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Collective excitations in Bose and Fermi gases

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Collective oscillations of superfluid atomic Bose and Fermi gases

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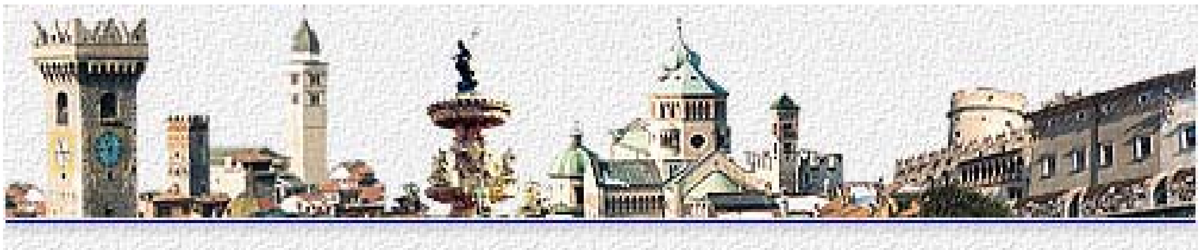
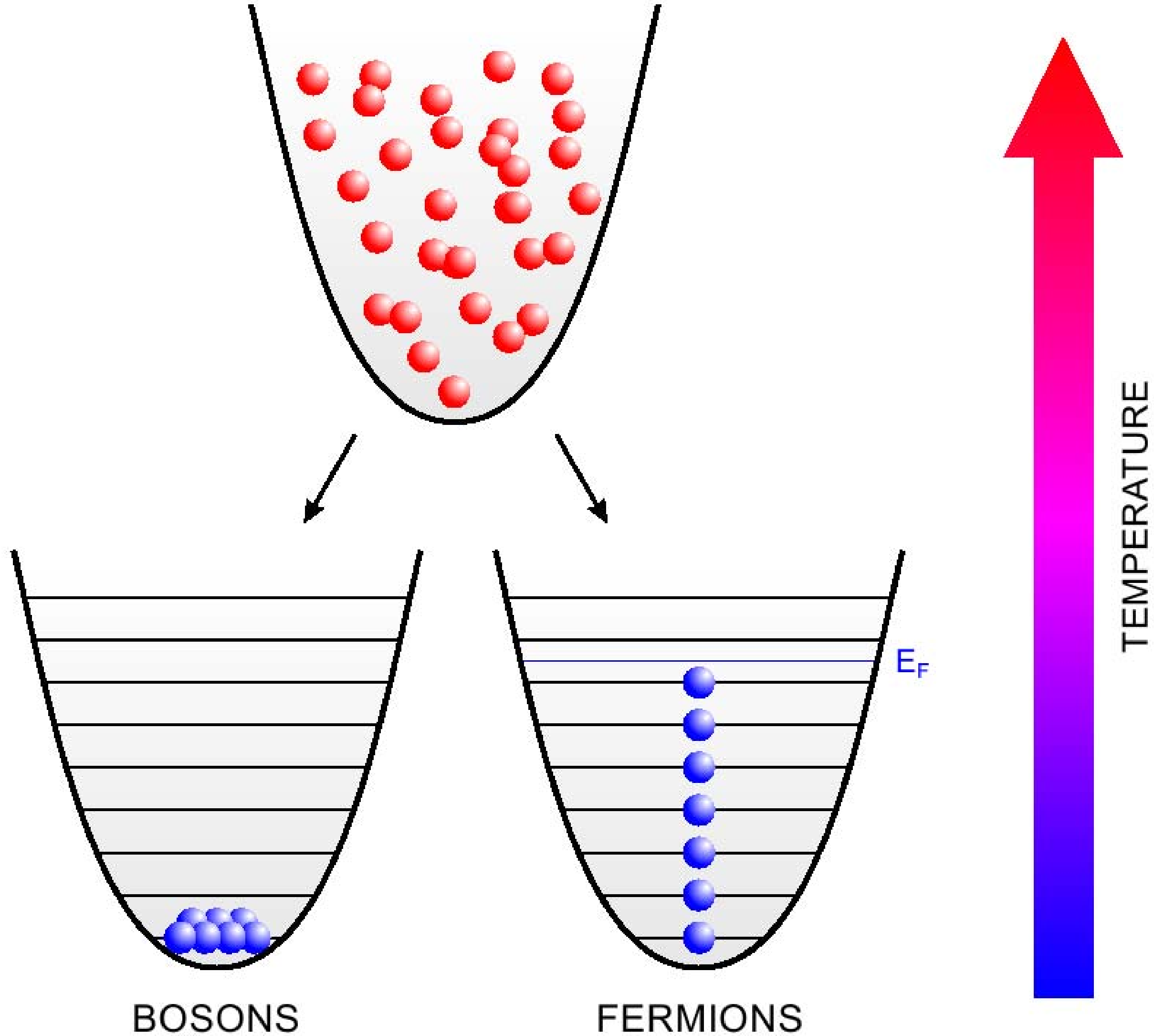


Quantum statistics and **temperature** scales



$$k_B T_C = 0.94\,\hbar\omega\, N^{1/3}$$

$$k_B T_F = \hbar\omega (6N)^{1/3}$$

Bosons

- When T tends to 0 a macroscopic fraction of **bosons** occupies a single particle state (**BEC**)

- Wave function of macroscopically occupied single particle state defines **order parameter**

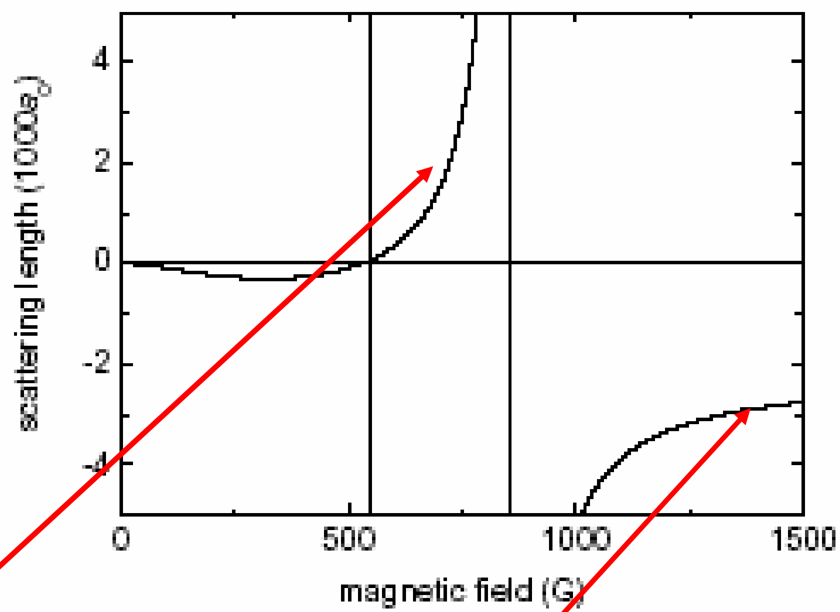
- Actual form of order parameter depends on two-body interaction (scattering length) and is described by Gross-Pitaevskii equation

Fermions

- In the absence of interactions the physics of **fermions** deeply differs from the one of bosons (consequence of **Pauli** principle)

- **Interactions** can change the scenario in a drastic way:
  - pairs of atoms can form a bound state (molecule) and give rise to **BEC**
  - pairing can affect the many-body physics also in the absence of two-body molecular formation (many-body or Cooper pairing) giving rise to **BCS** superfluidity.
  - In most cases the physics of interacting fermions is characterized by s-wave scattering length **a**

In the presence of Feshbach resonance
the value of **a** can be tuned
by adjusting the external magnetic field
At resonance a becomes infinite

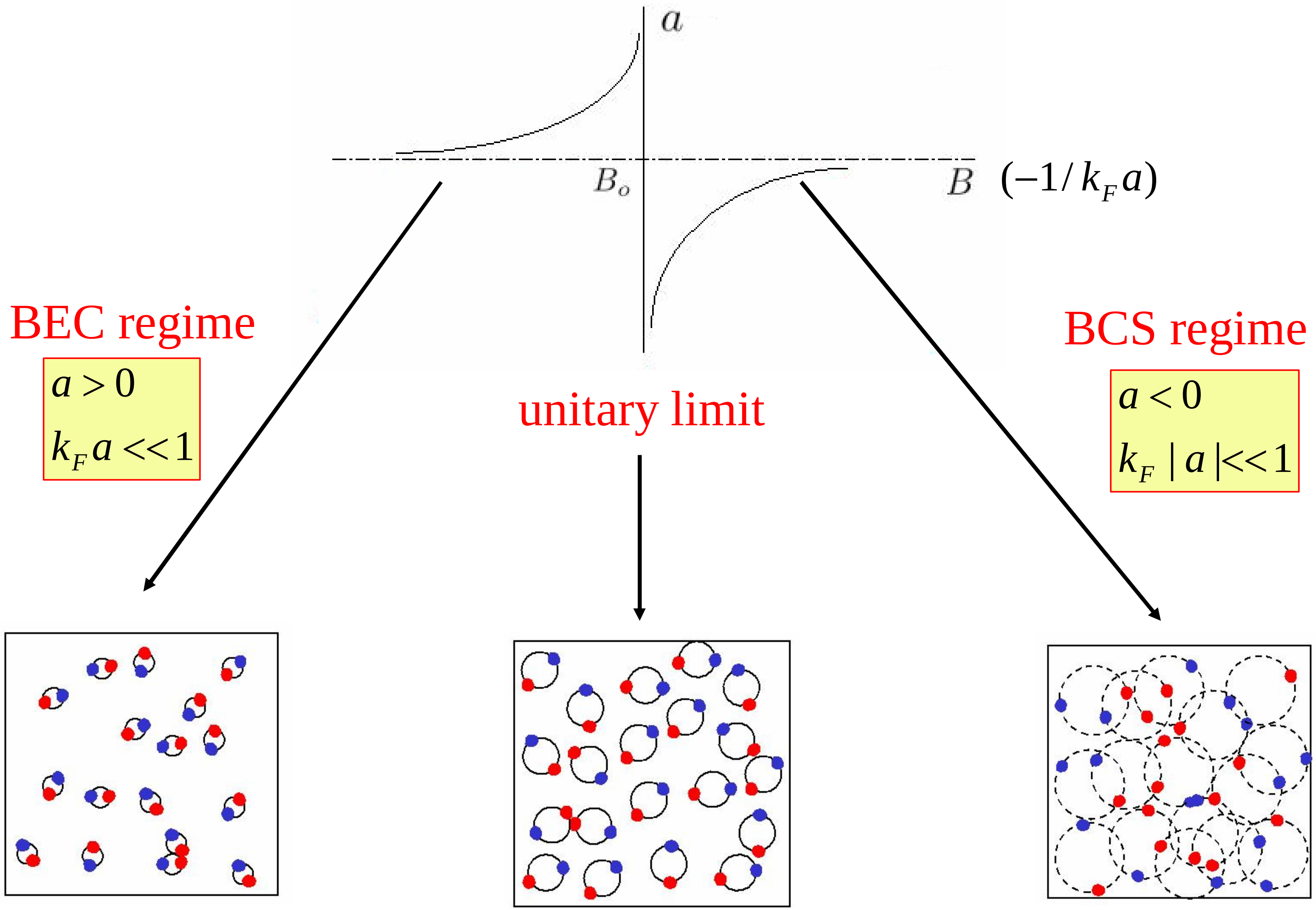


When scattering length **is positive** weakly **bound molecules** of size a and binding energy $\hbar^2 / ma^2$ are formed

If size of molecules is much smaller than average distance between molecules $k_F a << 1$ the gas is a **BEC** gas of molecules

In opposite regime of small small and **negative** values of **a** size of pairs is larger than interparticle distance (Cooper pairs, **BCS** regime)

Many-body aspects (BEC-BCS crossover)



Interactions provide a continuous **link** between the physics of **BEC and BCS superfluids**

Manifestations of superfluidity:

this talk

Macroscopic dynamic phenomena
(expansion, **collective oscillations**, moment of inertia)
are described by theory of **irrotational hydrodynamics**

More microscopic theories required to describe
other important superfluid phenomena
(quantized vortices, **Landau critical velocity**, pairing gap)

HYDRODYNAMIC THEORY OF SUPERFLUIDS

Basic assumptions:

- **Irrotationality** constraint
  (follows from the phase of order parameter)

- **Conservation laws**
  (equation of continuity, equation for the current)

Basic ingredient:

- **Equation of state**

Consequence of Galilean invariance: from the equation for the field operator to the hydrodynamic equations of superfluids

Heisenberg equation for the field operator in uniform systems (Bose field)

$$i\hbar\frac{\partial}{\partial t}\hat{\Psi}(r,t)=[\hat{\Psi}(r,t),H]=[-\frac{\hbar^2}{2m}\nabla^2+2\int dr'\hat{\Psi}^+(r',t)V(r-r')\hat{\Psi}(r',t)]\hat{\Psi}(r,t)$$

(similar equation for Fermi field operator)

If $\hat{\Psi}(r,t)$ is solution, $\hat{\Psi}(r-vt,t)\exp[\frac{i}{\hbar}(mrv-\frac{1}{2}mv^2t)]$

fluid at rest → $\hat{\Psi}(r,t)$; fluid moving with velocity v → $\hat{\Psi}(r-vt,t)$

is also solution (Galilean transformation with velocity v)

Order parameter $<\hat{\Psi}>$ ($<\hat{\Psi}\hat{\Psi}>$ in Fermi case)

acquires phase $$S(r,t)=[mrv-(\frac{1}{2}mv^2+\mu)t]/\hbar$$

$m\rightarrow 2m$ *in Fermi case*

Gradient of the phase ➡ $$v_S = \frac{\hbar}{m}\nabla S$$ **Superfluid velocity**

$m \to 2m$ in Fermi case

IRROTATIONALITY of flow is fundamental feature of superfluids: ⇨

- quenching of **moment of inertia**
- quantization of circulation and **quantized vortices**)

Time derivative of the phase ➡ $$\hbar\frac{\partial}{\partial t}S(r,t) = -(\frac{1}{2}mv_S^2 + \mu)$$

Relationship for **superfluid velocity** and **equation for the phase** are expected to hold also if order parameter varies **slowly** in space and time as well as in the presence of a smooth external potential. $\mu \to \mu + V_{ext}(r)$

HYDRODYNAMIC EQUATIONS AT ZERO TEMPERATURE

$$\frac{\partial}{\partial t} n + \nabla (vn) = 0$$

$$m \frac{\partial}{\partial t} v + \nabla (\frac{1}{2} m v^2 + \mu(n) + V_{ext}) = 0$$

irrotationality

Hydrodynamic equations of superfluids

Closed equations for **density** and superfluid **velocity** field

KEY FEATURES OF HD EQUATIONS OF SUPERFLUIDS

- Have **classical** form (do not depend on Planck constant)
- Velocity field is **irrotational**
- Are equations for the total density (not for the condensate density)
- Should be distinguished from **rotational hydrodynamics**.
- Applicable to **low energy, macroscopic**, phenomena
- Hold for both **Bose** and **Fermi** superfluids
- Depend on **equation of state** $\mu(n)$
  (sensitive to quantum correlations, statistics, dimensionality, ...)
- **Equilibrium** solutions (**v**=0) consistent with **LDA** $\mu(n)+V_{ext}(r)=\mu_0$

What do we mean by **macroscopic, low energy** phenomena ?

BEC superfluids

$$\lambda >> \frac{\hbar}{\sqrt{2mgn}}$$ (**healing length**)

$$\hbar\omega << gn$$

BCS Fermi superfluids

$$\lambda >> \frac{\hbar v_F}{\Delta}$$ (**size of Cooper pairs**)

$$\hbar\omega << \Delta$$

(Δ: **superfluid gap**)

more restrictive than in BEC

WHAT ARE THE HYDRODYNAMIC EQUATIONS USEFUL FOR ?

They provide quantitative predictions for

- **Expansion** of the gas follwowing sudden release of the trap

- **Collective oscillations** excited by modulating harmonic trap

Quantities of highest interest from both **theoretical** and **experimental** point of view

- **Expansion** provides information on **release energy**, sensitive to **anisotropy**

- **Collective frequencies** are measurable with **highest precision** and can provide accurate test of **equation of state**

Collective oscillations in trapped gases

Collective oscillations: unique tool to explore consequence of ***superfluidity*** *and test the* ***equation of state*** *of interacting quantum gases (both* ***Bose*** *and* ***Fermi****)*

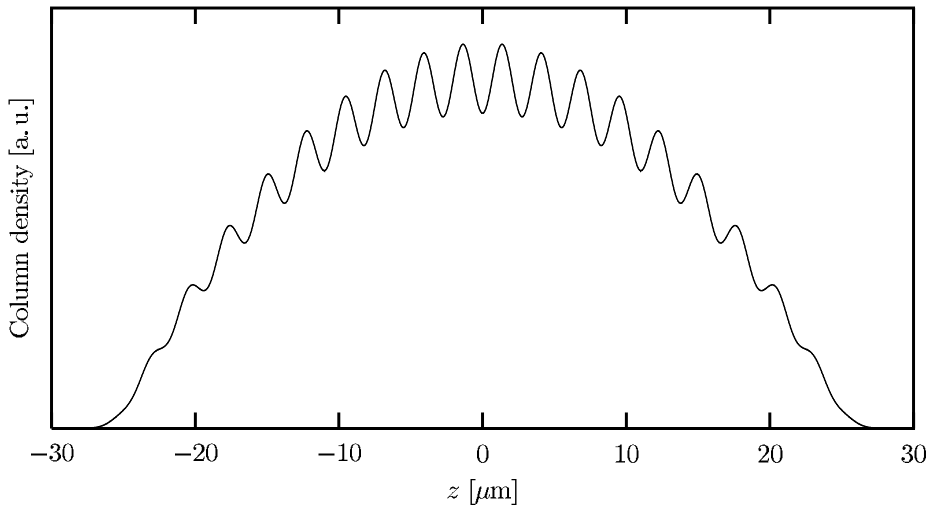
Experimental data *for collective frequencies are available with* ***high precision***
(recent application to the study of the Casimir force, Obrecht et al. 2007))

Propagation of sound in trapped gases

In **uniform** medium HD theory gives sound wave solution

$\delta n \propto e^{i(qz-\omega t)}$ with $\omega = cq\,;\;\; mc^2 = \partial\mu/\partial n$

In **trapped** gases sound waves can propagate if wave length is smaller than axial size of the condensate. Condition is easily satisfied in elongated condensates.



Propagation of sound in elongated traps

-If **wave length** is **larger** than **radial size** of elongated trapped gas sound has **1D** character

$$mc_{1D}^2 = n_1 \partial\mu_1 / \partial n_1$$

where $n_1 = \int ndxdy$ and n is determined by TF eq. $\mu(n) + V_{ext}(\vec{r}) = \mu_1$

one finds

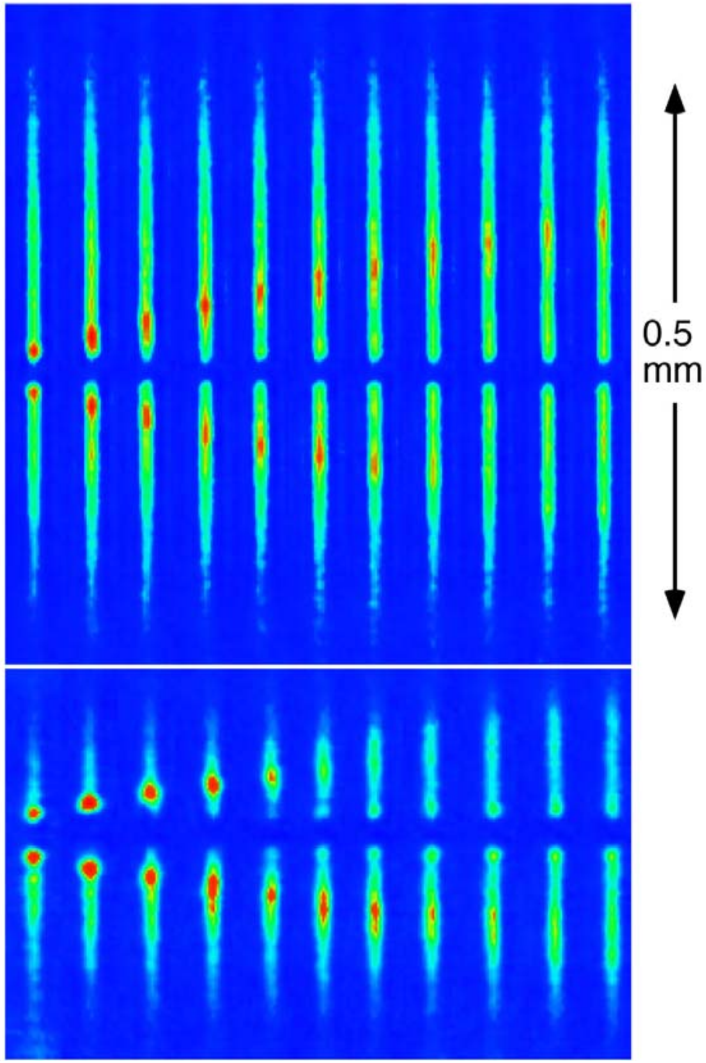
$$mc_{1D}^2 = \frac{\int ndxdy}{\int (\partial\mu / \partial n)^{-1} dxdy}$$

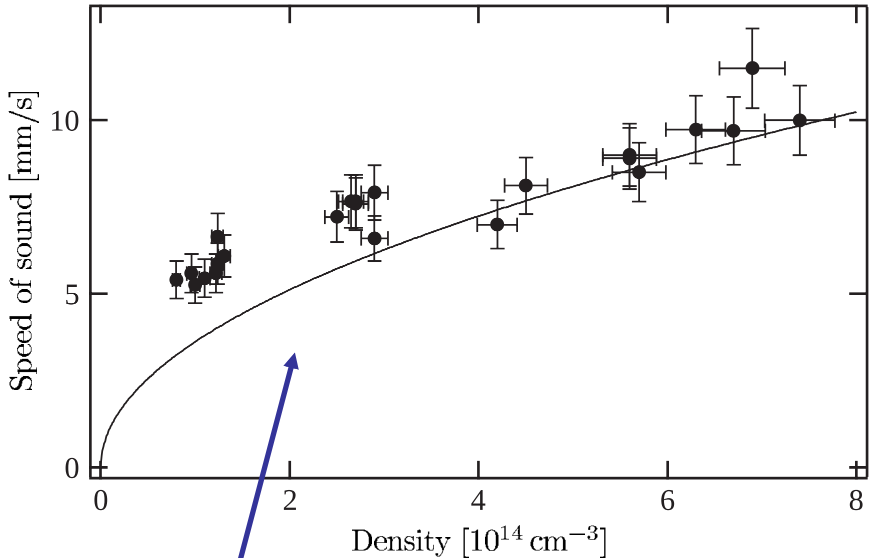
For BEC gas ($\mu \propto n$) $\quad c_{1D} = c_{bulk} / \sqrt{2}$ *(Zaremba, 1998)*

For unitary Fermi gas ($\mu \propto n^{2/3}$) $\quad c_{1D} = c_{bulk} \sqrt{3/5}$ *(Capuzzi et al, 2006)*

Bosons

Sound wave packets propagating in a BEC (Mit 97)

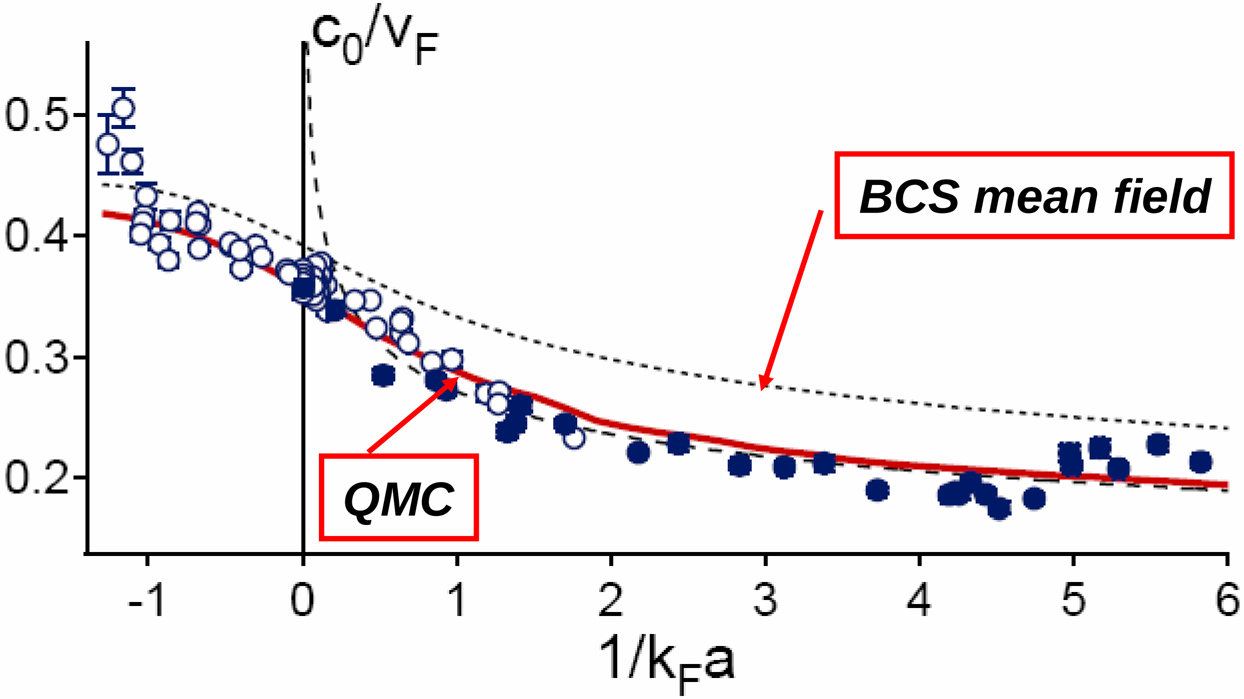




velocity of sound as a function of central density

Fermions

Sound wave packets propagating in an Interacting Fermi gas (Duke, 2006) behavior along the crossover



Difference bewteen BCS and QMC reflects:

-at unitarity: different value of β *in eq. of state* $\mu \propto (1+\beta)n^{2/3}$

-On BEC side different molecule-molecule scattering length

Collective oscillations in harmonic trap

When wavelength is of the order of the size of the atomic cloud sound is no longer a useful concept. Solve linearized 3D HD equations

$$\frac{\partial^2}{\partial t^2}\delta n = \nabla(n_0 \nabla(\frac{\partial \mu}{\partial n}\delta n))$$

$n_0(\vec{r})$ *is non uniform equilibrium Thomas Fermi profile*

$$m\frac{\partial}{\partial t}v = -\nabla(\frac{\partial \mu}{\partial n}\delta n)$$

Solutions of HD equations in harmonic trap predict both **surface** and **compression** modes (first investigated in dilute BEC gases (Stringari 96)

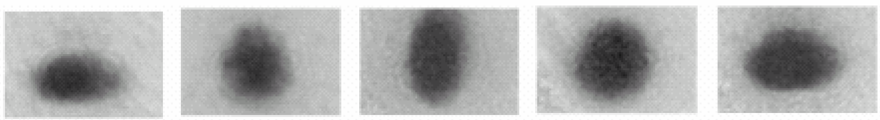
t

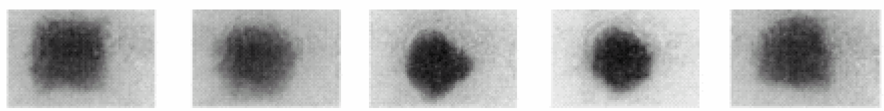
Surface modes

- **Surface** modes are **unaffected** by equation of state
- For isotropic trap one finds $\omega = \sqrt{l}\,\omega_{ho}$ where l is angular momentum
- surface mode is driven by external potential, **not by surface tension**
- Dispersion law differs from ideal gas value $\omega = l\,\omega_{ho}$ (**interaction effect**)

Bosons

Surface modes in BEC's, Mit 2000





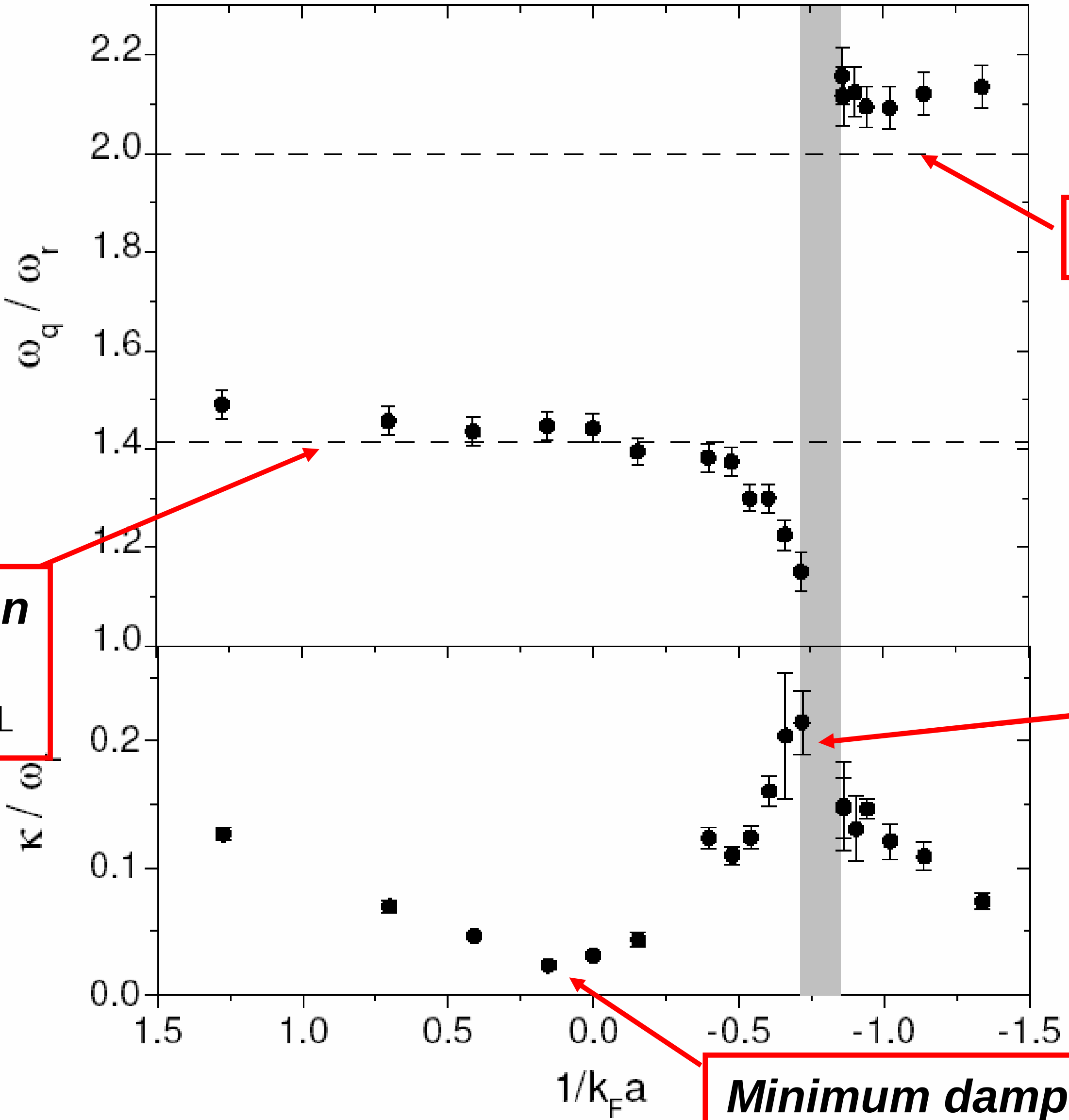
m=2

m=4

ℓ	ν_ℓ (Hz)	ν_ℓ/ν_1 (Expt.)	ν_ℓ/ν_1 (Theor.)
1	90.1 ± 0.5	...	...
2	130.5 ± 2.5	1.45 ± 0.04	$\sqrt{2}$
4	177 ± 5	1.96 ± 0.06	2

Fermions

l=2 Quadrupole mode measured on ultracold Fermi gas along the crossover (Altmeyer et al. 2007)



Ideal gas value

HD prediction

$$\omega = \sqrt{2}\omega_\perp$$

Enhancement of damping

Minimum damping near unitarity

Fermions

- Experiments on collective oscillations show that on the BCS side of the resonance superfluidity is broken for relatively small values of $1/k_F|a|$ ***(where gap is of the order of radial oscillator frequency)***

\- ***Deeper in BCS regime frequency takes collisionless value***

- Damping is minimum near resonance

Compression modes

- Sensitive to the **equation of state**

-**analytic** solutions for collective frequencies available for **polytropic** equation of state $\mu \propto n^{\gamma}$

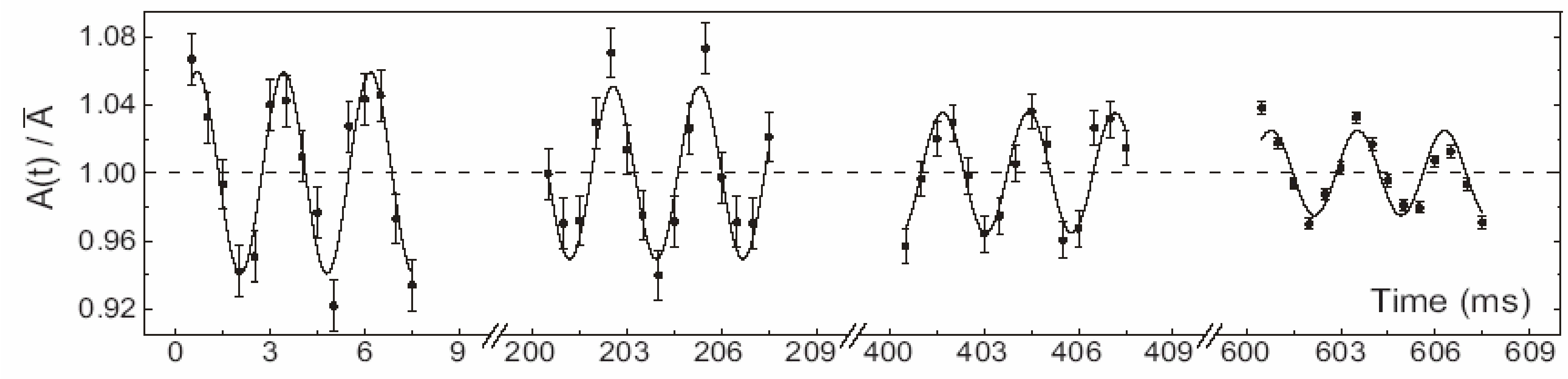
- Example: **radial compression** mode in cigar trap $\omega = \sqrt{2(\gamma+1)}\omega_{\perp}$

- At unitarity ($\gamma = 2/3$) one predicts universal value $\omega = \sqrt{10/3}\omega_{\perp} \approx 1.83\omega_{\perp}$

- For a BEC gas one finds $\omega = 2\omega_{\perp}$

Bosons

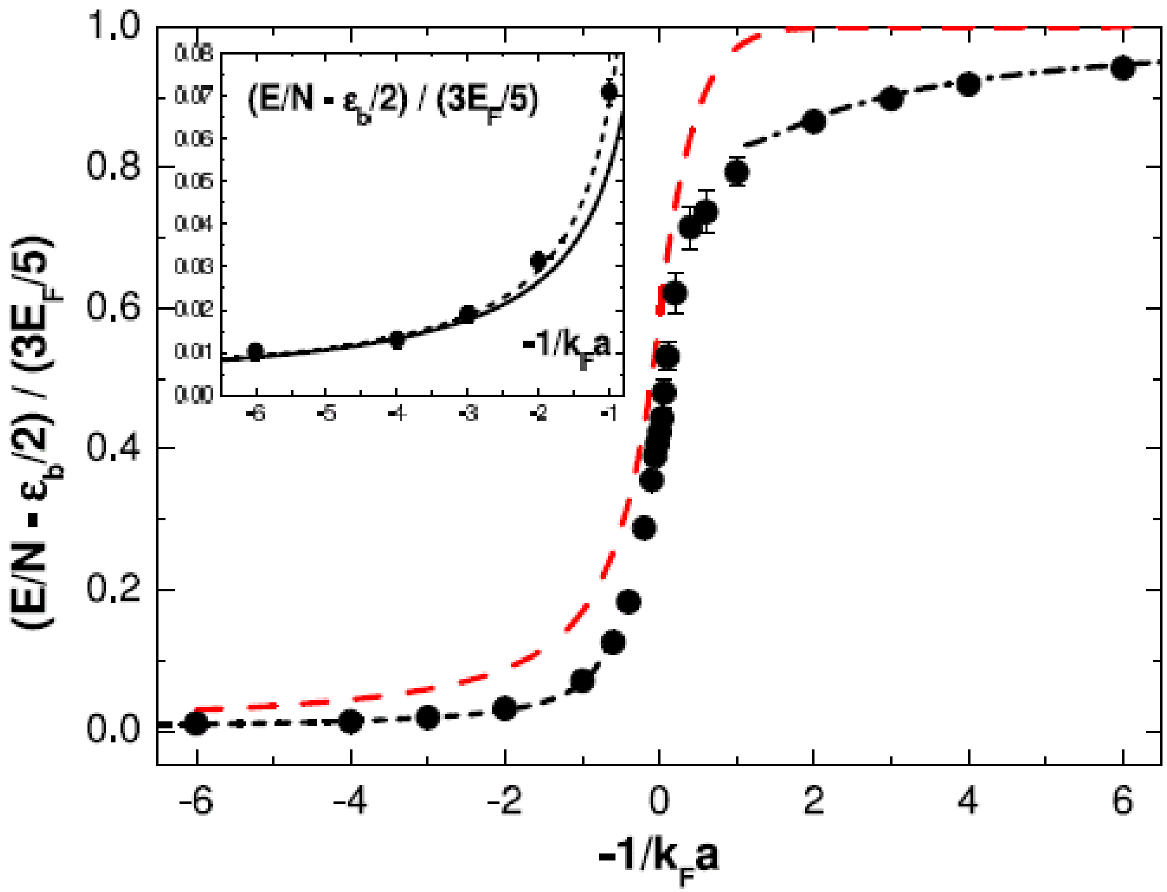
m=0 radial compression mode at T=0 (Ens 2001) exp: $\omega = 2.07\ \omega_z$ theory: $\omega = 2\ \omega_z$



Fermions

Equation of state along BCS-BEC crossover

- Fixed Node Diffusion MC (Astrakharchick et al., 2004)
- Comparison with mean field BCS theory (- - - - -)

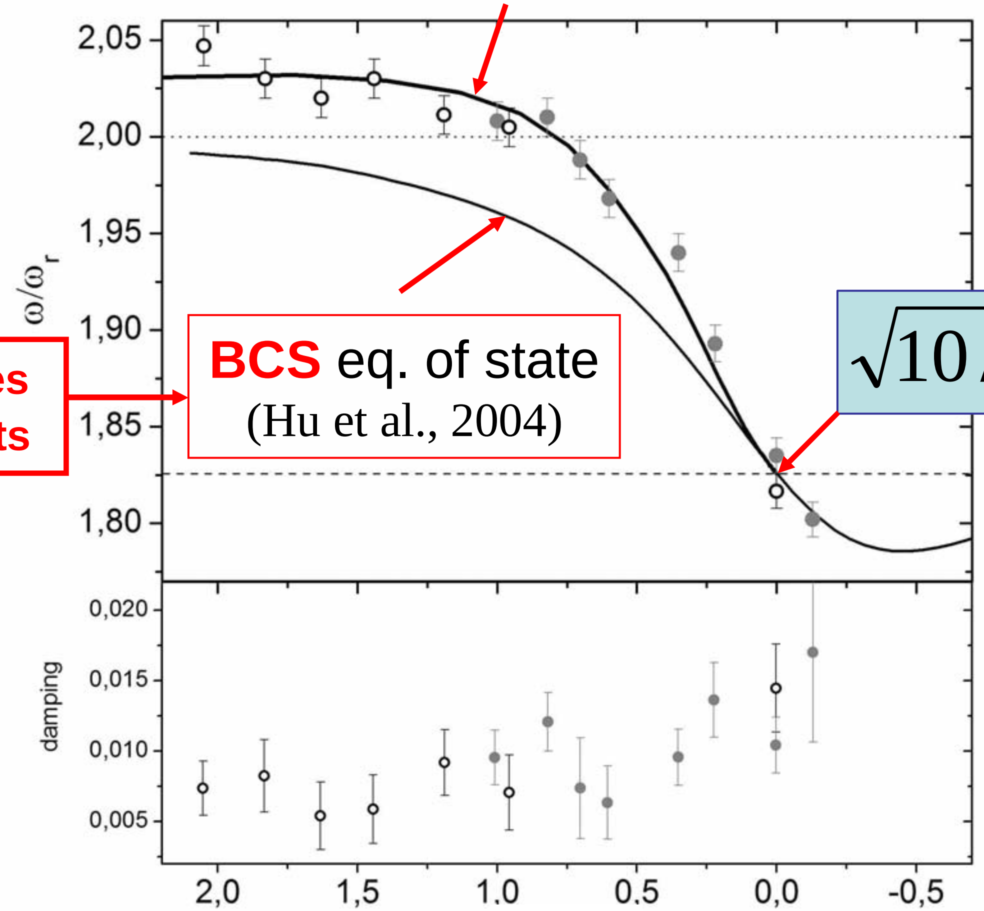


Fermions

Radial breathing mode at Innsbruck (Altmeyer et al., 2007)

MC equation of state (Astrakharchick et al., 2005)

includes beyond mf effects



$1/k_F^0 a$

does not includes beyond mf effects

BCS eq. of state (Hu et al., 2004)

$$\sqrt{10/3} = 1.83$$

universal value at unitarity

Measurement of collective frequencies provides accurate test of equation of state !!

Fermions

Main conclusions concerning the m=0 radial compression mode in superfluid Fermi gases

- **Accurate confirmation** of the **universal HD value** $\sqrt{10/3}\omega_{\perp}$ predicted at unitarity.

- Accurate confirmation of **QMC equation of state** on the BEC side of the resonance.

- **First evidence for Lee Huang Yang effect** (enhancement of frequency with respect to BEC value (role of quantum fluctuations)

Landau's critical velocity (beyond HD)

While in BEC gas ***sound velocity*** *provides critical velocity,*
in a Fermi BCS superfluid critical velocity is fixed
by pair breaking mechanisms (role of the ***gap****)*

Landau's critical velocity

$$v_{cr} = \min_p \frac{\varepsilon(p)}{p}$$

Dispersion law of elementary excitations

- **Landau's criterion for superfluidity** (**metastability**):
  fluid moving with velocity **smaller** than critical velocity cannot decay (**persistent current**)

- Ideal Bose gas and ideal Fermi gas one has $v_{cr} = 0$

- In interacting Fermi gas one predicts two limiting cases:

BEC (Bogoliubov dispersion)

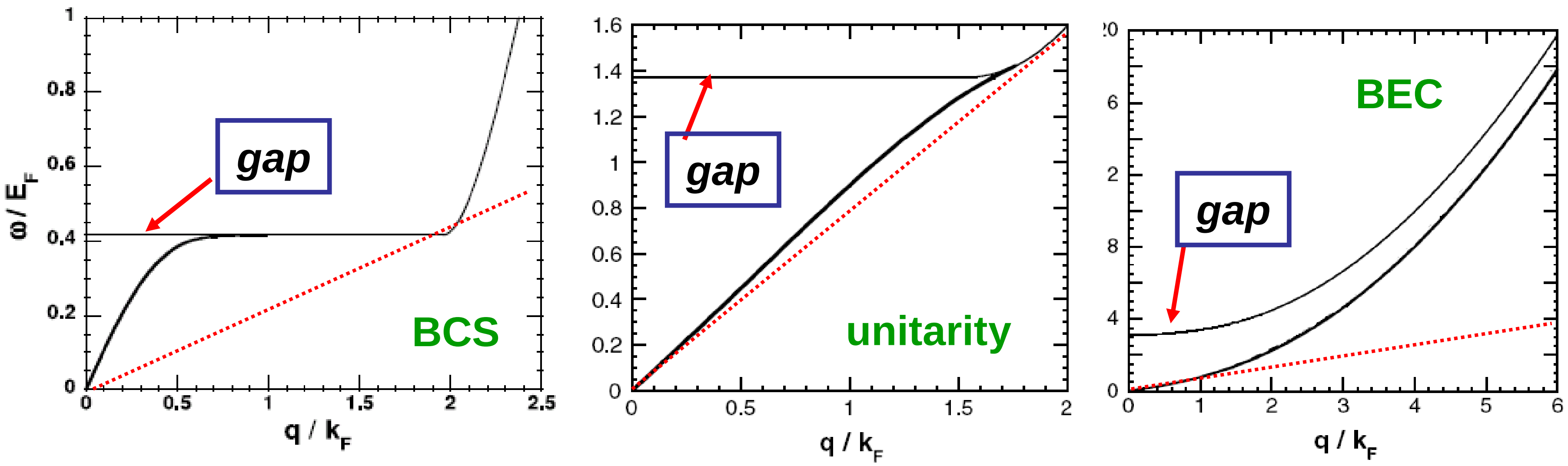
$$v_{cr} = c \propto \sqrt{a}$$

(sound velocity)

BCS (role of the gap)

$$v_{cr} = \Delta / p_F \propto \exp(\pi / 2k_F a)$$

Dispersion law of density excitations along BCS-BEC crossover



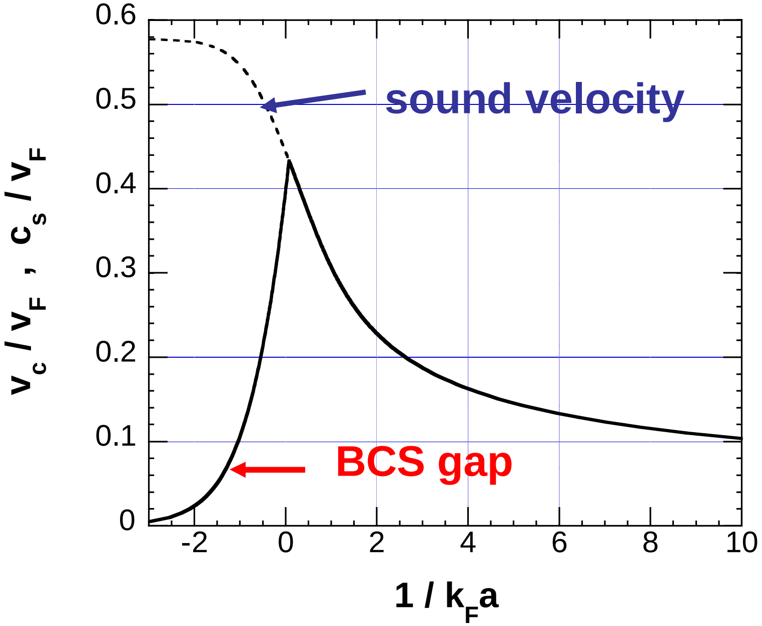
(Combescot, Kagan and Stringari 2006)

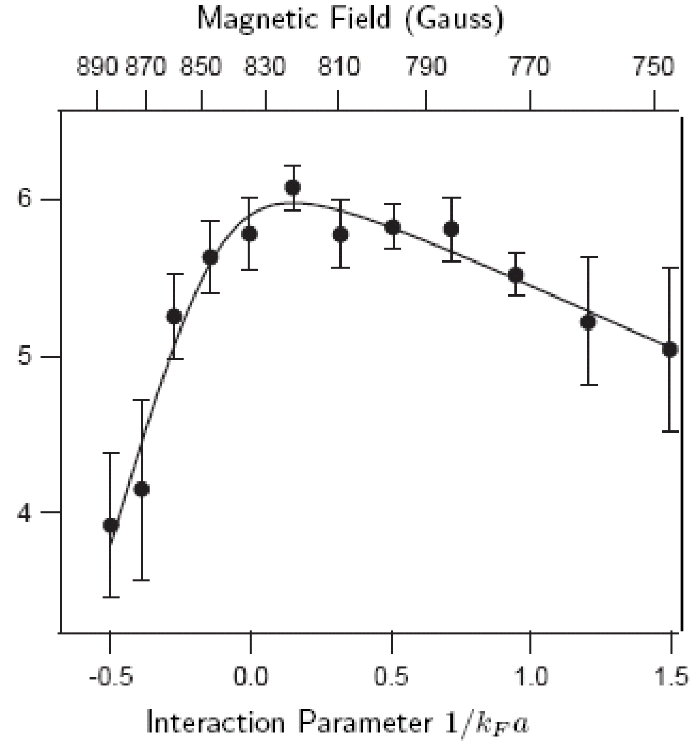
Landau's critical velocity

Theory (BCS mean field)

Experiment (MIT)

$$\frac{v_{cr}}{v_F}$$





(Combescot, Kagan and Stringar, 2006)

Miller et al. cond-mat/07072354

Landau's critical velocity is highest near unitarity !!

Some conclusions

- **Hydrodynamic** theory provides **excellent description** of **macroscopic dynamics** in trapped superfluid gases

- **Superfluidity** in Fermi gases is particularly robust at **unitarity** ($1/k_F a = 0$):
  - **small damping** of collectivemodes;
  - **high Landau's critical velocity**

Recent reviews on ultracold Bose and Fermi gases

- Ultracold atomic Fermi gases
Proceedings of 2006 Varenna Summer School
W. Ketterle, M. Inguscio, and Ch. Salomon (in press)

- **Many-body physics with ultracold atomic gases**
I. Bloch, J. Dalibard and W. Zwerger
cond-mat/0704.3011 (submitted to Rev.Mod.Phys.)

- **Theory of Ultracold Fermi gases**
S. Giorgini, L. Pitaevskii and S. Stringari
cond-mat/0706.3360 (submitted to Rev.Mod.Phys.)