The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific and
Cultural Organization

IAEA
International Atomic Energy Agency

1859-6

Summer School on Novel Quantum Phases and Non-Equilibrium Phenomena in Cold Atomic Gases

27 August - 7 September, 2007

Introduction to experiments in optical lattices - Part I

Immanuel Bloch
University of Mainz

Ultracold Atoms in Optical Lattices

Simon Fölling, Stefan Trotzky, Patrick Cheinet,
Artur Widera, Fabrice Gerbier, Michael Feld,
Ute Schnorrberger, Robert Saers, Torben Müller,

Thorsten Best, Ulrich Schneider,
Sebastian Will, Dries van Oosten, Lucia
Hackermüller, Martin Zwierlein, Christoph Clausen,

Stefan Kuhr, Jacob Sherson,
Christof Weitenberg, Manuel Endres,
Oliver Loesdau

Marionna Morena, Belén Paredes

Markus Greiner, Olaf Mandel,

I.B.

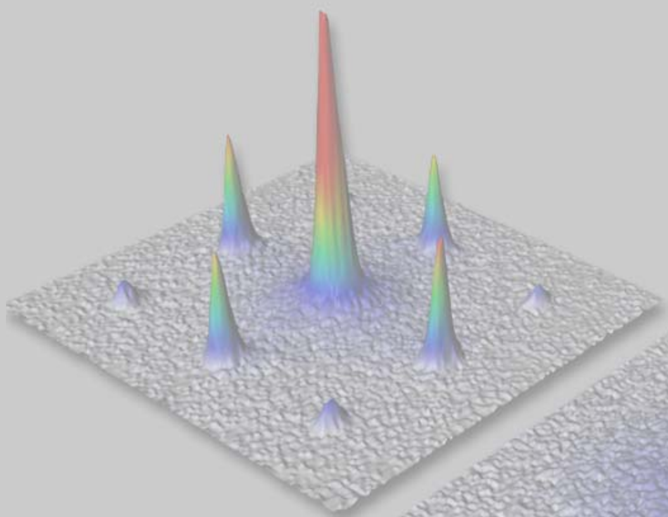
Many inspiring discussions:
E. Demler, M.D. Lukin,
A. Polkovnikov, A.-M. Rey, V. Gritsev

funding by
€ DFG, European Union,
$ AFOSR

www.quantum.physik.uni-mainz.de

Johannes Gutenberg-Universität, Mainz

Neutral Atoms in Optical Lattices



Part 1

- Introduction
- Experimental Setup
- Loading a BEC into an Optical Lattice
- From a Superfluid to a Mott Insulator
  - Coherence, Atom Number Statistics,
  - Shell Structure, Finite Temperature Effects, ...

Part 2

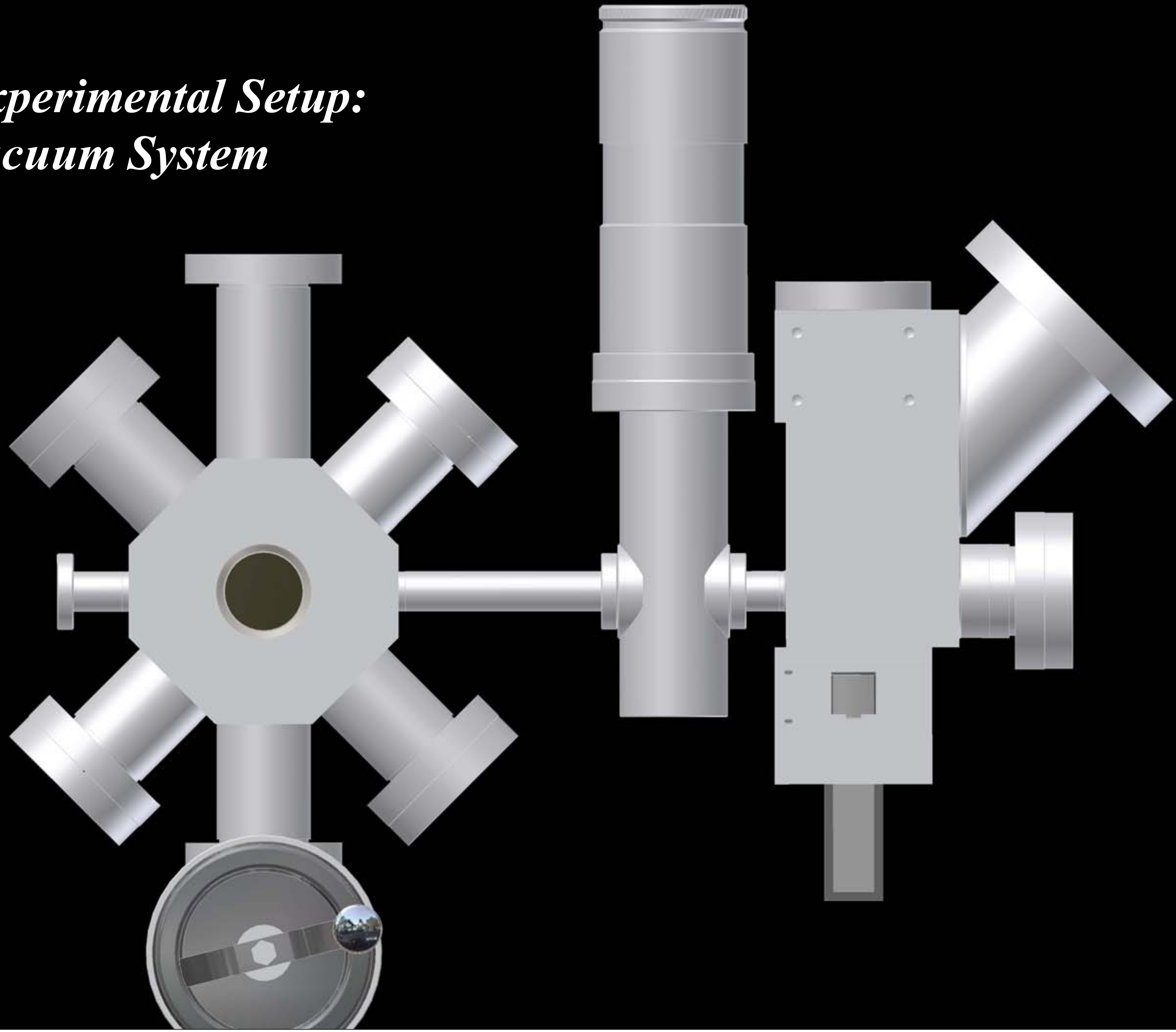
- Tonks-Girardeau Gas
- Multi Orbital Mott Insulator Physics
- Spin Dependent Lattice Potentials
- Generation of Multiparticle-Entangled States
- Novel Quantum Information Schemes for Ultracold Atoms in Optical Lattices
- Generating Nonclassical Field States

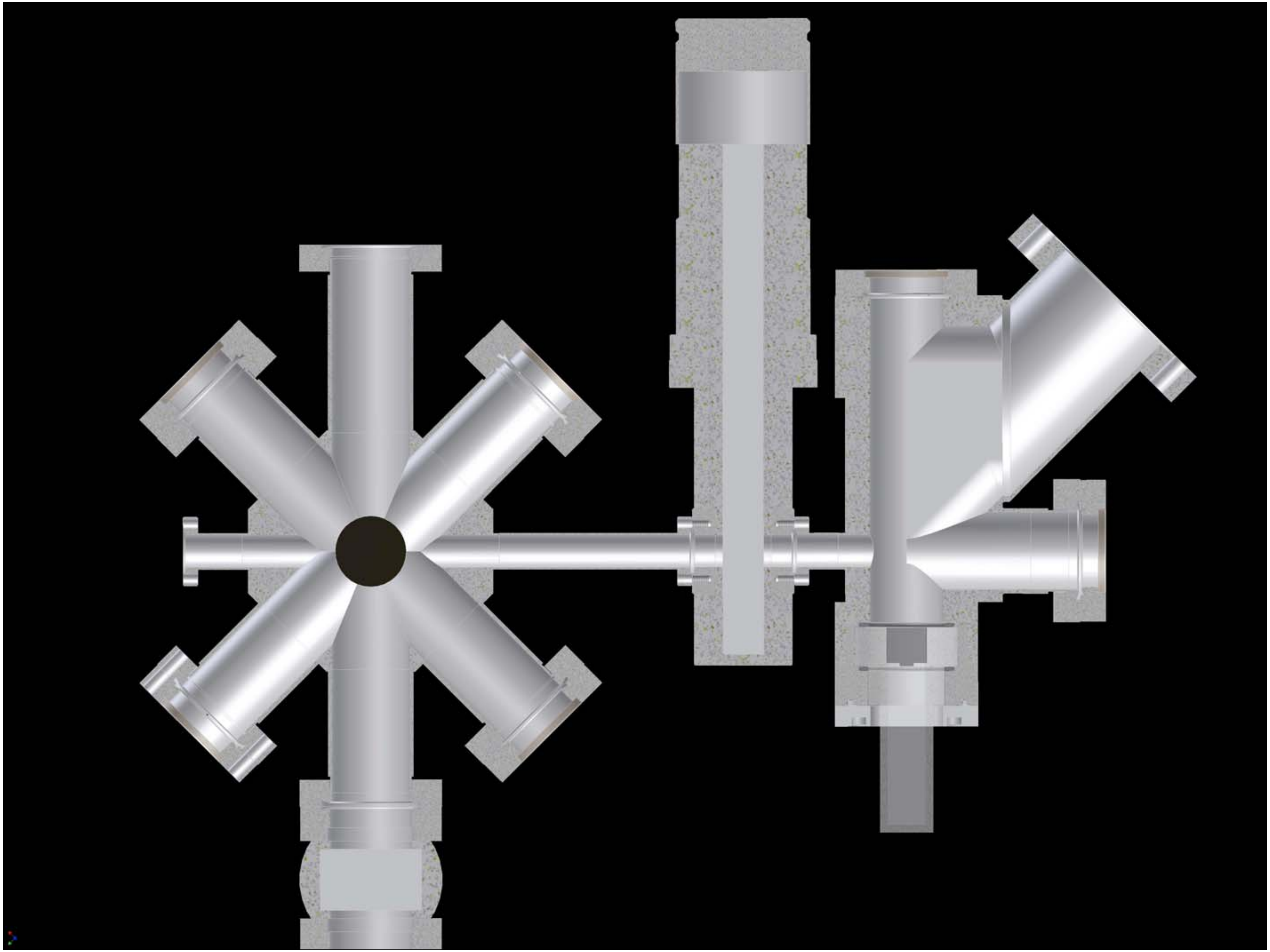
Part 3

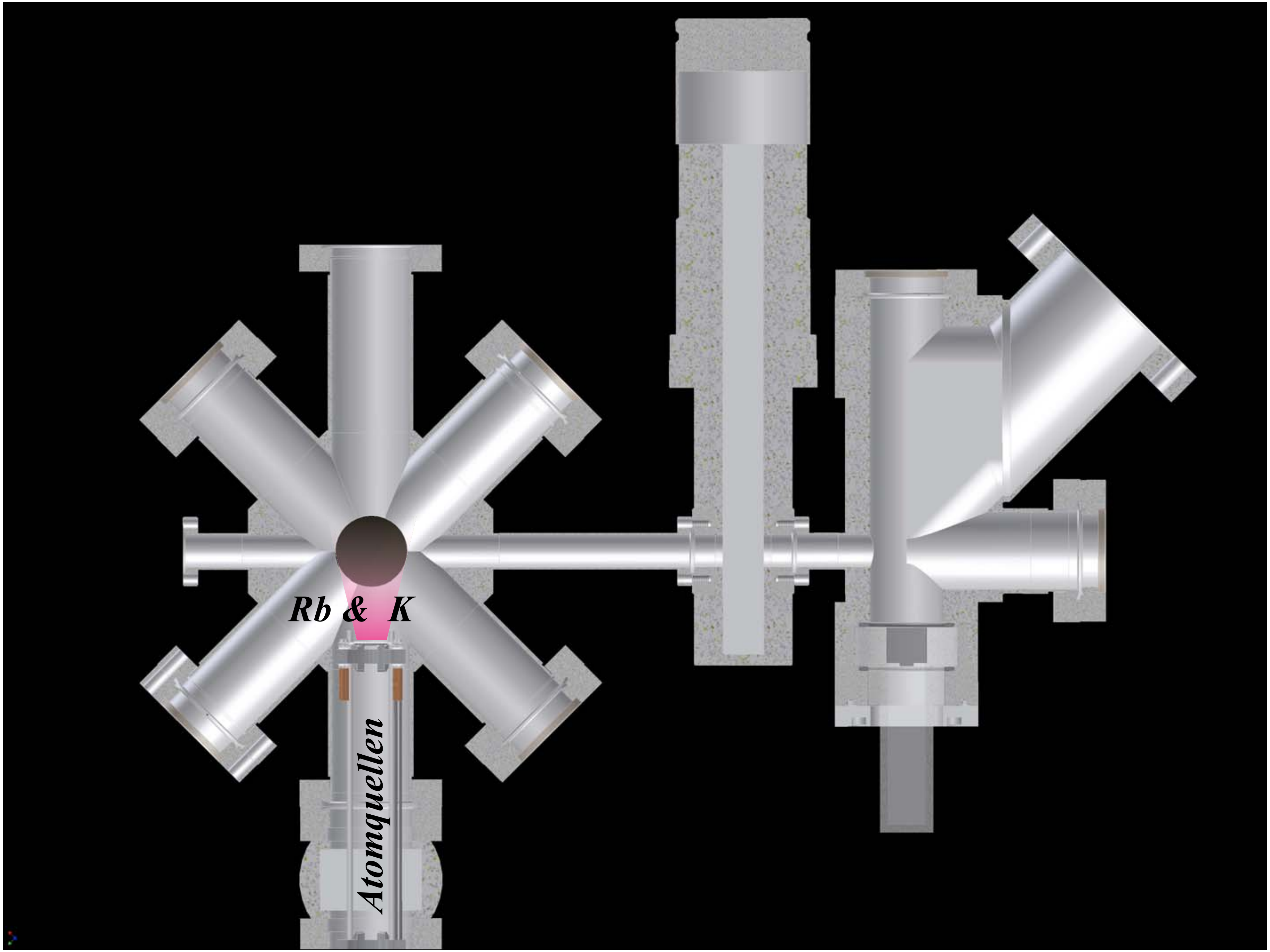
- Quantum Noise Correlations
- Optical Superlattices
- Correlated Atom Pair Tunnelling
- Detection & Control of Superexchange Interactions
- Outlook

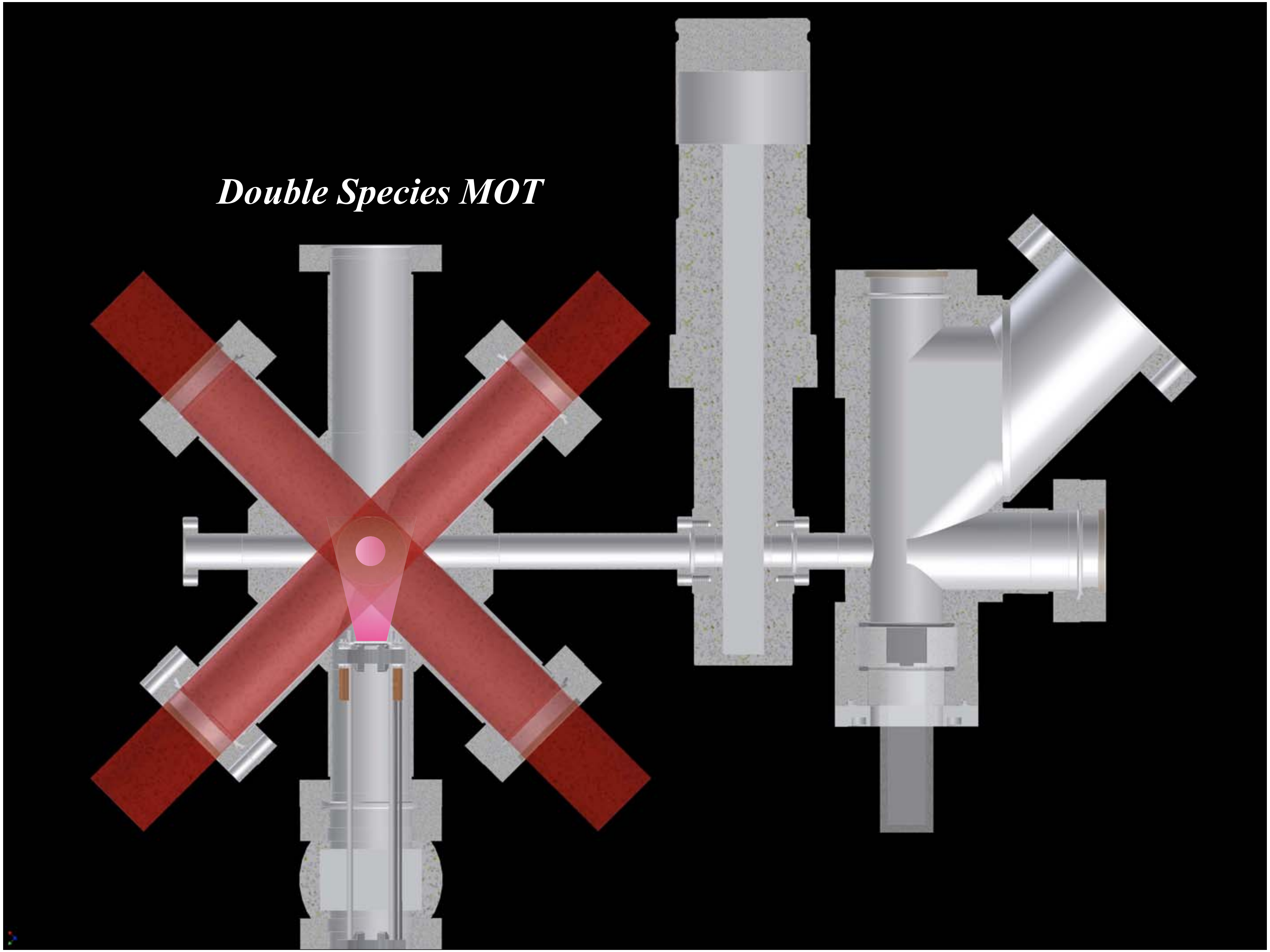
JOHANNES GUTENBERG UNIVERSITÄT MAINZ

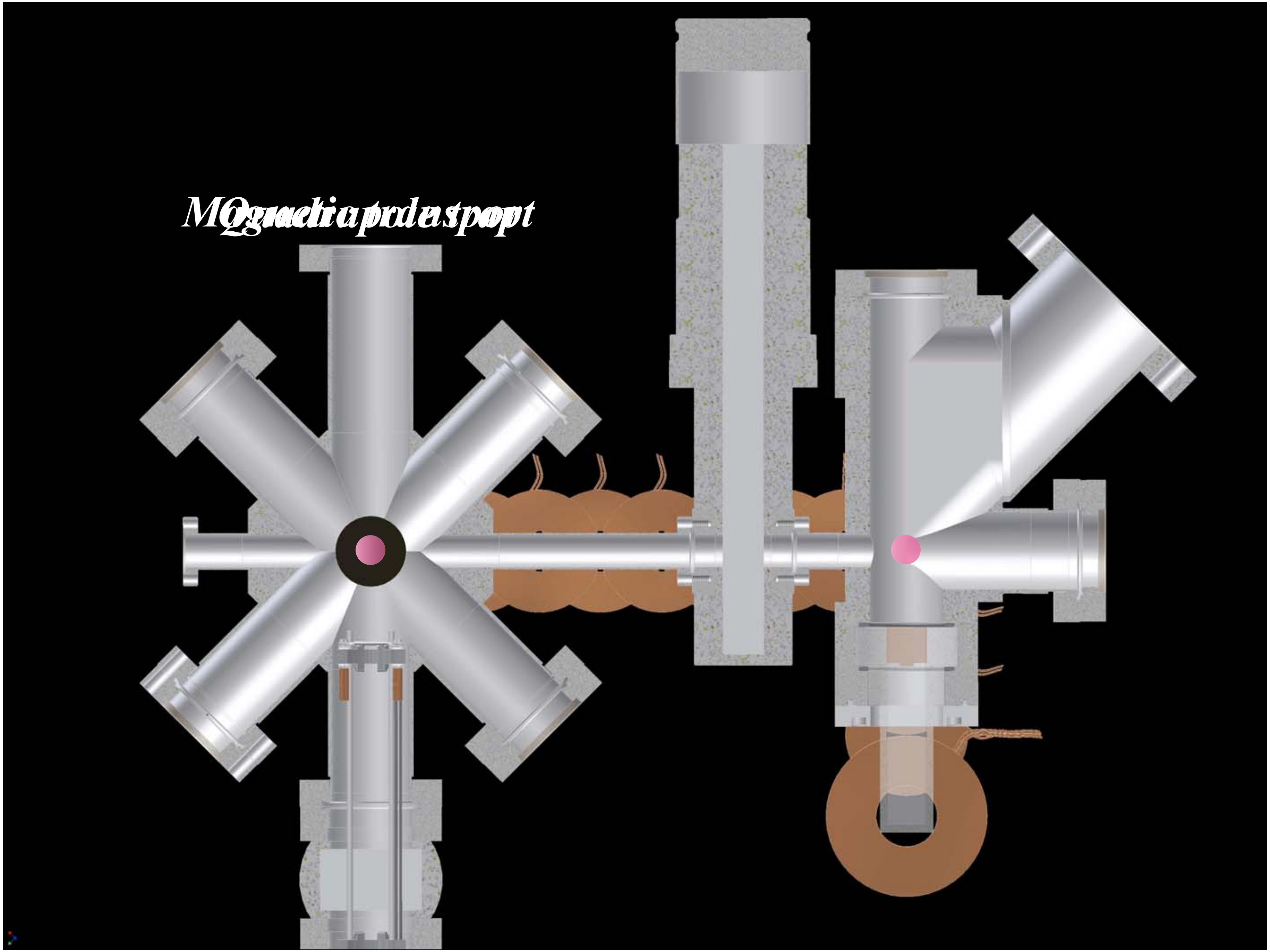
Experimental Setup:
Vacuum System

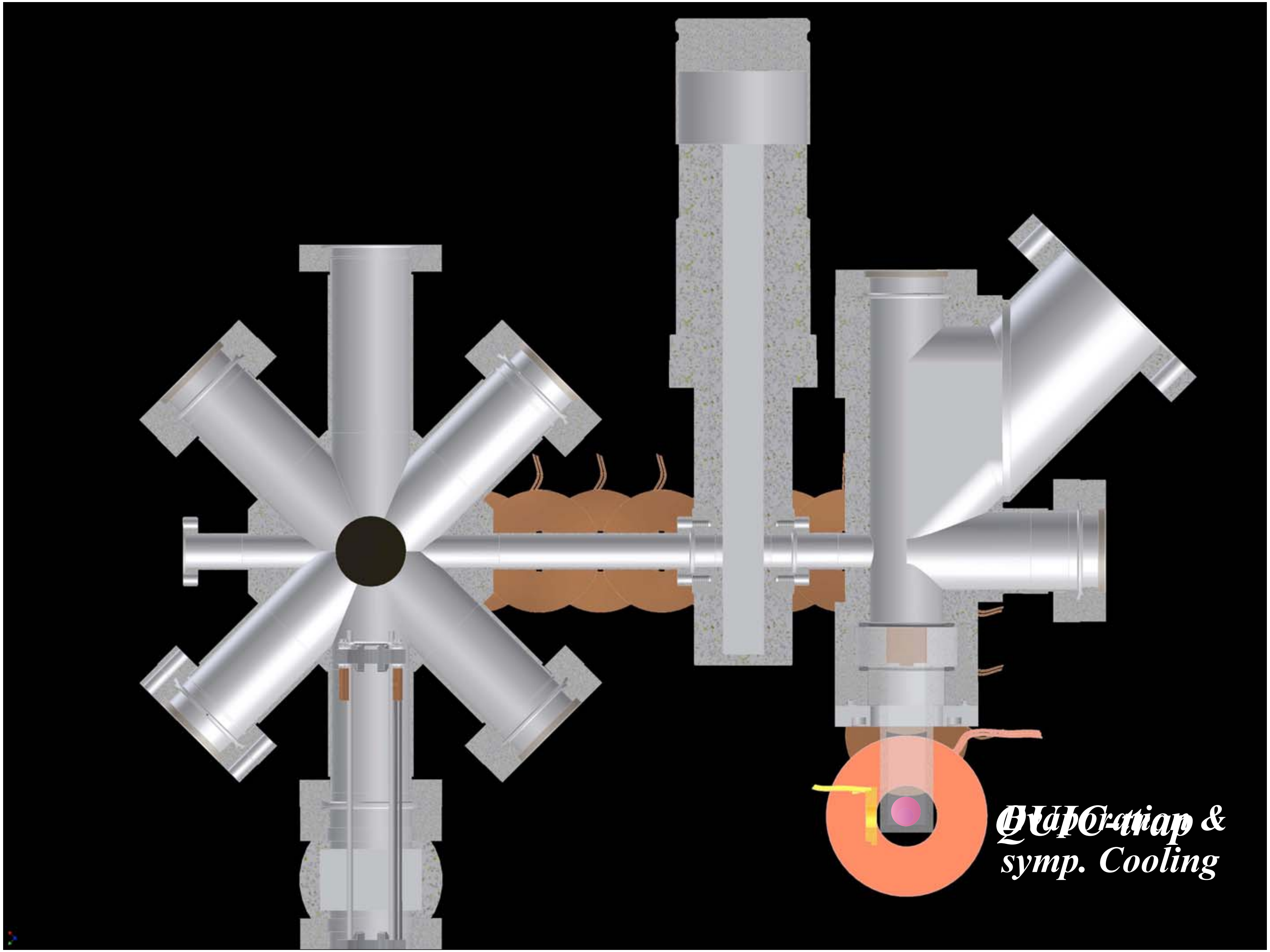


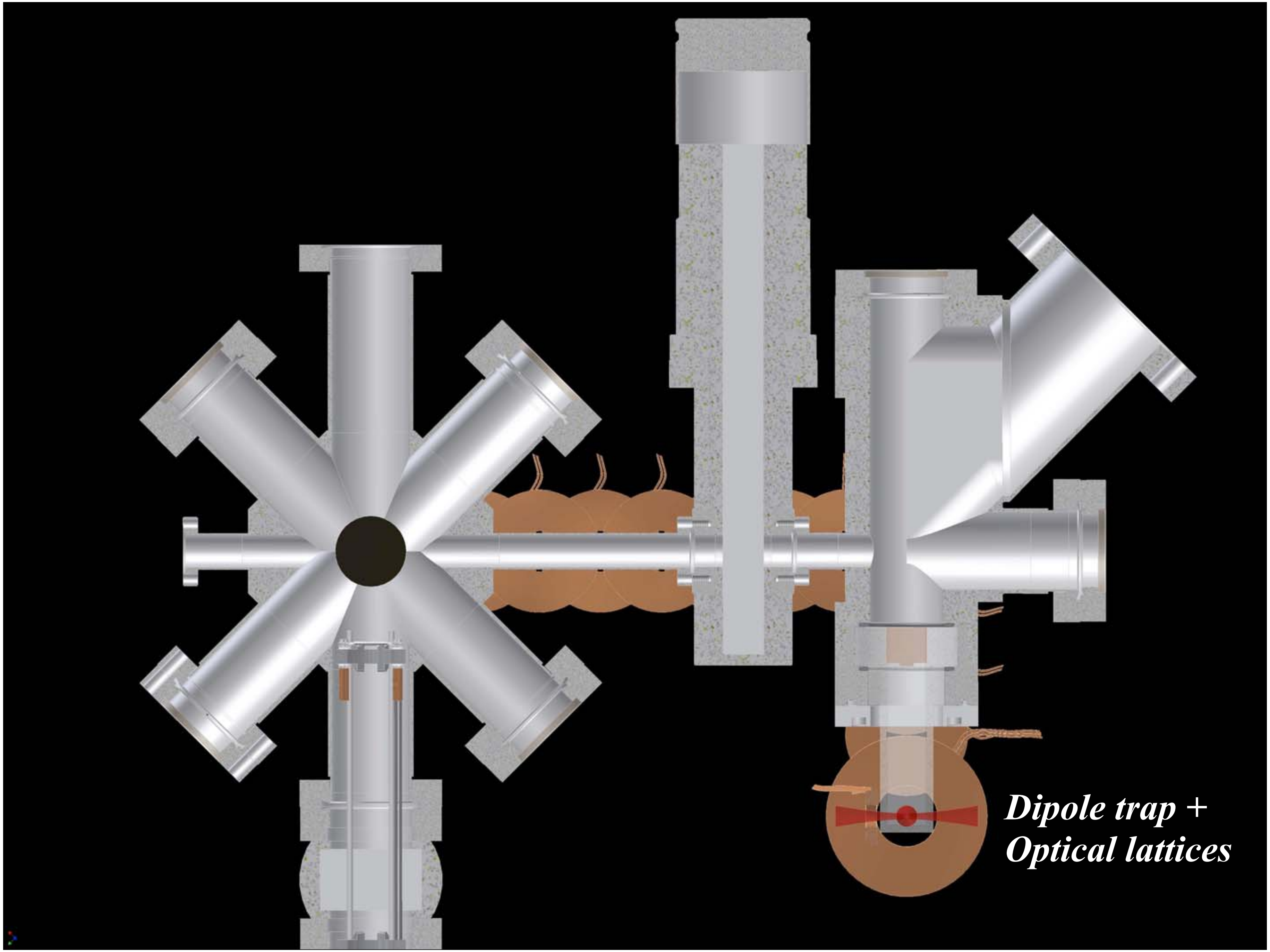


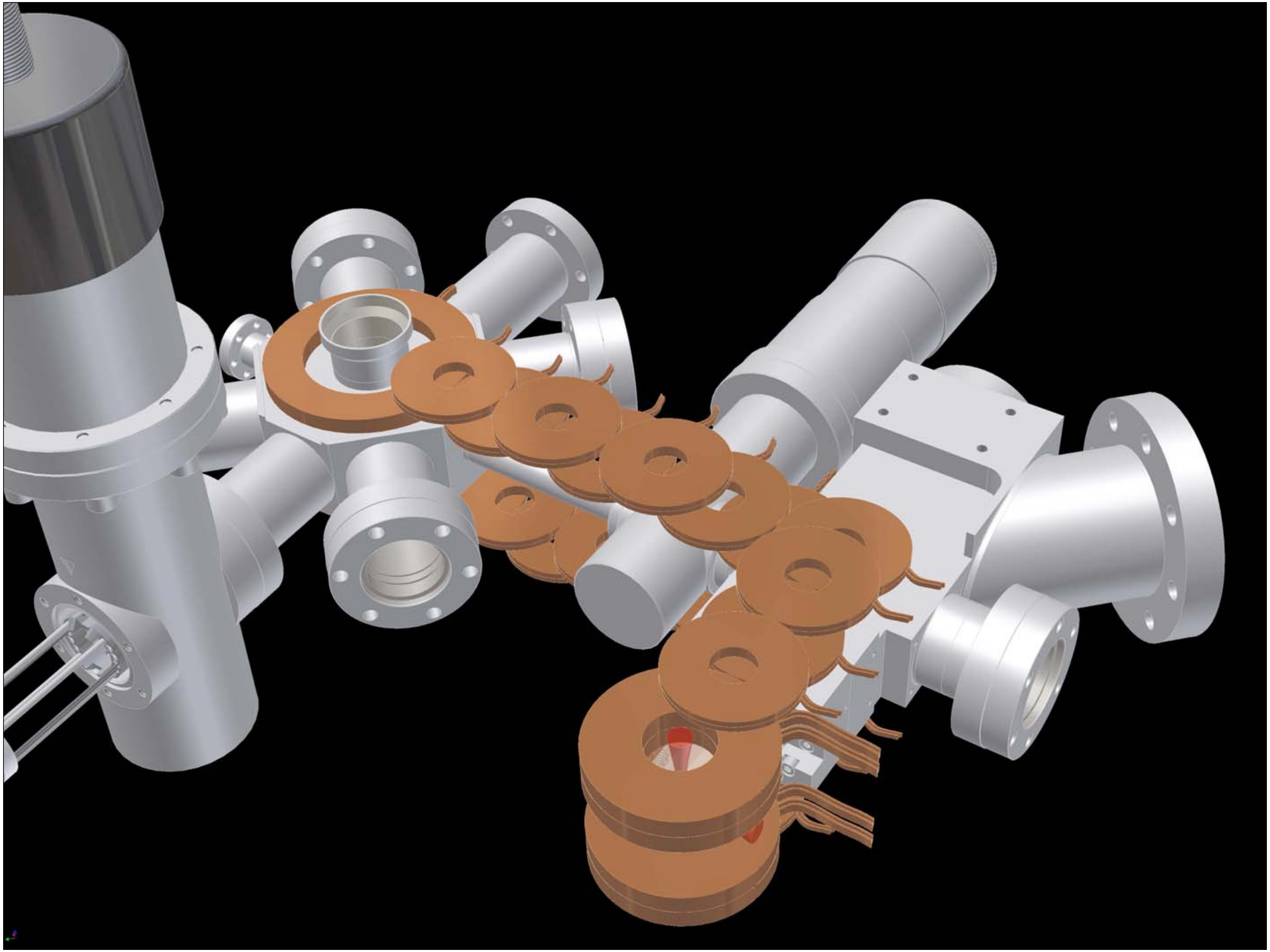

Rb & K
Atomquellen

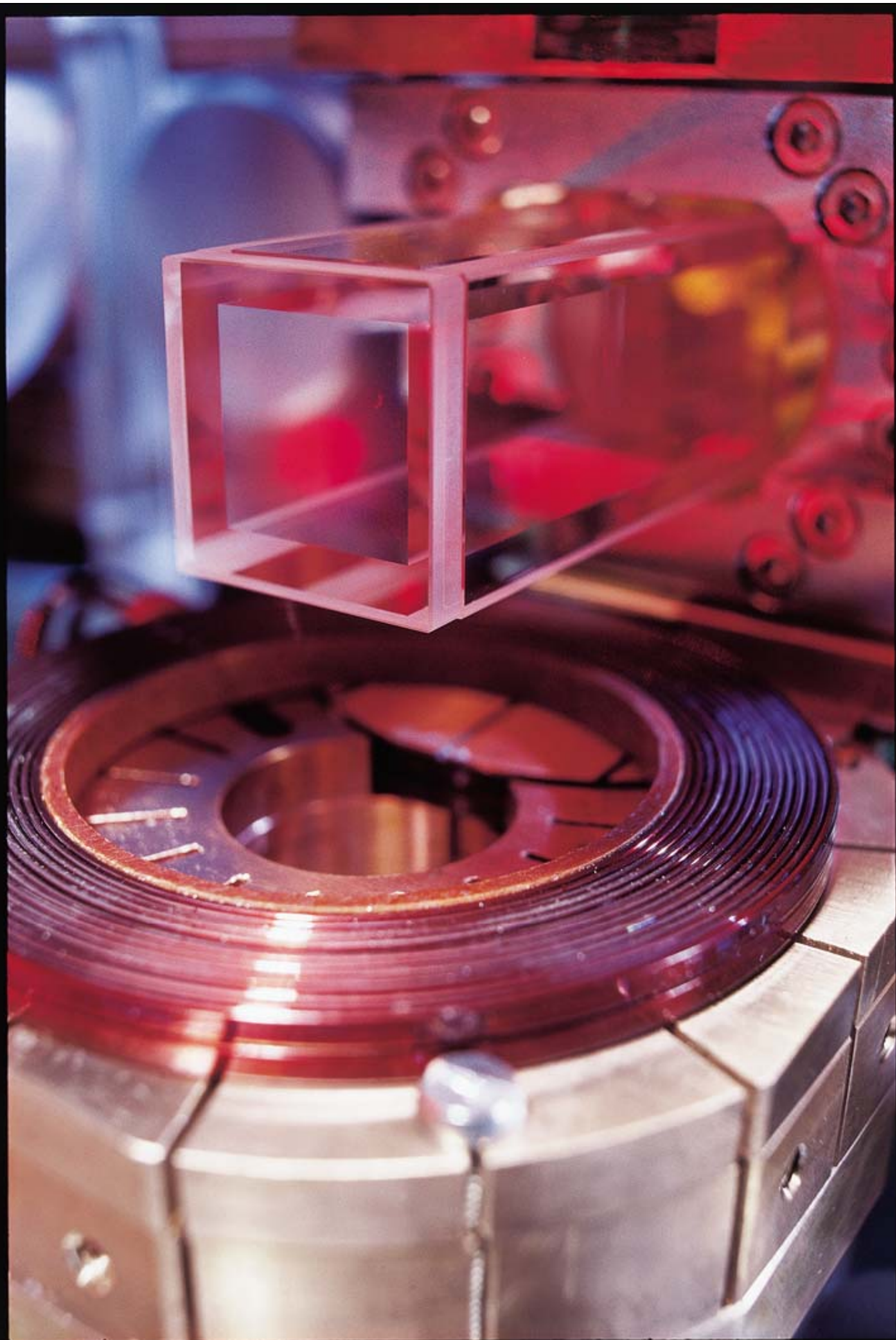

Double Species MOT


Magnetic transport
Quadrupole transport

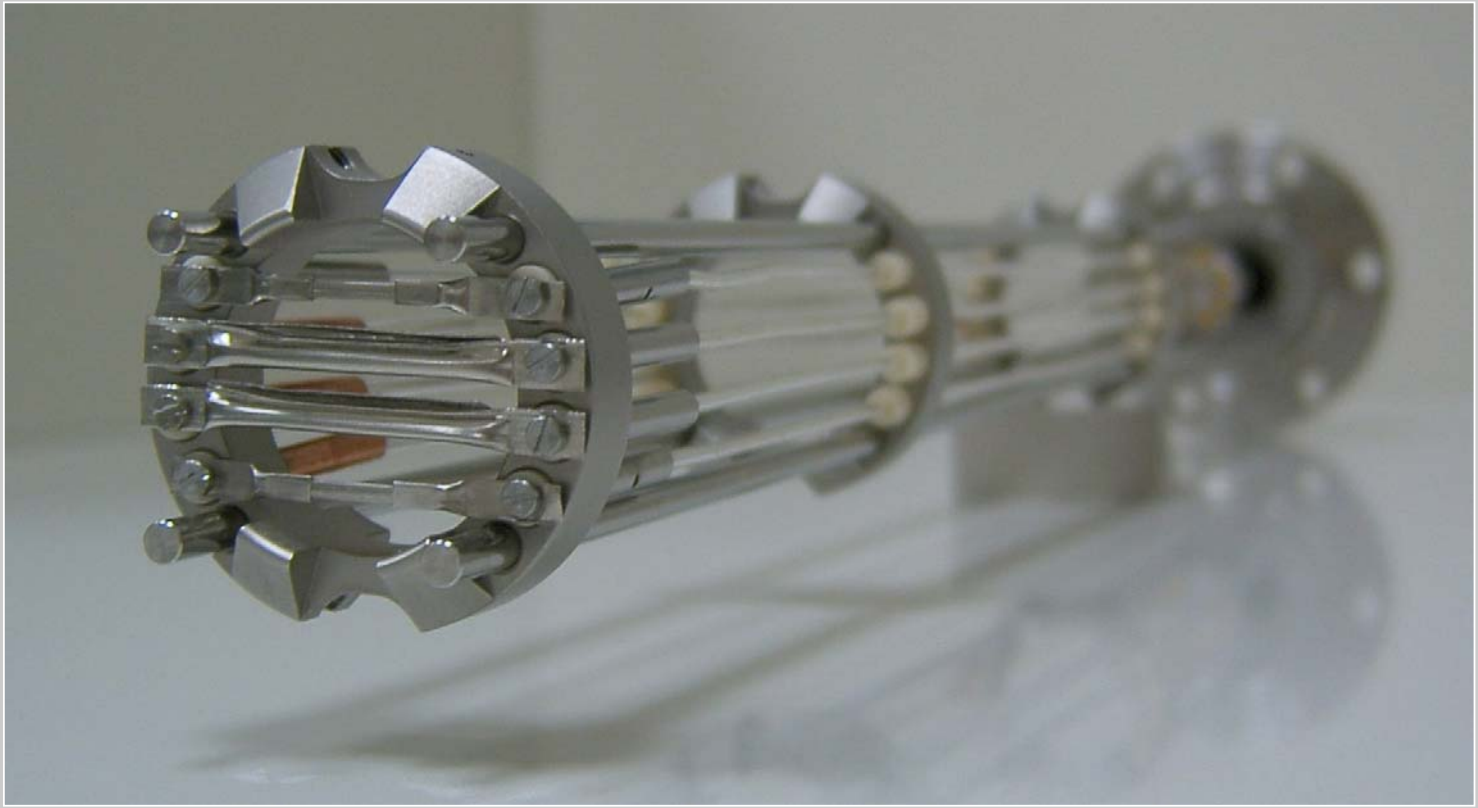

Evapor-trap &
symp. Cooling
QUPC-trap


Dipole trap +
Optical lattices



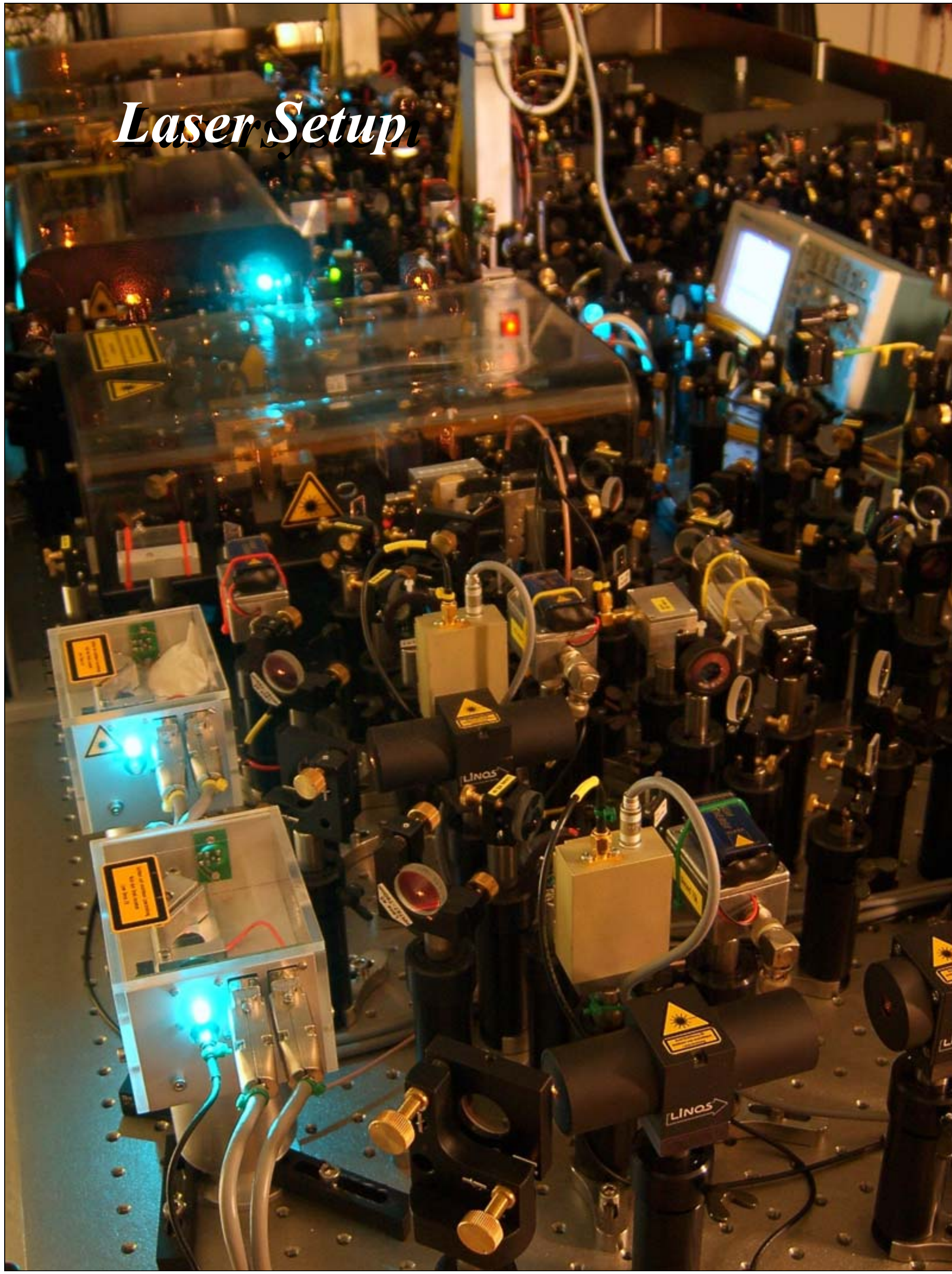


Atomic Sources

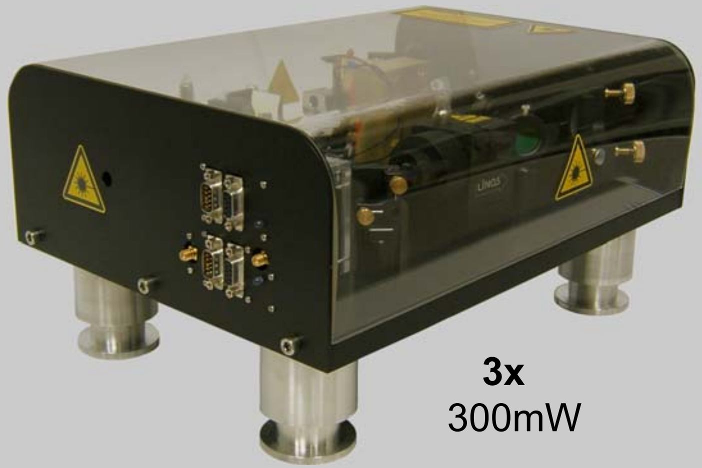


JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Laser Setup



780nm for Rb und 767nm for K



3x
300mW

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

How do we detect these quantum gases ?

Trapping Atoms in Light Field - Optical Dipole Potentials

Energy of a dipole in an electric field:

$$U_{dip} = -\mathbf{d} \cdot \mathbf{E}$$

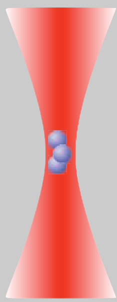
An electric field induces a dipole moment:

$$\mathbf{d} = \alpha \mathbf{E}$$

$$U_{dip} \propto -\alpha(\omega) I(\mathbf{r})$$

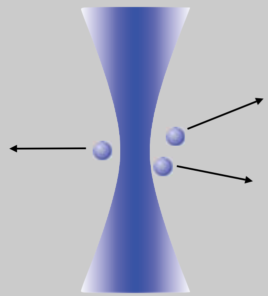
Red detuning:

Atoms are trapped in the intensity maxima



Blue detuning:

Atoms are repelled from the intensity maxima

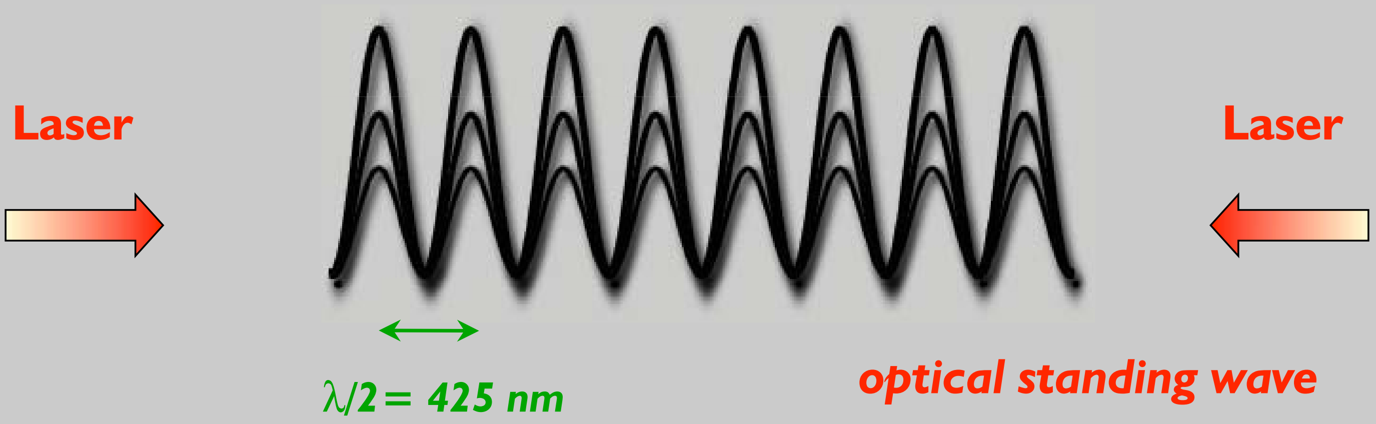


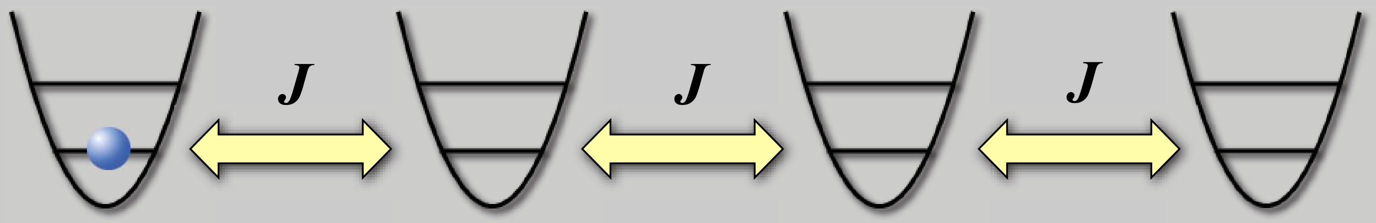
See R. Grimm et al., Adv. At. Mol. Opt. Phys. 42, 95-170 (2000).

Pioneering work by Steven Chu

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Optical Lattice Potential

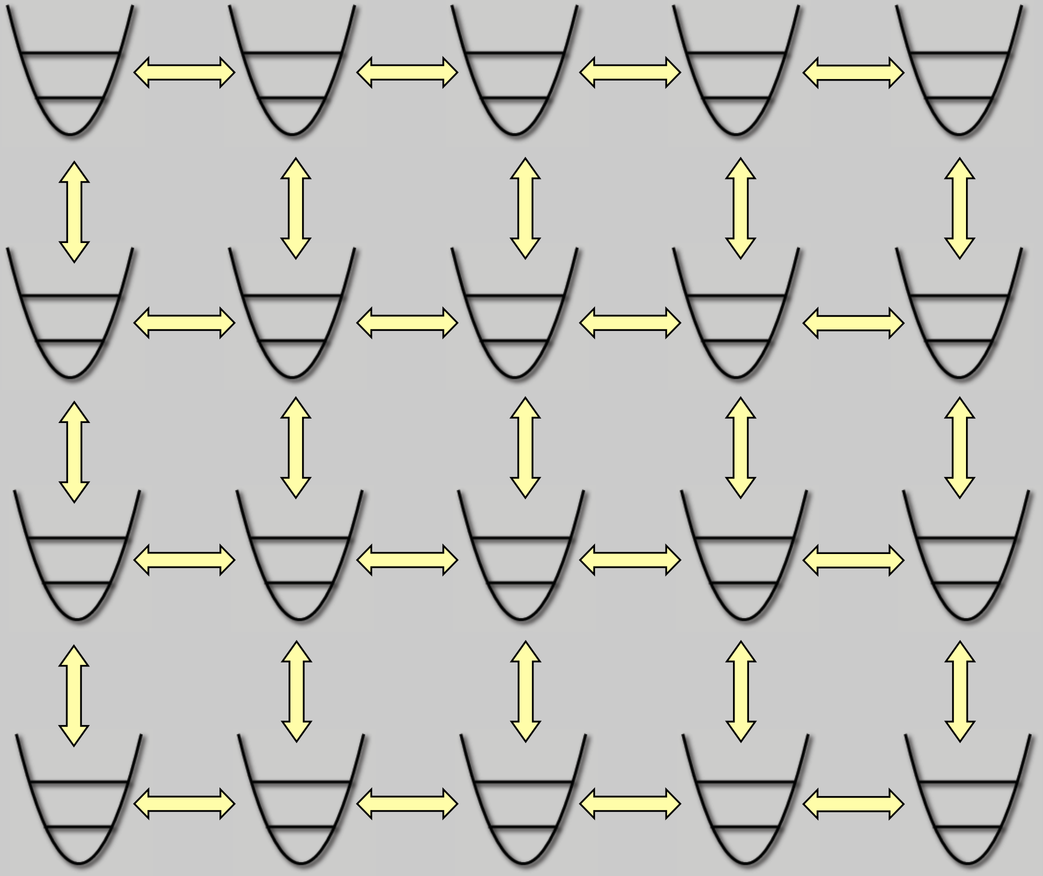




Effectively: Harmonic Oscillators Coupled via Quantum Mechanical Tunneling

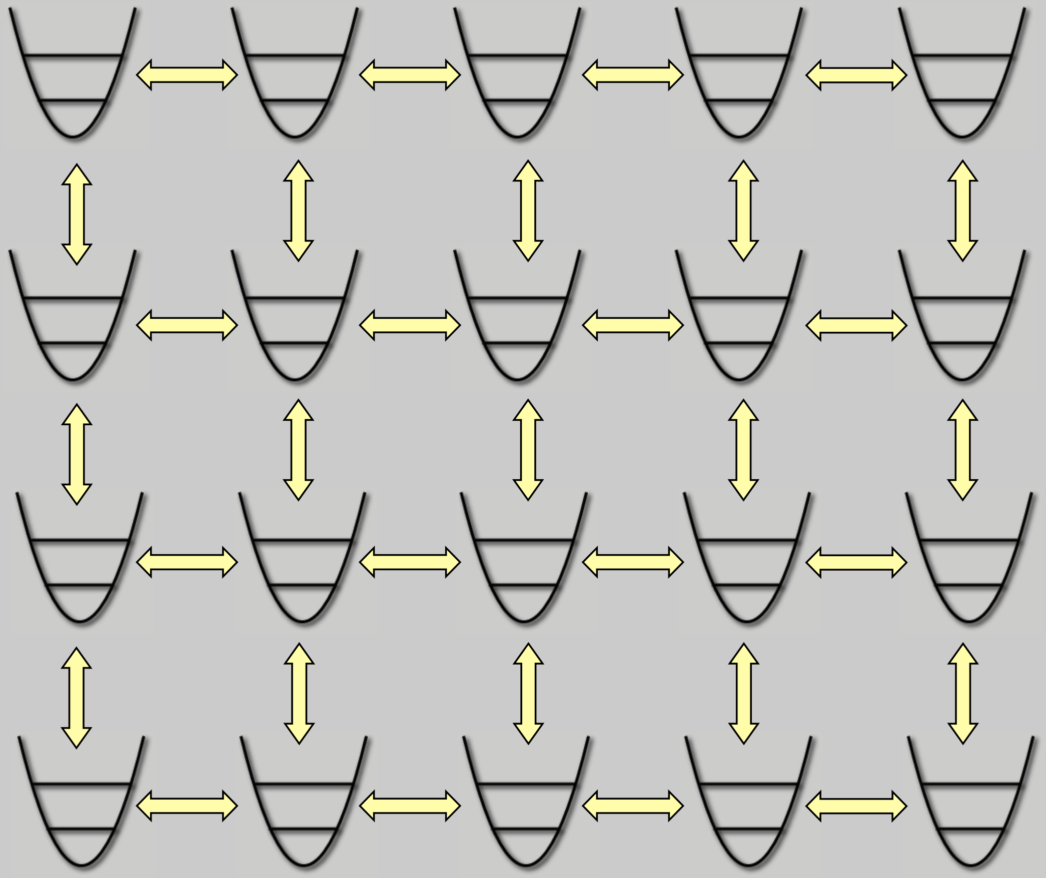
JOHANNES GUTENBERG UNIVERSITÄT MAINZ

…and in Higher Dimensions



Tunnel Coupling Tunable!

…and in Higher Dimensions



Tuning the Dimensionality

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

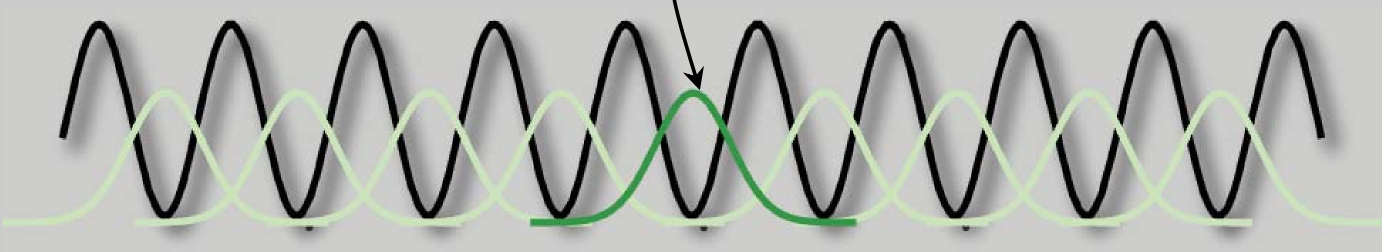
Macroscopic Wave Function of a BEC in an Optical Lattice

Number of atoms on j^{th} lattice site

Phase of wave function on j^{th} lattice site

$$\Psi(x) = \sum_i A(x_j) \cdot w(x - x_j) \cdot e^{i\phi(x_j)}$$

Localized wave function on J^{th} lattice site



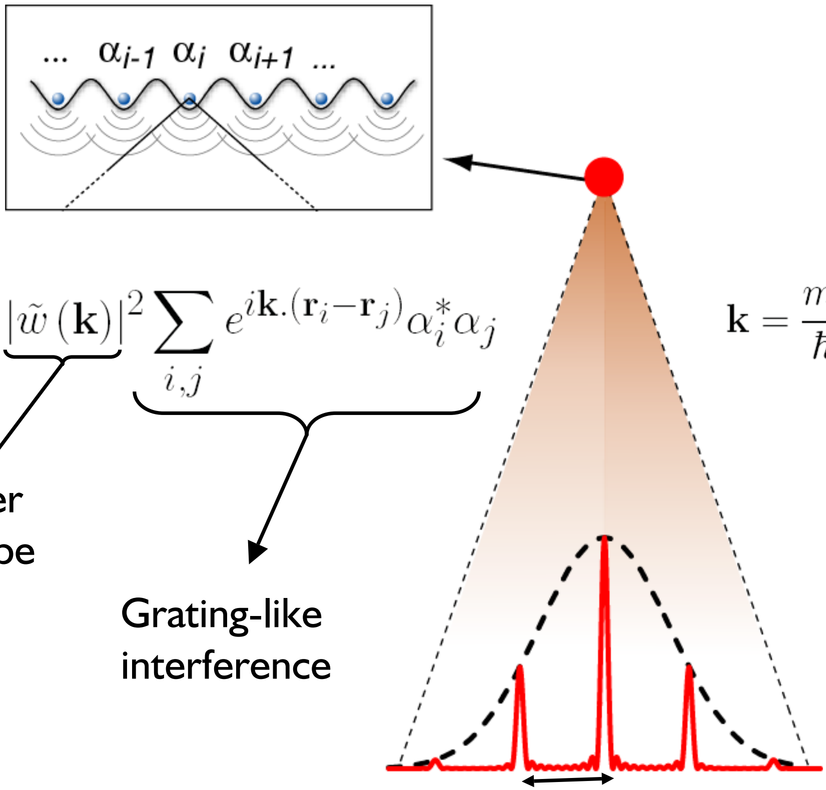
If there is a constant phase shift $\Delta\phi$ between lattice sites, the state is an eigenstate (Bloch wavefunction) of the lattice potential!

Quantum number characterizing these Bloch waves:

Crystal (Quasi-) momentum $$q = \frac{2\hbar}{\lambda}\Delta\phi$$

Time of flight interference pattern

- **Interference between all waves coherently emitted from each lattice site**



$$\tilde{n}(\mathbf{k}) = |\tilde{w}(\mathbf{k})|^2 \sum_{i,j} e^{i\mathbf{k}.(\mathbf{r}_i-\mathbf{r}_j)} \alpha_i^* \alpha_j$$

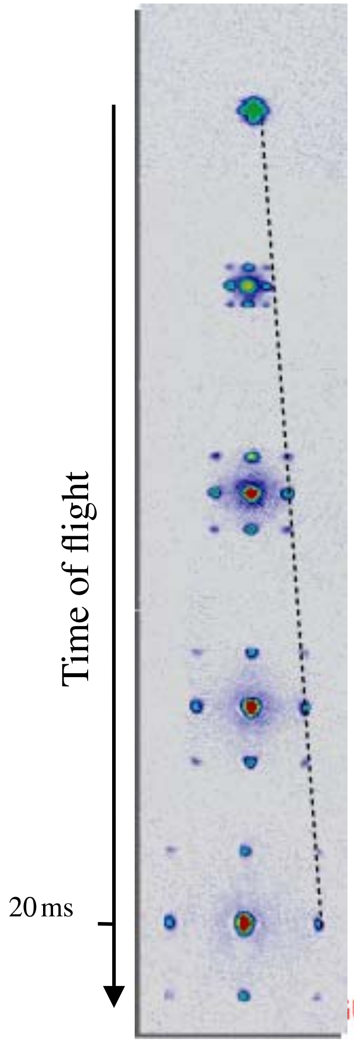
$$\mathbf{k} = \frac{m\mathbf{r}}{\hbar t}$$

Wannier envelope

Grating-like interference

Periodicity of the reciprocal lattice

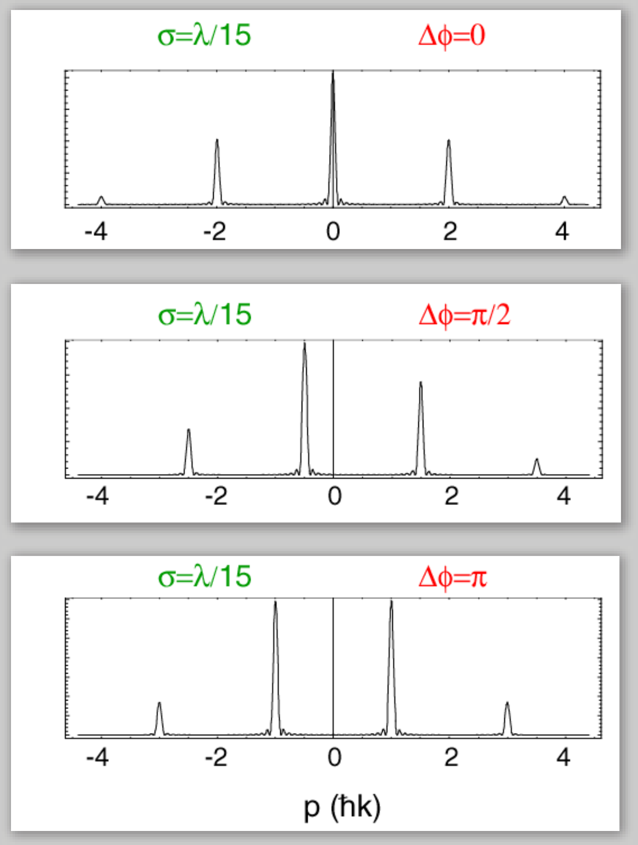
$$l = \frac{2\hbar k_L t}{m}$$



Momentum Distributions – 1D

Momentum distribution can be obtained by Fourier transformation of the macroscopic wave function.

$$\Psi(x) = \sum_i A(x_j) \cdot w(x - x_j) \cdot e^{i\phi(x_j)}$$



JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Single Particle in a Periodic Potential - Band Structure (1)

$$H\phi_q^{(n)}(x) = E_q^{(n)}\phi_q^{(n)}(x) \qquad \text{with} \qquad H = \frac{1}{2m}\hat{p}^2 + V(x)$$

Solved by Bloch waves (periodic functions in lattice period)

$$\phi_q^{(n)}(x) = e^{iqx} \cdot u_q^{(n)}(x)$$

q = Crystal Momentum or Quasi-Momentum
n = Band index

Plugging this into Schrödinger Equation, gives:

$$H_B\, u_q^{(n)}(x) = E_q^{(n)} u_q^{(n)}(x) \qquad \text{with} \qquad H_B = \frac{1}{2m}(\hat{p}+q)^2 + V_{lat}(x)$$

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Single Particle in a Periodic Potential - Band Structure (2)

Use Fourier expansion

$$V(x) = \sum_r V_r e^{i2rkx} \qquad \text{and} \qquad u_q^{(n)}(x) = \sum_l c_l^{(n,q)} e^{i2lkx}$$

yields for the potential energy term

$$V(x) u_q^{(n)}(x) = \sum_l \sum_r V_r e^{i2(r+l)kx} c_l^{(n,q)}$$

and the kinetic energy term

$$\frac{(\hat{p}+q)^2}{2m} u_q^{(n)}(x) = \sum_l \frac{(2\hbar kl + q)^2}{2m} c_l^{(n,q)} e^{i2lkx}.$$

In the experiment standing wave interference pattern gives

$$V(x) = V_{lat} \sin^2(kx) = -\frac{1}{4}\left(e^{2ikx} + e^{-2ikx}\right) + \text{c.c.}$$

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Single Particle in a Periodic Potential - Band Structure (3)

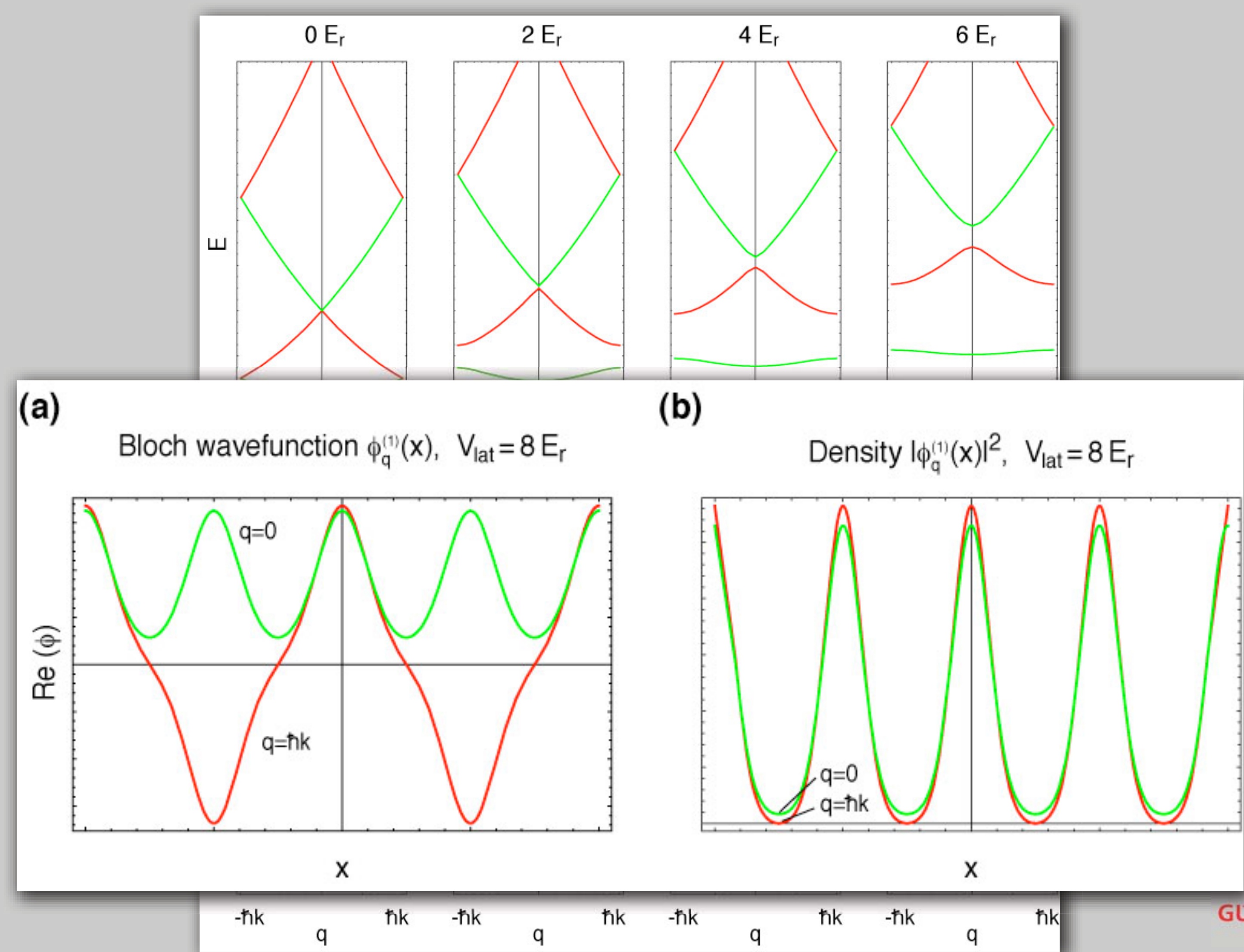
Use Fourier expansion

$$\sum_l H_{l,l'} \cdot c_l^{(n,q)} = E_q^{(n)} c_l^{(n,q)} \qquad \text{with} \qquad H_{l,l'} = \begin{cases} (2l + q/\hbar k)^2 E_r & \text{if } l = l' \\ -1/4 \cdot V_0 & \text{if } |l - l'| = 1 \\ 0 & \text{else} \end{cases}$$

$$\begin{pmatrix} (q/\hbar k)^2 E_r & -\frac{1}{4}V_0 & 0 & 0 & \cdots \\ -\frac{1}{4}V_0 & (2+q/\hbar k)^2 E_r & -\frac{1}{4}V_0 & 0 & \\ 0 & -\frac{1}{4}V_0 & (4+q/\hbar k)^2 E_r & -\frac{1}{4}V_0 & \\ & & -\frac{1}{4}V_0 & \ddots & \\ & & & & \end{pmatrix} \begin{pmatrix} c_0^{(n,q)} \\ c_1^{(n,q)} \\ c_2^{(n,q)} \\ \\ \vdots \end{pmatrix} = E_q^{(n)} \begin{pmatrix} c_0^{(n,q)} \\ c_1^{(n,q)} \\ c_2^{(n,q)} \\ \\ \vdots \end{pmatrix}$$

Diagonalization gives us Eigenvalues and Eigenvectors!

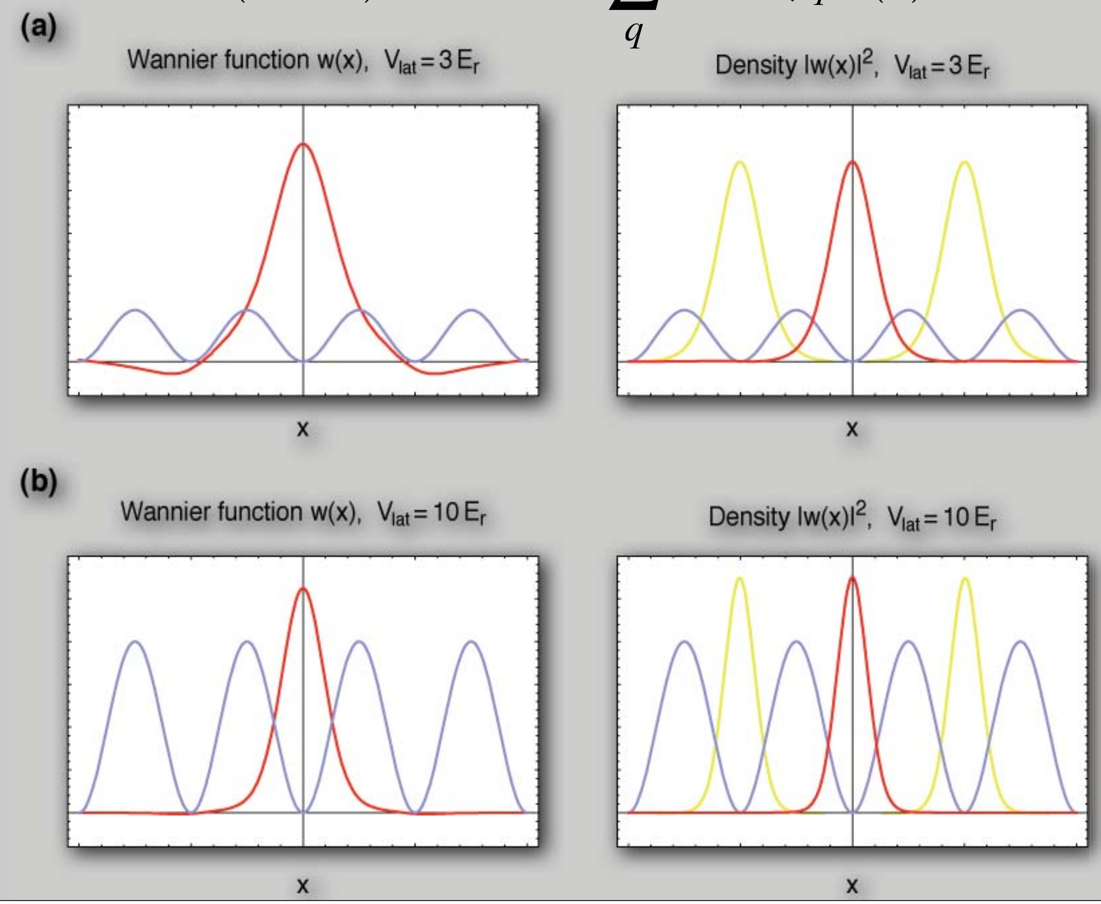
Bandstructure - Blochwaves



Wannier Functions

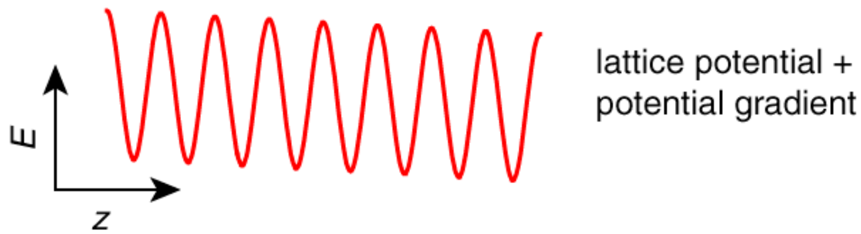
An alternative basis set to the Bloch waves can be constructed through localized wave-functions: Wannier Functions!

$$w_n(x - x_i) = \mathcal{N}^{-1/2} \sum_q e^{-iqx_i} \phi_q^{(n)}(x)$$



JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Preparing Arbitrary Phase Differences Between Neighbouring Lattice Sites

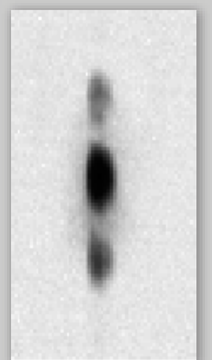


$$\phi_j = E_j \cdot t / \hbar$$

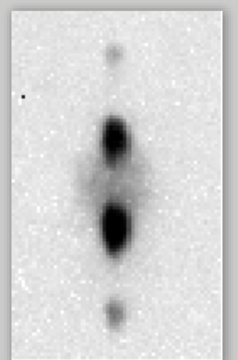
Phase difference between neighboring lattice sites

$$\Delta\phi_j = (V'\lambda/2)\,\Delta t$$

(cp. Bloch-Oscillations)



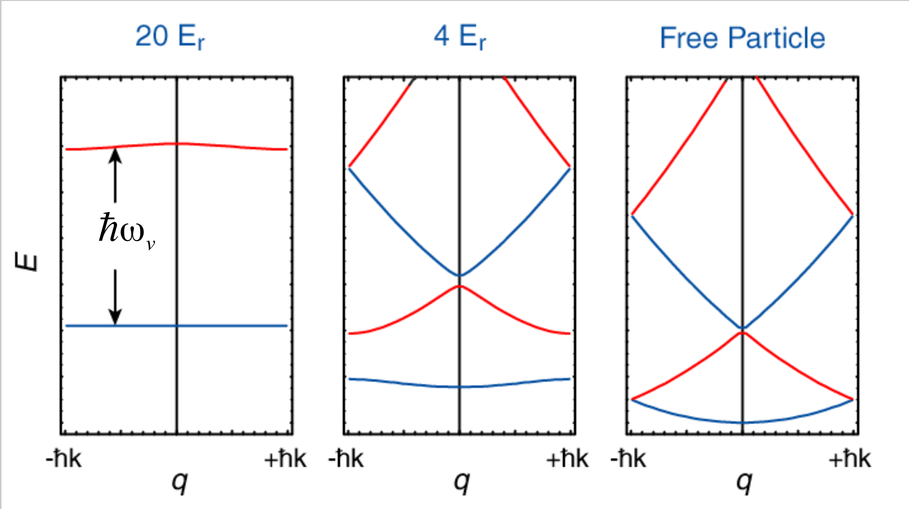
$$\Delta\phi = 0$$



$$\Delta\phi = \pi$$

But: dephasing if gradient is left on for long times !

Mapping the Population of the Energy Bands onto the Brillouin Zones

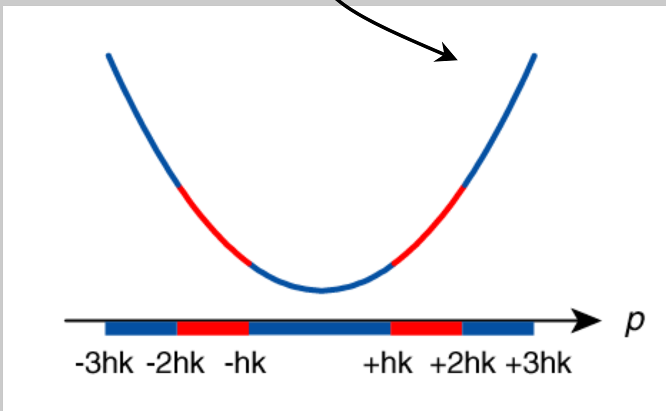


Crystal momentum

Crystal momentum is conserved while lowering the lattice depth adiabatically !

Population of n^{th} band is mapped onto n^{th} Brillouin zone !

A. Kastberg et al. PRL 74, 1542 (1995)
M. Greiner et al. PRL 87, 160405 (2001)

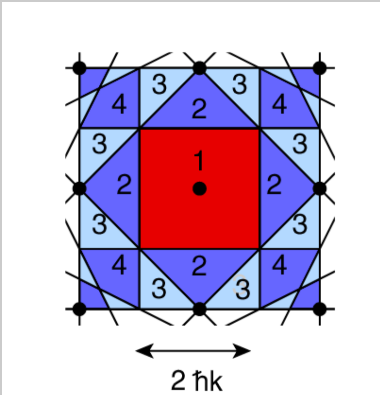


Free particle momentum

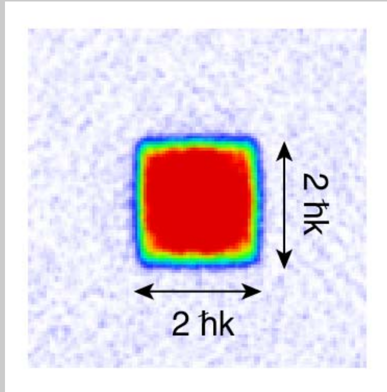
Experimental Results

Piet Mondrian

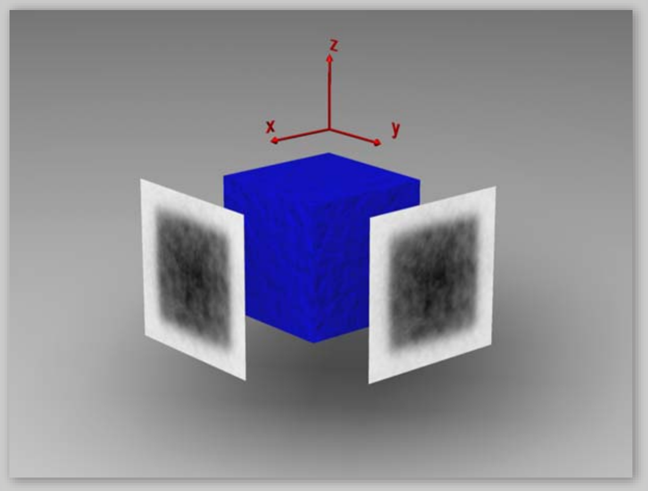
Brillouin Zones in 2D



Momentum distribution of a dephased condensate after turning off the lattice potential adiabtically



2D

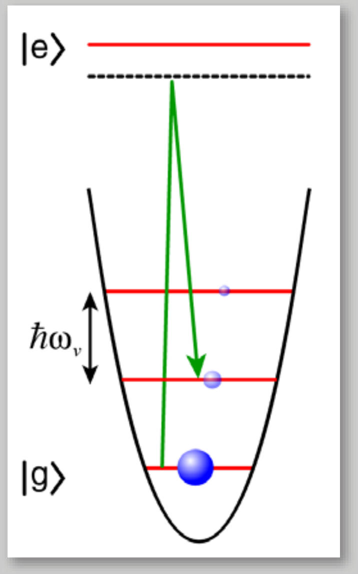


3D

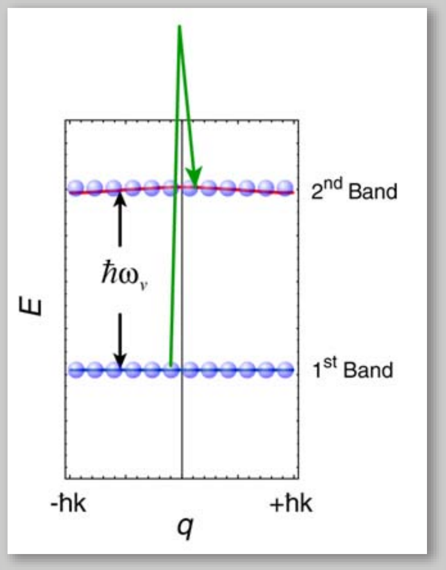
JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Populating Higher Energy Bands

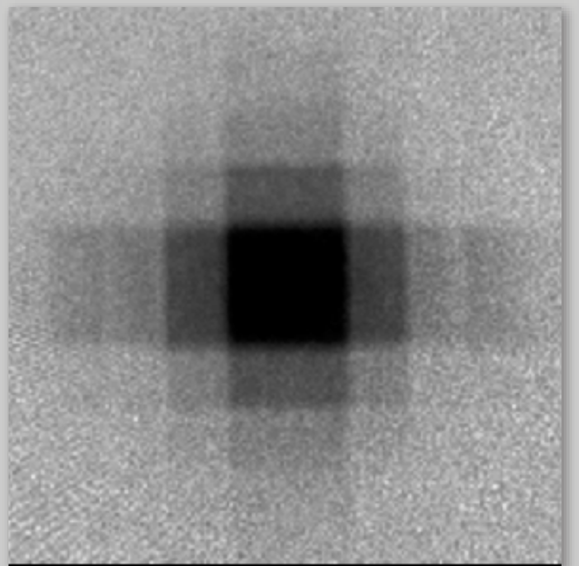
Single lattice site



Energy bands



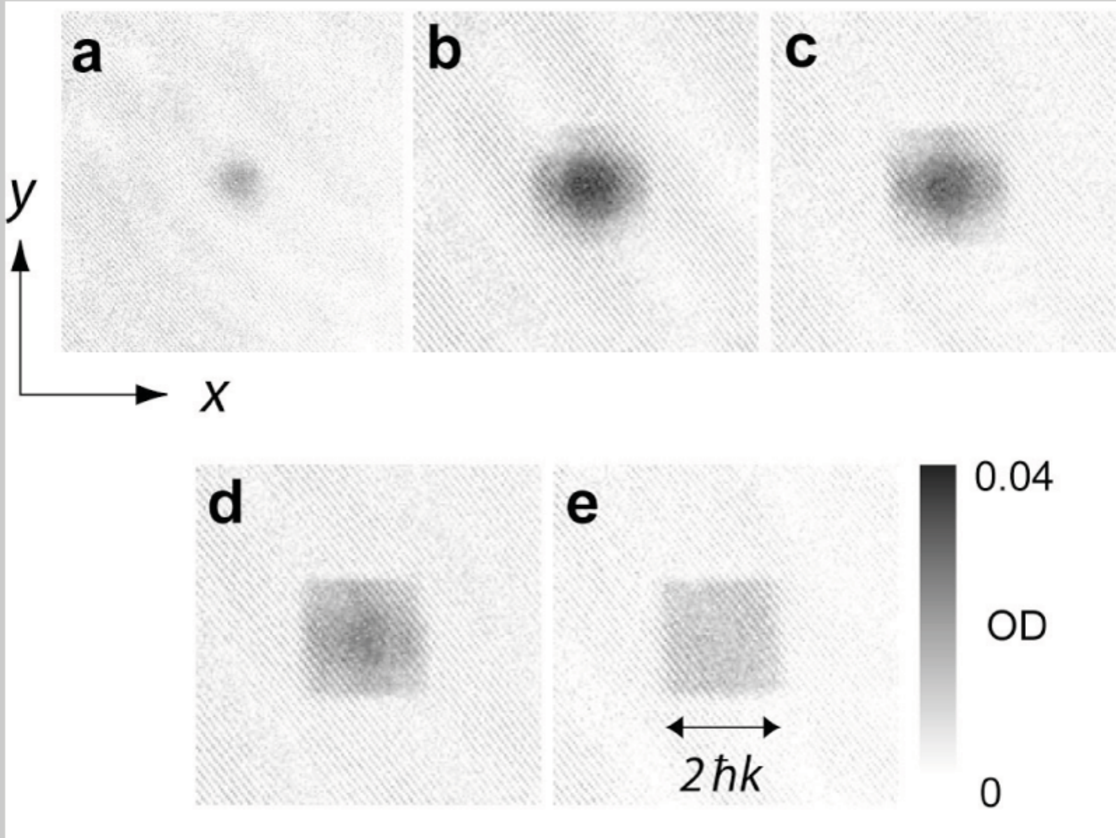
Stimulated Raman transitions between vibrational levels are used to populate higher energy bands.



Measured Momentum Distribution !

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

From a Conductor to a Band Insulator



Fermi Surfaces become directly visible!

M. Köhl et al. PRL (2005)

Bose-Hubbard Hamiltonian

Expanding the field operator in the Wannier basis of localized wave functions on each lattice site, yields :

$$\hat{\psi}(\boldsymbol{x}) = \sum_i \hat{a}_i w(\boldsymbol{x} - \boldsymbol{x}_i)$$

Bose-Hubbard Hamiltonian

$$H = -J \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j + \sum_i \varepsilon_i \hat{n}_i + \frac{1}{2} U \sum_i \hat{n}_i (\hat{n}_i - 1)$$

Tunnelmatrix element/Hopping element

$$J = -\int d^3x\, w(\mathbf{x} - \mathbf{x}_i) \left(-\frac{\hbar^2}{2m} \Delta + V_{lat}(\mathbf{x}) \right) w(\mathbf{x} - \mathbf{x}_j)$$

Onsite interaction matrix element

$$U = \frac{4\pi\hbar^2 a}{m} \int d^3x |w(\mathbf{x})|^4$$

M.P.A. Fisher et al., PRB 40, 546 (1989); D. Jaksch et al., PRL 81, 3108 (1998)

Mott Insulators now at: NIST, ETHZ, MIT, Innsbruck, Florence, Garching...

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Describing the Phase Transition (1)

$$H = -J \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j + \frac{1}{2} U \sum_i \hat{n}_i (\hat{n}_i - 1) - \mu \sum_i \hat{n}_i$$

Usual Bogoliubov replacement does NOT capture SF-MI transition!
(However can describe Quantum Depletion due to interactions)

$$\hat{a} = \psi + \Delta \hat{a}$$

Self consistent mean field approximation (decoupling approx.)

$$\begin{aligned} a_i^\dagger \hat{a}_j &= \langle \hat{a}_i^\dagger \rangle \langle \hat{a}_j \rangle + \langle \hat{a}_i^\dagger \rangle \Delta \hat{a}_j + \Delta \hat{a}_i^\dagger \langle \hat{a}_j \rangle \\ &= \langle \hat{a}_i^\dagger \rangle \hat{a}_j + \hat{a}_i^\dagger \langle \hat{a}_j \rangle - \langle \hat{a}_i^\dagger \rangle \langle \hat{a}_j \rangle \end{aligned}$$

$$\langle \hat{a}_i \rangle = \sqrt{n_i} = \psi$$

K. Sheshadri et al., EPL **22**, 257 (1993)
D. van Oosten, P. van der Straten & H. Stoof, PRA **63**, 053601 (2001)

Describing the Phase Transition (2)

$$H = -zJ\psi \sum_i \left(\hat{a}_i^\dagger + \hat{a}_i\right) + zt\psi^2 N_s + \frac{1}{2}\sum \hat{n}_i(\hat{n}_i - 1) - \mu \sum_i \hat{n}_i$$

Is diagonal in site index i, so we can use an effective on-site Hamiltonian

$$H_i = \frac{1}{2}\bar{U}\hat{n}_i(\hat{n}_i - 1) - \bar{\mu}\hat{n}_i - \psi\left(\hat{a}_i^\dagger + \hat{a}_i\right) + \psi^2$$

$$\bar{U} = U/zJ$$

$$\bar{\mu} = \mu/zJ$$

Can diagonalize Hamiltonian in occupation number basis!
or use perturbation theory with tunnelling term to find phase diagram analytically....

$$H = H^{(0)} + \psi V$$

$$H^{(0)} = \frac{1}{2}\bar{U}\hat{n}(\hat{n} - 1) - \bar{\mu}\hat{n} + \psi^2$$

$$V = -(\hat{a}^\dagger + \hat{a})$$

D. van Oosten, P. van der Straten & H. Stoof, PRA **63**, 053601 (2001)

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Describing the Phase Transition (3)

For our initial state (with fixed particle number), only second order perturbation gives a first correction.

$$E_n^{(2)} = \psi^2 \sum_{n' \neq n} \frac{|\langle n|V|n'\rangle|^2}{E_n^{(0)} - E_{n'}^{(0)}}$$

$$E_n^{(2)} = \frac{n}{\bar{U}(n-1) - \bar{\mu}} + \frac{n+1}{\bar{\mu} - \bar{U}n}$$

$$E_g(\psi) = a_0 + a_2\psi^2 + \mathcal{O}(\psi^4)$$

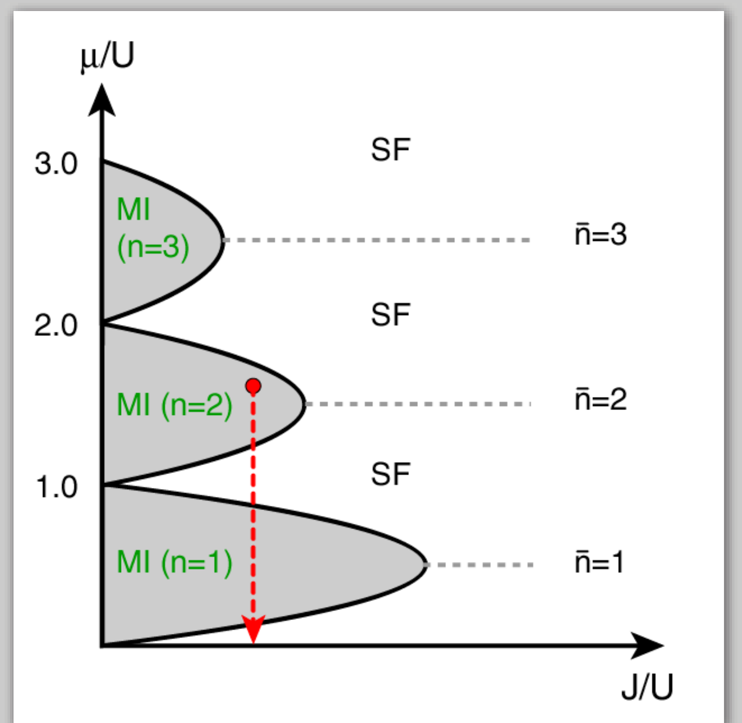
$$a_2 > 0 \rightarrow \psi = 0$$
$$a_2 < 0 \rightarrow \psi \neq 0$$

Phase transition for $a_2 = 0 \Rightarrow U/zJ \approx n \times 5.83$

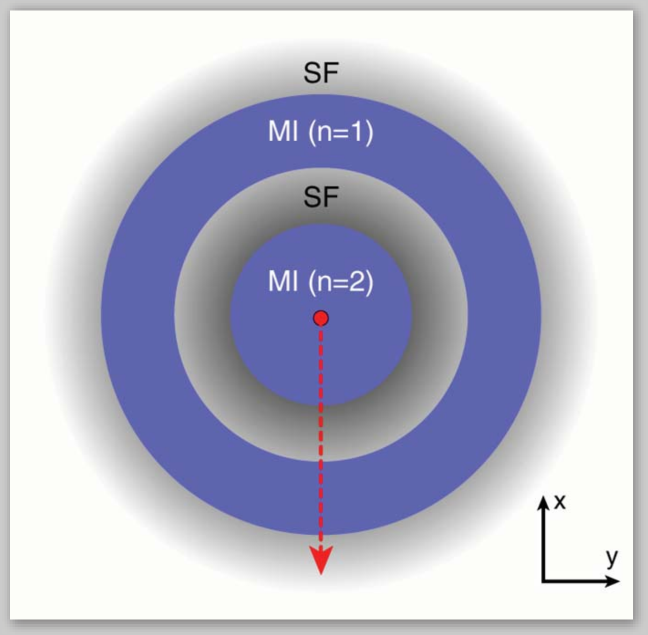
D. van Oosten, P. van der Straten & H. Stoof, PRA **63**, 053601 (2001)

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Superfluid – Mott-Insulator Phase Diagram



Jaksch et al. PRL 81, 3108 (1998)



For an inhomogeneous system an effective local chemical potential can be introduced

$$\mu_{loc} = \mu - \varepsilon_i$$

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Superfluid Limit

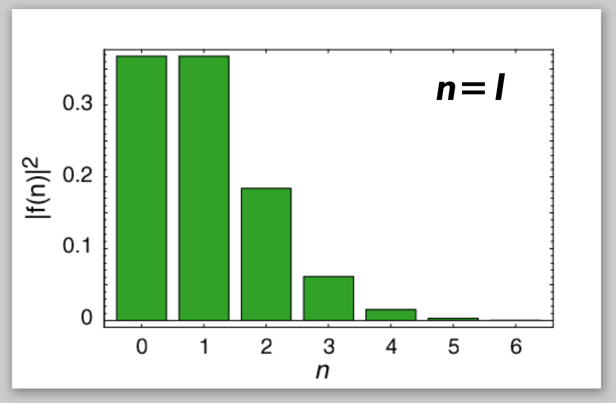
$$H = -J\sum_{i,j} \hat{a}_i^\dagger \hat{a}_j + \frac{1}{2}U\sum_i \hat{n}_i(\hat{n}_i - 1)$$

Atoms are delocalized over the entire lattice !
Macroscopic wave function describes this state very well.

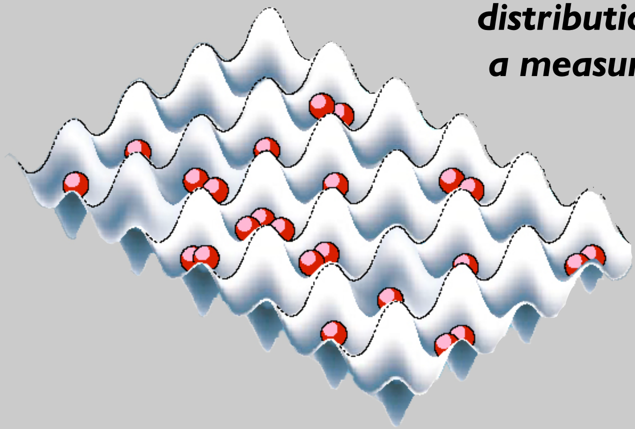
$$|\Psi_{SF}\rangle_{U=0} = \left(\sum_{i=1}^{M} \hat{a}_i^\dagger\right)^N |0\rangle$$

$$\langle \hat{a}_i \rangle_i \neq 0$$

Poissonian atom number distribution per lattice site



Atom number distribution after a measurement



JOHANNES GUTENBERG UNIVERSITÄT MAINZ

“Atomic Limit“ of a Mott-Insulator

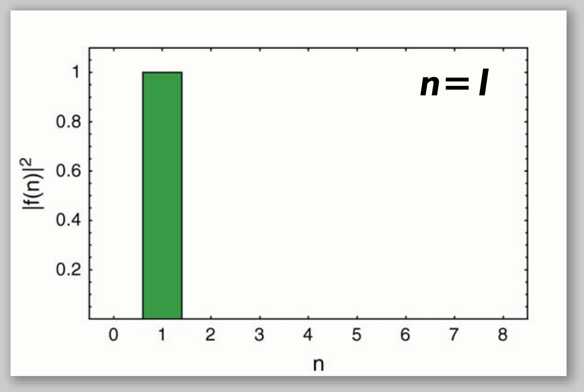
$$H = -J\sum_{i,j} \hat{a}_i^\dagger \hat{a}_j + \frac{1}{2}U\sum_i \hat{n}_i(\hat{n}_i - 1)$$

Atoms are completely localized to lattice sites !

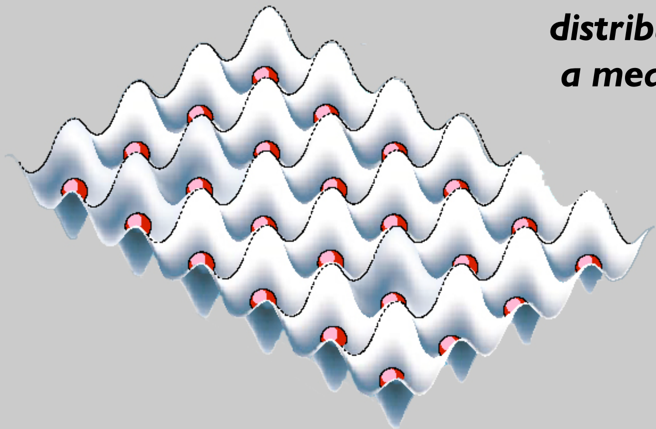
$$|\Psi_{Mott}\rangle_{J=0} = \prod_{i=1}^{M} \left(\hat{a}_i^\dagger\right)^n |0\rangle$$

$$\langle \hat{a}_i \rangle_i = 0$$

Fock states with a vanishing atom number fluctuation are formed.

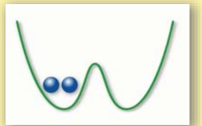
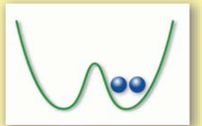
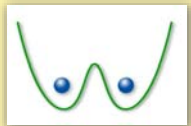
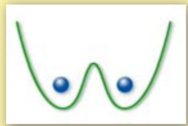


Atom number distribution after a measurement

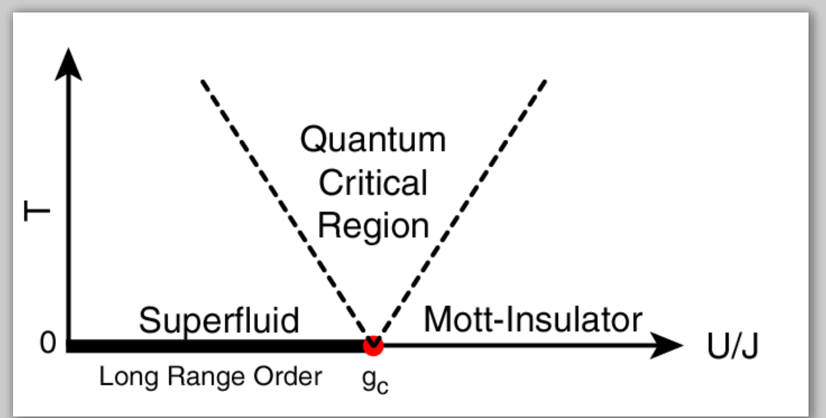


JOHANNES GUTENBERG UNIVERSITÄT MAINZ

The Simplest Possible "Lattice": 2 Atoms in a Double Well

	Superfluid State	***MI State***
	$\frac{1}{\sqrt{2}}(\phi_l + \phi_r) \otimes \frac{1}{\sqrt{2}}(\phi_l + \phi_r)$	$\frac{1}{\sqrt{2}}\phi_l \otimes \phi_r + \frac{1}{\sqrt{2}}\phi_r \otimes \phi_l$
	0.25 x  **+** ***0.25 x***  **+** ***0.5 x*** 	
Average atom number per site:	***<n> = 1***	***<n> = 1***
Average onsite Interaction per site:	$\langle E_{int} \rangle = \frac{1}{2} U$	$\langle E_{int} \rangle = 0$

Quantum Phase Transition (QPT) from a Superfluid to a Mott-Insulator



At the critical point g_c the system will undergo a phase transition from a superfluid to an insulator !

This phase transition occurs even at T=0 and is driven by quantum fluctuations !

Characteristic for a QPT

- ***Excitation spectrum is dramatically modified at the critical point.***
- ***U/J < g_c (Superfluid regime)***
  Excitation spectrum is gapless
- ***U/J > g_c (Mott-Insulator regime)***
  Excitation spectrum is gapped

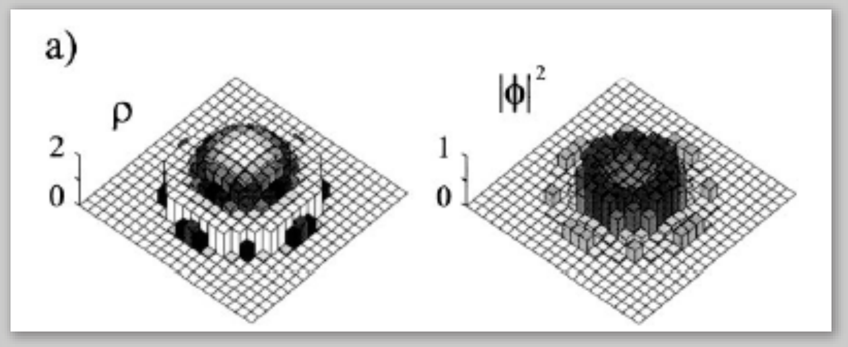
Critical ratio for:

U/J = z 5.8

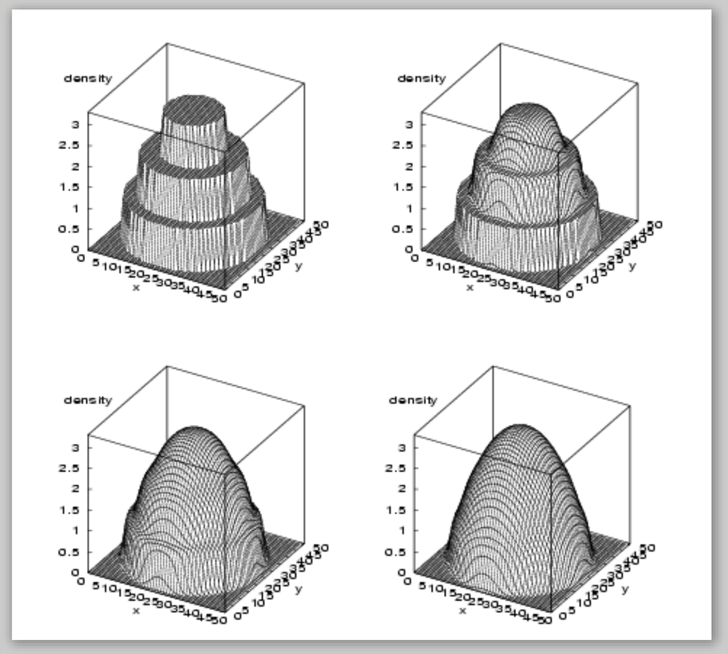
see Subir Sachdev, Quantum Phase Transitions, Cambridge University Press

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Ground State of an Inhomogeneous System



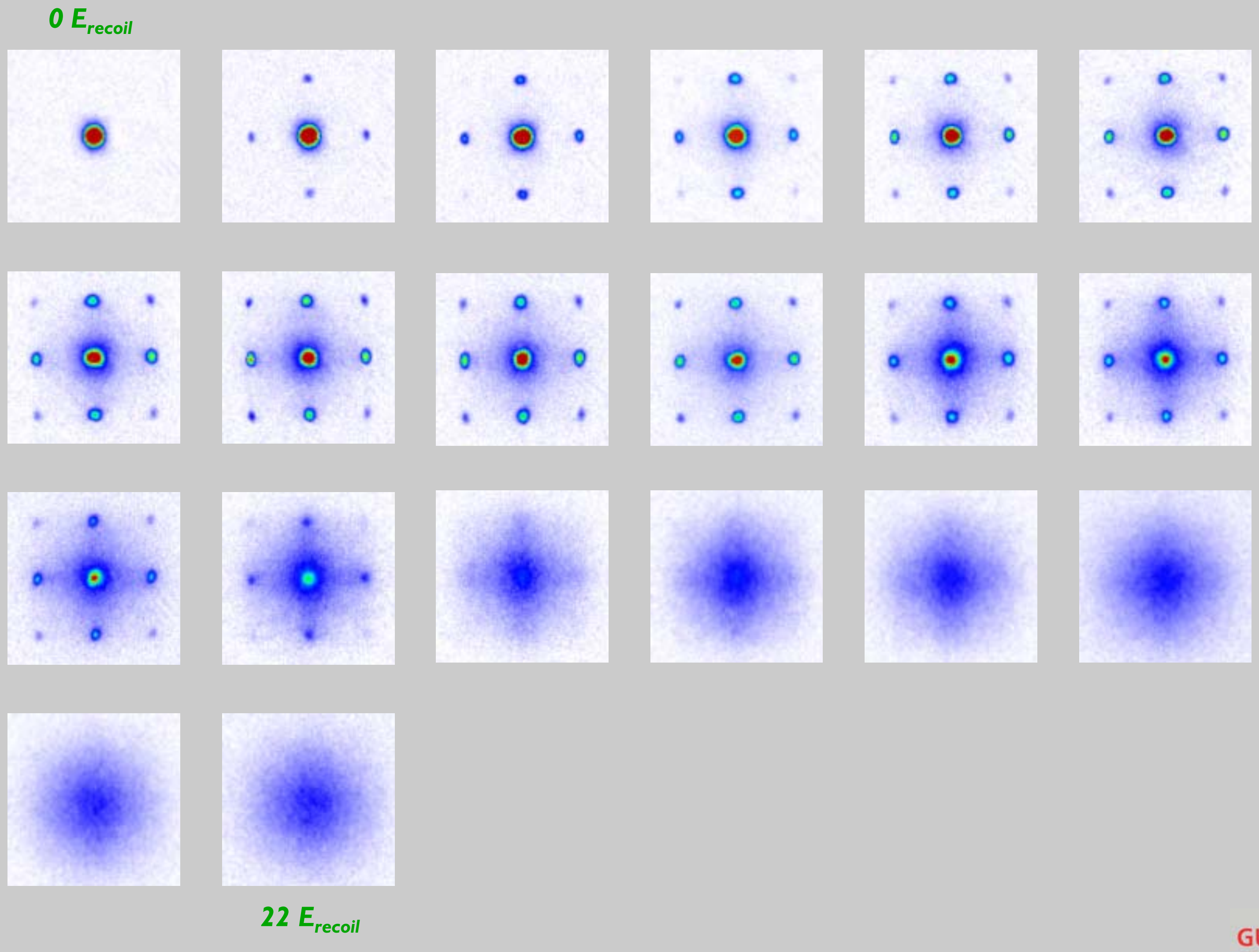
From Jaksch et al. PRL 81, 3108 (1998)



From M. Niemeyer and H. Monien (private communication)

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Momentum Distribution for Different Potential Depths



Phase coherence of a Mott insulator

Does a Mott insulator produce an interference pattern ?

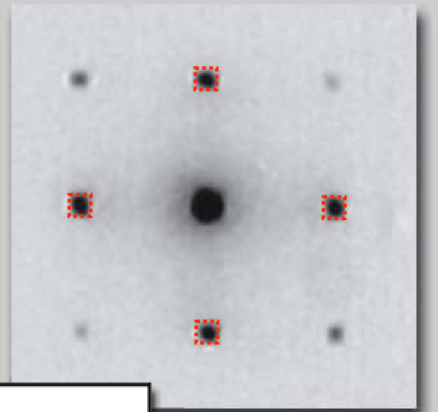
F. Gerbier *et al.*, PRL (2005)

Theory : V. N. Kashurnikov *et al.*, PRA **66**, 031601 (2002).
R. Roth & K. Burnett, PRA **67**, 031602 (2003).

Quantitative Analysis of Interference Pattern

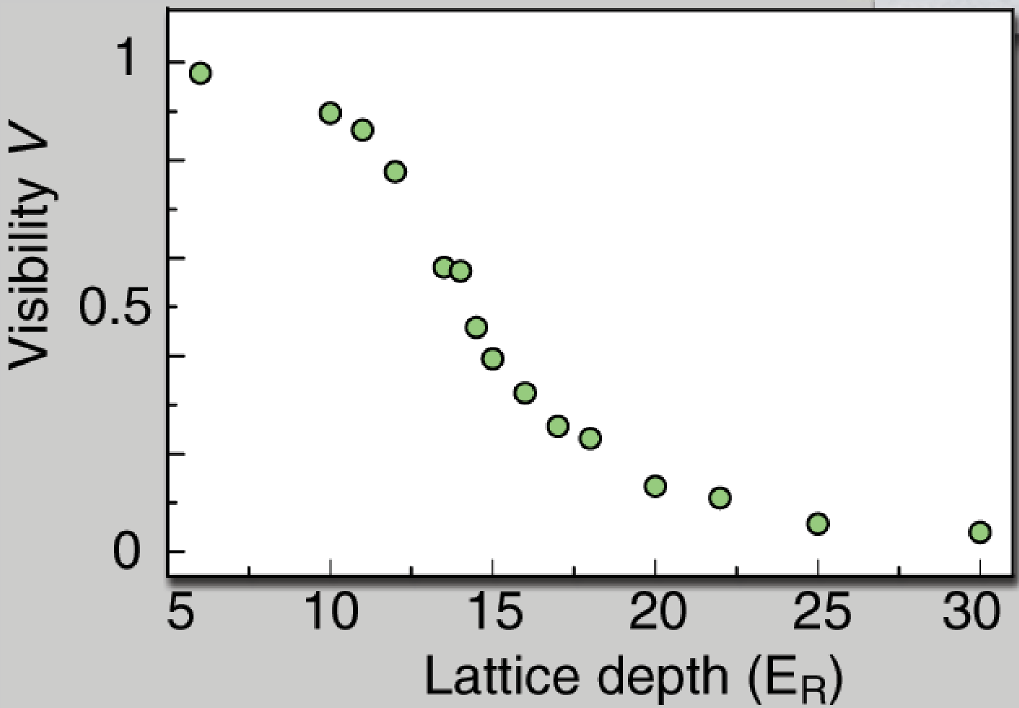
$$V = \frac{n_{max} - n_{min}}{n_{max} + n_{min}}$$

Visibility measures coherence



n_{max}

n_{min}



Visibility decays slowly with increasing lattice depth!

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Excitations in the zero tunneling limit

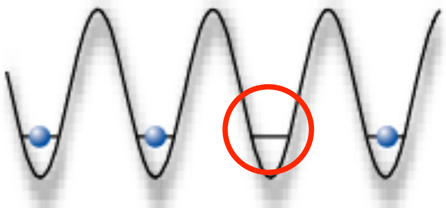
Energy E_0

Perfect Mott insulator ground state

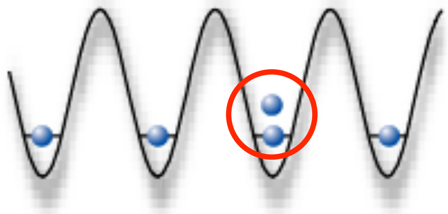
$$|\Psi_0\rangle = \prod_i |n_0\rangle_i$$

- Low energy excitations :

n$_0$: filling factor
Here n_0=1

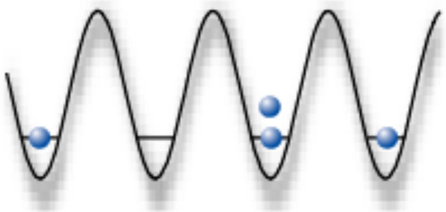


$$|\Psi\rangle_{\mathrm{hole}} \propto \sum_i \hat{a}_i |\Psi_0\rangle$$



$$|\Psi\rangle_{\mathrm{particle}} \propto \sum_i \hat{a}_i^\dagger |\Psi_0\rangle$$

- **Particle/hole pairs** couples to the ground state :



$$|\Psi\rangle_{\mathrm{ph}} \propto \sum_{\langle i,j\rangle} \hat{a}_i^\dagger \hat{a}_j |\Psi_0\rangle$$

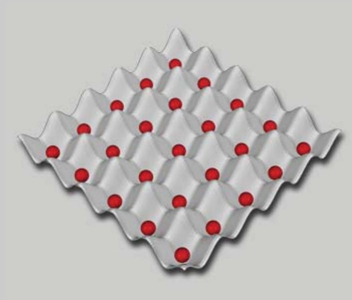
Energy E_0+U, separated from the ground state by an interaction gap U

Deviations from the perfect Mott Insulator

Ground state for J=0 :

``perfect´´ Mott insulator

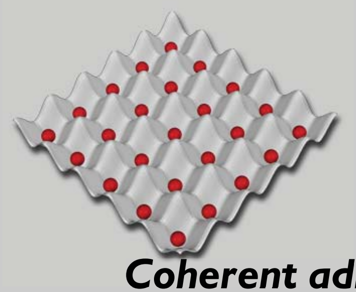
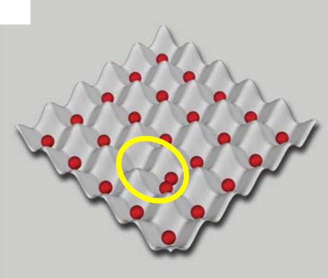
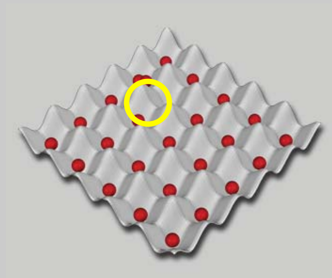
$$|\Psi_0\rangle = \prod_i |n_0\rangle_i$$



Ground state for finite J<<U :

treat the hopping term $\boldsymbol{H_{hop}}$ in 1st order perturbation

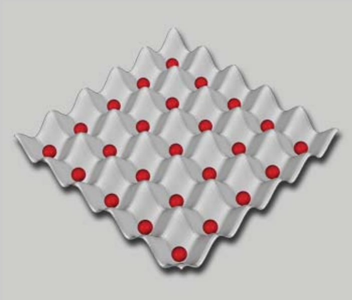
$$|\Psi_1\rangle = -\sum_{n\neq g} \frac{H_{\mathrm{hop}}}{E_g^{(0)} - E_n^{(0)}} |\Psi_0\rangle$$

$=$  $+\ \frac{J}{U}$  $+\ \frac{J}{U}$  $+\ldots$

Coherent admixture of particle/holes at finite J/U

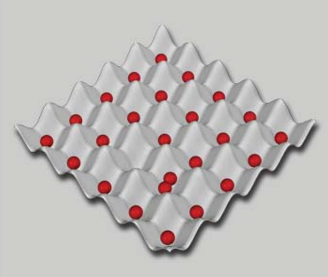
Predictions for the visibility

Perfect MI



$$V = 0$$

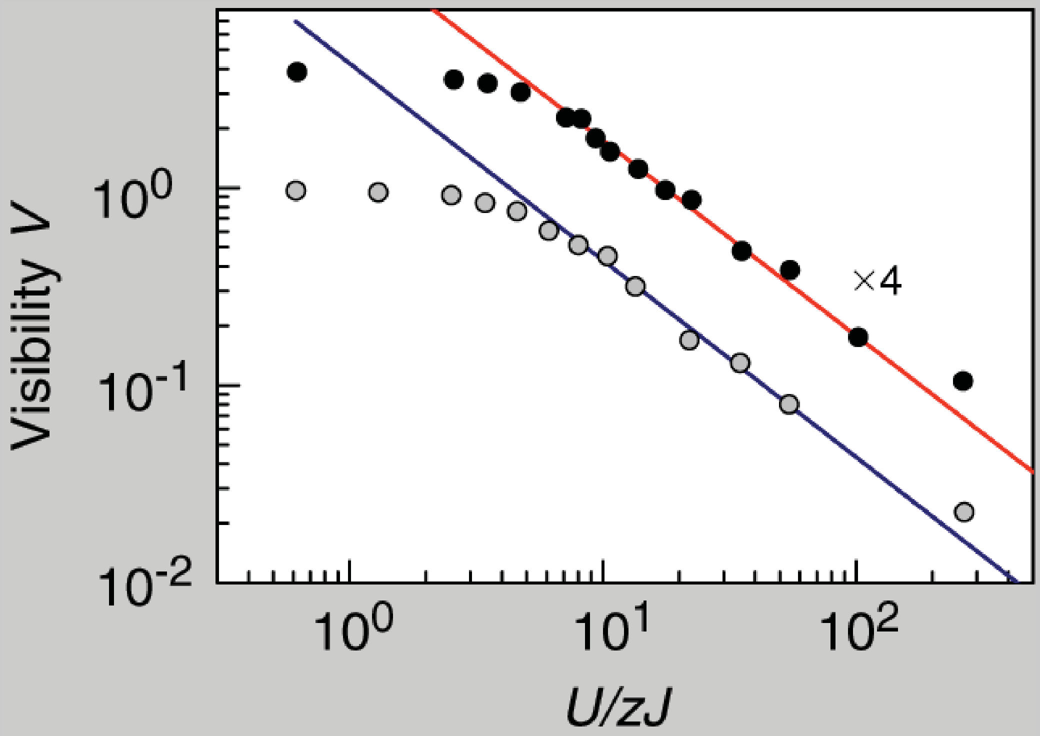
MI with
particle/hole pairs



$$V \approx \frac{4}{3}(n_0 + 1)\frac{zJ}{U}$$

Perturbation approach predicts a **finite visibility, scaling as (U/J)$^{-1}$**

Comparison with experiments



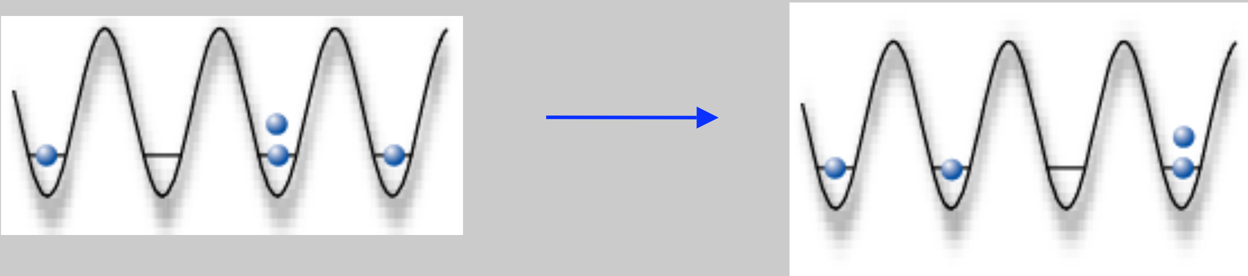
Average slope measured to be -0.97(7)

A more careful theory

Many-body calculation for the homogeneous case

- 1st order calculation : admixture of particle/hole pairs to the MI bound to neighboring lattice sites

- Higher order in J/U : particle/holes excitations become mobile



Dispersion relation of the excitations is still characterized by an interaction gap.

One can obtain analytically the interference pattern (momentum distribution) for a given n_0.

More details in :

D. van Oosten ***et al.*** PRA **63**, 053601 (2001) and following papers

D. Gangardt ***et al.***, cond-mat/0408437 (2004)

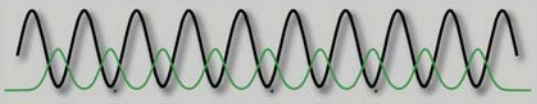
K. Sengupta and N. Dupuis, PRA **71**, 033629 (2005)

F. Gerbier et al., PRA 72, 53606 (2005)

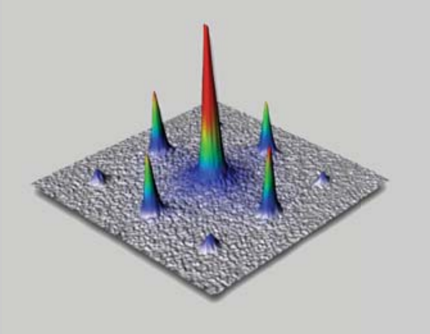
JOHANNES GUTENBERG UNIVERSITÄT MAINZ

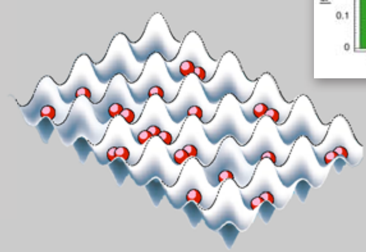
From a Superfluid to a Mott Insulator

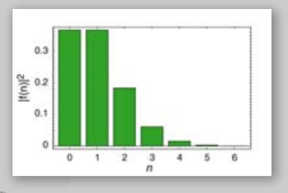
Delocalized particles

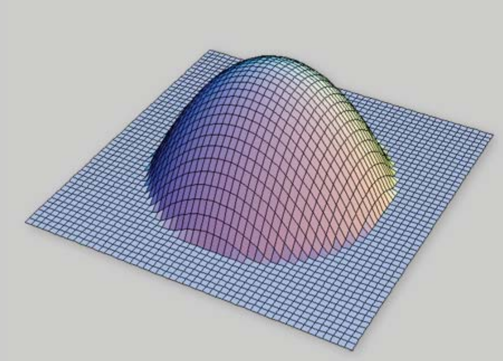


Superfluid State $U / J << 1$

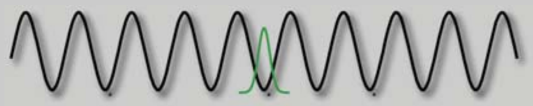




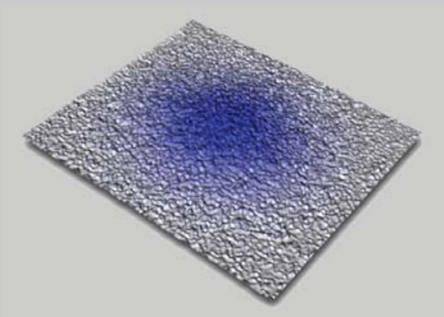


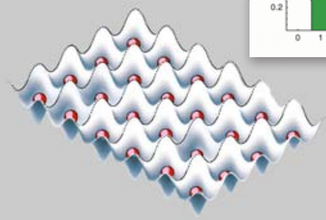


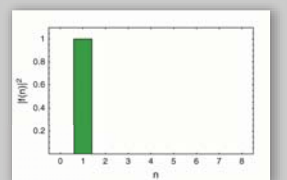
Localized particles

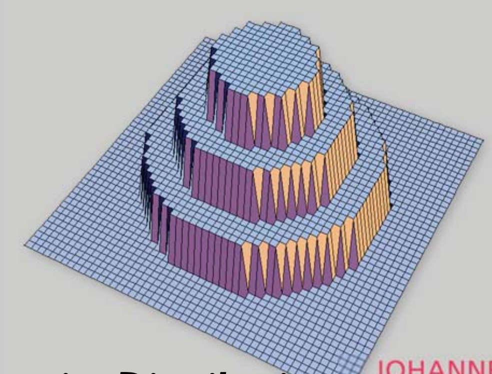


Mott Insulator State $U / J >> 1$









Phase coherence **Number statistics** **Density Distribution**

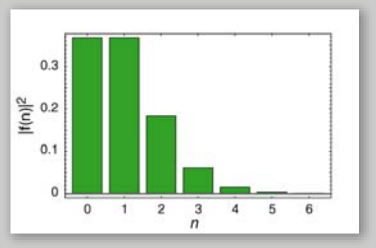
JOHANNES GUTENBERG UNIVERSITÄT MAINZ

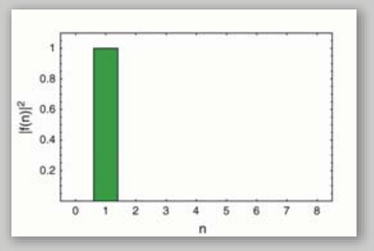
How can we Probe the Number Statistics?

We want to know:

1) **How many sites with 1 atom**
2) **How many sites with 2 atoms**
3) **How many sites with 3 atoms**
4) **…**

For a weakly interacting BEC, one would obtain Poissonian type number distribution (e.g. coherent states on each lattice site)





F. Gerbier et al., PRL **96**, 090401 (2006)
G.Campbell et al, Science **313**, 649 (2006)

Spin Changing Collisions

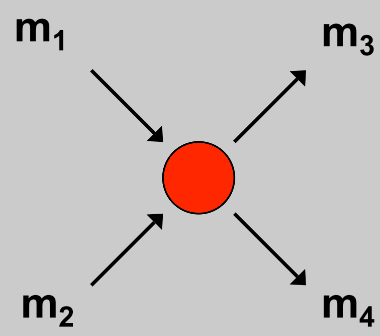
Spin-independent case

s-wave collisions

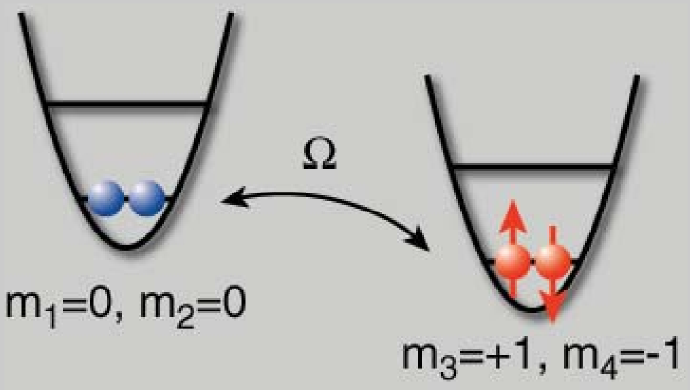
Spin-dependent case

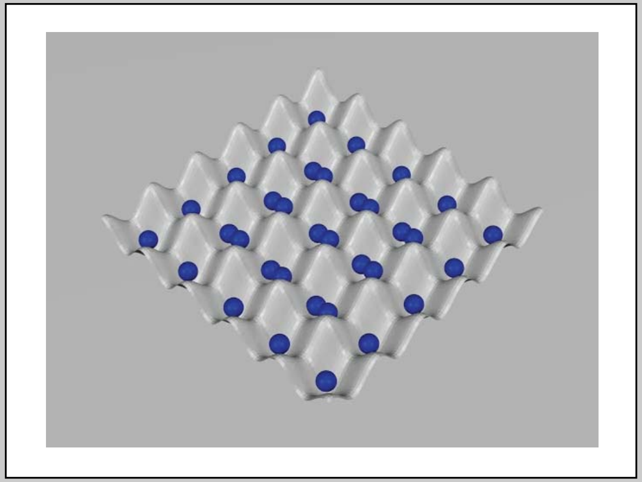
$$V(\vec{r}-\vec{r}\,') = \frac{4\pi\hbar^2}{M} \times \Delta a^{m1,m2}_{m3,m4} \times \delta(\vec{r}-\vec{r}\,')$$

Spin-dependent interaction strength



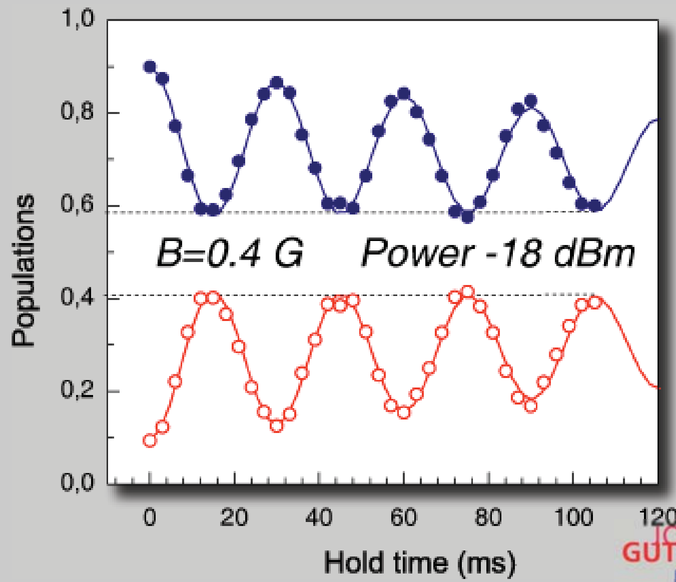
Spin Changing Collisions in an Optical Lattice





Collisionally induced
„Rabi-Type“ Oscillations

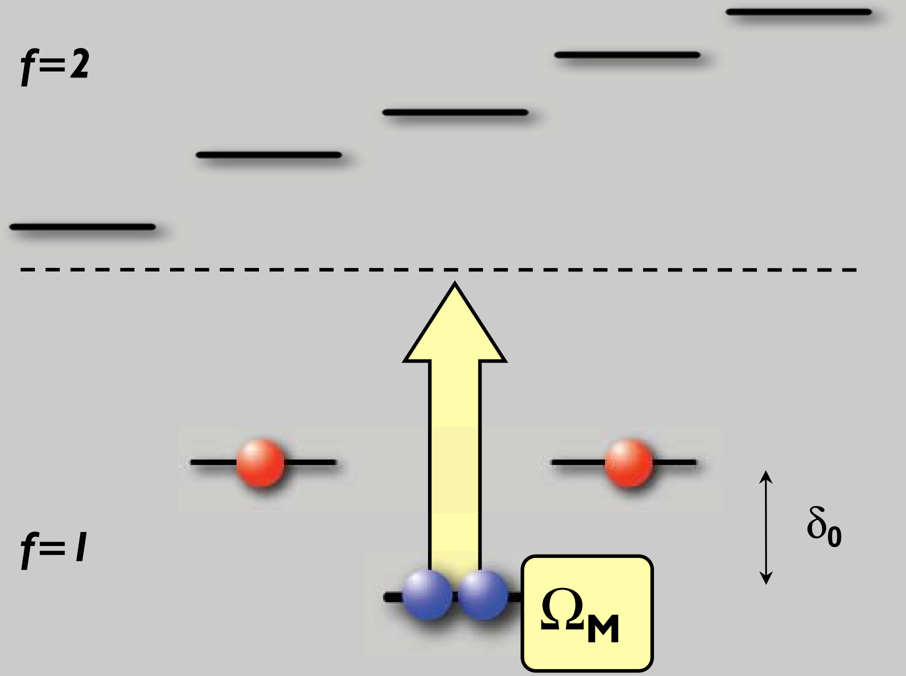
$$|0,0\rangle \leftrightarrow \left(|\uparrow,\downarrow\rangle + |\downarrow,\uparrow\rangle\right)/\sqrt{2}$$



JOHANNES GUTENBERG UNIVERSITÄT MAINZ

AC-“Stark“ shift control of the resonance frequency

Spin-1 two-level system at zero magnetic field



Detuning δ_0 is present even at zero magnetic field

Energy shift due to microwave field can bring levels into resonance.

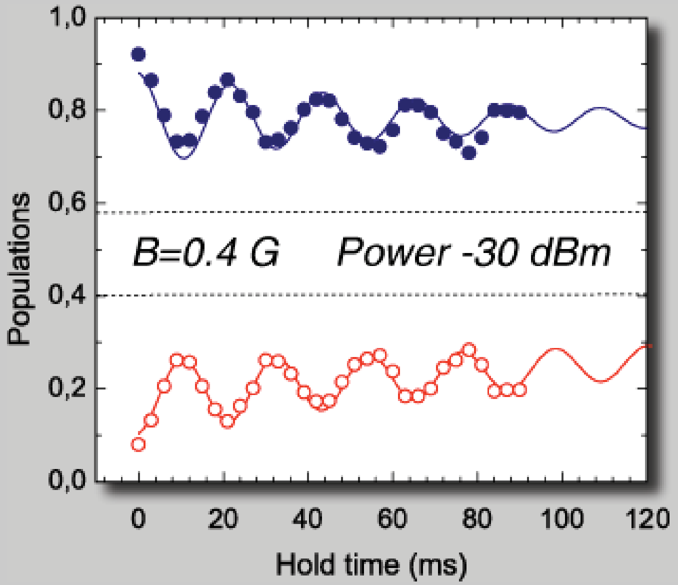
H. Pu and P. Meystre PRL 2000 and
Duan, Sorensen, Cirac, Zoller PRL 2000

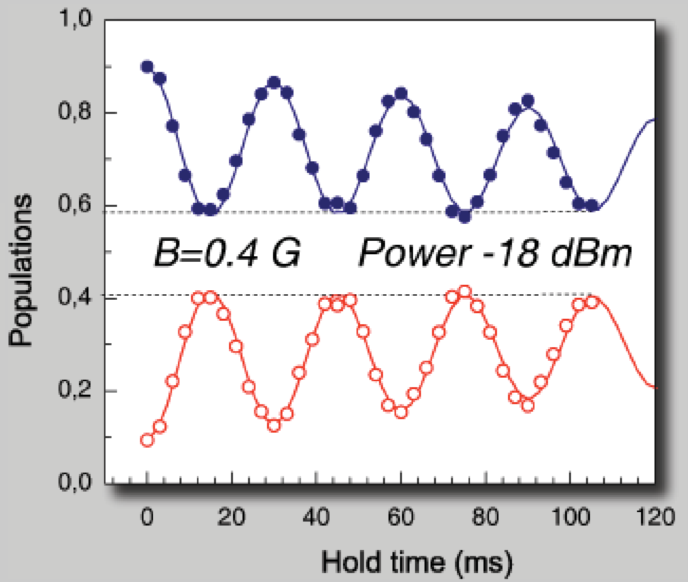
JOHANNES GUTENBERG UNIVERSITÄT MAINZ

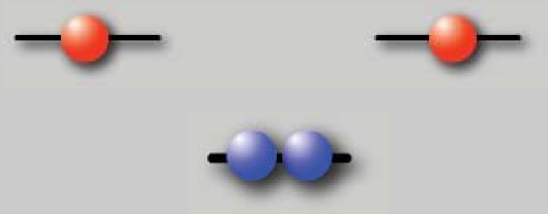
AC-Stark shift control of the resonance frequency

Energy shift can be tuned by power of the microwave and detuning

$$\Delta E \propto \frac{\Omega_M^2}{4\Delta}$$



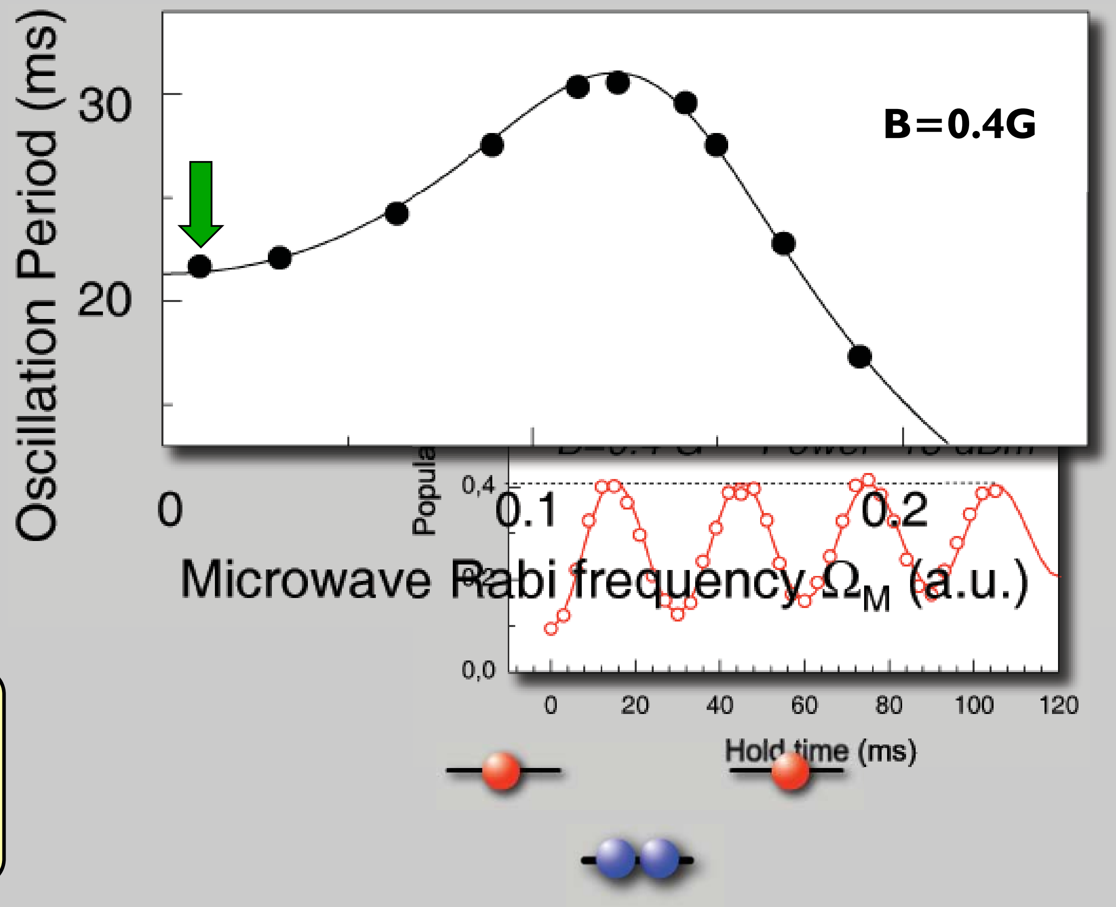




$\delta_0 + \delta(\mathbf{B}^2)$

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

AC-Stark shift control of the resonance frequency



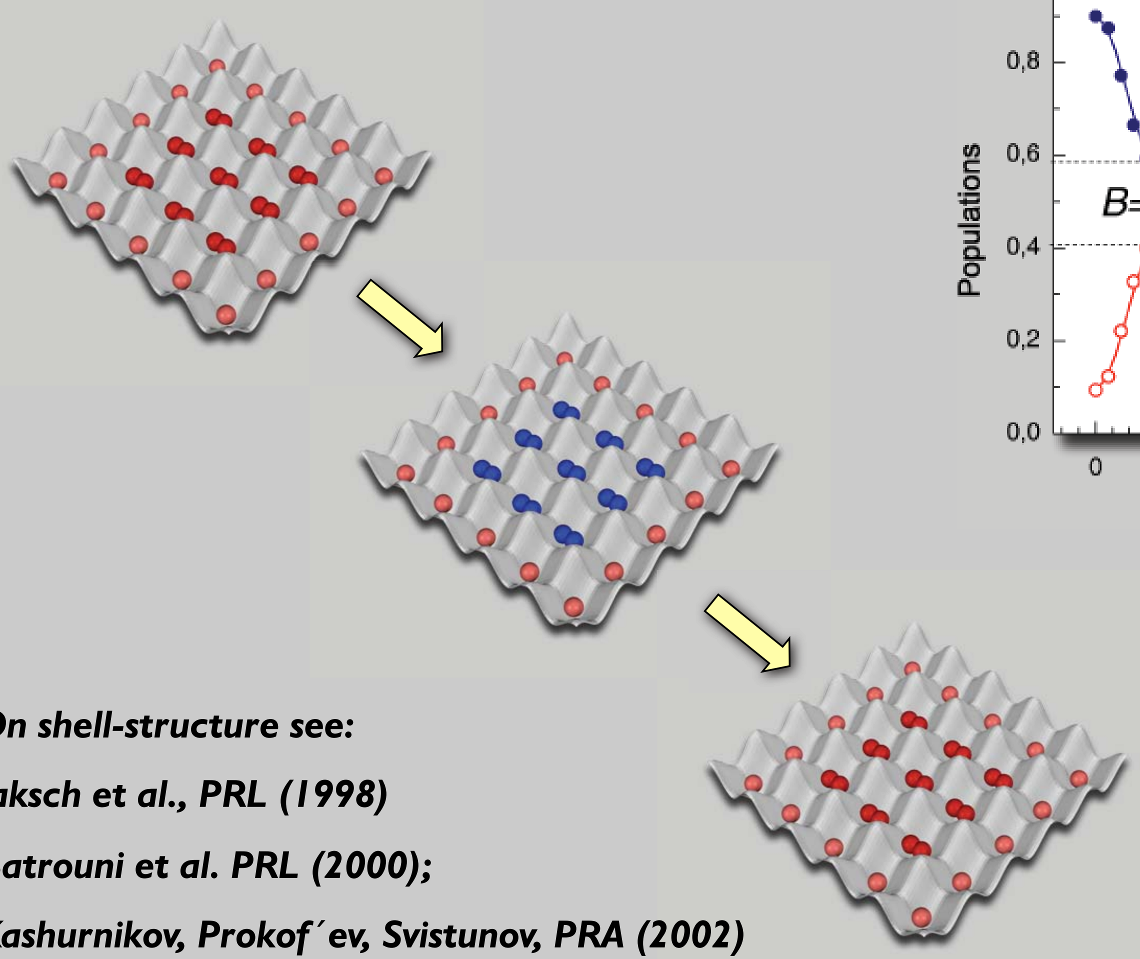
$$\frac{N_{+1}+N_{-1}}{N_{tot}} \propto \left(\frac{\Omega_0}{\Omega'}\right)^2$$

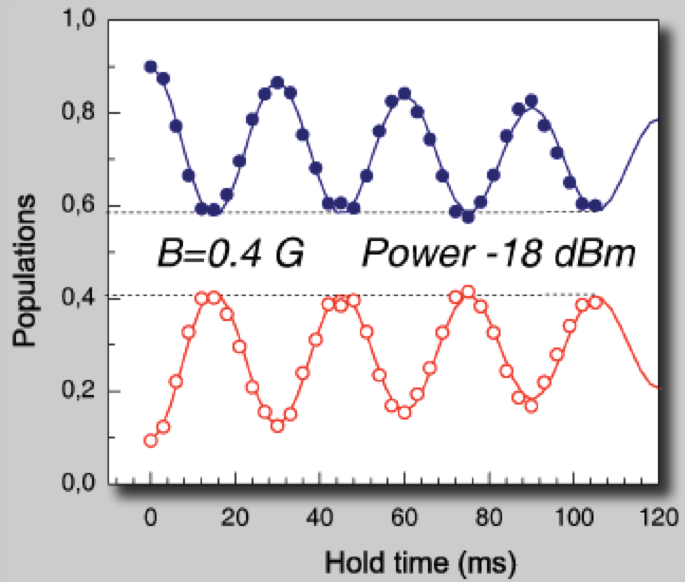
F. Gerbier et al., PRA **73**, 041602 (2006)

$\delta = \delta_0 + \delta(B^2)$

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Amplitude decrease due to single site spectators





Sensitive and non-destructive detector for doubly occupied lattice sites

On shell-structure see:

Jaksch et al., PRL (1998)

Batrouni et al. PRL (2000);

Kashurnikov, Prokof´ev, Svistunov, PRA (2002)

Alet et al., PRA (2004), recent work P. Denteneer

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Quantum Spin Oscillations as Non-Destructive Probe of Atom Number Statistics

***Classical field* (mean field) limit (continuous frequencies)**

$$\Omega(n) \square c_2 n$$

***Quantum limit* (discrete frequencies)**

$$\Omega_{N_{at}} = \Omega_0 \sqrt{N_{at}\left(N_{at} - 1\right)} \qquad \Omega_2 = \sqrt{2}\Omega_0 \quad \Omega_3 = \sqrt{6}\Omega_0 \quad \Omega_4 = \sqrt{12}\Omega_0 \quad \ldots$$

Leads to quantum dynamics beyond mean field!
Collapse & Revivals, Cat states, etc.

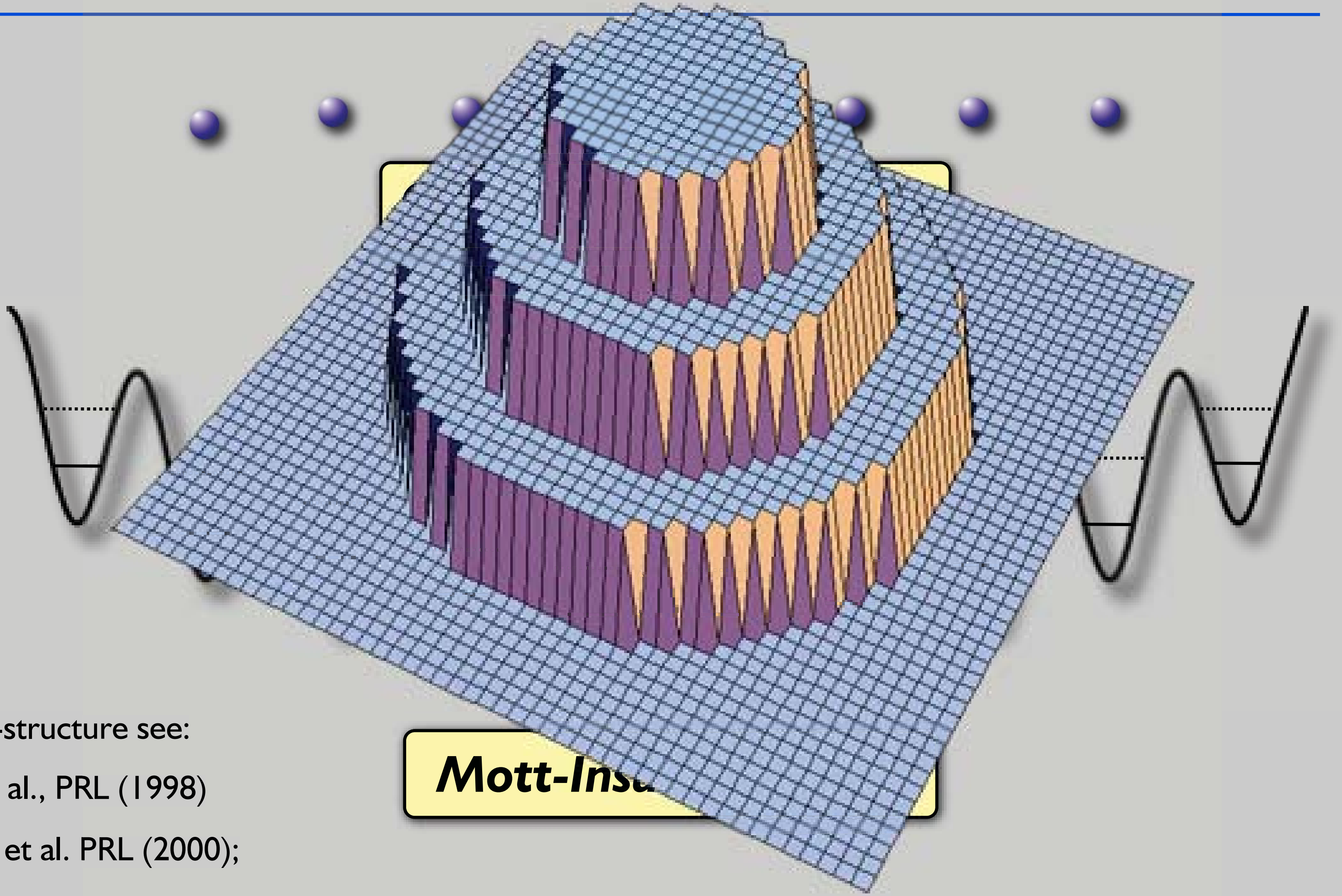
Cf. Work of L. You, J. Ho,…

⇒ ***Amplitude of Spin-Changing Oscillations at Freq Ω_{Nat}*** ⇔ ***Number of sites with N_{at} atoms***

Resembles exp. in Cavity QED to reveal photon number statistics (Haroche, Walther)
see also work of G. Campbell et al. (MIT)

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

What is the atom number distribution in a lattice?



On shell-structure see:

Jaksch et al., PRL (1998)

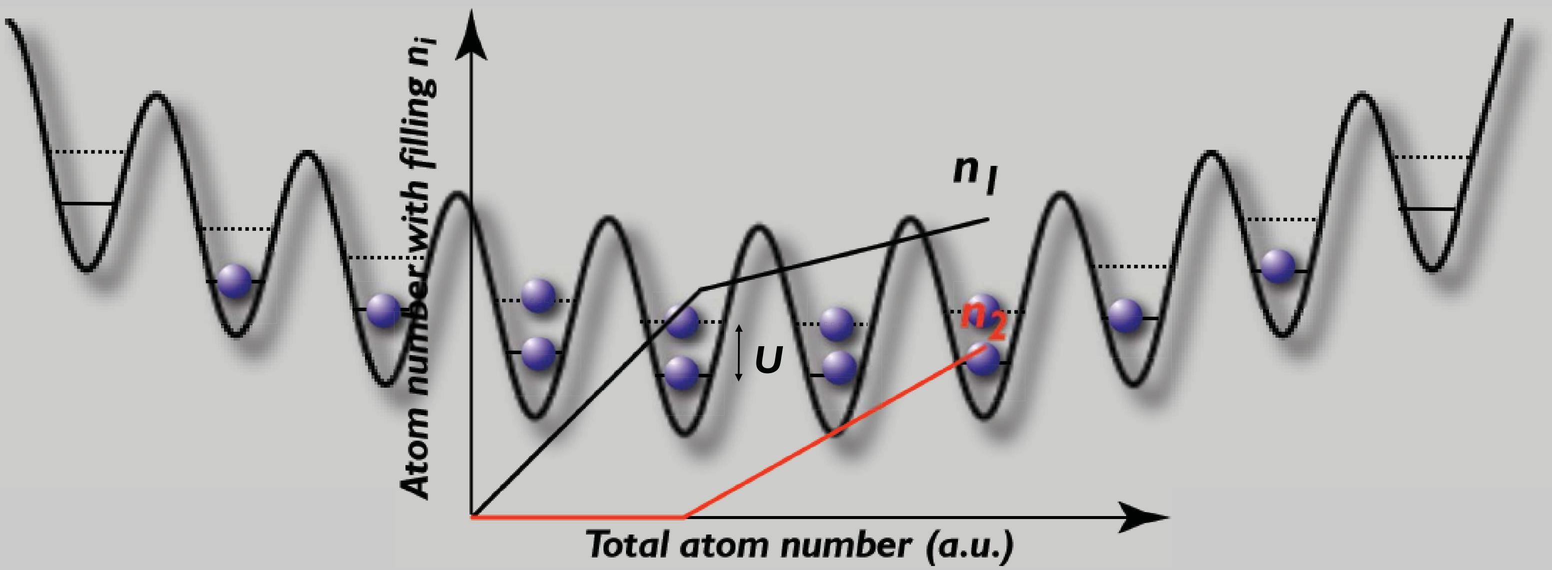
Batrouni et al. PRL (2000);

Kashurnikov, Prokof´ev, Svistunov, PRA (2002)

Alet et al., PRA (2004)

What is the number distribution in a lattice??

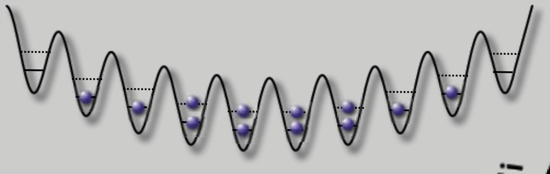
Mott insulator

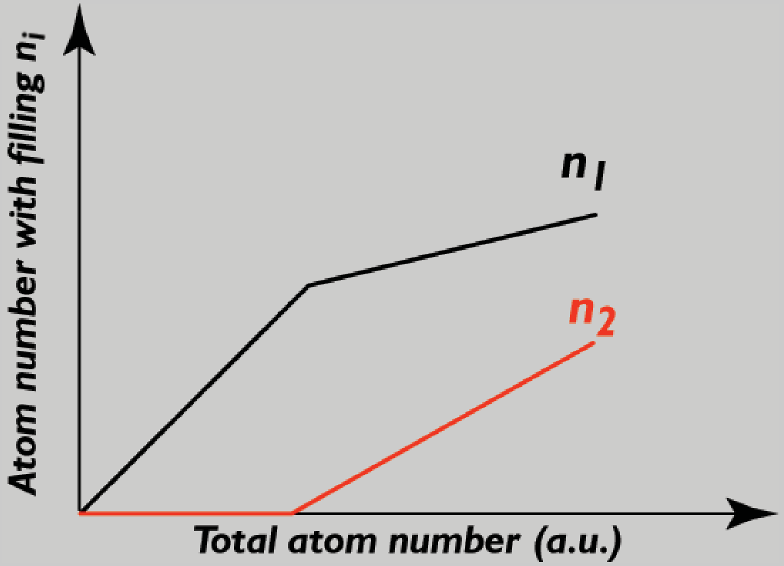


JOHANNES GUTENBERG UNIVERSITÄT MAINZ

What is the number distribution in a lattice??

Mott insulator

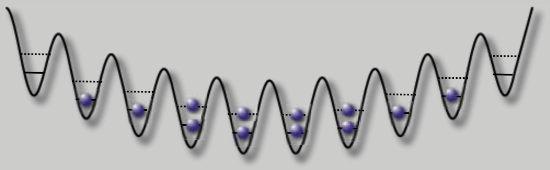


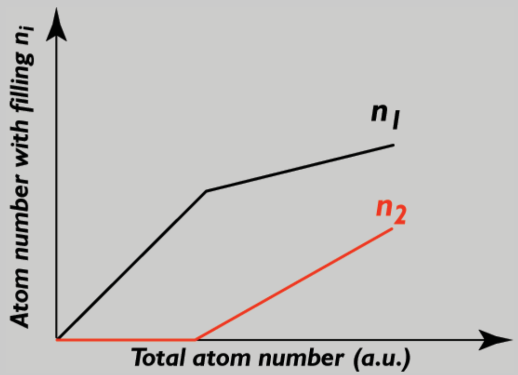


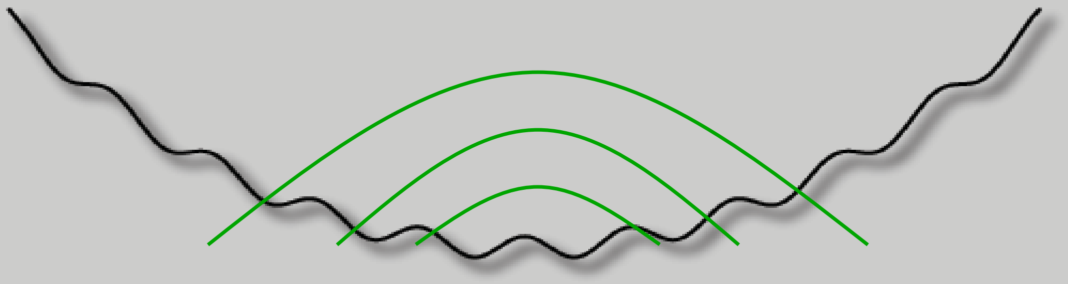
Strong suppression of n=2 sites for low atom numbers

What is the number distribution in a lattice??

Mott insulator





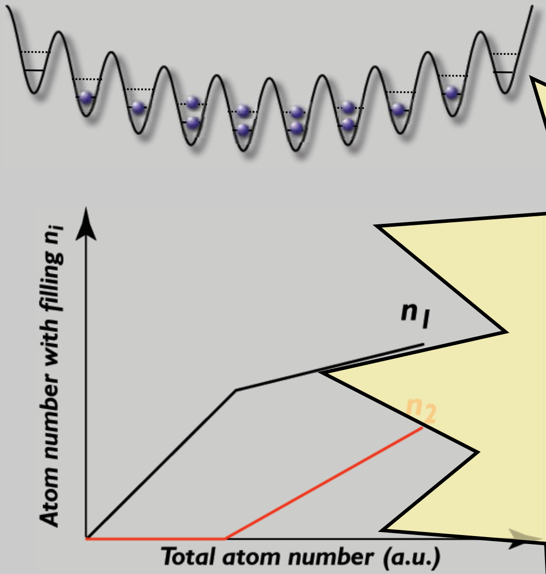


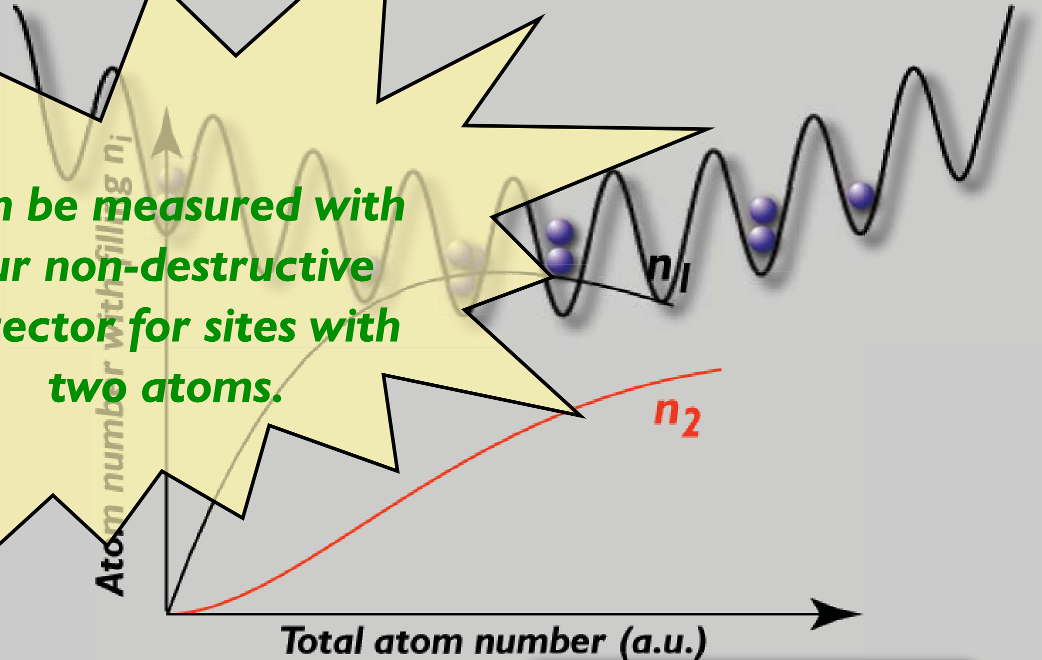
Formation of Mott-Shells

What is the number distribution in a lattice??

Mott insulator

Superfluid





Can be measured with our non-destructive detector for sites with two atoms.

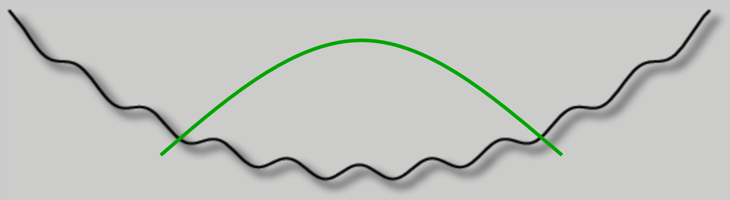
Formation of Mott-Shells

Poissonian number distribution

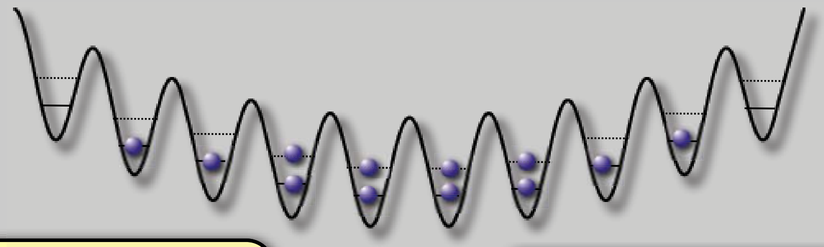
Related scheme proposed: D.C. Roberts, K. Burnett, PRL (2003)

What is the number distribution in a lattice??

Prepare the system at a certain lattice depth and atom number:



Quickly increase lattice depth in order to preserve atom number statistics:

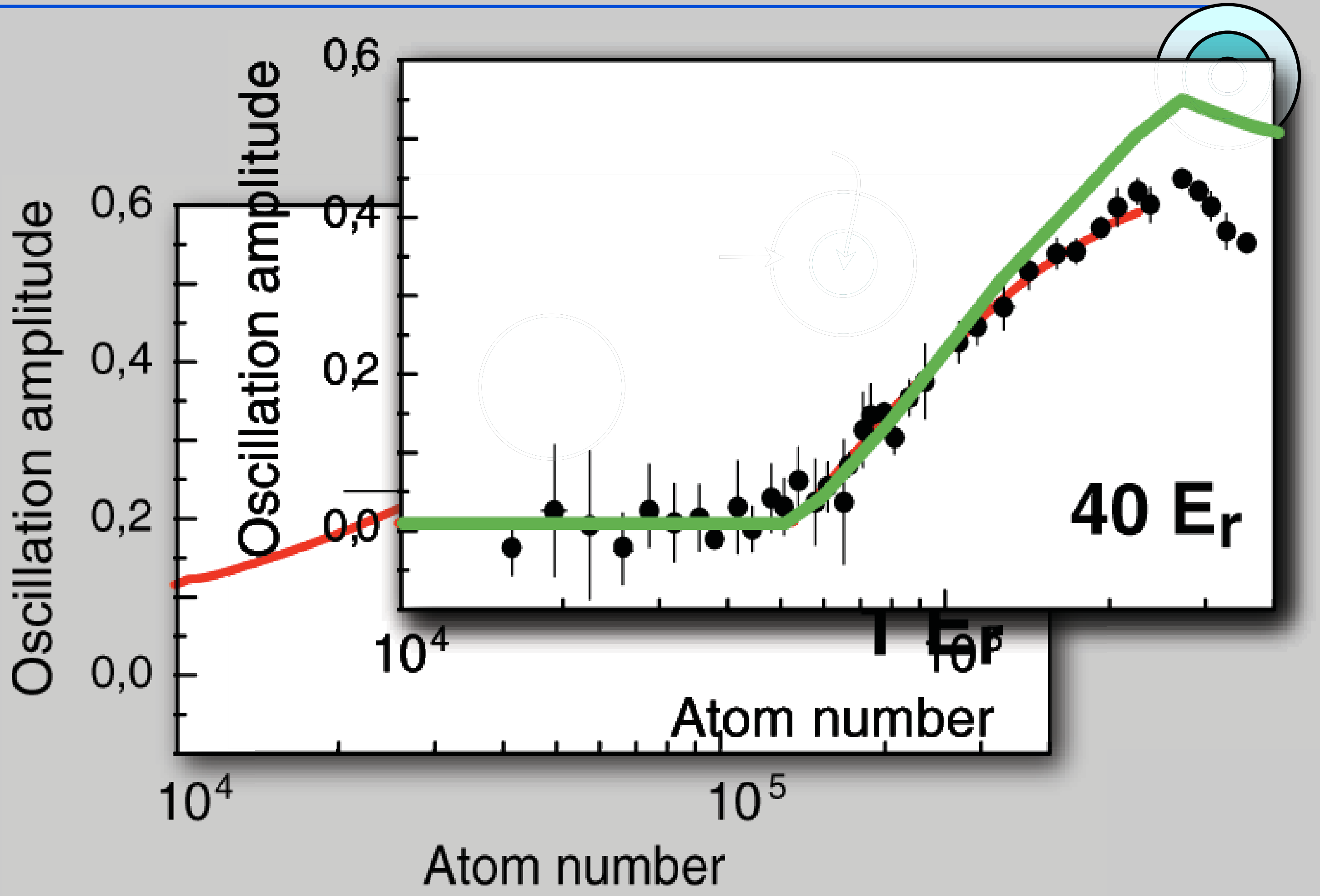


Amplitude of coherent spin-oscillations ⟺ ***Measure of sites with two or more atoms per site***

F. Gerbier et al., PRL (2006)

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

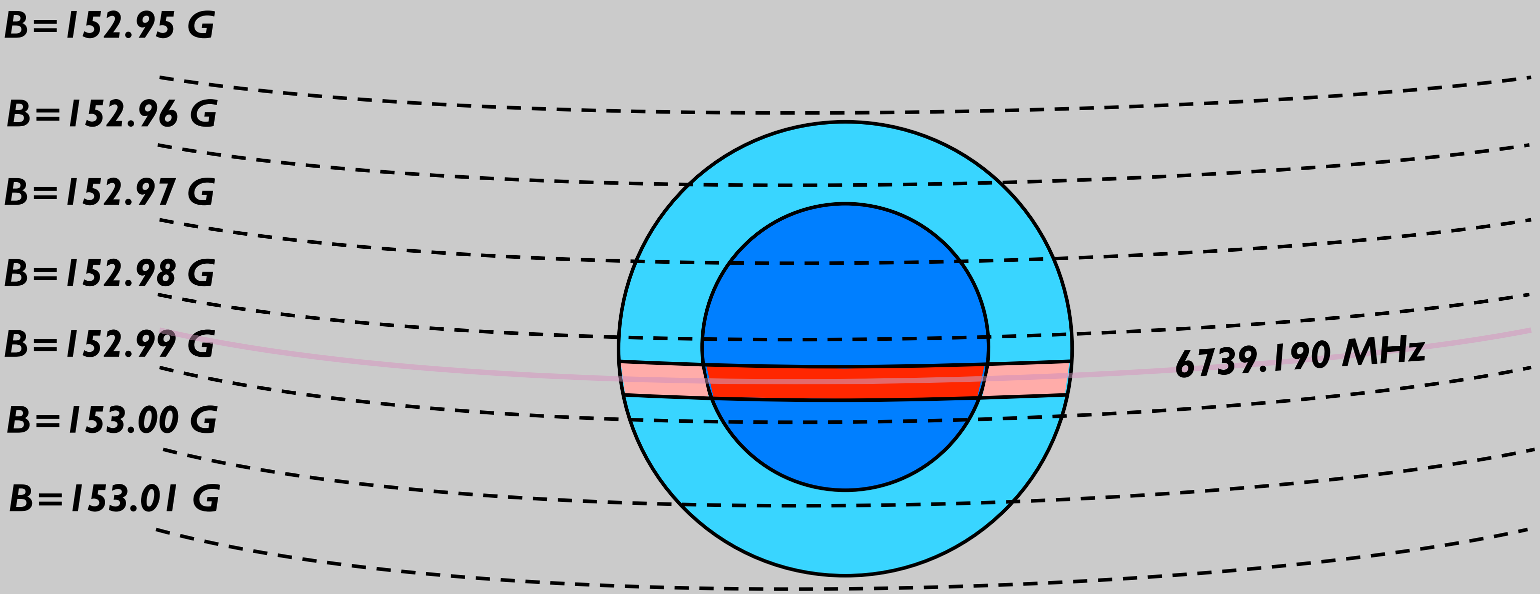
Atom number statistics…N=2 sites vs Total Atom Number



F. Gerbier et al., PRL **96**, 090401 (2006)


Dissecting a Mott insulator

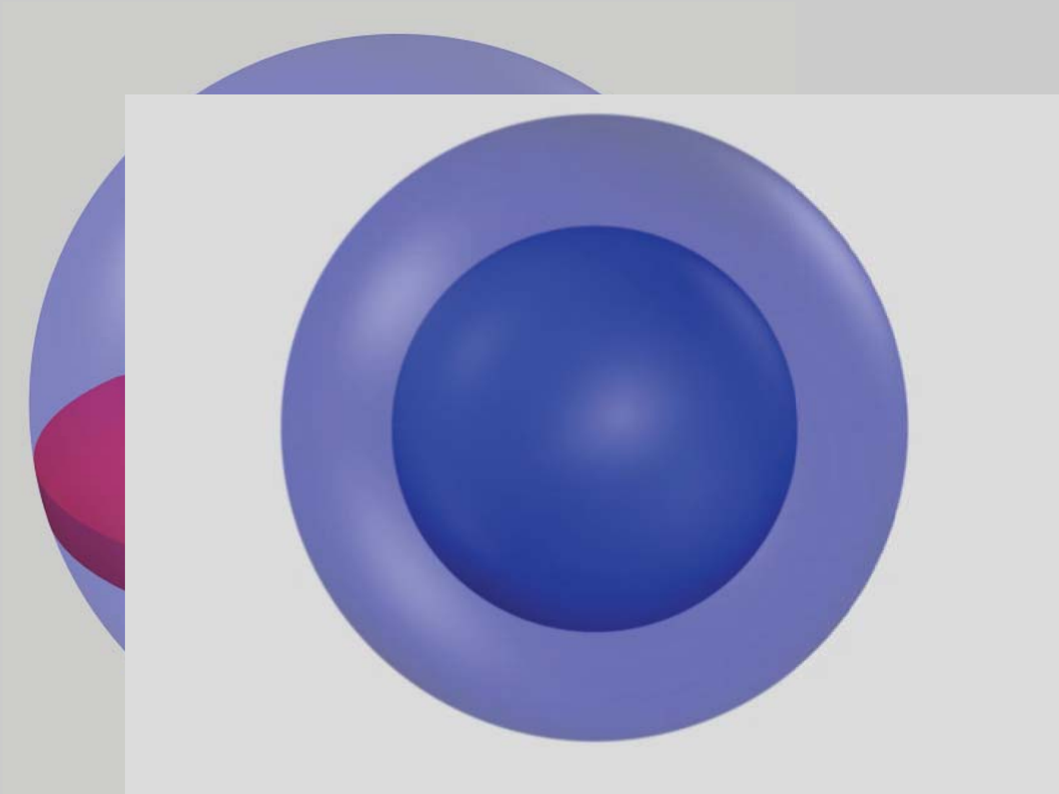
Probing the density distribution



Atoms on the line of resonance are transferred to another hyperfine state!

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Dissecting a Mott Insulator

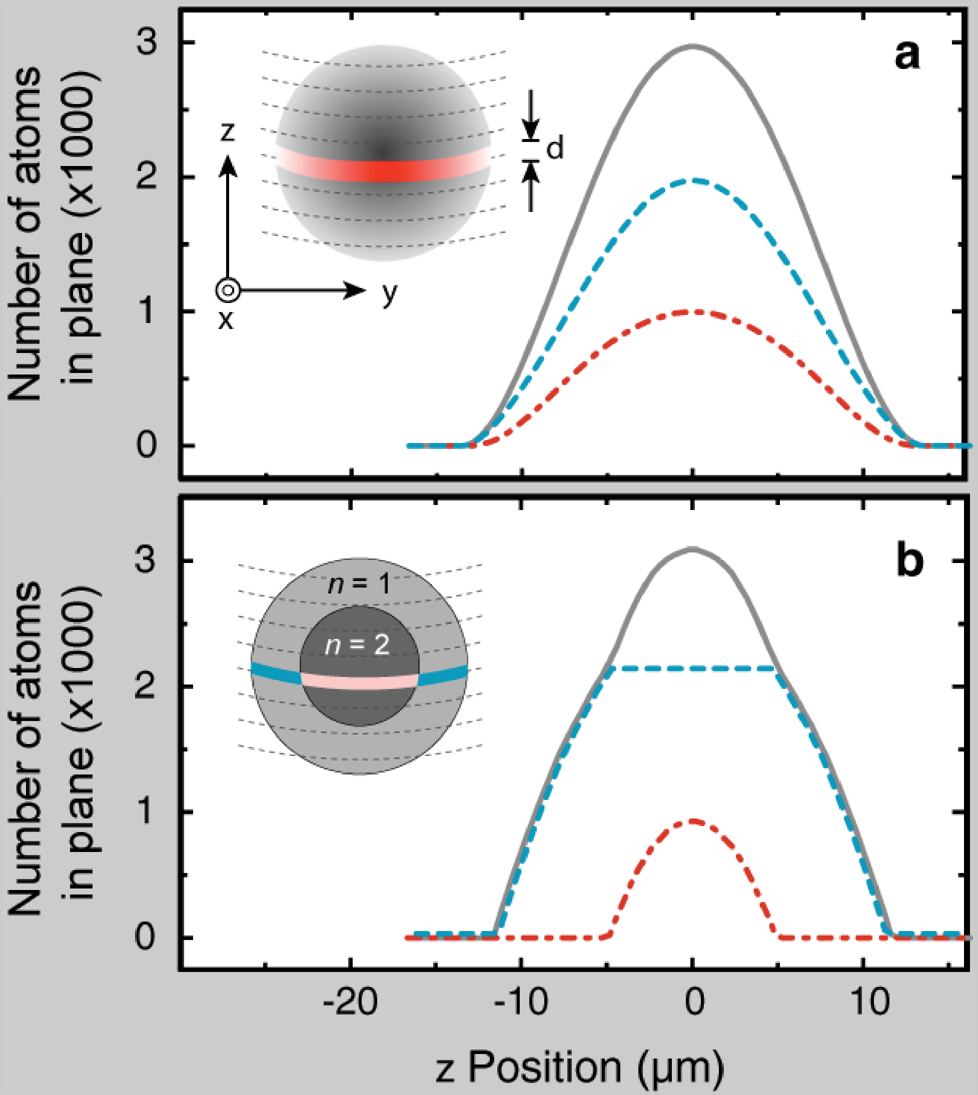


High spatial resolution of up to 1 μm can be achieved!

S. Fölling, PRL **97**, 060403 2006

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

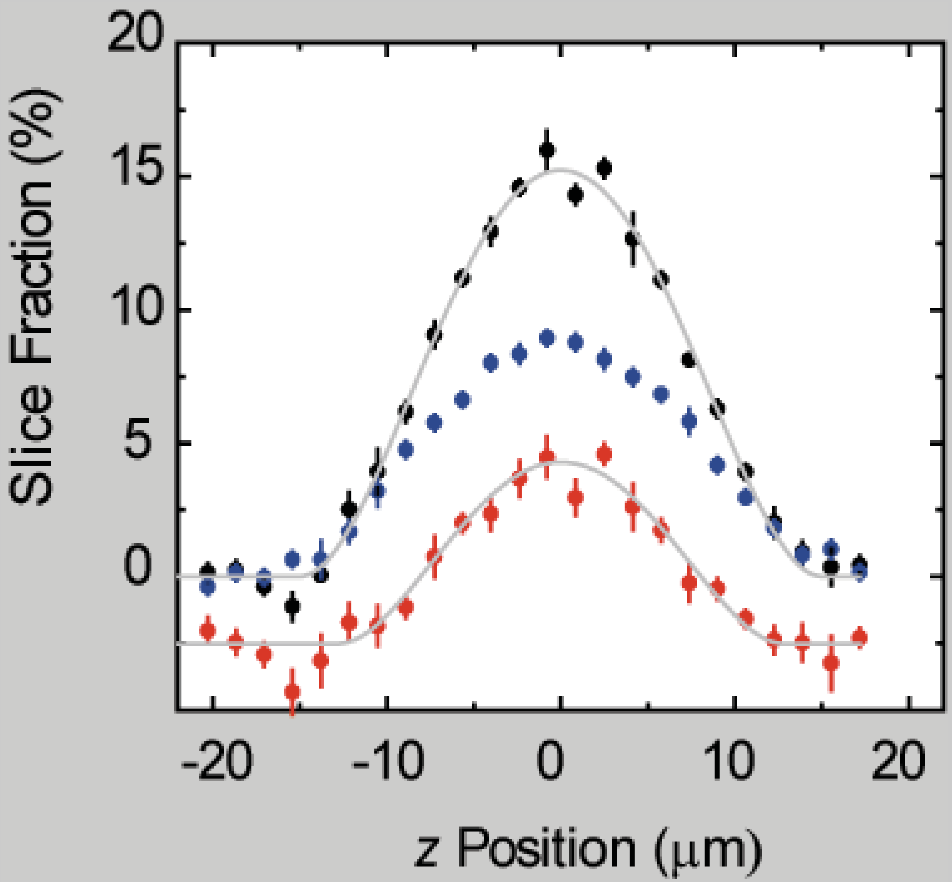
What to expect…



BEC

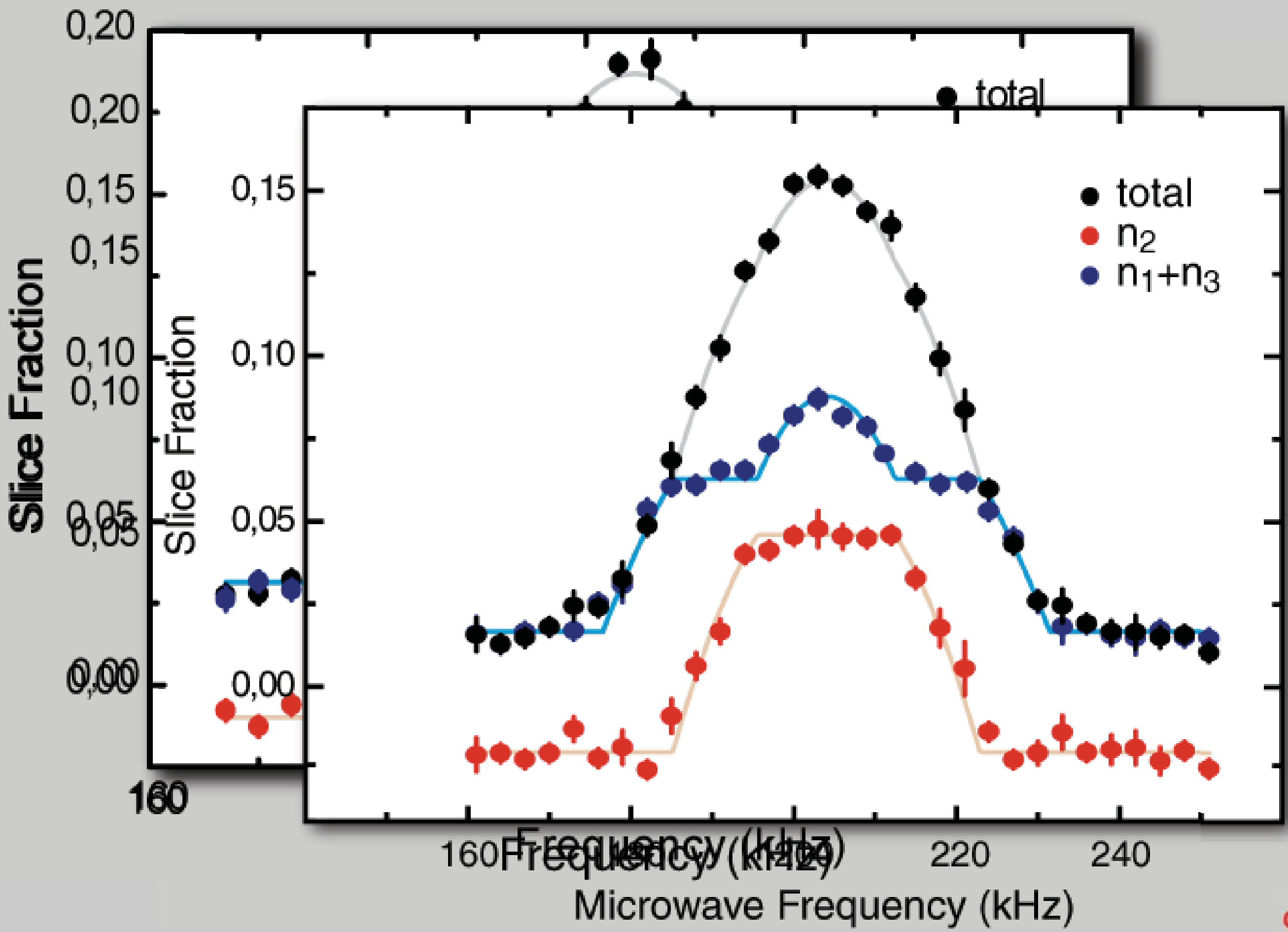
Mott insulator

Density Profile in the SF Regime



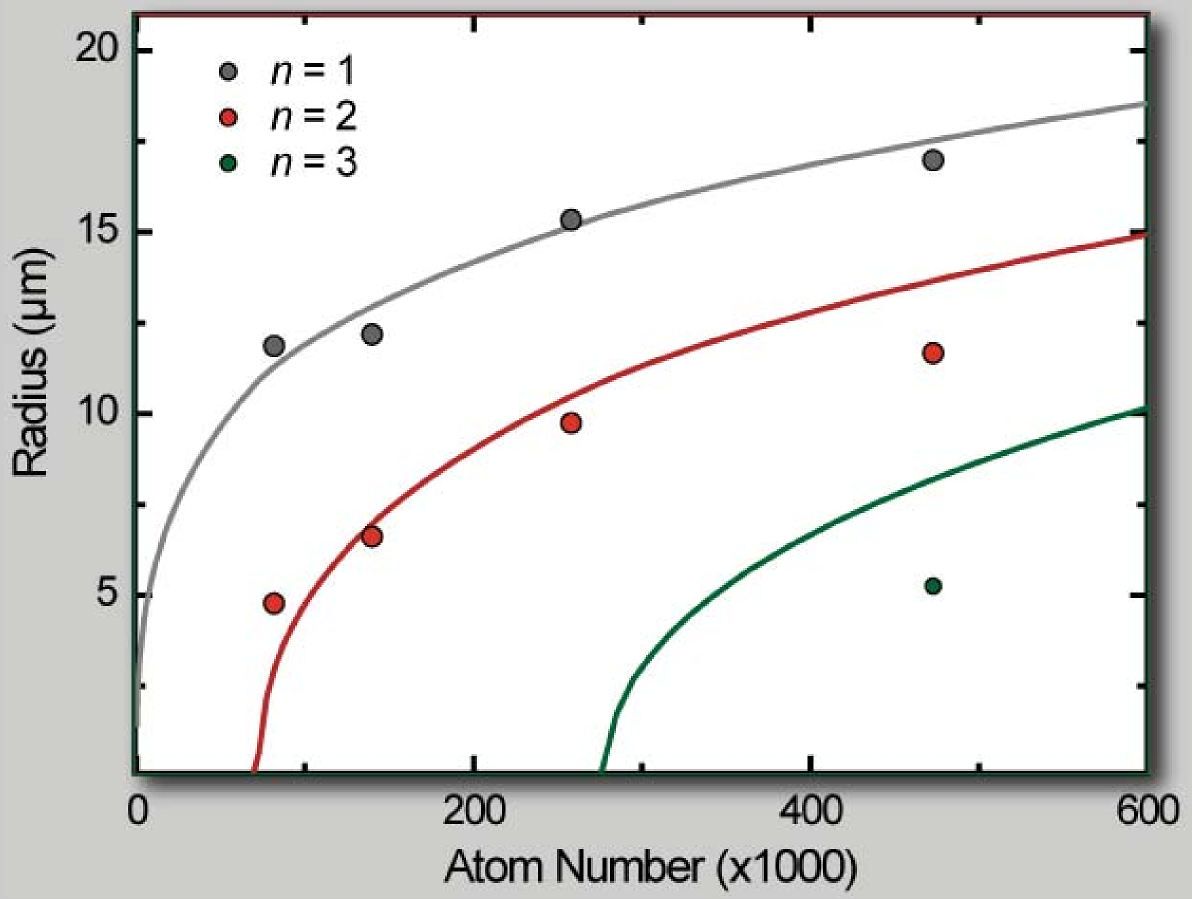
JOHANNES GUTENBERG UNIVERSITÄT MAINZ

In Trap Atom Number Resolved Profiles - MI



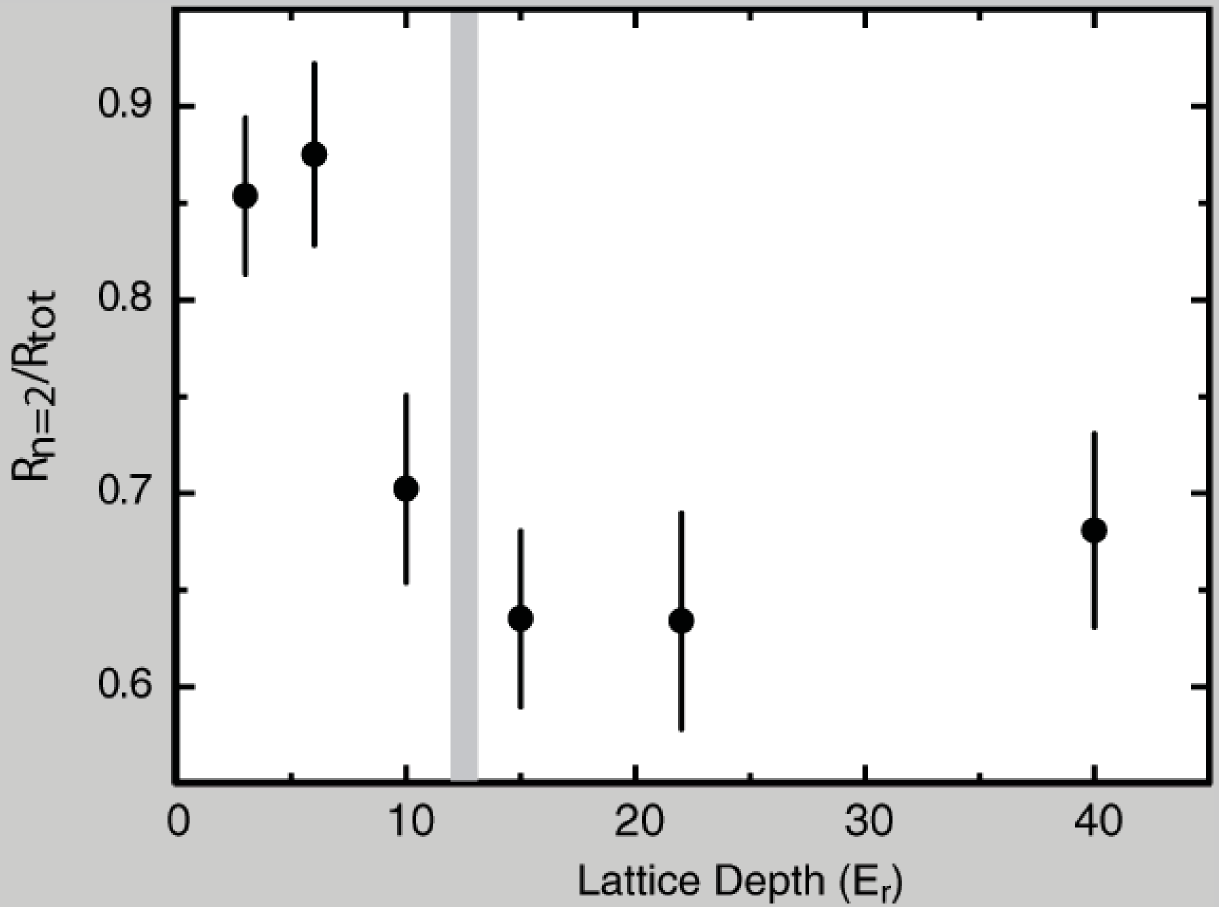
JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Mott Insulator Shell Radii



Good agreement with ab-initio T=0 theory!

In Trap Observation of the Transition from a Compressible SF to an Incompressible MI



JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Estimating Finite Temperature Effects

Let us consider isolated wells (J=0): $n_h(\mu, T) \qquad s_h(\mu, T)$

Particle & Entropy densities

Work in local density approximation $\mu_{loc}(\mathbf{r}) = \mu - V_T(\mathbf{r})$

Lowest lying excited states within Mott domains

$$U_0 n - \mu$$

Free energy cost for adding a particle

$$\mu - U(n_0 - 1)$$

Free energy cost for removing a particle

Higher lying excitations cost at least energy U and are suppressed by $e^{-\beta U}$!

Restrict States to $n_0 - 1, n_0, n_0 + 1$

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Onsite Thermodynamics

Onsite Partition Function: $$z_0 = \sum_n e^{-\beta(E(n)-\mu n)}$$

with $$E(n) = \frac{1}{2} U n(n-1)$$

We obtain

$$\bar{n}_0 \approx n_0 + \left(B^{(+)} - B^{(-)}\right)/z_0$$

$$Var(n)_0 \approx \left(B^{(+)} + B^{(-)}\right)/z_0^2$$

with $$B^{(+)} = e^{\beta(\mu - U n_0)} \quad B^{(-)} = e^{\beta(U(n_0-1)-\mu)}$$

$$z_0 = 1 + B^{(+)} + B^{(-)}$$

being the Boltzmann factors for the addition/subtraction of a particle from the „background“ value n0.

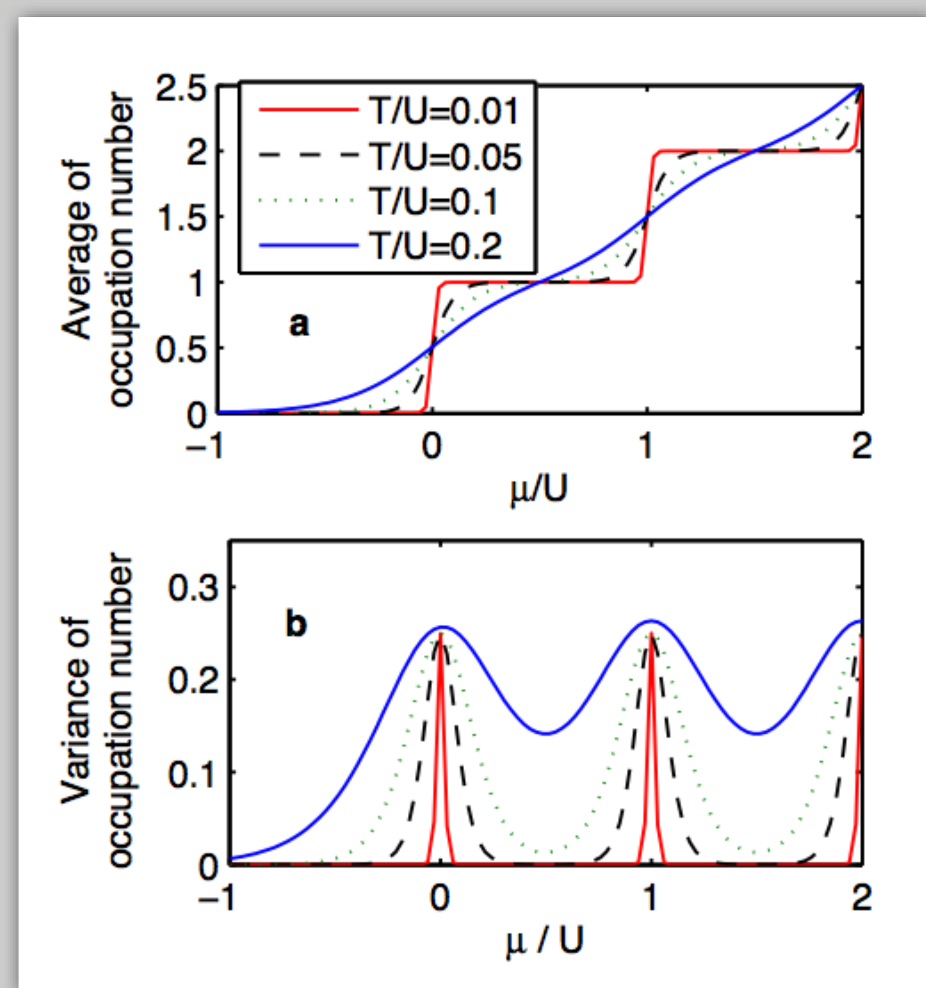
F. Gerbier arXiv:0705.3956

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Thermal Effects on Shell Structure

Shell structure is completely destroyed by thermal defects for:

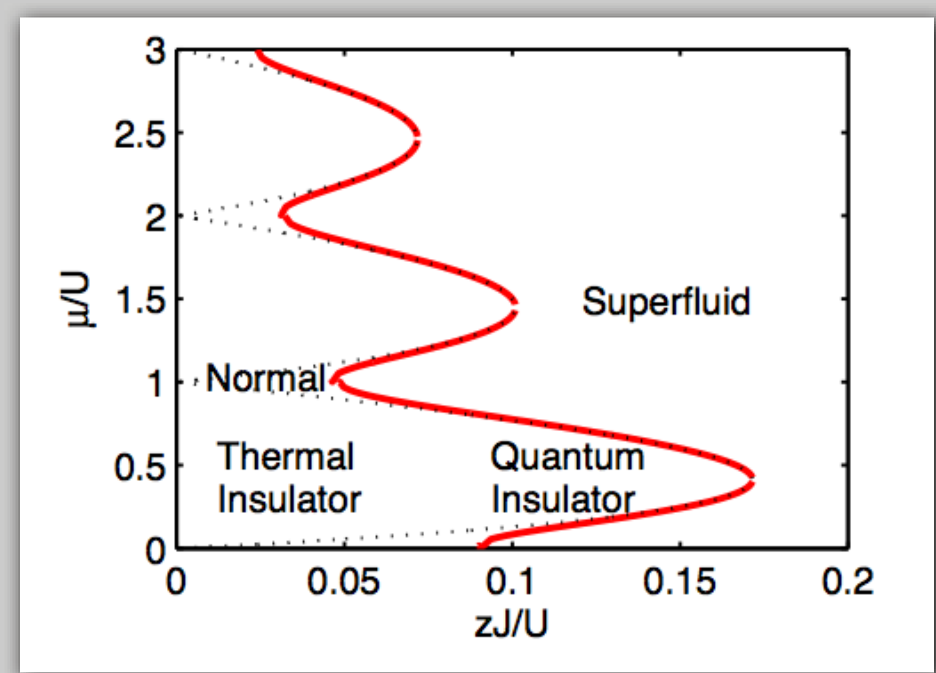
$$T^* \approx 0.2U/k_B$$



F. Gerbier arXiv:0705.3956

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Thermal Effects for Finite J



Superfluid shells will turn normal for

$$k_B T_c \approx zJ(n_0+1)/2$$

with

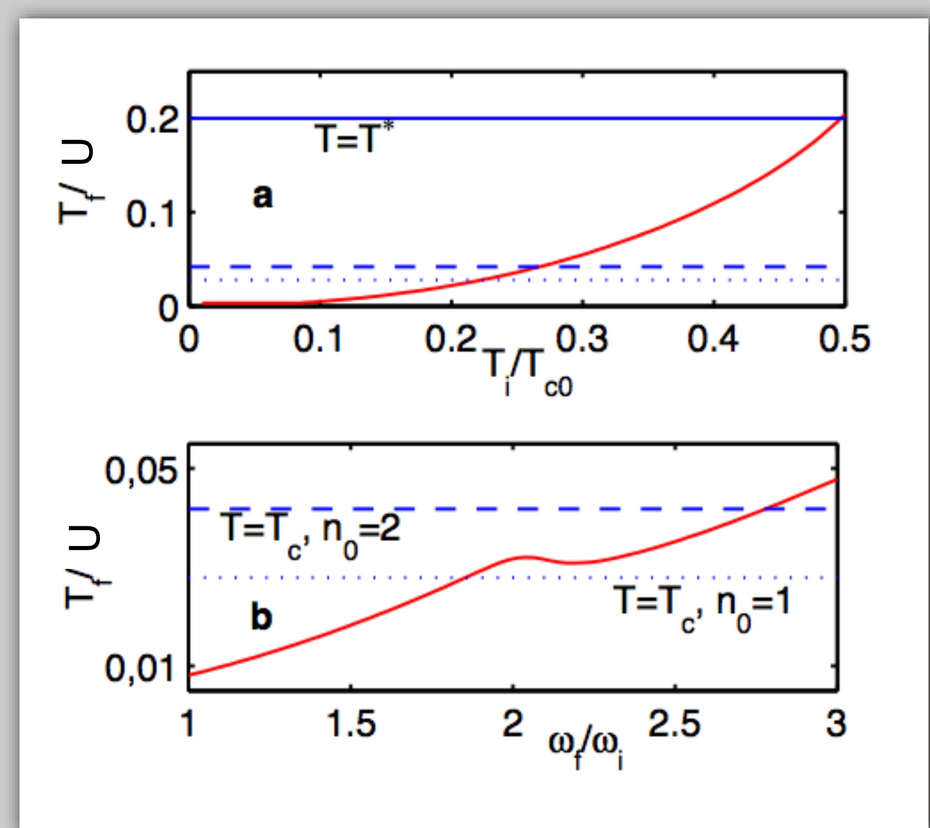
$$T_c \ll T^*$$

F. Gerbier arXiv:0705.3956,
see also: T.-L. Ho cond-mat/0703169

What can we hope to reach?

For perfect adiabatic loading, entropy in BEC equals entropy in MI state!

Load into 20 E_r lattice.



For fixed trap frequency

$$\omega_f = 2\pi \times 70\text{Hz}$$

For fixed initial temperature

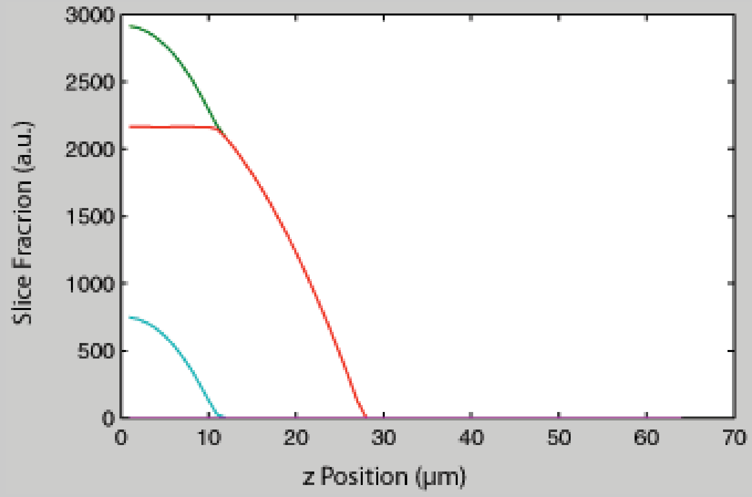
T_i=0.3 T_{c0}

F. Gerbier arXiv:0705.3956

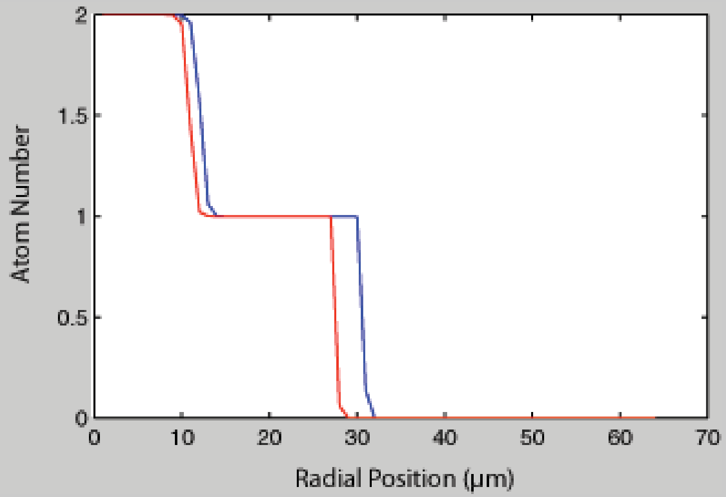
JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Finite Temperature Effects (1)

Integrated Profiles



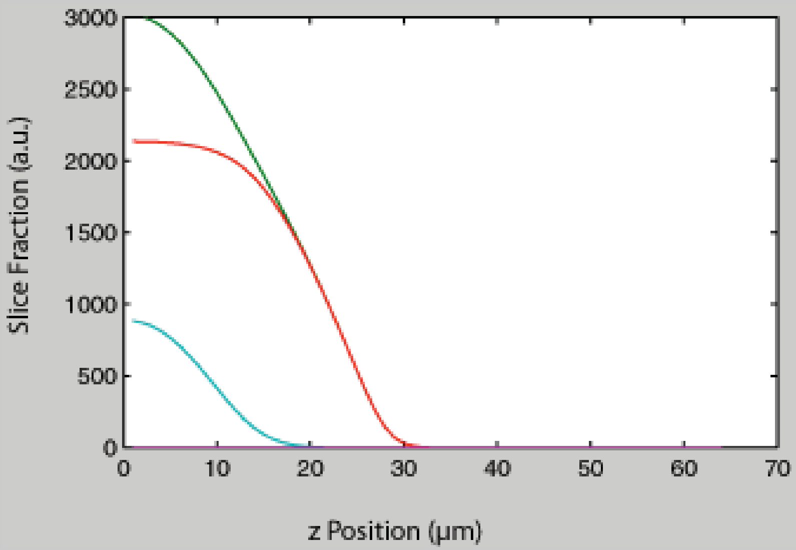
Radial Profiles



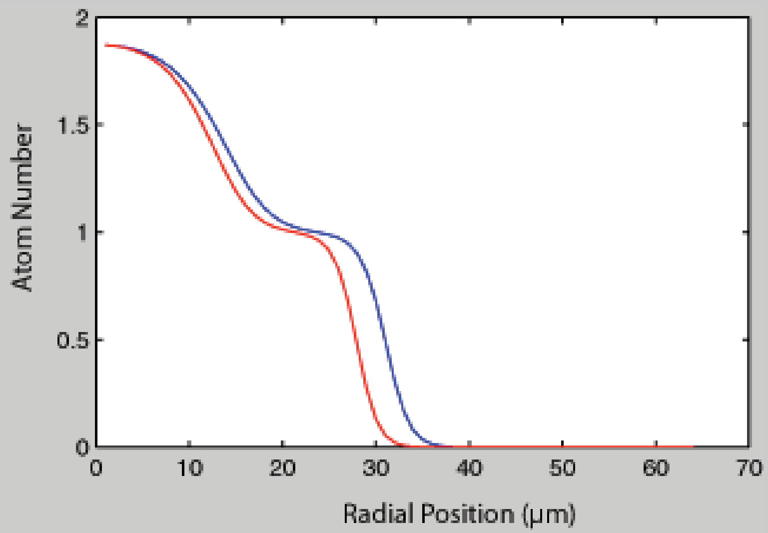
$k_BT=0.01U$

Finite Temperature Effects (2)

Integrated Profiles



Radial Profiles

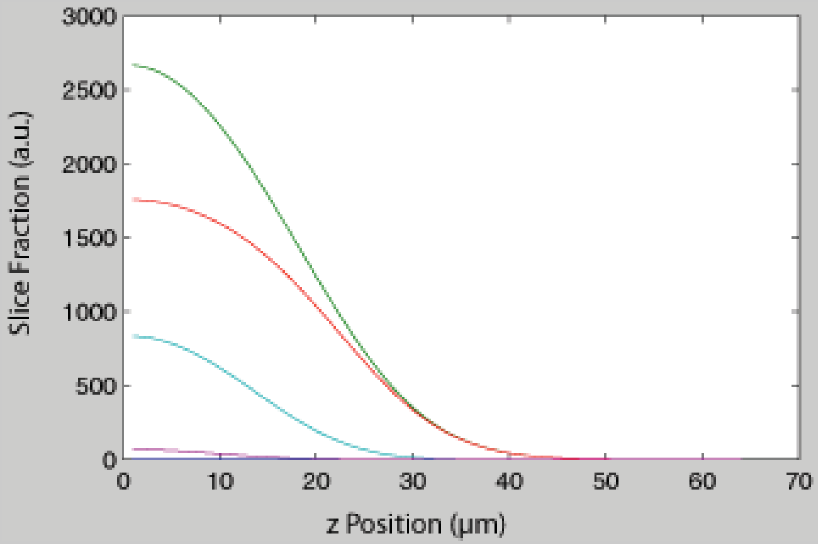


$k_B T = 0.1 U$

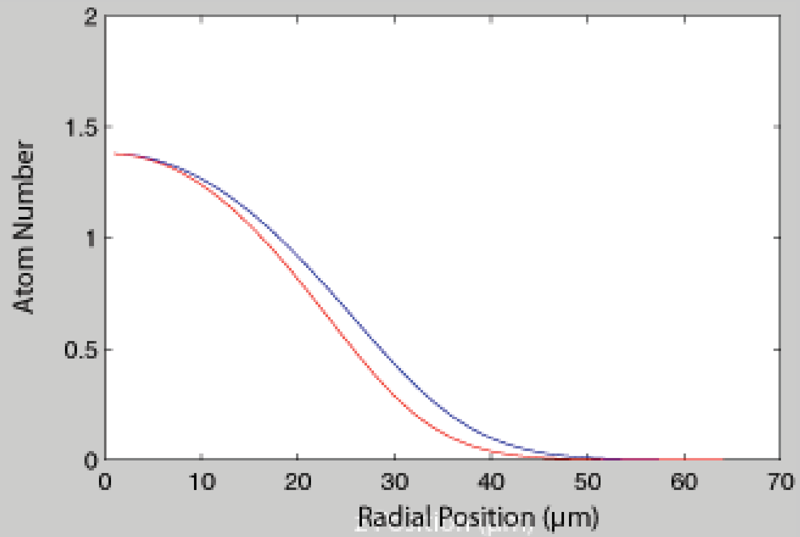
JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Finite Temperature Effects (3)

Integrated Profiles



Radial Profiles



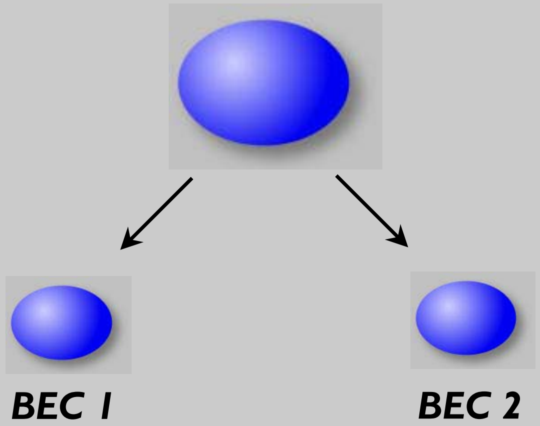
$k_B T = 0.5U$

Comparing with our measured integrated profiles, we find that

$k_B T_{exp} < 0.1U$

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

What Happens to the Relative Phase of two Quantum Liquids over Time ?



Start with a single Bose-Einstein condensate

Split it into two BECs with a constant relative phase ϕ

Relative phase ϕ

Fundamental question arises:

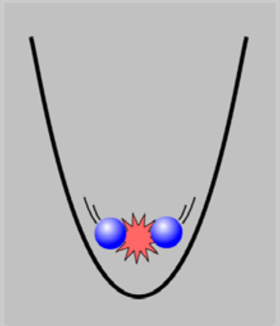
What happens to the relative phase between the two condensates over time ?

What happens to the individual wave functions of the two BECs over time ?

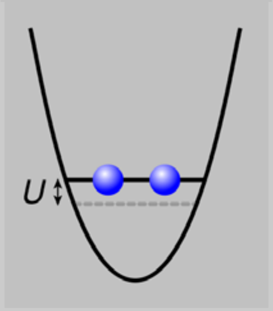
M. Greiner, O. Mandel, T. W. Hänsch and I. Bloch
Nature, 419 (6901), 2002

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Dynamical Evolution of a Many Atom State due to Cold Collision



How do collisions affect the many body state in time ?



Phase evolution of the quantum state of two interacting atoms:

Collisional phase

$$|2\rangle(t) = |2\rangle \times e^{-iUt/\hbar}$$

- *Phase shift is coherent !*
- *Can be used for quantum computation (see Jaksch, Briegel, Cirac, Zoller schemes)*
- *Leads to dramatic effects beyond mean-field theories !*

Collisional phase of n-atoms in a trap:

$$E_n t/\hbar = \frac{1}{2} U n (n-1) t/\hbar$$

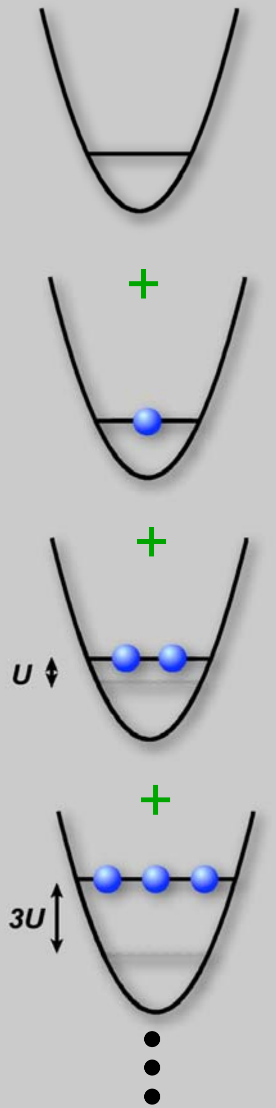
JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Time Evolution of a Coherent State due to Cold Collisions

Coherent state in each lattice site !

$$|\Psi\rangle_i = e^{-|\alpha|^2/2} \sum_n \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

=



1

1

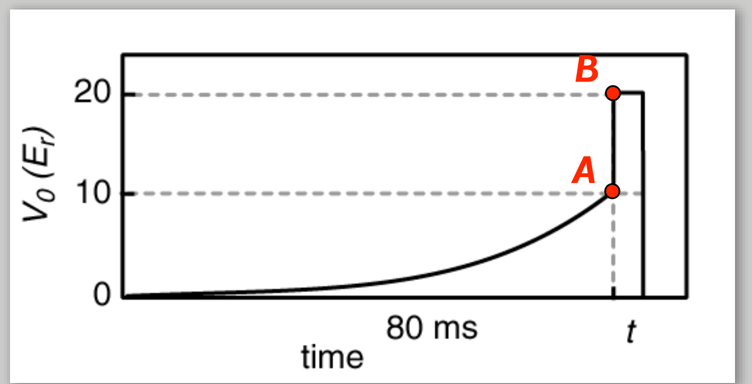
$e^{-iUt/\hbar}$

$e^{-i3Ut/\hbar}$

1. ***Here α = amplitude of the coherent state***
2. ***Here $|\alpha|^2$ = average number of atoms per lattice site***

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Freezing Out Atom Number Fluctuations



Ramp up lattice fast from the superfluid regime (A) to the MI regime (B), such that atoms do not have time to tunnel !

Atom number fluctuations at (A) are "frozen" !

$$\left|\Psi(0)\right\rangle_i = e^{-|\alpha|^2/2}\sum_n \frac{\alpha^n}{\sqrt{n!}}\left|n\right\rangle$$

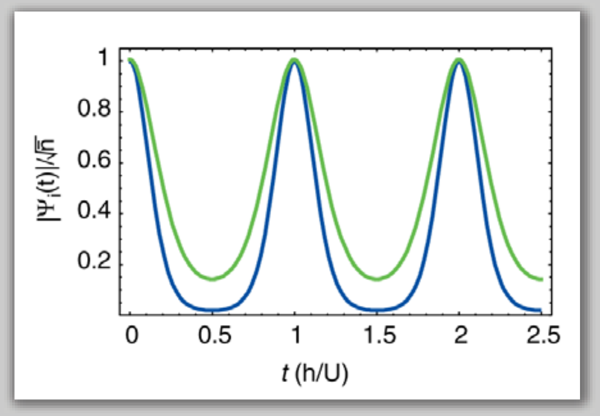
Collapse and Revival of the Matter Wave Field due to Cold Collisions

Quantum state in each lattice site (e.g. for a coherent state)

$$\left|\Psi(t)\right\rangle_i = e^{-|\alpha|^2/2} \sum_n \frac{\alpha^n}{\sqrt{n!}} e^{-i\frac{1}{2}U n(n-1)t/\hbar} \left|n\right\rangle$$

Matter wave field on the i^{th} lattice site

$$\Psi_i(t) = {}_i\left\langle \Psi(t) \right| \hat{a}_i \left| \Psi(t) \right\rangle_i$$



1. ***Matter wave field collapses but revives after times multiple times of h/U !***
2. ***Collapse time depends on the variance σ_N of the atom number distribution !***

Yurke & Stoler, 1986, F. Sols 1994; Wright et al. 1997; Imamoglu, Lewenstein & You et al. 1997, Castin & Dalibard 1997, E. Altman & A. Auerbach 2002, G.-B. Jo et al 2006

Similiar to Collapse and Revival of Rabi-Oscillations in Cavity QED !

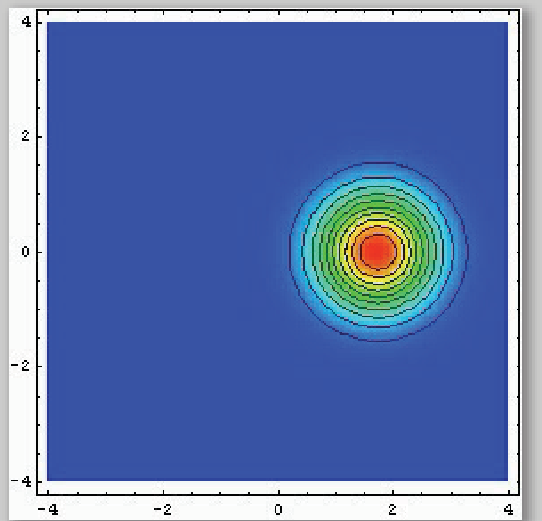
JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Dynamical Evolution of a Coherent State due to Cold Collisions

The dynamical evolution can be visualized through the Q-function

$$Q = \frac{\left|\langle \alpha | \psi_i(t) \rangle\right|^2}{\pi}$$

Characterizes overlap of our input state with an arbitrary coherent state $|\alpha\rangle$

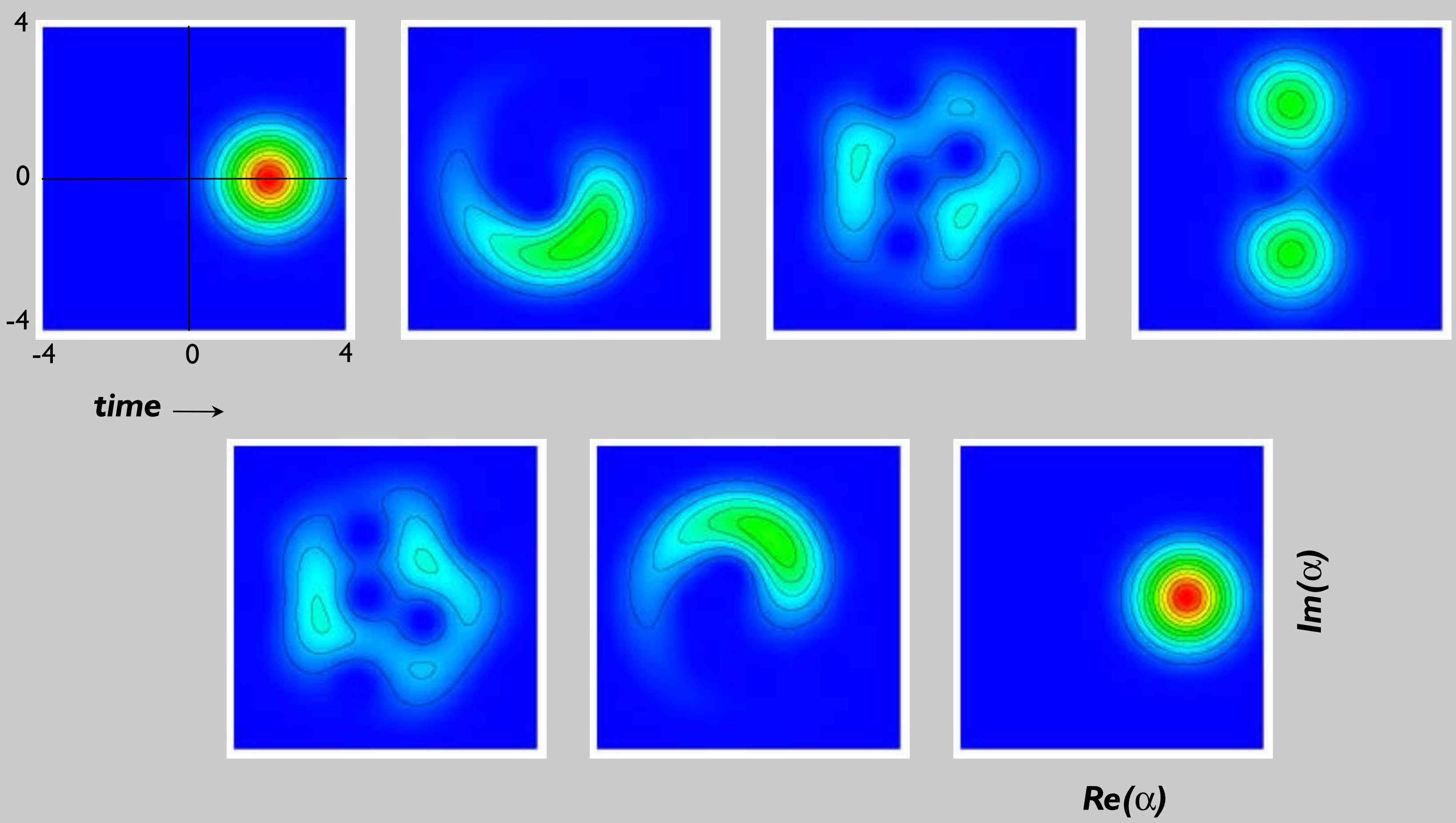


Im(α)

Re(α)

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

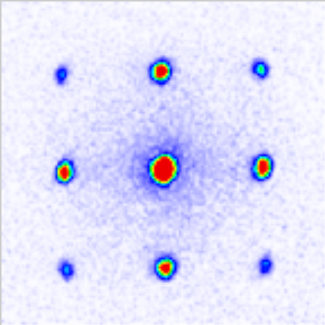
Dynamical Evolution of a Coherent State due to Cold Collisions



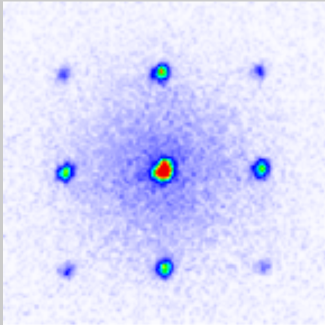
G.J. Milburn & C.A. Holmes PRL 56, 2237 (1986);
B. Yurke & D. Stoler PRL 57, 13 (1986)

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

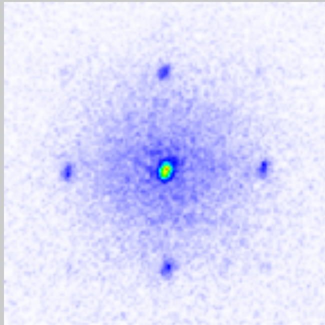
Dynamical Evolution of the Interference Pattern



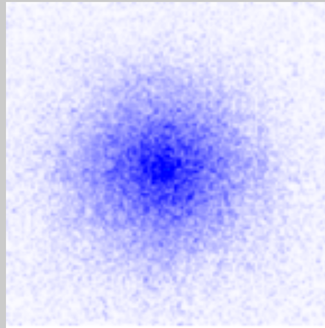
t=50 μs



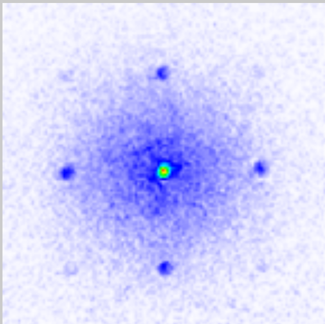
t=150 μs



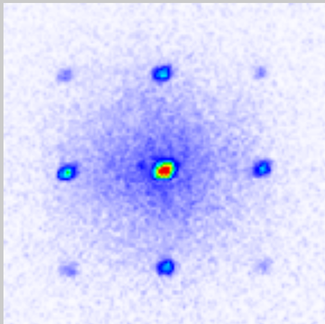
t=200 μs



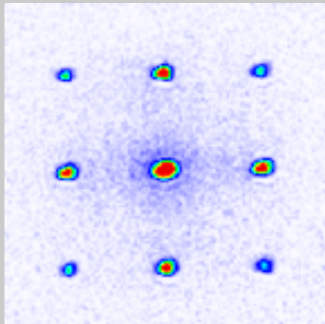
t=300 μs



t=400 μs



t=450 μs

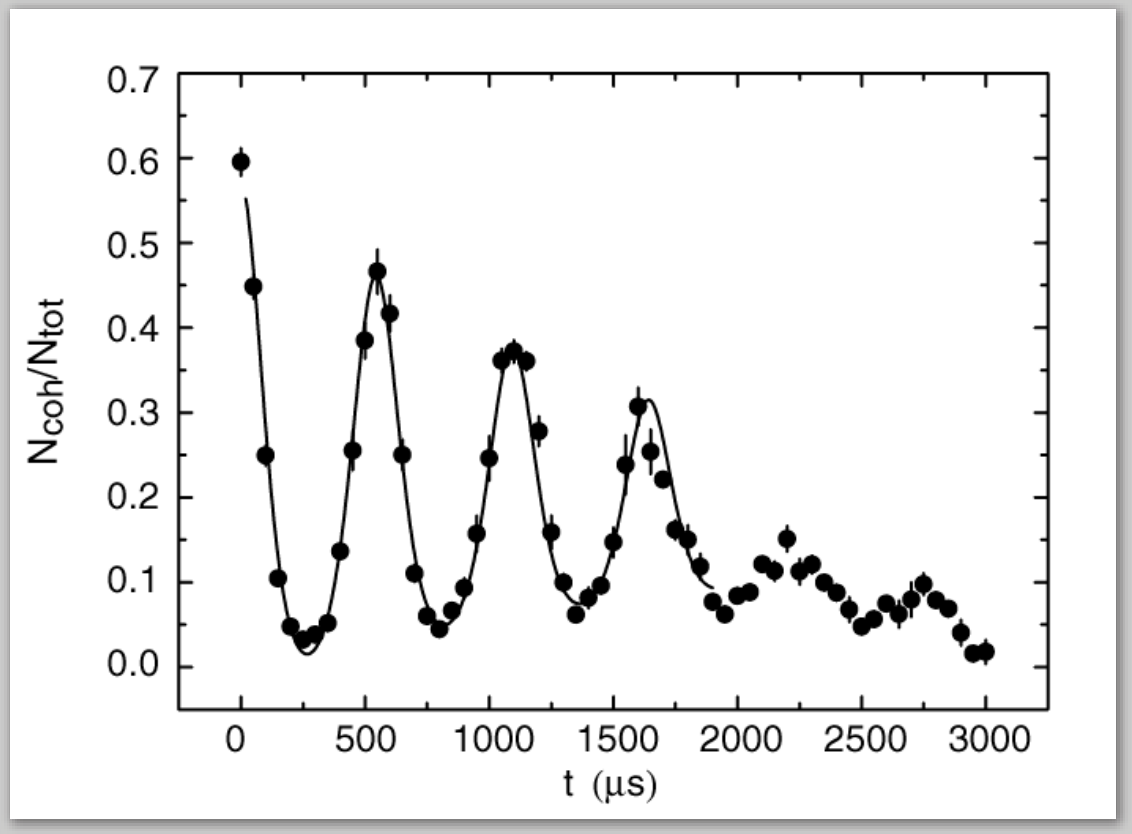


t=600 μs

After a potential jump from $V_A=8E_r$ to $V_B=22E_r$.

Collapse and Revival N_{coh}/N_{tot}

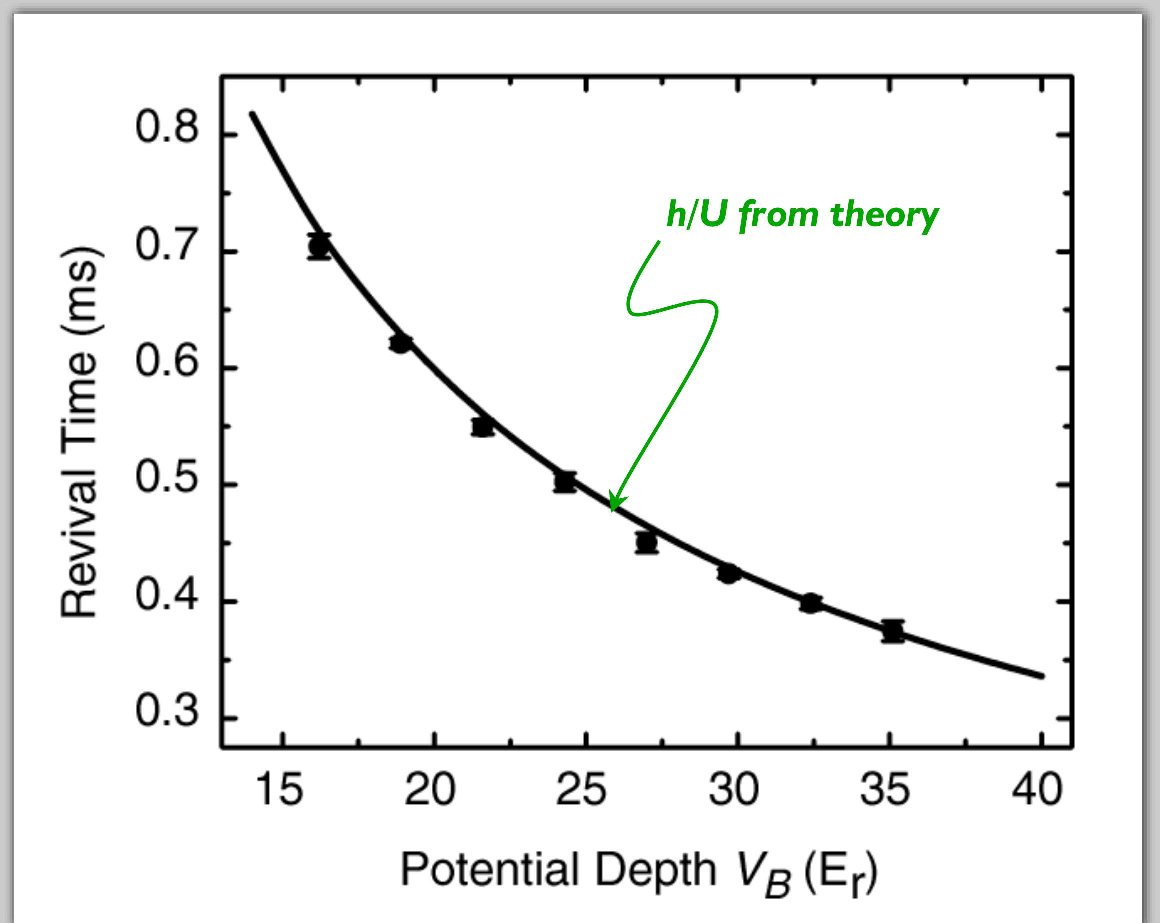
Oscillations after lattice potential jump from 8 E_{recoil} to 22 E_{recoil}



Up to 5 revivals are visible !

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

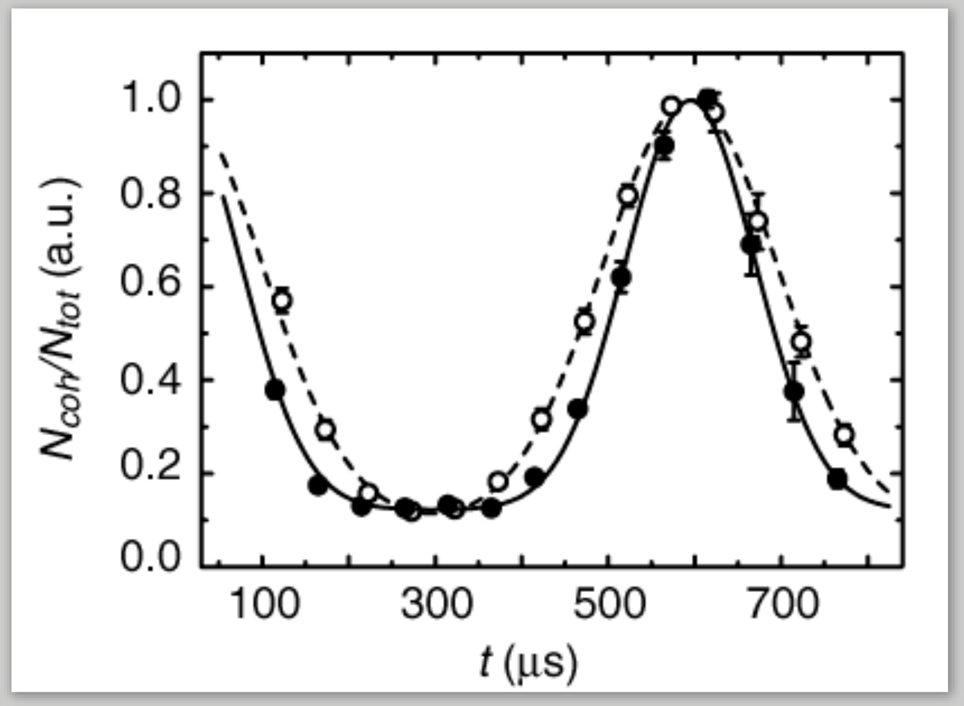
Revival Frequency vs. Lattice Potential Depth

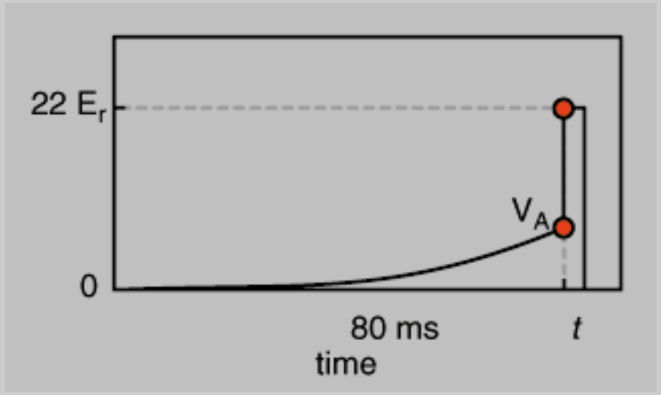


JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Influence of the Atom Number Statistics on the Collapse Time

Final potential depth $V_B=22E_r$





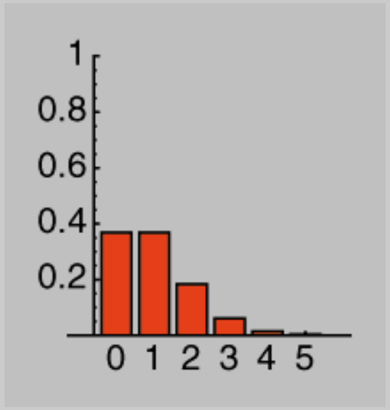
○ $V_A=11E_r$

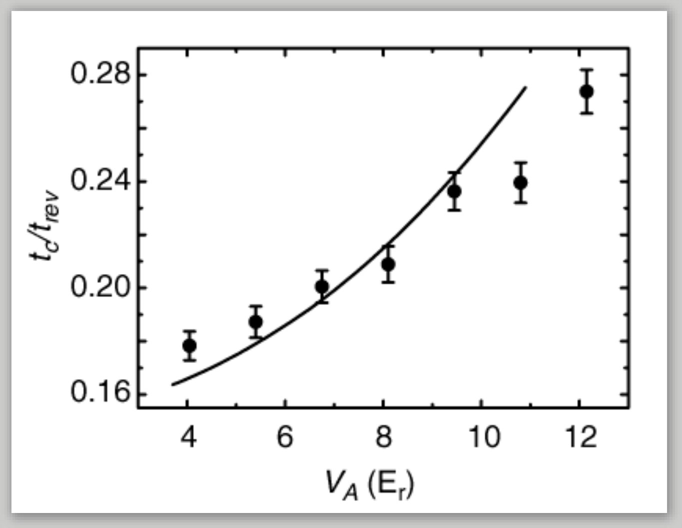
● $V_A=4E_r$

t_c/t_{rev} for Different Initial Potential Depths

Atom Number Statistics

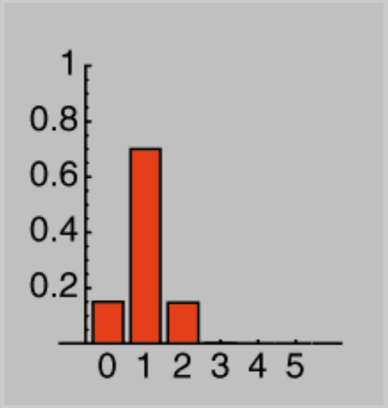
***n*=1, U/J≈0**





Atom Number Statistics

***n*=1, U/J=17**



Independent proof of sub-Poissonian atom number statistics for finite U/J !

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

The End....
...for today...