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Summer School on Novel Quantum Phases and Non-Equilibrium Phenomena in Cold Atomic Gases

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From few-body to many-body physics in cold Fermi gases - Part I

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From few-body to many-body physics in ultracold Fermi gases

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- Feshbach resonance. Molecules in Fermi gases
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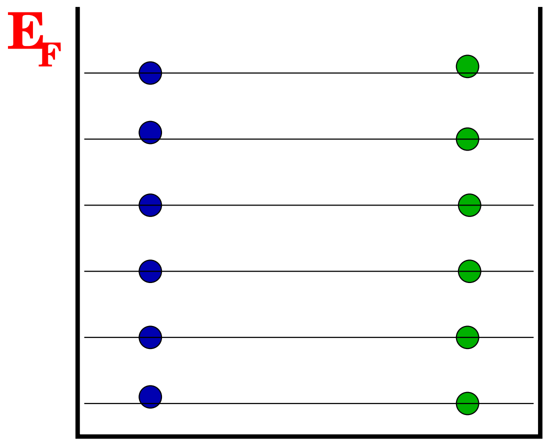
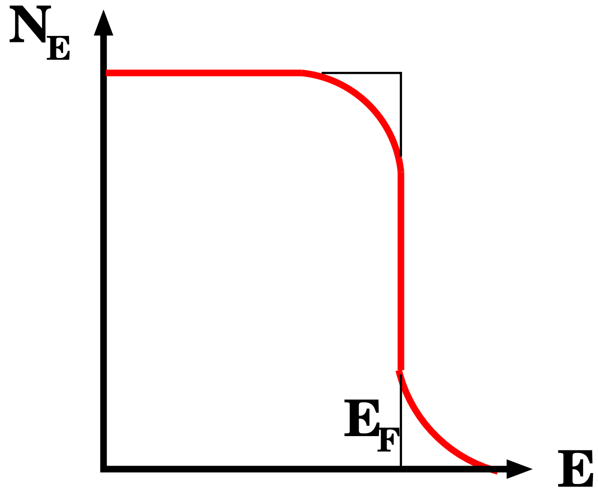
Two-component trapped Fermi gas





$$\int_0^{k_F} \frac{4\pi V k^2 dk}{(2\pi)^3} = \frac{V k_F^3}{6\pi^2} = \frac{N}{2}$$

$$k_F = (3\pi^2 n)^{1/3}; \qquad n = \frac{N}{V}; \qquad E_F = \frac{\hbar^2 k_F^2}{2m}$$

trapped gas $\Rightarrow E_F \sim N^{1/3}\hbar\omega$

Weakly interacting ultracold limit

Weakly interacting gas

$$|a| \ll n^{-1/3}$$

$$n|a|^3 \ll 1 \text{ or } k_F|a| \ll 1$$

Ultracold limit

$$\Lambda_T = \left(\frac{2\pi\hbar^2}{mT}\right)^{1/2} \gg R_e \quad \Rightarrow \text{s-wave scattering}$$

Interspecies interaction only

What does the interaction do?

$a < 0$ →Interspecies attraction →Cooper pairing at low T

Superfluid BCS transition → $T_c \sim E_F \exp\{-\pi/2k_F|a|\}$

$T_c \ll 0.1 E_F$ for ordinary a Very hard to reach

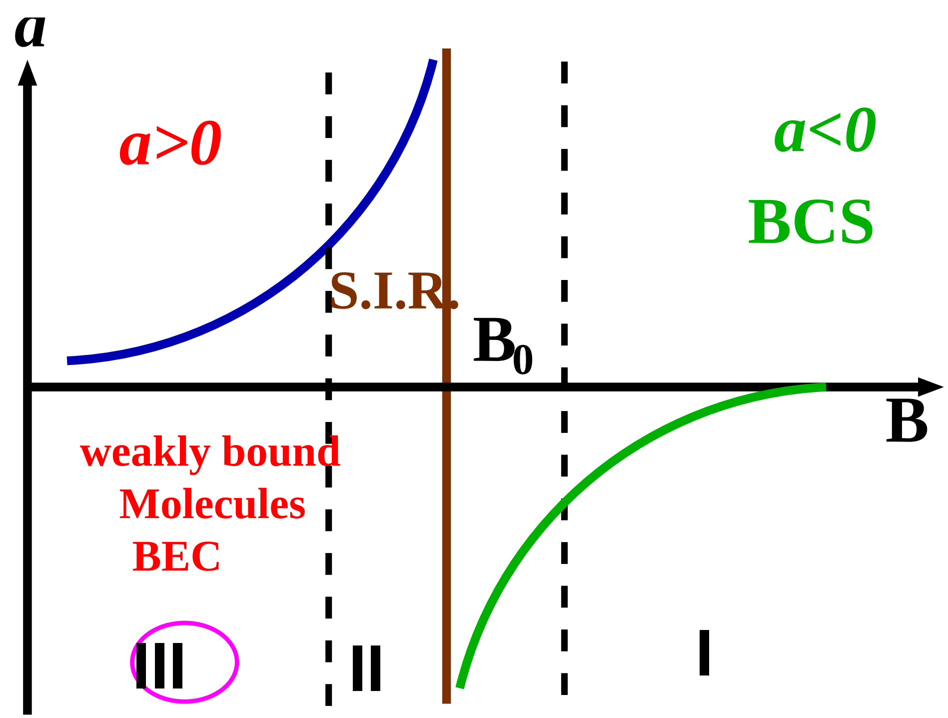
Experiments ^{40}K ^{6}Li

Dilute limit $nR_e^3 \ll 1$ Ultracold limit $\Lambda_T \gg R_e$

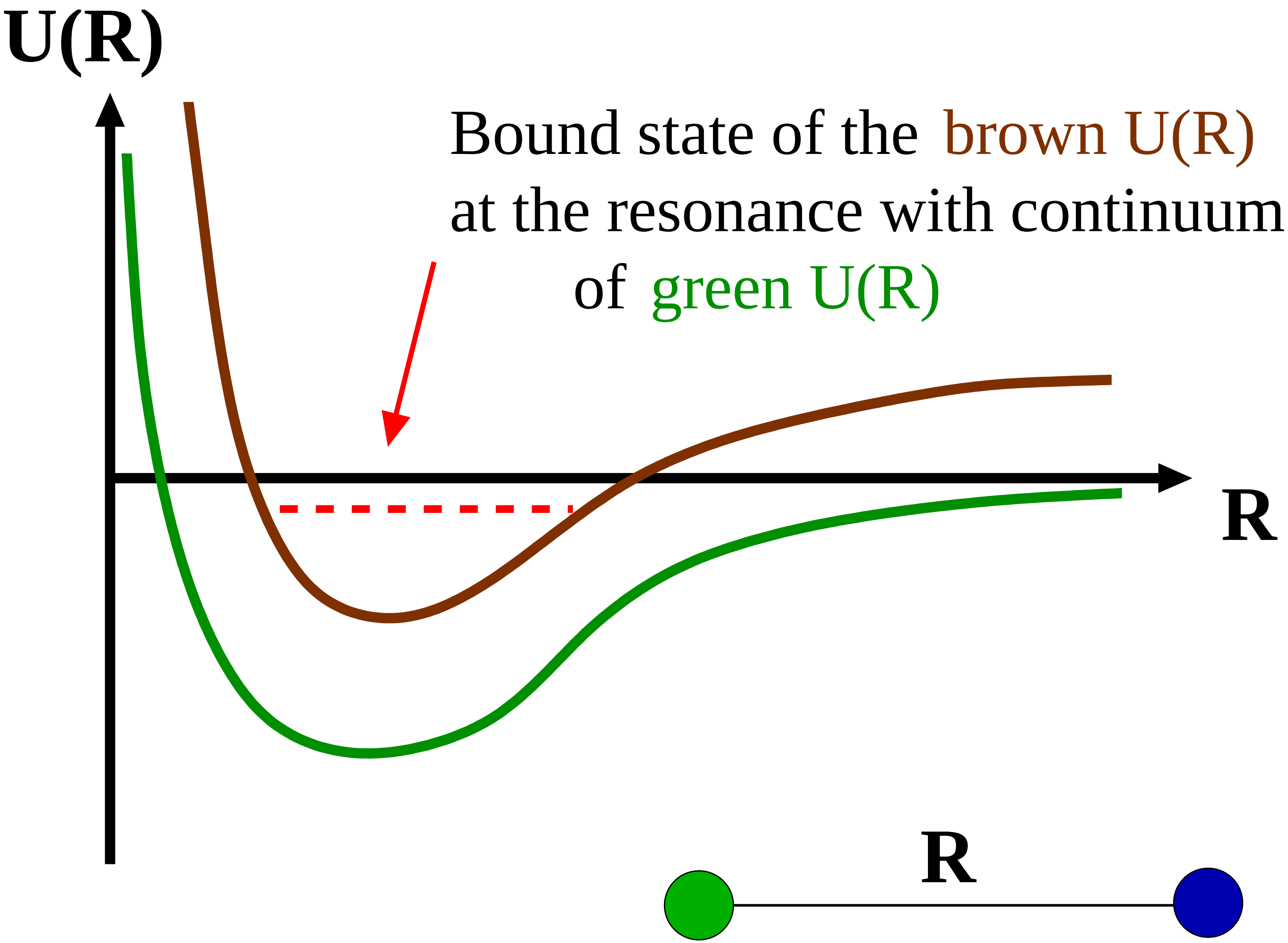
Quantum degeneracy $\rightarrow$ JILA 1998 ^{40}K

At present $n \sim 10^{13} - 10^{14}\mathrm{cm}^{-3}$; $T \sim 1\mu\mathrm{K}$

JILA, LENS Innsbruck,MIT,ENS,Rice,Duke, ETH, Hamburg, Tuebingen, Toronto

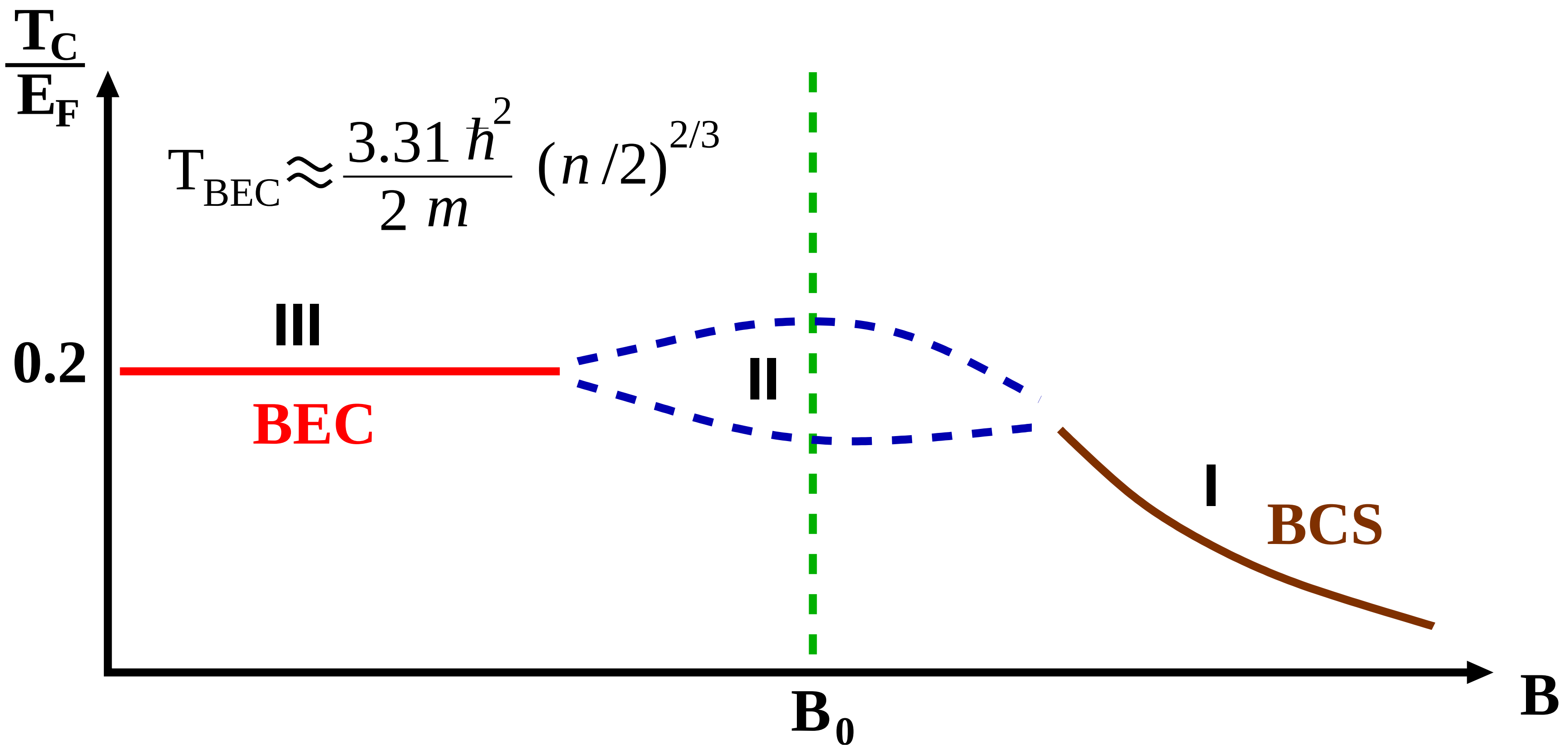


Feshbach resonance



Superfluid regimes

I	$k_F\|a\| \ll 1$	$\rightarrow$	**BCS**
II	$k_F\|a\| > 1$	$\rightarrow$	**Strongly interacting regime**
III	$na^3 \ll 1$	$\rightarrow$	**Gas of bosonic molecules**
	$a \gg R_e$	$\rightarrow$	BEC of weakly bound molecules



BCS-BEC crossover: Leggett, Nozieres-Schmitt-Rink

Scattering amplitude and bound state

$$f = \frac{1}{a^{-1}+ik+k^2R_*} \quad \Rightarrow \text{ scattering amplitude}$$

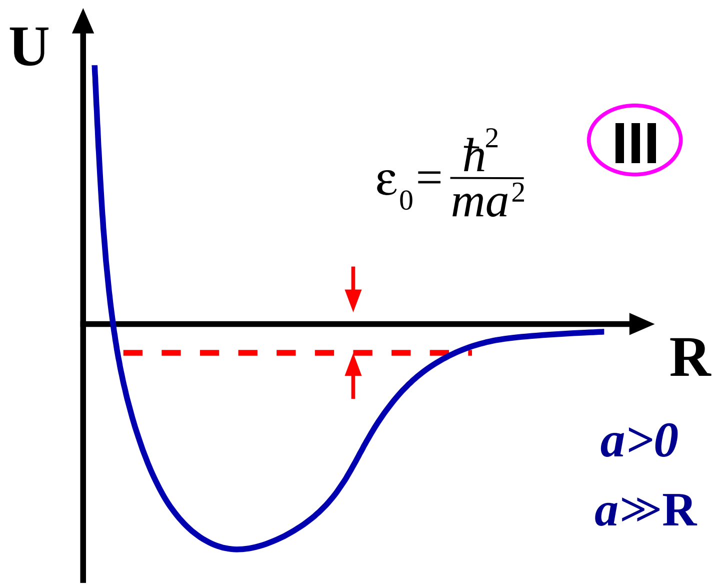
Bound state $\Rightarrow$ pole of f

$$k = i\eta \quad \Rightarrow \eta^2 R_* + \eta - a^{-1} = 0; \quad \eta = -\frac{1}{2R_*} + \left(\frac{1}{4R_*^2} + \frac{1}{aR_*}\right)^{1/2}$$

$a > 0$ and $R_* \ll a \quad \Rightarrow \eta = a^{-1}$

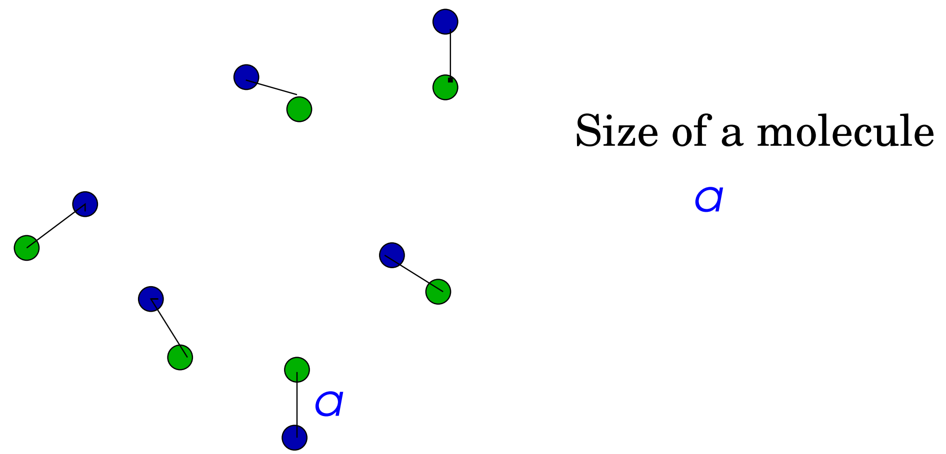
Binding energy $\Rightarrow \quad \epsilon_0 = \frac{\hbar^2\eta^2}{m} = \frac{\hbar^2}{ma^2}$

$R_* \ll a \quad \Rightarrow$ wide resonance/single-channel model



Gas of bosonic molecules (dimers)

Region III $(a > 0) \Rightarrow$ gas of weakly bound bosonic molecules



$a \ll n^{-1/3}$ or $na^3 \ll 1 \quad \Rightarrow$ weakly interacting Bose gas

Interaction energy $E_{int} = \frac{N(N-1)}{2}\varepsilon_{int}$

$$\varepsilon_{int} = \frac{g}{V}; \qquad g = ?$$

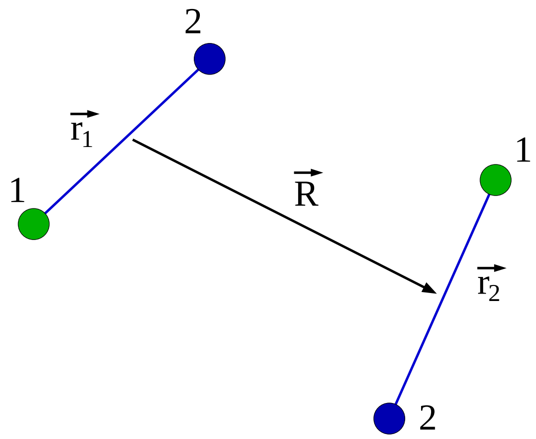
$g < 0 \Rightarrow$ collapse of a Bose-Einstein condensate

$g > 0 \Rightarrow$ stable BEC

Molecule-molecule elastic interaction

Interaction constant $g = 4\pi\hbar^2 a_{dd}/2m$ "Old answer" $\rightarrow 2a$

4-body problem **Exact solution for** $a \gg R_e$ (Petrov et al 2003)



$\Psi \rightarrow$ 9 variables

$a \gg R_e \Rightarrow$ Zero-range approximation

$$\Psi_{r_1 \rightarrow 0} \rightarrow f(\vec{r}_2, \vec{R})(1/4\pi r_1 - 1/4\pi a)$$

$$R \gg a \quad \Psi = \phi_0(r_1)\phi_0(r_2)(1 - a_{dd}/R); \quad \phi_0(r) = \frac{1}{\sqrt{2\pi a}} \exp(-r/a)$$

$$R \gg a \quad f(\vec{r}, \vec{R}) = (2/rR) \exp(-r/a)(1 - a_{dd}/R);$$

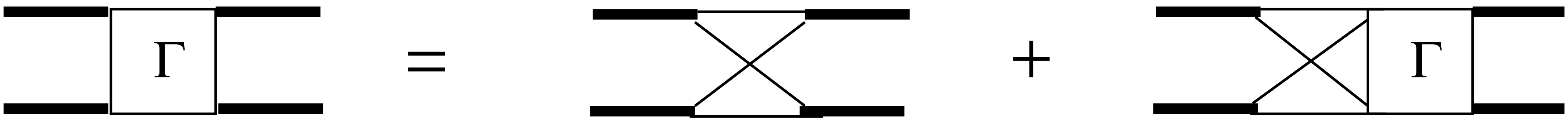
Derivation of a_{dd}

Limit $r_1 \to 0$ **Integral equation for f** (3 variables)

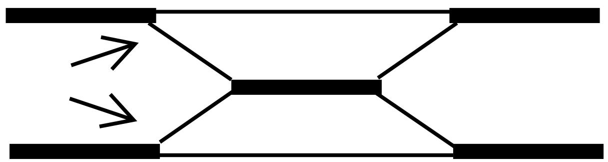
$$a_{dd} = 0.6a$$

Monte Carlo (Giorgini/Astracharchik, 2004)

Diagrammatic approach (M.Kagan et al, 2005; V. Gurarie et al, 2006)

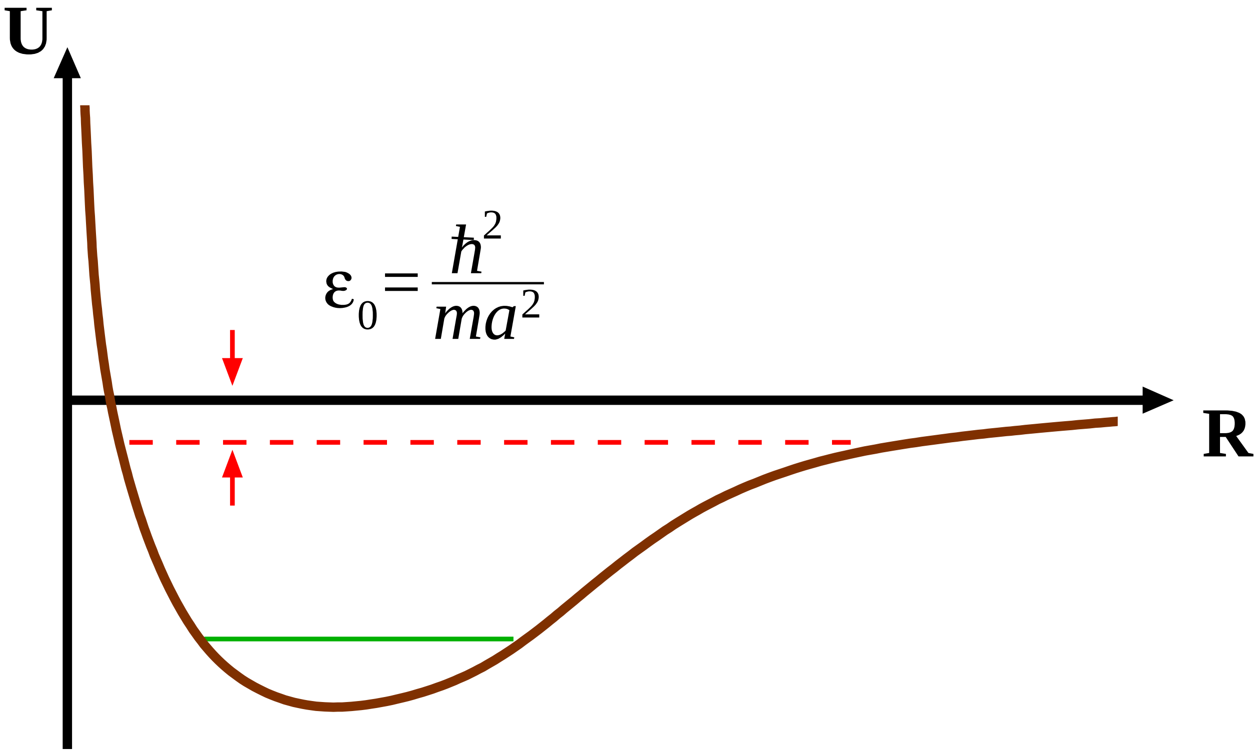


$$\Gamma = 0.75a$$



Weakly bound dimers

Weakly bound dimers $\rightarrow$ The highest rovibrational state of the diatomic molecule



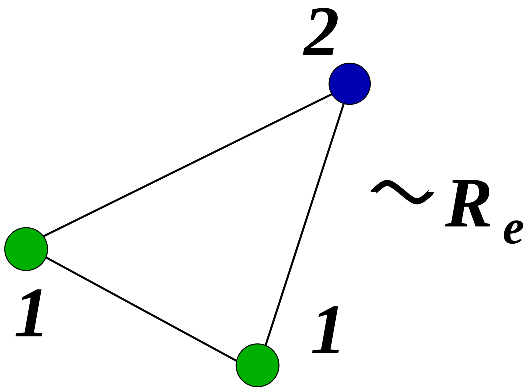
Collisional relaxation to deep bound states
(~ 1ms for Rb_2 at $n \sim 10^{13}$cm^{-3})

Atom-dimer collisions. Physical picture

Weakly bound dimer $\sim a$

Size $\rightarrow$

Deep bound state $\sim R_e$ (50 Å) $\ll a$



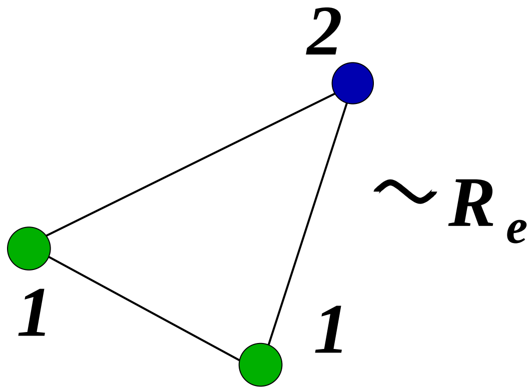
2 particles are identical fermions

Pauli principle

$$\alpha_{rel} \sim (k_{eff} R_e)^{2?} \sim (R_e/a)^{2?}$$

Atom-dimer collisions. Relaxation rate

The released binding energy is $\sim \hbar^2/mR_e^2$
Establish the dependence of α_{rel} on a



2 particles are identical fermions

$r \sim R_e \quad \Rightarrow \quad \Psi = A(a)\tilde{\psi}$ $\rightarrow$ valid at any $r \ll a$

$R_e \ll r \ll a \quad \Rightarrow$ zero-range approximation
as well as at any $r \gg R_e$

The only distance scale is $a \quad \Rightarrow \quad , \Psi = B(a)F(\vec{r}_i/a)$

$\rho = \sqrt{x^2 + y^2} \ll a \quad \Rightarrow \quad \Psi \approx B(a)(\rho/a)^\gamma \Phi_\gamma(\Omega)$
$A(a) = B(a)a^{-\gamma} \qquad \gamma =\approx 0.1662$

Atom-dimer collisions. Relaxation rate

Long distance behavior $\Rightarrow$

$$\Psi = \left(1 - \frac{a_{ad}}{R}\right) \frac{1}{\sqrt{2\pi} a^{3/2}(r/a)} \exp\left(-r/a\right)$$

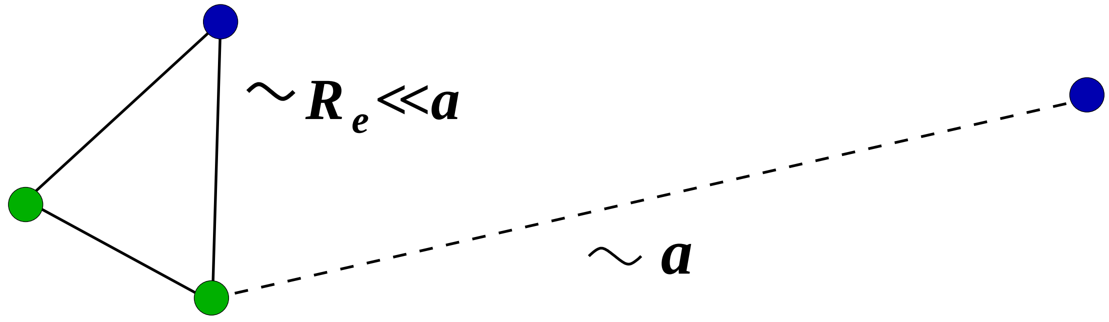
$\Rightarrow B \propto a^{-3/2}$. Hence, $A(a) \propto a^{-3/2-\gamma}$

$\alpha_{\rm rel} \propto A^2(a) \quad \Rightarrow \quad \alpha_{rel} \propto a^{-3-2\gamma} = a^{-3.33}$

$$\alpha_{rel} = C \left(\frac{\hbar R_e}{m}\right) \left(\frac{R_e}{a}\right)^s ; \qquad s = 3.33$$

Strong decrease of relaxation on approach to resonance

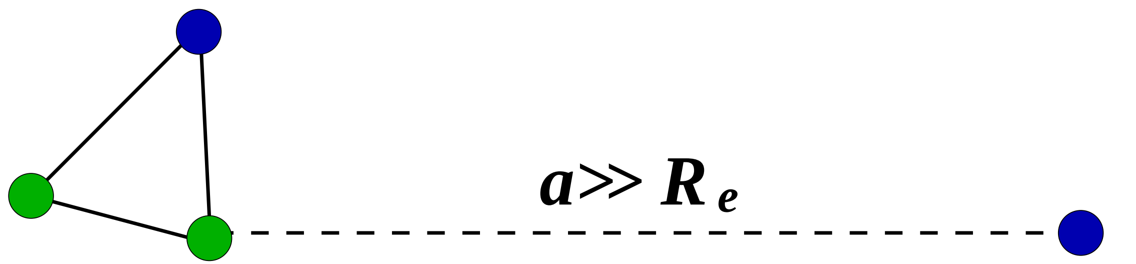
Molecule-molecule relaxation collisions



$$\alpha_{rel} = C\frac{\hbar R_e}{m}\left(\frac{R_e}{a}\right)^s; \quad s = 2.55$$

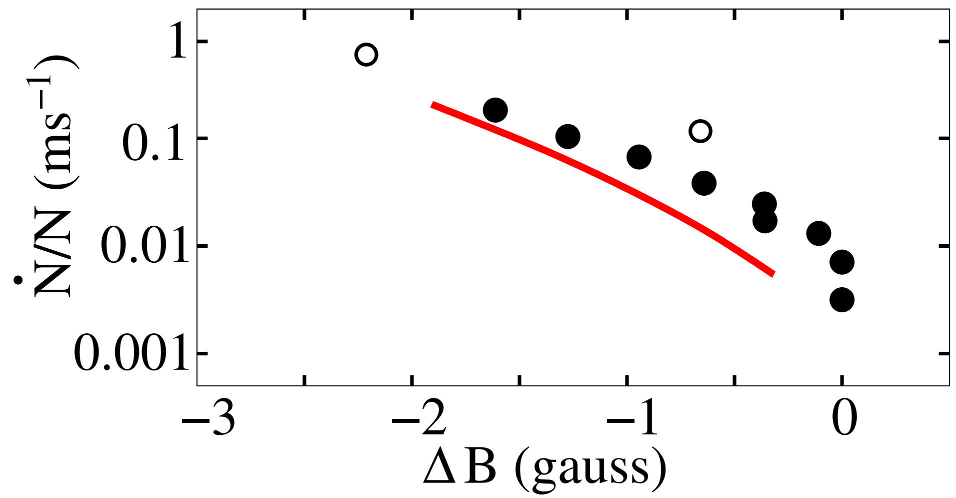
$$\tau \sim (\alpha_{rel} n)^{-1} \sim \quad \text{seconds}$$

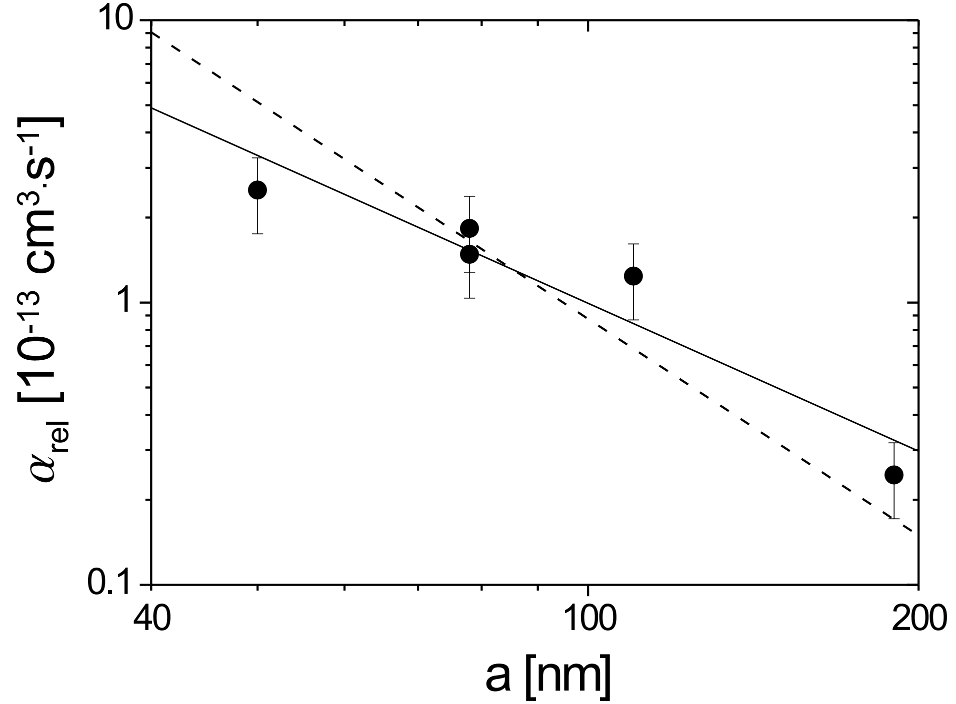
Molecules of bosonoc atoms



Resonant enhancement $\alpha_{rel} \sim \hbar a/m$ $\tau < 1$ms

Suppressed collisional relaxation





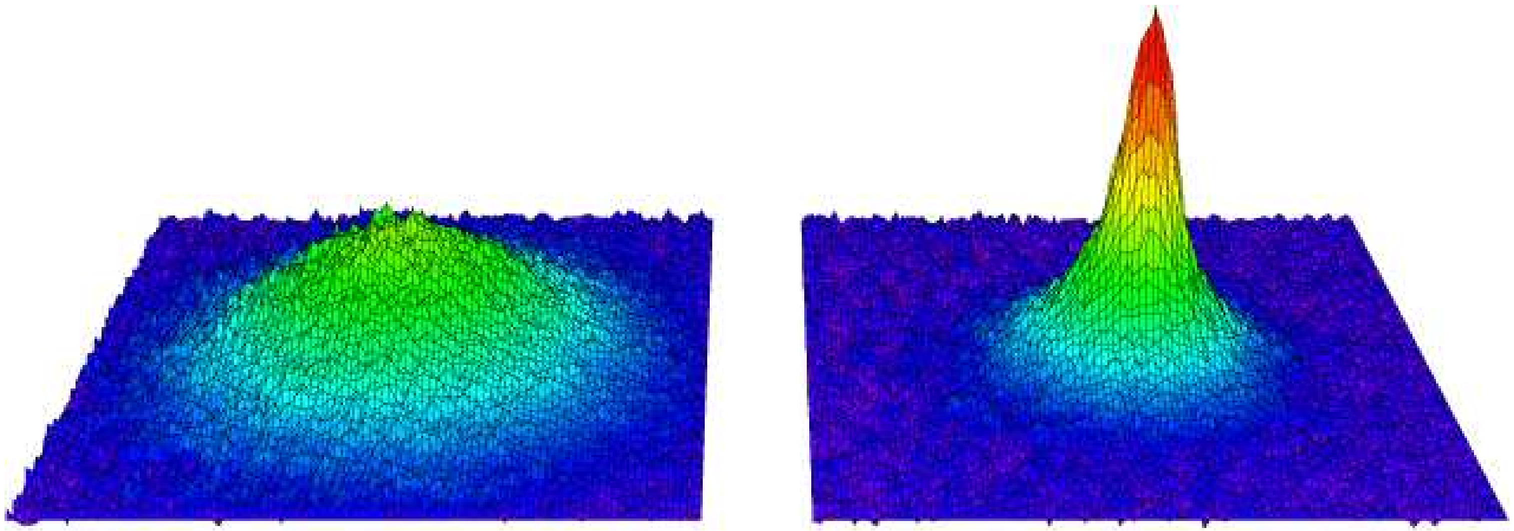
Bose-Einstein condensates of molecules

Suppressed relaxation **Fast elastic collisions** $a_{dd} = 0.6a$

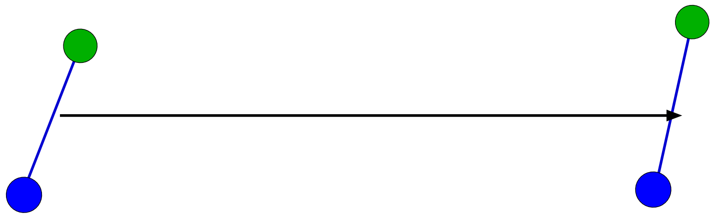
$$^6\mathrm{Li}_2 \rightarrow \frac{\alpha_{rel}}{\alpha_{el}} \leq 10^{-4}$$

Efficient evaporative cooling $\rightarrow$ BEC

JILA, Innsbruck, MIT, ENS, Rice

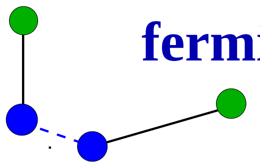


Composite bosons



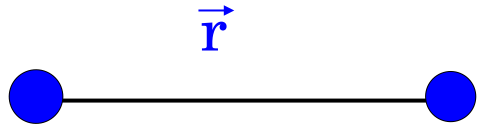
Bosonic behavior at large separations **BEC**

Small separations ?



fermionic correlations!

$$\Psi \propto (\exp\{i\vec{k}\vec{r}\} - \exp\{-i\vec{k}\vec{r}\})$$
$$\rightarrow 2\vec{k}\vec{r} \quad \text{for } kr \ll 1$$



$$|\Psi|^2 \propto k^2$$

$$\Psi \propto (\exp\{i\vec{k}\vec{r}\} + \exp\{-i\vec{k}\vec{r}\})$$
$$\rightarrow \text{const for } \; kr \ll 1$$

bosons