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$1/f^\alpha$ noise as a source of the Earth's fluctuations

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Abstract. – In the absence of earthquakes the Earth undergoes continuous oscillations, mainly caused by oceanic/atmospheric fluctuations. For the frequency range of 0.04 Hz to 0.3 Hz, these fluctuations are known as microseismic activity and are characterized by a spectrum that depends on the site and on atmospheric storms. However, at its lowest level the microseismic activity appears to be quite stable worldwide and can be considered as an observational invariant which is not explained by standard theories. We interpret this invariant, that appears when the energy input is at a minimum, as Earth's equilibrium fluctuations in an elastic wavefield, independent of the external sources. A phenomenological model has been derived, in which the source of energy is represented as $1/f^\alpha$ noise that accounts for a superposition of relaxation transients.

It was well established from the early twentieth century, that in the absence of earthquakes the Earth undergoes continuous oscillations [1,2] often known as Earth hum. An analysis of its power spectrum reveals that it is generated as a superposition of different phenomena: in the frequency range of 2–7 mHz it is mainly due to atmospheric fluctuations [2], in the frequency range 0.04–0.3 Hz it is due to microseismic activity (see below), and at higher frequencies it is attributed to site response and local random noise. When the microseism activity is at its lower level (as, for example, in low atmospheric activity), the analysis of seismic records reveals the existence of wave field fluctuations, what we call the *equilibrium fluctuations* in the time domain and *equilibrium spectrum* in the frequency domain.

Microseismic activity, often contemplated as background noise, has been observed for more than a century [1, 3]. It is considered to be *noise* because it masks the arrival of coherent seismic phases. From the early days it was recognized that the source of well-developed

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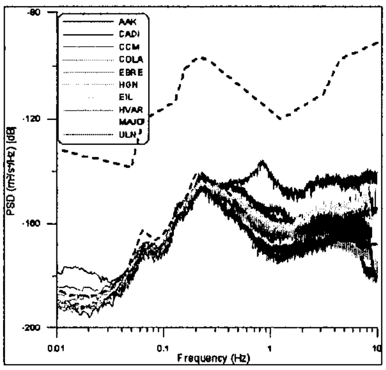

Fig. 1 – Power spectra of the minimum energy time series recorded at several continental seismic stations located in the Northern Hemisphere. It can clearly be seen that in the frequency interval of the microseismic activity, between 0.04 Hz and 0.3 Hz, all spectra coalesce to an observational invariant. For frequencies in the interval 0.03 Hz to 1 Hz, the discrepancies are due to the different local seismic responses, and at higher frequencies the spectra are dominated by local meteorological conditions. At frequencies lower than 0.04 Hz, the spectra are not comparable due to the USGS's High Noise Model (NHNM) and Low Noise Model (NLNM) [9].

microseismic activity is atmospheric storms at sea that generate quasi-stationary perturbations propagating as normal modes, mainly as Rayleigh waves [4]. Two main mechanisms of microseismic generation have been proposed: the surf mechanism [1] generating primary microseisms with a period equal to the storm wave period, and the interference mechanism [5], generating secondary microseisms as the result of fluctuations of pressure caused by standing waves along the sea bed, with a period equal to a half that of the storm waves. As a general rule, spectral characteristics of microseismic activity are interpreted in terms of source mechanism [5], irrespective of medium response [6]. Standard models centered on spectral characteristics are unable to reproduce its dynamic evolution in phase space. Another approach consists in the development of phenomenological models. In a previous paper [7] we proposed a phenomenological model that reproduces, at a given observation point, the highly variable microseismic activity as a non-linear forced oscillation, in which the external forces represent the atmosphere-energy contribution, and the oscillator accounts for the medium properties. Their spectral characteristics display the same variability as the observed microseismic activity.

In the present study our goal is to retrieve the underlying characteristics of the equilibrium fluctuations, masked by the presence of microseismic activity. To do so, a search has been performed through a large set of seismograms recorded worldwide, looking for the spectrum that displays minimum energy, in the sense of the minimum area in the power spectrum for the frequency interval 0.04 Hz to 0.3 Hz (the interval of microseismic activity), that constitutes the best approximation to the background equilibrium fluctuations (that is, the wavefield in the absence of sources). A set of spectra of minimum energy computed from records of seismic stations located in the Northern Hemisphere is displayed in fig. 1.

In [8,9] a worldwide set of average spectra of observations in continental seismic stations is presented. All reported spectra display similar characteristics as those of fig. 1: a main peak located at about 0.2 Hz, a second peak located at about 0.07 Hz and very often a third peak located at about 0.016 Hz. From the set of reported spectra, as well as those presented in fig. 1, it can clearly be seen that the frequency ratio between the primary and the secondary peaks is closer to the ratio (1 : 3). After an exhaustive search we have found that the computed minimum spectra determined from the seismic stations presented in [9] and in fig. 1 reveal that the frequency location of the largest spectral peak is an observational invariant: its amplitude is the same for all analyzed continental stations. Taking into account that we are dealing with spectra of minimum energy, this finding constitutes strong evidence in favor of the spectral characteristics as a global medium property (in oceanic stations located at the Southern Hemisphere the location of the peaks is the same, with some variation in the amplitude, probably due to large scale structural variations). Moreover, from this point of view, the spectra of well-developed microseismic activity can be interpreted as fluctuations of the equilibrium spectrum.

In order to derive a model to explain the characteristics of the minimum spectra, we hypothesize that the main source of energy responsible for the excitation of the fluctuations is provided by the presence of coda waves, originated by the continuous occurrence of earthquakes of different magnitude and at different places, along with the coda of Rayleigh waves generated by atmospheric fluctuations [2] and atmospheric storms [5], all of them defining an extended source. Coda waves are the result of a multiple scattering process. In the course of its propagation, three different regimes can be distinguished [10]: the ballistic regime (associated with non-scattered energy), the diffusion regime (characterized by a diffusive decay with time due to multiple scattering) and the equipartioning regime (for which the flux of energy falls to zero, but the average energy is still above the background level, and for which the wavefield [11] has lost any preferable propagation direction). Whereas the well-developed microseismic activity occurs in the ballistic regime, the equilibrium fluctuations are placed in the equipartitioned regime. We understand equipartioning regime as that regime for which the energy is separated into multiple wave packets due to multiple scattering, where initial coherent wave-fronts are broken and re-radiated as in Huygens reflections. After some interactions, the energy has lost any propagation direction, the information about the source is totally diffused [12] and what remains is the medium response that can be represented in terms of normal modes [10, 11].

It was shown in [13] that the power spectrum of coda waves follow a power law that behaves as $1/f^\alpha$ noise, and we hypothesize that, in a first approximation, this behavior can be extended to the codas of Rayleigh waves generated by heavy storm fluctuations, see [2]. Moreover, $1/f^\alpha$ noise can be interpreted [14, 15] as a superposition of transients. This interpretation also agrees with the classical coda model of Aki [16]. In the present study we assume the multiple scattered coda waves are the only source of energy (we emphasize we want to model the equilibrium fluctuations).

The present study is devoted to the vertical component of the seismogram, well represented by a scalar equation. To obtain the gross features of microseismic activity, in [7] a nonlinear analysis of the recorded time series was reported, revealing that the recorded time series is non-stationary, stochastic and displays nonlinear behavior. Figure 2 displays the evolution of the microseismic time series in phase space, clearly displaying chaotic behavior. The phenomenological model devised in [7] to simulate the observed microseismic displacement field $q(t)$ was derived from the point of view of the observer, *i.e.*, to simulate the oscillations recorded by the seismometer, hence the use of a nonlinear oscillator. Moreover, as shown in [17], travelling-wave solutions of nonlinear partial differential equations $u(x,t)$ of the kind $u_{tt} + au_{xx} + bu + cu^3 = 0$, can be transformed into nonlinear ordinary differential equations $u(\xi)$, $\xi = k(x - \lambda t)$, where k and λ are constants. The model reads:

$$\begin{aligned} p &= \dot{q}, \\ \dot{p} + \frac{\partial V(q)}{\partial q} + \delta p &= \sum_{i=1}^{2} \gamma_i \cos(\omega_i t) + \Gamma \varepsilon(t), \end{aligned} \tag{1}$$

where V is a mono-potential defined as $V = -\nu(t)\frac{q^2}{2} + \beta\frac{q^4}{4}$ and $\varepsilon(t)$ is random noise, assumed to be white noise. δ is the coefficient of damping, β the coefficient of nonlinearity, γ_i the

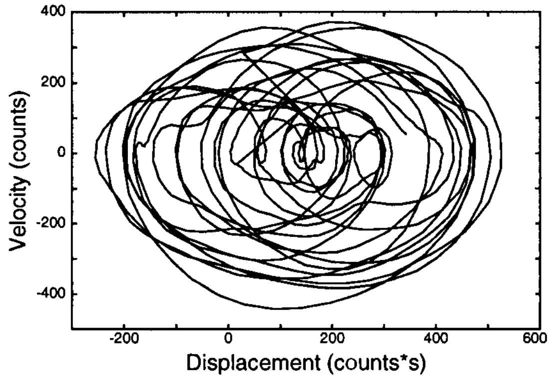

Fig. 2 – Evolution of the microseismic time series in phase-space: the motion follows well-defined trajectories, similar to those of a particle bouncing irregularly in a potential well, consisting of a superposition of loops of different mean radius (*i.e.* motion with different frequencies) with the centre of the loops displaying separate irregular oscillations, over a well-defined path. The corresponding motion is random in the sense that it is not possible to predict either the time evolution of the center of the loops or its mean radius.

amplitudes of two external harmonic forces of frequency ω_i and Γ the noise amplitude. In order to reproduce the spectral shape of the microseism correctly, a parametric resonance was introduced by defining a time-dependent coefficient for the linear term as $\nu(t) = \nu_0(1 + \eta\cos\omega_0 t)$, where η is the amplitude and ω_0 the frequency of the parametric resonance. As can readily be seen, eq. (1) constitutes a generalization of the Duffing equation, that in turn can also be interpreted as a generalization of the Longet-Higgins model [5]. Our previous model [7] is thus compatible with the observed fact [2] that, in the presence of large energy sources (such as atmospheric storms or fiord resonances, represented by the two harmonic forces) coherently travelling Rayleigh waves will develop.

However, in the equipartitioned regime these forces will be negligible. To take into account coda waves as the main source of energy, our model equation (1) needs to be modified. As already reported, for well-developed coda waves the power spectrum behaves as $1/f^\alpha$ noise, but, on the other hand, as we minimize high-frequency fluctuations we can set in eq. (1) the terms γ_i, η and Γ equal to zero. Hence, as a working hypothesis we assume $1/f^\alpha$ is the only source term. As a first approximation, and from a pragmatic point of view, following [14] we interpret the $1/f^\alpha$ spectrum of coda waves in the diffusive/equipartioned regime as a simple exponential-relaxation process $N(t) = N_0 e^{-\lambda t}$ for $t \geq 0$ and $N(t) = 0$ for $t < 0$. As coda waves are continuously generated, we will consider a summation of exponential processes, with the inter-event time following a Poisson distribution. Because on average, the excitation level of hum is equivalent to that of magnitude 5.7–6.0 earthquakes, see [2], we assume an initial amplitude N_0 constant for each exponential process, and finally, a random phase has been added. Thus, our model reads

$$\begin{aligned} p &= \dot{q}, \\ \dot{p} + \frac{\partial V(q)}{\partial q} + \delta p &= N_0 \sum_{i=1}^{n} e^{-\lambda(t-t_i)}, \end{aligned} \tag{2}$$

where $V(q)$ is as in eq. 1) but for $\nu(t) = \nu_0$, and the sub-index i stands for each of coda wave-packets.

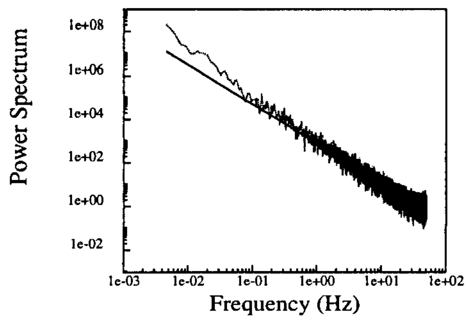

Fig. 3 – Representation of the source term, $1/f^{\alpha}$ noise, of the proposed model for the minimum energy spectrum. The source term consists of the summation of exponential relaxation processes $N(t) = N_0 e^{-\lambda(t-t_i)}$. The following values of the model parameters have been used: $\lambda = 0.01$, $N_0 = 5.0$, and for the Poisson distribution $P(t_i - t_{i-1} = \exp[-1.0(t_i - t_{i-1})]$. The plot clearly displays a linear trend with an exponent $\alpha = 1.8$.

Figure 3 displays the spectrum corresponding to $\sum_{i=1}^{n} e^{-\lambda(t-t_i)}$: clearly a $1/f^{\alpha}$ noise has been obtained for the source term, with $\alpha = 1.8$. Figure 4 displays the spectrum predicted by the model, eq. (2), and fig. 5 its corresponding evolution in phase space. These numerical simulations follow very closely the observations presented in figs. 1 and 2. As this is a first approximation, no attempt has been made to match observations, although the influence of model parameters deserve some comments, based on model simulations. The values of the parameters δ, N_0 and λ are irrelevant from the point of view of the conclusions of the study. δ and N_0 control the amplitude of the oscillations, whereas λ controls the decay of the relaxation. We have simply chosen δ to be small and the values of N_0 and λ are rather arbitrary. The main concern is with the coefficient of nonlinearity β. This is the parameter that really controls the shape and location of the predominant frequency of the spectral peak, as shown in fig. 6 for different values of β. The influence of the nonlinearity will be further studied. The origin of the nonlinearity in the real Earth can be understood in terms of wave propagation through irregular wave guides, see [18] for example. A value of $\beta = 0.05$ has been chosen to account for smooth irregularities.

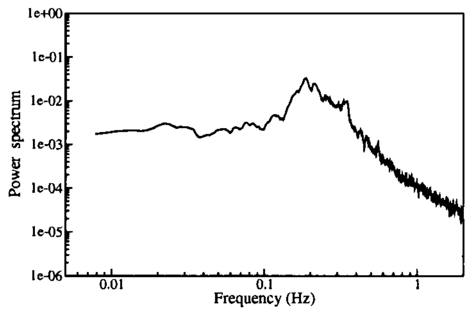

Fig. 4 – Power spectrum simulated by our proposed model. The values of the model parameters are $\nu = 4$, $\beta = 0.05$, $\delta = 0.01$. The spectrum clearly displays a main peak, located at about 0.3 Hz, and two secondary peaks located at about 0.2 Hz and 0.07 Hz. These secondary peaks appear as subharmonics of the main peak.

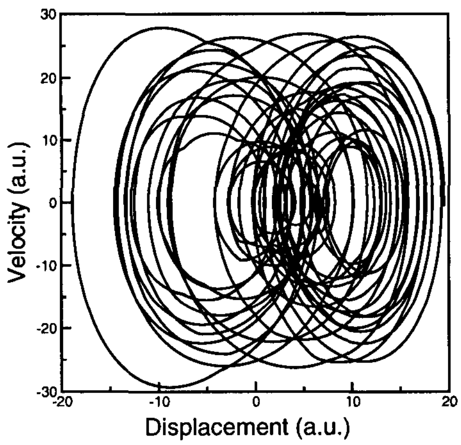

Fig. 5 – Evolution of the simulated time series (corresponding to the power spectrum of fig. 4) in phase space. Clearly, in a qualitative way this evolution is quite similar to that of the observed data, see fig. 2.

The results obtained so far clearly show that our model is able to simulate the spectrum and motion in phase space of observations (figs. 4 and 5) of equilibrium fluctuations, as well as the time series (not shown). As a consequence, the above results provide strong evidence in favor of our interpretation of background equilibrium fluctuations as representative of the Earth's response (resonance) due to internal and external excitation, a global medium property.

From a generic point of view, we can compare the recorded equilibrium wave field to black-body radiation. Black-body radiation refers to a system which absorbs all radiation incident upon it and re-radiates energy which is a characteristics of this radiating system only, not depending upon the type of radiation which is incident upon it. The energy absorbed by the Earth is that of the atmosphere, the oceans and earthquakes. When observing the minimum energy spectrum, the input of energy can be considered as quasi-stationary and isotropic. In the actual case, instead of temperature we can talk about stress. As the Earth

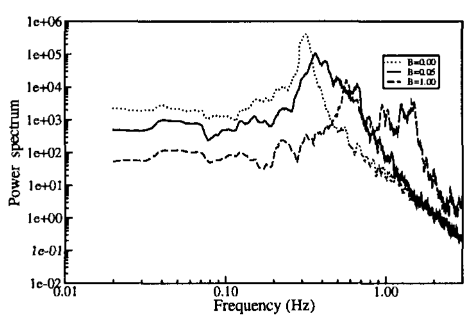

Fig. 6 – Evolution of the power spectrum as a function of the coefficient β of nonlinearity, which clearly controls the frequency contents of the main peak as well as its location. For $\beta = 0$ the main peak is located at *ca.* 0.3 Hz, progressively shifting to higher frequencies as β increases. Two subharmonics located, respectively at *ca.* 0.2 Hz and 0.07 Hz are always present, becoming stronger as β increases, probably due to a competition between the two nonlinear terms, δ and $1/f^{\alpha}$. Note that although the main peak strongly depends on β, the subharmonics are invariant.

is an open system, the incoming energy is re-radiated via a scattering process, giving rise to the background fluctuations with a well-defined reference spectrum that corresponds to the medium response, that can be associated, for a fixed point, to a chaotic oscillator in the presence of noise. We are, thus, in the presence of the resonance of the medium in the frequency interval of the microseism activity at its lower level.

As a main conclusion, we can say that the observed fluctuation spectrum, masked by the presence of well-developed microseism activity, is a medium property that corresponds to a nonlinear resonance of the heterogeneous medium. This medium response, a global property, is well defined by the minimum energy spectrum mainly excited by coda waves. The similarity between the equilibrium fluctuation spectra and that of the well-developed microseism activity can be interpreted in the sense that the latter consists on fluctuations of the former caused by the competition between the external forces (atmospheric storms) and the resonant peak of the medium.

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