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Synchronization in coupled complex Systems

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Synchronization in coupled complex systems

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Toolbox TOCSY

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Outline

- Introduction
- Complex synchronization in simple geometry
  - Phase-coherent complex systems
  - Applications
- Synchronization in non-phase coherent systems: **phase and/vs. generalized synchronization**
  - Concepts of curvature and recurrence
  - Applications
- Conclusions

Synchronization

Greek origin:

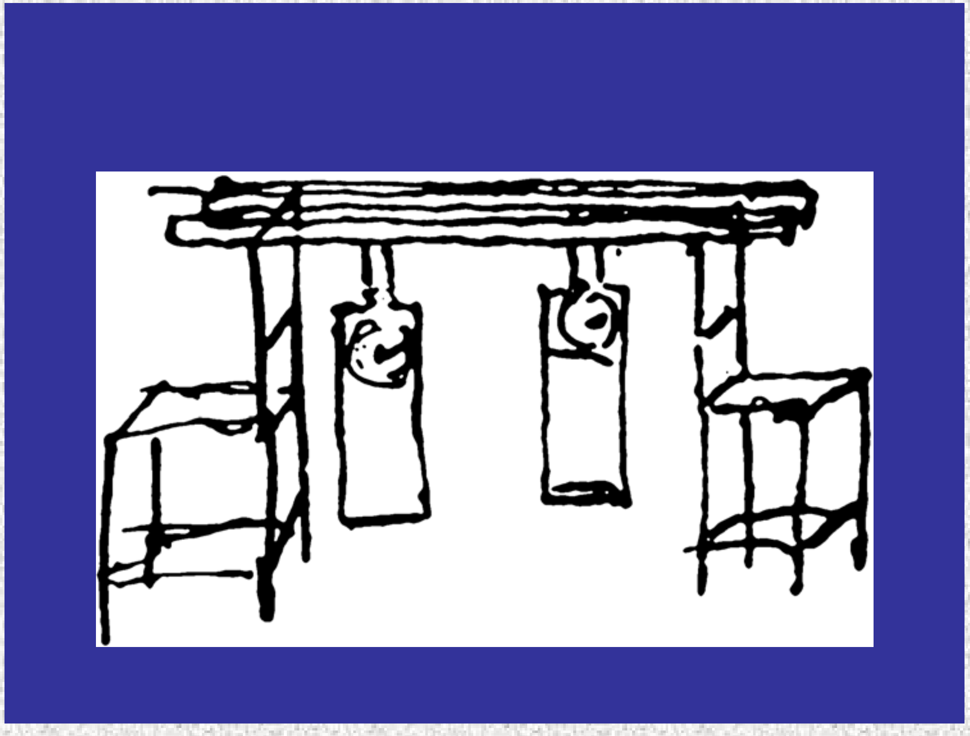
Σὺγ χρόνος – sharing a common
property in time

Nonlinear Sciences

Start in 1665 by Christiaan Huygens:

Discovery of **phase synchronization,** called **sympathy**

Huygens´-Experiment



Pendulum Clocks

- Christiaan Huygens:

  Pendelum clocks hanging at the same wooden beam (half-timber house)

  It is quite worth noting that when we suspended two clocks so constructed from two hooks imbedded in the same wooden beam, the motions of each pendulum in opposite swings were so much in agreement that they never receded the last bit from each other...Further, if this agreement was disturbed by some interference, it reastablished itself in a short time...after a careful examination I finally found that the cause of this is due to the motion of the beam, even though this is hardly perceptible (Huygens, 1673)

Modern Example: Mechanics

London´s Millenium Bridge

- pedestrian bridge
- 325 m steel bridge over the Themse
- Connects city near St. Paul´s Cathedral with Tate Modern Gallery

Big opening event in 2000 -- movie

Bridge Opening

- Unstable modes always there
- Mostly only in vertical direction considered
- Here: extremely strong unstable lateral Mode – If there are sufficient many people on the bridge we are beyond a threshold and synchronization sets in

  (Kuramoto-Synchronizations-Transition, book of Kuramoto in 1984)

Supplemental tuned mass dampers to reduce the oscillations

GERB Vibration Control Systems

GERB Schwingungsisolierungen GmbH, Berlin/Essen

Examples: Sociology, Biology, Acoustics, Mechanics

- Hand clapping (common rhythm)
- Ensemble of doves (wings in synchrony)
- Mexican wave
- Menstruation (e.g. female students living in one room in a dormitory)
- Organ pipes standing side by side – quenching or playing in unison (Lord Rayleigh, 19th century)
- Fireflies in south east Asia (Kämpfer, 17th century)
- Crickets and frogs in South India

Types of Synchronization in Complex Processes

- **phase synchronization**

  (Rosenblum, Pikovsky, Kurths, 1996)

  phase difference bounded, a zero Lyapunov exponent becomes negative (phase-coherent)

- **generalized synchronization**

  (Abarbanel et al., 1995)

  a positive Lyapunov exponent becomes negative, amplitudes and phases interrelated

- **complete synchronization** (Fujisaka, 1984)

Necessary Conditions for Synchronization

- **Two Oscillators** (or more; best: self-sustaining)
- **Coupling**: Master – Slave, or mutually coupled
- **Starting**: (slightly) different systems

  (initial conditions, internal frequencies)
- **Goal**: becoming identical in a main property or sharing some important behaviour due to forcing or interaction

  (becoming identical, adjusting their phases…)

Phase Synchronization in Complex Systems

Most systems not simply periodic

➔ Synchronization in complex (non-periodic) systems

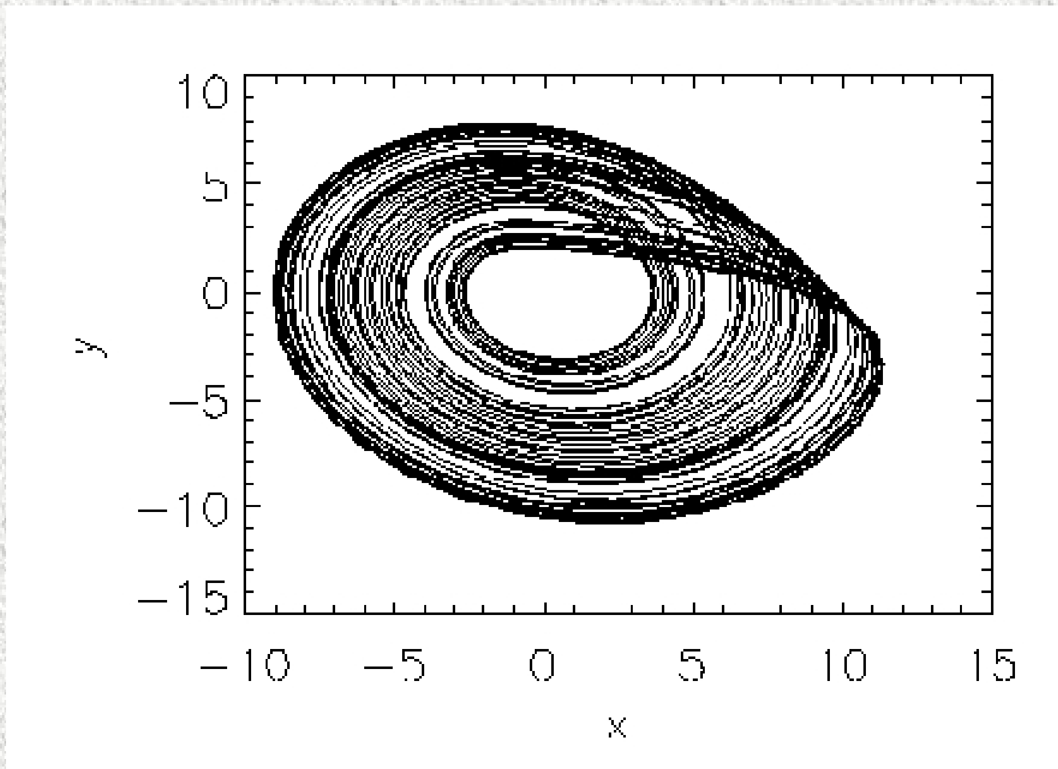
Interest in **Phase Synchronization**

How to retrieve a phase in complex dynamics?

Phase Definitions in Coherent Systems

Rössler Oscillator – 2D Projection

Phase-coherent (projection looks like a smeared limit cycle, low diffusion of phase dynamics)



Phase dynamics in periodic systems

- Linear increase of the phase

$$\varphi(t) = t\ \omega$$

$\omega = 2\,\Pi / T$ – frequency of the periodic dynamics

T – period length

➔ $\varphi(t)$ increases $2\,\Pi$ per period

$$d\,\varphi(t) / d\,t = \omega$$

Phase Definitions

Analytic Signal Representation (Hilbert Transform)

$$\psi(t) = s(t) + \mathrm{j}\tilde{s}(t) = A(t)\mathrm{e}^{\mathrm{j}\phi(t)}$$

$$\tilde{s}(t) = \frac{1}{\pi}\,\mathrm{P.V.}\int_{-\infty}^{\infty} \frac{s(\tau)}{t-\tau}\,\mathrm{d}\tau$$

Direct phase

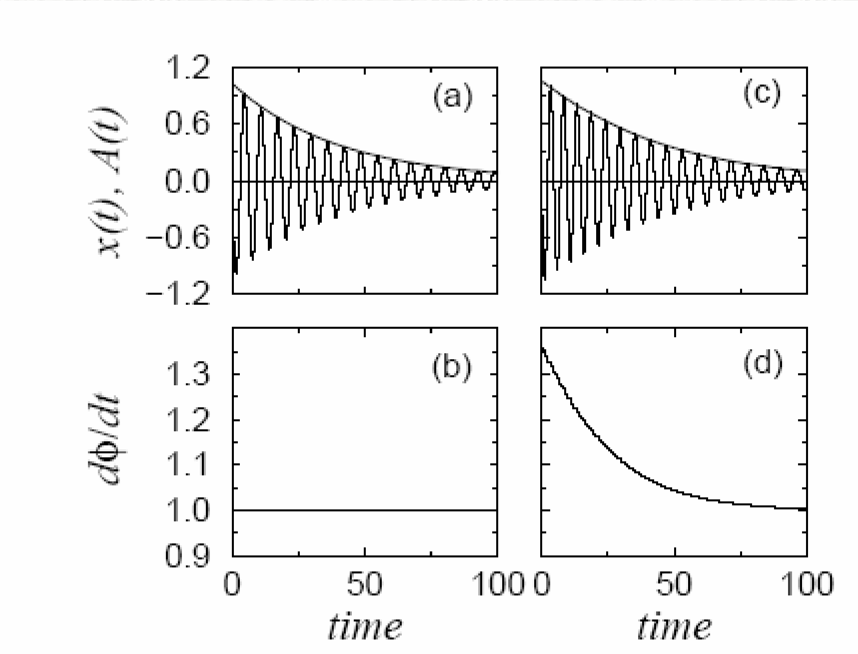
$$\phi(t) = \arctan\left(y(t)/x(t)\right)$$

Phase from Poincare´ plot

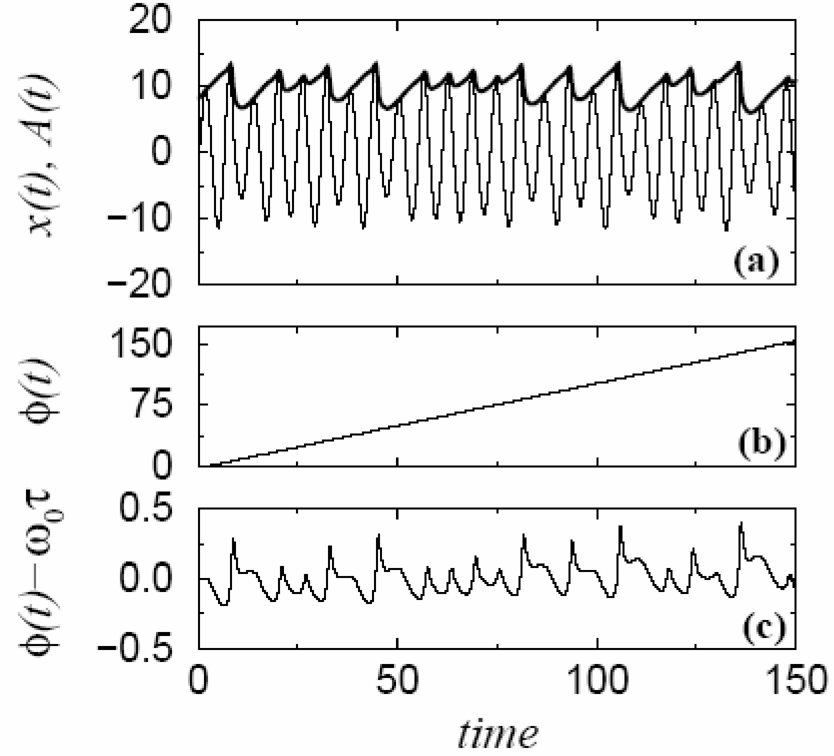
$$\phi(t) = 2\pi k + 2\pi \frac{t-\tau_k}{\tau_{k+1}-\tau_k} \qquad (\tau_k < t < \tau_{k+1}).$$

(Rosenblum, Pikovsky, Kurths, Phys. Rev. Lett., 1996)

Hilbert transform for periodic signals



Hilbert transform for chaotic signals



Phase for coherent chaotic oscillators

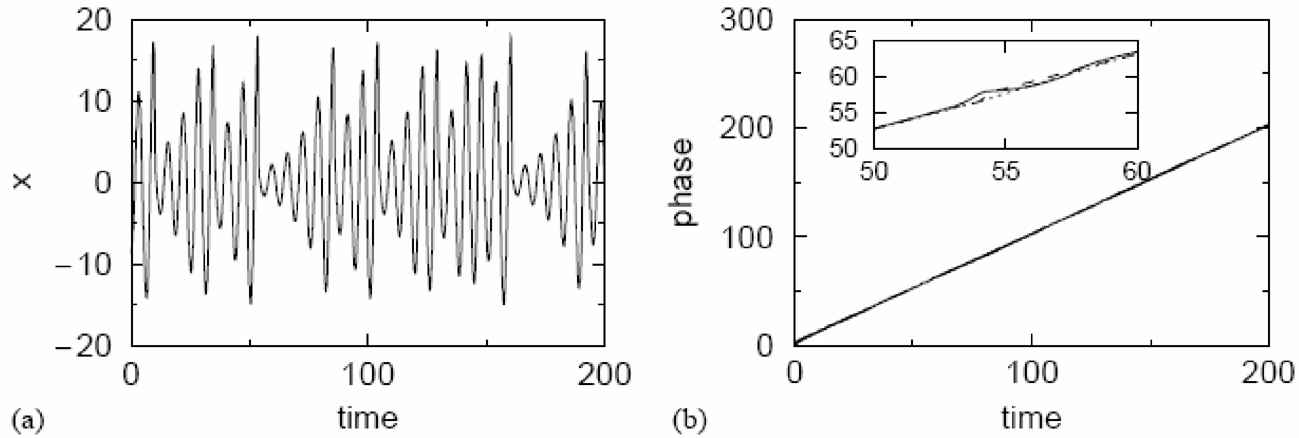


Fig. 3.3. (a) Chaotic signal $x(t)$ of the chaotic Rössler oscillator. (b) Phase of the chaotic signal. Solid line: phase of Eq. (3.5); dashed line: phase of Eq. (3.7); and dotted line: phase of Eq. (3.8).

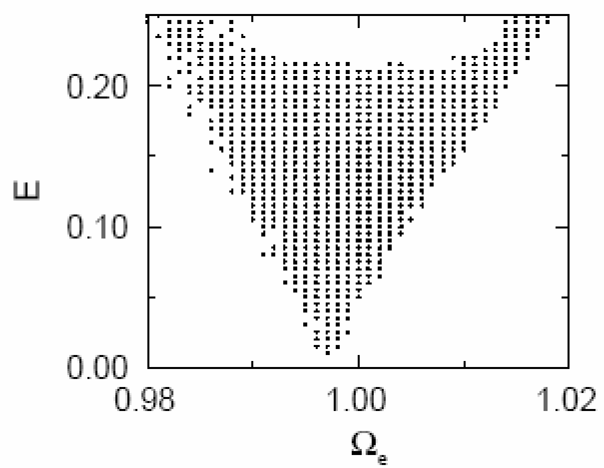
Phase dynamics and phase synchronization phenomena very similar in periodic and phase-coherent chaotic systems,
e.g. one zero Lyapunov exponent becomes negative

Synchronization due to periodic driving

$$\dot{x} = -\omega y - z + E\sin(\Omega_e t)$$

$$\dot{y} = \omega x + a y\ ,$$

$$\dot{z} = f + z(x - c)$$



Synchronization due to periodic driving

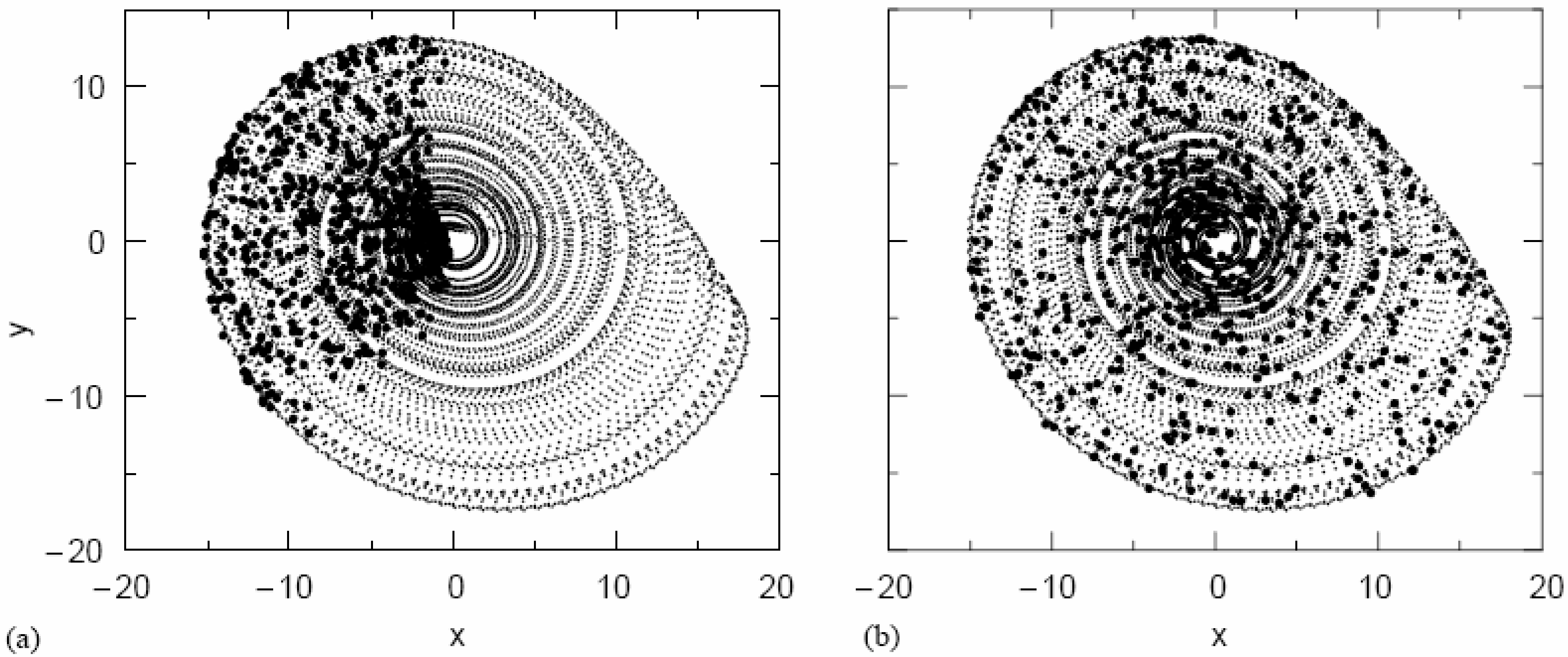
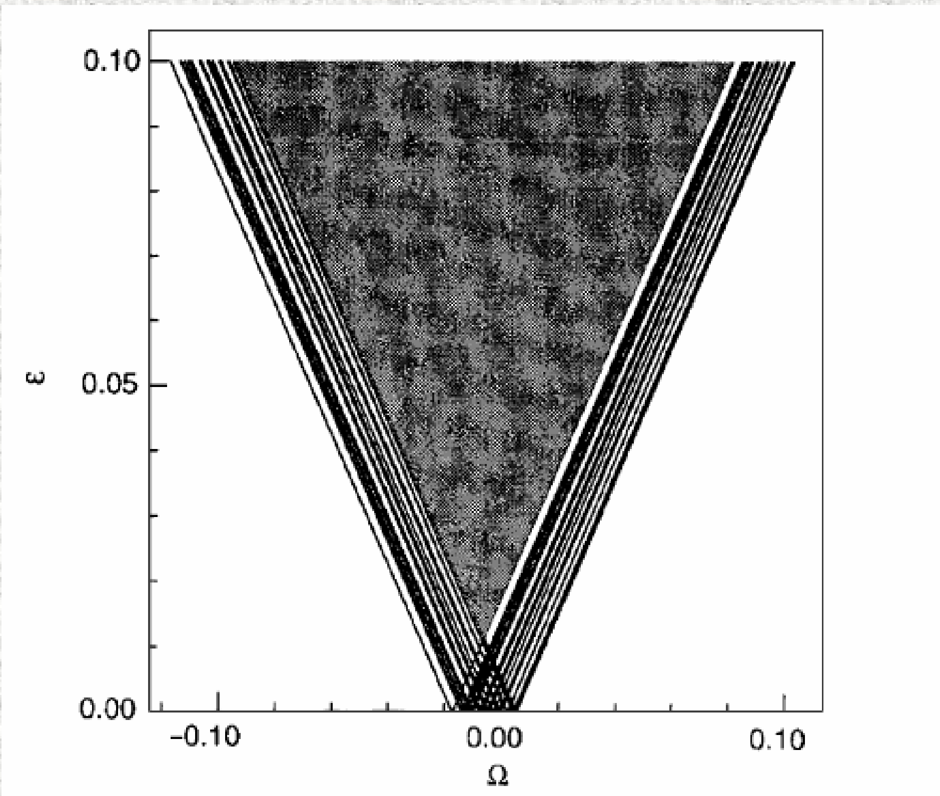


Fig. 3.5. Stroboscopic plot of the Rössler system state (x, y) (filled cycles) at each period of the driving signal (Eq. (3.4)). The dotted background is the unforced chaotic attractor. (a) $E = 0.15$, $\Omega_e = 1.0$, phase is synchronized. (b) $E = 0.15$, $\Omega_e = 1.02$, phase is not synchronized.

Understanding synchronization by means of unstable periodic orbits



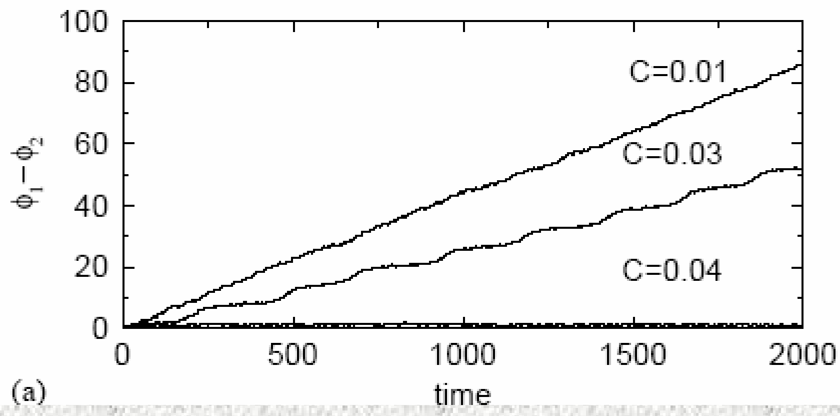
Phase-locking regions for periodic orbits with periods 1-5; overlapping region – region of full phase synchronization (dark, = natural frequency of chaotic system – ext force)

Synchronization of two coupled non-identical chaotic oscillators

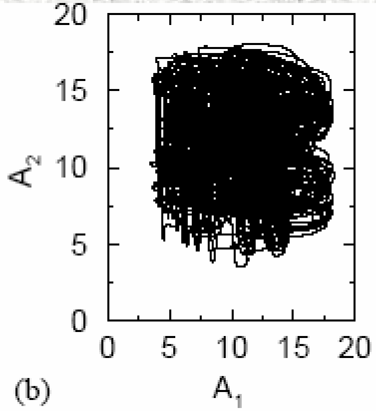
$$\dot{x}_{1,2} = -\omega_{1,2} y_{1,2} - z_{1,2} + C(x_{2,1} - x_{1,2}) ,$$

$$\dot{y}_{1,2} = \omega_{1,2} x_{1,2} + a y_{1,2} ,$$

$$\dot{z}_{1,2} = f + z_{1,2}(x_{1,2} - c) ,$$



(a)



(b)

Phases are synchronized BUT Amplitudes almost uncorrelated

Two coupled non-identical oscillators

$$\phi = \arctan(y/x), \quad A = (x^2 + y^2)^{1/2} ,$$

we get

$$\dot{A}_{1,2} = aA_{1,2}\sin^2\phi_{1,2} - z_{1,2}\cos\phi_{1,2} + C(A_{2,1}\cos\phi_{2,1}\cos\phi_{1,2} - A_{1,2}\cos^2\phi_{1,2}) ,$$

$$\dot{\phi}_{1,2} = \omega_{1,2} + a\sin\phi_{1,2}\cos\phi_{1,2} + z_{1,2}/A_{1,2}\sin\phi_{1,2}$$
$$- C(A_{2,1}/A_{1,2}\cos\phi_{2,1}\sin\phi_{1,2} - \cos\phi_{1,2}\sin\phi_{1,2}) ,$$

$$\dot{z}_{1,2} = f - cz_{1,2} + A_{1,2}z_{1,2}\cos\phi_{1,2} .$$

Equation for the slow phase θ: $\phi_{1,2} = \omega_0 t + \theta_{1,2}$

Averaging yields (Adler-like equation, phase oscillator):

$$\frac{\mathrm{d}}{\mathrm{d}t}(\theta_1 - \theta_2) = 2\Delta\omega - \frac{C}{2}\left(\frac{A_2}{A_1} + \frac{A_1}{A_2}\right)\sin(\theta_1 - \theta_2)$$

Synchronization threshold

Fixed point solution (by neglecting amplitude fluctuations)

$$\theta_1 - \theta_2 = \arcsin \frac{4\Delta\omega A_1 A_2}{C(A_1^2 + A_2^2)}$$

Fixed point stable (synchronization) if coupling is larger than

$$C_{\mathrm{PS}} = 4\Delta\omega A_1 A_2/(A_1^2 + A_2^2).$$

Applications in various fields

Lab experiments:

- Electronic circuits (Parlitz, Lakshmanan, Dana...)
- Plasma tubes (Rosa)
- Driven or coupled lasers (Roy, Arecchi...)
- Electrochemistry (Hudson, Gaspar, Parmananda...)
- Controlling (Pisarchik, Belykh)
- Convection (Maza...)

Natural systems:

- Cardio-respiratory system (Nature, 1998...)
- Parkinson (PRL, 1998...)
- Epilepsy (Lehnertz...)
- Kidney (Mosekilde...)
- Population dynamics (Blasius, Stone)
- Cognition (PRE, 2005)
- Climate (GRL, 2005)
- Tennis (Palut)

Cardio-respiratory System

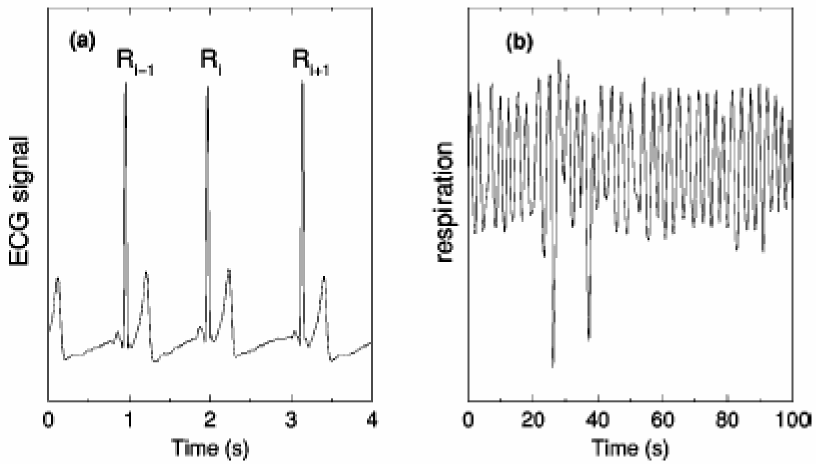


FIG. 1. Short segments of an electrocardiogram with the R peaks marked (a) and of a respiratory signal (b); both signals are in arbitrary units.

Analysis technique: **Synchrogram**

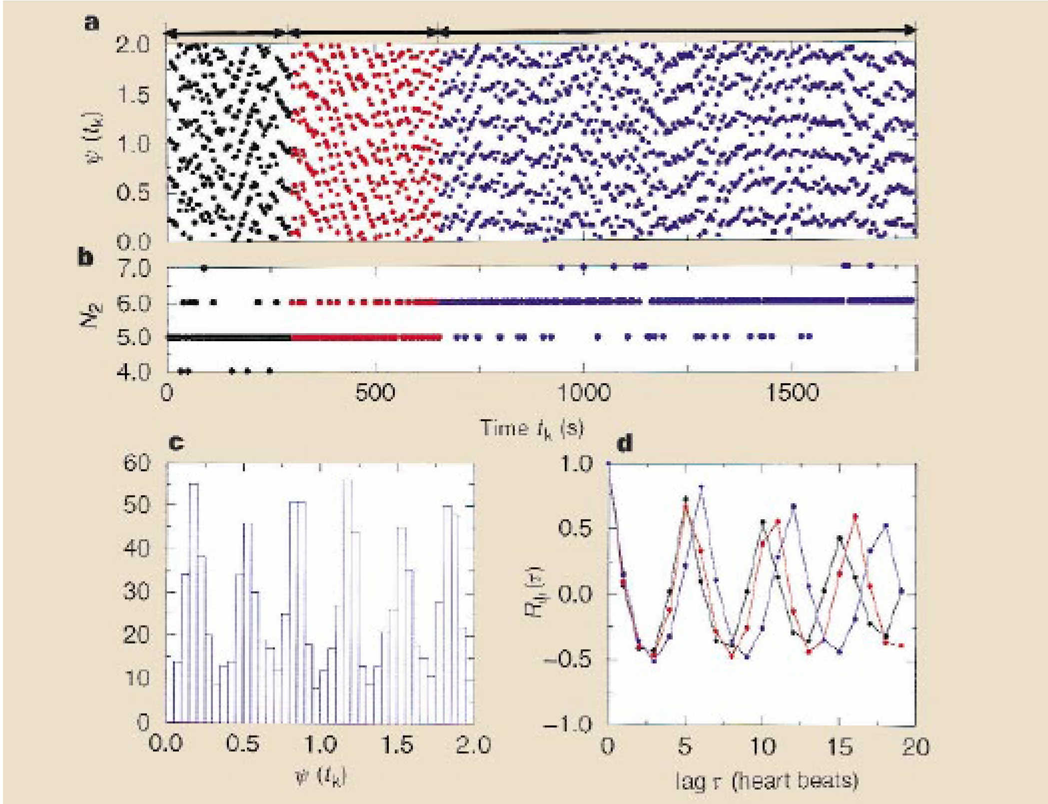


Figure 1 Analysis of cardiorespiratory cycles. a, Cardiorespiratory synchrogram, showing the transition (red) from 5:2 frequency locking (black) to 3:1 phase locking (blue). Each point shows the normalized relative phase of a heartbeat within two adjacent respiratory cycles $\psi(t_k)=(\phi_r(t_k) \bmod 4\pi)/2\pi$. b, Number of heartbeats within two adjacent respiratory cycles. c, Histogram of phases. The six horizontal stripes in the blue region of the CRS result in six well-pronounced peaks in the distribution of phases. d, Autocorrelation function of phases $R_\psi(\tau)=\Sigma_k(\psi(t_k)-<\psi>)(\psi(t_{k+\tau})-<\psi>)/\Sigma_k(\psi(t_k)-<\psi>)^2$. The coloured curves correspond to respective regions.

Schäfer, Rosenblum, Abel, Kurths: Nature, 1998

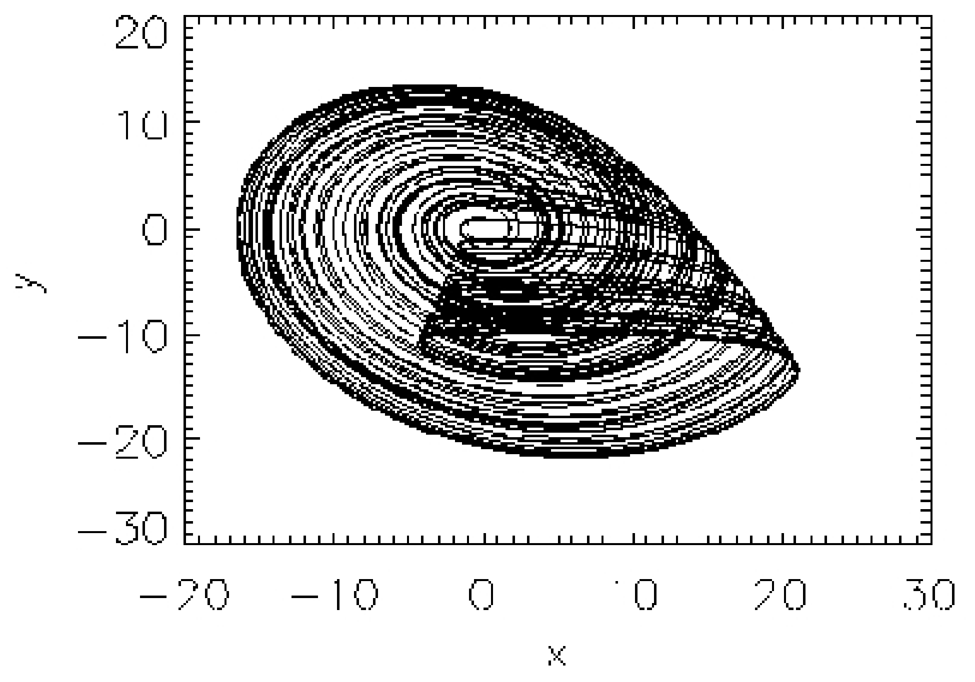
Synchronization in more complex topology

- Systems are often non-phase-coherent (e.g. funnel attractor – much stronger phase diffusion)
- How to study phase dynamics there?
- 1st Concept: **Curvature**

  (Osipov, Hu, Zhou, Ivanchenko, Kurths: Phys. Rev. Lett., 2003)

$$\phi = \arctan \frac{\dot{y}}{\dot{x}}.$$

Roessler Funnel – Non-Phase coherent



Phase basing on curvature

curve $\vec{r}_1 = (u, v)$ the angle velocity at each point is

$$\nu = \frac{ds}{dt}/R,$$

where

$$ds/dt = \sqrt{\dot{u}^2 + \dot{v}^2}$$

is the speed along the curve and

$$R = (\dot{u}^2 + \dot{v}^2)^{3/2}/[\dot{v}\ddot{u} - \ddot{v}\dot{u}]$$

is the radius of the curvature. If $R > 0$ at each point, then

$$\nu = \frac{d\phi}{dt} = \frac{\dot{v}\ddot{u} - \ddot{v}\dot{u}}{\dot{u}^2 + \dot{v}^2},$$

is always positive and therefore the variable ϕ defined as

$$\phi = \int \nu dt = \arctan \frac{\dot{v}}{\dot{u}},$$

Dynamics in non-phase-coherent oscillators

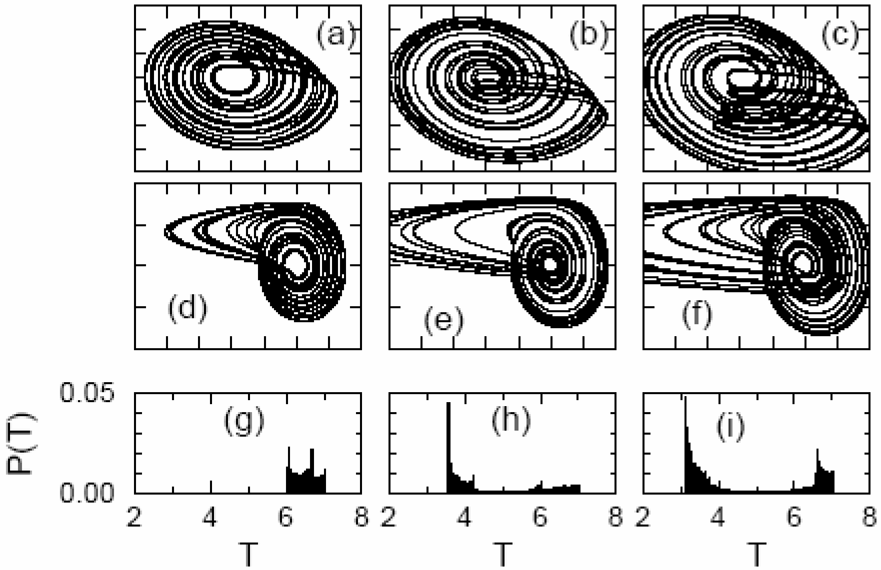
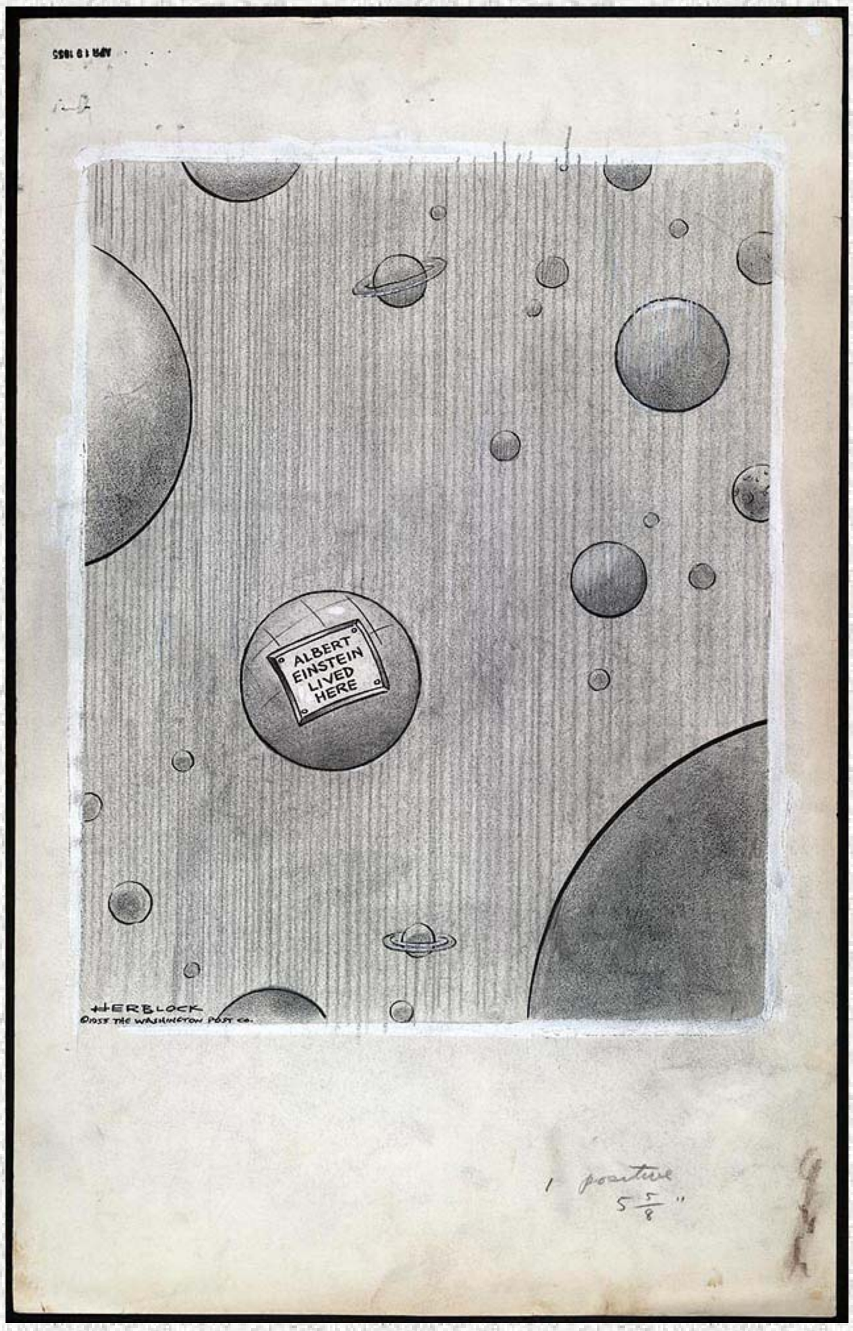


FIG. 1: Upper panel (a,b,c): projections of the attractors of the Rössler systems (1) onto the plane (x, y); middle panel: (d,e,f): projections onto $(\dot{x}, \dot{y})$; lower panel (g,h,i): distribution of the return times T. The parameters are $\omega = 0.98$ and $a = 0.16$ (a,d,g), $a = 0.22$ (b,e,h) and $a = 0.28$ (c,f,i).

Washington Post
19. April 1955

Warum gerade
Albert Einstein?



Explanation of Brownian motion

- A. Einstein: Über die von der molekularkinetischen Theorie der Wärme geforderte Bewegung von in ruhenden Flüssigkeiten suspendierten Teilchen
  (Annalen der Physik 4. Folge, Band 17, 1905, Seite 549-560)

Results

1) Mean squared movement of the particles after a time t:

$$\boldsymbol{<\Delta\, x^2> \sim T\, t \,/\, (\, \acute{\eta}\, r\,) \sim t}$$

T – temperature, ή – viscosity of the fluid,

r – radius of the particles

„Die mittlere Verschiebung ist also proportional der Quadratwurzel aus der Zeit“ (Einstein)

Hence, variance is **not a constant**, but increases with time!!!

2) Diffusion process with $D = <\Delta\, x^2> / (\, 2\, t\,)$

3) estimation of the number of the molecules in the fluid

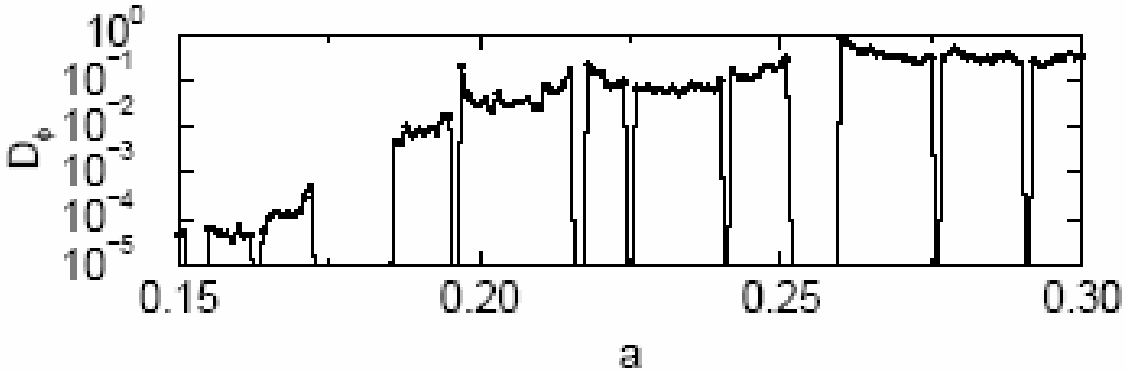


FIG. 2: Phase diffusion coefficient D_ϕ (3) vs a. $\omega = 0.98$.

Three types of transition to phase synchronization

- **Phase-coherent**: one zero Lyapunov exponent becomes negative (small phase diffusion); phase synchronization to get for rather weak coupling, whereas generalized synchronization needs stronger one
- **Weakly non-phase-coherent**: inverse interior crises-like
- **Strongly non-phase-coherent**: one positive Lyapunov exponent becomes negative (strong phase diffusion) – also amplitudes are interrelated

Application: El Niño vs. Indian monsoon

- El Niño/Southern Oscillation (ENSO) – self-sustained oscillations of the tropical Pacific coupled ocean-atmosphere system
- Monsoon - oscillations driven by the annual cycle of the land vs. Sea surface temperature gradient
- ENSO could influence the amplitude of Monsoon – Is there phase coherence?
- Monsoon failure coincides with El Niño

- (Maraun, Kurths, Geophys Res Lett (32, 15709, 2005))

El Niño vs. Indian Monsoon

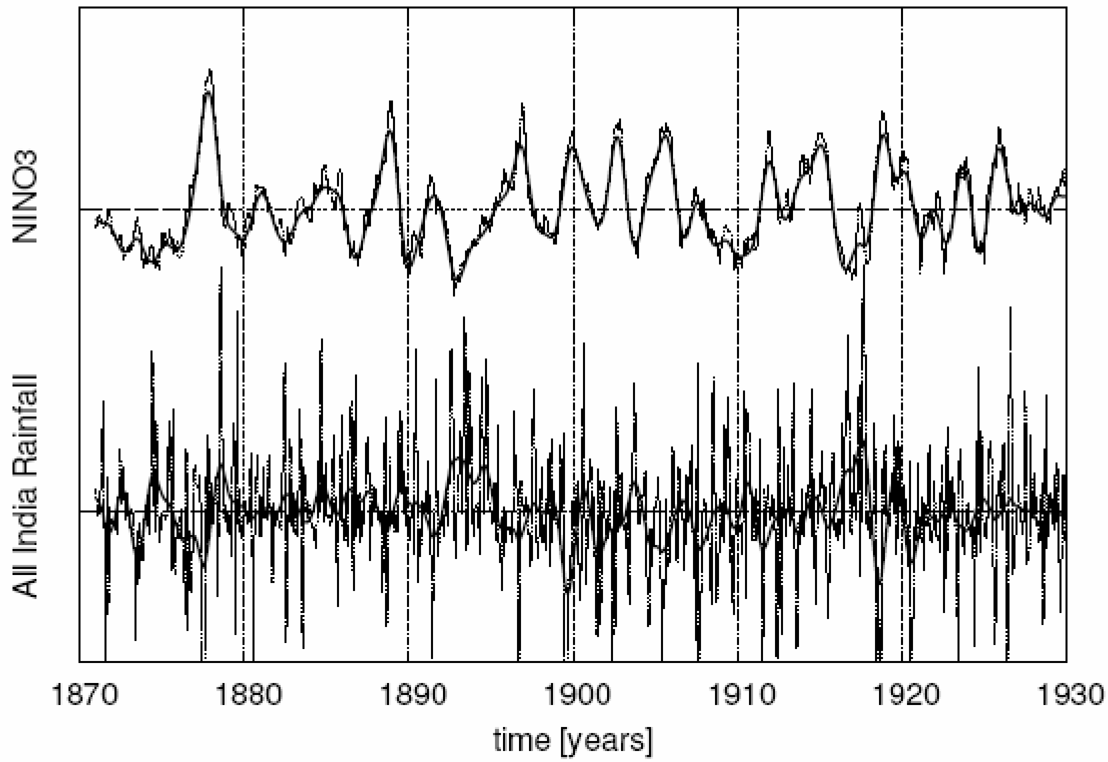


Figure 1. Section of the NINO3 (upper graph) and AIR anomalies (lower graph) time series. The dotted lines depict the raw data, **the solid lines show** the low-pass filtered data used for the further analysis.

El Niño – non phase-coherent

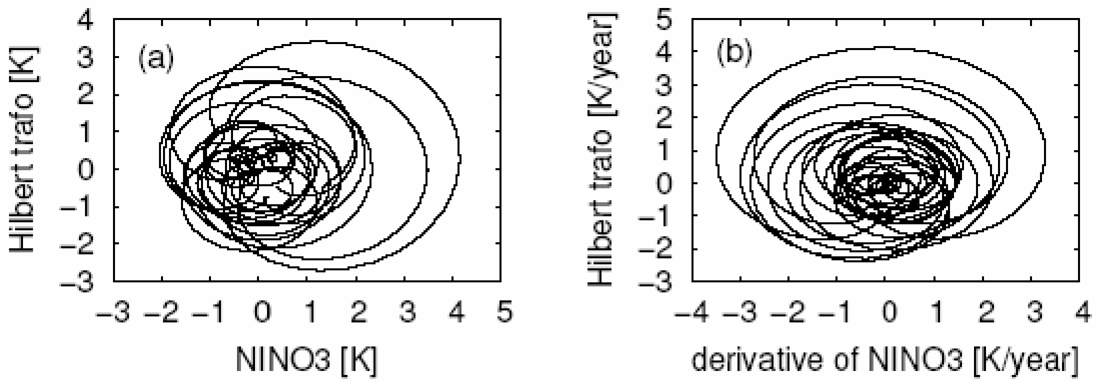


Figure 2. (a) Embedding of low-pass filtered NINO3 time series by Hilbert transformation. Many oscillations are not centered around a common center. (b) The same, but for the time derivative of the NINO3 time series. All pronounced oscillations circle around the origin.

Phase coherence between El Niño and Indian monsoon

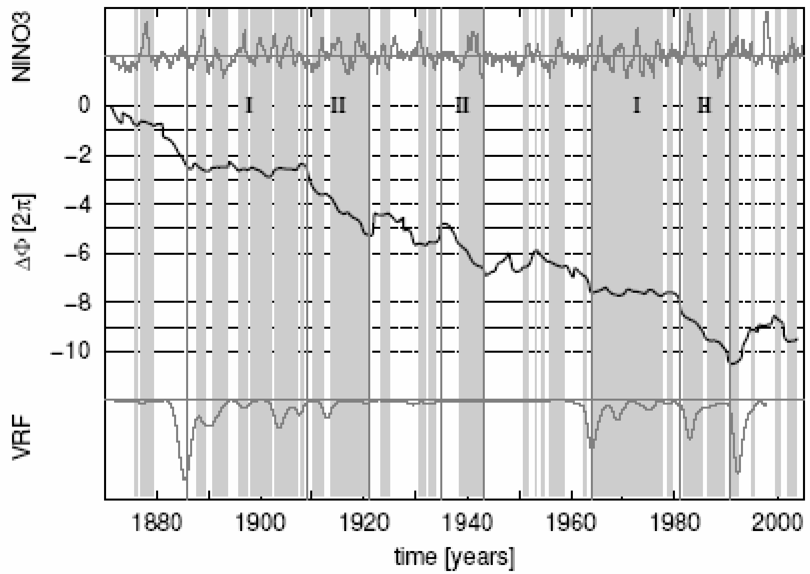


Figure 5. Phase difference of ENSO and Monsoon (black). Grey shading marks intervals of jointly well defined phases. 1886-1908 and 1964-1980 (I): plateaus indicate phase coherence. 1908-1921, 1935-1943 and 1981-1991 (II): Monsoon oscillates with twice the phase velocity of ENSO. During these intervals, both systems exhibit distinct oscillations (NINO3 time series, upper graph). 1921-1935 and 1943-1963: phases are badly defined, both processes exhibit irregular oscillations of low variance (upper graph). Lower graph shows volcanic radiative forcing index (VRF).

Concept of Recurrence

- Curvature ➔ new theoretical insights and also applications, but sometimes problems with noisy data

- Other concept is necessary:

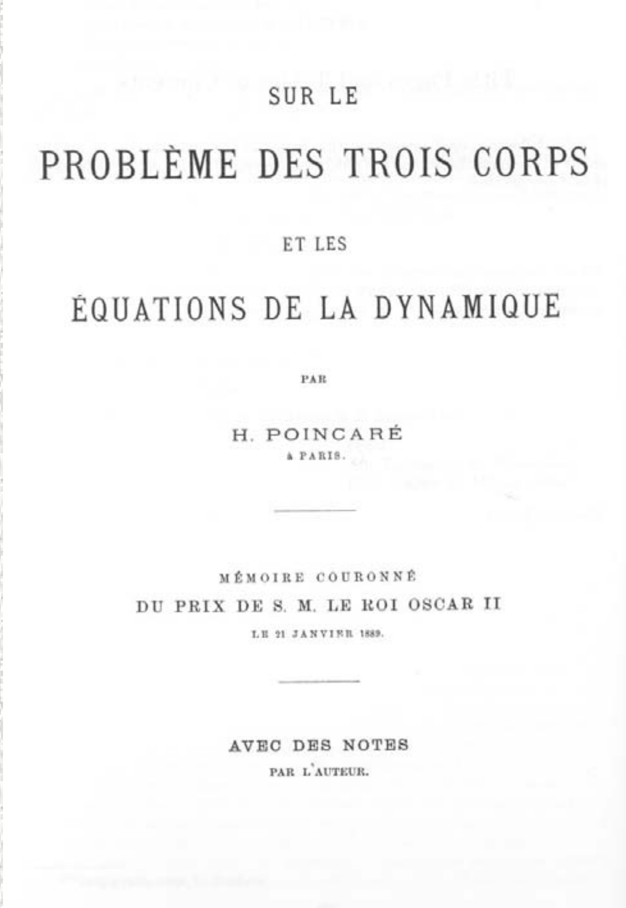
Recurrence

What is CHAOS?

Mathematical price to celebrate the 60th birthday of Oskar II, king of Norway and Sweden, 1889:

„Is the solar system stable?"

- Henri Poincaré (1854-1912)



SUR LE

PROBLÈME DES TROIS CORPS

ET LES

ÉQUATIONS DE LA DYNAMIQUE

PAR

H. POINCARÉ

À PARIS.

MÉMOIRE COURONNÉ

DU PRIX DE S. M. LE ROI OSCAR II

LE 21 JANVIER 1889.

AVEC DES NOTES

PAR L'AUTEUR.

H. Poincare

If we knew exactly the laws of nature and the situation of the universe at the initial moment, we could predict exactly the situation of that same universe at the succeeding moment.

but even if it were the case that the natural laws had no longer any secret for us, we could still only know the initial situation approximately. If that enabled us to predict the succeeding situation with the same approximation, that is all we require, and we should say that the phenomenon had been predicted, that it is governed by laws.

But it is not always so; it may happen that **small differences in the initial conditions produce very great ones in the final phenomena**. A small error in the former will produce an enormous error in the latter. Prediction becomes impossible, and we have the fortuitous phenomenon.

(1903 essay: Science and Method)

Weak Causality

Concept of Recurrence

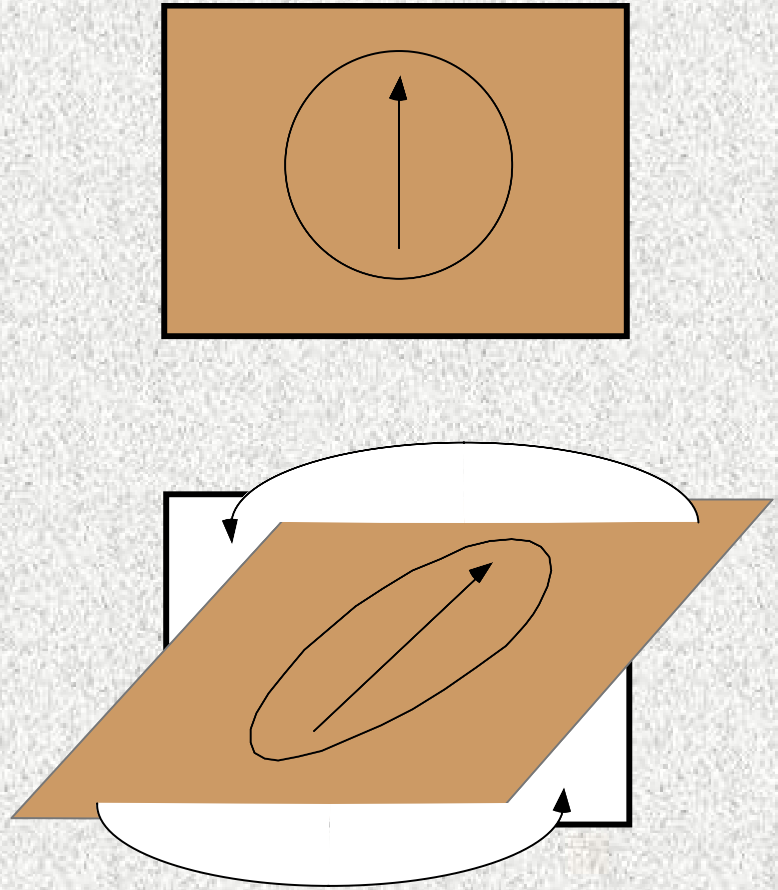
Recurrence theorem:

Suppose that a point P in phase space is covered by a conservative system. Then there will be trajectories which traverse a small surrounding of P infinitely often.
That is to say, in some future time the system will return arbitrarily close to its initial situation and will do so infinitely often.
(Poincare, 1885)

Poincaré‘s Recurrence

Crutchfield 1986,
Scientific American

Arnold‘s cat map



Recurrence plot analysis

- **Recurrence plot**

  $R(i, j) = \Theta(\varepsilon - |x(i) - x(j)|)$

Θ – Heaviside function

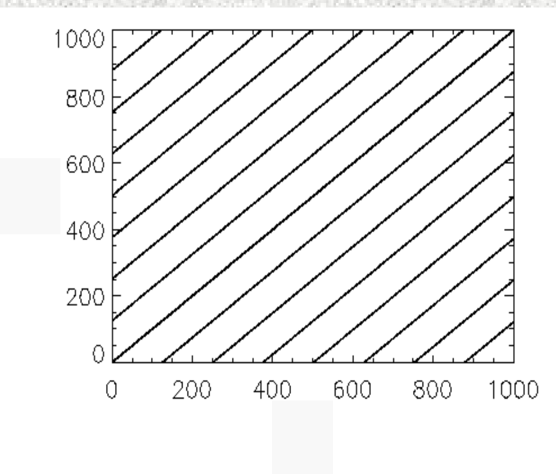
ε – threshold for neighborhood (recurrence to it) - (Eckmann et al., 1987

Generalization:

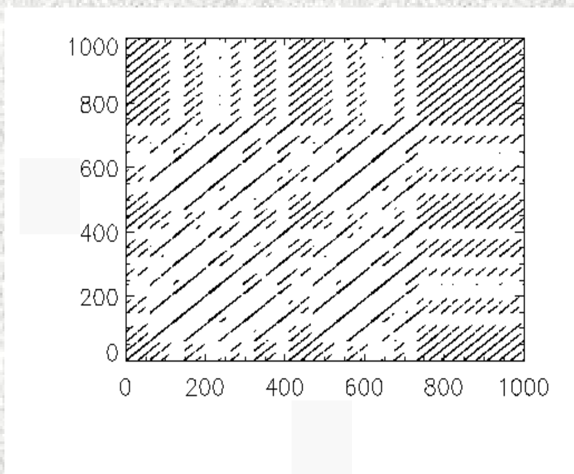
Statistical properties of all side diagonals

➔ Measures of complexity (2002...)

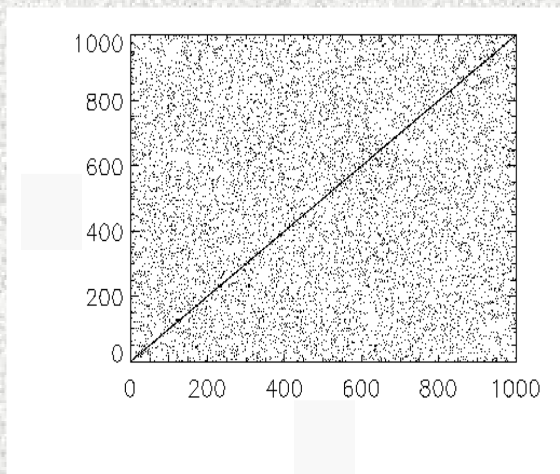
Sine



Rössler oscillator



white noise



Predictability
Long diagonals

Unpredictability
Short diagonals

Distribution of the Diagonals

$$P_\varepsilon(l) \approx \varepsilon^{D_2} \exp(-\tau K_2 l)$$

The following parameters can be estimated by means of RPs (Thiel, Romano, Kurths, CHAOS, 2004):

Correlation Entropy:

$$\hat{K}_2(\varepsilon, l) = \frac{1}{l\tau} \ln\left(\frac{1}{N^2} \sum_{s,t=1}^{N} \prod_{m=0}^{l-1} R_{t+m,s+m} \right)$$

Correlation Dimension:

$$\hat{D}_2(\varepsilon, l) = \ln\left(\frac{P_\varepsilon(l)}{P_{\varepsilon+\Delta\varepsilon}(l)} \right) \Big/ \left(\frac{\varepsilon}{\varepsilon + \Delta\varepsilon} \right)$$

Mutual Information:

$$\hat{I}_2(\varepsilon, \tau) = -2\ln\left[\frac{1}{N^2} \sum_{i,j=1}^{N} R_{i,j} \right] + \ln\left[\frac{1}{N^2} \sum_{i,j=1}^{N} R_{i,j} R_{i+\tau,j+\tau} \right]$$

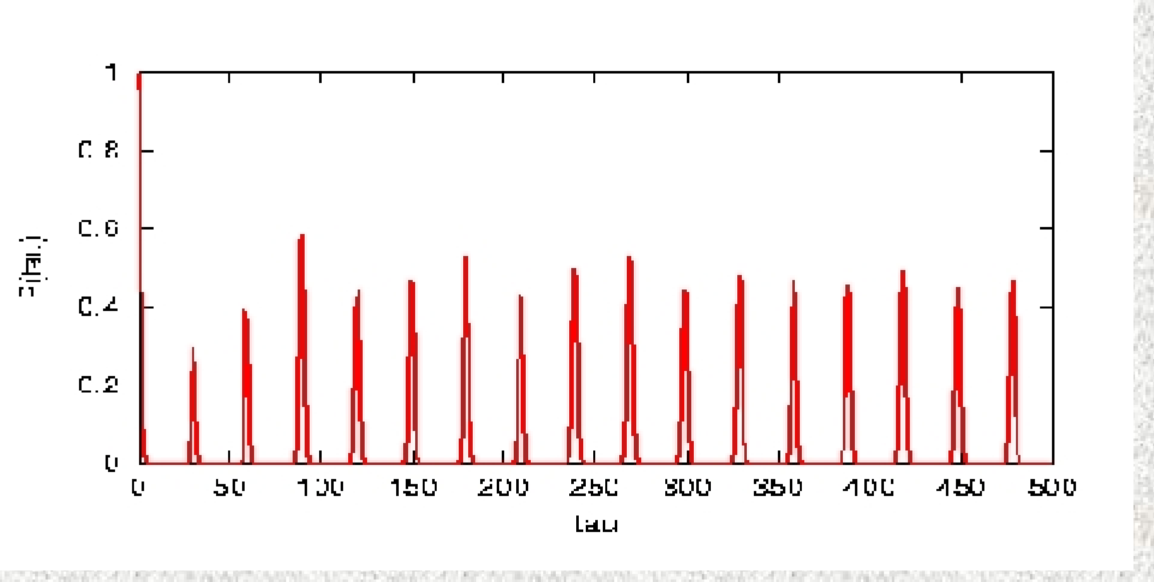
Probability of recurrence after a certain time

- Generalized auto (cross) correlation function

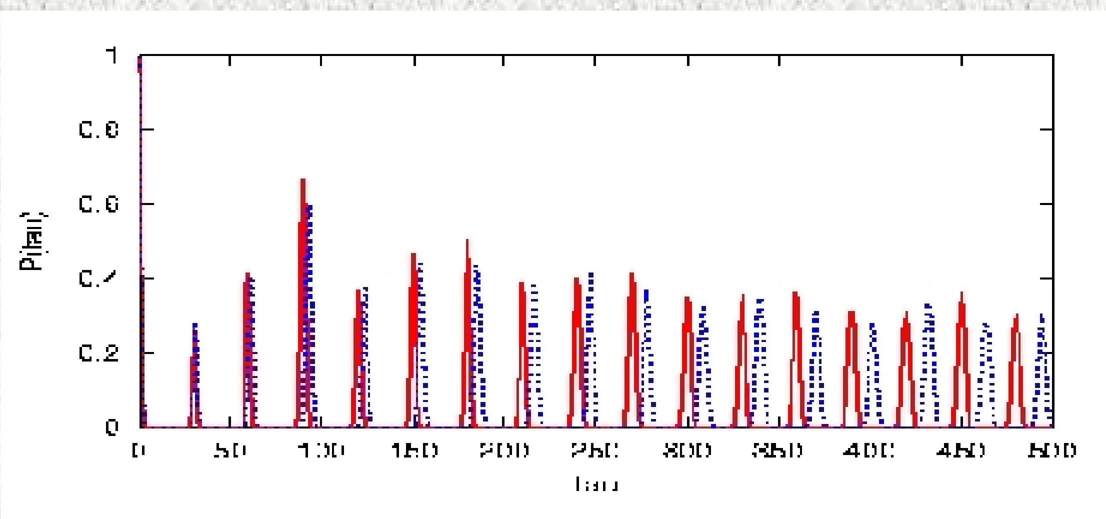
$$\hat{P}^{(\varepsilon)}(\tau) = \frac{\sum_{i=1}^{N-\tau} \Theta(\varepsilon - \|\mathrm{x}_i - \mathrm{x}_{i+\tau}\|)}{N-\tau} = \frac{\sum_{i=1}^{N-\tau} R_{i,i+\tau}^{(\varepsilon)}}{N-\tau}.$$

(Romano, Thiel, Kurths, Kiss, Hudson
Europhys. Lett. 71, 466 (2005))

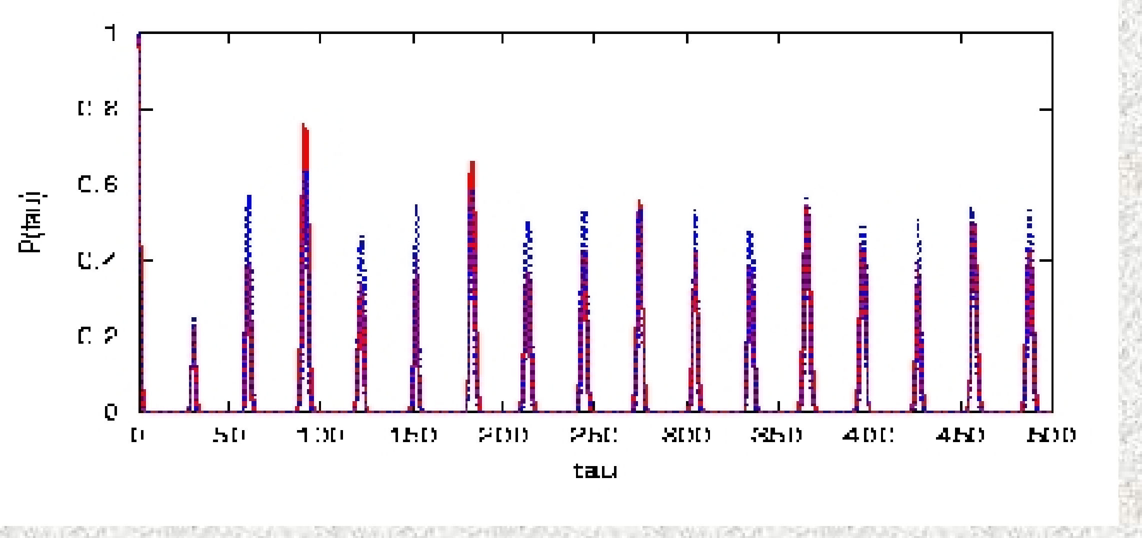
Recurrence Rate Roessler (phase coherent)



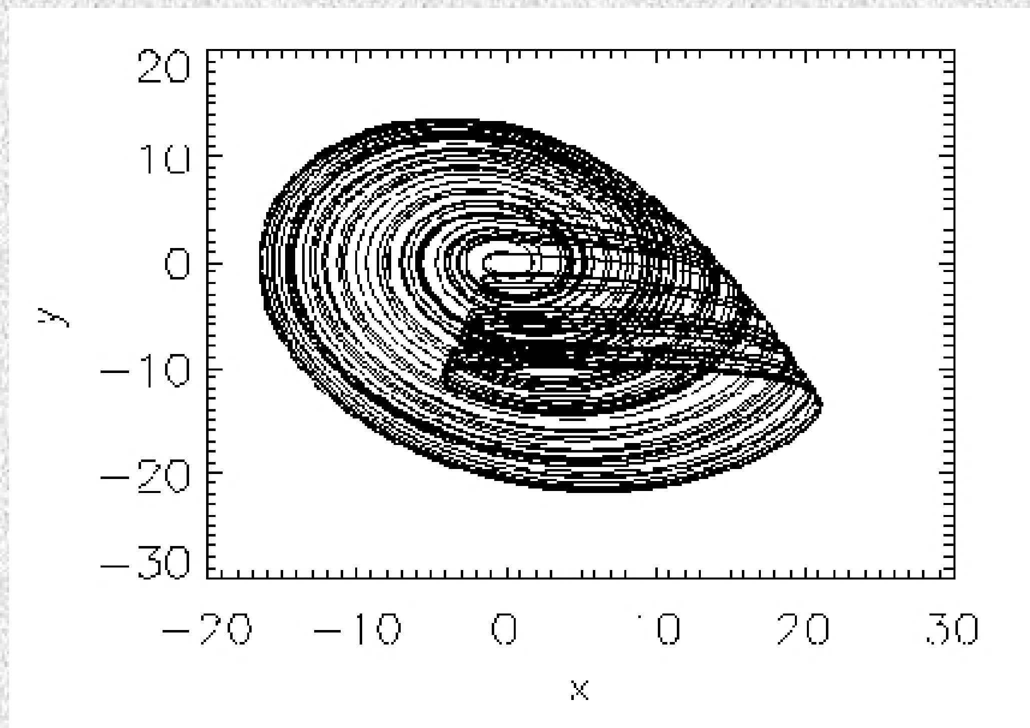
Two coupled Roessler oscillators - Non-synchronized



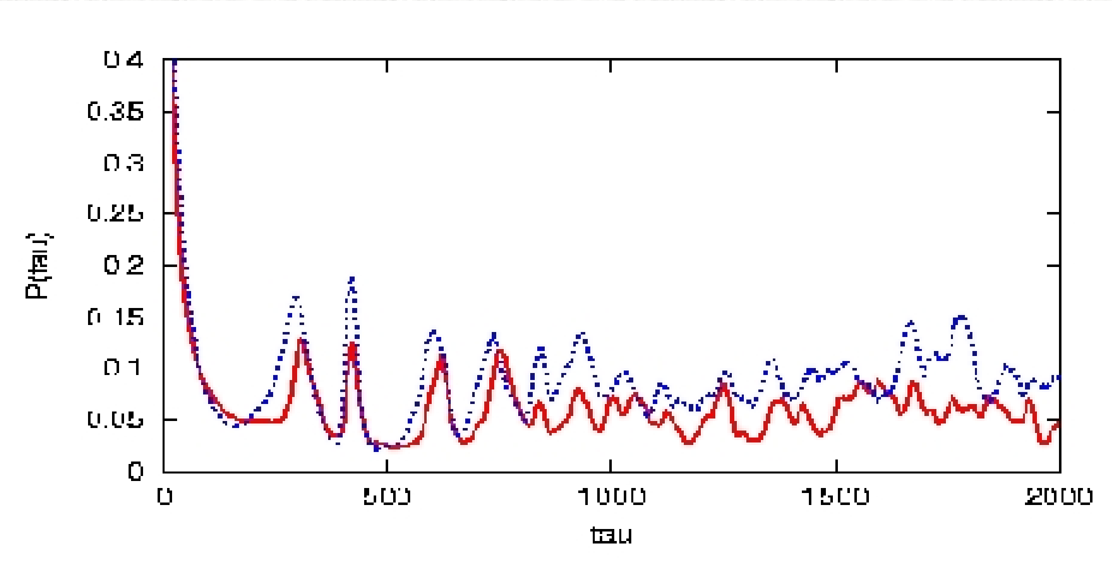
Two coupled Roessler oscillators - Phase-synchronized



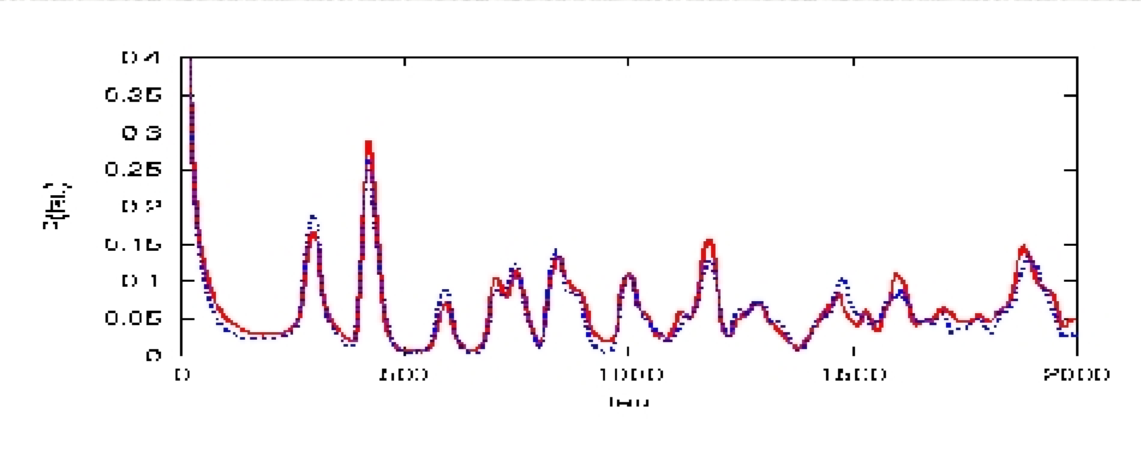
Roessler Funnel – Non-Phase coherent



Two coupled Funnel Roessler oscillators - Non-synchronized



Two coupled Funnel Roessler oscillators – Phase and General synchronized



Cross-Synchronization Analysis

Cross-Phase-Recurrence

$$CPR = \langle \bar{P}_1(\tau) \bar{P}_2(\tau) \rangle / (\sigma_1 \sigma_2)$$

Analysis of Generalized Synchronization

JPR - Joint probability of recurrence

$$JPR = \max_{\tau} \frac{S(\tau) - RR}{1 - RR}$$

RR – average probability of recurrence

S(τ) - Similarity function between x and y with time lag

$$S(\tau) = \frac{\frac{1}{N^2}\sum_{i,j}^{N} \Theta(\varepsilon_{\boldsymbol{x}}^{i} - ||\boldsymbol{x}_i - \boldsymbol{x}_j||)\Theta(\varepsilon_{\boldsymbol{y}}^{i} - ||\boldsymbol{y}_{i+\tau} - \boldsymbol{y}_{j+\tau}||)}{RR}$$

Phase and generalized synchronization analysis

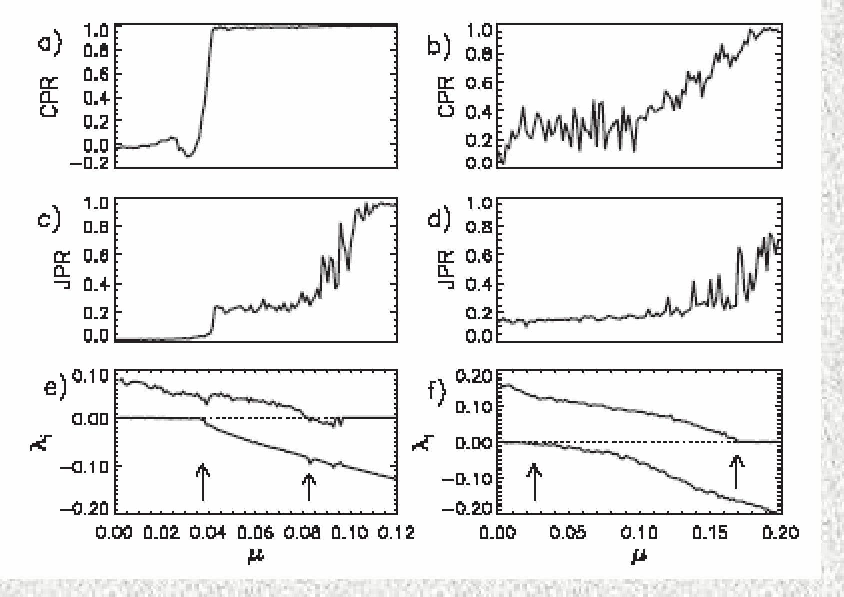


Fig. 2 – CPR index, JPR index and λ_2 and λ_4 as functions of the coupling strength μ for two mutually coupled Rössler systems in phase-coherent regime (a,c,e) and in funnel regime (b,d,f). The dotted zero line in (e) and (f) is plotted to guide the eye. Here, we choose ε corresponding to 10% recurrence points in each RP.

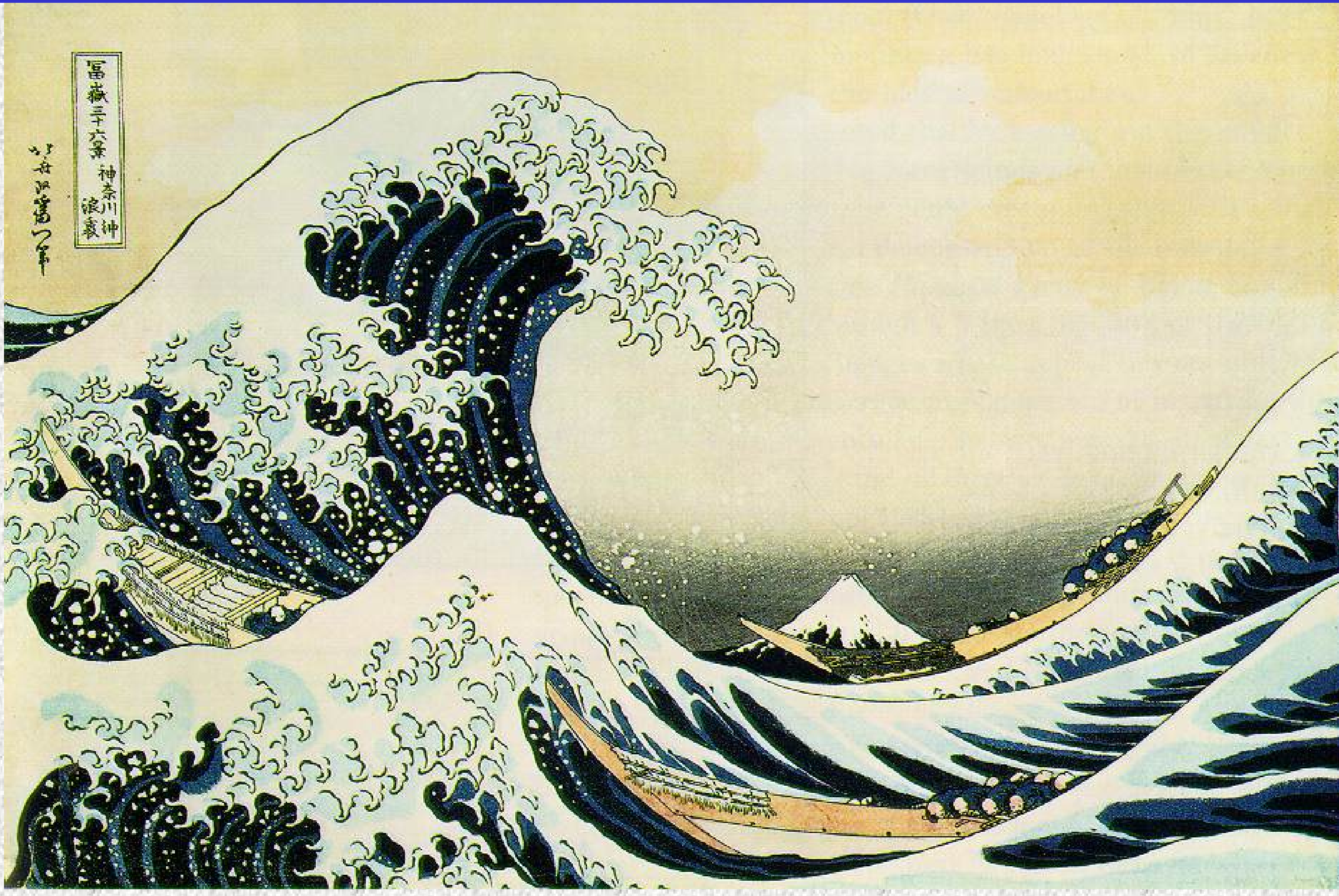
Generalizations and Applications

Extension to chains, lattices and multivariate data

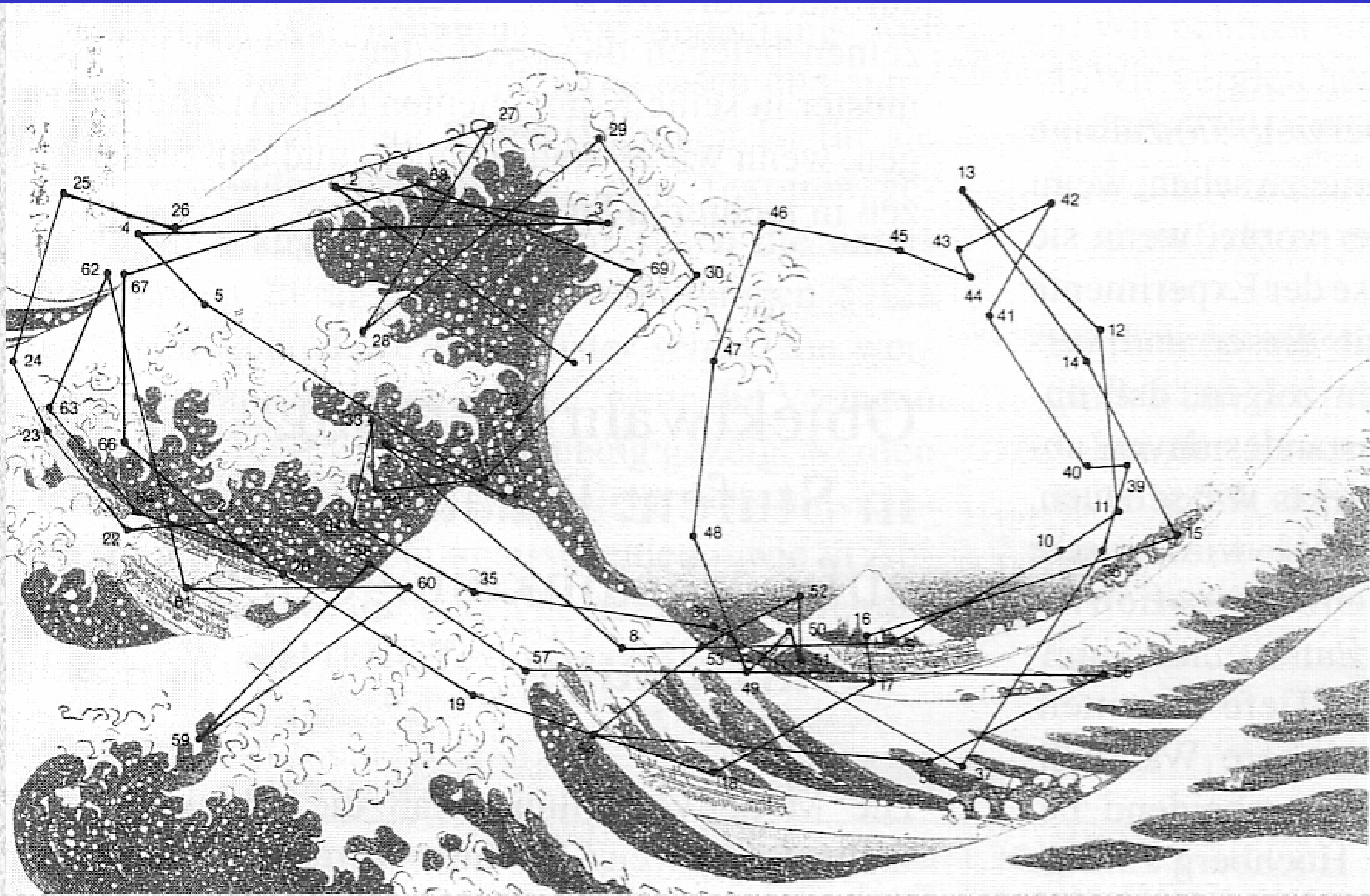
Applications:

- electrochemical experiments
- Eye movement during visual perception

The Great Wave by Katsushika Hokusai (1760-1850)

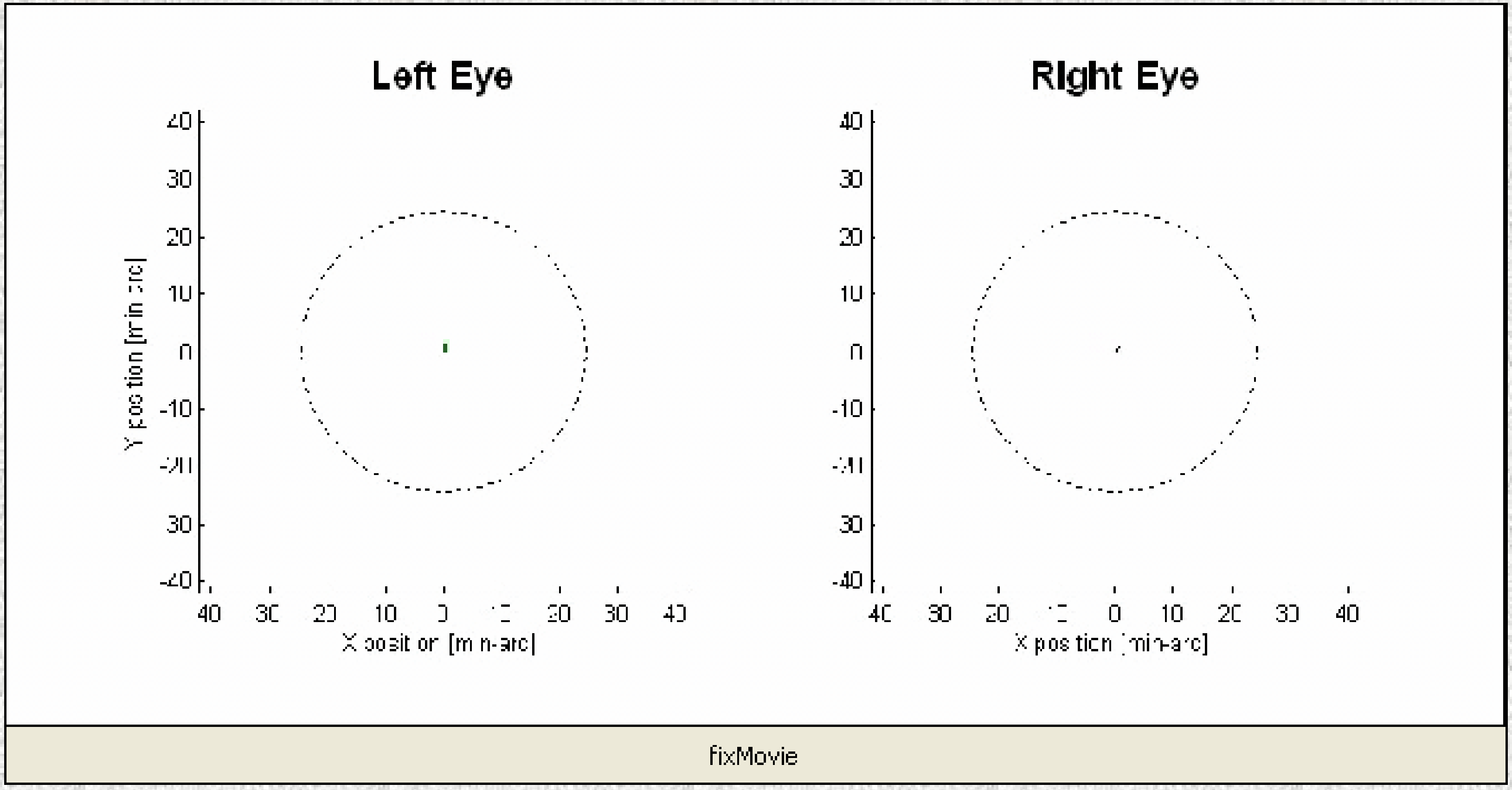


The Great Wave by Katsushika Hokusai (1760-1850)



(from: Buswell, 1935)

Eyes directed to one point ➔ Mikrosaccades



Results:

- Fixational movements of the left and right eye are phase synchronized

- Hypothesis: there might be one center only in the brain that produces the fixational movement in both eyes

Take home messages:

- Synchronization is **not a state** but a **process** of adjusting rhythms due to interaction.
- When subsystems (e.g. people, animals, cells, neurons) **synchronize**, they also can **communicate**.

Co-Workers and Cooperation

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Recurrence plots for the analysis of complex systems

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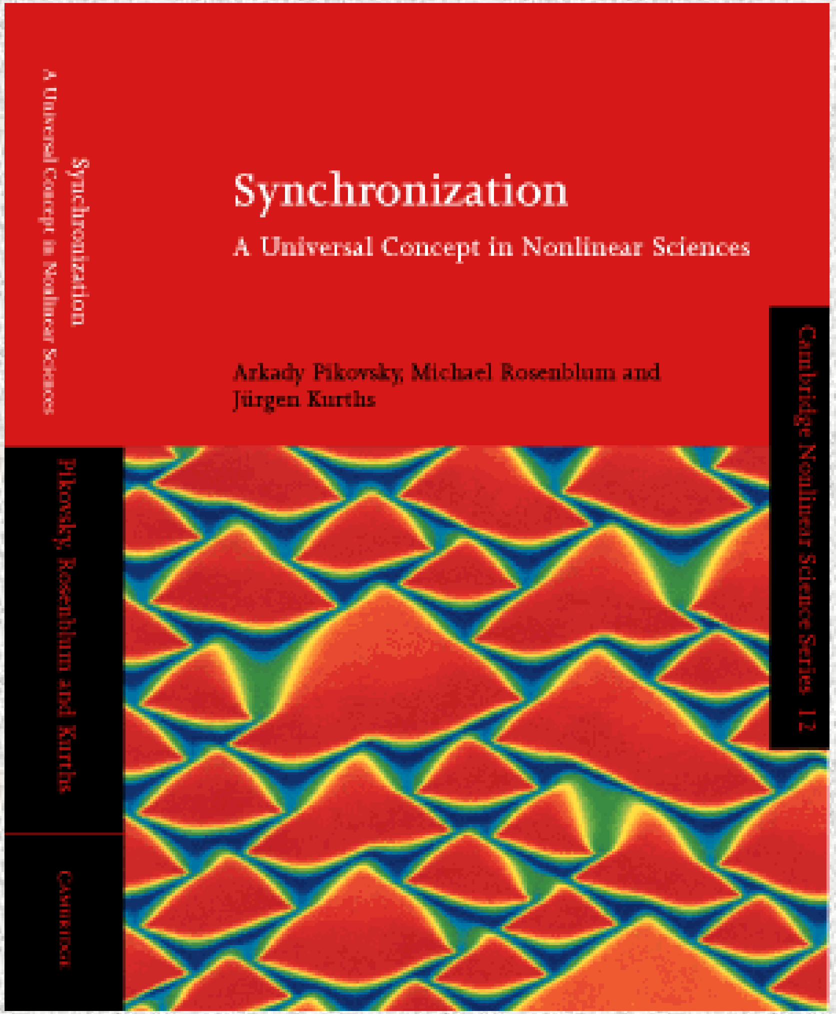
Abstract

Recurrence is a fundamental property of dynamical systems, which can be exploited to characterise the system's behaviour in phase space. A powerful tool for their visualisation and analysis called *recurrence plot* was introduced in the late 1980's. This report is a comprehensive overview covering recurrence based methods and their applications with an emphasis on recent developments. After a brief outline of the theory of recurrences, the basic idea of the recurrence plot with its variations is presented. This includes the quantification of recurrence plots, like the recurrence quantification analysis, which is highly effective to detect, e. g., transitions in the dynamics of systems from time series. A main point is how to link recurrences to dynamical invariants and unstable periodic orbits. This and further evidence suggest that recurrences contain all relevant information about a system's behaviour. As the respective phase spaces of two systems change due to coupling, recurrence plots allow studying and quantifying their interaction. This fact also provides us with a sensitive tool for the study of synchronisation of complex systems. In the last part of the report several applications of recurrence plots in economy, physiology, neuroscience, earth sciences, astrophysics and engineering are shown. The aim of this work is to provide the readers with the know how for the application of recurrence plot based methods in their own field of research. We therefore detail the analysis of data and indicate possible difficulties and pitfalls.

PACS: 05.45; 07.05.Kf; 07.05.Rm; 91.25.–r; 91.60.Pn

Keywords: Data analysis; Recurrence plot; Nonlinear dynamics


Synchronization
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Jürgen Kurths
Cambridge Nonlinear Science Series 12
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