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Summer School on Novel Quantum Phases and Non-Equilibrium Phenomena in Cold Atomic Gases

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From few-body to many-body physics in cold Fermi gases - Part II

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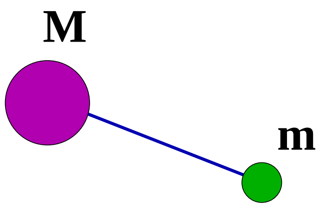
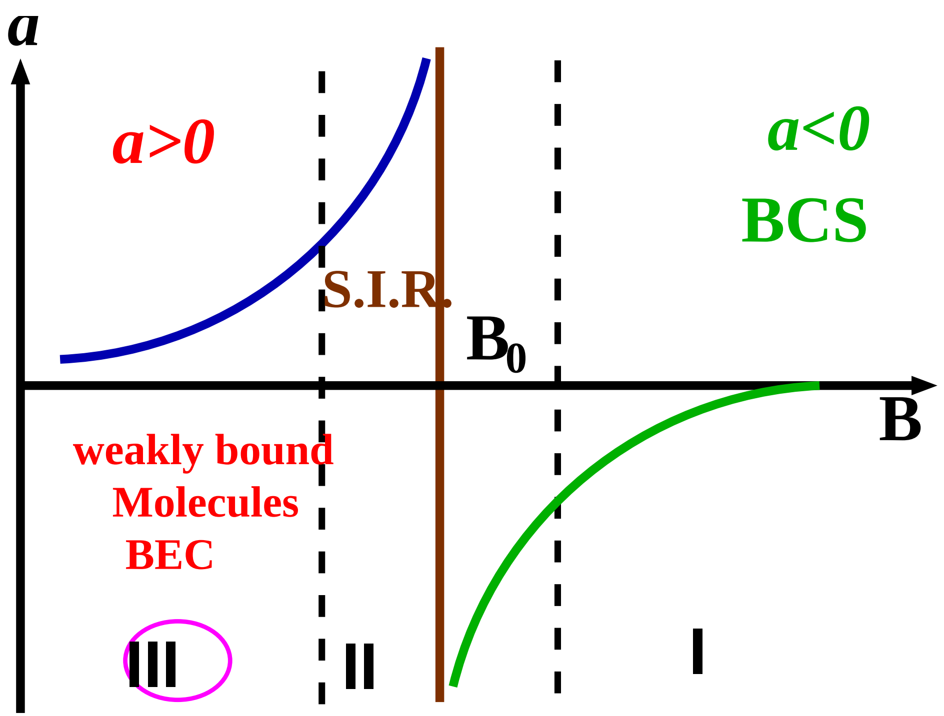
Lecture 2. Molecular regimes in Fermi-Fermi mixtures

Contents

Mixtures of Fermi gases

Molecules of different fermionic atoms $^{6}\mathrm{Li}^{40}\mathrm{K}$ $^{6}\mathrm{Li}^{87}\mathrm{Sr}$



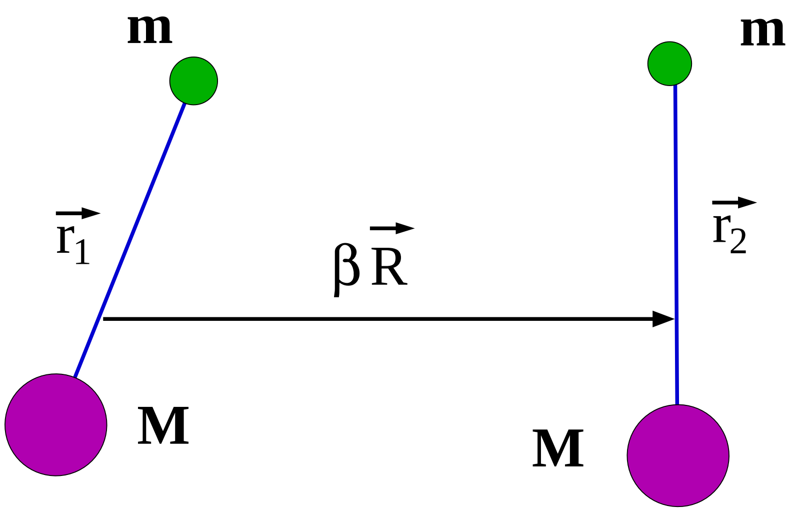


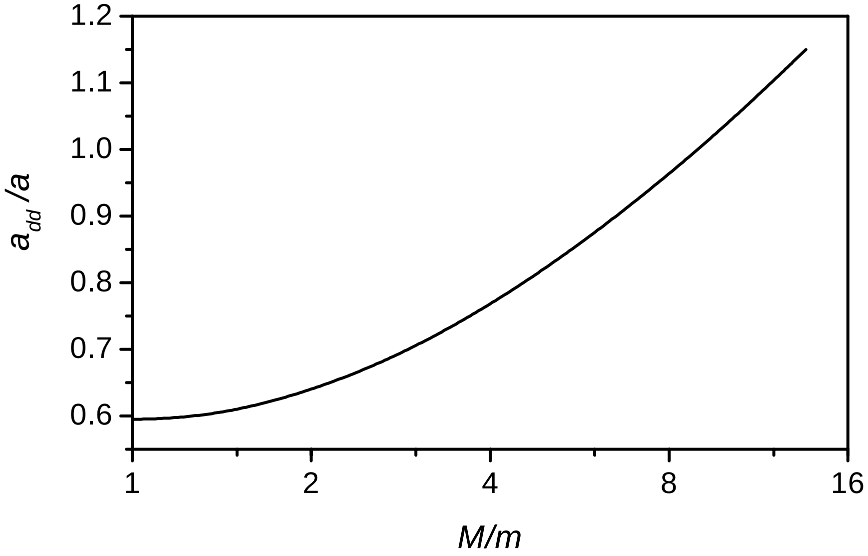
What happens with collisional stability and molecular BEC?

Is there something else interesting ?

Molecule-molecule interaction

Interaction between the molecules (a_{dd}) Petrov et al 2005





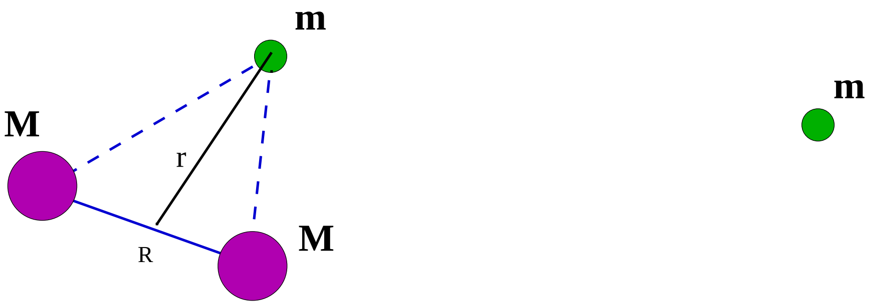
$$\beta = \sqrt{2Mm}/(M+m)$$

Nothing dramatic happens, but why I stop at $M/m = 13.6$?

Collisional relaxation. Mediated potential

Exact solution for the dependence on a and M/m

$M \gg m \quad \rightarrow \quad$ Born-Oppenheimer picture



$r \ll a \rightarrow$ One bound state of a light atom with two fixed heavy ones

$$\psi(\vec{r}) \propto \left(\frac{\exp(-\lambda|\vec{r}-\vec{R}/2|/R)}{|\vec{r}-\vec{R}/2|} + \frac{\exp(-\lambda|\vec{r}+\vec{R}/2|/R)}{|\vec{r}+\vec{R}/2|} \right)$$

Bethe-Peierls boundary condition

$|\vec{r} \pm \vec{R}/2| \rightarrow 0 \quad \psi \propto (1 - a/|\vec{r} \pm \vec{R}/2|)$

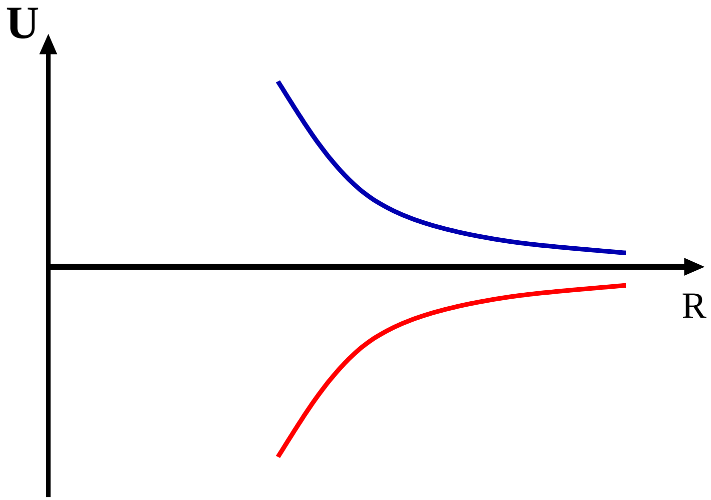
$\lambda \approx 0.567 \quad \varepsilon(R) = U(R) = -\hbar^2\lambda^2/2mR^2$

Mediated attractive potential $\quad U(R) \approx -0.16\hbar^2/mR^2$

Effective potential

Pauli principle $\Rightarrow$ Centrifugal potential $U_c = 2\hbar^2/MR^2$

Mediated attraction competes
with Pauli principle



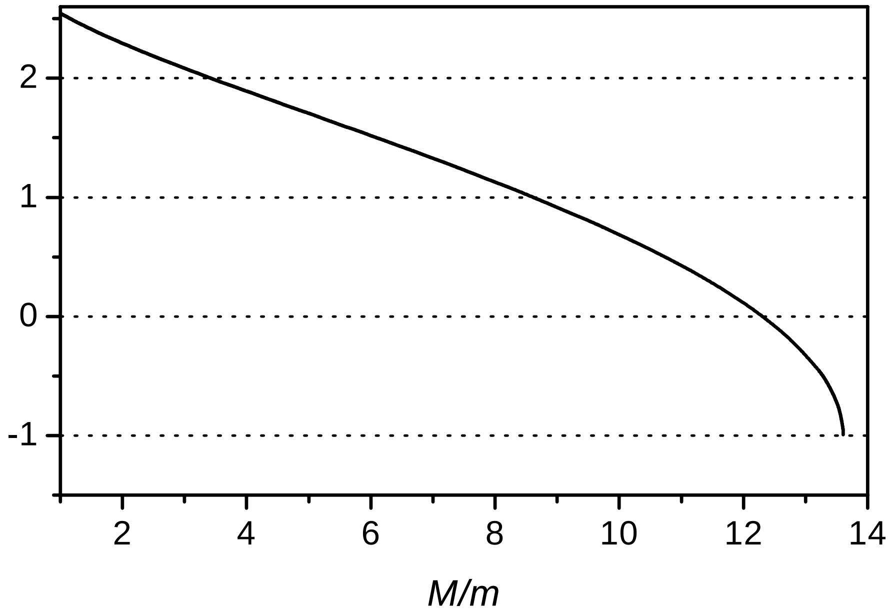
$$U_{eff}(R) = U(R) + U_c(R)$$
$$= -0.16\hbar^2/mR^2 + 2\hbar^2/MR^2$$

$M/m > 13.6 \quad \rightarrow$ fall into center short-range physics

Many nodes of the wavefunction. Many (trimer) bound states

$M/m < 13.6 \quad \rightarrow \quad \alpha_{rel} \sim a^{-s}$

Relaxation rate



$$\alpha_{rel} \sim a^{-s}$$

$M/m > 12.33 \rightarrow \;\; U_{eff} > 0, \;\; \rightarrow s > 0$

$M/m > 12.33 \rightarrow \;\; U_{eff} < 0, \; s < 0 \rightarrow \alpha_{rel}$ increases with a

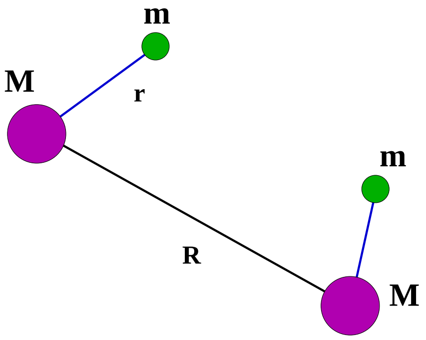
$M/m = 13.6 \;\rightarrow\; s = -1 \;\rightarrow\; \alpha_{rel} \sim a$

$M/m > 13.6 \;\rightarrow$ fall into center short-range physics

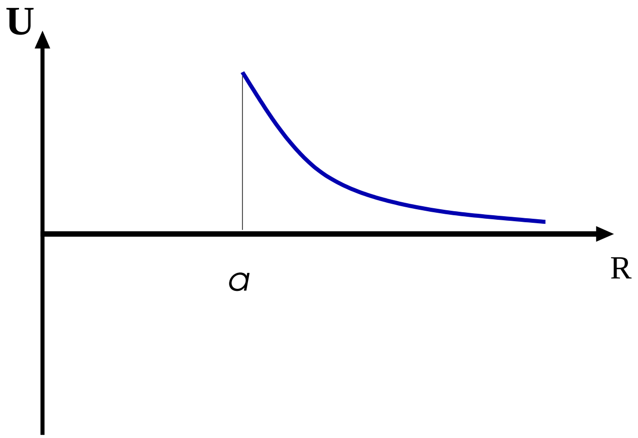
Long-range intermolecular repulsion

Molecules of heavy and light fermions Born-Oppenheimer picture

R ≫ a



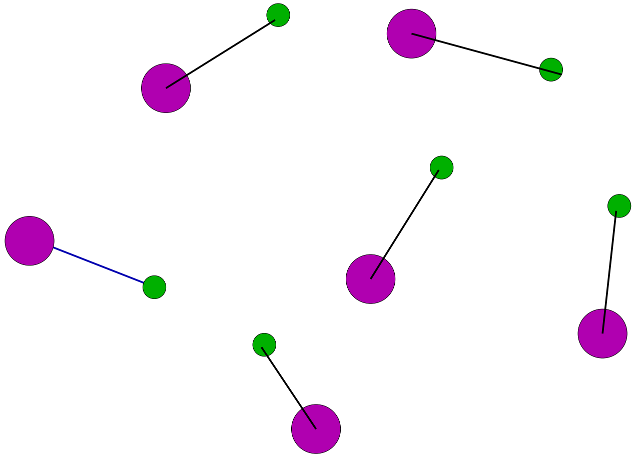
$$U(R) = 2\left(\frac{\hbar^2}{maR}\right)\exp(-2R/a)$$



$$P \sim \exp\left(-0.9\sqrt{\frac{M}{m}}\right)$$

$M >>> m \rightarrow$ Collisional stability independent of a

Many-body system of molecules



Petrov, Astrakharchik,
Papoular, Salomon, GS

No interaction between light fermions

Born-Oppenheimer approach N lowest single-particle states for a light atom

Zero-range appr. for light-heavy interaction. Large inter-heavy distances $\Rightarrow$

Narrow band of N light-atom states, by $\sim \epsilon_0$ below the continuum

Total energy $E = -N\epsilon_0 + (1/2)\sum_{i,j} U(R_{ij})$

$\epsilon_0 = \hbar^2\kappa_0^2/2m \Rightarrow$ molecular binding energy, $\kappa_0^{-1} \rightarrow$ molecular size

$$U_{3D}(R) = 4\epsilon_0[1 - 2(\kappa_0 R)^{-1})]\exp(-2\kappa_0 R);\ \ (1/\kappa_0 R)\exp(-\kappa_0 R) \ll 1$$

$$U_{2D}(R) = 4\epsilon_0[\kappa_0 R K_0(\kappa_0 R)K_1(\kappa_0 R) - K_0^2(\kappa_0 R)];\quad K_0(\kappa_0 R) \ll 1$$

$R \approx 2/\kappa_0$ or larger

Phase diagram

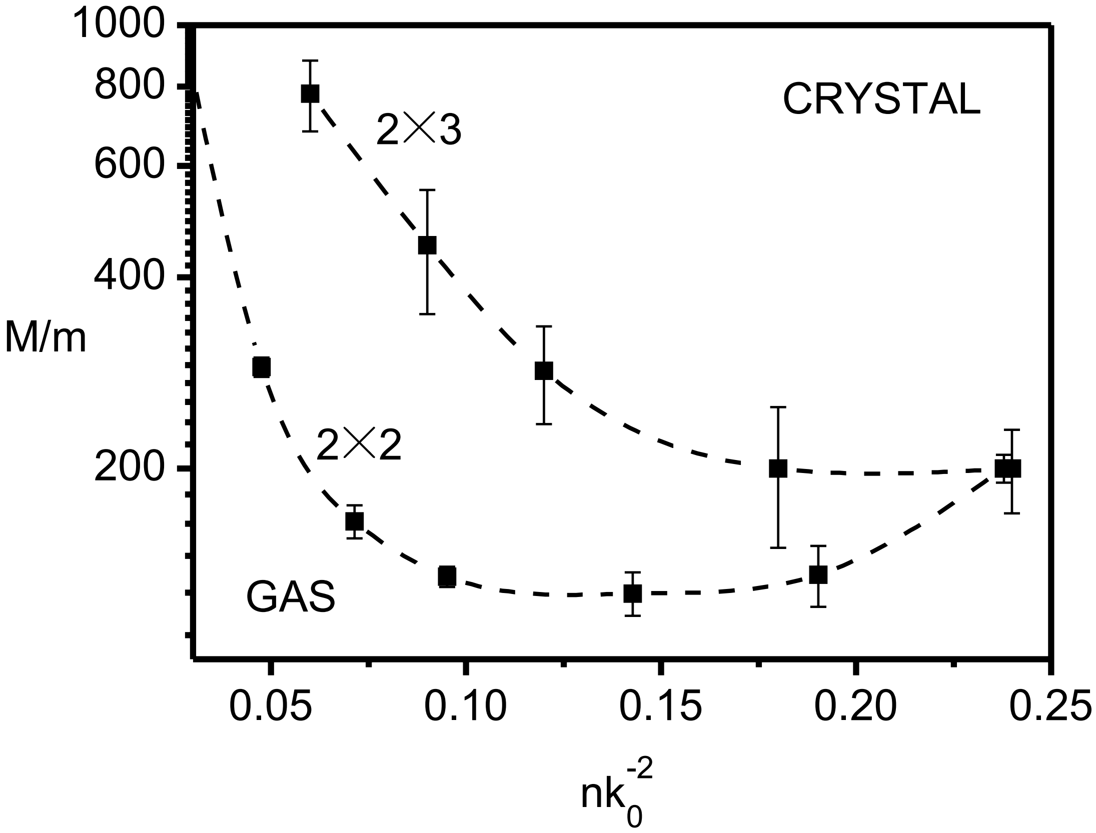
2D motion of heavy atoms

$$H = -(\hbar^2/2M)\sum_i \Delta_{R_i} + (1/2)\sum_{i,j} U(R_{i,j})$$

$(M/m) > (M/m)_c \rightarrow$ crystalline phase

2D motion of light atoms $\Rightarrow$ $(M/m)_c = 120$ triangular lattice

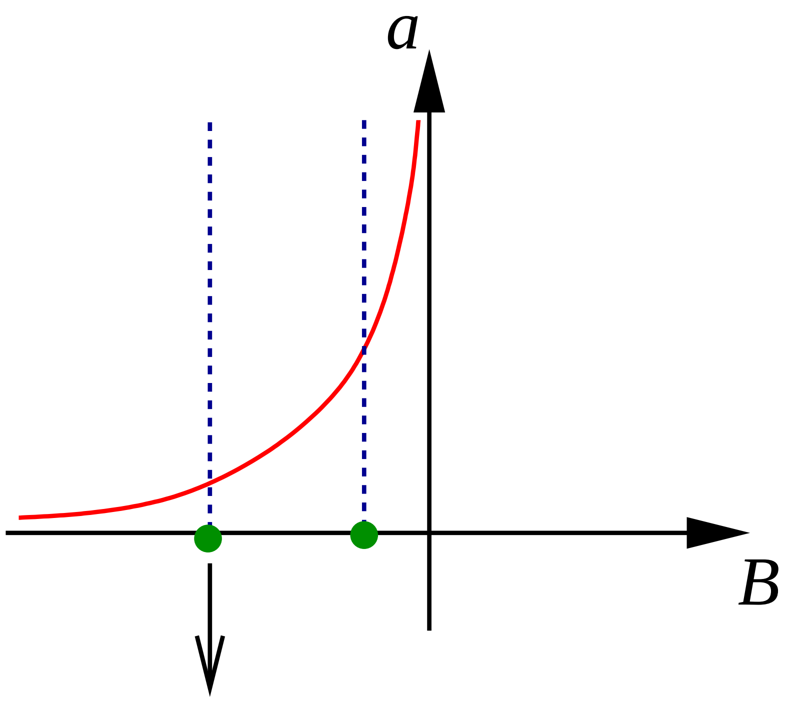
3D motion of light atoms $\Rightarrow$ $(M/m)_c = 200$ triangular lattice



Quantum transitions

$$\frac{M}{m} > \left(\frac{M}{m}\right)_c \qquad \text{and } n \text{ fixed}$$

Increase a



depends on $\frac{M}{m}$ but always $na^3 \ll 1$

first-order transition

Realization of the crystalline phase

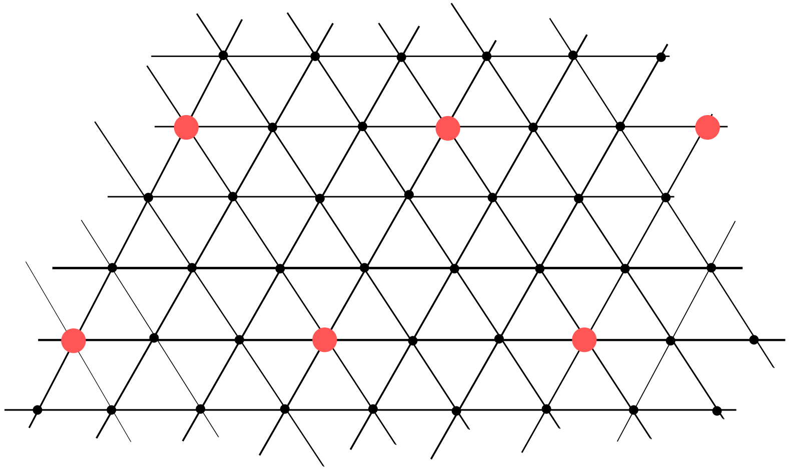
$$\frac{M}{m} \approx 200 \quad \text{or} \quad \frac{M}{m} \approx 100 \quad \rightarrow \text{no gas phase possible}$$

How to obtain the crystalline phase?

Optical lattice for heavy fermions

Small filling factor $\Rightarrow$ Increase of M/m

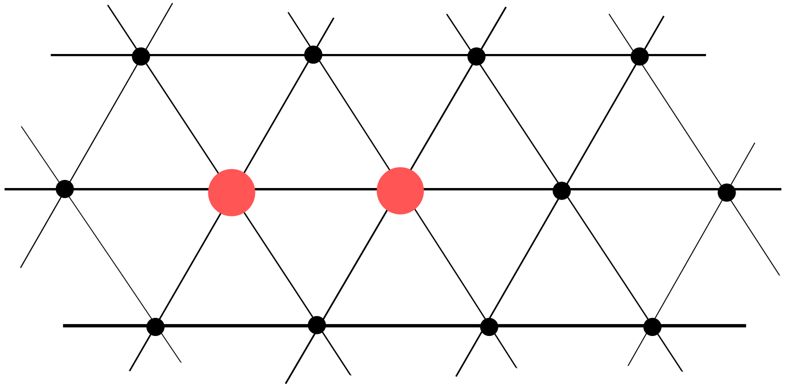
Increase of M by a factor of 20 or more is possible



Formation of a superlattice

Stability of the crystalline phase

Relaxation into deep bound states



heavy atoms in neighboring sites $\Rightarrow P_1 \sim nL^2 \exp(-\sqrt{M_*/m})$

jump to the same site $\Rightarrow P_2 \sim (t/U_0)^2; \;\; t = \hbar^2/M_* L^2, \;\; U_0 \sim \hbar\omega_l = \hbar^2/Ml^2$

undergo relaxation process $\Rightarrow \tau_0^{-1} \sim (\hbar/Ml)(1/l^3)$ at worst

Relaxation rate $\tau^{-1} \sim P_1 P_2 \tau_0^{-1} \sim nL^2 (M/M_*)^2 (l/L)^2 (\hbar/M) \exp(-\sqrt{M_*/m})$

τ exceeds 10s even for $n \sim 10^9$cm^{-2}

Formation of trimer states (2 heavy and 1 light atom)

4-body problem in a lattice $\Rightarrow$ τ can range from 0.1 to 100s for $n \sim 10^9$ cm^{-2}

Conclusions

- Remarkable physics of weakly bound molecules in cold Fermi gases
- Novel physics of molecular collisional stability in mixtures of Fermi gases
- Possibilities to create new macroscopic quantum systems