Application of the Shear- and Curvature- Vorticity Equations to the Mid-level Mesocyclogenesis

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Motivation and Background

The motivation for this study is the question of how the interaction of an isolated updraft with a vertically-sheared flow results in the formation of a vortex about a vertical axis. Though extensive literature exists on the evolution of the vorticity within an incipient supercell, the evolution of a vortex apparently has not been considered.

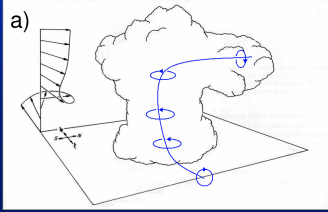
Whenever an isolated updraft exists in a sheared environment, two processes, that happen to be closely entwined with one another, occur: The horizontal vorticity of the ambient air is tilted into the vertical by the updraft, and a non-hydrostatic pressure field develops in and around the updraft. The development of vertical vorticity in the updraft has been shown to be due to the tilting of horizontal shear vorticity into the vertical (e.g., Davies-Jones, 1984). The presence of the perturbation pressure field is tied to rotation (*spin forcing*), deformation (*splat forcing*), and the vertical buoyancy gradient (e.g., Rotunno and Klemp, 1982; Bradshaw and Koh, 1981; Davies-Jones, 2002).

The well-known display of the vortex lines that are tilted into the vertical (Fig. 1a, b), albeit correct from a vorticity perspective, disguises the process of how the initially vertically sheared flow is evolving into a vortex. Also note that in this picture, the azimuthal vorticity surrounding the updraft, is neglected.

Usually, two extremes are considered when analyzing the development of rotation in a supercell thunderstorm. These are an inflow containing purely streamwise vorticity (helical inflow), associated with a (semi-) circular hodograph, and an inflow containing purely crosswise vorticity, associated with a straight-line hodograph.

Davies-Jones (1984) has shown that only in the streamwise-vorticity case, the vorticity will be tilted in such a way that the updraft center and the vertical-vorticity center coincide. In case of crosswise vorticity, the vorticity is accumulated at the flanks of the cell. This behavior can be visualized with the aid of vortex lines that are tilted upward by the updraft (Fig 1 a,b). However, whether the resultant vertical vorticity is associated with shear, curvature, or a coherent vortex cannot be inferred from this analysis.

a)



b)

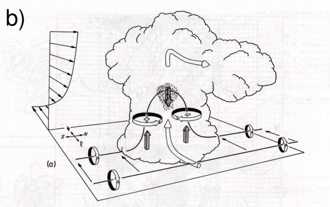


Fig. 1: Vortex lines in streamwise-vorticity (a) and crosswise-vorticity (b) cases. Adapted and modified from Klemp (1987).

Vortex Characteristics

A truly vertical flow is characterized by pure vorticity, zero deformation, and zero divergence. It can readily be shown that such a flow represents a vortex which features shear and curvature vorticity, both contributing to equally large parts. Such a flow is said to exhibit „solid-body“ rotation. The cores of most natural vortices strongly resemble that of a „solidly“ rotating fluid. In any event, a vortex requires the presence of both, shear and curvature vorticity. I.e., mere shear or mere curvature vorticity are not associated with a vortex. Sheared motion is a superposition of shearing deformation and solid-body rotation.

Also note that vorticity cannot be used to diagnose a vortex. Vorticity will be present even if there is mere wave motion (curvature vorticity), or if there is sheared motion (shear vorticity). The formal definition and diagnosis of a vortex happens to be quite complicated (e.g., Haller, 2005).

Shear and curvature vorticity

Vorticity can be decomposed into shear and curvature the components, as mentioned above. With the assumption of no baroclinic generation of vertical vorticity, and a vanishing Coriolis parameter, the tendency equations for the vertical shear and curvature vorticity in height coordinates are given by:

$$\frac{D\zeta_c}{Dt} = -\zeta_c(\nabla_h\cdot\mathbf{v}) + \boldsymbol{\omega}_{sw}\cdot\nabla_h w - c_p\theta\frac{\partial^2\pi}{\partial n\partial s} + c_p\theta\frac{1}{V}\frac{\partial V}{\partial s}\frac{\partial\pi}{\partial n}$$

$$\frac{D\zeta_s}{Dt} = -\zeta_s(\nabla_h\cdot\mathbf{v}) + \boldsymbol{\omega}_{cw}\cdot\nabla_h w + c_p\theta\frac{\partial^2\pi}{\partial n\partial s} - c_p\theta\frac{1}{V}\frac{\partial V}{\partial s}\frac{\partial\pi}{\partial n}$$

$\zeta_c = \frac{V}{r_c}$	*curvature vorticity*
$\zeta_s = -\frac{\partial V}{\partial n}$	*shear vorticity*
$\mathbf{v} = (u,v)$	*horizontal velocity vector*
$\boldsymbol{\omega}_{sw} = -V\frac{\partial\alpha}{\partial s}\mathbf{s}$	*streamwise vorticity*
$\boldsymbol{\omega}_{cw} = \frac{\partial V}{\partial z}\mathbf{n}$	*crosswise vorticity*
$\pi = \left(\frac{p}{p_0}\right)^{\kappa}$	*Exner function*
$V = \lVert\mathbf{v}\rVert$	*velocity magnitude*
$w = \frac{Dz}{Dt}$	*vertical velocity*

See table on the right for the meaning of the variables. (s,n) are the unit vectors tangential and normal to the streamlines, respectively. These equations are similar to the common vorticity equation, in that they contain divergence terms (1st terms of the rhs) and tilting terms (2nd terms on the rhs). The last two terms only differ in the signs, which identifies them as *interchange* or *conversion* terms.

Important conclusions are that

- Convergence cannot change the nature of the vorticity (shear, curvature)
- Vertical shear vorticity results if crosswise vorticity is tilted
- Vertical curvature vorticity results if streamwise vorticity is tilted.

This implies that neither in the streamwise, nor in the crosswise vorticity case, a vertical vortex results from tilting alone.

Interpretation

With the aid of these results, the details of the tilting process can be elucidated. Assume an unstable stratification (decreasing entropy with height) and let the isentropes be perturbed by an axisymmetric convective updraft, as showd in Fig. 2. On every isentropic surface, the flow is going straight atop the humps in the individual isentropes (as in Davies-Jones, 1984; Davies-Jones, 2000).

Making a horizontal cross section through the stack of perturbed isentropes (Fig. 2) yields an onion-like structure, with increasing entropy towards the center.

In case of a straight-line hodograph, only the velocity magnitude varies on the different isentropes. This doesn't change as the flow is passing over the isentropic "humps". Looking at the horizontal velocity components in the cross section, it becomes apparent that all vertical vorticity is manifest as shear (blue arrows in Fig. 3a). If the ambient flow is helical, i.e., if the wind veers with height from easterly to westerly directions, the cross section reveals that only curvature vorticity is created by tilting (Fig. 3b), as predicted by the shear- and curvature-vorticity equations. The blue color indicates cyclonic vorticity, the red color indicates anticyclonic vorticity. These results are entirely consistent with Davies-Jones's helicity concept from 1984.

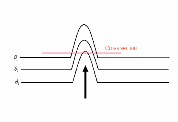
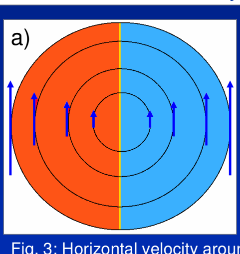


Fig. 2: Horizontal cross section through isentropes perturbed by a convective updraft.

a)



b)

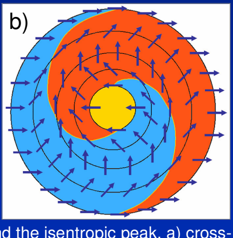


Fig. 3: Horizontal velocity around the isentropic peak. a) cross-wise Vorticity environment; b) streamwise-vorticity environment.

Conclusion and Further Research

In order to sustain a vortex in the supercell's updraft, it is concluded that part of the shear vorticity has to be converted to curvature (crosswise case) and that part of the curvature vorticity has to be converted to shear (streamwise case). This implies that the conversion terms are instrumental in the genesis of a vortex in the incipient Supercell. It is suggested that the sign of the conversion terms in the right-moving storm split member in the straight-line case is opposite to that in the single right mover growing in a streamwise-vorticity environment. Soon, results of numerical simulations which are currently being carried out, will be presented, showing the behavior of the conversion terms in both cases.

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References

Bradshaw, P., Y. M. Koh, 1981: A note on Poisson's equation in a turbulent flow, *phys. Fluids*, **24**, 777; Davies-Jones, R. P., 1984: Streamwise vorticity: The origin of udraft rotation in supercell storms, *J Atmos. Sci.*, **41**, 2991-3006; Davies-Jones, R. P., 2000: A Lagrangian model for the baroclinic genesis of mesoscale vortices. Part I: Theory, *J. Atmos. Sci.*, **57**, 715-763; Davies-Jones, R. P., 2002: Linear and nonlinear propagation of supercell storms, *J. Atmos. Sci.*, **59**, 3178-3205; Haller, G., 2005: An objective definition of a vortex, *J. Fluid Mech.*, **525**, 1-26; Klemp, J. B., 1987: Dynamics of tornadic thunderstorms, *Annu. Rev. Fluid Mech.*, **19**, 369-402; Rotunno, R., J. B. Klemp, 1982: The influence of the shear-induced pressure gradient on thunderstorm motion, *Mon. Weather Rev.*, **110**, 136-151