



*The Abdus Salam
International Centre for Theoretical Physics*



1859-20

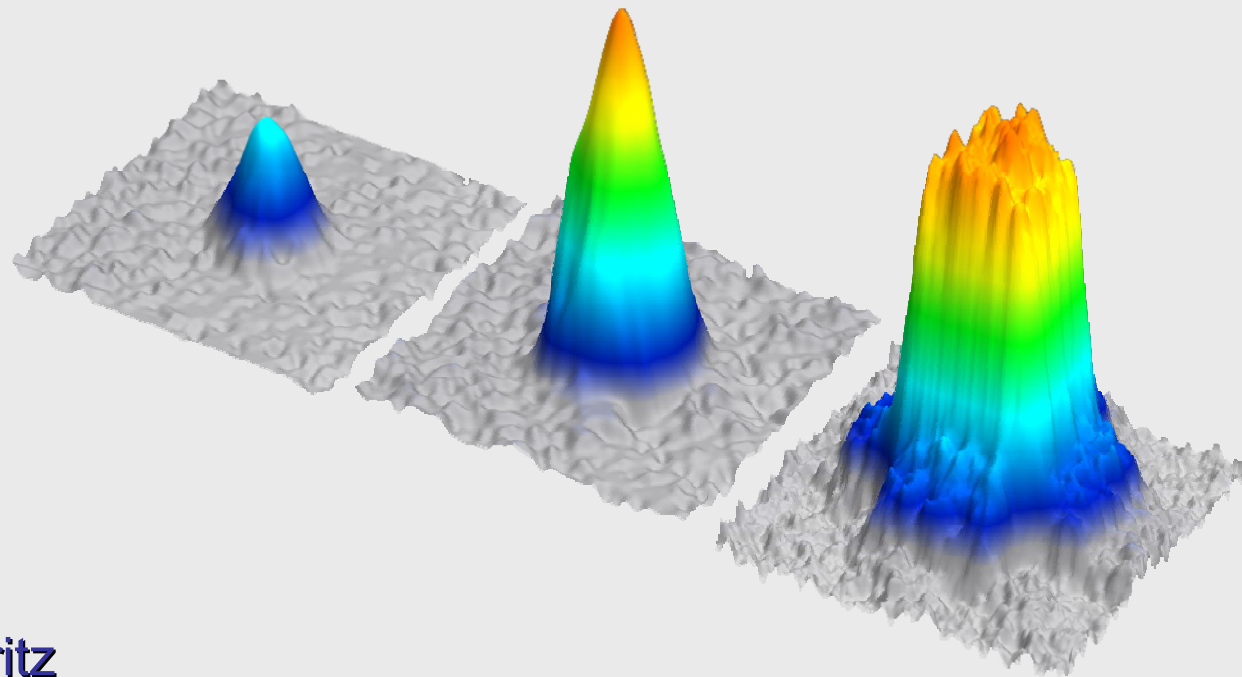
**Summer School on Novel Quantum Phases and Non-Equilibrium
Phenomena in Cold Atomic Gases**

27 August - 7 September, 2007

Fermions in optical lattices

Henning Moritz
ETHZ Zurich

Fermions in Optical Lattices



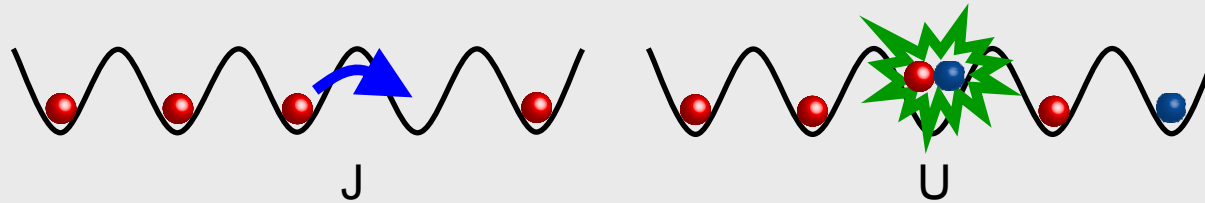
Henning Moritz

N. Strohmaier, R. Jördens, K. Günter

Y. Takasu, T. Stöferle, M. Köhl, T. Esslinger

ETH Zürich

Fermi-Hubbard model



$$H = -J \sum_{i,\sigma} (\hat{c}_{i\sigma}^\dagger \hat{c}_{i+1\sigma} + h.c.) + U \sum_i \hat{n}_{i\uparrow} \hat{n}_{i\downarrow} - \sum_{i,\sigma} (\mu - \varepsilon_i) \hat{n}_{i\sigma}$$



Interaction $U \rightarrow$ molecule formation



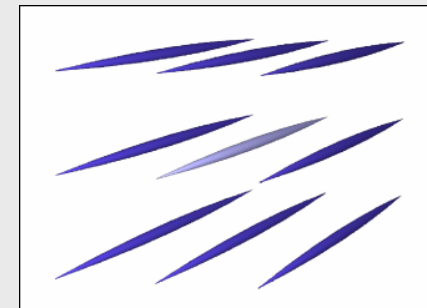
Tunneling J



Dimensionality



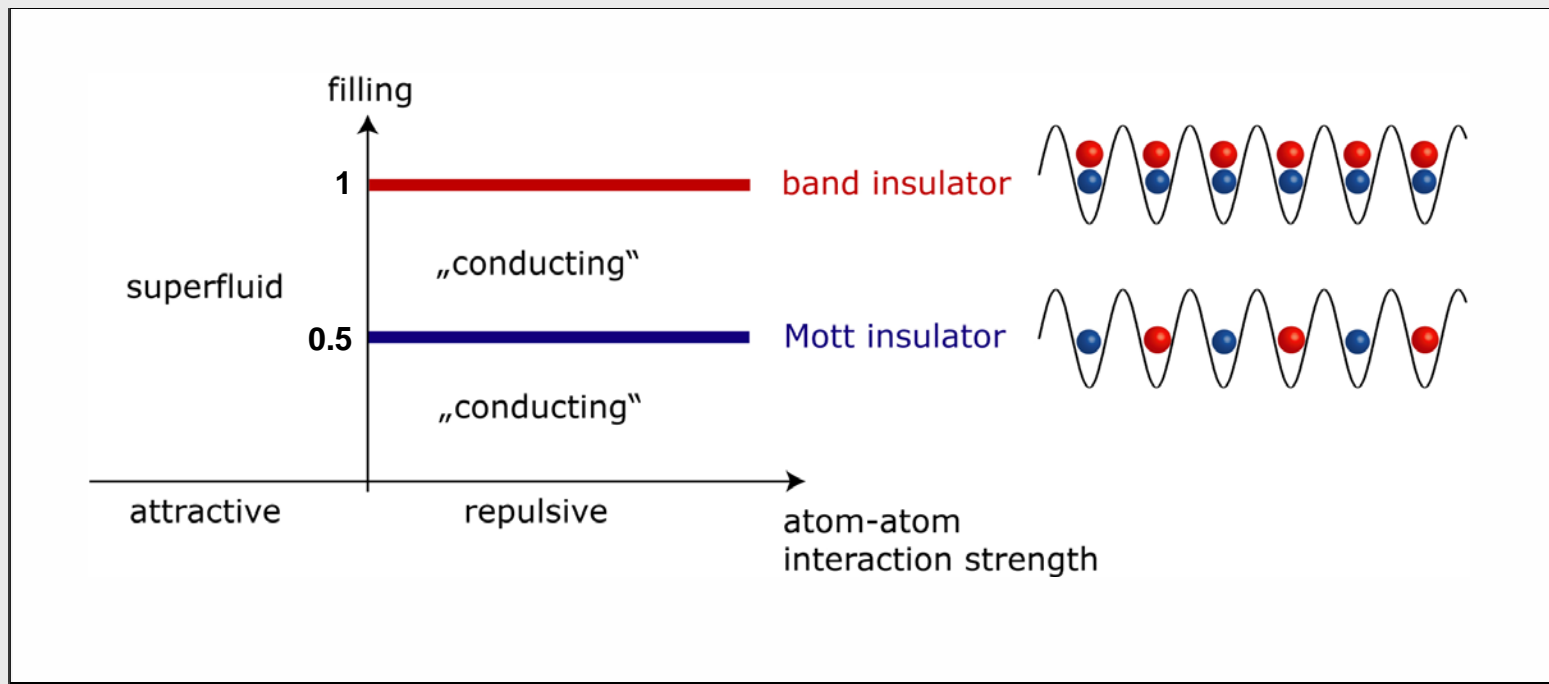
Filling



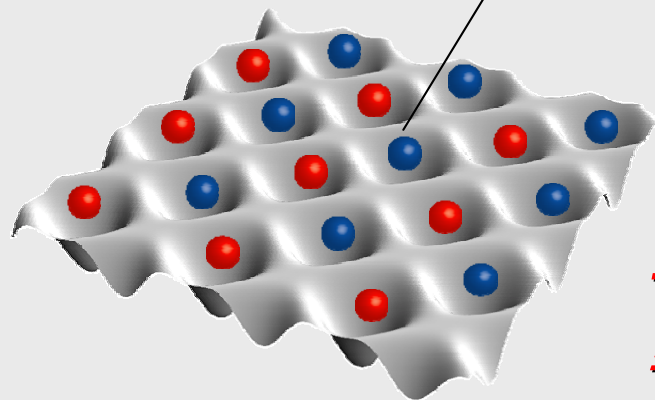
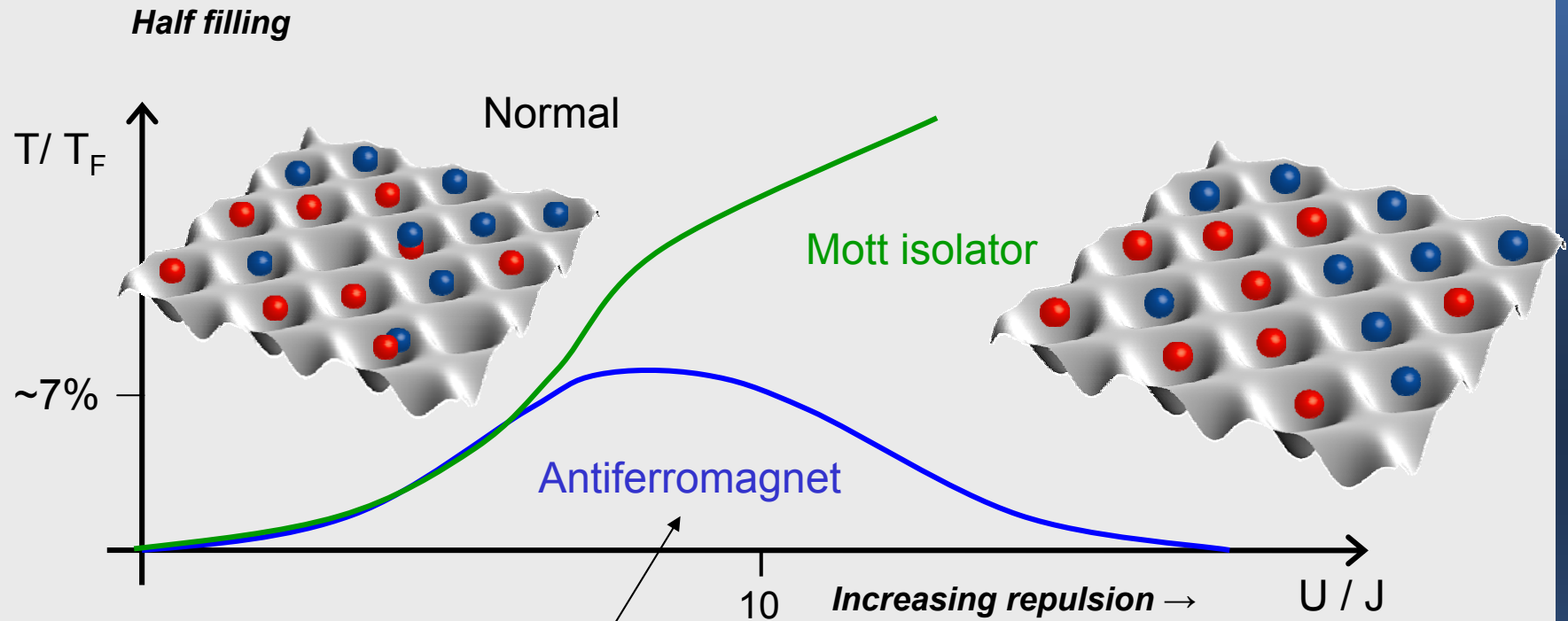
D. Jaksch et al., PRL 81, 3108 (1998).

W. Hofstetter et al., PRL 89, 220407 (2002).

Quantum phases in the lattice



Repulsive Phases



Antiferromagnetic order favorable because atoms can virtually hop to next site (paying U). This delocalisation minimises kinetic energy.

**Take away a few particles
 \Rightarrow High T_c Superconductivity ?**

Outline

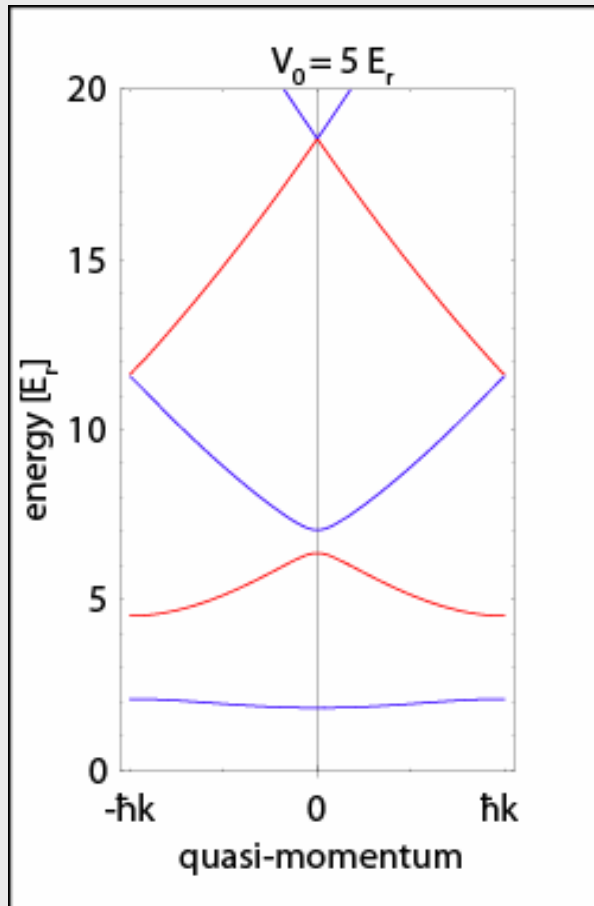
Ideal Fermi gas in a 3D lattice

Strong interactions in the lattice

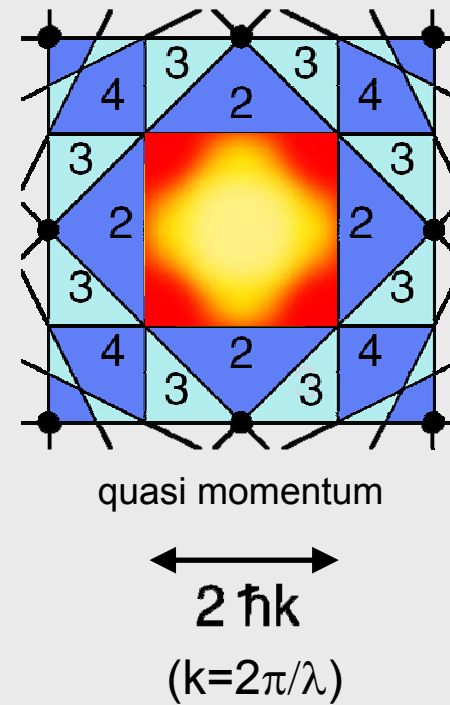
Transport of interacting fermions

1D Fermi gas

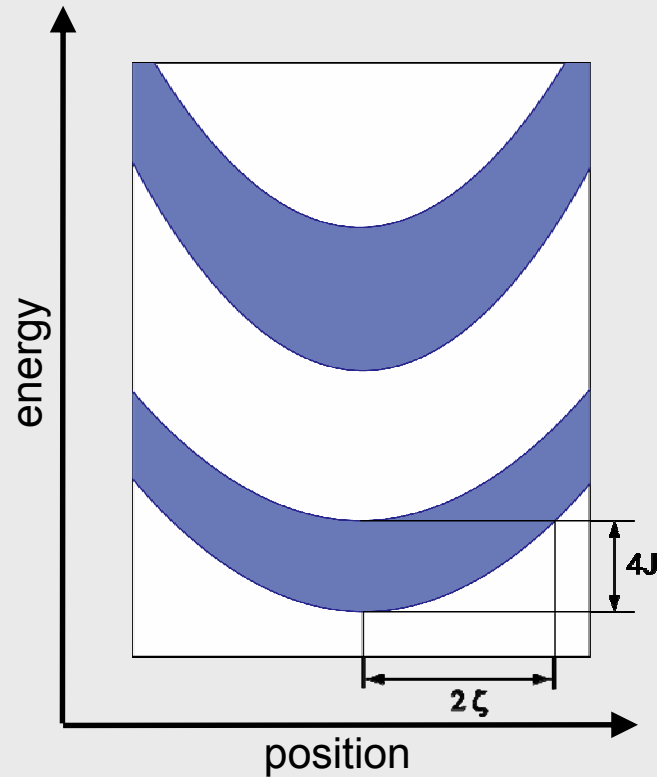
Filling the lattice



Brillouin zones of a square lattice



The inhomogeneous lattice

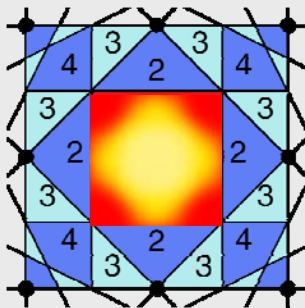


characteristic filling:

$$\frac{1}{2}m\omega^2\zeta^2 = J$$

$$\rho_c = \frac{N}{(\zeta/d)^3}$$

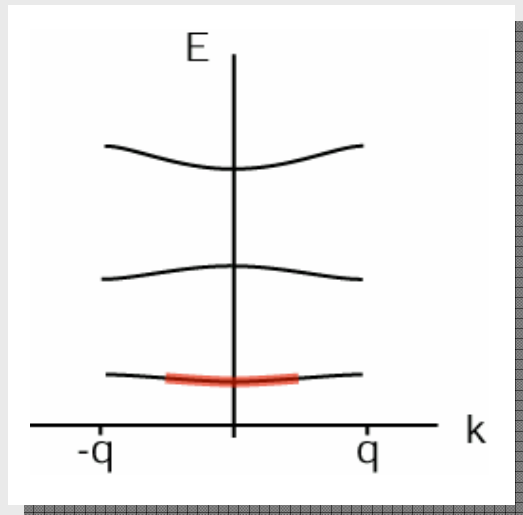
- atom number N
- external confinement ω
- tunneling J



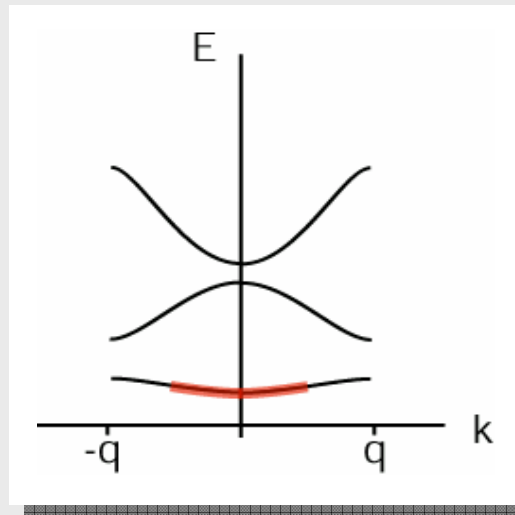
M. Rigol and A. Muramatsu PRA 70,043627 (2004)

Adiabatic Expansion

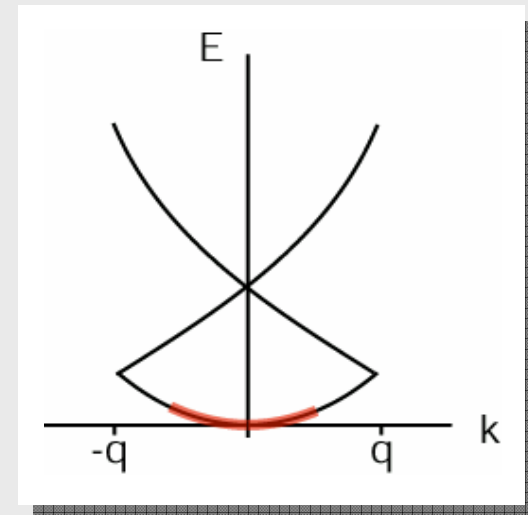
Deep lattice



weak lattice

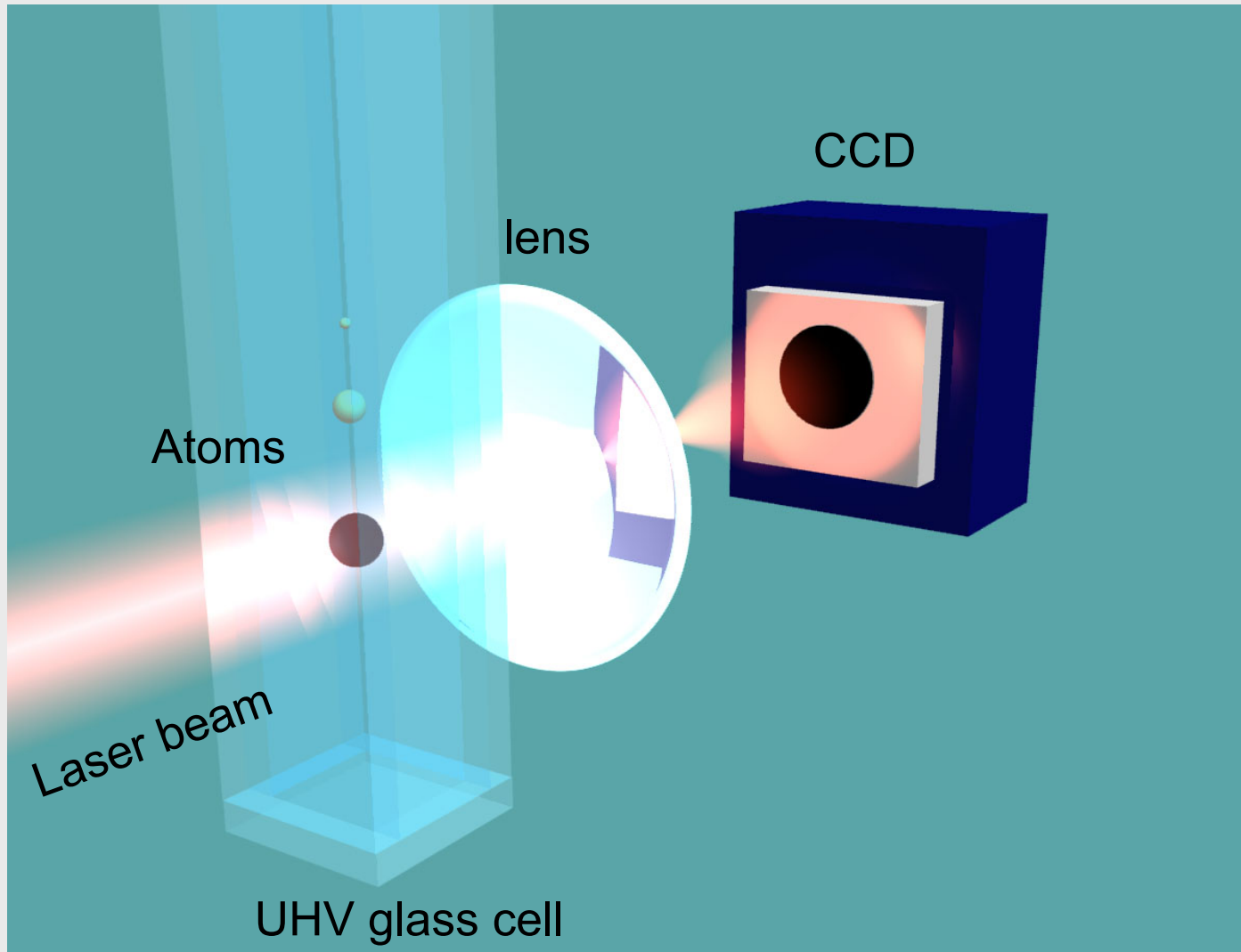


free atoms

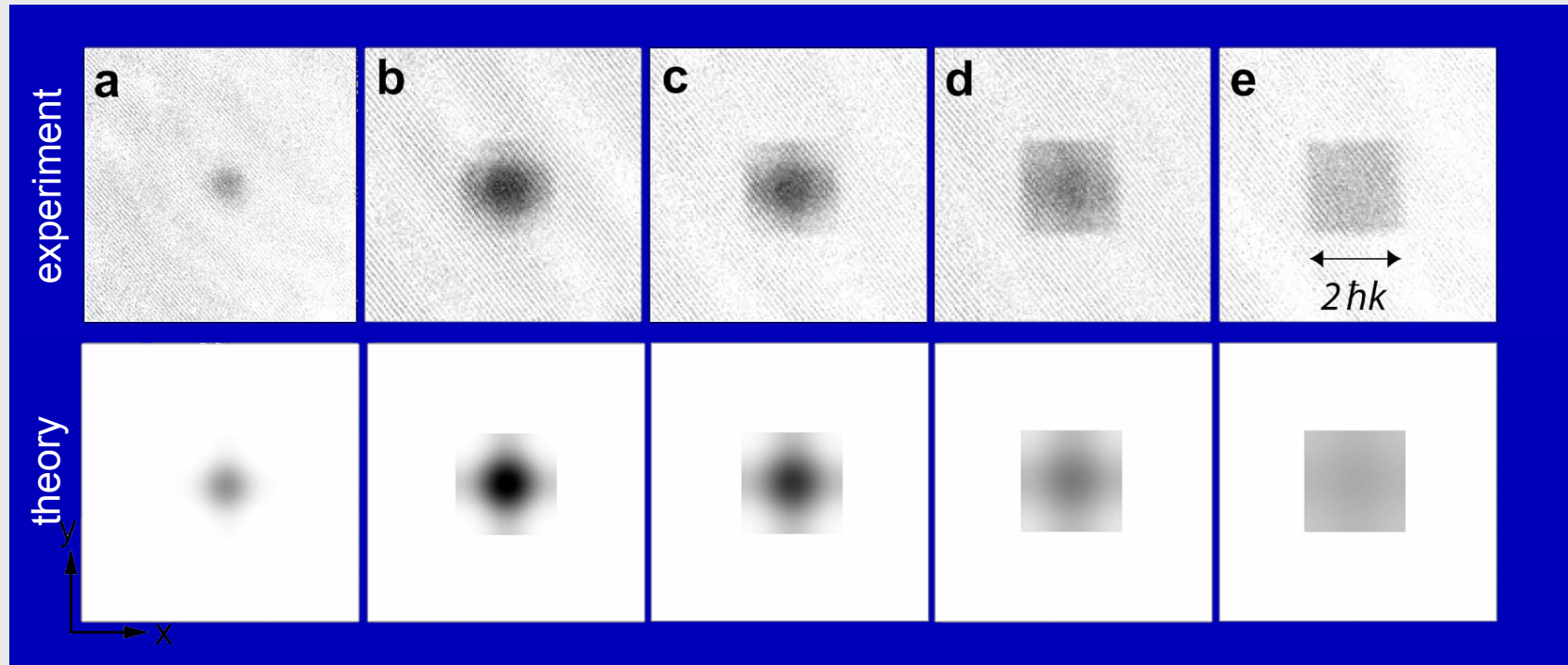


- no transitions to higher bands
- quasi-momentum conserved (nearly)
- not adiabatic for many-body wavefunction

Absorption Imaging



Observed Fermi surfaces



“conductive state“



characteristic
filling

“band insulator“

Ideal Fermi gas in a 3D lattice

Strong interactions in the lattice

Transport of interacting fermions

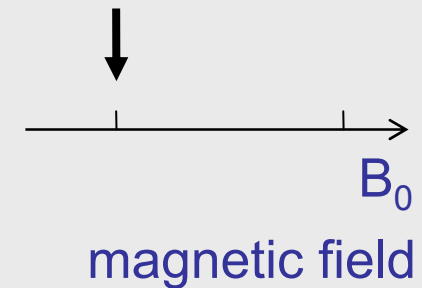
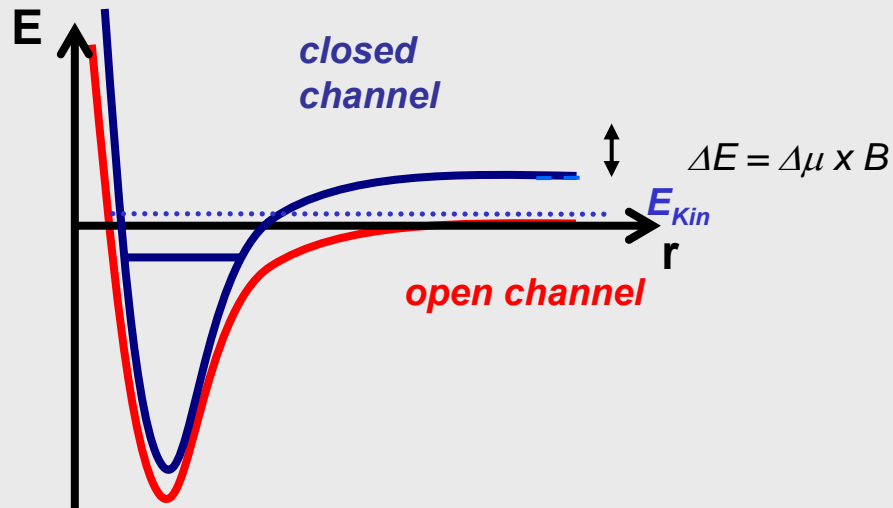
1D Fermi gas

Feshbach resonance



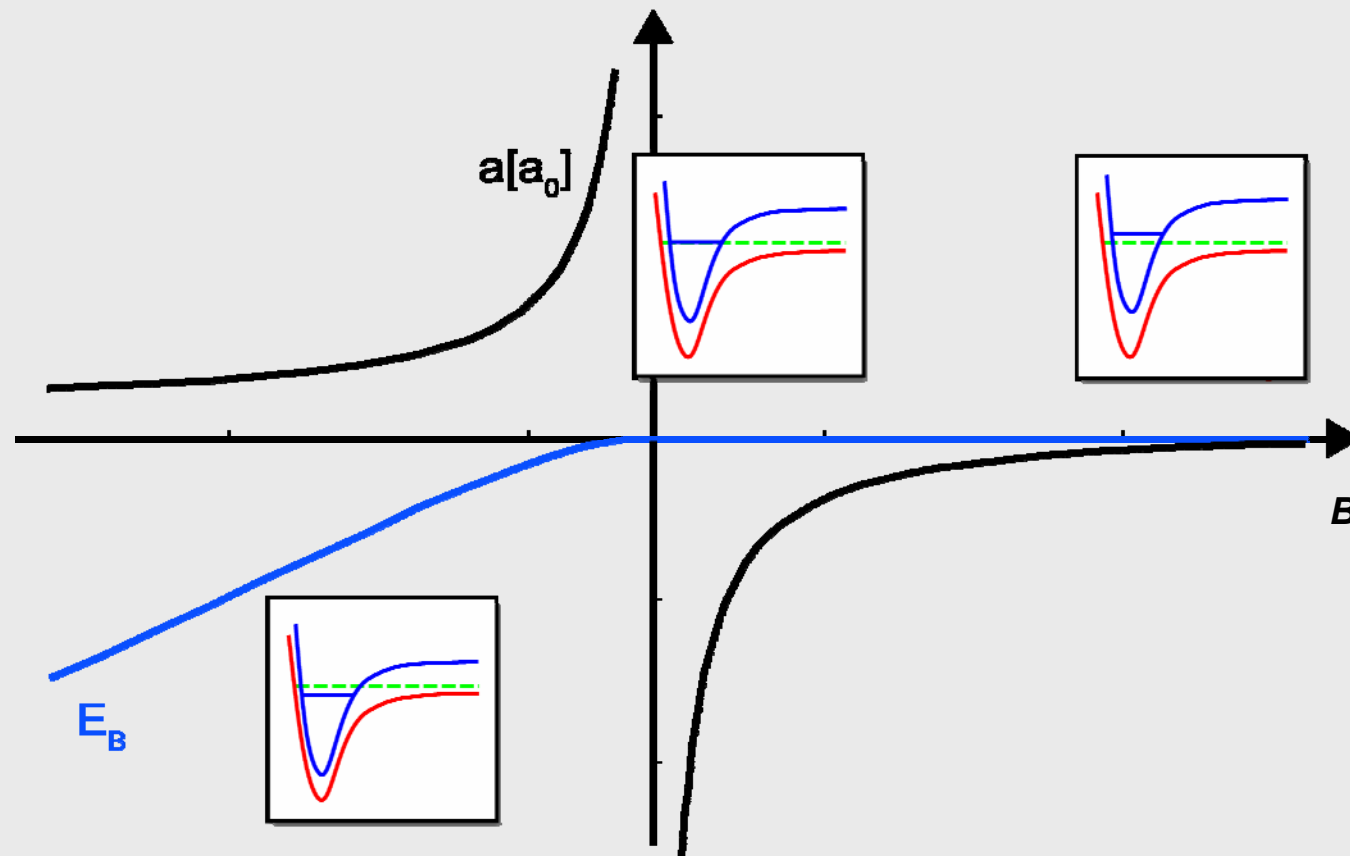
Interaction between spins of atoms at close distance may couple states m_{tot} conserved

- Tune bound state of a “closed channel” into degeneracy with the continuum
- Convenient tuning by mag. field: the two states have different mag. moment



Free space: Feshbach resonance

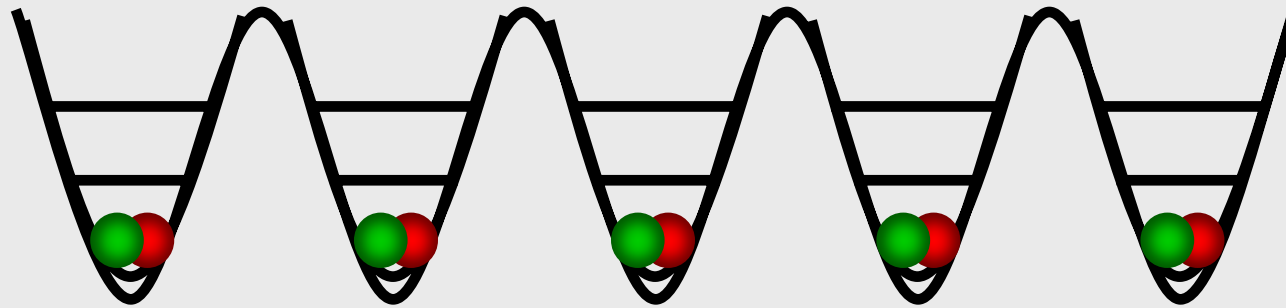
ETH



Composite Bosons

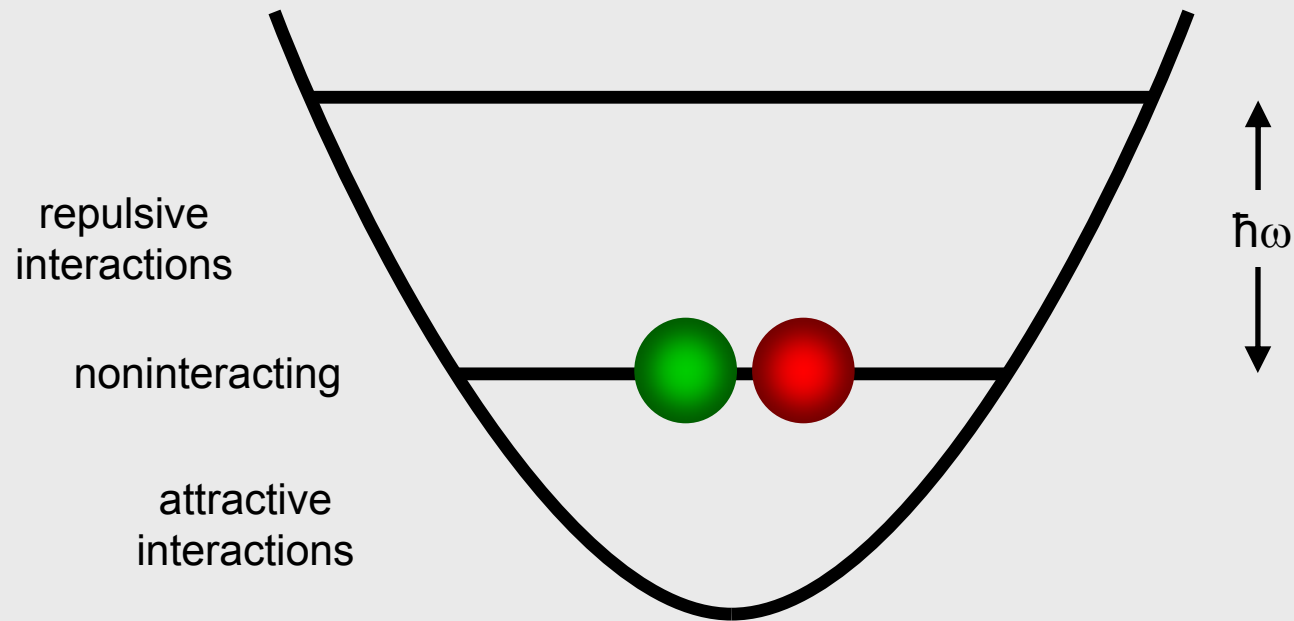
Free Fermions

Interactions in the lattice

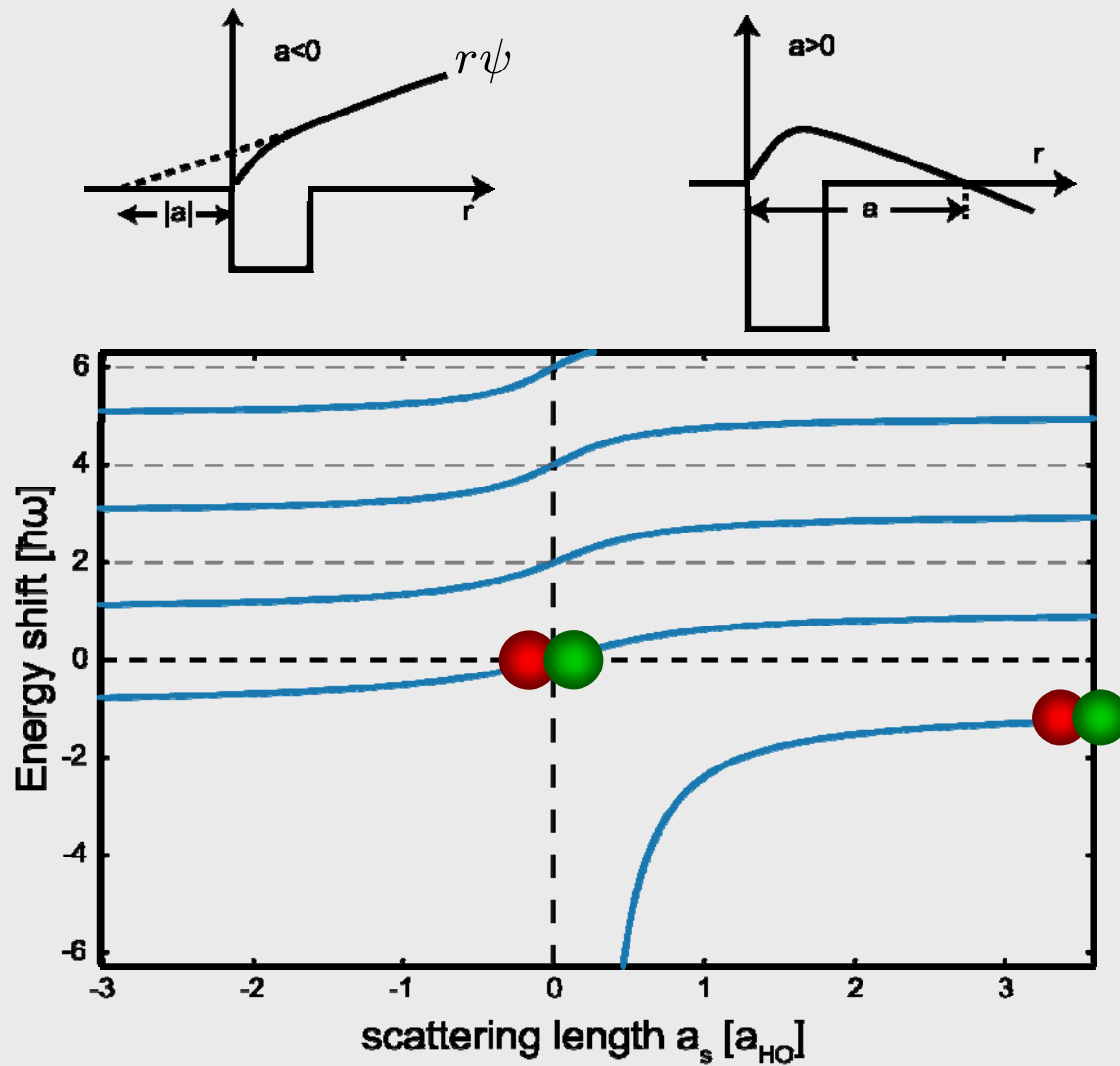


deep lattice = array of harmonic oscillators

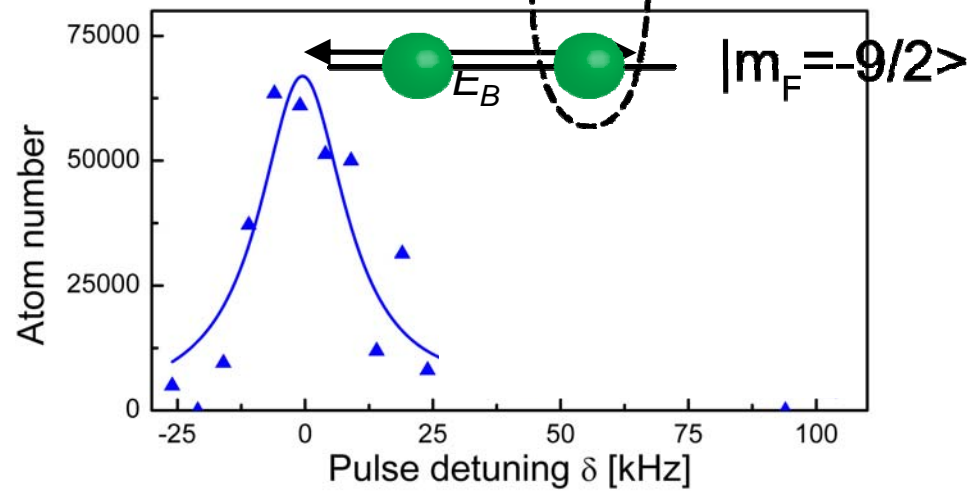
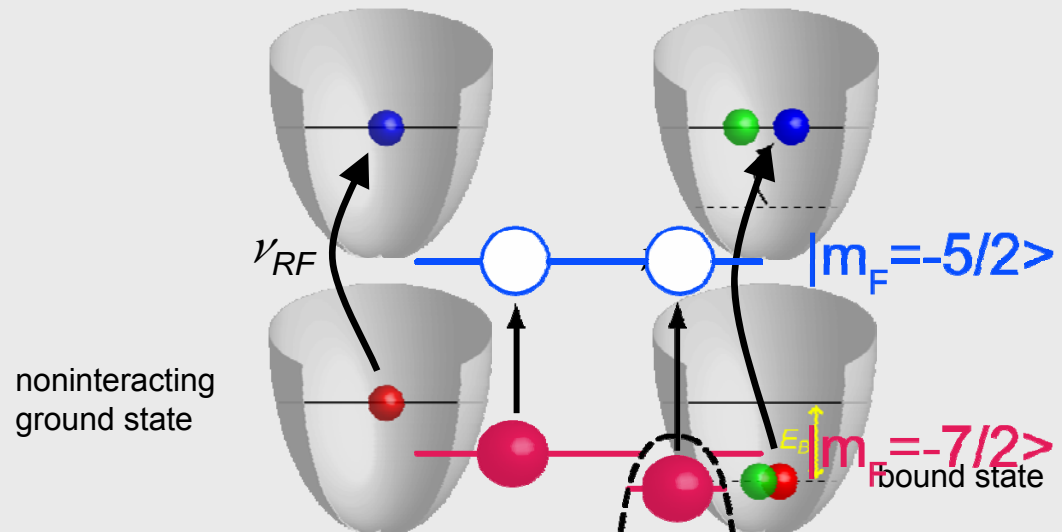
Interacting harmonic oscillator



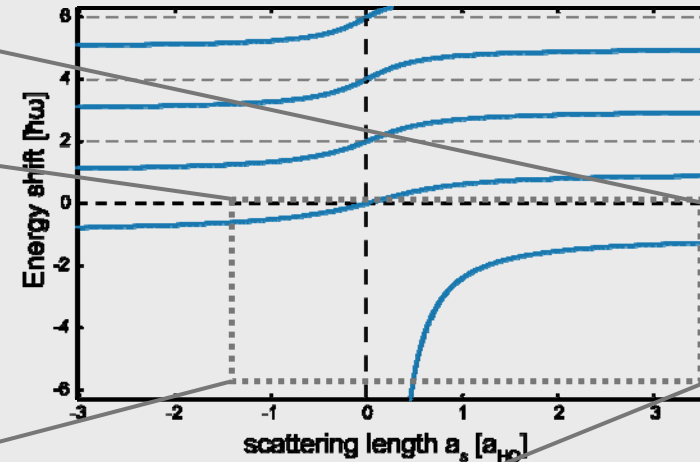
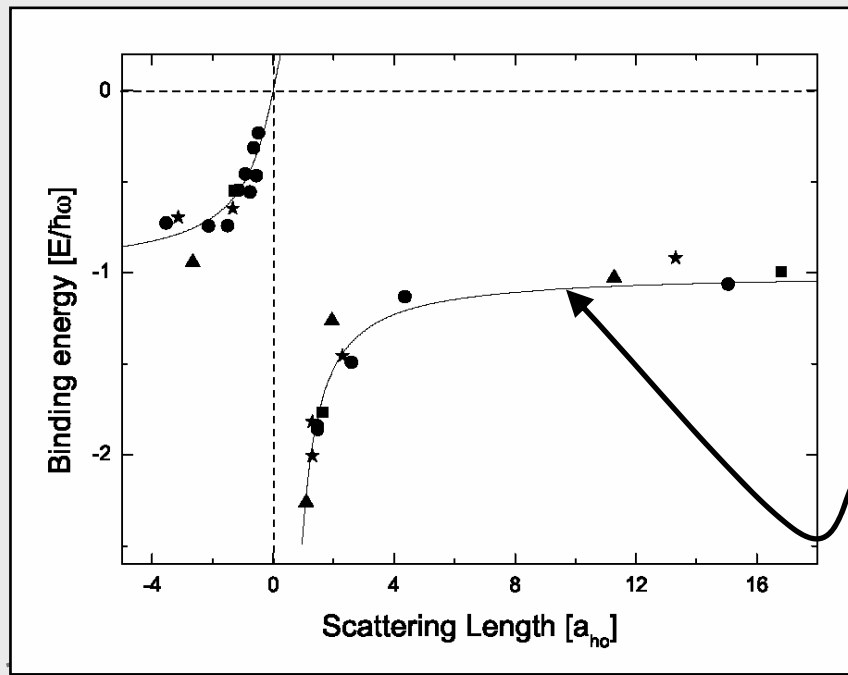
Harmonic oscillator spectrum



RF spectroscopy in the lattice



Measuring the binding energy



T. Busch et al., Found. Phys. 28, 549 (1998)

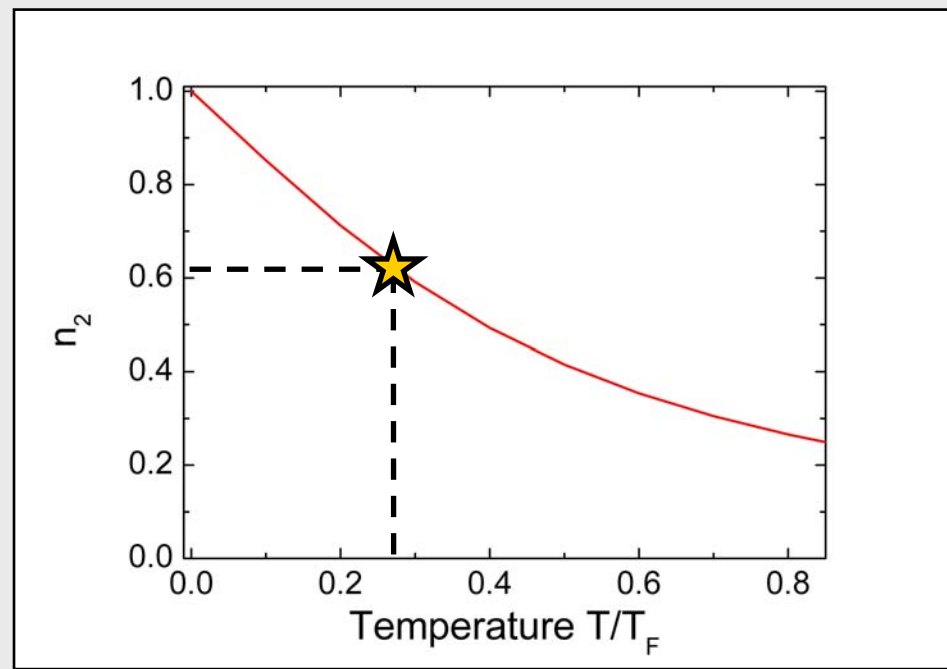
*Exact theory
(no free parameters)*

Fermionic atoms transform into bosonic molecules!

T Stöferle, H. M., K. Günter, M. Köhl, T. Esslinger, Phys. Rev. Lett. 96, 040301 (2006)

Thermometry in the lattice

Temperature determines the fraction of doubly occupied lattice sites n_2 .

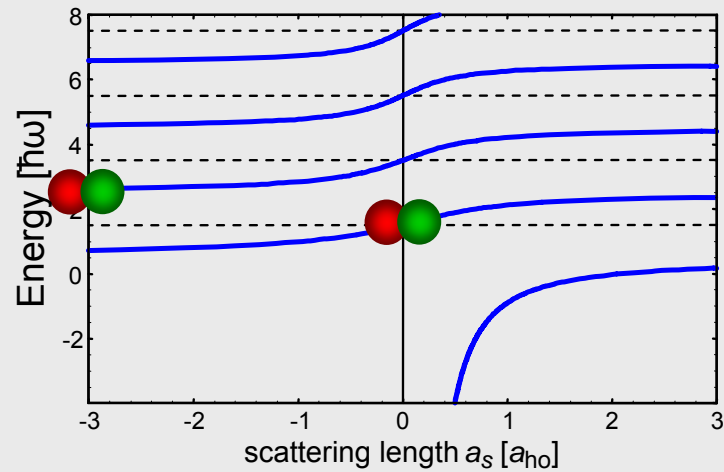


M. Köhl, cond-mat/0510567.

see also: H. G. Katzgraber et al., cond-mat/0510194 and for bosons: G. Pupillo et al., cond-mat/0407075.

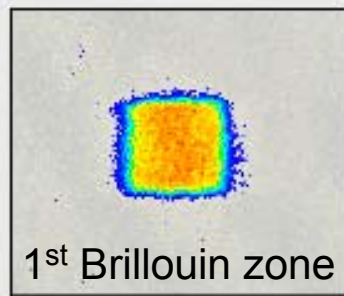
Going the other direction ...

Two interacting particles
in a harmonic oscillator

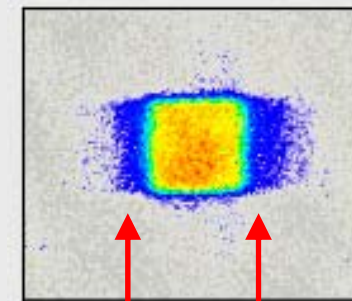
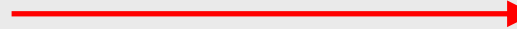


Physics beyond the single
band Hubbard model.

noninteracting



sweep across
Feshbach resonance

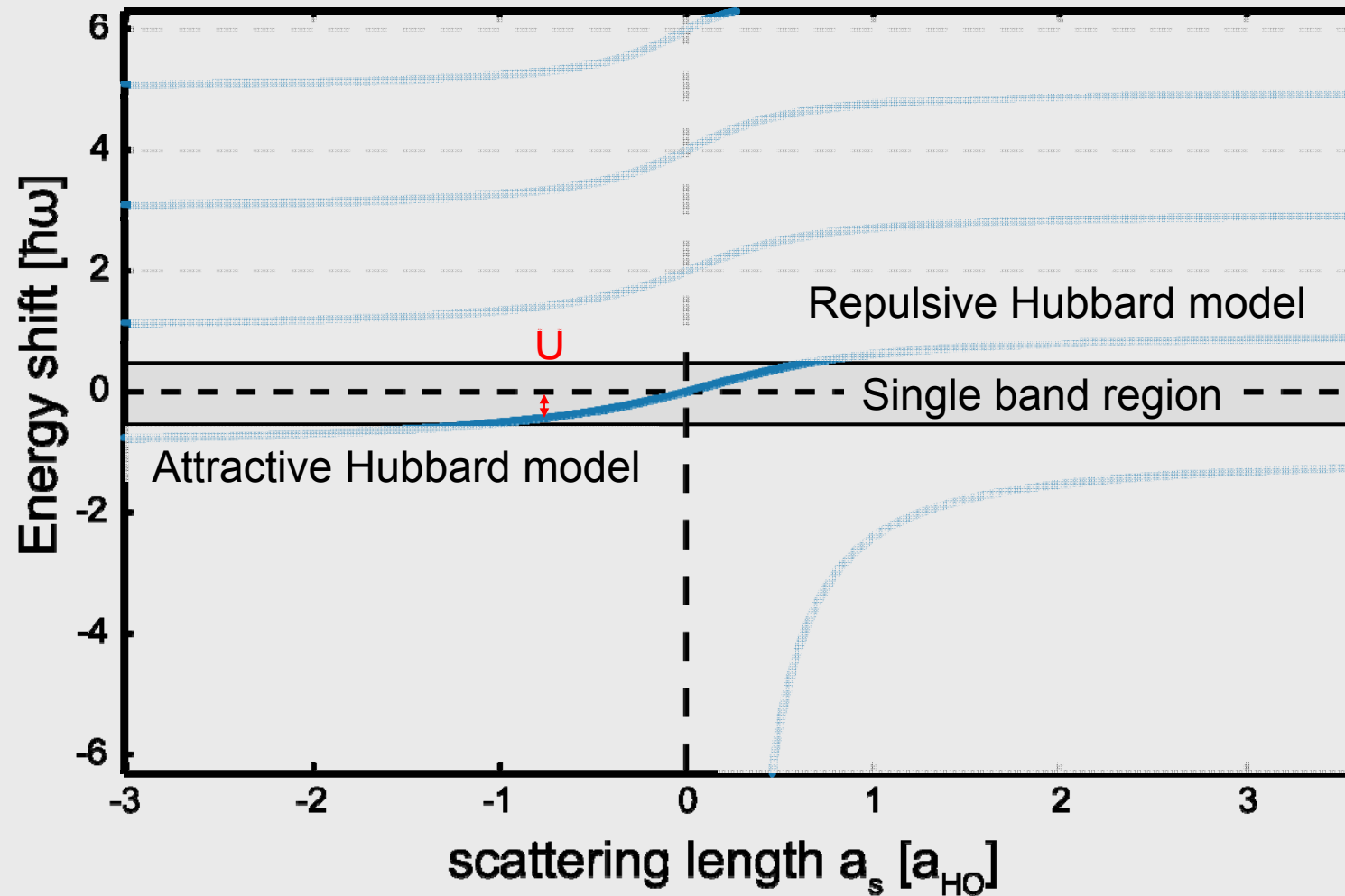


observe atoms in
higher bands

M. Köhl et al., Phys. Rev. Lett. 94, 080403 (2005).

Theory: Diener & Ho, PRL 96, 010402 (2006), H. G. Katzgraber et al., PRA 74, 043602 (2006).

Connection to Hubbard model



Ideal Fermi gas in a 3D lattice

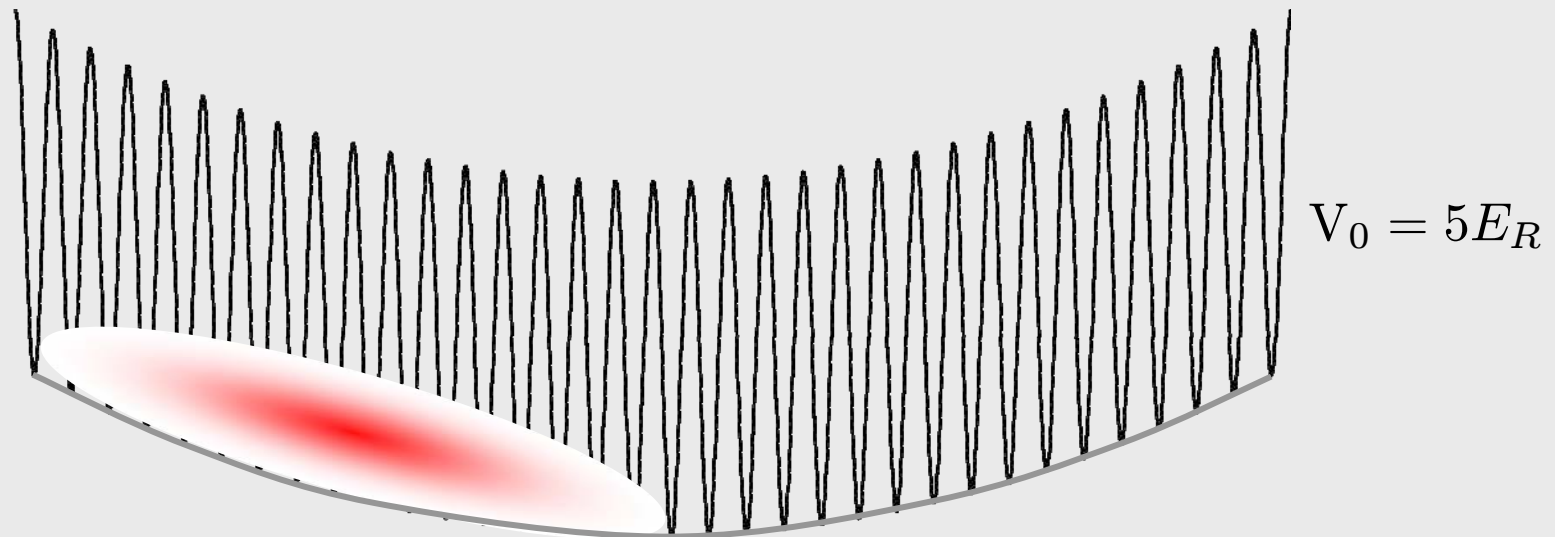
Strong interactions in the lattice

Transport of interacting fermions

1D Fermi gas

Observing transport

ETH

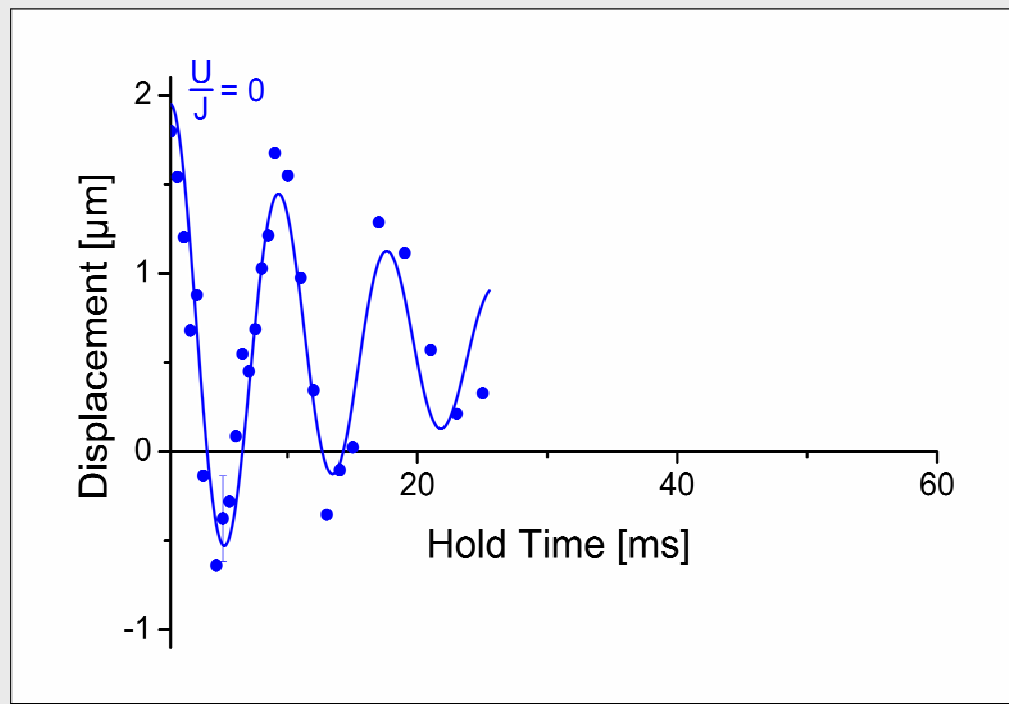


C. D. Fertig et. al, Phys. Rev, Lett, 92, 120403 (2005)

H. Ott, E. de Mirandes, F. Ferlaino, G. Roati, G. Modugno, and M. Inguscio, Phys. Rev, Lett, 92, 160601 (2004)

Observing transport

Lattice depth $5 E_R$, \sim half filling in the center

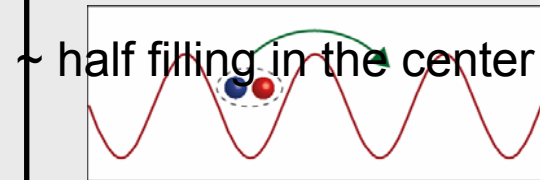
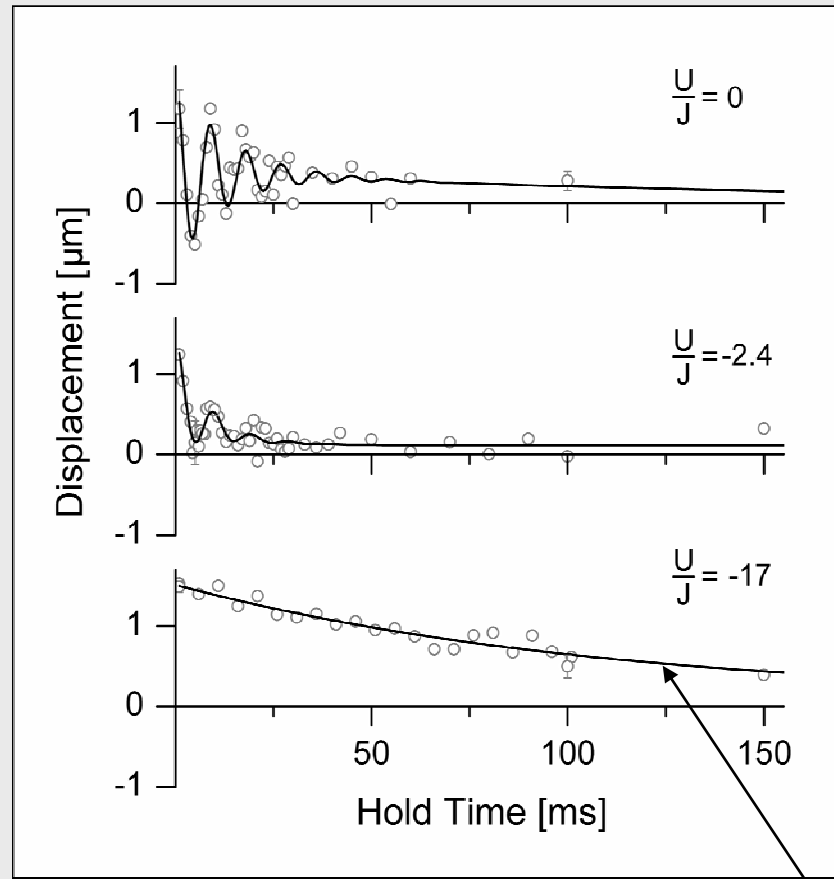


Inhibited Transport

No interaction

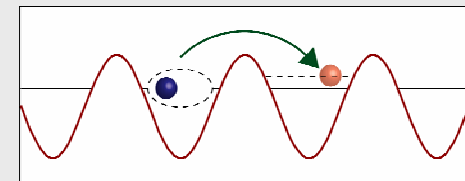


Strong attraction



pair tunneling

$$J_{pair} \sim \frac{J}{60}$$

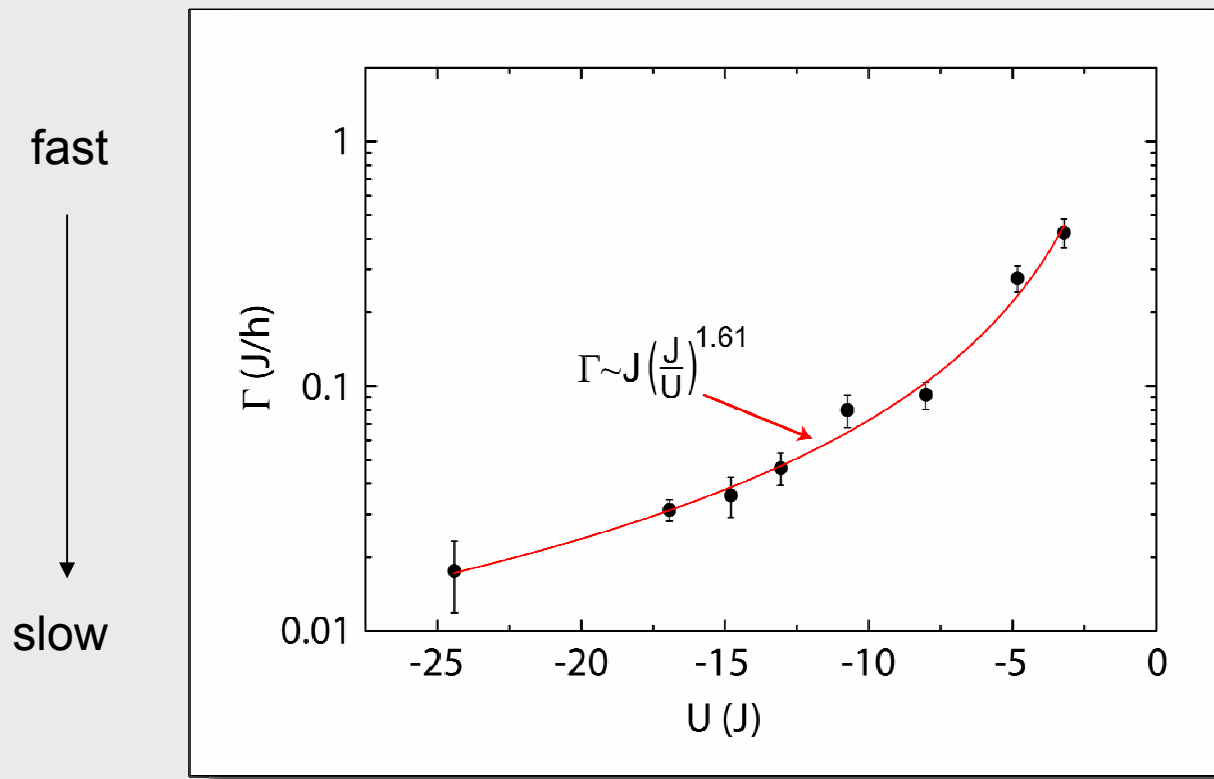


effective tunneling

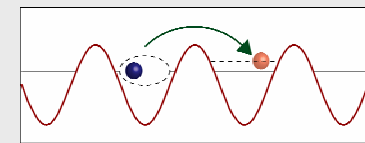
$$J_{eff} = 2J \cdot \frac{J}{U}$$

$$y(t) = y_0 + A e^{-\Gamma \cdot t}$$

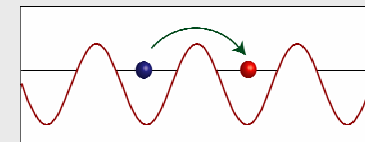
Drift Rate



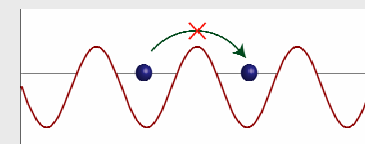
← Increasing attraction



$$J_{eff} = 2J \cdot \frac{J}{U}$$

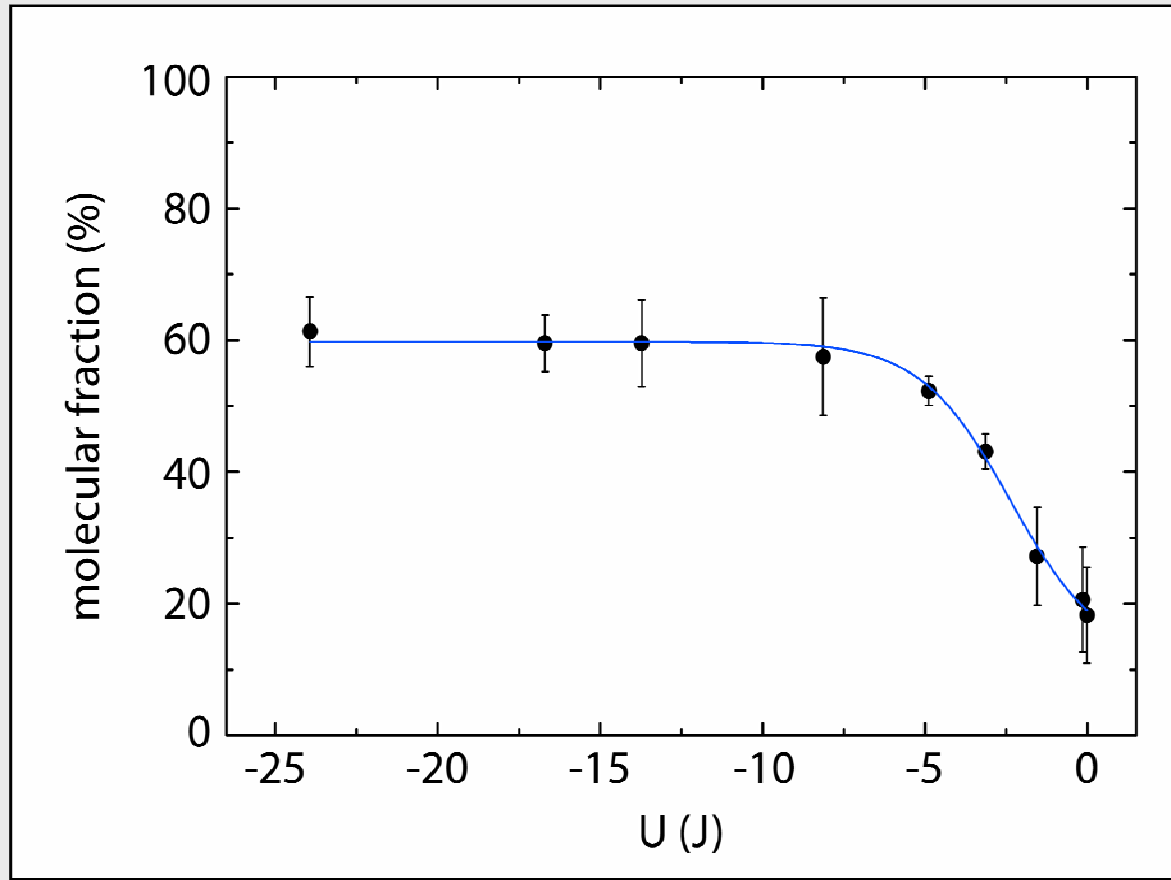


Normal tunneling J



No tunneling

Pair formation increases



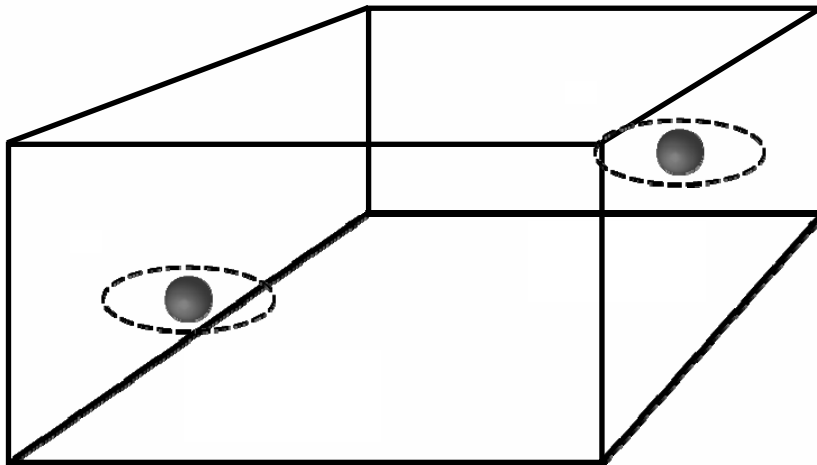
Ideal Fermi gas in a 3D lattice

Strong interactions in the lattice

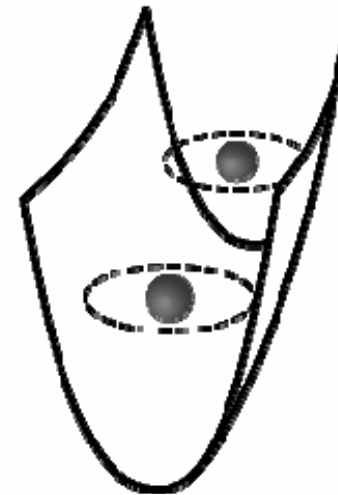
Transport of interacting fermions

1D Fermi gas

Scattering and confinement

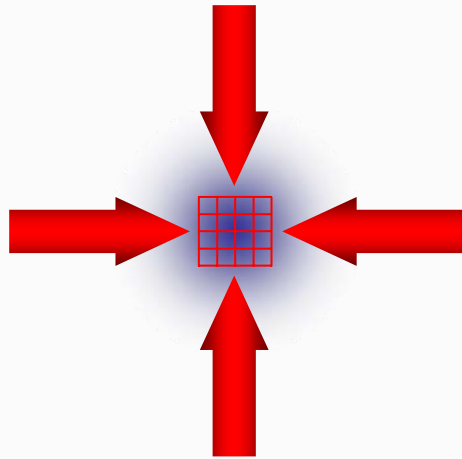




*Bound state for $a > 0$
(Wigner, 1933)*

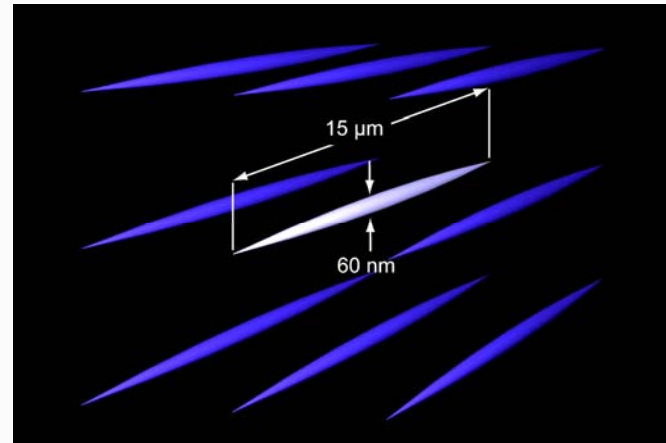


*1D is different!
(Olshanii, 1998)*

1D Fermi Gas

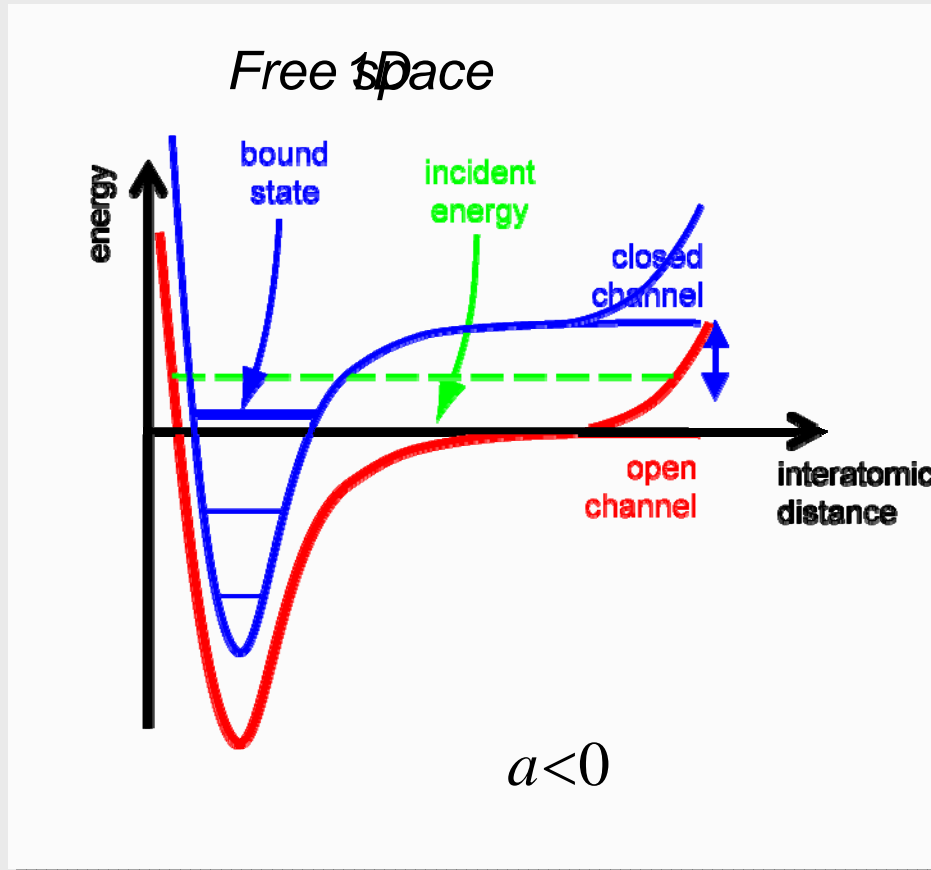


-  fermionic ^{40}K
-  red-detuned standing waves
 $V = 25 E_R$, $\lambda = 826 \text{ nm}$



- aspect ratio $\omega_{\perp} / \omega_z \approx 270$
- $E_F = N\hbar\omega_z \ll \hbar\omega_{\perp}$
- $k_B T < E_F$

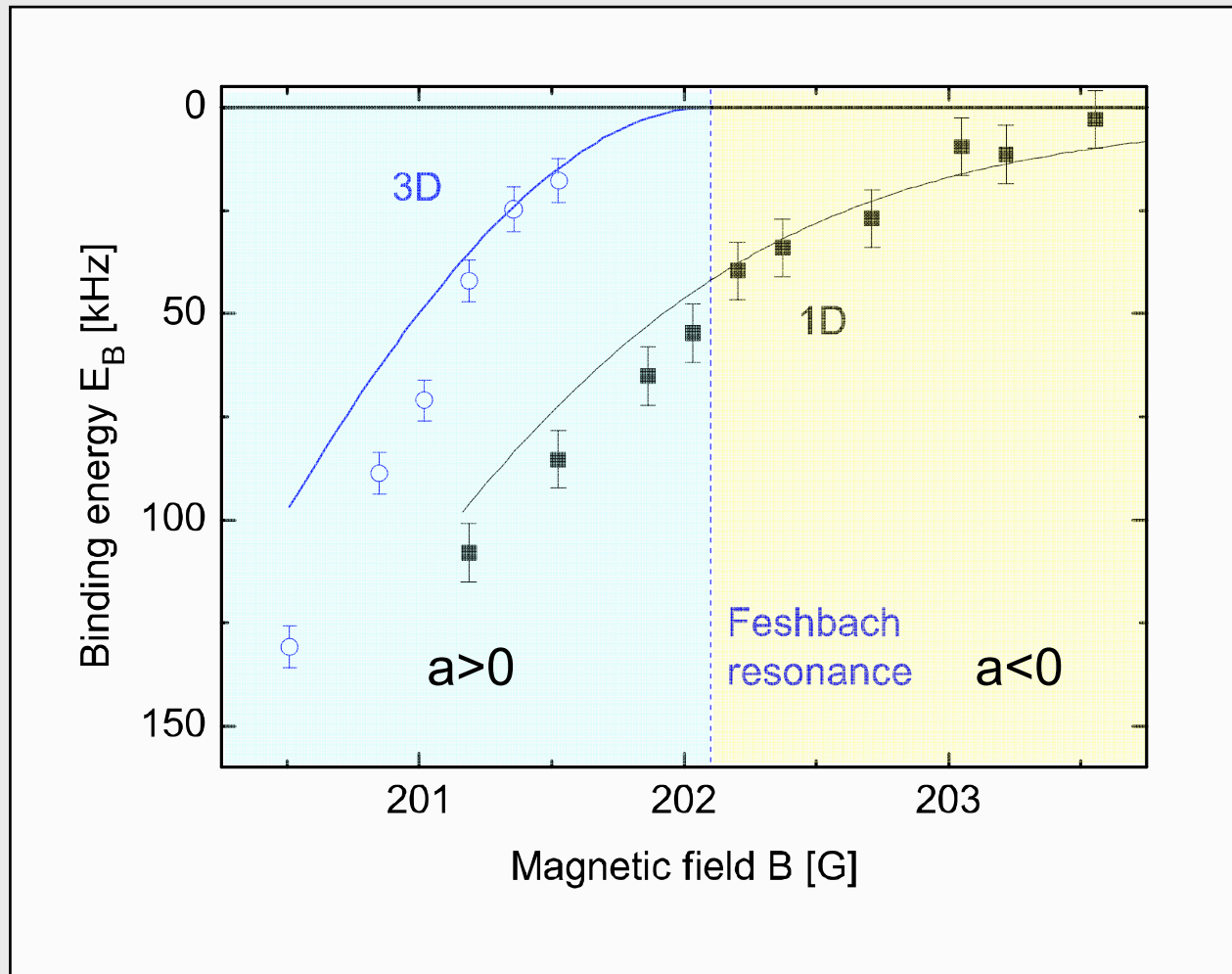
Molecules in 1D



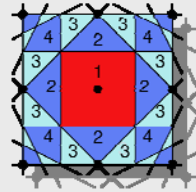
$$\frac{a}{l_{\perp}} = -\frac{\sqrt{2}}{\Gamma(1/2, -E_B/2\hbar\omega_{\perp})}$$

T. Bergemann, M.G. Moore, M. Olshanii
Phys. Rev. Lett. 91, 163201 (2003)

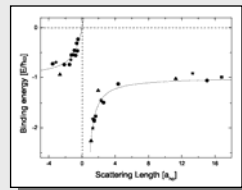
1D versus 3D



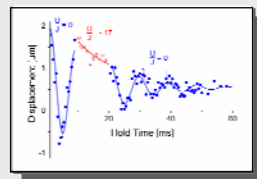
Conclusions



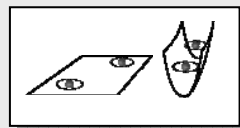
Fermi surfaces



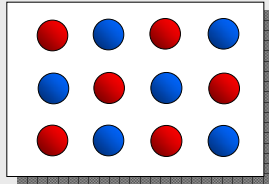
Strong interactions & molecules



Transport of interacting fermions



1D Fermi Gas

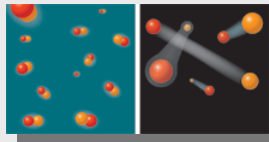


Mott insulating & antiferromagnetic phase

W. Hofstetter et al., PRL 89, 220407 (2002)

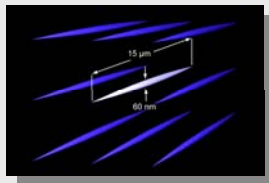
F. Werner et al., PRL 95, 056401 (2005)

E. Altman et al., PRA 70, 013603 (2004)



Superfluidity in the lattice

W. Hofstetter et al., PRL 89, 220407 (2002)



Low-dimensional systems

Exactly solvable BEC-BCS Crossover

J.N. Fuchs et al., PRL 93, 090408 (2004)

I.V. Tokatly, Phys. PRL 93, 090405 (2004)