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Summer School on Novel Quantum Phases and Non-Equilibrium Phenomena in Cold Atomic Gases

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Dissipationless Higgs Boson in Fermionic condensates

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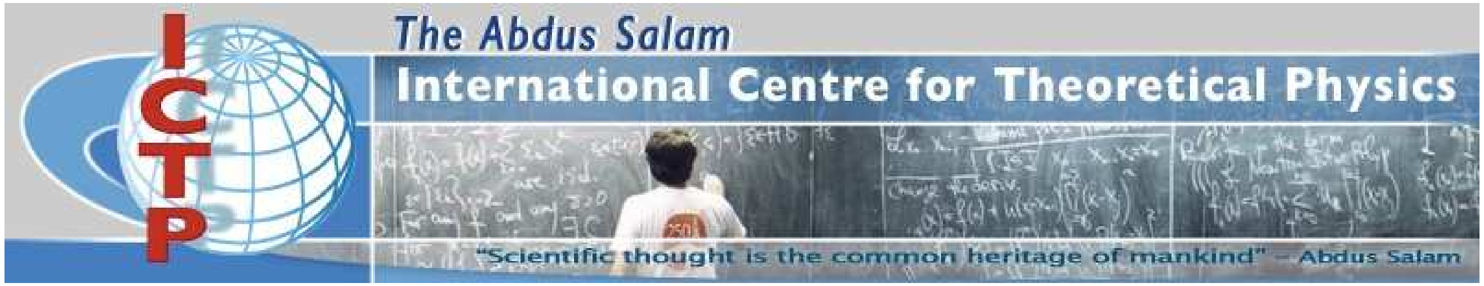
Dissipationless Higgs Boson in Fermionic Condensates

7 Sept. 2007



Outline

- Collective modes in Bose and Fermi superfluids
- Amplitude mode (Higgs boson): different approaches
- Dynamical regimes of pairing: role of initial states
- The dissipationless Higgs mode
- Relevance to experiments
- Open questions and future directions
- Conclusion

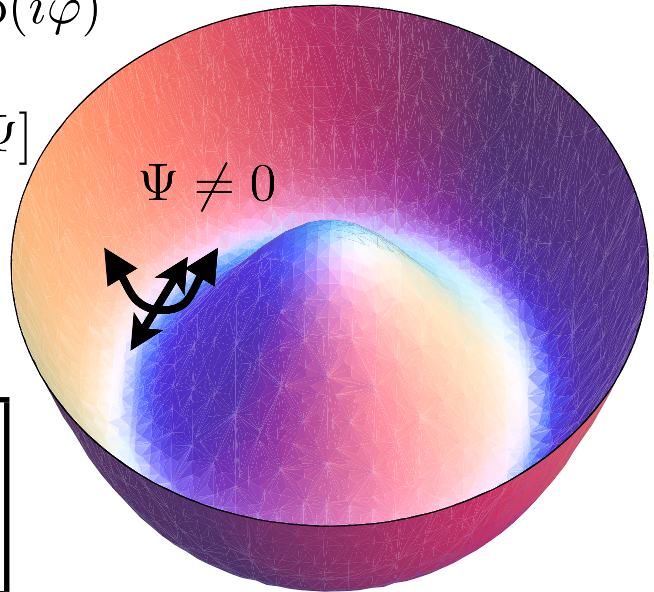
Collective modes in superfluids $T = 0$

complex order parameter $\Psi = |\Psi| \exp(i\varphi)$

Landau-Ginzburg free energy $F = F[\Psi]$

Ground state breaks $U(1)$ symmetry

Phase and amplitude modes



φ azimuthal angle

	Gapless Goldstone mode	Massive Higgs mode
Bosons	Sound	no
Fermions	Sound (Bogoliubov-Anderson mode)	Amplitude mode

Phase is conjugate to the number of particles

Higgs Boson

Charged systems: sound is gapped (plasmon)

$\Psi = \sqrt{n_s} \exp(i\varphi)$ $\quad$ $\Psi = \langle \hat{\psi}_\uparrow(r) \hat{\psi}_\downarrow(r) \rangle = |\Delta| \exp(i\varphi)$

Probing excitation spectrum of Fermi superfluids

$$\mathcal{H} = \sum_{\mathbf{p}\sigma} \xi_{\mathbf{p}} a^\dagger_{\mathbf{p}\sigma} a_{\mathbf{p}\sigma} - \lambda(t) \sum_{\mathbf{pqd}} a^\dagger_{\mathbf{p+d/2}\uparrow} a^\dagger_{-\mathbf{p+d/2}\downarrow} a_{-\mathbf{q+d/2}\downarrow} a_{\mathbf{q+d/2}\uparrow}$$

$$\xi_{\mathbf{p}} = \frac{\mathbf{p}^2}{2m} - \mu \qquad \Delta(t) = \lambda \sum_{\mathbf{p}} \langle a_{\mathbf{p}\uparrow}(t) a_{-\mathbf{p}\downarrow}(t) \rangle$$

- Single-particle excitations, by RF spectroscopy
- Collective modes
  - Bogoliubov-Anderson mode, by perturbation of the density $\delta\mu(t)$
  - Amplitude mode (no experiments so far), by changing the coupling at Feshbach resonance

$$\lambda(t) = \lambda_s \theta(-t) + \lambda_f \theta(t)$$

Formulation of the problem

$$\mathcal{H} = \sum_{\mathbf{p}\sigma} \xi_{\mathbf{p}} a^\dagger_{\mathbf{p}\sigma} a_{\mathbf{p}\sigma} - \lambda(t) \sum_{\mathbf{pqd}} a^\dagger_{\mathbf{p+d/2}\uparrow} a^\dagger_{\mathbf{-p+d/2}\downarrow} a_{\mathbf{-q+d/2}\downarrow} a_{\mathbf{q+d/2}\uparrow}$$

$$\xi_{\mathbf{p}} = \mathbf{p^2}/2m - \mu$$

$$\lambda(t) = \lambda_s \theta(-t) + \lambda_f \theta(t)$$

- Initial condition: BCS or normal state
- Dynamics induced by non-adiabatic switching of attractive interactions
- Goal: Time-dependence of the pairing amplitude

$$\Delta(t) = \lambda(t) \sum_{\mathbf{p}} \langle a_{\mathbf{p}\uparrow}(t) a_{-\mathbf{p}\downarrow}(t) \rangle$$

Formulation of the problem (cont.)

$$\mathcal{H} = \sum_{\mathbf{p}\sigma} \xi_{\mathbf{p}} a^\dagger_{\mathbf{p}\sigma} a_{\mathbf{p}\sigma} - \lambda(t) \sum_{\mathbf{pqd}} a^\dagger_{\mathbf{p+d/2}\uparrow} a^\dagger_{\mathbf{-p+d/2}\downarrow} a_{\mathbf{-q+d/2}\downarrow} a_{\mathbf{q+d/2}\uparrow}$$

$$\xi_{\mathbf{p}} = \mathbf{p^2}/2m - \mu$$

- Time-dependent Ginzburg-Landau approach

Applicability: $T_c - T \ll T_c$, $\tau_\epsilon \ll \tau_\Delta$

e.g. gapless superconductors

Abrahams, Tsuneto 1966,
Gor'kov, Eliashberg 1968

Quasiparticles are always in equilibrium, i.e. follow the pairing amplitude

- Kinetic equation approach

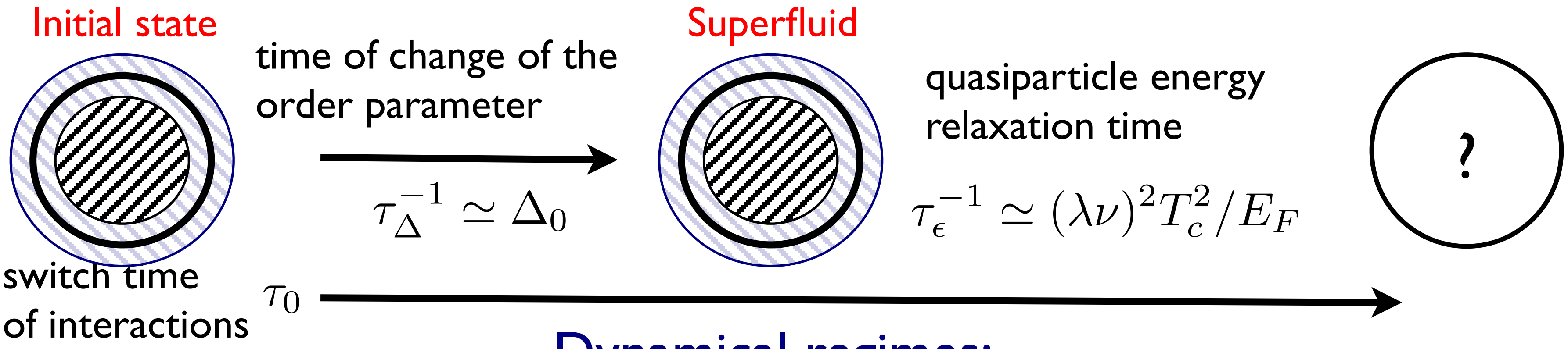
Applicability: $\hbar\omega \ll \Delta^2/T_c \qquad \hbar\omega \ll \Delta, \quad q\xi \ll 1$

Aronov, Galperin, Gurevich, Kozub 1981

Pairing amplitude follows the quasi-particles

Both approaches fail in our case

Time scales in the problem



Dynamical regimes:

- Adiabatic regime (equilibrium with the instantaneous Hamiltonian) $\tau_\Delta, \tau_\epsilon \ll \tau_0$
- Tuning the initial state (non-equilibrium) $\tau_\Delta \ll \tau_0 \ll \tau_\epsilon$
- TDGL regime $\tau_\epsilon \ll \tau_\Delta$

NEW regime possible in atomic gases

$$\tau_0 \ll \tau_\Delta \ll \tau_\epsilon$$

Dissipationless dynamics

Bogoliubov-de Gennes equations

Reduced BCS Hamiltonian

$$\mathcal{H} = \sum_{\mathbf{p}\sigma} \xi_{\mathbf{p}} a^\dagger_{\mathbf{p}\sigma} a_{\mathbf{p}\sigma} - \lambda(t) \sum_{\mathbf{pq}} a^\dagger_{\mathbf{p}\uparrow} a^\dagger_{-\mathbf{p}\downarrow} a_{-\mathbf{q}\downarrow} a_{\mathbf{q}\uparrow}$$

zero-momentum pairs - spatially uniform dynamics

Truncated Hilbert space: BCS wave function

$$|\Phi(t)\rangle = \prod_{\mathbf{p}} \left(u_{\mathbf{p}}(t) + v_{\mathbf{p}}(t) a^+_{\mathbf{p}\uparrow} a^+_{-\mathbf{p}\downarrow} \right) |0\rangle$$

$$i\partial_t \begin{pmatrix} u_{\mathbf{p}} \\ v_{\mathbf{p}} \end{pmatrix} = \begin{pmatrix} \xi_{\mathbf{p}} & \Delta \\ \Delta^* & -\xi_{\mathbf{p}} \end{pmatrix} \begin{pmatrix} u_{\mathbf{p}} \\ v_{\mathbf{p}} \end{pmatrix}$$

Self-consistency condition

$$\Delta(t) = \lambda(t) \sum_{\mathbf{p}} u_{\mathbf{p}}(t) v^*_{\mathbf{p}}(t)$$

$\tau_0 \ll t \ll \tau_\epsilon$ $\quad$ $\nabla_{\mathbf{r}} \Delta = 0$

↘

Dissipationless, spatially uniform and non-linear dynamics

Pseudospin representation of BCS problem

P.W. Anderson, 1958

$$\mathcal{H} = -\sum_{\mathbf{p}} 2\epsilon_{\mathbf{p}} s^z_{\mathbf{p}} - \lambda(t) \sum_{\mathbf{p},\mathbf{q}} s^-_{\mathbf{p}} s^+_{\mathbf{q}}$$

Mean-field $\mathbf{r_p} = \mathbf{2}\langle \mathbf{s_p} \rangle$ unit vector

$$\mathbf{r_p} = (r_1, r_2, r_3)_{\mathbf{p}} \qquad r_1 + ir_2 = 2u_{\mathbf{p}} v^*_{\mathbf{p}} \qquad r_3 = |u_{\mathbf{p}}|^2 - |v_{\mathbf{p}}|^2$$

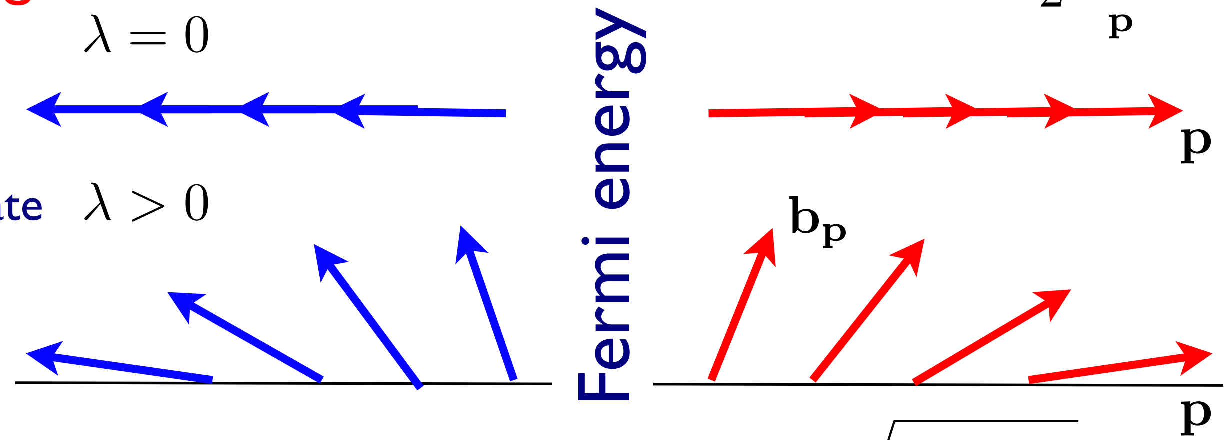
Mean-field dynamics (Bogoliubov - De Gennes equations)

Bloch equations: $$\frac{d\mathbf{r_p}}{dt} = 2\mathbf{b_p} \times \mathbf{r_p}$$

Effective 'magnetic field' $\mathbf{b_p} = -(\Delta', \Delta'', \xi_{\mathbf{p}})$ $$\Delta = \frac{\lambda}{2} \sum_{\mathbf{p}} (r_1 + ir_2)_{\mathbf{p}}$$

Normal state $\lambda = 0$

Superfluid state $\lambda > 0$



Excited pair states - spin flip $$E_{\mathbf{p}} = 2\sqrt{\Delta_0^2 + \epsilon_{\mathbf{p}}^2}$$

Non-equilibrium initial state without dynamics

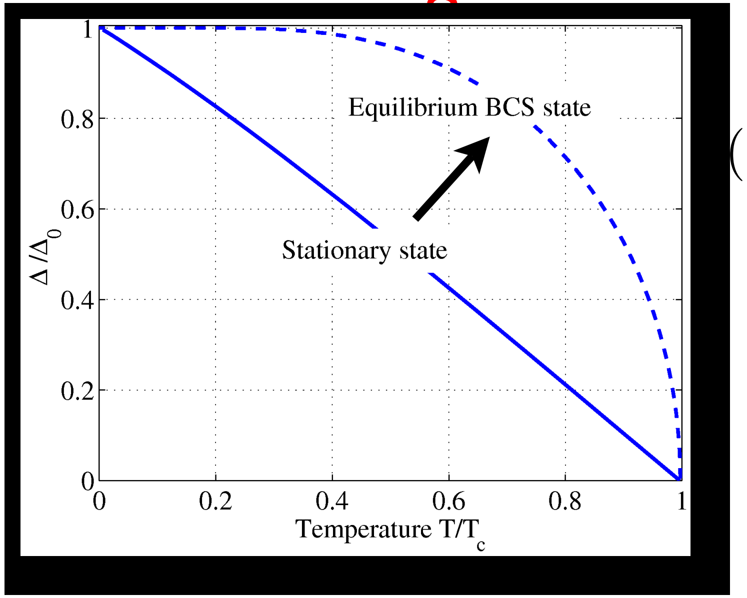
$$\tau_\Delta \ll \tau_0 \ll \tau_\epsilon$$

Bloch equations:

$$\frac{d\mathbf{r_p}}{dt} = 2\mathbf{b_p} \times \mathbf{r_p} = \mathbf{0}$$

Effective 'magnetic field'

$$\mathbf{b_p} = -(\Delta', \Delta'', \xi_\mathbf{p}) \qquad \Delta = \frac{\lambda}{2}\sum_\mathbf{p}(r_1 + ir_2)_\mathbf{p}$$



$$(r_1 + ir_2)_\mathbf{p} = \frac{\Delta}{\sqrt{\xi_\mathbf{p}^2 + |\Delta|^2}} \qquad r_{3\mathbf{p}} = \frac{\xi_\mathbf{p}}{\sqrt{\xi_\mathbf{p}^2 + |\Delta|^2}}$$

$$1 = \frac{\lambda}{2}\sum_\mathbf{p}\frac{\tanh(\beta|\xi_\mathbf{p}|/2)}{\sqrt{\xi_\mathbf{p}^2 + |\Delta|^2}}$$

Dissipative dynamics between the stationary and the equilibrium state

System out of equilibrium

- Initial BCS state + small variation of interaction

$$\lambda(t) = \lambda_0 \pm \delta\lambda\, \theta(t)$$

- Initial normal state + turning on the interaction

$$\lambda(t) = \lambda\theta(t)$$

- Initial BCS state + switching off the interaction

$$\lambda(t) = \lambda\theta(-t)$$

Our problem:

Parametric range of all initial BCS-like states

step-like interaction $\lambda(t) = \lambda_s\theta(-t) + \lambda_f\theta(t)$

Excitation by small variation of interaction $\lambda(t) = \lambda_0 \pm \delta\lambda\,\theta(t)$

Volkov, Kogan 1974; Yuzbashyan, Tsyplyatyev, Altshuler, 2006

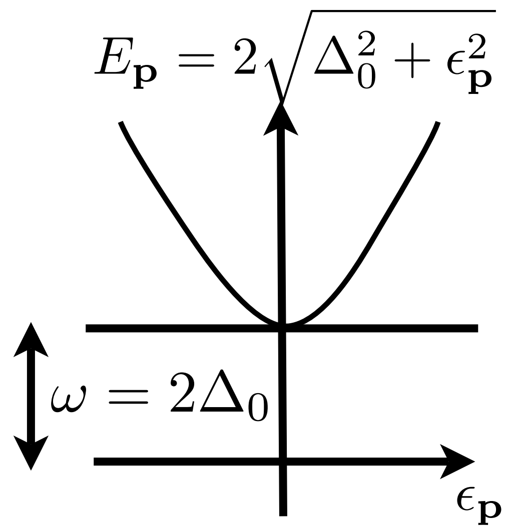
$$\mathcal{H} = \sum_{\mathbf{p}\sigma} \xi_{\mathbf{p}} a^{\dagger}_{\mathbf{p}\sigma} a_{\mathbf{p}\sigma} - \lambda(t) \sum_{\mathbf{pq}} a^{\dagger}_{\mathbf{p}\uparrow} a^{\dagger}_{-\mathbf{p}\downarrow} a_{-\mathbf{q}\downarrow} a_{\mathbf{q}\uparrow}$$

$$\Delta(t) = \lambda \sum_{\mathbf{p}} r^{+}_{\mathbf{p}}(t)$$

Linear analysis of the Bloch (BdG) equations

$$\Delta(t) = \Delta_a + A(t)\sin(2\Delta_a t + \alpha), \quad A(t) \propto t^{-1/2}.$$

collisionless "Landau damping"



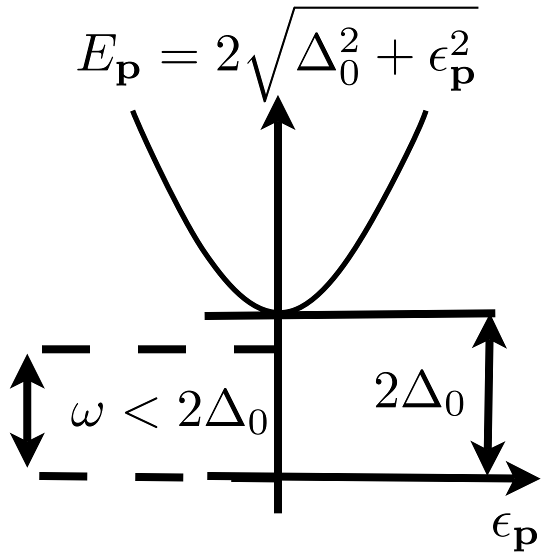
Higgs mode decays algebraically due to coupling to the continuum of excited pair states

How to make Higgs non-decaying?

- Energy dependence of the interaction

$$\mathcal{H} = -\sum_{\mathbf{p}} 2\epsilon_{\mathbf{p}} s^z_{\mathbf{p}} - \sum_{\mathbf{pq}} \lambda_{\mathbf{pq}}(t) s^-_{\mathbf{p}} s^+_{\mathbf{q}}$$

Amplitude mode enters the gap

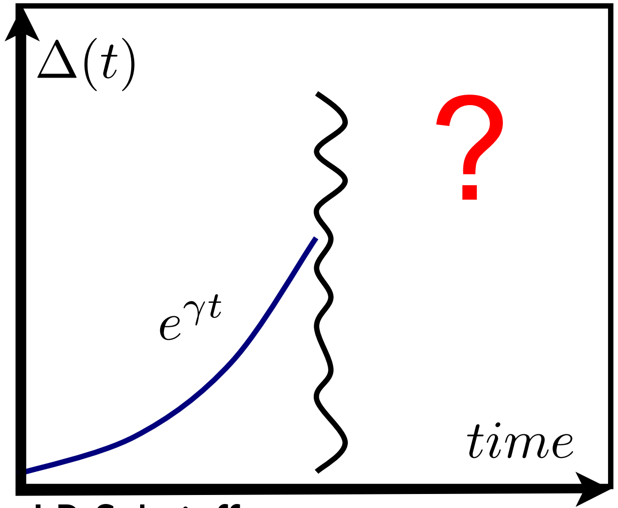


- Large perturbation

Synchronization of paired states may stabilize Higgs mode

"constructive interference" $\Delta(t) = \lambda \sum_{\mathbf{p}} r^+_{\mathbf{p}}(t)$

Linear dynamics of pairing (initial state close to normal)



J.R.Schrieffer,
Theory of Superconductivity

$$\mathcal{H} = \sum_{\mathbf{p}\sigma} \xi_{\mathbf{p}} a^\dagger_{\mathbf{p}\sigma} a_{\mathbf{p}\sigma} - \lambda(t) \sum_{\mathbf{pq}} a^\dagger_{\mathbf{p}\uparrow} a^\dagger_{-\mathbf{p}\downarrow} a_{-\mathbf{q}\downarrow} a_{\mathbf{q}\uparrow}$$

$$|\Phi(t)\rangle = \prod_{\mathbf{p}} \left(u_{\mathbf{p}}(t) + v_{\mathbf{p}}(t) a^+_{\mathbf{p}\uparrow} a^+_{-\mathbf{p}\downarrow} \right) |0\rangle$$

Linear analysis of the Bloch equations:

Eigen-modes $$1 = \lambda \sum_{\mathbf{p}} \frac{1 - 2n_{\mathbf{p}}}{2\xi_{\mathbf{p}} - \zeta}$$

Complex roots indicate instability $\zeta = \omega + i\gamma$

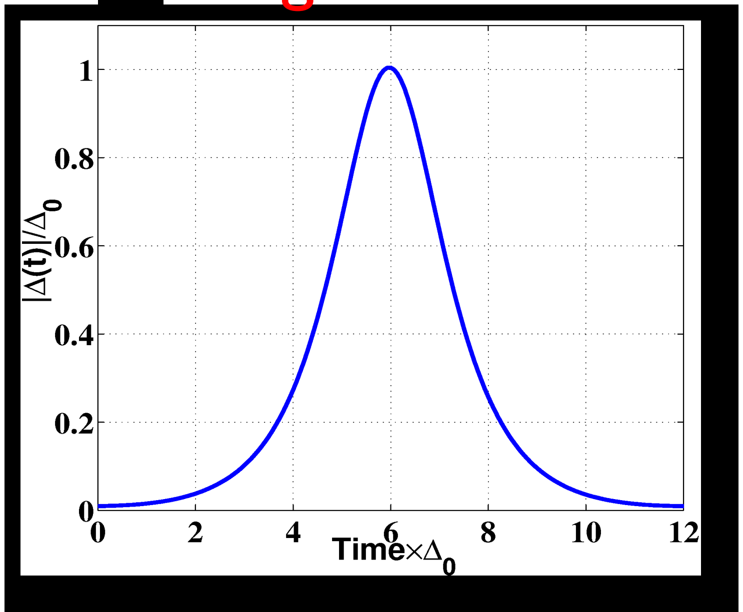
Exponential growth $\Delta(t) \propto e^{\gamma t - i\omega t}$

Time scale $\tau_\Delta = \gamma^{-1} \simeq \Delta_0^{-1}$ $\omega = 0$ (p-h symmetry)

Dynamics at long times (synchronized)

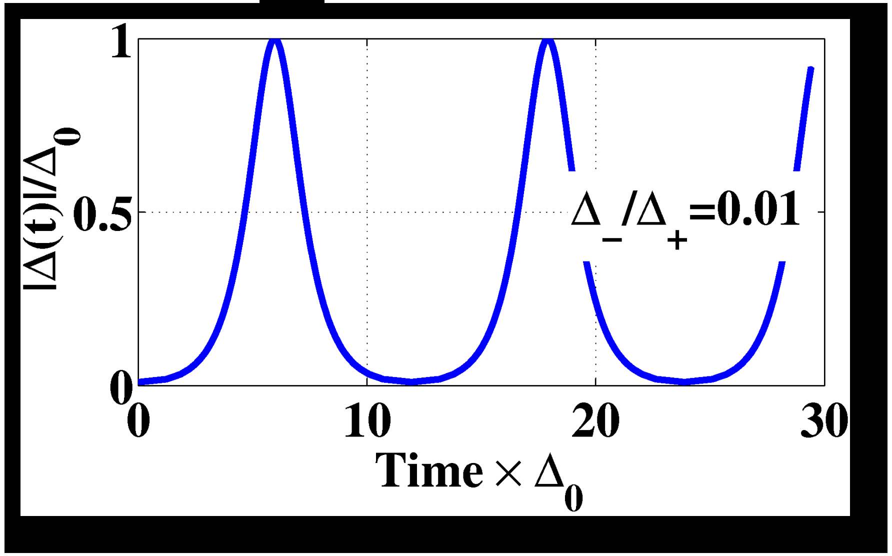
RB, Levitov, Spivak 2004, 2006

Single soliton



$$\Delta(t) = \frac{\Delta_+}{\cosh(\Delta_+ t)}$$

Soliton trains



$$\Delta(t) = \Delta_+ \mathrm{dn}\left[\Delta_+(t - t_0), k\right]$$
$$k^2 = 1 - \Delta_-^2/\Delta_+^2$$

Indication of synchronization
of energy states

$$\Delta = \frac{\lambda}{2}\sum_{\mathbf{p}}(r_1 + ir_2)_{\mathbf{p}}$$

Synchronized regime (analytics)

Initial normal state: turning on the interaction

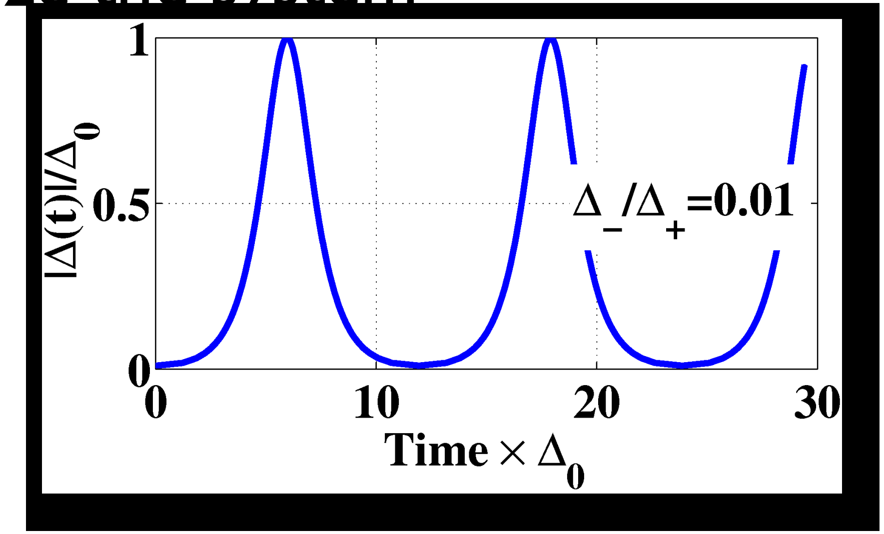
$$\lambda(t) = \lambda\theta(t)$$

Trivial solution $\Delta(t) = 0$ is unstable!

Fluctuations destabilize the system

$$\Delta(t) = \Delta_+ \mathrm{dn}\left[\Delta_+(t - t_0), k\right]$$

$$k^2 = 1 - \Delta_-^2/\Delta_+^2$$



Limiting solution

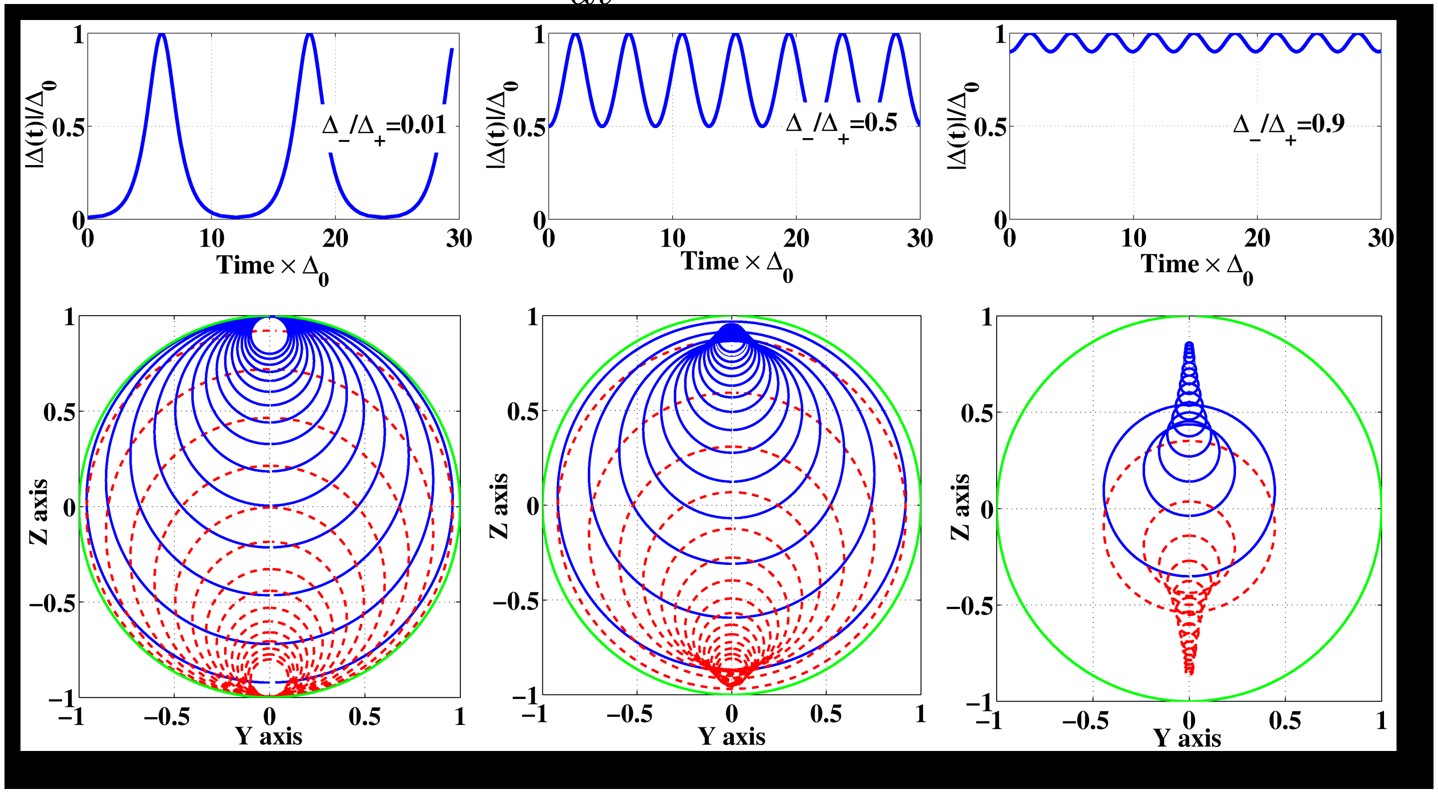
$$\Delta(t) = \Delta_0/\cosh[\Delta_0(t - t_0)]$$

Physical solution depends on the strength of fluctuations, e. g. through temperature

It corresponds to soliton trains

Pseudospin dynamics (ansatz)

$$\frac{d\mathbf{r_p}}{dt} = 2\mathbf{b_p} \times \mathbf{r_p}$$



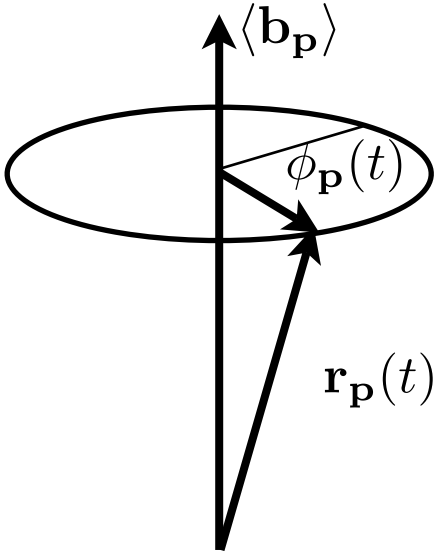
Synchronization of energy states

Syncronized regime (numerics)

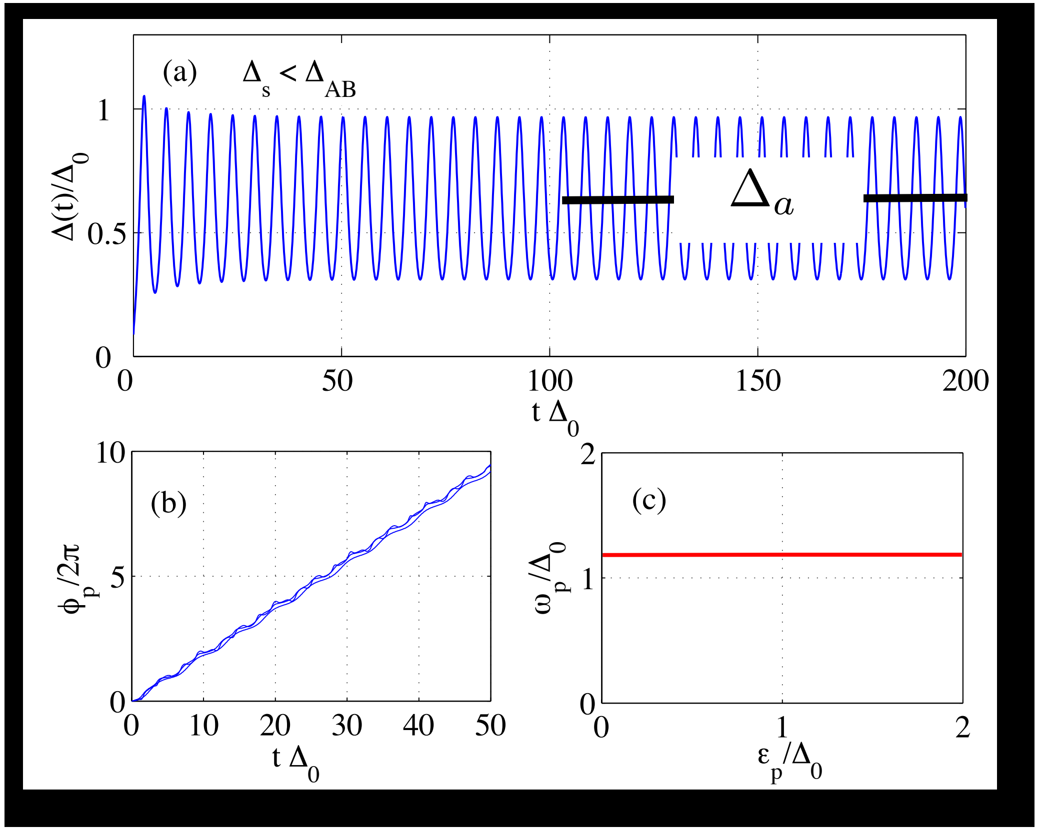
Time dependence of individual states: rotation around local magnetic field

$$\langle \mathbf{b_p} \rangle = -(\Delta_a, 0, \xi_{\mathbf{p}})$$

$$\Delta = \frac{\lambda}{2} \sum_{\mathbf{p}} (r_1 + i r_2)_{\mathbf{p}}$$



RB, Levitov, 2006



Average frequency

$$\omega_{\mathbf{p}} = \langle d\phi_{\mathbf{p}}/dt \rangle = (\phi_{\mathbf{p}}(\tau) - \phi_{\mathbf{p}}(0))/\tau$$

No energy dispersion!

Dephased regime (analytics)

Volkov, Kogan 1974; Yuzbashyan, Tsyplyatyev, Altshuler, 2006

- Small variation of interaction

$$\lambda(t) = \lambda_0 \pm \delta\lambda\, \theta(t)$$

$$\Delta(t) = \Delta_a + A(t) \sin(2\Delta_a t + \alpha), \quad A(t) \propto t^{-1/2}.$$

collisionless "Landau damping"

- Switching off the interaction - initial BCS state

RB, Levitov, 2006; Yuzbashyan, Dzero, 2006

$$\lambda(t) = \lambda\theta(-t)$$

$$\Delta(t \gg \Delta_s^{-1}) \propto (\Delta_s t)^{-1/2} e^{-2\Delta_s t}.$$

"over-heating" the system

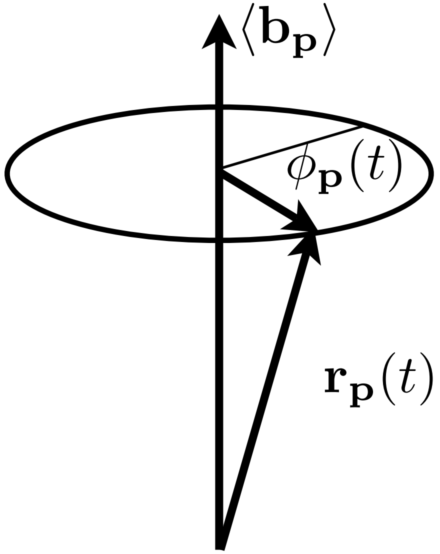
Dephased regime: damped and overdamped

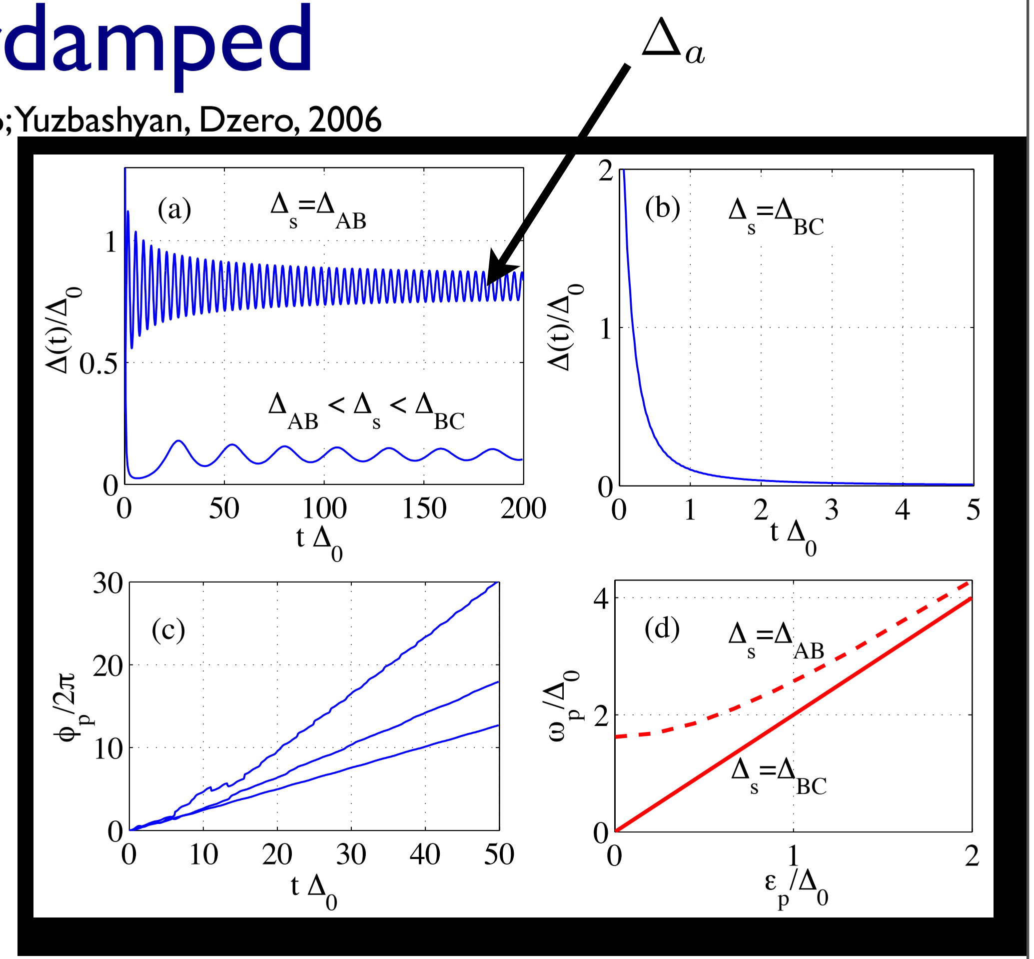
RB, Levitov, 2006; Yuzbashyan, Dzero, 2006

Rotation around local magnetic field

$$\langle \mathbf{b_p} \rangle = -(\Delta_a, 0, \xi_\mathbf{p})$$

$$\Delta = \frac{\lambda}{2} \sum_\mathbf{p} (r_1 + i r_2)_\mathbf{p}$$





$$\omega_\mathbf{p} = \langle d\phi_\mathbf{p}/dt \rangle = (\phi_\mathbf{p}(\tau) - \phi_\mathbf{p}(0))/\tau \qquad \omega_\mathbf{p} = 2(\epsilon_\mathbf{p}^2 + \Delta_a^2)^{1/2}$$

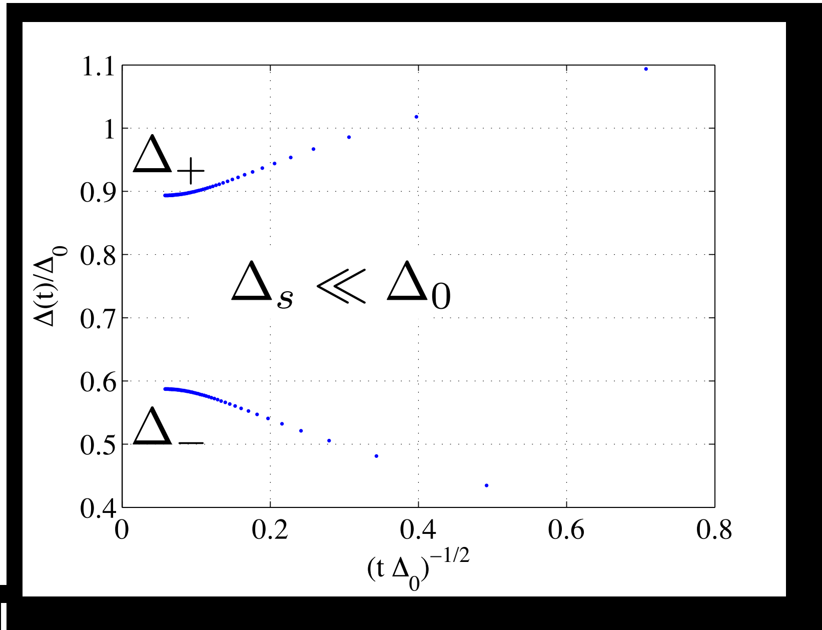
Energy dispersion - Bogoliubov spectrum of excited pairs

Asymptotic diagram from numerics

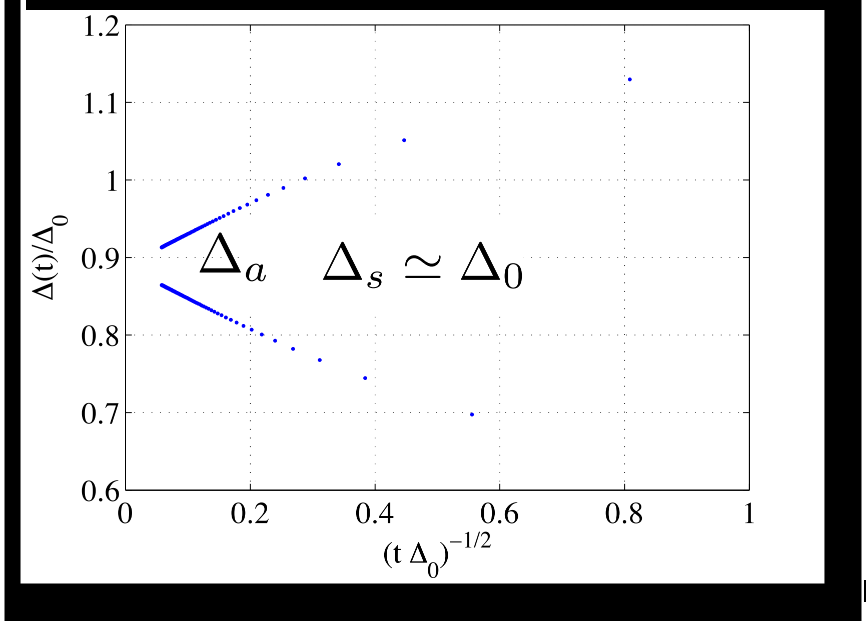
RB, Levitov, 2006

Extract asymptotics:

- Synchronized regime:
  two values Δ_+ and Δ_-



- Dephased regime: Δ_a
  one value (lines intersect)



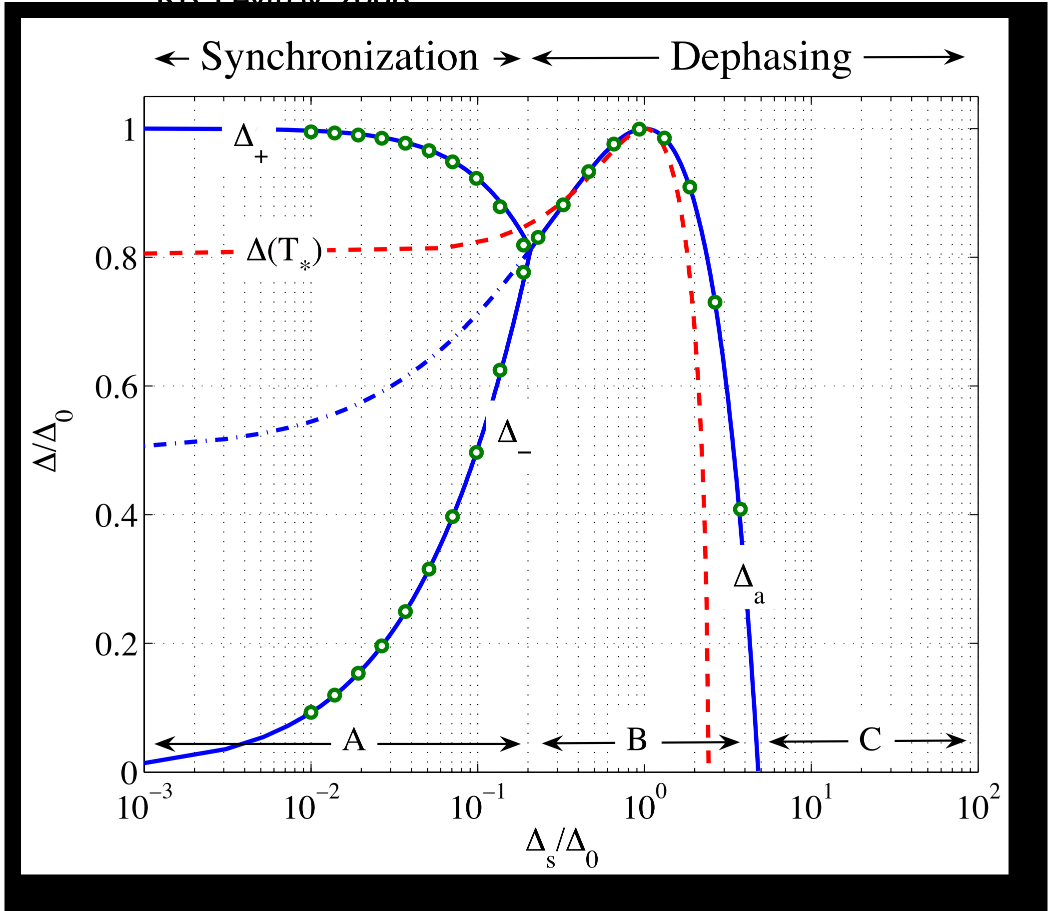
Different regimes of pairing: syncronized and dephased

RB Levitov 2006

Phase boundaries (numerics)

AB $\Delta_s \approx 0.21\Delta_0$

BC $\Delta_s \approx 4.8\Delta_0$



$D_A = \Delta_+ - \Delta_- \qquad D_B = \Delta_a$

Two order parameters vanishing at critical points

Regimes of pairing: analytics

RB, Levitov, 2006 based on work by Yuzbashyan, Tsyplyatyev, Altshuler, 2006

Integrals of motion and Lax vector

Richardson, Sherman 1964;
Cambiaggio, Rivas, Saraceno, 1997;
Yuzbashyan, Altshuler, Kuznetsov, Enolskii, 2005

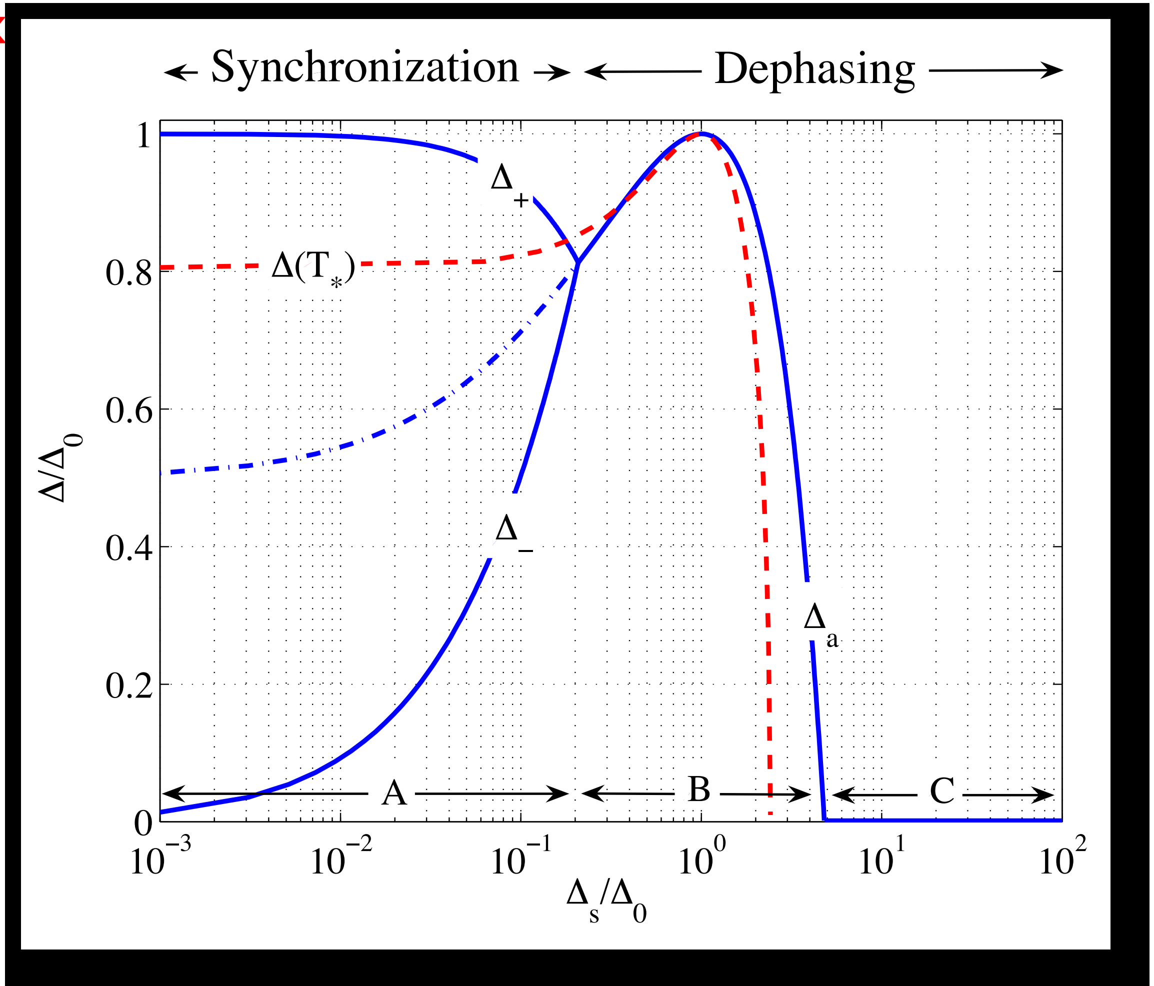
$$R_{\mathbf{p}} = \mathbf{L}_{\mathbf{p}} \mathbf{s}_{\mathbf{p}}$$

$$\mathbf{L}_{\mathbf{p}} = \hat{\mathbf{z}} + \lambda \sum_{\mathbf{p}' \neq \mathbf{p}} \frac{\mathbf{s}_{\mathbf{p}'}}{\epsilon_{\mathbf{p}} - \epsilon'_{\mathbf{p}}}$$

$$\mathbf{L}^2(\epsilon_{\mathbf{p}}) = \mathbf{0}$$

Number of complex roots defines classes of solutions

Yuzbashyan, Tsyplyatyev, Altshuler, 2006



RB, Levitov, 2006; Yuzbashyan, Dzero, 2006

Synchronized: two zeros (A) Δ_+ Δ_-

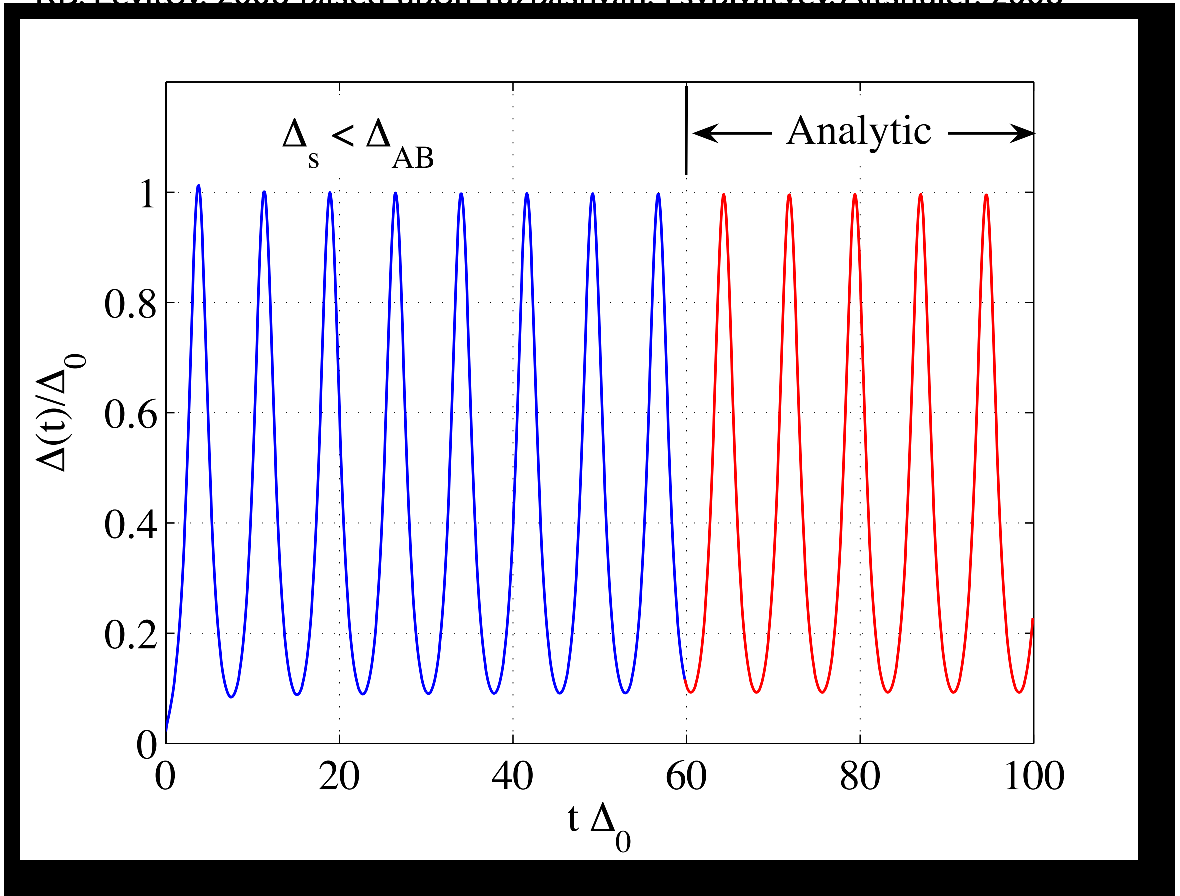
Dephased: one (B) and zero (C) Δ_a

$$\Delta_{AB} = e^{-\pi/2}\Delta_0 \qquad \Delta_{BC} = e^{\pi/2}\Delta_0$$

$$D_{A,B} = \Delta_a \propto \ln\left(\frac{\Delta_s}{\Delta_0}\right) \pm \frac{\pi}{2}$$

Asymptotic dynamics (synchronized)

RB, Levitov, 2006 based upon Yuzbashyan, Tsyplyatyev, Altshuler, 2006

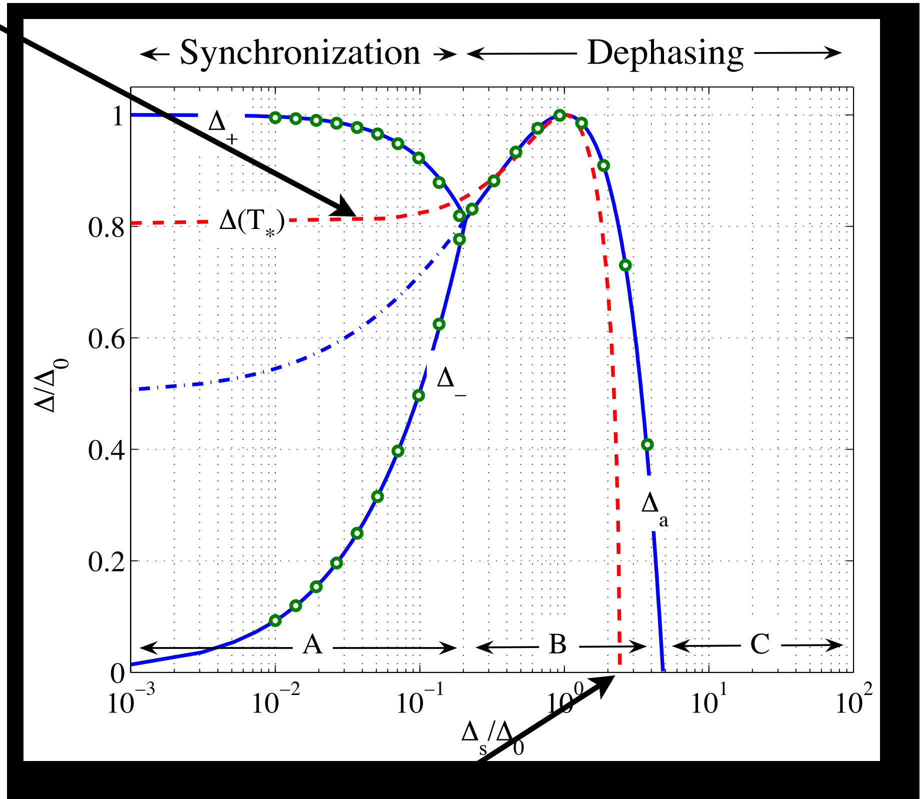


$$\Delta(t) = \Delta_+ \mathrm{dn}\left(\Delta_+(t-\tau_0), k\right), \quad k = 1 - \Delta_-^2/\Delta_+^2$$

All parameters are defined analytically from exact solution

Long-time equilibrium states

RB, Levitov, 2006



Energy is conserved

$$E_{t\gg\tau_{\rm el}}(T_*) = E_0$$

$$E_0 = \sum_{\mathbf{p}} (\epsilon_{\mathbf{p}} - \tilde{\epsilon}_{\mathbf{p}}) + \frac{\Delta_s^2}{\lambda_s}\left(2 - \frac{\lambda}{\lambda_s}\right)$$

$$n_{\mathbf{p}} = 1/\left(1 + e^{\tilde{\epsilon}_{\mathbf{p}}(\Delta_*)/T_*}\right)$$

$$E_{t\gg\tau_{\rm el}}(T_*) = \sum_{\mathbf{p}} \left(\epsilon_{\mathbf{p}} - (1 - 2n_{\mathbf{p}})\,\tilde{\epsilon}_{\mathbf{p}}(\Delta_*)\right) + \frac{\Delta_*}{\lambda}$$

Final state turns normal

$$\Delta_s \geq f(g)\Delta_0$$

$$f(g) = 2.2 + 0.86g + O(g^2)$$

Finite temperature phase diagram

RB, Levitov, unpublished

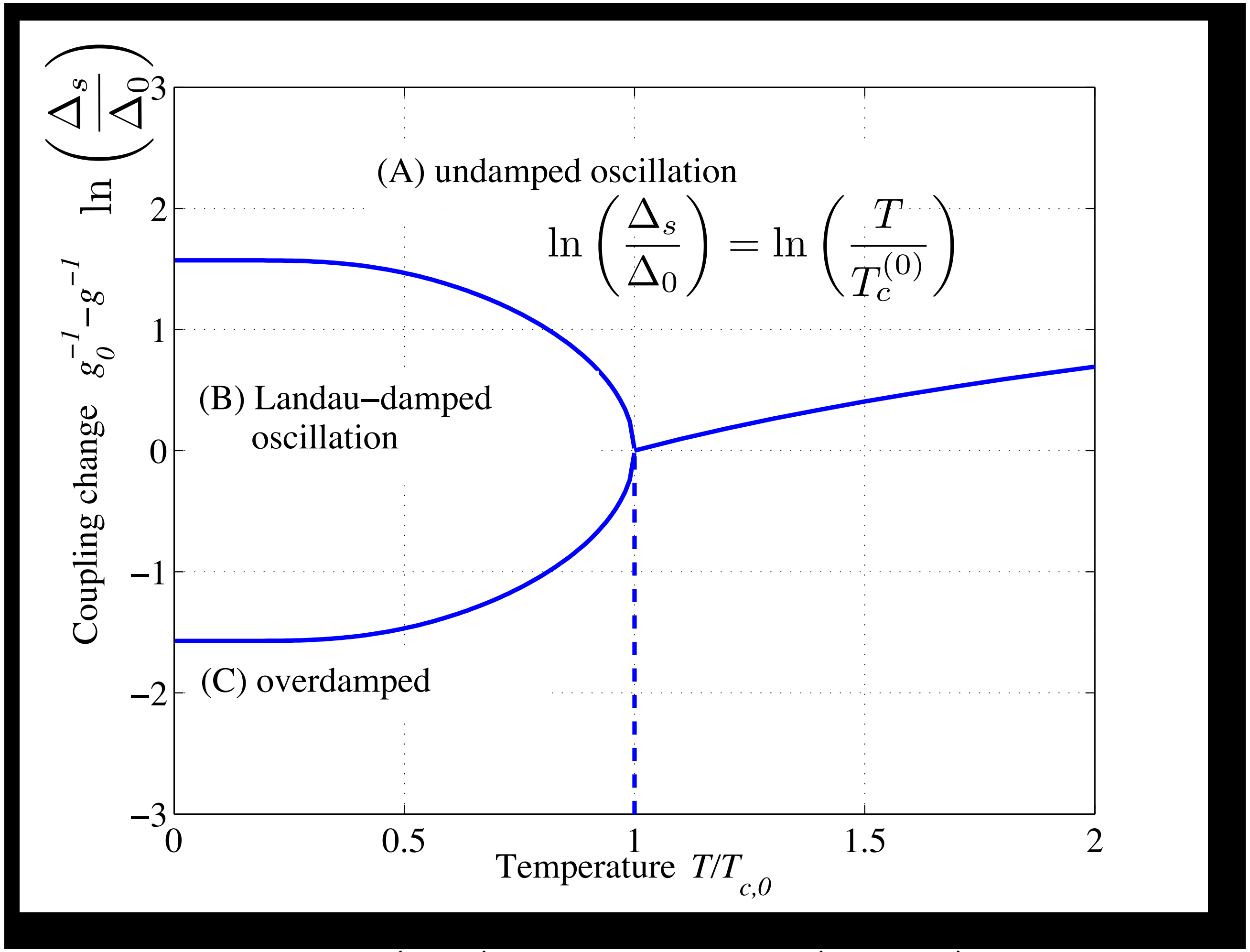
BCS gap
(weak interactions)

$$\Delta = 0.49 E_F e^{-1/g}$$

$$g = \frac{2}{\pi} k_F |a|$$

Variation of the scattering length

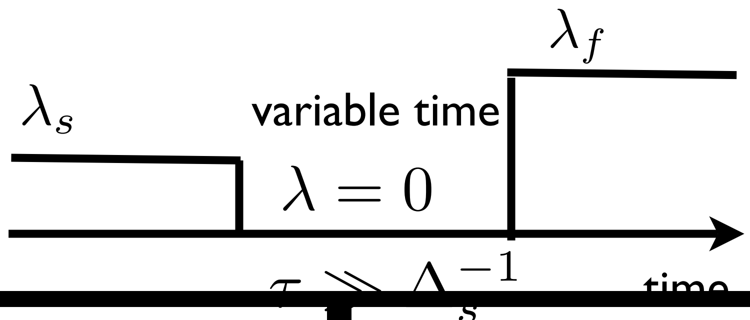
$$\delta a / a \approx \pm k_F a$$

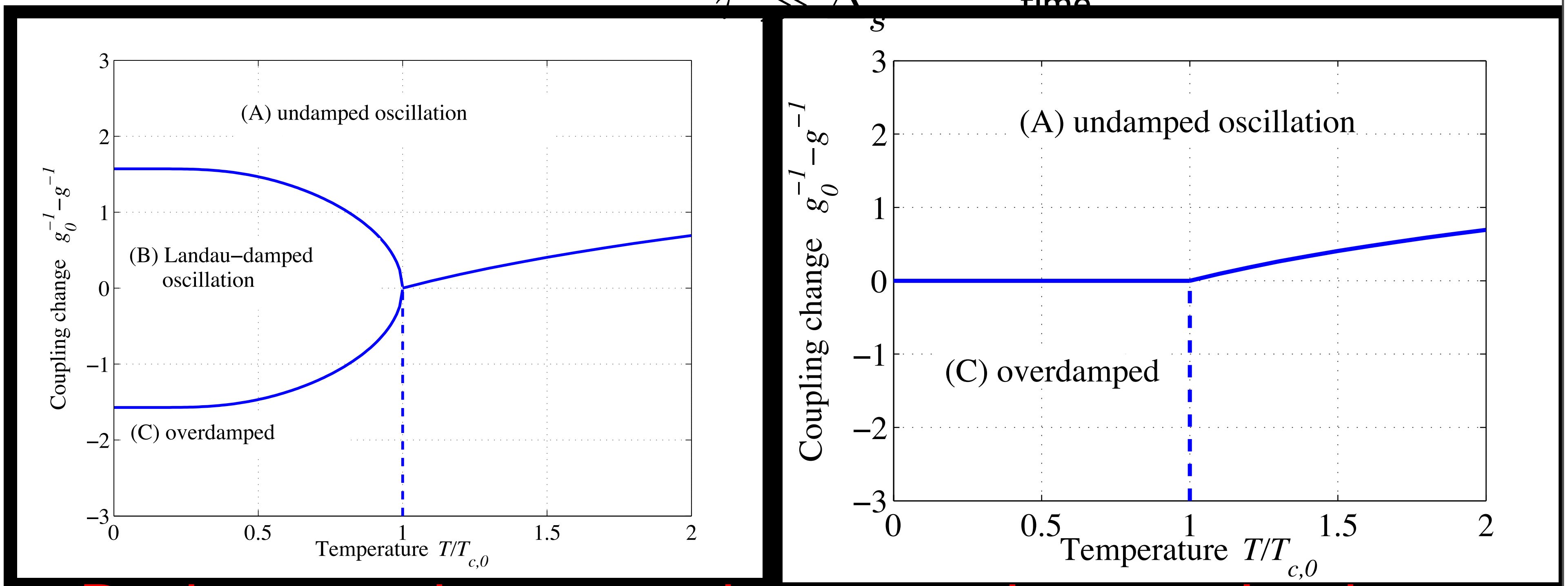


$$\ln\left(\frac{\Delta_s}{\Delta_0}\right) = \pm \frac{\pi}{2} \tanh\left(\frac{\Delta(T)}{2T}\right)$$

Playing with the phases: eliminating Landau damping

RB, Levitov, unpublished





Dephasing in the intermediate state eliminates Landau-damping

Role of energy-dependent interaction

$$\mathcal{H} = -\sum_{\mathbf{p}} 2\epsilon_{\mathbf{p}} s^z_{\mathbf{p}} - \sum_{\mathbf{pq}} \lambda_{\mathbf{pq}}(t) s^-_{\mathbf{p}} s^+_{\mathbf{q}}$$

$$\lambda_{\mathbf{pq}}(t) = \lambda(t)\,(a_1 + a_2 v_{\mathbf{p}} v_{\mathbf{q}}) \qquad v_{\mathbf{p}} = \frac{\gamma}{\sqrt{\gamma^2 + \epsilon^2_{\mathbf{p}}}}$$

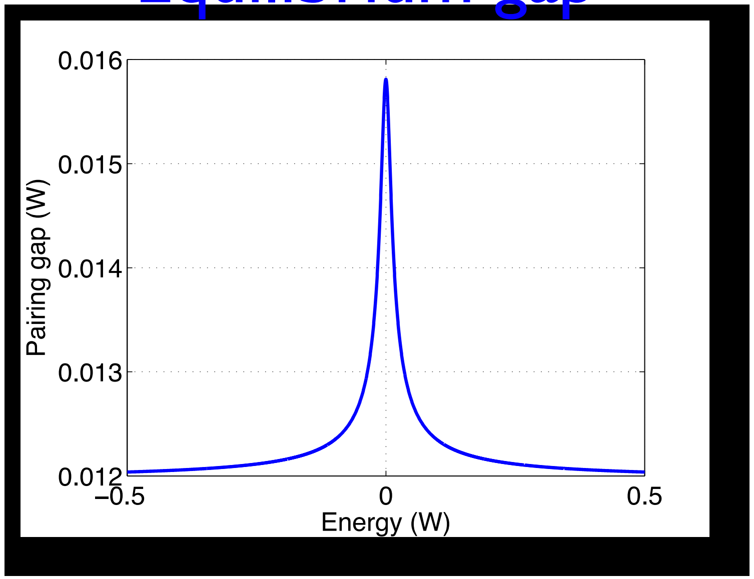
Two limits:

- constant interaction $a_1 = 1,\, a_2 = 0$
- separable interaction $a_1 = 0,\, a_2 = 1$

Role of integrability in the pairing dynamics

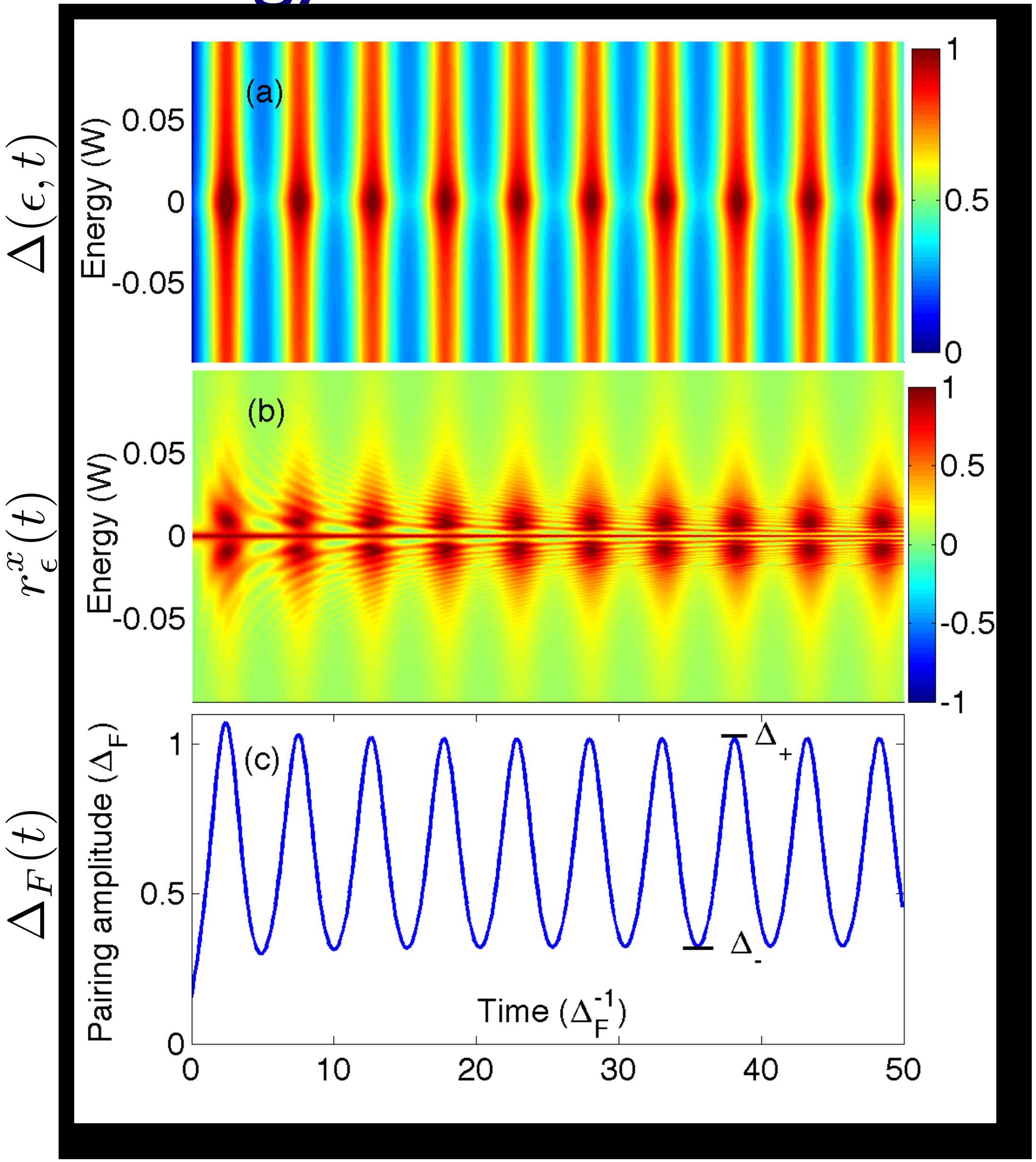
Individual energy states

Equilibrium gap



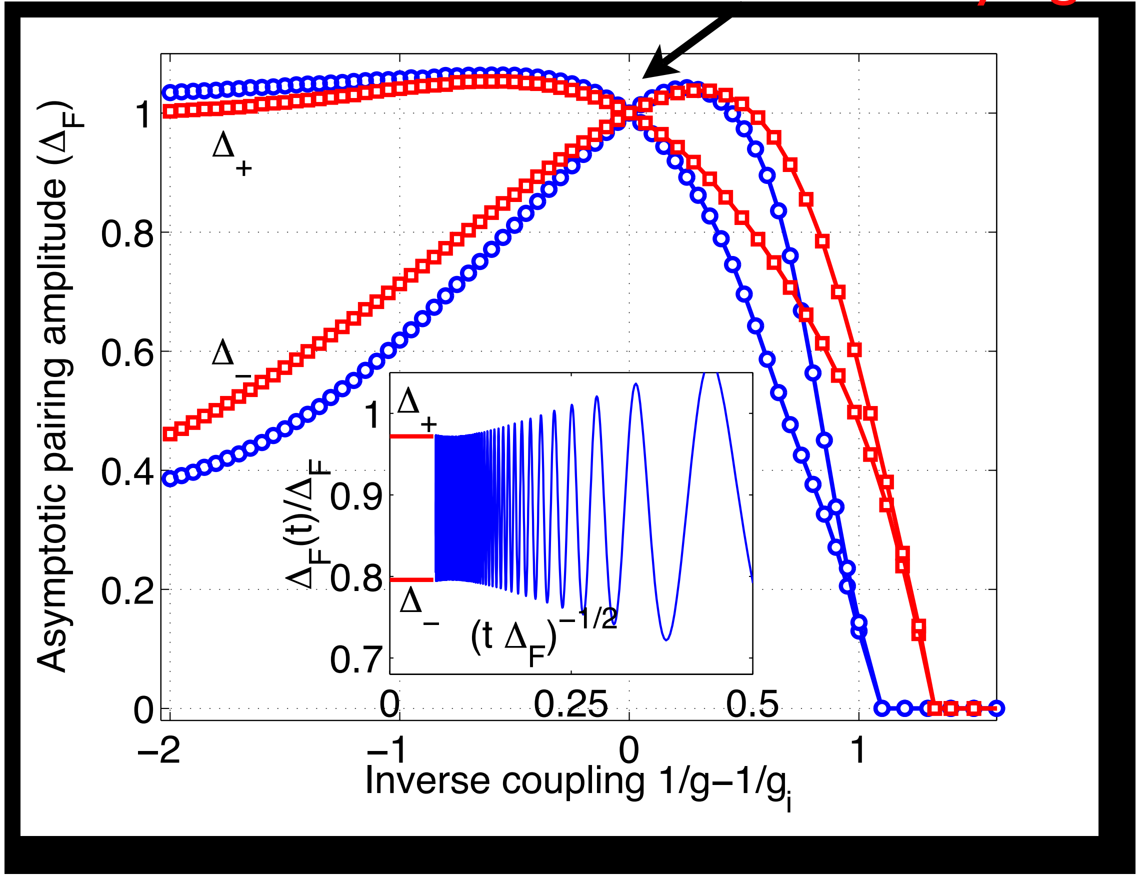
$$a_1 = a_2 = 0.5, \gamma/W = 0.01$$

Synhronized phase is robust



Asymptotic diagram

Non-decaying Higgs mode

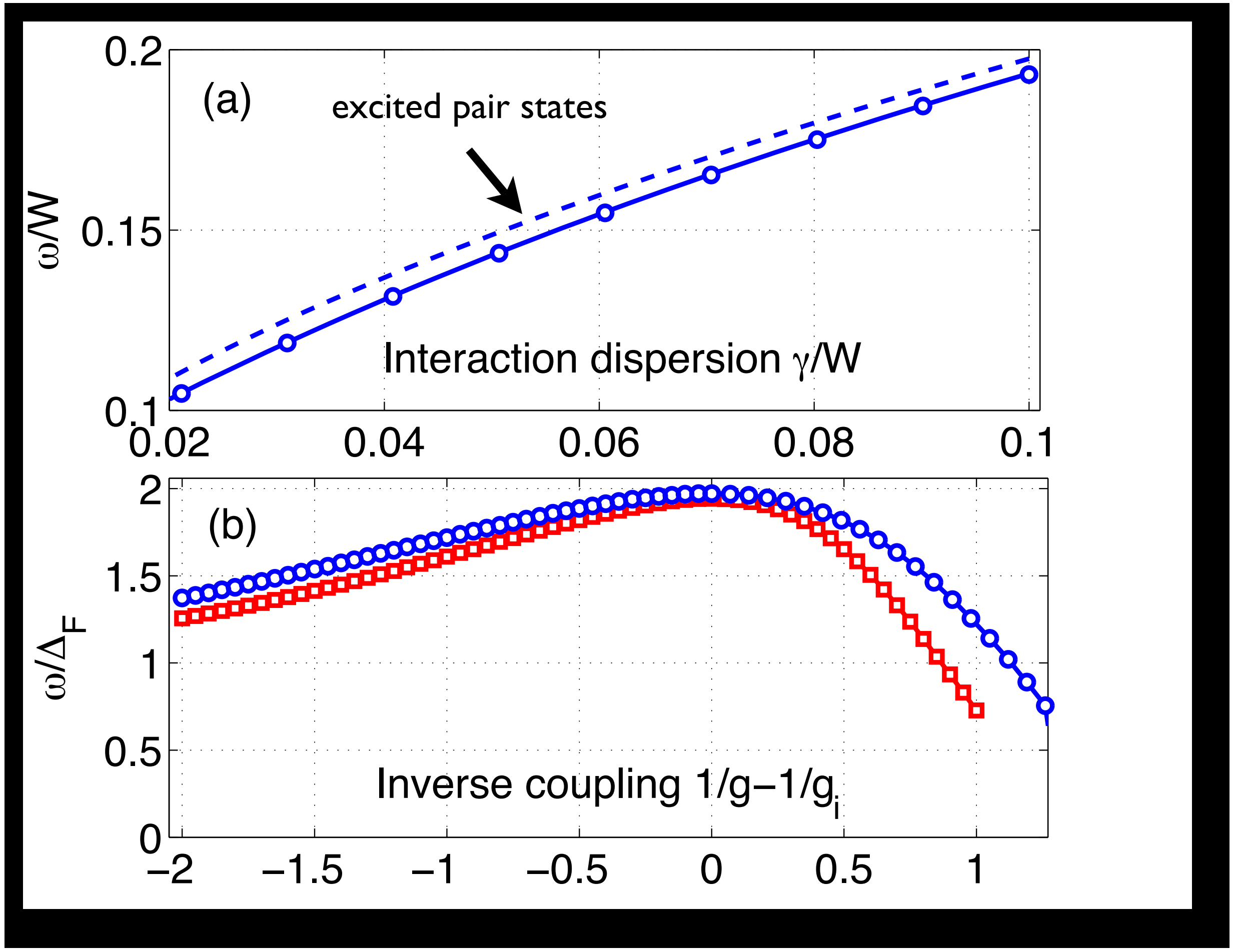


Circles (non-separable interaction) $a_1 = a_2 = 0.5, \gamma/W = 0.01$

Squares (separable) $a_1 = 0, a_2 = 1, \gamma/W = 0.01$

The dissipationless Higgs mode

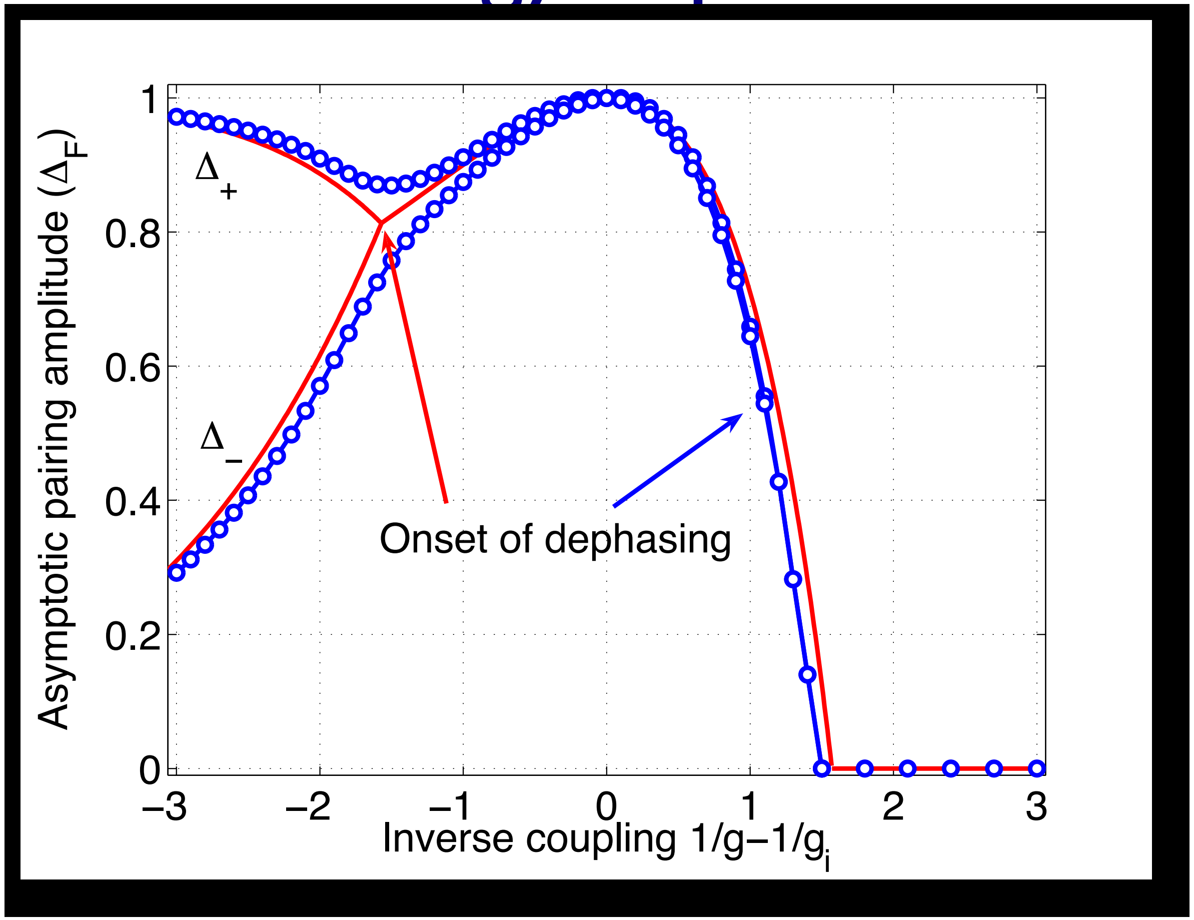
Collective mode frequency



Eigen-equation for the collective mode

$$\delta\Delta^x_{\mathbf{p}\omega} = \frac{1}{2}\sum_{\mathbf{q}} \frac{\lambda_{\mathbf{pq}}\delta\Delta^x_{\mathbf{q}\omega}}{\sqrt{\epsilon^2_{\mathbf{q}}+\Delta^2_{\mathbf{q}}}} \frac{\epsilon^2_{\mathbf{q}}}{\epsilon^2_{\mathbf{q}}+\Delta^2_{\mathbf{q}}-\omega^2/4}$$

Smooth energy dependence



Circles $a_1 = a_2 = 0.5, \gamma/W = 0.1$

Red line $a_1 = 1, a_2 = 0$

Where to observe? Feshbach resonance

Atomic scattering length near FR $$g = \frac{2}{\pi} k_F |a|$$

Dynamical control of interactions $$\Delta = 0.49 E_F e^{-1/g}$$

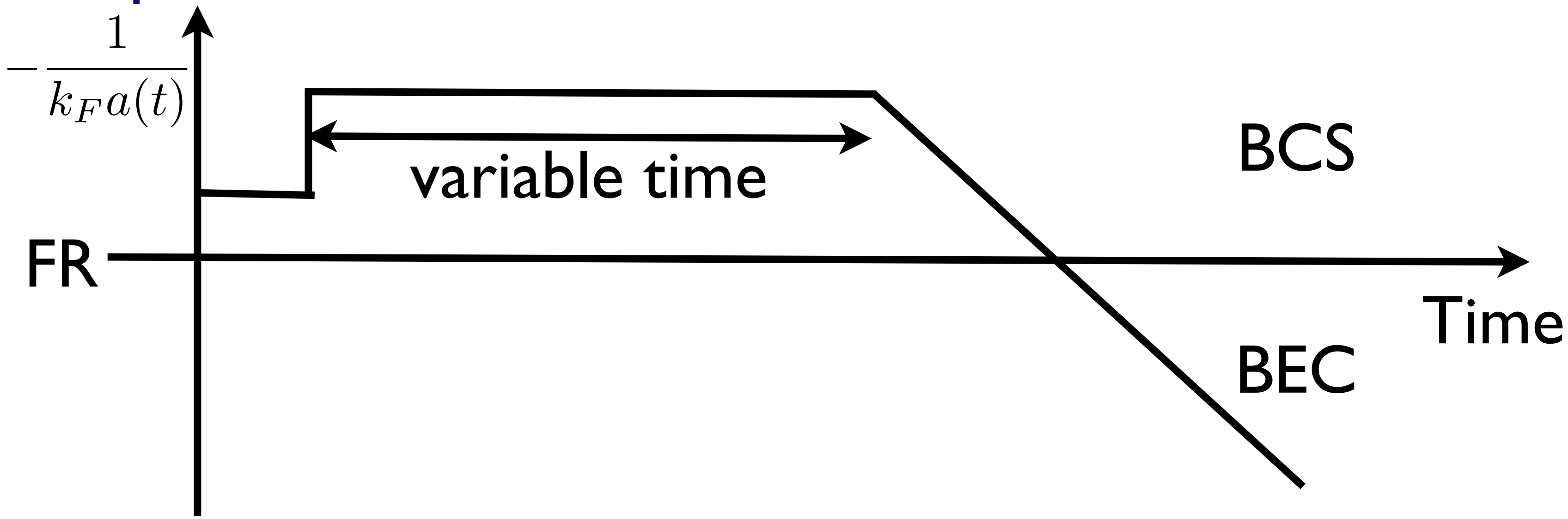
How to observe?

Fast projection: probe of pairing correlations

proposed and realized by JILA and MIT groups (2004)

Condensate fraction of molecules - pairing correlations

Proposed method: $\delta a / a \approx \pm k_F a$ ← Phase boundaries



Physical parameters ^{6}Li $B_{FR} \approx 850\,\mathrm{G}$

- **Typical sample:**
  - Particle density $n \approx 10^{13}\mathrm{cm}^{-3}$
  - Fermi energy $E_F \approx 0.5\,\mu\mathrm{K}$
- **Interaction:** Scattering length of atoms $|a| \gtrsim 100\,\mathrm{nm}$

At lower densities marginally weak BCS (?)

Weak coupling BCS $g = \nu\lambda = \frac{2}{\pi}k_F|a| \approx 0.3$

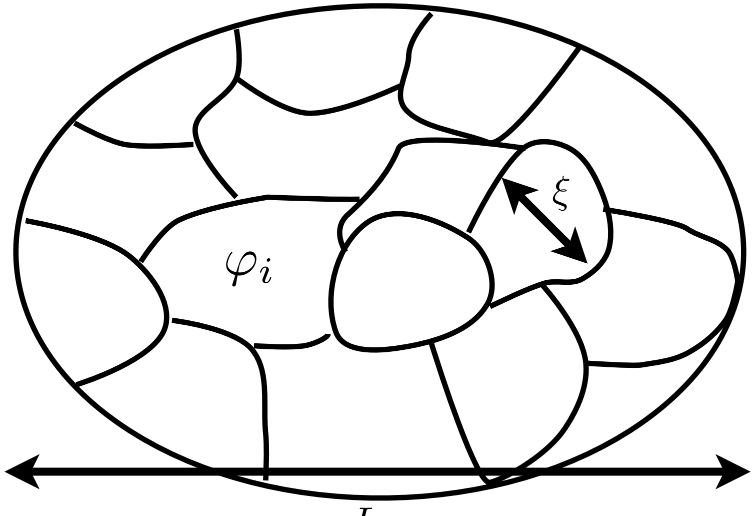
(strong in experiments!) $T_c \approx 0.3 E_F e^{-1/\lambda} \approx 0.01 E_F$

- **Time scales:**
  - period $\tau_\Delta \simeq \hbar/\Delta_0 \approx 2\,\mathrm{ms}$
  - quasi-particle relaxation time $\tau_{\xi\simeq\Delta} \simeq \hbar E_F/\Delta_0^2 \approx 200\,\mathrm{ms}$
- **Spatial scales:**
  - Correlation length $\xi = \hbar^2 k_F/m\Delta_0 \simeq 24\,\mu\mathrm{m}$
  - Sample size $L \simeq 20\,\mu\mathrm{m}$

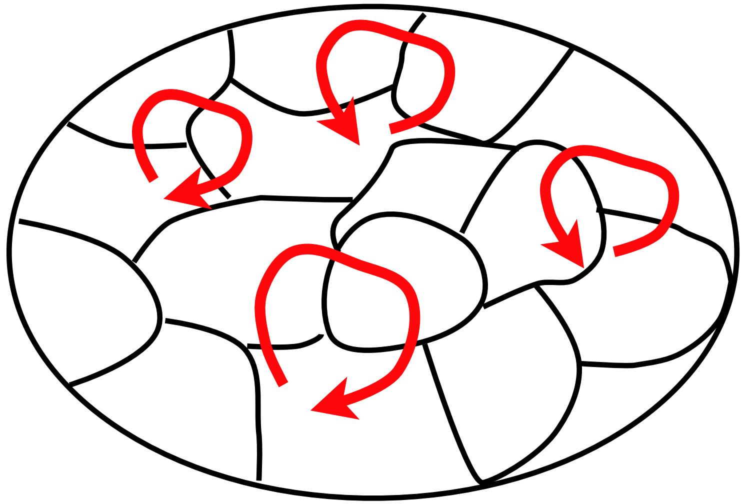
It may be possible to observe the pairing dynamics

Long-time dynamics in space

Patches of size $\simeq \xi$ develop different phases



Vortices

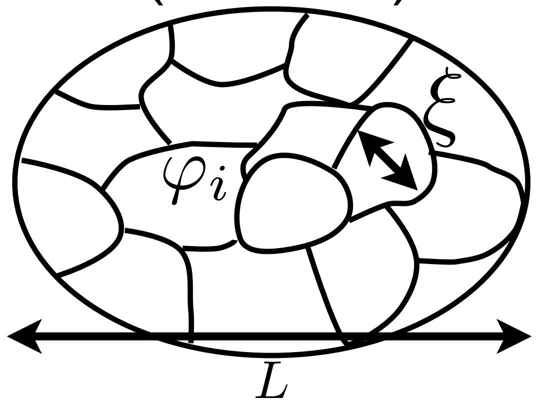


$$\Delta\varphi_i = 2\pi$$

Possible sources of dissipation

Uniform geometry:

- Small correlation length at unitarity, and thus inhomogeneous dynamics $\Delta \simeq E_F$ $\xi \simeq k_F^{-1}$
- Quasiparticle relaxation and Higgs frequency are close ($\simeq \hbar / E_F$) at unitarity



- At unitarity sound and Higgs mode couple

and also in trapped geometry:

- Spatial inhomogeneity of the order parameter

Conclusions:

- The pairing amplitude mode (Higgs boson) is characteristic for Fermi superfluids - no analog of this mode in Bose systems
- Excitation of the Higgs boson in fermionic superfluids:
  - Nonlinear excitation induced by changing the interaction strength
    - Dephased and synchronized regime
    - Two dynamical transitions between underdamped, damped and overdamped dynamics
    - The dynamics is robust with respect to the energy dependent interaction
  - Small perturbation (linear) at the FR
    - Higgs mode is Landau-damped (algebraic decay) for constant interaction
    - Higgs mode may become non-decaying for the energy-dependent interaction

Open questions and future directions

- Uniform dynamics:
  - Formation of a superfluid "seed" (quantum flucts.)
  - Role of integrability in the dynamics
  - Dynamics in the regime of strong interactions for realistic models
- Spatial dynamics:
  - Formation of a paired state in space (topological defects, Kibble-Zurek mechanism)
  - Dissipation
  - Response to non-linear external drive
- Experiments: cold atoms