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Complex Systems (an overview)

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COMPLEX SYSTEMS
(an overview)

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(October 2007)

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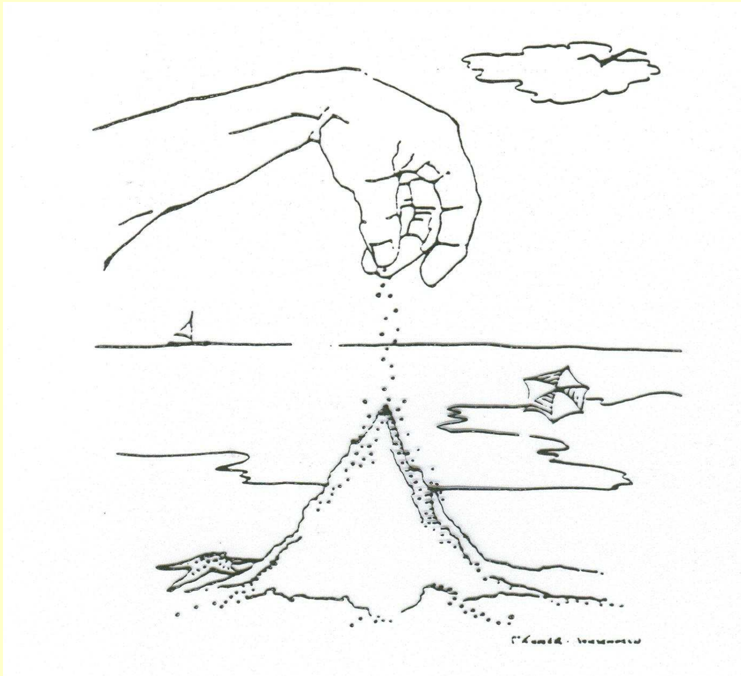
NATURAL PROCECESS

NATURAL PROCECESS

OBSERVATIONS



PRERDICTIONS

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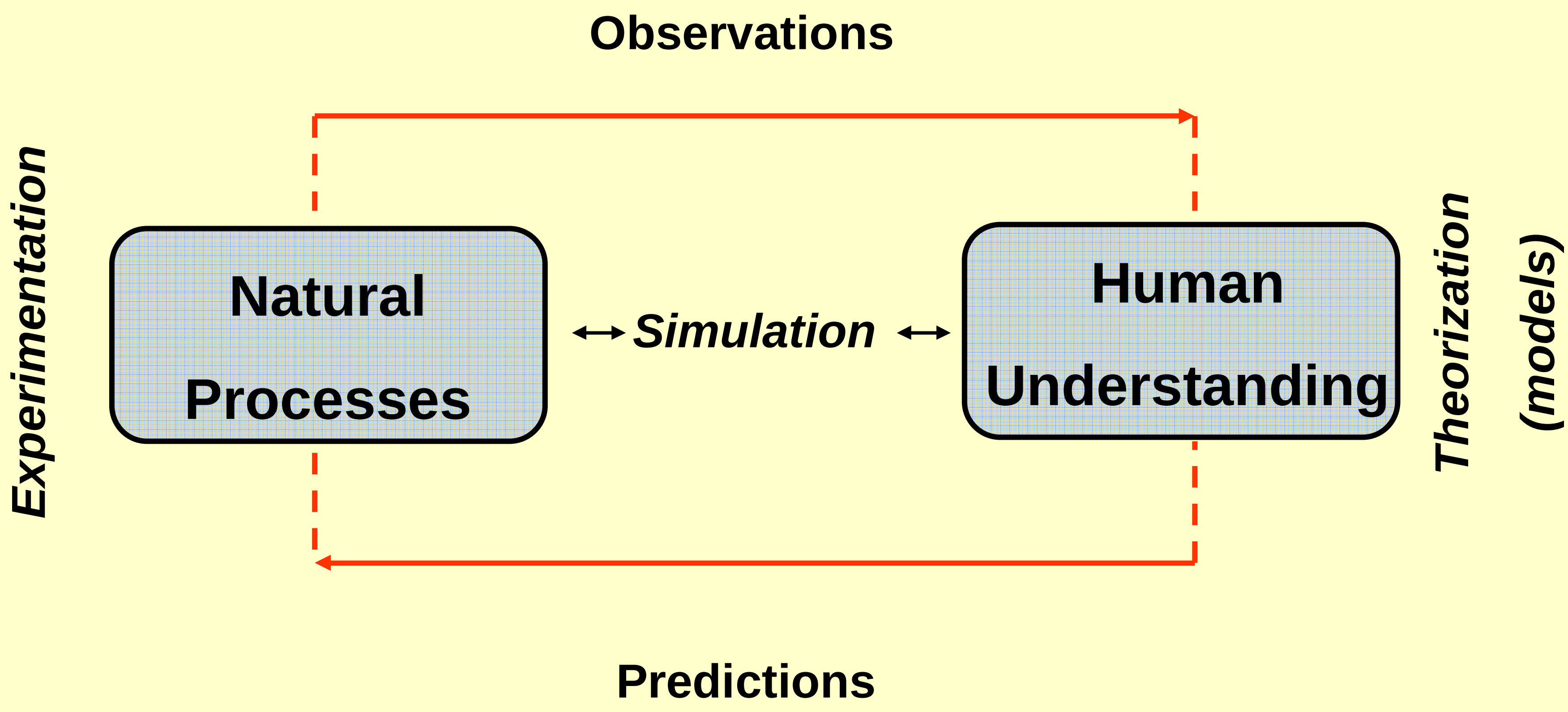
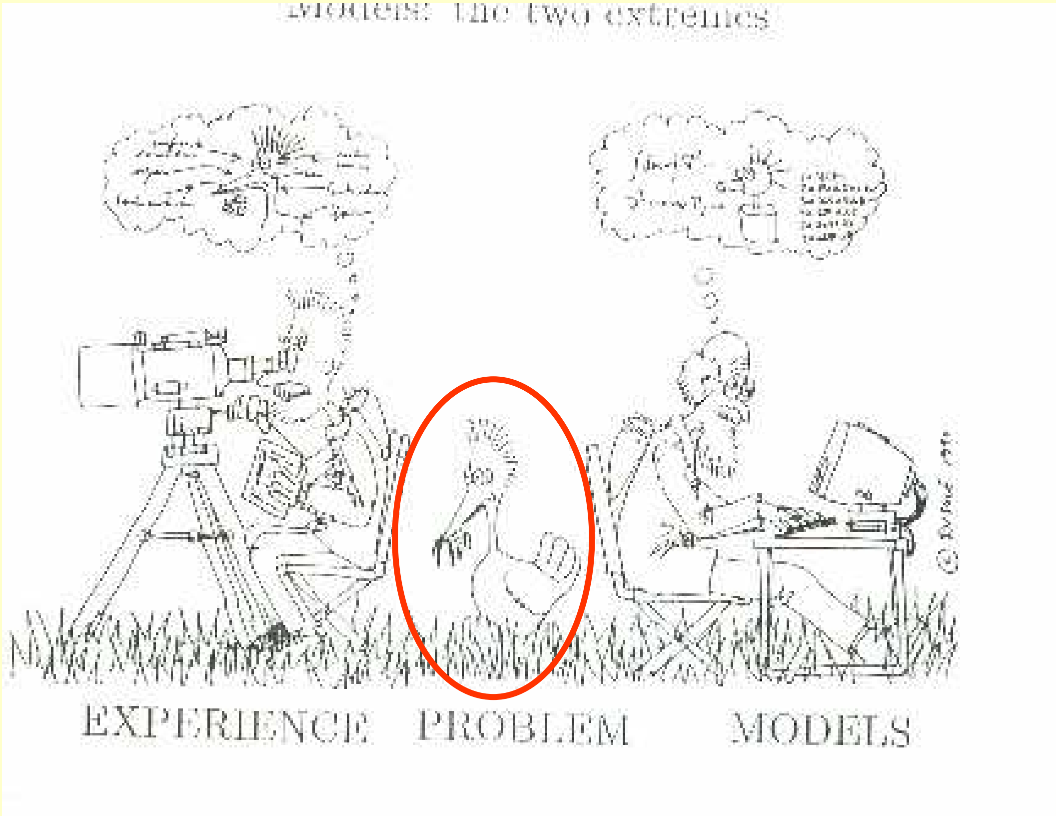
OBSERVATIONS / PREDICTIONS



OBSERVATIONS / MODELS

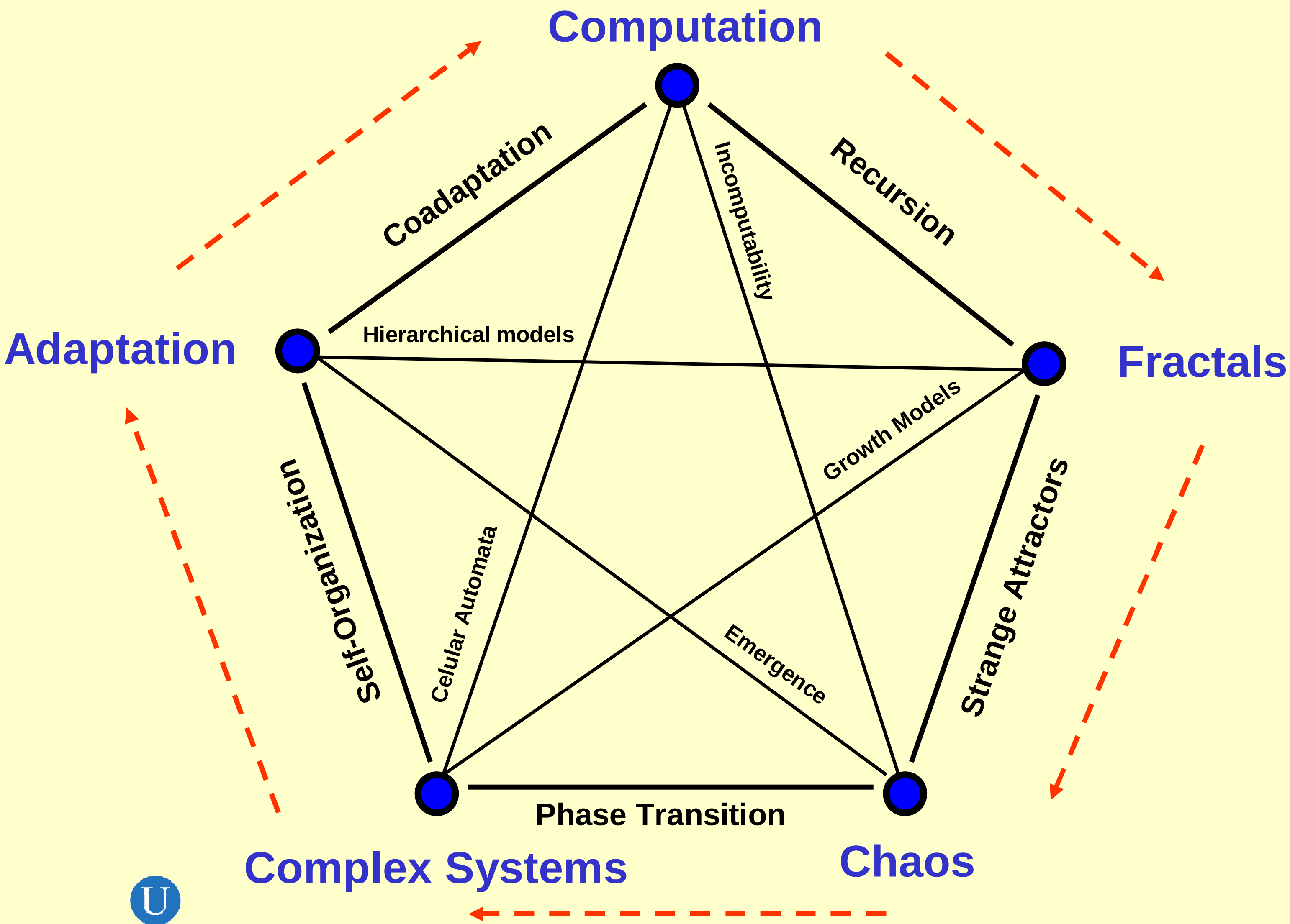


COMPLEX ADAPTIVE SYSTEM

The word **complex** comes from the Latin "complexus" and means **a whole made up of many interacting , interwoven parts.**

A **complex system** is said to be **adaptive** if the system can change its behavior under external influences.

COMPLEXITY AND RELATED FIELDS



(Adapted from Flake (1998))

SOME DESCRIPTIONS. 1

Computation What are the limits of computers and what does it mean to compute? The theory of computation yields a surprisingly simple definition of what it means to compute: one can construct higher mathematical functions with only a very small set of primitive computational functions as a starting place.

Fractals Beautiful images that can be efficiently produced by iterating equations. They are often found in natural systems.

Chaos A Chaos makes the future behavior of deterministic systems impossible to predict in the long term, but predictable in the short term. Often associated with chaos, there is a special type of fractal known as a *strange attractor*.

SOME DESCRIPTIONS. 2

Complex Systems Consisting of many very simple units that interact. The amount of interaction among agents (components) partially determines the overall behavior of the whole system. On one extreme, systems with little interaction fall into static patterns, while on the other extreme, overactive systems boil with chaos. Between the two extremes is an region of criticality.

Adaptation Due to changes of the environment (the action of external forces), the complex systems may change, adapt, learn, and evolve.

(Flake (1998))

FEEDBACK

Is the most frequent mechanism of interaction among the different components of natural systems.

Consists in the interaction of a signal with itself, as for example when the output of a signal interacts with the input.

Feedback can be **negative** (tend to reduce the output), **positive** (tend to amplify the output) or **bipolar** (may amplify or reduce the output).

Bipolar feedback is present in many natural phenomena and is related to its regulation. **Negative** feedback keps the stability of a system against external variations, whereas **positive** feedback increase the possibilities of divergence of the system and thus opening the possibilities of the system to new equilibrium states.

In natural phenomena, **multiplicative noise** may be an important contribution to feedback.

LINEAR AND NONLINEAR SYSTEMS

Linear systems are characterized by the property:

$$f\left(ax+by\right)=af\left(x\right)+bf\left(y\right).$$

For them applies the **superposition principle**:

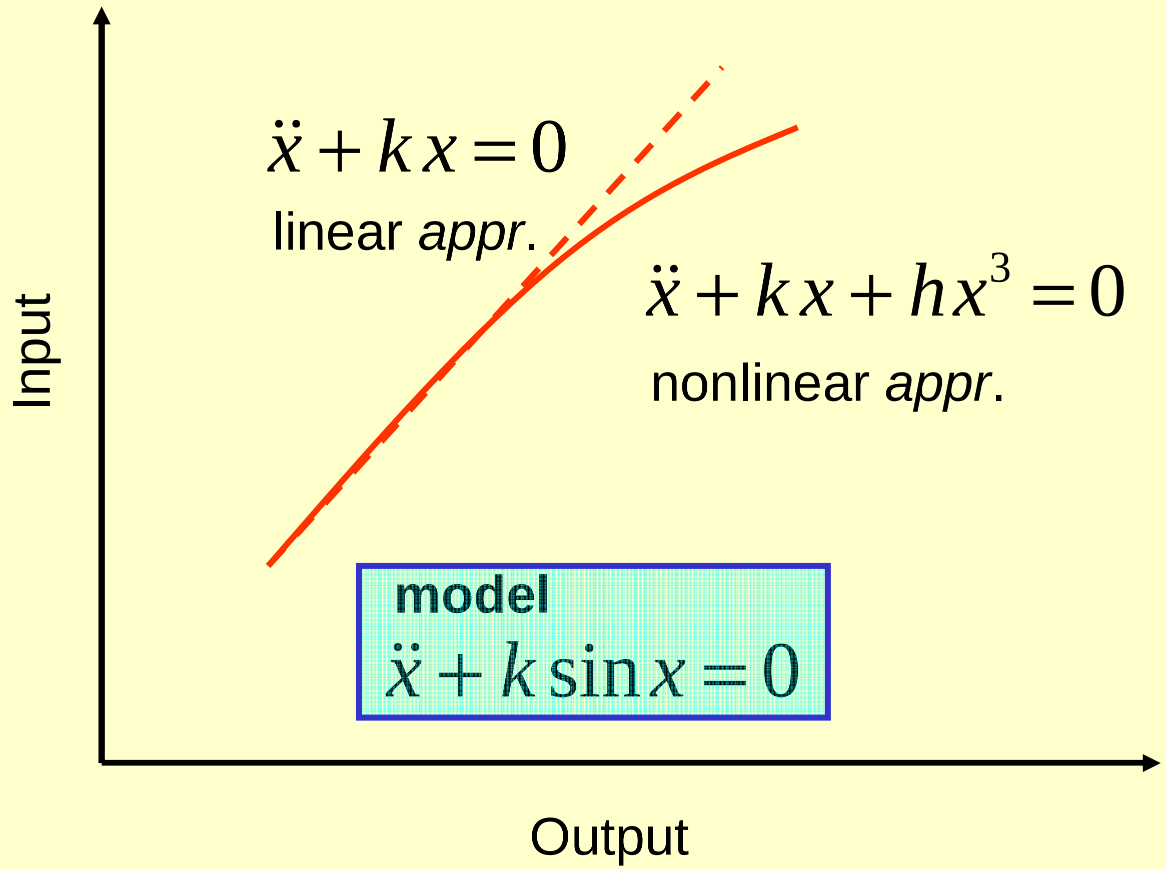
$$x(t)=\sum_{n=-\infty}^{\infty}c_n e^{in\omega t}.$$

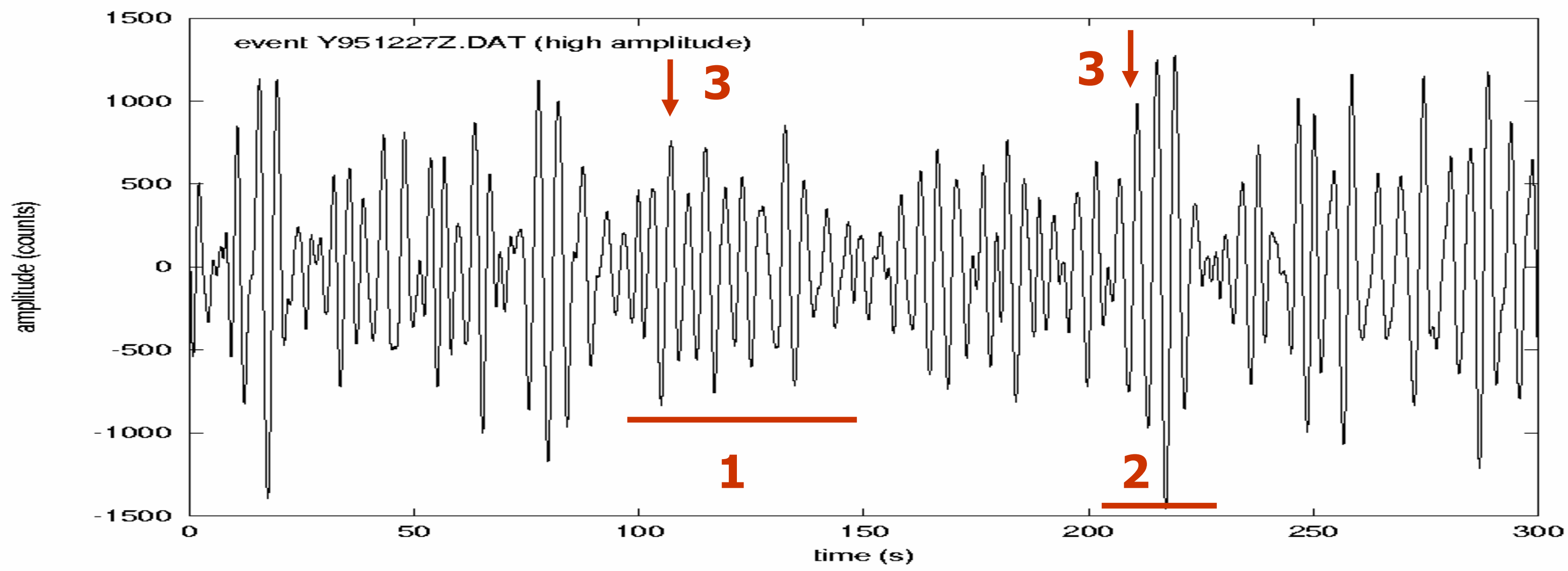
For a **nonlinear system** the first relation does not apply. For example,

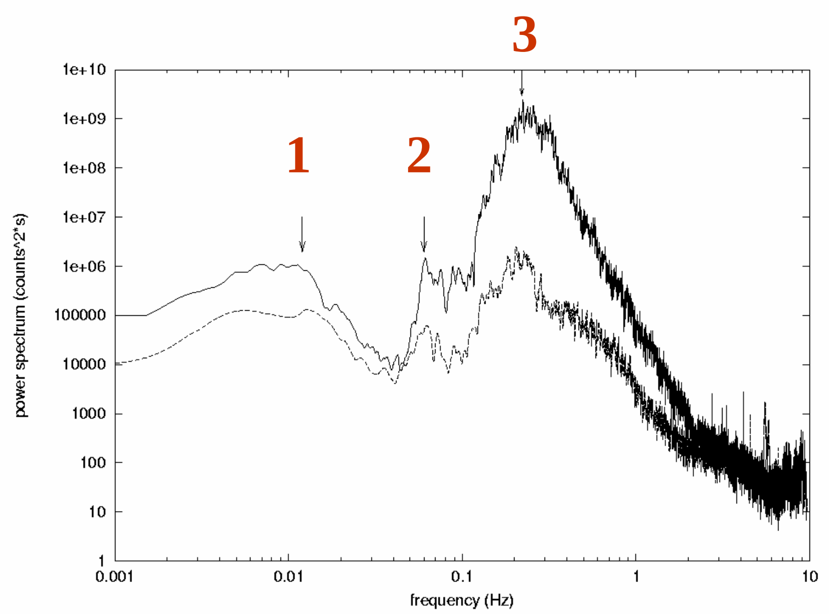
$$\sin\left(x+y\right)\neq\sin\left(x\right)+\sin\left(y\right)$$

As well, the superposition principle is no longer valid, being substituted by a **competition** between the different components.

LINEARITY - NONLINEARITY





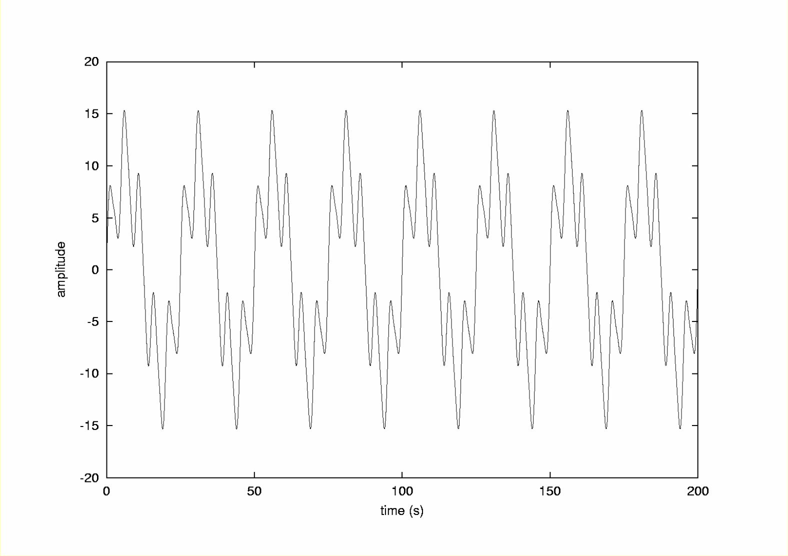


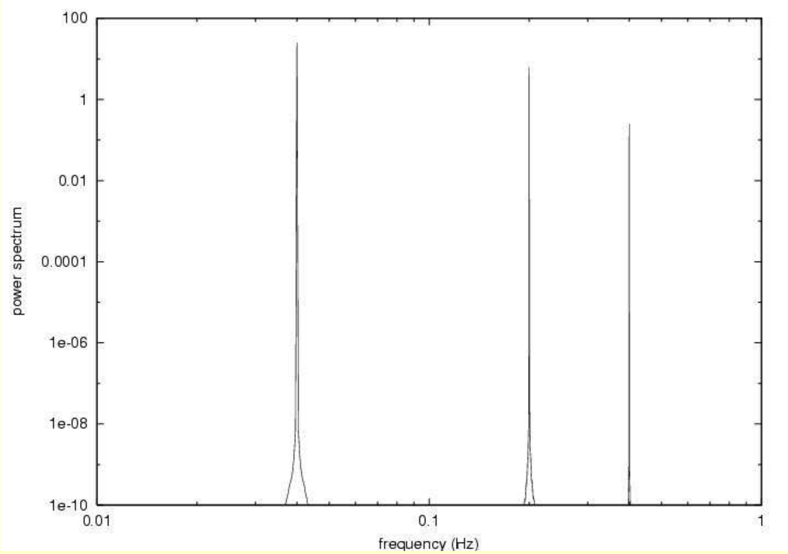
Microseism activity:

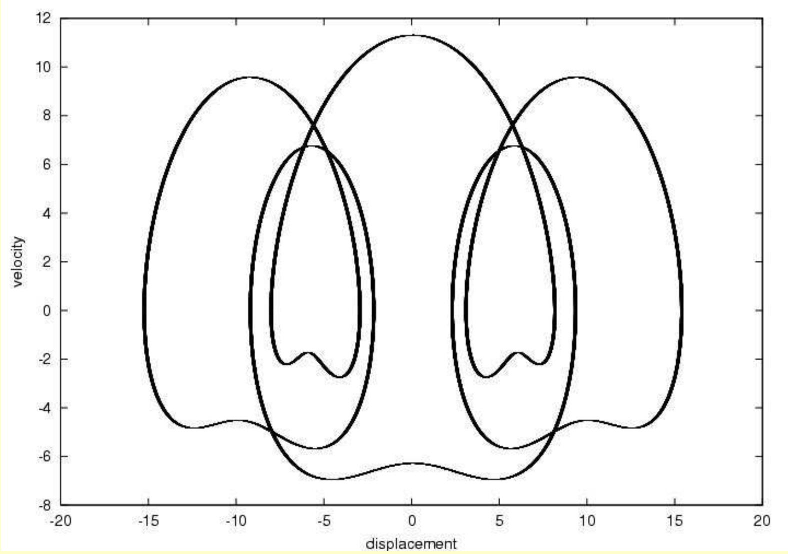
an example of a natural nonlinear system

Linear oscillator

$$\frac{d^2X}{dt^2} + 2\gamma\varepsilon_0 \frac{dx}{dt} + \omega_0^2 = \sum_i F_i(t)$$





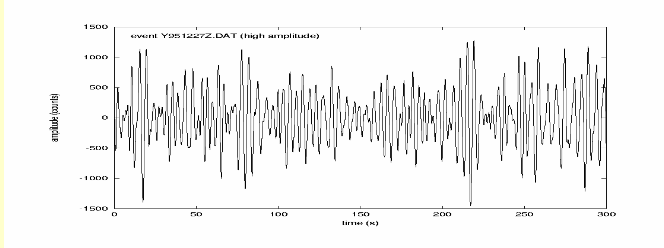


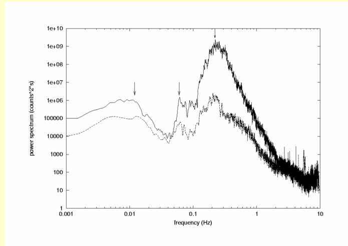
Nonlinear oscillator

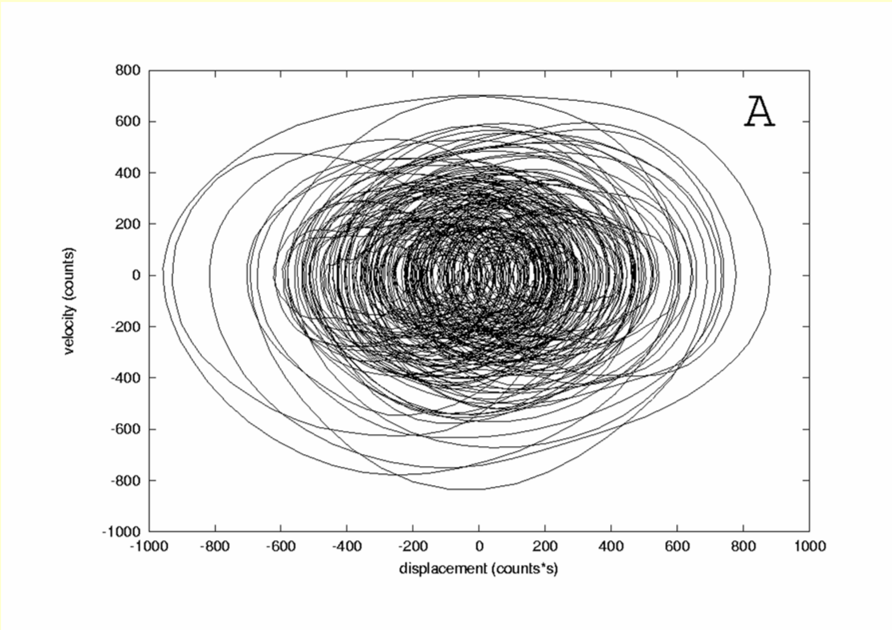
$$\frac{d^2q}{dt^2}+\frac{dV_0}{dq}+\delta\frac{dq}{dt}=\sum_{i=1}^{2}\gamma_i\cos(\omega_i t)+\varepsilon F(t)$$

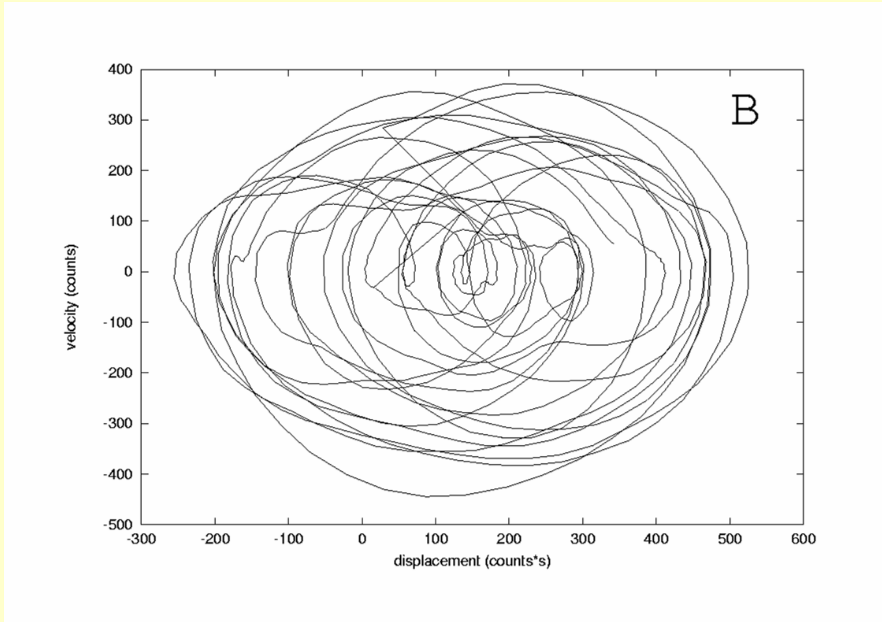
$$V_0(q)=-\alpha\frac{q^2}{2}+\beta\frac{q^4}{4}$$

$$\alpha=\alpha_0+\eta f(t)$$









COMPLICATED *vs*. COMPLEX. 1

- A **complicated** system may consist in many simple systems, each one acting independently. The global behavior is the sum of the different contributions.
- A **complex** system is that consisting of interconnected or interwoven parts interacting continuously.

The difference of a system behaving as complicated or complex lies precisely in the way the different parts may interact (no interaction, linear interaction, nonlinear interaction)

COMPLICATED *vs*. COMPLEX. 2

The elements and their connections are equally important	+	Simple algorithms originate responses simple and predictible	+	The elements of the response are fully determined

= **complicated**

Connections are critical, individual elements no	+	Simple rules originate responses complex and adaptable	+	The elements have capacity of response inside the rules

= **complex**

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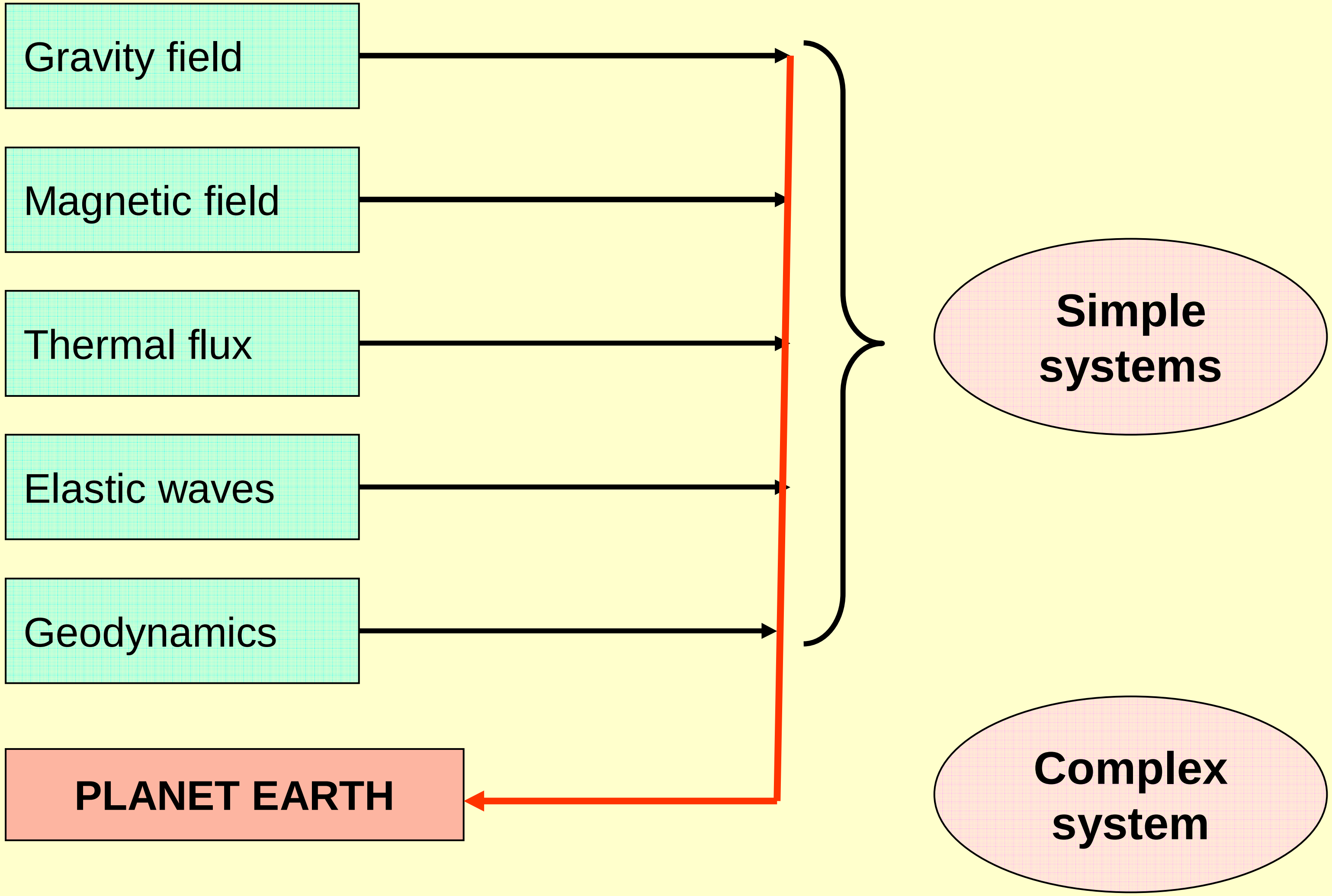
PHYSICS OF THE EARTH



CHARACTERIZATION OF COMPLEX SYSTEMS

- External manifestations
- Spatial structures
- Temporal evolution of the system (often defining cycles)
- Relation of the system with its environment
- Ability of the system to ***self-organize*** (due to interactions among its elements) and/or to ***adapt*** (accommodation of the system to variations of the environment)
- Appearance of ***emergent*** structures and behaviors

QUANTIFICATION OF COMPLEX SYSTEMS

We should be able to identify

- Its elements
- How the elements interact
- How the complex systems emerge
- How the complex systems evolve and their predominant spatio-temporal scales
- The interaction of the system with the environment

THE RISE OF COMPLEXITY

- **Emergence**
- **Complexity**
- **Self-organization**
- **Self-Organization vs entropy**
- **Patterns**
- **Adaptation**

EMERGENCE

Emergence is...

1. ... what parts of a system do together that they would not do by themselves: collective behavior.
   How behavior at a larger scale of the system arises from the detailed structure, behavior and relationships on a finer scale.
2. ... what a system does by virtue of its relationship to its environment that it would not do by itself: e.g. its function.
   Emergence refers to all the properties that we assign to a system that are really properties of the relationship between a system and its environment.
3. … the act or process of becoming an emergent system.

In short: the term "emergence" refers to a process by which a system of interacting elements acquires qualitatively new pattern and structure that cannot be understood simply as the superposition of the individual contributions. → HOLISTIC appr.

COMPLEXITY (of a complex system)

- **Complexity is** …(the abstract notion of complexity has been captured in many different ways. Most, if not all of these, are related to each other and they fall into two classes of definitions):
  - 1) ...**the (minimal) length of a description of the system.**
  - 2) …**the (minimal) amount of time it takes to create the system**
- The length of a description is measured in *units of information*. The former definition is closely related to Shannon information theory and algorithmic complexity, and the latter is related to computational complexity.

SELF-ORGANIZATION. 1

The most robust and unambiguous examples of self-organizing systems are from physics, although it is also relevant in chemistry, and is central to the description of biological systems.

There are also examples of "self-organizing" behavior in the literature of many other disciplines, both in the natural sciences and the social sciences such as economics or anthropology, and has also been observed in mathematical systems such as cellular automata (CA).

Self-organizing systems are systems in which pattern and structure at the global level arises solely from interactions among the lower-level components of the system. The rules specifying interactions among system's components are executed using only local information, without reference to the global pattern.

The pattern is an emergent property of the system, rather than a property imposed on the system by an external influence.

SELF-ORGANIZATION. 2

Self-organization usually relies on four basic ingredients:

1. **Positive feedback**
2. **Negative feedback**
3. **Balance of exploitation and exploration**
4. **Multiple interactions**

SELF-ORGANIZATION *vs*. ENTROPY

Self-organization ("*order*") challenges an earlier paradigm of ever-decreasing order, based on the second law of thermodynamics: entropy is a measure of the statistical "*disorder*" at a microstate level. However, **at the microscopic or local level, the two need not be in contradiction**: it is possible for a system to reduce its entropy by transferring it to its environment.

In open systems, it is the flow of matter and energy through the system that allows the system to self-organize, and to exchange entropy with the environment. This is the basis of the theory of dissipative structures. Ilya Prigogine noted that self-organization can only occur far away from thermodynamic equilibrium.

PATTERNS. 1

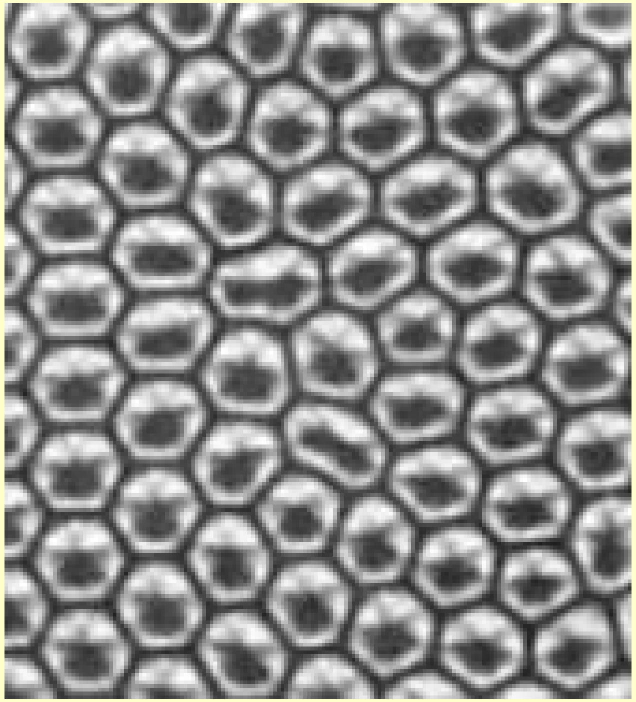
Pattern refers not only to a particular arrangement of objects in **space**, but also to structure and organization in **time**.

In self-organizing systems, pattern and organization develop through interactions internal to the system, that is, without the intervention of external influences. **The pattern is an emergent property of the system itself**.

PATTERNS. 2

Based on physical and chemical properties

1. Belousov-Zhabotinsky reaction
2. Bénard convection cells
3. Sand dune ripples
4. Glass cracks
5. Mud cracks



Benard convection cells

Non-equilibrium Pattern Formation vs Equilibrium Phase Transitions. 1

Natural phenomena can be considered as open (dissipative) systems far from equilibrium, whereas phase transition (as for example the Ising model) are related to equilibrium systems (see Appendix).

The Benard convective cells illustrates some features of non equilibrium pattern formation (thus order creation) that are similar to those of second-order phase transitions in equilibrium systems:

1. In both cases the pattern formation depends on a parameter crossing some value (compare the Ising model with the Benard cell).
2. There is a change of symmetry as the threshold is crossed.
3. Dissipative structures show order at a length scale that is much larger than the intrinsic microscopic length scale: a large correlation length emerges that allows the system to organize itself at a collective level. This is similar to what happens near a second order phase transition.

Non-equilibrium Pattern Formation vs Equilibrium Phase Transitions. 2

There are also some differences:

1. In non-equilibrium systems it is impossible in general to determine the exact pattern that will be formed, as the system is usually sensitive to various details and noise. By contrast, the state of an equilibrium system is uniquely determined by extremal principles, like the minimization of free energy. There is, thus, more variety of patterns in non-equilibrium systems, as Nature proves.

2. The final (away from the transition point) order of equilibrium systems shows a characteristic length scale similar to that of the underlying level. However, the patterns in non-equilibrium systems have no relation to the underlying structure. In fact, ***the large correlation length of dissipative structures is a classic example of an emergent property***.

DISSIPATIVE STRUCTURES

The order and complexity in complex systems is possible if they are open systems that are far from equilibrium.

By contrast to closed systems in equilibrium, which evolve towards a state of “boring” uniformity where differences and irregularities are smoothened out, open systems that are out-of-equilibrium can evolve towards states that display macroscopic order and patterns. This order is dynamic, instead of the static order of the equilibrium systems.

The term dissipative structures is often used to describe non-equilibrium open systems which take in matter or energy, also dissipate matter and/or energy, and which display ,macroscopic structure or order not inherent at the microscopic level.

Note that the **inflow** is necessary to maintain the system out of equilibrium, while the **dissipation** is required for decreasing the entropy of the system (but of course increasing that of the environment much more, as well as to maintain stability (remove the excess of energy or mass).

PATTERNS. 3

A far from equilibrium system can self-organize in emergent structures, known as patterns. This property allows us to shorten its description: we need to describe only one simple structure. This structure may repeat in space (Benard cells) or in time (climatic seasons).

The structures (patterns) can also be viewed as **prototype**s, that can be repeated *at infinitum.*

The relation between a structure as a repetition and a structure as a prototype is just the relation between to types of emergent properties: those that arise from a relation in the components of a system for one side (**self-organization**), and those that arise from a relation between the system and its environment (**adaptation**).

ADAPTATION

An adaptive system is a system that is able to adapt its behavior according to changes in its environment or in parts of then system itself, as for example a human being.

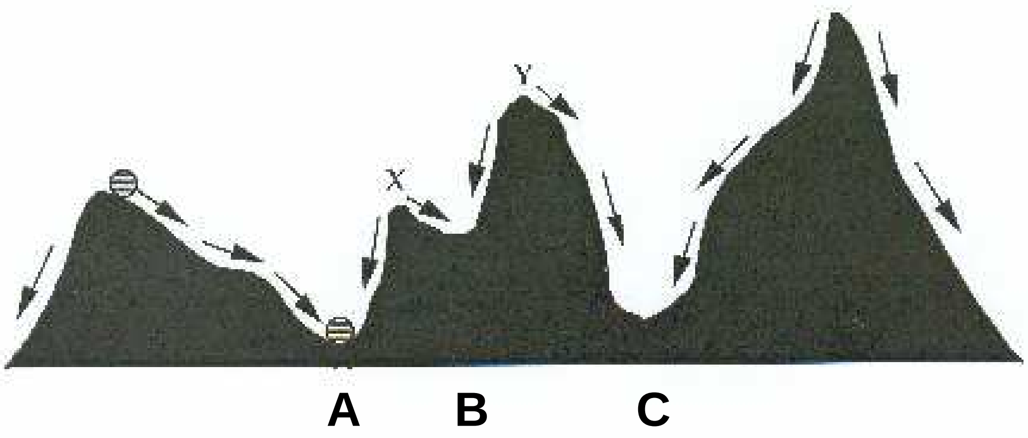


Figure 4: a fitness landscape: the arrows denote the directions in which the system will move. The height of a position corresponds to its potential or to its lack of fitness. Thus, A has a higher fitness (or lower potential) than B. The bottoms of the valleys A, B and C are local minima of the potential, i.e. attractors. The peaks X and Y delimit the basin of the attractor B. X separates the basins of A and B. Y separates the basins of B and C.

ADAPTATION AS FIT

A configuration of a system may be called “fit” if is is able to maintain or grow given the specific configuration of its environment.

An unfit configuration is one that spontaneously disintegrate under new boundary conditions.

Different configurations can be compared as to their degree of fitness, or likeliness to survive under the given conditions imposed by the environment.

Adaptation can be conceived as achieving a *fit* between system and environment.

It follows that every self-organizing system adapts to its environment: **self-organization implies adaptation**.

Systems may be called adaptive if they can adjust to changes in boundary conditions while keeping their organization.

REGULATION AND THE EDGE OF CHAOS

Adaptation can be modeled as a problem of r*egulation* or *control*, minimizing deviations from a given configuration. This means that the system must be able to

1. produce a sufficient ***variety*** of actions to cope with each of the possible perturbations

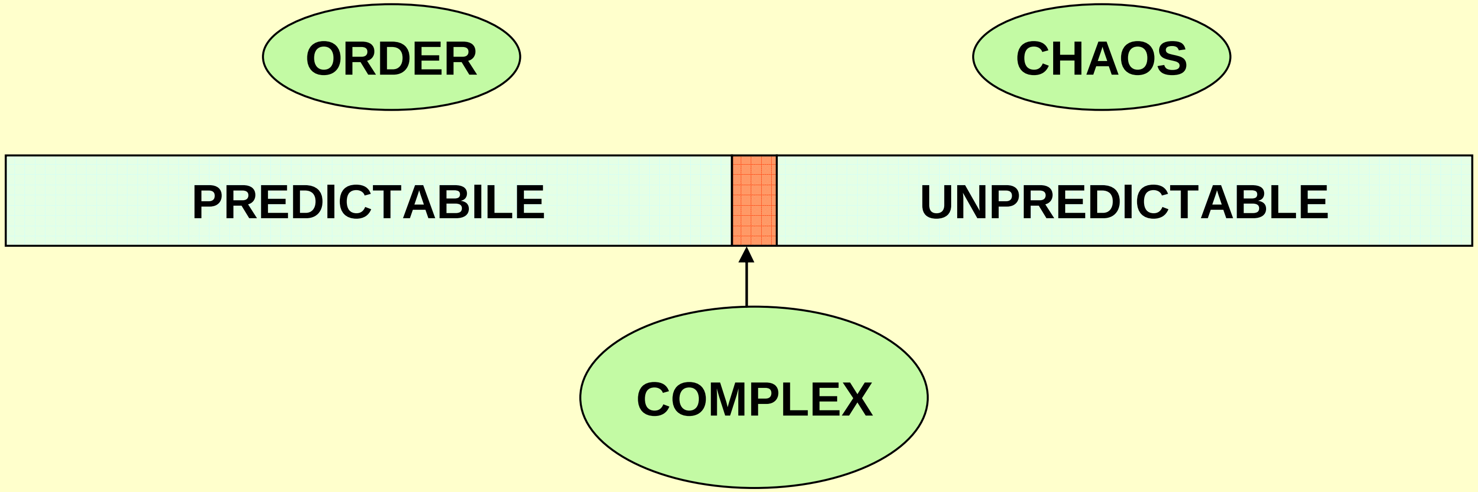
2. select the most adequate counteraction for a given perturbation.

Variety can be fostered by keeping the system suficiently far from equilibrium so that it has plenty of stationary states to choose from.

Selectivity requires that these configurations be sufficiently small in number and sufficiently stable to allow an appropriate one to be "chosen" without danger of losing the overall organization.

Complex adaptive systems tend to reside on the "edge of chaos", the narrow domain between frozen constancy (equilibrium) and turbulent, chaotic activity.

EDGE OF CHAOS



Regulation and the edge of chaos

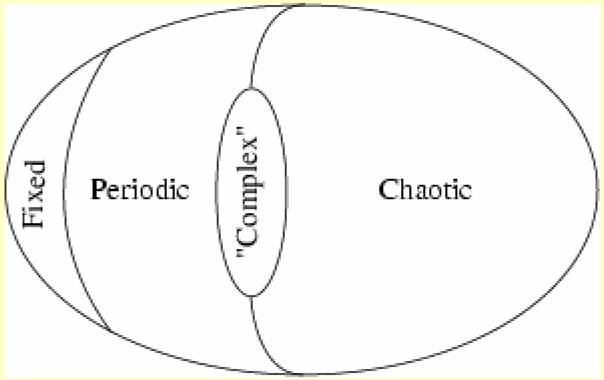
- Self Organized Criticality
- Control parameters

Modeling of natural phenomena

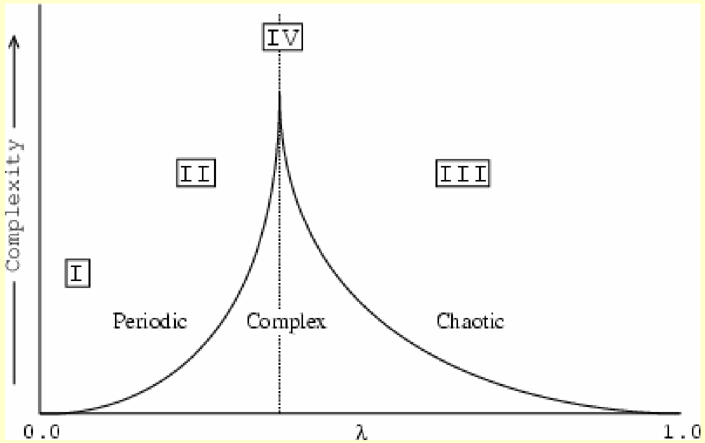
CHAOS *vs*. COMPLEX

- Chaos deals with deterministic systems whose trajectories diverge exponentially over time
- Models of chaos generally describe the dynamics of one (or a few) variables which are real. Using these models some characteristic behaviors of their dynamics can be found
- Complex systems do not necessarily have these behaviors. Complex systems have many degrees of freedom: many elements that are partially but not completely independent
- Complex behavior $\rightarrow$ "high dimensional chaos"

EDGE OF CHAOS IN CELLULAR AUTOMATA



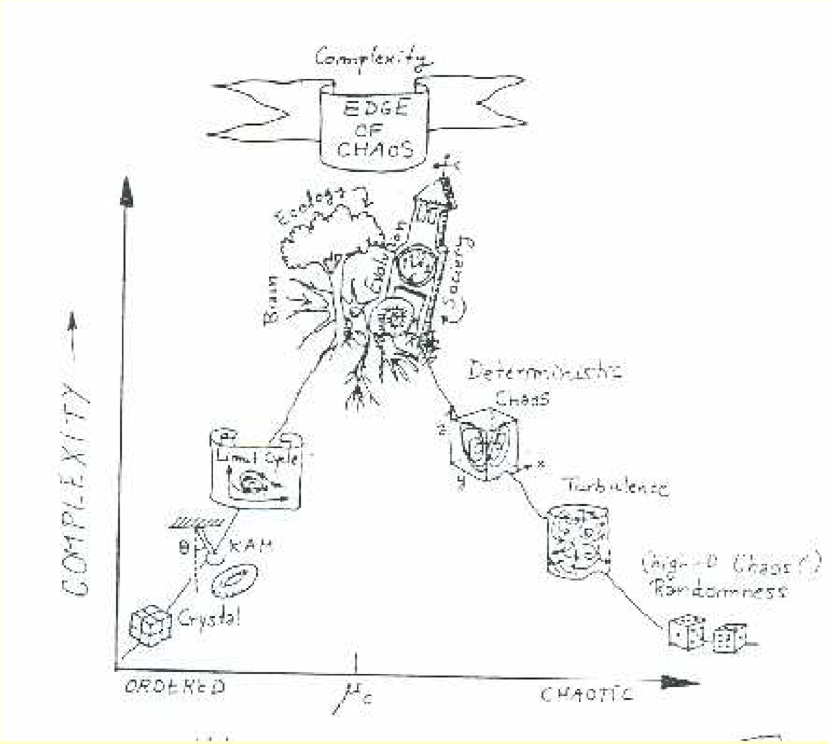
There are two primary regimes of rules, *periodic* (Class II) and *chaotic* (Class III), separated by a third, *transition regime* (Class IV)
(Langton's egg diagram)

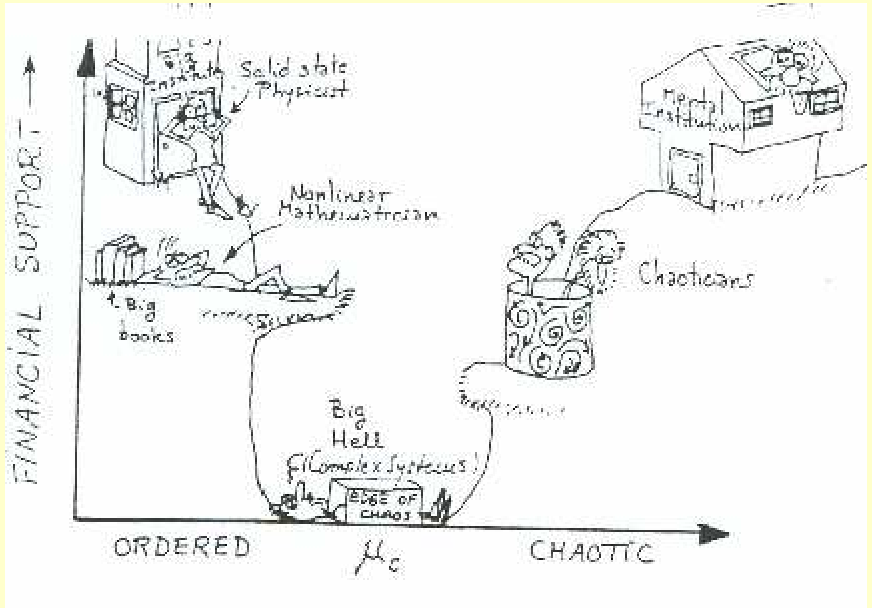


Schematic drawing of CA rule space showing the relationship between the Wolfram classes and the underlying phase-transition structure.

Jeremy Avnet (2000). Computation, Dynamics and the Phase-Transition. http://www.theory.org/complexity/cdpt/html/cdpt.html (last visited 24 June 2007)

EDGE OF CHAOS: NOT ALWAYS MAXIMUM





(Vicens Solé)

Self-Organized Criticality (SOC). 1

A working definition of self-organized criticality is that a slowly driven system will evolve into a statistical state that exhibits scale invariance without any parameter tuning. The system fluctuates about a quasi-equilibrium state.

- **Input to system is steady.**
- **Minor events start chain reactions affecting any number of elements in the system.**
- **Output series of 'avalanches' with power-law frequency-size statistics.**
- **Criticality: power-law scaling: $N(E) \sim E^{-b}$, $b \sim 1$**
- **SOC: Systems evolve spontaneously towards a critical state.**

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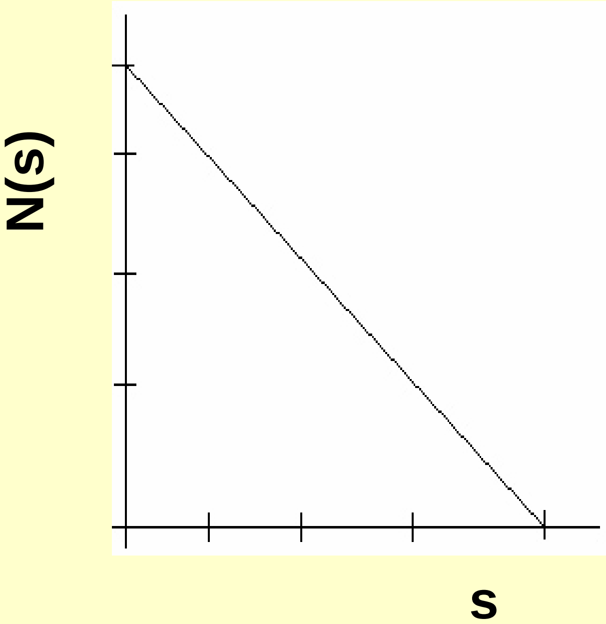
Self-Organized Criticality (SOC). 2

Power Law: graphed

$$N(s) = s^{-t}$$

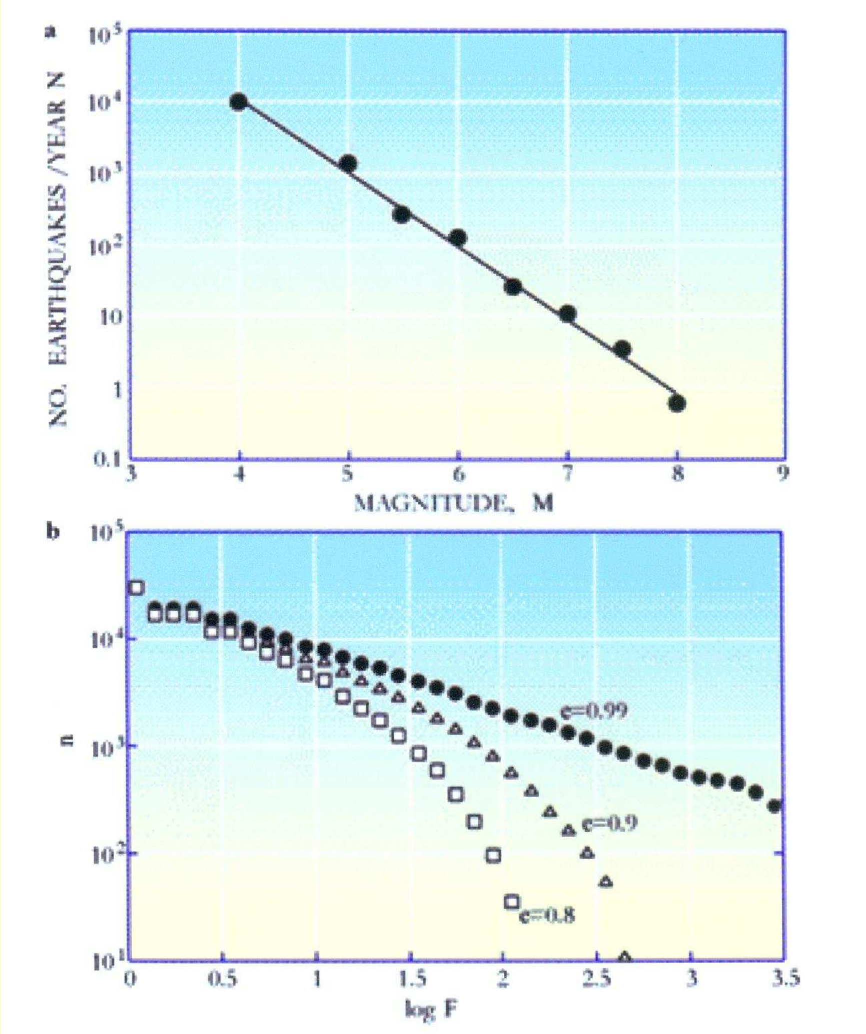
$$\log N(s) = -t \log s$$



CONTROL PARAMETERS

In natural systems the power law distribution is reached spontaneously.

In laboratory or in numerical simulations the power law distribution is reached by tuning some **control parameters**.



MODELING NATURAL SYSTEMS. 1

One of the most striking aspects of physics is the simplicity of its laws: see, for example, Maxwell's equations, Schrödinger equation, and Hamiltonian mechanics.

Every thing is simple and neat –except, of course, the world.

Complexity means that there appear structures that evolve with time. There is some natural tendency toward the formation of structures in the physical world.

Chaos, the sensitive dependence of a final result upon the initial conditions, is also found very frequently.

A complex world is interesting because it is highly structured. A chaotic world is interesting because we do not know what is coming next. But the world contains regularities as well.

MODELING NATURAL SYSTEMS. 2

For example, climate is very complex, but winter follows summer in a predictable pattern. Our world is both complex and chaotic.

Nature can produce complex structures in simple situations, and can obey simple laws even in complex situations

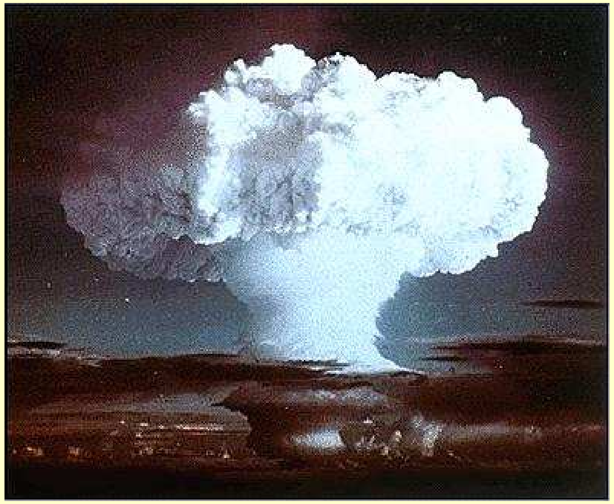
NATURAL & MAN-MADE ORGANIZED SYSTEMS

hurricanes

tornados

galaxy

thermonuclear explosion



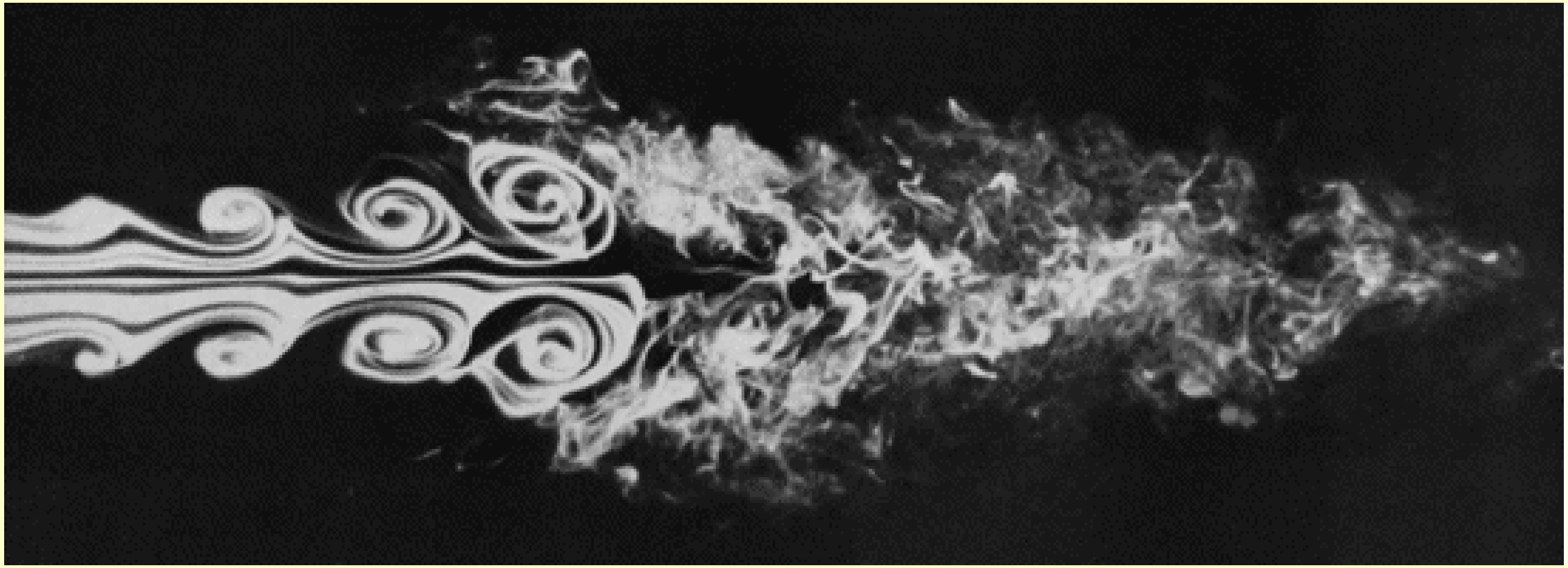
PHASE TRANSITION ORDER/DISORDER

The phase change corresponds to the evolution of a laminar to turbulent flux (Taylor-Couette convection) in an ascensional column of cigarette smoke.

Ford, J. *How random is acoin toss?* Physics Today, April 1983, 1

MODELING NATURAL SYSTEMS. 3

In complex systems the separation of scales is not possible becausen theynare coupled: we are entering the domain of chaos and fractal geometry (hierarchical systems). Chaos describe the coupling of the different scales, and fractals describe the relation among the components of the system at different scales.



Leonardo da Vinci's illustration of the swirling flow of turbulence.
(*The Royal Collection © 2004, Her Majesty Queen Elizabeth II*)

CREATING COMPLEXITY

Fluids frequently produce complex behavior, which can be either highly organized (think of a tornado) or chaotic (like a highly turbulent flow).

What is seen often depends on the size of the observer.

The equations that describe how the fluid velocity at one point in space affects the velocity at other points in space are derived from three basic ideas:

- ***Locality***. A fluid contains many particles in motion. A particle is influenced only by other particles in its immediate neighborhood.
- ***Conservation***. Some things are never lost, only moved around, such as particles and momentum.
- ***Symmetry***. A fluid is isotropic and rotationally invariant.

Nature has been kind enough to provide us (a lot of times) with a convenient separation of length, energy and time scales.

UNDERSTANDING COMPLEXITY

To extract physical knowledge from a complex system, one must focus on the right level of description.

There are three modes of investigation of systems like this:

- experimental
- computational
- theoretical

Experiment is best for exploration, because experimental techniques can scan large ranges of data very efficiently.

Computer simulations are often used to check our understanding of a particular physical process or situation. Use the right level of description to catch the phenomena of interest.

The same applies to ***theoretical work***. The modeling should be driven by asking “What are the simplest nonlinearities that should be present”.

DESCRIBING COMPLEXITY. 1

Traditional physics is based on reductionism: the distinct contributions of a phenomenon can be dealt with separately. **The reductionist approach is valid when the characteristic scales of the physical processes can be separated.**

The idea of the separation of scales is that there are three types of processes: **slow**, **dynamic** and **fast**. Each of these processes uses a different approach.

The **slow processes** are considered to be static. All of the parameters describing these slow processes are fixed (frozen).

The **dynamic processes** are the ones we treat using Newtonian laws of physics, logical laws of computers, *etc*.

The **fast processes** are averaged over.

DESCRIBING COMPLEXITY. 2

The problem with the idea of separation of scales in complex systems is that the different scales of behavior become coupled.

This is captured in a simple through the study of **chaos** and **fractals**.

Chaos describes the coupling of different scales through the time evolution of the dynamics of the system (**amplification or dissipation of differences**).

Fractals describe the **causal/logical relationship between behaviors of the system at different scales**. Since the behavior of the system at different scales are related, our descriptions should include these relationships.

The use of separation of scales in the real world is to average the fast degrees of freedom (thermodynamics) and discuss their influence on the dynamic degrees of freedom (Newtonian Mechanics) while keeping fixed the slow degrees of freedom.

EPILOGUE

A complex system is exactly that: there are many things going on simultaneously. If you search carefully, you can find your favorite toy: fractals, chaos, self-organized criticality, phase transition analogies, Lotka-Volterra predator-prey oscillations, etc., in some corner, in a relatively well-developed and isolated way. But do not expect any single insight to explain it all.

(Rolf-Landauer)

(resist over-enthusiastic attempts at universalization!)

APPENDIXES

APPENDIX I. EMERGENCE OF STRUCTURES AS PHASE TRASNSITIONS

1. EQUILIBRIUN SYSTEMS

2. FAR FROM EQUILIBRIUM SYSTEMS

    2.1. Emergence of patterns: interactions of elements

    2.2. Beginning of turbulence

    2.3. Extended systems

    2.4. Far from equilibrium systems (formation of patterns) with respect to equilibrium systems (phase transitions)

APPENDIX II. EXAMPLE OF A SIMPLE ADAPTABLE SYSTEM

APPENDIX III. EXAMPLE OF DYNAMIC SELFORGANIZATION: SELF-ORGANIZED CRITICALLITY

PHASE CHANGE ORDER-DISORDER. 1

Equilibrium systems – The Ising model

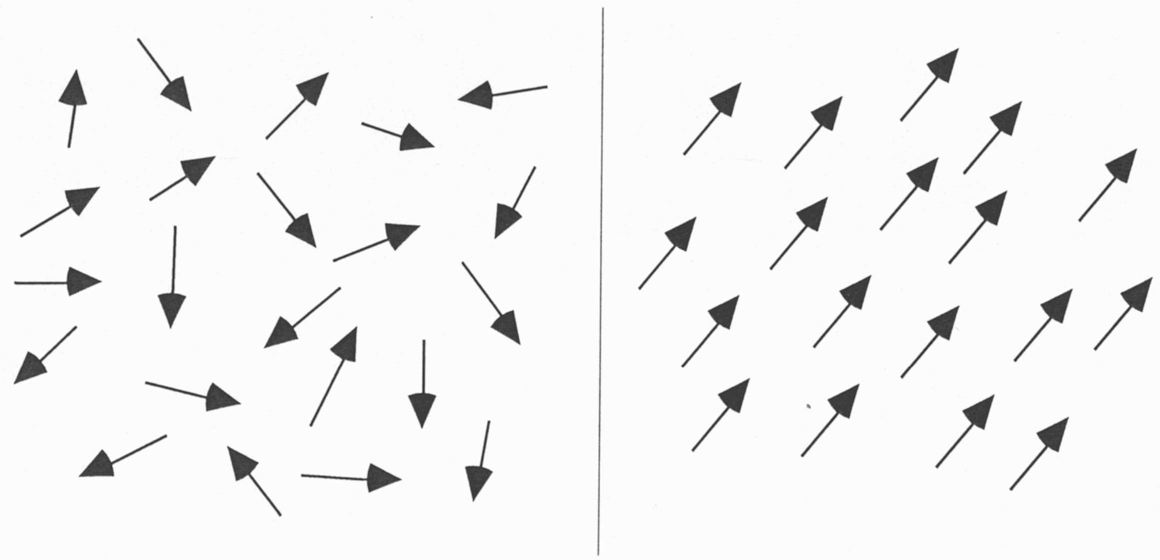


Figure 1: two arrangement of spins: disordered (left) and ordered (right)

Second order phase transition for a magnetic material. The arrows represent the spin of the particles.

THE ISING MODEL

Consider a sheet of metal:

It has the property that at low temperatures it is magnetized, but as the temperature increases, the magnetism “melts away”.

We would like to model this behavior. We make some simplifying assumptions to do so.

– The individual atoms have a “spin”, i.e., they act like little bar magnets, and can either point up (a spin of +1), or down (a spin of –1).

– Neighboring atoms with different spins have an interaction energy, which we will assume is constant.

– The atoms are arranged in a regular lattice.

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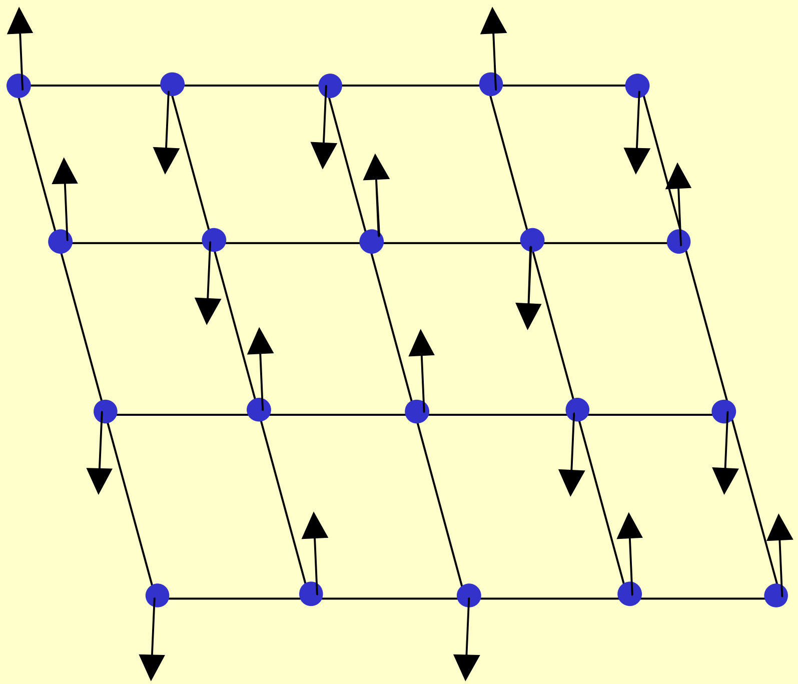
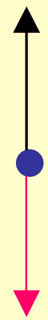
One possible state of the lattice

A choice of ‘spin’ at each lattice point.





q = 2

Ising Model has a choice of **two possible spins** at each point

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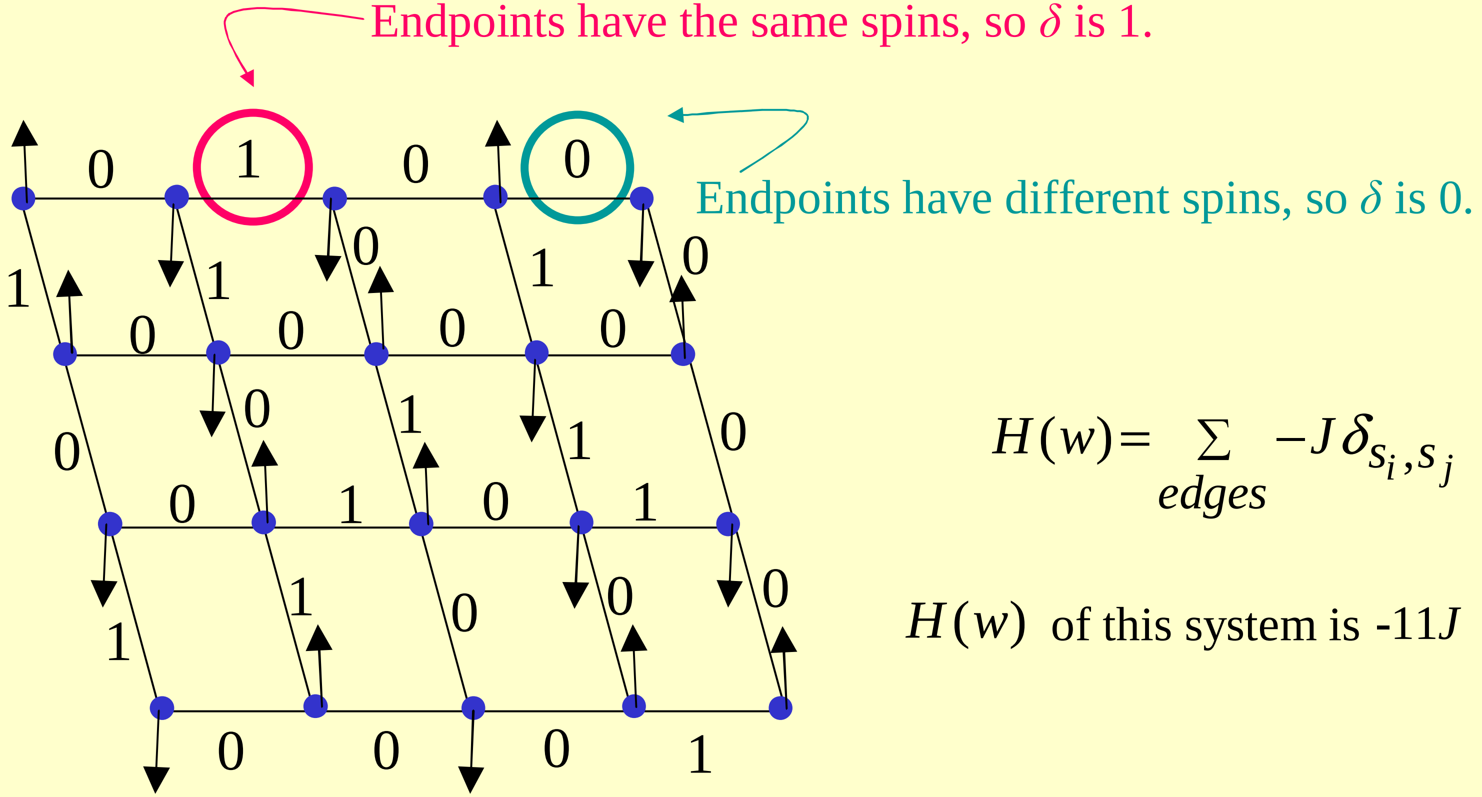
The energy (Hamiltonian) of the state



$$H(w) = \sum_{edges} -J\,\delta_{s_i, s_j}$$

$H(w)$ of this system is -11J

A state w with the value of δ marked on each edge.

THE ISING MODEL

Appendix I

The Spins interact according to a Hamiltonian

$$H = -J \sum_{\substack{\langle i\, j \rangle \\ (edge)}} s_i s_j - H' \sum_i s_i = -J \sum_{\langle ij \rangle} \delta s_i, s_j - H' \sum_i s_i .$$

Where s_i is the spin variable. The first term is responsible for the cooperative behavior and the possibility of a phase transition. *J* is the exchange of energy: positive j favors parallel alignment and negative *J* antiparallel alignment of the spins. <*ij*> denotes a sum over nearest neighbors spins (edges).

For *J* = *0* the equation is the Hamiltonian of a paramagnet. The only influence ordering the spins is the field *H'.* They do not interact, there are no cooperative effect and hence no phase transition.

ORDER PARAMETER AND CORRELATION LENGTH

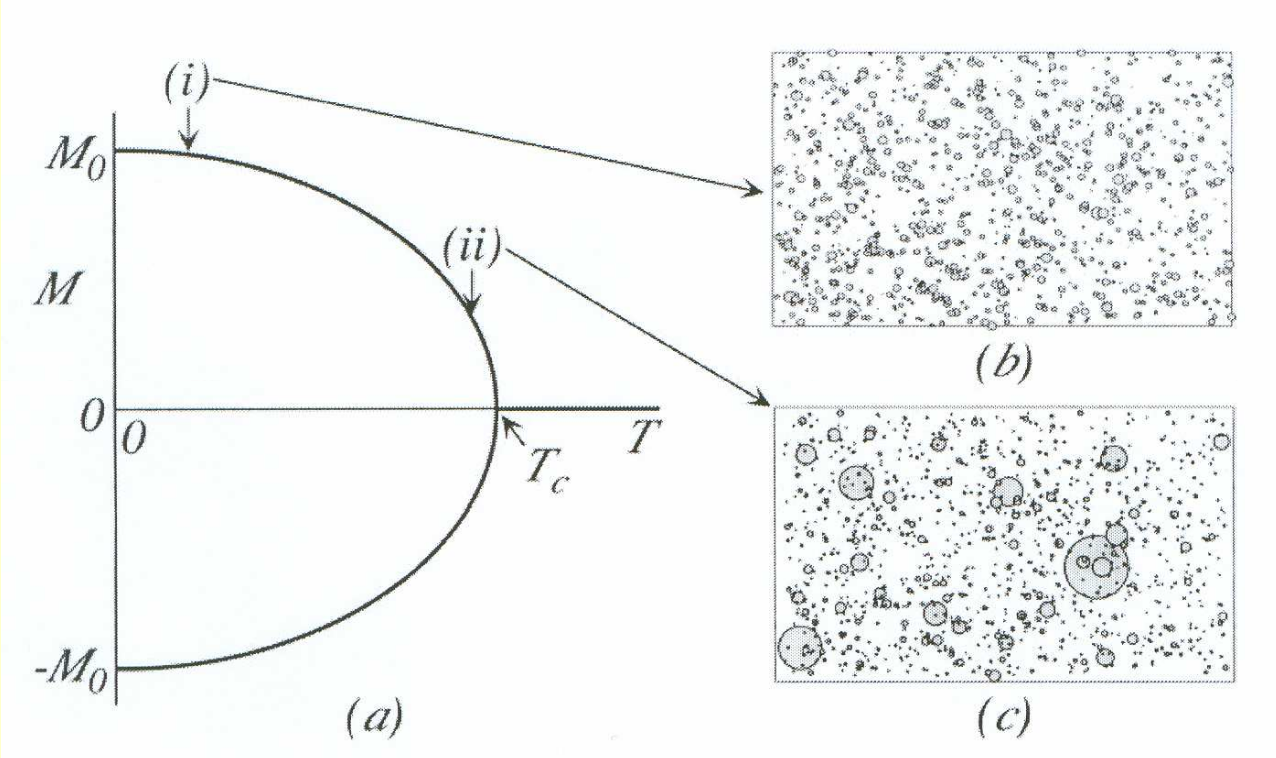
The **order parameter Q** can be defined as the difference of a state variable characterizing the system in a given state (for example, the difference in densities in a fluid/vapor continuous phase transition). In earthquake occurrence it could be defined as the difference between the stress concentration and the resistance of the material, the critical stress $\boldsymbol{\sigma_c}$. (**Unfortunately, this parameter is not an observable, so that some other relater parameter should be found**).

The medium is inhomogeneous, so that the stress concentration will also be inhomogeneous. Consider the medium subdivided in patches and assume the patches characterized as **+1** if the stress concentration is ~ σ_c or **-1** otherwise. (in this way we reduce this model to an Ising one). The **correlation length ξ** can be roughly defined as the linear dimension of the largest correlated spatial structure (i.e. the size of the largest +1 or -1 island). From another point of view, it can be defined as the distance over which the effects of a disturbance spread.

Close to the critical point the order parameter and the correlation length show a power law behavior

$$Q \propto |\sigma_r|^{\beta}, \qquad \xi \propto |\sigma_r|^{-\nu}, \qquad \sigma_r = (\sigma - \sigma_c)/\sigma_c$$

Dependence of the magnetic moment on the temperature



Patches size: exponential distribution

Patches size: potential distribution

(c): $M \propto (T_c - T)^{\alpha}$ $\xi \propto (T_c - T)^{-\gamma}$

POTTS' MODEL

Let's consider a system of interacting spins, which can be parallel or antiparallel. The spins are located on a planar grid.

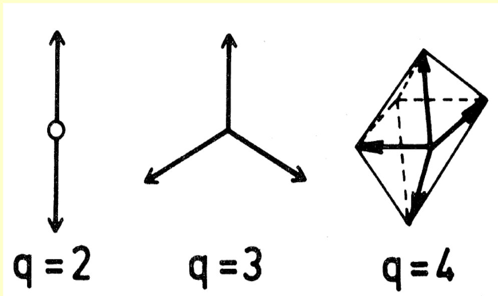
The direction of the spins may be those defined along one of the *q* possible values (ground states): $\theta_n = 2\pi n / q,\ n = 0,1,\ldots,n-1.$

In its more general form the interactions among the nearest neighbors depend only on the relative angle between the two vectors

$$\mathrm{H} = -\sum_{\langle ij \rangle} J\left(\theta_{ij}\right),\ \theta_{ij} = \theta_{ni} - \theta_{nj}\,.$$

As temperature increases, a paramagnetic phase transition appears, continuous for *q* ≤ 4 and first order (in two dimensions) for *q* > 4.

Examples



Ising model

PHASE CHANGE ORDER-DISORDER. 2

Far from equilibrium systems

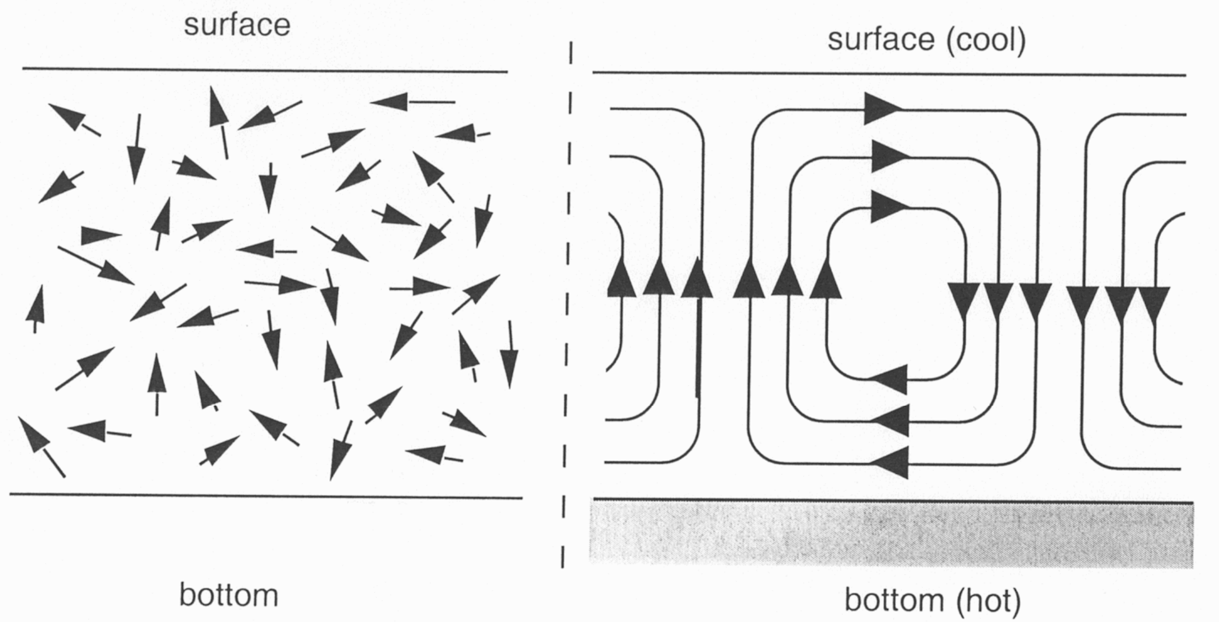
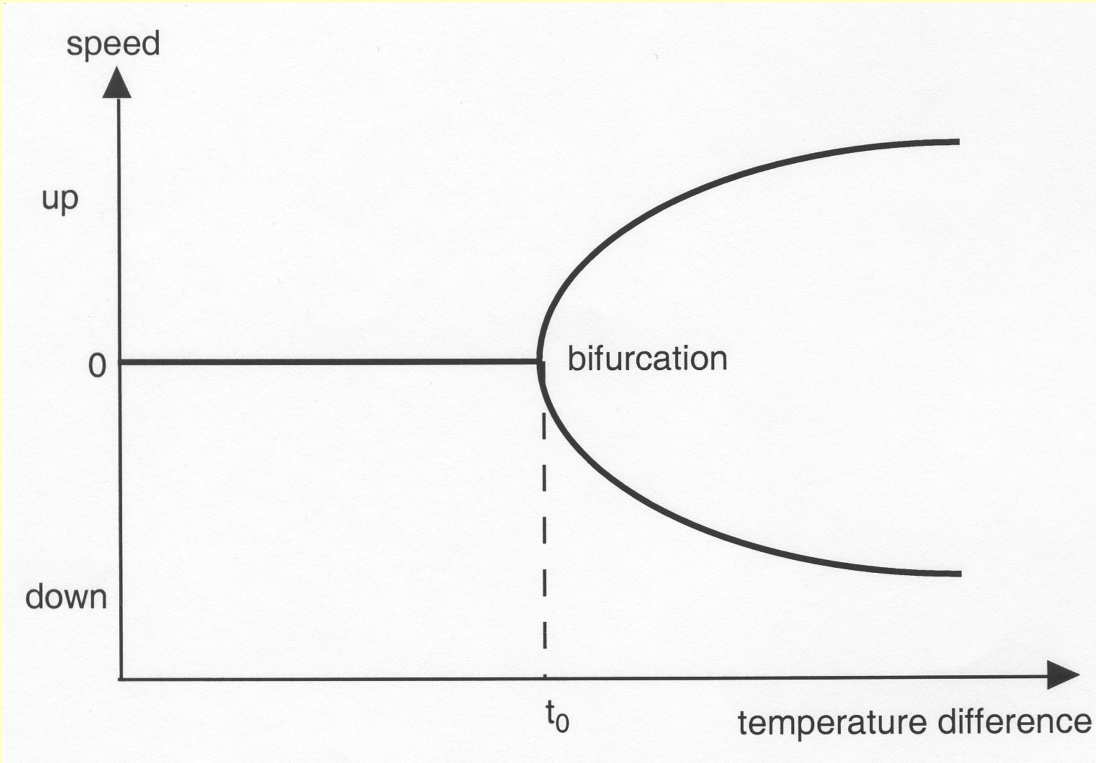


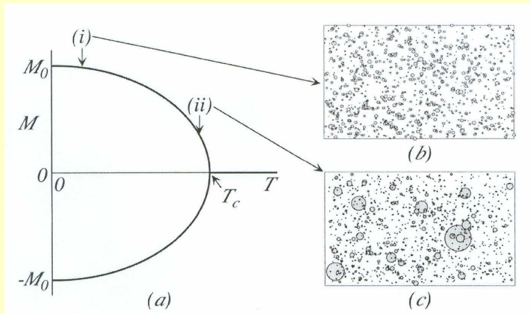
Figure 2: two types of movements of liquid molecules: random (left) and in the form of Bénard rolls (right), caused by a difference of temperature between the bottom and the surface of the container.

Rayleigh-Benard convection

BIFURCATION POINT

Appendix I





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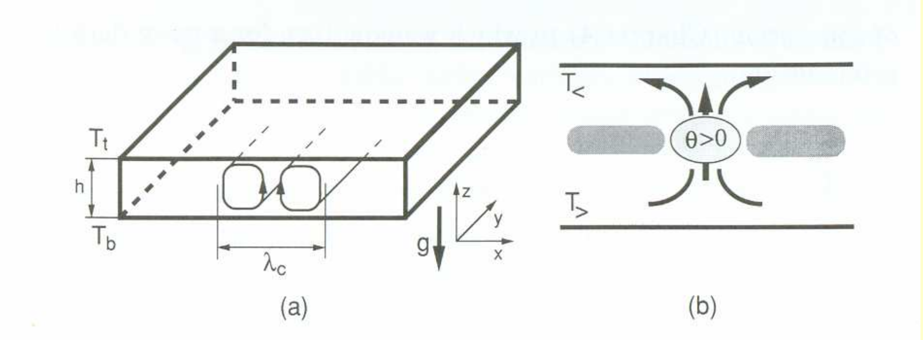
EMERGENCY OF PATTERNS. 1



Temperature gradient:

$$\beta = \Delta T / h,\ \Delta T = T_b - T_t$$

Temperature field

$$T_0(z) = T_b - \beta z$$

Let θ (> 0) be a temperature fluctuation with respect to the unperturbed profile.

Upward buoyancy force : as the density at the bottom is lower than that at the top (ρ decreases with increasing *T),* a drop of fluid experiences an upward force and tends to rise. As well, the fluid at the top, with higher density, tends to fall.

Dissipative forces:

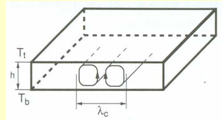
- Friction (damping by viscosity)
- Heat diffusion (the warmer drop looses its heat)

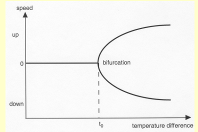
The instability develops only if the drop is accelerated sufficiently to overcome the stabilizing processes.

EMERGENCY OF PATTERNS. 2

Appendix I

Control parameter: β **Threshold:** β_c





Above the threshold a specific structure of organized convection cells develop.

Temperature field at mid-height of the cell: $\theta\left(z'=\frac{h}{2},x,t\right)\sim A(t)\cos\left(k_c x\right),$

For $\beta \sim \beta_c$ and A small enought (*A*: amplitude of the perturbation, the *order parameter*)

$$\boxed{\frac{dA}{dt}=sA}$$

$$s=r/\tau_0 \qquad r=\left(\beta-\beta_c\right)/\beta_c$$

τ_0: characteristic time scale

Beyond the threshold, *A* cannot grow indefinitely. Substituting s by

$$s_{ef}=\left(r-gA^2\right)/\tau_0 \qquad \boxed{\tau_0\frac{dA}{dt}=rA-gA^3}$$

This is a nonlinear evolution equation for the amplitude which correcly describes the bifurcation point.

EXTENDED SYSTEMS. 1

Appendix I

A(t) → **A(x,t)**

$$\theta \sim A\cos\left[k_c\left(x-\underline{x_0}\right)\right]$$

$$\boxed{\theta \sim A(t)\cos\left(k_c x\right)} \longrightarrow \boxed{\theta \sim \frac{1}{2}\left[A\exp\left(ik_c x\right)+c.c.\right]}$$

$$A=\left|A\right|\exp\left(i\phi\right), \quad \phi=-k_c x_0$$

Equation for the evolution of the amplitude |A| (now complex)**:**

$$\boxed{\tau_0\frac{dA}{dt}=rA-g\left|A^2\right|}$$

$$\left\{ \begin{aligned} &\tau_0\frac{d\left|A\right|}{dt}=r\left|A\right|-g\left|A^3\right| \\ &\tau_0\left|A\right|\frac{d\phi}{dt}=0. \end{aligned} \right.$$

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EXTENDED SYSTEMS. 2

The modulations with respect to a reference state may be described by means of an amplitude with spatial dependence $A \rightarrow A(x,t)$, known as ***envelope***.

Assume that the coherency involved in the mechanism of instability will be the origin of the inhomogeneities of diffusive relaxations. Thus, we can complete the amplitude equation with the term $\xi_0^2 \partial_{x^2} A$:

$$\boxed{\tau_0 \partial_t A = rA + \xi_0^2 \partial_{x^2} A - g|A|^2 A.}$$

This is the Ginzburg-Landau equation.

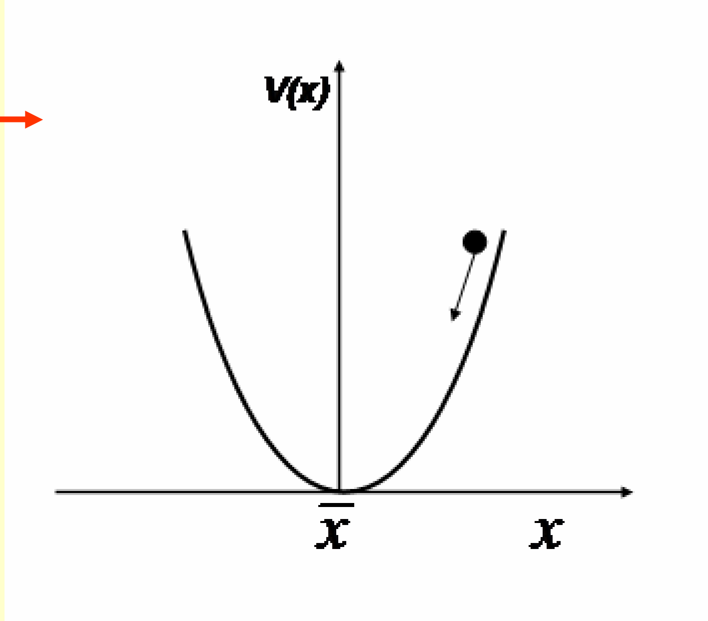
A SIMPLE ADAPTIVE SYSTEM. 1

Consider the branch of a tree which adapts its angle *x* under the weight of snow. If we call *D* the constant of elasticity of the branch, the equation of motion in the limit of strong friction (i.e., neglecting inertia)

$\boxed{\dot{x} = -Dx + F\,,}$ *F*: a constant external force that mimics the snow.

Introduce the potential $V(x) = \tfrac{1}{2}(x - \bar{x})^2 \quad \bar{x} = F/D \quad \tau = 1/D$

$$\boxed{\tau\,\dot{x} = (x - \bar{x}) = -\frac{\partial}{\partial x}V(x) \quad (1)}$$



The figure shows that the above equation describes also the motion of a particle sliding by the basin of a parabolic potential.

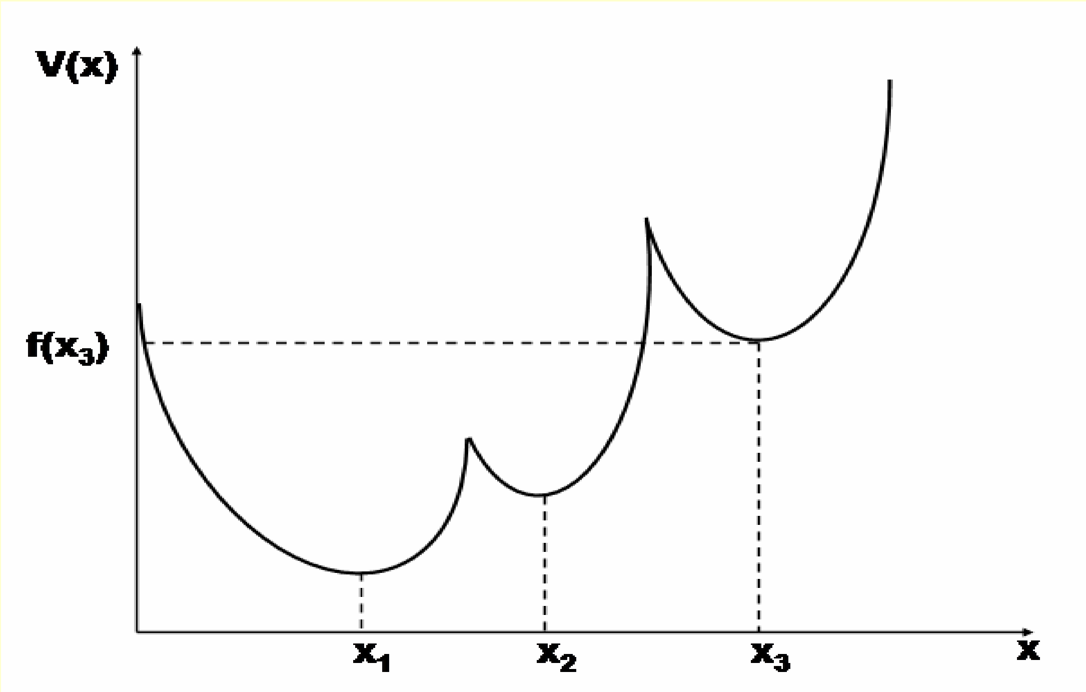
A SIMPLE ADAPTIVE SYSTEM. 2

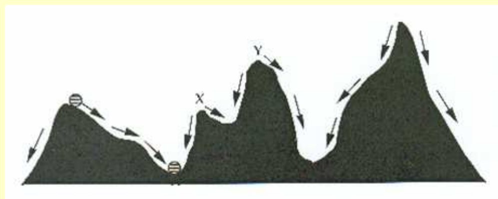
Appendix II

The potential in equation (1) can take a more general form, as for example

$$V(x) = -\frac{1}{\beta}\log\sum_{i=1}^{n}\exp\left[-\beta f(x_i)\right]\exp\left[-\beta\left(x_i - x\right)^2\right]$$

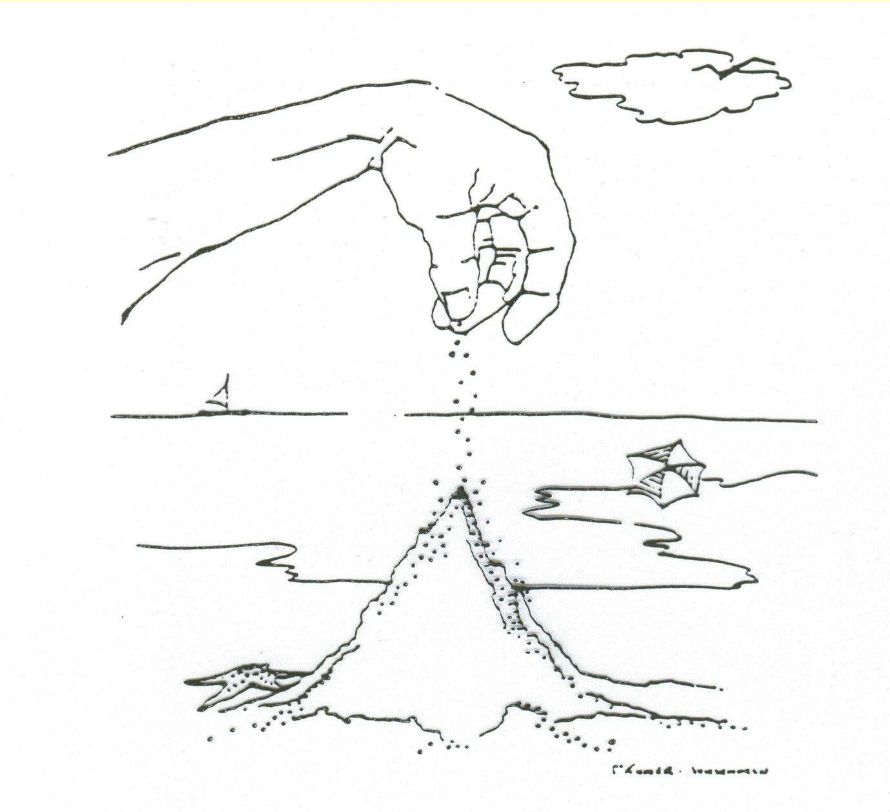
Which is represented in the figure for $\beta >> 1$ and $n = 3$.





SELF-ORGANIZED CRITICALLITY

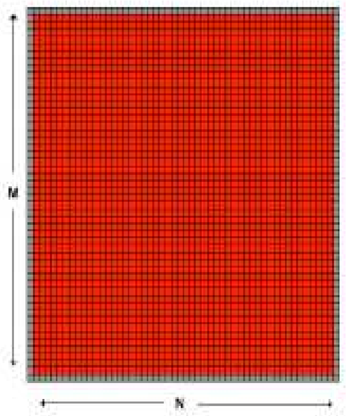
Paradigmatic example: **the sand pile**



Upon a slow loading, the pile spontaneously self-organizes to a critical state, in which the addition of a single grain may trigger an avalanche of any size. Once the pile is unloaded, the process starts again, describing cycles.

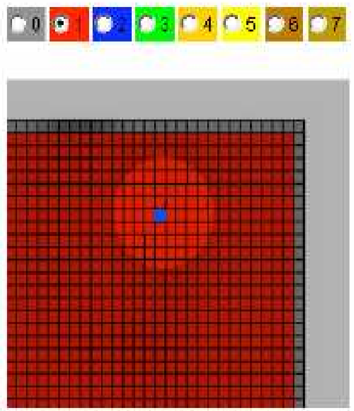
Appendix III

THE SAND PILE. 1



- Grey border represents the edge of the pile
- Each cell, represents a column of sand

Adding Sand to pile

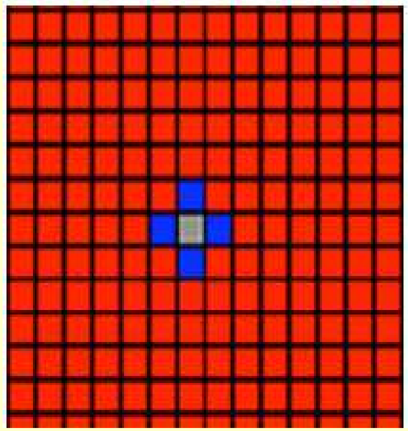


- Chose Random (x,y) position on grid
- Increment that cell
  - Z(x,y) → Z(x,y)+1
- Number of sand grains indicated by colour code

Model Rules

- Drop a single grain of sand at a random location on the grid
  - Random (x,y)
  - Update model at that point: Z(x,y) → Z(x,y)+1
- If Z(x,y) > Threshold, spark an avalanche
  - Threshold = 3

Avalanches



- When threshold has been exceeded, an avalanche occurs
- If Z(x,y) > 3
  - Z(x,y) → Z(x,y) – 4
  - Z(x+-1,y) → Z(x+-1,y) +1
  - Z(x,y) → Z(x,y+-1) +1

Duong (2003)

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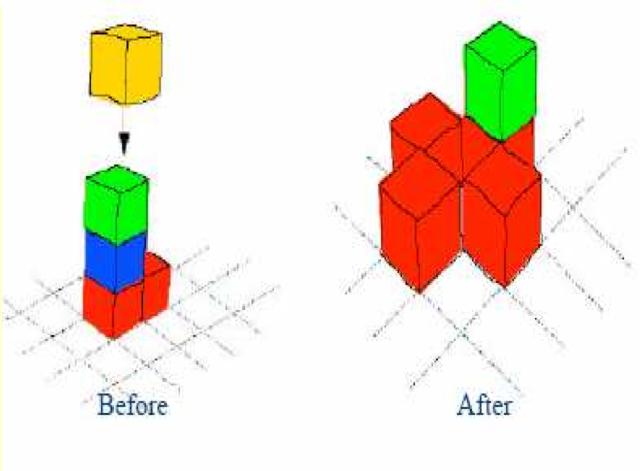
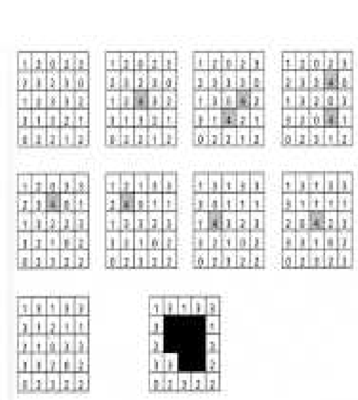
THE SAND PILE. 2

Appendix III

Before and After

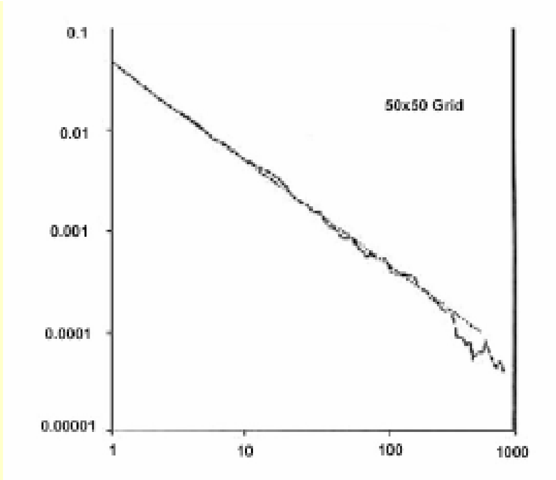


Domino Effect



- Avalanches may propagate

Size Distribution of Avalanches



Duong (2003)

Observances

- Transient/stable phase
- Progresses towards Critical phase
  - At which avalanches of all sizes and durations
- Critical state was robust
  - Various initial states. Random, not random
- Measured events follow the desired Power Law

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