The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA International Atomic Energy Agency

1859-22

Summer School on Novel Quantum Phases and Non-Equilibrium Phenomena in Cold Atomic Gases

27 August - 7 September, 2007

From laser cooling to quantum gases:
Part 1 - Laser manipulation of atoms
Part 2 - Rotating condensates
Part 3 - Cold atoms in Flatland

Jean Dalibard
ENS Paris

From laser cooling to quantum gases

Review article:
I. Bloch, J. Dalibard, W. Zwerger,
Many-Body Physics with Ultracold Gases
arXiv:0704.3011
to appear in Rev. Mod. Phys.

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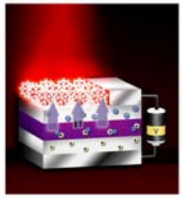
Goal of these lectures

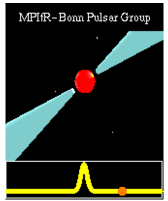
Start with basic concepts in atom manipulation and address some aspects of the physics of quantum low dimensional and/or rotating systems

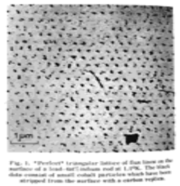
Quantum wells and MOS structures



High T_c superconductivity

Neutron stars





Superconductor in a magnetic field

VORTICES !

Outline of the three lectures

1. A brief introduction to laser cooling of atoms

   Doppler, Sisyphus, subrecoil cooling

2. Cold atoms in Flatland

   Vortices and the Berezinskii-Kosterlitz-Thouless mechanism

   Experiments with flat gases

3. Rotating gases

   From the single vortex case to strongly correlated states

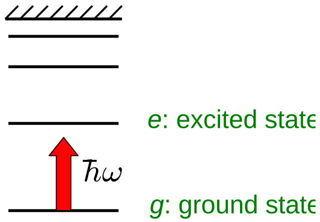
Part 1.

A brief introduction to laser cooling

Doppler, Sisyphus, subrecoil cooling

The basic process in atom-light interaction

Quasi-resonant photons: $\hbar\omega \simeq E_e - E_g$



In each absorption or emission event,
the atom velocity changes by:

$$v_{\text{rec}} = \frac{\hbar k}{m}$$

Recoil velocity: 3 cm/s for sodium (m=23), 3 mm/s for cesium (m=133)

Possible repetition rate?

→ radiative lifetime Γ^{-1} of an atomic excited state: 10 to 100 ns

The radiation pressure force

The atom undergoes a succession of absorption – spontaneous emission cycles. Each cycle changes the atom velocity by v_{rec}

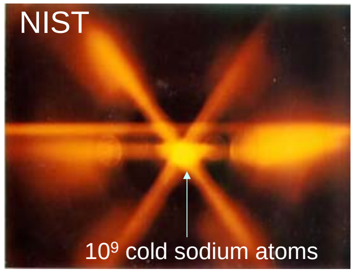
At large laser intensity, the time interval between two cycles is $2\Gamma^{-1}$

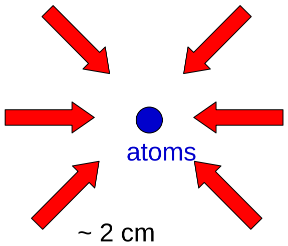
Atom acceleration: $a_{\max} = v_{\text{rec}}\,\Gamma/2$

For sodium atoms, $a_{\max}$ is 100 000 times larger than gravity

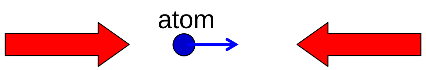
➡ Atoms moving at 100 m/s can be stopped over 1cm!

Optical molasses (Doppler cooling)





$$\hbar\omega < E_e - E_g$$



The atom moving to the right (resp. left) interacts more with the wave coming from the right (resp. left)

$\vec{F} = -\alpha\vec{v}$ → cooling

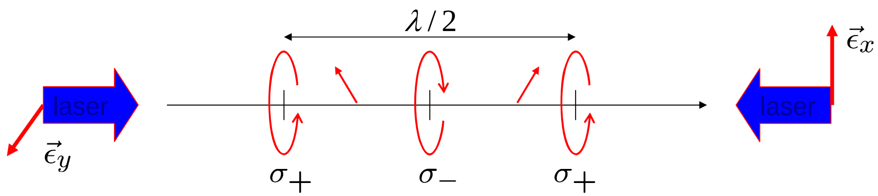
Sisyphus cooling (1)

The Doppler cooling limit is

$$k_B T > \frac{\hbar\Gamma}{2} \sim \text{a few hundred microkelvins}$$

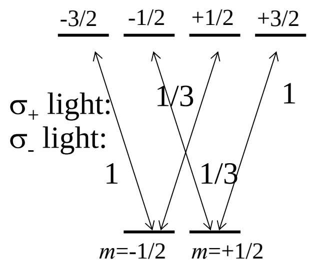
However the temperature measured in optical molasses can be as low as a few microkelvins. What is missing?

The laser field in a 3D optical molasses has not only phase and intensity gradients, but also polarisation gradients. A model 1D system:



Periodic modulation of the polarization of the light

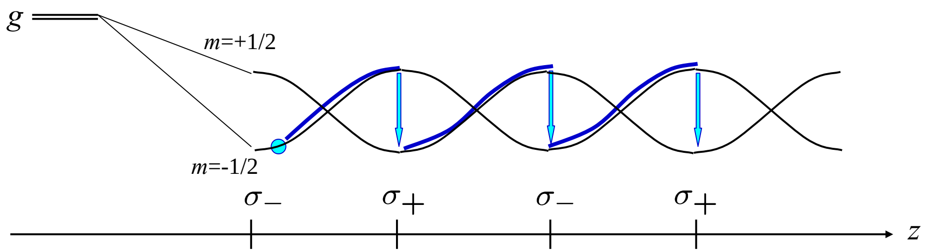
Sisyphus cooling (2)



Atom with a degenerate ground and excited levels, e.g. J_g =1/2, J_e=3/2

choose $\omega < \omega_A$

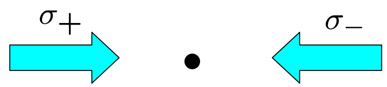
→ Periodic modulation of the atomic levels

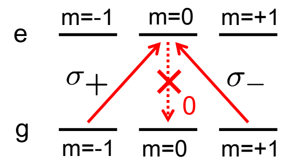


$k_B T \propto \frac{\text{intensity}}{\text{detuning}}$ until the recoil limit is reached $k_B T \sim \frac{\hbar^2 k^2}{2M}$

Light as a Maxwell demon: subrecoil cooling

Consider a g,J=1 ⟷ e,J=1 atomic transition in a 1D laser configuration





Closed Λ system

Atoms trapped in the state

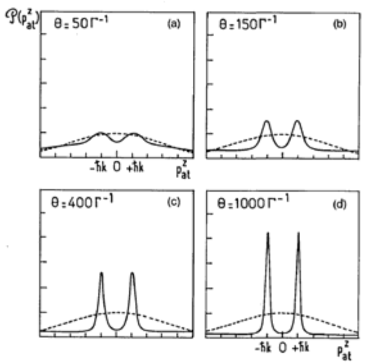
$$\frac{1}{\sqrt{2}}\left(|g, m=-1, p=-\hbar k\rangle \; - \; |g, m=+1, p=+\hbar k\rangle\right)$$

will remain in this state for ever, because

- of the destructive interference in the transitions of the only possible excited state $|e, m=0, p=0\rangle$
- it is an eigenstate of the kinetic energy $E = \hbar^2 k^2/(2m)$

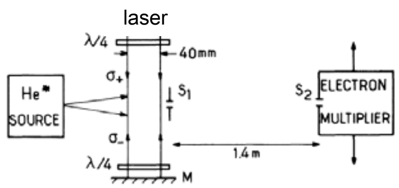
The Maxwell demon in action

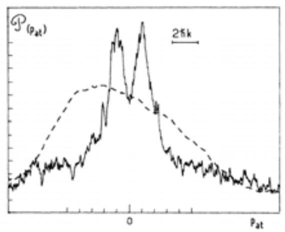
Time evolution of the atomic momentum distribution



A. Aspect, E. Arimondo, R. Kaiser, N. Vansteenkiste, and C. Cohen-Tannoudji, J. Opt. Soc. Am. B **6**, 2112 (1989).

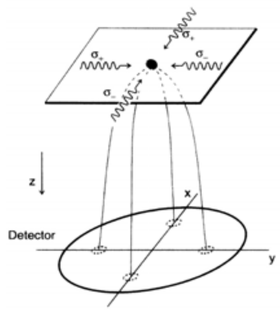
Experimental evidence





A. Aspect, E. Arimondo, R. Kaiser, N. Vansteenkiste, and C. Cohen-Tannoudji, Phys. Rev. Lett. **61**, 826 (1988).

Subrecoil cooling in 2 or 3 dimensions



J. Lawall et al, PRL 73, 1915 (1994)

Olshanii and Minogin (1992): for any monochromatic laser configuration acting on a g, J=1 ⟷ e, J=1 configuration, there is a (generally unique) state

$$\psi(\vec{r}) = \sum_{m=-1}^{+1} C_m(\vec{r}) |g, m\rangle$$

that stays uncoupled to laser light forever.

Part 2.

Cold atoms in FlatLand

Simple theoretical aspects:

BEC or not BEC?

the Berezinskii-Kosterlitz-Thouless mechanism

The ideal Bose gas in the uniform case

In 3D: when the phase space density reaches $n\lambda^3 = 2.612$, BEC occurs

$$\lambda^2 = 2\pi\hbar^2/(mk_BT)$$

In 2D: for any phase space density $n\lambda^2$, the occupation of the various states in non singular, and no BEC is expected.

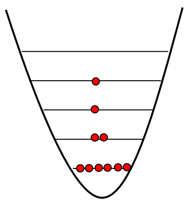
$$n\lambda^2 = -\ln\left(1 - e^{\mu/k_BT}\right)$$

μ varies from $-\infty$ to 0

The absence of BEC (i.e. long range order) at finite temperature remains true in presence of repulsive interactions (Hohenberg-Mermin-Wagner)

The ideal Bose in a harmonic potential

In 3D, BEC occurs when $N = 1.2\left(\frac{k_BT}{\hbar\omega}\right)^3$



In 2D, BEC occurs when $N = 1.6\left(\frac{k_BT}{\hbar\omega}\right)^2$

Bagnato – Kleppner (1991)

Does harmonic trapping make 2D and 3D equivalent?

What about interaction?

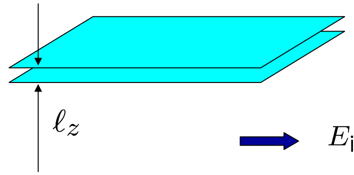
How a 3D pseudopotential is transformed in 2D ?

3D interaction energy: $E_{\text{int}} = \frac{N(N-1)}{2}\, g^{(3D)} \int |\psi(\vec{r})|^4\, d^3r$

$$g^{(3D)} = \frac{4\pi\hbar^2 a}{m}$$

Going to 2D, trial wave functions:

$$\psi(x,y,z) = \Phi(x,y)\,\frac{e^{-z^2/2\ell_z^2}}{(\pi\ell_z^2)^{1/4}}$$



$$\Rightarrow E_{\text{int}} = \frac{N(N-1)}{2}\, g^{(2D)} \int |\Phi(\vec{r})|^4\, d^2r$$

$$g^{(2D)} = \frac{\hbar^2}{m}\,\tilde{g} \qquad \tilde{g} = \sqrt{8\pi}\,\frac{a}{\ell_z}$$

To go beyond this simple reasoning: Petrov-Holzmann-Shlyapnikov

Problem with 2D BEC in a harmonic trap

Treat the interactions at the mean field level: $V_{\text{eff}}(r) = \frac{m\omega^2 r^2}{2} + 2g n_{\text{mf}}(r)$

where the mean field density is obtained from the self-consistent equation

$$n_{\text{mf}}(r)\lambda^2 = -\ln\left(1 - e^{(\mu - V_{\text{eff}}(r))/kT}\right)$$

Two remarkable results

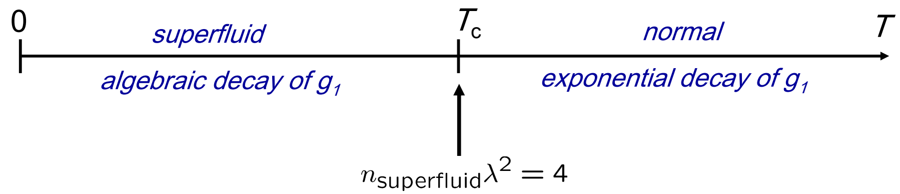
- One can accommodate an arbitrarily large atom number. Badhuri et al
- The effective frequency deduced from $V_{\text{eff}}(r) \simeq m\omega_{\text{eff}}^2 r^2/2$ tends to zero when $\mu \to 2g n_{\text{mf}}(0)$ Holzmann et al.

Similar to a 2D gas in a flat potential…

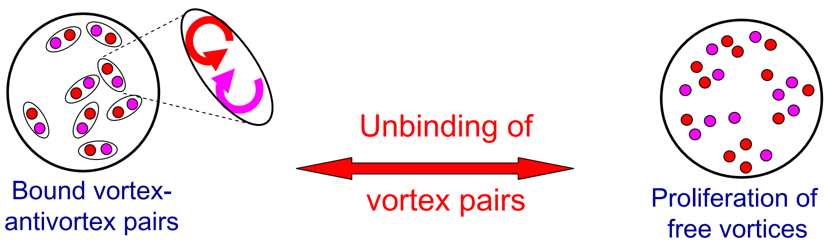
However there is more than just (no) BEC in a 2D interacting gas…

Existence of a phase transition in a 2D Bose fluid

Berezinski and Kosterlitz –Thouless 1971-73
Nelson Kosterlitz 1977



$$g_1(|\vec{r} - \vec{r}\,'|) = \langle \psi^*(\vec{r})\psi(\vec{r}\,')\rangle$$



A vortex in a Bose gas

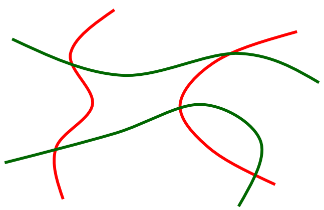
Consider a gas described by the macroscopic wave function

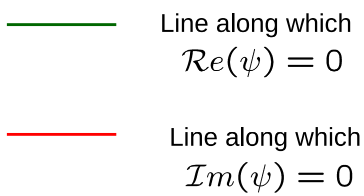
$$\psi(\vec{r}) = \sqrt{n(\vec{r})}\, e^{i\theta(\vec{r})}$$

A vortex is a point (or a line in 3D) where the density vanishes and around which the phase varies by $\eta \times 2\pi$ (η non-zero integer)

η is the "charge" of the vortex

Vortices with charge $\eta = \pm 1$ appear naturally in a gas with a fluctuating wave function



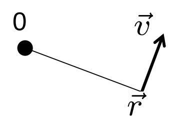


Energy of a vortex

The velocity field associated with a wave function is proportional to the gradient of the wave function

$$\vec{v}(\vec{r}) = \frac{\hbar}{m}\vec{\nabla}\theta(\vec{r}) \qquad \oint \vec{v}\cdot d\vec{r} = \frac{2\pi\hbar}{m}$$

Single vortex in 2D



$$v = \frac{\hbar}{mr}$$

BIOT-SAVART

Kinetic energy: $E_K = \frac{m}{2}\int n_s(r)\, v^2(r)\, d^2r \;= \frac{m n_s}{2}\frac{\hbar^2}{m^2}\int \frac{1}{r^2}\, d^2r$

Lower bound: size of the vortex core ξ Upper bound: size of the gas R

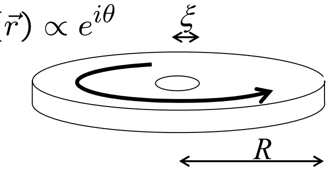
$$E_K = \frac{\pi\hbar^2 n_s}{m}\ln(R/\xi)$$

The KT mechanism for pedestrians

Probability for a vortex to appear as a thermal excitation?

One has to calculate the vortex free energy *E-TS* and compare it with *kT*

$\psi(\vec{r}) \propto e^{i\theta}$



$$v = \frac{\hbar}{mr} \qquad n_s = \frac{N}{\pi R^2}$$

Energy: $E = \frac{\pi \hbar^2}{m} n_s \log(R/\xi)$

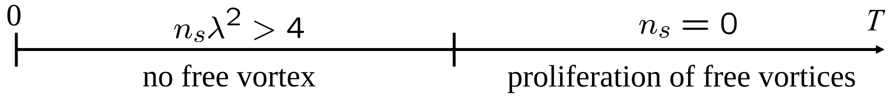
Entropy: $S = k \ \log(W) \sim k \ln(R^2/\xi^2) = 2k \log(R/\xi)$

Free energy of a vortex: $\frac{E - TS}{kT} \sim \frac{1}{2}\left(n_s \lambda^2 - 4\right) \ln(R/\xi)$

BKT transition for pedestrians (2)

$$\frac{E - TS}{kT} \sim \frac{1}{2}\left(n_s \lambda^2 - 4\right) \ln(R/\xi)$$

Thermodynamic limit: $R \to +\infty$



The criterion $n_s\lambda^2 = 4$ is implicit, because the superfluid density depends on temperature and it is usually unknown.

Calculation of the total density needed at a given temperature:
Analytics by Fisher & Hohenberg, Monte-Carlo by Prokof'ev et al

$$n_{\text{total}}\lambda^2 = \ln\left(\frac{C}{\tilde{g}}\right) \qquad C = 380 \pm 3 \qquad \tilde{g} = \frac{m g^{(2D)}}{\hbar^2}$$

dimensionless interaction strength

Part 2.

Cold atoms in FlatLand,

Experiments with flat gases

Z. Hadzibabic, P. Krüger

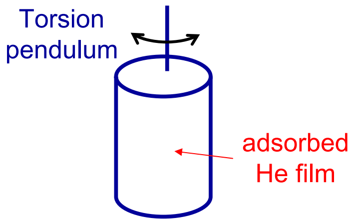
B. Battelier, M. Cheneau, S.-P. Rath, S. Stock

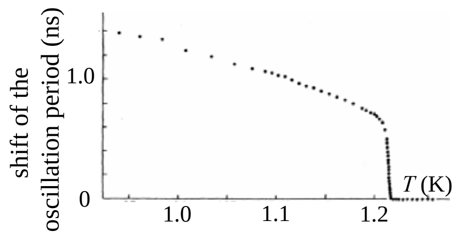
Phys. Rev. Lett. **95**, 190403 (2005)
Nature **441**, 1118 (2006)
Phys. Rev. Lett. **99**, 040402 (2007)

Theory: Shlyapnikov-Gangardt-Petrov, Holtzman *et al.*, Kagan *et al.*, Stoof *et al.*, Mullin *et al.*, Simula-Blackie, Hutchinson *et al.* Polkovnikov-Altman-Demler

Superfluidity in 2 dimensions

A 2D film of helium becomes superfluid at sufficiently low temperature (Bishop and Reppy, 1978)





"universal" jump to zero of superfluid density at $T = T_c$

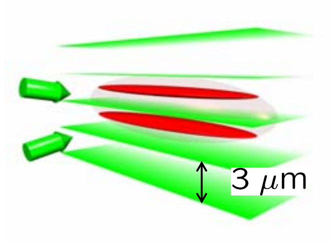
$$\rho_s(T_c)\lambda^2 = 4 \longrightarrow \rho_s = 0$$

Also Safonov et al, 1998: variation of the recombination rate in a H film

How to make a cold 2D gas?

Take a 3D condensate and slice it

superposition of a harmonic magnetic potential
+
periodic potential of a laser standing wave



2 independent planes with 10^5 Rb atoms in each

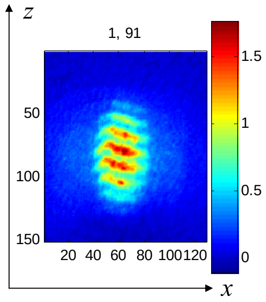
$$\hbar\omega_z > kT, \ \mu$$

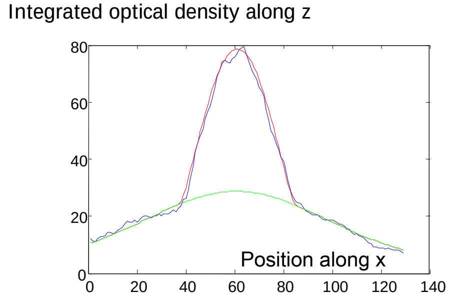
other 2D exp. at MIT, Innsbruck, Oxford, Florence, Heidelberg, NIST, Boulder, etc.

Why 2 planes? The crucial properties of a 2D Bose gas sit in its coherence function $g_1(x,y) = \langle \psi^*(x,y)\,\psi(0) \rangle$

→ accessible in an interference experiment

Overlap between expanding planar gases





Interference fringes

Central bright component
Halo in the periphery
} Characteristic of each gas individually

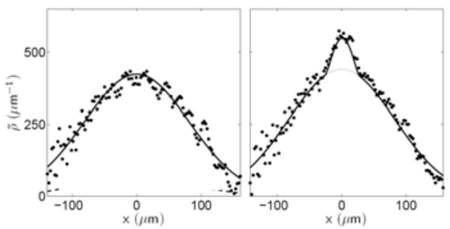
Bimodal fit:
Gauss + Thomas-Fermi give info on numbers, sizes, temperature …

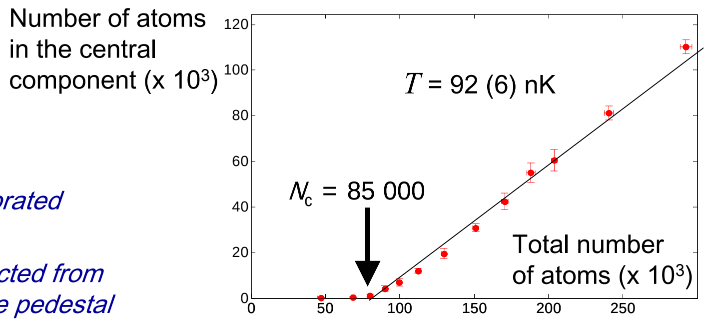
TF profile ⇏ true BEC

Transition in a planar atomic gas

We maintain a constant temperature and vary the atom number

Apparition of a bi-modal distribution above a critical atom number

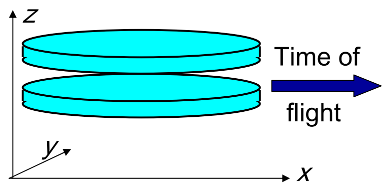


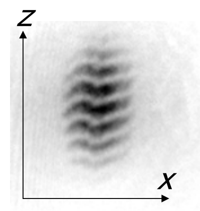


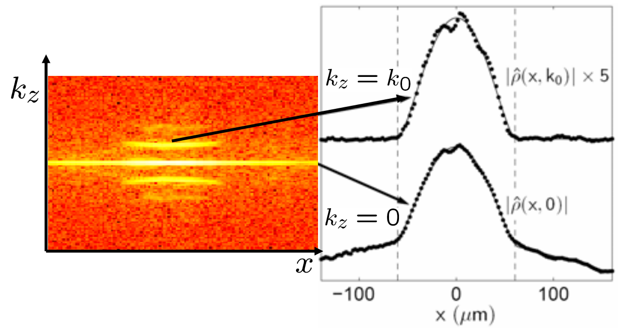
Atom number calibrated using 3D BEC

Temperature extracted from a gaussian fit of the pedestal

Interference between two planar gases

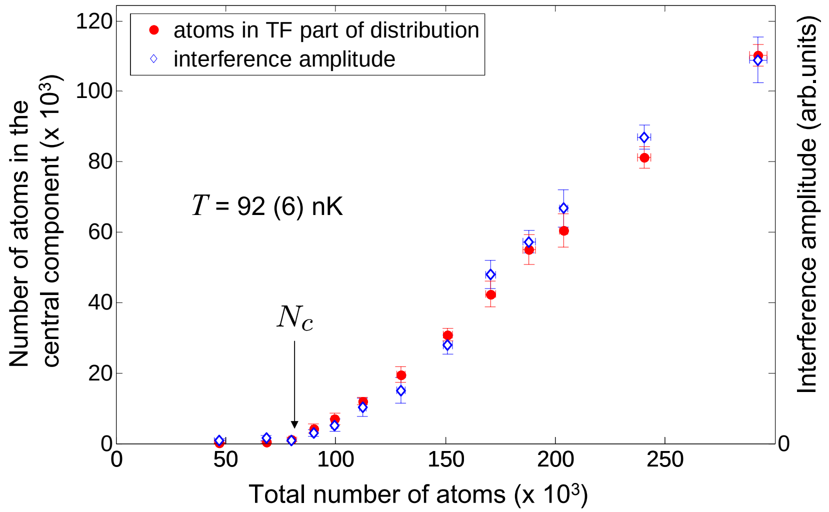






The "interfering" part coincides with the central part of the bimodal distribution

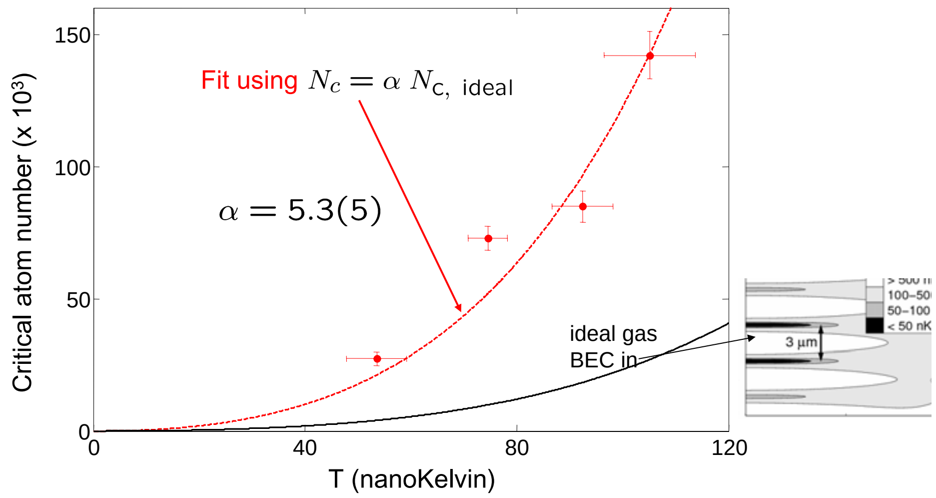
Bimodality and interferences



Within our accuracy, onset of bimodality and interference agree

Variation of the critical atom number with T

$\hbar\omega_z = 150$ nK



5.3 larger than the ideal gas prediction

Can it be the Kosterlitz-Thouless transition point?

For a uniform system with our interaction strength, the KT transition is expected to occur for

$$n_{\text{total}}\lambda^2 = \ln\left(\frac{C}{\tilde{g}}\right) \qquad C = 380 \pm 3, \quad \tilde{g} = 0.13 \quad \Rightarrow \quad n_{\text{total}}\lambda^2 \simeq 8.0$$

Local density approximation + gaussian density profile (as seen experimentally)

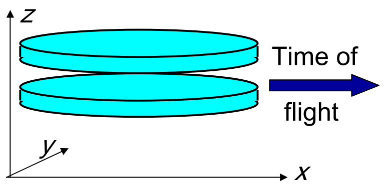
$$n_{\text{total}}(0)\lambda^2 = 8.0 \qquad n(\vec{r}) = n(0)\ e^{-V(\vec{r})/kT}$$

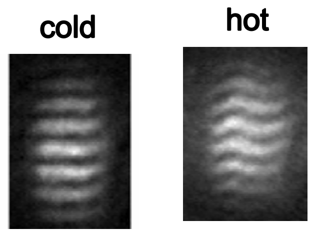
$$N_{\text{c, KT}} = 4.9\ N_{\text{c, ideal}}$$

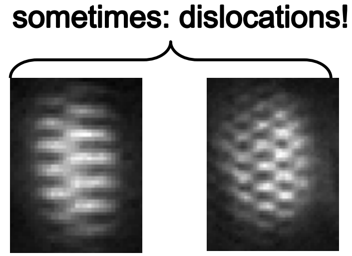
This factor 4.9 has to be compared with the experimental factor 5.3: not bad…

The central component investigated using matter-wave interferometry

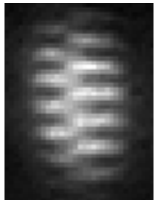
From now on: $N > N_c$

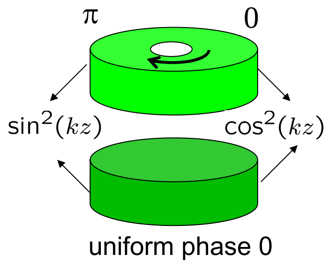






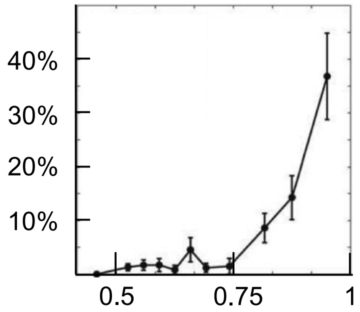
Dislocation = evidence for free vortices





Similar results at NIST

fraction of images showing a dislocation



temperature control (arb.units)

0 = full contrast at center

1 = no contrast at center

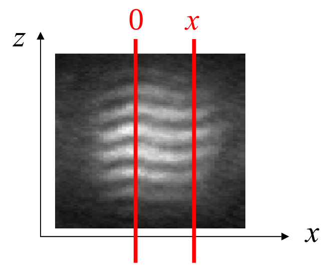
Investigation of the long range order: $g_1(\vec{r}) = \langle \psi^*(\vec{r})\, \psi(0) \rangle$

The interference signal between $\psi_a(x,y)$ and $\psi_b(x,y)$ gives

$$|\psi_a|^2 + |\psi_b|^2 + \psi_a^*\psi_b\, e^{ikz} + \psi_a \psi_b^*\, e^{-ikz}$$

$\psi_a^*\psi_b \rightarrow \kappa(x,y)$

$$\langle \kappa(x,y)\kappa^*(0)\rangle = \langle \psi_a^*(x,y)\psi_b(x,y)\psi_a(0)\psi_b^*(0)\rangle$$
$$= \langle \psi_a^*(x,y)\psi_a(0)\rangle\, \langle \psi_b(x,y)\psi_b^*(0)\rangle = |g_1(x,y)|^2$$

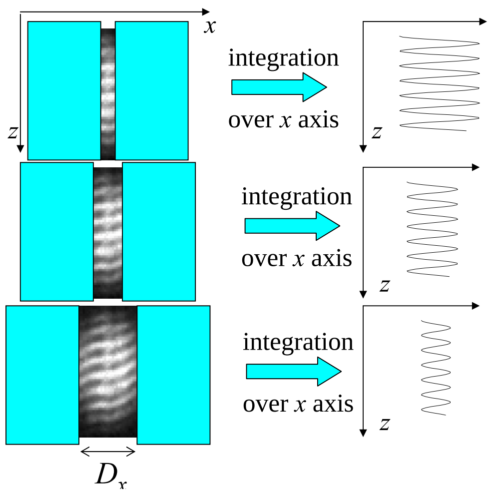


Investigation of the long range order (2)

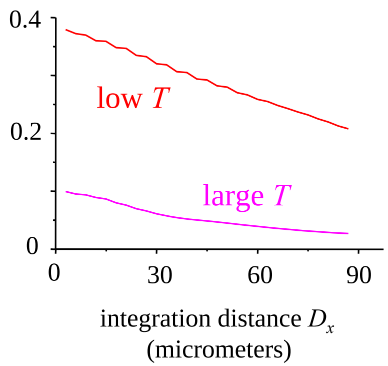
Integrated contrast: $C(D_x) = \dfrac{1}{D_x}\displaystyle\int_0^{D_x} \kappa(x)\, dx$

Polkovnikov, Altman, Demler

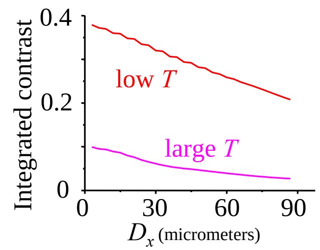
average over many images: $\langle C^2(D_x)\rangle \sim \dfrac{1}{D_x}\displaystyle\int_0^{D_x} (g_1(x,0))^2\, dx$



Contrast after integration



Investigation of the long range order (3)



Polkovnikov, Altman, Demler

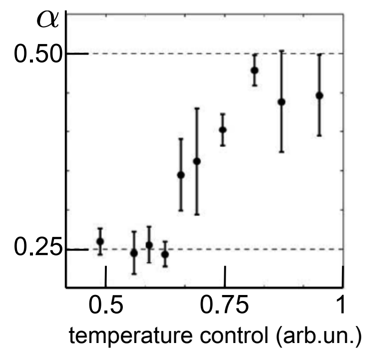
fit $\langle C^2(D_x)\rangle$ by $\dfrac{1}{(D_x)^{2\alpha}}$

- Just below the transition, $g_1(r)$ decays algebraically like $r^{-1/4}$

  $\alpha = 1/4$

- Just above the transition, $g_1(r)$ decays exponentially

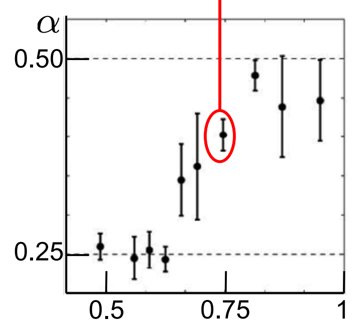
  $\alpha = 1/2$

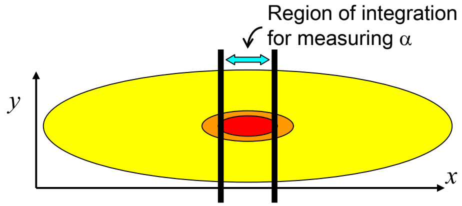


The inhomogeneous density profile in the trap

$N > N_c(T)$

$n_{\text{superfluid}}(0)\ \lambda^2 = 6(2)$





Red: $n_{\text{superfluid}}\lambda^2 > 4$

Orange : $n_{\text{superfluid}}\lambda^2 < 4$

Yellow : $n_{\text{superfluid}} = 0$

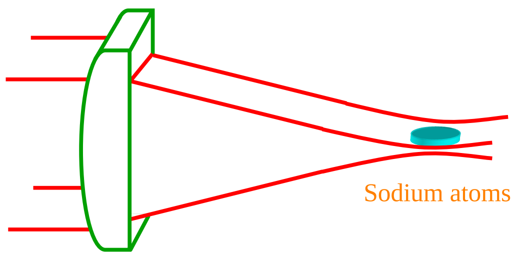
Results from other labs working on 2D/KT physics

NIST Gaithersburg

JILA, Boulder

Study of a single 2D Bose gas at NIST

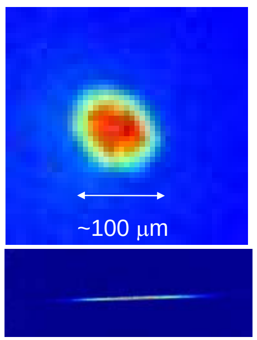
By courtesy of Kristian Helmerson (preliminary results)



400 mW ⟶ ~1 kHz & ~20 Hz

aspect ratio of 50

In situ images



from above

from side

Number of atoms: 10^5 to $2\ 10^5$

Interaction coefficient: $\tilde{g} \sim 0.02$

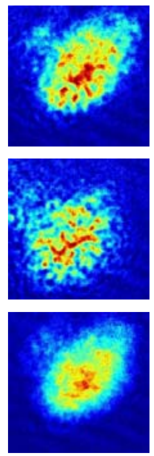
Excitations of the 2D gas (NIST)

The BEC is displaced by an external force

Center of mass motion damps in less than 100 ms

⟶ ripples, density fluctuations

BEC at different times
in the trap

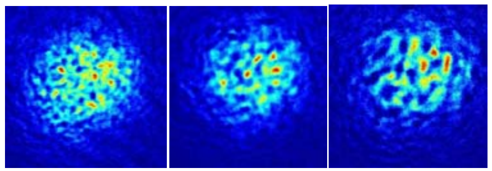


200 ms

500 ms

5 s

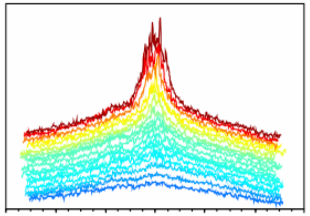
2D gas for different Time-of-Flights



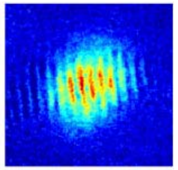
5 ms   8 ms   10 ms

Excitations are vortex/anti-vortex
(they merge after TOF)

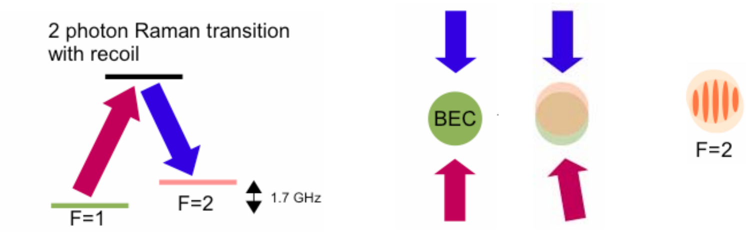
Current studies at NIST



Bi-modal distributions, when the number of atoms increases at a fixed temperature

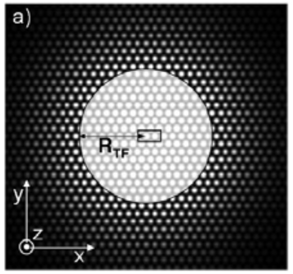


Study of quasi-long range coherence, by interfering the gas with a displaced copy of itself



Another approach to the BKT mechanism in Boulder

V. Schweikhard, S. Tung, and E. A. Cornell, PRL **99,** 030401 (2007)



Array of parallel tubes at finite temperature T, coupled together by tunneling J

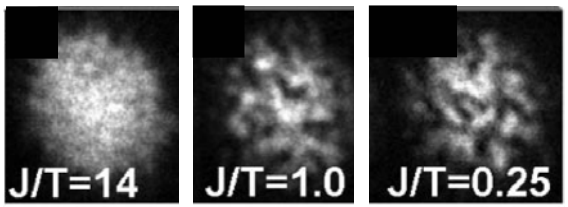
Equivalent to an array of Josephson junctions

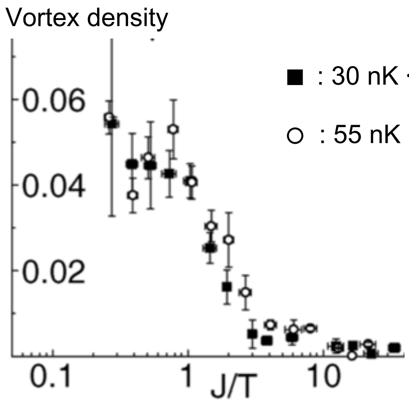
Implementation of the discrete x-y model

When the lattice is shut down and the various gases overlap, are there phase defects (i.e. vortices) trapped in the expanding cloud ?

The Boulder experiment (II)

The relevant parameter is the ratio J / T





■ : 30 nK < T < 40 nK

○ : 55 nK < T < 70 nK

Increasing the ramp-down time of the lattice allows to selectively probe vortices at increasing spatial scales

Nice confirmation of the BKT mechanism

The 2D atomic Bose gas at present…

Several signatures of a Kosterlitz-Thouless cross-over have been identified

- Apparition of a bi-modal structure (superfluid core?)
- proliferation of vortices
- loss of long range order

The observed phenomena do not match the ideal Bose gas condensation

Good agreement with quantitative predictions of the KT mechanisms

One important remaining question: direct test the superfluidity of the core

Part 3.

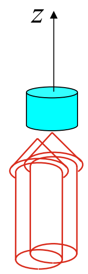
Rotating Bose gases

Connection with other rotating quantum systems

Rotating bucket experiment with liquid helium, neutrons stars, rotating nuclei, superconductor in a magnetic field

Connection with quantum Hall physics

Physics in a rotating frame



Cylindrically symmetric trap potential in the xy plane:

$$V(\vec{r}) = \frac{1}{2} m\omega^2 r^2 \qquad r^2 = x^2 + y^2$$

Stir at frequency Ω (with a rotating laser beam for example)

$$\delta V(\vec{r}, t) = \frac{\epsilon}{2} m\omega^2 \left(X^2 - Y^2\right) \qquad \epsilon \sim \text{ a few \%}$$

Hamiltonian in the rotating frame:

$$H = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 r^2 - \Omega L_z + \xcancel{\frac{\epsilon}{2} m\omega^2 (x^2 - y^2)}$$

$$= \frac{\left(\vec{p} - \vec{A}\right)^2}{2m} + \frac{1}{2} m \left(\omega^2 - \Omega^2\right) r^2 \qquad \vec{A} = m\,\vec{\Omega} \times \vec{r}$$

Same physics as charged particles in a magnetic field + harmonic confinement

Classical vs. quantum rotation

Rotating classical gas

velocity field of a rigid body $\vec{v} = \vec{\Omega} \times \vec{r}$ $\longrightarrow$ $\vec{\nabla} \times \vec{v} = 2\vec{\Omega}$

Rotating a quantum macroscopic object

macroscopic wave function: $\psi(\vec{r}) = \sqrt{\rho(\vec{r})}\; e^{i\phi(\vec{r})}$

In a place where $\rho(\vec{r}) \neq 0$, irrotational velocity field: $\vec{v} = \frac{\hbar}{m} \vec{\nabla} \phi$

The only possibility to generate a non-trivial rotating motion is to nucleate quantized vortices (points in 2D or lines in 3D)

$$\oint \vec{v} \cdot d\vec{r} = \frac{nh}{m}$$

Feynman, Onsager, Pitaevskii

Outline of the lecture

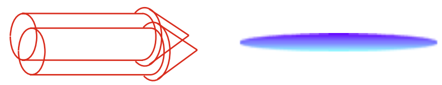
1. From the single vortex to vortex arrays
2. The fast rotation regime: Lowest Landau Level physics
3. Towards fractional quantum Hall effect
4. Topological gauge potential for neutral atoms

1.

From the single vortex to vortex arrays

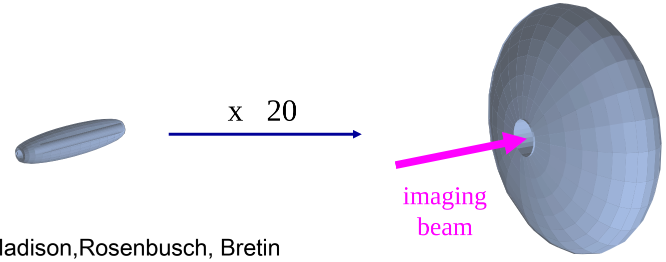
Vortices in a stirred condensate

Cylindrical trap
+
stirring

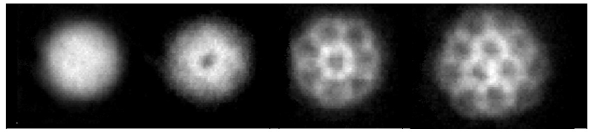


ENS, Boulder, MIT, Oxford

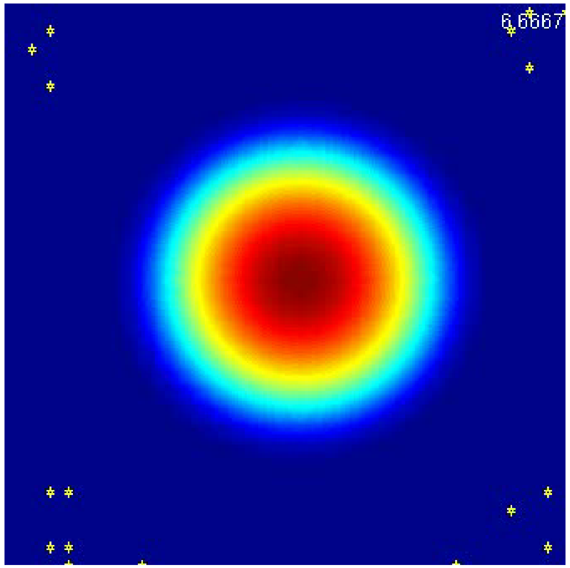
Time of flight analysis (25 ms)



ENS 2000: Chevy, Madison, Rosenbusch, Bretin



Nucleation of vortices: a simulation using Gross-Pitaevskii equation

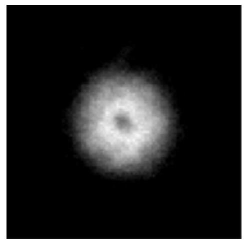


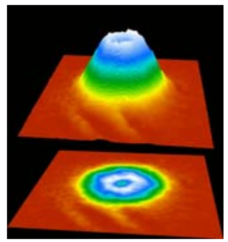
C. Lobo,
A. Sinatra,
Y. Castin,
Phys. Rev. Lett. **92**,
020403 (2004)

also M. Tsubota,
K. Kasamatsu,
and M. Ueda,
Phys. Rev. A. 65,
023603 (2002)

The single vortex case

After time-of-flight expansion:





Questions which have been answered:

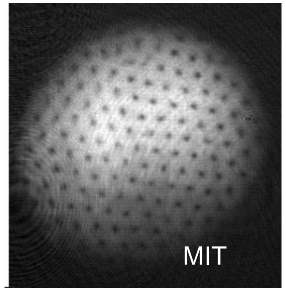
- Total angular momentum $N\hbar$ (i.e. $\hbar$ per particle) ?
- Is the phase pattern varying as $e^{i\theta}$?
- What is the shape of the vortex line?
- Can this line be excited (as a guitar string)? Kelvin mode

The intermediate rotation regime

The number of vortices is notably larger than 1.

However one keeps the rotation frequency Ω notably below ω

core size ξ << vortex spacing



What gives the vortex surface density ?

Uniform surface density of vortices n_v with

$$\Omega = \frac{\pi \hbar}{m} n_v$$

Coarse-grain average for the velocity field

$$\vec{v} = \vec{\Omega} \times \vec{r}$$

2.

The fast rotation regime: Lowest Landau Level physics

(mean field)

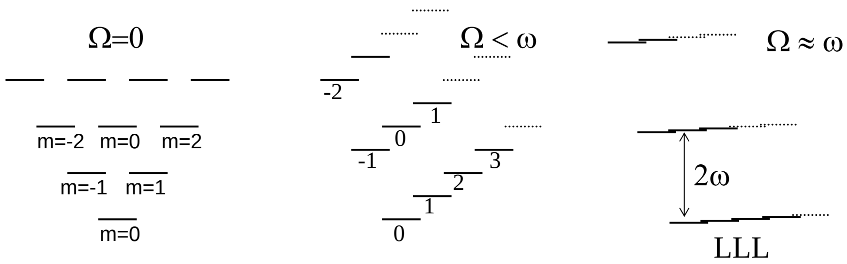
Is there a "Hc2" equivalent for rotating condensates ?

Ho, Baym, Fischer, Watanabe *et al*, Komineas *et al,* Aftalion *et al*

Landau levels for a rotating gas

Isotropic harmonic trapping in the xy plane with frequency ω

Hamiltonian in the rotating frame: $H - \Omega L_z$

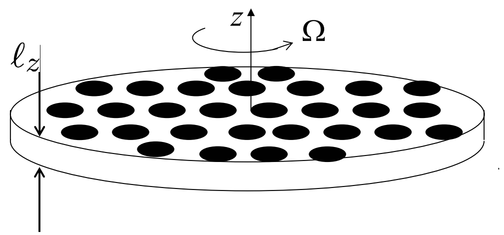


When $\Omega = \omega$, macroscopically degenerate ground state for the one-body hamiltonian (Rohshar, Ho)

$$H = \frac{\left(\vec{p} - \vec{A}\right)^2}{2m} \qquad \vec{A} = \vec{\Omega} \times \vec{r}$$

Reaching the lowest Landau level

Assume that the z direction is frozen, with a extension ℓ_z

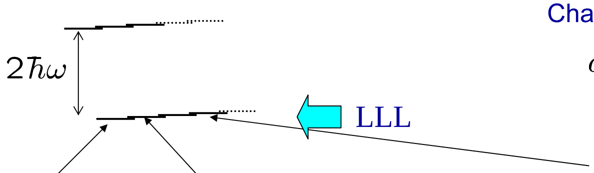


If the chemical potential and the temperature are much smaller than $2\hbar\omega$, the physics is restricted to the lowest Landau level

This LLL regime corresponds to $1 - \frac{\ell_z}{Na} < \frac{\Omega}{\omega}$ a: scattering length

Typically: N=10 000 atoms, ℓ_z=0.5 μm, a =5 nm ⇨ $\Omega > 0.99\ \omega$

The lowest Landau level



$e^{-r^2/2a_0^2}$ $\quad (x+iy)e^{-r^2/2a_0^2}$ $\quad (x+iy)^n e^{-r^2/2a_0^2}$

Characteristic length scale

$$a_0 = \sqrt{\hbar/m\omega}$$

General one-particle state in the LLL:

$$e^{-r^2/2a_0^2}\, P(x+iy) \longrightarrow e^{-r^2/2a_0^2} \prod_{j=1}^{n} (u - u_j)$$

Polynomial or analytic function

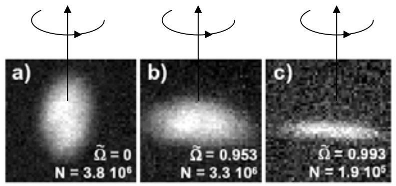
$u = x + iy$ $\quad u_j$: vortices

➡ The size of the vortices is comparable to their spacing

➡ The atom distribution is entirely determined from the vortex position

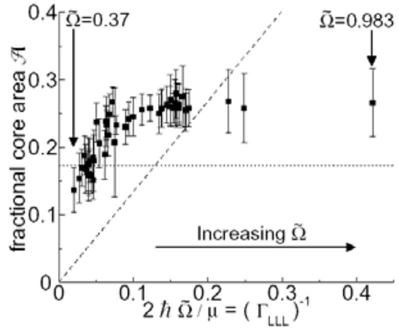
Reaching the LLL experimentally: Boulder and ENS

Evaporative spin-up method (Boulder): first rotate the gas at a moderate frequency and then evaporate of atoms with small angular momentum.



side view of the rotating BEC

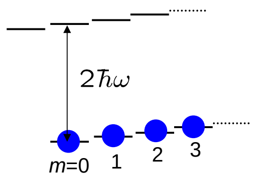
V. Schweikhard *et al.*, PRL **92**, 040404 (2004)



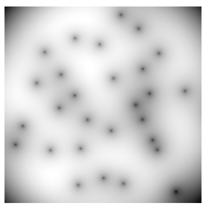
What about the ideal gas case?

The LLL is still a good approximation for fast rotation if $kT \ll 2\hbar\omega$

State of the gas: superposition of independent BEC's in the various orbitals with random complex amplitudes C_m



A realization of the experiment corresponds to a particular choice of the C_m's.

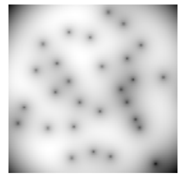


$$\psi(x,y) = \sum_m C_m \frac{(x+iy)^m}{\sqrt{m!}}\, e^{-r^2/2a_0^2}$$

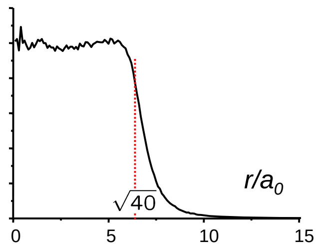
Vortices are still present and clearly visible: zeroes of this polynomial

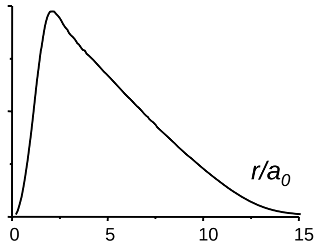
Each vortex correspond to a destructive interference between the BEC's

Vortex position in the ideal gas case



Equal weight for $\langle |C_m| \rangle$ up to m=40





Vortices (i.e. zeroes of a random polynomial) repel each other (as the eigenvalues of a random matrix).

Atom interactions provide long range order for the Abrikosov lattice

Y. Castin , Z. Hadzibabic, S. Stock, J. Dalibard, and S. Stringari, Phys. Rev. Lett. **96**, 040405 (2006)

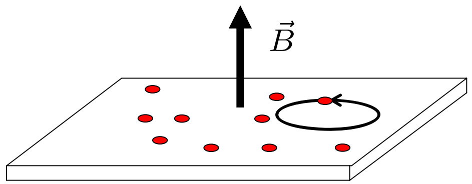
3.

Towards fractional quantum Hall effect

Cooper, Wilkin & Gunn, Ueda *et al.*, Regnault and Jolicoeur, Cazalilla *et al.*, Paredes *et al.*, Read, MacDonald *et al.*, Stoof *et al.*

The quantum Hall effect (in a few words)

2D electron fluid in a magnetic field



cyclotron motion: $m_e v = qBr$

Heisenberg: $m_e \Delta r \; \Delta v \geq \hbar$

↓

"natural" orbit size $r_0^2 = \hbar/(qB)$

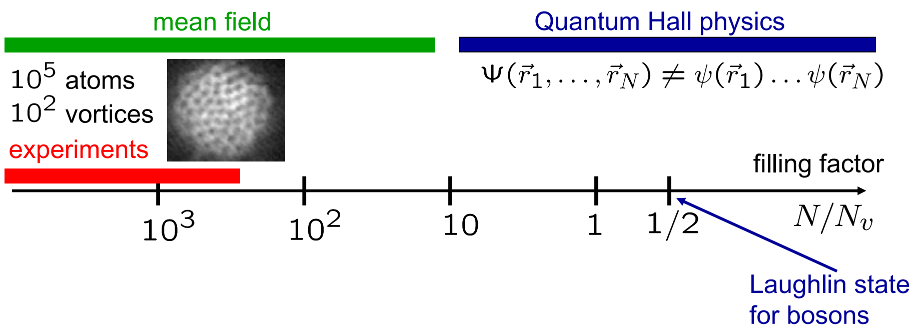
Filling factor: $$f = \frac{2\pi r_0^2 \; N_{\text{electron}}}{\text{total area}}$$

For specific values of the filling factor (1,2,3,…, 1/3, 2/5,…) the electrical resistance of the fluid drops to zero

Quantum Hall effect with rotating Bose gases?

The 'filling factor' is the ratio $\frac{N}{N_v}$ ← N: number of atoms, N_v: number of vortices

For $N \gg N_v$ the mean field description is valid and the gas forms a Bose-Einstein condensate $\Psi(\vec{r}_1,\ldots,\vec{r}_N) \simeq \psi(\vec{r}_1)\ldots\psi(\vec{r}_N)$

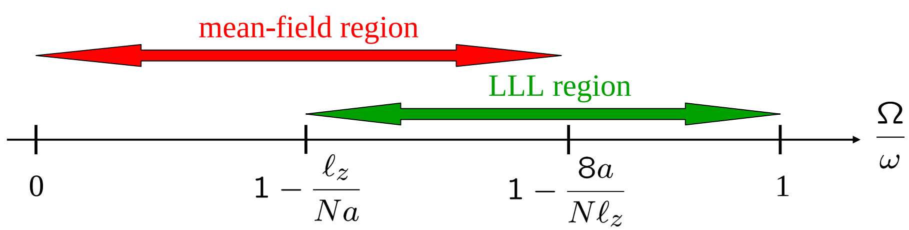


When does the mean field approach break down ?

The number of vortices N_v becomes similar to the atom number N when

$$\frac{\Omega}{\omega} \geq 1 - \frac{8a}{N\ell_z}$$

Total angular momentum: $L_z \sim N^2\hbar$ ($N\hbar$ per particle)



Note that $a < \ell_z$ for a 3D description of the binary collisions

Beyond mean field: Laughlin states and more

For some specific filling factors $\nu = N/N_v$ the ground state is separated from the excited states by an energy gap

incompressible states, analogous to those of Quantum Hall effect

Example: $\nu = 1/2 \quad L = N(N-1)$

$$\Psi(\vec{r}_1, \ldots, \vec{r}_N) = \left[\prod_{j<k} (u_j - u_k)^2 \right] e^{-\sum_j r_j^2 \,/\, 2a_0^2} \qquad u_j = x_j + i y_j$$

Eigenstate of the 1-body Hamiltonian in the LLL

Eigenstate of the interaction Hamiltonian: $V = g \sum_{j<k} \delta(\vec{r}_j - \vec{r}_k)$

4.

Topological gauge potential for neutral atoms

Dum & Olshanii, 1996
Ho & Shenoy, 1996
Jaksch and Zoller, 2003
Juzeliunas, Ohberg, et al, 2004

Mueller, 2004
Soerensen et al, 2005
Osterloh et al, 2005
Ruseckas et al, 2005

Gauge fields for atomic gases

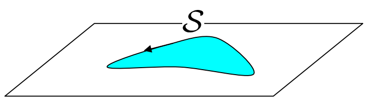
The passage in the rotating frame corresponds to adding a vector potential into the atomic hamiltonian (+ centrifugal force)

$$\hat{H} = \frac{\left(\vec{p} - \vec{A}\right)^2}{2m} + \frac{1}{2} m \left(\omega^2 - \Omega^2\right) r^2 \qquad \vec{A} = m\,\vec{\Omega} \times \vec{r}$$

This vector potential corresponds physically to the accumulation of the phase

$$\delta\varphi = 2m\Omega\mathcal{S}/\hbar$$

along the contour encircling the area $\mathcal{S}$



Sagnac effect

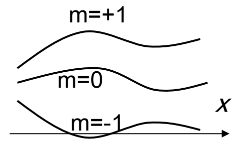
Any other accumulated 'topological' phase can be used in order to mimic the effect of a magnetic field on charged particles

Berry's phase is a good candidate !

Using Berry's phase for generating a gauge field

Suppose that the atoms have several possible internal states, for example a ground level with angular momentum 1

⟶ 3 Zeeman sublevels: m = -1,0,+1



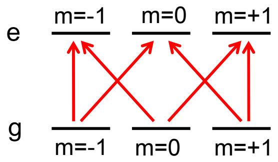
We label $\{|m(\vec{r})\rangle\}$ the local energy basis of the atomic ground level in $\vec{r}$

We suppose that an atom initially prepared in the internal state $|m(\vec{r})\rangle$ will follow adiabatically this state while moving (like in a magnetic trap)

The Berry's phase accumulated during this motion gives rise to the vector potential

$$\vec{A}_m(\vec{r}) = i\hbar \, \langle m(\vec{r}) | \vec{\nabla} m(\vec{r}) \rangle$$

Use of a J=1 dark state to generate a gauge potential

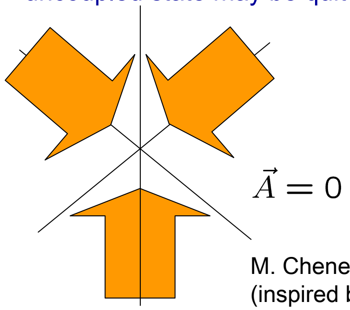


Olshanii and Minogin (1992): for any monochromatic laser configuration, there exists a state

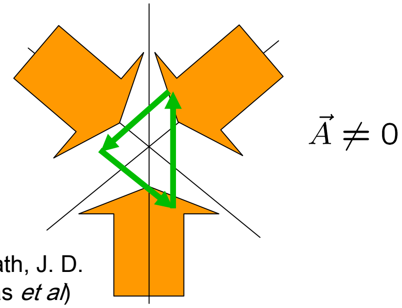
$$\psi(\vec{r}) = \sum_{m=-1}^{+1} C_m(\vec{r})|g,m\rangle$$

that stays uncoupled to laser light forever.

The Berry's phase associated with the adiabatic following of this uncoupled state may be quite large.



M. Cheneau, S.-P. Rath, J. D.
(inspired by Juzeliunas *et al*)



Conclusions

Low dimensional systems are not simpler than 3D systems!

➡ Several questions are still open concerning the static case, in particular in connection with superfluidity

➡ Concerning the rotating case, the analogy with the fractional quantum Hall effect has not yet been explored experimentally.

It is an experimental challenge to achieve vortex numbers similar to atom numbers, but the outcomes would be remarkable

The use of other topological gauge potentials may provide another access to fractional Quantum Hall physics