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Rapidly rotating atomic gases

Nigel Cooper
Cavendish Laboratory, University of Cambridge

Strada Costiera 11, 34014 Trieste, Italy - Tel. +39 040 2240 111; Fax +39 040 224 163 - sci_info@ictp.it, www.ictp.it

Rapidly Rotating Cold Atomic Gases

Nigel Cooper
Theory of Condensed Matter Group,
Cavendish Laboratory, University of Cambridge

School on Cold Atomic Gases
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Mike Gunn, Nicola Wilkin, Nick Read, Stavros Komineas, Ed Rezayi, Steve Simon, Kareljan Schoutens, Miguel Cazalilla, Duncan Haldane, Gunnar Möller.

EPSRC
Engineering and Physical Sciences
Research Council

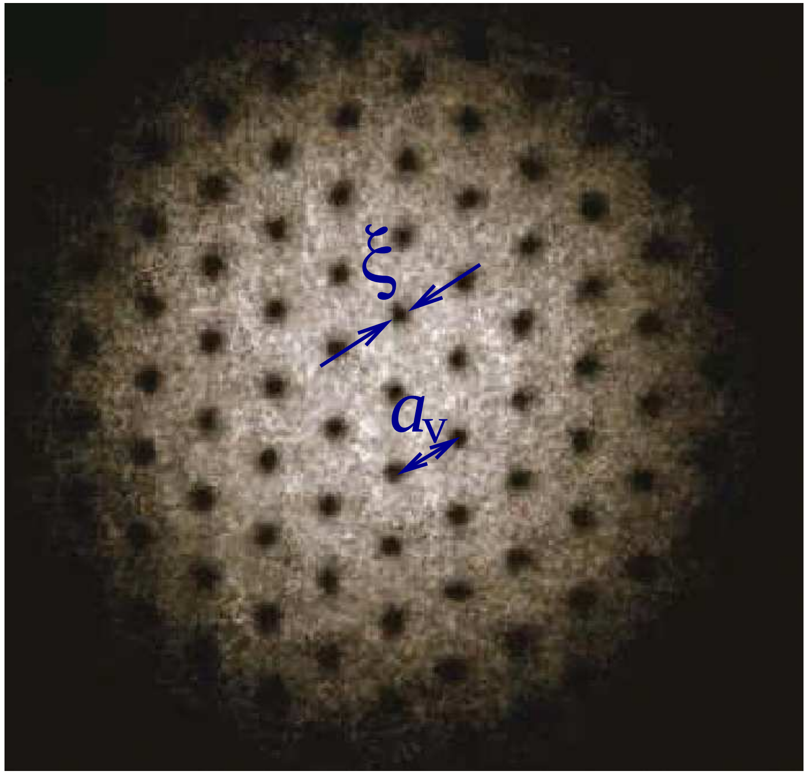
Outline

- Rapidly Rotating Atomic Bose Gases
- Mean-Field Theory
- Quantum Fluctuations & Quantum Melting
- Strongly-Correlated Phases

(• Rapidly Rotating Fermi Gases)

- Summary

Rapidly Rotating Atomic Bose Gases



Healing length, $\xi = \frac{1}{\sqrt{8\pi\bar{n}a_{\rm s}}} \sim 0.5\mu$m

Vortex spacing, $a_{\rm v} = \sqrt{\frac{\hbar}{M\Omega}} \sim 2\mu$m

[Coddington *et al.* [JILA], PRA **70**, 063607 (2004)]

Imaging, Dynamics, Phase imprinting, Multiple components, Optical lattices, Tunable interactions. . .

▷ *Weakly interacting* $\xi \gtrsim a_{\rm v}$ [Wilkin, Gunn & Smith, PRL **80**, 2265 (1998)]

Formulation of the Problem

$$H = \sum_{i=1}^{N} \left[\frac{p_i^2}{2m} + \frac{1}{2} m\omega_\perp^2 (x_i^2 + y_i^2) + \frac{1}{2} m\omega_\parallel^2 z_i^2 \right] + \eta \sum_{i<j} \delta(\vec{r_i} - \vec{r_j}) \qquad \left[\eta = \frac{4\pi\hbar^2 a_s}{m} \right]$$

Rotating Frame: $H_\Omega = H - \vec{\Omega} \cdot \vec{L}$

One-Body Terms

$$\begin{aligned} H_\Omega^{(1)} &= \frac{|\vec{p}|^2}{2m} + \frac{1}{2} m\omega_\perp^2 (x^2 + y^2) + \frac{1}{2} m\omega_\parallel^2 z^2 - \vec{\Omega} \cdot \vec{r} \times \vec{p} \\ &= \frac{|\vec{p} - m\vec{\Omega} \times \vec{r}|^2}{2m} + \frac{1}{2} m(\omega_\perp^2 - \Omega^2)(x^2 + y^2) + \frac{1}{2} m\omega_\parallel^2 z^2 \end{aligned}$$

$$\boxed{q^* \vec{B}^* = 2m\vec{\Omega}}$$

$+$ Interactions

$\eta\bar{n} \ll \hbar\omega_{\|}, \hbar\omega_{\perp} \Rightarrow$ Single particle states restricted to 2D & the lowest Landau level [Wilkin, Gunn & Smith (1998)]

$$\langle x, y | m \rangle \propto z^m e^{-|z|^2/2} \qquad \left[z \equiv (x + iy)/a_{\perp}; a_{\perp} \equiv \sqrt{\frac{\hbar}{m\omega_{\perp}}} \right]$$

What is the groundstate at given N and $L = \sum_{i=1}^{N} m_i$?

Kinetic + Potential energies: $E = \left(\hbar\omega_{\perp} + \frac{1}{2}\hbar\omega_{\|}\right) N + \hbar\omega_{\perp} L$

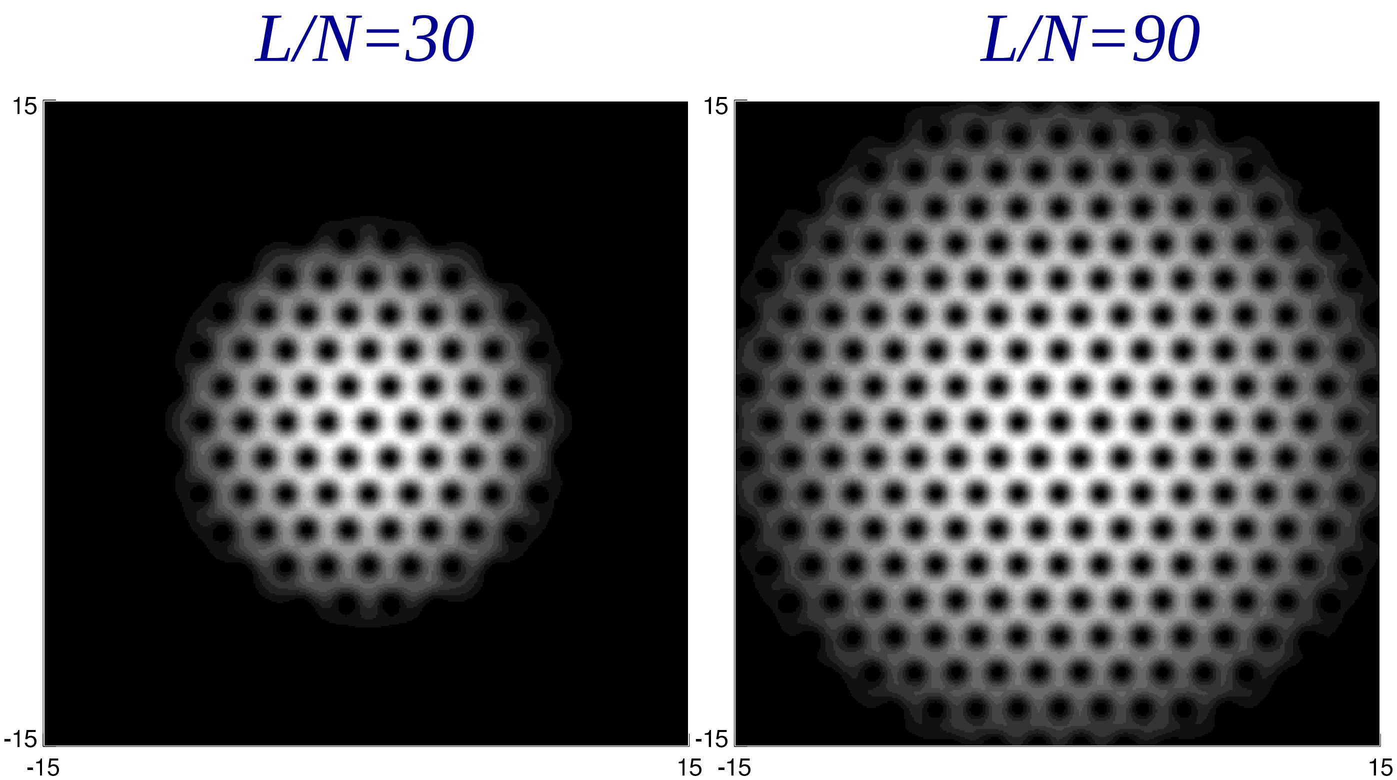
The groundstate is completely determined by interactions.

Gross-Pitaevskii Mean-Field Theory

[Butts & Rohksar, Nature **397**, 327 (1999)]

$$\Psi(\{\vec{r_i}\}) = \prod_{i=1}^{N} \psi(\vec{r_i}) \qquad \psi(\vec{r_i}) \propto \prod_{\alpha=1}^{N_\mathrm{v}} (z - Z_\alpha) \quad \left[\times e^{-|z|^2/2}\right]$$

Minimise the expectation value of the interaction energy, for fixed N and L.



[NRC, Komineas & Read, PRA **70**, 033604 (2004)]

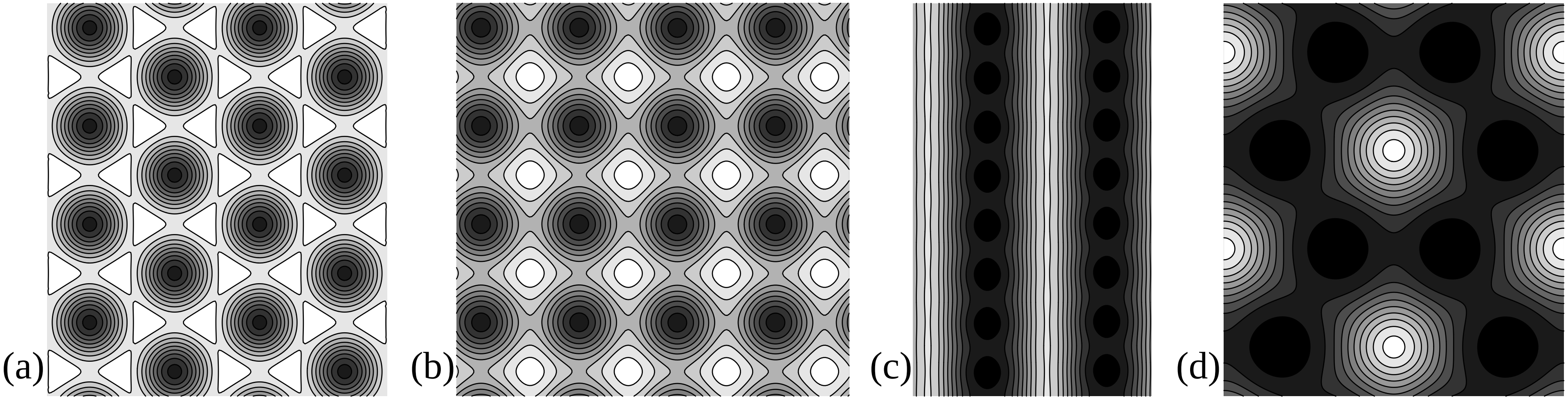
Aside: Vortex Lattices in Dipolar Bose Gases

[NRC, E.H. Rezayi & S.H. Simon, PRL **95**, 200402 (2005)]

$a_\parallel / a_\perp \ll 1$ $\gamma \equiv \frac{V_2}{V_0}$

$$V_0 = \sqrt{\frac{2}{\pi}} \frac{\hbar^2 a_s}{M a_\perp^2 a_\parallel} + \sqrt{\frac{2}{\pi}} \frac{C_d}{a_\perp^2 a_\parallel} - \sqrt{\frac{\pi}{2}} \frac{C_d}{a_\perp^3}$$

$$V_2 = \sqrt{\frac{\pi}{2}} \frac{1}{8} \frac{C_d}{a_\perp^3}$$



$\gamma = 0 \rightarrow 0.20$ $0.20 \rightarrow 0.24$ $0.24 \rightarrow 0.60$ $0.60 \rightarrow$

Stripe crystal "Bubble" crystals (q vortices per bubble)

Vortex Lattice vs. "Vortex Liquid"

[NRC, Wilkin & Gunn, PRL **87**, 120405 (2001)]

The Filling Factor

FQHE: $\nu = n_{\mathrm{e}} \frac{h}{eB}$

Here: $\nu = n_{2\mathrm{d}} \frac{h}{q^* B^*} = n_{2\mathrm{d}} \frac{h}{2m\Omega}$

$$\boxed{\nu = \frac{n_{2\mathrm{d}}}{n_{\mathrm{v}}} \simeq \frac{N}{N_{\mathrm{v}}}}$$

Quantum fluctuations

[Fetter, Phys. Rev. **162**, 143 (1967); Haldane & Wu, PRL **55**, 2887 (1985)]

Magnus force dynamics for a 2D vortex:

$$\begin{aligned} \rho_s \kappa_0 \dot{Y} + F_X^{\mathrm{ext}} &= 0 \\ -\rho_s \kappa_0 \dot{X} + F_Y^{\mathrm{ext}} &= 0 \end{aligned} \qquad \left[\rho_s \kappa_0 = (n_{\mathrm{2d}} m)\frac{h}{m} = n_{\mathrm{2d}} h\right]$$

Lagrangian:

$$L = -n_{\mathrm{2d}} h \, \dot{X} Y - V(X,Y) \qquad \left[\vec{F}^{\mathrm{ext}} = -\vec{\nabla} V\right]$$

Quantise:

$$\begin{aligned} \Pi_X &\equiv \frac{\partial L}{\partial \dot{X}} = -n_{\mathrm{2d}} h \, Y \\ [\hat{X}, \hat{\Pi}_X] = i\hbar &\Rightarrow [\hat{X}, \hat{Y}] = -\frac{i}{2\pi n_{\mathrm{2d}}} \\ \Delta X \Delta Y &\geq \frac{1}{4\pi n_{\mathrm{2d}}} \\ \Delta X^2 + \Delta Y^2 &\geq \frac{1}{2\pi n_{\mathrm{2d}}} \end{aligned}$$

Lindemann criterion for (quantum) melting:

$$\sqrt{\Delta X^2 + \Delta Y^2} \sim \frac{1}{\sqrt{n_{\rm 2d}}} \quad \geq \quad c_{\rm L} \times a_{\rm v} \sim c_{\rm L} \frac{1}{\sqrt{n_{\rm v}}}$$

$\Rightarrow$ Filling factor $\quad \nu \equiv \dfrac{n_{\rm 2d}}{n_{\rm v}} \quad \leq \quad \nu_c \quad \simeq 14$

Exact diagonalisations, $\nu_c \simeq 6$ [NRC, Wilkin & Gunn PRL **87**, 120405 (2001)]

Strongly Correlated Regime, $\nu < \nu_c$

Laughlin State

[Wilkin, Gunn & Smith, PRL **80**, 2265 (1998)]

$$\Psi_{\mathrm{L}}(\{z_i\}) = \prod_{i<j}^{N} (z_i - z_j)^2 \qquad \left[\times e^{-\sum_i |z_i|^2/2}\right]$$

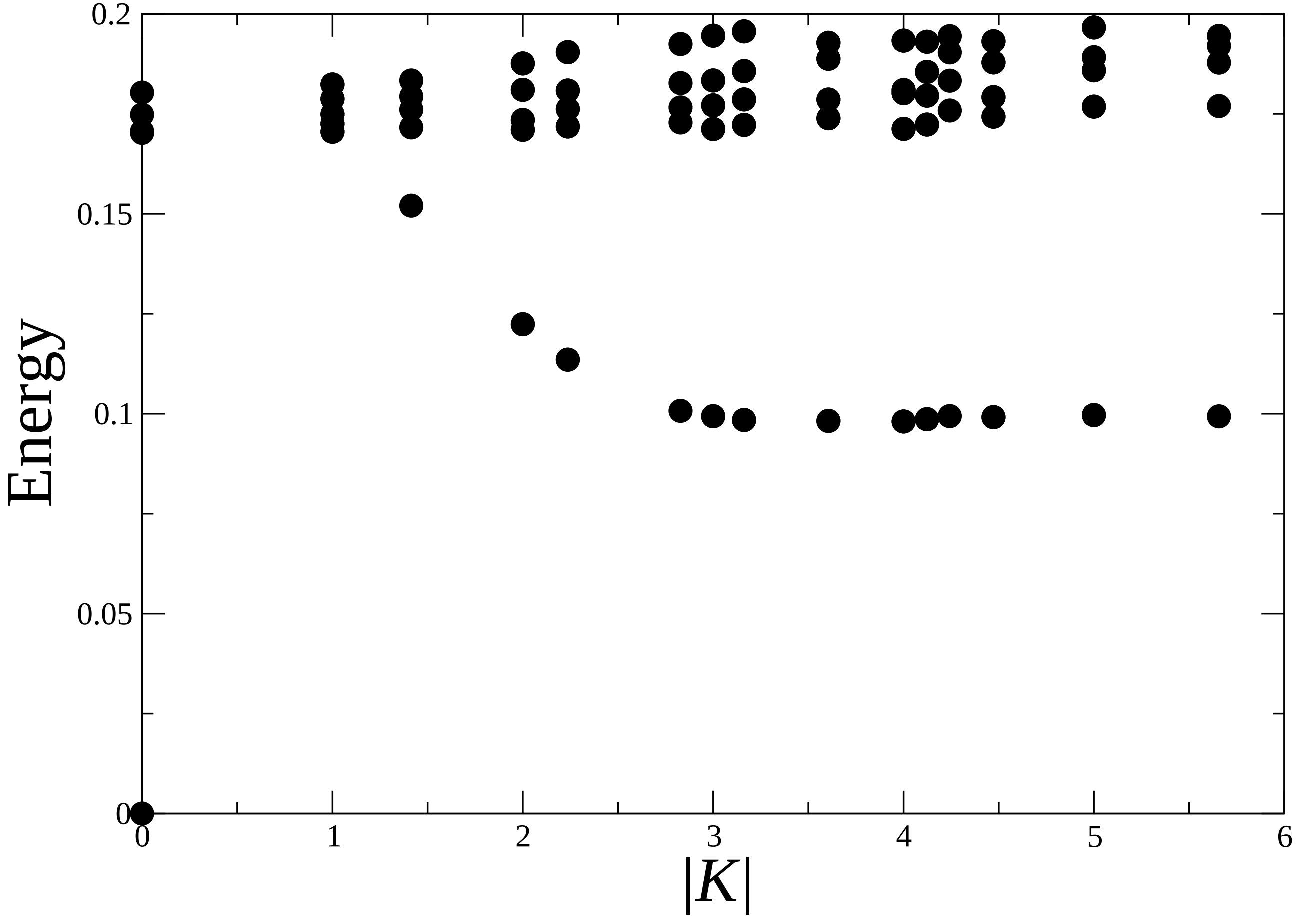
Exact groundstate with total angular momentum $L = N(N-1)$.

Describes a filling factor $\nu = \frac{N}{N_{\mathrm{v}}} = 1/2$.

An incompressible fluid – gapped collective excitations in the bulk.
Particle-like excitations have: fractional particle number;
fractional statistics ("anyons").

[See articles by Laughlin & Haldane in "The Quantum Hall Effect", eds. Prange & Girvin]

$\nu = 1/2$



$(N = 8, N_{\mathrm{v}} = 16)$

Composite Fermion (Hierarchy) States

[NRC & Wilkin, PRB **60**, R16279 (1999)]

$$\boxed{\text{Composite Fermion} = \text{Boson} + \text{Vortex}}$$

[Girvin, Read, Zhang, Jain...]

$$\Psi(\{\vec{r}_k\}) = \mathcal{P}_{\text{LLL}} \prod_{i<j} (z_i - z_j) \psi_{\text{CF}}(\{\vec{r}_k\})$$

Treat the composite fermions as *non-interacting* $\Rightarrow \nu = \frac{p}{p \pm 1}$

Gapped FQH fluids at $\nu = (1/2), 2/3, 3/4$.

[Regnault & Jolicoeur, PRL **91**, 030402 (2003), PRB **69**, 235309 (2004); Chang *et al.*, PRA **72**, 013611 (2005)]

Moore-Read "Pfaffian", and Read-Rezayi "Parafermion" states, $\nu = k/2$

[Moore & Read, Nucl. Phys. B **360**, 362 (1991); Read & Rezayi, PRB **59**, 8084 (1999)]

$$\Psi^{(k)}(\{z_i\}) = \mathcal{S}\left[\prod_{i<j\in A}^{N/k}(z_i - z_j)^2 \prod_{l<m\in B}^{N/k}(z_l - z_m)^2 \ldots\right]$$

[Cappelli *et al.*, Nucl. Phys. B **599**, 499 (2001)]

$$\nu^{(k)} = \frac{k}{2}$$

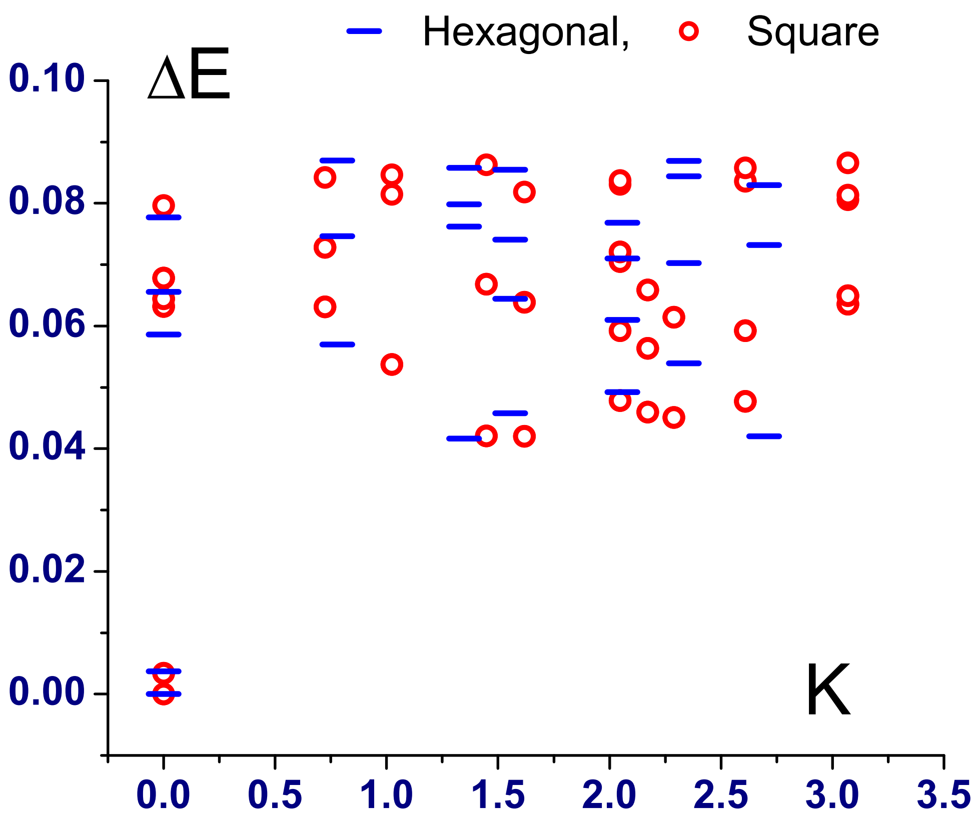
$k = 1$: Laughlin state ($\nu = 1/2$)
$k = 2$: Moore-Read ("Pfaffian") state ($\nu = 1$)
$k \geq 3$: Read-Rezayi ("Parafermion") states ($\nu = k/2$).

Incompressible states, with quasiparticle excitations which obey *non-abelian exchange statistics*.

Incompressible Liquid Phase at $\nu = 3/2$

[E. H. Rezayi, N. Read & NRC, PRL **95**, 160404 (2005)]

A small amount of dipolar interaction improves the overlap with the $k = 3$ Read-Rezayi state.



$N = 18, N_{\mathrm{v}} = 12$
$\gamma = V_2/V_0 = 0.380$

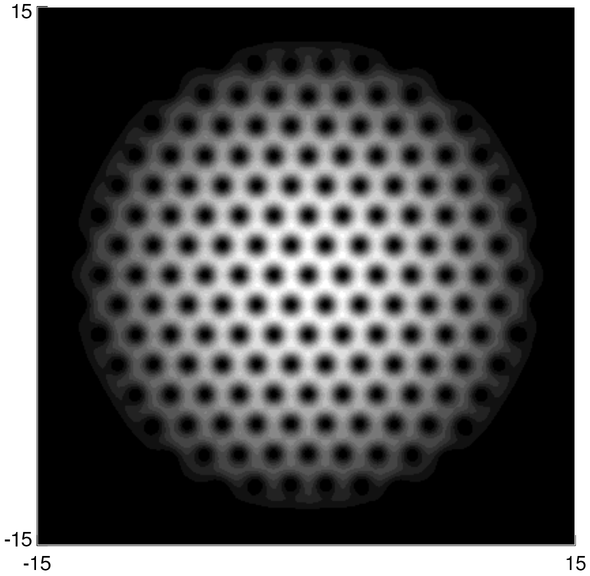
The two-fold degeneracy of the groundstate on the torus is expected for the RR state.

Rapidly Rotating Bosons: Overview of Results

The Filling Factor $\boxed{\nu = \frac{n_{2\mathrm{d}}}{n_{\mathrm{v}}} \simeq \frac{N}{N_{\mathrm{v}}}}$ [NRC, Wilkin & Gunn, PRL **87**, 120405 (2001)]

$\nu > \nu_{\mathrm{c}} \simeq 6$: Vortex Lattice

Gross-Pitaevskii mean-field theory accurate.



[Butts & Rohksar, Nature **397**, 327 (1999); . . .
NRC, Komineas & Read, PRA **70**, 033604 (2004); . . .]

$\nu < \nu_{\mathrm{c}} \simeq 6$: Vortex Lattice "melts"

Laughlin, Hierarchy/Composite Fermion, Moore-Read and Read-Rezayi states.
[Wilkin, Gunn & Smith, PRL **80**, 2265 (1998); NRC & Wilkin, PRB **80**, 16279 (1999); NRC, Wilkin & Gunn, PRL **87**, 120405 (2001); Regnault & Jolicoeur, PRL **91**, 030402 (2003); Chang *et al.*, PRA **72**, 013611 (2005)]

Competing "smectic" phase at intermediate ν. [NRC & Rezayi, PRA **75**, 013627 (2007)]

Experimental Status

$\mu/(2\hbar\omega_\perp) \lesssim 1 \Rightarrow$LLL
$\nu \gtrsim 500 \Rightarrow$vortex lattice

[Schweikhard *et al.*[JILA], PRL **92**, 040404 (2004)]

It will require further special efforts to achieve $\nu \lesssim 10$.

The challenge: the interaction scale at $\nu \sim 1$ is small $\sim \frac{\hbar^2 a_\mathrm{s}}{M}\bar{n} \sim \frac{a_\mathrm{s}}{a_\|}\hbar\omega_\perp$

Artificial gauge fields

Lattice models: [Jaksch & Zoller, NJP **5**, 56 (2003); Mueller, PRA **70**, 041603 (2004); Sørensen, Demler & Lukin, PRL **94**, 086803 (2005)]

Continuum: [Juzeliūnas, Ruseckas, Öhberg & Fleischhauer, PRA **72**, 025602 (2006)]

Summary

- Rapidly rotating atomic BECs allow access to a regime of high vortex density, $\xi \gtrsim a_{\mathrm{v}}$. The atoms are restricted to states in the lowest Landau level.

- For contact interactions, mean-field theory predicts a smooth crossover from $\xi \lesssim a_{\mathrm{v}}$ to $\xi \gtrsim a_{\mathrm{v}}$ (triangular vortex lattice).

- The parameter controlling the degree of quantum fluctuations of the vortices is the filling factor $\nu = N/N_{\mathrm{v}}$.

- The vortex lattice is stable for $\nu > \nu_c \simeq 6$; for $\nu < \nu_c \simeq 6$, the vortex lattice is replaced by strongly-correlated states. These include the conventional Laughlin and hierarchy/composite fermion states, as well as non-abelian Moore-Read and Read-Rezayi states.