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Summer School on Novel Quantum Phases and Non-Equilibrium Phenomena in Cold Atomic Gases

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The normal state of the polarized Fermi gas at unitarity

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Normal State of a Polarized Fermi Gas at Unitarity

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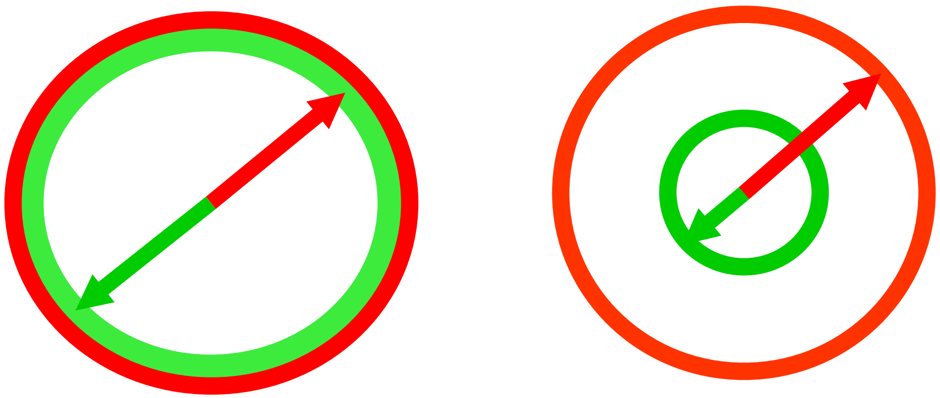
Phys. Rev. Lett. **97**, 200403 (2006)

An unsolved puzzle:

What happens when we polarize a Fermi superfluid?

“polarize” = create an imbalance between the densities of spin up and of spin down electrons/atoms

Up and **down**
Fermi spheres



A bit of history…

Theory:

Chandrasekhar, Clogston (1962) The Pauli limit

Fulde and Ferrell, Larkin and Ovchinnikov (1964) Exotic states with SC/M coexistence

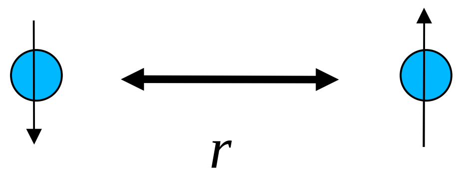
and many others since …

Experiments in superconductors:

SC destroyed for high enough magnetic fields

No conclusive evidence for new states

The interaction in cold atomic gases



Only s-wave collisions at low T

For fermions:
- only opposite spin interactions
- same spin noninteracting

radius of interatomic potential (R) ~ few tens Å
s-wave scattering length (a) ~ few hundred Å
interatomic distance ($n^{-1/3}$) ~ few thousand Å

Changing the interactions: ***Interatomic potential depends on the magnetic field. By changing it we can modify the s-wave phase shift of the potential (with a Feshbach resonance)***

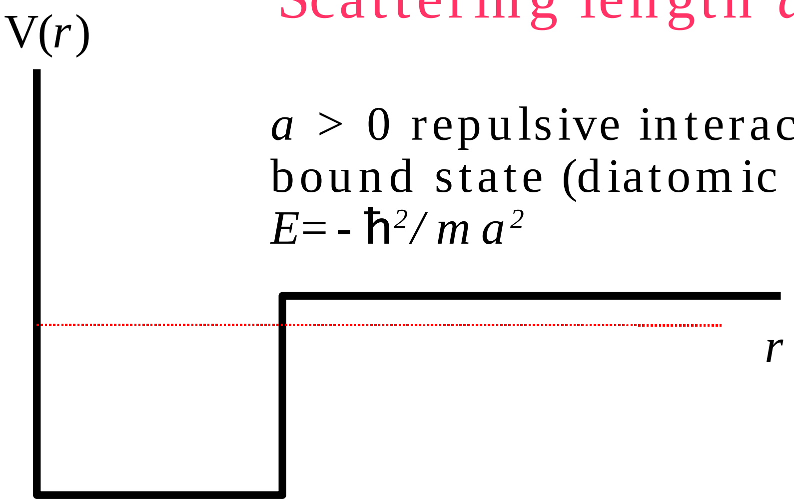
Provides us with a "knob" to tune the interactions!

If the s-wave scattering length → ∞ (phase shift → $\pi/2$) then the atoms are strongly interacting – the unitary limit

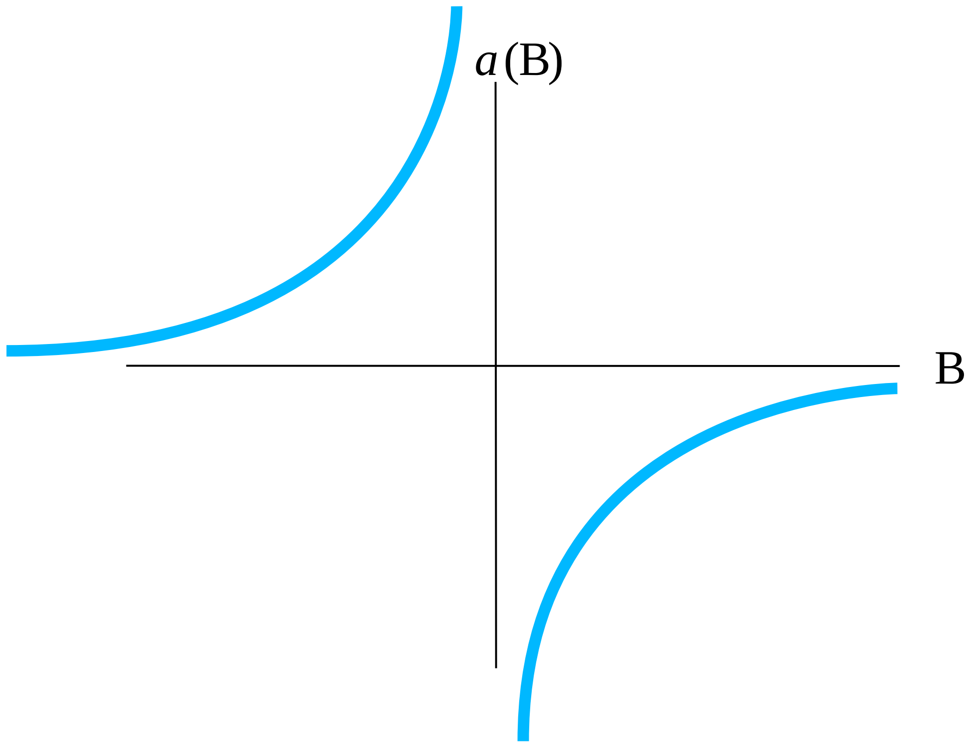
Changing the interaction with a Feshbach resonance

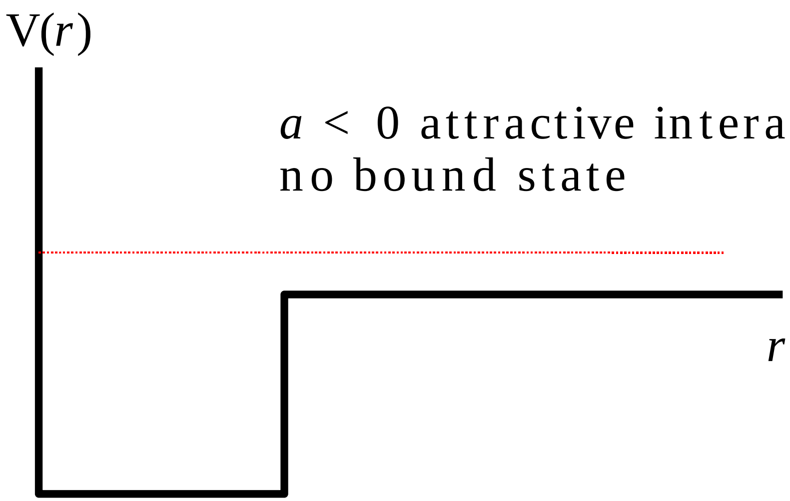
Scattering length *a* tunable with an external magnetic field B

V(*r*) – potential between two atoms



$a > 0$ repulsive interaction
bound state (diatomic molecule)
$E = -\hbar^2/ma^2$





$a < 0$ attractive interaction
no bound state

Many-body case: the BEC-BCS crossover

Eagles 1969
Leggett 1981 (T=0)

Unitarity

BEC of diatomic molecules ←

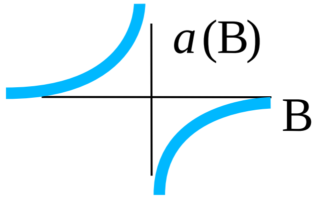
(2-body bound state)

(no 2-body bound state)

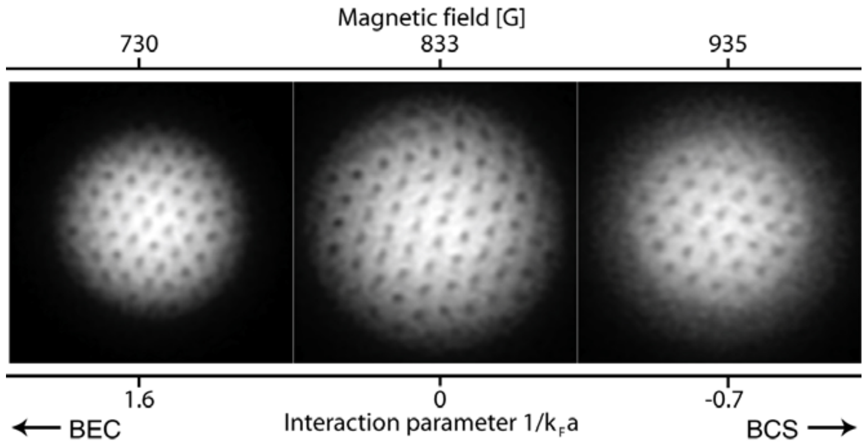
BCS (Cooper pairs) →

0 $-1/k_F a \quad (\approx -1/n^{1/3}a)$

Condensate of diatomic molecules goes smoothly (crossover) into a "condensate" of Cooper pairs



Molecular binding energy
→ BCS gap



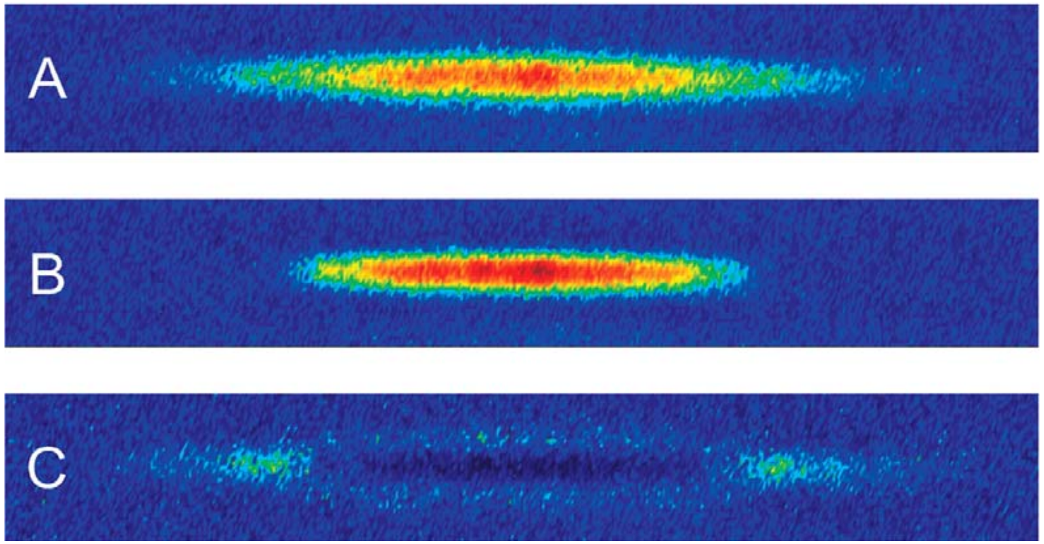
Superfluidity preserved across the resonance (MIT 2005)

What happens to a polarized Fermi superfluid? (I)

unitary superfluid

The Rice experiment:

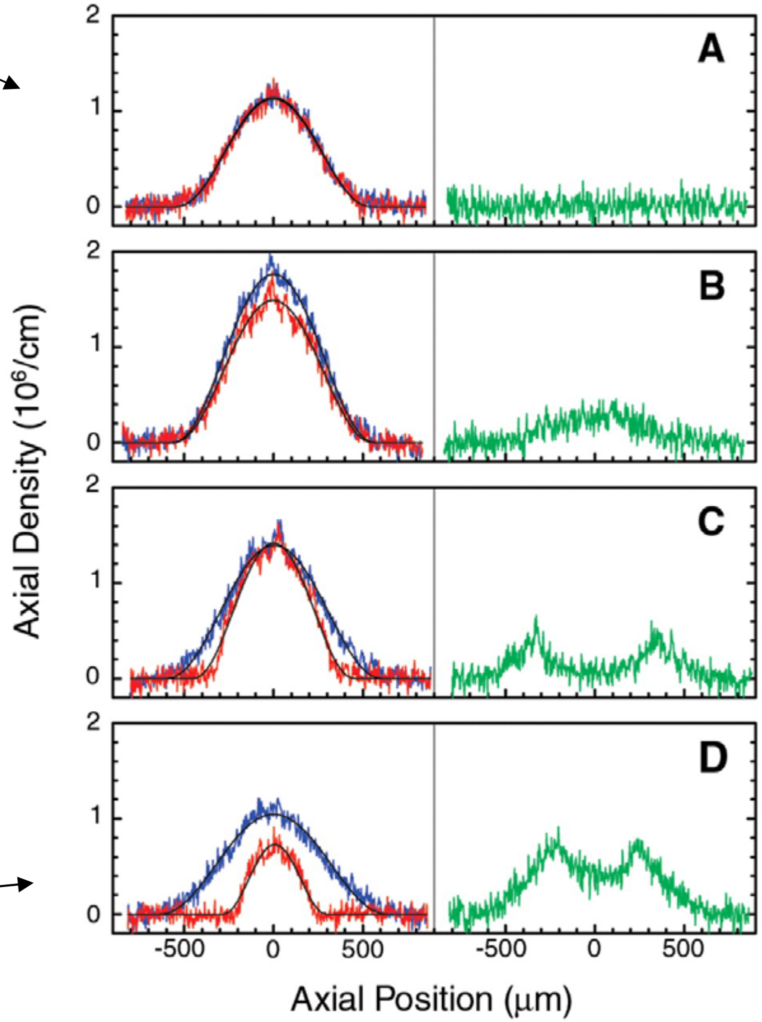
Partridge et al, Science 503 **311** (2006)



Majority species

Minority species

A–B

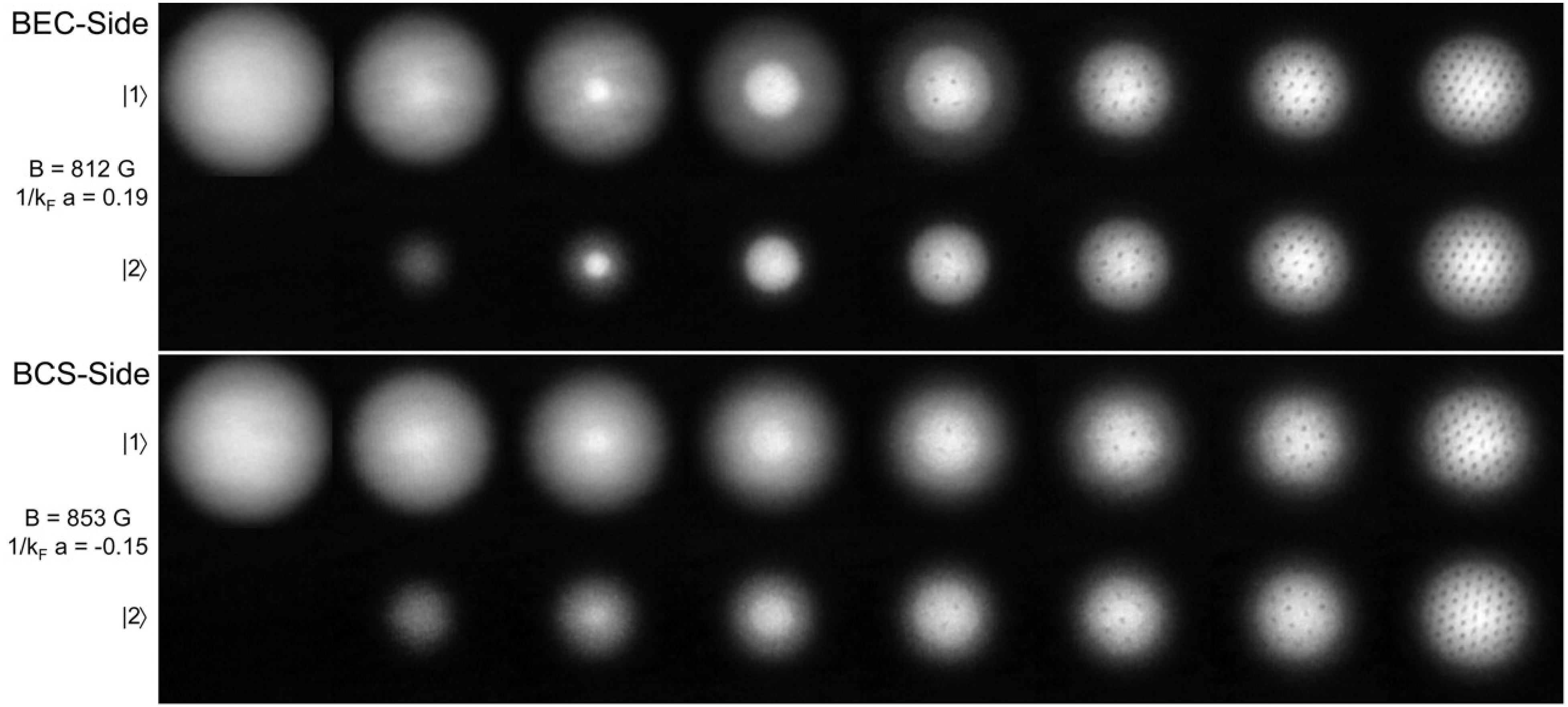


Phase separated unitary SF/ fully polarized ideal gas

What happens to a polarized Fermi superfluid? (II)

The MIT experiment:

M. W. Zwierlein et al, Science 492 **311** (2006)



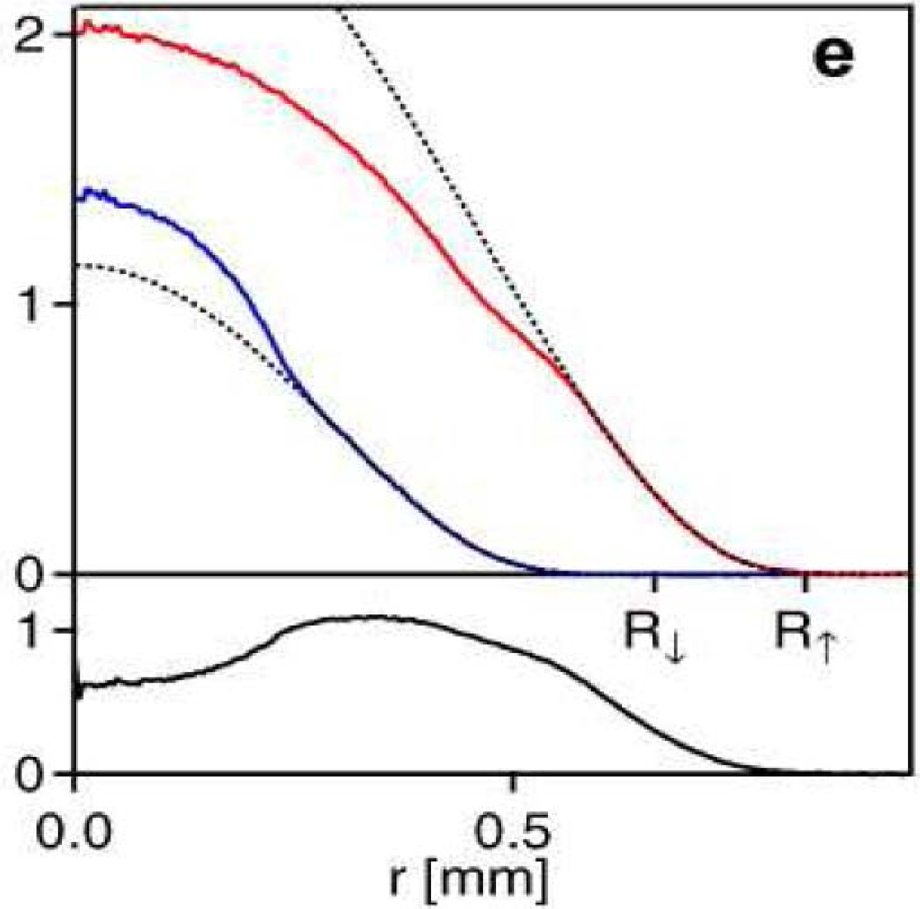
Current experiments in the unitary regime (Rice, MIT):

Evidence for 3 types of phases (LDA)

Unpolarized ($n_\uparrow = n_\downarrow$) : unitary SF

Fully polarized phase ($n_\downarrow = 0$) : ideal gas

Partially polarized phase ($n_\uparrow > n_\downarrow > 0$) : ?



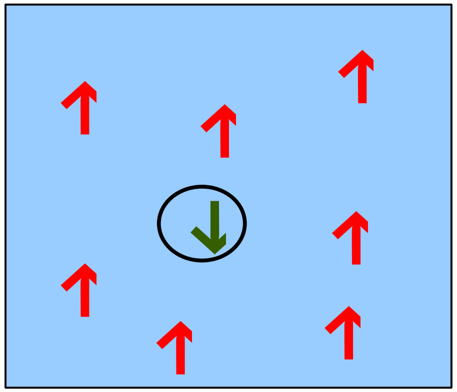
Column integrated densities
M.W. Zwierlein, C.H. Schunck, A. Schirotzek, W. Ketterle, Nature 442, 54-58 (2006)

A problem for theory:

What is the nature of the partially polarized region?
What is its equation of state as a function of concentration? – energy($x=n_\downarrow/n_\uparrow$)?

Assumption: It is a normal (non SF) phase

Idea: energy must have a small x expansion – start from the x=0 (ideal ↑ gas) and add one ↓ atom – how does the energy change (homogeneous system)?



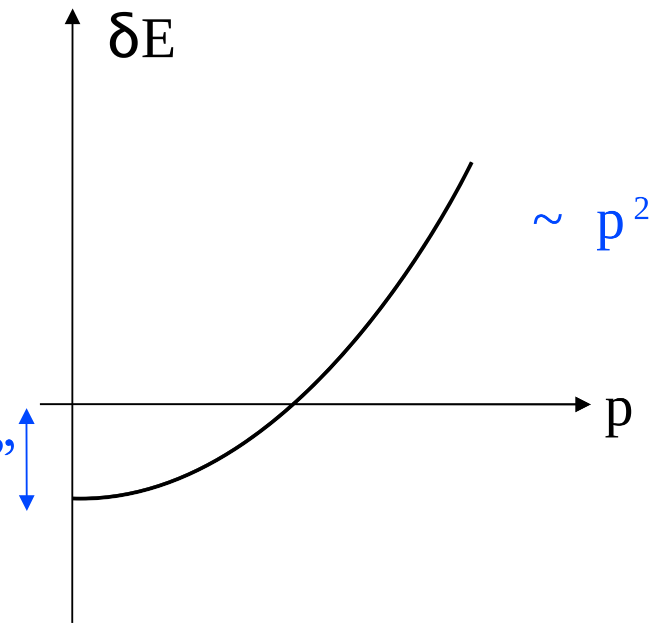
Similar problem in dilute solutions of ^3He in ^4He

– formation of a quasiparticle

$$\delta E = \frac{p^2}{2m^*} - \frac{3}{5} E_{F\uparrow} A$$

effective mass

“binding energy”



So, for a small amount of atoms $x = n_{\downarrow}/n_{\uparrow} \ll 1$

$$\frac{E(x)}{N_{\uparrow}} = \frac{3}{5} E_{F\uparrow} \left(1 - Ax + \frac{m}{m^*} x^{5/3} \right)$$

Ideal gas

Binding energy

Kinetic energy

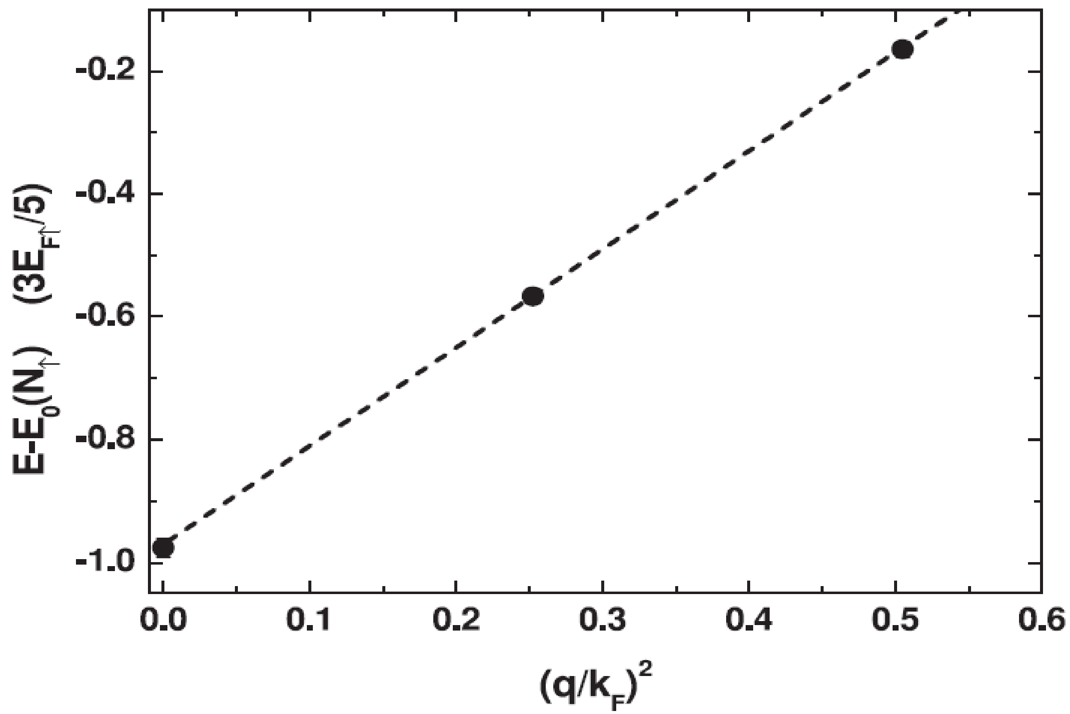
Allows us to predict density profiles and frequency of collective modes

How do we determine the parameters A and m^*?

Can be done using a fixed node diffusion Monte Carlo approach (FN-DMC) – finds the state with lowest energy whose nodal surfaces are those of a T=0 ideal Fermi gas

Results: A = 0.97(2)
m^*/m = 1.04(3)

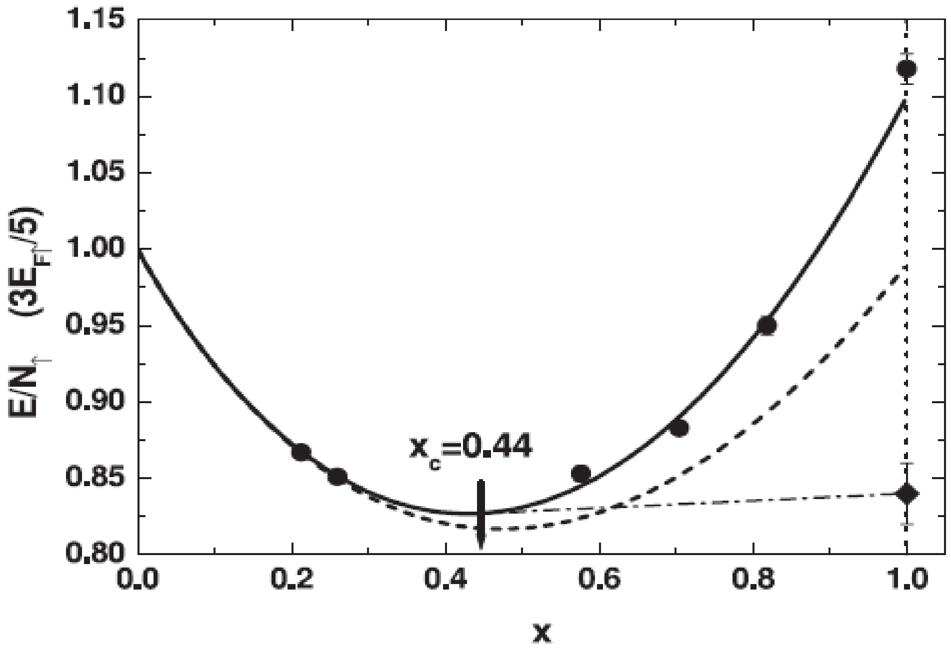
Also: F. Chevy
M. Forbes, A. Bulgac
J. Carlson
R. Combescot et al
S. Pilati, S. Giorgini
N. Prokof'ev et al



Even though the system is strongly interacting, the effective mass is close to the bare mass!

But we can go beyond and compare our equation of state with a Monte Carlo calculation at finite values of x

Again, nodes of the wave function are those of the ideal gas – Incompatible with off-diagonal long range order i.e. cannot describe a superfluid state



Continuous – fit to MC points

Dot-dashed – normal/SF coexistence

Dashed: $$\frac{E(x)}{N_\uparrow} = \frac{3}{5}E_{F\uparrow}\left(1 - Ax + \frac{m}{m^*}x^{5/3}\right)$$

Excellent agreement with energy expression for small x

– importance of the $x^{5/3}$ term

Predicts :

– a fully polarized/partially polarized 2nd order transition at $\mu_\downarrow / \mu_\uparrow = -3/5\ A$

– a partially polarized/superfluid 1st order transition at $x=0.44$

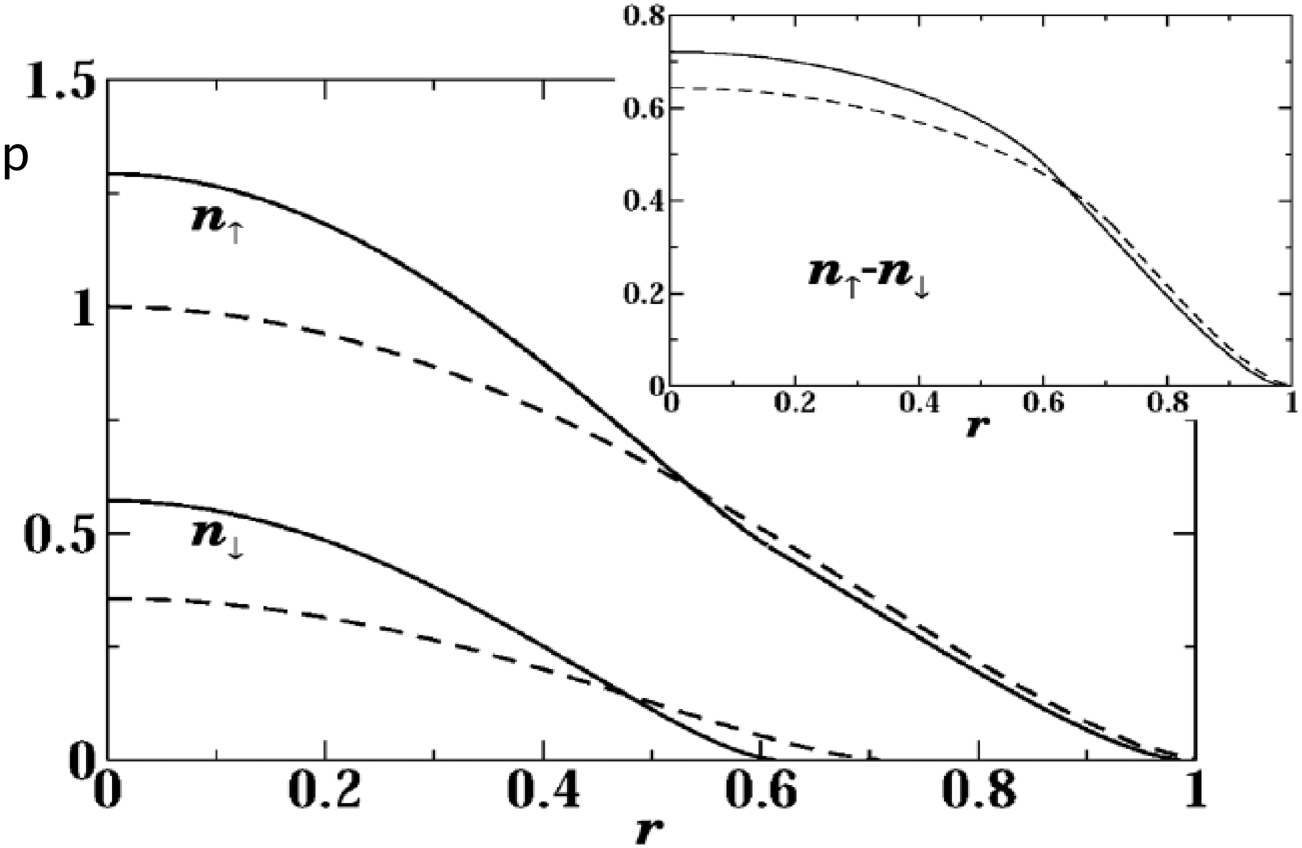
At unitarity a highly polarized gas behaves as a weakly interacting gas of quasiparticles associated with the ↓ atoms.

Trapped case

P – total polarization of the gas

$P = (N_{\uparrow} - N_{\downarrow})/(N_{\uparrow} + N_{\downarrow}) > P_{critical}$:

Density distributions in a trap for interacting and ideal Fermi gases

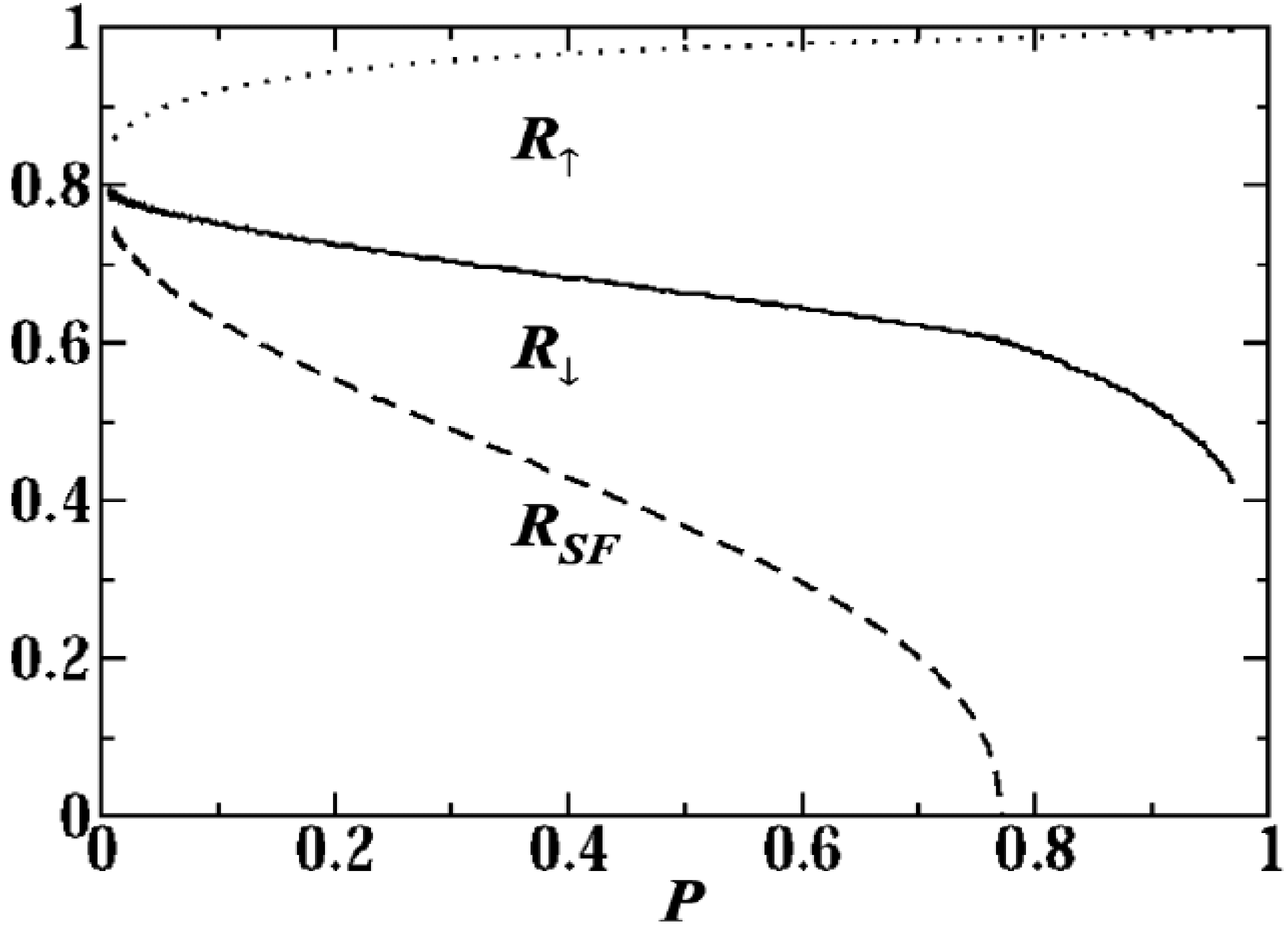


Formation of domains (LDA) of fully polarized, partially polarized and SF

$P < P_{critical}$: formation of SF central core surrounded by normal fluid

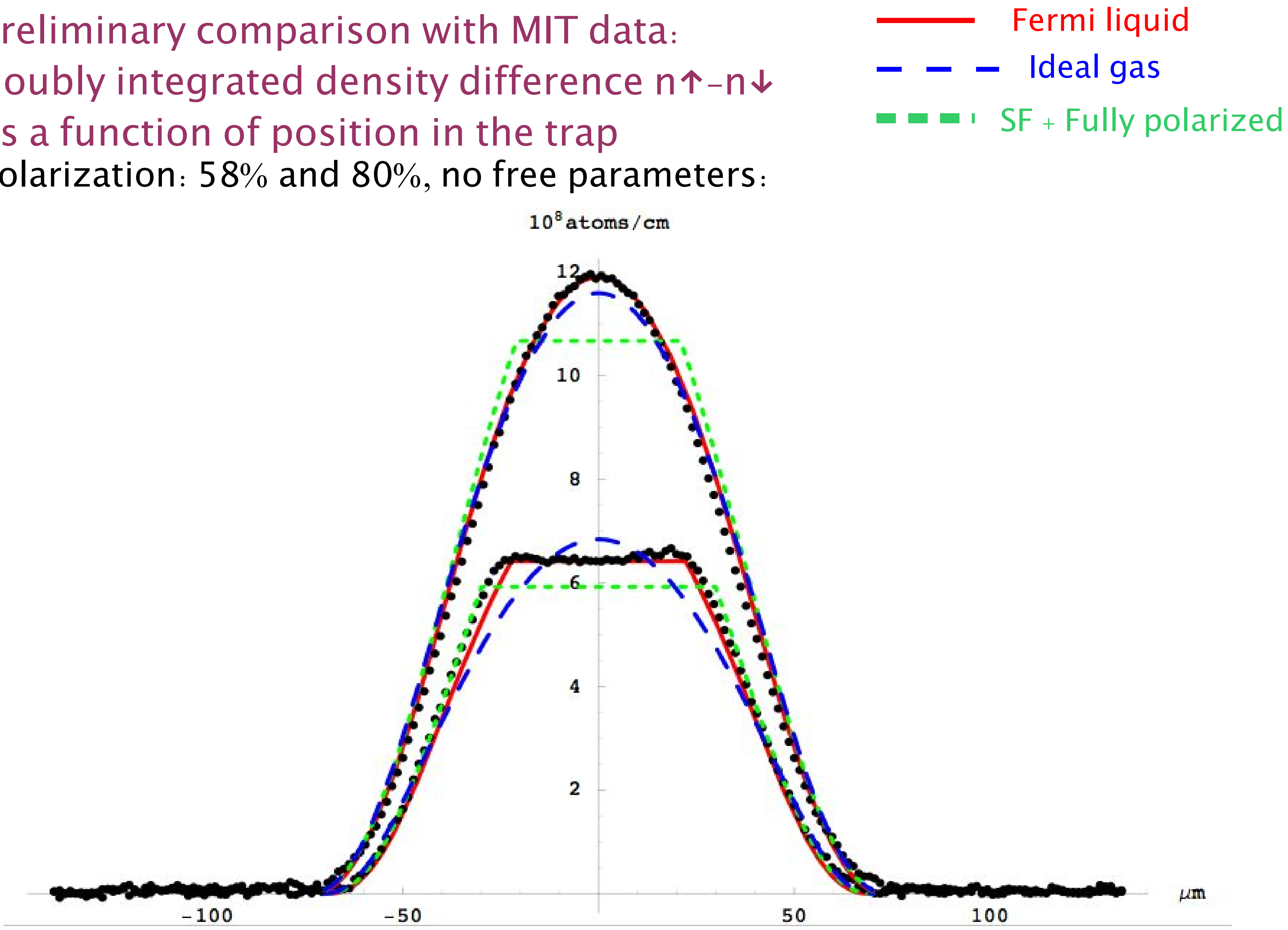
$P_{critical} = 0.77$ in agreement with MIT expts

Thomas-Fermi radii of phases in a trap

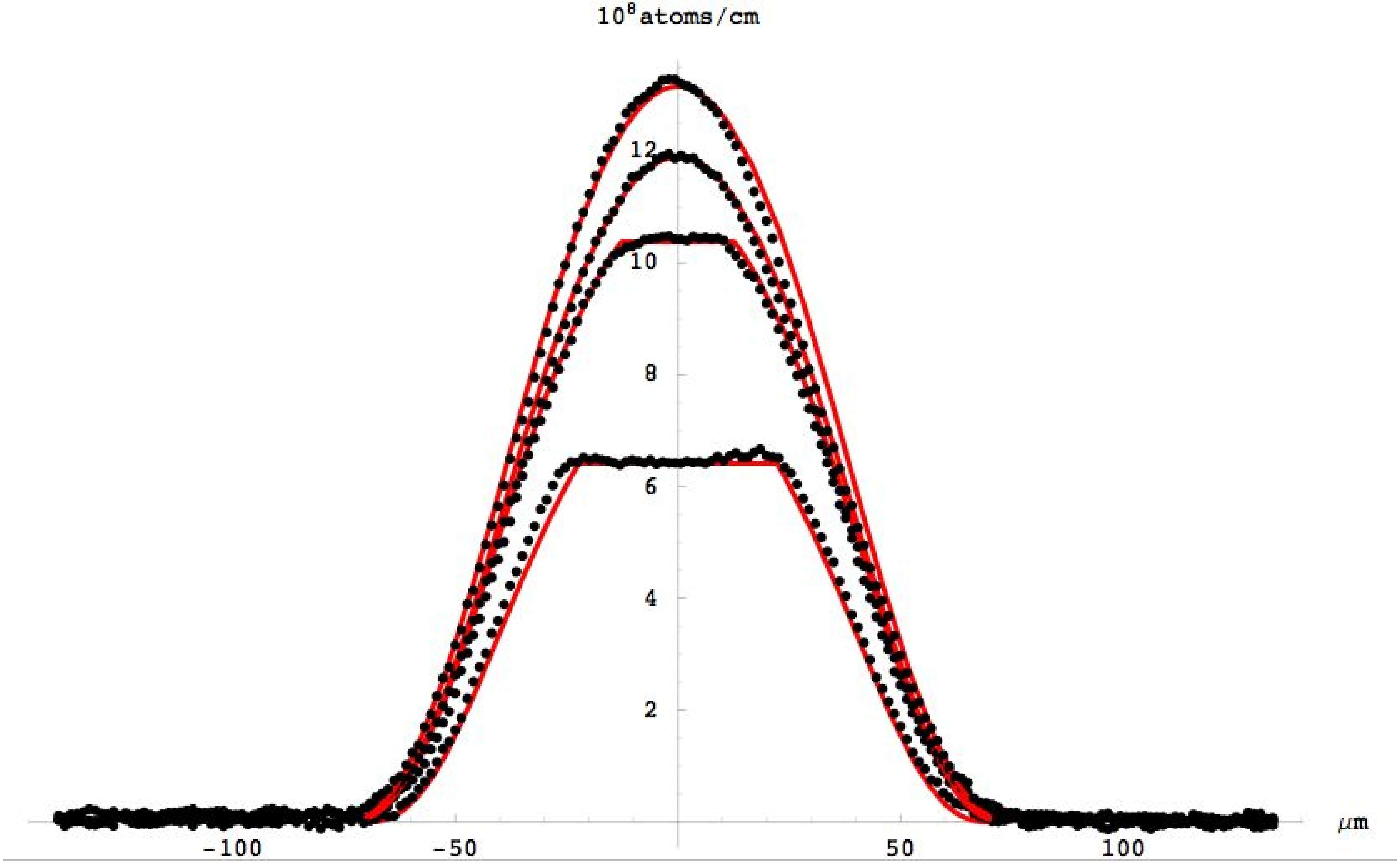


Preliminary comparison with MIT data: doubly integrated density difference n↑–n↓ as a function of position in the trap

Polarization: 58% and 80%, no free parameters:



Further comparison with MIT data for a range of polarizations P(%)=58, 73, 80, 92
As before, no free parameters.



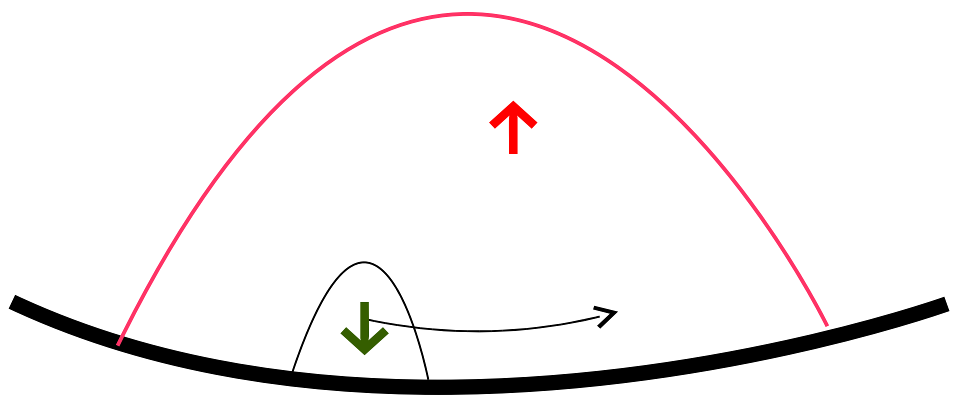
Predictions for collective modes

$$\delta E = \frac{p^2}{2m^*} - \frac{3}{5} E_{F\uparrow} A$$

Quasiparticle sees a modified trapping potential V (1+3/5 A)

Spin dipole oscillation frequency is changed:

$$\omega_i^{\text{dip}} = \omega_i \sqrt{(1 + 3/5A)\frac{m}{m^*}} \simeq 1.23\,\omega_i$$



(needs spin dependent driving force)

Predictions for collective modes (cont)

Quadrupole oscillation (spin independent driving force)

Quadrupole drive excites two modes,
at 2w, dominated by the majority atoms
at 2.46w, mainly oscillation of minority atoms

The amplitude of the motion of the minority atoms will be the same for both modes - beating of 2w and 2.46w easily visible

Damping of the modes:
(ongoing collab. with C. Pethick)

$$\frac{\hbar}{\tau} \propto \frac{T^2}{T_{F\uparrow}}$$

Our theory:

We propose that the unitary Fermi gas at high polarizations is a normal Fermi liquid composed of weakly interacting quasiparticles associated with the ↓ atoms.

We predict –

their binding energy and effective mass as well as the full equation of state as a function of concentration.

a 1st order normal/SF phase transition at $x=0.44$ corresponding to a total polarization of $P_{critical}=0.77$

a shift in the frequency of the spin dipole mode of 1.23 and a minority quadrupole mode at 2.46w

We don't yet know if there are polarized superfluids or other exotica
....experiments ongoing....

But – a new direction for research for cold atoms:

Problem of an single impurity in a Fermi liquid

Related to – x-ray edge singularity in metals
– mobility of ions in ^{3}He

Effective mass and binding energy in the crossover
R. Combescot, A. Recati, C.L., F. Chevy, PRL `07
Pilati, Giorgini
Prokof'ev et al
– what is a molecule in a medium?

Unequal masses