The Abdus Salam
International Centre for Theoretical Physics

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1859-9

Summer School on Novel Quantum Phases and Non-Equilibrium Phenomena in Cold Atomic Gases

27 August - 7 September, 2007

Transport in cold atomic systems: theory and experiments

Qian Niu
University of Texas at Austin

Transport of Cold Atoms in Optical Lattices

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University of Texas at Austin

Collaborators (1995-2005) :
Biao Wu, Jie Liu, Dae-Il Choi, M. G. Raizen,

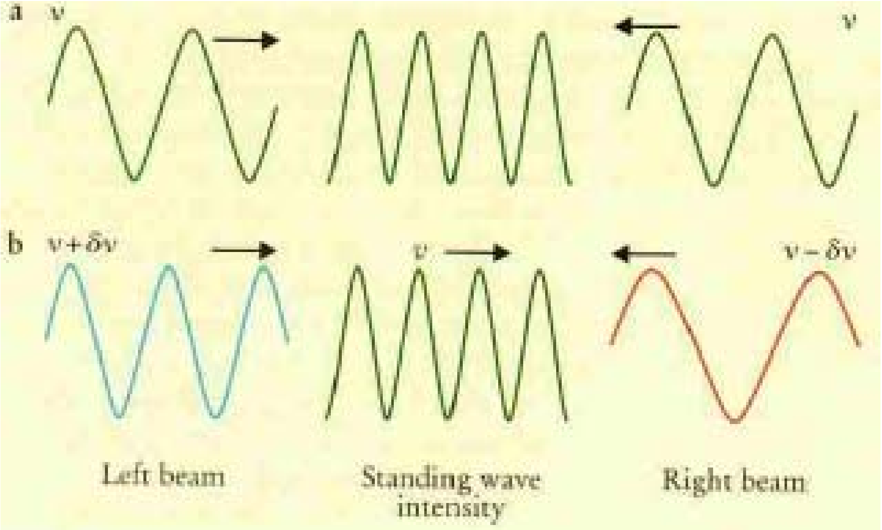
Support: NSF, Welch

Outline

1. Optical lattice and transport
2. Non-interacting cold atoms
3. Interacting BEC atoms
4. Conclusion

Optical Lattice and Motion

- Stationary



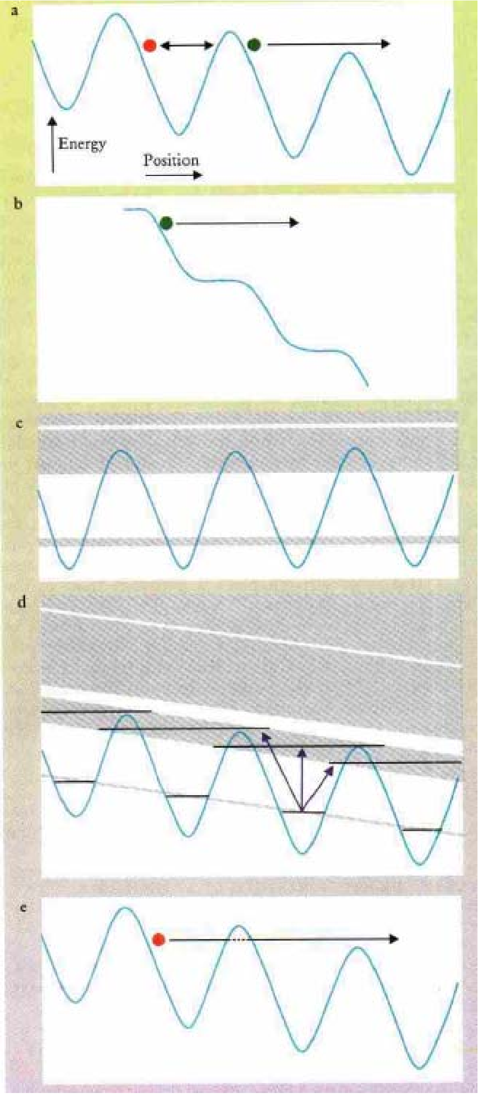
- Constant Motion

  $V = \delta\nu\,\lambda$

- Constant force
  - Gravity
  - Inertial force from acceleration

Classical and Quantum Effects

- Optical lattice + constant force = tilted lattice potential
- Classical: trapped or untrapped
- Quantum: tilted Bloch bands
  - Bloch Oscillations
  - Wannier Stark Ladders
  - Zener tunneling



Raizen, Salomon, Niu,
Physics Today, 1996.

Interacting BEC atoms

Screening of lattice potential by mean field interaction

--Choi and Niu, PRL 1999

Enhanced Landau-Zener tunnelling due to nonlinear effects

--Wu and Niu, PRA 2000; Liu, Wu, Niu PRL 2003

Dynamical and Landau instabilities of BEC Bloch state

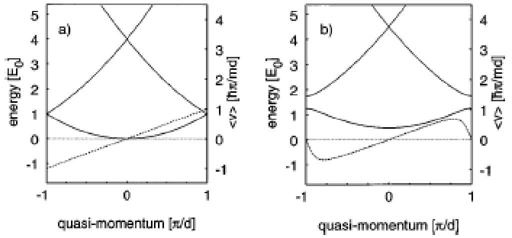
--Wu and Niu, PRA 2001

Non-interacting cold atoms

- Pre-BEC, laser cooled atoms
- Low density
  - – non-interacting. No distinction between bosons and fermions. Single particle physics.
- Cold
  - – Kinetic energy of atoms are comparable to photon recoil energy
  - – Quantum dynamics in optical lattice

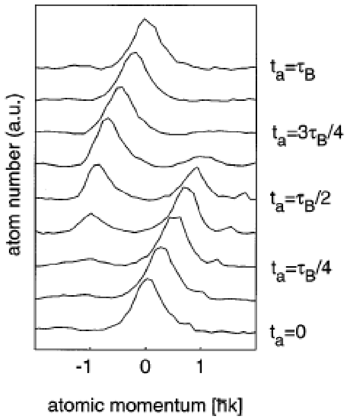
Bloch Oscillations

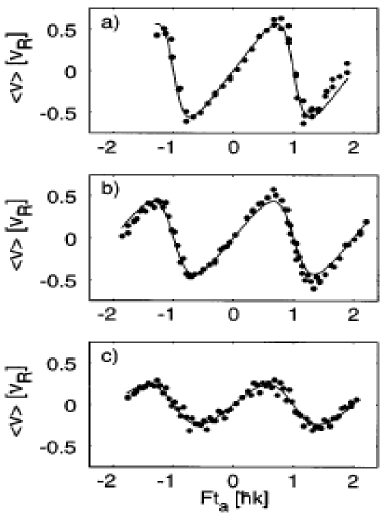
- Bloch bands



- Crystal momentum

$$dq/dt = F / \hbar$$





- Velocity

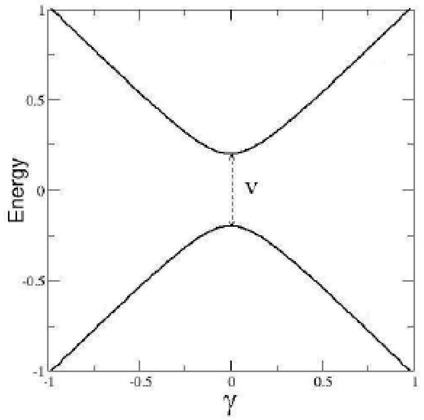
$$v = dE_n/dq$$

Bahan et al, PRL 1996

Landau-Zener Tunneling

- Two level system

$$i\frac{d}{dt}\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}=\frac{1}{2}\begin{pmatrix}\gamma & \mathrm{v}\\ \mathrm{v} & -\gamma\end{pmatrix}\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix},\quad \gamma=\alpha t$$



- Tunneling probability

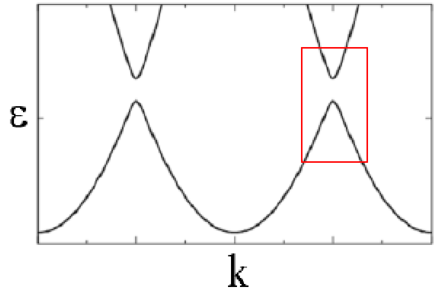
$$r=\exp\left(-\frac{\pi \mathrm{v}^2}{2\alpha}\right)$$

Tunneling between bands

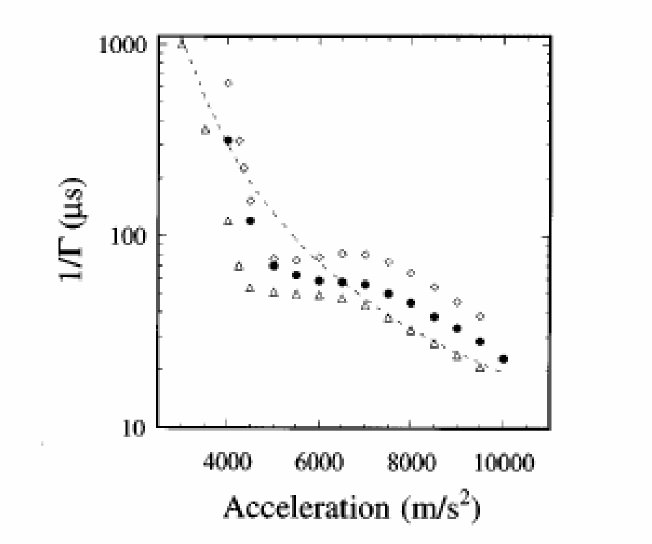
- Tunneling rate across a gap

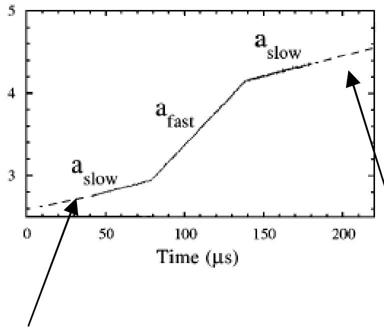
$$\gamma = \alpha \exp(-\alpha_c/\alpha), \qquad \alpha_c = \pi\Delta^2/K$$

  - Niu et al, PRL 1996



- Experiment
  - Bharucha et al, PRA 1997





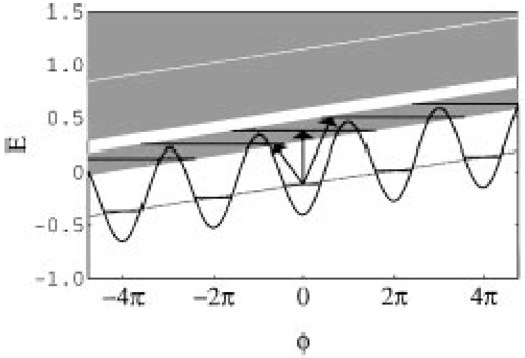
Clean up atoms above gap

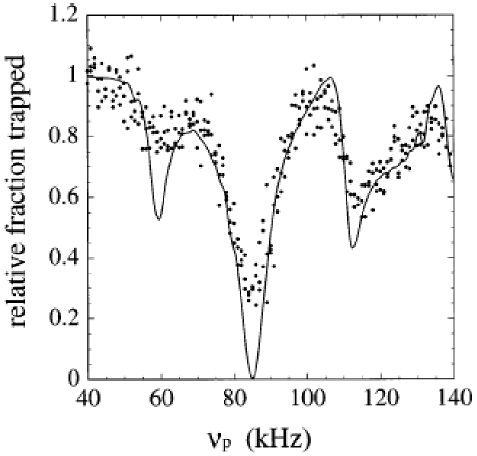
Wannier-Stark Ladders

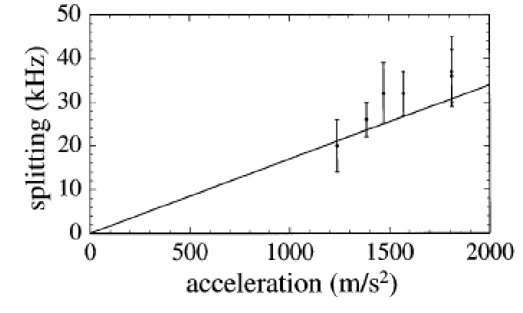
- Tilted Bloch bands and ladders

$$\delta E = h/\tau_B$$

  - Niu et al PRL 1996
- Experiment
  - Wilkinson et al PRL 1996



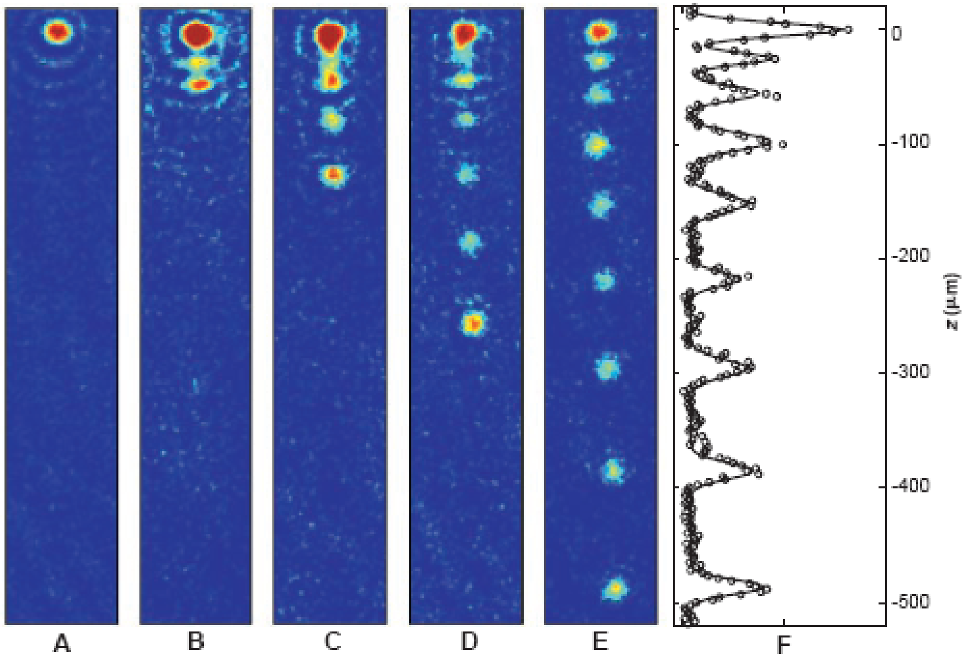




Splitting = $\delta E / h$

Bloch Oscillation and Zener Tunneling of BEC

- BEC in an stationary optical lattice under gravity – Anderson et al, Science 1998



Interaction Effects

- Mott insulator at strong interaction
- Interaction effects already manifest well within the weak coupling regime
  - Focus of this talk

Screening effects

- GP equation $$i\frac{\partial \phi}{\partial t} = -\frac{1}{2}\frac{\partial^2 \phi}{\partial x^2} + V_0 \cos(x)\phi + C|\phi|^2\phi$$
- Effective potential $$V_{\mathrm{eff}} = V_0/(1 + 4C).$$
- Experiment:
  - Morsch et al PRL 2001

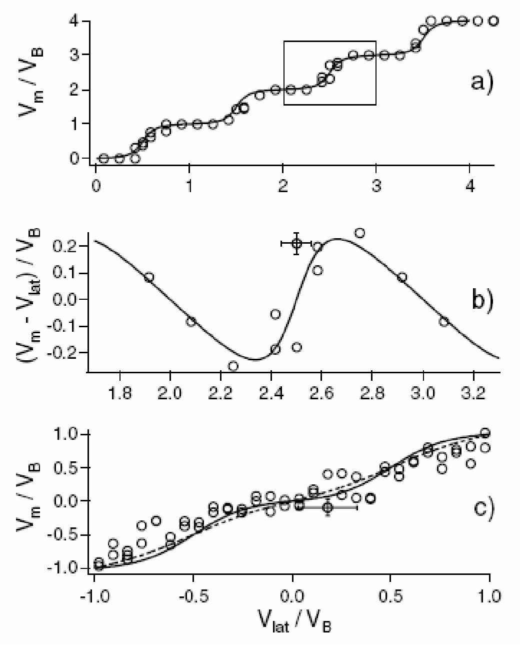


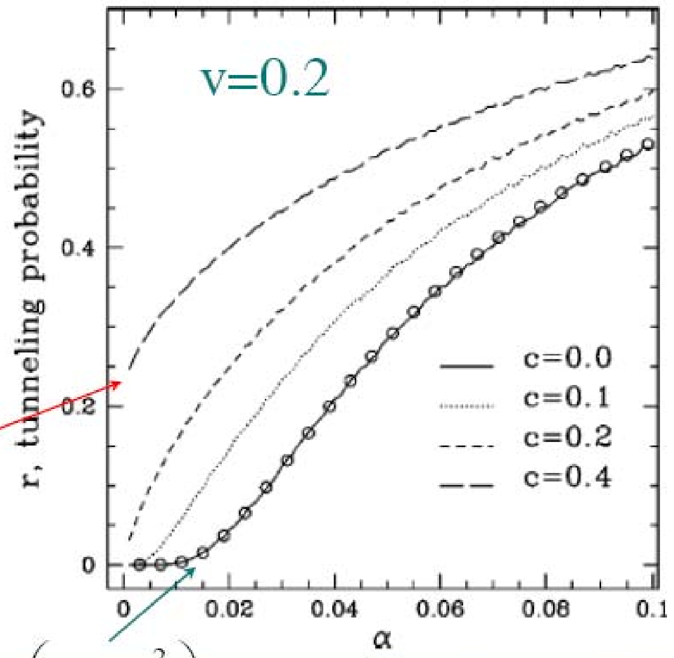
FIG. 2. Bloch oscillations of the condensate mean velocity v_m in an optical lattice. (a) Acceleration in the counterpropagating lattice with $d = 0.39\ \mu\mathrm{m}$, $U_0 \approx 0.29E_{\mathrm{B}}$, and $a = 9.81\ \mathrm{m\,s^{-2}}$. Solid line: theory. (b) Bloch oscillations in the rest frame of the lattice, along with the theoretical prediction (solid line) derived from the shape of the lowest Bloch band. (c) Acceleration in a lattice with $d = 1.56\ \mu\mathrm{m}$, $U_0 \approx 1.38E_{\mathrm{B}}$, and $a = 0.94\ \mathrm{m\,s^{-2}}$. In this case, the Bloch oscillations are much less pronounced. Dashed and solid lines: theory for $U_0 = 1.38E_{\mathrm{B}}$ and $U_{\mathrm{eff}} \approx 0.88E_{\mathrm{B}}$.

Nonlinear Landau-Zener tunneling

Wu & Niu, PRA 2000

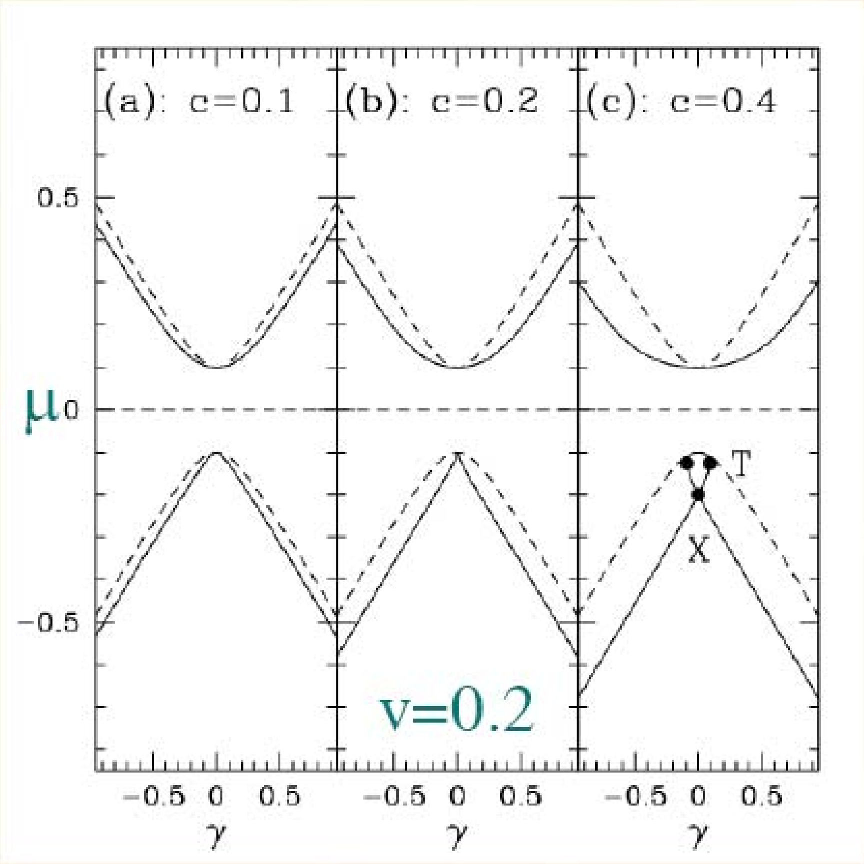
Overall, repulsive nonlinearity enhances tunneling

When c>v,
non-zero tunneling
in the adiabatic limit



$$r = \exp\left(-\frac{\pi v^2}{2\alpha}\right)$$

Loop in the band structure



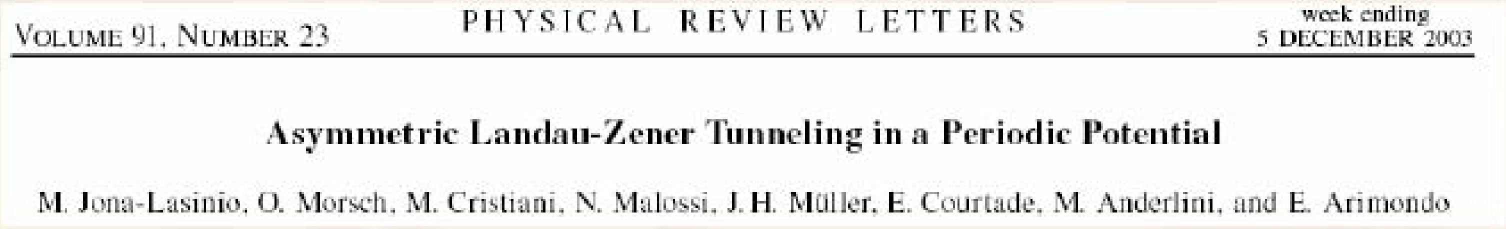
Wu&Niu PRA (2000)

Eigen-equation

$$\mu\begin{pmatrix}a\\ b\end{pmatrix}=\frac{1}{2}\begin{pmatrix}\gamma+\xi & \mathrm{v}\\ \mathrm{v} & -\gamma-\xi\end{pmatrix}\begin{pmatrix}a\\ b\end{pmatrix}$$

$$\xi = c(|\,b\,|^2 - |\,a\,|^2)$$

Experiment on
Nonlinear LZ tunneling



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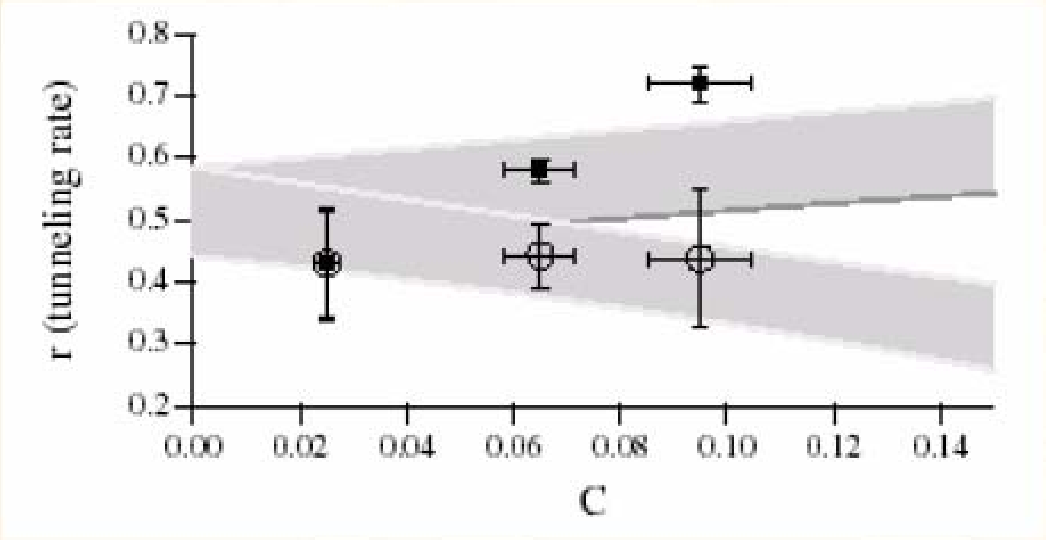
5 DECEMBER 2003

Asymmetric Landau-Zener Tunneling in a Periodic Potential

M. Jona-Lasinio, O. Morsch, M. Cristiani, N. Malossi, J. H. Müller, E. Courtade, M. Anderlini, and E. Arimondo



C<V

C >V?

Adiabatic theorem revisited

PRL 2003,2005 with Liu and Wu

- Linear quantum system
  - Condition: non-degeneracy of levels
  - Probability on each level is conserved
  - Dynamic phase + Berry phase
  - Superposition principle
- Nonlinear quantum system
  - Eigenstate: stationary point
  - General state: periodic, quasiperiodic, or chaotic orbits
  - Conservation of `classical action' or AA phase
  - Concept of Berry phase is also generalized
  - Adiabatic condition: dynamical stability

Stability of BEC states

- **Perturbation in the wave function**

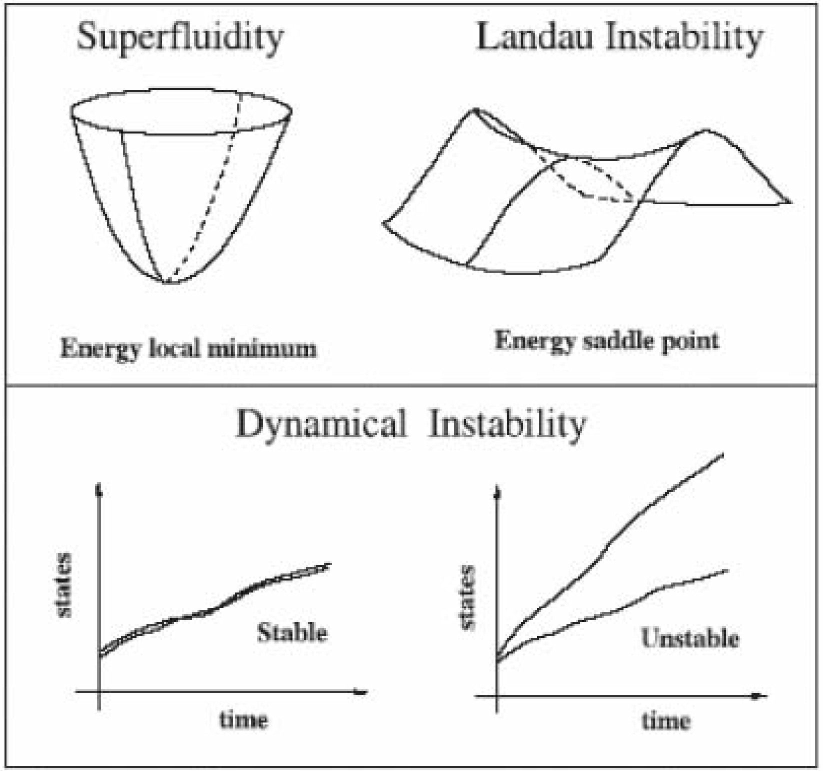
$$\psi = \psi_k + \delta\psi$$

- **Energy repsonse:**

$$E = E_k + \delta E$$

- **Dynamical response :**

Bogoliubov spectrum



Stability phase diagram

q: mode of perturbation (momentum of Excitation)

k: Bloch wavenumber or quasimomentum

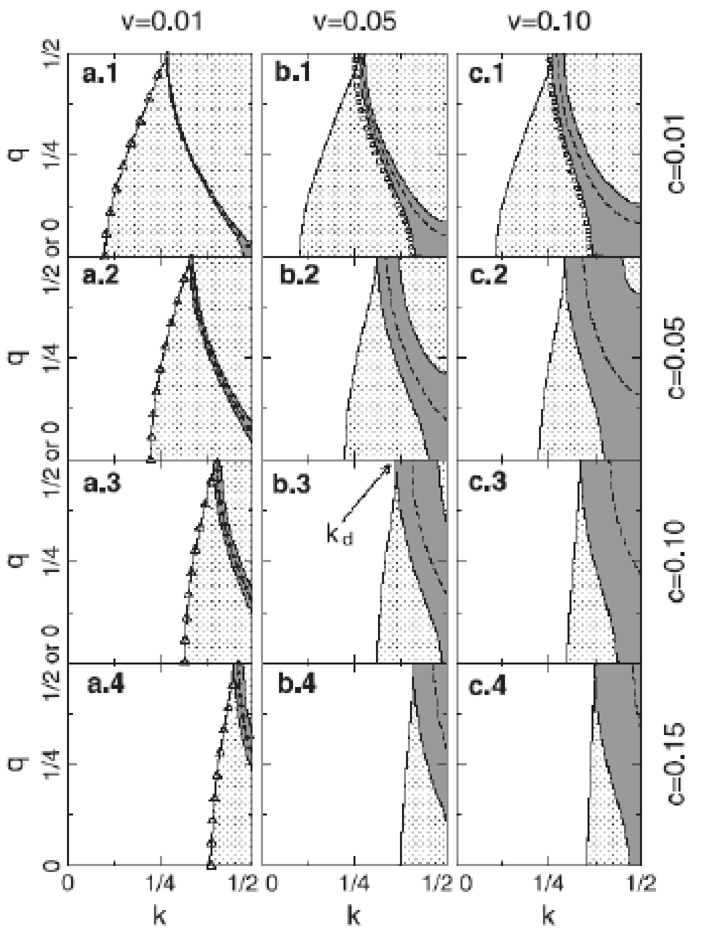
dark shaded area :

dynamical instability

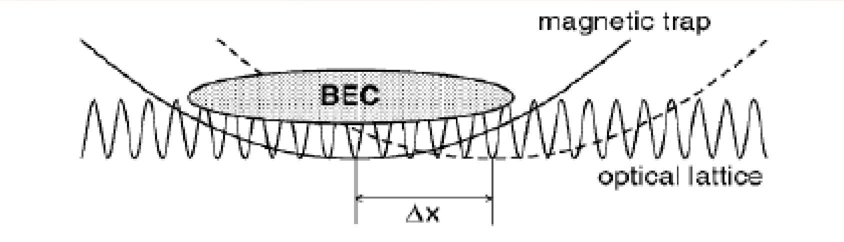
shaded area (dark or light):

Landau instability

Wu and Niu, PRA **64**, 061603(2001)



Experiment I



PRL **86** (2001)4447

$\Delta x = 30\,\mu\,\mathrm{m}$

$\Delta x = 60\,\mu\,\mathrm{m}$

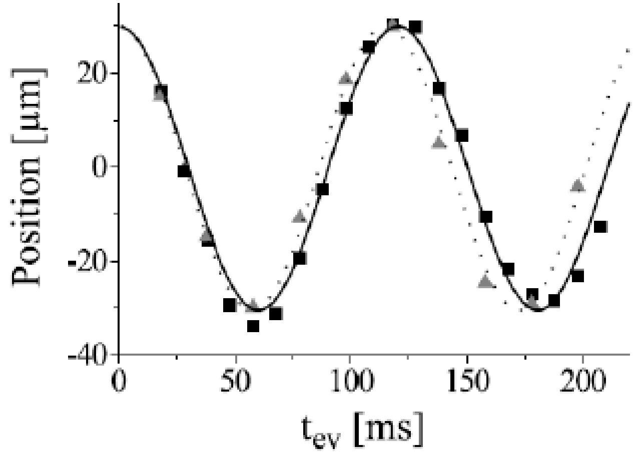
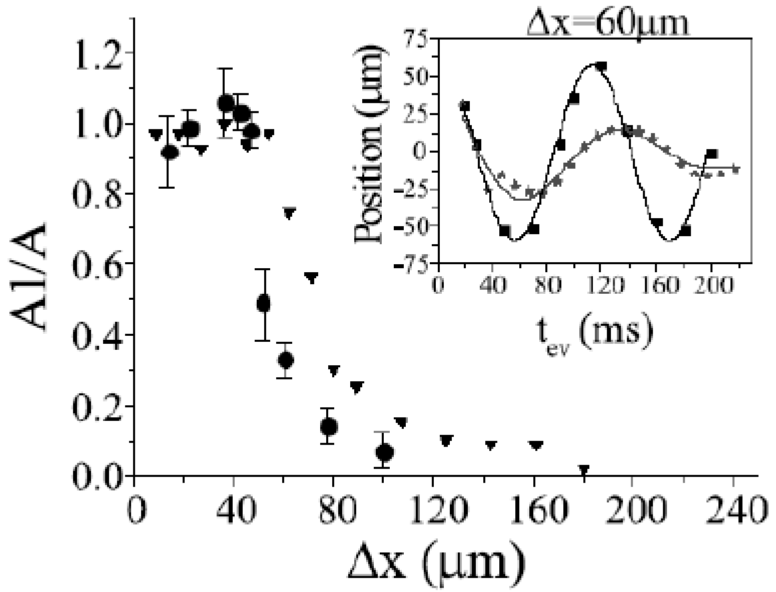
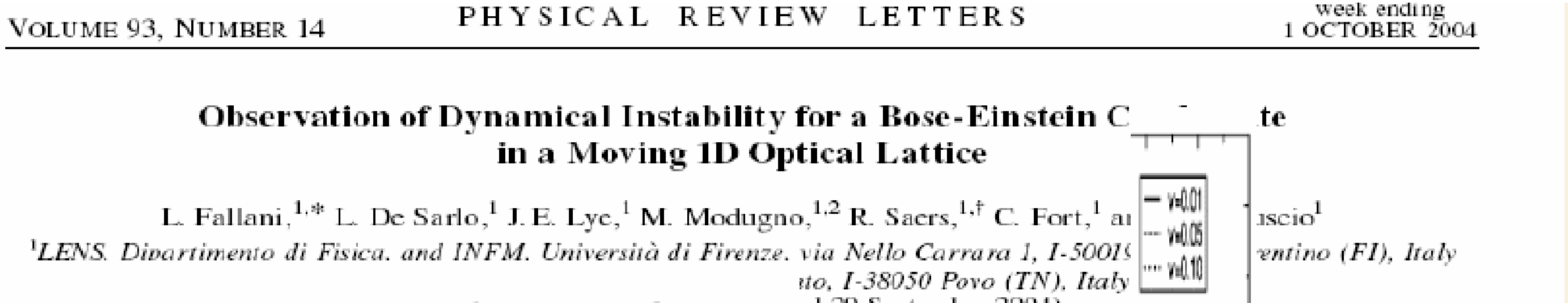


FIG. 2. Superfluid oscillations of a BEC in the presence of an optical lattice potential of height $V_0/k_B \simeq 270$ nK (squares) and in a purely magnetic trap (triangles), for initial displacement $\Delta x = (31 \pm 3)\ \mu$m. The lines give results from a numerical simulation of the 1D GPE at the experimental parameters.



Experiment II



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Observation of Dynamical Instability for a Bose-Einstein C te in a Moving 1D Optical Lattice

L. Fallani,[1,*] L. De Sarlo,[1] J. E. Lye,[1] M. Modugno,[1,2] R. Saers,[1,†] C. Fort,[1] a ıscio[1]

[1]*LENS, Dipartimento di Fisica, and INFM, Università di Firenze, via Nello Carrara 1, I-5001 entino (FI), Italy*

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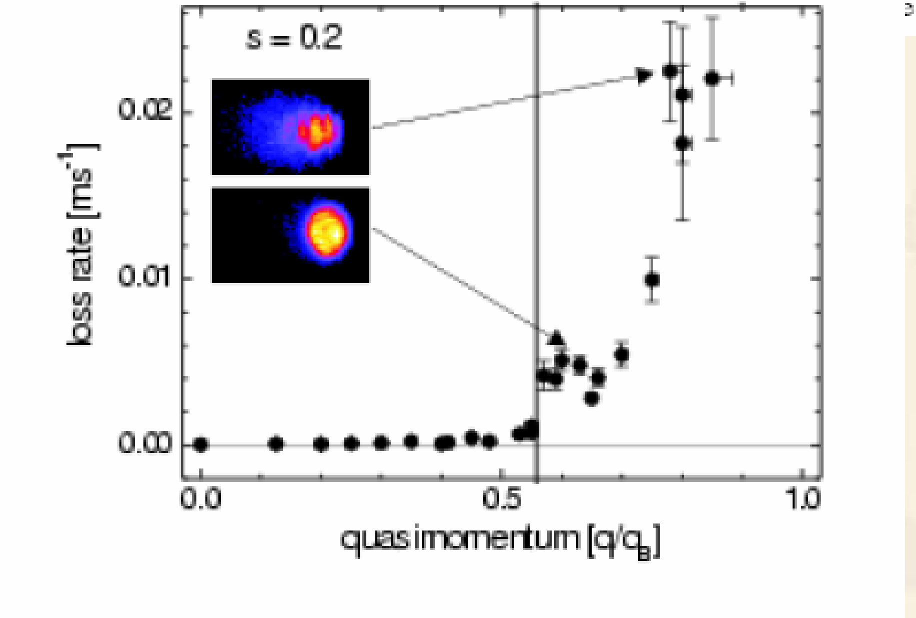
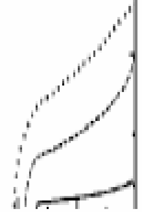
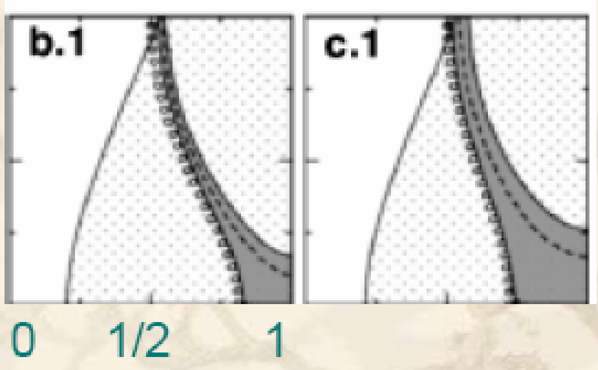
ed 29 September 2004)

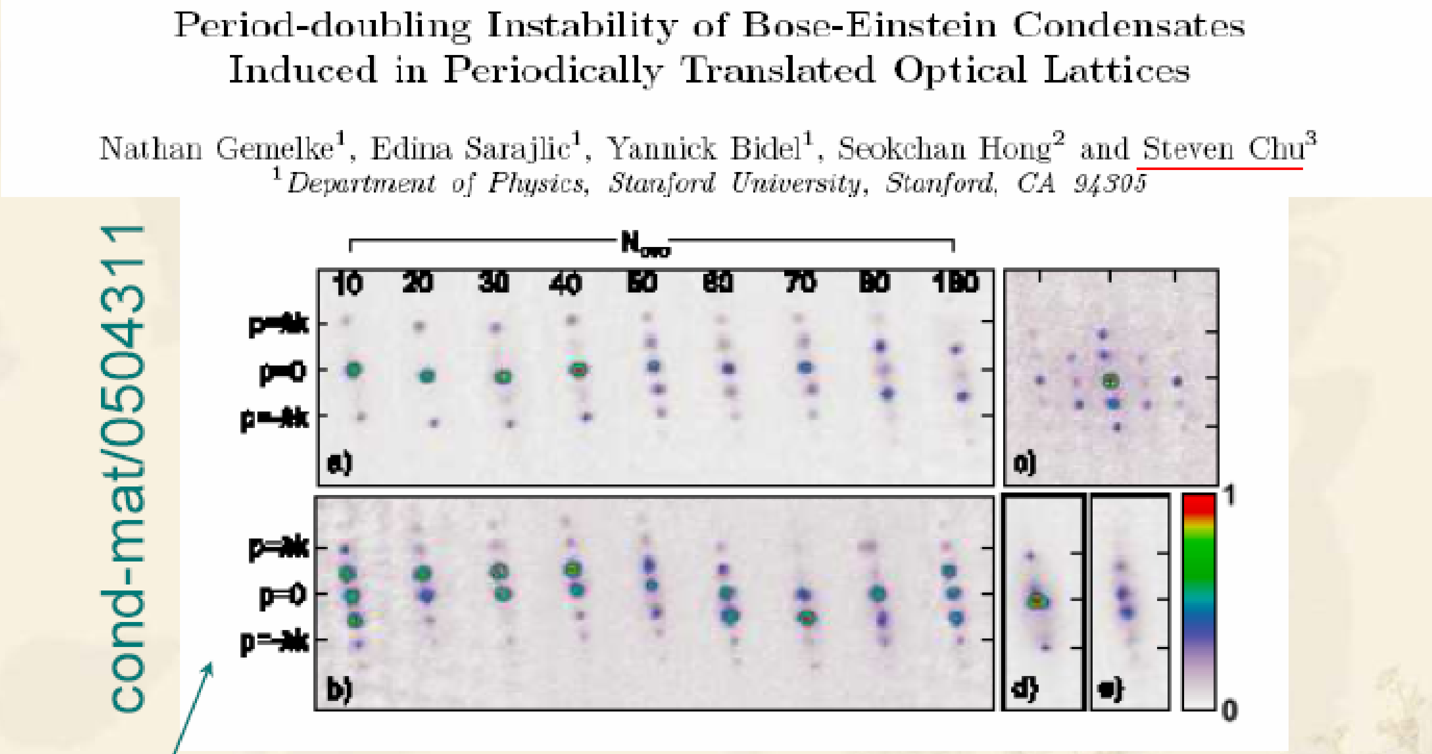






our theoretical result

Experiment III



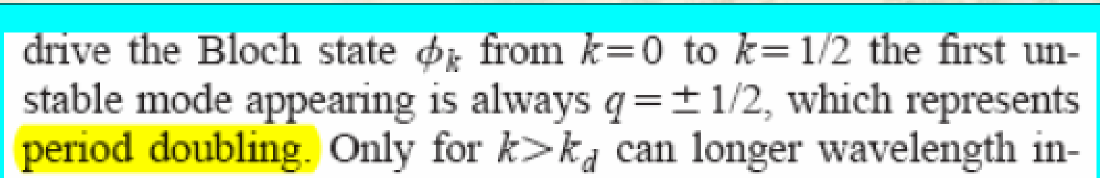
Period-doubling Instability of Bose-Einstein Condensates Induced in Periodically Translated Optical Lattices

Nathan Gemelke[1], Edina Sarajlic[1], Yannick Bidel[1], Seokchan Hong[2] and Steven Chu[3]

[1] *Department of Physics, Stanford University, Stanford, CA 94305*

cond-mat/0504311

76th cycle



drive the Bloch state ϕ_k from $k=0$ to $k=1/2$ the first unstable mode appearing is always $q=\pm 1/2$, which represents period doubling. Only for $k>k_d$ can longer wavelength in-

Wu and Niu, PRA **64**, 061603(2001)

Conclusion

- Very interesting quantum phenomena even for non-condensed cold atoms:
  - Bloch oscillations, LZ tunneling, WS ladders
- BEC and interaction lead to much richer behaviors
  - Screening, looping spectrum, dynamical instability, …
- More theoretical and experimental studies are needed on transport in strong interaction regime and near Mott transition.