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1859-8

Summer School on Novel Quantum Phases and Non-Equilibrium Phenomena in Cold Atomic Gases

27 August - 7 September, 2007

Introduction to the physics of low dimensional systems

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Systems of reduced dimensionality

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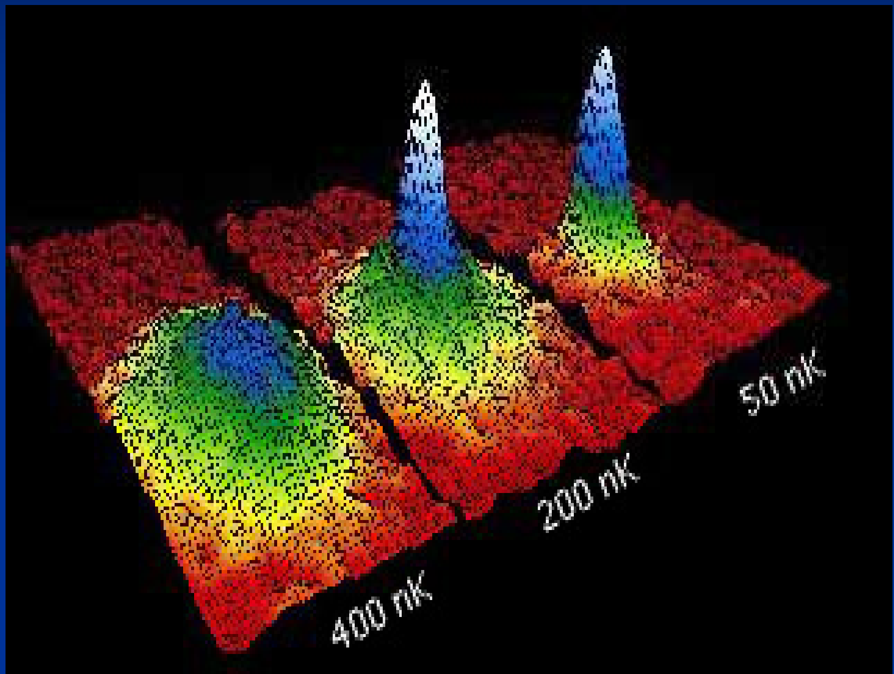
BEC in cold atomic gases



2001: Cornell, Ketterle, Wieman

SO WHAT !

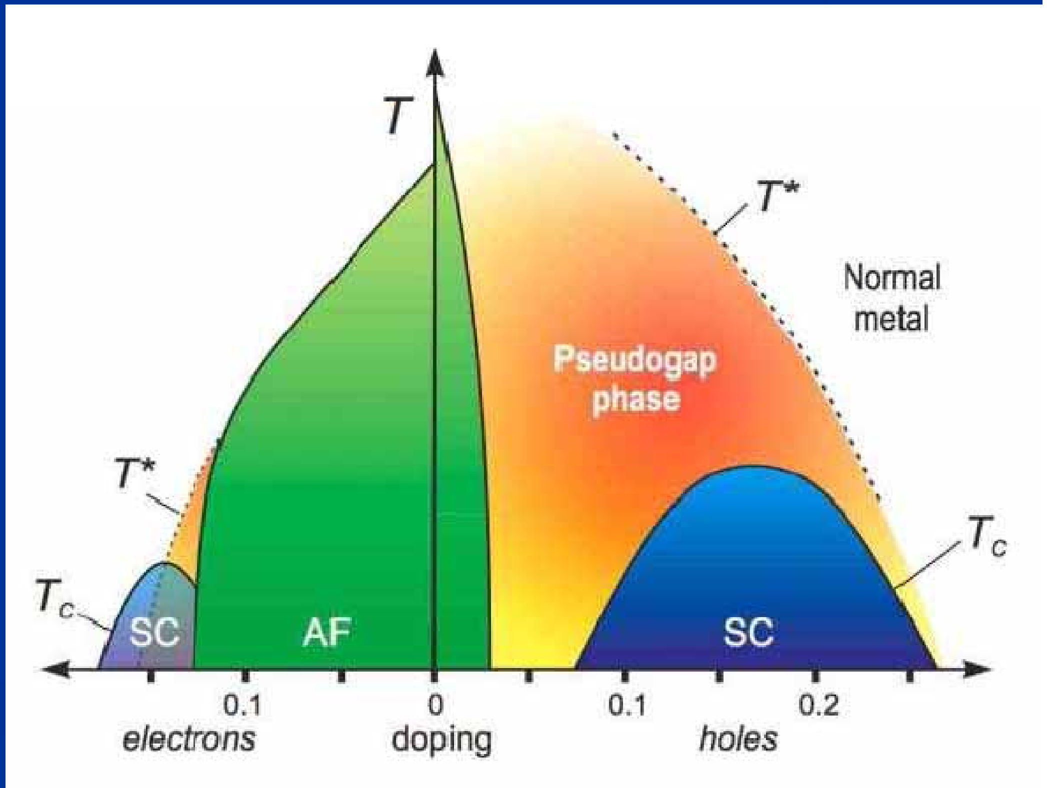
1924: predicted by Bose and Einstein

Strong correlations

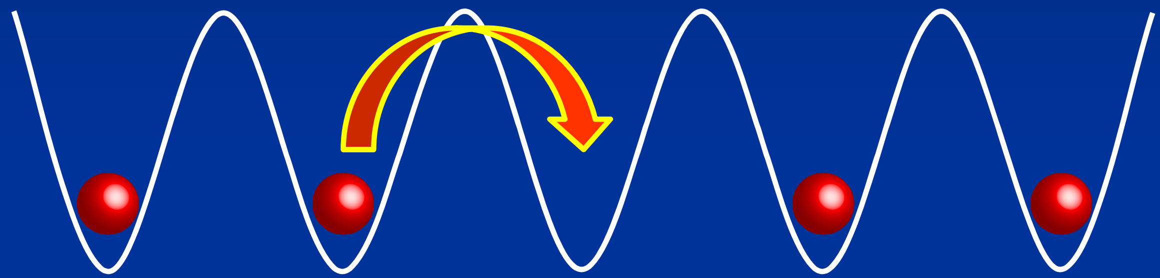
- Condensed matter:

$$E_{cin} = E_{coul}$$

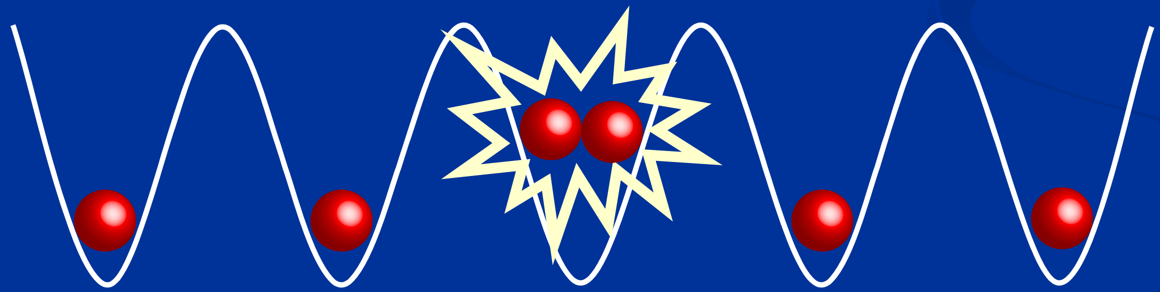
Strong correlations !



Atoms in a lattice

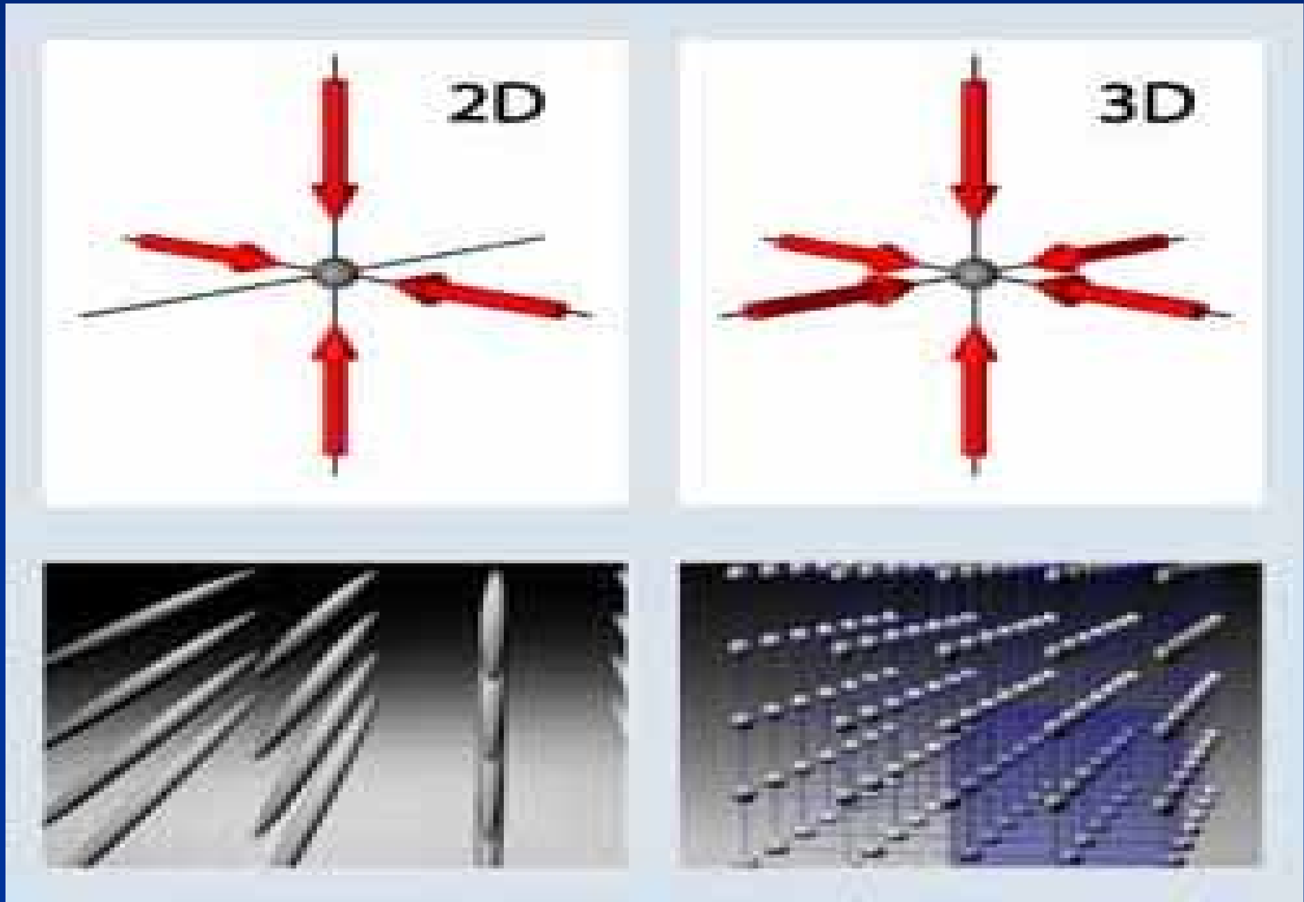


Tunnelling



Short range interaction

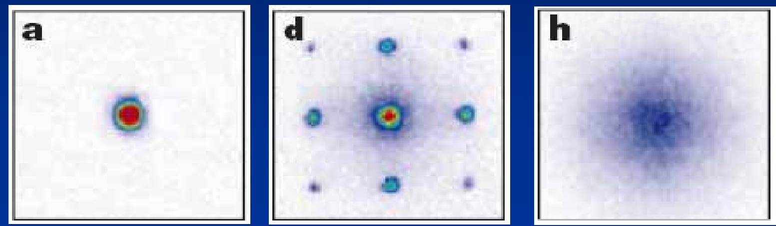
Optical lattices: control kinetic energy



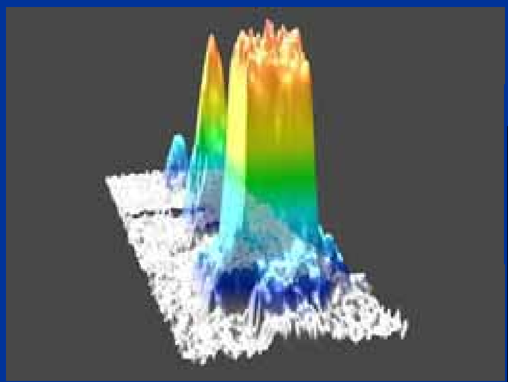
Greiner et al. (2002);

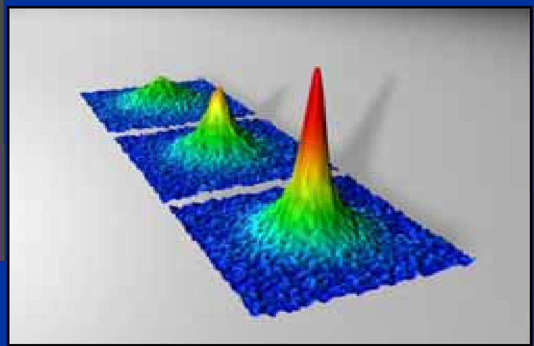
Quantum simulators !

Interactions

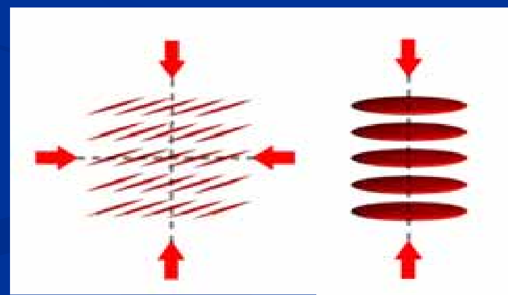


Statistics





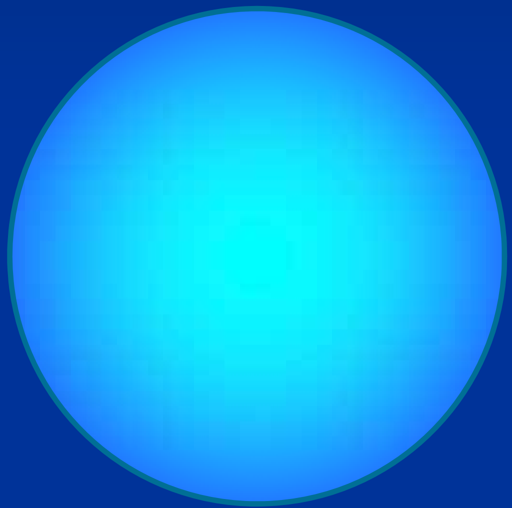
Dimensionality

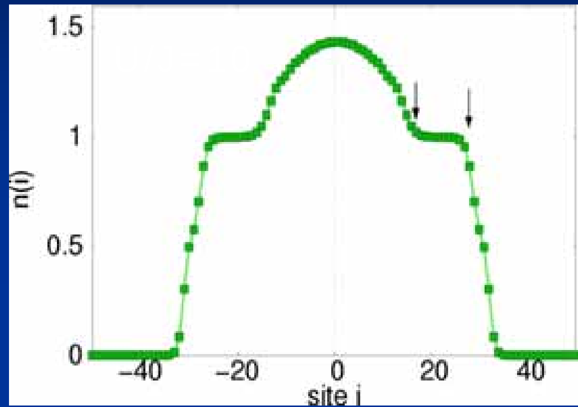


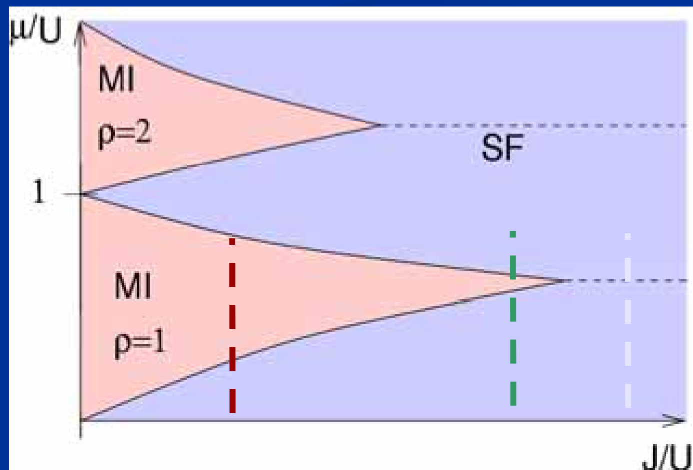
ENS, ETH, LENS, Mainz, MIT,
NIST, Penn State,

But... not so idilic !

Confining potential







$$H = \int r^2 \rho(r)$$

- No homogeneous phase !

Can change physics drastically

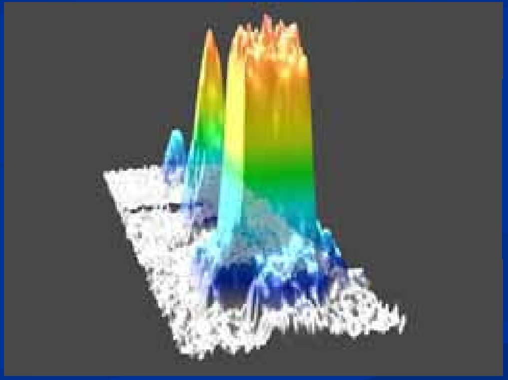
M.A. Cazalilla, A. F. Ho, TG, PRL 95 226402 (2005)

Probes !

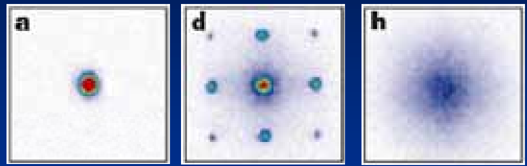
Atoms are neutral !

n(k) (time of flight) useless for fermions !

Need to probe correlations !

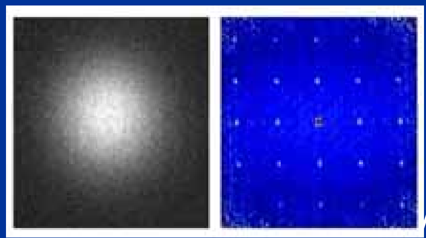


time-of-flight measurement
-> momentum distribution



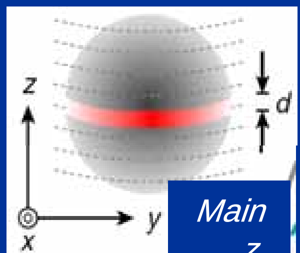
München

noise measurement:
-> density-density correlations



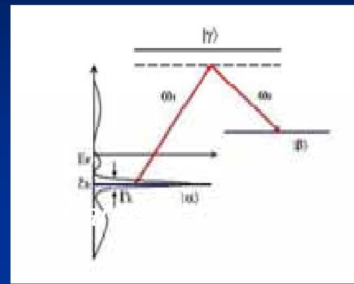
Mainz

microwave spin-changing
transitions
density spatially resolved

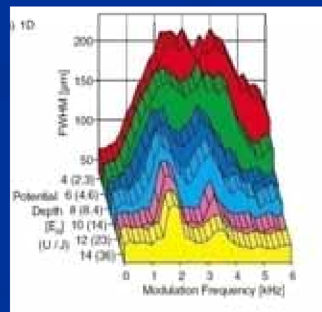


*Main
z*

proposed: Raman spectroscopy
->Green's function, Fermi
surface



periodic lattice
modulation



Zurich

molecule formation
binding energy
doubly occupied
sites

So why reduced dimensionality ?

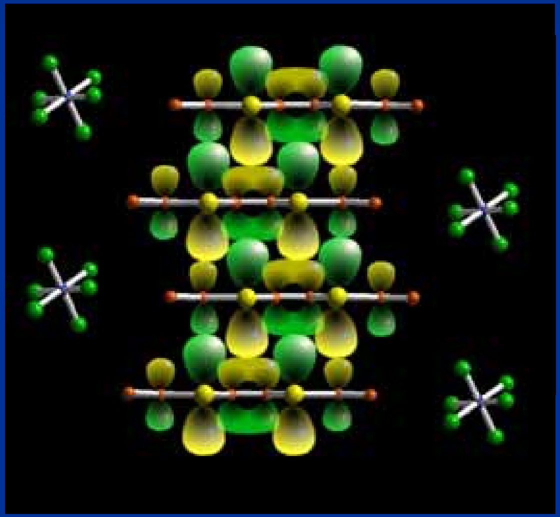
- Not easy to realize in condesed matter
- Effect of interactions at their strongest
- Novel physics !

This is where the fun is ☺

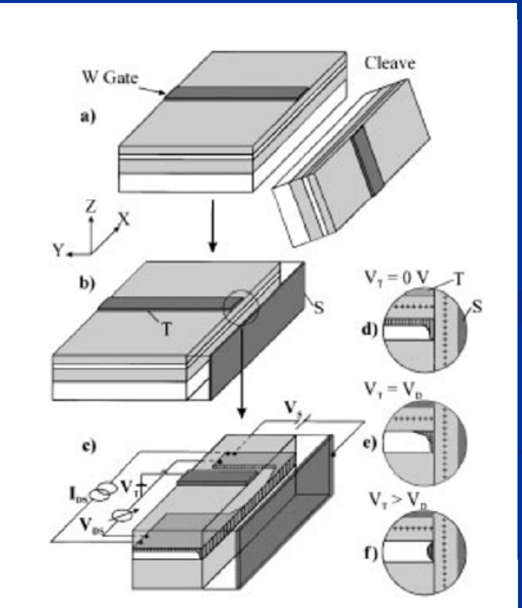
Let us start with 1D

Does one dimension exists ?

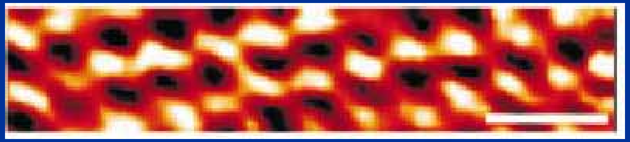
Hard to realize in condensed matter



Organic conductors



Quantum wires

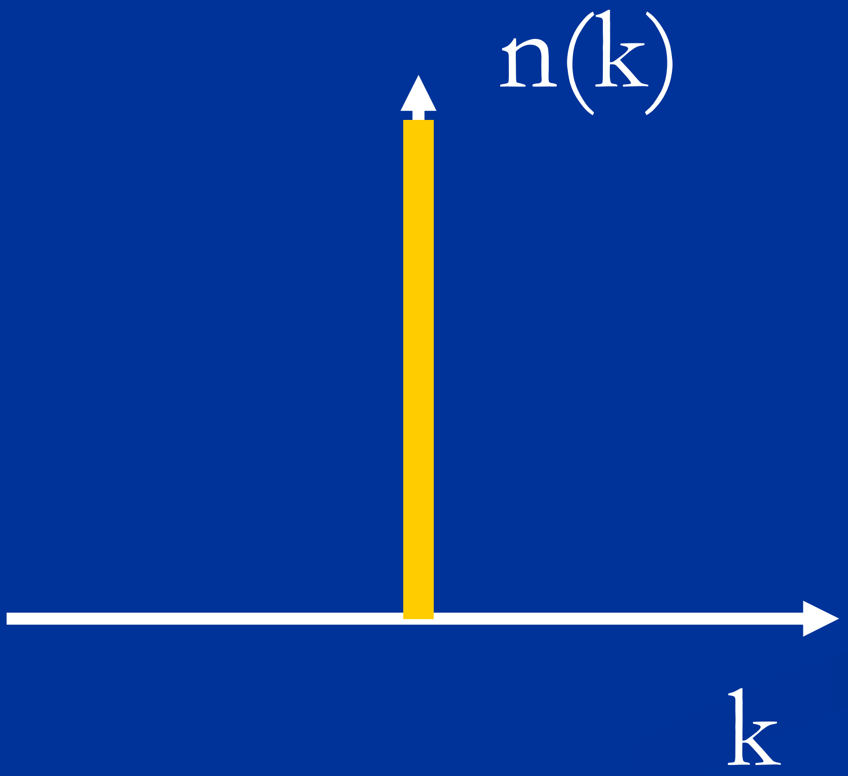


Nanotubes

- Josephson junctions
- Ladders
- Edge states in FQHE

Free bosons : crash course

- Free particles: condensation in k=0 state

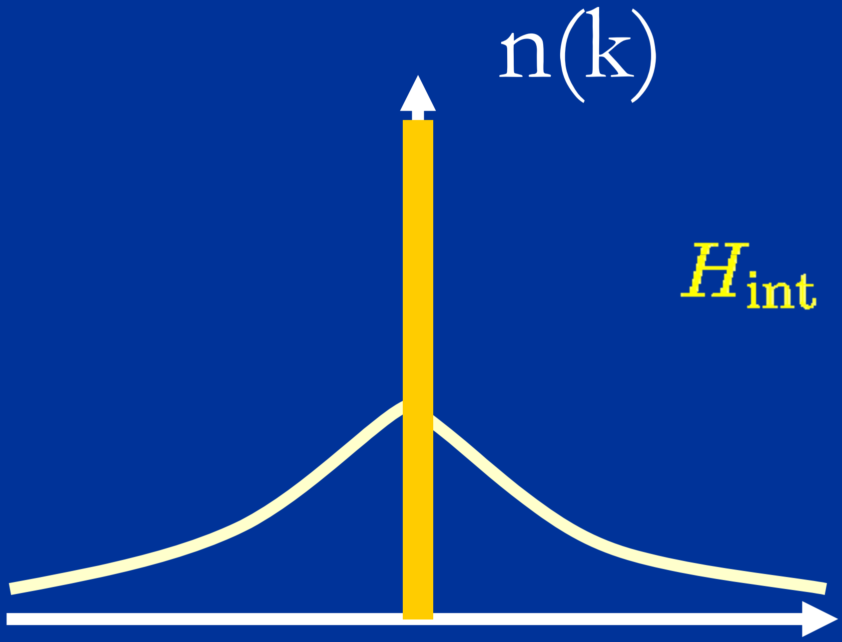


$$H_0 = \sum_k \xi_k b_k^\dagger b_k$$

$$n(k) = \langle 0 | b_k^\dagger b_k | 0 \rangle$$

Excitations: single particles with momentum k

- Do interactions change this ?

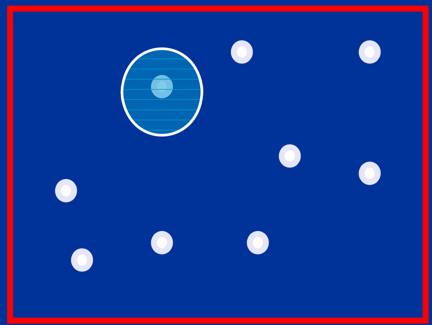


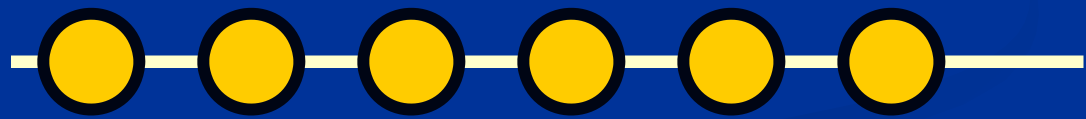
$$H_{\text{int}} = \frac{1}{\Omega} \sum_{k,k',q} V(q) b^\dagger_{k+q} b^\dagger_{k'-q} b_{k'} b_k$$

Not much !

One dimension is different

- No individual excitation can exist (only collective ones)





- Strong quantum fluctuations

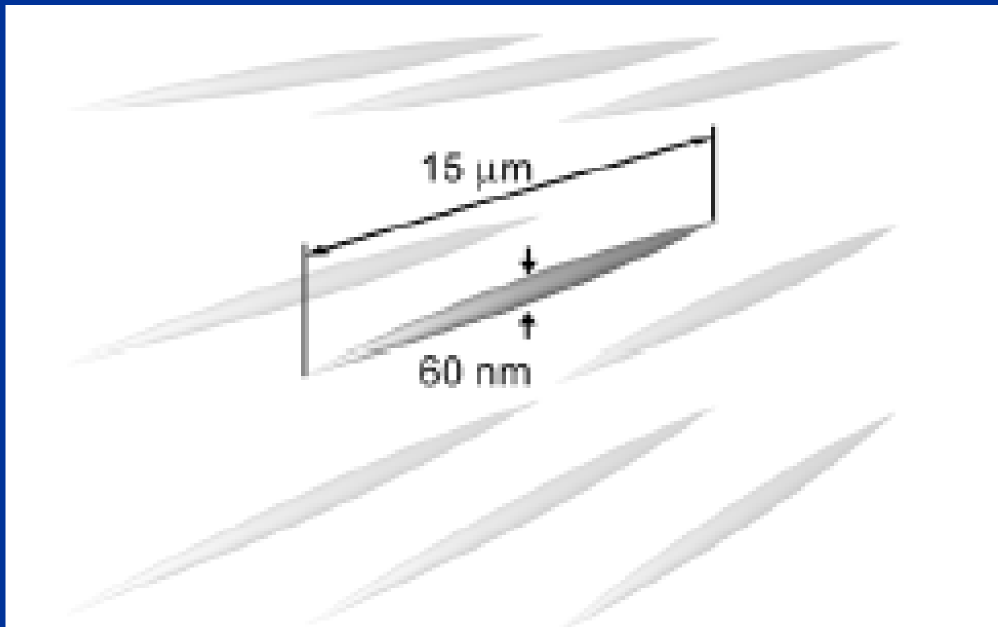
$$\psi \rightarrow \psi e^{i\theta}$$

Continuous symmetry

Cold atoms: ideal systems

Optical lattices, Chips

$N_0 \sim 10$ to 10^3 atoms



T. Stoferle *et al.* PRL **92** 130403 (2004)

M. Greiner et al. PRL (2001)

W. Haensel et al. Nature (2002)

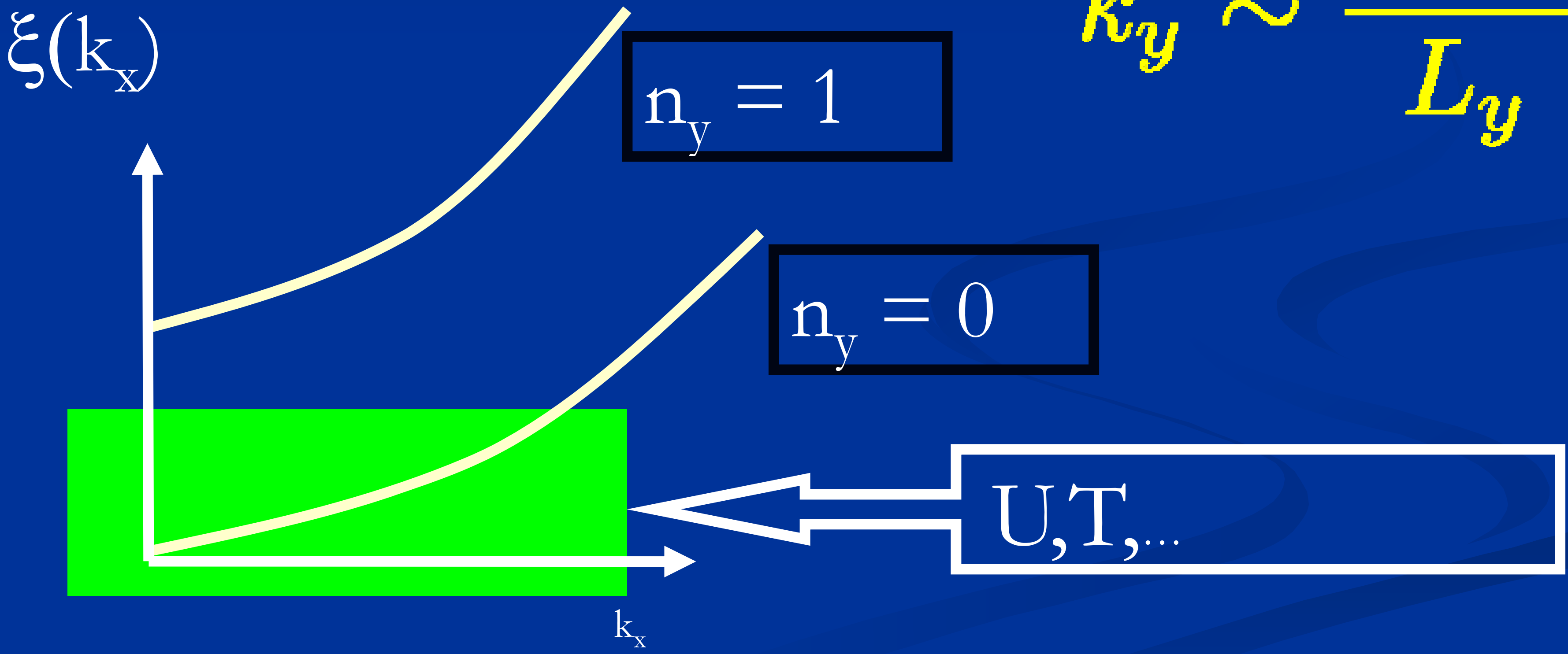
……

What does 1D means ?

$$k_x \sim \frac{2\pi n_x}{L_x}$$

$$\xi(k) = \frac{k_x^2}{2m} + \frac{k_y^2}{2m}$$

$$k_y \sim \frac{2\pi n_y}{L_y}$$

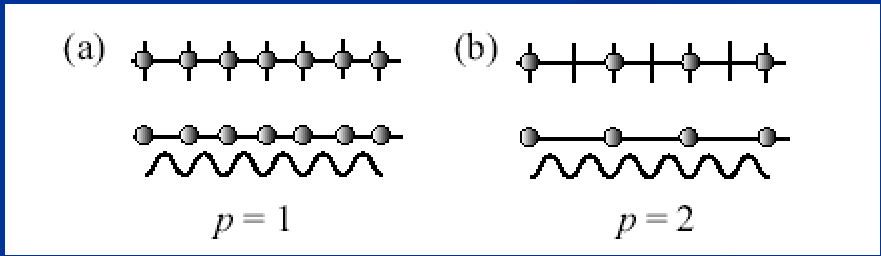


Models

- Continuum:

$$H = \int dx \frac{(\nabla\psi)^\dagger(\nabla\psi)}{2M} + \frac{1}{2}\int dx\, dx'\, V(x-x')\rho(x)\rho(x') - \mu\int dx\, \rho(x)$$

- Lattice:



$$H = -J\sum_{\langle i,j\rangle} b_i^\dagger b_j + U\sum_i n_i(n_i-1) - \mu\sum_i n_i$$

Questions

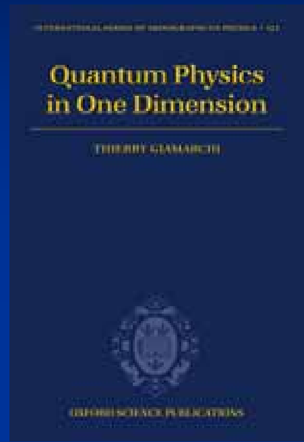
- How to deal with interactions/quantum fluctuations

- What is the new physics in 1D ?
  - Change of nature of the « particles »
  - New phases ?

- How to go from 1D to higher dimensions

General references on 1D

• Will follow closely:



TG, Quantum physics in one dimension, Oxford (2004)

TG, cond-mat/0605472 (Salerno lectures)

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- Solyom, J. (1979). Adv. Phys., 28, 209.
- Schulz, H. J. (1995). *Les Houches LXI* pp. 533. Elsevier.
- Voit, J. (1995). Rep. Prog. Phys., 58, 977.
- Gogolin, A. O., Nersesyan, A. A., and Tsvelik, A. M. (1999). Bosonization and Strongly Correlated Systems. Cambridge University Press, Cambridge.

- M.A. Cazalilla, J. Phys B, 37 S1 (2004)

How to study

- Exact methods (Bethe Ansatz)

Exact

spectrum; limited to very special models

- Numerics

``Exact”

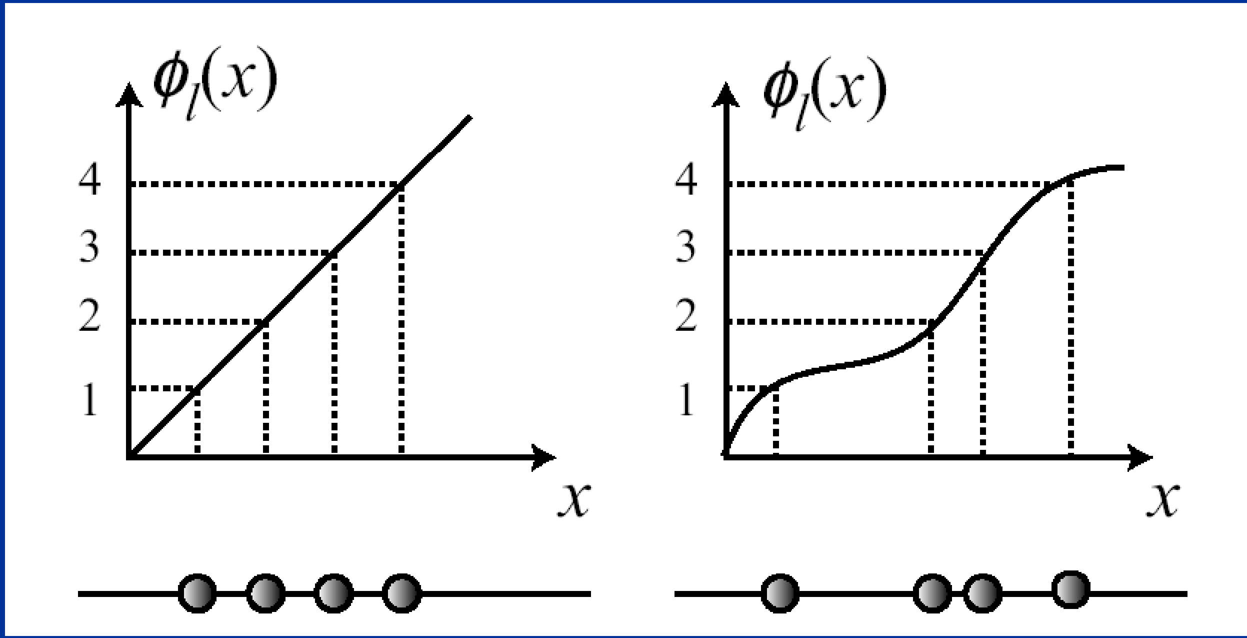
special models, size limitations, quantities specific to models

- Low energy methods

Labelling the particles

$$\rho(x) = \sum_i \delta(x - x_i)$$
$$= \sum_n |\nabla \phi_l(x)| \delta(\phi_l(x) - 2\pi n)$$

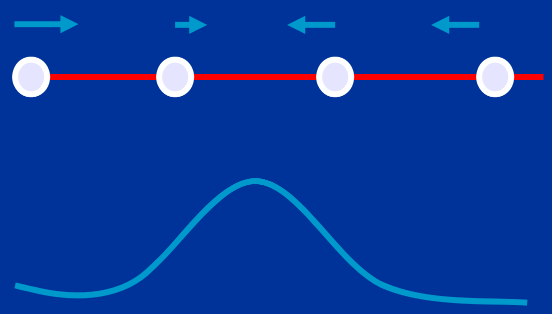
1D: unique way of labelling

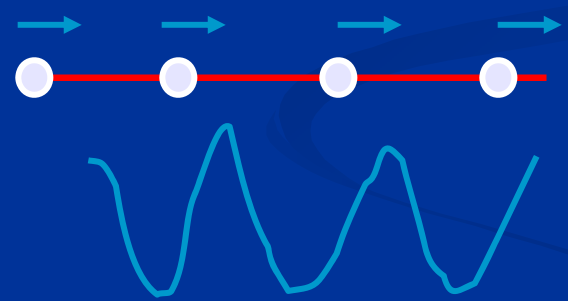


$$\phi_l(x) = 2\pi\rho_0 x - 2\phi(x)$$

$$\rho(x) = \left[\rho_0 - \frac{1}{\pi}\nabla\phi(x)\right]\sum_p e^{i2p(\pi\rho_0 x - \phi(x))}$$

$\phi(x)$ varies slowly





CDW

$q \sim 0$ $q \sim 2\pi\rho_0$

$$\psi^\dagger(x) = [\rho(x)]^{1/2} e^{-i\theta(x)}$$

θ: superfluid phase

$$[\frac{1}{\pi}\nabla\phi(x), \theta(x')] = -i\delta(x - x')$$

Quantum fluctuations

$$\psi_B^\dagger(x) = [\rho_0 - \frac{1}{\pi}\nabla\phi(x)]^{1/2} \sum_p e^{i2p(\pi\rho_0 x - \phi(x))} e^{-i\theta(x)}$$

All short distance properties: form of the operators. ϕ, θ: smooth fields

Hamiltonian

$$\int dx \frac{(\nabla\psi)^\dagger(\nabla\psi)}{2M} \rightarrow \frac{\rho_0}{2M}\int d_x(\nabla\theta(x))^2$$

$$\frac{1}{2}\int dx\, dx'\, V(x-x')\rho(x)\rho(x') \rightarrow \frac{U}{2\pi^2}\int d_x(\nabla\phi(x))^2$$

$$H = \frac{\hbar}{2\pi}\int dx[\frac{uK}{\hbar^2}(\pi\Pi(x))^2 + \frac{u}{K}(\nabla\phi(x))^2]$$

Luttinger liquid concept

- How much is perturbative

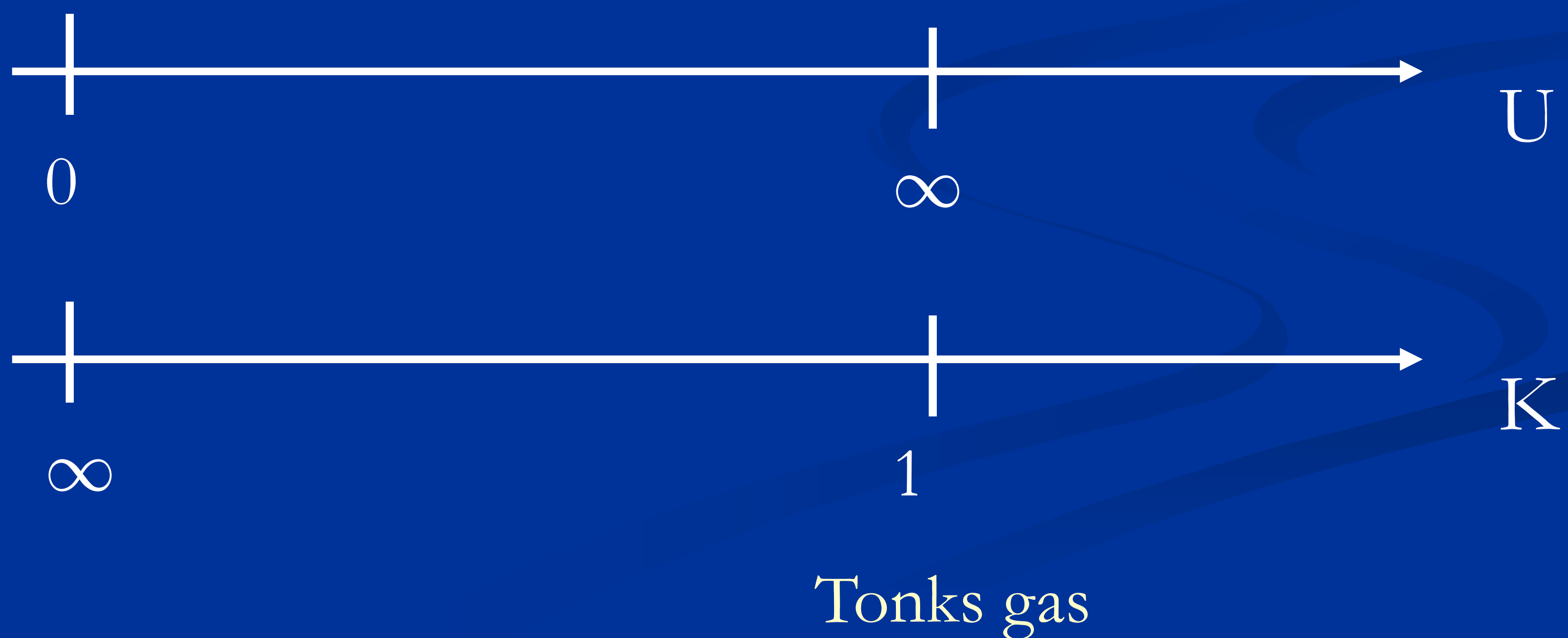
- Nothing provided the correct u,K are used (Haldane)

- Low energy properties: Luttinger liquid (fermions, bosons, spins…)

Luttinger parameters

u: velocity of collective excitations

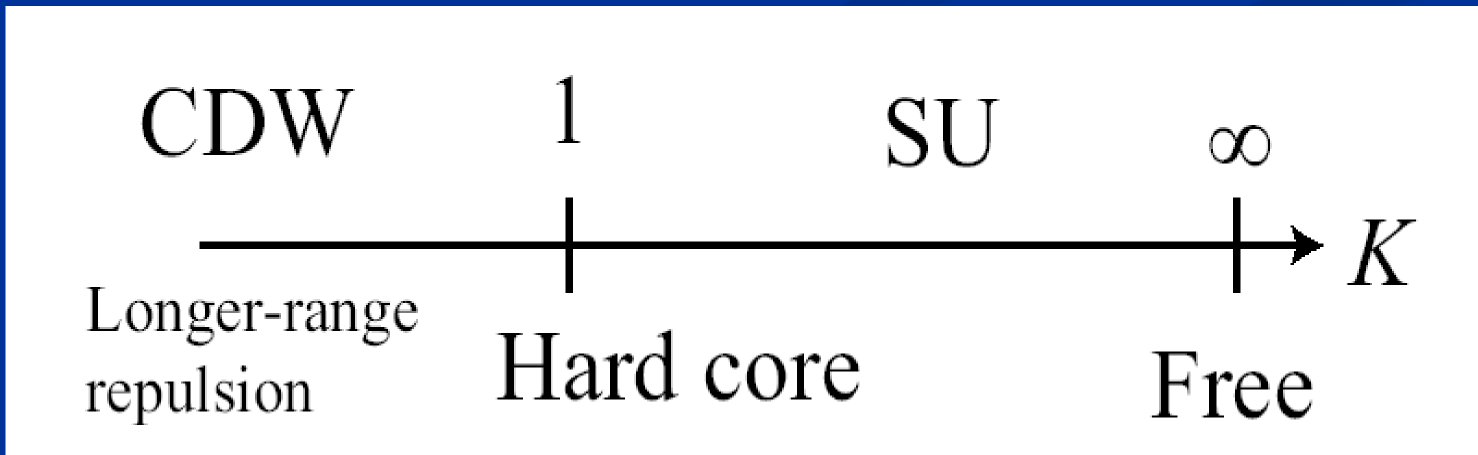
K: dimensionless parameter

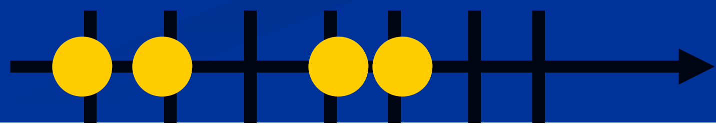


Correlations

$$\langle \psi(r)\psi^{\dagger}(0)\rangle = A_1 \left(\frac{\alpha}{r}\right)^{\frac{1}{2K}} + \cdots$$

$$\langle \rho(r)\rho(0)\rangle = \rho_0^2 + \frac{K}{2\pi^2}\frac{y_\alpha^2 - x^2}{(y_\alpha^2 + x^2)^2} + A_3 \cos(2\pi\rho_0 x)\left(\frac{1}{r}\right)^{2K} + \cdots$$

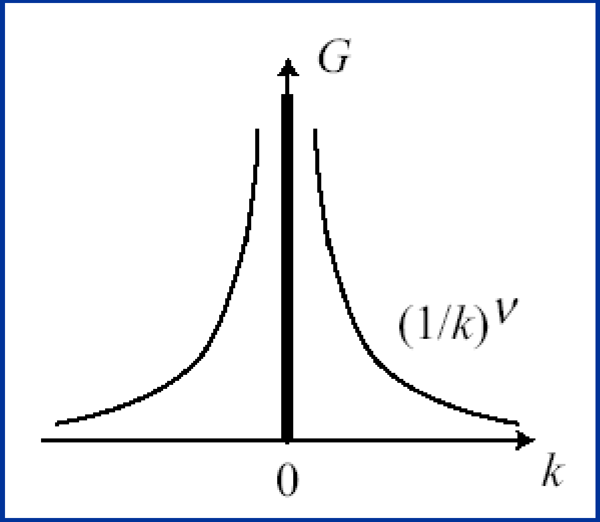




Condensate ?

$$G(x,\tau) = -\langle T_\tau \psi(x,\tau)\psi^\dagger(0,0)\rangle$$

$$n(k) = -\int dx e^{ikx} G(x,\tau = 0^-)$$

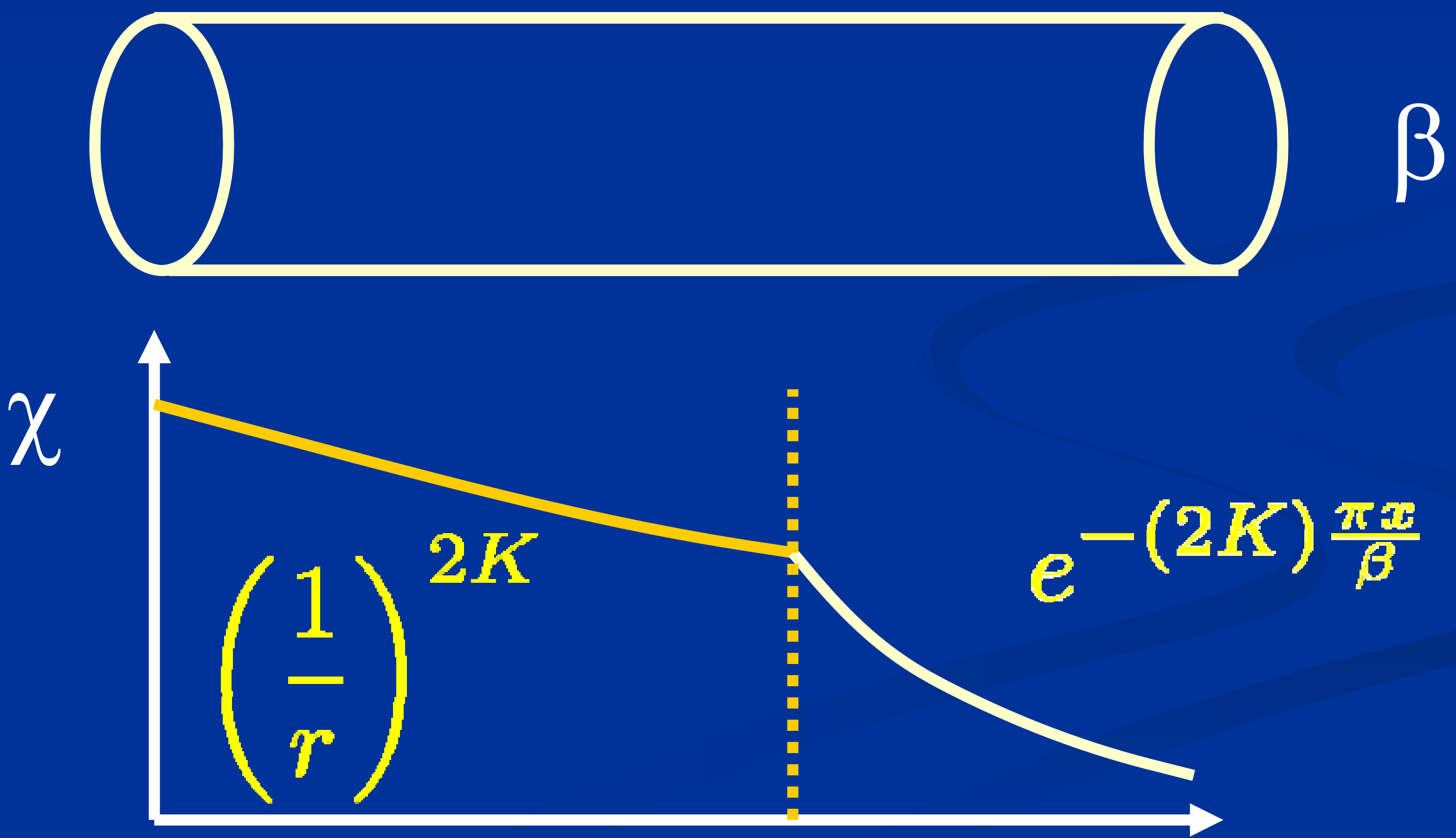


$$\nu = \frac{1}{2K} - 1$$

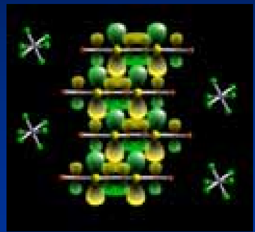
No true condensate
($K \neq 0$)

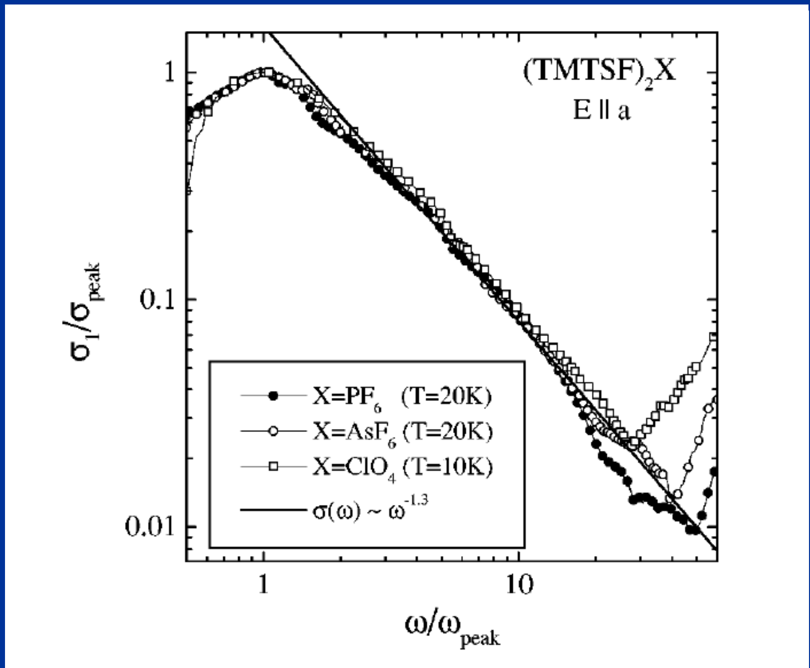
Finite temperature

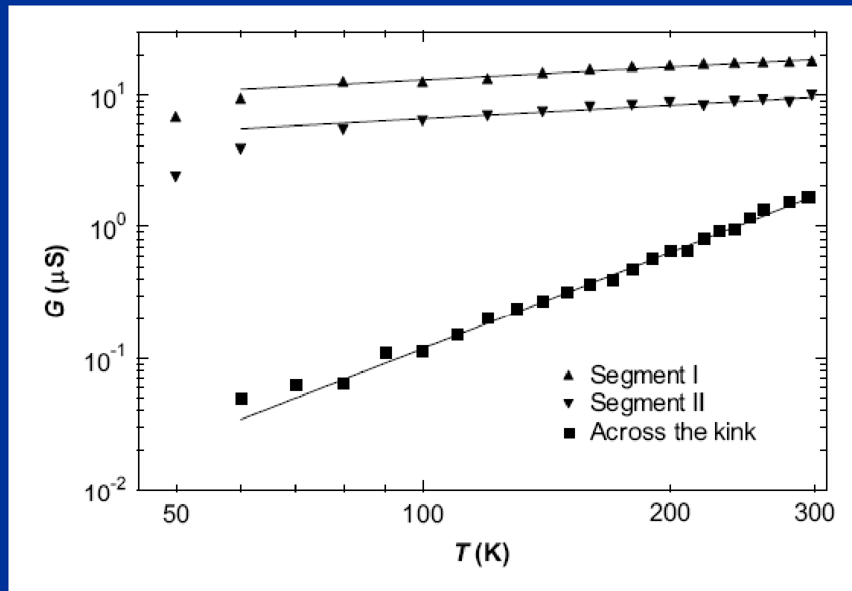
Conformal theory



Check for the powerlaws







A. Schwartz et al. PRB 58
B. 1261 (1998)

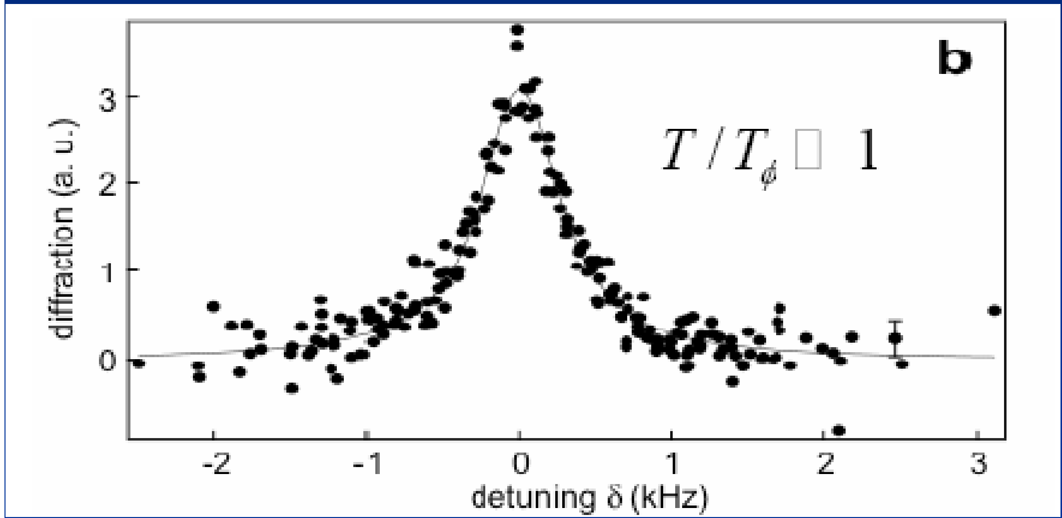
Z. Yao et al. Nature 402 273 (1999)

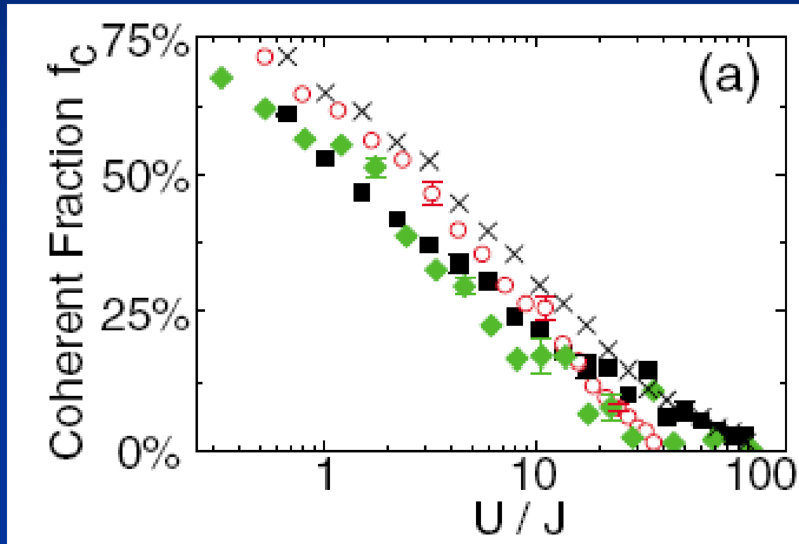
Cold atoms ?

- Difficult ! (trap!)

- Need probes !

Quantum depletion of condensate



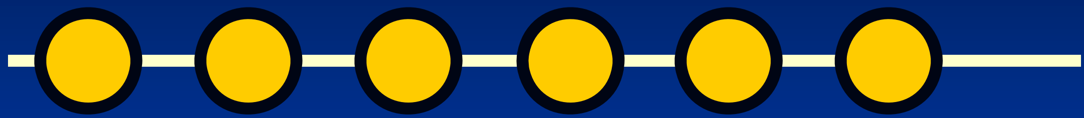


P. Bouyer *et al.* (2004)

T. Stoferle *et al.* (2004)

Good qualitative agreement

Tonks limit



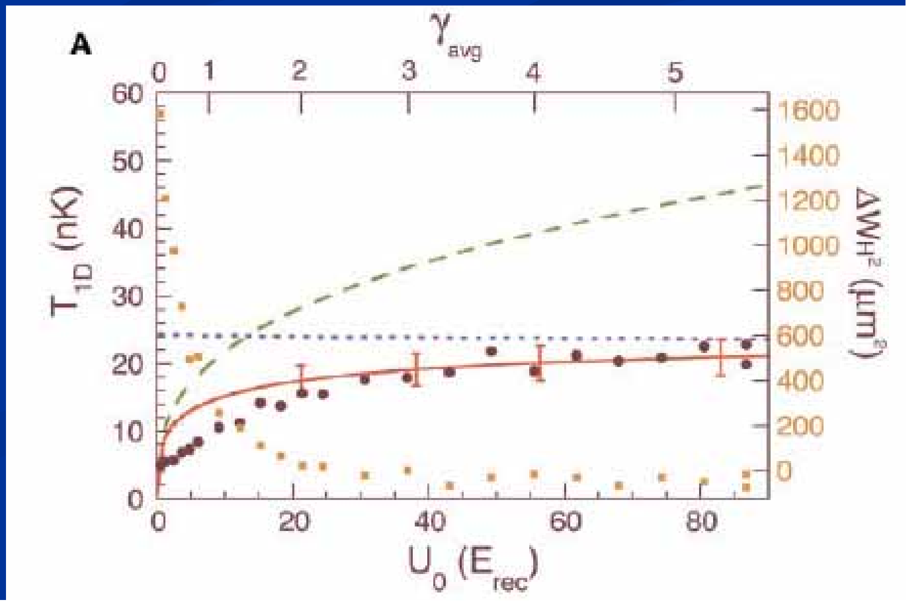
U = ∞ : spinless fermions

Not for n(k) : $\psi_F \neq \psi_B$

B. Paredes et al Nature (2004)

T. Kinoshita et al. Science (2004)

M. Kohl et al. PRL (2004)



Systems with « spins »

Luttinger liquid

Same treatment

$$\rho_\uparrow \to \nabla\Phi_\uparrow \qquad \rho_\downarrow \to \nabla\Phi_\downarrow$$

More convenient

$$\rho = \frac{1}{\sqrt{2}}(\rho_\uparrow + \rho_\downarrow) \qquad \sigma = \frac{1}{\sqrt{2}}(\rho_\uparrow - \rho_\downarrow)$$

$$H_{kin} = H_\uparrow + H_\downarrow = H_\rho + H_\sigma$$

$$H_{\text{int}} = U\sum_i \rho_\uparrow \rho_\downarrow = U(\rho+\sigma)(\rho-\sigma)$$

$$= U(\rho\rho - \sigma\sigma)$$

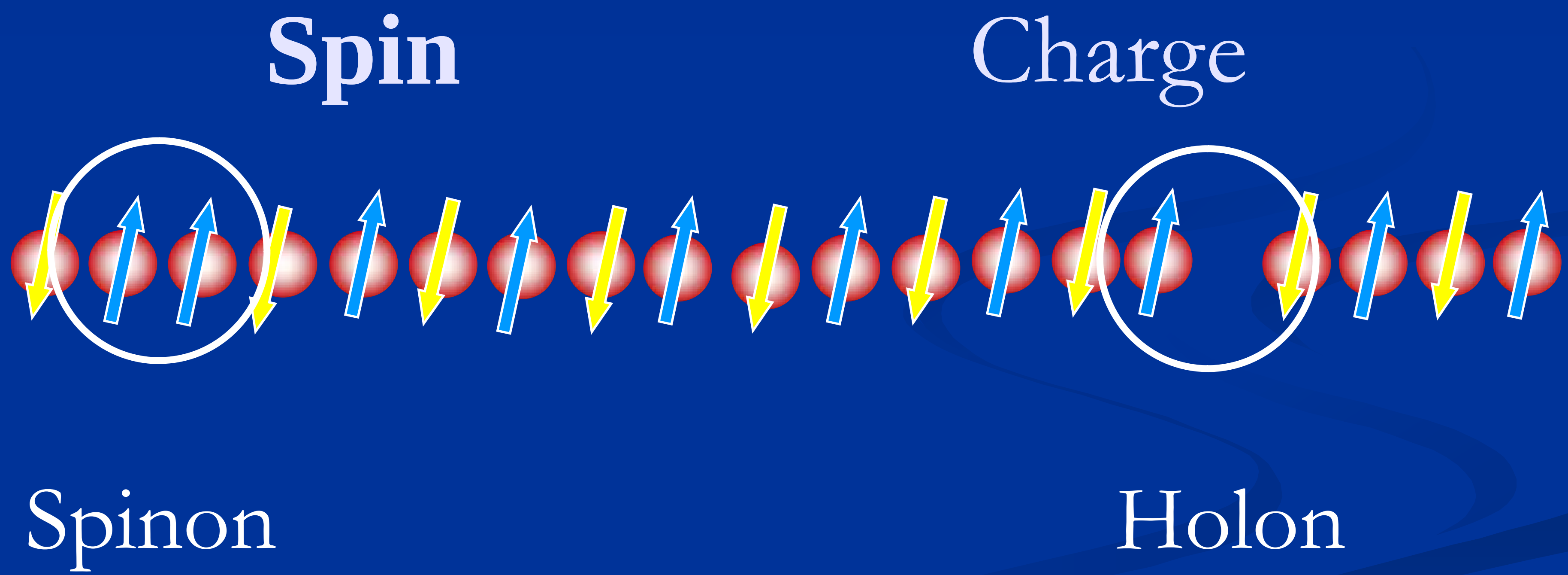
$$H = H_\rho + H_\sigma$$

(u_ρ, K_ρ) Charge excitations

(u_σ, K_σ) Spin excitations

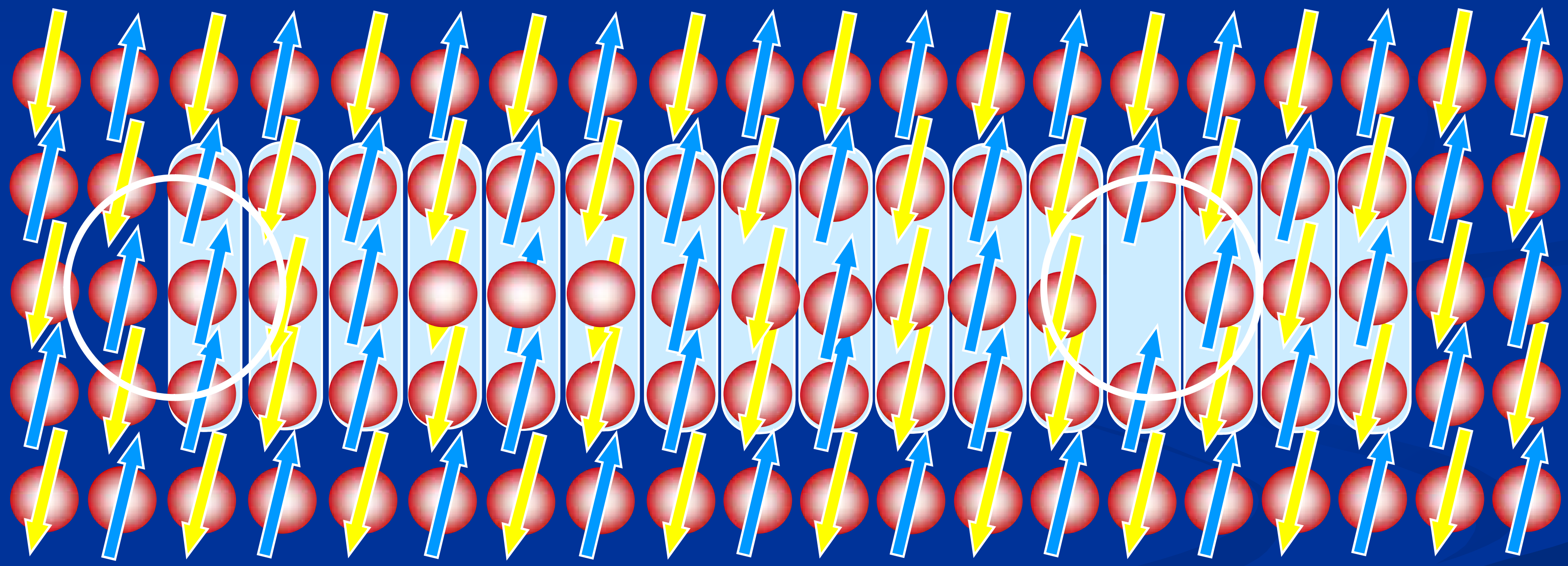
Charge-spin separation

Spin-Charge Separation



Spin-Charge Separation higher D ?

Spin Charge



Energy increases with spin-charge separation

Confinement of spin-charge: « quasiparticle »

Can one observe spin-charge separation ?

- Condensed matter: difficult
- One serious experiment: Yacoby et al.

Two component Bosons

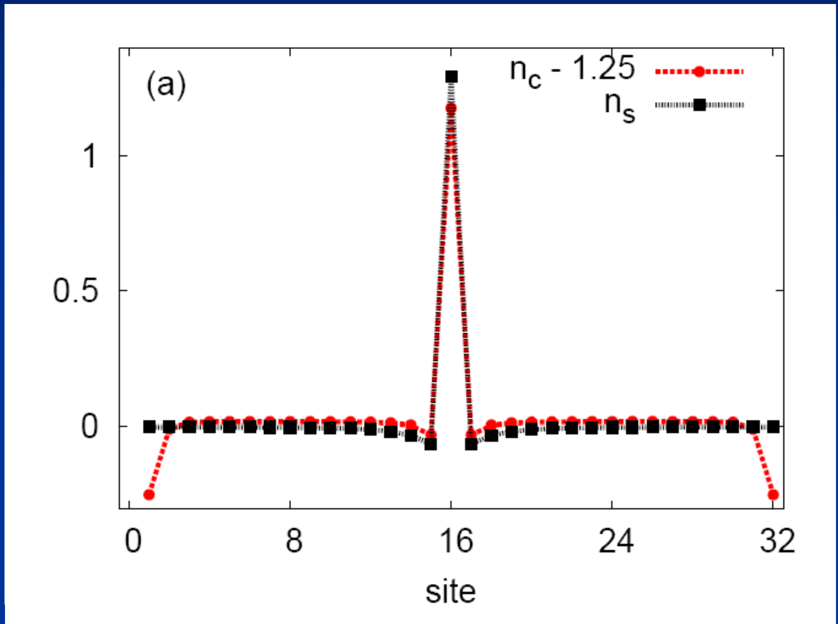
A. Kleine et al. cond-mat/0706.0709

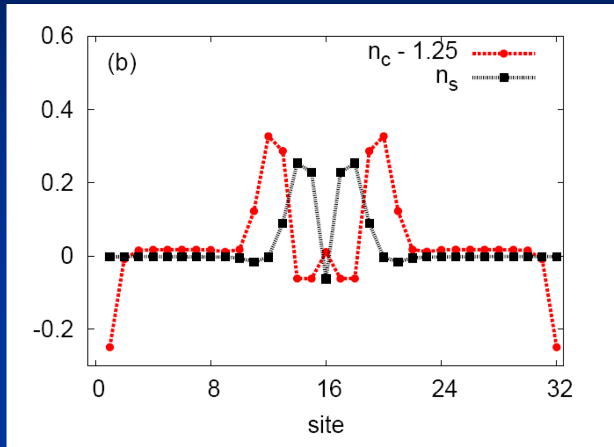
- e.g. ^{87}Rb (two hyperfine states)

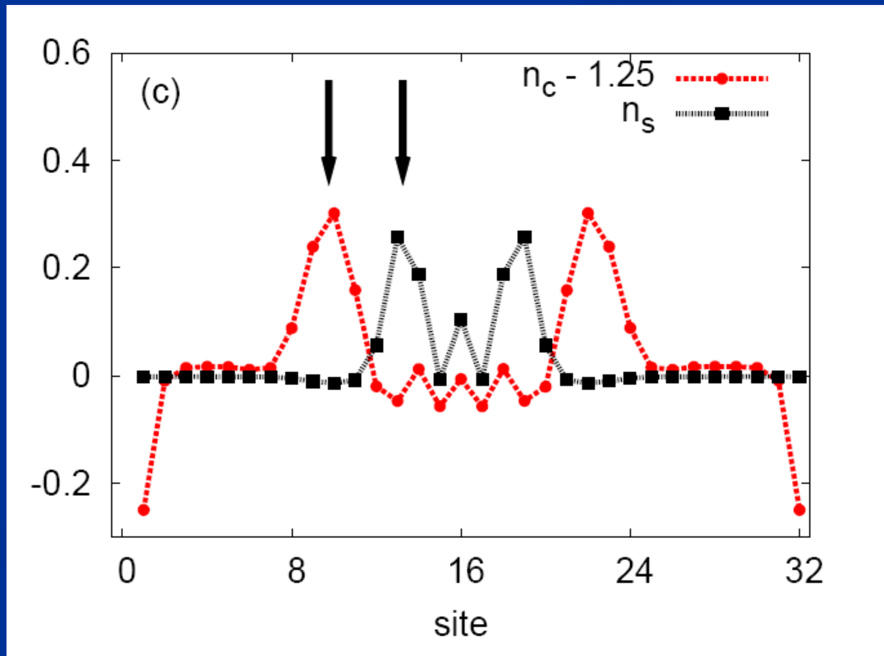
$$
\begin{aligned}
H = \; & -J\sum_{j,\nu}\left(b^{\dagger}_{j+1,\nu}b_{j,\nu}+h.c.\right)+\sum_{j,\nu}\frac{U_{\nu}\hat{n}_{j,\nu}(\hat{n}_{j,\nu}-1)}{2} \\
& +U_{12}\sum_{j}\hat{n}_{j,1}\hat{n}_{j,2}+\sum_{j,\nu}\varepsilon_{j,\nu}\hat{n}_{j,\nu}
\end{aligned}
$$

$$
\begin{aligned}
v_{c,s} \quad & = v_0\sqrt{1\pm(g_{12}K)/(\pi v_0)} \\
\text{and } K_{c,s} \quad & = K/\sqrt{1\pm(g_{12}K)/(\pi v_0)}.
\end{aligned}
$$

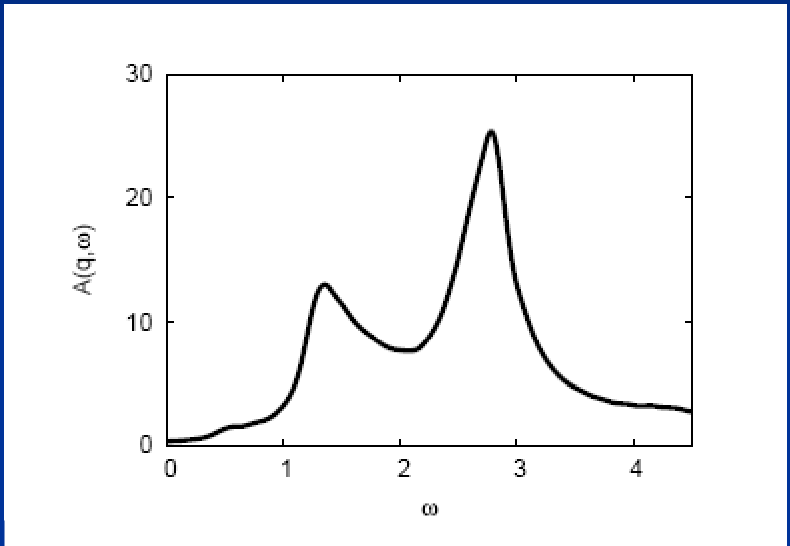
t-DMR study

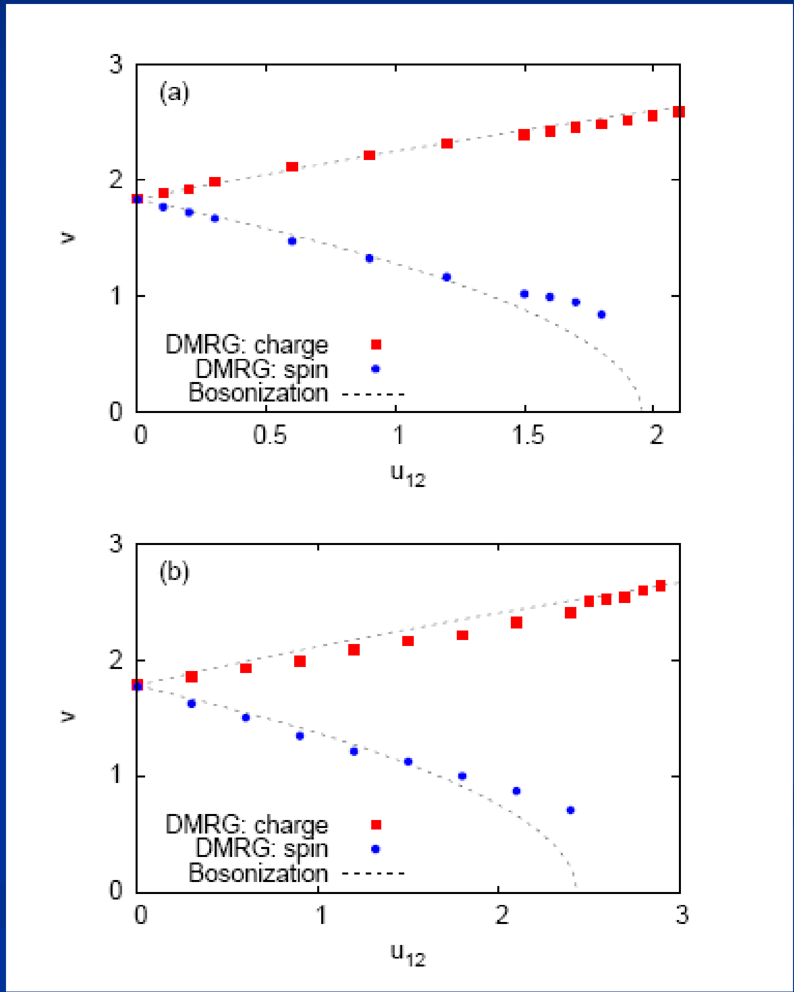

(a)
n_c - 1.25
n_s
1
0.5
0
0
8
16
24
32
site


(b)
n_c - 1.25
n_s
0.6
0.4
0.2
0
-0.2
0
8
16
24
32
site


(c)
n_c - 1.25
n_s
0.6
0.4
0.2
0
-0.2
0
8
16
24
32
site

Parameters and Spectral functions

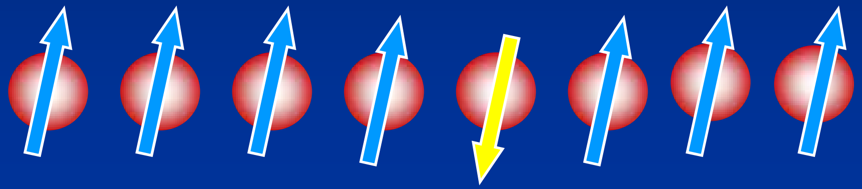




Spectral function by bosonization:
A. Iucci, G. Fiete, TG PRB 75, 205116 (2007))

Other effects for bosons ?

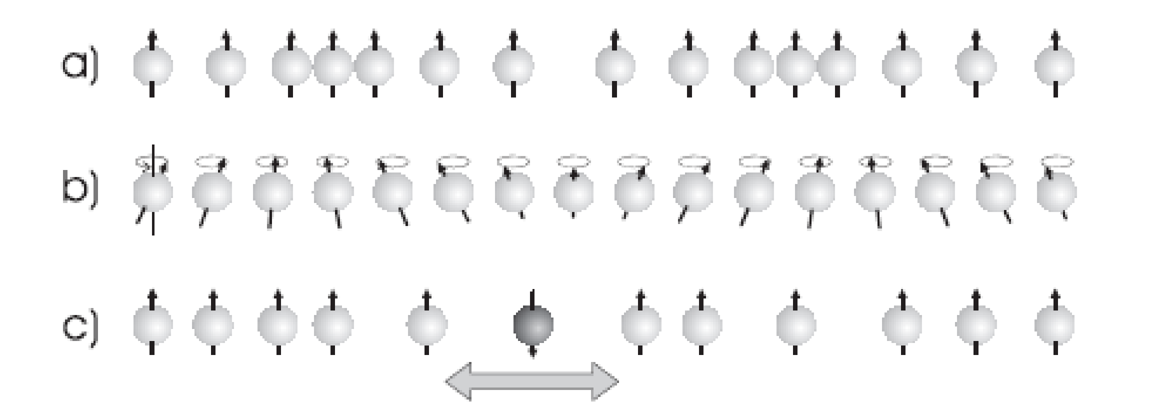
Ferromagnetism



(M. Zvonarev, V. Cheianov, TG cond-mat/0708.3638)

Excitations: k^2 not k

Not a LL !!



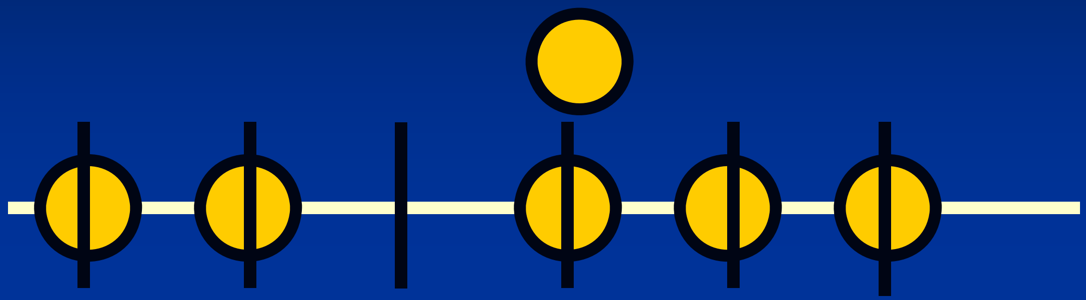
$$G_{\parallel}(x,t) = \langle \Uparrow | s_z(x,t) s_z(0,0) | \Uparrow \rangle$$
$$G_{\perp}(x,t) = \langle \Uparrow | s_+(x,t) s_-(0,0) | \Uparrow \rangle$$

$$\alpha = 2, \qquad \beta = \frac{K}{2k_F^2}$$

$$G_{\perp}(x,t) \sim t^{-\alpha} \left[\beta \ln\left(\frac{t}{t_0}\right) + \frac{it}{2m_*} \right]^{-1/2} \times \exp\left\{ \frac{i m_* x^2}{2t - 4i\beta m_* \ln(t/t_0)} \right\}.$$

Mott transition

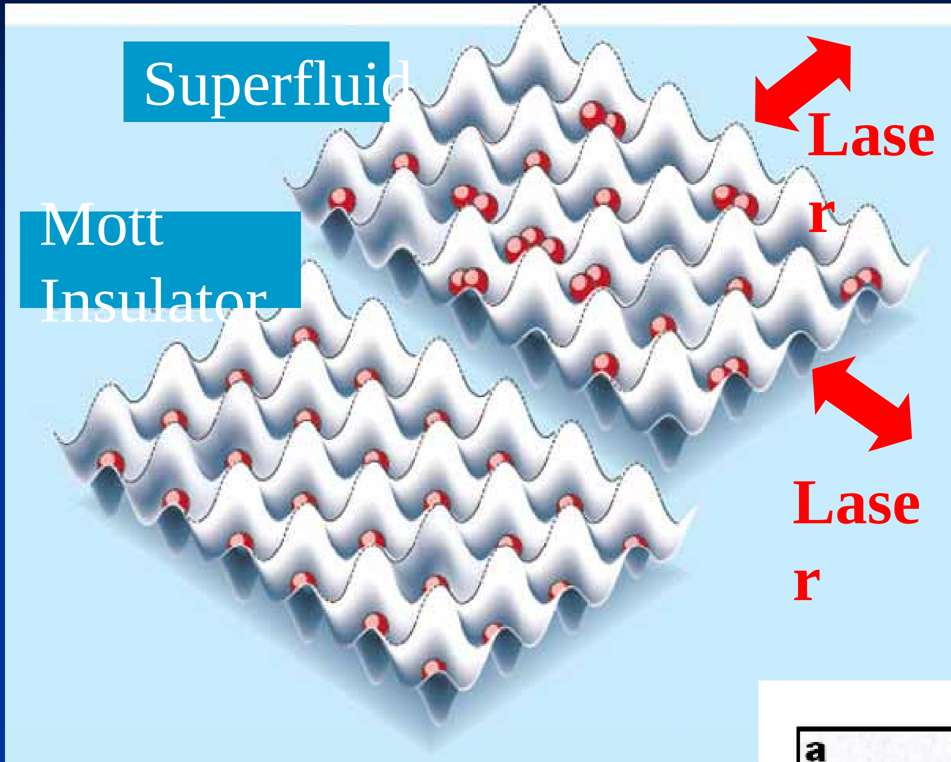
Lattice: Mott transition

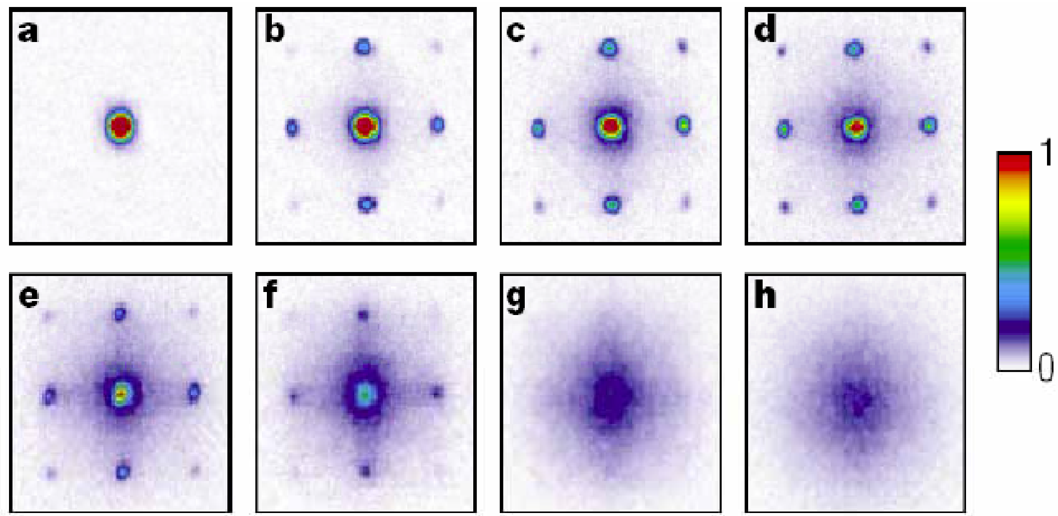


Costs U

Quantum phase transition

Mott transition and cold atoms

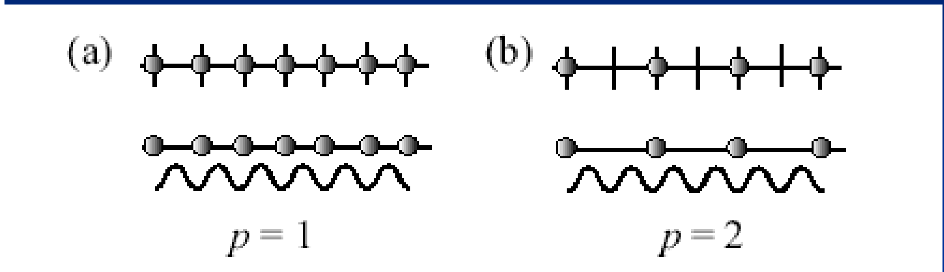




Superfluid to Mott insulator transition in a 3D optical lattice

[M Greiner *et al.* Nature, 415 (200

How to treat?



$$H_L = \int dx V(x)\rho(x)$$

$$= \int dx V(x) e^{i2p(\pi\rho_0 x - \phi(x))}$$

- Incommensurate: $Q \neq 2\pi\rho_0$

$$H_L = \int dx \cos(2\phi(x) + \delta x)$$

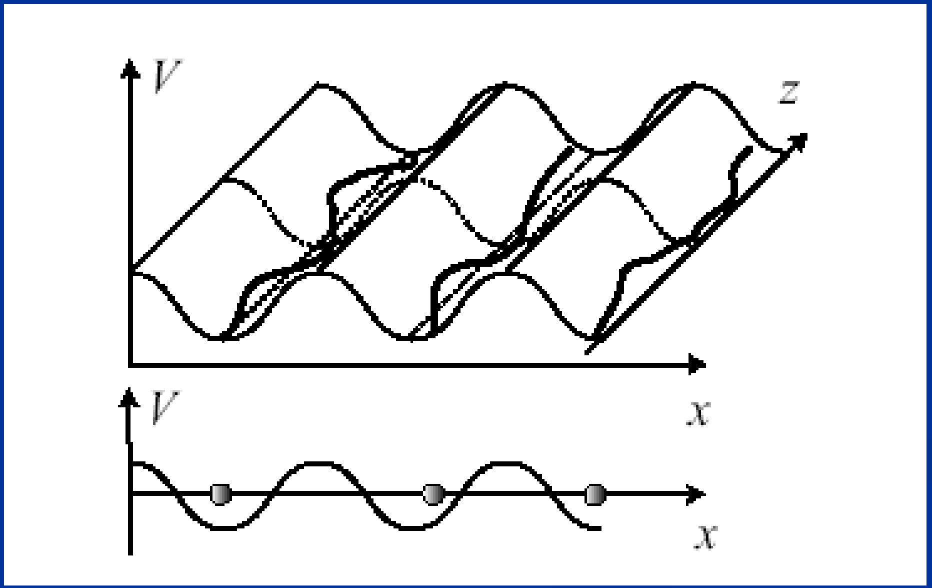
- Commensutate: $Q = 2\pi\rho_0$

$$H_L = \int dx \cos(2\phi(x))$$

Competition

$$S=\int\frac{dxd\tau}{2\pi K}[\tfrac{1}{u}(\partial_\tau\phi(x,\tau))^2+u(\partial_x\phi(x,\tau))^2]$$

$$S_{\rm lat}=\int dxd\tau V\cos(2\phi(x,\tau))$$

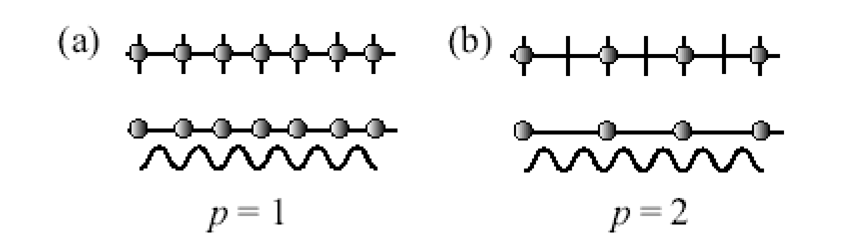


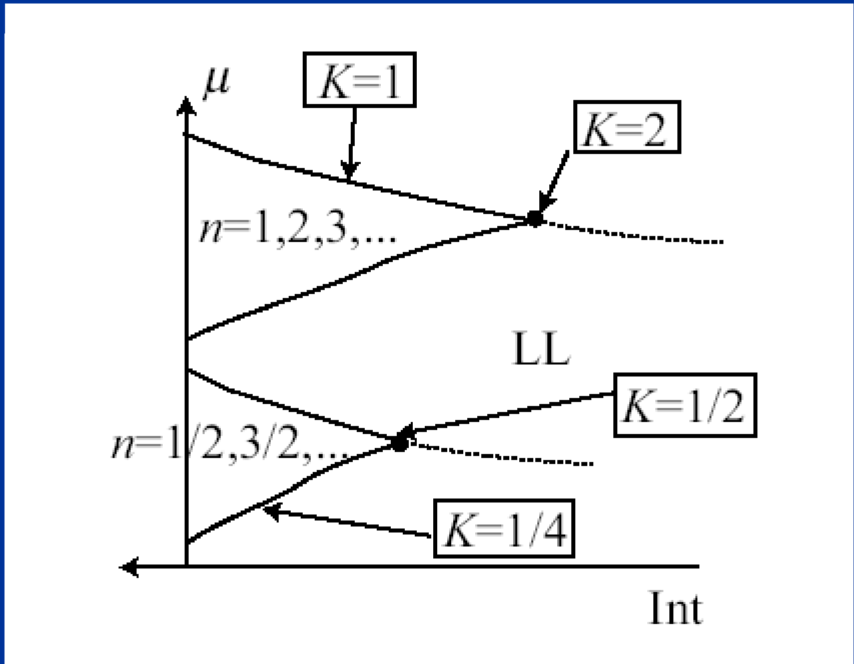
Beresinskii-Kosterlitz-Thouless transition

K=2

Lattice

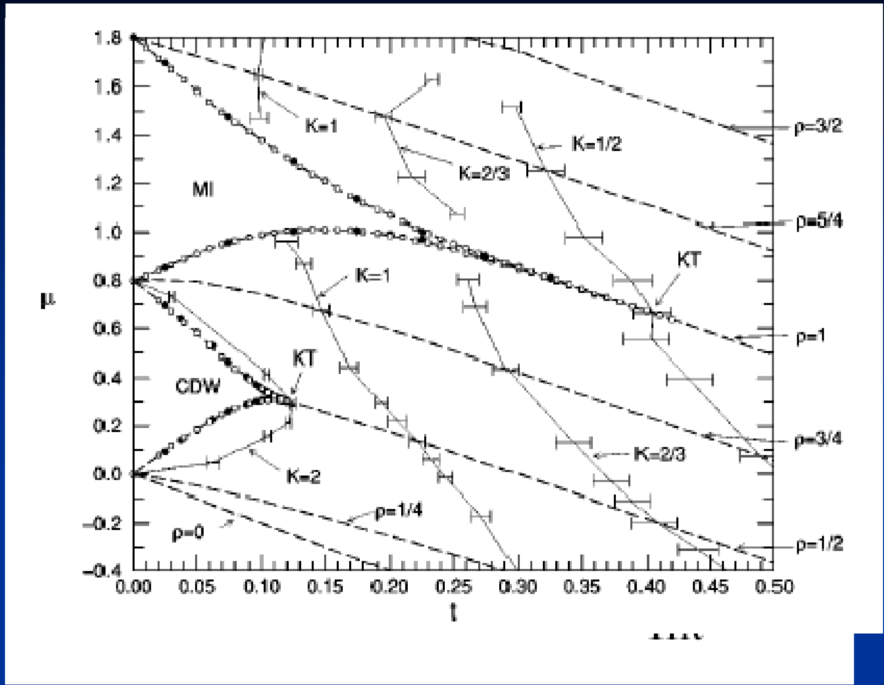
$$H_L \propto V_n^0 \int dx \;\cos(2p\phi(x))$$





Mott insulator:

ϕ is locked



TG, Physica B 230 975 (97)

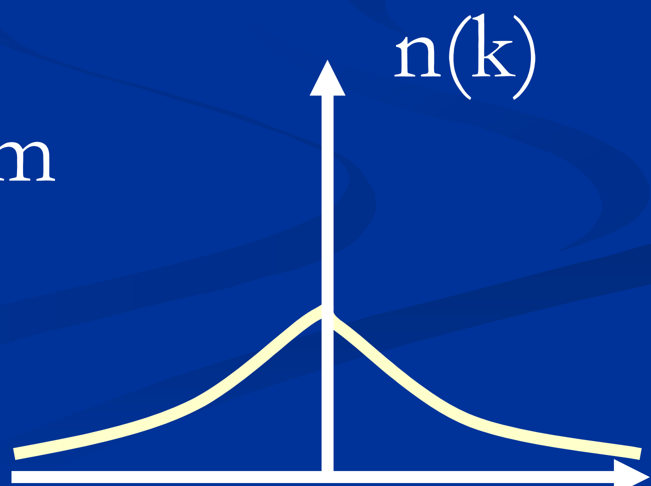
Mott insulator:
ϕ is locked

Density is fixed

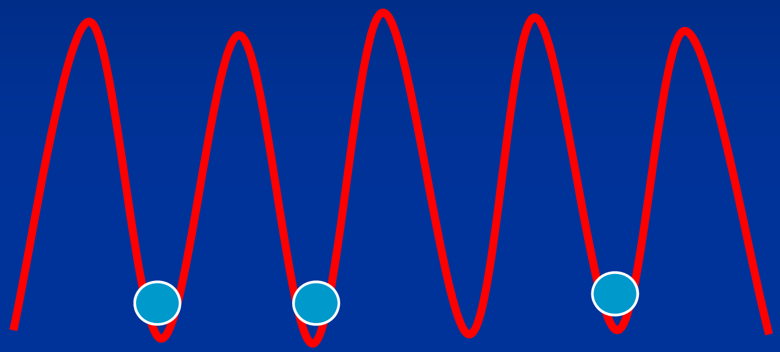
T. Kuhner et al. PRB 61 12474 (2000)

Gap in the excitation spectrum

$$G(x) \propto \exp[-|x|/\xi]$$

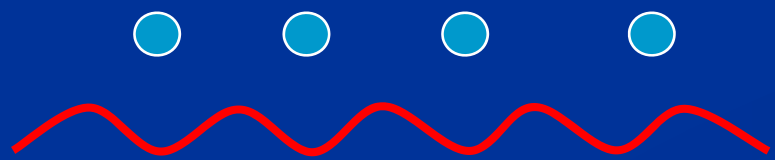


Important for 1D



$J \to 0$ ``trivial'' MI

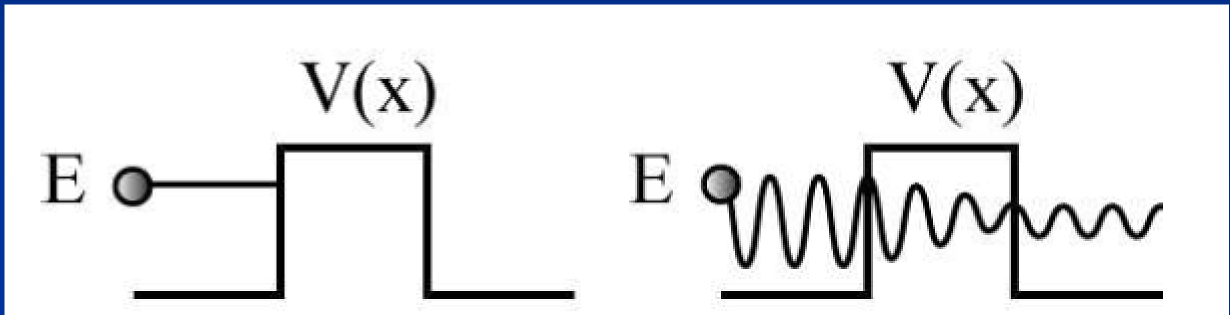
$J \gg V$ but $K < 2$



MI !!

Disorder

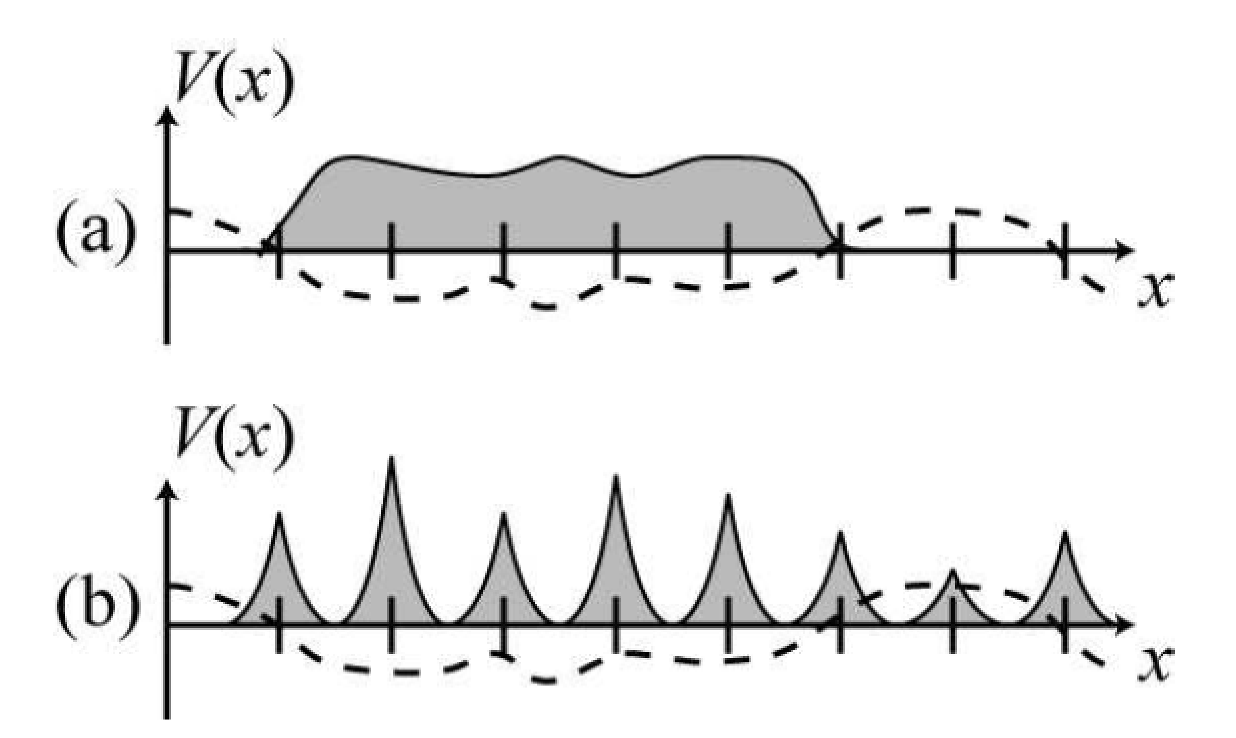
Disorder and quantum systems



disorder less important ??

No !! (Anderson localization):
interferences

Bosons



Free bosons: pathological (rare events)

How to treat

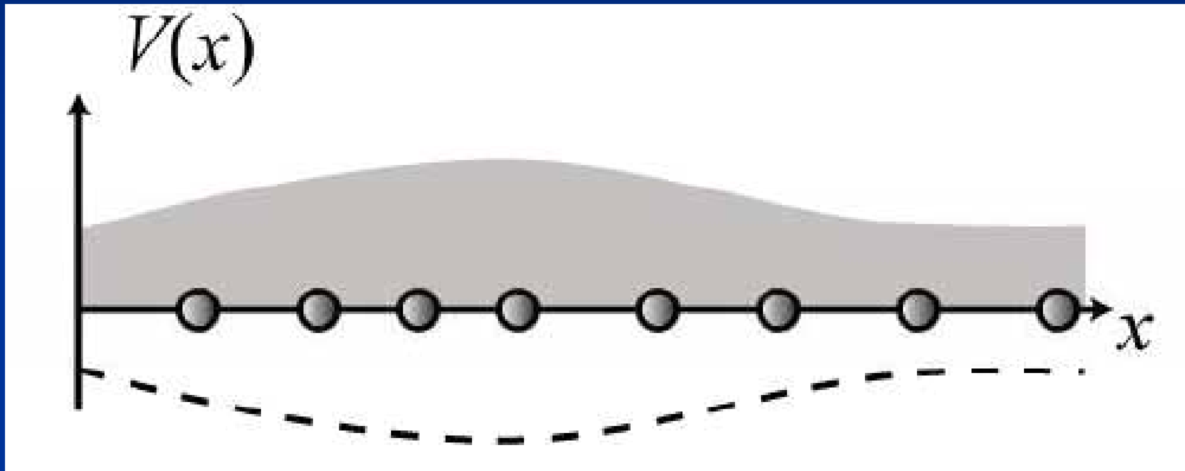
TG + H. J. Schulz PRB 37 325 (1988)

$$H_{\text{dis}} = \int dx\, V(x) \rho(x)$$

$$H_{\text{dis}} = \int dx\, V(x) [-\frac{1}{\pi} \nabla \phi(x) + \rho_0 (e^{i(2\pi\rho_0 x - 2\phi(x))} + \text{h.c.})]$$

``Two” fourier components of disorder

Forward scatting (q ~ 0)

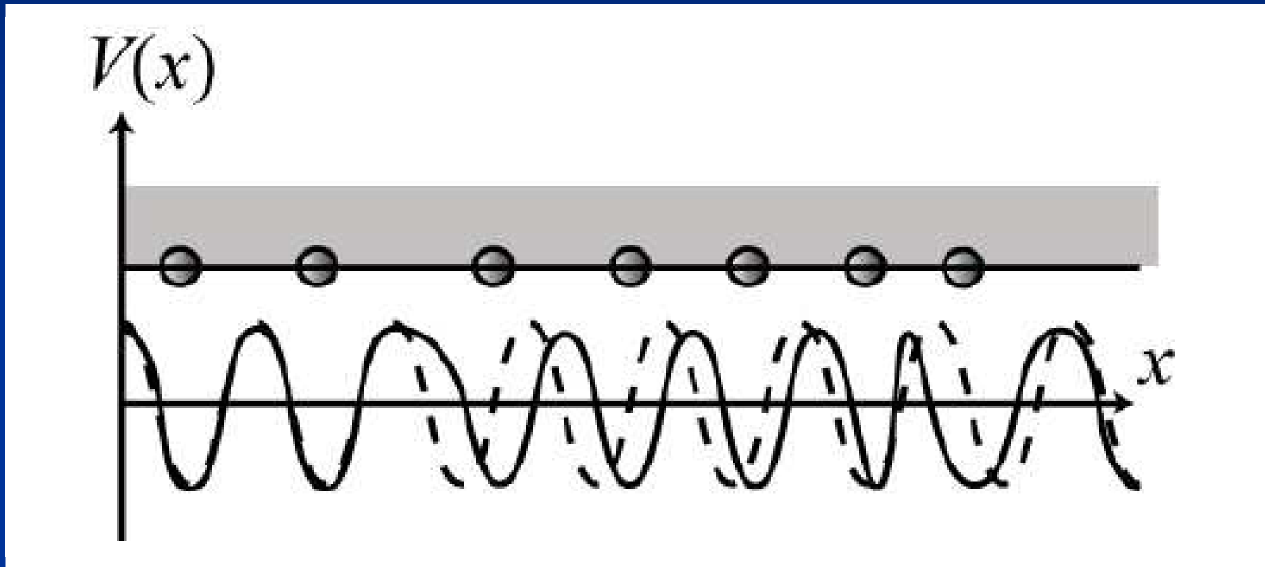


Random (smooth) chemical potential

No localization

Can break commensurate phases

Backward scattering ($q \sim 2 \pi \rho_0$)



Relevant for K < 3/2

Localized !!

Bose glass phase

1D : TG + H. J. Schulz PRB 37 325 (1988)

Superfluid – Localized (Bose glass) transition for $K < 3/2$

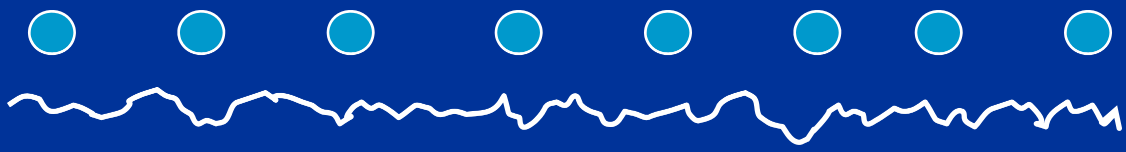
BKT like transition

Higher dimensions: M.P.A. Fisher et al. PRB 40 546 (1989)

Bose glass also exists

continuous transition

Weak disorder (strong interactions) is enough in 1D



Localized for $K < 3/2$ even if $V \ll \mu$

Quantum effect: destructive interferences

Bosons 1D

- Luttinger liquid physics

- New phases : Mott insulator/Bose glass
- Good qualitative agreement with exp.
- Correlation functions !

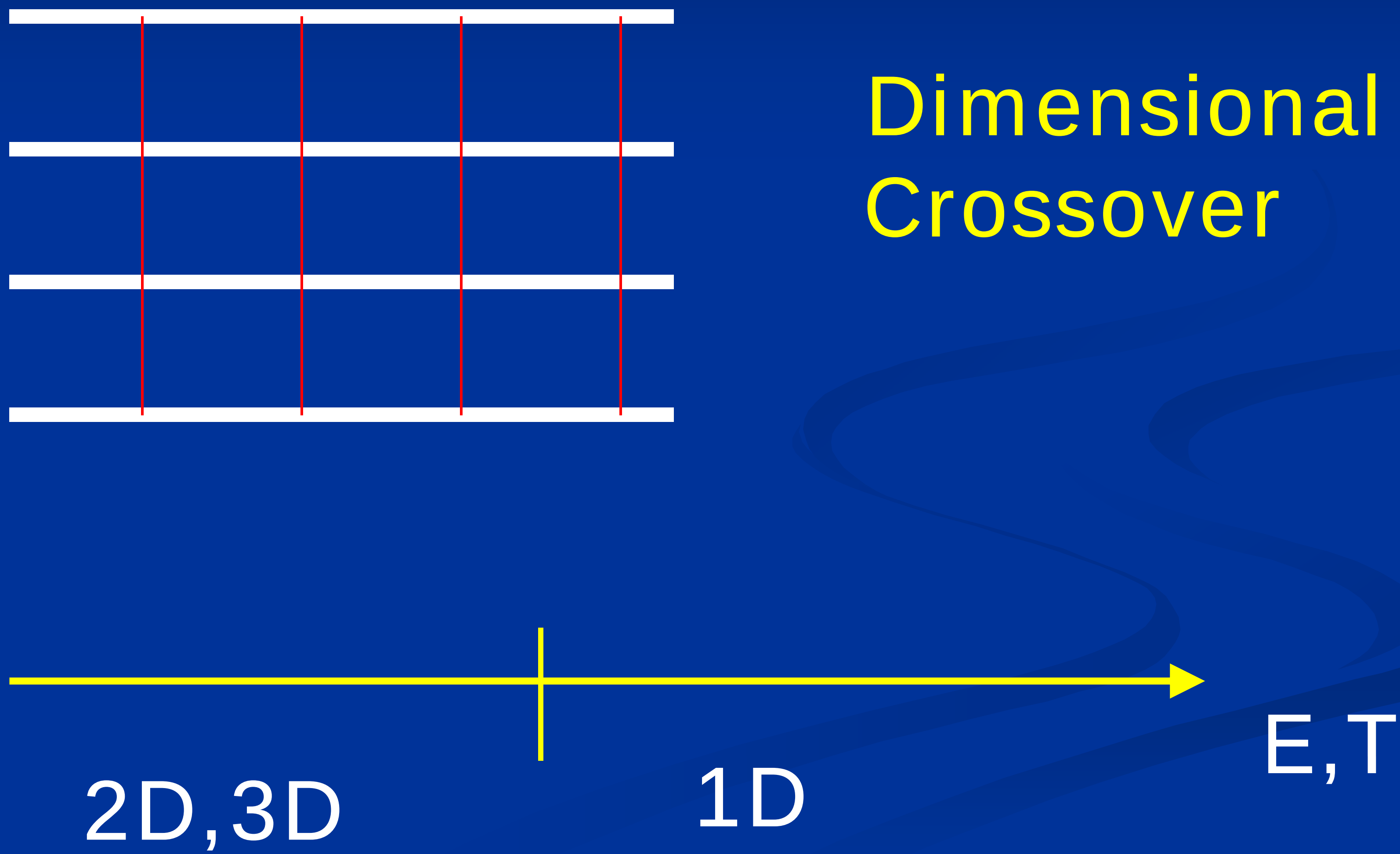
Many open points

- Confining potential
- Dynamics
- Disorder
- Etc……

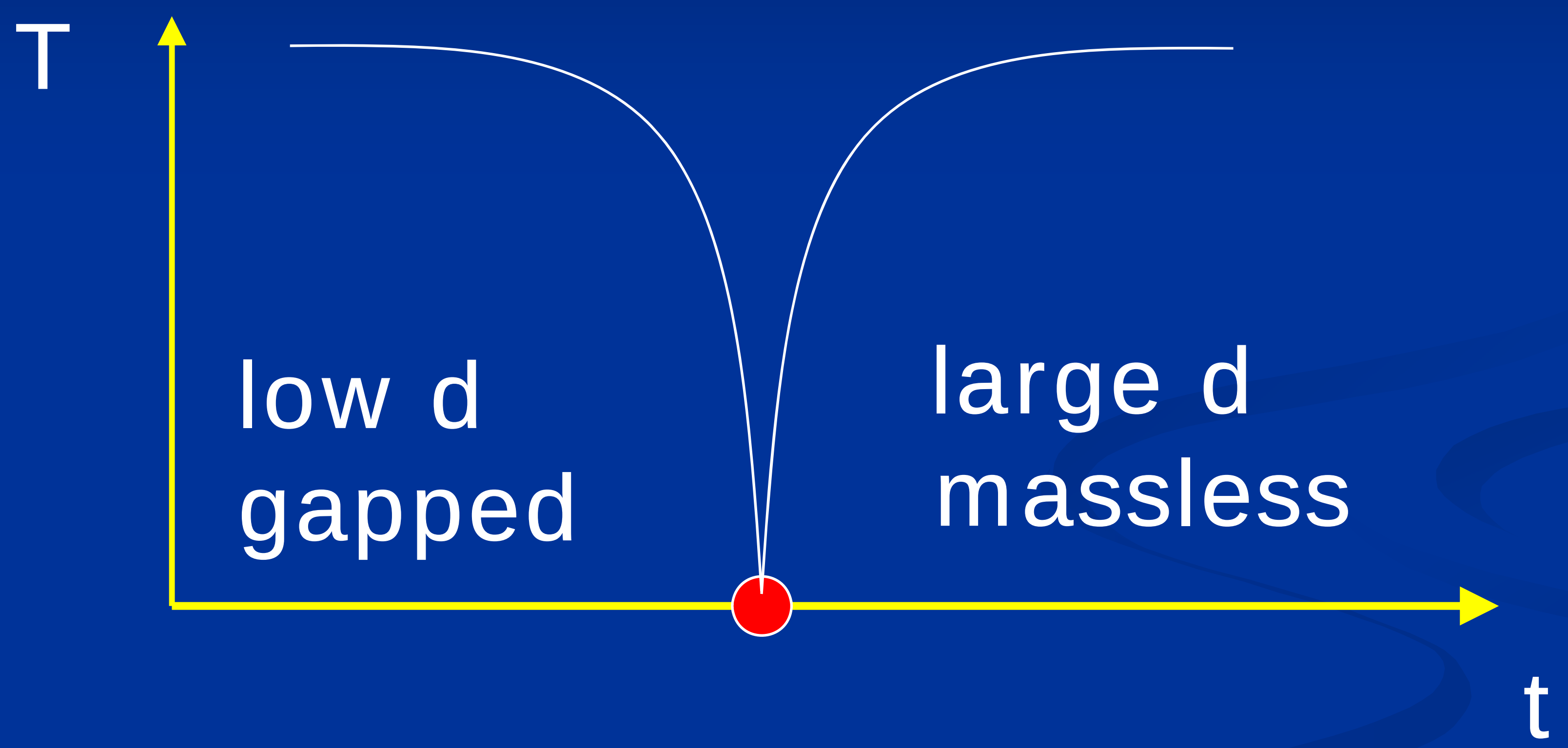
To higher dimensions…

And beyond……..

• Interaction effects vary enormously with dimension



• Even more interesting : lower dimensional phase gapped



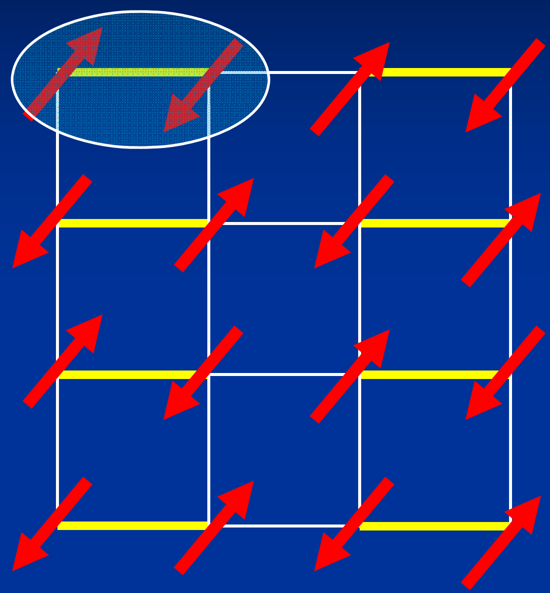
Quantum phase transition : deconfinement

Generic scenario for many systems

Questions :

- Nature and position of the transition ?
- Physical properties in the critical regime
- Impact of the low-d phase on the massless phase

Spin systems

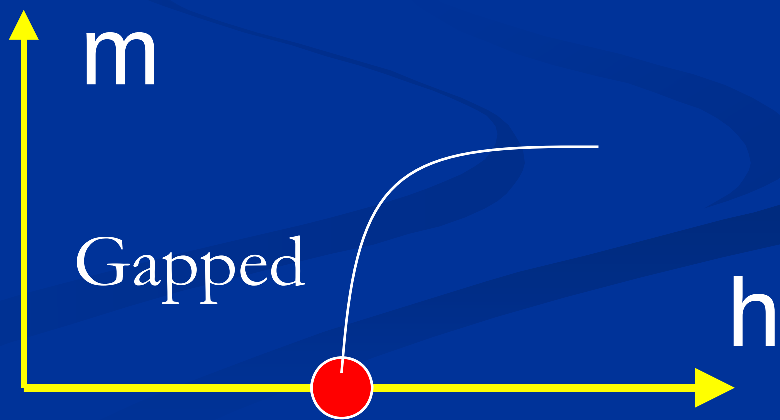


- Singlet phase of dimers (zero dimensional)
- J is irrelevant (gapped phase)

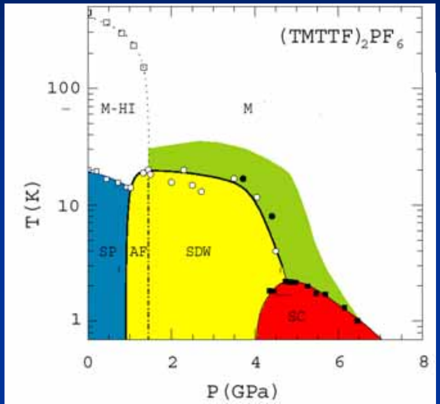
Break the gap: H

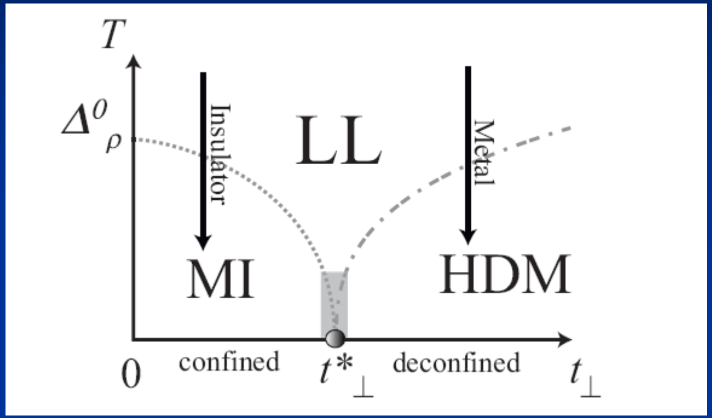
TG and A. M. Tsvelik
PRB 59 11398 (1999)

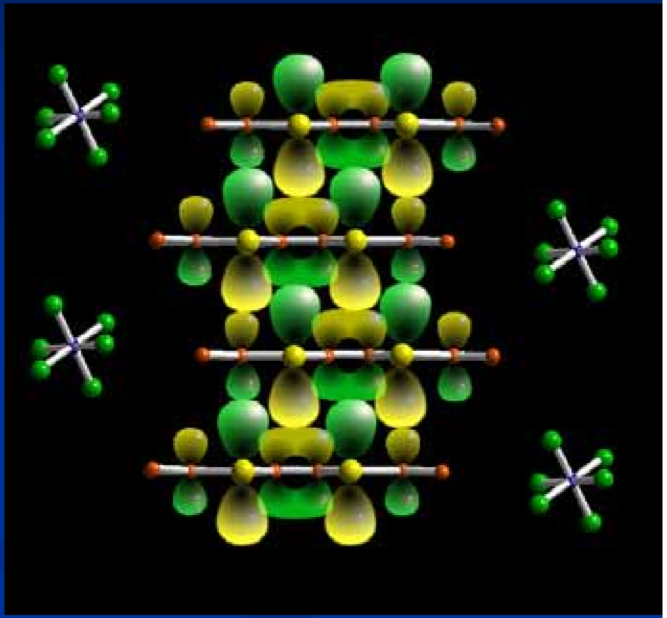
BEC

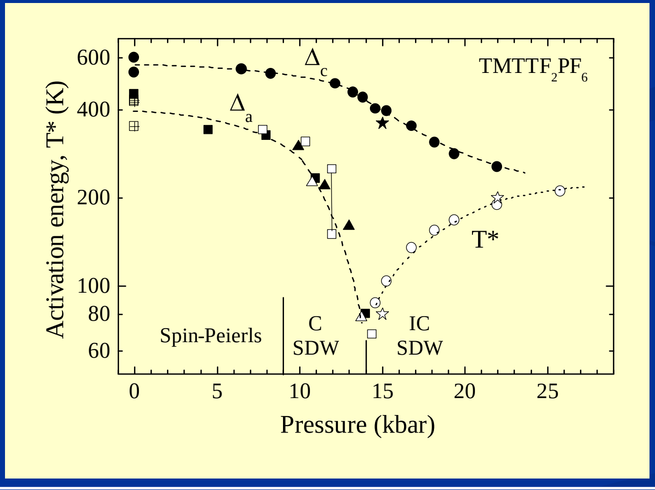


Deconfinement







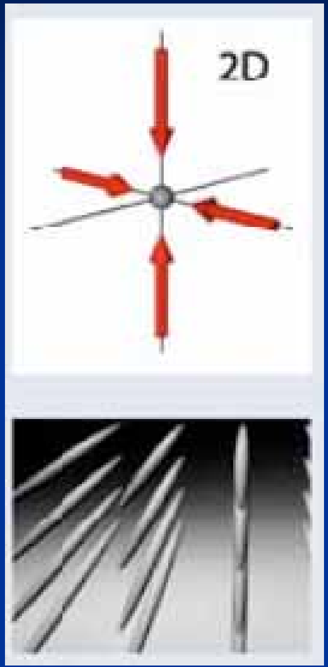


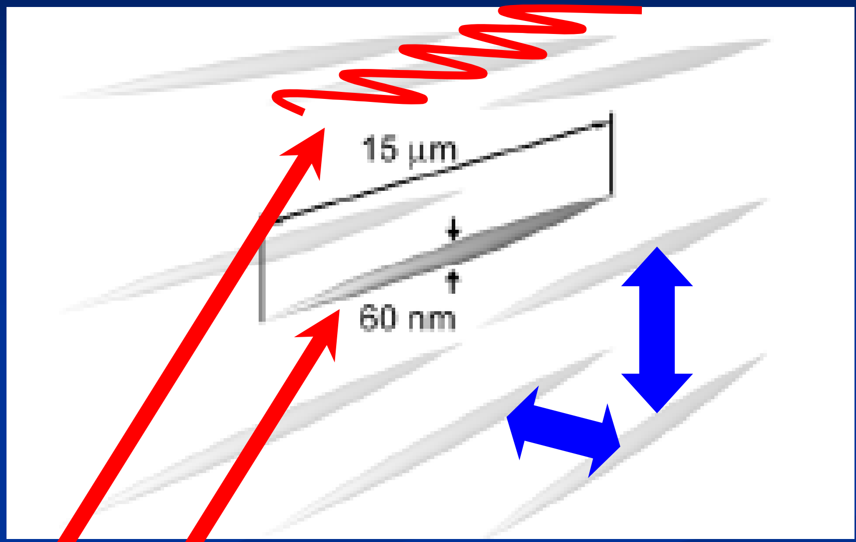
TG Physica B 230 975 (97); Chem Rev 104 5037 (2004); cond-mat/0702565

P. Auban-Senzier, D. Jérome, C. Carcel and J.M. Fabre J de Physique IV, (2004)

D. Jaccard et al., J. Phys. C, 13 L89 (2001)

Deconfinement





T. Stoferle *et al.*
PRL **92** 130403 (2004)

1D Mott insulator

1D physics (Luttinger Liquids)

Bosons

[^{87}Rb]

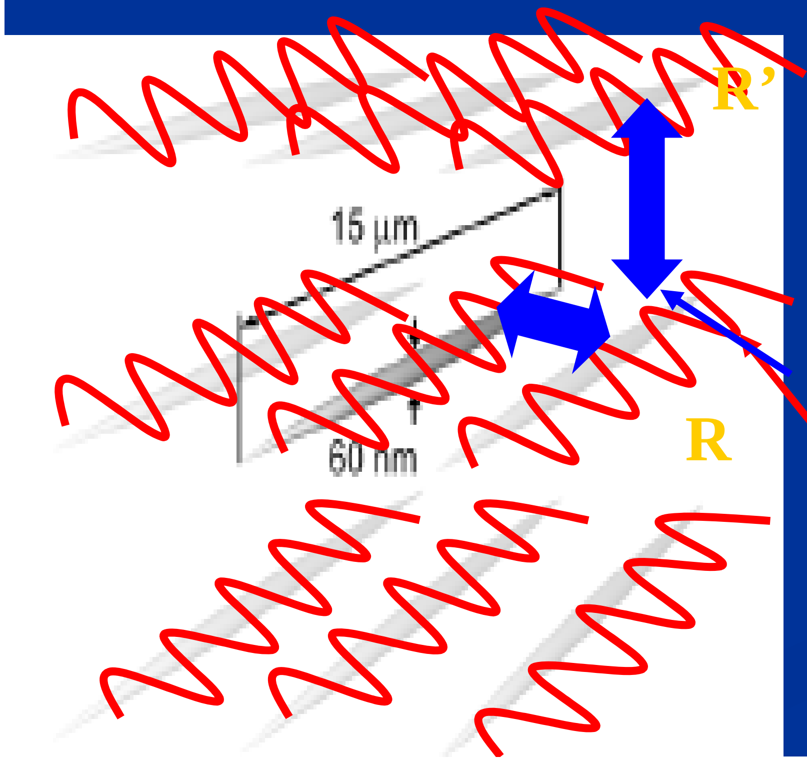
A. F. Ho, M. A. Cazalilla, TG PRL 92 130405 (2003)
M. A. Cazalilla, A. F. Ho, TG, NJP 8 158 (2006)

Mott vs. Josephson

$$H_{\rm eff} = \frac{\hbar v_s}{2\pi}\sum_{\mathbf{R}}\int_0^L dx\left[\frac{1}{K}\left(\partial_x\phi_{\mathbf{R}}(x)\right)^2 + K\left(\partial_x\theta_{\mathbf{R}}(x)\right)^2\right]$$

$$- \frac{\hbar v_s g_J}{2\pi a^2}\sum_{\langle \mathbf{R},\mathbf{R}'\rangle}\int_0^L dx\,\cos\left(\theta_{\mathbf{R}}(x) - \theta_{\mathbf{R}'}(x)\right)$$

$$+ \frac{\hbar v_s g_u}{2\pi a^2}\sum_{\mathbf{R}}\int_0^L dx\,\cos\left(2\phi_{\mathbf{R}}(x) + \delta\pi x\right)$$



Josephson coupling: delocalizes atoms

"Mott" potential: localizes atoms

Methods

- RG

$$
\begin{aligned}
\frac{dg_F}{d\ell} &= \frac{g_J^2}{K}, \\
\frac{dg_J}{d\ell} &= \left(2 - \frac{1}{2K}\right) g_J + \frac{g_J g_F}{2K}, \\
\frac{dg_u}{d\ell} &= (2 \quad K)\, g_u, \\
\frac{dK}{d\ell} &= 4 g_J^2 - g_u^2 K^2,
\end{aligned}
$$

Gives phase boundary

- Mean Field

$$
\begin{aligned}
H_{\text{eff}}^{\text{MF}} = {} & \frac{\hbar v_s}{2\pi} \int_0^L \left[K \left(\partial_x \theta(x)\right)^2 + K^{-1} \left(\partial_x \phi(x)\right)^2 \right] \\
& + 2\rho_0 u_0 \int_0^L dx \, \cos 2\phi(x) - 2 J z_C \sqrt{\mathcal{A}_B \rho_0} |\psi_c| \int_0^L dx \, \cos \theta(x) + J z_C L |\psi_c|^2 .
\end{aligned}
$$

- Mapping on spin chain

$$H^{MF} = J_0 \sum_m \mathbf{S}_m \cdot \mathbf{S}_{m+1} + \mathbf{h} \cdot \sum_m (-1)^m \, \mathbf{S}_m + J z_C L |\psi_c|^2,$$

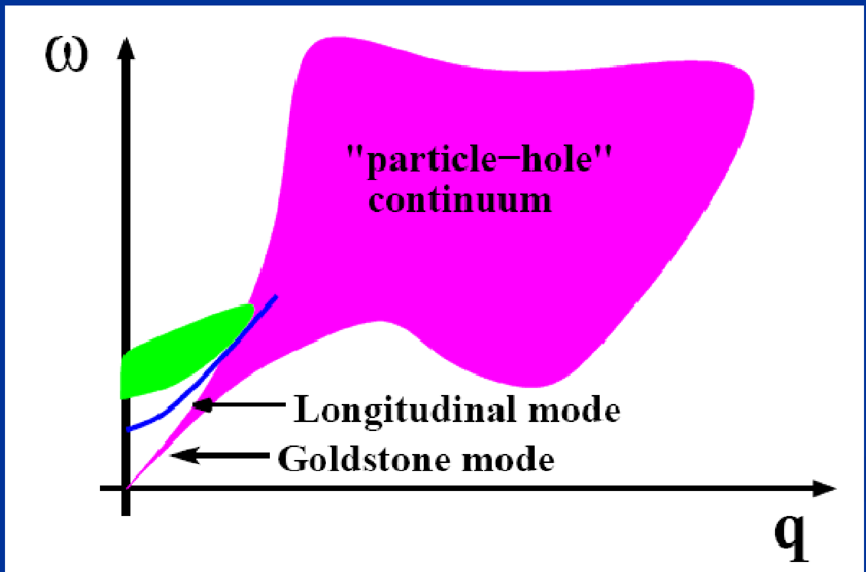
- Critical properties

Universality class of 4d XY model

Phase and amplitude mode

$$\omega_-^2(q, \mathbf{Q}) \simeq (v_\parallel q)^2 + (v_\perp^{(-)}\mathbf{Q})^2,$$

$$\omega_+^2(q, \mathbf{Q}) \simeq \Delta_{(+)}^2 + (v_\parallel q)^2 + (v_\perp^{(+)}\mathbf{Q})^2,$$



$$\mathcal{L}_{\mathrm{GP}}(x,\mathbf{R},t) = \hbar Z_1 \left[\mathrm{i}\Psi_c^*(x,\mathbf{R},t)\partial_t\Psi_c(x,\mathbf{R},t) - \frac{\hbar}{2M}|\partial_x\Psi_c(x,\mathbf{R},t)|^2 - \frac{y_\perp}{2}|\nabla_{\mathbf{R}}\Psi_c(x,\mathbf{R},t)|^2 \right] - \frac{\hbar\lambda}{2}\left(|\Psi_c(x,\mathbf{R},t)|^2 - |\psi_c|^2\right)^2, \quad (43)$$

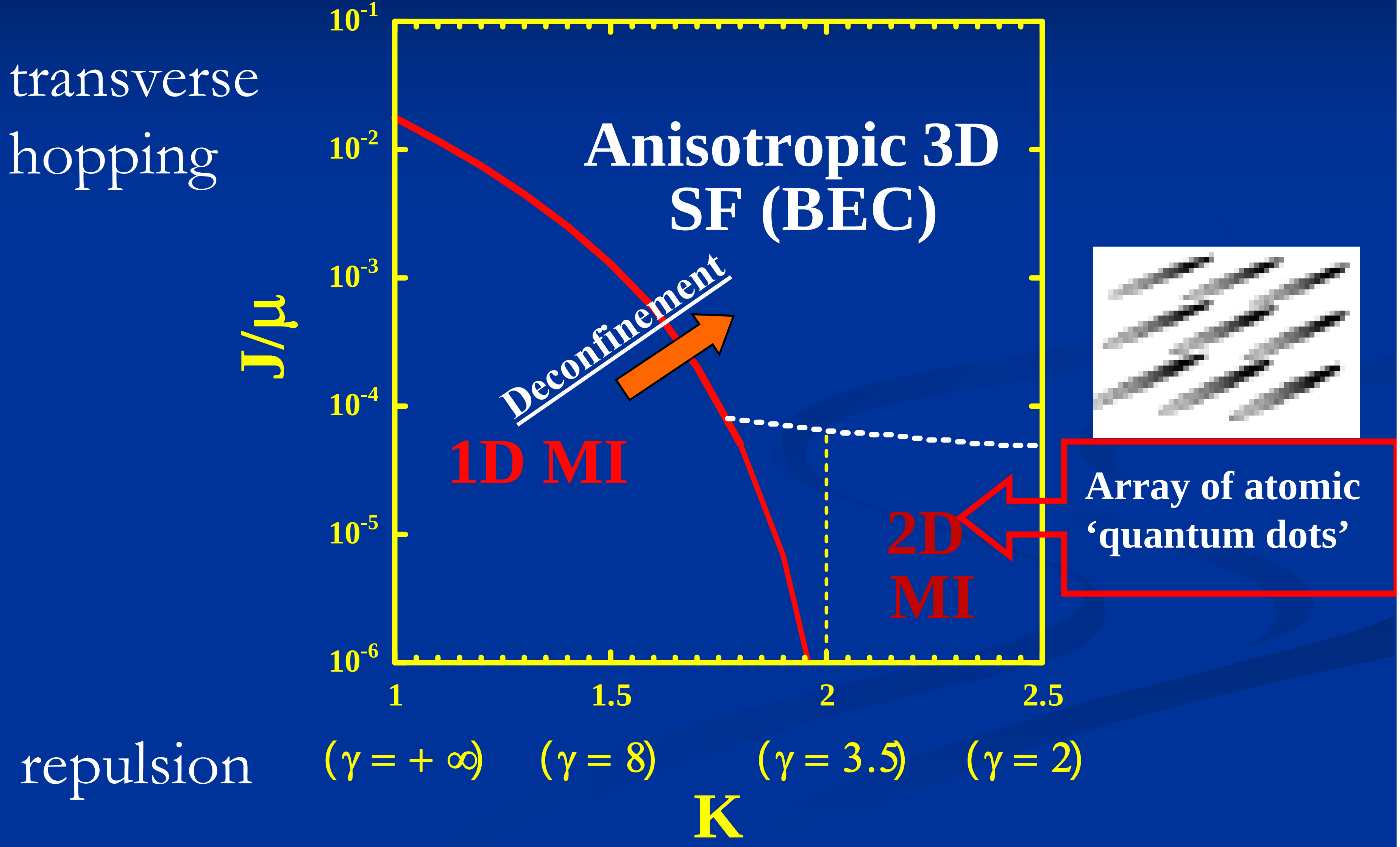
GP: only Goldstone mode

How to recover amplitude mode

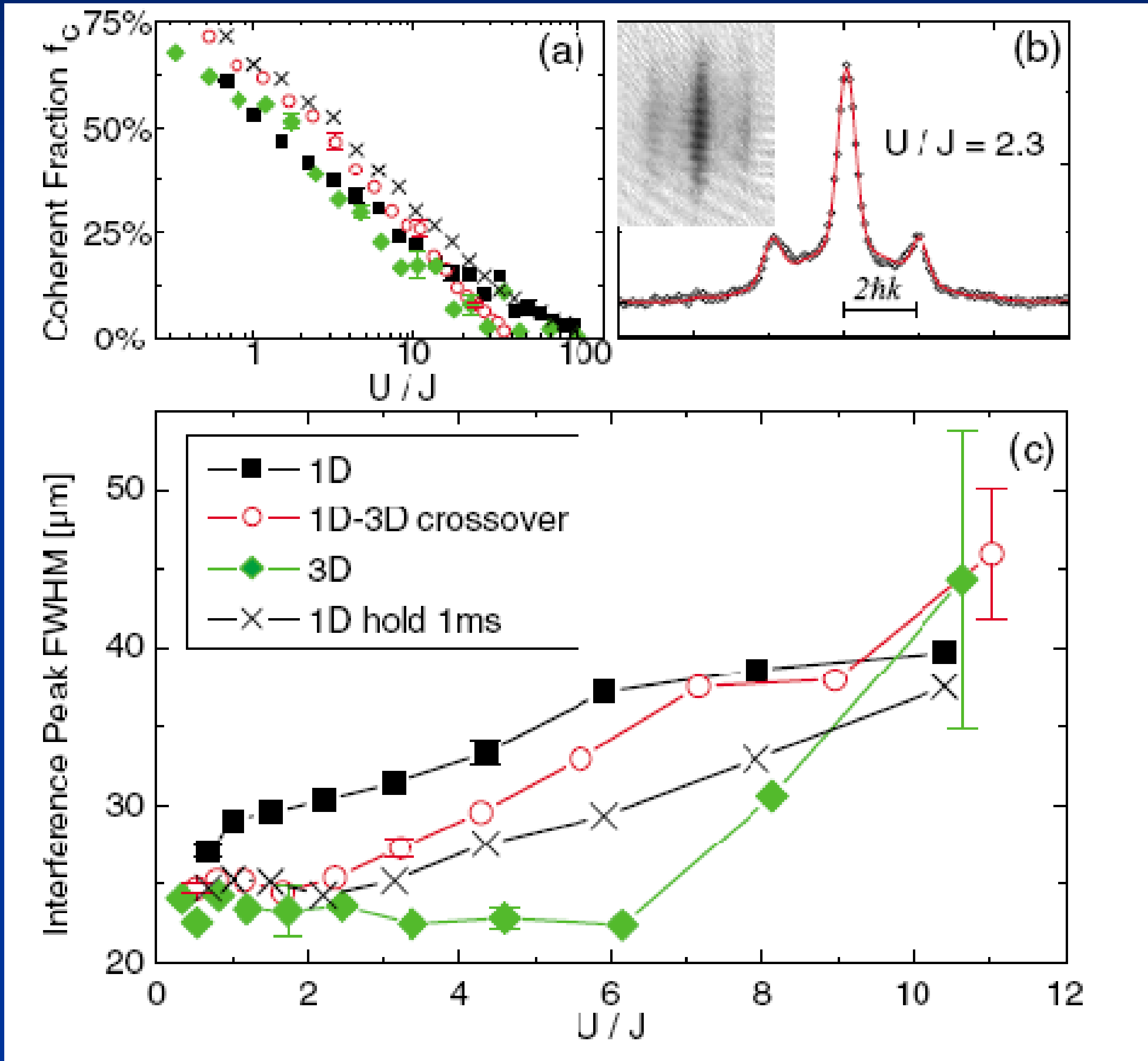
$$\mathcal{L}'_{\mathrm{GL}}(x,\mathbf{R},t) = \mathrm{i}\hbar Z_1 \Psi_c^* \partial_t \Psi_c + \frac{\hbar Z_2}{2}\left[|\partial_t\Psi_c(x,\mathbf{R},t)|^2 - v_\parallel^2|\partial_x\Psi_c(x,\mathbf{R},t)|^2 - v_\perp^2|\nabla_{\mathbf{R}}\Psi_c(x,\mathbf{R},t)|^2 \right] - \frac{\hbar\lambda}{2}\left(|\Psi_c(x,\mathbf{R},t)|^2 - \psi_c^2\right)^2, \quad (47)$$

$$\left[\omega^2 - \omega_+^2(q,\mathbf{Q})\right]\left[\omega^2 - \omega_-^2(q,\mathbf{Q})\right] + \frac{4Z_1^2}{Z_2^2}\omega^2 = 0.$$

Phase diagram



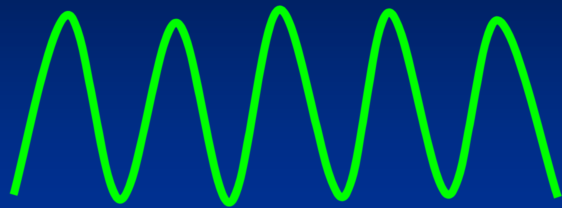
Experiments

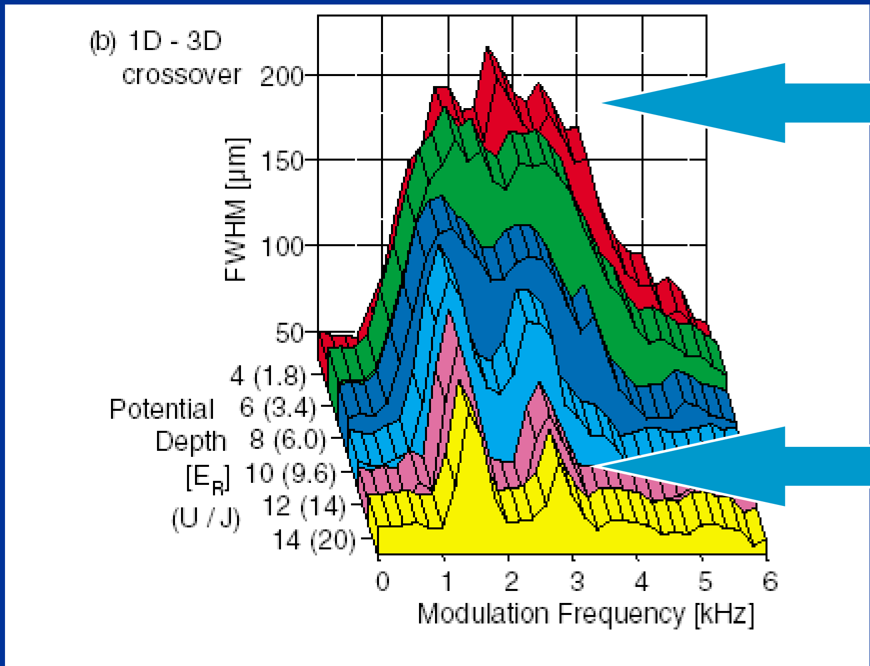


T. Stoferle *et al.*
PRL **92** 130403 (2004)

Shaking of the lattice

T. Stoferle *et al.* PRL **92** 130403 (2004)





3D superfluid

Mott ins.

A. Iucci, M.A. Cazalilla, AF Ho, TG, PRA **73**, 041608R (2006);
C. Kollath, A. Iucci, TG, W. Hofstetter, U. Schollwock, PRL 97 050402 (06)

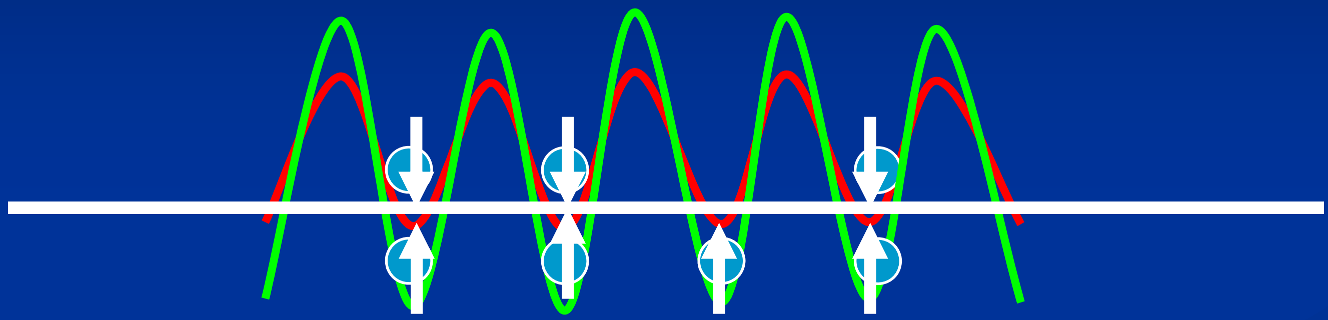
Not so simple !

Fermions

[^{6}Li or ^{40}K]

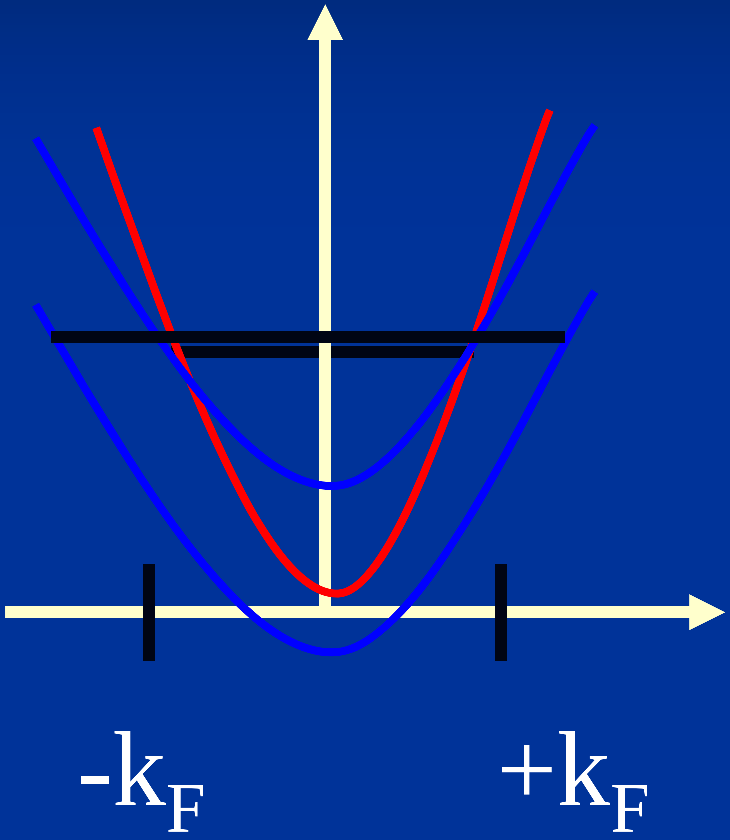
M.A. Cazalilla, A. F. Ho, TG, PRL 95 226402 (2005)

Fermionic tubes



- 2 different hoppings t (optical lattice)
- Local interaction U (Feshbach resonnance)
- $N_\uparrow = N_\downarrow$

$$H = -\sum_{\sigma,m} t_\sigma \left(c^\dagger_{\sigma m} c_{\sigma m+1} + \mathrm{H.c.} \right) + U \sum_m n_{\uparrow m} n_{\downarrow m}$$



- RG

$$\begin{aligned} \dot y^\sigma_{2||} &= r_{-\sigma} y^2_{1\perp}, \qquad \dot y_{2\perp} = -y^2_{1\perp}, \\ \dot y_{1\perp} &= \left(r_\uparrow y^\uparrow_{2||} + r_\downarrow y^\downarrow_{2||} - 2 y_{2\perp} \right) y_{1\perp}, \end{aligned}$$

- Strong coupling

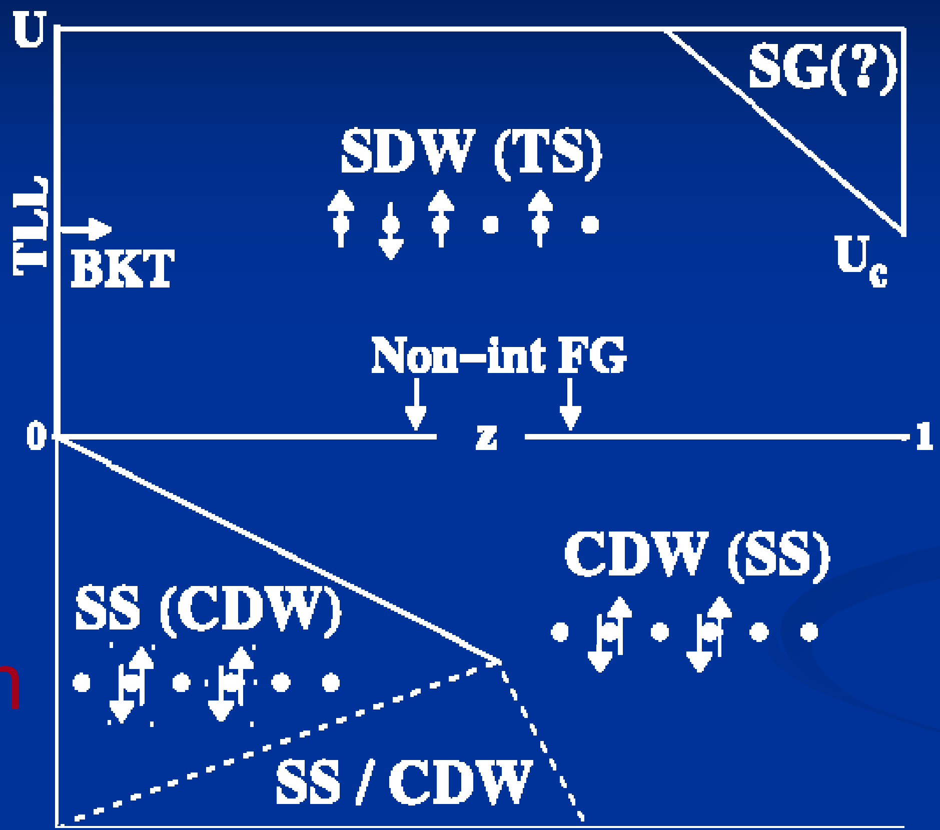
$$H_{\mathrm{XXZ}} = J \sum_m \left[\mathbf{S}_m \cdot \mathbf{S}_{m+1} + \gamma\, S^z_m S^z_{m+1} \right]$$

$$\gamma = (t_\uparrow - t_\downarrow)^2 / 2 t_\uparrow t_\downarrow$$

M.A. Cazalilla, AF Ho, TG, PRL 96 225402 (2005); cond-mat/0604525

1D: phase diagram

repulsion



attraction

Falikov-Kimball

$z = |t_\uparrow - t_\downarrow|/(t_\uparrow + t_\downarrow)$

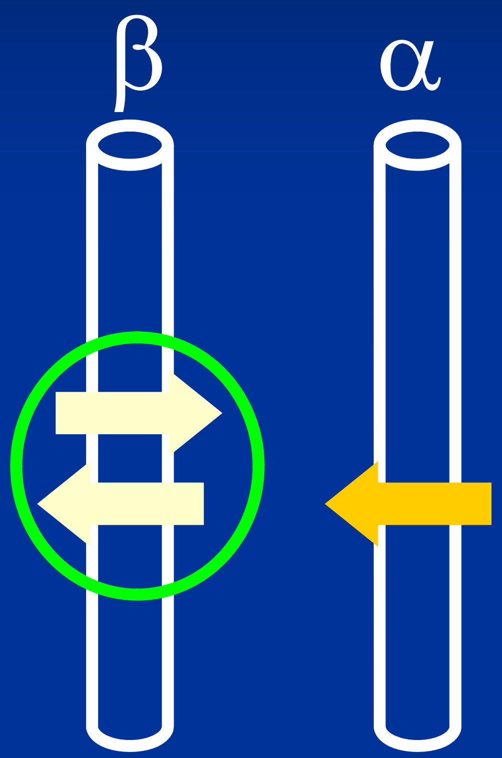
- Spin gap: Raman transitions

$S^\perp(q \sim 0, \omega)$

$\sqrt{(\hbar\omega)^2 - (2\Delta_s)^2}$

Trap is good (for once) !

Coupled tubes with Spin gap



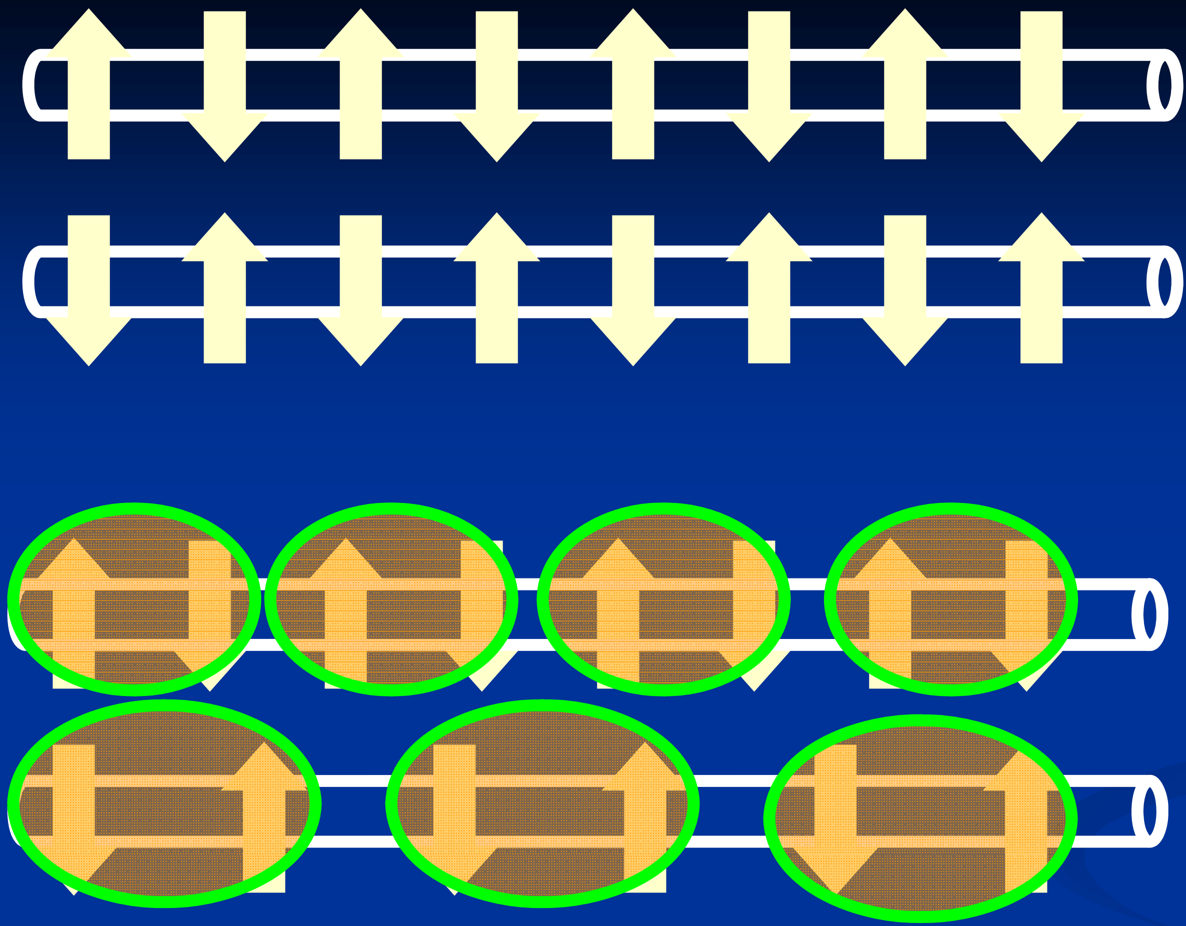
~~$t_\perp \psi^\dagger_{\alpha,\uparrow}(x)\psi_{\beta,\uparrow}(x)$~~

$$t_\perp^2 \psi^\dagger_{\alpha,\uparrow}(x)\psi_{\beta,\uparrow}(x)\psi^\dagger_{\beta,\downarrow}(x)\psi_{\alpha,\downarrow}(x)$$

$$t_\perp^2[\vec{S}_\alpha(x)\cdot\vec{S}_\beta(x)+\rho_\alpha(x)\rho_\beta(x)]$$

$$t_\perp^2 \psi^\dagger_{\alpha,\uparrow}(x)\psi_{\beta,\uparrow}(x)\psi^\dagger_{\alpha,\downarrow}(x)\psi_{\beta,\downarrow}(x)$$

$$t_\perp^2[O^\dagger_{\alpha,\mathrm{SU}}O_{\beta,\mathrm{SU}}]$$



AF Order

μ_1

μ_2

Triplet superconductivity (repulsive interactions)

Low dimensions

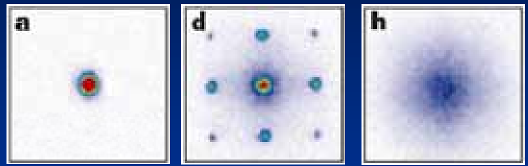
- Optical Lattices: possibility to study crossover from one to three dimensional physics
- Boson tubes: deconfinement transition from 1D Mott insulator to 3D superfluid
- Fermions tubes: spin gap due to spin-dependent hopping;
- triplet superconductivity for repulsive interactions

- Shaking of lattice. Efficient measurement to probe phases; efficient theory to compare to.
- Effects of trap
- Out of equilibrium

But to observe all that....

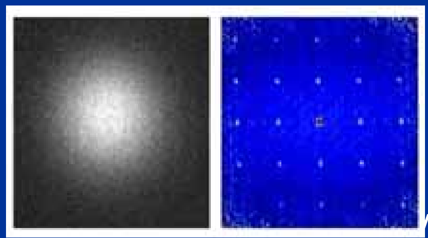
Probes would be good..

time-of-flight measurement
-> momentum distribution



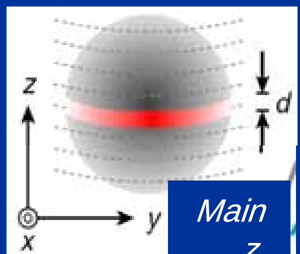
München

noise measurement:
-> density-density correlations



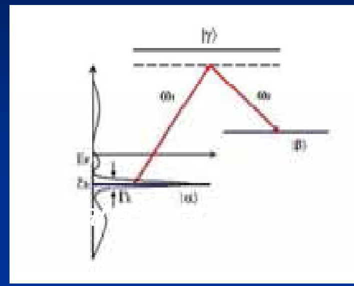
Mainz

microwave spin-changing
transitions
density spatially resolved

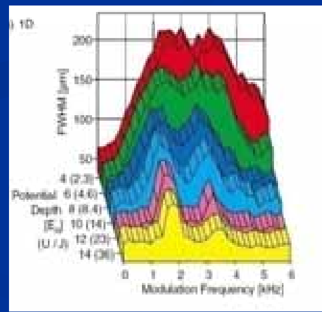


Main
z

proposed: Raman spectroscopy
->Green's function, Fermi surface

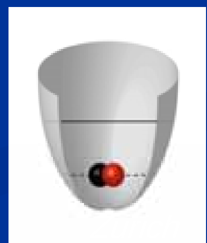


periodic lattice modulation

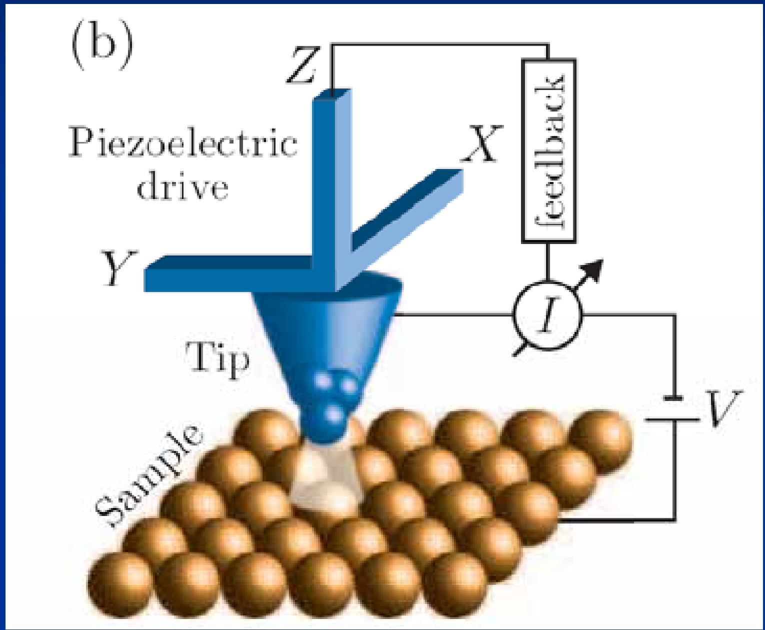


Zurich

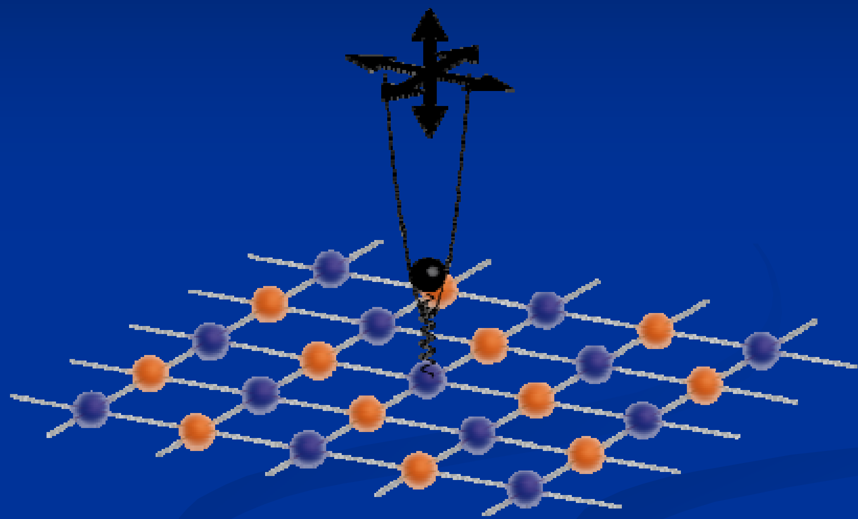
molecule formation
binding energy
doubly occupied sites



Need local probes !



STM

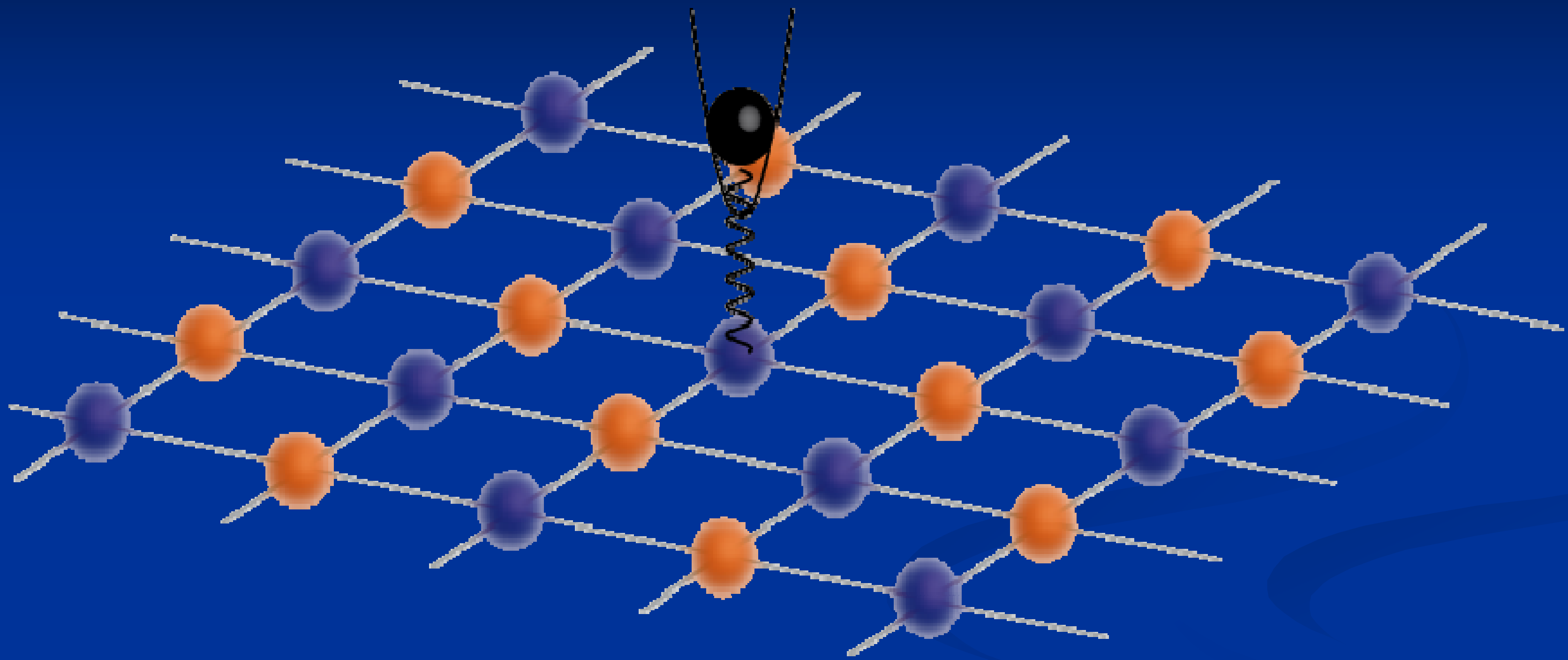


CAT

C. Kollath, M. Koehl, TG cond-mat/0704.1283; physics world (2007).

Conclusions

- Cold atoms/condensed matter: complementary
- Cold atoms: quantum simulators
- Tunability and local interactions. Ideal to explore low dimensional physics.
- Inhomogeneous phases
- Probes



The sky is the limit !

Let’s have fun !