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On Oscillations & Oscillators:
New Insights for Old Problems.

Equilibrium Fluctuations
&
Volcanic Tremors

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ON OSCILLATIONS & OSCILLATORS: NEW INSIGHTS FOR OLD PROBLEMS.

Equilibrium fluctuations
&
Volcanic tremors

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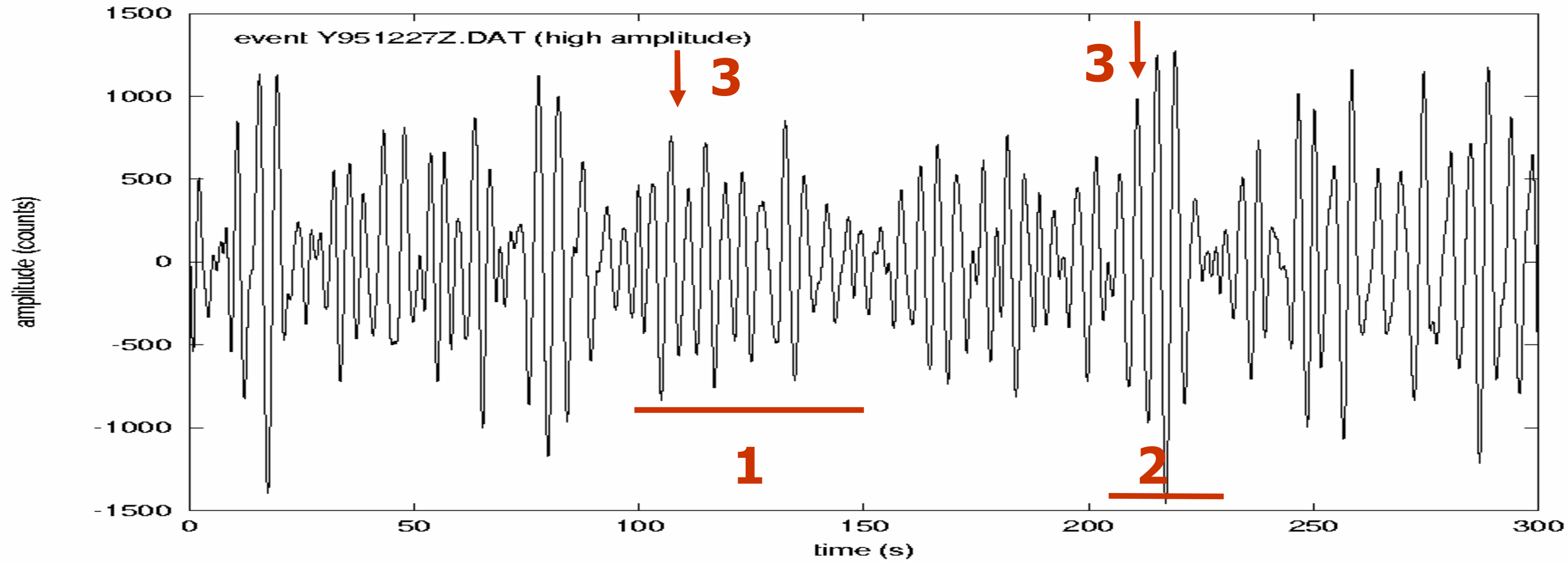
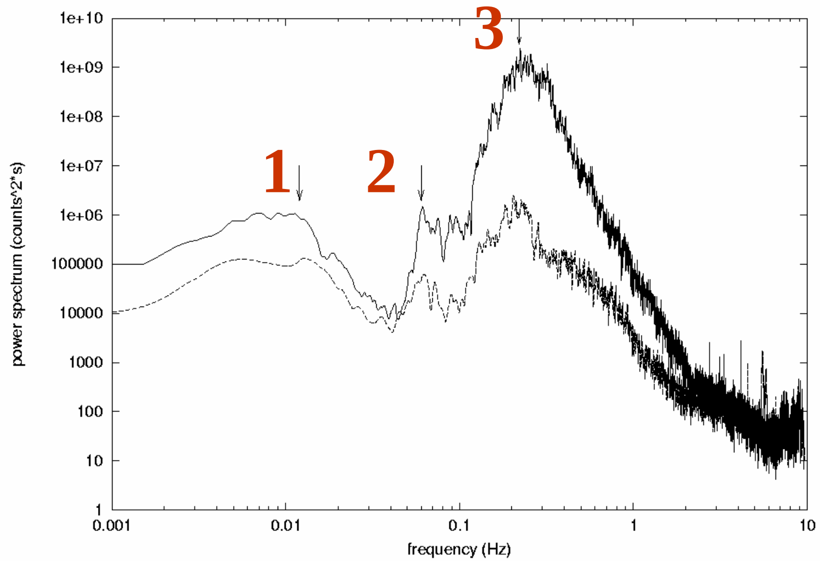
1. FROM MICROSEISM ACTIVITY TO EQUILIBRIUM FLUCTUATIONS

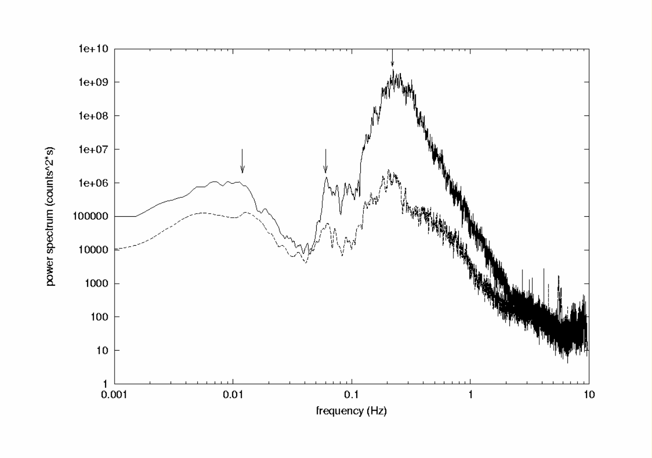




Microseism activity

MAIN FEATURES

- The central frequency of the main spectral peak may suffer important shifts, oscillating around a mean value, due to variations in the amplitude of the different sources.
- It is widely accepted that the source of microseism activity are storms at sea that generates quasi-stationary perturbations that propagate as normal modes, mainly as Rayleigh waves.
- Two main mechanisms have been proposed:
  - the surf mechanism generating primary microseism with a period equal to the storm wave period
  - the interference mechanism, generating secondary microseisms as the result of fluctuations of pressure caused by standing waves along the sea-bed, with a period equal to a half of that of the storm waves.



TIME SERIES ANALYSIS

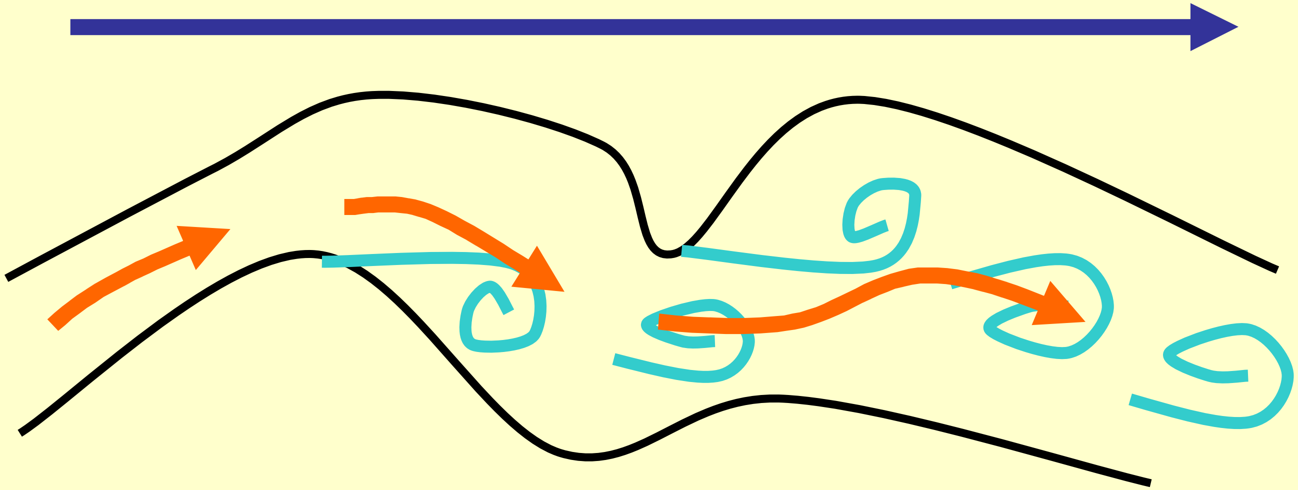
An analysis of the microseism time series (Ryabov *et al*., 2004) reveals that microseism activity is:

- Non-stationary
- Stochastic
- Non-linear

The **stochasticity** has been inferred because of the non-saturation of the correlation dimension for an embedding dimension of phase space up to 15; it could be due to a noise source component.
The **non-stationarity** has been inferred because of the no conservation of the first and second statistical moments, and can be due to processes of resonance in the stratified medium (irregular waveguides).
The **non-linearity** is inferred because otherwise it is not possible to simulate the evolution in phase space, and can be due to a nonlinear source process, non-linear medium or both.

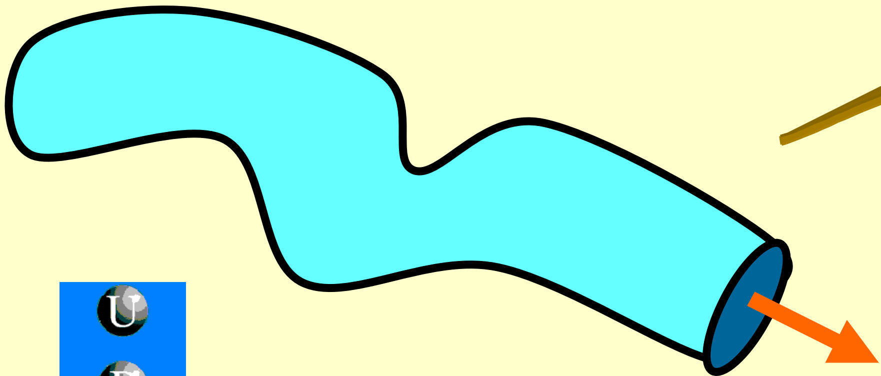
pressure drop

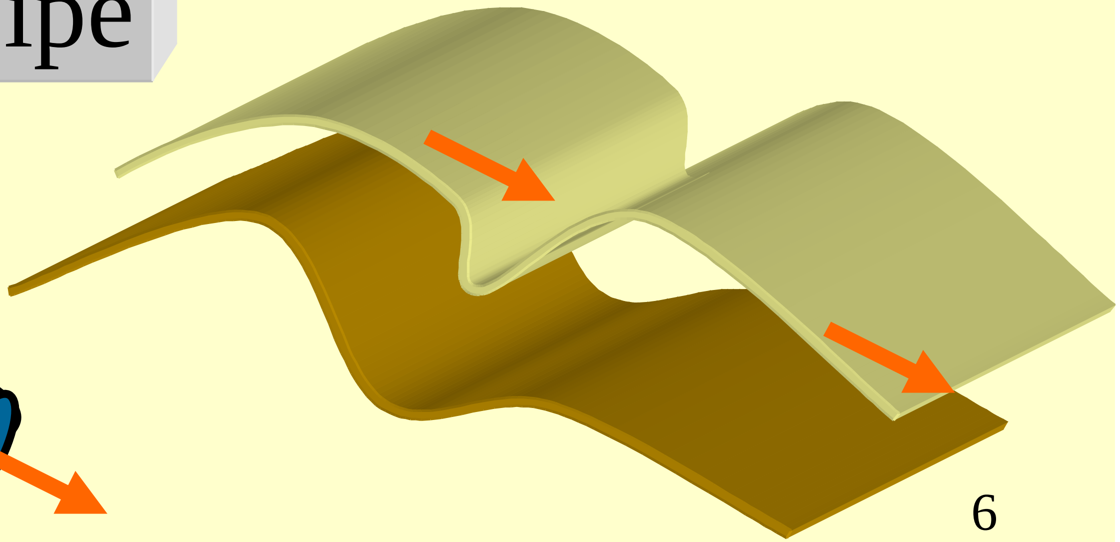
nonlinearity



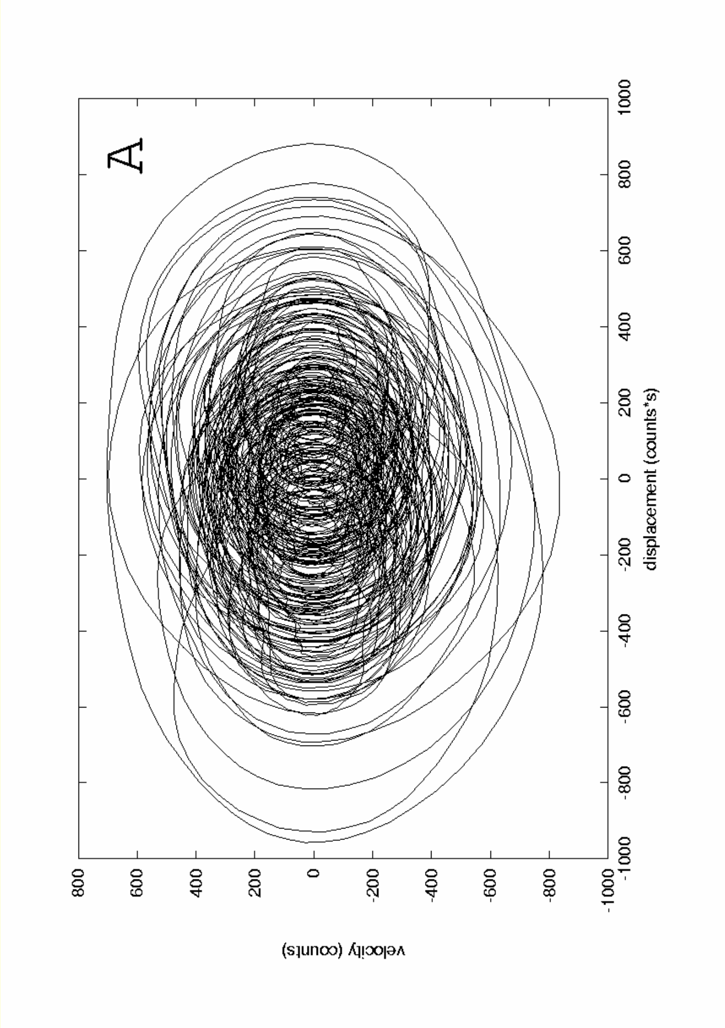
average flow speed

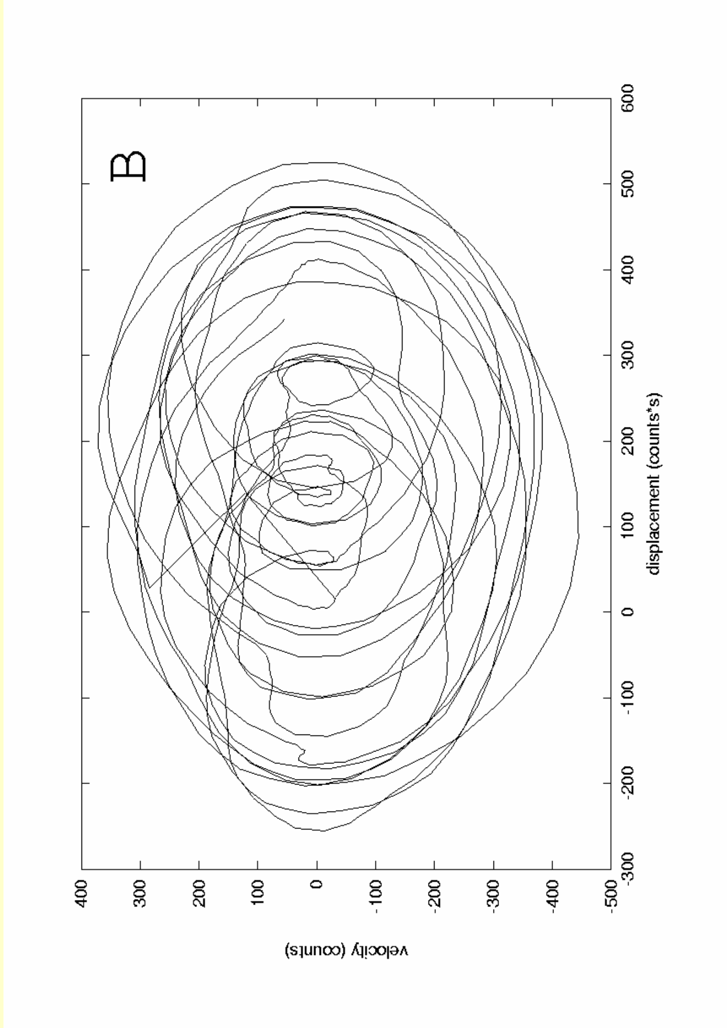
“random” pipe





PHASE SPACE EVOLUTION OF MICROSEISM TIME SERIES





MICROSEISMS' PHENOMENOLOGICAL MODEL

(Correig & Urquizu, 2002, following Longret-Higgins, 1950)

$$\dot{q} = p$$

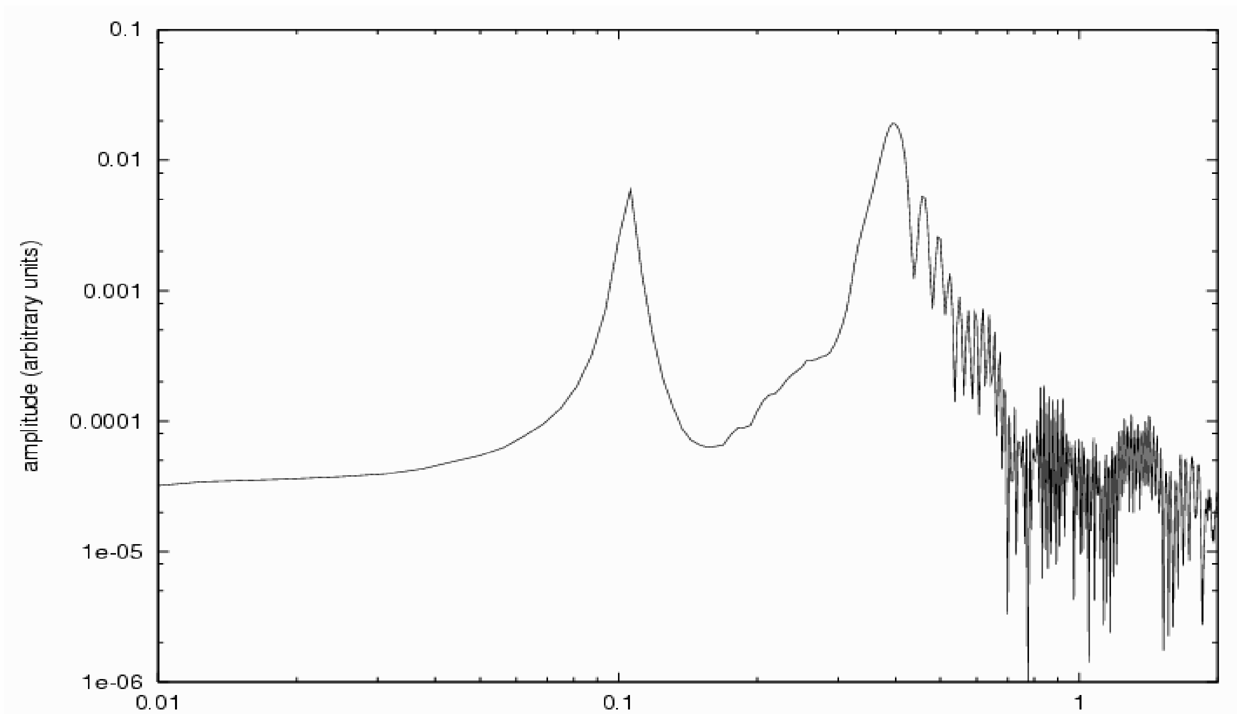
$$\dot{p} + \frac{\partial V_0(q)}{\partial q} + \delta p = \Sigma_{i=1}^{2} \gamma_i \cos(\omega_i t) + \varepsilon F(t)$$

$$V_0(q) = -\alpha \frac{q^2}{2} + \beta \frac{q^4}{4}$$

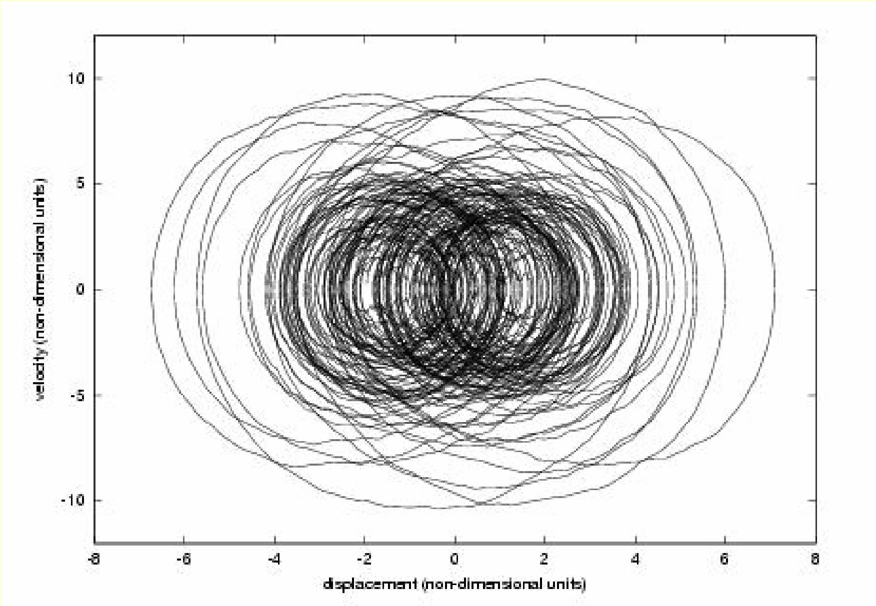
$$\alpha = \alpha_0 + \eta f(t)$$

The potential V_0 has the meaning of **medium response**

(a global property)



amplitude (arbitrary units)
0.1
0.01
0.001
0.0001
1e-05
1e-06
0.01
0.1
1



velocity (non-dimensional units)
10
5
0
-5
-10
-8
-6
-4
-2
0
2
4
6
8
displacement (non-dimensional units)

By similarity with the oscillatory model, we hypothesize:

1. **The main peak of microseism spectra can be interpreted in terms of the resonant response of the Earth's lithosphere.**
2. **The secondary peak, when present, can be interpreted as a subharmonic of the main peak.**
3. **The randomness of microseism time series can be due to medium lateral variations and to local, high-frequency, random 'noise' (local meteorological conditions and cultural activity) through an inverse cascading process.**
4. **Microseim activity, as a resonant response of the lithosphere lies between the high frequency local response of the medium (random) and the (linear) free oscillations low frequency response of the whole Earth.**

U B

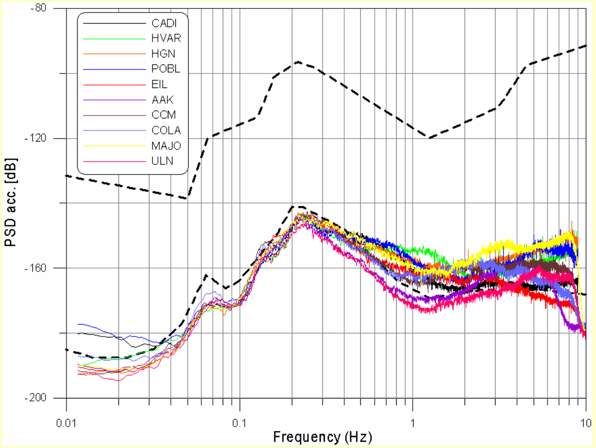
UNDERLYING DYMANICAL STRUCTURE OF MICROSEISM ACTIVITY

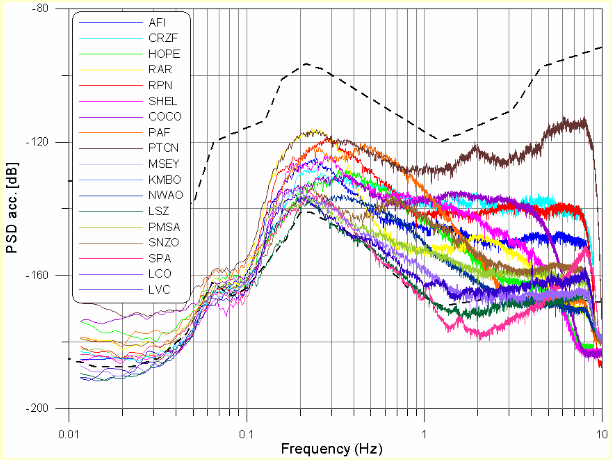
Microseism spectral peaks suffer strong fluctuations due to variations of the external forces. These fluctuations are well represented by the proposed phenomenological model.

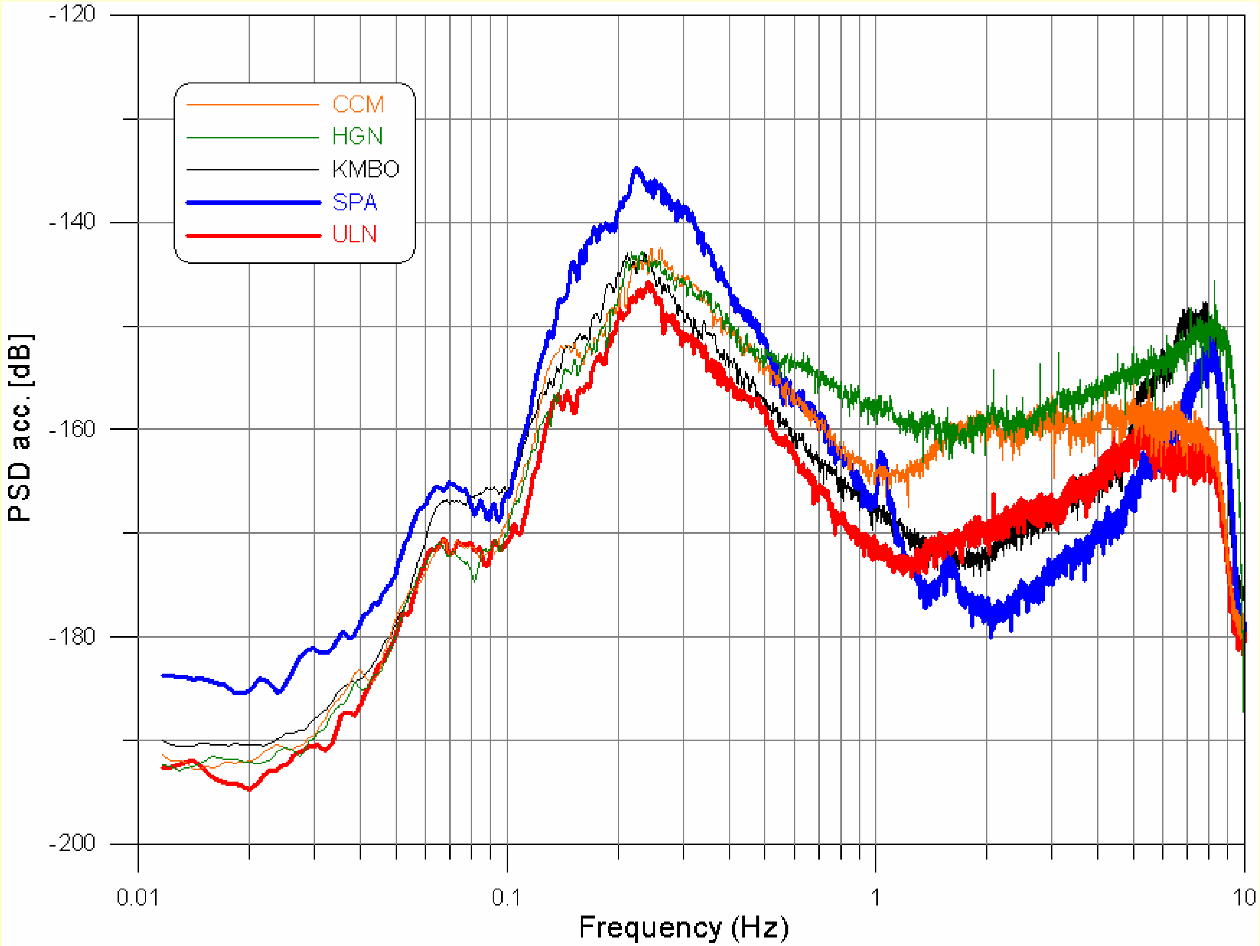
The influence of the variations of the external forces can be minimized looking at microseism time series in the absence of atmospheric and oceanic perturbations, that will be reflected by the lowest energy power spectrum.

The minimum energy spectrum can be obtained after analyzing the full set of all available records.

It is assumed that the minimum energy spectrum will be a good approximation of the Earth's equilibrium fluctuations.







Vila, Macià & Correig (2007). (submitted)

ON OSCILLATIONS & OSCILLATORS

Note that:

1. At the present we are not dealing with traveling waves, but with the record an observer would obtain at a fixed point; hence, our emphasis is referred to the oscillatory motion of a particle (the seismogram). Moreover, traveling-wave solutions of nonlinear partial differential equations $u(x,t)$ of the kind

$$u_{tt} + au_{xx} + bu + cu^3 = 0$$

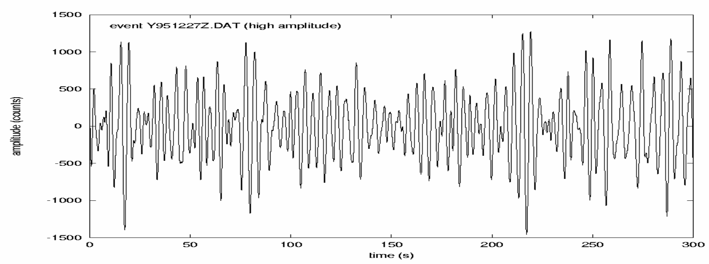
can be transformed into nonlinear ordinary differential equations

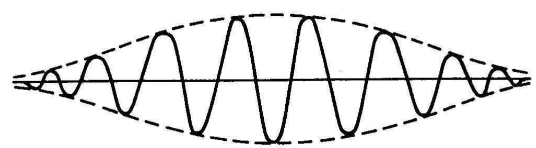
$u(\xi),\ \xi = k(x - \lambda t),$ where k and λ are constants.

2. The steady state wave field is a record of scattered waves that, in the course of its propagation, evolve in three different regimes: ballistic, diffuse and equipartioning.

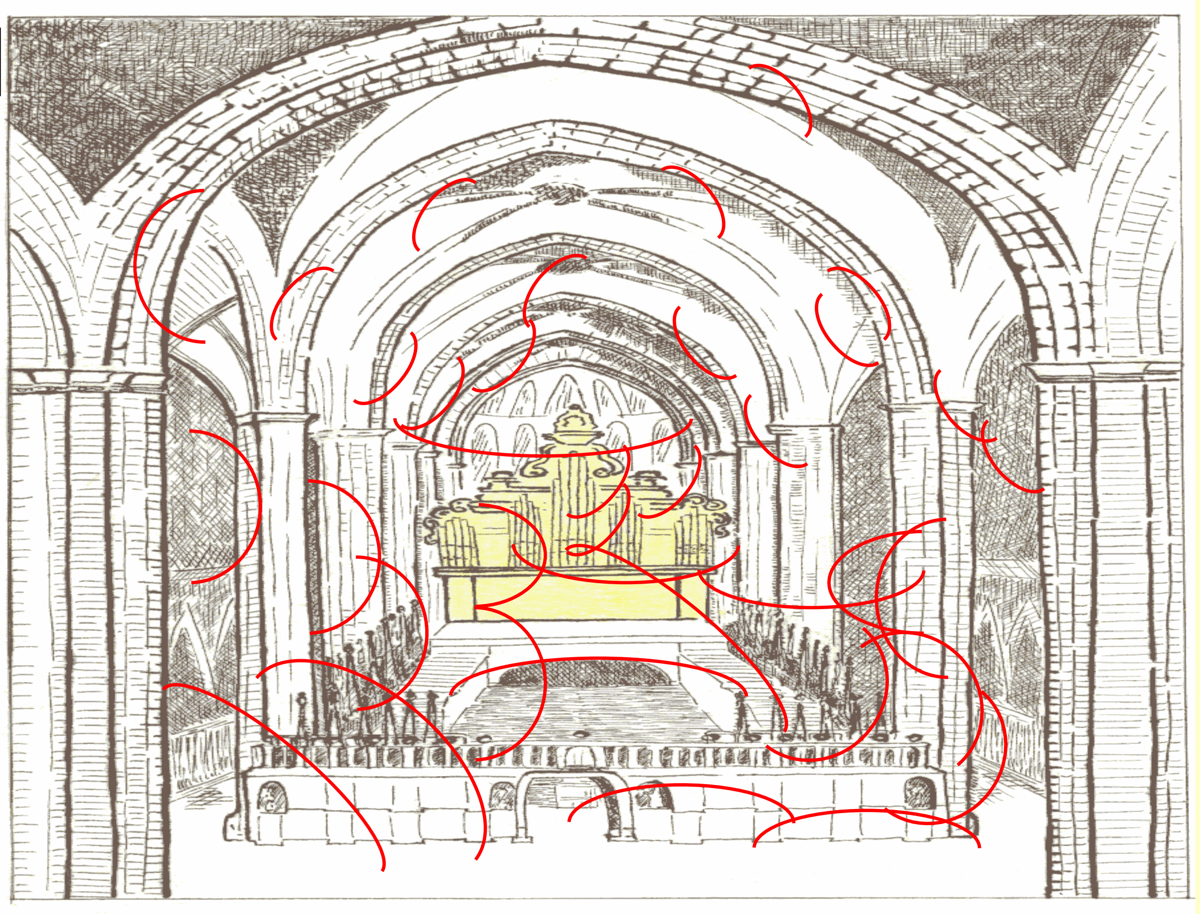
- **Ballistic regime**: is that associated with non-scattered or weakly scattered energy.
- **Diffuse regime**: is that characterized by a diffusive decay with time, due to multiple scattering.
- **Equipartioning regime**: is that for witch the flux of energy falls to zero, but the average energy is still above the background level, and the wave field has lost any preferable propagation direction.

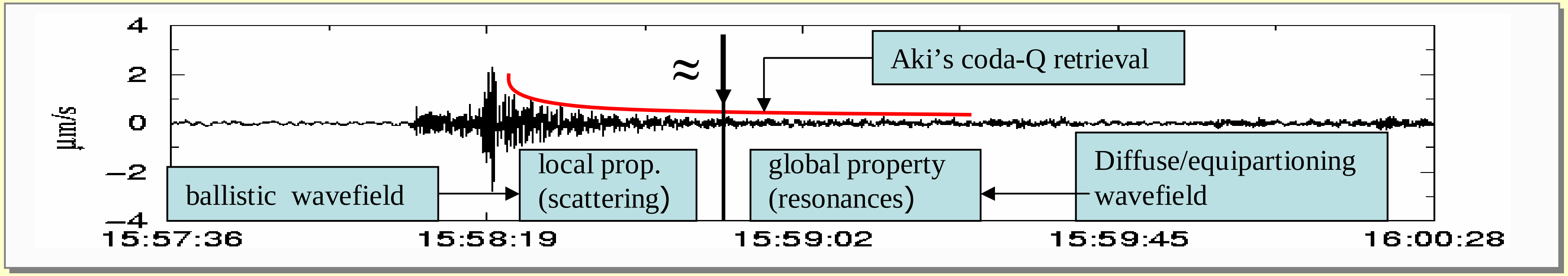
For long lasting microsism activity, seismic records correspond to **equipartioning regime**, that can be understood as that for which the energy is separated into multiple wave-packets, where initial coherent wave-fronts are broken and re-radiated as in Huygens reflections. The energy has lost any propagation direction, the information about the source is diffused and what remains is mainly the medium response, the medium **resonance.** Formally, this situation is similar to that of the long end tail of coda waves.

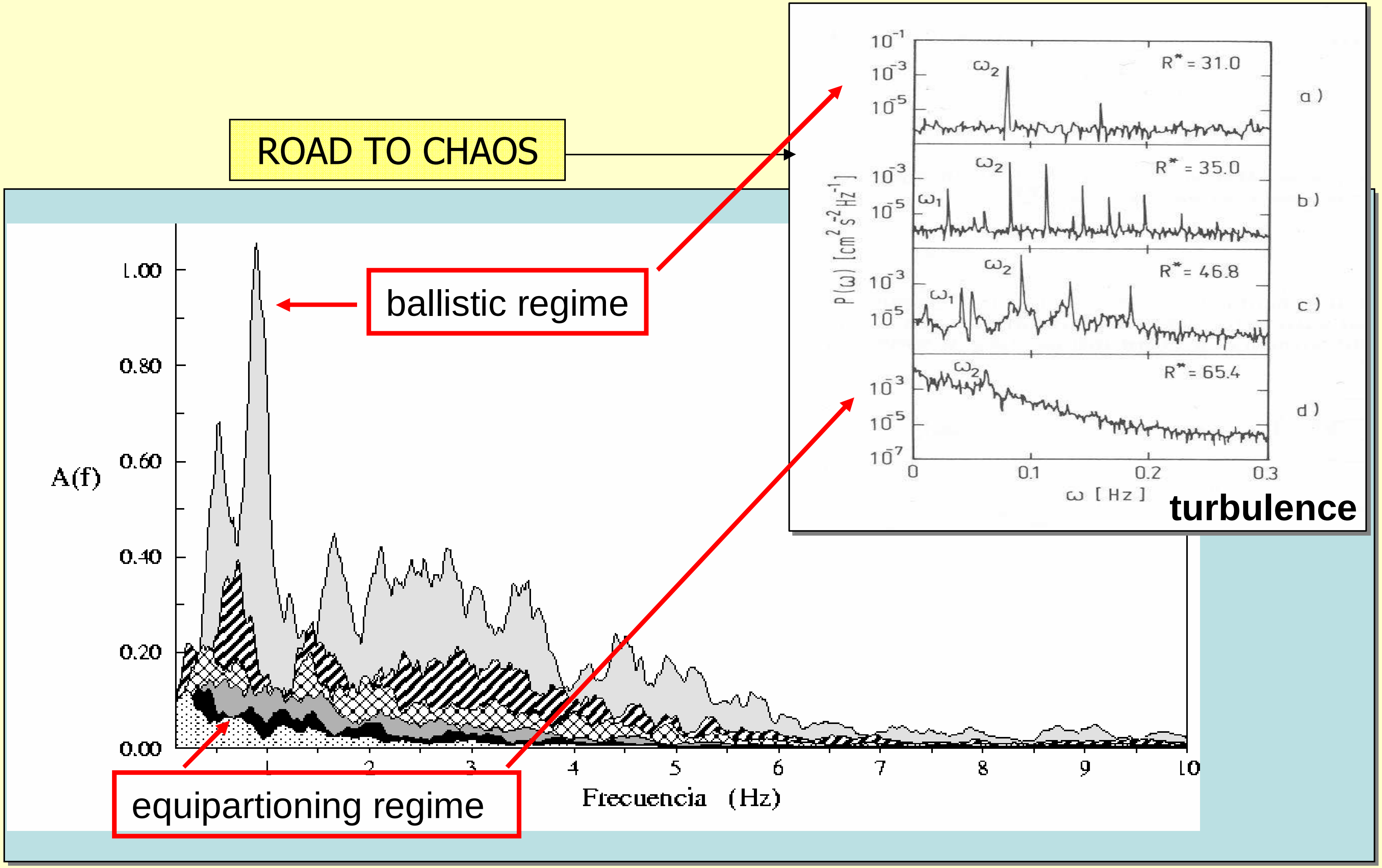



A group of waves whose envelope is a soliton.

Aki's coda model







Spectra of coda waves for several time windows

MINIMUM ENERGY SPECTRUM AND BLACKBODY RADIATION

- For the **minimum spectrum**, the source of energy would be that remaining after the main transient contributions, such as earthquakes or those of atmospheric and oceanic origin are suppressed, *i.e.,* we are dealing with fluctuations at all scales (equipartioning regime).
- The recorded equilibrium wave field is formally similar to the **blackbody radiation,** that refers to a system which absorbs all radiation incident upon it and re-radiate energy, being its spectrum a characteristic of the radiation system only. Actually, instead of temperature we would talk about stress.

SOURCE OF ENERGY

We hypothesize that the main source of energy, responsible for the excitation of the equilibrium fluctuations, is provided by the presence of **coda waves of body and surface waves** (excited by earthquakes and atmospheric storms)**,** in the diffuse/equipartitioned regime, originated by the continuous occurrence of earthquakes and storms of different magnitude and at different places, defining an extended source.

In the diffusive/equipartitioned regime, coda waves behave as $1/f^{\alpha}$ noise.

MAIN ASSUMPTIONS

- **The minimum energy spectrum, free of transient external forces, represents a global (medium) property.**
- **The minimum spectrum is generated by former transients in the diffusive-equipartitioned regimes, reached as a result of a multiple-scattered process.**
- **Coda waves, of a high frequency contents, in the diffusive-equipartitioned regimes behave as $1/f^{\alpha}$ noise**
- **As a first approximation, we extrapolate the $1/f^{\alpha}$ noise source term to the whole range of frequencies, thus accounting for coda waves as well as for meteorological and oceanic perturbations.**

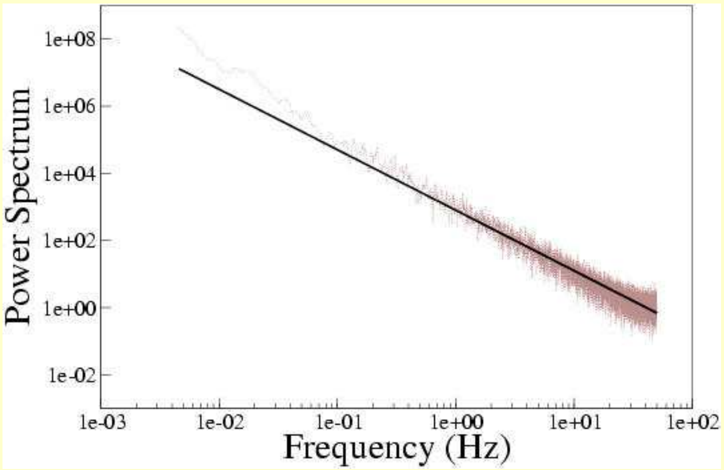
U B

MODEL OF 1/f$^{\alpha}$ SPECTRUM

The1/f$^{\alpha}$ spectrum that characterizes the source term can be modeled as a simple exponential relaxation process, defined as

$$N(t) = N_0 e^{-\lambda t} \text{ for } t \geq 0$$
$$N(t) = 0 \text{ for } t < 0$$

As coda waves and meteorological-oceanic perturbations are continuously generated, we will consider a summation of exponential processes, with the inter-event time following a Poisson distribution.

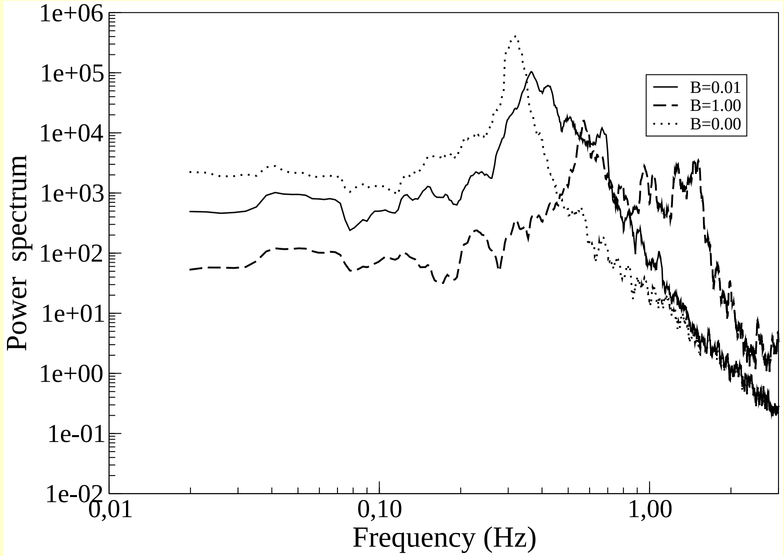


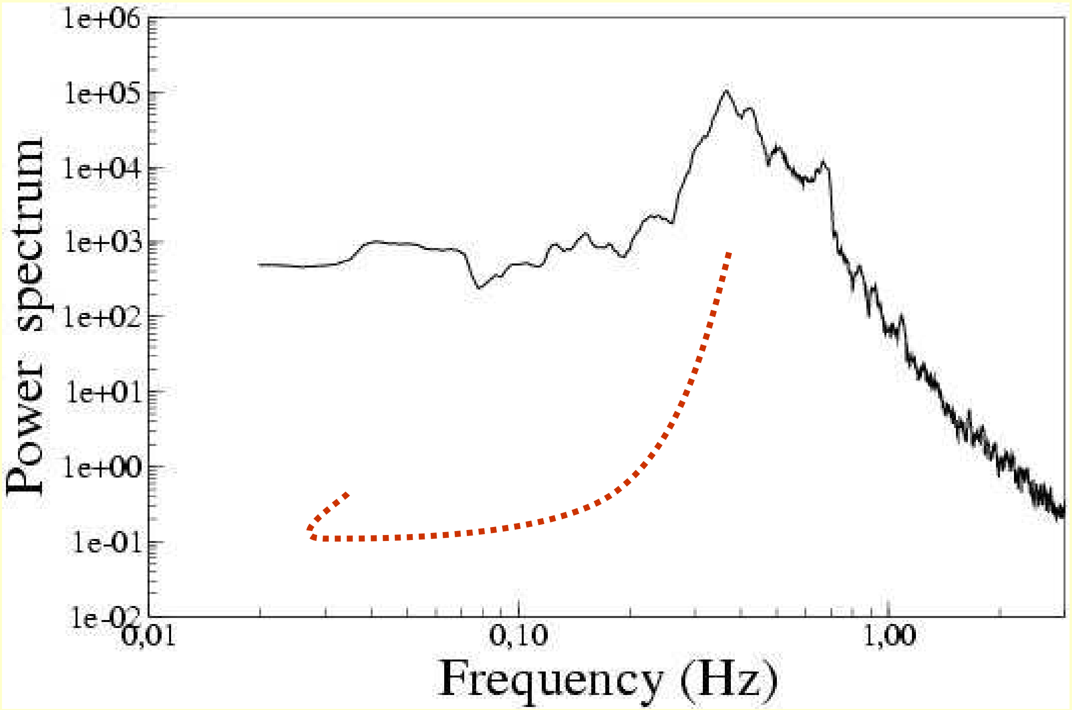
EQUILIBRIUM FLUCTUATION MODEL

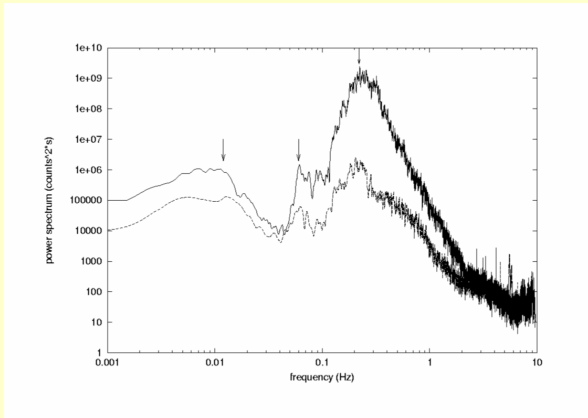
$$p = \dot{q}$$

$$\dot{p} + \frac{\partial V(q)}{\partial q} + \delta p = N_0 \sum_{i=1}^{n} e^{\lambda(t-t_i)}$$

$$V = -\alpha_0 \frac{q^2}{2} + \beta \frac{q^4}{4}$$


Power spectrum
Frequency (Hz)
1e+06
1e+05
1e+04
1e+03
1e+02
1e+01
1e+00
1e-01
1e-02
0,01
0,10
1,00
B=0.01
B=1.00
B=0.00


Power spectrum
Frequency (Hz)
1e+06
1e+05
1e+04
1e+03
1e+02
1e+01
1e+00
1e-01
1e-02
0,01
0,10
1,00


power spectrum (counts^2*s)
frequency (Hz)
0.001
0.01
0.1
1
10

CONCLUSIONS

The observed fluctuation spectrum, masked by the presence of well developed microseism activity, is a medium property that corresponds to a (nonlinear) resonance of the heterogeneous medium.

This medium response, a global property, is well defined by the minimum energy spectrum, mainly excited by $1/f^{\alpha}$ noise.

The origin of the nonlinearity in the real Earth can be understood in terms of wave propagation through irregular waveguides.

Similarities between the equilibrium fluctuation spectra and that of well-developed microseism activity: the latter consists on fluctuations of the former due to a **competition** between the external forces (atmospheric storms) and the resonant peak of the medium.

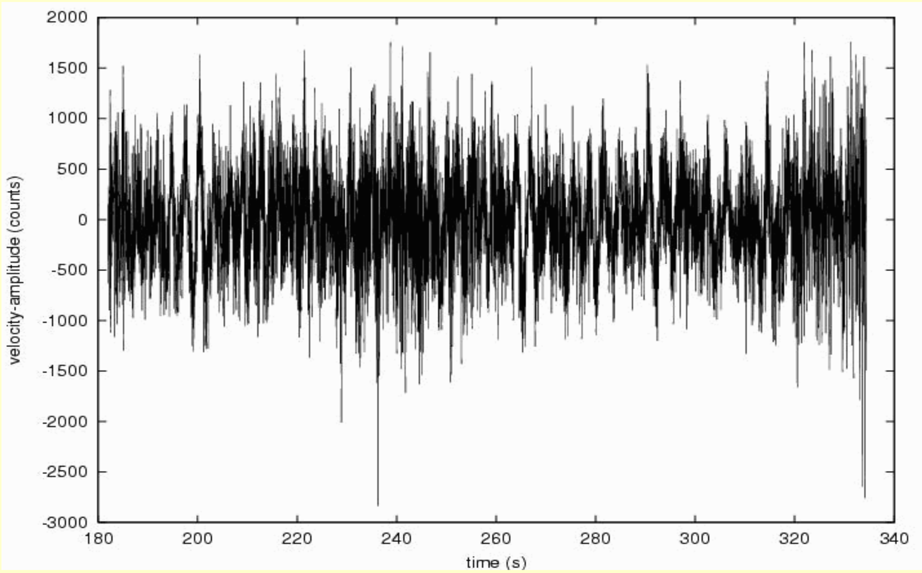
References

- Correig, A.M. and Urquizú, M., 1996. Chaotic behavior of coda waves in the eastern Pyrenees, *Geophys. J. Int.*, 126, 113-122.
- Correig, A.M. and Urquizú, M., 2002. Some dynamical aspects of microseism time series, *Geophys. J. Int.*, 149, 589-598.
- Correig, A.M., Urquizú, M., Macià, R. and Vila, J., 2005. $1/f^{\alpha}$ noise as a source of the Earth's fluctuations , *Europhys. Lett*. 74, 581-587.
- Ryabov, V.R., Correig, A.M., Urquizú, M. and Zaikin, A.A., 2003. Microseism oscillations: from deterministic to noise driven models, *Chaos, Solitons and Fractals*, 16, 195-210.

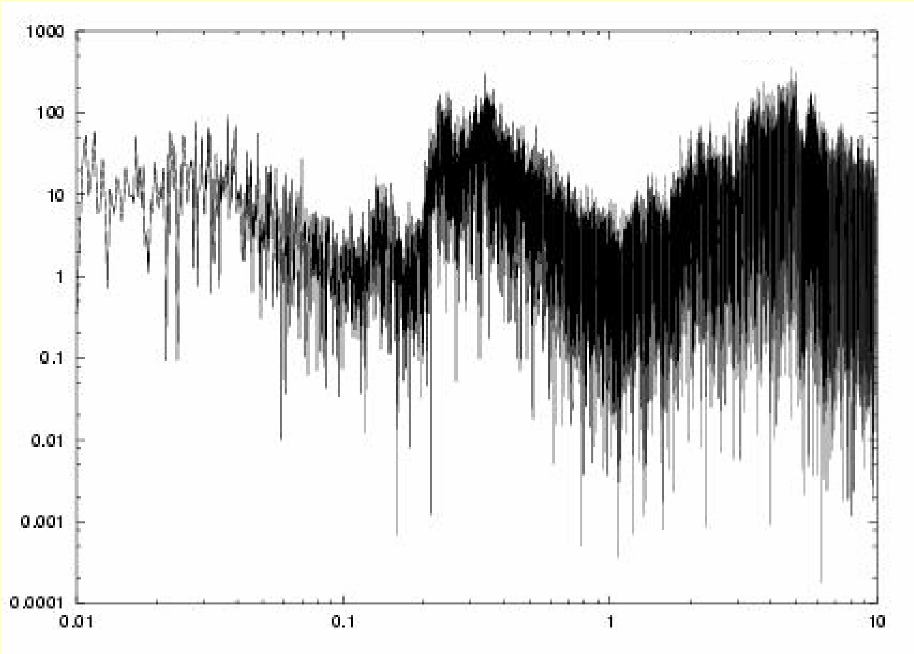
2. VOLCANIC TREMORS

- From the point of view of the observer, a **volcanic tremor** is a continuous flow of energy related to the transport of magma.
- A seismic record of a volcanic tremor consists of a continuous oscillation with a predominant frequency between 1 – 5 Hz, that may last from a few minutes to several months.
- A record of this continuous flow is distorted by another underlying and ubiquitous continuous flow of energy, the **microseism activity** or **seismic noise**.

VOLCANIC TREMOR OBSERVATIONS. 1



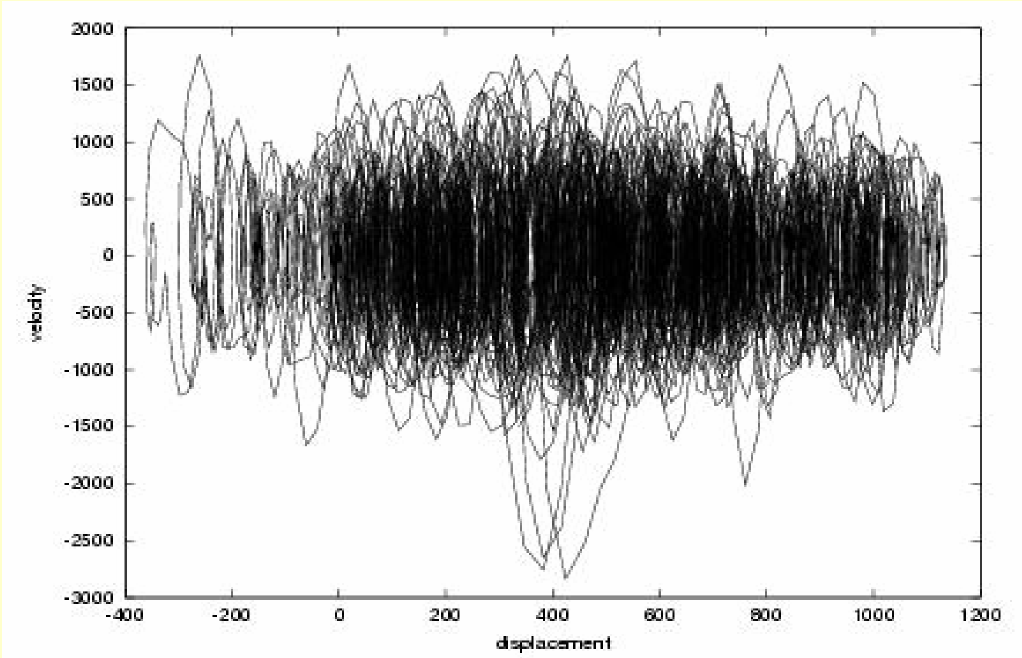
Volcanic tremor time series



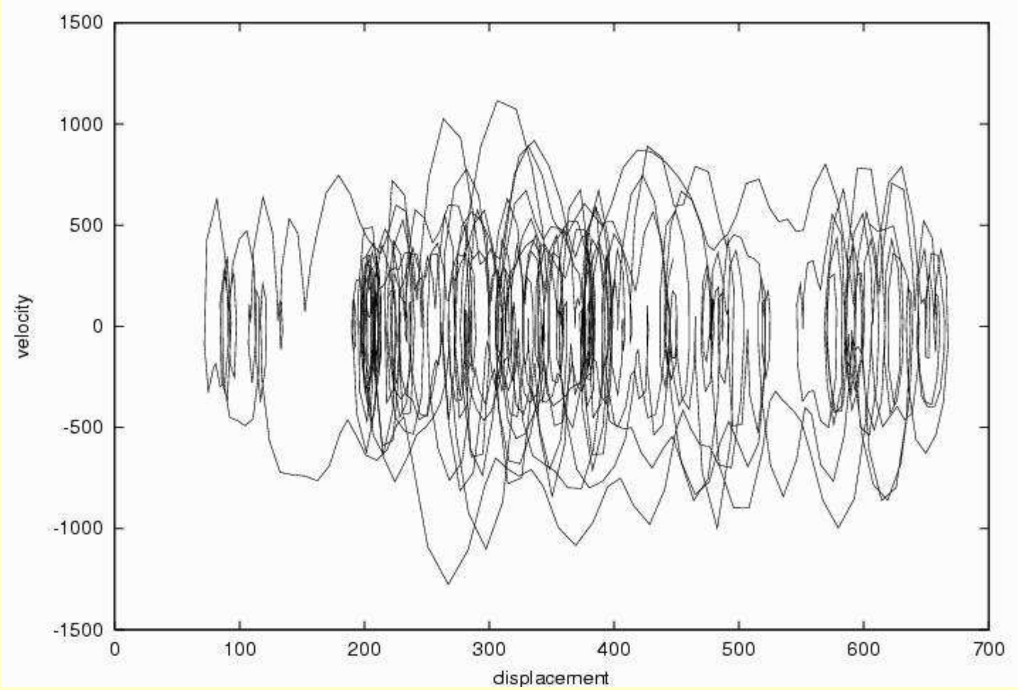
Volcanic tremor power spectrum. Clearly the spectrum is dominated by two broad peaks, one located in the interval 0.2 - 0.3 Hz and the other in the interval 1.0 – 10.0 Hz.

VOLCANIC TREMOR OBSERVATIONS. 2

A



B



Evolution of a volcanic tremor time series in phase space. (B) is a zoomed version of (A).

As in the case of microseism activity, an analysis of records of volcanic tremors reveals that their corresponding time series are:

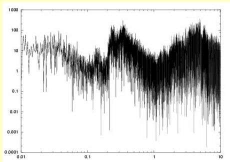
- non-stationary
- non-linear
- stochastic

Note that these characteristics **may not be universal**: they may apply to some specific volcanoes and not in others, and for a given volcano may apply in some episodes and not in the others.

Towards a numerical simulation of volcanic tremor activity

Records of volcanic tremor activity are composed of:

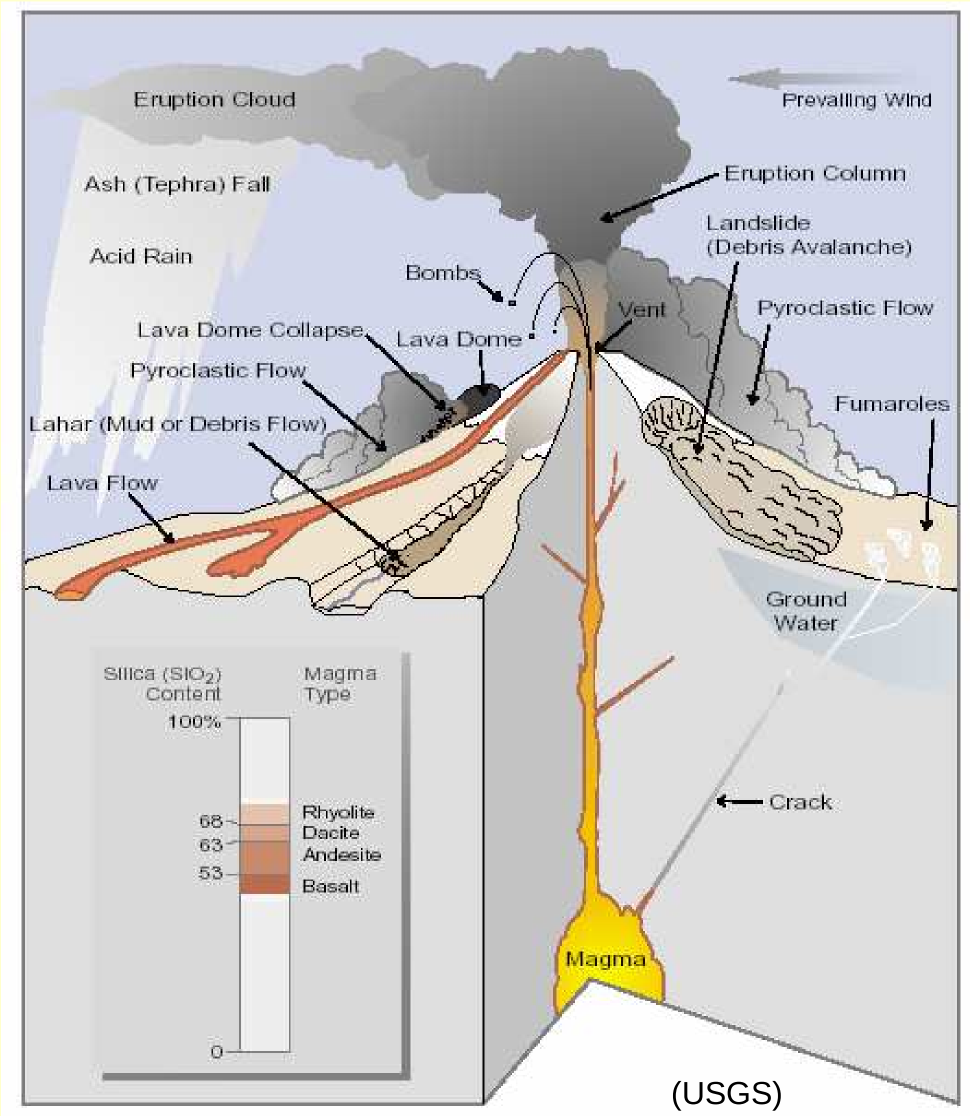
- Source term (magma flow, bobbles, etc., acting in the conduits)



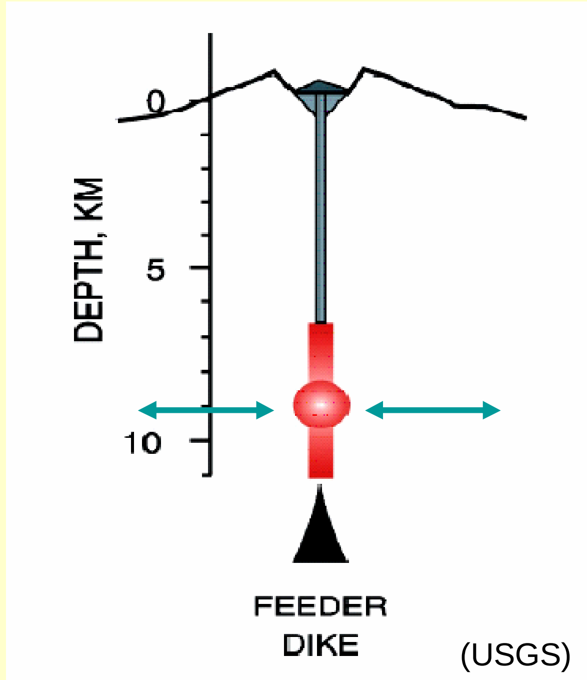
- The underlying fluctuations (microseism activity, with a spectral amplitude of the same order as that of the tremor, acting on the conduits)
- Propagation through the medium
- The geometric resonances of the volcanic structure (continuously excited by the long lasting oscillating source term and the fluctuations)

As the evolution of the system in phase space is chaotic, the model has to be nonlinear. The nonlinearity can be attributed to the source, to the underlying fluctuations, to the medium, or to a combination of some or all of them. Actually we know that the underlying fluctuations behave as a nonlinear oscillator.

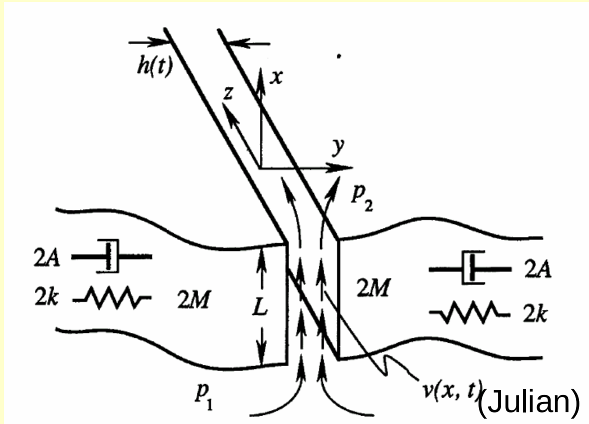
SOURCE TERM



(USGS)

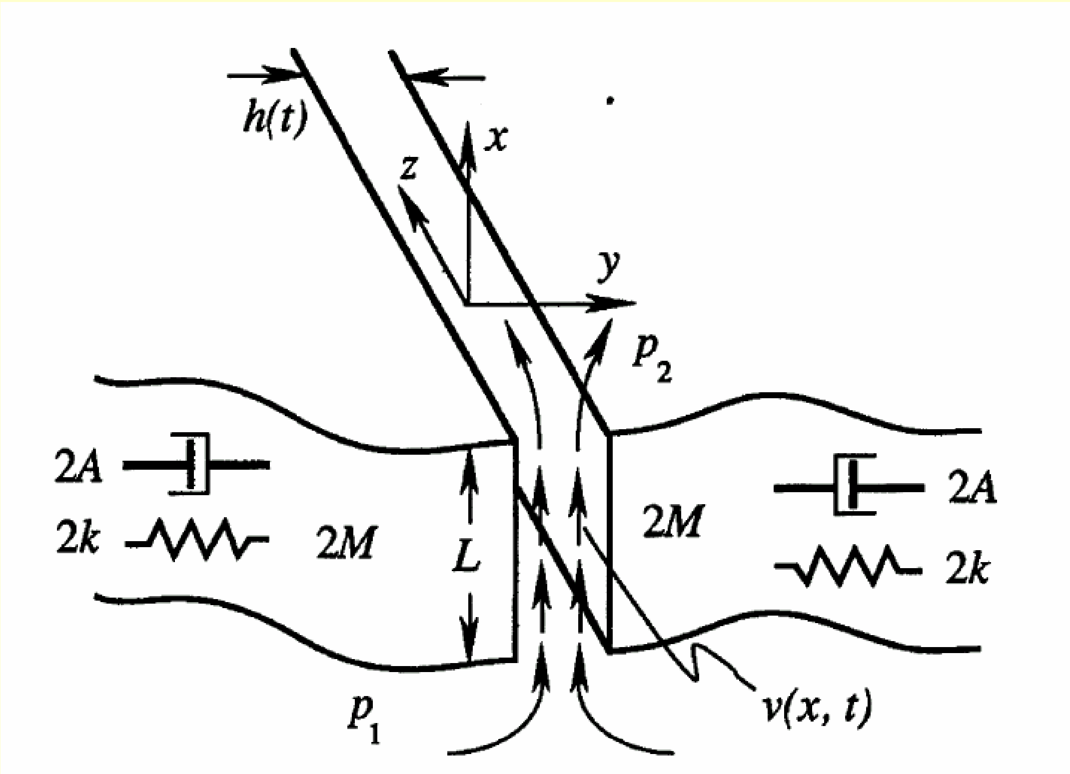


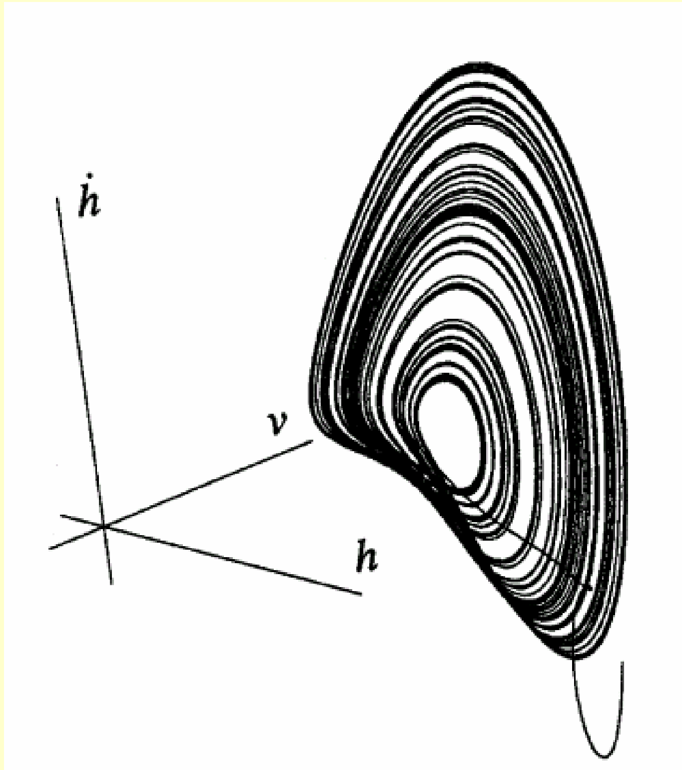
(USGS)



(Julian)

LUMPED-PARAMETER MODEL OF TREMOR





Julian (1994) developed a lumped-parameter model to account for the source of volcanic tremors, in terms of the oscillation of the walls of the magmatic conduits due to variations of the pressure of the magma motion and with the presence of damping. In phase space the attractor of the system is of the Rössler type.

LUMPED-PARAMETER MODEL OF TREMOR

$$\rho\frac{dv}{dt}+\frac{12\eta}{h^2}v=\frac{p_1-p_2}{L}$$

$$\left[M+\frac{\rho L^3}{12h}\right]\frac{d^2h}{dt^2}+\left[A+\frac{L^3}{12h}\left(\frac{12\eta}{h^2}-\frac{\rho}{2}\frac{dh/dt}{h}\right)\right]\frac{dh}{dt}+k\left(h-h_0\right)=L\left[\frac{p_1+p_2}{2}-\frac{\rho v^2}{2}\right]$$

Rössler Model

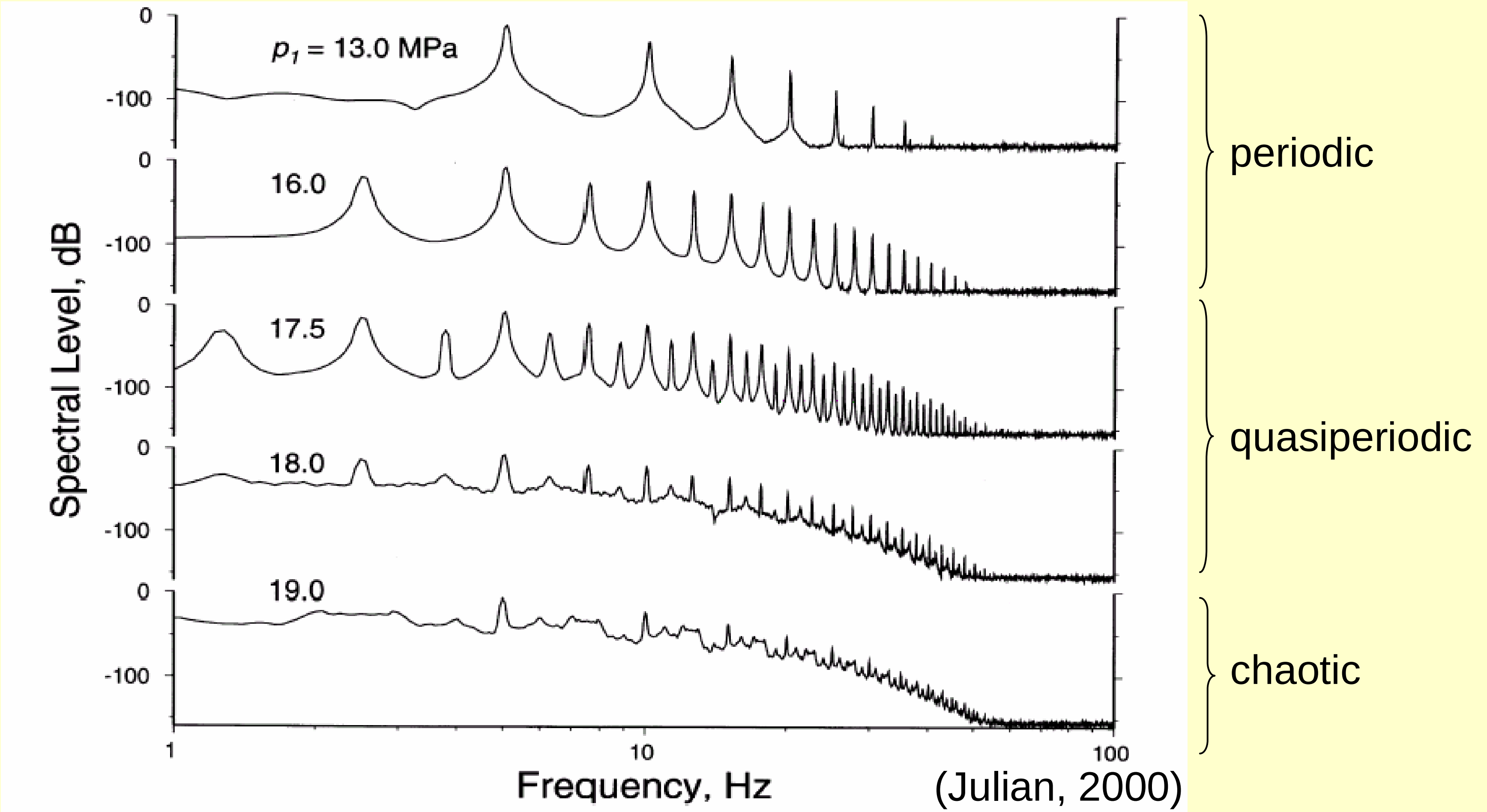
$$\frac{dZ}{dt}+\left(c+aY-\frac{dY}{dt}\right)Z=b$$

$$\frac{d^2Y}{dt^2}-a\frac{dY}{dt}+Y=-Z$$

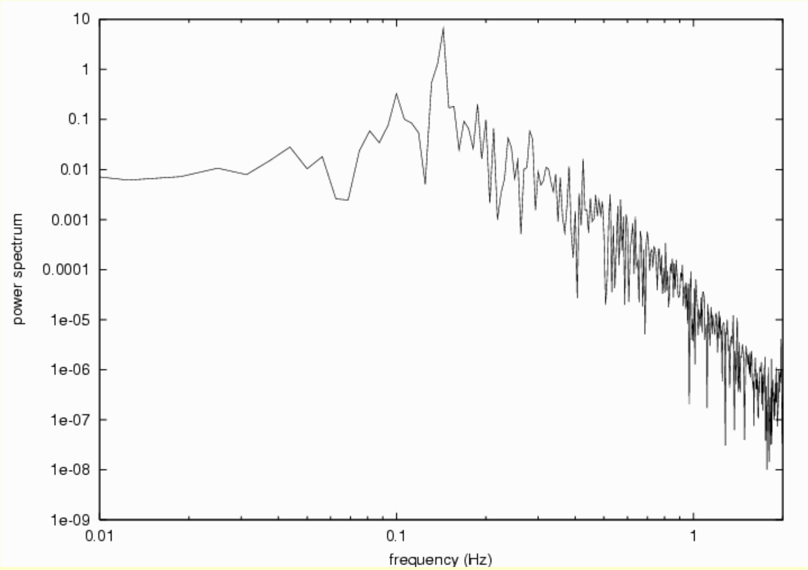
Linear oscillator

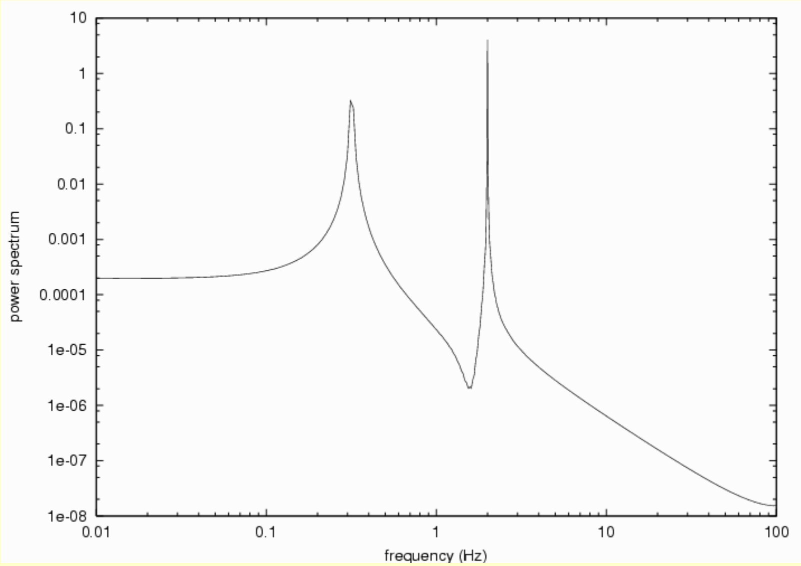
$$\frac{d^2X}{dt^2}+2\gamma\varepsilon_0\frac{dx}{dt}+\omega_0^2=F(t)$$

LUMPED-PARAMETER MODEL OF TREMOR



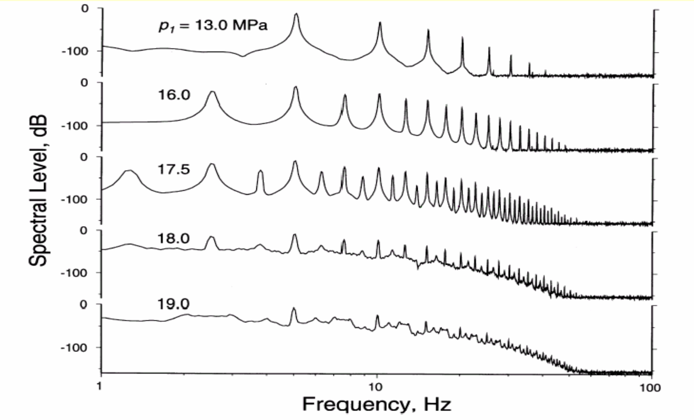
(Julian, 2000)





Power spectrum of the **Rössler oscillator.**

Power spectrum of a **linear oscillator** with one external force.



Usually the **medium** is assumed to be homogeneous and/or stratified and its influence upon wave propagation is through Snell's law, absorption and geometrical spreading. As it is assumed that these factors are well known and the pertinent corrections can be easily accounted for, we can consider the medium as "transparent", in the sense that from the seismograms we can retrieve the source properties.

However, **things are not so simple**:

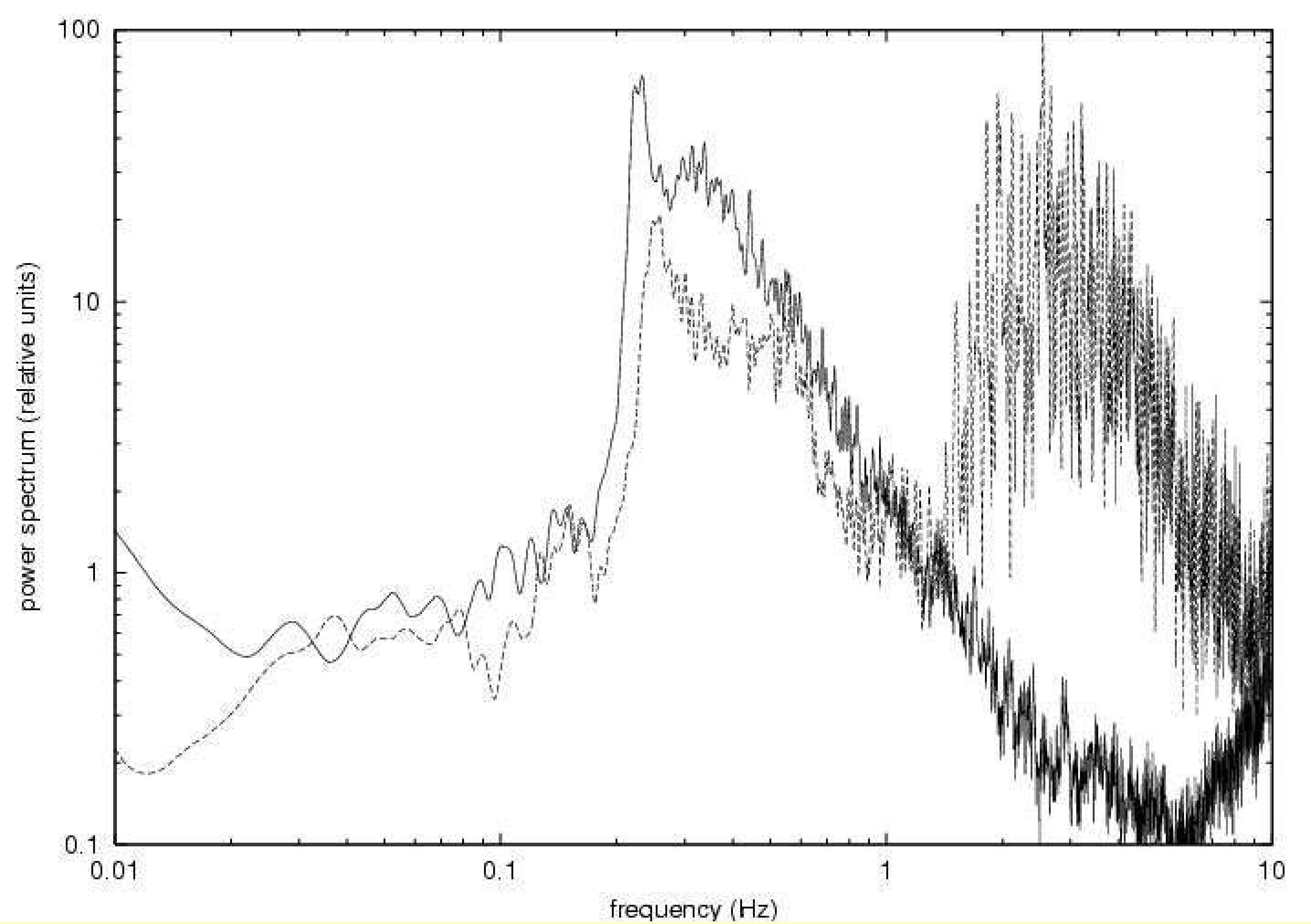
- Urquizú and Correig (1999) showed that the spectrum of volcanic tremors can be synthesized as a medium response.
- For the case of homogeneous layers but with variable cross-section, by a suitable mapping they can be transformed into an uniform cross section obeying ray theory, but the medium has been transformed to a nonlinear one.
- In the equipartitioned regime the effects of the source vanish.
- As in the case of microseisms, for long lasting volcanic tremors the seismogram is composed of wave packets of variable duration.

MODELLING THE MEDIUM

As shown by Urquizú and Correig (2004), a stratified medium is equivalent to an oscillator.

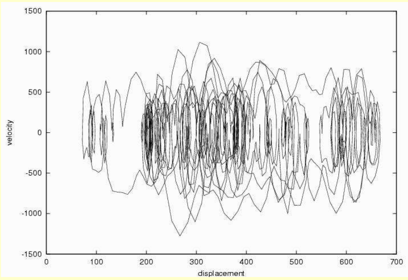
PHENOMENOLOGICAL MODEL

Taking into account that we pretend simulate the record of volcanic tremors at a fixed point in the ***equipartioning regime***, we will model the observed volcanic tremors as a ***system of coupled oscillators***.


100
10
1
0.1
0.01
0.1
1
10
power spectrum (relative units)
frequency (Hz)

PHENOMENOLOGICAL MODEL FOR VOLCANIC TREMOR ACTIVITY

The volcanic tremor time series is strongly influenced by the microseism activity, that may act as an external additive source term.

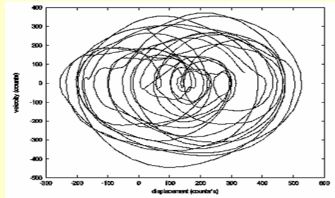


$\left(q(t),\dot{q}(t)\right)$

$$\frac{d^2q}{dt^2}+\delta\frac{dq}{dt}+\frac{dV}{dq}=\underline{\gamma F(t)}+\underline{U_{ms}(t)}$$

Chaotic external force?

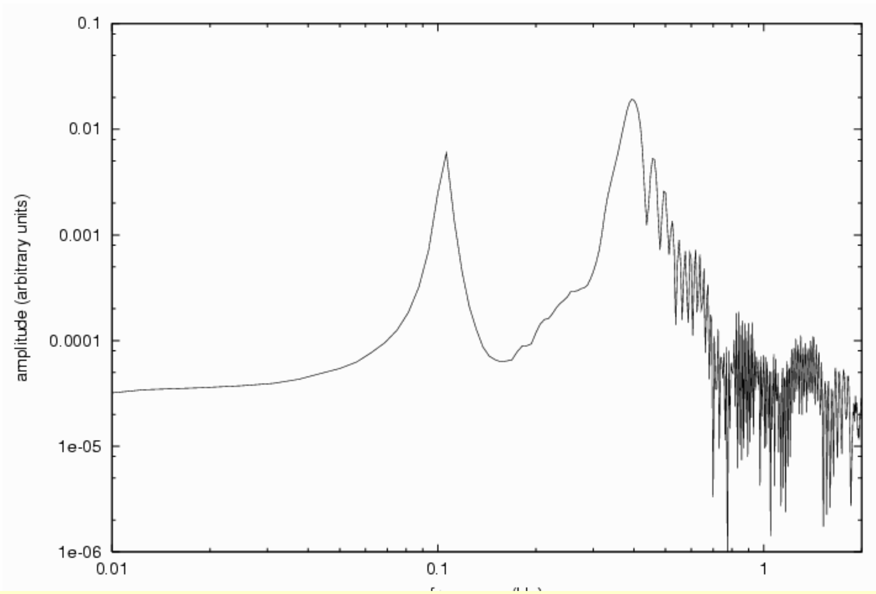
Additive noise

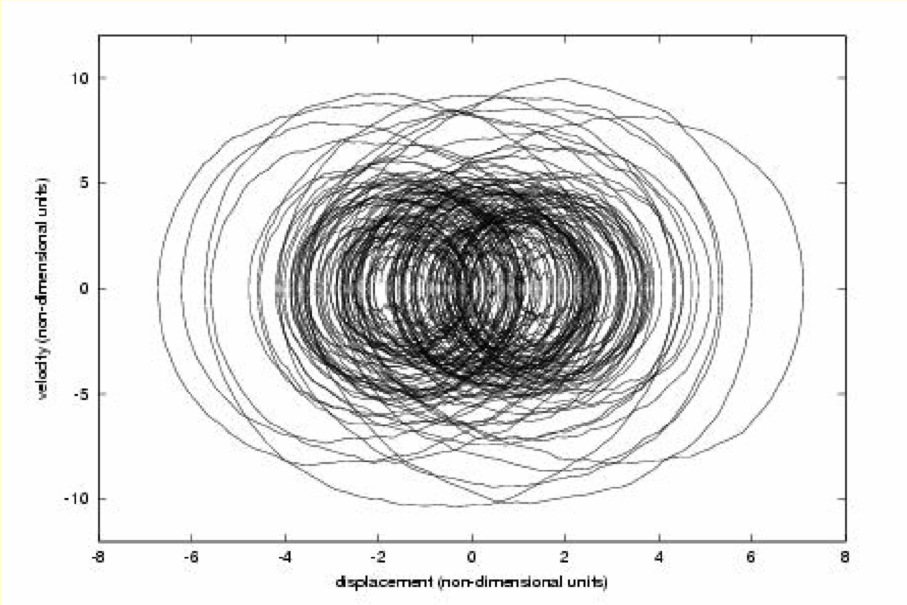


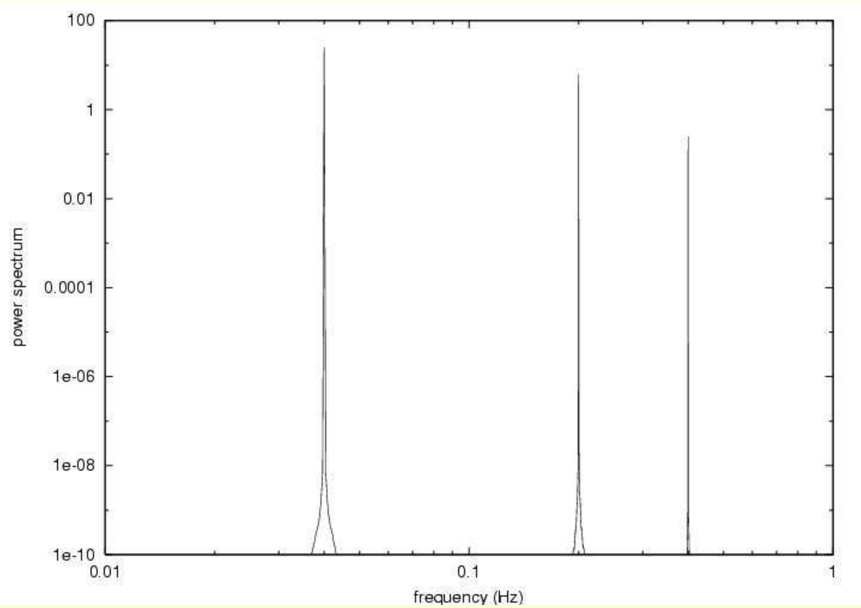
$\left(U(t),\dot{U}(t)\right)$

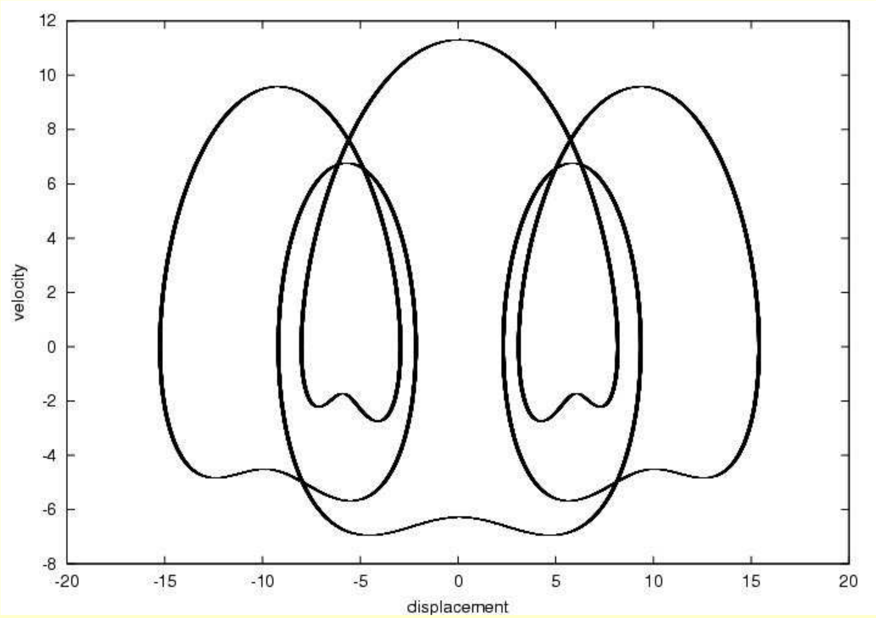
$$V(q)=-\alpha\frac{q^2}{2}+\beta\frac{q^4}{4},\quad \alpha<0.$$

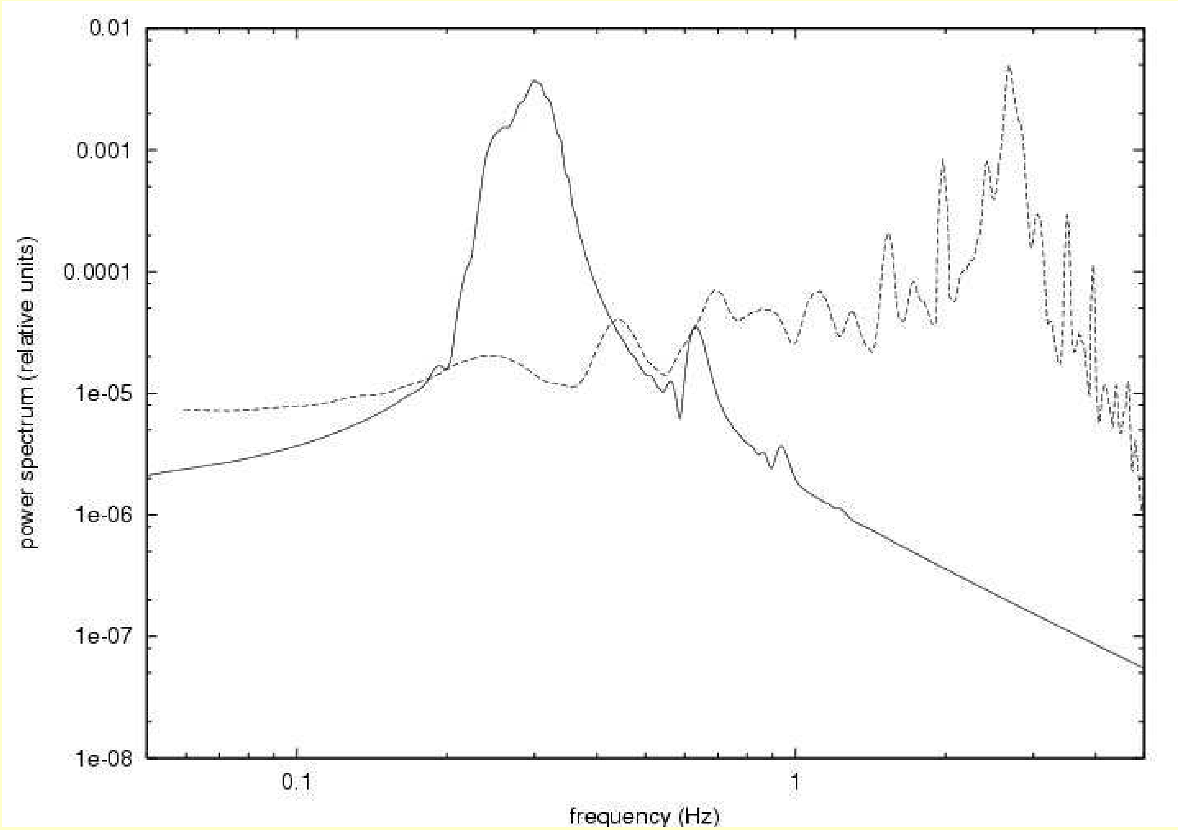
F(t) stands for a ***chaotic atractor*** like that of Rössler (Julian, 1994) and U_{ms} stands for the ***microseism activity additive noise term*** (this term may consist in a recorded microseism time series). The evolution of the system in phase space suggest that *V* may be assimilated to a potential consisting on a quadratic and a quartic terms accounting for the ***medium response***.


amplitude (arbitrary units)
0.1
0.01
0.001
0.0001
1e-05
1e-06
0.01
0.1
1

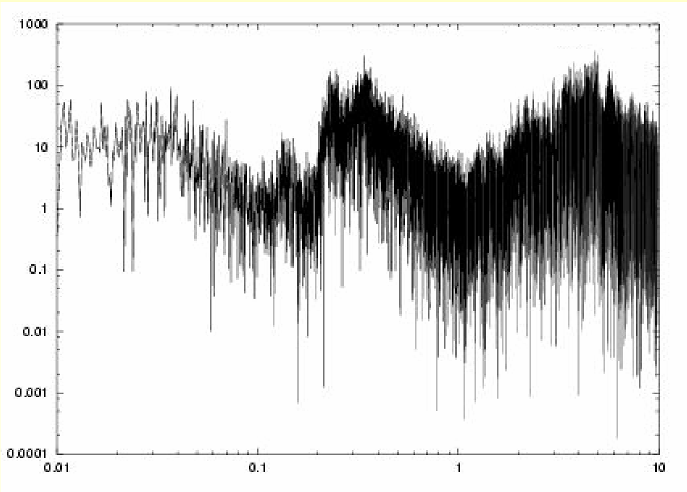

velocity (non-dimensional units)
displacement (non-dimensional units)
10
5
0
-5
-10
-8
-6
-4
-2
0
2
4
6
8

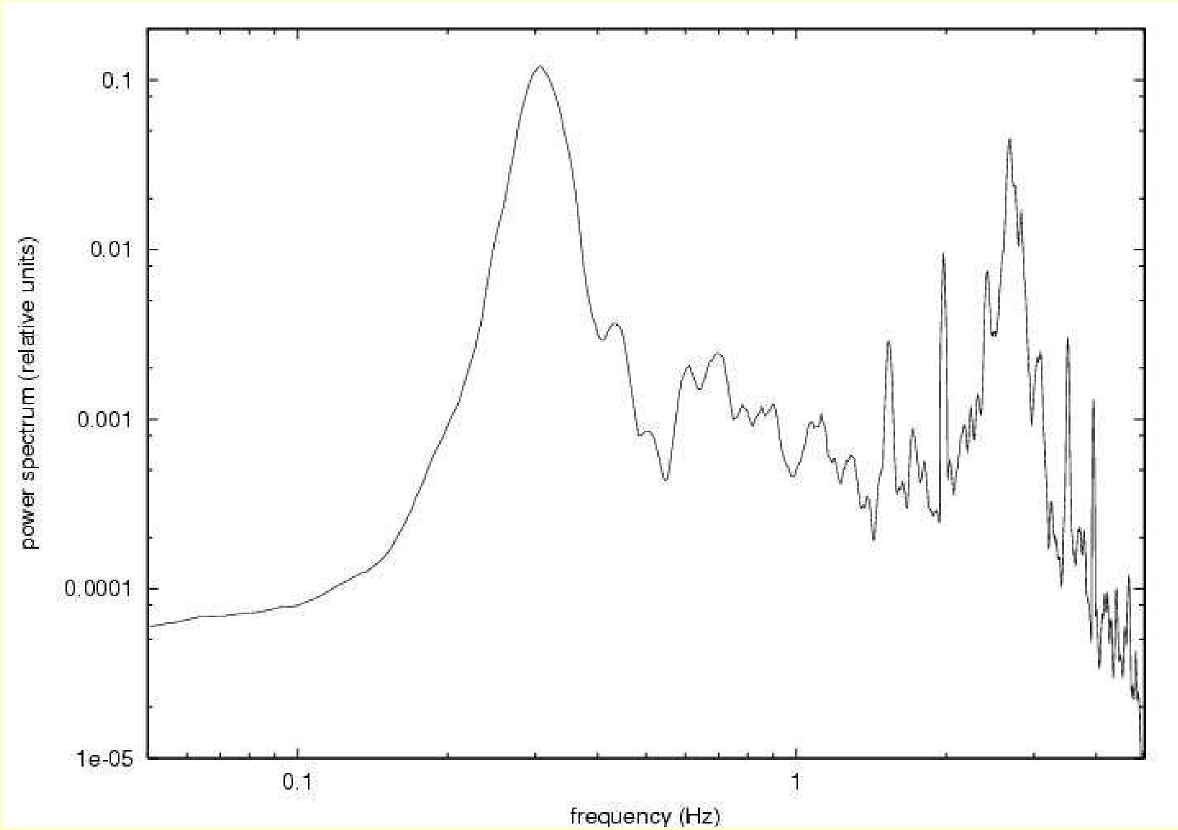

power spectrum
frequency (Hz)
100
1
0.01
0.0001
1e-06
1e-08
1e-10
0.01
0.1
1


velocity
displacement
12
10
8
6
4
2
0
-2
-4
-6
-8
-20
-15
-10
-5
0
5
10
15
20

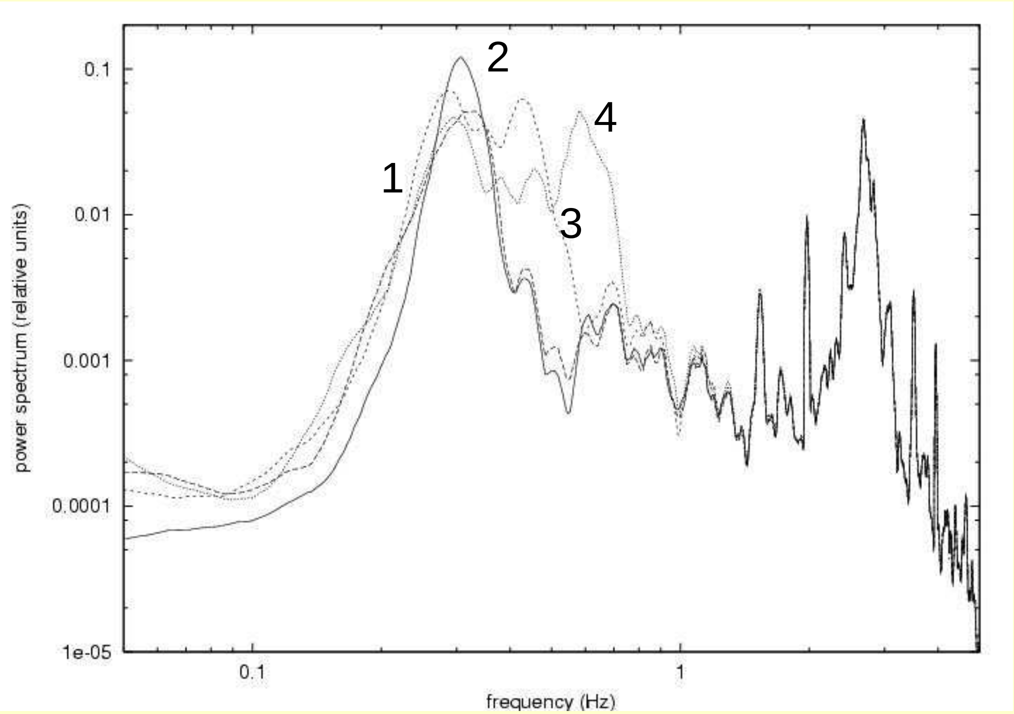


Microseism peak (solid line) and Rössler peak (dashed line). The Rössler peak has been generated with the following values: a = b = 0.2 and c = 5.7, and the microseism peak with $\delta = 0.05$, $\alpha_0 = -0.24$, $\beta = 0.05$, $\gamma_1 = 50.0$, $\gamma_2 = 0.0$, $f_{par} = 0.025$ and $A_{par} = 0.00025$.

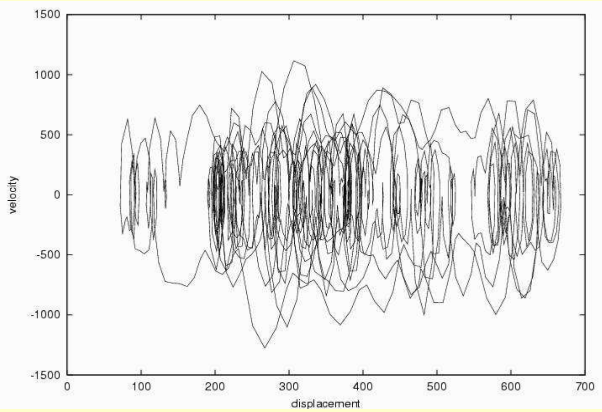


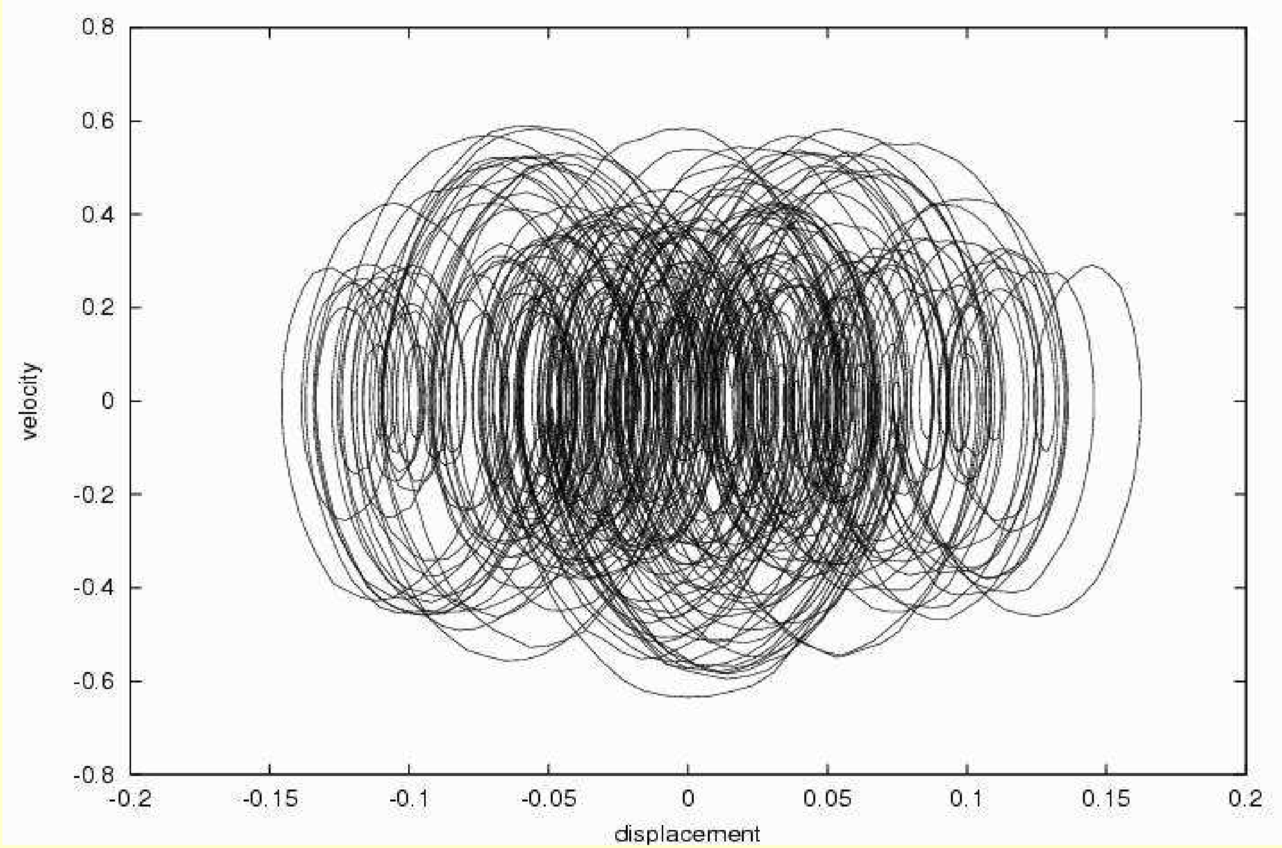


Simulation of an observed record accounting for microseism and tremor activities. The microseism has been generated with the following values of the parameters: $\delta = 0.05$, $\alpha_0 = -0.24$, $\beta = 0.05$, $f_1 = 0.05$, $\gamma_1 = 5.0$, $f_{par} = 0.025$, $A_{par} = 0.00025$.

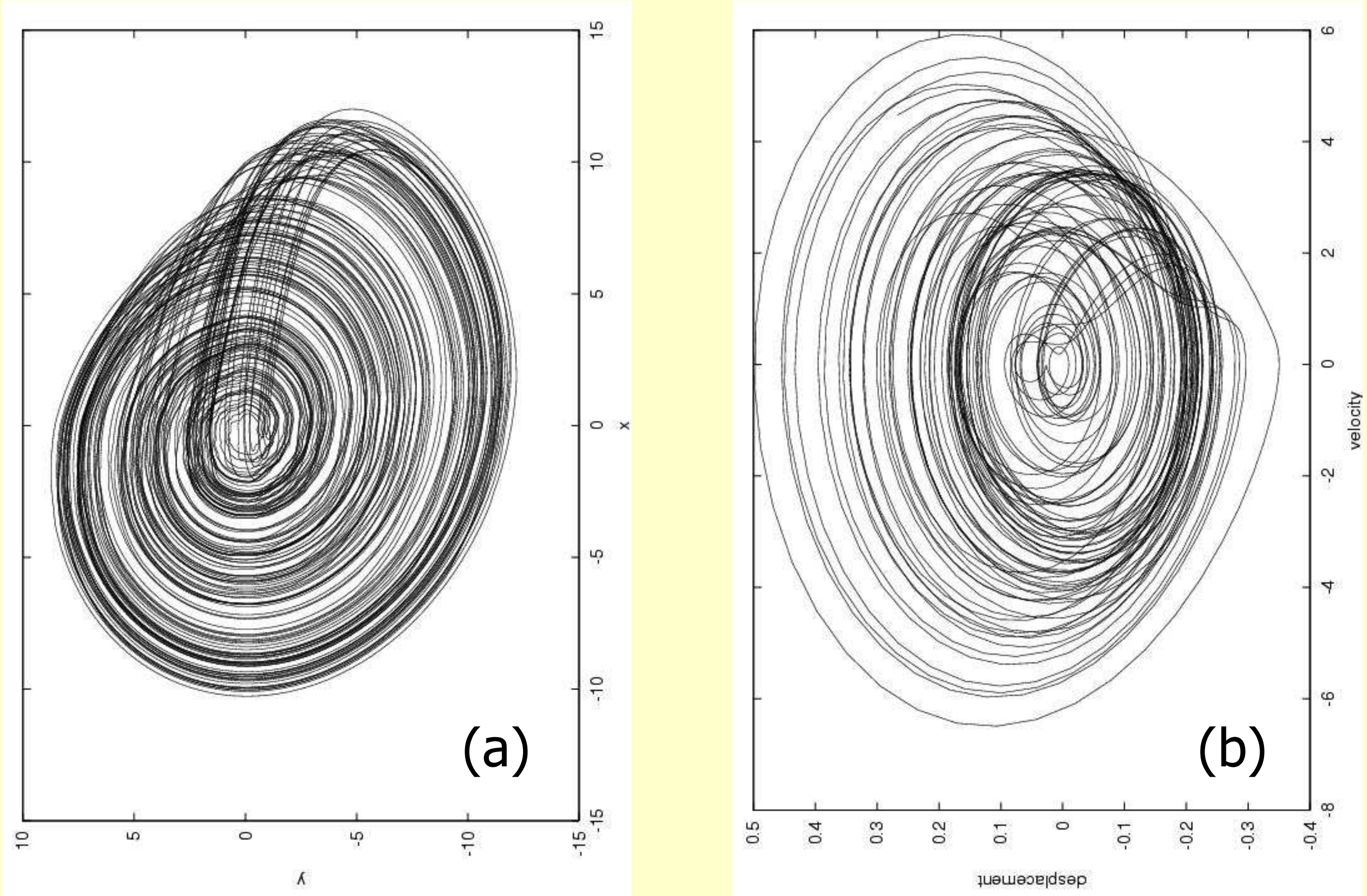


Broadening of the microseism peak due to variations of the amplitude of the parametric resonance. The values of the model parameters are $\delta = 0.05$, $\alpha_0 = -0.24$, $\beta = 0.05$, $f_1 = 0.05$, $\gamma_1 = 5.0$, $f_{par} = 0.025$. The values of the parametric resonance are **(1) = 0.00001, (2) = 0.00025, (3) = 0.0005 and (4) = 0.0075.** In all cases the peak of the tremor remain invariant, suggesting it can be recovered by filtering.

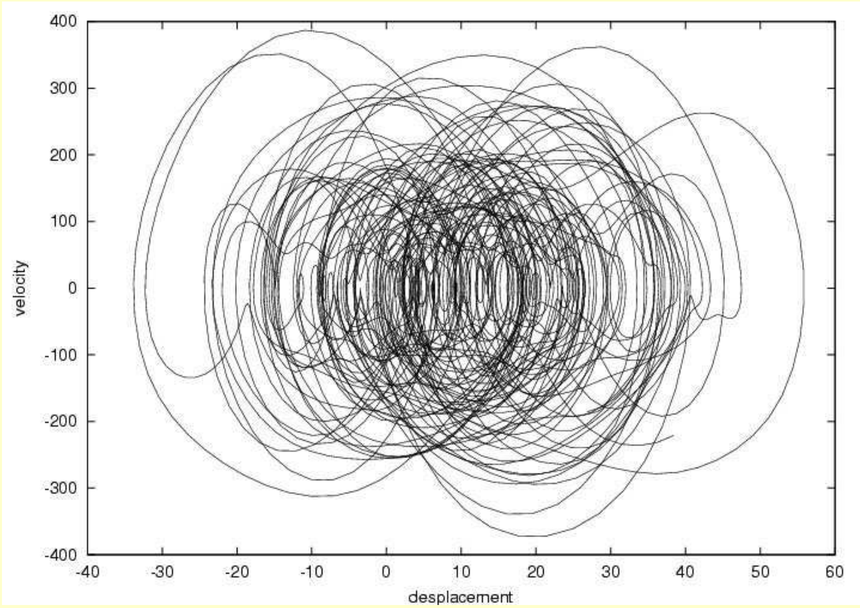


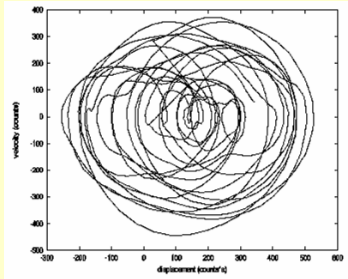


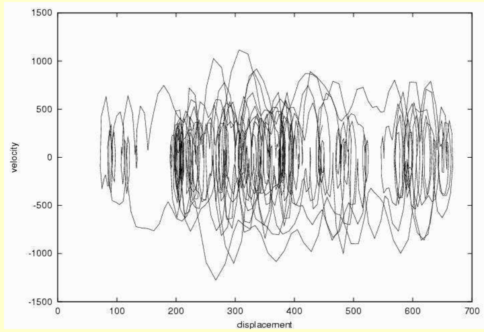
Numerical simulation of the evolution of the phenomenological model of volcanic tremor in phase space for the following values of the parameters: $\alpha_0 = -0.32$, $\delta = 0.05$, $\eta = 0.0$, $\beta = 0.05$, $\gamma_1 = 4.0$, $\gamma_2 = 0.0$, $\varepsilon = 1.5$.



The Rössler oscillator (in phase space) used as a source of the volcanic tremor (a) and its recovering (b) after a band pass filtering (Butterworth filter) of the simulated volcanic tremor.







volcanic tremor microseism

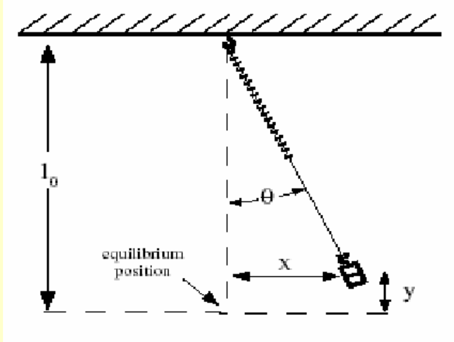
Source term (in phase space) recovered after a band pas filtering of a recorded volcanic tremor. It is **similar to the unfiltered one**. **Several possibilities arise**:

- the medium prevails over the source
- the dynamics of the source is similar to the dynamics of microseisms
- the model is not correct

Simplifying the problem, we are dealing with **two coupled oscillators**, one corresponding to volcanic tremors, and the other corresponding to microseism activity. Both have different time scales:

- Microseisms corresponds to a slow subsystem, with a s characteristic time of the order ~ 1/0.2 = 5 s.
- Tremors correspond to a fast subsystem, with a characteristic time of the order of ~ 1/2 = 0.5 s.

A SIMPLE (MATHEMATICAL) MODEL OF COUPLED OSCILLSATORS: THE MATHIEU EQUATION.



Let us consider the Mathieu equation

$$\frac{d^2\theta}{dt^2} + \omega_0{}^2[1 \underbrace{+ h\cos(2\omega t)}_{\mathbf{MN}})]\theta = 0. \qquad (1)$$

This equation ca be viewed as the equation of a frictionless pendulum for which the pivot is subjected to a vertical motion, *i.e.*, as a parametric pendulum, a pendulum whose parameters varies with time (for example, a variable gravity *g(t)).* This equation can also be derived from the equations of two coupled oscillators.

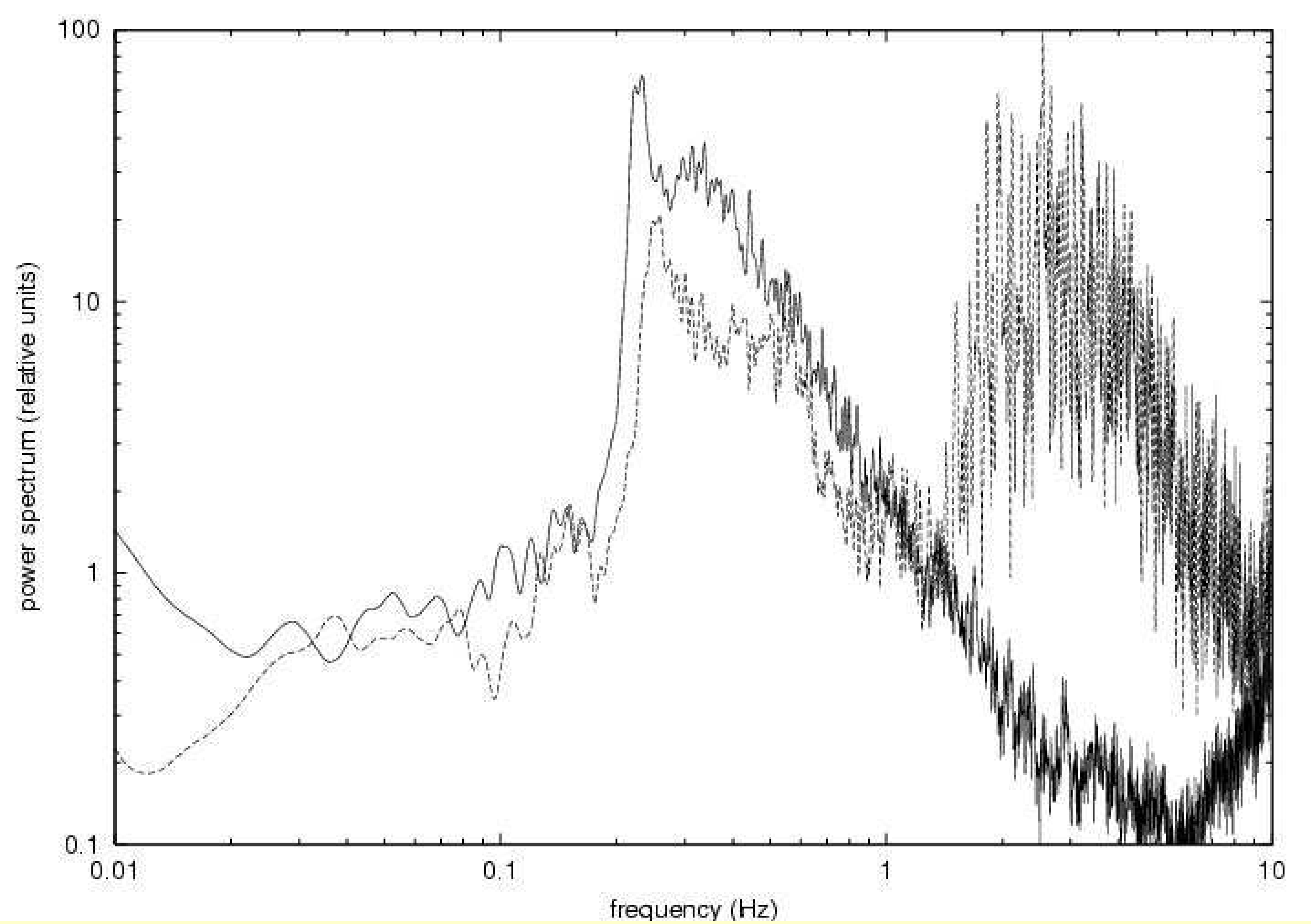
The term **MN** can also be interpreted as Multiplicative Noise.

Let us define $\alpha = h\cos(2\omega t)$ and consider the two following differential equations

$$\frac{d^2\theta}{dt^2} + \omega_0^2(1+\alpha)\theta = 0 \qquad (2a)$$

$$\frac{d^2\alpha}{dt^2} + 4\omega^2\alpha = 0, \qquad (2b)$$

which describe the behavior of two oscillators of amplitude θ and α. They are coupled by the term $\omega_0^2\alpha\theta$. By solving (2b) and substituting into (2a), eq. (1) is obtained. In this formulation it can be assumed, for example, that the external force, the gravity, is controlled by an external oscillator with amplitude proportional to α. The external oscillator must be insensitive to the motion of the parametric pendulum, which explain the absence of a θ-dependent term in the second equation. This is a specific form of coupling, in which the first oscillator is in fact forced by the second.


100
10
1
0.1
0.01
0.1
1
10
power spectrum (relative units)
frequency (Hz)

Note that, apart of the possible existence of external forces in equations (1) and (2), eq. (2b) can be considered as external noise (the microseism activity) coupled to the pendulum, eq. (2a) that could be assimilated to the motion of the walls of a dike due to the motion of magma.

Conduit magma wall

External excitation

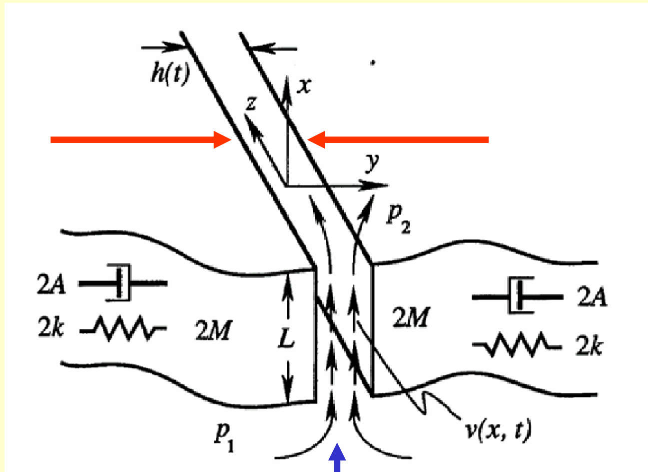
(microseism activity)

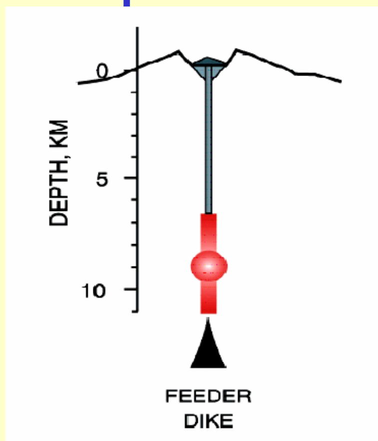
$$\frac{d^2\alpha}{dt^2} + 4\omega^2\alpha = 0$$

Internal oscillation

(magma motion)

$$\frac{d^2\theta}{dt^2} + \omega_0^2(1+\alpha)\theta = 0$$



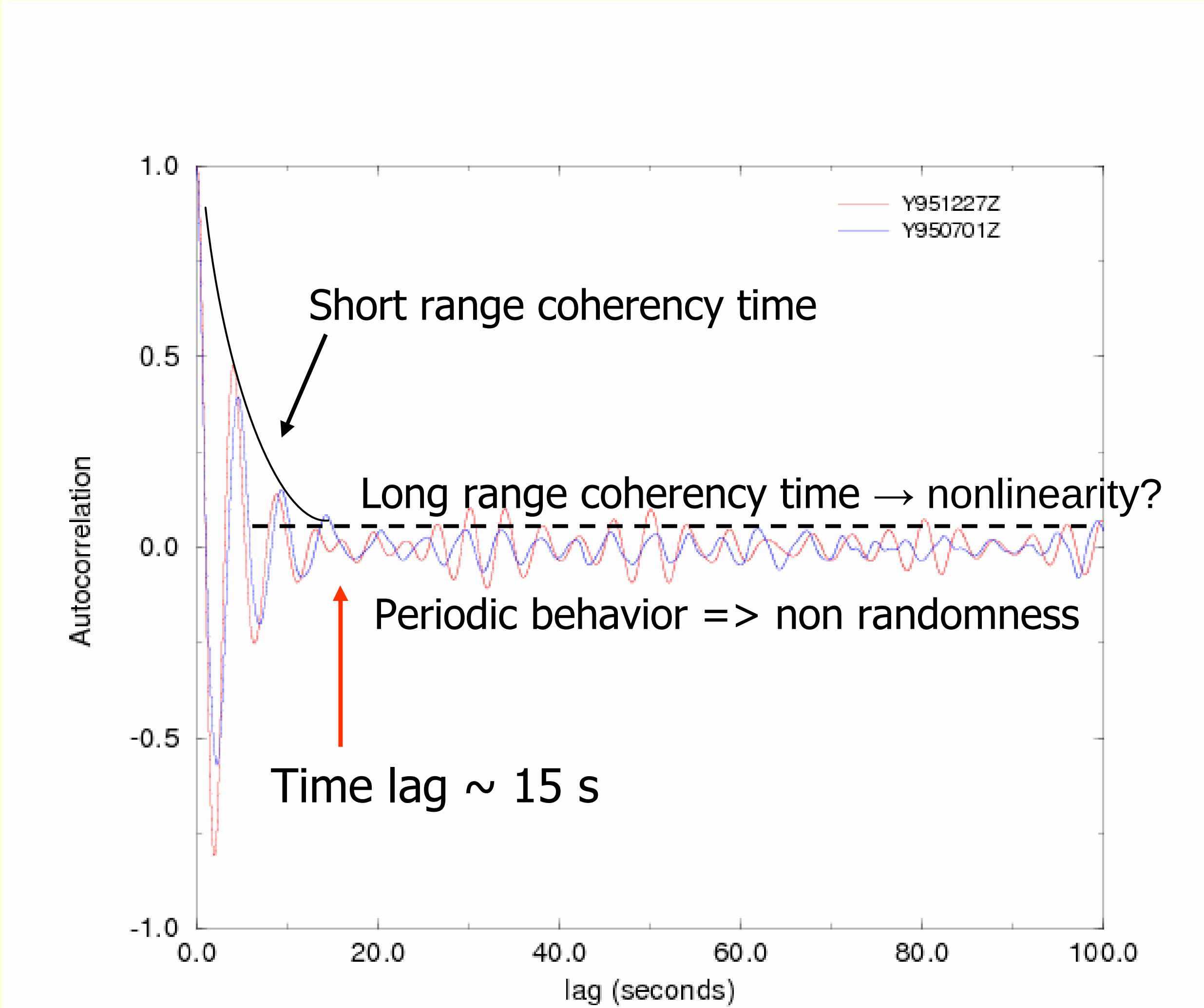


Whereas the microseism activity influences the oscillatory motion of the wall due to magma motion, magma motion does not influence microseism activity.

CHARACTERIZATION OF THE WAVE FIELD

- Power spectrum
- Evolution in phase space
- **Statistical characteristics: autocorrelation function**

Autocorrelation function of the microseism time series



Exponential decay of the autocorrelation function +

Periodicity of the autocorrelation function =>

The observations can be interpreted in terms of the Ornstein-Uhlenbeck narrow-band exponentially correlated noise, defined as

$$\langle \xi(t) \rangle = 0,$$
$$\langle \xi(t)\xi(t_1) \rangle = \sigma^2 \exp\left[-\lambda |t-t_1|\right] \cos(\omega_2 |t-t_1|).$$

It can be shown that for the exponential correlator

$$\langle \xi(t)\,\xi(t_1) \rangle = \sigma^2 \exp\left[-\lambda |t-t_1|\right],$$

its associated probability density has a Gaussian form defined by

$$S(t) = S_1(t)\cos\Omega t + S_2(t)\sin\Omega t$$

$S_1(t)$ and $S_2(t)$ are two independent and stationary Gaussian process with zero mean and correlations

$$\left\langle S_i(t)\,S_j(t)\right\rangle = \left\langle \xi^2 \right\rangle \delta_{ij} \exp\left[\frac{-\left|t-t_1\right|}{\tau}\right], \;\; i,\; j=1,2\,,$$

and τ is a correlation time. The corresponding power spectrum is

$$P(\omega)=\frac{1}{2}\left\langle \xi^2 \right\rangle \tau \left(\frac{1}{1+\tau^2\left(\Omega+\omega\right)^2} + \frac{1}{1+\tau^2\left(\Omega-\omega\right)^2} \right).$$

U
B

PROPOSED MODEL FOR THE OBSERVED WAVEFIELD

Assume a forced damped oscillator with multiplicative noise

$$\frac{d^2q}{dt^2}+\delta\frac{dq}{dt}+\alpha\left[1+\xi(t)\right]q+\eta q^3=\gamma F(t),$$

where the external force *F(t)* corresponds to the Rössler oscillator and $\xi(t)$ corresponds to seismic noise.

Two possibilities arise for the characterization of the seismic noise:

a) as a solution of the equation for the Earth Background

$$\frac{d^2\xi}{dt^2}+\frac{dV}{dt}+\delta'\frac{dq}{dt}=N_0\sum_{i=1}^{n}e^{-\lambda(t-t_i)}$$

$$V=-\alpha_0\frac{\xi^2}{2}+\frac{\xi^4}{4},\qquad \eta=0$$

b) by approximation the seismic noise as the Ornstein-Uhlenbeck narrow-band exponentially correlated noise with $\eta \neq 0$.

work in progress !!

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