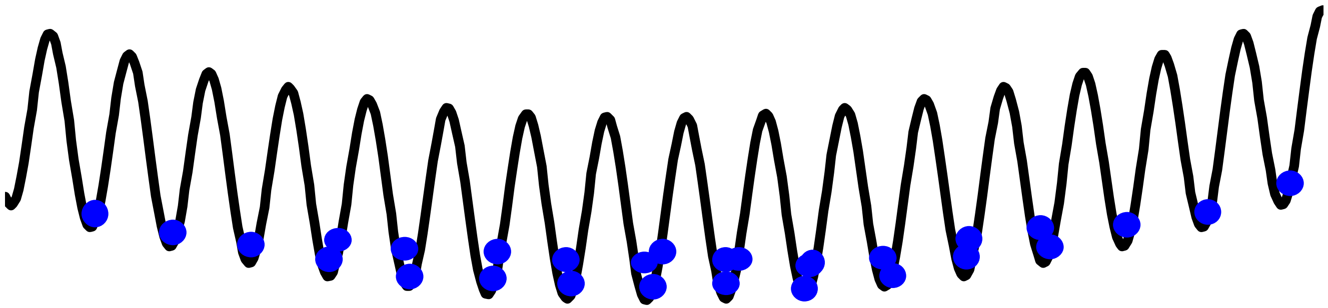
We look for stability against small fluctuations

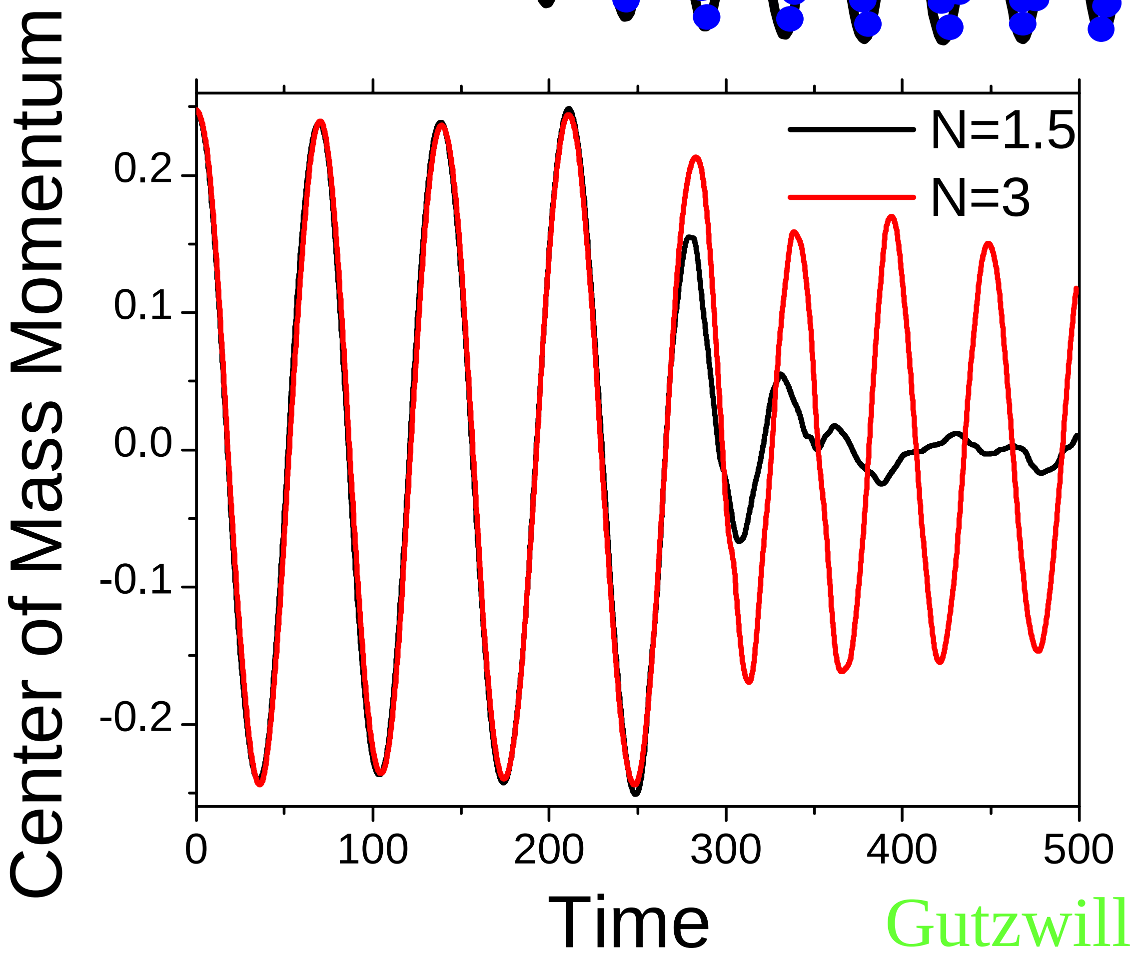
Phase diagram. Integer filling

Altman et al., PRL 95:20402 (2005)



Optical lattice and parabolic trap. Gutzwiller approximation





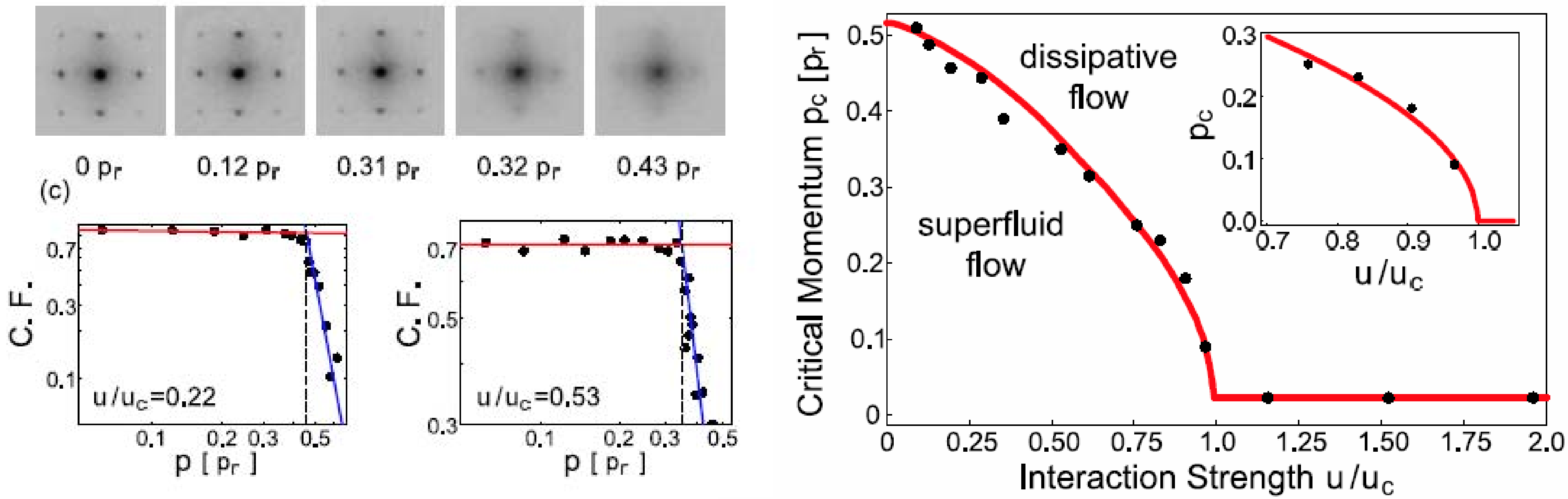
The first instability develops near the edges, where N=1

U=0.01 t
J=1/4

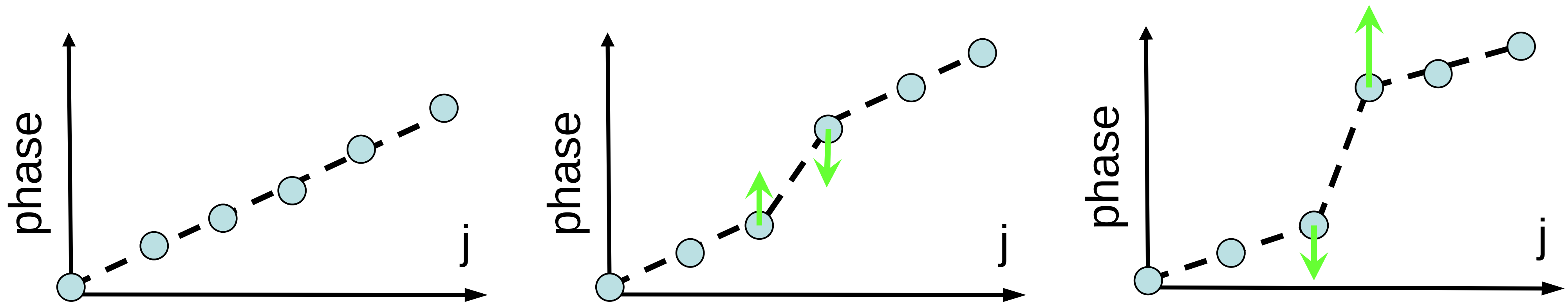
Gutzwiller ansatz simulations (2D)

Phase diagram for a Bose-Einstein condensate moving in an optical lattice

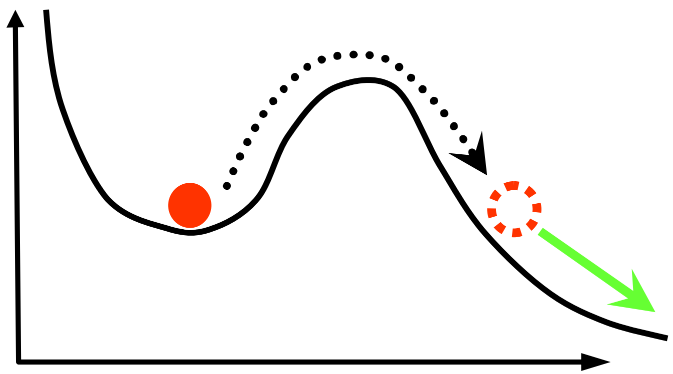
Jongchul Mun, Patrick Medley, Gretchen K. Campbell,* Luis G. Marcassa,† David E. Pritchard, and Wolfgang Ketterle
MIT-Harvard Center for Ultracold Atoms, Research Laboratory of Electronics, and Department of Physics, MIT, Cambridge, Massachusetts 02139, USA.



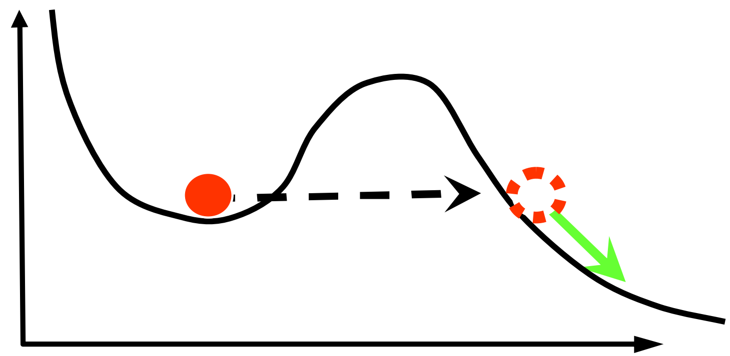
Beyond semiclassical equations. Current decay by tunneling



Current carrying states are metastable.
They can decay by thermal or quantum tunneling

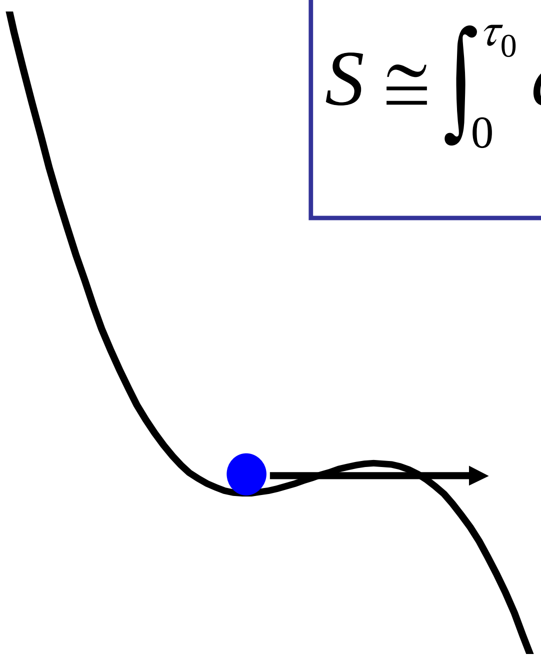


Thermal activation



Quantum tunneling

Decay rate from a metastable state. Example

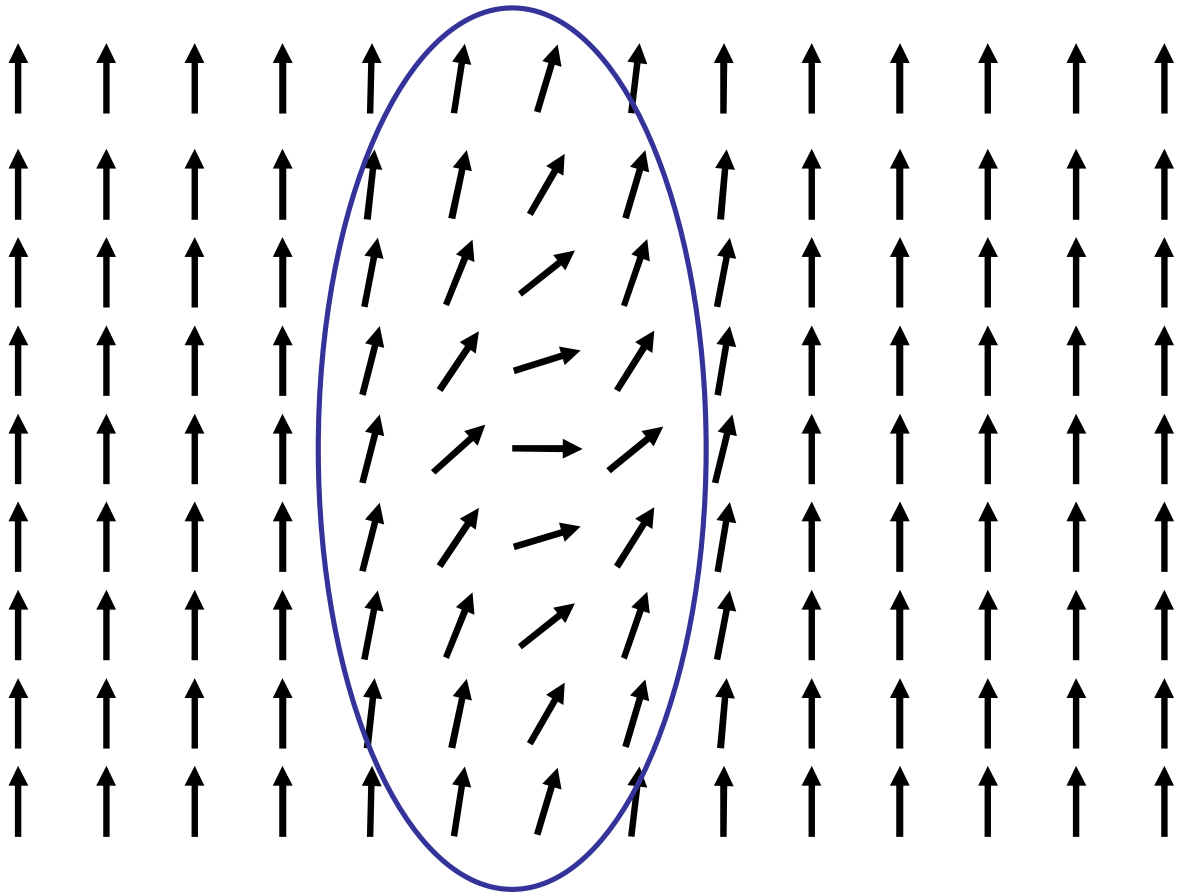


$$\boxed{S \cong \int_0^{\tau_0} d\tau \left(\frac{1}{2m} \left(\frac{dx}{d\tau} \right)^2 + \varepsilon x^2 - b x^3 \right) \qquad \varepsilon \propto (p_c - p) \to 0}$$

$$\Gamma \sim e^{-S}$$

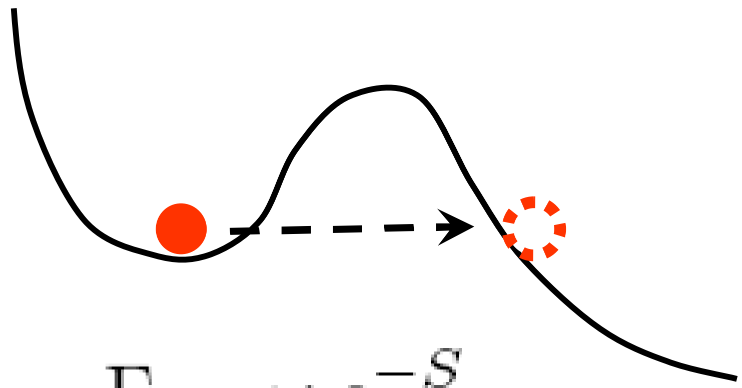
$$S \sim (p_c - p)^{5/2}$$

Need to consider dynamics of many degrees of freedom to describe a phase slip

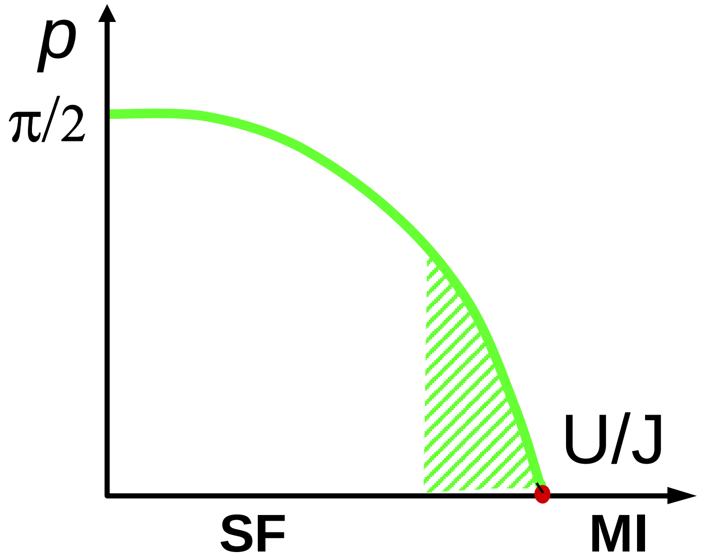


A. Polkovnikov et al., Phys. Rev. A 71:063613 (2005)

Strongly interacting regime. Vicinity of the SF-Mott transition
Decay of current by quantum tunneling



$$\Gamma = \omega\, e^{-S}$$



Action of a quantum phase slip in d=1,2,3

ξ - correlation length

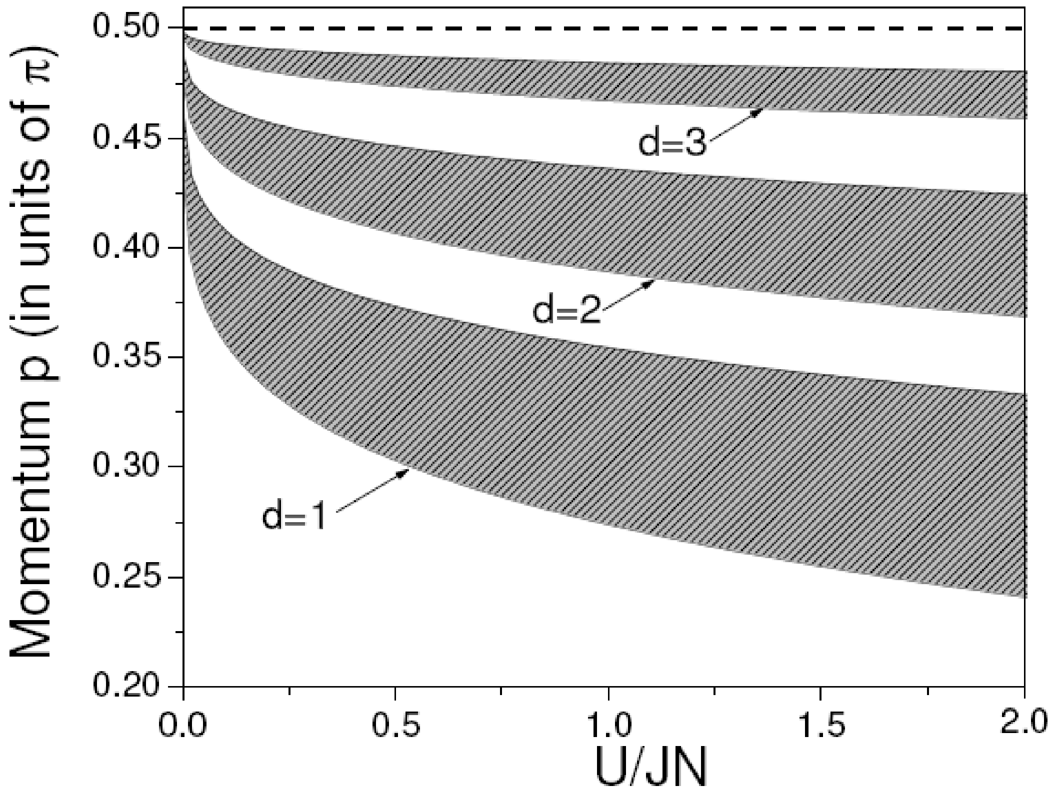
$$S_d \sim \frac{1}{\xi^2}\left(1 - \sqrt{3}\,p\,\xi\right)^{5/2-d}$$

$$\xi \sim (U_c - U)^{-1/2}$$

Strong broadening of the phase transition in d=1 and d=2

S_{3d} is discontinuous at the transition. Phase slips are not important.
Sharp phase transition

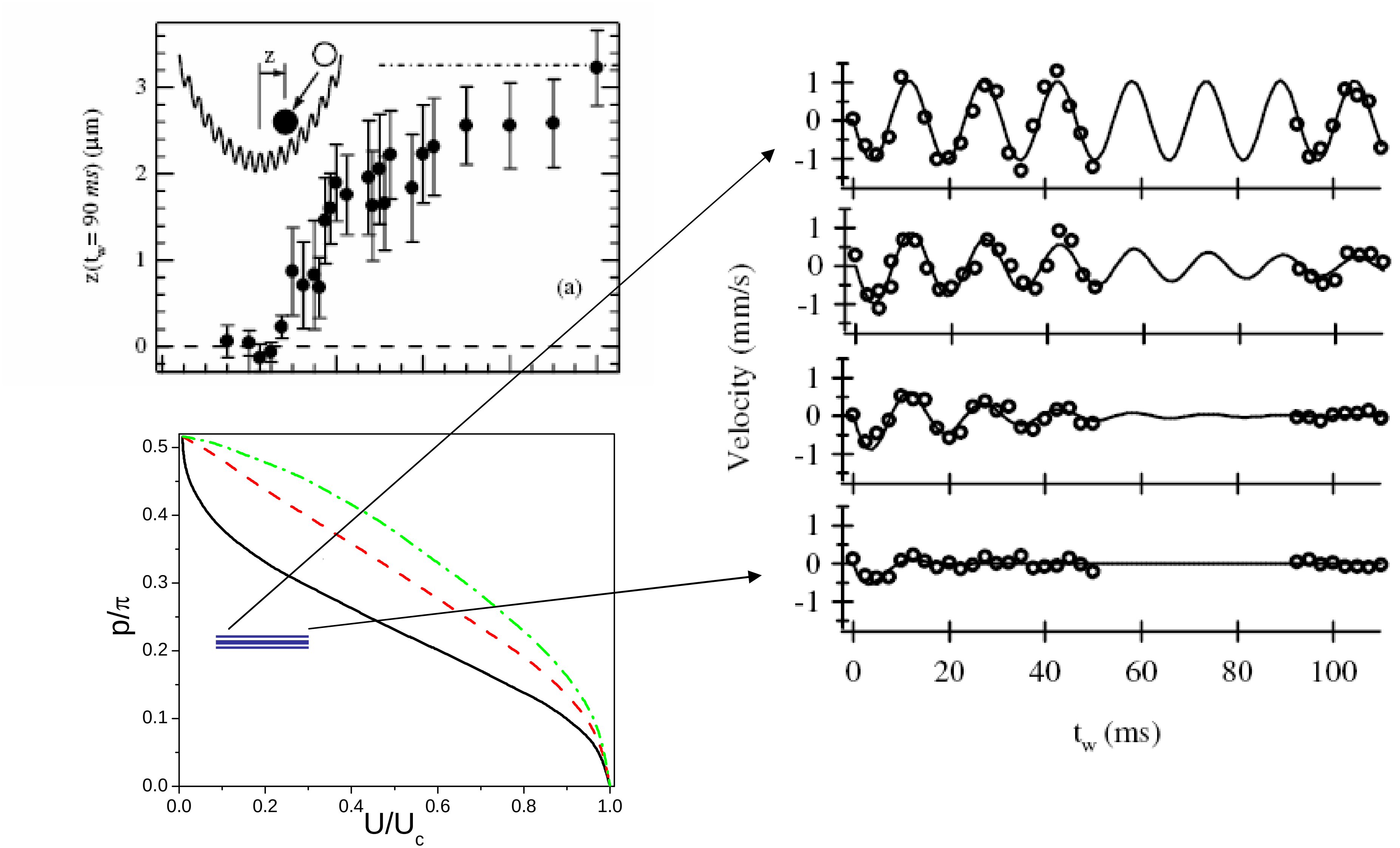
Decay of current by quantum tunneling



PRL **94,** 120403 (2005) PHYSICAL REVIEW LETTERS week ending 1 APRIL 2005

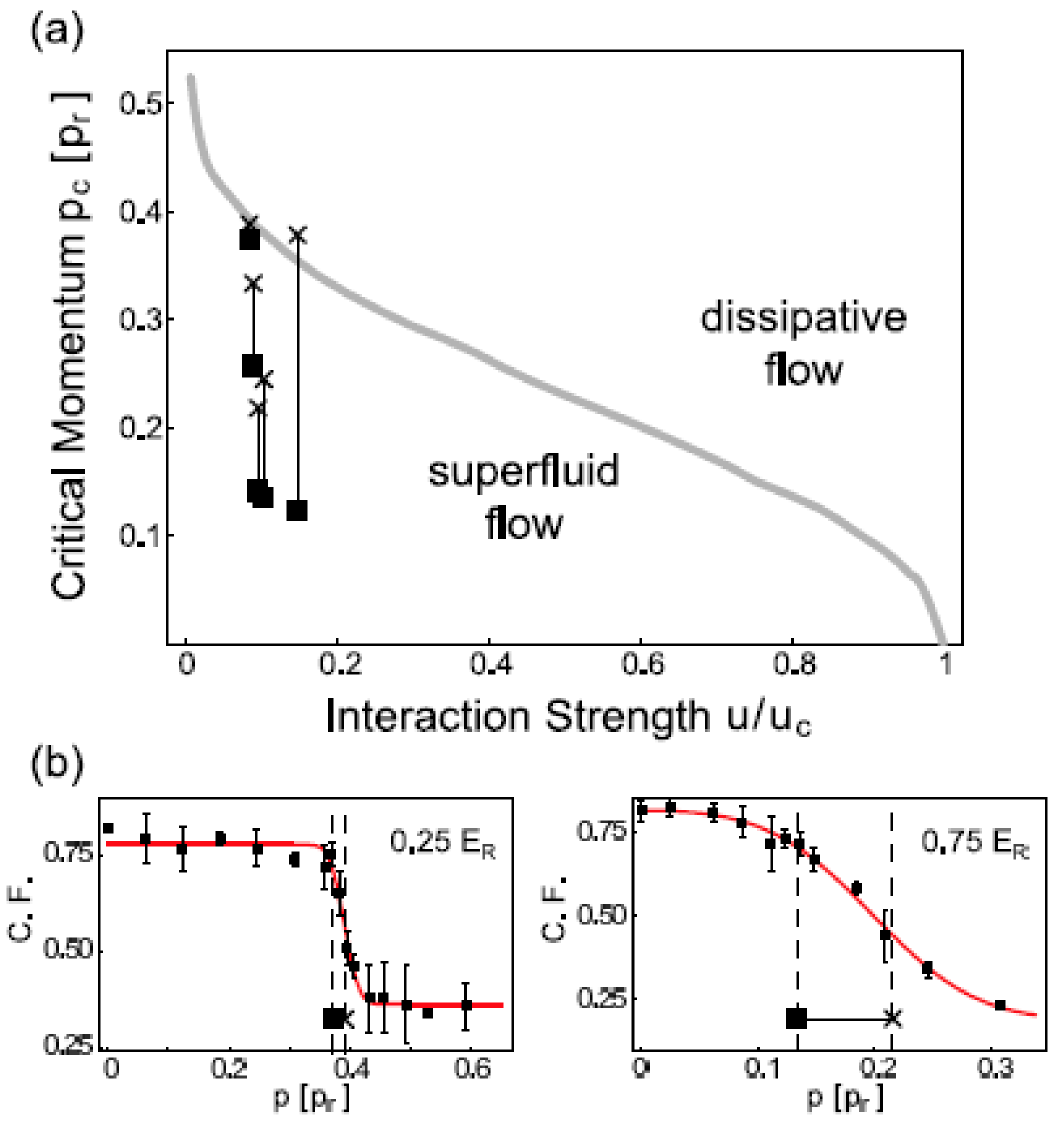
Strongly Inhibited Transport of a Degenerate 1D Bose Gas in a Lattice

C. D. Fertig,[1,2] K. M. O'Hara,[1,*] J. H. Huckans,[1,2] S. L. Rolston,[1,2] W. D. Phillips,[1,2] and J. V. Porto[1]

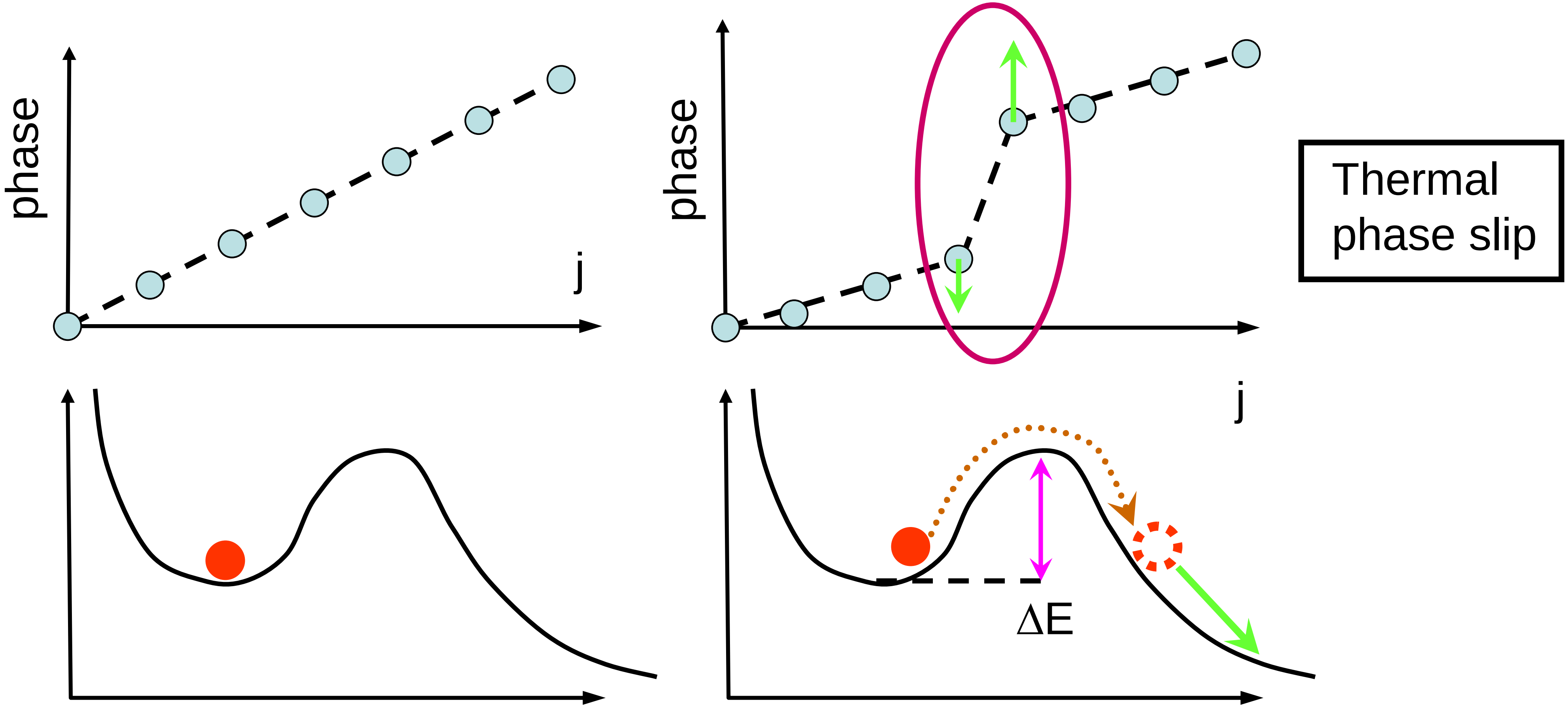


Phase diagram for a Bose-Einstein condensate moving in an optical lattice

Jongchul Mun, Patrick Medley, Gretchen K. Campbell,* Luis G. Marcassa,† David E. Pritchard, and Wolfgang Ketterle
MIT-Harvard Center for Ultracold Atoms, Research Laboratory of Electronics,
and Department of Physics, MIT, Cambridge, Massachusetts 02139, USA.



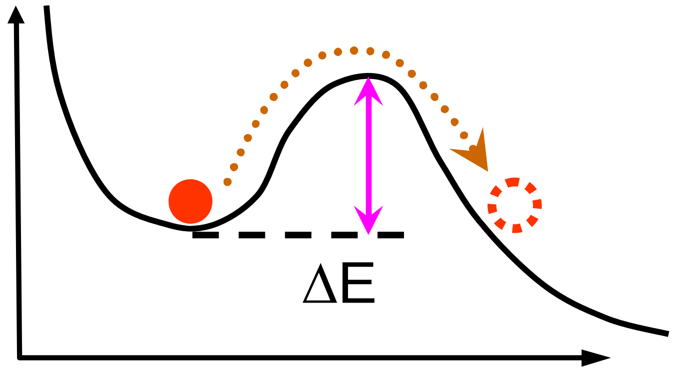
Decay of current by thermal activation



Escape from metastable state by thermal activation

$$\boxed{\Gamma \sim e^{-\Delta E/T}}$$

Thermally activated current decay. Weakly interacting regime



Thermal phase slip

$$\Gamma \sim e^{-\Delta E/T}$$

Activation energy in d=1,2,3

$$\Delta E_1 = 1.3\, J\, N\, (\frac{\pi}{2} - p)^3$$

$$\Delta E_2 = 10\, J\, N\, (\frac{\pi}{2} - p)^{5/2}$$

$$\Delta E_3 = 35\, J\, N\, (\frac{\pi}{2} - p)^2$$

Thermal fluctuations lead to rapid decay of currents

Crossover from thermal to quantum tunneling

$$T_Q \sim \sqrt{N\, J\, U \times (\frac{\pi}{2} - p)}$$

Decay of current by thermal fluctuations

Unstable regimes for a Bose-Einstein condensate in an optical lattice

L. De Sarlo*, L. Fallani, J. E. Lye, M. Modugno[1], R. Saers[†], C. Fort and M. Inguscio

Phys. Rev. Lett. (2004)

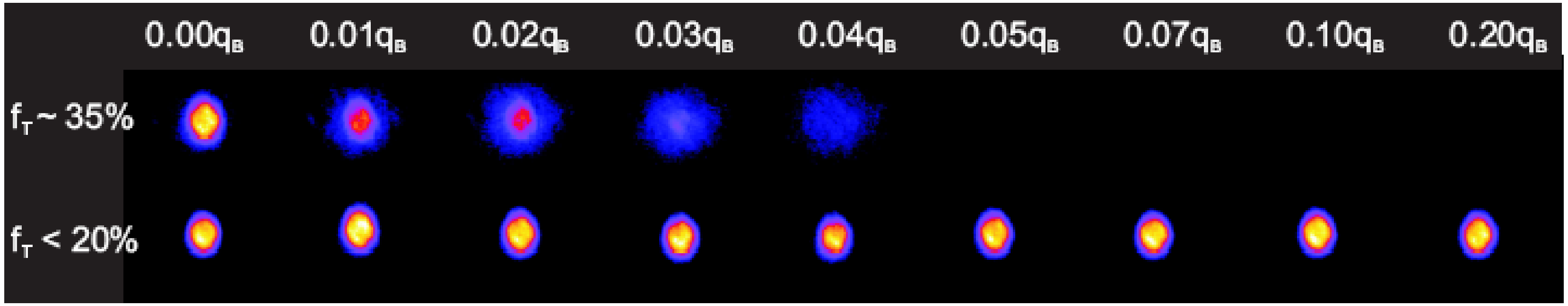
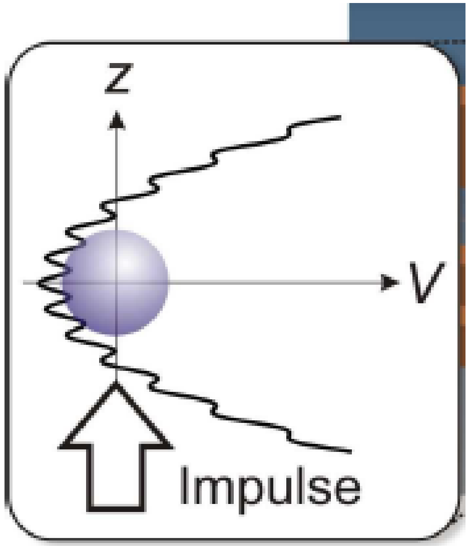
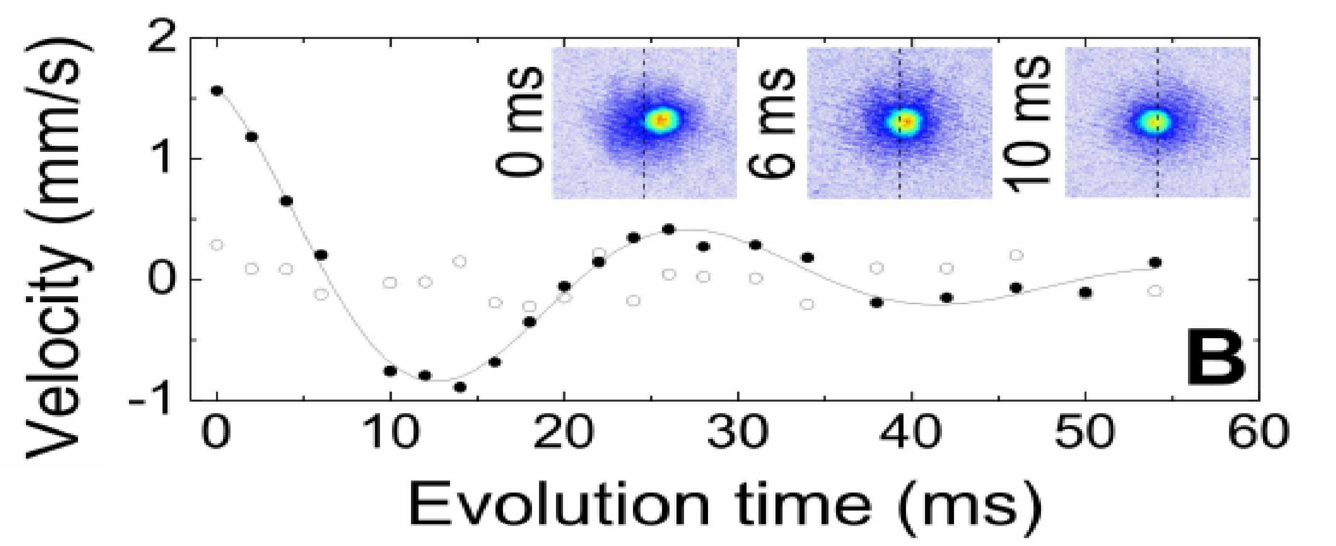


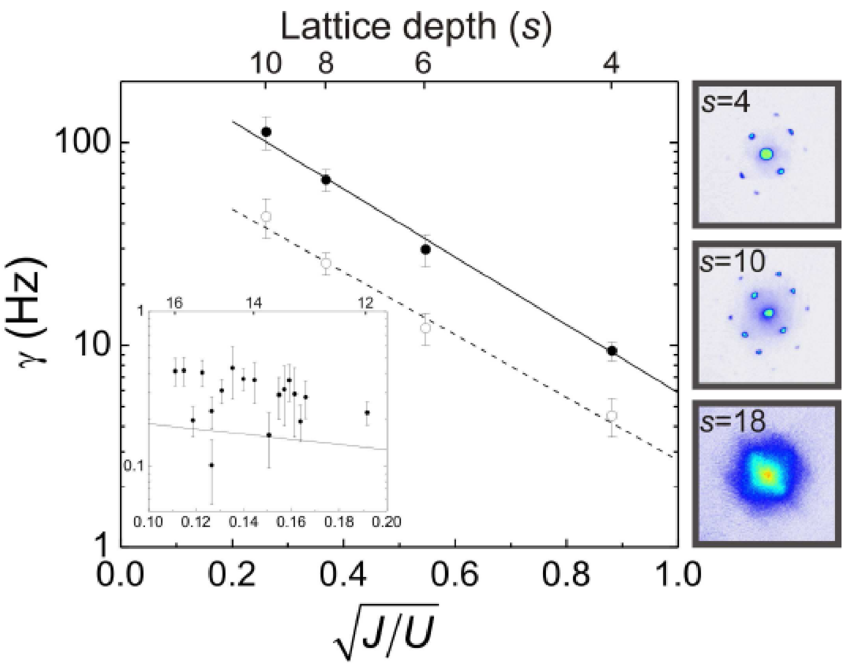
FIG. 2: Absorption images of the condensate interacting for $t = 15$s with a lattice with $s = 0.2$ for different values of quasimomentum ranging from 0 to $0.20\, q_B$ and for respectively a condensed fraction of about 65% (*top*) and no detectable thermal component (*bottom*).

Decay of current by thermal fluctuations



Experiments: Brian DeMarco et al., arXiv 0708:3074





Outline

- ✓ Introduction
- ✓ Basics of cold atoms in optical lattices

  Bose Hubbard model. Superfluid to Mott transition.
  Dynamical instability.
- Two component Bose mixtures

  Quantum magnetism
- Fermions in optical lattices. Bose-Fermi mixtures

  Pairing in systems with repulsive interactions. Polarons
- Low-dimensional Bose systems in and out of equilibrium

  Analysis of correlations beyond mean-field.

  Interference experiments with low dimensional condensates

Emphasis: detection and characterzation of many-body states

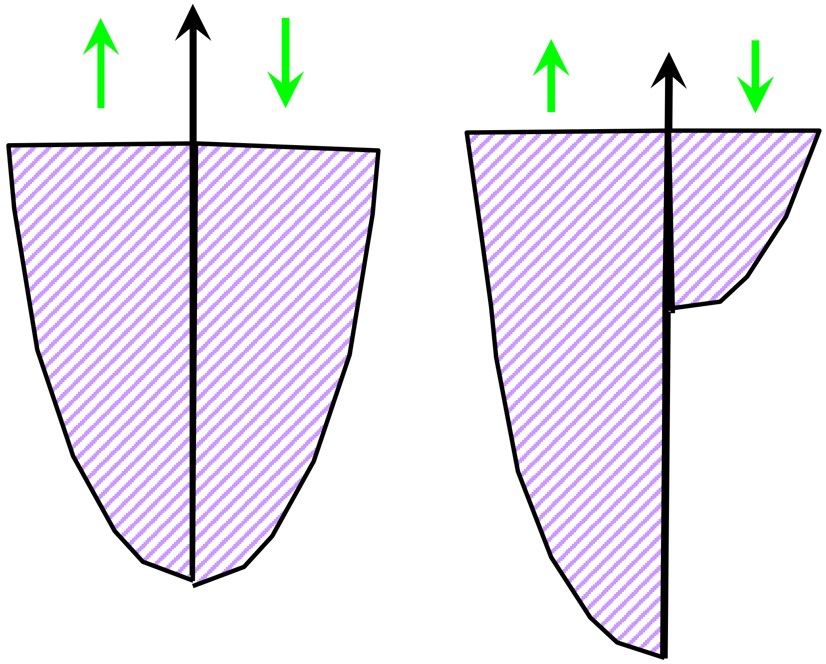
Magnetism in condensed matter systems

Ferromagnetism

Magnetic needle in a compass

Magnetic memory in hard drives.
Storage density of hundreds of
billions bits per square inch.

Stoner model of ferromagnetism



Spontaneous spin polarization decreases interaction energy but increases kinetic energy of electrons

Mean-field criterion

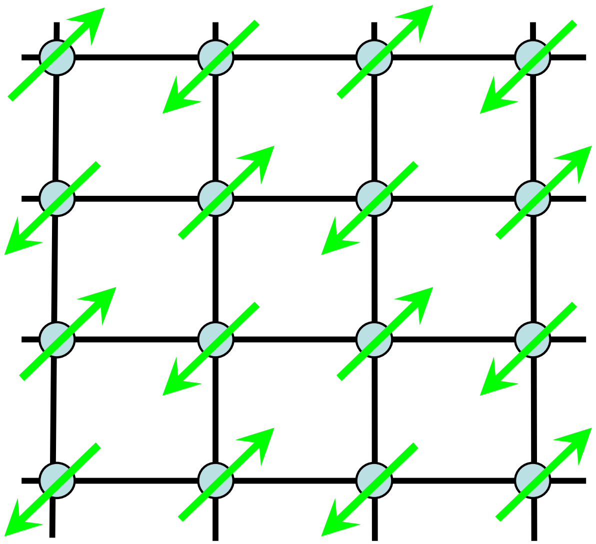
$$I\,N(0) = 1$$

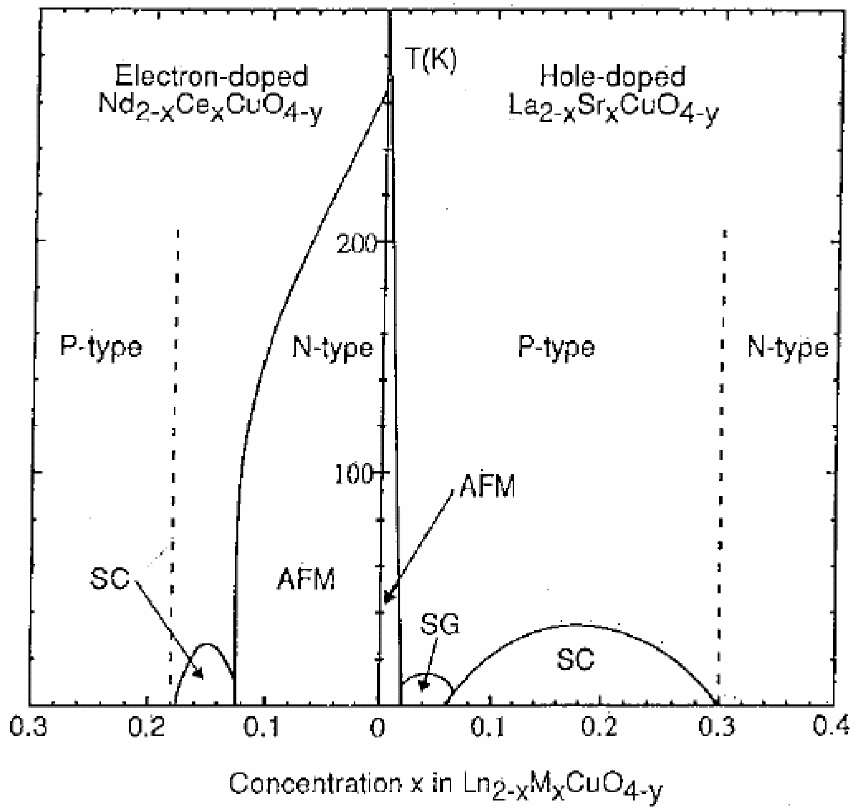
I – interaction strength

$N(0)$ – density of states at the Fermi level

Antiferromagnetism

Maple, JMMM 177:18 (1998)





High temperature superconductivity in cuprates is always found near an antiferromagnetic insulating state

Antiferromagnetism

Antiferromagnetic Heisenberg model $\mathcal{H} = J \sum_{\langle ij \rangle} \vec{S}_i \vec{S}_j$

$$|AF\rangle = |\uparrow \downarrow\rangle$$

$$|s\rangle = \frac{1}{\sqrt{2}} \left(|\uparrow \downarrow\rangle - |\downarrow \uparrow\rangle \right)$$

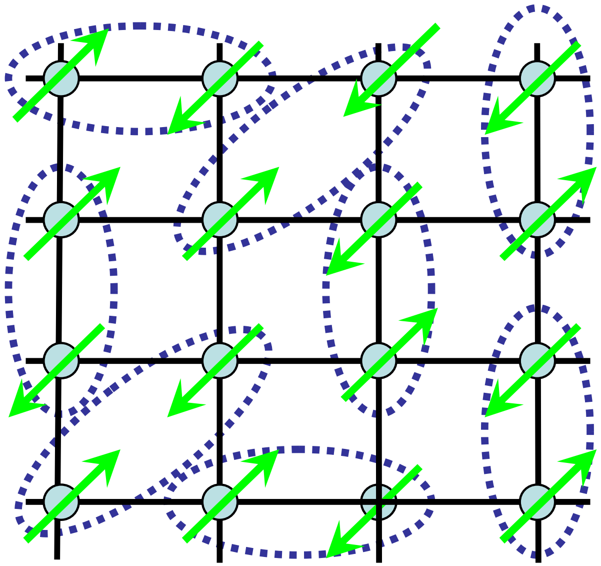
$$|t\rangle = \frac{1}{\sqrt{2}} \left(|\uparrow \downarrow\rangle + |\downarrow \uparrow\rangle \right)$$

$$\boxed{|AF\rangle = \frac{1}{\sqrt{2}} \left(|s\rangle + |t\rangle \right)}$$

Antiferromagnetic state breaks spin symmetry.
It does not have a well defined spin

Spin liquid states

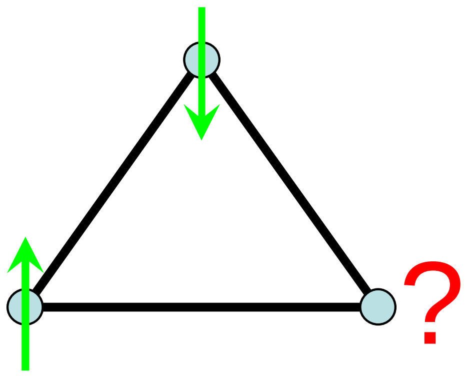
Alternative to classical antiferromagnetic state: spin liquid states



Properties of spin liquid states:

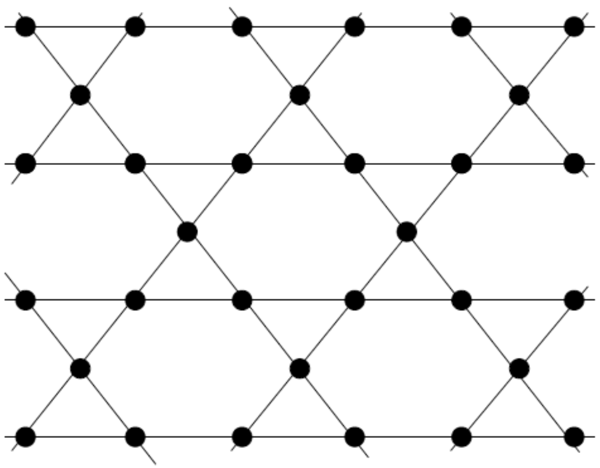
- fractionalized excitations
- topological order
- gauge theory description

Systems with geometric frustration



Spin liquid behavior in systems with geometric frustration

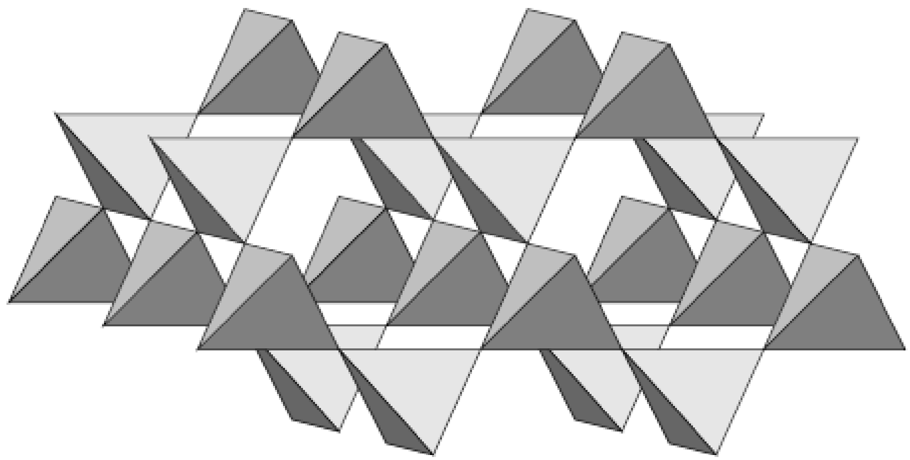
Kagome lattice



$SrCr_{9-x}Ga_{3+x}O_{19}$

Ramirez et al. PRL (90)
Broholm et al. PRL (90)
Uemura et al. PRL (94)

Pyrochlore lattice



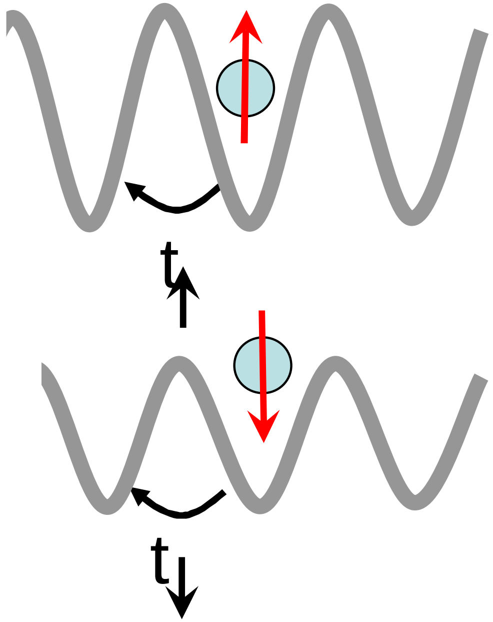
$ZnCr_2O_4$
$A_2Ti_2O_7$

Ramirez et al. PRL (02)

Engineering magnetic systems using cold atoms in an optical lattice

Two component Bose mixture in optical lattice

Example: $^{87}\mathrm{Rb}$.   Mandel et al., Nature 425:937 (2003)



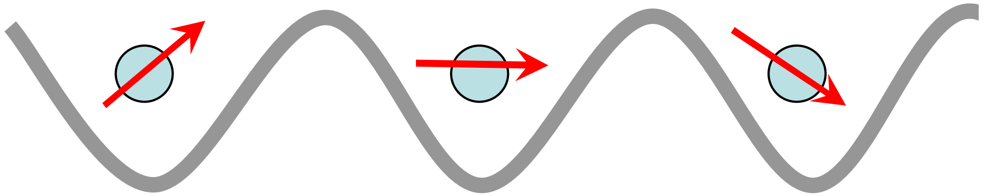
$$|\uparrow\rangle = |F=1, m_F=-1\rangle$$

$$|\downarrow\rangle = |F=2, m_F=-2\rangle$$

Two component Bose Hubbard model

$$\mathcal{H} = -\ t_\uparrow \sum_{\langle ij\rangle} b^\dagger_{i\uparrow} b_{j\uparrow} - t_\downarrow \sum_{\langle ij\rangle} b^\dagger_{i\downarrow} b_{j\downarrow} + U_{\uparrow\uparrow} \sum_i n_{i\uparrow}(n_\uparrow - 1)$$
$$+\ U_{\downarrow\downarrow} \sum_i n_{i\downarrow}(n_\downarrow - 1) + U_{\uparrow\downarrow} \sum_i n_{i\uparrow} n_\downarrow$$

Quantum magnetism of bosons in optical lattices



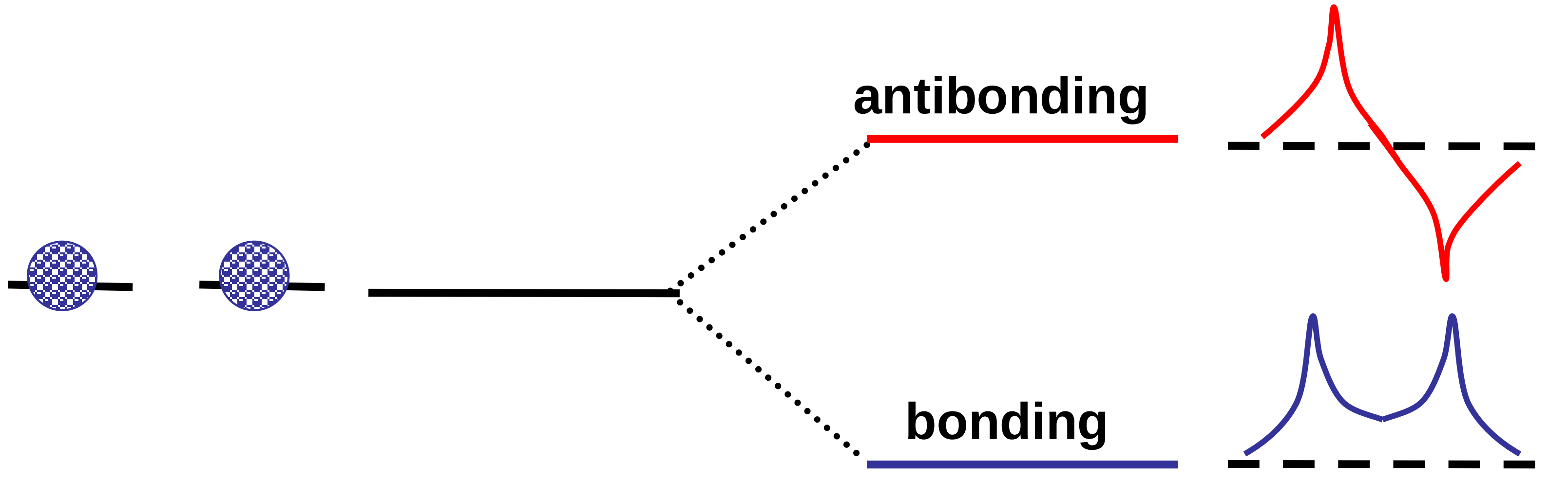
Duan, Demler, Lukin, PRL 91:94514 (2003)

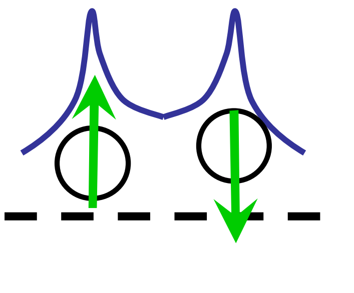
$$\mathcal{H} = J_z \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z + J_\perp \sum_{\langle ij \rangle} \left(\sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y \right)$$

$$J_z = \frac{t_\uparrow^2 + t_\downarrow^2}{2U_{\uparrow\downarrow}} - \frac{t_\uparrow^2}{U_{\uparrow\uparrow}} - \frac{t_\downarrow^2}{U_{\downarrow\downarrow}} \qquad J_\perp = -\frac{t_\uparrow t_\downarrow}{U_{\uparrow\downarrow}}$$

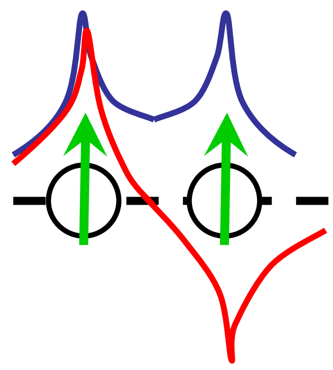
- Ferromagnetic $\quad U_{\uparrow\downarrow} >> U_{\uparrow\uparrow},\ U_{\downarrow\downarrow}$
- Antiferromagnetic $\quad U_{\uparrow\downarrow} << U_{\uparrow\uparrow},\ U_{\downarrow\downarrow}$

Exchange Interactions in Solids





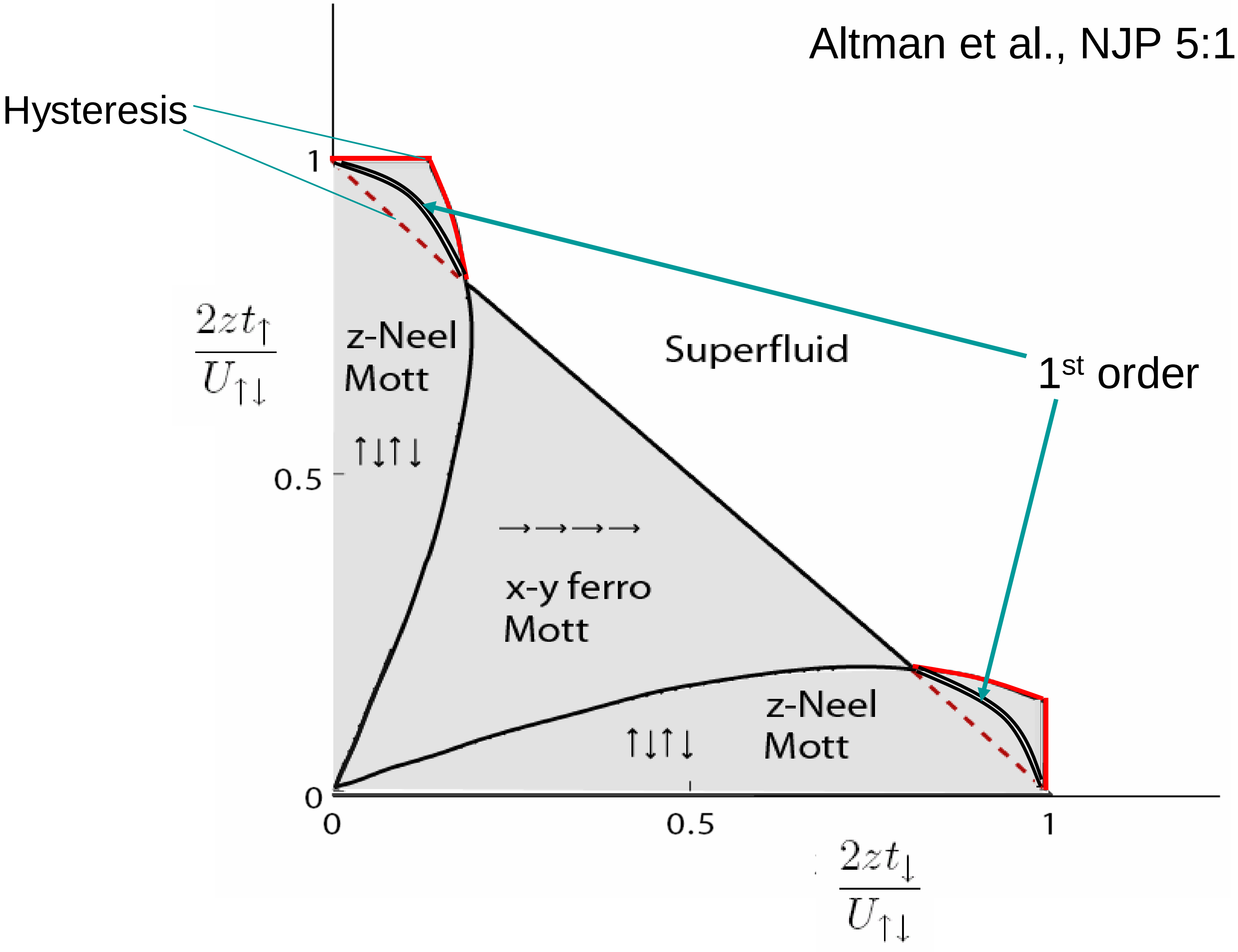
Kinetic energy dominates: antiferromagnetic state



Coulomb energy dominates: ferromagnetic state

Two component Bose mixture in optical lattice.
Mean field theory + Quantum fluctuations

Altman et al., NJP 5:113 (2003)



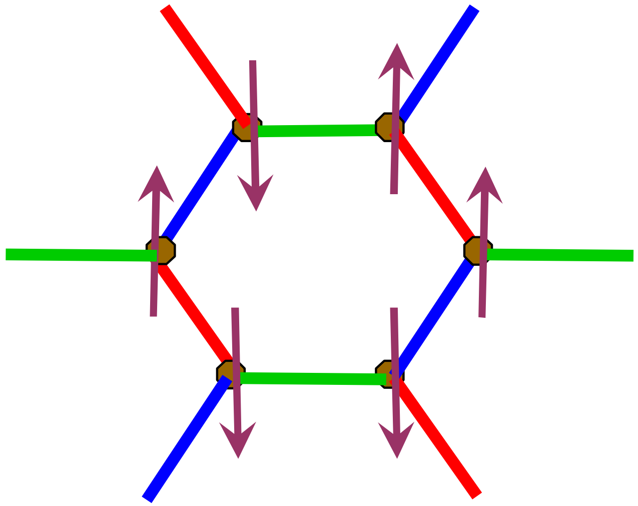
Realization of spin liquid using cold atoms in an optical lattice

Theory: Duan, Demler, Lukin PRL 91:94514 (03)

Kitaev model

Annals of Physics 321:2 (2006)

$$H = - J_x \sum \sigma_i^x \sigma_j^x - J_y \sum \sigma_i^y \sigma_j^y - J_z \sum \sigma_i^z \sigma_j^z$$



Questions:

Detection of topological order

Creation and manipulation of spin liquid states

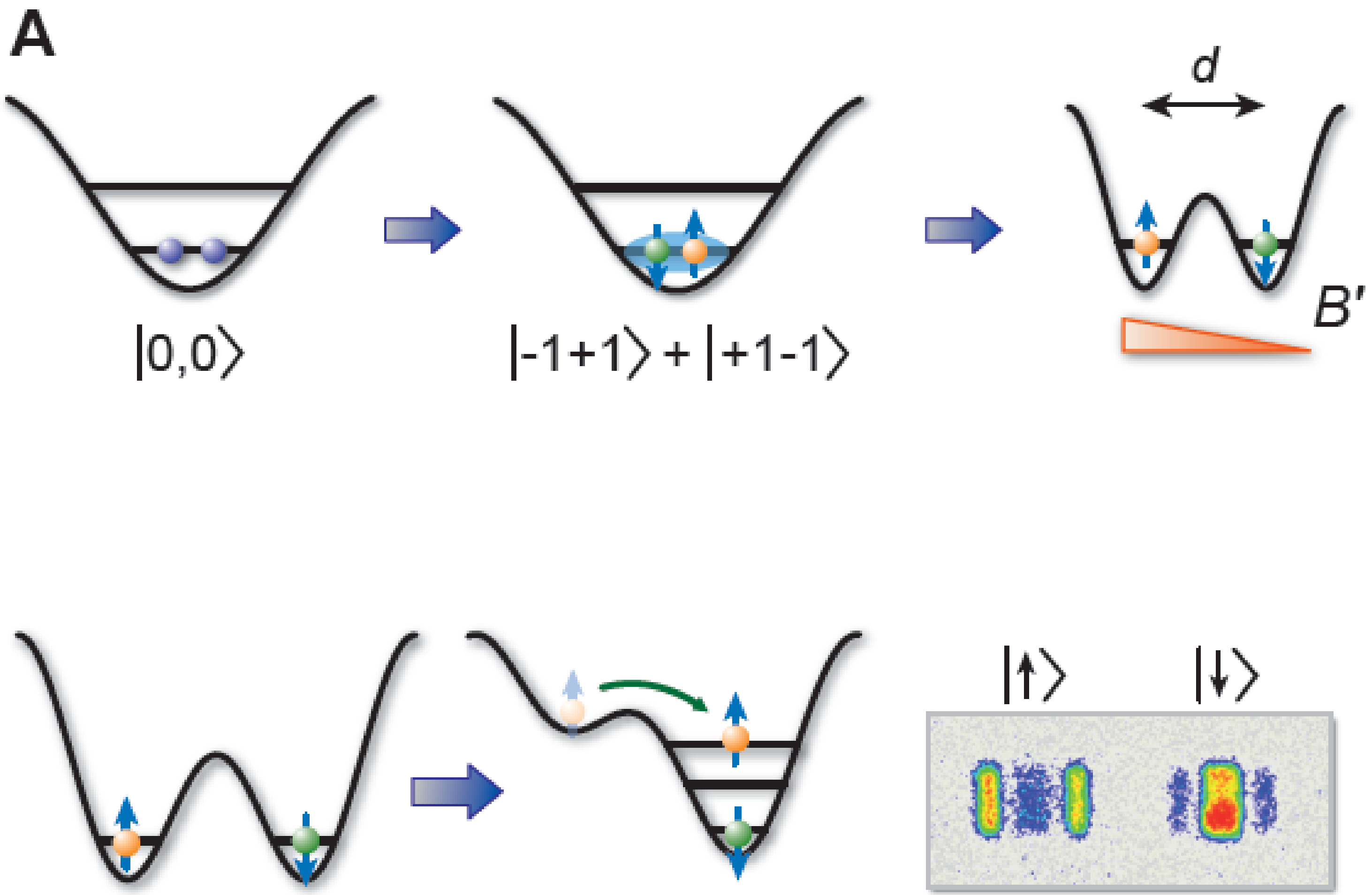
Detection of fractionalization, Abelian and non-Abelian anyons

Melting spin liquids. Nature of the superfluid state

Superexchange interaction
in experiments with double wells

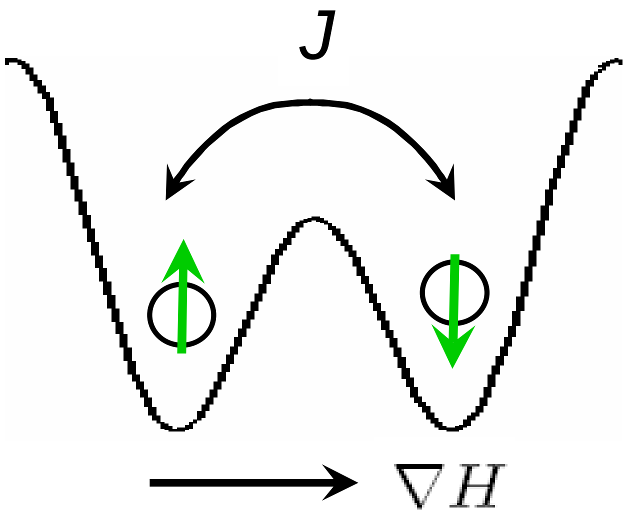
Immanuel Bloch et al.

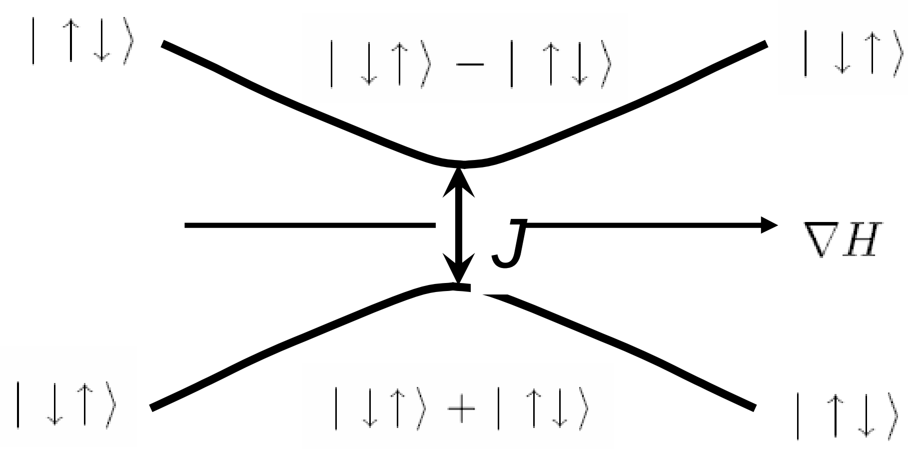
Preparation and detection of Mott states of atoms in a double well potential



Observation of superexchange in a double well potential

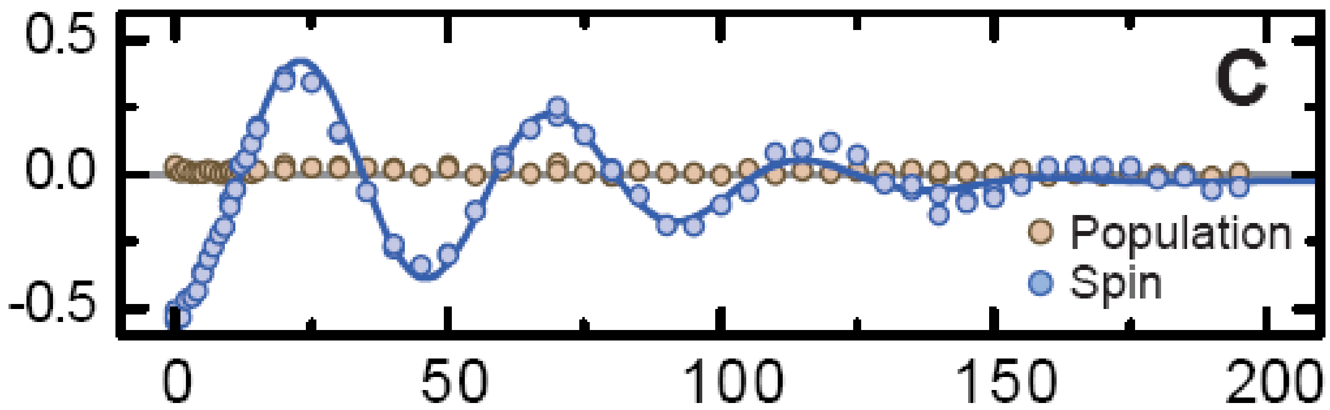
Theory: A.M. Rey et al., arXiv:0704.1413





Use magnetic field gradient to prepare a state $|\downarrow\uparrow\rangle$

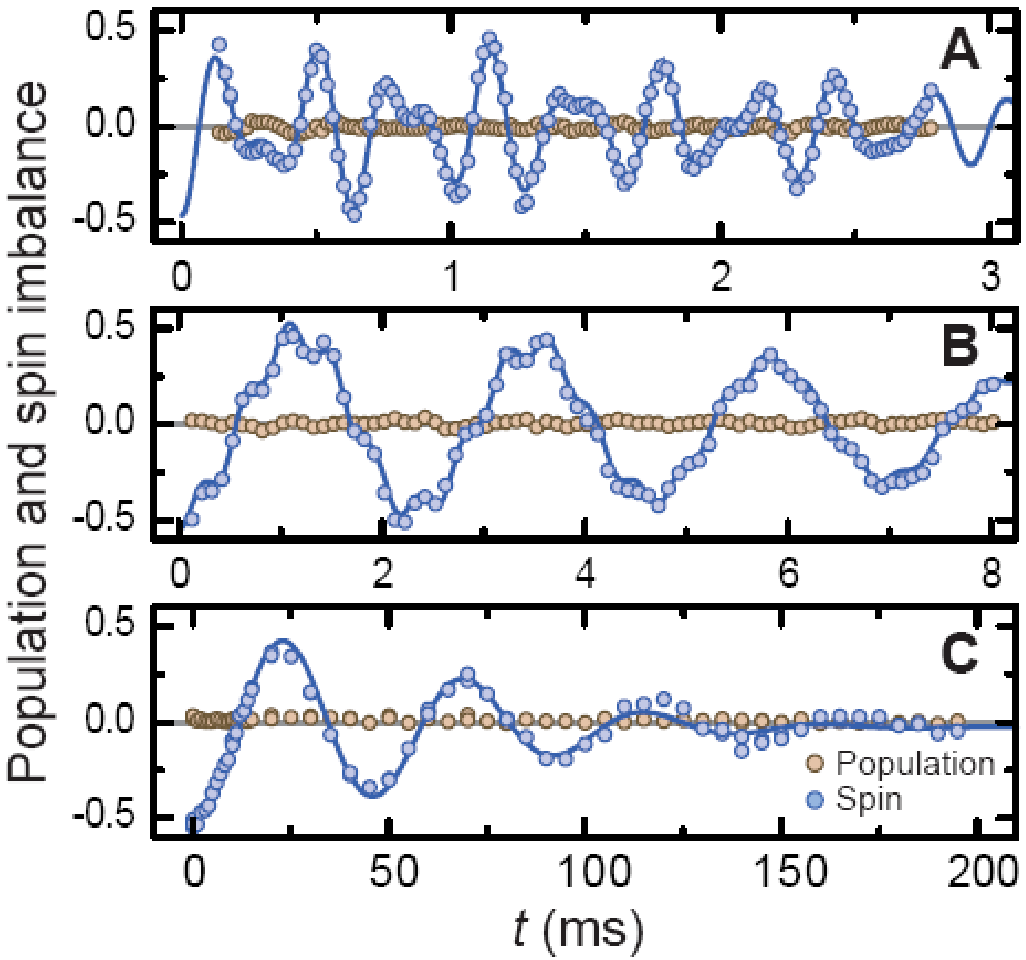
Observe oscillations between $|\downarrow\uparrow\rangle$ and $|\uparrow\downarrow\rangle$ states



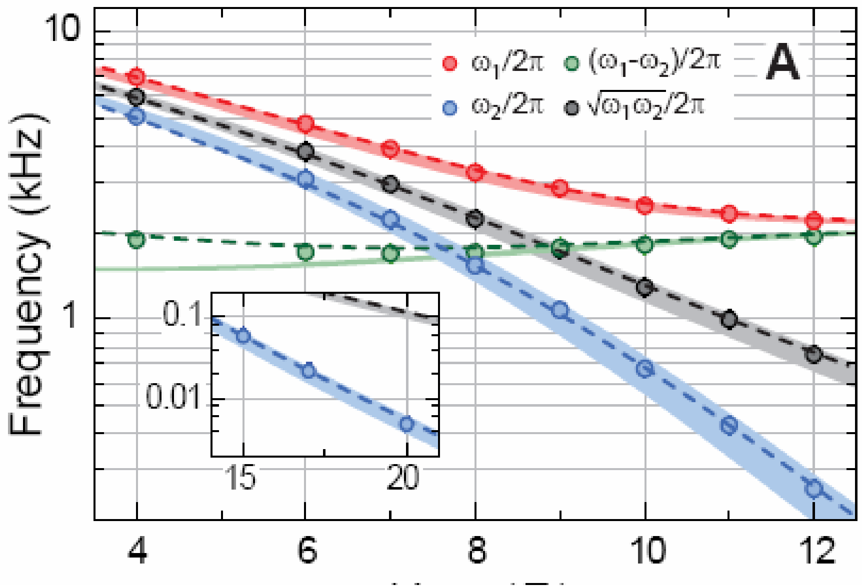
Experiments:
I. Bloch et al.

Comparison to the Hubbard model

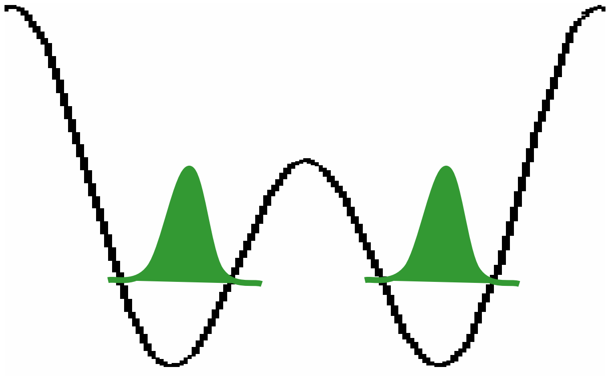
Experiments: I. Bloch et al.



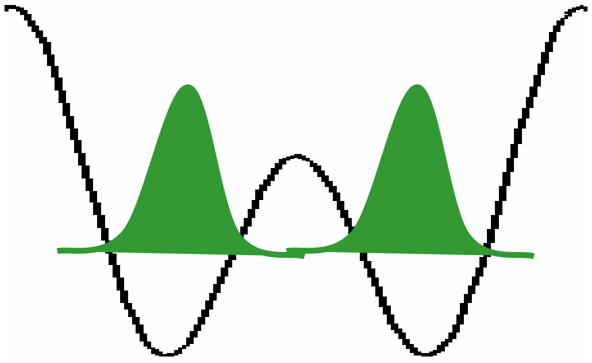
$$\hbar\omega_{1,2} = \frac{U}{2}\left(\sqrt{\left(\frac{4J}{U}\right)^2 + 1} \pm 1\right)$$



Beyond the basic Hubbard model



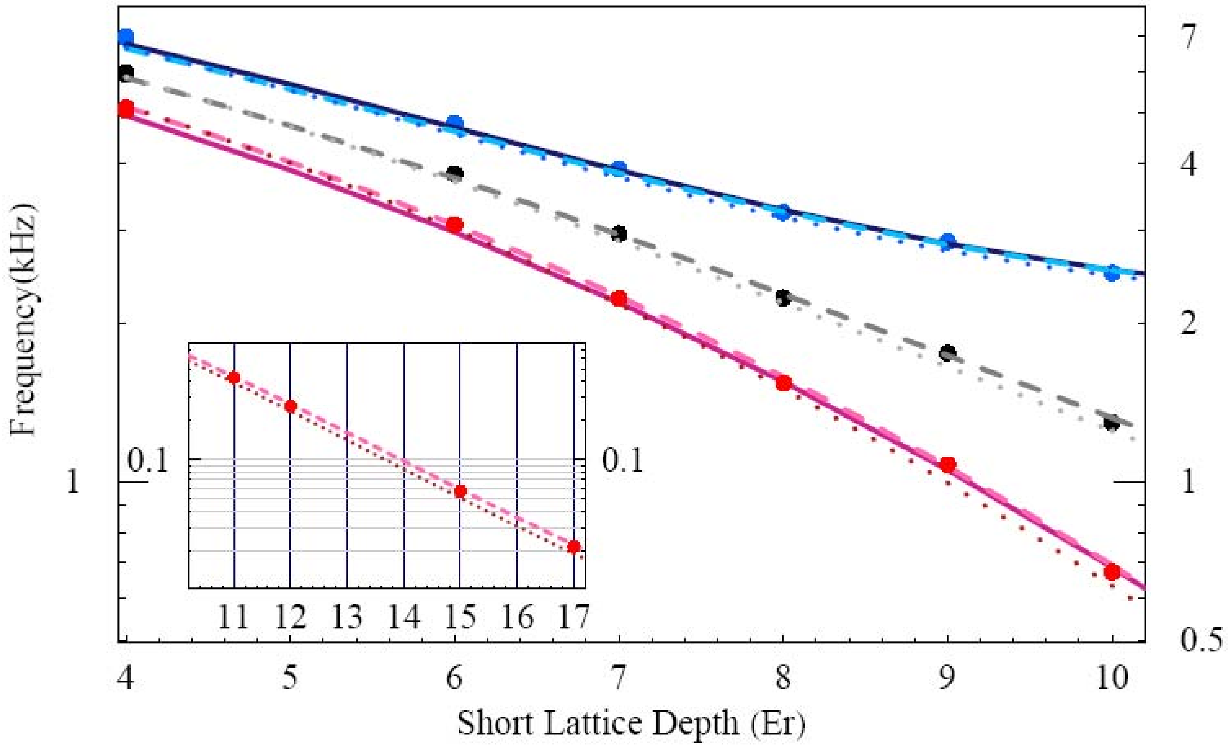
Basic Hubbard model includes only local interaction

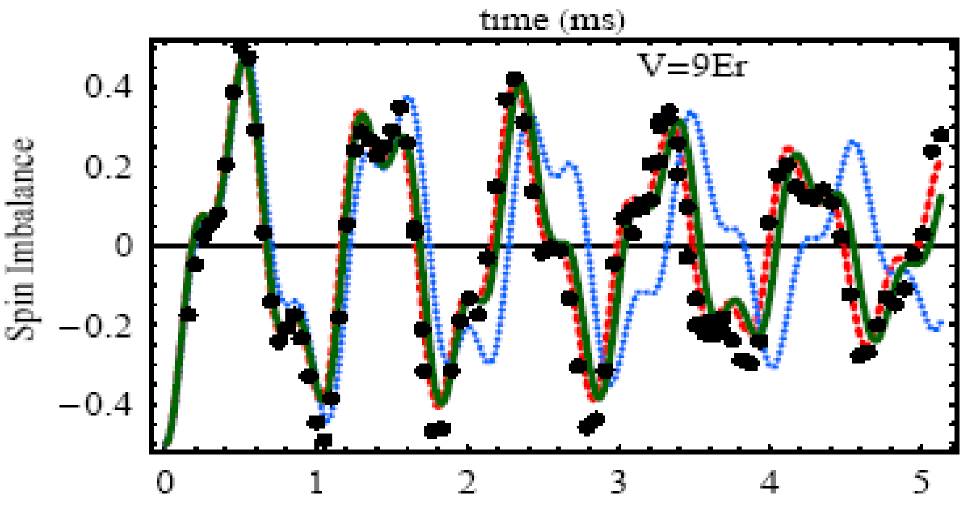


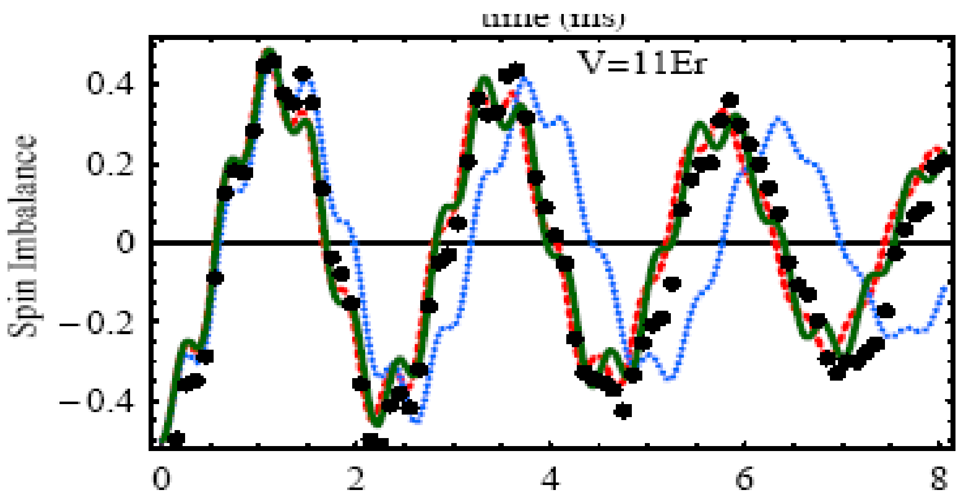
Extended Hubbard model takes into account non-local interaction

$$\begin{aligned}\hat{H}^{\mathrm{EHM}} &= \hat{H}^{\mathrm{HM}} - \Delta J \sum_{\sigma\neq\sigma'} \left(\hat{n}_{\sigma\mathrm{L}} + \hat{n}_{\sigma\mathrm{R}}\right)\left(\hat{a}^{\dagger}_{\sigma'\mathrm{L}}\hat{a}_{\sigma'\mathrm{R}} + \hat{a}^{\dagger}_{\sigma'\mathrm{R}}\hat{a}_{\sigma'\mathrm{L}}\right) \\ &+ U_{\mathrm{LR}} \sum_{\sigma\neq\sigma'} \Big(\hat{n}_{\sigma\mathrm{L}}\hat{n}_{\sigma'\mathrm{R}} + \hat{a}^{\dagger}_{\sigma\mathrm{L}}\hat{a}^{\dagger}_{\sigma'\mathrm{R}}\hat{a}_{\sigma'\mathrm{L}}\hat{a}_{\sigma\mathrm{R}} \\ &\qquad + \frac{1}{2}\hat{a}^{\dagger}_{\sigma\mathrm{L}}\hat{a}^{\dagger}_{\sigma'\mathrm{L}}\hat{a}_{\sigma'\mathrm{R}}\hat{a}_{\sigma\mathrm{R}} + \frac{1}{2}\hat{a}^{\dagger}_{\sigma\mathrm{R}}\hat{a}^{\dagger}_{\sigma'\mathrm{R}}\hat{a}_{\sigma'\mathrm{L}}\hat{a}_{\sigma\mathrm{L}}\Big) \,,\end{aligned}$$

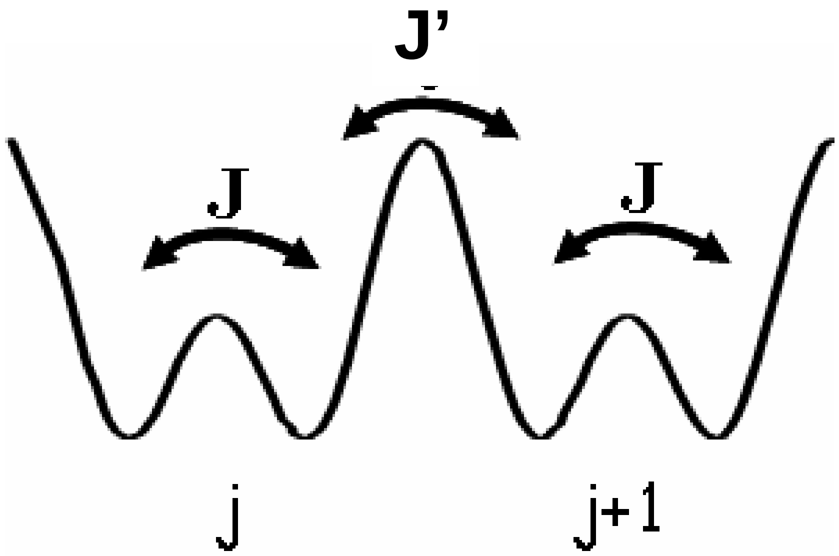
Beyond the basic Hubbard model



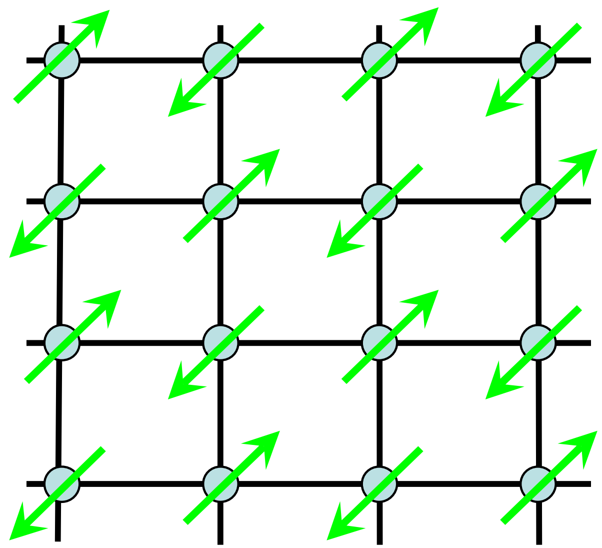




Connecting double wells …



Goal: observe antiferromagnetic order of cold atoms in an optical lattice!



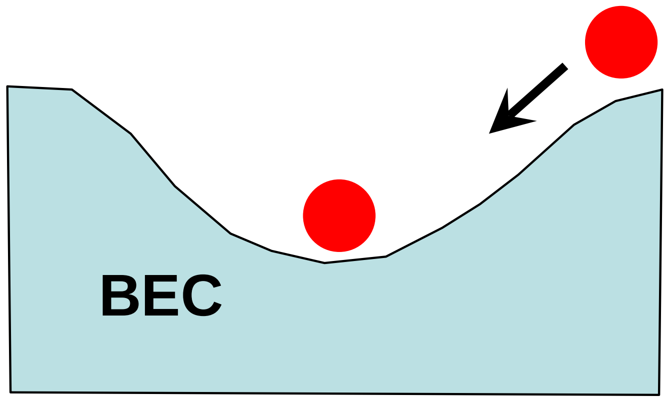
Detection: quantum noise, using superlattice (merging two wells into one), …

Boson Fermion mixtures

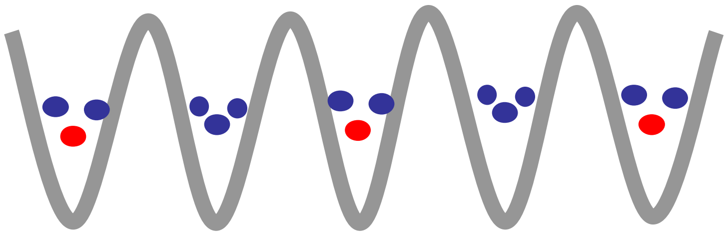
Fermions interacting with phonons.
Polarons. Competing orders

Boson Fermion mixtures

Experiments: ENS, Florence, JILA, MIT, ETH, Hamburg, Rice, …

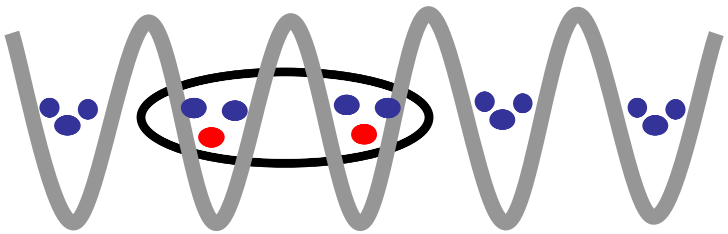


Bosons provide cooling for fermions and mediate interactions. They create non-local attraction between fermions



Charge Density Wave Phase

Periodic arrangement of atoms



Non-local Fermion Pairing

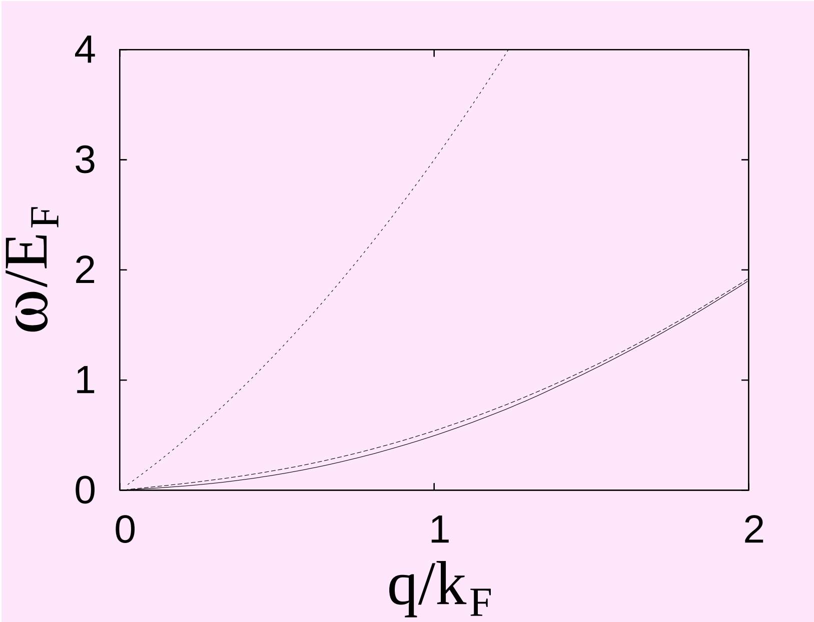
P-wave, D-wave, …

Boson Fermion mixtures

$$\begin{aligned}\mathcal{H} &= \mathcal{H}_{bb} + \mathcal{H}_{\rm ff} + \mathcal{H}_{bf}\\ \mathcal{H}_{bb} &= -t_b \sum_{\langle ij\rangle} b_i^\dagger b_j + U_{bb}\sum_i n_{bi}(n_{bi}-1)\\ \mathcal{H}_{ff} &= -t_f \sum_{\langle ij\rangle} f_i^\dagger f_j\\ \mathcal{H}_{bf} &= U_{bf}\sum_i n_{bi} n_{fi}\end{aligned}$$

"Phonons" :
Bogoliubov (phase) mode

Effective fermion-"phonon" interaction



$$\begin{aligned}\tilde{\mathcal{H}}_{bb} &= \sum_q \omega_q \beta_q^\dagger \beta_q\\ \tilde{\mathcal{H}}_{bf} &= \sum_{kq} g_q\,(\beta_q + \beta_{-q}^\dagger)\, f_{k+q}^\dagger f_k\end{aligned}$$

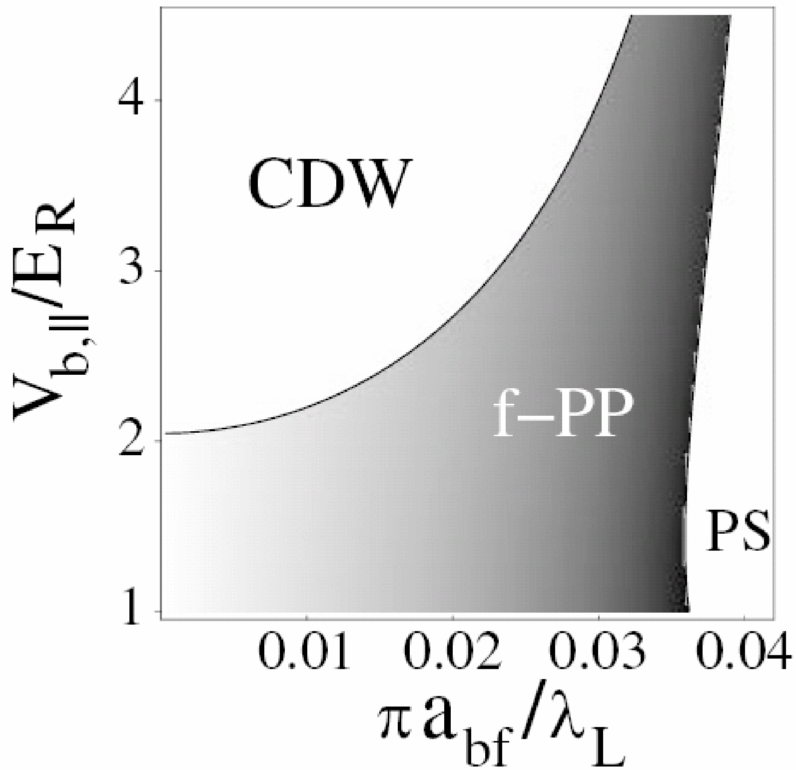
Fermion-"phonon" vertex $g_q \sim |q|$
Similar to electron-phonon systems

Boson Fermion mixtures in 1d optical lattices

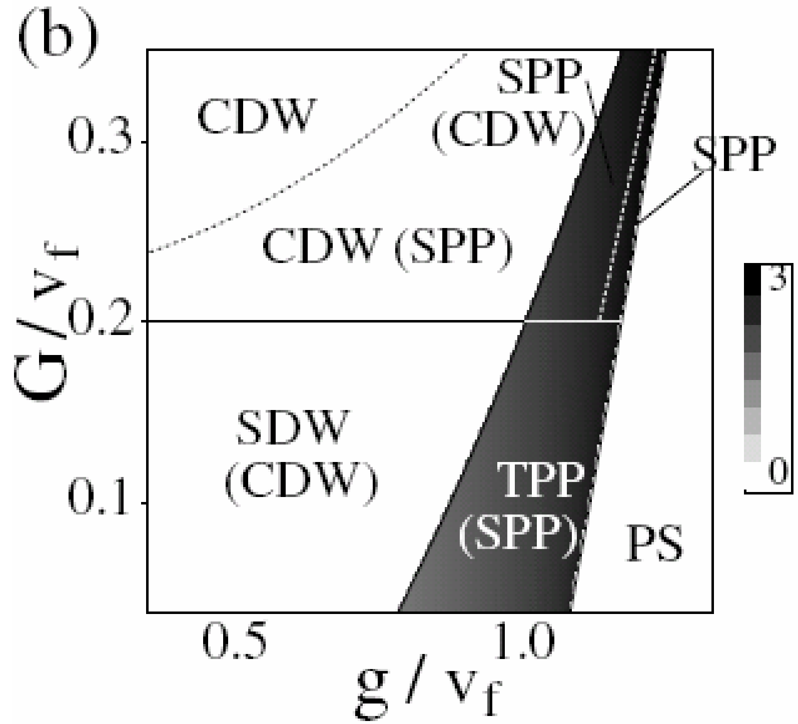
Cazalila et al., PRL (2003); Mathey et al., PRL (2004)

$$\mathcal{H} = -t_b \sum_{\langle ij \rangle} b_i^\dagger b_j - t_f \sum_{\langle ij \rangle} f_i^\dagger f_j - \mu_b \sum_i n_{bi} - \mu_f \sum_i n_{fi} + U_{bb} \sum_i n_{bi}^2 + U_{bf} \sum_i n_{bi} n_{fi}$$

Spinless fermions



Spin ½ fermions

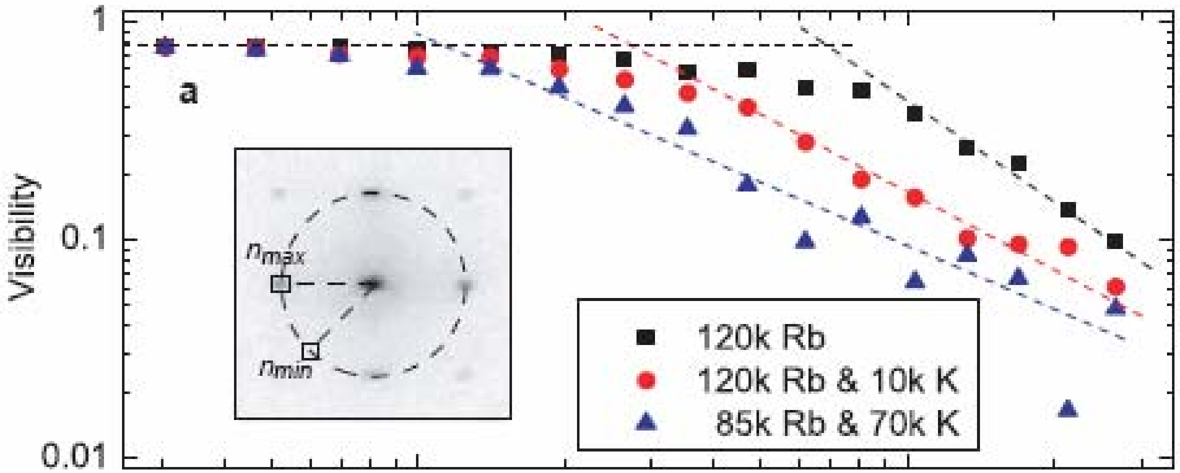


Note: Luttinger parameters can be determined using correlation function measurements in the time of flight experiments. Altman et al. (2005)

Bose-Fermi Mixtures in a Three-dimensional Optical Lattice

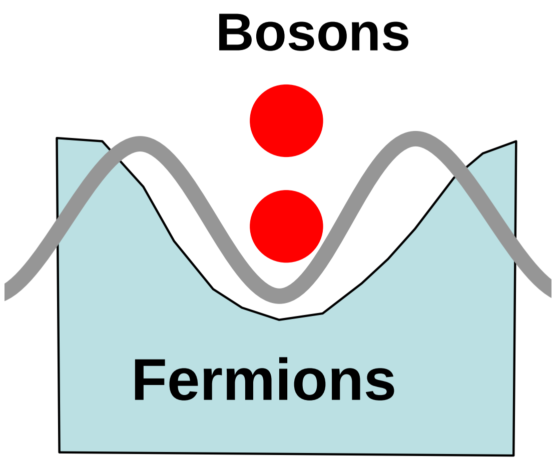
Kenneth Günter, Thilo Stöferle, Henning Moritz, Michael Köhl*, Tilman Esslinger
Institute of Quantum Electronics, ETH Zürich, CH–8093 Zürich, Switzerland
(Dated: May 24, 2007)

Suppression of superfluidity of bosons by fermions

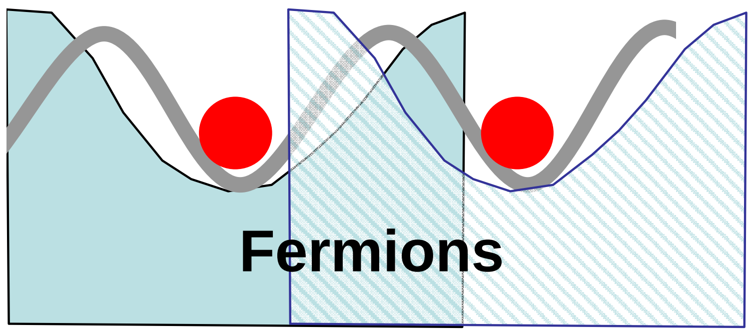


Fermion-Boson mixtures, see also Ospelkaus et al., cond-mat/0604179
Bose-Bose mixtures, see Catani et al., arXiv:0706.278

Competing effects of fermions on bosons

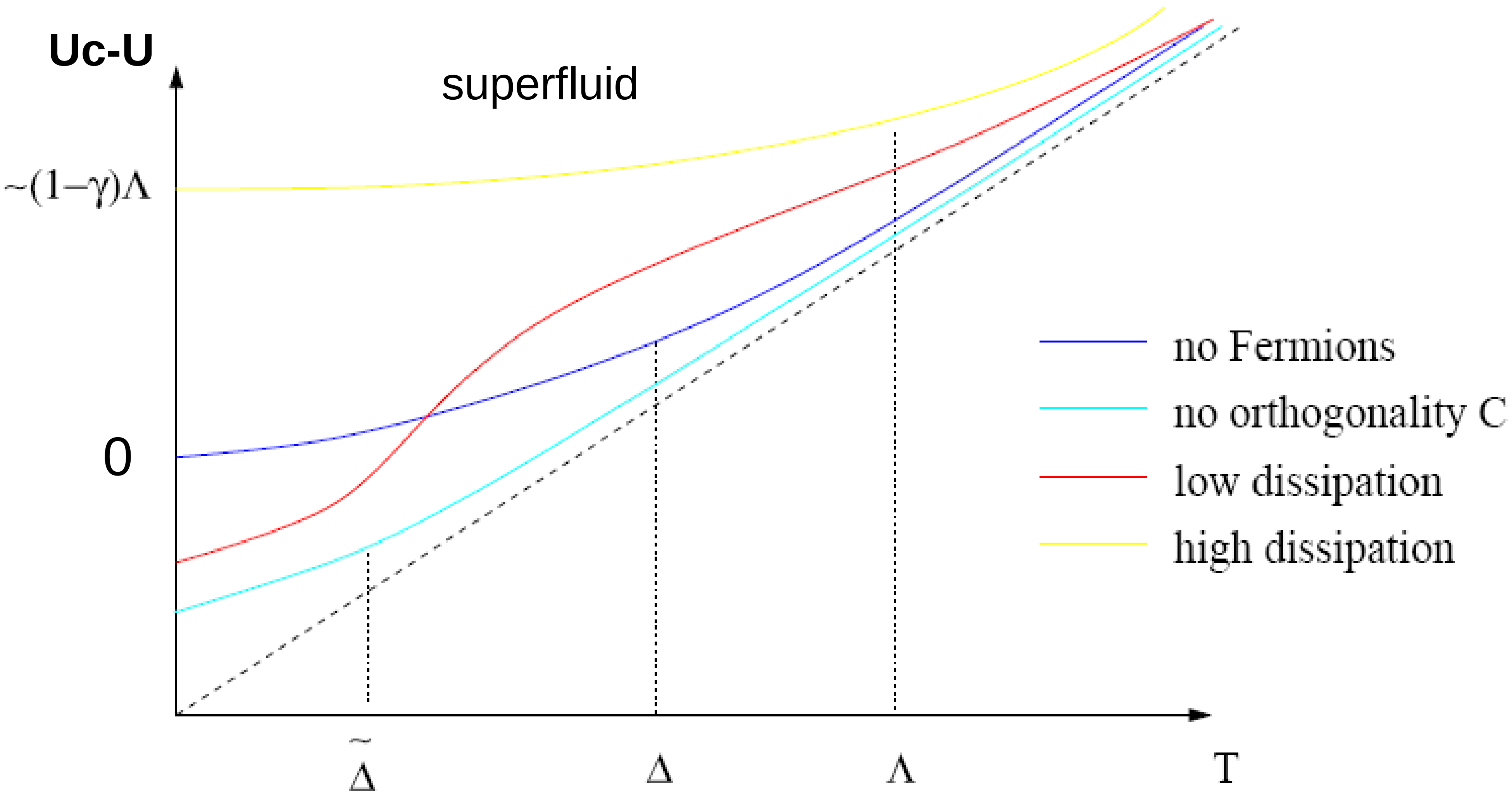


Fermions provide screening.
Favors SF state of bosons



Orthogonality catastrophy for fermions.
Favors Mott insulating state of bosons

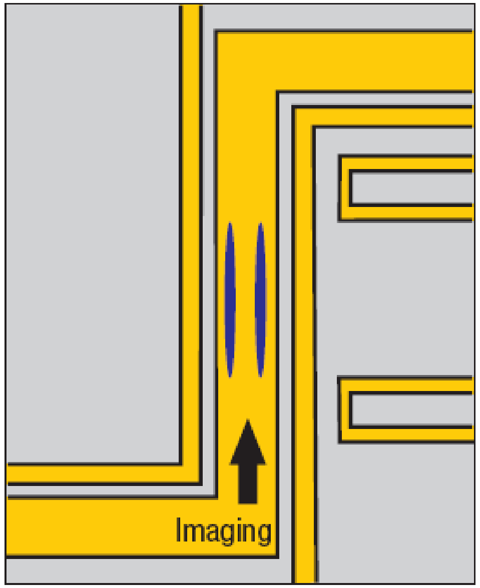
Competing effects of fermions on bosons

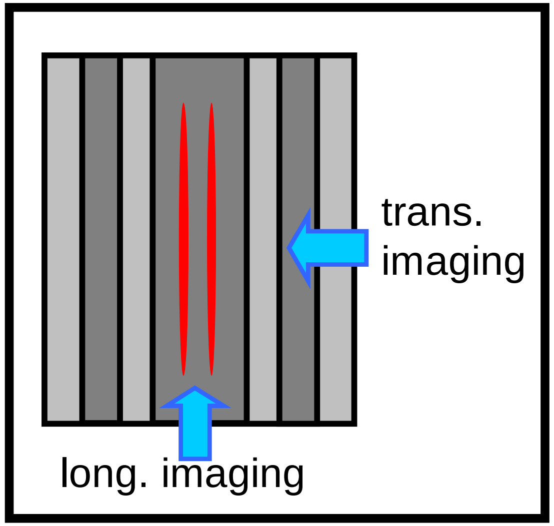


Interference as a probe of low dimensional condensates

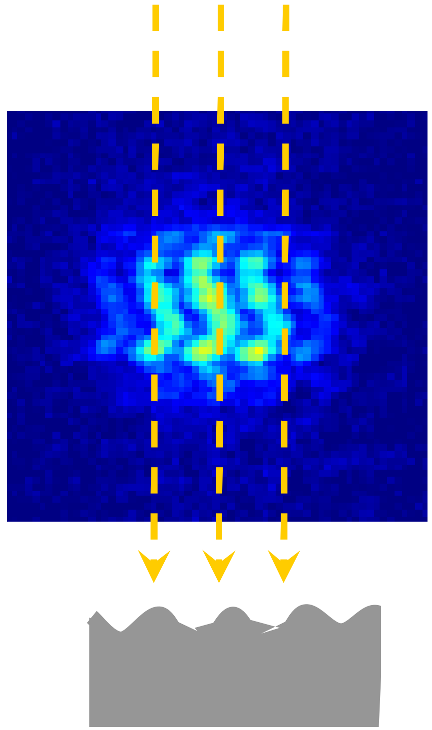
Interference of one dimensional condensates

Experiments: Schmiedmayer et al., Nature Physics (2005,2006)

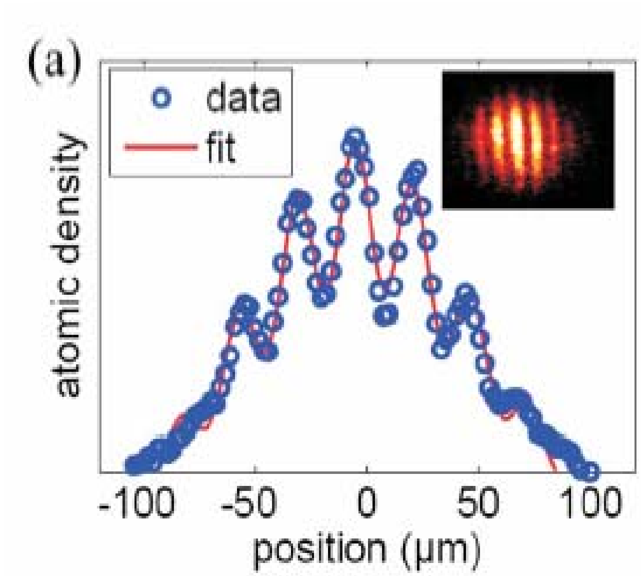




Transverse imaging



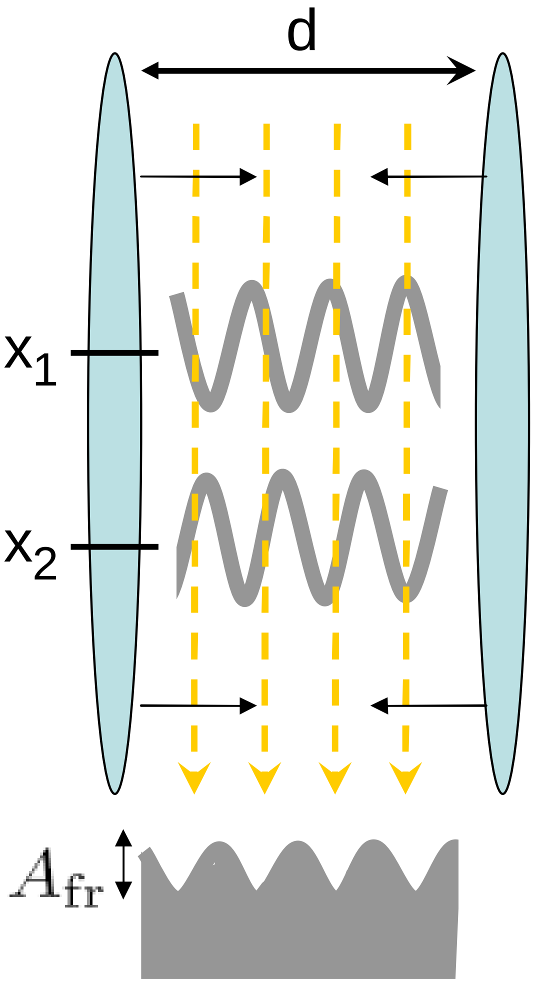
Longitudial imaging



Figures courtesy of J. Schmiedmayer

Interference of one dimensional condensates

Polkovnikov, Altman, Demler, PNAS 103:6125 (2006)



Amplitude of interference fringes, $A_{\rm fr}$

$$|A_{\rm fr}|\, e^{i\Delta\phi} = \int_0^L dx\; a_1^\dagger(x)\, a_2(x)$$

For independent condensates A_{fr} is finite but $\Delta\phi$ is random

$$\langle |A_{\rm fr}|^2 \rangle = \int_0^L \int_0^L dx\, dy\, \langle a_1^\dagger(x)\, a_2(x)\, a_2^\dagger(y)\, a_1(y) \rangle$$

$$\simeq L \int_0^L dx\, \langle\, a_1(x)\, a_1^\dagger(0)\,\rangle \langle\, a_2(0) a_2^\dagger(x)\,\rangle$$

For identical condensates $\quad \langle |A_{\rm fr}|^2 \rangle = L \int_0^L dx\, \left(G(x) \right)^2$

Instantaneous correlation function $\quad G(x) = \langle\, a(x)\, a^\dagger(0)\,\rangle$

Interference between Luttinger liquids

Luttinger liquid at T=0 $$G(x) \sim \rho \left(\frac{\xi_h}{x} \right)^{1/2K}$$

$$\langle |A_{\mathrm{fr}}|^2 \rangle \sim (\rho \xi_h)^{1/K} (L\rho)^{2-1/K}$$

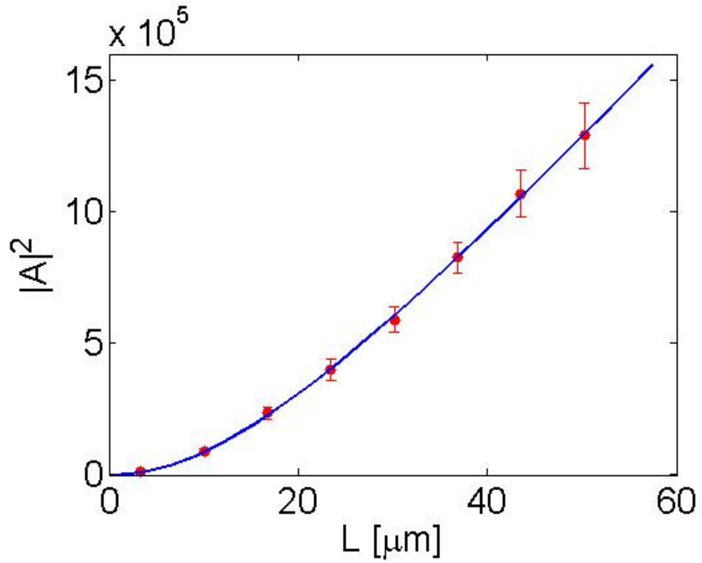
K – Luttinger parameter

For non-interacting bosons $K = \infty$ and $A_{\mathrm{fr}} \sim L$

For impenetrable bosons $K = 1$ and $A_{\mathrm{fr}} \sim \sqrt{L}$

Finite temperature

$$\langle |A_{\mathrm{fr}}|^2 \rangle \sim L \rho^2 \xi_h \left(\frac{\hbar^2}{m \xi_h^2} \frac{1}{T} \right)^{1-1/K}$$



Experiments: Hofferberth, Schumm, Schmiedmayer

$$n_{1d} = 60 \mu\mathrm{m}^{-1}$$

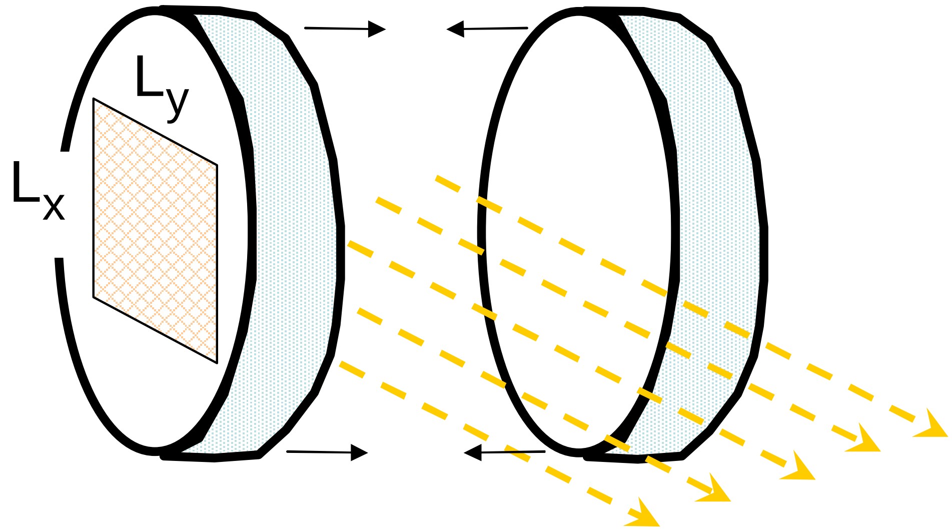
$$K = 47$$

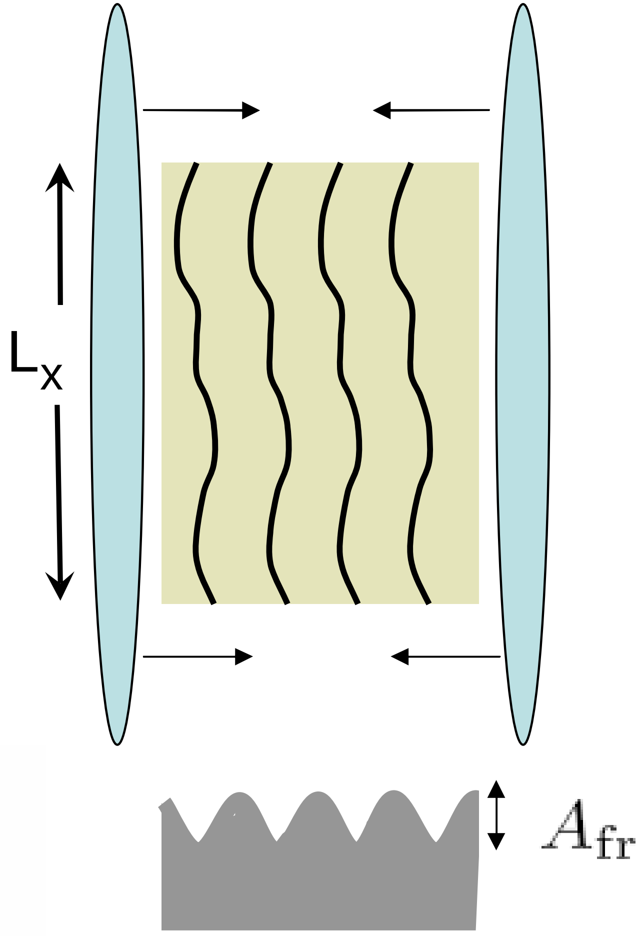
$$\boxed{T_{fit} = 84 \pm 22 \text{ nK}}$$

Interference of two dimensional condensates

Experiments: Hadzibabic et al. Nature (2006)

Gati et al., PRL (2006)





Probe beam parallel to the plane of the condensates

$$\langle\, |A_{\mathrm{fr}}|^2 \rangle \;=\; L_x\, L_y \int_0^{L_x} \int_0^{L_y} d^2\vec{r}\, \left(\, G(\vec{r}\,)\,\right)^2$$

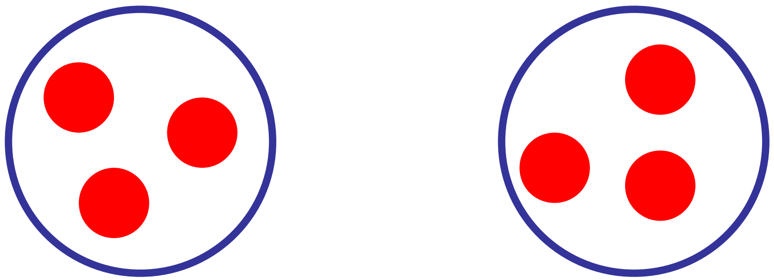
$$G(\vec{r}\,) \;=\; \langle\, a(\vec{r}\,)\, a^{\dagger}(0)\,\rangle$$

Observation of the BKT transition.
Talk by J. Dalibard

Fundamental noise in interference experiments

Amplitude of interference fringes is a quantum operator. The measured value of the amplitude will fluctuate from shot to shot. We want to characterize not only the average but the fluctuations as well.

Shot noise in interference experiments



Interference with a finite number of atoms. How well can one measure the amplitude of interference fringes in a single shot?

One atom:	No
Very many atoms:	Exactly
Finite number of atoms:	?

Consider higher moments of the interference fringe amplitude

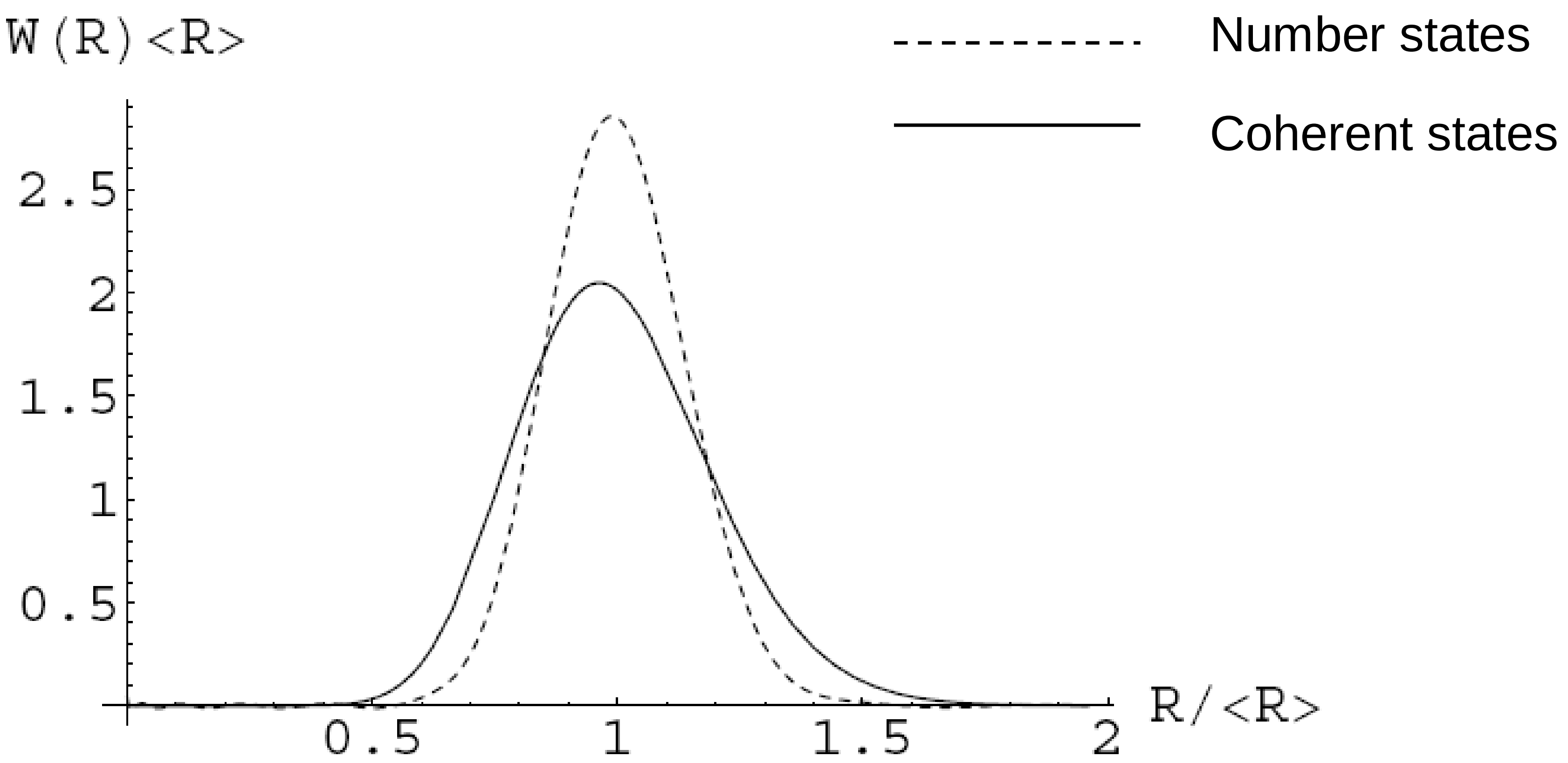
$\langle |A|^2 \rangle$, $\langle |A|^4 \rangle$, and so on

Obtain the entire distribution function of $|A|^2$

Shot noise in interference experiments

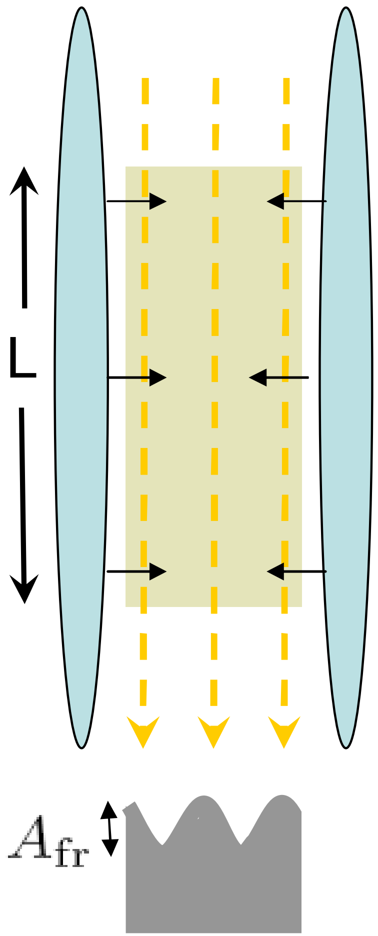
Polkovnikov, Europhys. Lett. 78:10006 (1997)
Imambekov, Gritsev, Demler, 2006 Varenna lecture notes, cond-mat/0703766

Interference of two condensates with 100 atoms in each cloud



Distribution function of fringe amplitudes for interference of fluctuating condensates

Gritsev, Altman, Demler, Polkovnikov, Nature Physics (2006)
Imambekov, Gritsev, Demler, cond-mat/0612011; c-m/0703766



$$|A_{\rm fr}|\, e^{i\Delta\phi} = \int_0^L dx\; a_1^\dagger(x)\, a_2(x)$$

$$\langle |A_{\rm fr}|^2 \simeq L \int_0^L dx\, \langle\, a_1(x)\, a_1^\dagger(0)\,\rangle\, \langle\, a_2(0) a_2^\dagger(x)\,\rangle$$

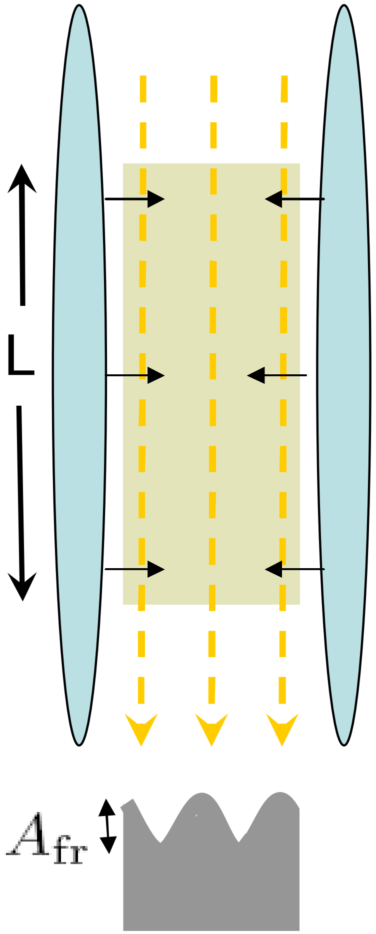
$A_{\rm fr}$ is a quantum operator. The measured value of $|A_{\rm fr}|$ will fluctuate from shot to shot.

$$\langle\, |A_{\rm fr}|^{2n} \rangle = \int_0^L dz_1 \ldots dz_n'\, |\langle\, a^\dagger(z_1) \ldots a^\dagger(z_n)\, a(z_1') \ldots a(z_n')\,\rangle|^2$$

Higher moments reflect higher order correlation functions

We need the full distribution function of $|A_{\rm fr}|$

Higher moments of interference amplitude



Method I: connection to quantum impurity model

Gritsev, Polkovnikov, Altman, Demler, Nature Physics 2:705 (2006)

Higher moments

$$\langle\, |A_{\rm fr}|^{2n} \rangle \;=\; \int_0^L dz_1 \,\ldots\, dz'_n \,|\langle\, a^\dagger(z_1) \,\ldots a^\dagger(z_n)\, a(z'_1) \,\ldots a(z'_n) \,\rangle|^2$$

Changing to periodic boundary conditions (long condensates)

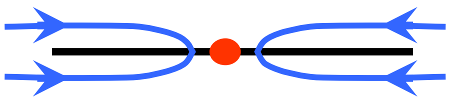
$$\langle\, |A_{\rm fr}|^{2n} \rangle \;=\; \langle\, |A_{\rm fr}|^2 \rangle^n \,\times\, Z_{2n}$$

$$Z_{2n} = \prod_{ij} \int_0^{2\pi} \cdots \int_0^{2\pi} \frac{du_i}{2\pi}\,\frac{dv_j}{2\pi} \left| \frac{\prod_{i<j} 2\,\sin(\frac{u_i - u_j}{2})\, \prod_{i<j} 2\,\sin(\frac{v_i - v_j}{2})}{\prod_{ij} 2\,\sin(\frac{u_i - v_j}{2})} \right|^{1/K}$$

Explicit expressions for Z_{2n} are available but cumbersome

Fendley, Lesage, Saleur, J. Stat. Phys. 79:799 (1995)

Impurity in a Luttinger liquid



$$S = \frac{\pi K}{2} \int dx\, d\tau \, [(\partial_\tau \phi)^2 + (\partial_x \phi)^2] + 2g \int d\tau \, \cos\phi\,(x=0,\tau)$$

Expansion of the partition function in powers of *g*

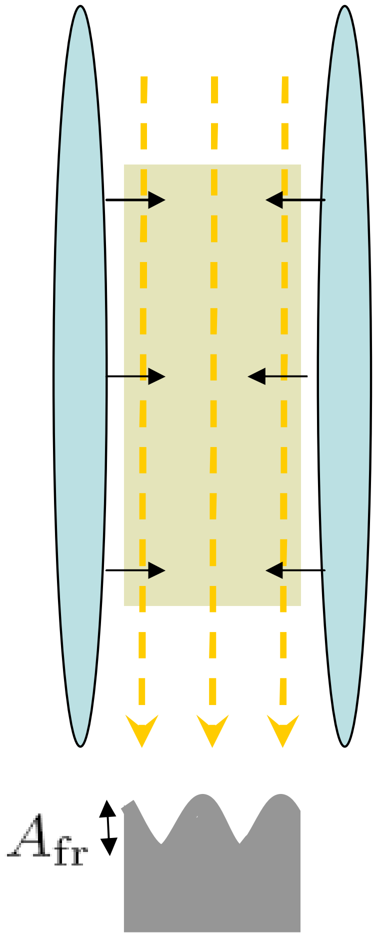
$$Z_{\text{imp}} = \sum_n \frac{g^{2n}}{(2n)!} \int d\tau_1 \ldots d\tau_n \,(e^{i\phi} + e^{-i\phi})_{\tau_1} \ldots (e^{i\phi} + e^{-i\phi})_{\tau_{2n}}$$

$$Z_{\text{imp}} = \sum \frac{g^{2n}}{(n!)^2} Z_{2n}$$

$$Z_{2n} = \prod_{ij} \int_0^{2\pi} \cdots \int_0^{2\pi} \frac{du_i}{2\pi} \frac{dv_j}{2\pi} \left| \frac{\prod_{i<j} 2\sin(\frac{u_i - u_j}{2}) \prod_{i<j} 2\sin(\frac{v_i - v_j}{2})}{\prod_{ij} 2\sin(\frac{u_i - v_j}{2})} \right|^{1/K}$$

Partition function of the impurity contains correlation functions taken at the same point and at different times. Moments of interference experiments come from correlations functions taken at the same time but in different points. Euclidean invariance ensures that the two are the same

Relation between quantum impurity problem and interference of fluctuating condensates



Normalized amplitude of interference fringes $\quad a^2 = |A_{\rm fr}|^2 / \langle\, |A_{\rm fr}|^2 \rangle$

Distribution function of fringe amplitudes $\quad W(\,K,\, a^2\,)$

Relation to the impurity partition function

$$Z_{\rm imp}(\,K,\, g\,) = \int_0^\infty da^2\, W(\,K\,,\, a^2\,)\, I_0(\,2g\, a\,)$$

Distribution function can be reconstructed from $Z_{\rm imp}(\,K,\, g\,)$ using completeness relations for the Bessel functions

$$\boxed{W(K, a^2) = 2\int_0^\infty g\, dg\, Z_{\rm imp}(K, ig) J_0(2ga^2)}$$

Bethe ansatz solution for a quantum impurity

$Z_{\text{imp}}(K, g)$ can be obtained from the Bethe ansatz following
Zamolodchikov, Phys. Lett. B 253:391 (91); Fendley, et al., J. Stat. Phys. 79:799 (95)
Making analytic continuation is possible but cumbersome

Interference amplitude and spectral determinant

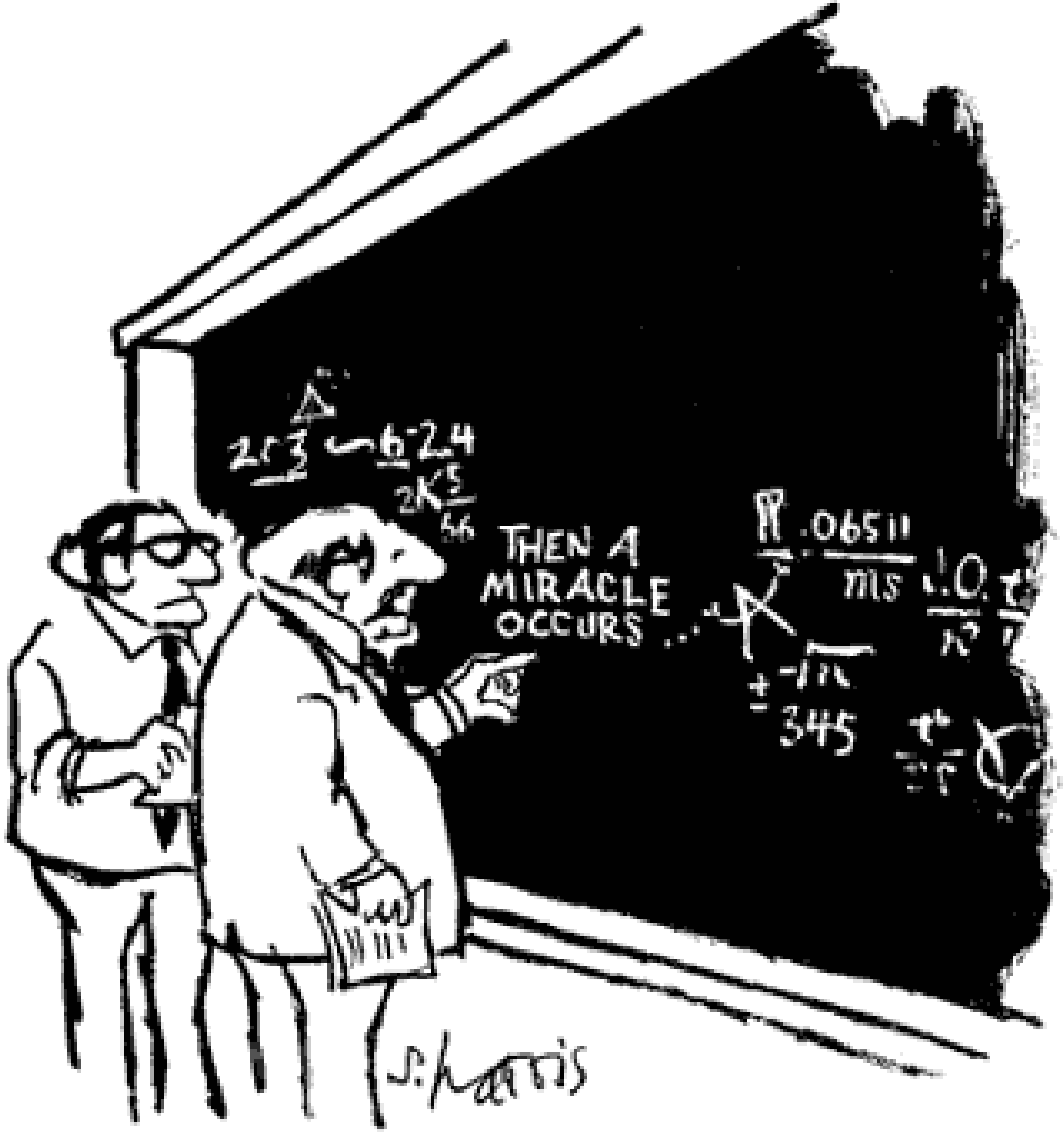
$Z_{\text{imp}}(K, ig)$ is related to the Schroedinger equation

Dorey, Tateo, J.Phys. A. Math. Gen. 32:L419 (1999)
Bazhanov, Lukyanov, Zamolodchikov, J. Stat. Phys. 102:567 (2001)

$$-\frac{d^2\Psi}{dx^2} + \left(x^{4K-2} + \frac{3}{4x^2} \right)\Psi = E\,\Psi$$

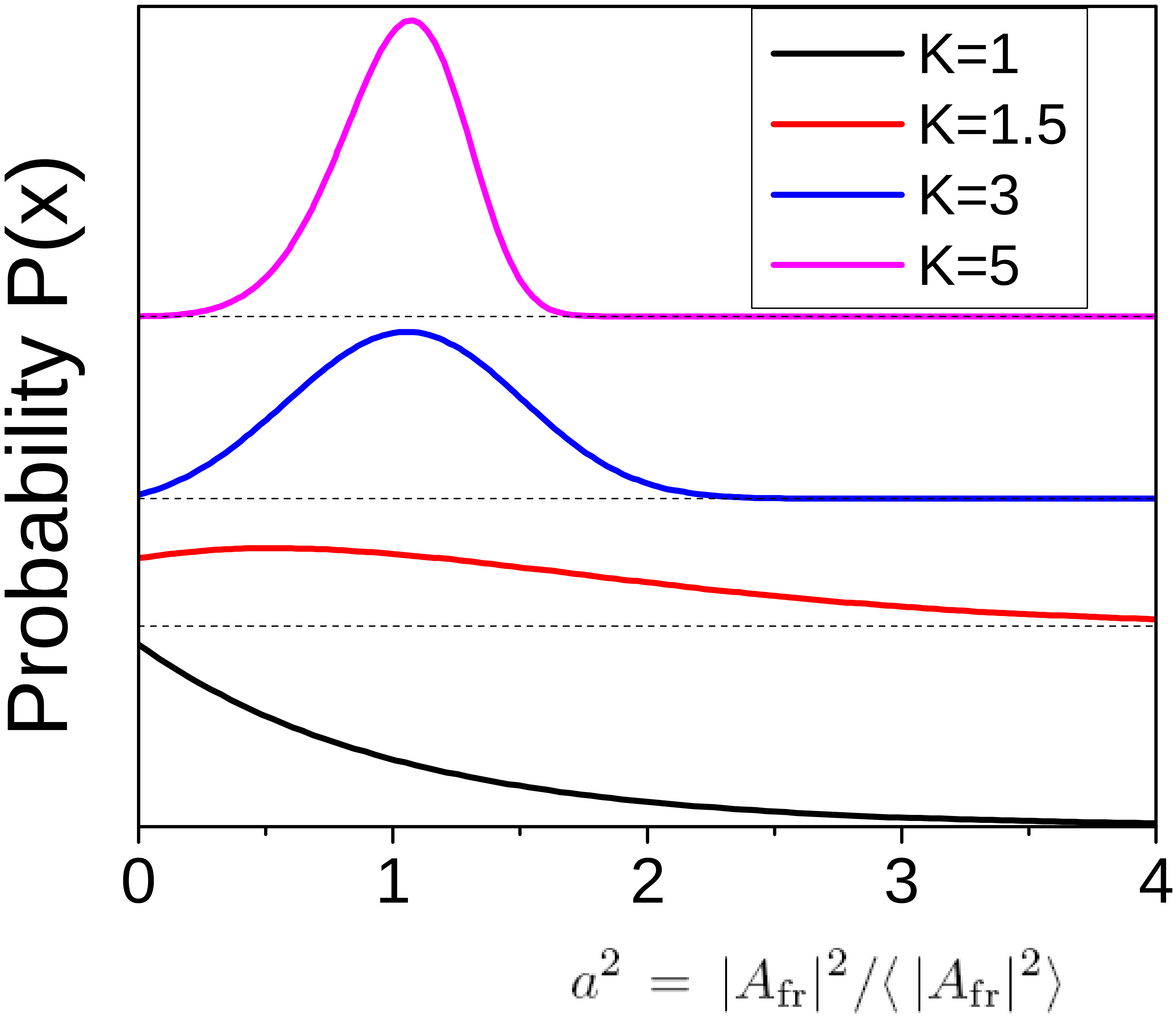
Spectral determinant $$D(E) = \prod_{n=1}^{\infty} \left(1 - \frac{E}{E_n}\right)$$

$$\boxed{Z_{\text{imp}}(K, ig) = D\left(\frac{g^2}{\pi^2} (4K)^{2-K^{-1}} \left[\Gamma(1 - \frac{1}{2K})\right]^2 \sin^2(\frac{\pi}{2K}) \right)}$$



"I think you should be more explicit here in step two."

Evolution of the distribution function



Narrow distribution for $K \to \infty$ Approaches Gumbel distribution.

Width $\sigma = \dfrac{\pi}{6K}$

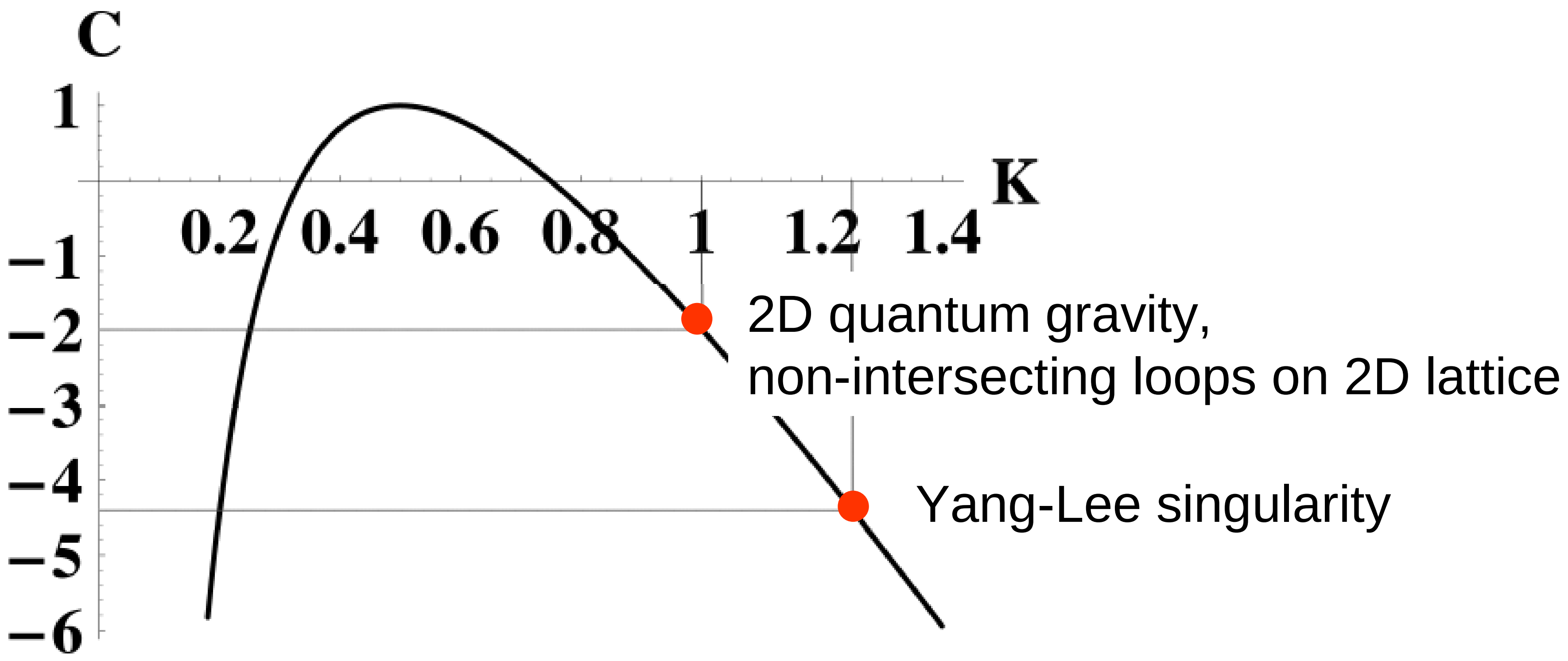
Wide Poissonian distribution for $K \to 1$

From interference amplitudes to conformal field theories

$Z_{\mathrm{imp}}(K, ig)$ correspond to vacuum eigenvalues of Q operators of CFT

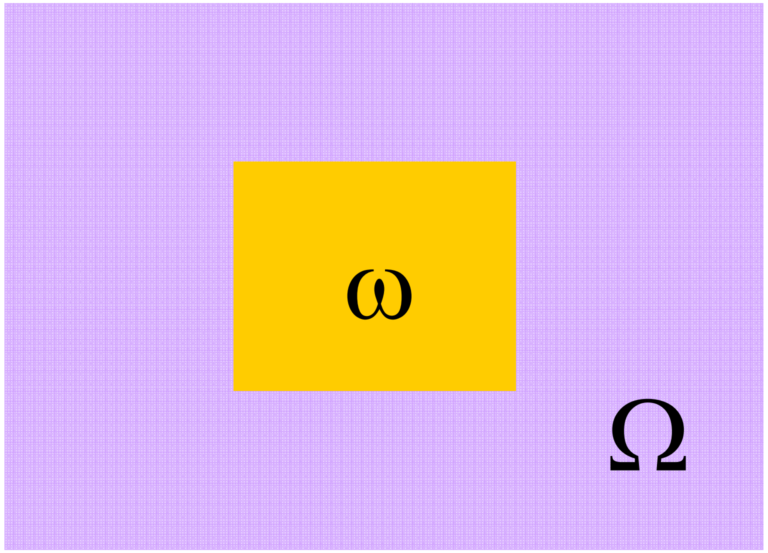
Bazhanov, Lukyanov, Zamolodchikov, Comm. Math. Phys.1996, 1997, 1999

When K>1, $Z_{\mathrm{imp}}(K, ig)$ is related to Q operators of CFT with c<0. This includes 2D quantum gravity, non-intersecting loop model on 2D lattice, growth of random fractal stochastic interface, high energy limit of multicolor QCD, …



How to generalize this analysis to 1d with open boundary conditions and 2d condensates?

Inhomogeneous Sine-Gordon models



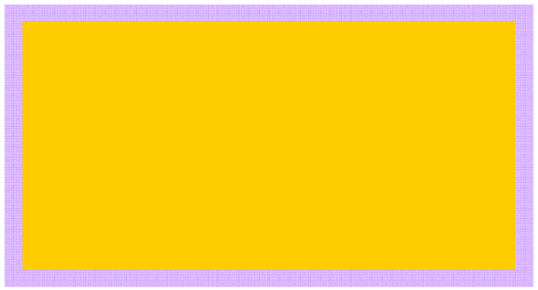
$$\boxed{S = \frac{K}{2}\int_{\Omega} d^2x\,(\nabla\phi)^2 \; + g\int_{\omega} d^2x\cos\phi}$$

$$Z(g) = \frac{\int \mathcal{D}\phi e^{-S(g)}}{\int \mathcal{D}\phi e^{-S(0)}}$$

Limiting cases

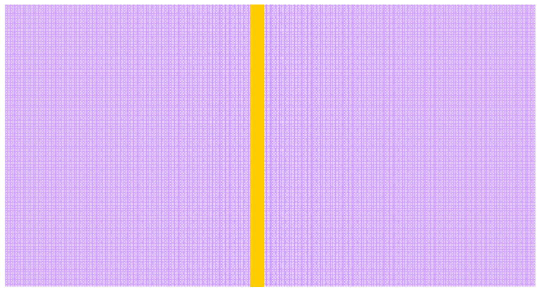
Bulk Sine-Gordon model

$\omega=\Omega$

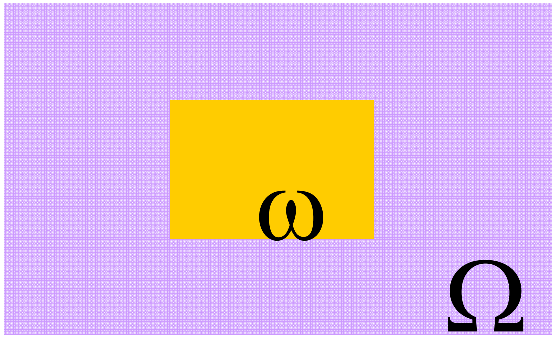


Boundary Sine-Gordon model

$\omega = \delta(x-x_0)$



Inhomogeneous Sine-Gordon models



$$S = \frac{K}{2} \int_\Omega d^2x \, (\nabla\phi)^2 \; + \; g \int_\omega d^2x \cos\phi$$

$$Z(g) \; = \; \frac{\int \mathcal{D}\phi e^{-S(g)}}{\int \mathcal{D}\phi e^{-S(0)}}$$

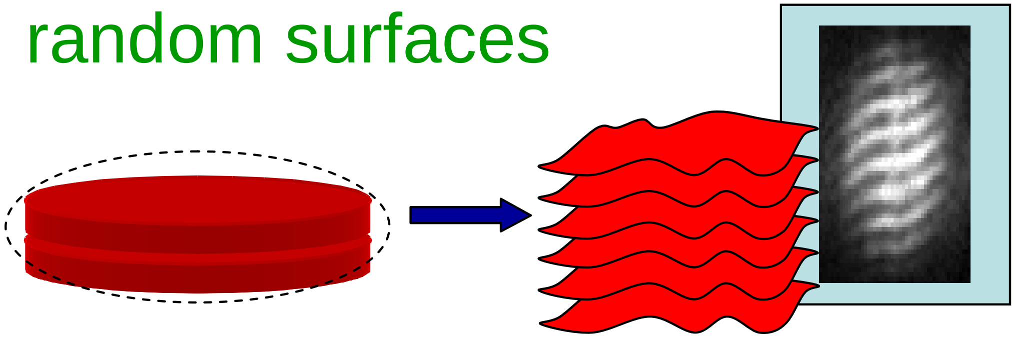
Expand in powers of g

$$Z(g) \; = \sum_{n=0}^{n=\infty} \frac{g^{2n}}{(n!)^2} Z_{2n}$$

$$Z_{2n} \; = \prod_{i=1}^{n} \int_\omega du_i \prod_{j=1}^{n} \int_\omega dv_j \, \frac{\prod_{ij} \langle e^{i\phi(u_i)} e^{-i\phi(v_j)} \rangle}{\prod_{i<i'} \langle e^{i\phi(u_i)} e^{-i\phi(u_{i'})} \rangle \, \prod_{j<j'} \langle e^{i\phi(v_j)} e^{-i\phi(v_{j'})} \rangle}$$

Higher moments of interference amplitude

Method II: connection to generalized sine-Gordon models and random surfaces

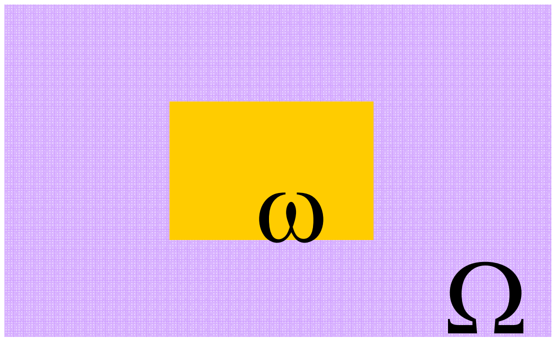


Imambekov, Gritsev, Demler, cond-mat/

Higher moments $\langle |A_{\rm fr}|^{2n} \rangle = \int_0^L dz_1 \ldots dz'_n \, |\langle a^\dagger(z_1) \ldots a^\dagger(z_n)\, a(z'_1) \ldots a(z'_n) \rangle|^2$

$$\langle |A_{\rm fr}|^{2n} \rangle = \langle |A_{\rm fr}|^2 \rangle^n \times Z_{2n}$$

$$Z_{2n} = \prod_{i=1}^{n} \int_\omega du_i \prod_{j=1}^{n} \int_\omega dv_j \, \frac{\prod_{ij} \langle e^{i\phi(u_i)} e^{-i\phi(v_j)} \rangle}{\prod_{i<i'} \langle e^{i\phi(u_i)} e^{-i\phi(u_{i'})} \rangle \prod_{j<j'} \langle e^{i\phi(v_j)} e^{-i\phi(v_{j'})} \rangle}$$



Example:
Interference of 2D condensates
Ω = entire condensate
ω = observation area

Coulomb gas representation

$$Z_{2n} = \prod_{i=1}^{n} \int_\omega du_i \prod_{j=1}^{n} \int_\omega dv_j \, \frac{\prod_{ij} \langle e^{i\phi(u_i)} e^{-i\phi(v_j)} \rangle}{\prod_{i<i'} \langle e^{i\phi(u_i)} e^{-i\phi(u_{i'})} \rangle \prod_{j<j'} \langle e^{i\phi(v_j)} e^{-i\phi(v_{j'})} \rangle}$$

$$Z_{2n} = \prod_{i=1}^{n} \int_\omega du_i \prod_{j=1}^{n} \int_\omega dv_j \, e^{\frac{1}{K}\left(\sum_{i<i'} f(u_i,u_{i'}) + \sum_{j<j'} f(v_i,v_{j'}) - \sum_{ij} f(u_i,v_j)\right)}$$

Diagonalize Coulomb gas interaction

$$\int_\omega f(x,y)\Psi_m(x)dx = f(m)\Psi_m(y) \qquad f(x,y) = \sum^{\infty} f(m)\Psi_m(x)\Psi_m(y)$$

Connection to the distribution function

$$\alpha^2 = \frac{|A_{\mathrm{fr}}|^2}{\langle |A_{\mathrm{fr}}|^2 \rangle}$$

$$W(\alpha) = \prod_m \frac{\int_{-\infty}^{\infty} dt_m \, e^{-t_m^2/2}}{\sqrt{2\pi}} \, \delta\left(\alpha - \int_\omega dx \, e^{h(x,t_m)+h_0(x)} \int_\omega dx \, e^{h(x,-t_m)+h_0(x)}\right)$$

$$h(x,t_m) = \sum_m t_m \sqrt{\frac{f(m)}{K}} \Psi_m(x) \qquad h_0(x) = -\sum_m \frac{f(m)}{2K} \Psi_m(x)^2$$

From SG models to fluctuating surfaces

$$\boxed{W(\alpha)=\prod_m \frac{\int_{-\infty}^{\infty} dt_m\, e^{-t_m^2/2}}{\sqrt{2\pi}}\,\delta\left(\alpha-\int_\omega dx\, e^{h(x,t_m)+h_0(x)}\int_\omega dx\, e^{h(x,-t_m)+h_0(x)}\right)}$$

$$h(x,t_m)=\sum\nolimits_m t_m\sqrt{\frac{f(m)}{K}}\Psi_m(x) \qquad h_0(x)=-\sum\nolimits_m \frac{f(m)}{2K}\Psi_m(x)^2$$

Simulate by Monte-Carlo!

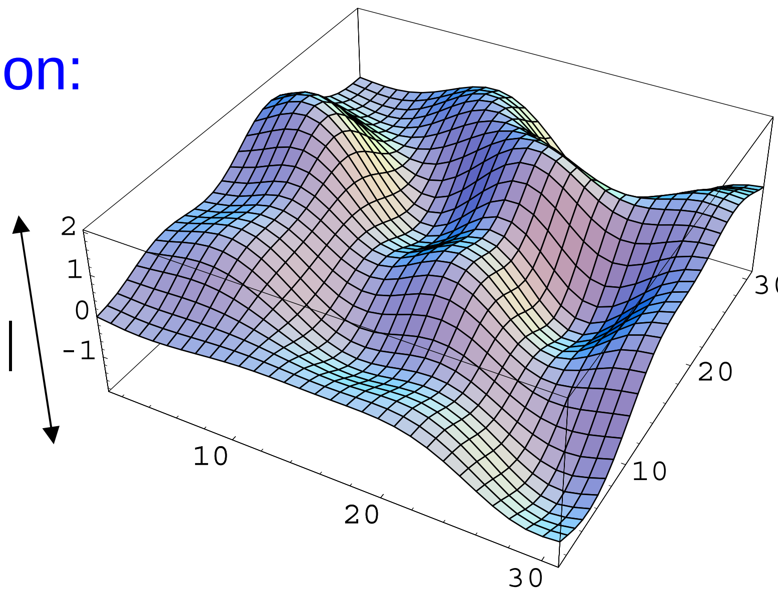
Random surfaces interpretation:

$h(x,t_m)$ fluctuating surface

t_m "noise" variables

$\left.\begin{array}{ll}\Psi_m(x) & \text{eigenmodes} \\ |f(m)| & \text{"noise" power}\end{array}\right\}$ determined by $f(x,y)$

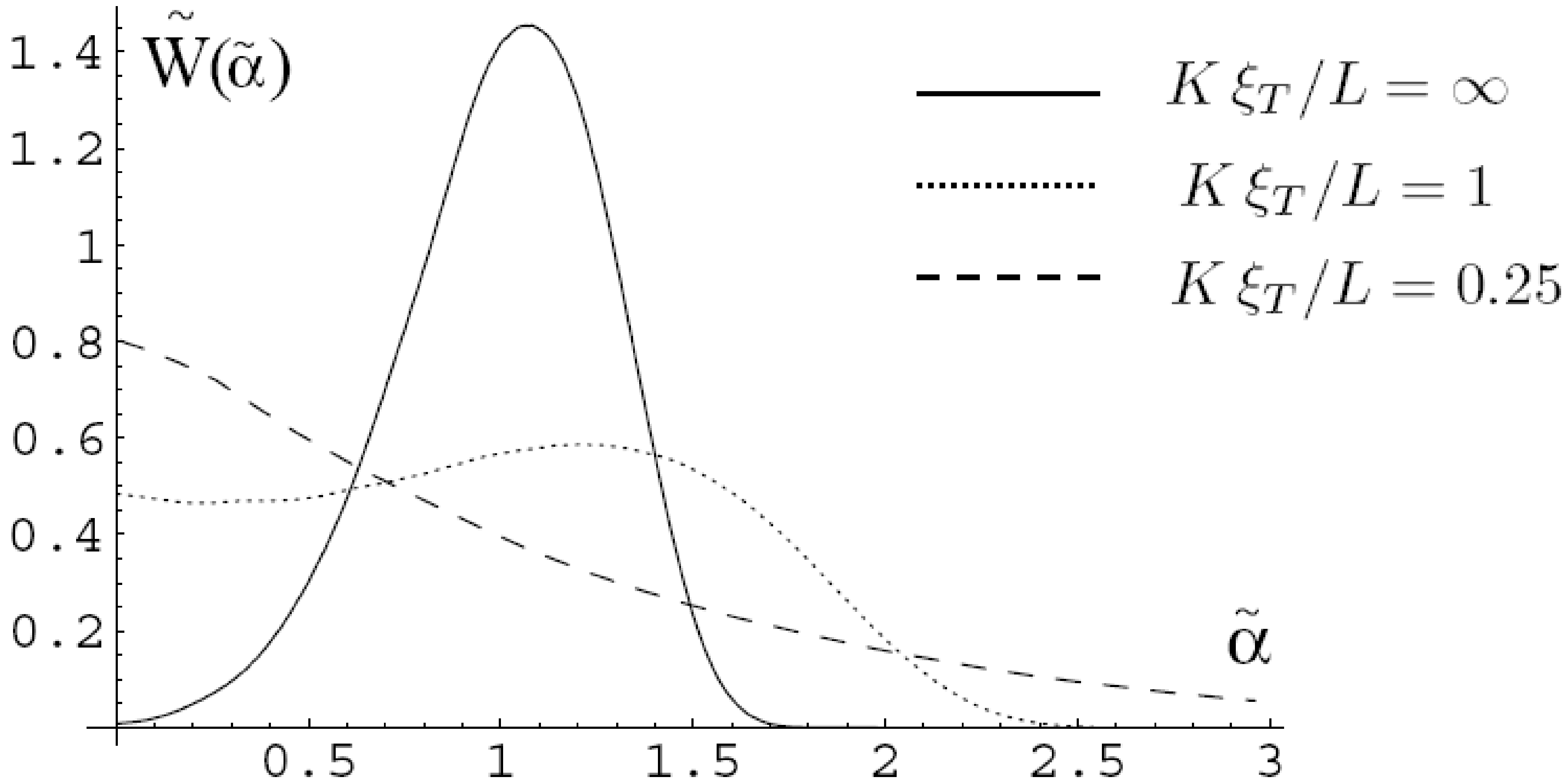
$|h(x,t_m)|$



This method does not rely on the existence of the exact solution

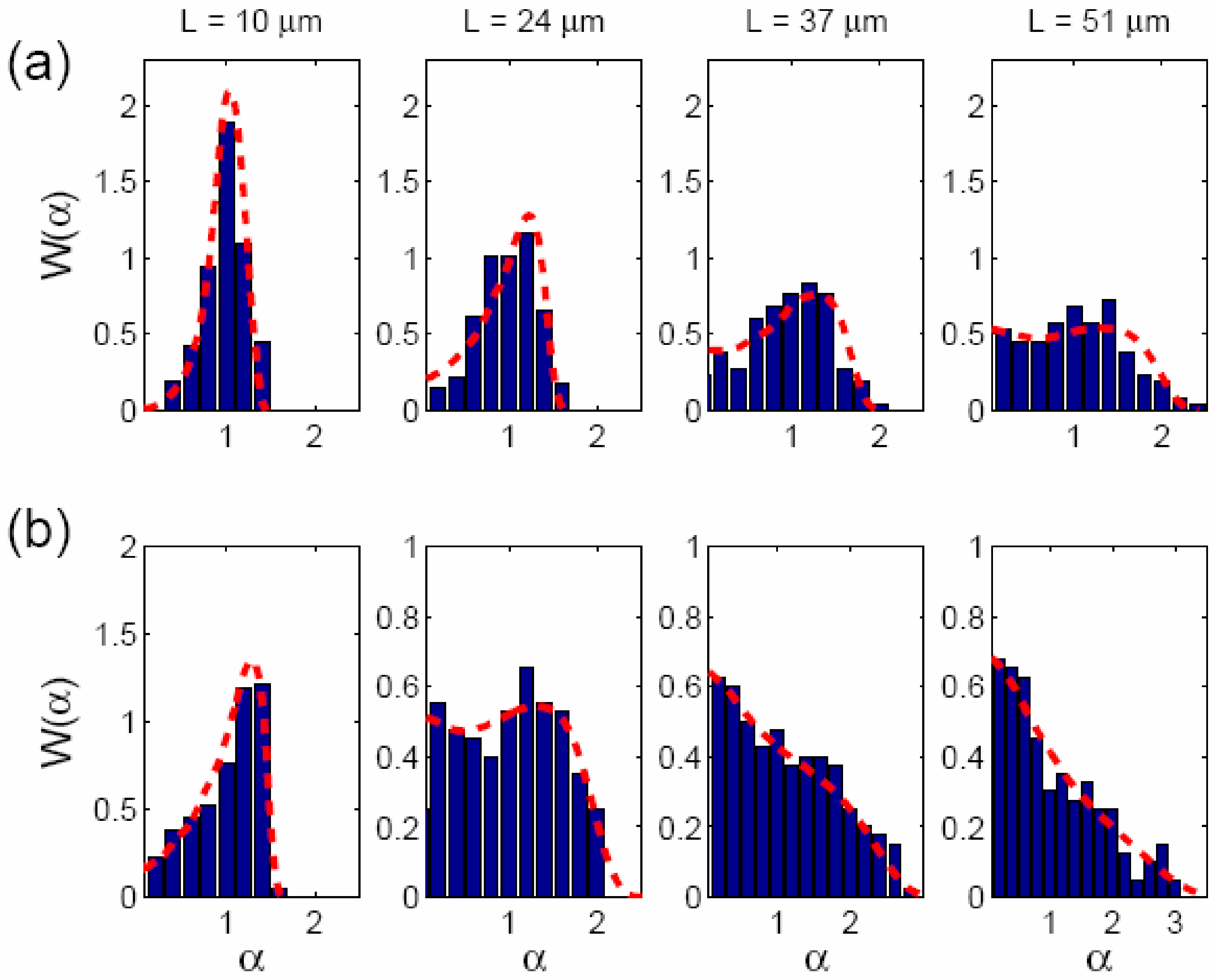
Interference of 1d condensates at finite temperature. Distribution function of the fringe contrast

Luttinger parameter *K=5*



Interference of 1d condensates at finite temperature. Distribution function of the fringe contrast

Experiments: Hofferberth, Schumm, Schmiedmayer et al.



T=30nK

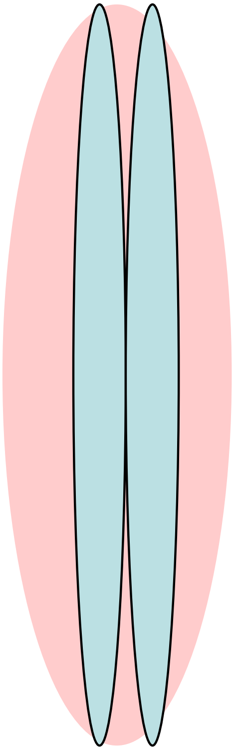
ξ_T=0.9μm

T=60nK

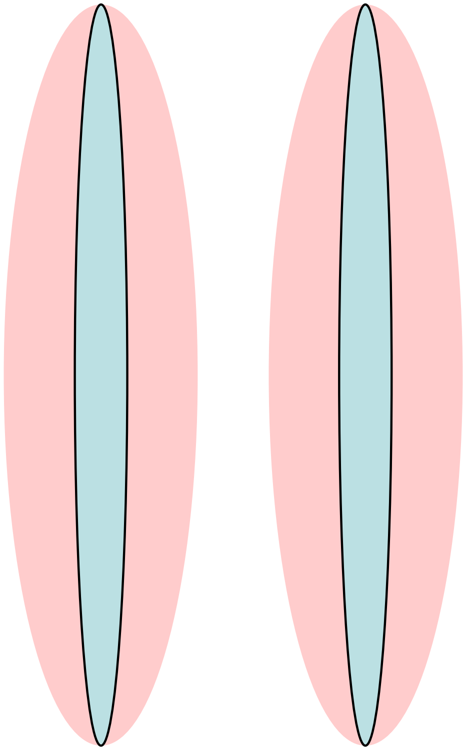
ξ_T=0.45μm

Non-equilibrium coherent dynamics of low dimensional Bose gases probed in interference experiments

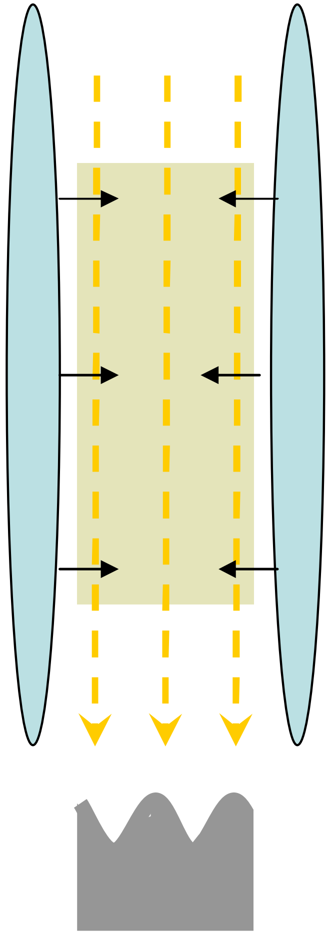
Studying dynamics using interference experiments



Prepare a system by
splitting one condensate

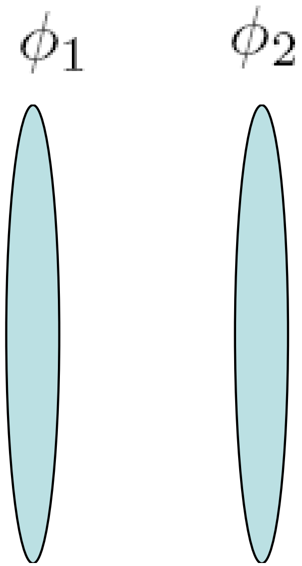


Take to the regime of
zero tunneling



Measure time evolution
of fringe amplitudes

Relative phase dynamics



Bistrizer, Altman, PNAS (2007)
Burkov, Lukin, Demler, PRL 98:200404 (2007)

$$\mathcal{H} = \int d^d r \left[\frac{g}{2} (\Delta n)^2 + \frac{\rho}{2} (\nabla \phi)^2 \right]$$

$$\phi = \phi_1 - \phi_2$$

$$\Delta n = (n_1 - n_2)/2$$

Conjugate variables

Hamiltonian can be diagonalized in momentum space

A collection of harmonic oscillators with $\omega_q = \sqrt{g\rho}\,|q|$

Need to solve dynamics of harmonic oscillators at finite T

Coherence $\langle \Psi(t) | e^{i\phi} | \Psi(t) \rangle = e^{-\frac{1}{2}\sum_q \langle \phi_q^2(t) \rangle}$

Relative phase dynamics

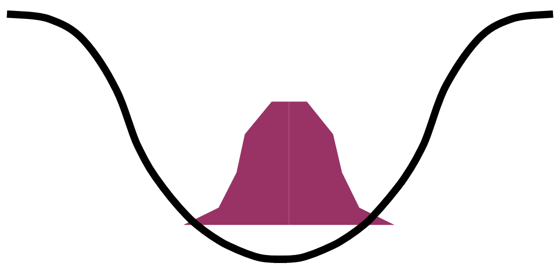
High energy modes, $\hbar\omega_{\mathrm{osc}} > k_{\mathrm{B}} T$, quantum dynamics

Low energy modes, $\hbar\omega_{\mathrm{osc}} < k_{\mathrm{B}} T$, classical dynamics

Combining all modes

$$t < \frac{h}{k_{\mathrm{B}} T}$$ Quantum dynamics

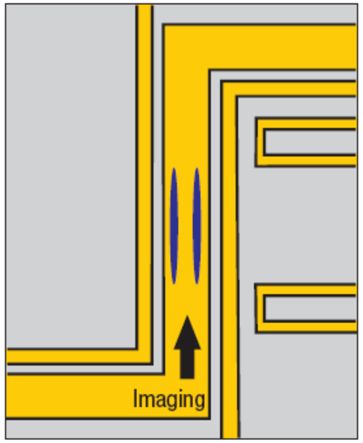
$$t > \frac{h}{k_{\mathrm{B}} T}$$ Classical dynamics



For studying dynamics it is important to know the initial width of the phase

Relative phase dynamics

Burkov, Lukin, Demler, cond-mat/0701058

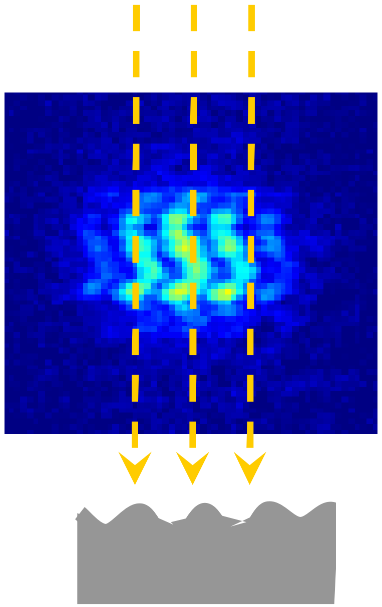


Quantum regime $\frac{h}{\mu} < t < \frac{h}{k_B T}$

1D systems $\langle e^{i\phi(t)} \rangle \sim e^{-\frac{\mu t^2}{2N\tau_s}} e^{-t/2\pi K \tau_s}$

2D systems $\langle e^{i\phi(t)} \rangle \sim e^{-\frac{\mu t^2}{2N\tau_s}} \left(\frac{t_0}{t} \right)^{1/16 T_{KT} \tau_s}$

Different from the earlier theoretical work based on a single mode approximation, e.g. Gardiner and Zoller, Leggett



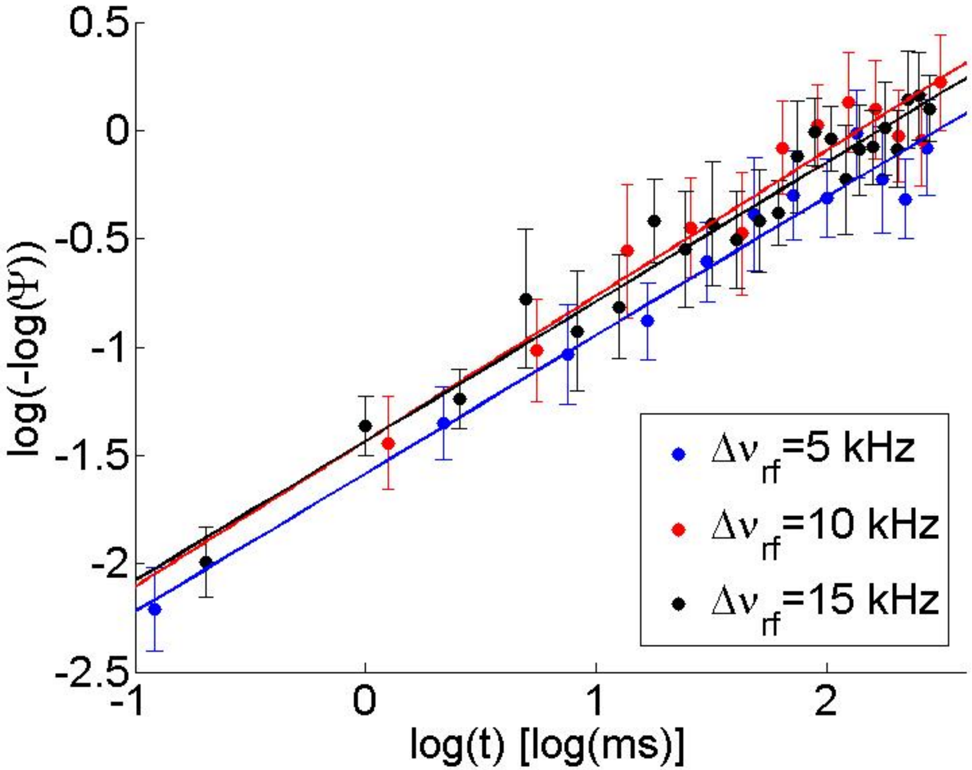
Classical regime $t > \frac{h}{k_B T}$

$$\boxed{\text{1D systems} \quad \langle e^{i\phi(t)} \rangle \sim e^{-\left(\frac{t}{t_T}\right)^{2/3}}} \qquad t_T \sim \frac{\mu K}{T^2}$$

2D systems $\langle e^{i\phi(t)} \rangle \sim \left(\frac{t_0}{t} \right)^{\frac{T}{8 T_{KT}}}$

1d BEC: Decay of coherence

Experiments: Hofferberth, Schumm, Schmiedmayer, arXiv:0706.2259



$$\Psi(t) \propto \exp[-(t/t_0)^{2/3}]$$

double logarithmic plot of the coherence factor

slopes:

$$\boxed{\begin{array}{c} 0.64 \pm 0.08 \\ 0.67 \pm 0.1 \\ 0.64 \pm 0.06 \end{array}}$$

get t_0 from fit with fixed slope 2/3 and calculate T from

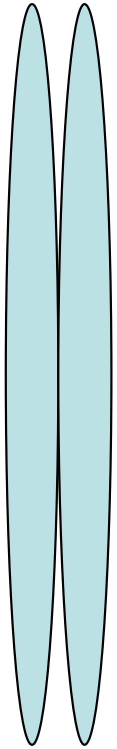
$$t_0 = 2.61\pi K/T^2$$

T_5 = 110 ± 21 nK

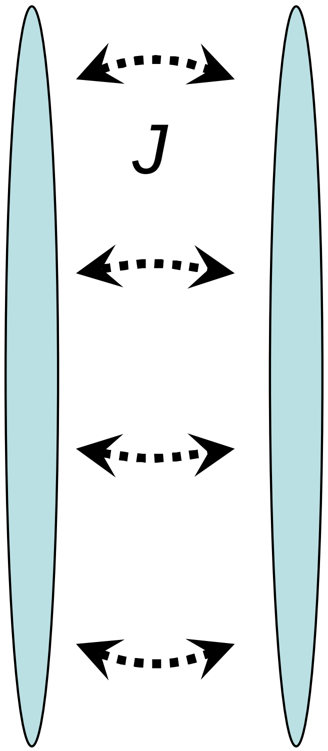
T_{10} = 130 ± 25 nK

T_{15} = 170 ± 22 nK

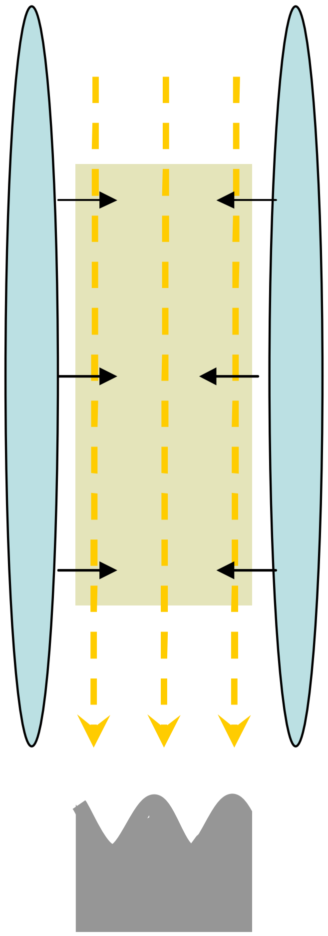
Quantum dynamics of coupled condensates. Studying Sine-Gordon model in interference experiments



Prepare a system by splitting one condensate

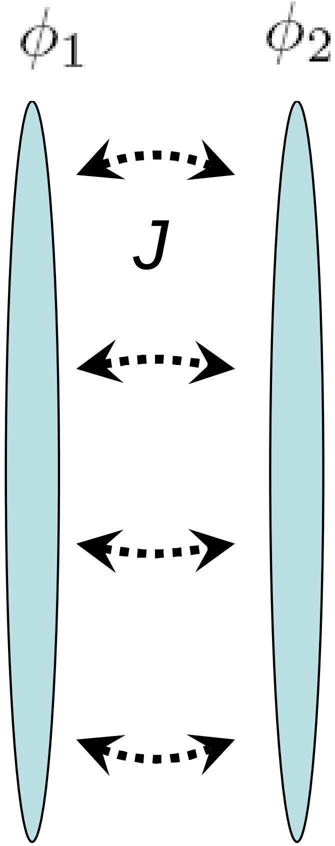


Take to the regime of finite tunneling. System described by the quantum Sine-Gordon model



Measure time evolution of fringe amplitudes

Coupled 1d systems



$$\mathcal{H}_0 = \int dx\,\left[g\,n_1^2(x) + \rho\,(\partial_x\phi_1)^2\right] + \int dx\,\left[g\,n_2^2(x) + \rho\,(\partial_x\phi_2)^2\right]$$

Interactions lead to phase fluctuations
within individual condensates

$$\mathcal{H}_{tun} = -J\int dx\,\cos(\phi_1 - \phi_2)$$

Tunneling favors aligning of the two phases

$$\phi = \phi_1 - \phi_2$$
$$\Delta n = (\,n_1 - n_2\,)/2$$

Interference experiments measure the relative phase

$$\mathcal{H}[\phi] = \int dx\,d\tau\left[\frac{1}{2K}(\Delta n)^2 + \frac{K}{2}\,(\partial_x\phi)^2\right] - J\int dx\,d\tau\,\cos\phi$$

Quantum Sine-Gordon model

Hamiltonian

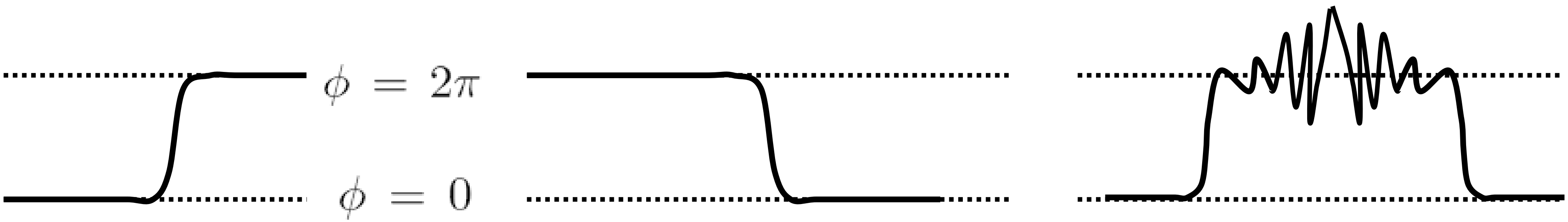
$$\mathcal{H}[\phi] = \int dx\, d\tau \left[\frac{1}{2K} (\Delta n)^2 + \frac{K}{2} (\partial_x \phi)^2 \right] - J \int dx\, d\tau\, \cos\phi$$

Imaginary time action

$$\mathcal{S}[\phi] = \frac{K}{2} \int dx\, d\tau \left[(\partial_\tau \phi)^2 + (\partial_x \phi)^2 \right] + J \int dx\, d\tau\, \cos\phi$$

Quantum Sine-Gordon model is exactly integrable

Excitations of the quantum Sine-Gordon model



soliton antisoliton many types of breathers

Dynamics of quantum sine-Gordon model

Hamiltonian formalism

$$\mathcal{H}[\phi] = \int dx\, d\tau \left[\frac{1}{2K}(\Delta n)^2 + \frac{K}{2}(\partial_x\phi)^2\right] - J\int dx\, d\tau\, \cos\phi$$

Initial state $\phi(t=0) = 0$

Quantum action in space-time

$$\mathcal{S}[\phi] = \frac{K}{2}\int dx\, d\tau\, \left[(\partial_\tau\phi)^2 + (\partial_x\phi)^2\right] + J\int dx\, d\tau\, \cos\phi$$

Initial state provides a boundary condition at *t=0*

Solve as a boundary sine-Gordon model

Boundary sine-Gordon model

Exact solution due to Ghoshal and Zamolodchikov (93)
Applications to quantum impurity problem: Fendley, Saleur, Zamolodchikov, Lukyanov,…

$$S = \int_{x\tau} \left[\frac{K}{2} \left(\frac{\partial \phi}{\partial \tau}\right)^2 + \frac{K}{2} \left(\frac{\partial \phi}{\partial x}\right)^2 - m \cos \phi \right] - M \int_{\tau} cos \frac{\phi(x=0)}{2}$$

Limit $M \to \infty$ enforces boundary condition $\phi(x=0) = 0$

Sine-Gordon
+ boundary condition in space

quantum impurity problem

Boundary
Sine-Gordon
Model

space and time
enter equivalently

Sine-Gordon
+ boundary condition in time

two coupled 1d BEC

Boundary sine-Gordon model

Initial state is a generalized squeezed state

$$| \, \psi(t=0) \, \rangle = e^{\{\sum_\gamma g_\gamma A^\dagger_\gamma(\theta=0) + \sum_{\alpha\beta} \int_\theta K_{\alpha\beta}(\theta) A^\dagger_\alpha(-\theta) A^\dagger_\beta(\theta)\}} | \, \mathrm{vac} \, \rangle$$

$A^\dagger_\alpha(\theta)$ creates solitons, breathers with rapidity θ

$A^\dagger_\gamma(\theta = 0)$ creates even breathers only

Matrix $K_{\alpha\beta}(\theta)$ and g_γ are known from the exact solution of the boundary sine-Gordon model

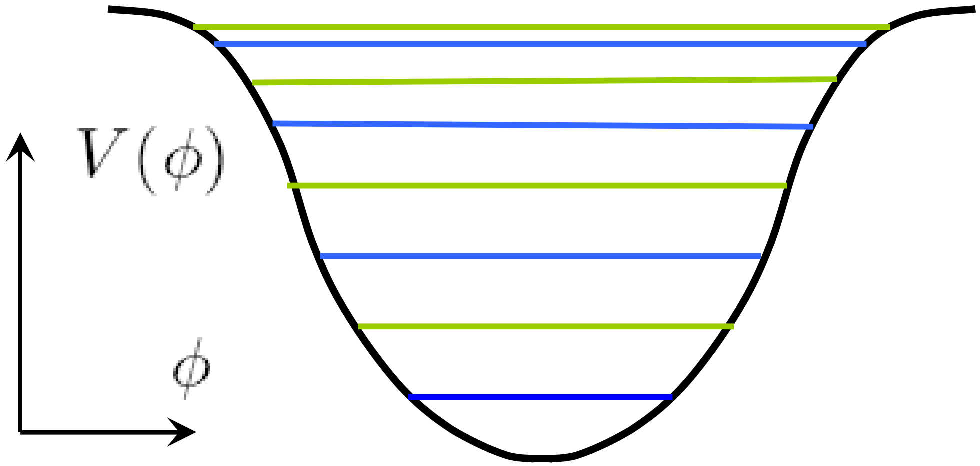
Time evolution $A^\dagger_\alpha(\theta, t) = A^\dagger_\alpha(\theta) \, e^{-iE_\alpha(\theta)t}$

Coherence $\langle \, \psi(t) \, | \, e^{i\phi} \, | \, \psi(t) \, \rangle$

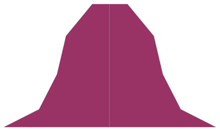
Matrix elements can be computed using form factor approach
Smirnov (1992), Lukyanov (1997)

Quantum Josephson Junction

$$\boxed{\mathcal{H} = \frac{g}{2} n^2 - J \cos\phi}$$



Limit of quantum sine-Gordon model when spatial gradients are forbidden

Initial state  $$|\,\psi(t=0)\,\rangle = \sum_n C_{2n}\,|\,2n\,\rangle$$

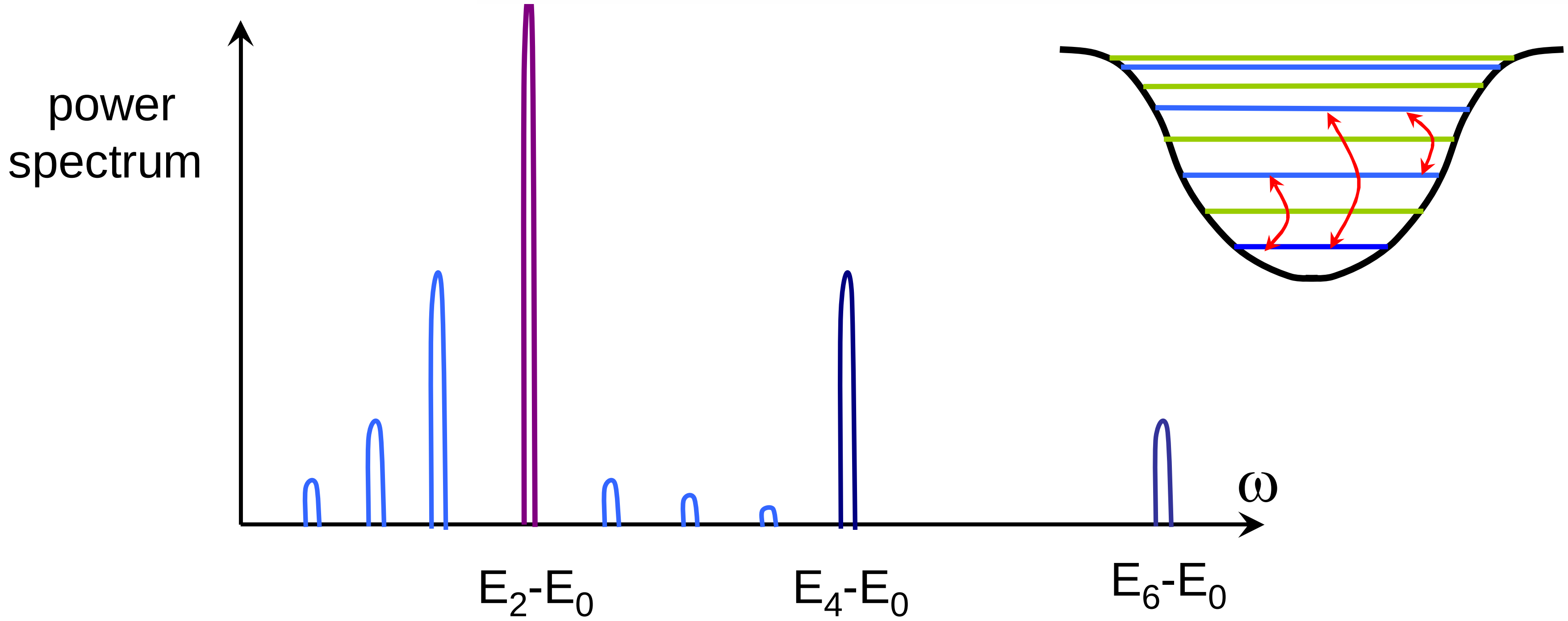
Eigenstates of the quantum Jos. junction Hamiltonian are given by Mathieu's functions

Time evolution $$|\,\psi(t)\,\rangle = \sum_n C_{2n}\, e^{-iE_{2n}t}\,|\,2n\,\rangle$$

Coherence $$\langle\,\psi(t)\,|\,e^{i\phi}\,|\,\psi(t)\,\rangle$$

Dynamics of quantum Josephson Junction

Power spectrum $$P(\omega) = \left| \int_t e^{i\omega t} \langle \psi(t) \,|\, e^{i\phi} \,|\, \psi(t) \rangle \right|^2$$



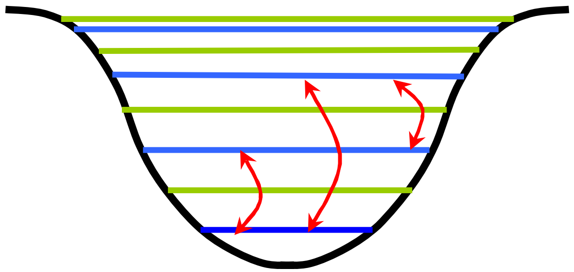
Main peak $\omega = E_2 - E_0$

"Higher harmonics" $\omega = E_4 - E_0\,,\ E_6 - E_0\,,\ \ldots$

Smaller peaks $\omega = E_{2n+2} - E_{2n}\,,\ E_{2n+4} - E_{2n}\,,\ \ldots$

Dynamics of quantum sine-Gordon model

$$|\,\psi(t)\,\rangle = e^{\{\sum_\gamma g_\gamma A^\dagger_\gamma(\theta=0)+\sum_{\alpha\beta}\int_\theta K_{\alpha\beta}(\theta)A^\dagger_\alpha(-\theta)A^\dagger_\beta(\theta)\,\}}|\,\mathrm{vac}\,\rangle$$



Coherence $\langle\,\psi(t)\,|\,e^{i\phi}\,|\,\psi(t)\,\rangle$

Main peak $\int_\theta \langle\,\mathrm{vac}\,|\,e^{i\phi}\,|\,\mathrm{B}_1(\theta)\,\mathrm{B}_1(-\theta)\,\rangle$

"Higher harmonics" $\int_\theta \langle\,\mathrm{vac}\,|\,e^{i\phi}\,|\,\mathrm{B}_n(\theta)\,\mathrm{B}_n(-\theta)\,\rangle$

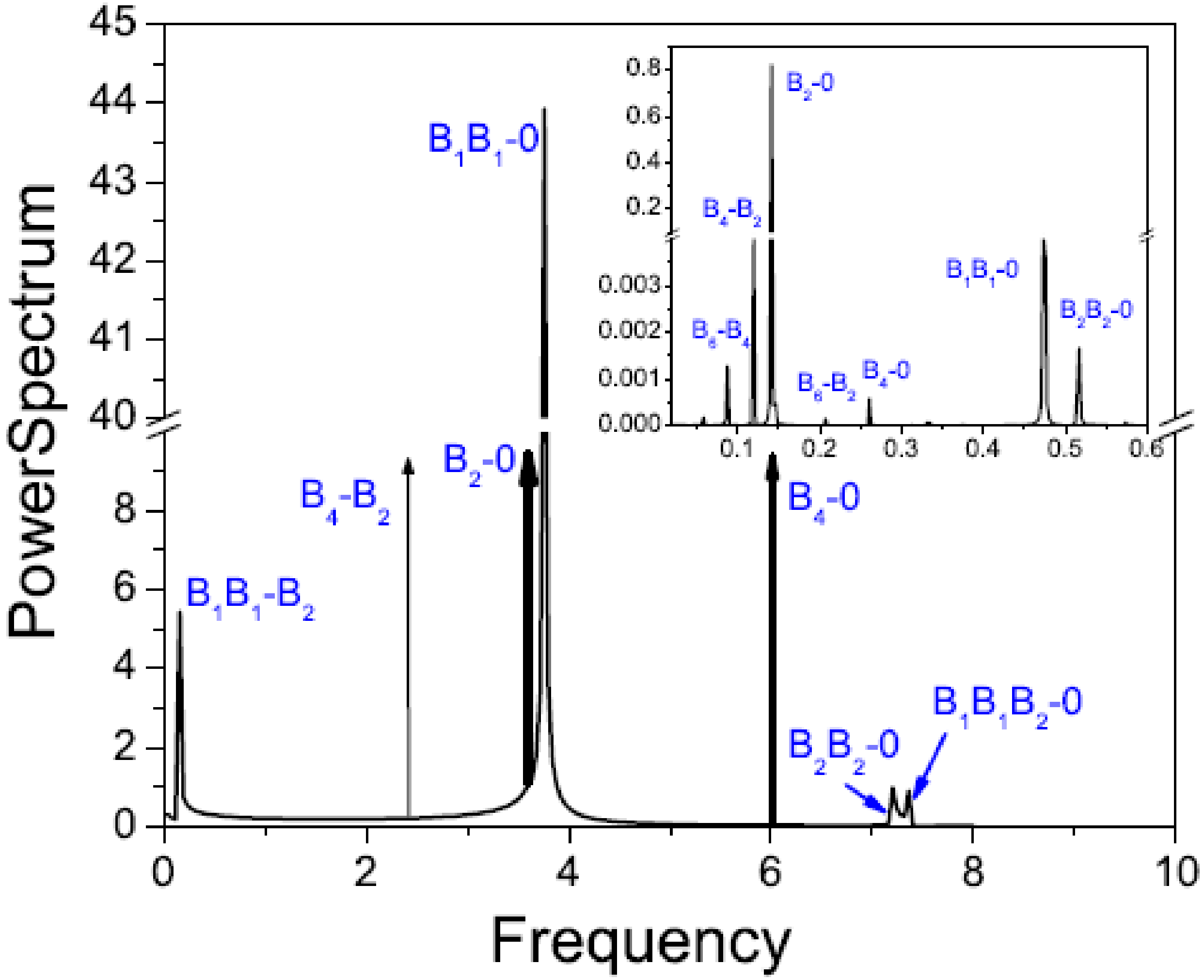
Smaller peaks $\int_{\theta\theta'} \langle\,\mathrm{B}_m(\theta')\,\mathrm{B}_m(-\theta')\,|\,e^{i\phi}\,|\,\mathrm{B}_n(\theta)\,\mathrm{B}_n(-\theta)\,\rangle$

Sharp peaks $\langle\mathrm{vac}\,|\,e^{i\phi}\,|\,B_{2n}(\theta=0)\,\rangle$

Dynamics of quantum sine-Gordon model

Gritsev, Demler, Lukin, Polkovnikov, cond-mat/0702343

Power spectrum $P(\omega) = \left| \int_t e^{i\omega t} \langle \psi(t) \,|\, e^{i\phi} \,|\, \psi(t) \rangle \right|^2$



A combination of broad features and sharp peaks. Sharp peaks due to collective many-body excitations: breathers

Decoherence of Ramsey interferometry

Interference in spin space

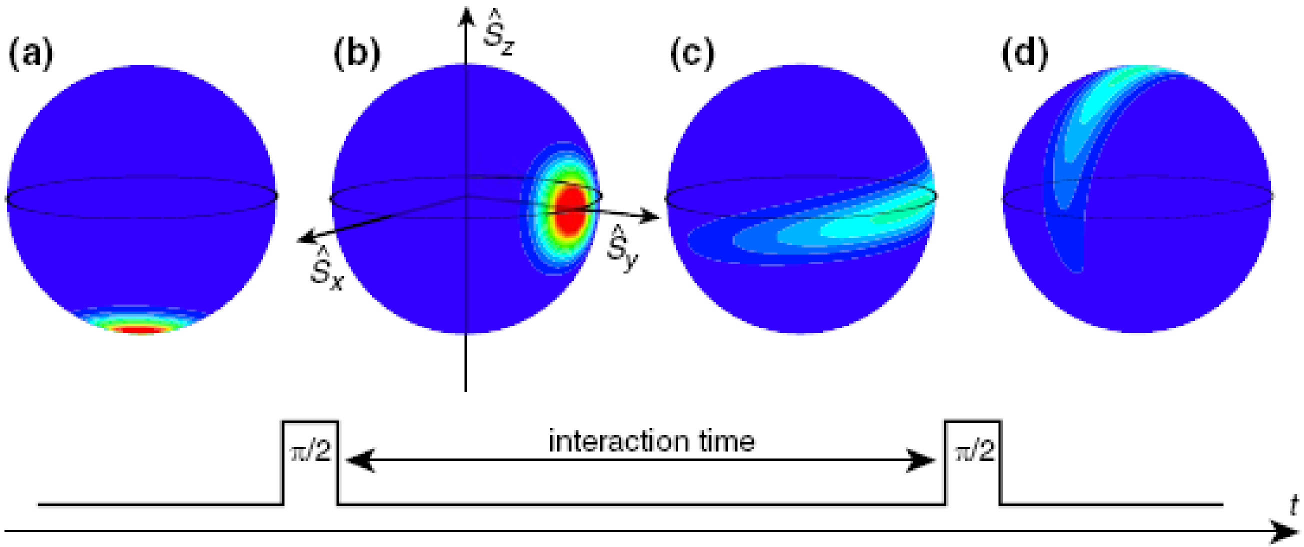
Squeezed spin states for spectroscopy

Motivation: improved spectroscopy. Wineland et. al. PRA 50:67 (1994)

Generation of spin squeezing using interactions.
Two component BEC. Single mode approximation

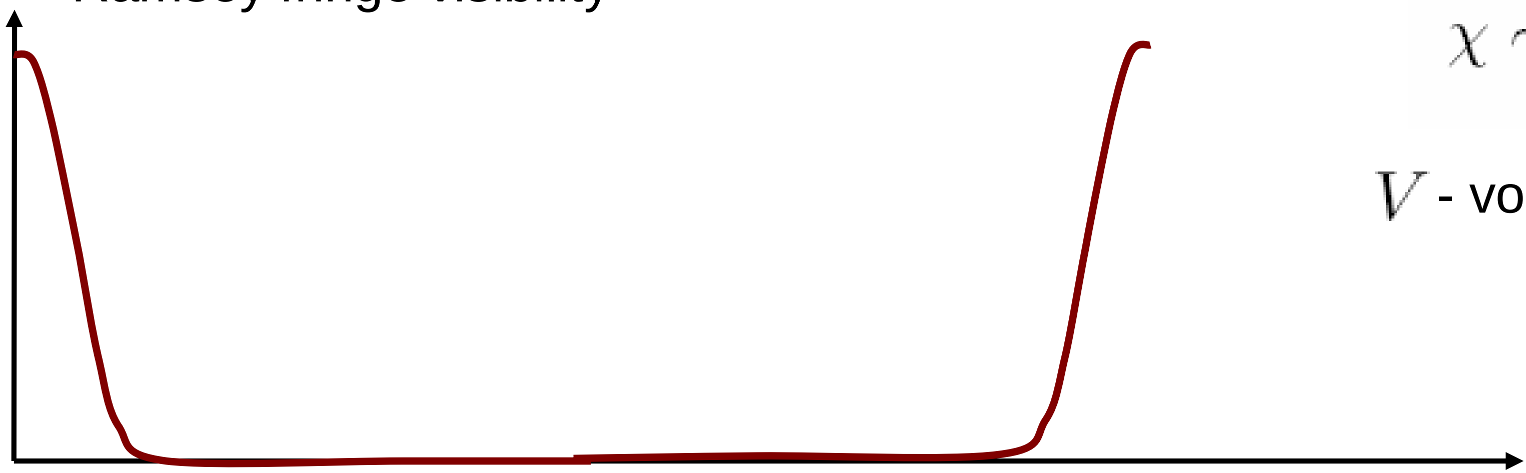
$$\mathcal{H} = \chi_s \left(S^z_{\mathrm{tot}}\right)^2$$

Kitagawa, Ueda, PRA 47:5138 (1993)



Interaction induced collapse of Ramsey fringes

Ramsey fringe visibility



time

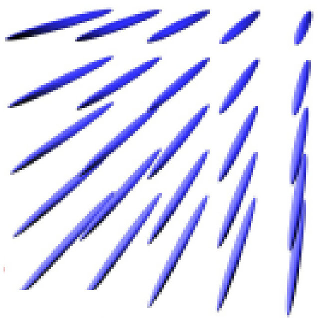
$$\chi \sim \frac{\hbar^2 a_s}{M V}$$

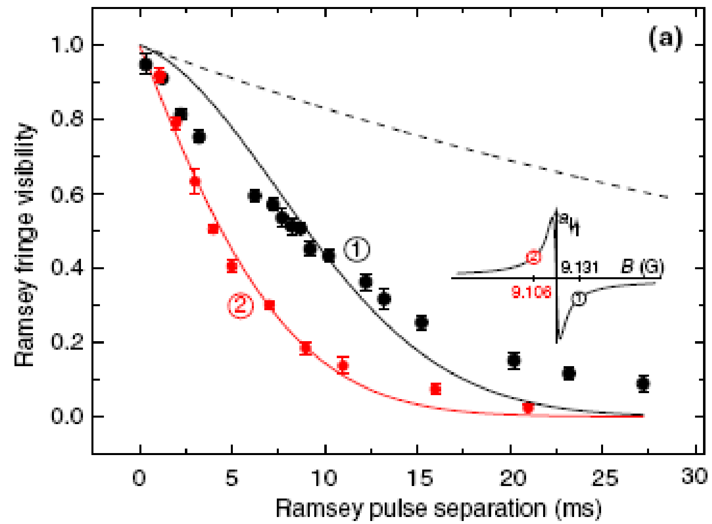
V - volume of the system

$$t_{\text{collapse}} \sim \frac{1}{\chi\sqrt{N}} \qquad t_{\text{revival}} \sim \frac{1}{\chi}$$

Experiments in 1d tubes:

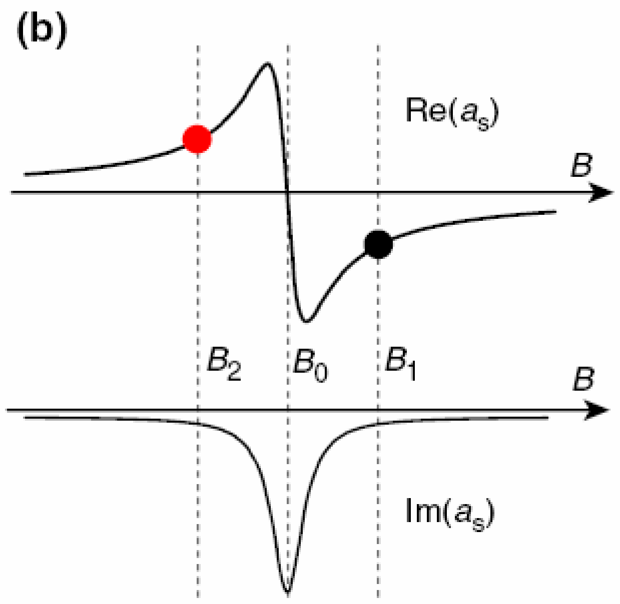
A. Widera, I. Bloch et al.



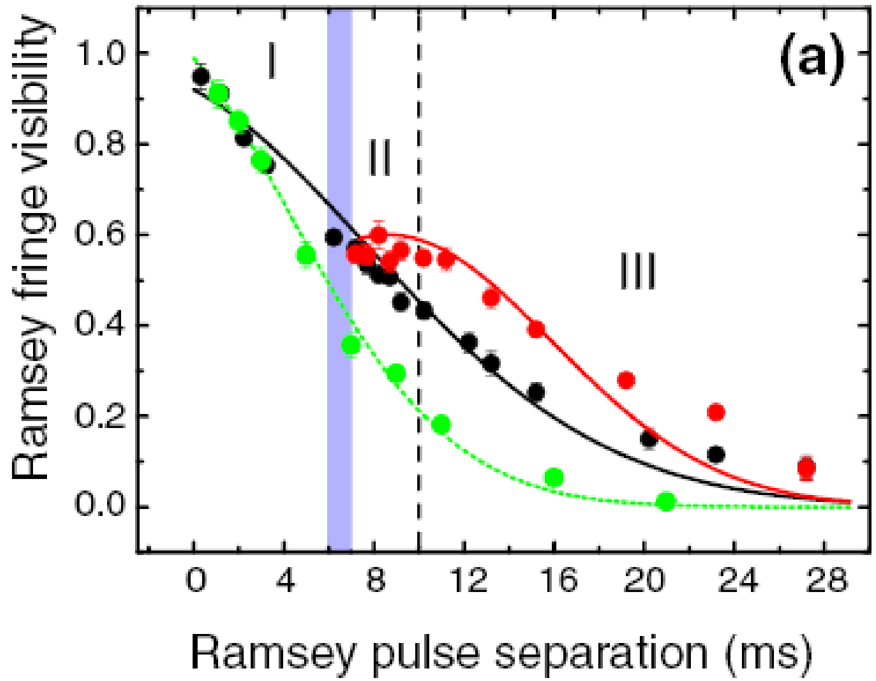


Spin echo. Time reversal experiments

$$a_s \rightarrow -a_s$$



Expts: A. Widera, I. Bloch et al.



In the single mode approximation

$$\mathcal{H} \rightarrow -\mathcal{H}$$

$$e^{i\int_T^{2T} \mathcal{H}(t)dt} \times e^{i\int_0^T \mathcal{H}(t)dt} = 1$$

No revival?

Related earlier theoretical work:
Kuklov et al., cond-mat/0106611

Interaction induced collapse of Ramsey fringes. Multimode analysis

Experiments done in array of tubes. Strong fluctuations in 1d systems

Bosonized Hamiltonian (Luttinger liquid approach)

$$S^+(x,t) \sim e^{i\phi_s(x,t)} \qquad [S^z(x), \phi_s(x')] = -i\delta(x-x')$$

$$\mathcal{H}_s = \int_0^L dx \left[g_s (S^z)^2 + \frac{\rho}{2m} (\nabla \phi_s)^2 \right]$$

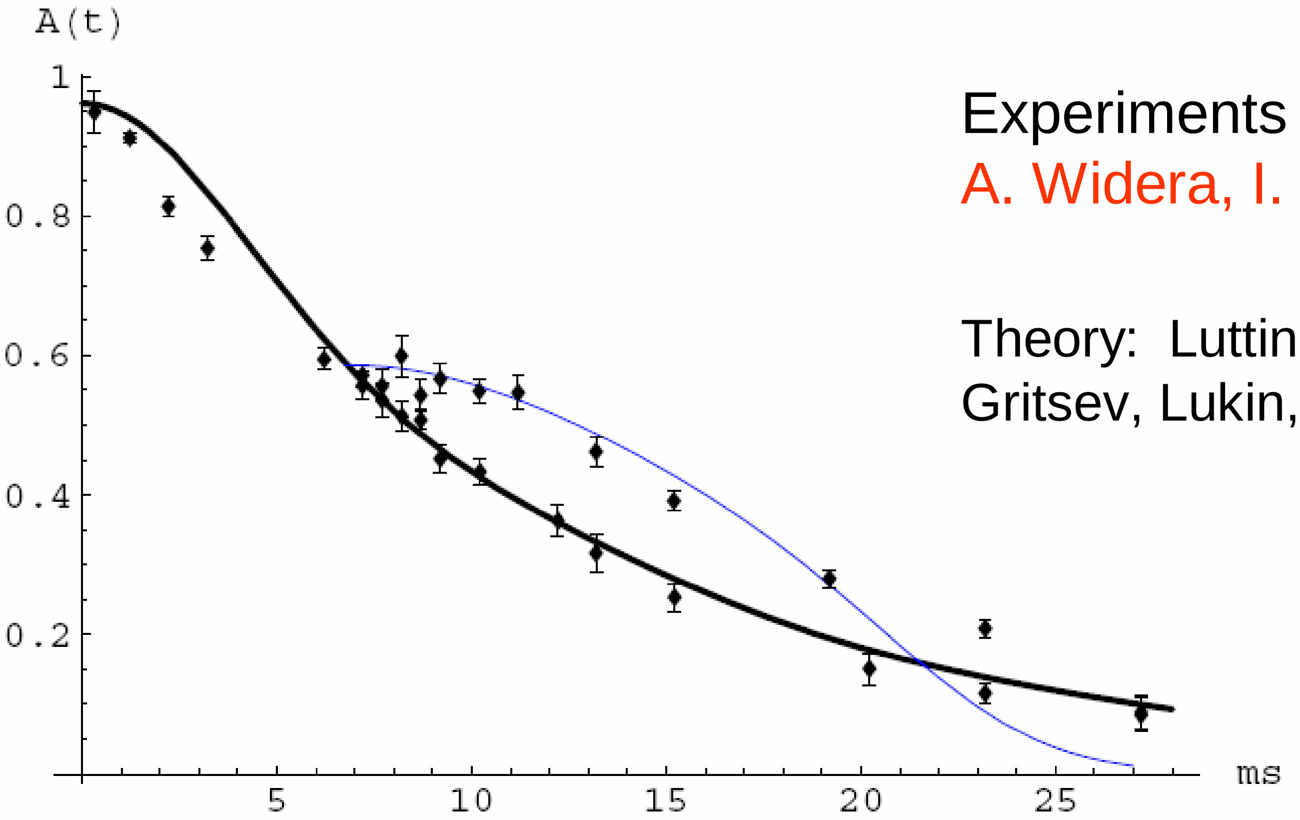
Changing the sign of the interaction reverses the interaction part of the Hamiltonian but not the kinetic energy

$$\mathcal{H}_s = \sum_q \left[g_s(t) S_q^z S_q^{z*} + \frac{\rho\, q^2}{m} \phi_{sq} \phi_{sq}^* \right]$$

$$[S_{q'}^z, \phi_{sq}] = -i\delta_{qq'}$$

Time dependent harmonic oscillators can be analyzed exactly

Interaction induced collapse of Ramsey fringes in one dimensional systems



Experiments in 1d tubes:
A. Widera, I. Bloch et al.

Theory: Luttinger liquid analysis
Gritsev, Lukin, Demler

Fundamental limit on Ramsey interferometry