The Abdus Salam International Centre for Theoretical Physics

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IAEA International Atomic Energy Agency

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Summer School on Novel Quantum Phases and Non-Equilibrium Phenomena in Cold Atomic Gases

27 August - 7 September, 2007

Dipolar BECs with spin degrees of freedom

Yuki Kawaguchi
Tokyo Institute of Technology

Dipolar BECs with Spin Degrees of Freedom

YUKI KAWAGUCHI
Tokyo Institute of Technology, Japan

Collaborators:
Hiroki Saito (Univ. of Elec.-Comm., Japan)
Masahito Ueda (Tokyo Tech., ERATO-JST, Japan)

Outline

Introduction : dipole-dipole interaction

What's new in a spinor dipolar BEC

- Einstein-de Haas effect

  YK, H. Saito, and M. Ueda, PRL **96**, 080405 (2006)

- Ground-state circulation

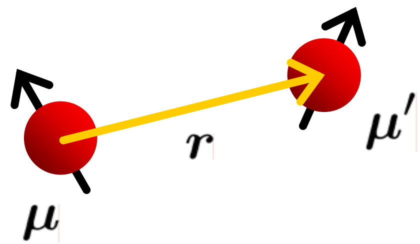
  YK, H. Saito, and M. Ueda, PRL **97**, 130404 (2006)

- Possible experimental scheme

  YK, H. Saito, and M. Ueda, PRL **98**, 110406 (2007)

Dipole-Dipole Interaction

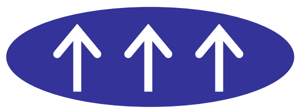
$$V_{\rm dd} = \frac{\mu_0}{4\pi}\frac{\boldsymbol{\mu}\cdot\boldsymbol{\mu}' - 3(\boldsymbol{\mu}\cdot\hat{\boldsymbol{r}})(\boldsymbol{\mu}'\cdot\hat{\boldsymbol{r}})}{r^3}$$



- anisotropic
- long-range

geometry dependence

attractive



repulsive

- tensor force

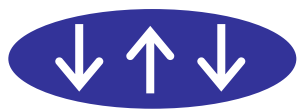
$$V_{\rm dd} = \frac{\mu_0}{4\pi}\,\mu_\alpha Q_{\alpha\beta}(\boldsymbol{r})\mu'_\beta$$

$$Q_{\alpha\beta} = \frac{1}{r^3}\left(\delta_{\alpha\beta} - 3\hat{r}_\alpha\hat{r}_\beta\right)$$

+ spin degrees of freedom

spin relaxation

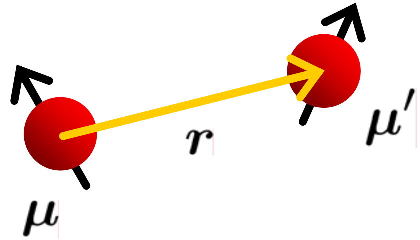
spin texture formation



?

Dipole-Dipole Interaction

$$V_{\rm dd} = \frac{\mu_0}{4\pi}\frac{\boldsymbol{\mu}\cdot\boldsymbol{\mu}' - 3(\boldsymbol{\mu}\cdot\hat{\boldsymbol{r}})(\boldsymbol{\mu}'\cdot\hat{\boldsymbol{r}})}{r^3}$$



- Magnetic dipole-dipole interaction

$$V_{\rm dd} \sim \frac{\mu_0}{4\pi}\frac{\mu_{\rm B}^2}{d^3} = \frac{1}{4}\alpha^2 E_{\rm Ryd}\left(\frac{a_0}{d}\right)^3 \sim 10 \times \left(\frac{a_0}{d}\right)^3 [\mathrm{K}] \sim 1\ [\mathrm{nK}]$$

weak

fine-structure const. $\sim 1/137$

Rydberg energy $\sim 3 \times 10^5$ K

mean atomic distance ~ 100 nm $\simeq 2000 a_0$

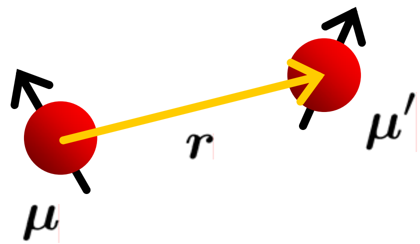
solid-state ferromagnets
$d \sim a_0 \longrightarrow V_{\rm dd} \sim 1$ [K]

- Electric dipole-dipole interaction

$$V_{\rm dd} \sim \frac{1}{4\pi\epsilon_0}\frac{(ea_0)^2}{d^3} = E_{\rm Ryd}\left(\frac{a_0}{d}\right)^3 \sim 10\ [\mu\mathrm{K}]$$

Dipole-Dipole Interaction

$$V_{\mathrm{dd}} = \frac{\mu_0}{4\pi}\frac{\boldsymbol{\mu}\cdot\boldsymbol{\mu}' - 3(\boldsymbol{\mu}\cdot\hat{\boldsymbol{r}})(\boldsymbol{\mu}'\cdot\hat{\boldsymbol{r}})}{r^3}$$



+ spin degrees of freedom

	dipole moment μ	dipolar energy V_{dd}	energy ratio $V_{\mathrm{dd}} / E_{\mathrm{con}}$
Alkali atom (spin-1)	μ_B / 2	0.1 nK	0.1 %
^{52}Cr atom	6 μ_B	10 nK	10 % → 100 %
heteronuclear molecule	∝ external electric field E	10 μK	> 100 %

(×12 from μ_B / 2 to 6 μ_B; ×144 from 0.1 nK to 10 nK)

Th. Lahaye et. al., Nature **448**, 672 (2007).

Polarized Dipolar BEC

Stability of the ground state

Santos, Shlyapnikov, Zoller and Lewenstein, PRL **85**, 1791 (2000)

O'Dell, Giovanazzi, and Eberlein, PRL **92**, 250401 (2004)

Collective mode

Yi and You, PRA **66**, 013607 (2002)

Goral and Santos, PRA **66**, 023613 (2002)

Optical lattice – Supersolid

Goral, Santos, and Lewenstein, PRL **88**, 170406 (2002)

Roton-maxon excitation

O'Dell, Siovanazzi, and Kurizki, PRL **90**, 110402 (2003)

Santos, Shlyapnikov, and Lewenstein, PRL **90**, 250403 (2003)

Rotating Dipolar BEC

Cooper, Rezayi, and Simon, PRL **95**, 200402 (2005)

Zhang and Zhai, PRL **95**, 200403 (2005)

Yi and Pu, PRA **73**, 061602(R) (2006)

1D and 2D dipolar gases

Pedri and Santos, PRL **95**, 200404 (2005)

H. P. Büchler, et. al., PRL **96**, 060404 (2007)

R. Citro, et. al., PRA **75**, 051602 (2007)

Spinor Dipolar BECs

Dipole Ordering in 1D and 2D Mott-Insulator

→ SUPERFLOW

Pu, Zhang, and Meystre, PRL **87**, 140405 (2001)

Gross et. al., PRA **66**, 033603 (2002)

Ground State Phase with Single-Mode Approximation

→ SPIN TEXTURE

Yi, You, and Pu, PRL 93, 040403 (2004)

Santos and Pfau, PRL **96**, 190404 (2006)

Diener and Ho, PRL **96**, 190405 (2006)

Spin dinamics in Optical Lattice

Sun, Zhang, Chapman, and You, PRL **97**, 123201 (2006)

Molecular BEC in Optical Lattice

Barnett, Petrov, Lukin, and Demler, PRL **96**, 190401 (2006)

Micheli, Brennen, and Zoller, Nature Physics **2**, 341 (2006)

Einstein-de Haas Effect

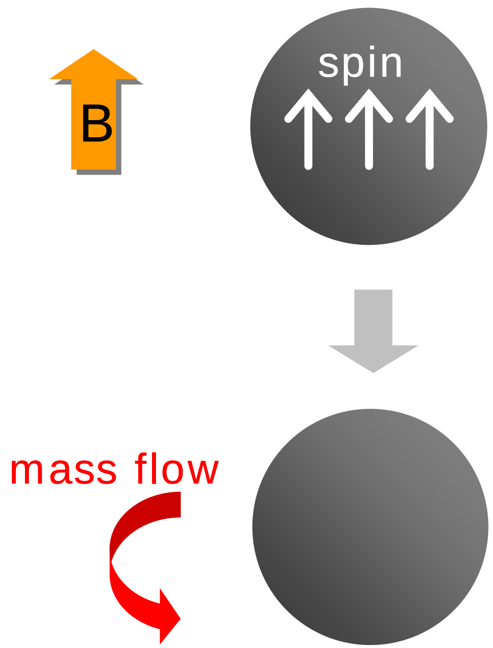
YK, Saito, and Ueda, PRL **96**, 080405 (2006)
See also, Santos & Pfau, PRL **96**, 190404 (2006)

Dipolar interaction

$$V_{\mathrm{dd}} = \frac{\mu_0}{4\pi}\frac{\boldsymbol{\mu}\cdot\boldsymbol{\mu}' - 3(\boldsymbol{\mu}\cdot\hat{\boldsymbol{r}})(\boldsymbol{\mu}'\cdot\hat{\boldsymbol{r}})}{r^3}$$

SPIN ⟷ ORBTAL

→ BEC starts rotating



Einstein-de Haas Effect

Non-local Gross Pitaevskii equation

$$i\hbar\frac{d\psi_m(\boldsymbol{r})}{dt} = \left[-\frac{\hbar^2\nabla^2}{2M} + U_{\rm trap}(\boldsymbol{r})\right]\psi_m(\boldsymbol{r})$$

$$+g_0 n(\boldsymbol{r})\psi_m(\boldsymbol{r}) + g_1\sum_\mu\sum_n f_\mu(\boldsymbol{r})(F_\mu)_{mn}\psi_n(\boldsymbol{r}) + \cdots$$

$$+c_{\rm dd}\sum_{\mu\nu}\sum_n\int d\boldsymbol{r}' f_\mu(\boldsymbol{r}')Q_{\mu\nu}(\boldsymbol{r}-\boldsymbol{r}')(F_\nu)_{mn}\psi_n(\boldsymbol{r})$$

local density: $n(\boldsymbol{r}) = \sum_m \psi_m^*(\boldsymbol{r})\psi_m(\boldsymbol{r})$

spin matrix

local magnetization: $\boldsymbol{f}(\boldsymbol{r}) = \sum_{m,n}\psi_m^*(\boldsymbol{r})\boldsymbol{F}_{mn}\psi_n(\boldsymbol{r})$

kernel of the dipolar interaction: $Q_{\mu\nu}(\boldsymbol{r}) = \frac{1}{r^3}(\delta\mu\nu - 3\hat{r}_\mu\hat{r}_\nu)$

Einstein-de Haas Effect

Non-local Gross Pitaevskii equation

$$i\hbar\frac{d\psi_m(\boldsymbol{r})}{dt} = \left[-\frac{\hbar^2\nabla^2}{2M} + U_{\rm trap}(\boldsymbol{r})\right]\psi_m(\boldsymbol{r})$$

$$+g_0 n(\boldsymbol{r})$$

$$+c_{\rm dd}\sum_{\mu\nu}$$

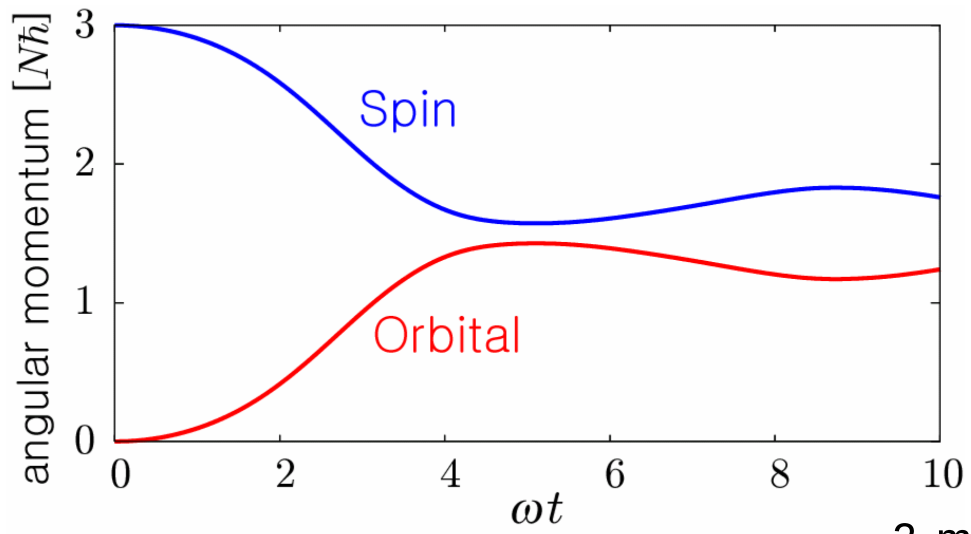
local density:

local magnetiz

kernel of the d

Numerical results for a ^{52}Cr BEC

spin-3 : 7-component GP eq.



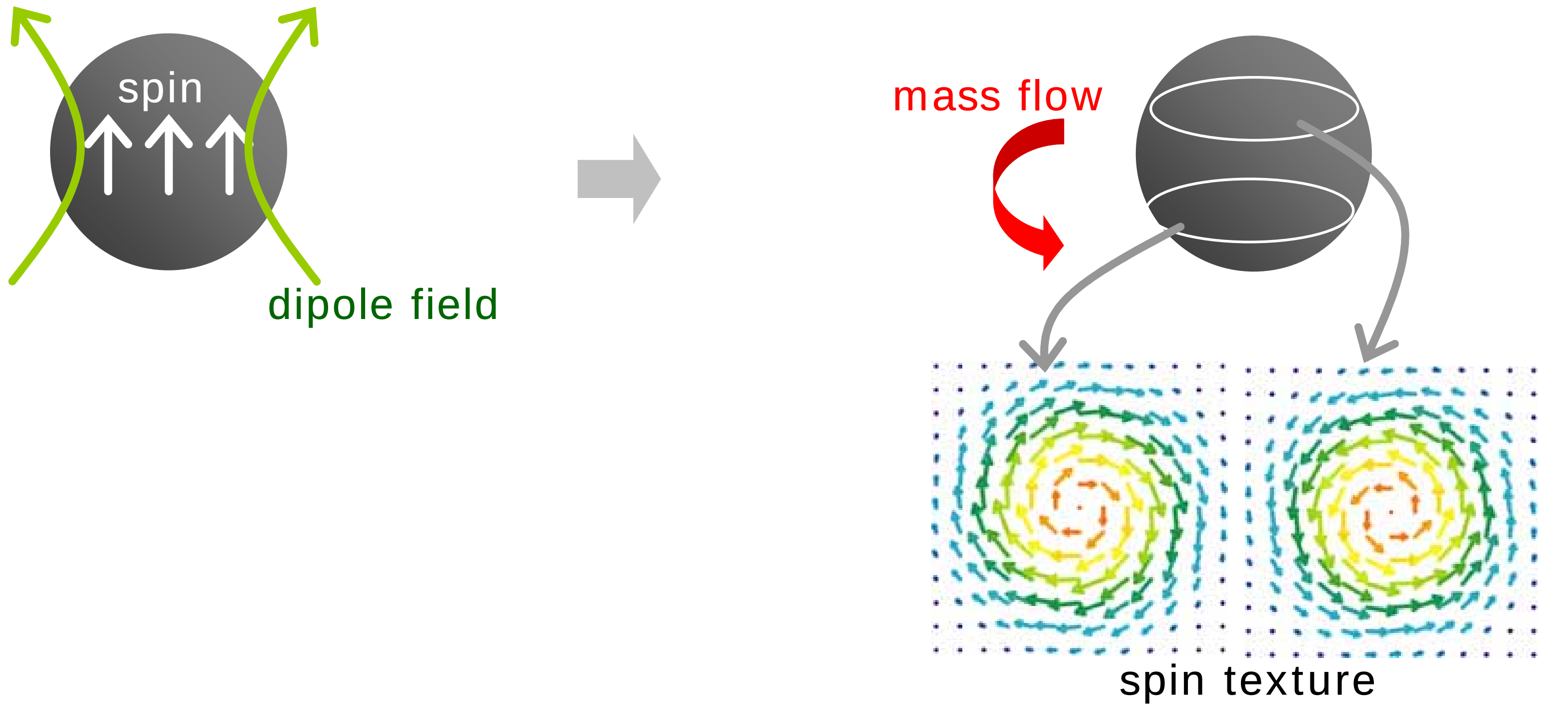
2 ms

Einstein-de Haas Effect

Spin relaxation

↑

Larmor precession around the dipole field



What is the ground state of
a spinor dipolar BEC ???

Ground-State Circulation

flux-closure constraint

solid-state ferromagnets → domain (spin) structure

ferromagnetic BEC → spin texture

spin-gauge symmetry

↓

mass current

↓ then…

spontaneous mass current
in the ground state ???   **YES !!!**

Flux-Closure Constraint

Domain structure in 2D ferromagnets
Landau & Lifshitz, 1935.

Dipolar interaction energy

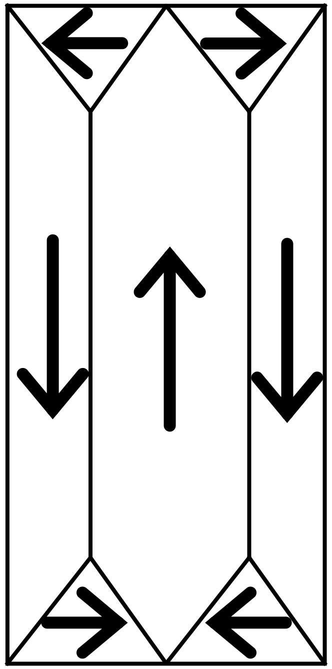
||

Energy of the static magnetic field induced by the magnet

$$\nabla \cdot \boldsymbol{B} = \nabla \cdot (\boldsymbol{H} + \boldsymbol{M}) = 0$$

$$\nabla \cdot \boldsymbol{H} = \underline{-\nabla \cdot \boldsymbol{M}}$$

source



flux-closure structure

Flux-closure constraint:

ferromagnets	BEC (no rigid boundary)
$\nabla \cdot \boldsymbol{M} = 0$	$\nabla \cdot \boldsymbol{f} = 0$
$\boldsymbol{M}//$ boundary	

$\longrightarrow$ structure formation

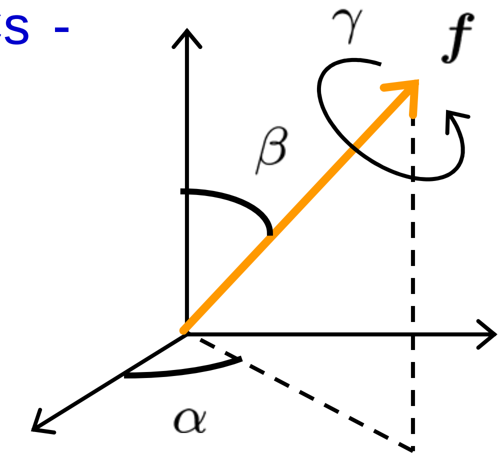
Spin-Gauge Symmetry

- ferromagnetic or locally spin-polarized BECs -

- order parameter (spin 1)

$$\begin{pmatrix}\psi_1\\ \psi_0\\ \psi_{-1}\end{pmatrix} = \sqrt{n}e^{i\theta}\mathcal{U}(\alpha,\beta,\gamma)\begin{pmatrix}1\\0\\0\end{pmatrix}$$

$$= \sqrt{n}e^{i(\theta-\gamma)}\begin{pmatrix}e^{-i\alpha}\cos^2\frac{\beta}{2}\\ \sqrt{2}\cos\frac{\beta}{2}\sin\frac{\beta}{2}\\ e^{i\alpha}\sin^2\frac{\beta}{2}\end{pmatrix}$$



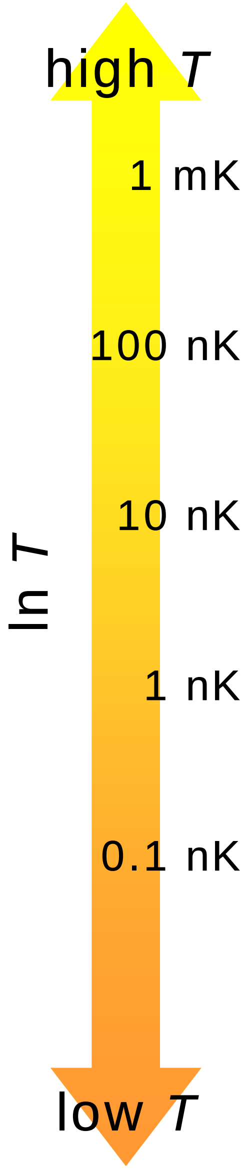
$$\boldsymbol{f} = n\begin{pmatrix}\cos\alpha\sin\beta\\ \sin\alpha\sin\beta\\ \cos\beta\end{pmatrix}$$

- superfluid velocity

$$n\boldsymbol{v}_s \equiv \frac{\hbar}{M}\mathrm{Im}[\sum_m \psi_m^*\nabla\psi_m] = -\frac{n\hbar}{M}[\nabla(\gamma-\theta)+\cos\beta\nabla\alpha]$$

Texture $\longrightarrow$ mass current
Circulation not quantized
Circulation + topological phase quantized

Energy Scales



Zeeman energy @ 100 mG

spin-independent mean-field interaction $g_0 n$
+ harmonic trap $\hbar\omega$
$\rightarrow$ density distribution: n

spin-exchange interaction $|g_1|n$
$\rightarrow$ magnetism (form of the order parameter)

dipolar interaction $c_{\rm dd} n$
$\rightarrow$ spin texture

Characteristic Length Scales

spin-indep. mf + trap Thomas-Fermi radius	$R_{\rm TF} = 2\sqrt{\frac{g_0 n_0}{M\omega^2}}$	1~100 μm	
spin-exchange spin healing length	$\xi_{\rm sp} = \frac{\hbar}{\sqrt{2M\|g_1\|n_0}}$	^{87}Rb 2 μm	^{52}Cr 0.5 μm
dipole dipole healing length	$\xi_{\rm dd} = \frac{\hbar}{\sqrt{2Mc_{\rm dd}n_0}}$	6 μm	0.6 μm

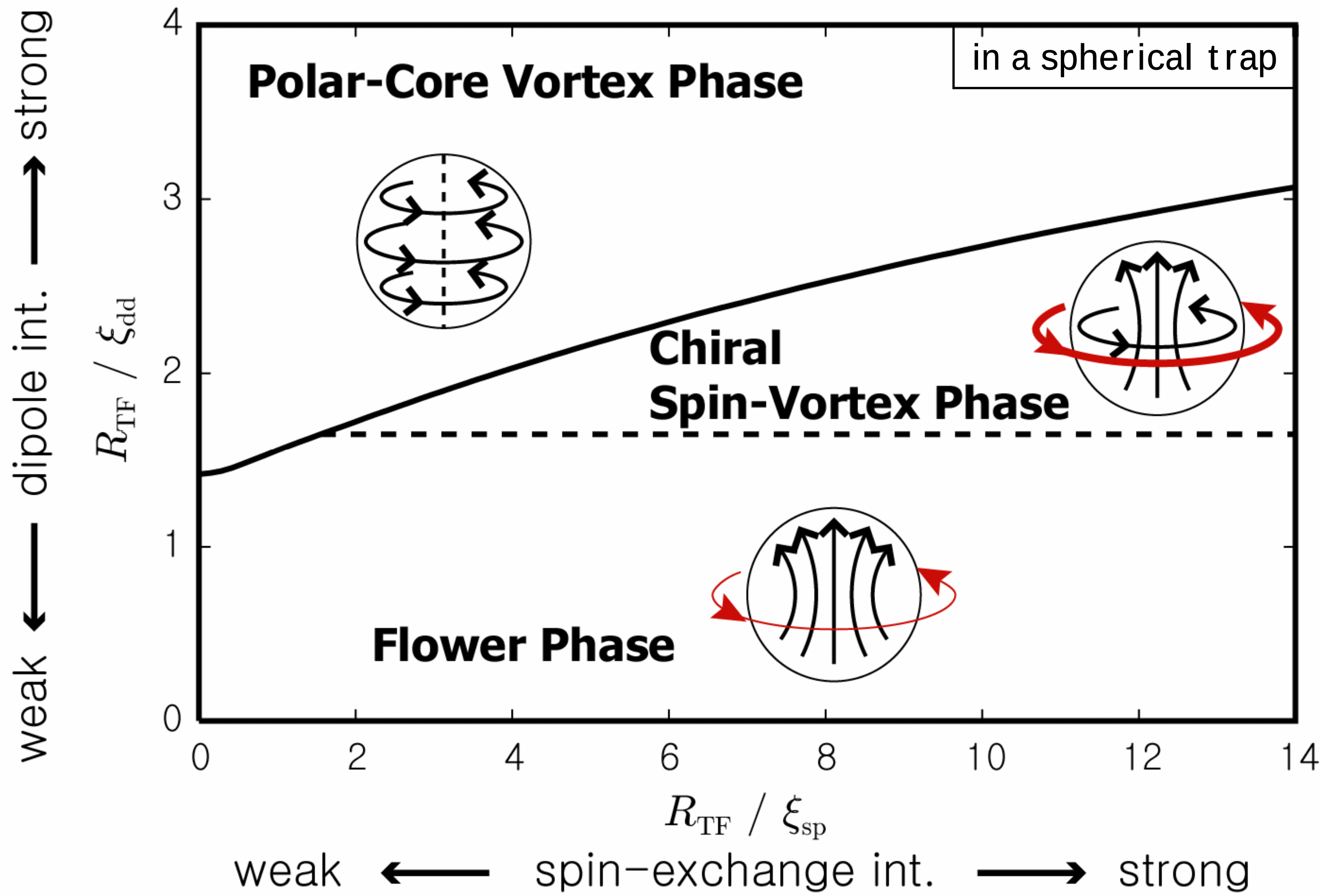
@ n~5x10^{14} cm^{-3}

ω : trap frequency
n_0 : TF peak density

Spin texture can develop when $R_{\rm TF} > \xi_{\rm dd}$.

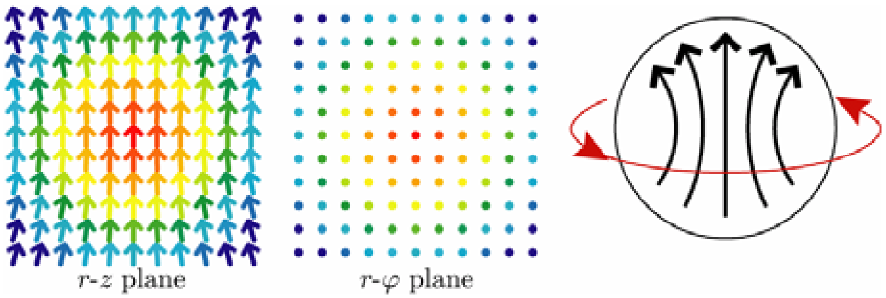
Phase Diagram
- ferromagnetic BEC -

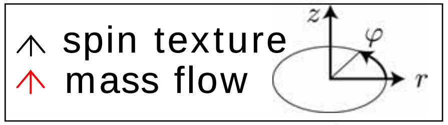
YK, Saito, and Ueda, PRL **97**, 130404 (2006)
See also, Yi & Pu, PRL **97**, 020401 (2006)

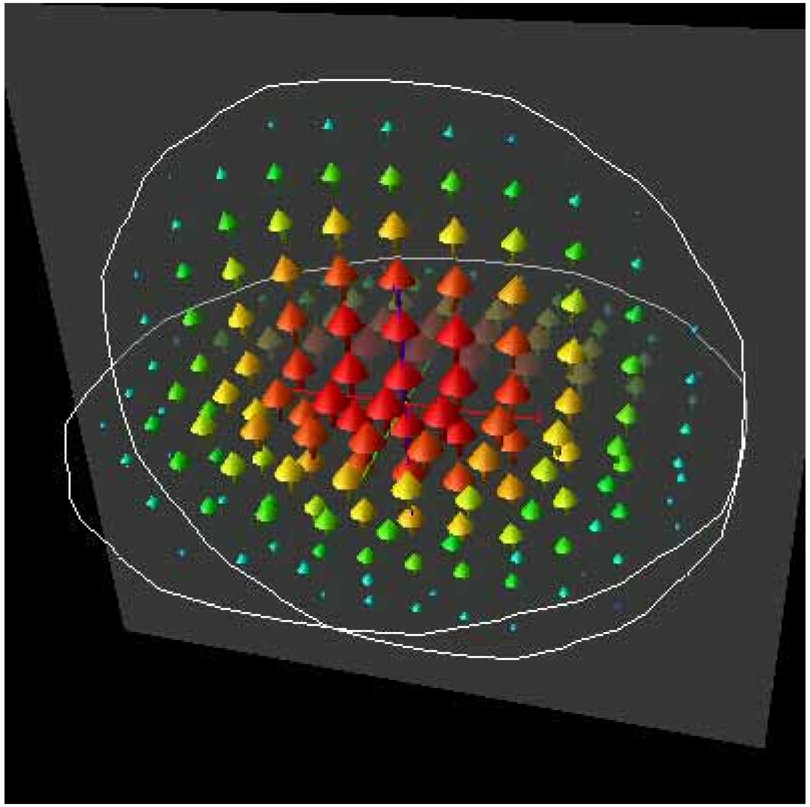


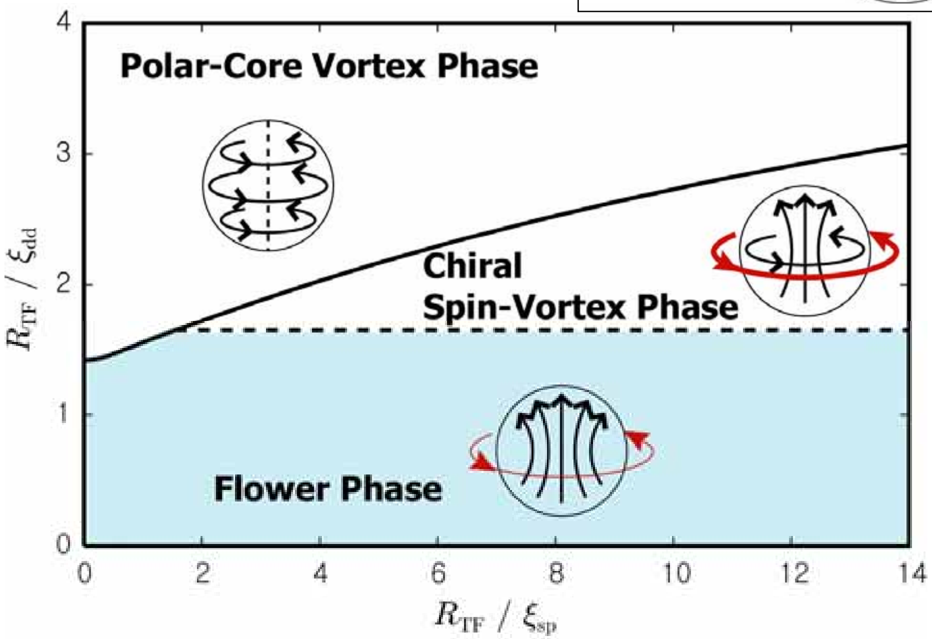
Flower Phase

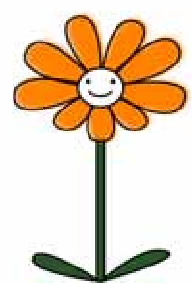
almost spin polarized





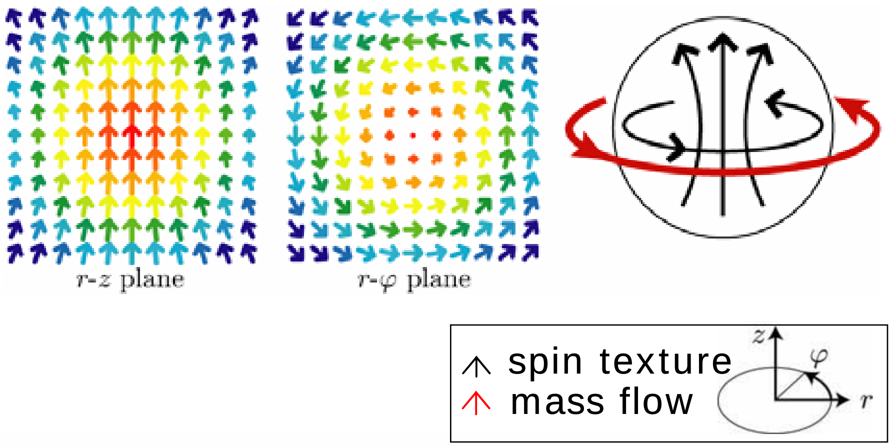


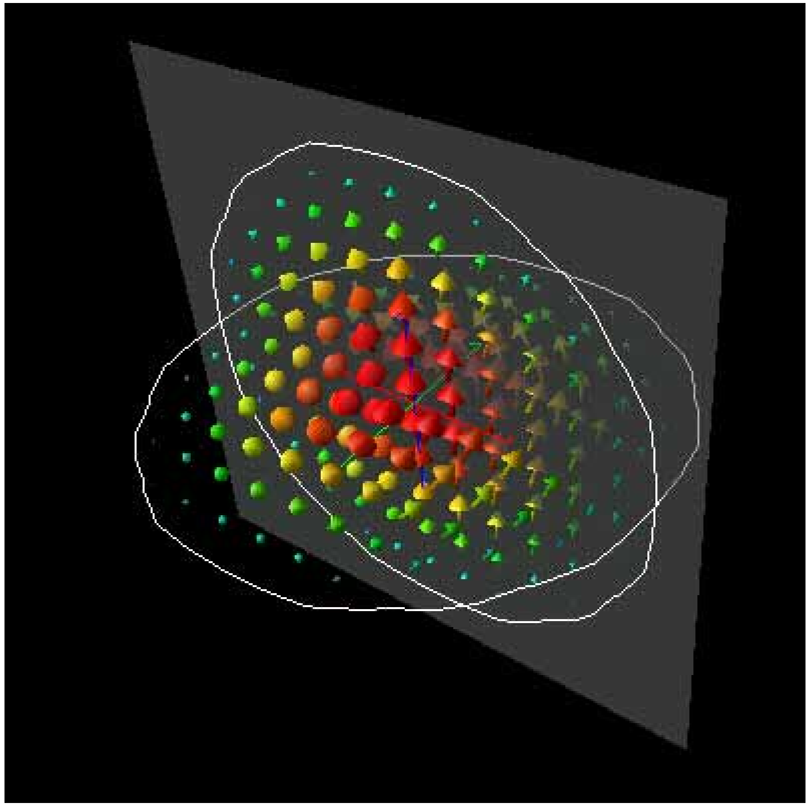


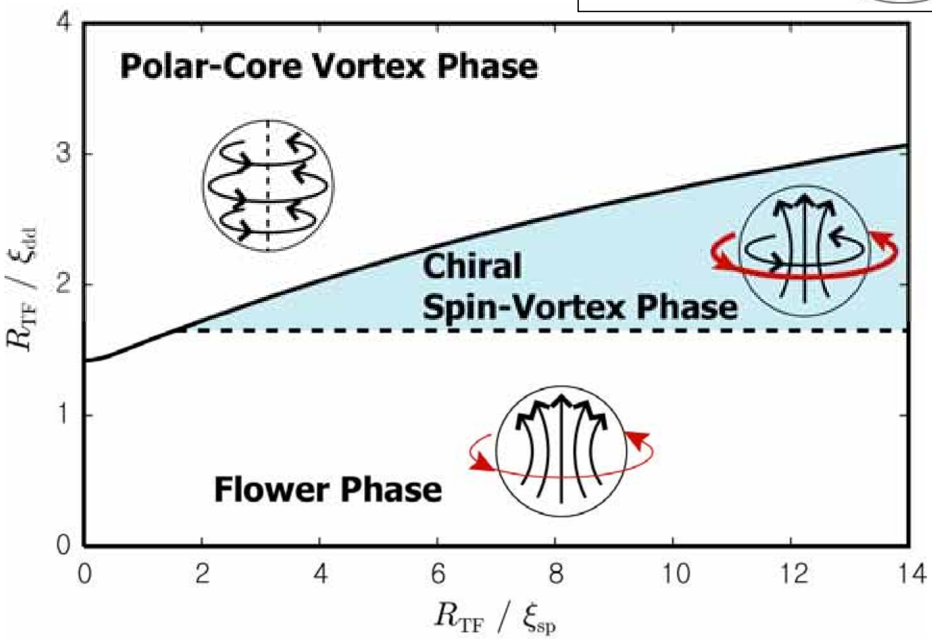


Chiral Spin-Vortex Phase

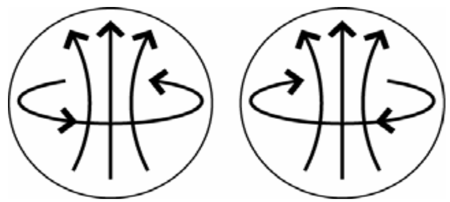
chiral symmetry breaking state





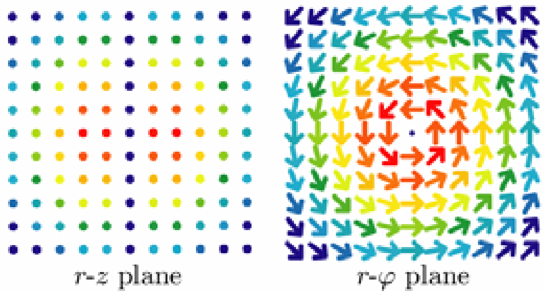


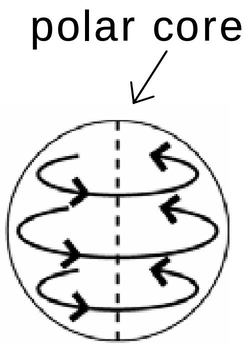
chirality

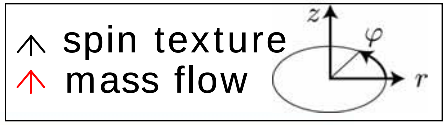


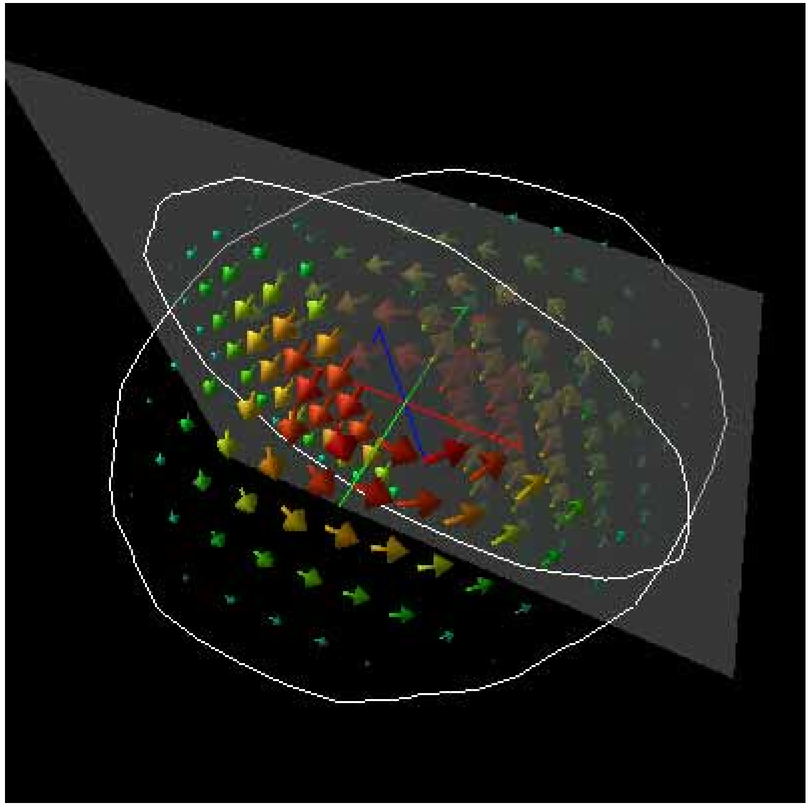
Polar-Core Vortex Phase

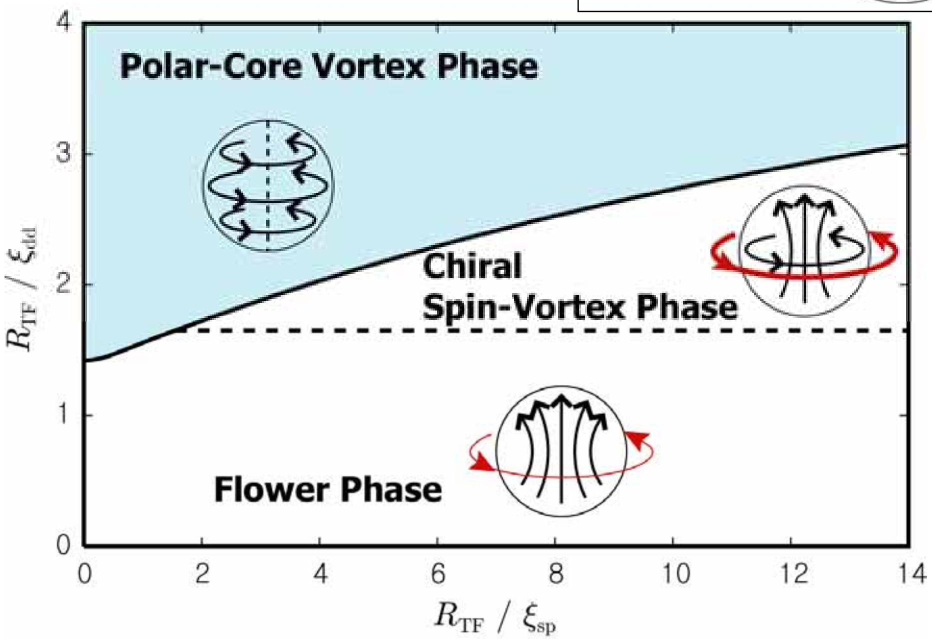
flux-closure structure
spin defect (polar core)



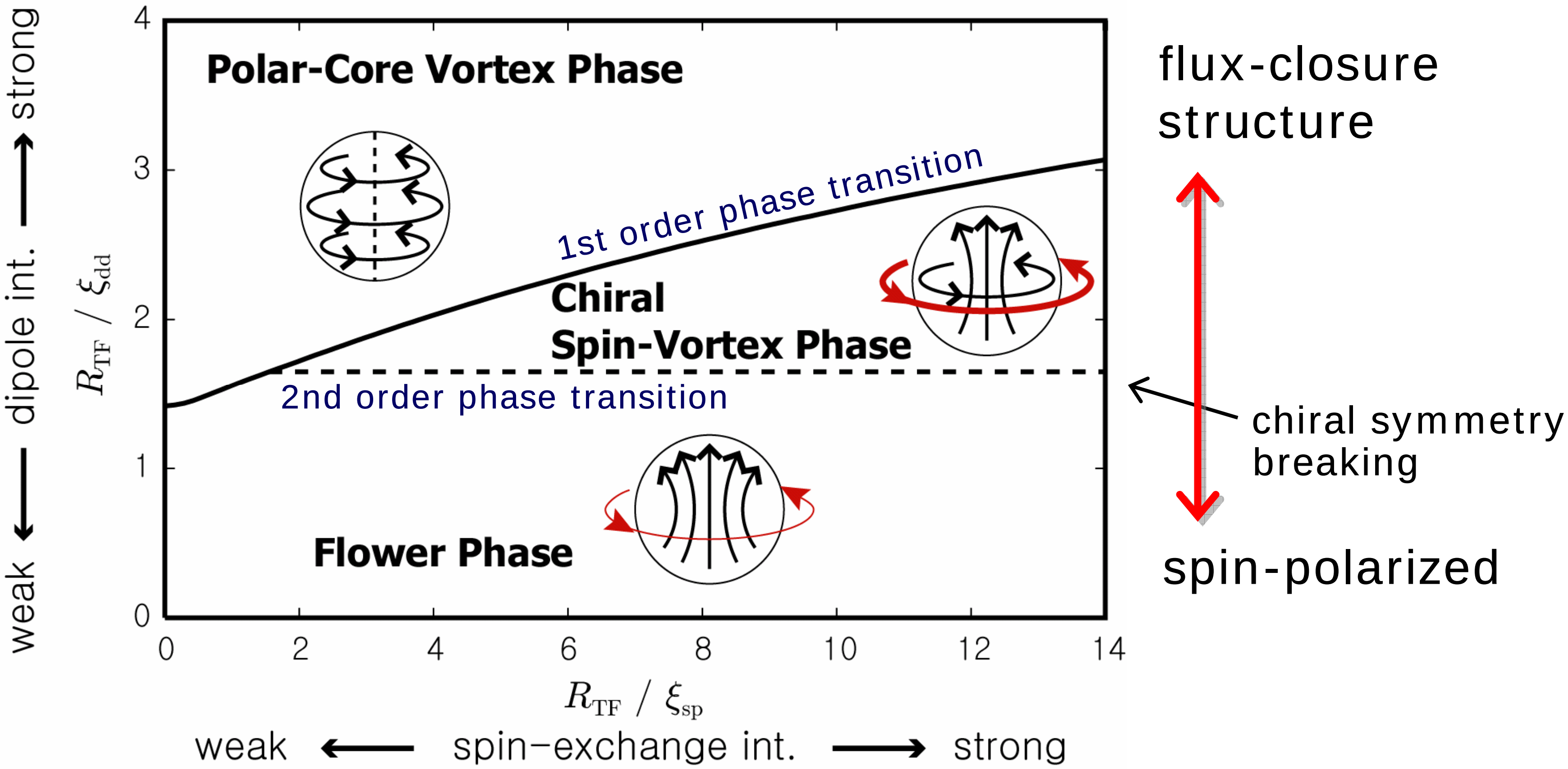




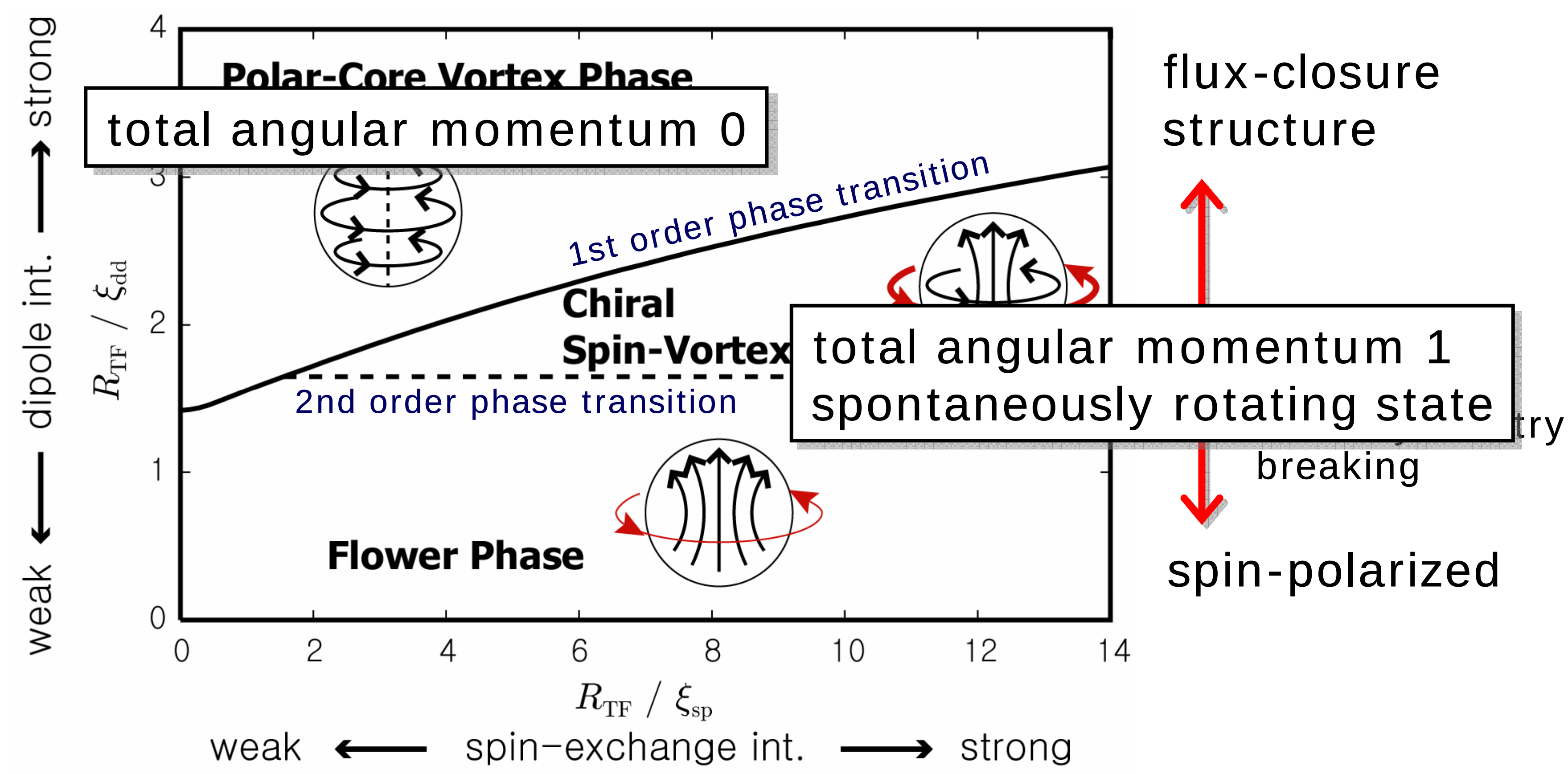




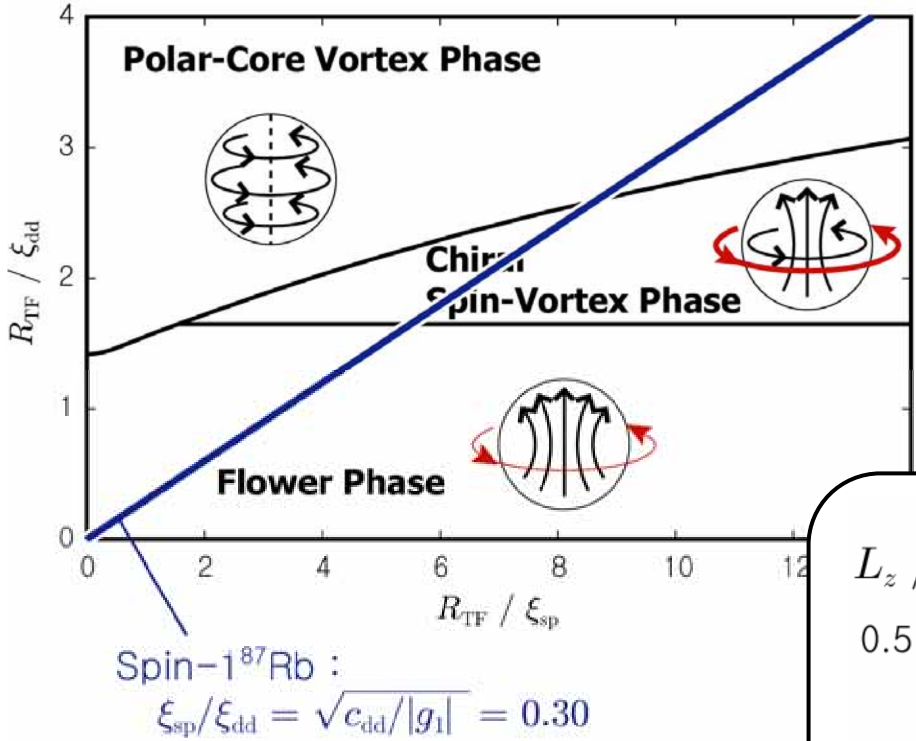
Phase Diagram



Phase Diagram

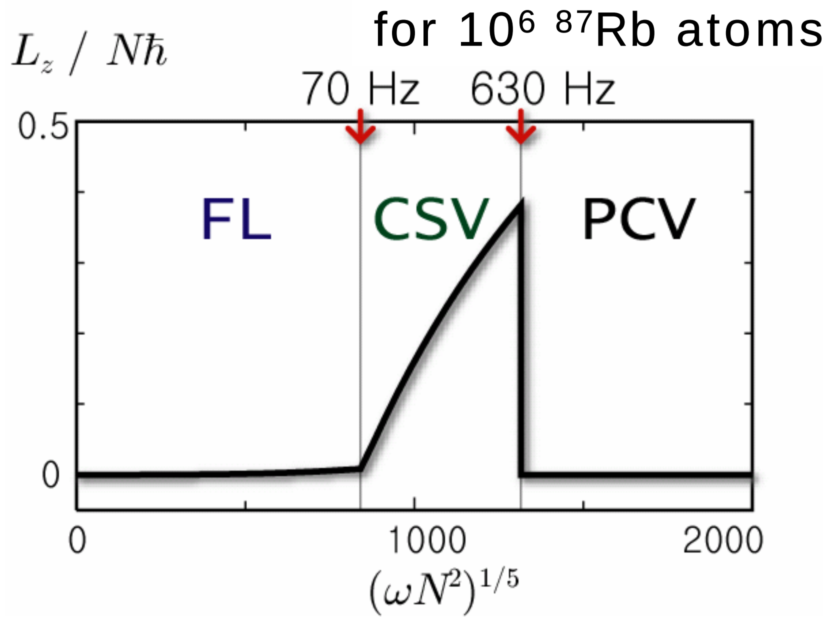


Ground-State Circulation



flux-closure constraint
→ spin texture

spin-gauge symmetry
→ ground-state circulation



Dipolar Interaction vs. Zeeman Energy

Zeeman : $g_F \mu_{\rm B} B$

Dipole : $\frac{4\pi}{3} c_{\rm dd} n_0, \quad c_{\rm dd} = \frac{\mu_0}{4\pi}(g_F \mu_{\rm B})^2$

⇩

Critical field $B_{\rm c} = \frac{\mu_0}{3} g_F \mu_B n_0$

^{87}Rb : $B_{\rm c} \sim 10\ \mu{\rm G}$ @ $n_0 = 10^{15}\ {\rm cm}^{-3}$

^{52}Cr : $B_{\rm c} \sim 0.1\ {\rm mG}$

Then…

is the spinor dipolar effect observable

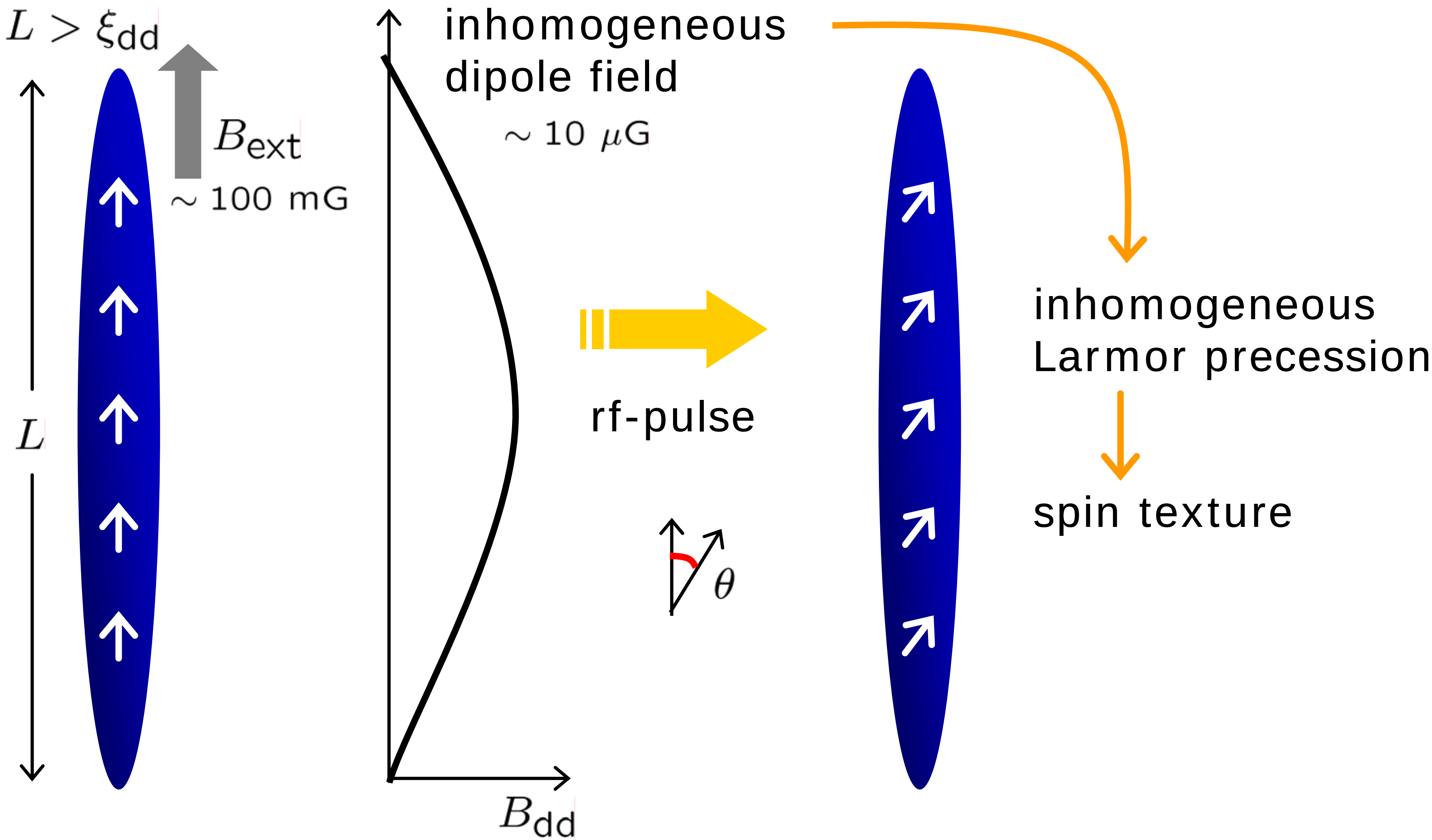
only when the external magnetic field is extremely weak ?

NO!!!

There is an observable effects even though

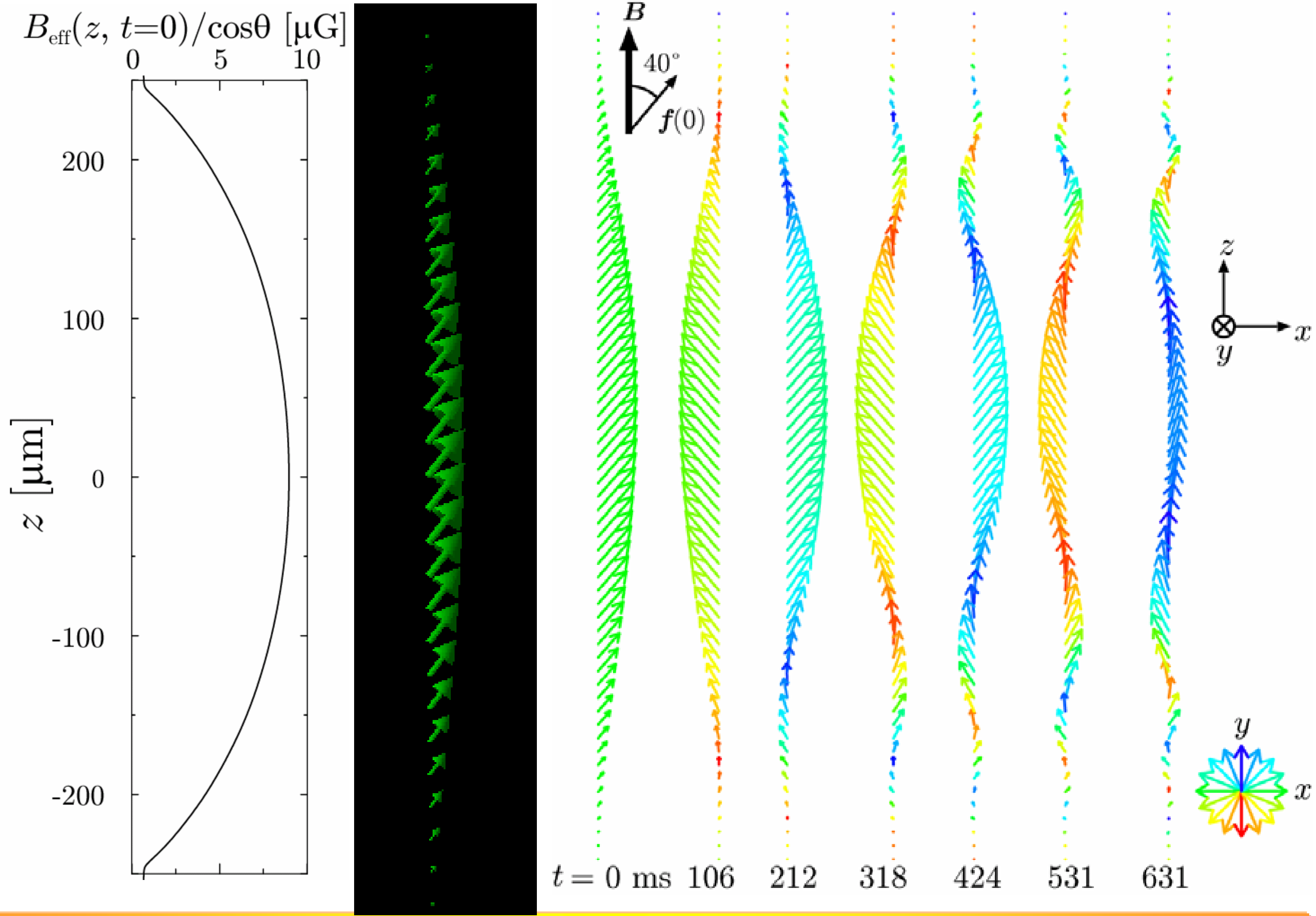
the Zeeman energy dominates the system.

Possible Experimental Scheme

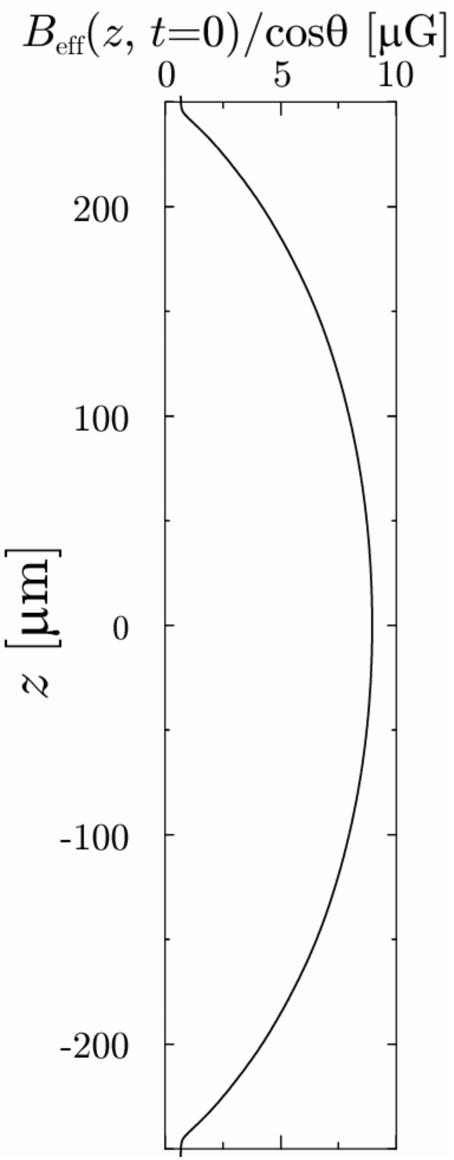


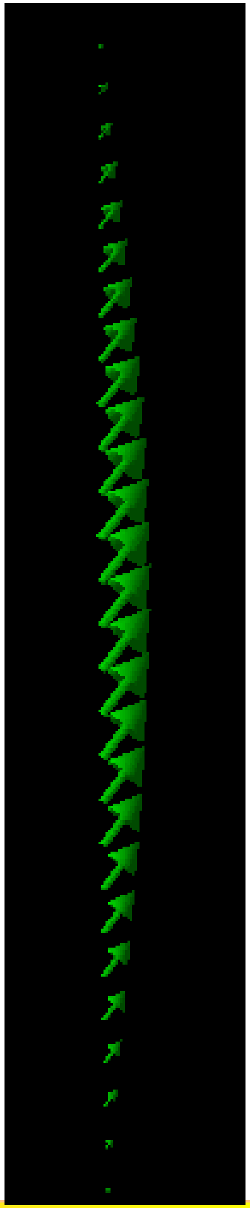
YK, H. Saito, and M. Ueda, PRL **98**, 110406 (2007)

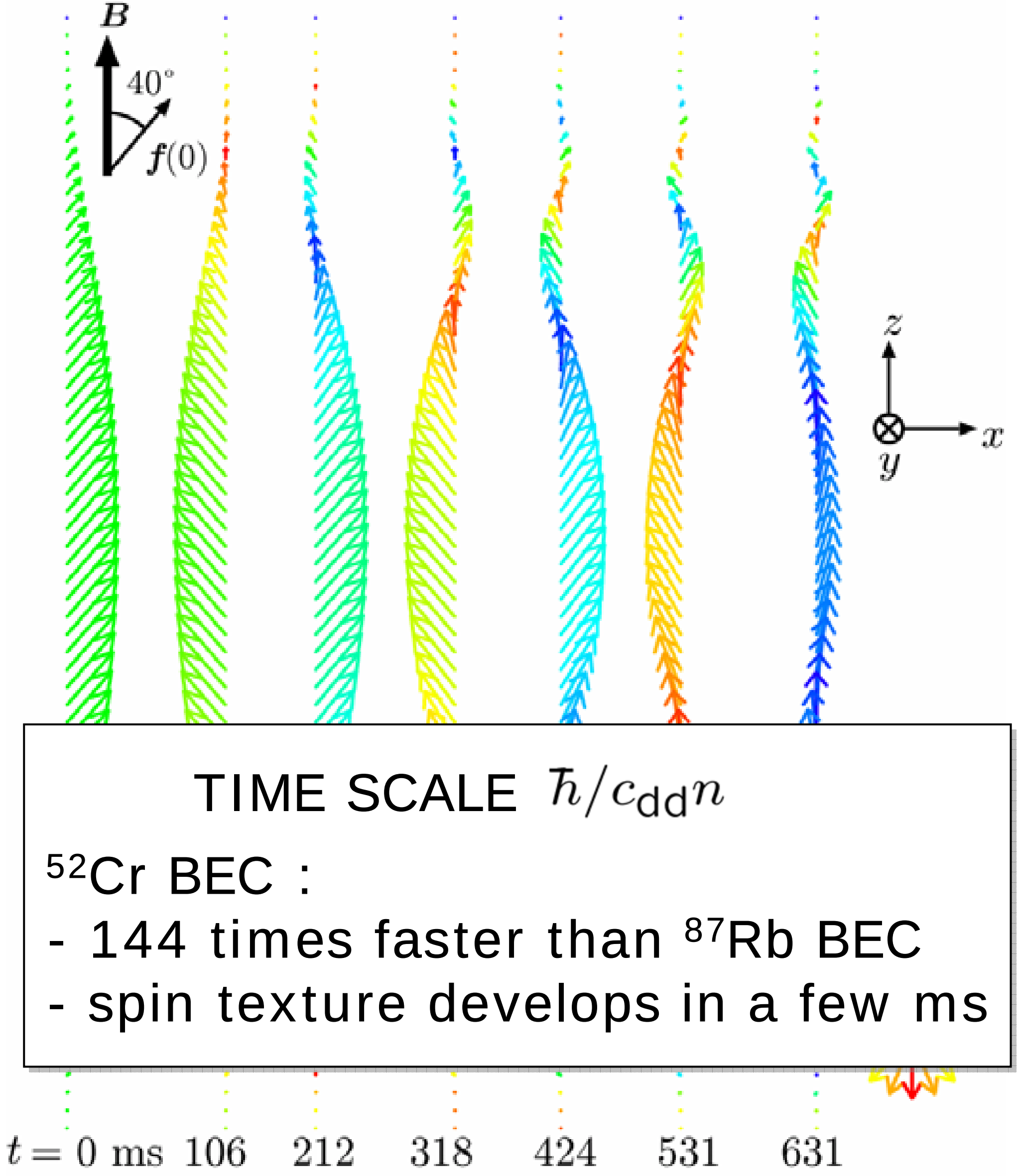
Numerical results for ^{87}Rb BEC



Numerical results for ^{87}Rb BEC







TIME SCALE $\hbar/c_{\rm dd}n$

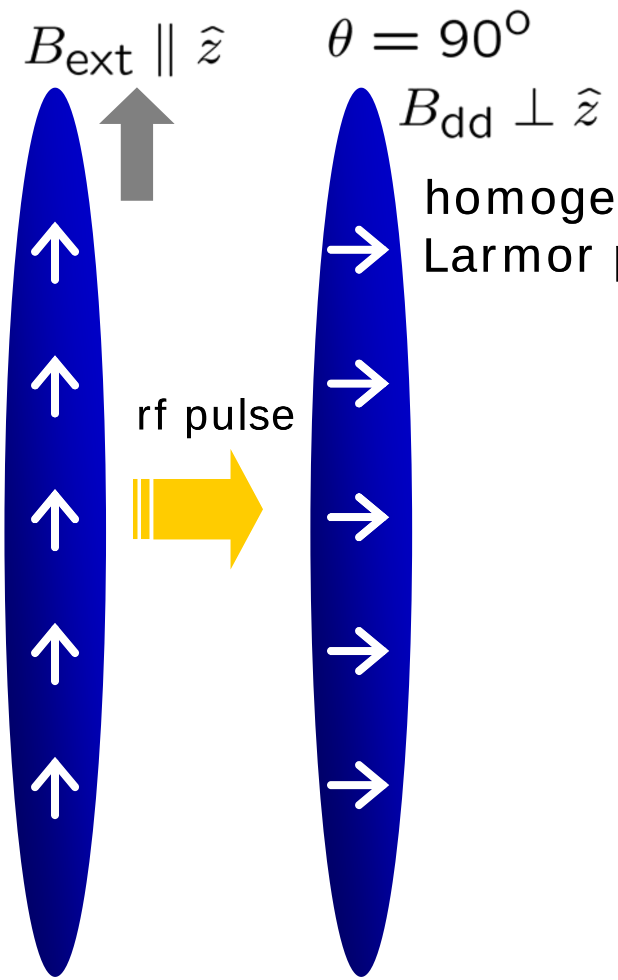
^{52}Cr BEC :

- 144 times faster than ^{87}Rb BEC
- spin texture develops in a few ms

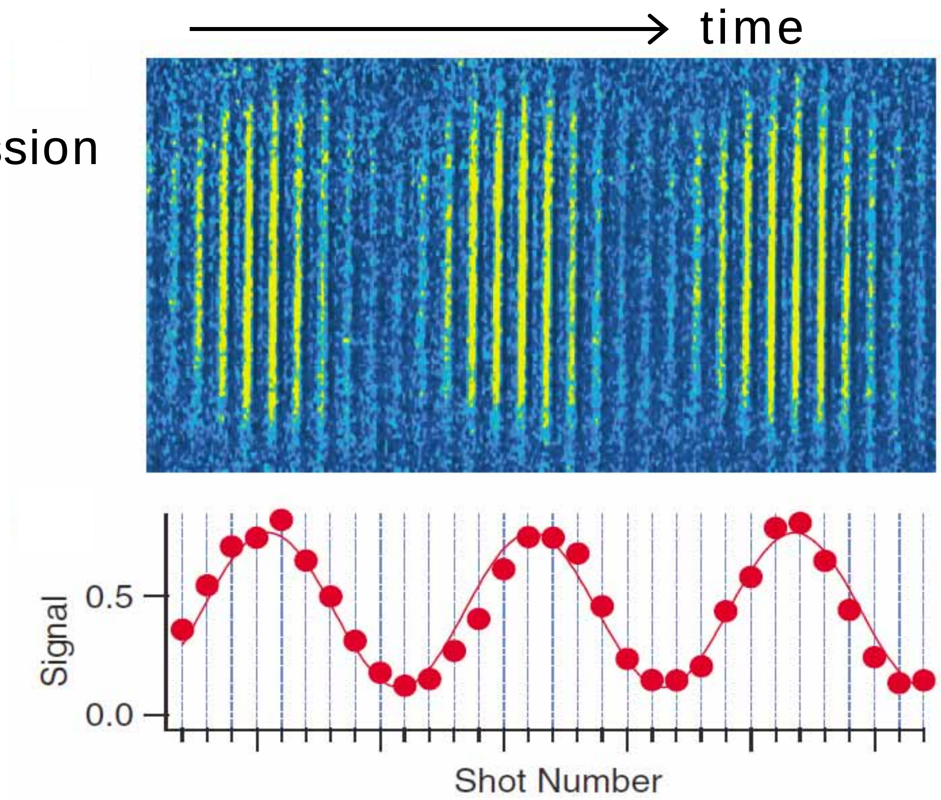
Berkeley Experiment

J. M. Higbie, et. al., PRL **95**, 050401 (2005).

Observation of the Larmor precession.



Phase contrast imaging
Time development of f_y

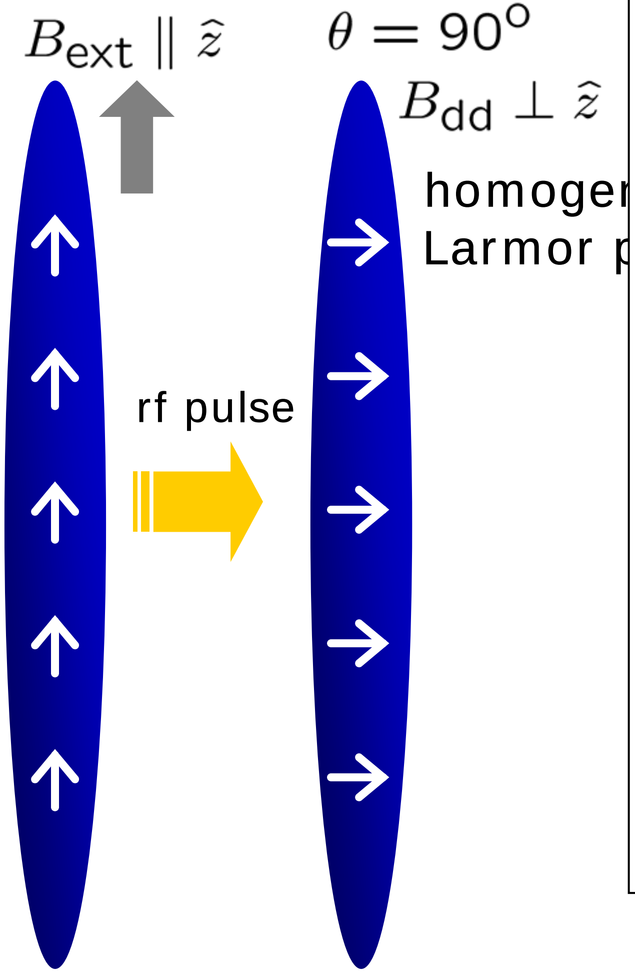


beat { sampling rate : 20 kHz
Larmor precession : 28 kHz

Berkeley Experiment

J. M. Higbie, et. al., PRL **95**, 050401 (2005).

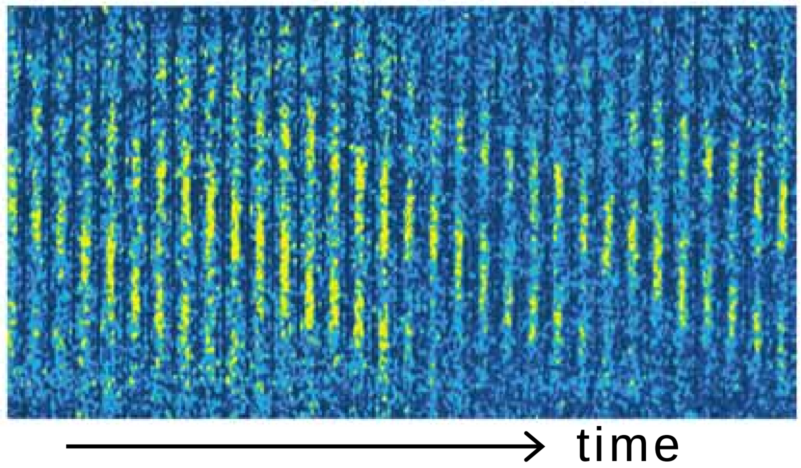
Observation of the Larmor precession.



homogen
Larmor p

Phase contrast imaging

helical structure
under field gradient



→ When $\theta \neq 90^o$
helical structure will appear
without field gradient

beat { sampling rate : 20 kHz
Larmor precession : 28 kHz

Summary

- Einstein-de Haas effect

  YK, H. Saito, and M. Ueda, PRL **96**, 080405 (2006)

- Ground-state circulation

  YK, H. Saito, and M. Ueda, PRL **97**, 130404 (2006)

- Possible experimental scheme

  YK, H. Saito, and M. Ueda, PRL **98**, 110406 (2007)

↓

Unique feature of a spinor dipolar BEC

DIPOLAR : flux-closure constraint

SPINOR : spin-gauge symmetry

Dipolar BEC becomes more exciting

when it has internal degrees of freedom!