1866-15

School on Pulsed Neutrons: Characterization of Materials

15 - 26 October 2007

Superconductivity and flux lattice structures

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(High-Tc) Superconductivity and Vortex Lattice Structures

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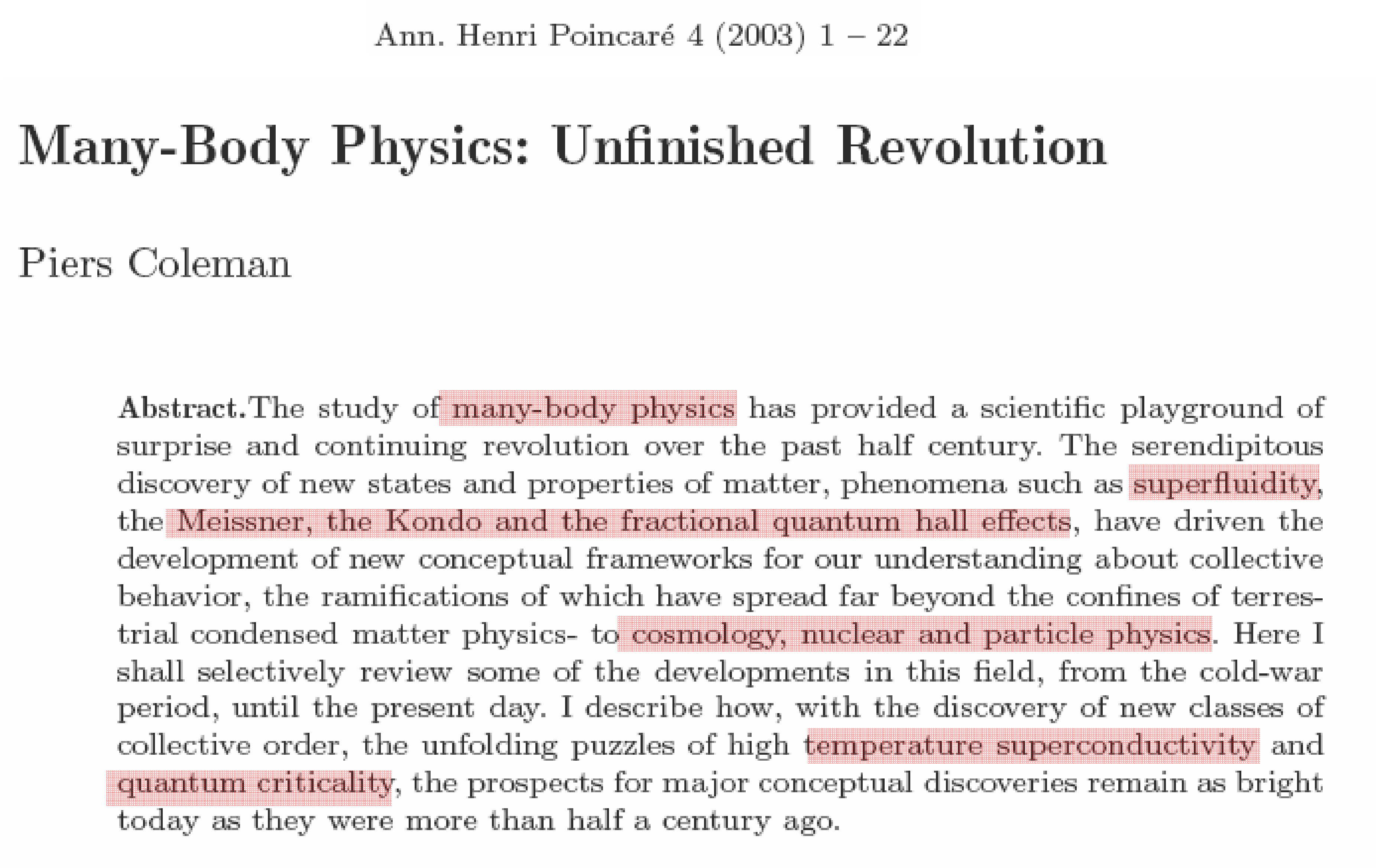
http://lns.web.psi.ch/
http://www.psi.ch/

1. Introduction
2. Low-T_c superconductors
3. High-T_c cuprate superconductors
4. Electronic and magnetic excitations
5. The Abrikosov phase



Ann. Henri Poincaré 4 (2003) 1 – 22

Many-Body Physics: Unfinished Revolution

Piers Coleman

Abstract. The study of many-body physics has provided a scientific playground of surprise and continuing revolution over the past half century. The serendipitous discovery of new states and properties of matter, phenomena such as superfluidity, the Meissner, the Kondo and the fractional quantum hall effects, have driven the development of new conceptual frameworks for our understanding about collective behavior, the ramifications of which have spread far beyond the confines of terrestrial condensed matter physics- to cosmology, nuclear and particle physics. Here I shall selectively review some of the developments in this field, from the cold-war period, until the present day. I describe how, with the discovery of new classes of collective order, the unfolding puzzles of high temperature superconductivity and quantum criticality, the prospects for major conceptual discoveries remain as bright today as they were more than half a century ago.

Measuring excitations: what for?

Macroscopic Measurements

Transport, Specific heat, Magnetisation

…

$$M_\alpha = \frac{1}{k_B T}\frac{\partial \ln Z}{\partial H_\alpha} = g\mu_B \sum_i p_i \langle \Gamma_i | J_\alpha | \Gamma_i \rangle$$

$$C_v = \left(\frac{\partial U}{\partial T}\right)_V = k_B \left[\sum_i \left(\frac{E_i}{k_B T}\right)^2 p_i - \left(\sum_i \left(\frac{E_i}{k_B T}\right) p_i \right)^2 \right]$$

Z=partition function

$U = F - T(\partial F/\partial T)_V$ $\qquad$ $F = -k_B T \ln Z$

Theoretical Model

Hamiltonian

$H=H_e+H_{mag}+H_{ph}+H_{e\text{-}ph}+\ldots.$

Eigenvalues E_i

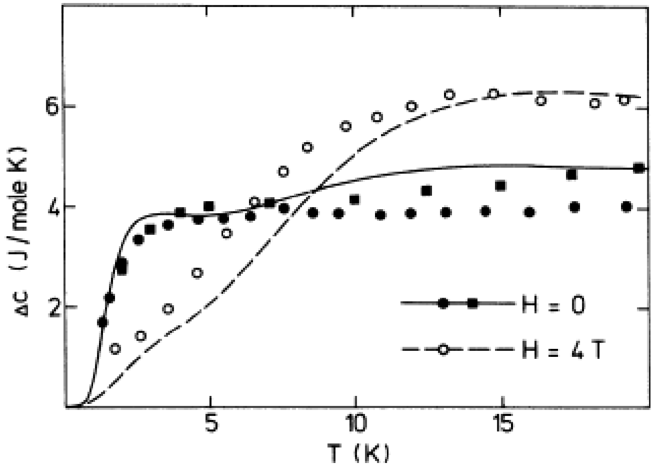
Eigenstates $|\Gamma_i>$

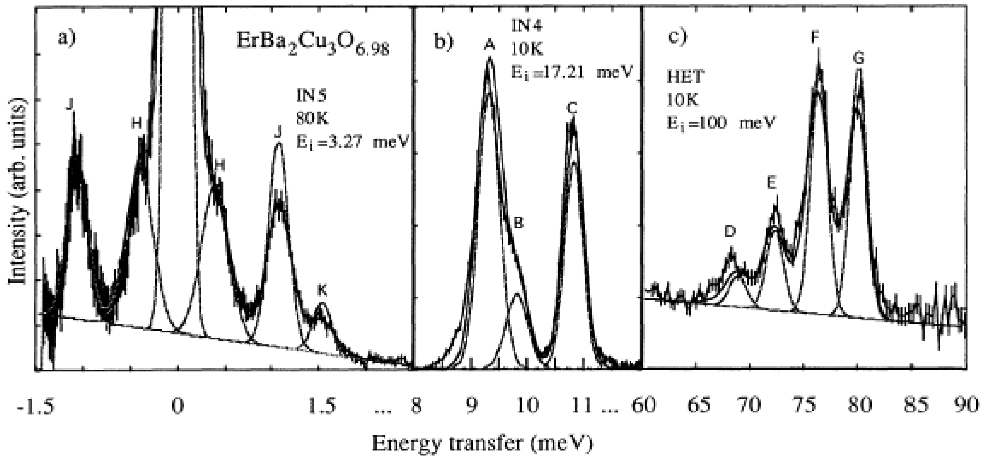
Neutron scattering, Photoemission

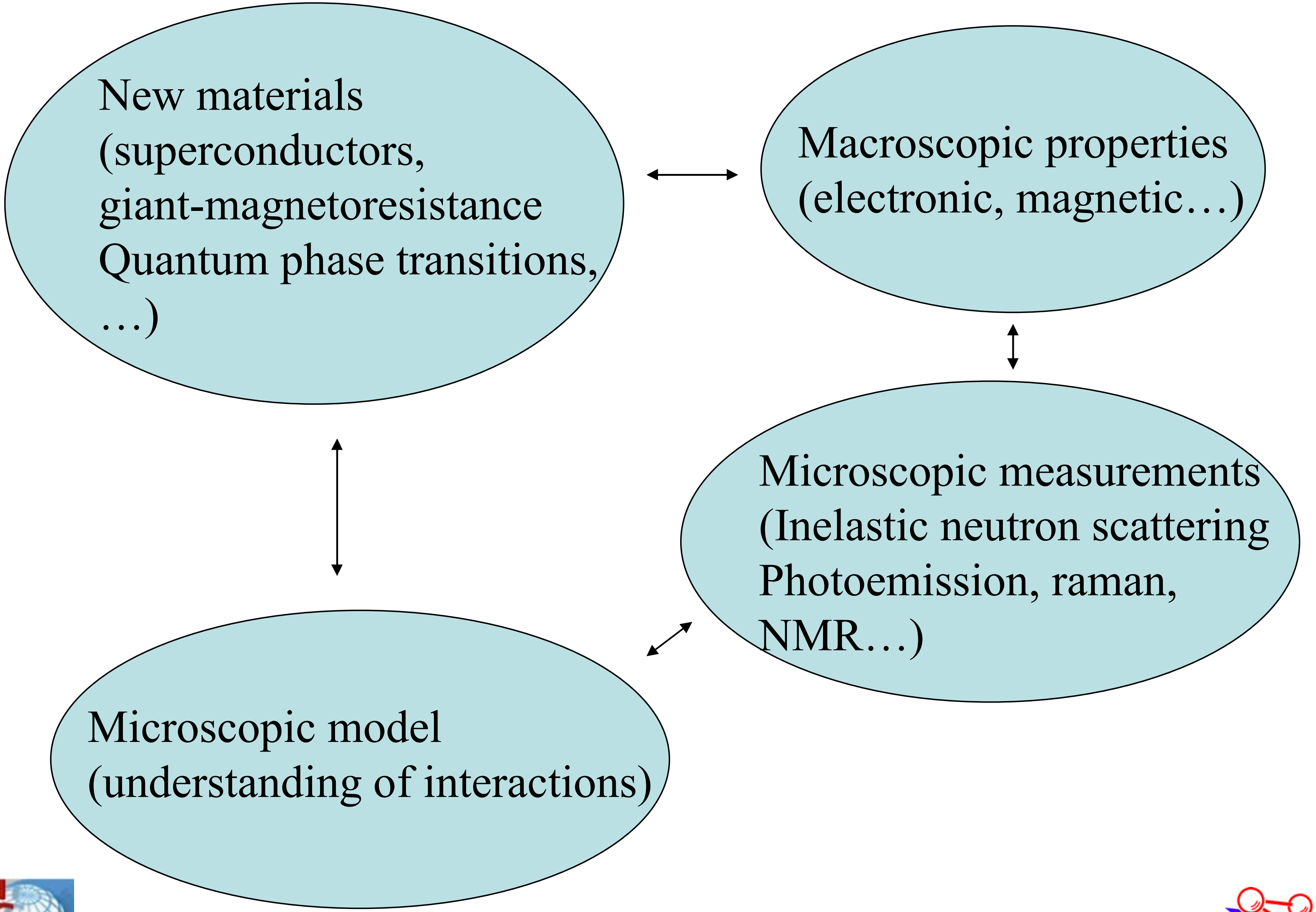
yield direct information about eigenvalues and wave functions !

<u>Neutrons</u>

$$\frac{d^2\omega}{d\Omega d\omega} \approx \left| \left\langle \Gamma_m \middle| \hat{\mathbf{J}}_\perp \middle| \Gamma_n \right\rangle \right|^2 \delta(\hbar\omega + E_{\Gamma_n} - E_{\Gamma_m}) \ ,$$







New materials
(superconductors,
giant-magnetoresistance
Quantum phase transitions,
…)

Macroscopic properties
(electronic, magnetic…)

Microscopic measurements
(Inelastic neutron scattering
Photoemission, raman,
NMR…)

Microscopic model
(understanding of interactions)

Normal metals

resistance = losses = bad efficiency

Normal State

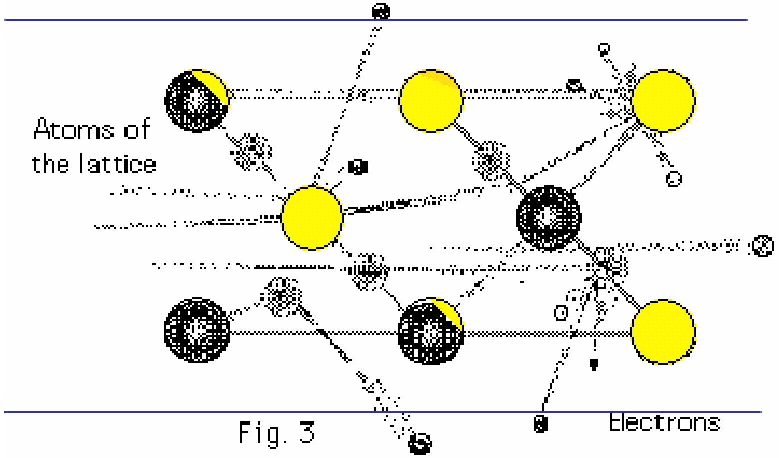
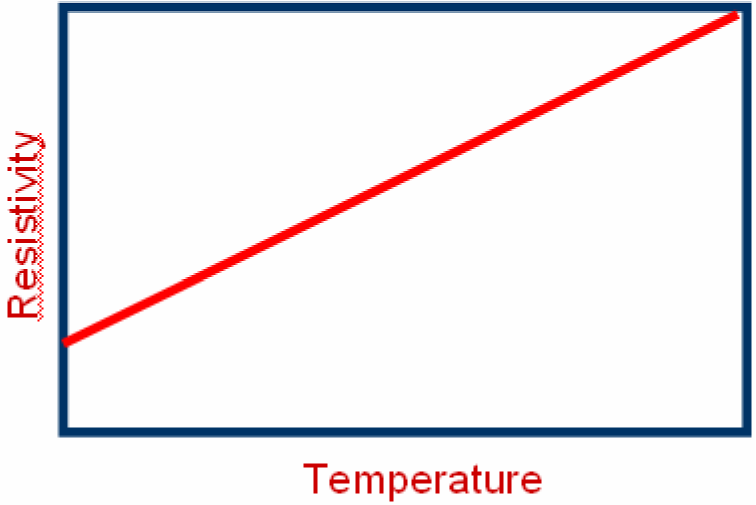
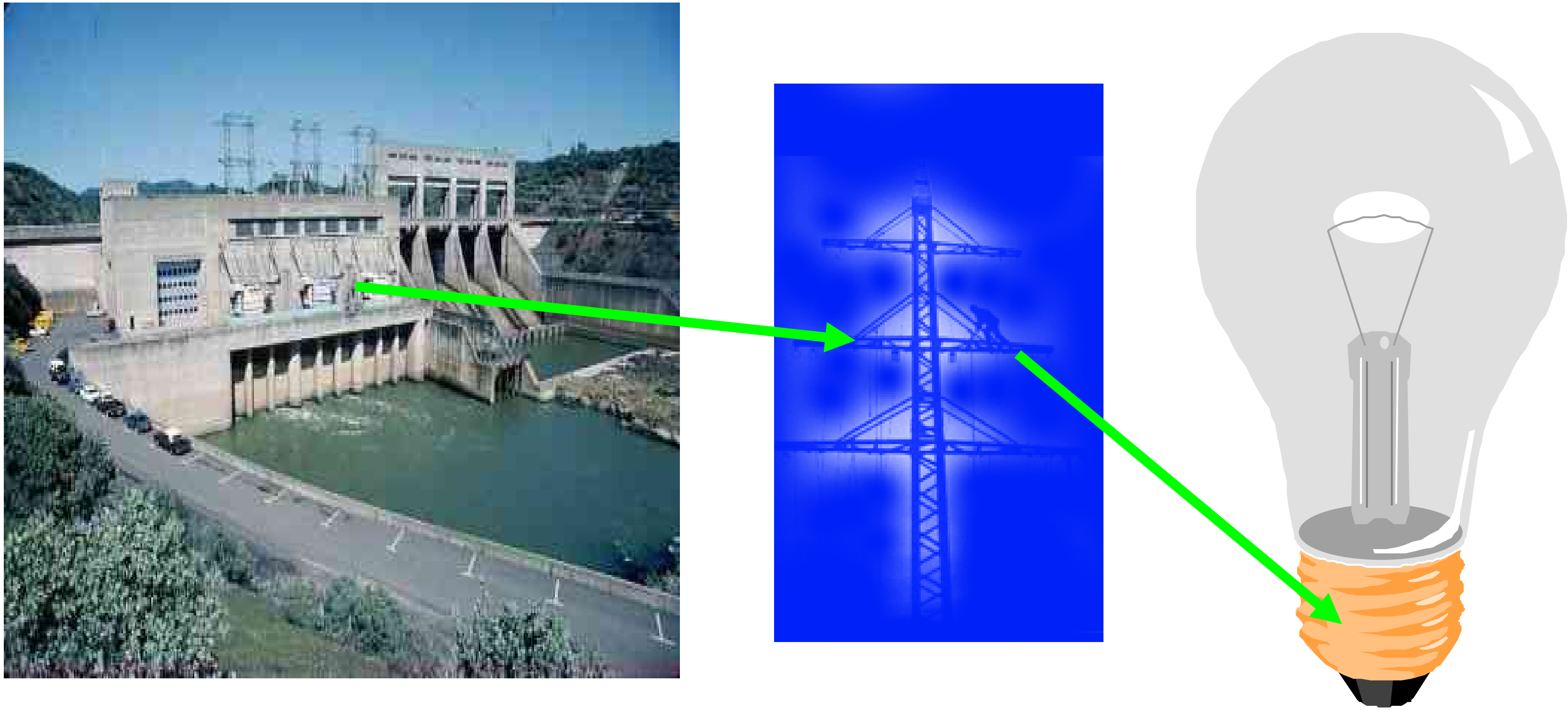


Fig. 3

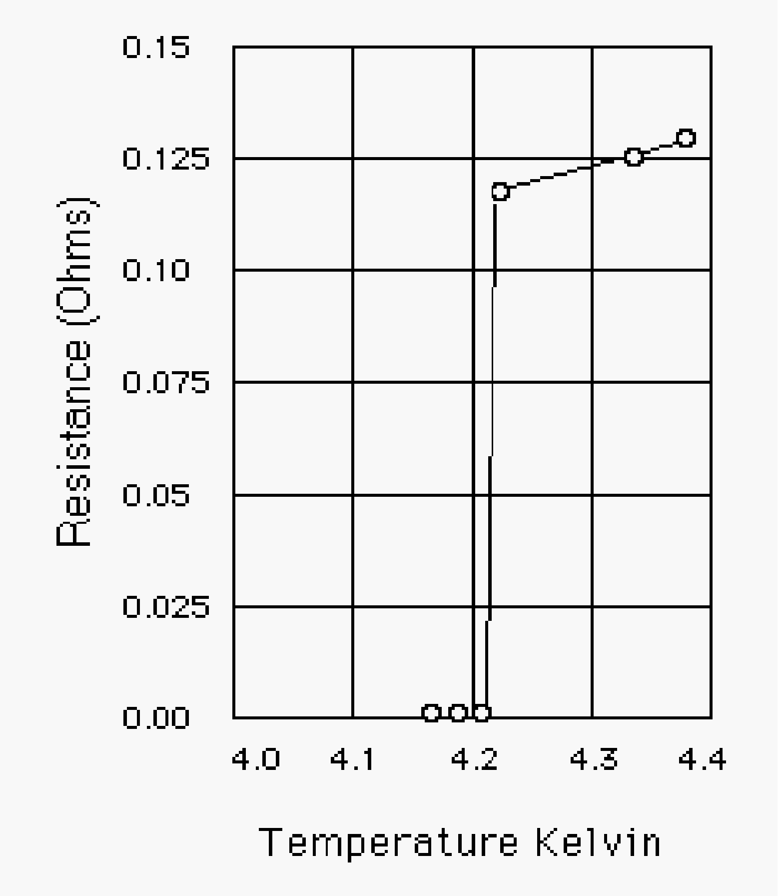


Normal conductors

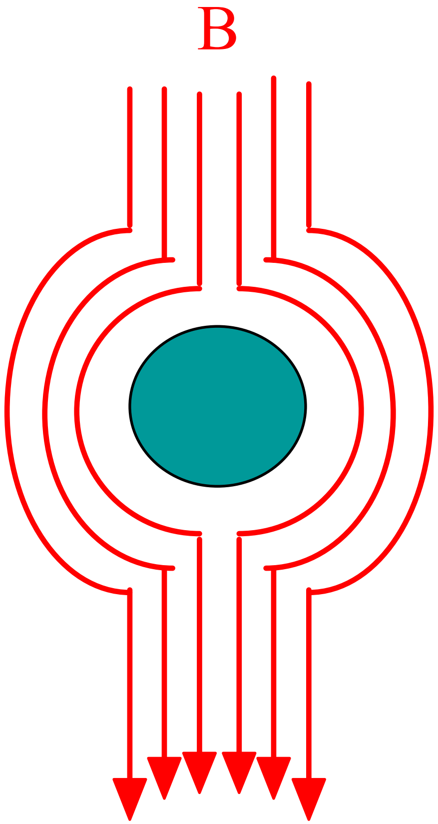


Superconductivity

Kamerlingh-Onnes (1911)



Meissner effect (1933)



Meissner Effect
(Levitation)

Levitation: examples


浮いた
土佐ノ海
TOSANOUMI
(Sumo Wrestler)
Height of Tosanoumi 186cm
Weight of Tosanoumi 142kg
Weight of disk 60kg
Total weight 202kg
As of February '96
協力：財団法人　日本相撲協会

But what is the mechanism producing superconductivity

Isotope effect

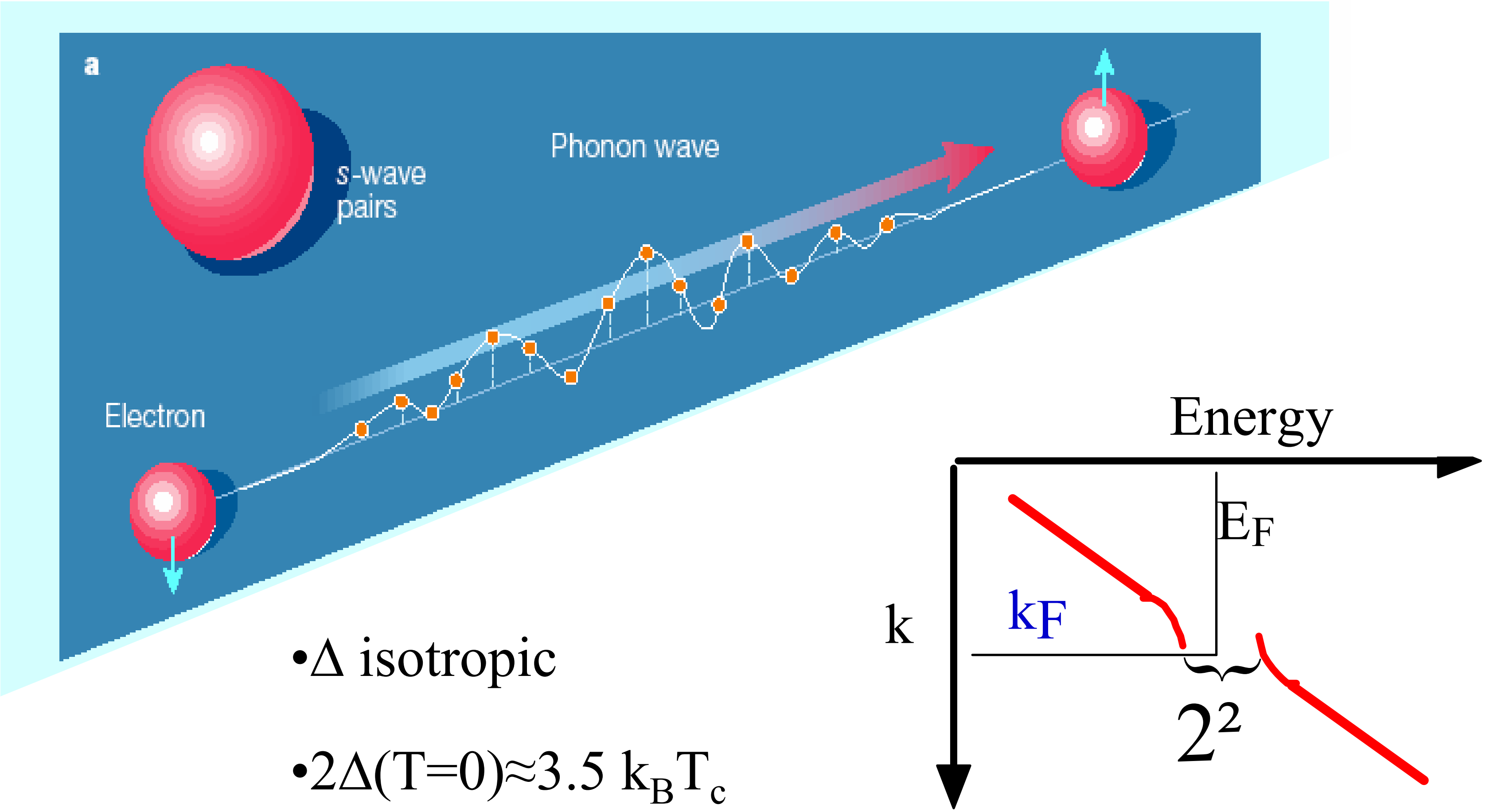
$T_c \approx M^{-\alpha}$ ---> phonons are involved ($\omega_{ph} \approx M^{-0.5}$)

Table 1: measured coefficients α of the isotopic effect T_c - $M^{-\alpha}$

	α		α
Hg	0.50±0.03	Cd	0.50±0.10
Tl	0.50±0.10	Mo	0.33±0.05
Sn	0.47±0.02	Ru	0.00±0.10
Pb	0.48±0.01	Os	0.20±0.05

?

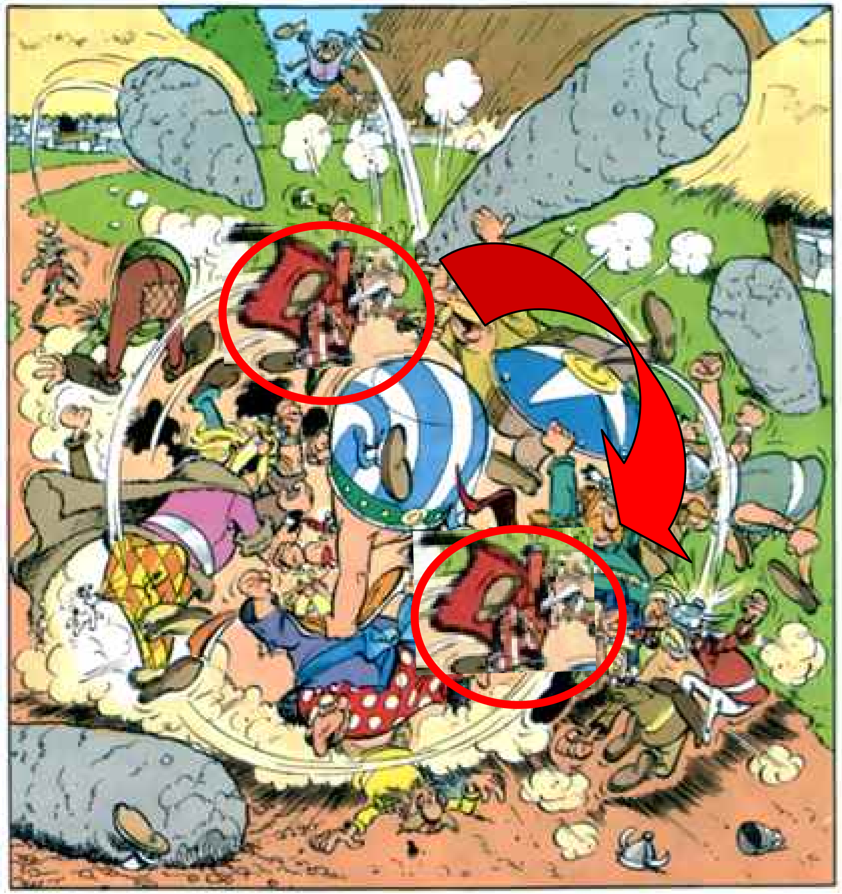
BCS Theory (1957)



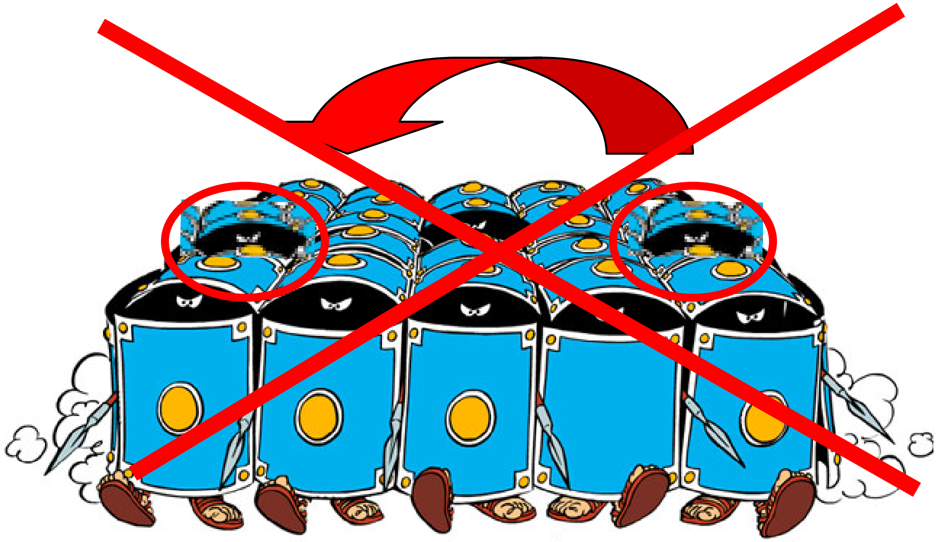
- •Δ isotropic
- •$2\Delta(T=0)\approx 3.5\ k_B T_c$
- • Cooper pairs (bosons) condense into a macroscopic coherent ground-sate

Normal vs Superconducting electrons

"normal state"



"Superconducting state"



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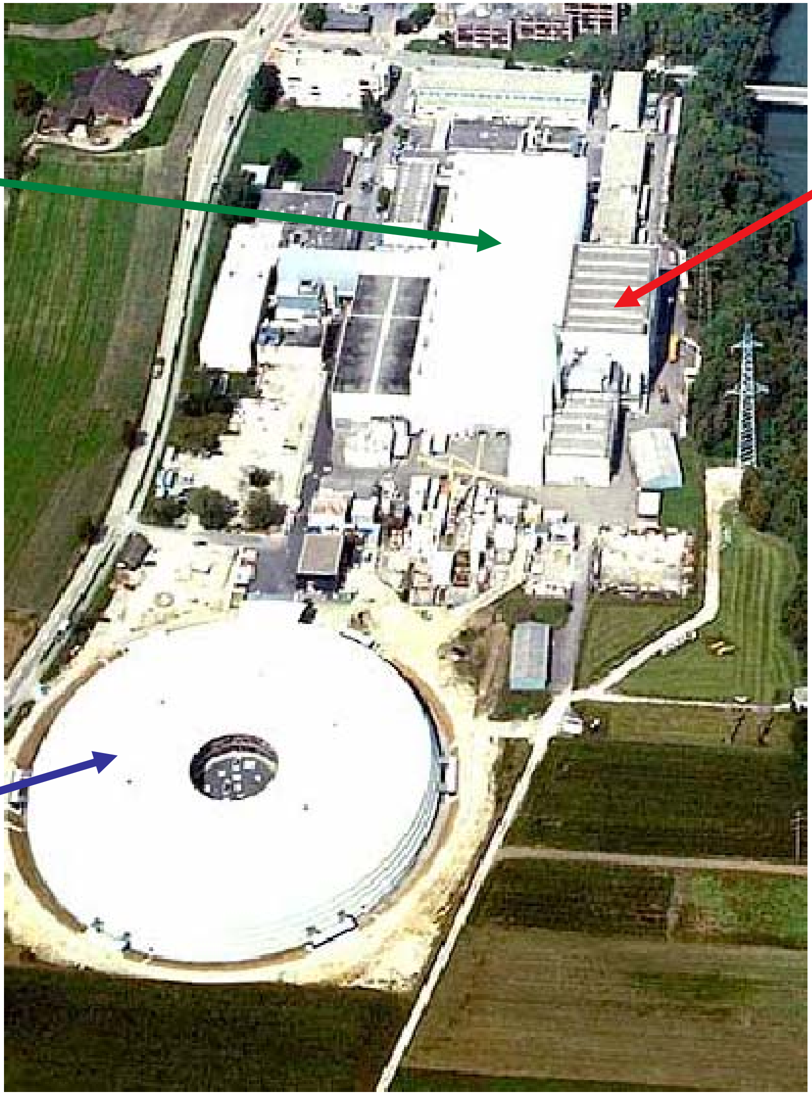
How can we experimentally prove the e-phonon interaction?

Study the dynamics of:

-electrons --> tunneling spectroscopy, photoemission

-lattice (phonons) ---> neutron scattering

Neutrons+Photons+Muons @ PSI



Muons
SμS

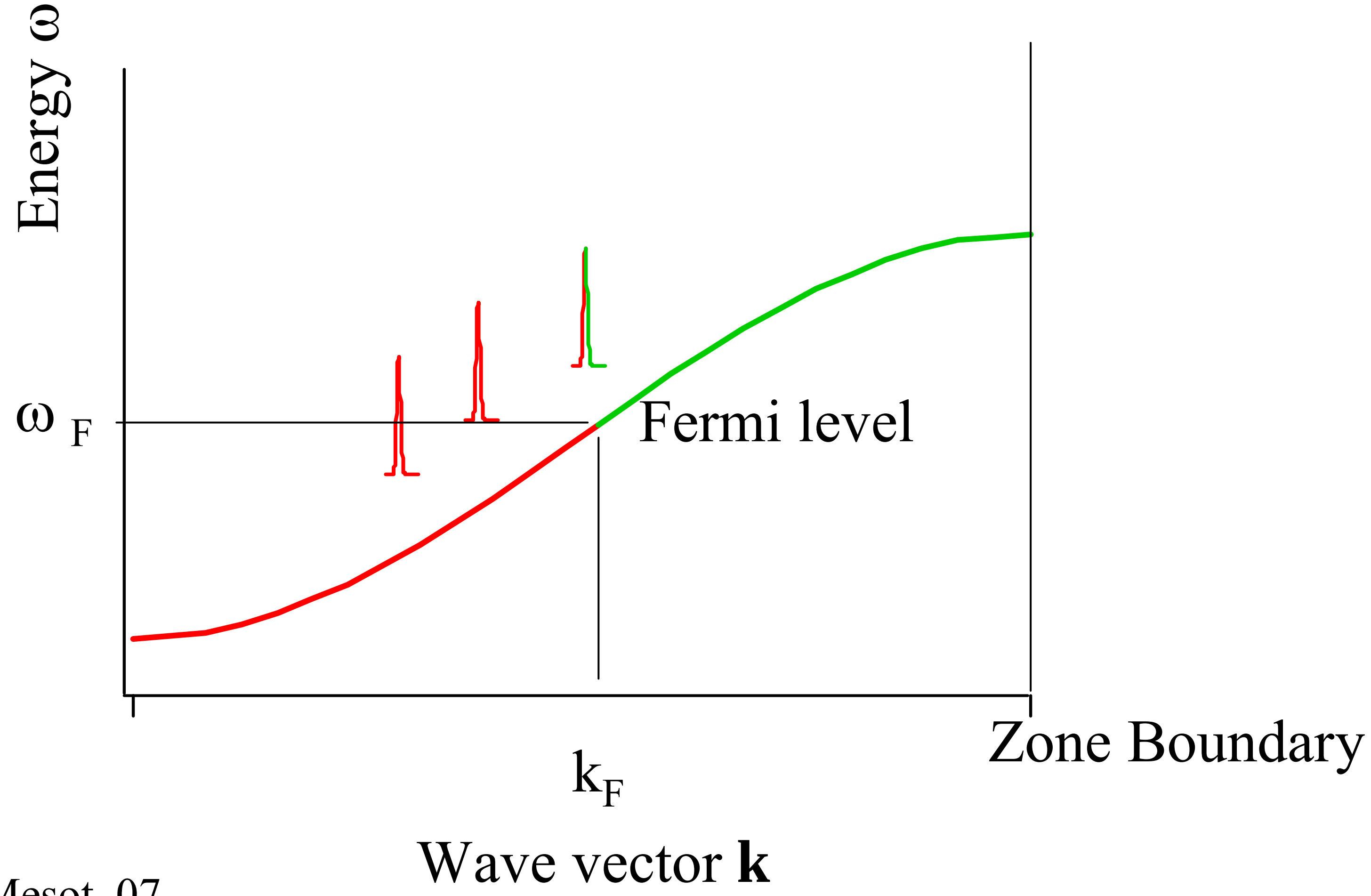
Neutrons
SINQ

Photons
SLS

Electrons in a periodic potential (metals)

(Free electron: $E=1/2mv^2=k^2/2m$)

a) No interactions --> delta function



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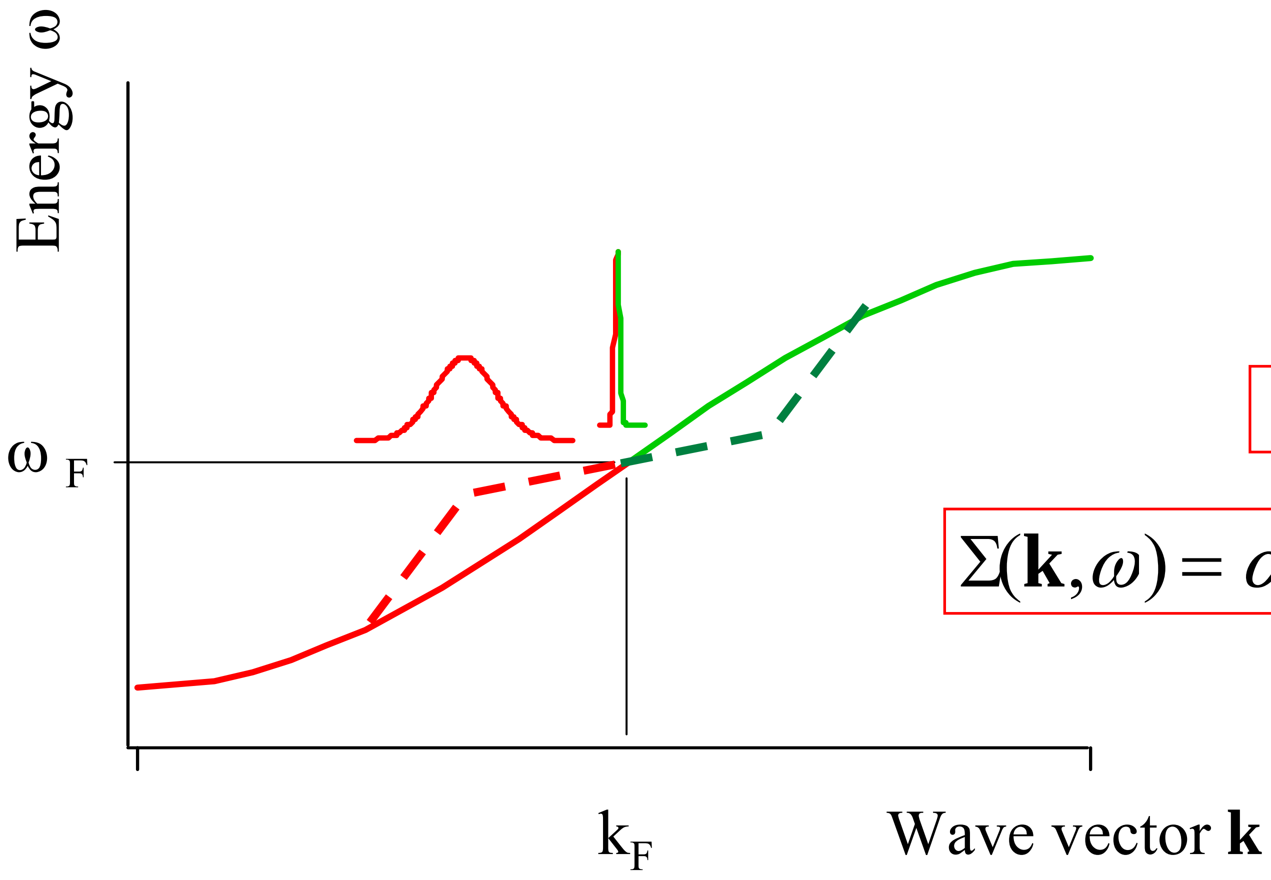
b) Weak interactions--> Renormalized quasiparticles

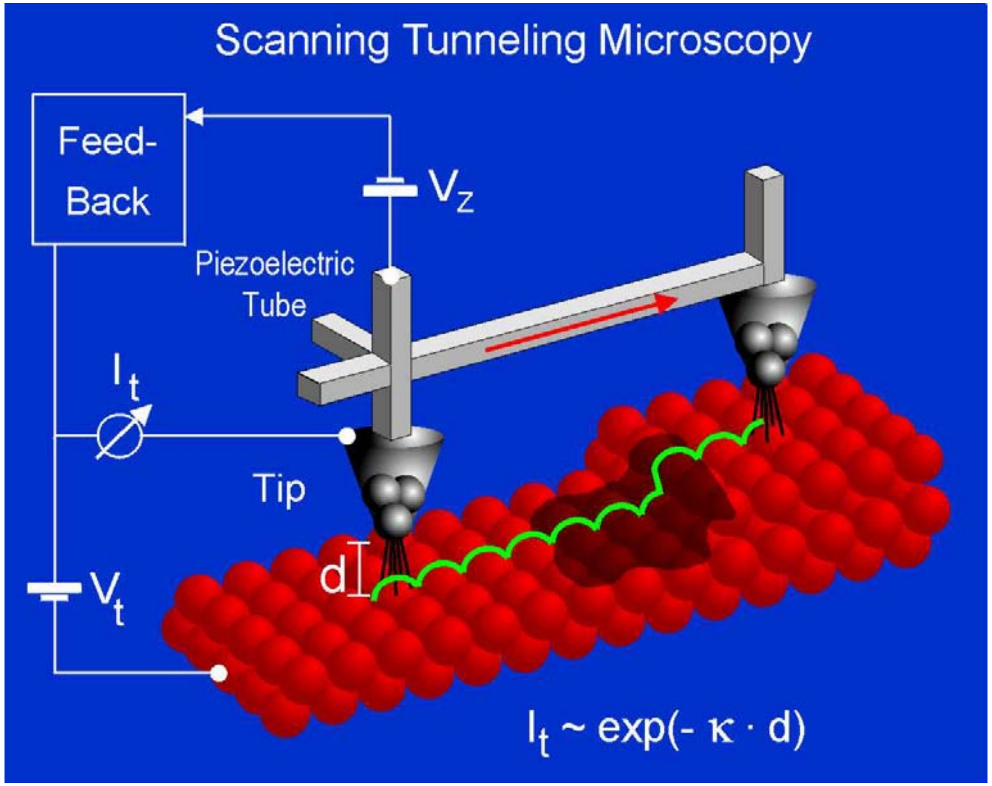
$$m^*,\ \omega(\mathbf{k}) = \omega_0(\mathbf{k}) + \Sigma'(\omega,\mathbf{k}) \qquad \tau(\mathbf{k},\omega) \sim 1/_{\Sigma''(\mathbf{k},\omega)}$$

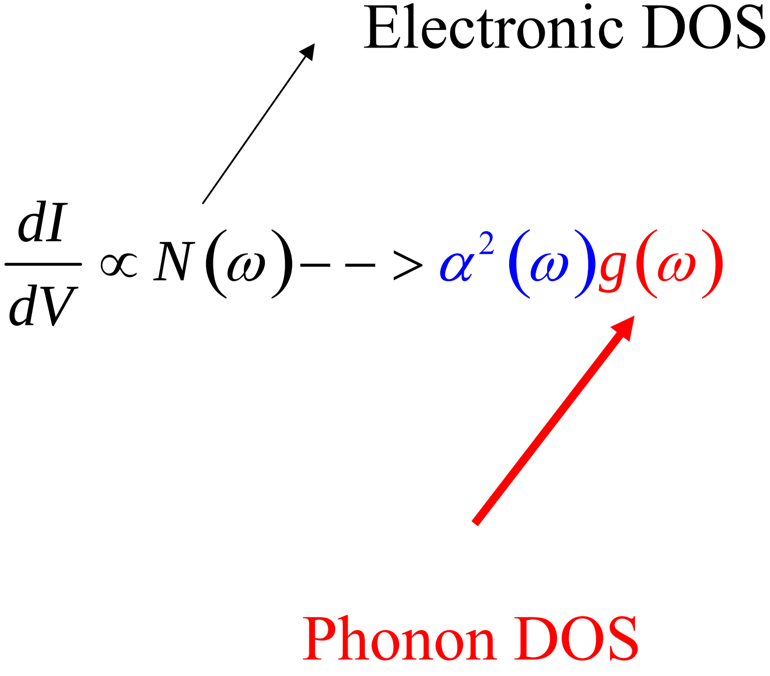


e-e interaction

$$\Sigma(\mathbf{k},\omega) = \alpha\omega + i(\beta\omega^2 + \gamma T^2)$$

Evidence for e-Phonon Interaction: Tunneling Spectroscopy





Electronic DOS

$$\frac{dI}{dV} \propto N(\omega) --> \alpha^2(\omega) g(\omega)$$

Phonon DOS

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Nuclear, Inelastic, Coherent Neutron Scattering

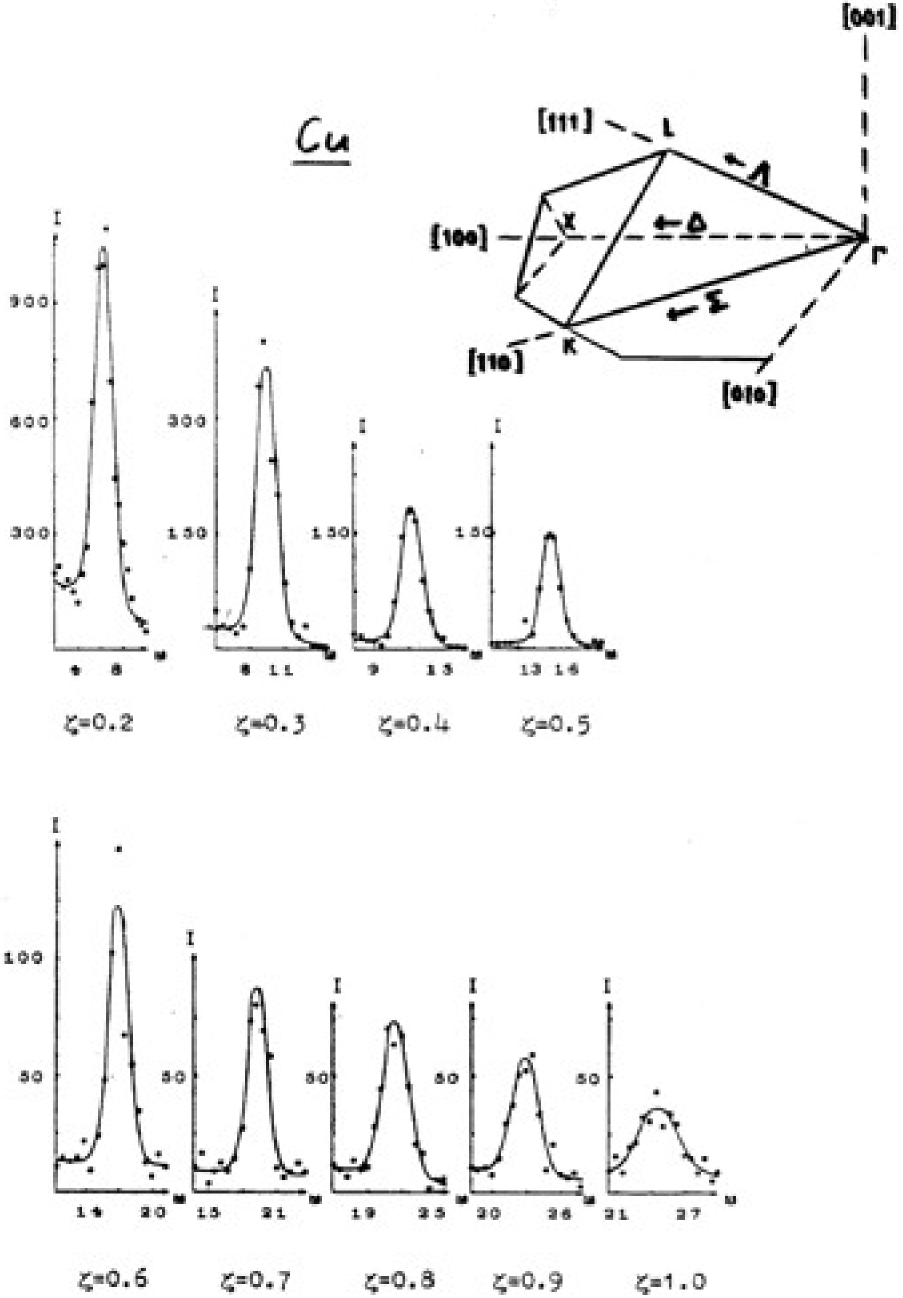
$$\frac{d^2\sigma}{d\Omega d\omega} = \frac{4\pi^3}{v_o}\cdot\frac{k'}{k}\sum_{s,\mathbf{q}} \frac{1}{\omega_s(\mathbf{q})}\left|\sum_{\mathbf{d}} \frac{<b_{\mathbf{d}}>}{\sqrt{M_{\mathbf{d}}}} e^{-W_{\mathbf{d}}(\mathbf{Q})} e^{i\mathbf{Q}\cdot\mathbf{d}}\left[\mathbf{Q}\cdot\mathbf{e}_{\mathbf{d},s}(\mathbf{q})\right]\right|^2$$

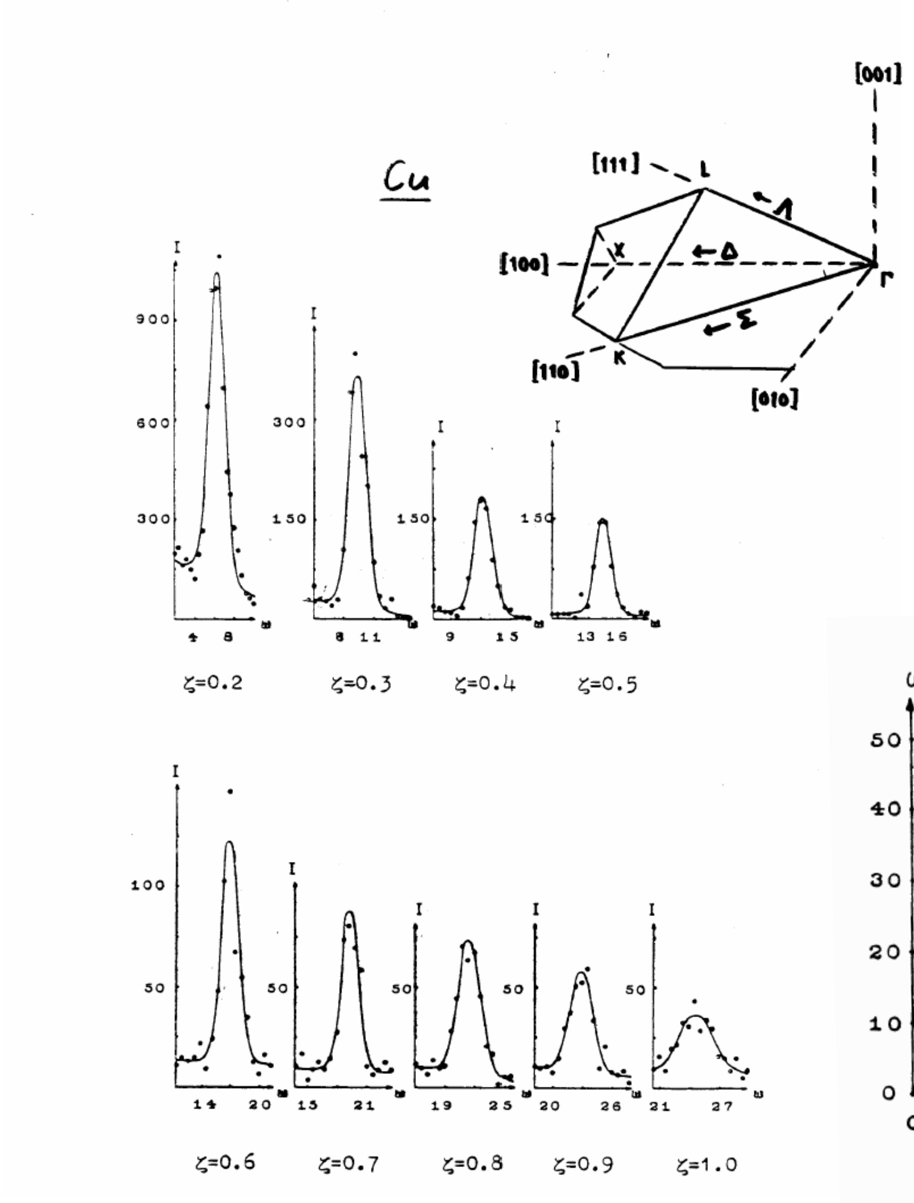
$$\left\{ \, [n_s(\mathbf{q})+1]\,\delta\{\omega-\omega_s(\mathbf{q})\}\sum_{\tau}\delta(\mathbf{Q}-\mathbf{q}-\tau)\right.$$

$$\left. +\, n_s(\mathbf{q})\,\delta\{\omega+\omega_s(\mathbf{q})\}\sum_{\tau}\delta(\mathbf{Q}+\mathbf{q}-\tau) \,\right\}$$

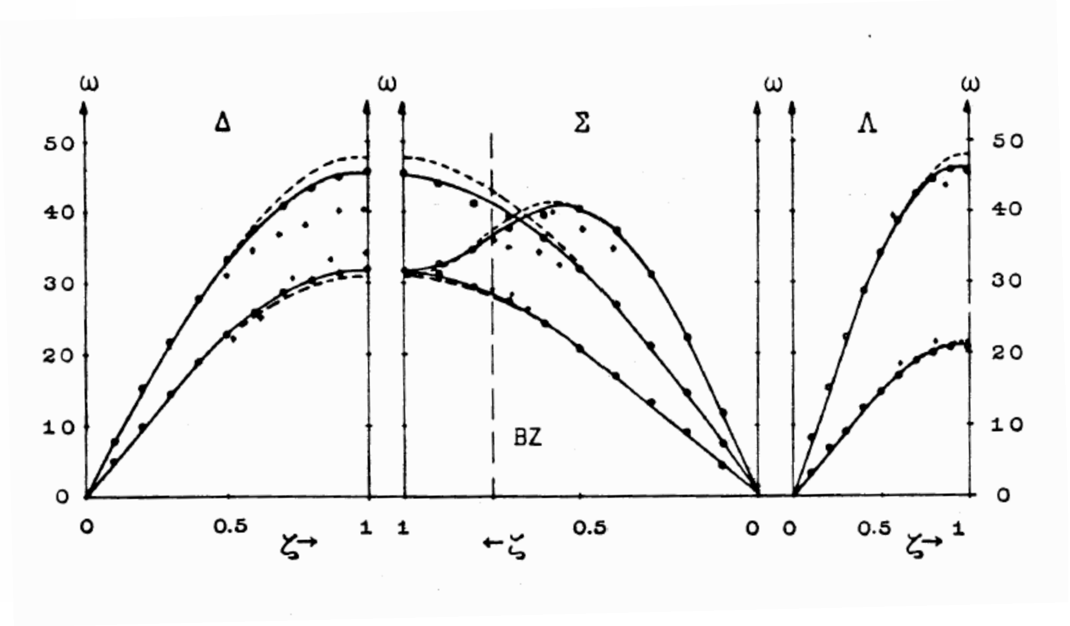
$$n_s(\mathbf{q}) = \left[\exp\left\{\frac{\hbar\omega_s(\mathbf{q})}{k_B T}\right\} - 1\right]^{-1}.$$

Bose-Einstein statistics


Cu
[001]
[111]
L
Λ
[100]
X
Δ
Γ
Σ
K
[110]
[010]
I
900
600
300
4
8
ω
ζ=0.2
150
300
8
11
ζ=0.3
150
9
13
ζ=0.4
150
13
16
ζ=0.5
100
50
14
20
ζ=0.6
50
15
21
ζ=0.7
50
19
25
ζ=0.8
50
20
26
ζ=0.9
50
21
27
ζ=1.0



Phonons in Cu



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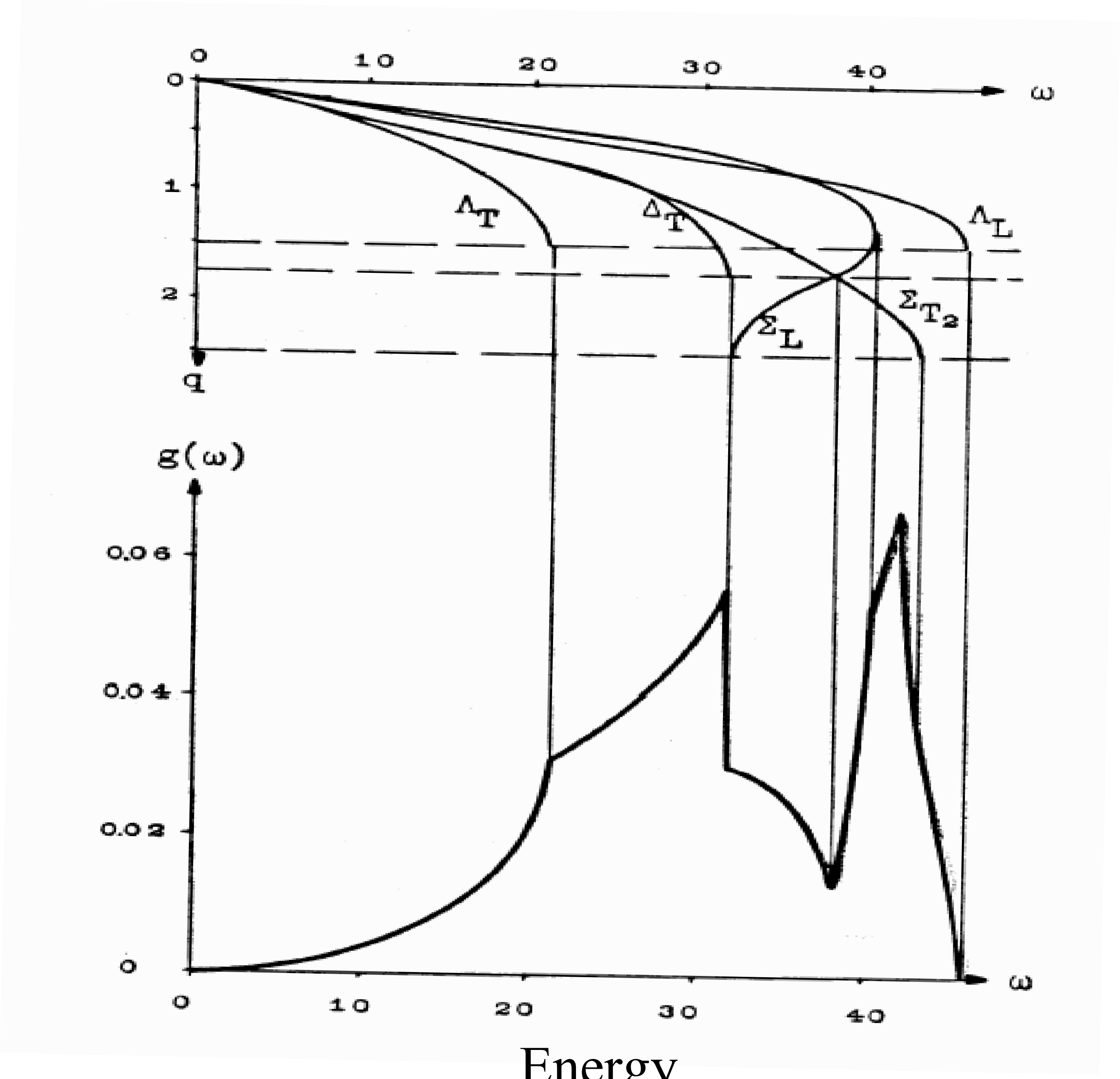
Phonon density-of-state



Energy

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Nuclear Incoherent Neutron Scattering

$$\frac{d^2\sigma}{d\Omega d\omega} = \frac{1}{4M} \cdot \frac{k_f}{k_i}\left[< b^2 > - < b >^2 \right] e^{-2W(\mathbf{Q})} \left\langle \left(\mathbf{Q} \cdot \mathbf{e}_s(\mathbf{q}) \right)^2 \right\rangle \frac{g(\omega)}{\omega} \left\{ \coth\left(\frac{\hbar\omega}{2k_B T} \right) \pm 1 \right\}$$

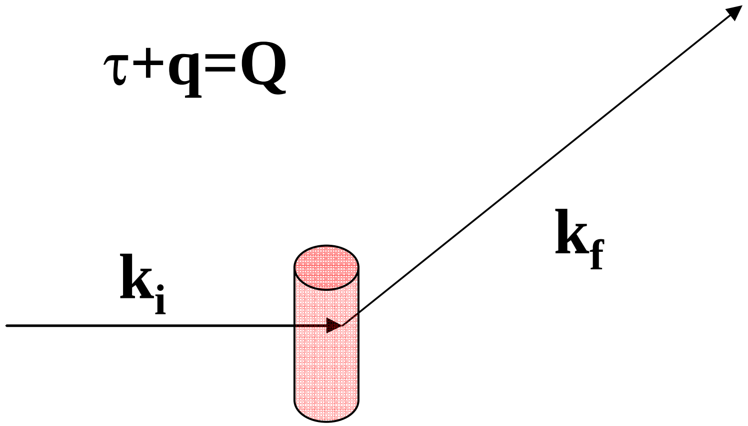
$\mathbf{Q}=\mathbf{k}_i-\mathbf{k}_f$= momentum transfer

$\hbar\omega = E_i - E_f$ = energy transfer

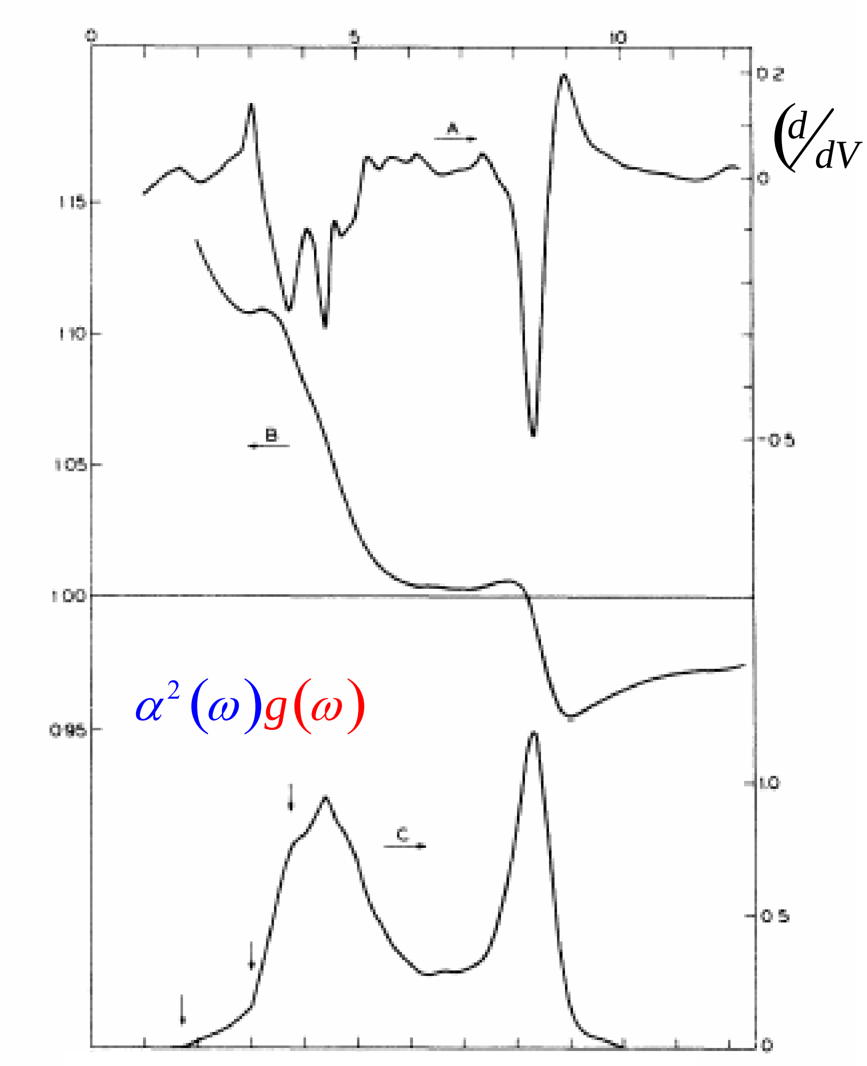
b= scattering length

$\mathbf{e}_s(\mathbf{q})$= phonon eigenvector

W($\mathbf{Q}$)=Debye-Waller factor



McMillan PRL 65- Pb-I-Pb tunneling



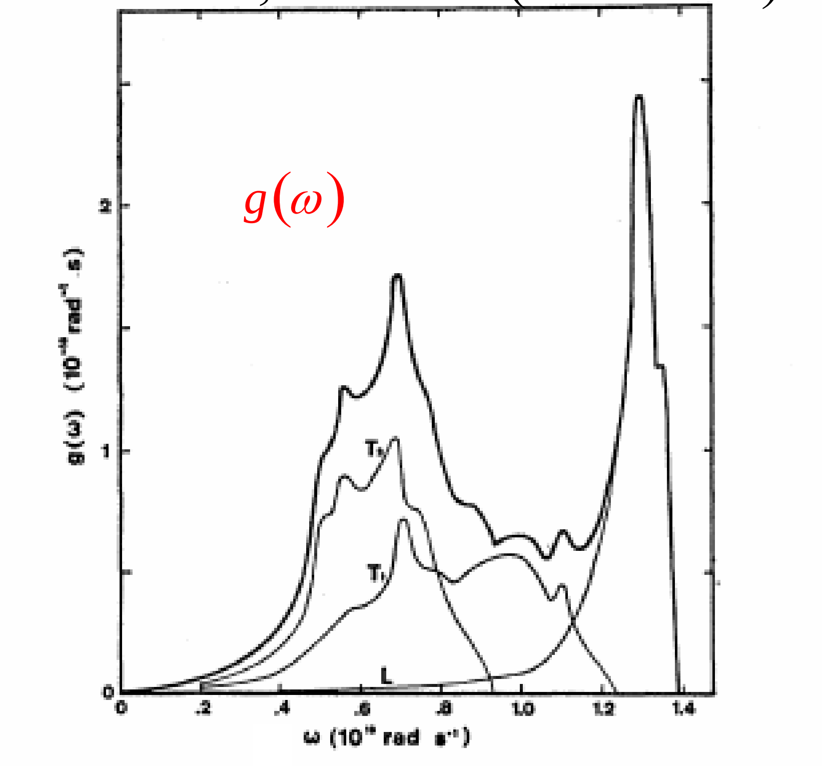
$$\left(d/dV\right)\left[\left(\frac{dI}{dV}\right)_n\Big/\left(\frac{dI}{dV}\right)_s\right]$$

$\alpha^2(\omega)g(\omega)$

0 5 10

Energy meV

Stedman, PRB 67 (Neutrons)



$g(\omega)$

FIG. 2. The pho n-frequency distribution of lead.

10^{13} rad/s=6.6 meV

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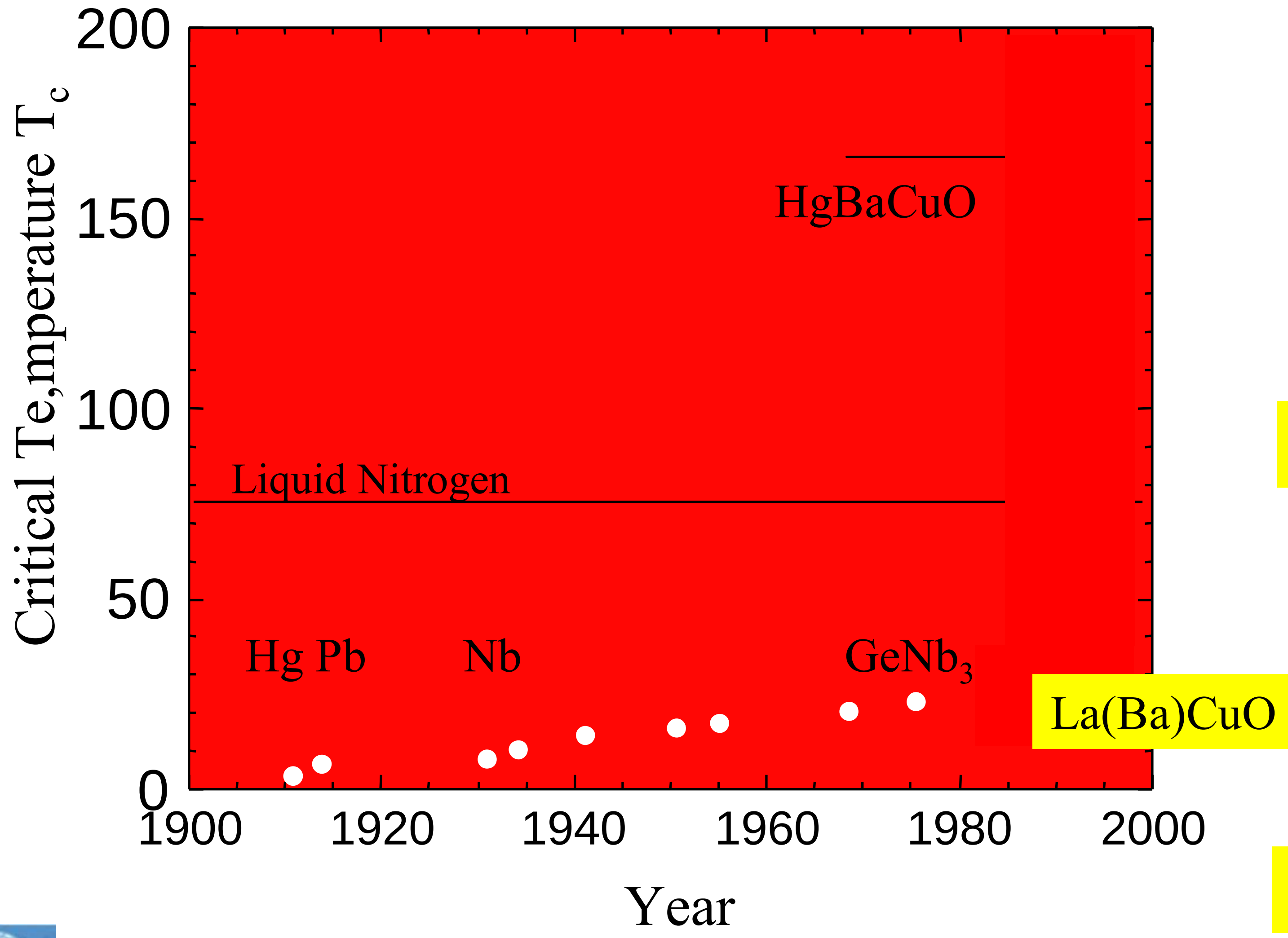
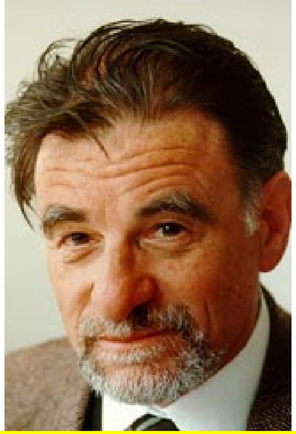
Full understanding of superconductivity (1986)?

J. M. Ziman (1972)

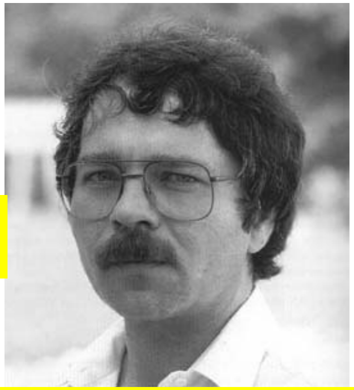
"SC was long considered the most extraordinary and mysterious of the properties of metals; but the theory of Bardeen, Cooper and Schrieffer –the BCS theory- has explained so much that we can say that we now understand the superconducting state almost as well as we do the normal 'state'."

The 1986 revolution





K. A. Müller



J. G. Bednorz

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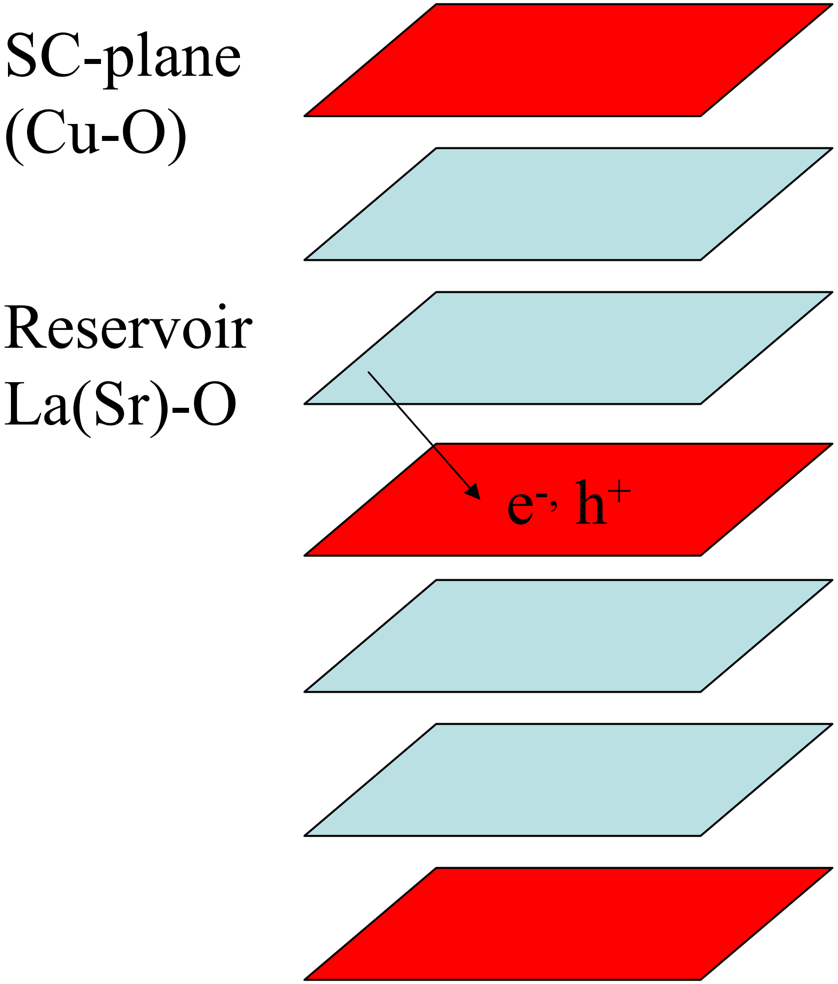
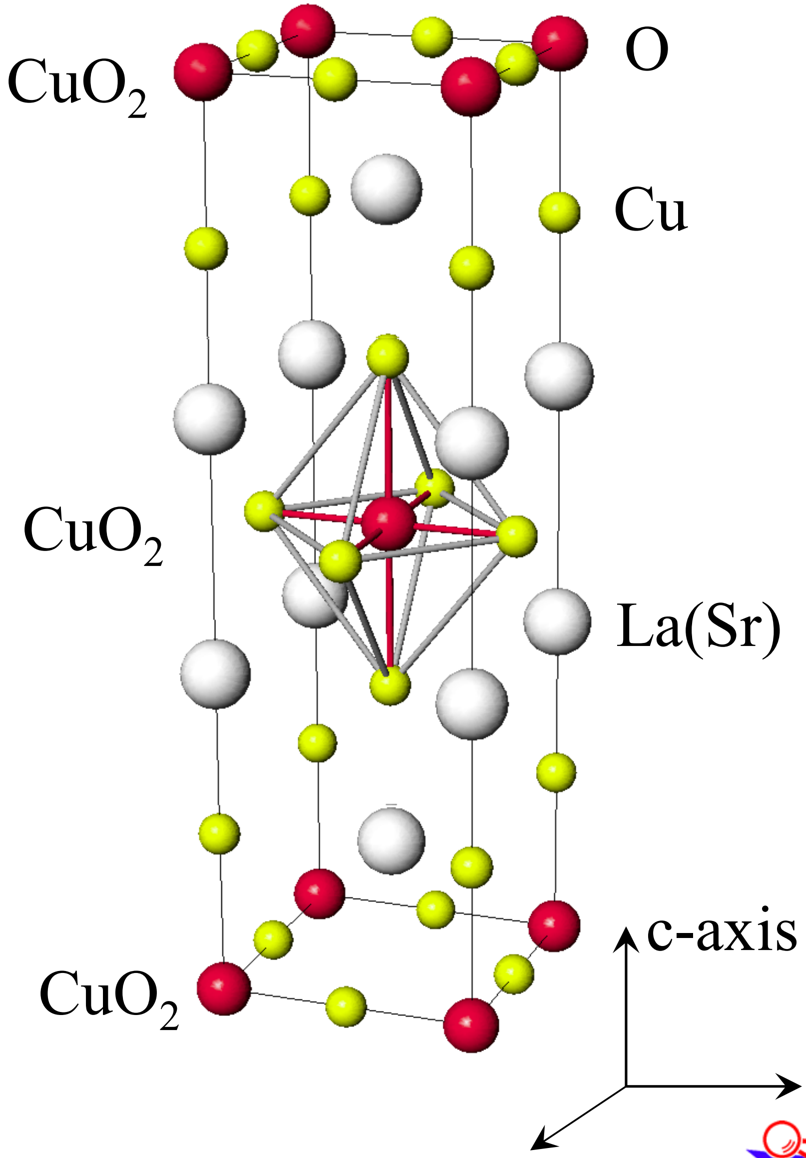
2D-Structure La(2-x)Sr(x)CuO(4)





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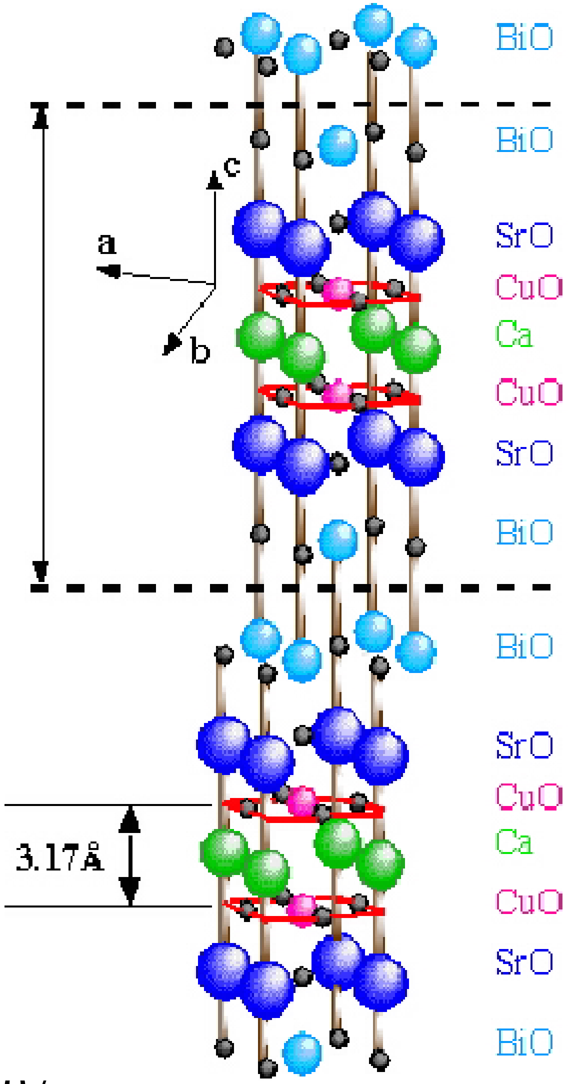
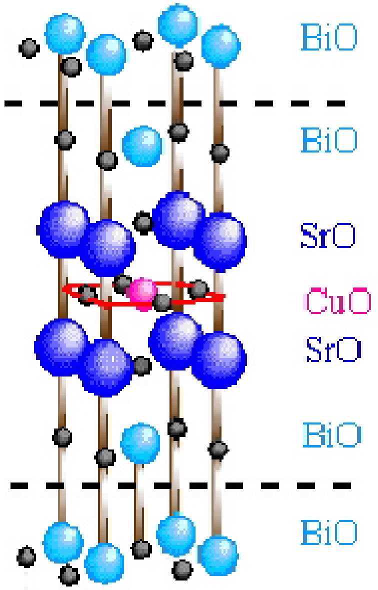
$Bi_2Sr_2Ca_nCu_{1+n}O_{6+2n}$ (n=0,1…)

Bi2212 (n=1)

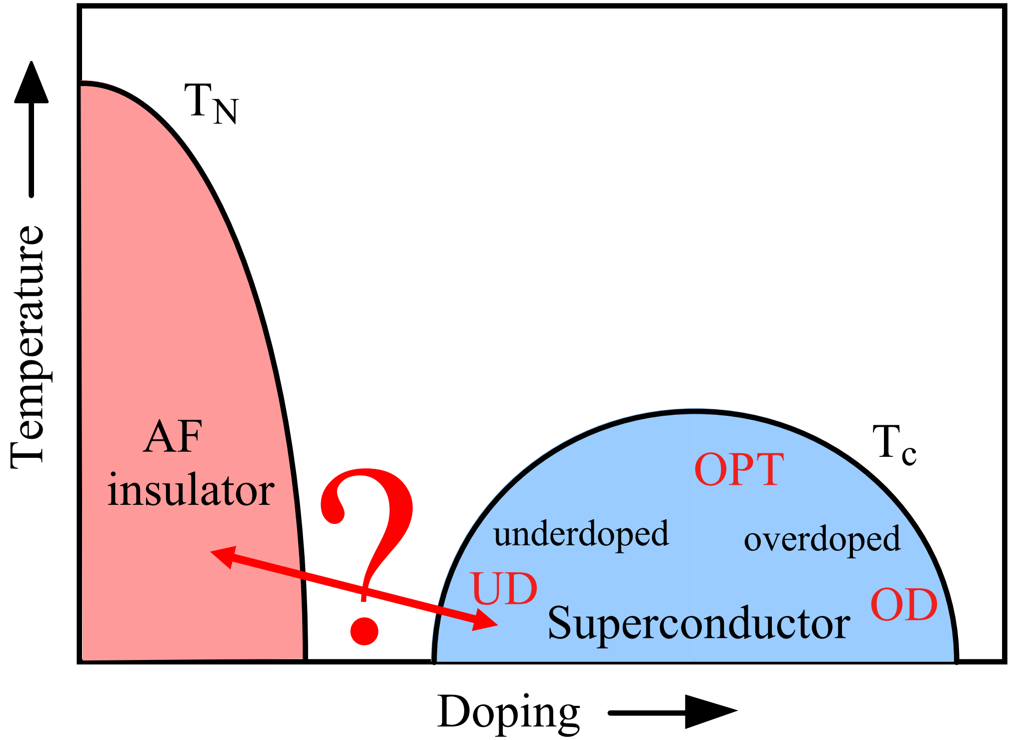


Bi2201 (n=0)



Phase Diagram of HTSC

HTSC: a doped antiferromagnet
Undoped Cu^{2+}: $3d^9$--> 1 hole/1 spin.



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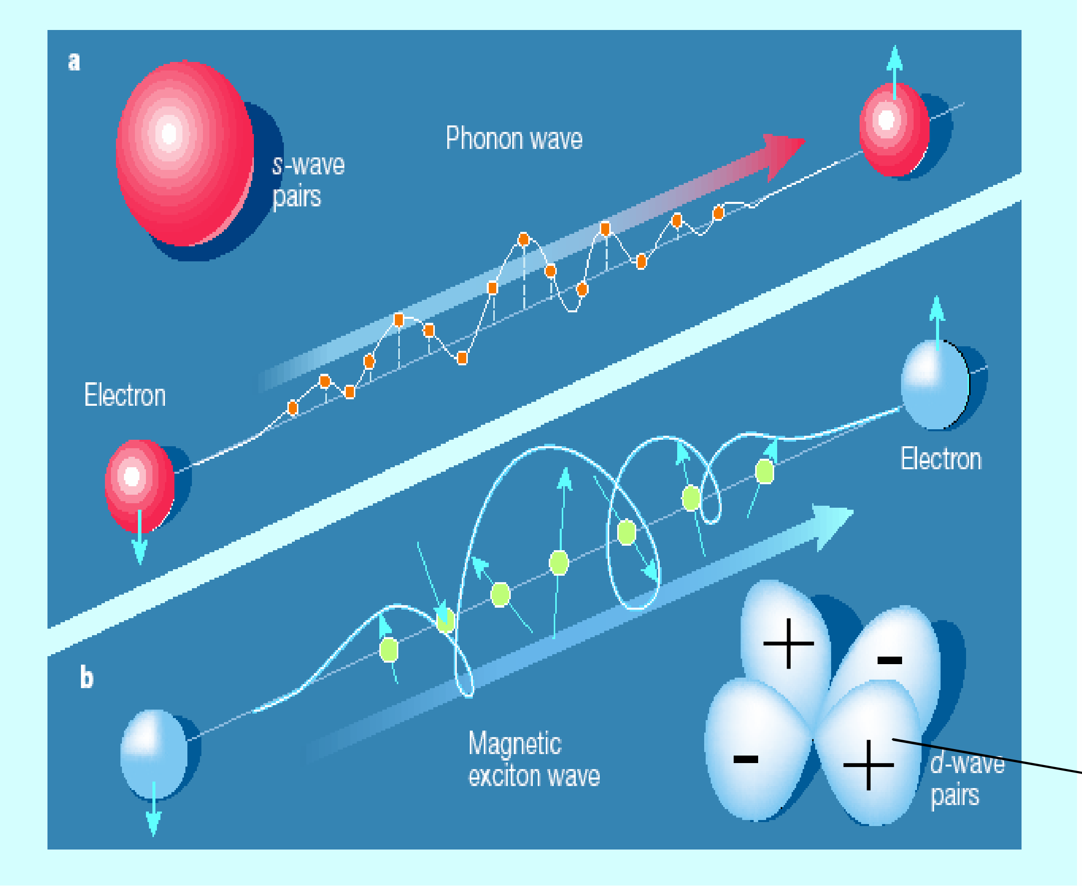
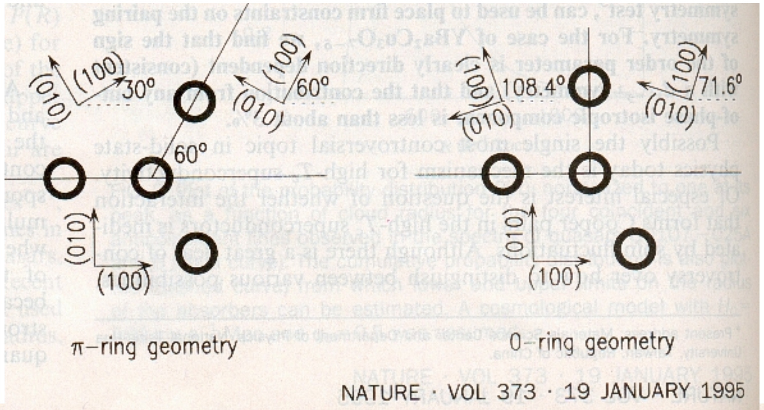
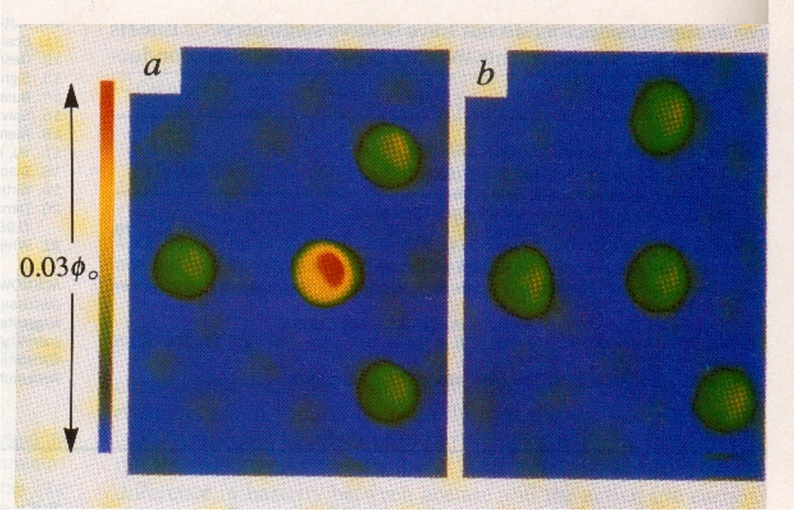
d-wave gap?

$2\Delta(0)=3.5\ k_BT_c$





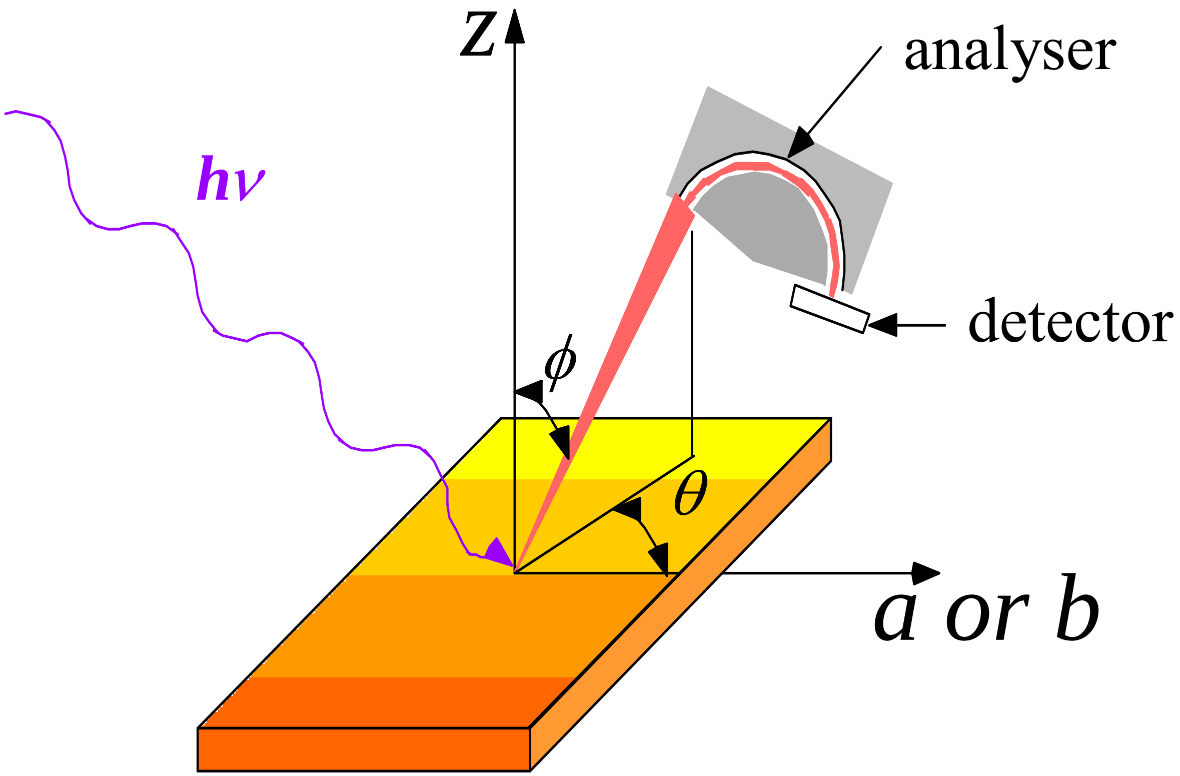


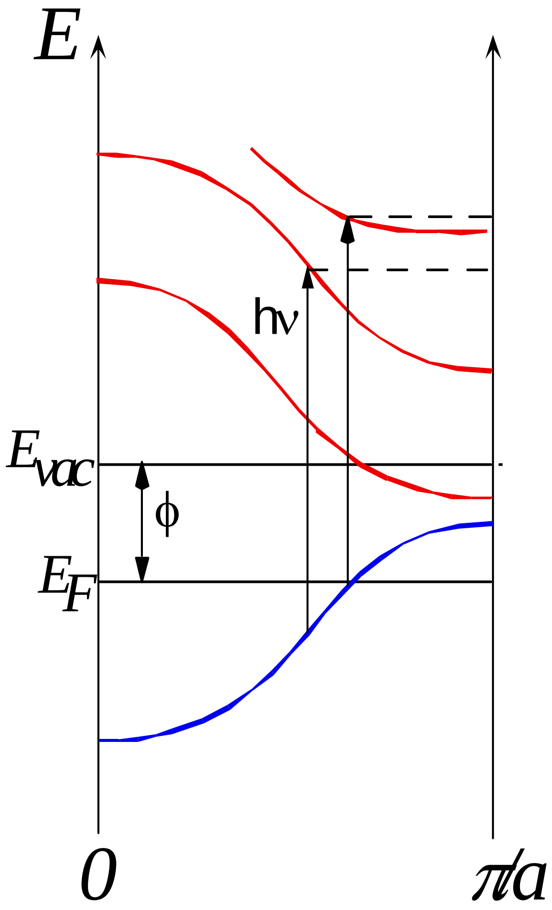
Phase sensitive experiments:

$\Delta(k)=\Delta_0\ [\cos k_y-\cos k_y]$

Tsuei et al. Nature **373** (1995) 225

Angle Resolved Photoemission (ARPES)





$$k^{f}_{//} = k^{i}_{//} = \sqrt{\frac{2mE}{\hbar^{2}}}\,\sin(\phi)$$

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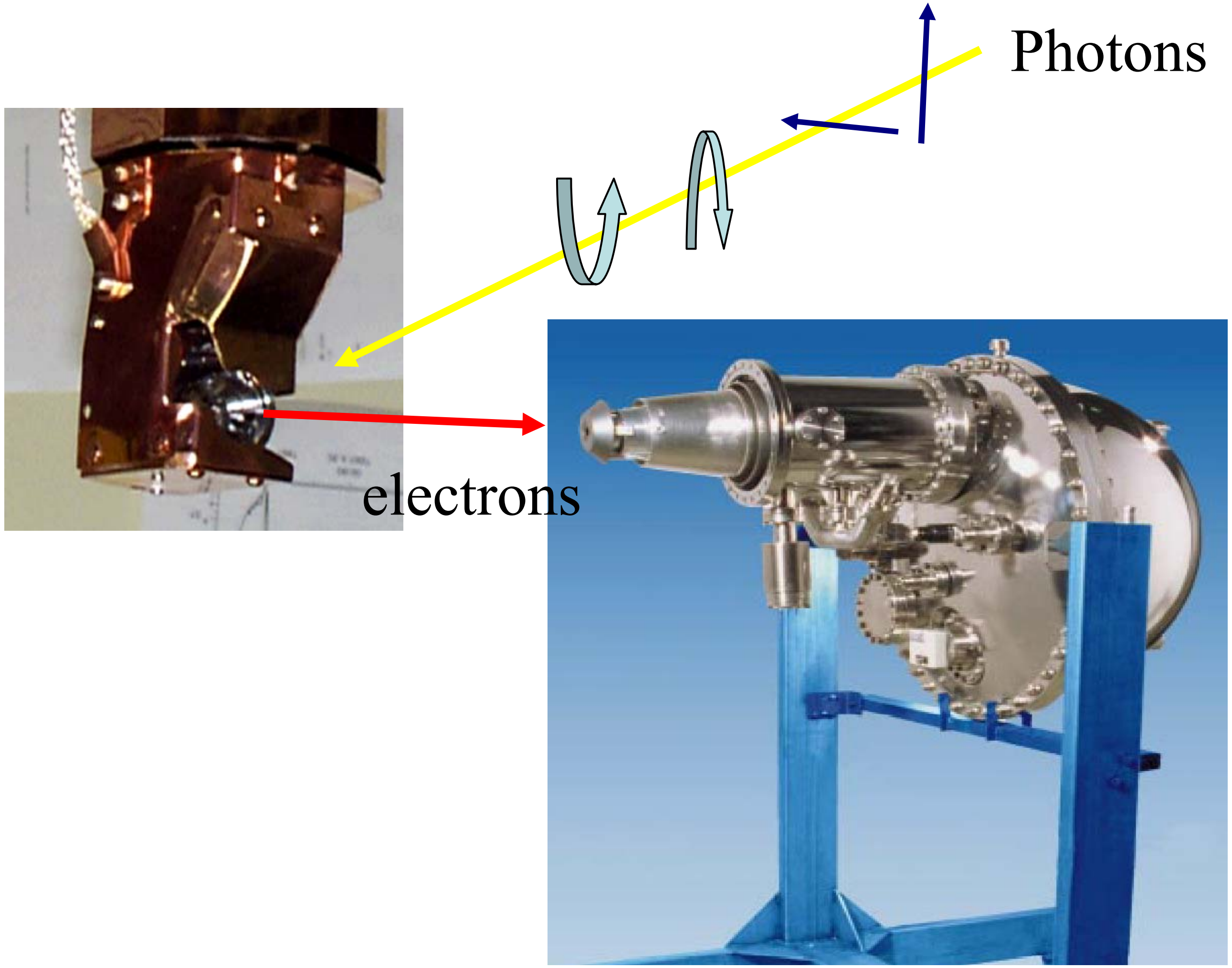
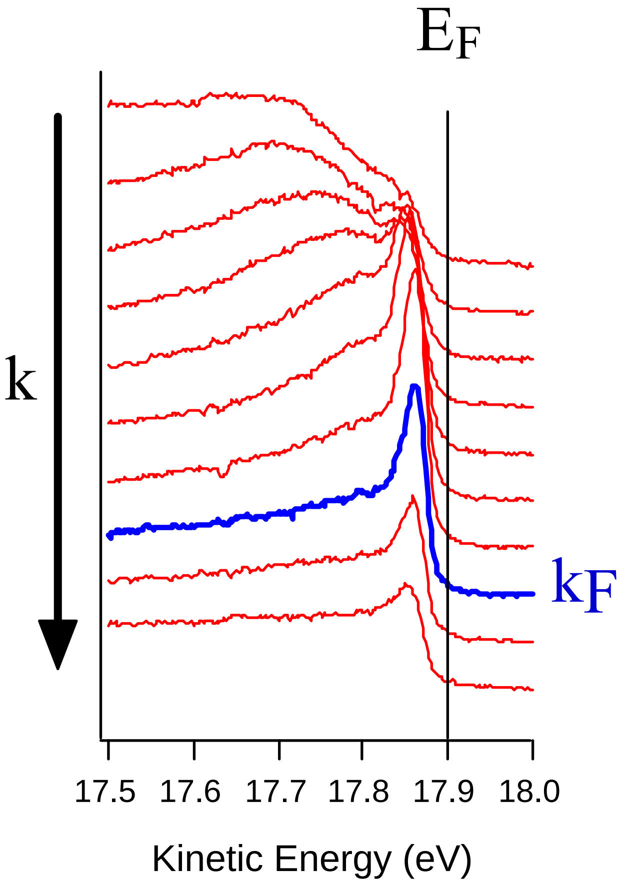
Surface-Interface-Spectroscopy beamline @ SLS-PSI



Photons

electrons

Spectral function



$I(\mathbf{k},\omega)=M(\mathbf{k},\omega)f(\omega)A(\mathbf{k},\omega)$

$A(\mathbf{k},\omega)$ = spectral function

$f(\omega)$ = Fermi function

$M(\mathbf{k},\omega)$ = matrix elements

Self-energy $\Sigma(\mathbf{k},\omega)$

$$A(\mathbf{k},\omega)=\frac{1}{\pi}\frac{|\Sigma''(\mathbf{k},\omega)|}{[\omega-\varepsilon_{\mathbf{k}}-\Sigma'(\mathbf{k},\omega)]^2+[\underbrace{\Sigma''(\mathbf{k},\omega)}_{\tau^{-1}(\mathbf{k},\omega)}]^2}$$

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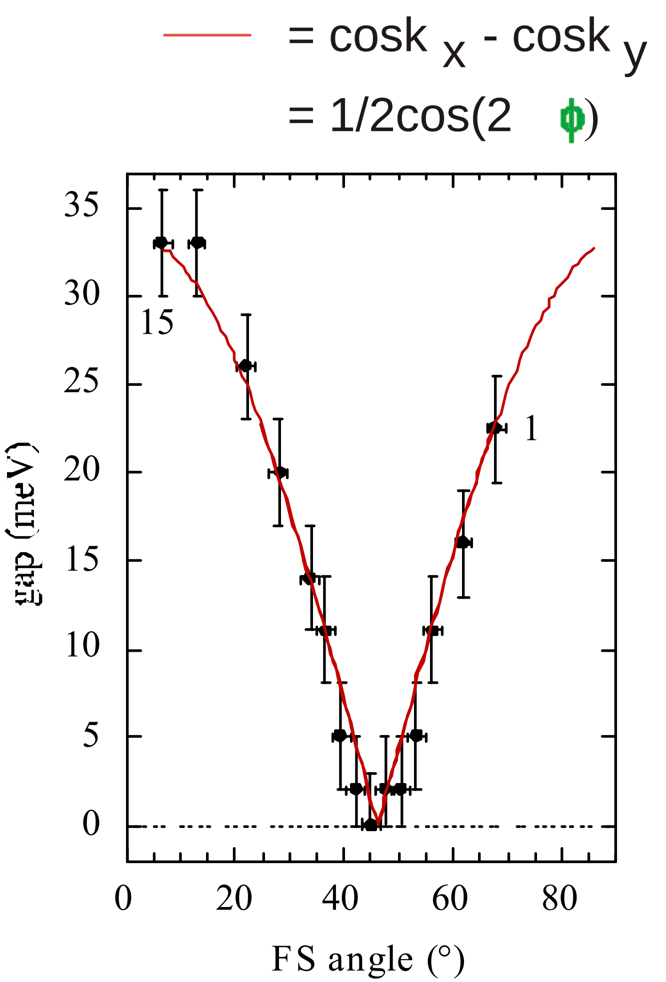
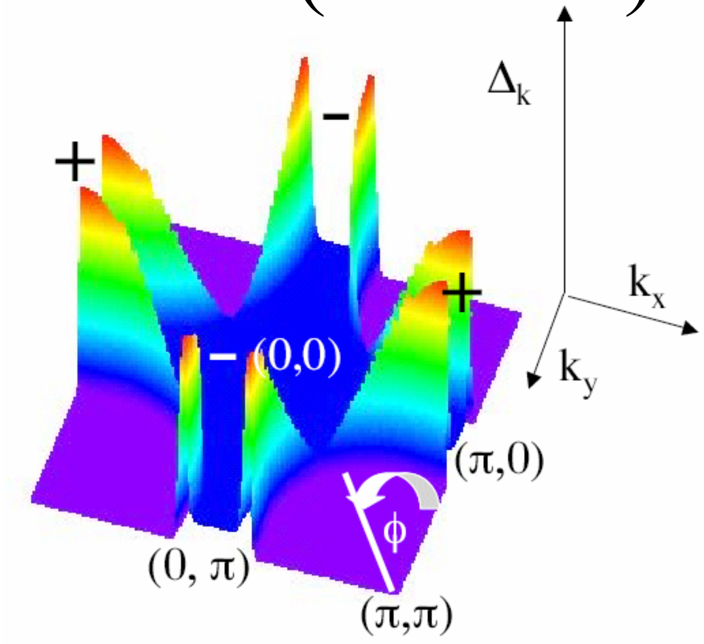
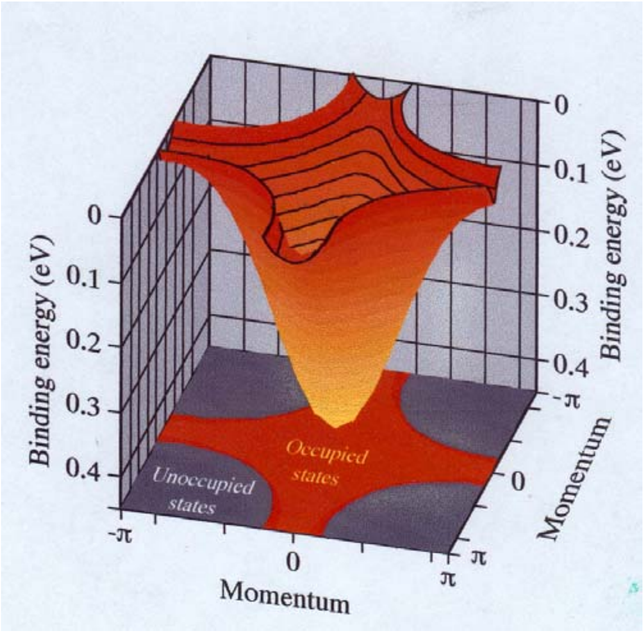
Angle Resolved Photoemission (ARPES)







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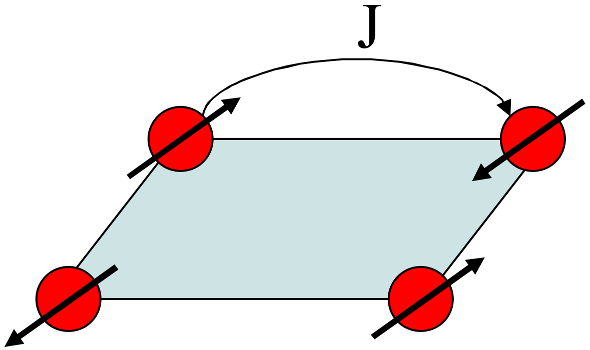
Can we demonstrate that in HTSC the glue binding the electrons/holes is of magnetic origin?

Magnetism: undoped HTSC

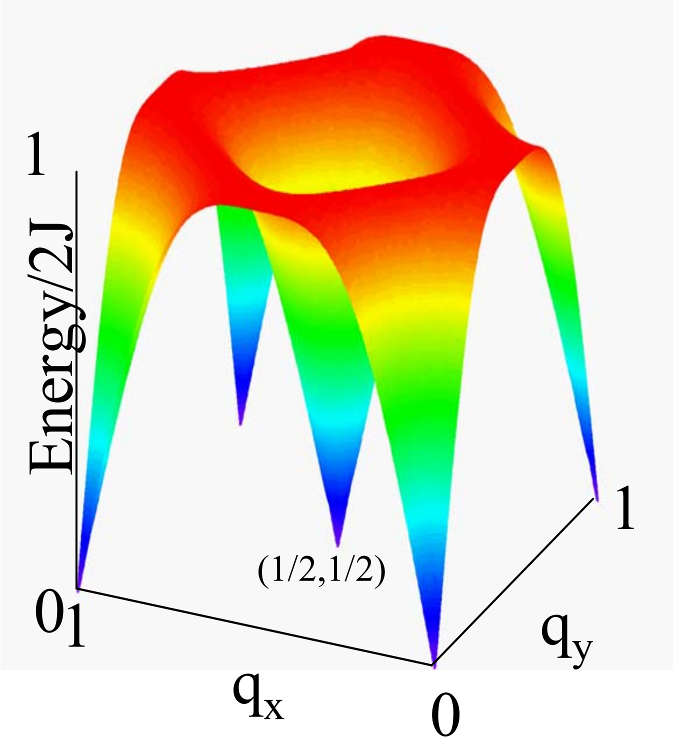
2D-square lattice ---> Heisenberg Hamiltonian $$H = \sum_{\langle ij \rangle} J\mathbf{S}_i\mathbf{S}_j$$



$$\hbar\omega(\mathbf{q}) = 2J\left[1 - \gamma^2(\mathbf{q})/4\right]^{1/2}$$

$$\gamma(\mathbf{q}) = \cos(q_x a) + \cos(q_y a)$$

Kittel, Quantum Theory of Solids (Wiley, NY, 1963)



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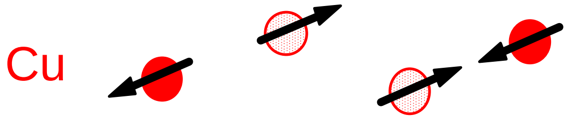
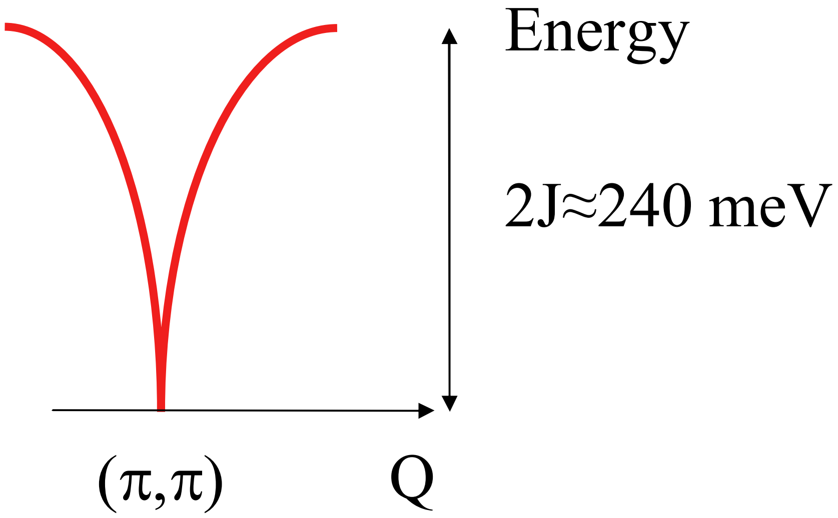
Doped Cuprates

Undoped

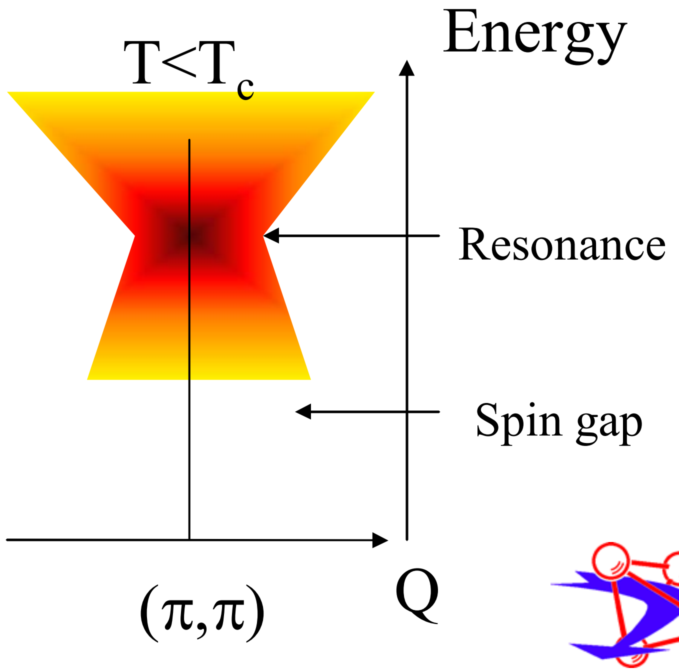


Ordre AF, **k**=(π, π)=(1/2,1/2)



Doped

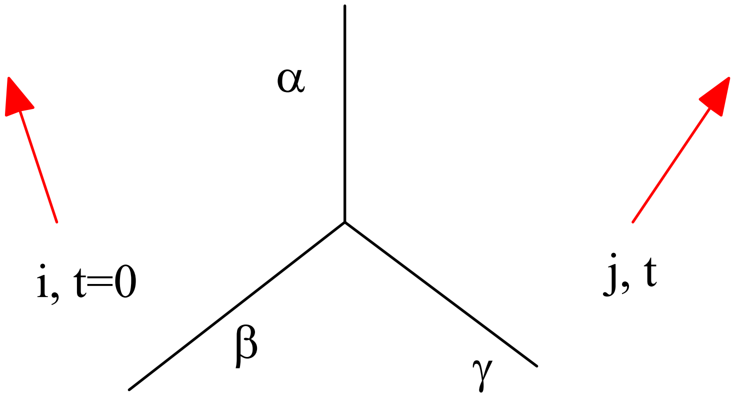
Fluctuations centered around (π,π)



Inelastic Neutron Scattering: Magnetic Cross-Section

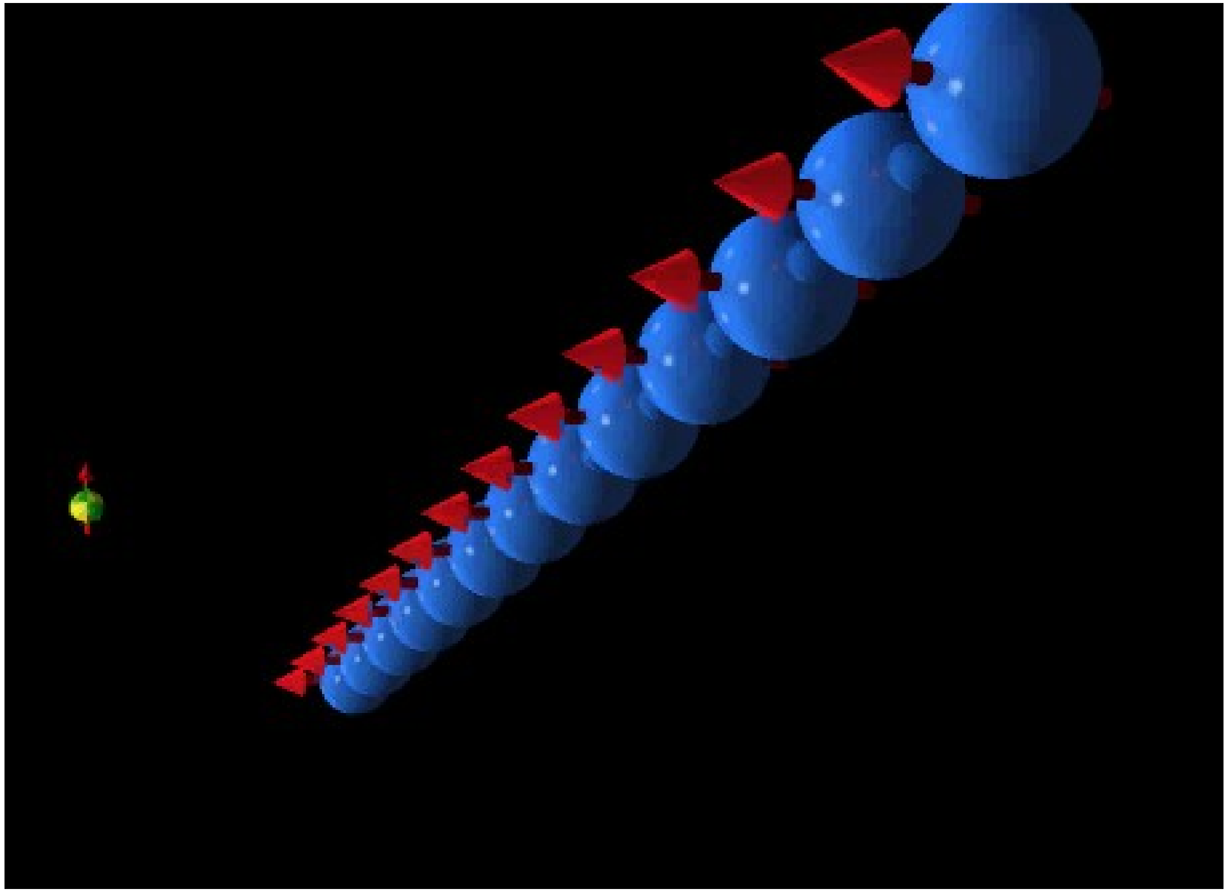
$$I(\mathbf{Q},\omega) = r_0^2 \frac{k_f}{k_i} f^2(\mathbf{Q}) e^{-2W(\mathbf{Q})} \sum_{\alpha\beta}\left(\delta_{\alpha\beta} - \frac{Q_\alpha Q_\beta}{Q^2}\right) \underline{S_{\alpha\beta}(\mathbf{Q},\omega)}$$

$$\cancel{S_{\alpha\beta}(\mathbf{Q},\omega)} \propto \sum_{i,j}\int dt\, e^{i\left(\mathbf{Q}\mathbf{R}_{ij}-\omega t\right)}\left\langle S_i^{\alpha} S_j^{\beta}(t)\right\rangle$$



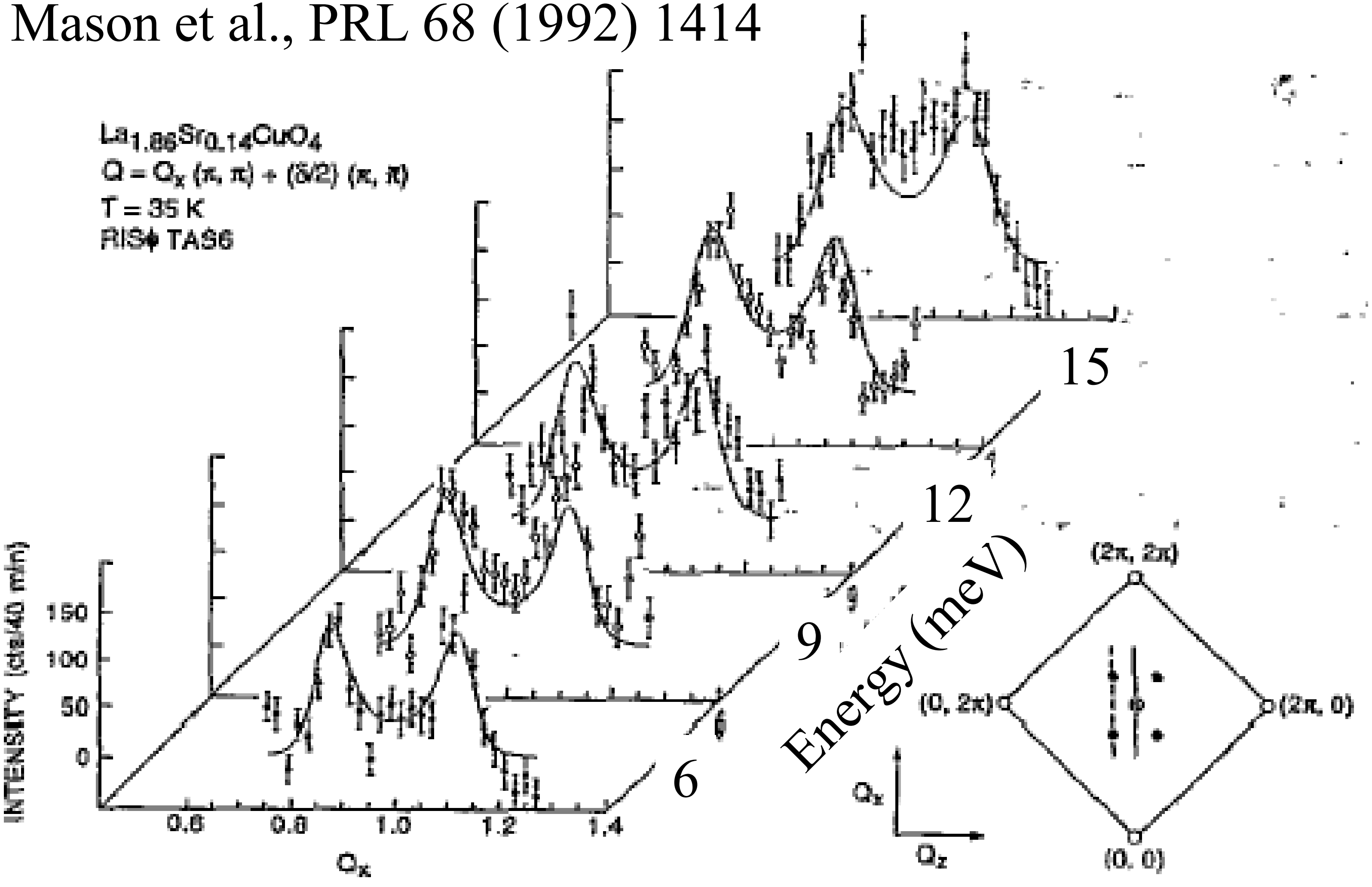
Fluctuation-dissipation theorem

$$S_{\alpha\beta}(\mathbf{Q},\omega) = \frac{1+n(\omega)}{\pi\left(\gamma\mu_B\right)^2}\chi''_{\alpha\beta}(\mathbf{Q},\omega)$$



$La_{1.86}Sr_{0.14}CuO_4$ (normal state)

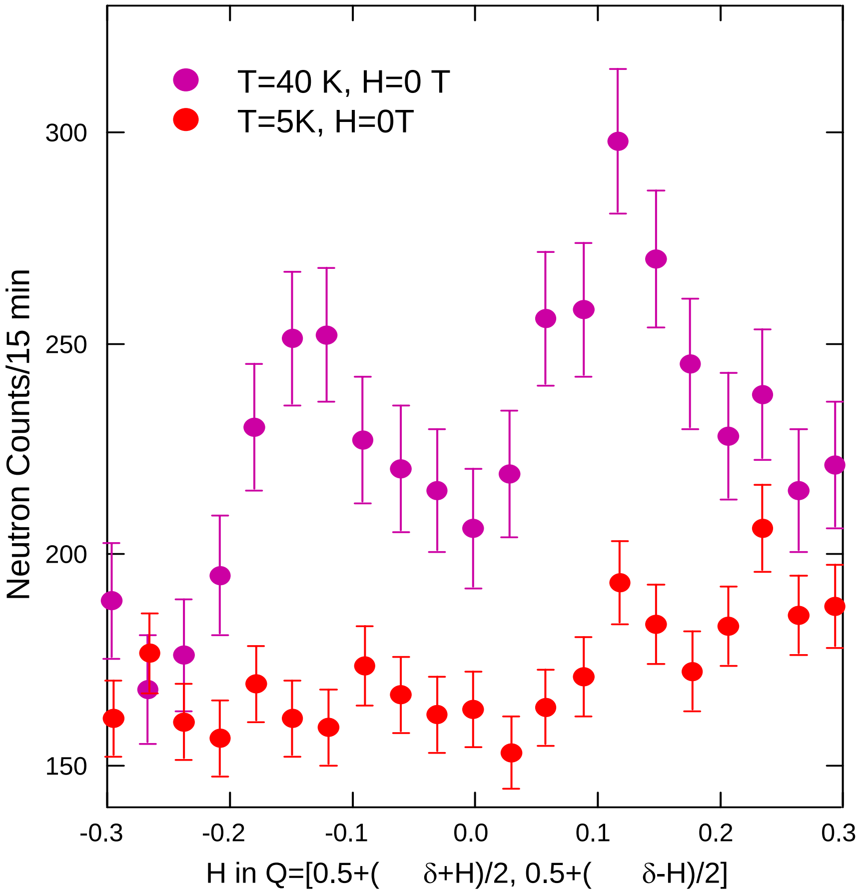
Mason et al., PRL 68 (1992) 1414

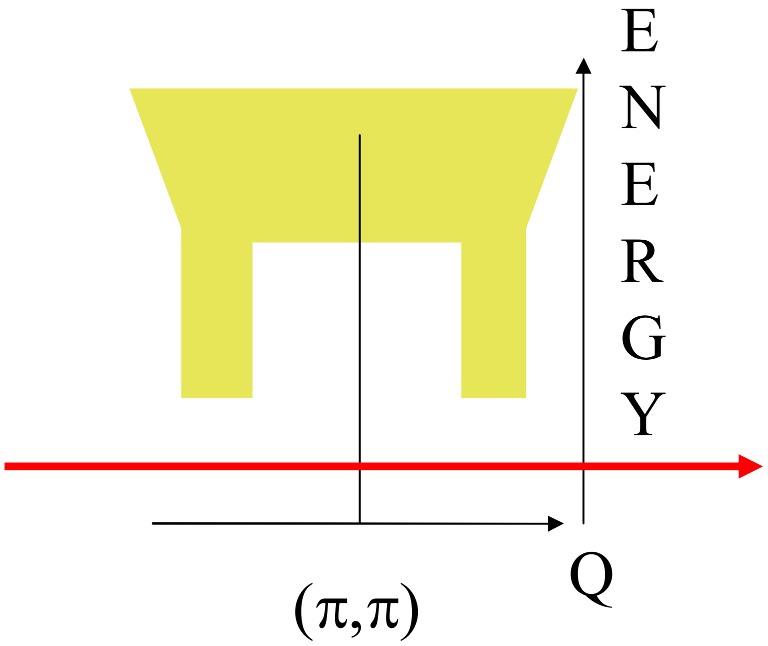


Q_x

Overdoped LSCO (x=0.17, T_c=37 K)

Q-scans, ΔE=4 meV





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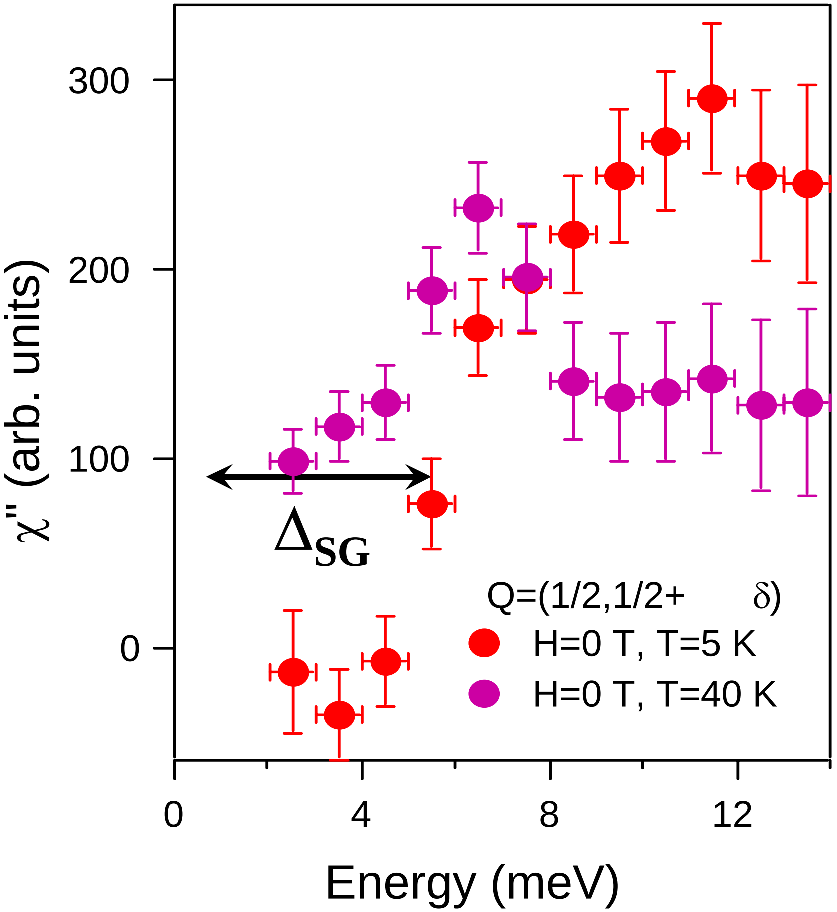
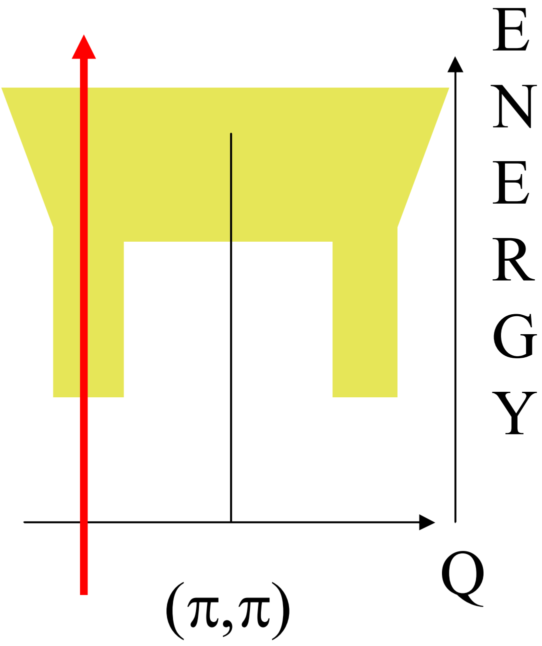
Overdoped LSCO (x=0.17, T_c=37 K)

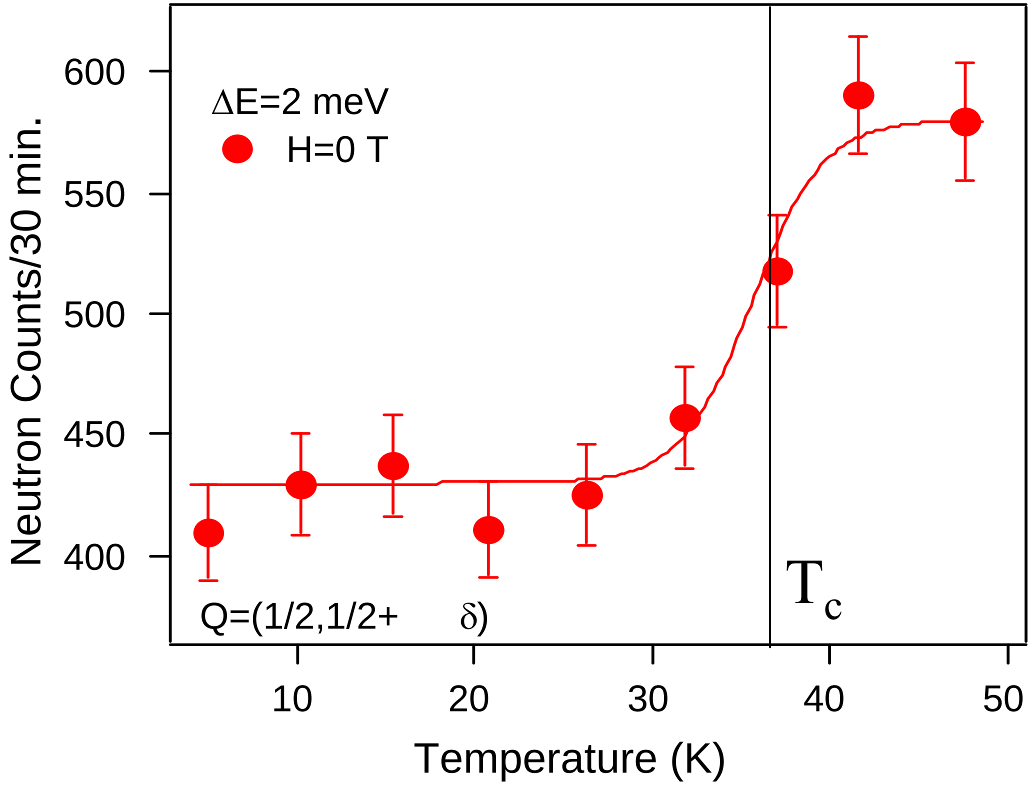
E-scans, Q=(1/2,1/2+δ)

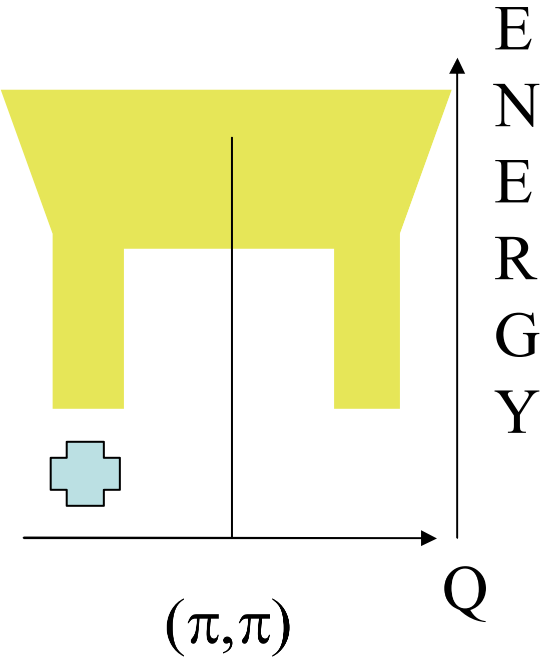




Spin gap: temperature scans at ΔE=2 meV

Overdoped (x=0.17, T_c=37 K)





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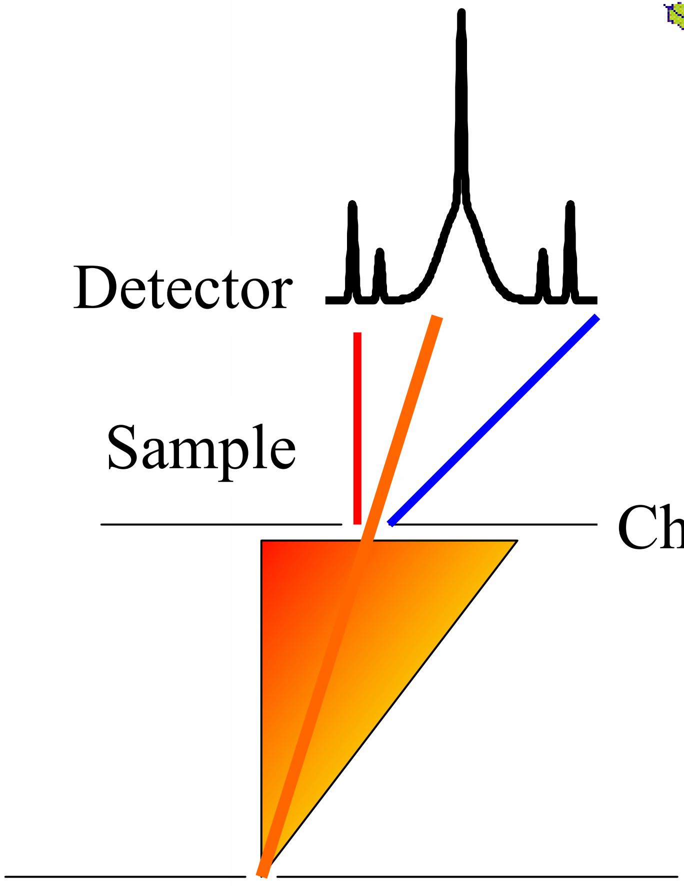
MAPS @ ISIS -RAL

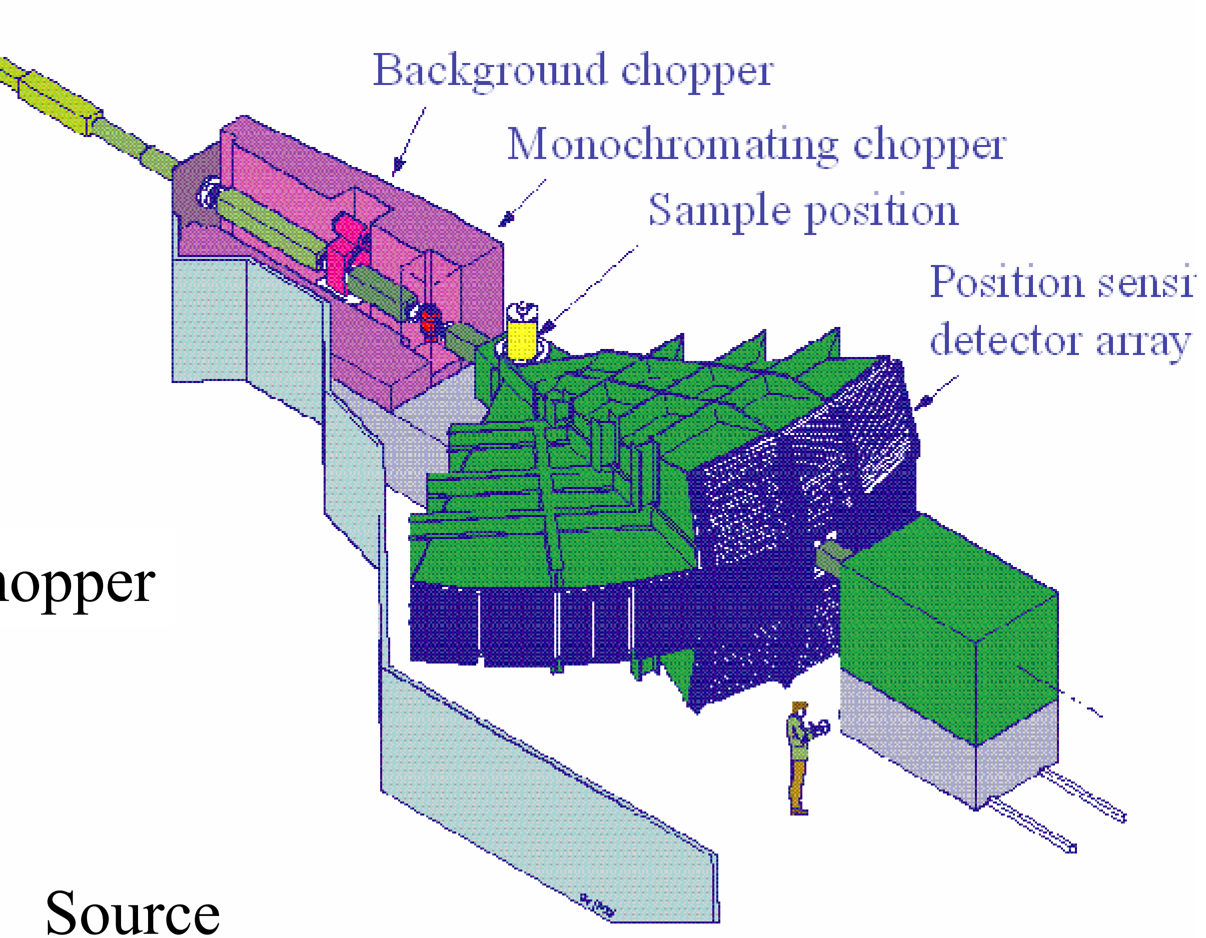


Detector

Sample

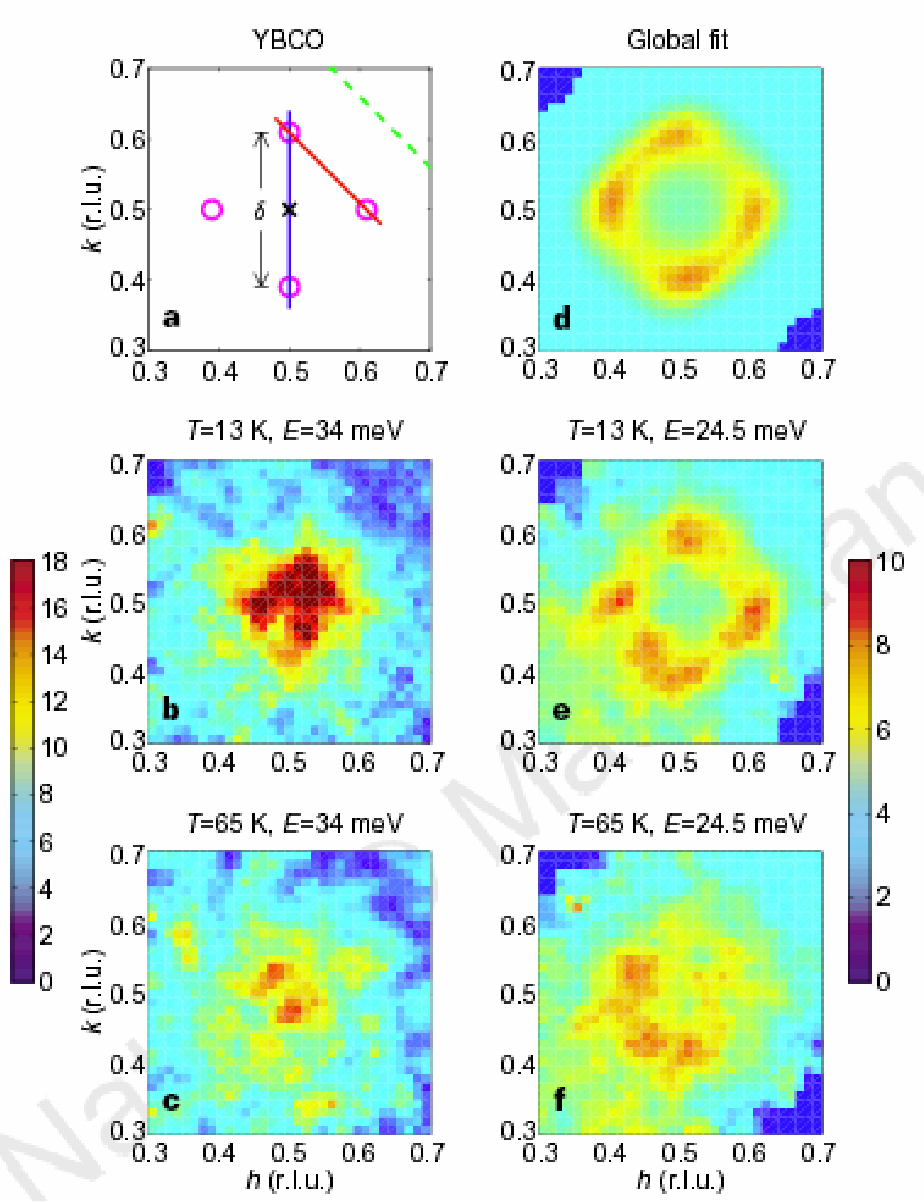
Chopper

Source



Y123 x=6.6 / TOF

Mook *et al.*,
Nature **395** (1998) 580



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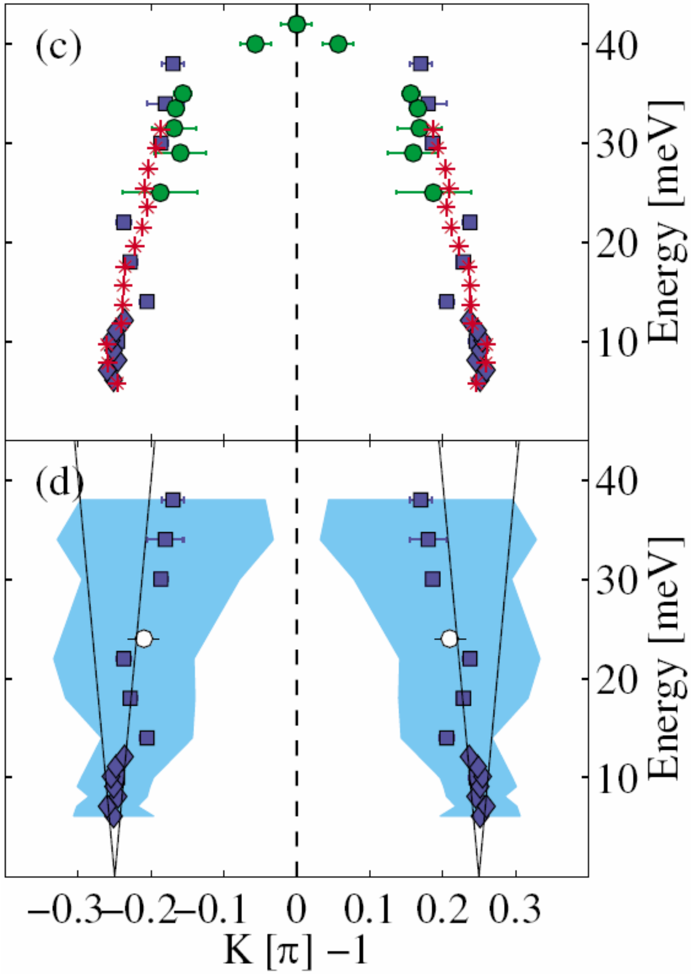
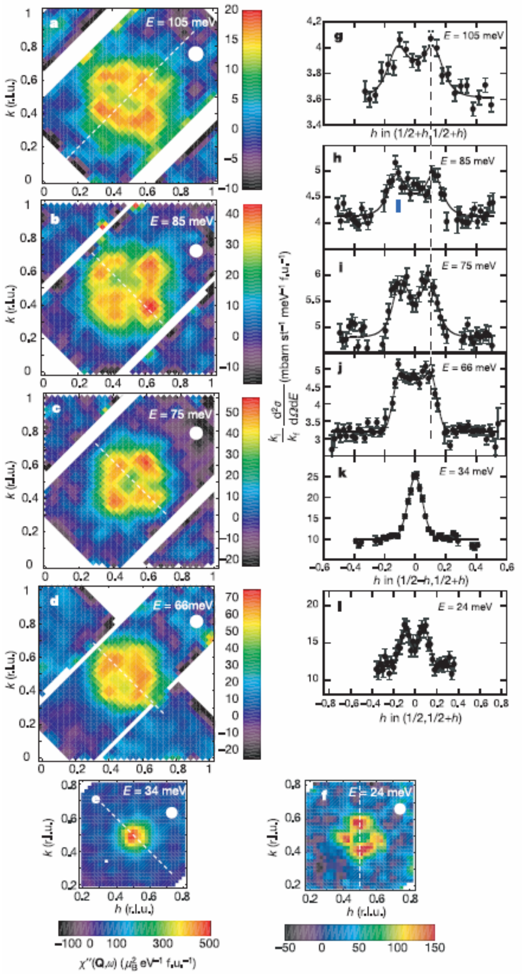
High-Energy (2004)

Opt LSCO /YBCO



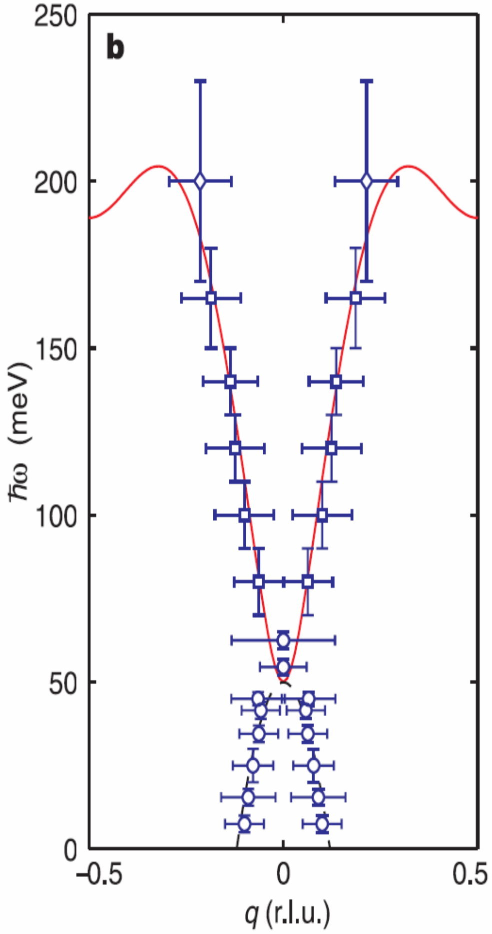
Christensen, *et al.* Phys. Rev. Lett.

Underdoped YBCO



Hayden *et al.* Nature

non-SC LBCO



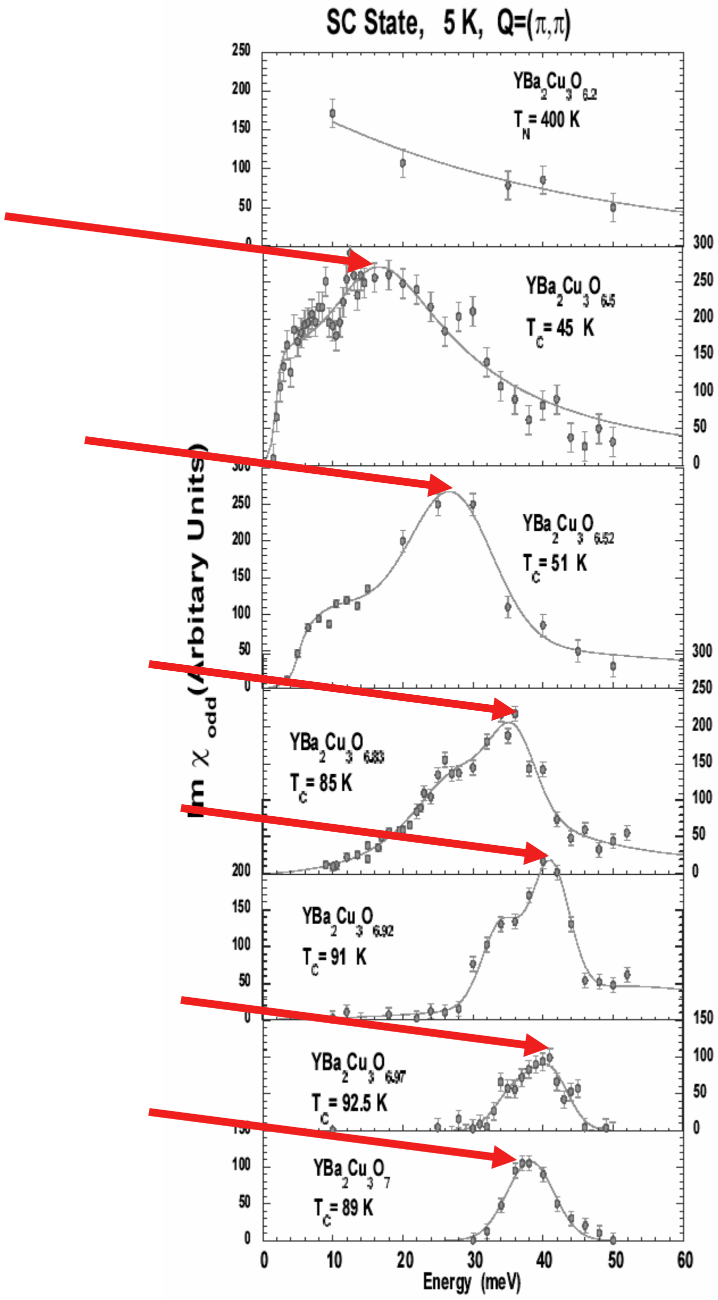
Tranquada *et al.* Nature

J. Mesot, 07

Resonance:
doping dependence

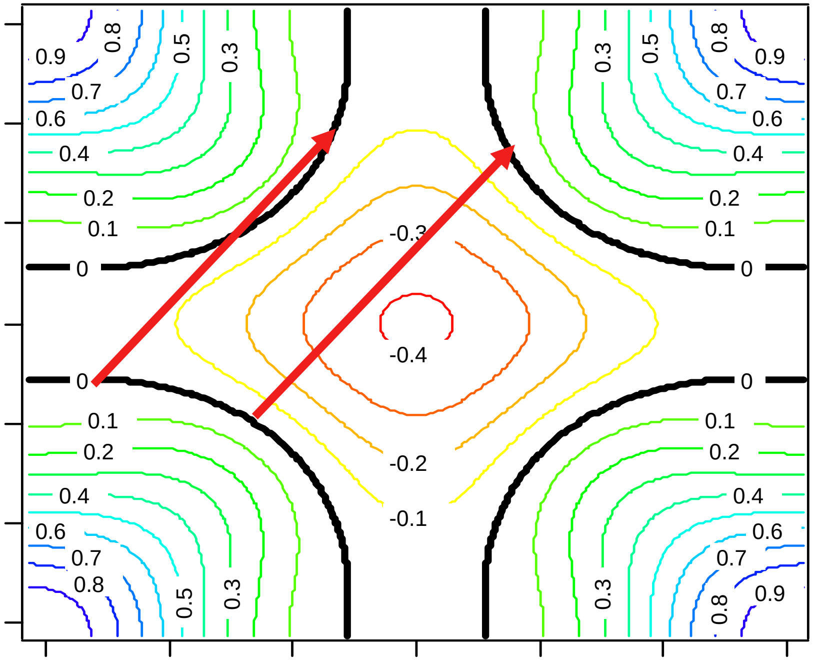
Ph. Bourges cond/mat (9901333)

$YBa_2Cu_3O_{6+x}$

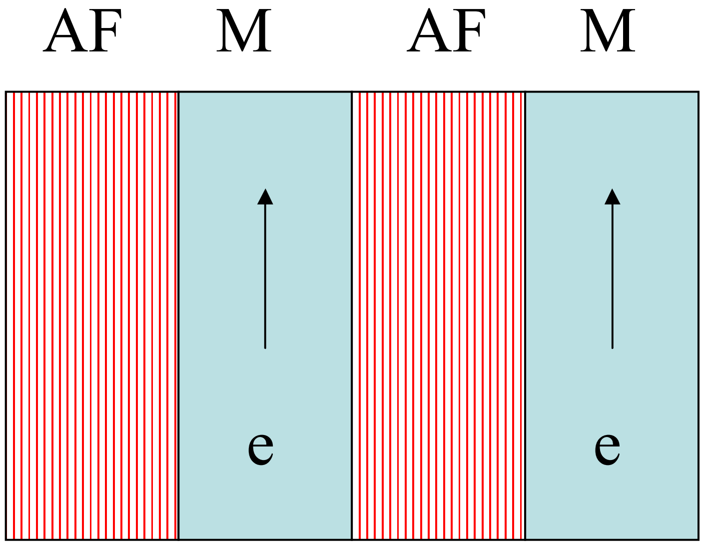


What is the nature of the spin excitations?

Fermi nesting



Stripes



Fermi Nesting Scenario

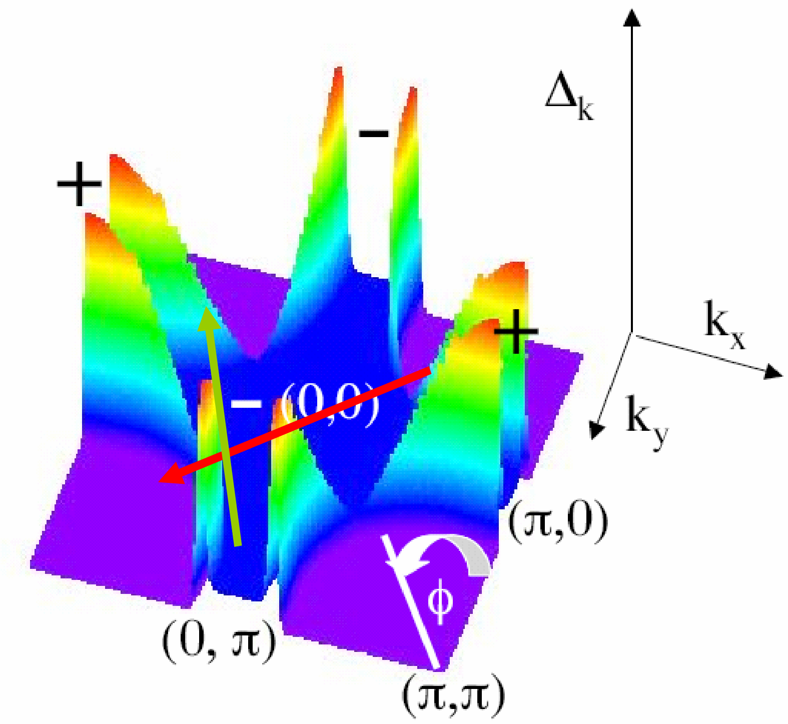
$$\chi_0(q,\omega)=\sum_k \left(1-\underbrace{\frac{\varepsilon_k\varepsilon_{k+q}+\Delta_k\Delta_{k+q}}{E_kE_{k+q}}}_{}\right)\left(\frac{f\left(E_{k+q}\right)+f\left(E_k\right)-1}{\omega-\left(E_{k+q}+E_k\right)+i\delta}\right)$$

$$E_k=\sqrt{\varepsilon_k^2+\Delta_k^2}$$

$\varepsilon_k=0$ --> $1-\mathrm{sign}(\Delta_k)\,\mathrm{sign}(\Delta_{k+q})$

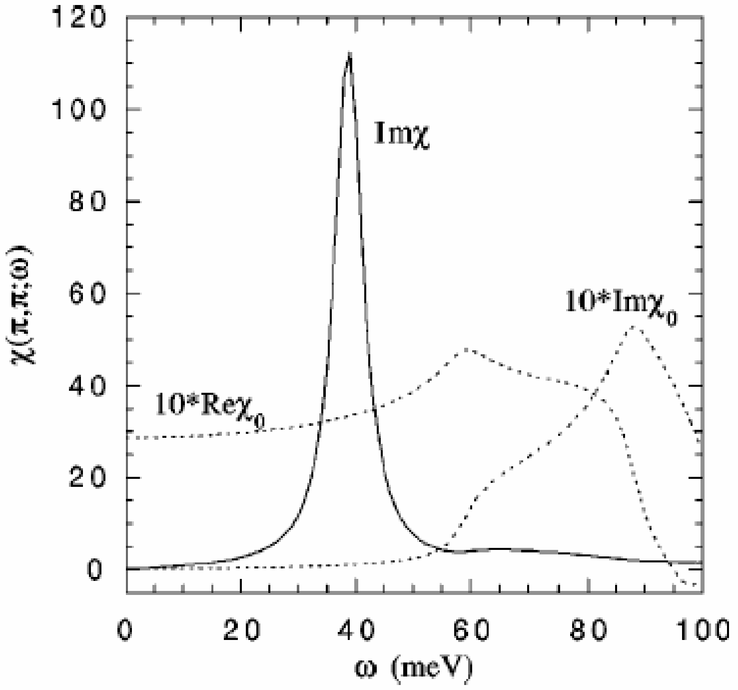
$q=(\pi,\pi)$--> 0 if Δ_k even (s)
2 if Δ_k odd (d)

$q=(\pi,\pi)$ ---> $q=(\pi+\delta,\pi)$



Renormalized Susceptibility

M. Lavagna PRB **49** (94) 4235, D. Z. Liu, PRL **75** (95) 4130, N. Bulut, PRB **53** (96) 5149, J. Brinckmann, PRL **82** (99) 2915, Norman, PRB **61** (00) 14751



Norman, PRB **61** (00) 14751

+interactions (RPA)

$$\chi(q,\omega)=\frac{\chi_0(q,\omega)}{1-J(q)\,\chi_0(q,\omega)}$$

$$J(q)=J(\cos(q_x a)+\cos(q_y a))/2$$

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Fermi nesting approach:

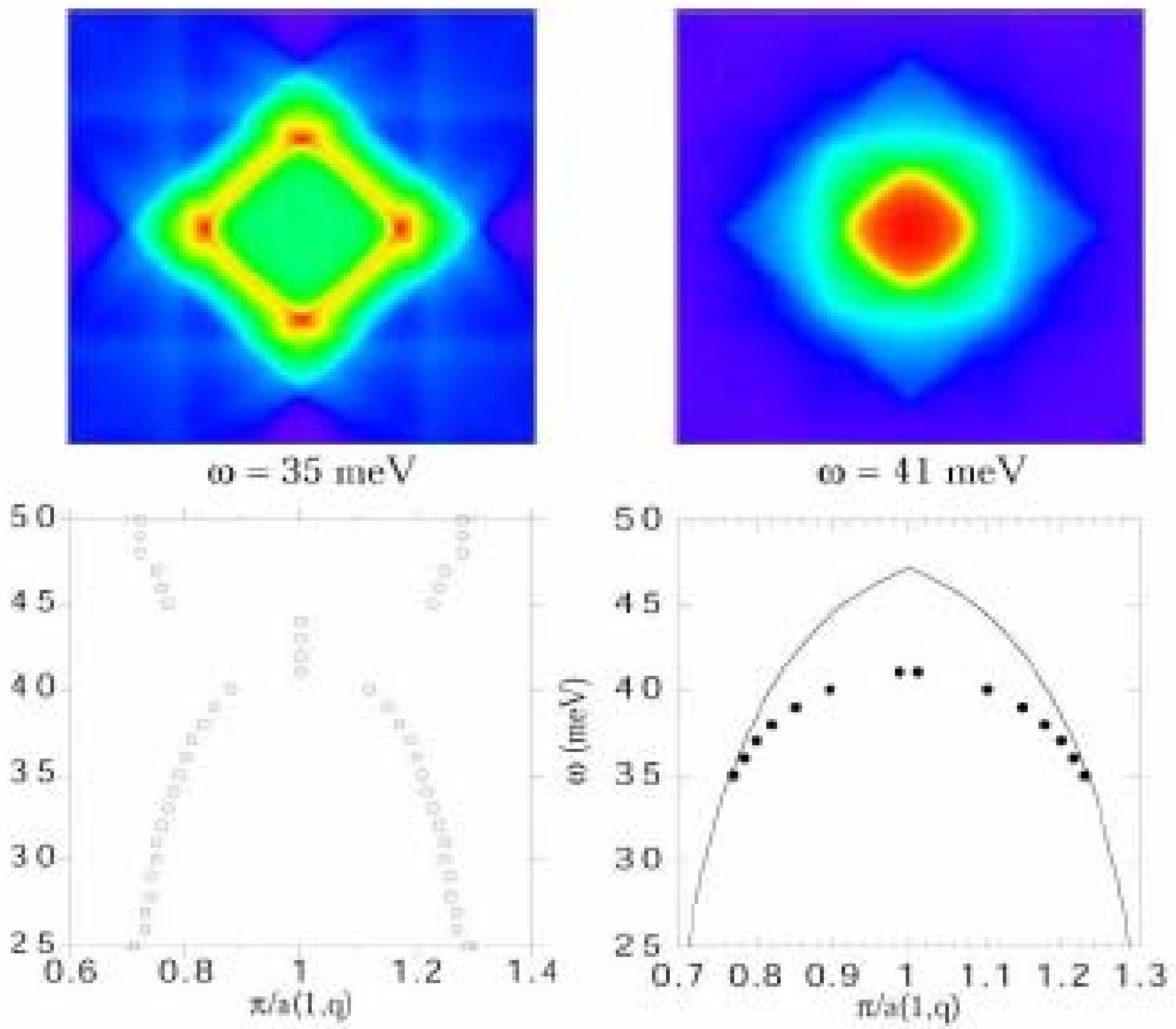
YBCO

Norman, Pépin

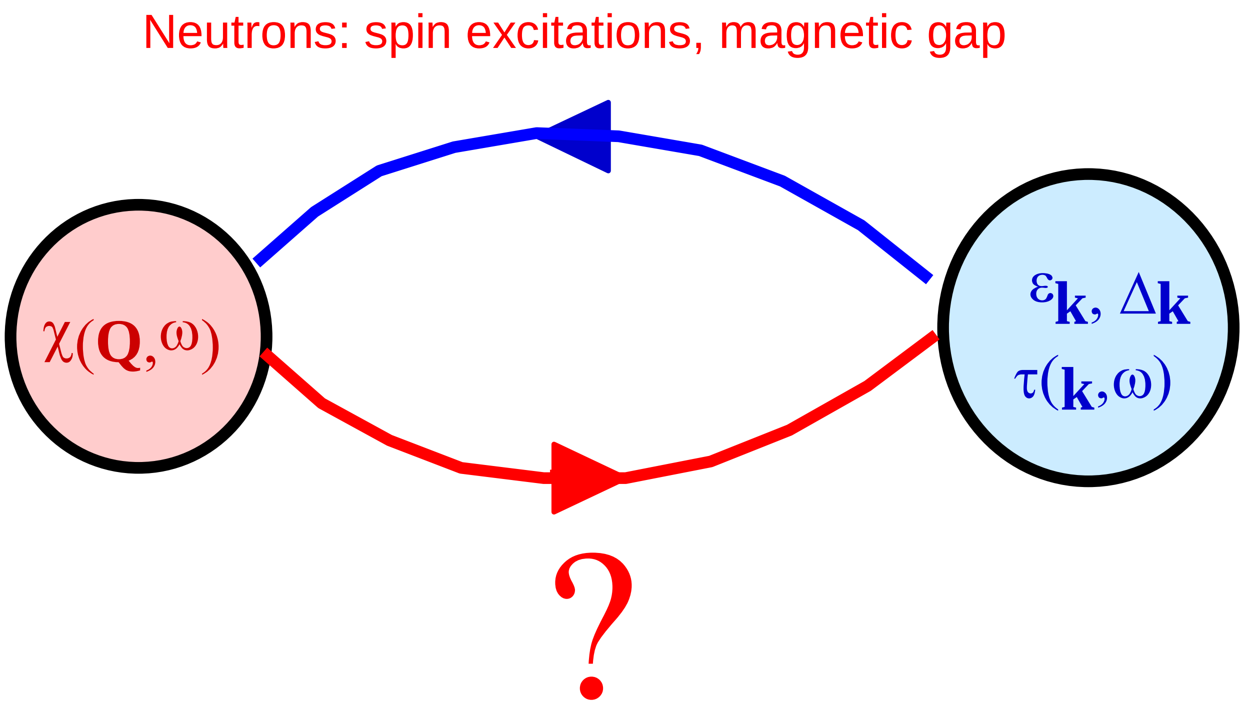
Cond-mat/0302347

Schneider et al.

Phys. Rev. B. 2004



Is magnetism relevant for HTSC?



Magnetism

Neutrons: spin excitations, magnetic gap

$\chi(\mathbf{Q},\omega)$

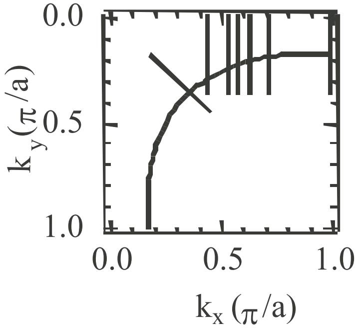
$\varepsilon_{\mathbf{k}}, \Delta_{\mathbf{k}}$

$\tau(\mathbf{k},\omega)$

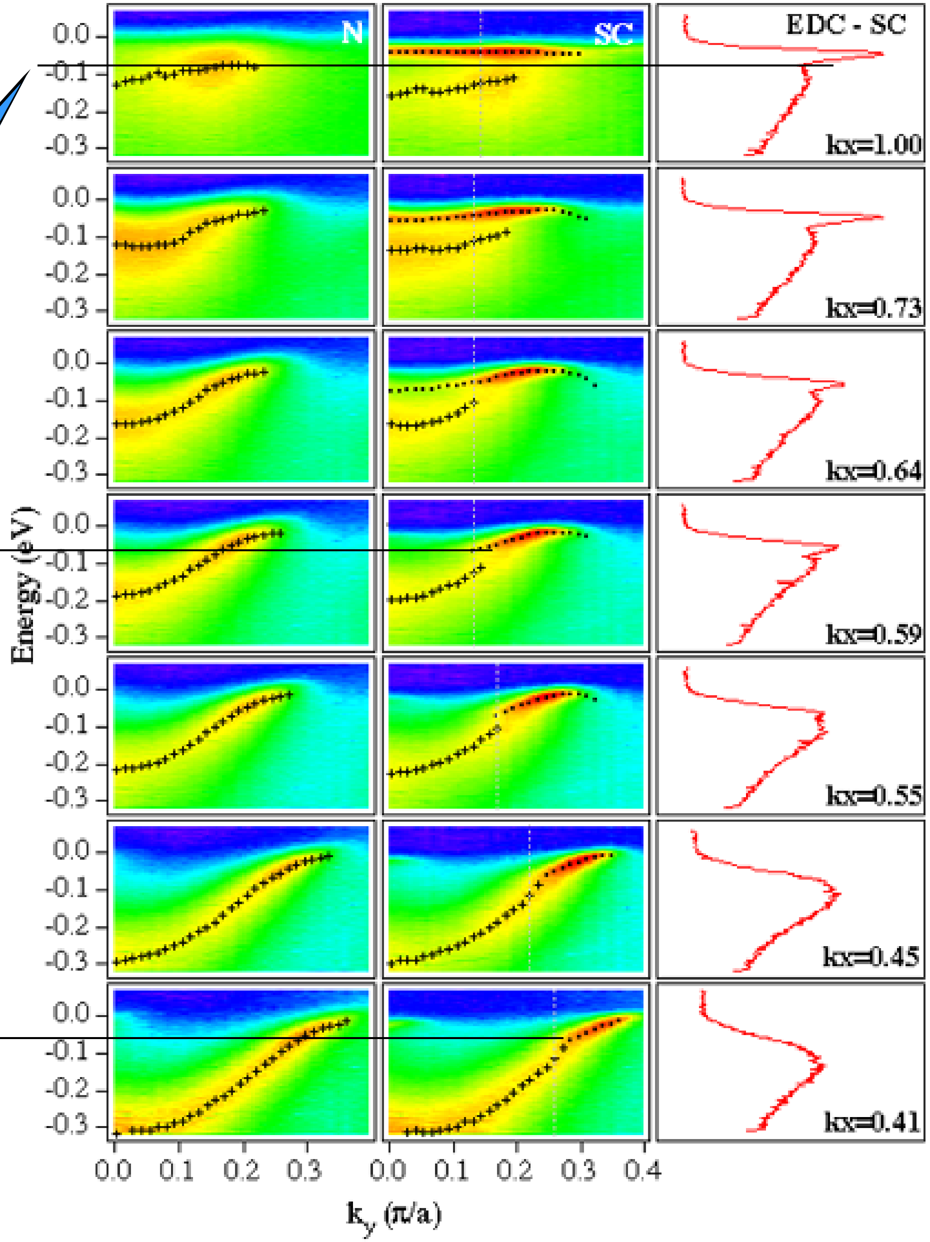
?

Transport

Electronic renormalization (ARPES)?



The effect is present
at all k-points



Bi2212 Opt

J. Mesot, 07

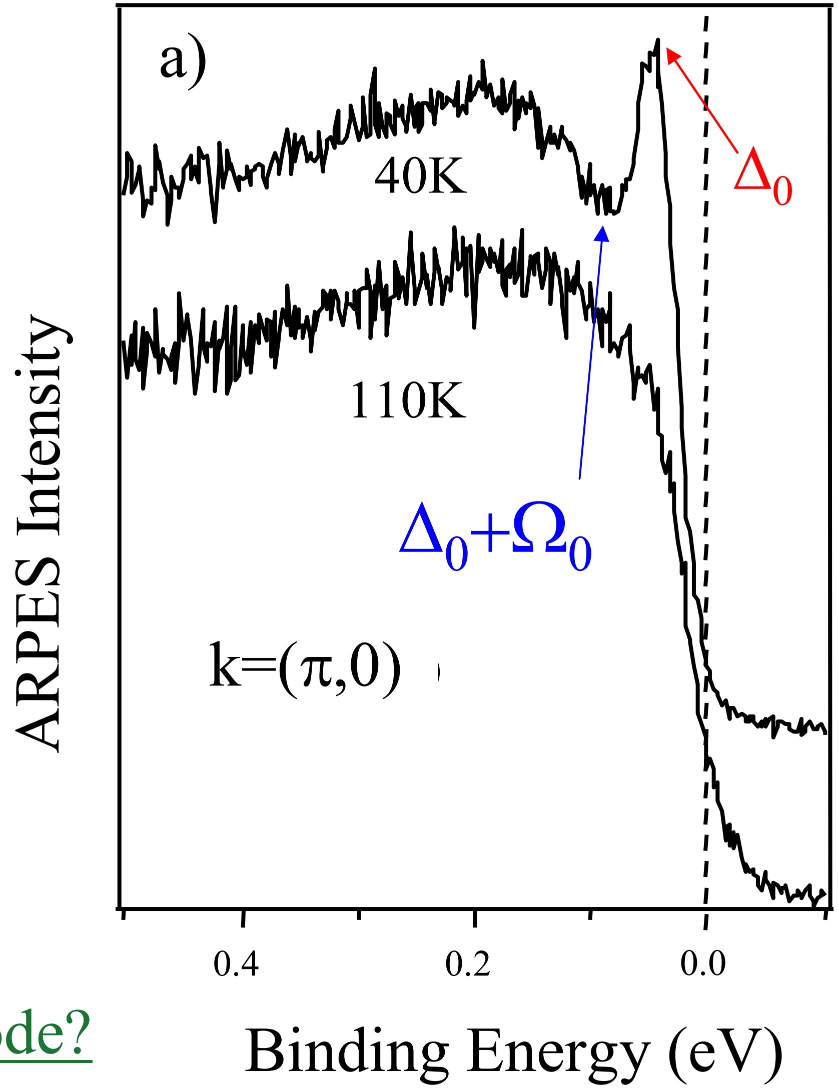
Origin of the electronic renormalization?

Interaction of the electrons with a collective mode of energy $\Omega_0 < 2\Delta$

-Norman *et al.* PRL 79 (1997) 3506
-Abanov *et al.* PRL 83 (1999) 1652
-Dahm *et al.* PRB 58 (1999) 12454

$$A(\mathbf{k},\omega) = \frac{\operatorname{Im}\Sigma(\mathbf{k},\omega)}{|\omega - E_{\mathbf{k}} - \operatorname{Re}\Sigma(\mathbf{k},\omega)|^2 + |\operatorname{Im}\Sigma(\mathbf{k},\omega)|^2}$$

$$(\text{at } E_F) \rightarrow \frac{1}{|\operatorname{Im}\Sigma(\mathbf{k},\omega)|} \approx \tau\,(\text{life time})$$



What is the nature of the collective mode?

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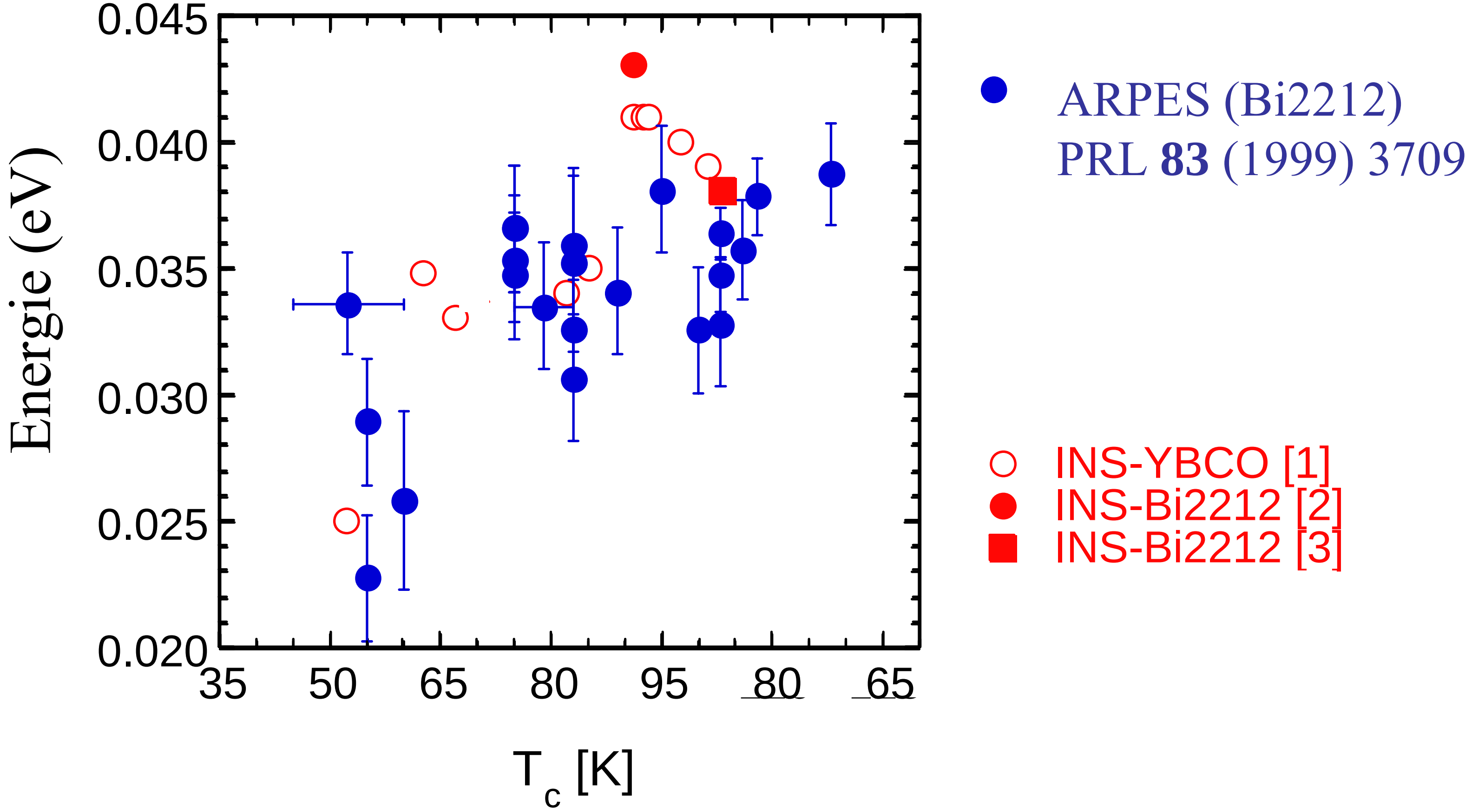
Collective mode = resonance ?



[1] Ph. Bourges, cond-mat/9901333
[2] H. Fong, Nature **398** (99) 588
[3] H. He, cond-mat/0002013

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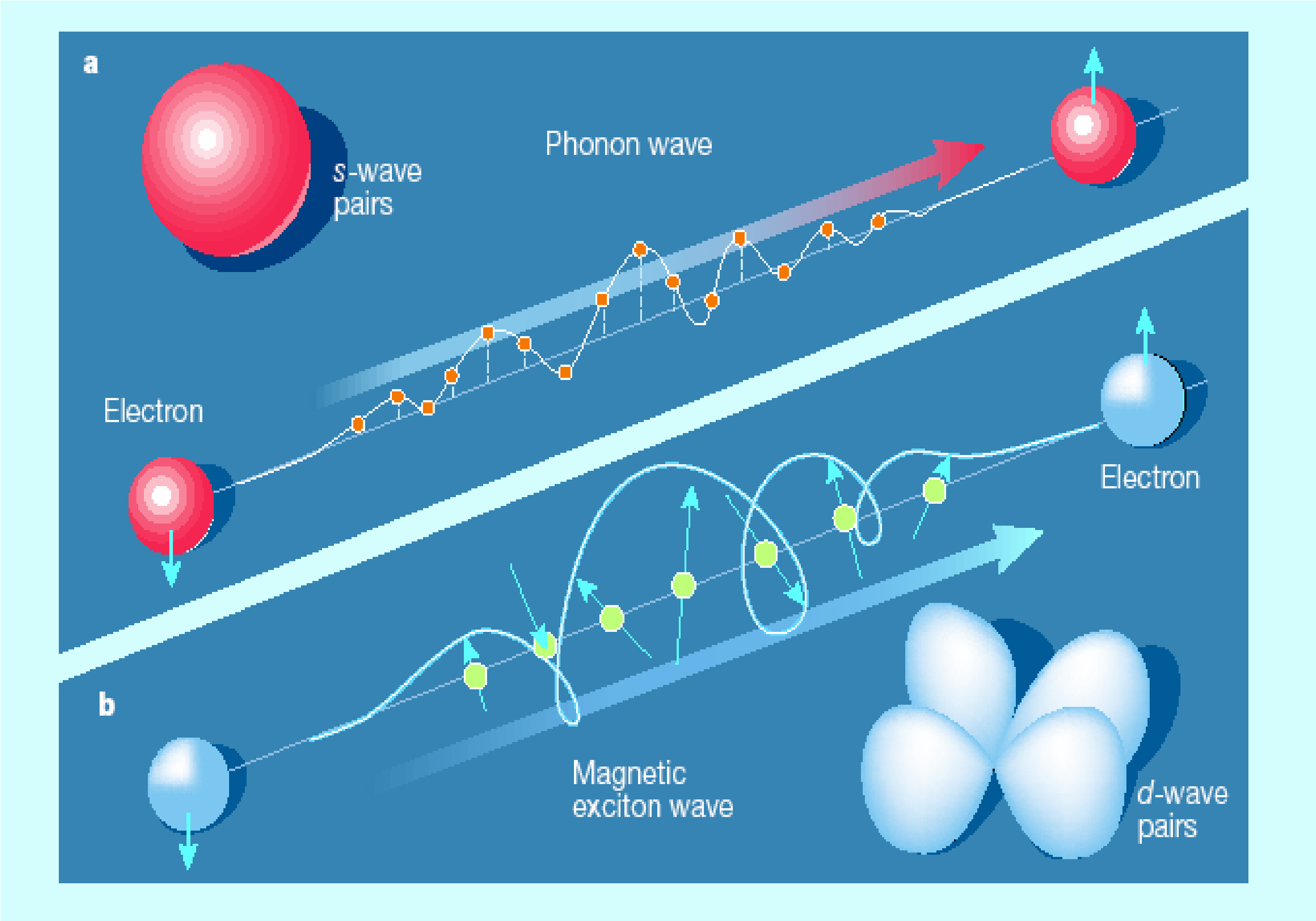
Low- vs high-T_c Superconductors



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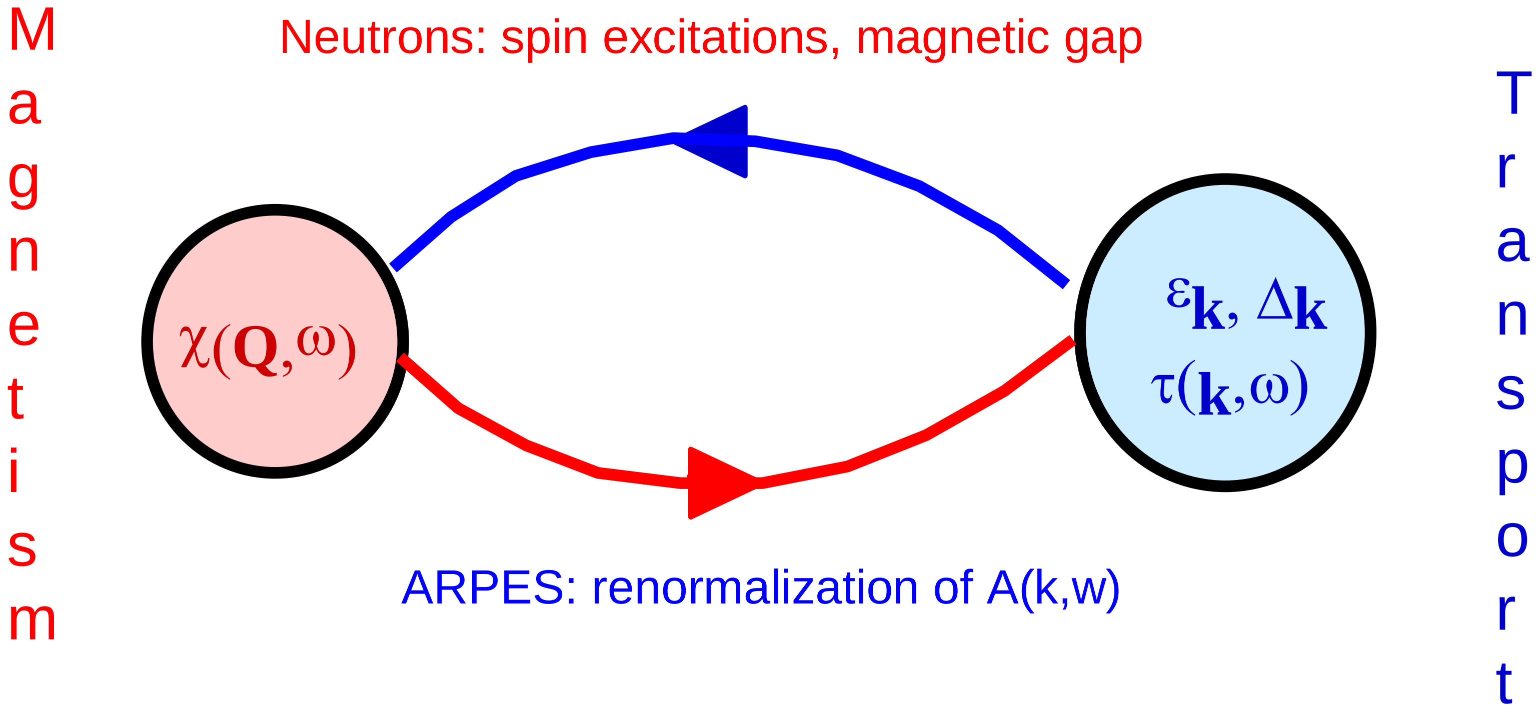
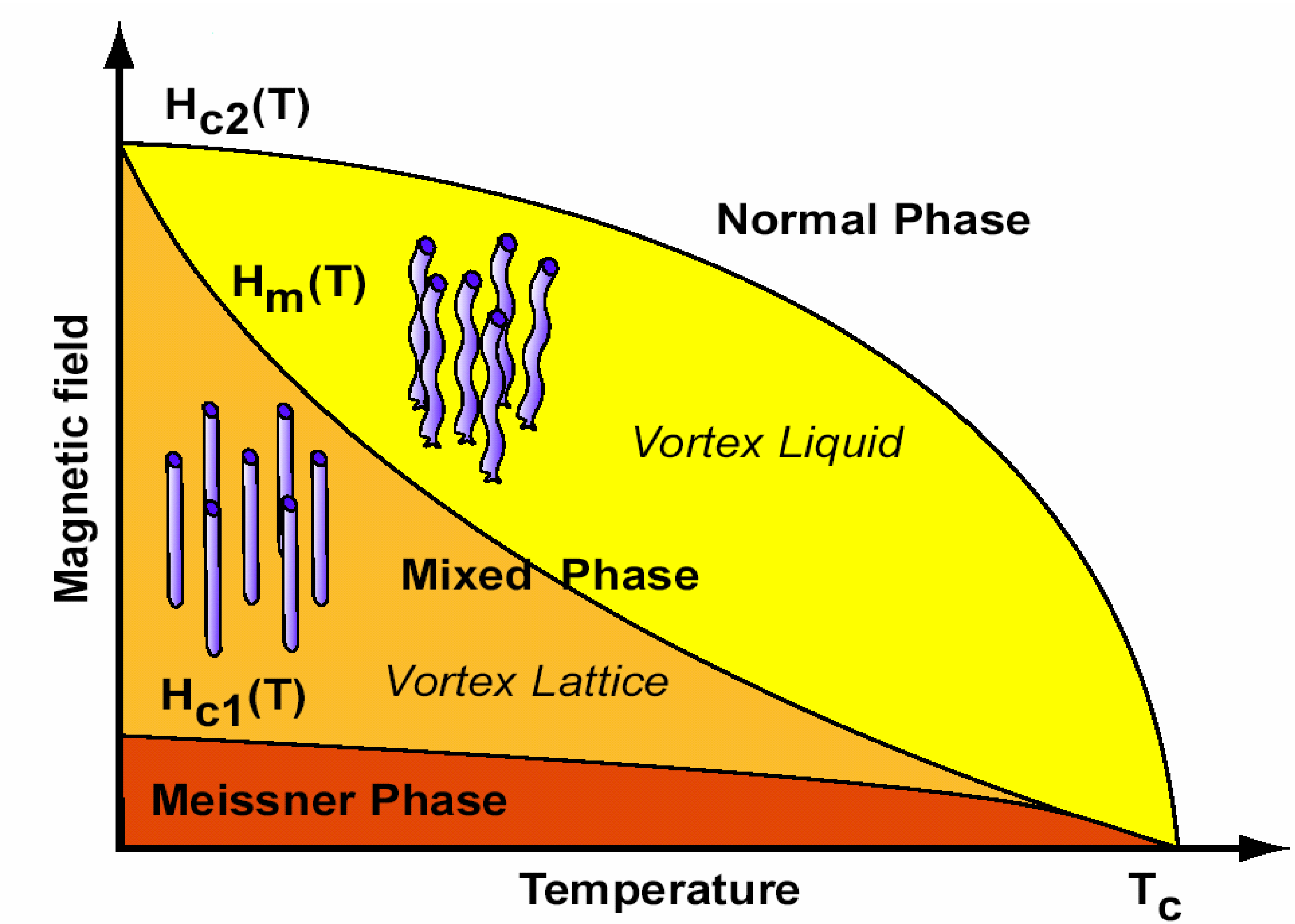
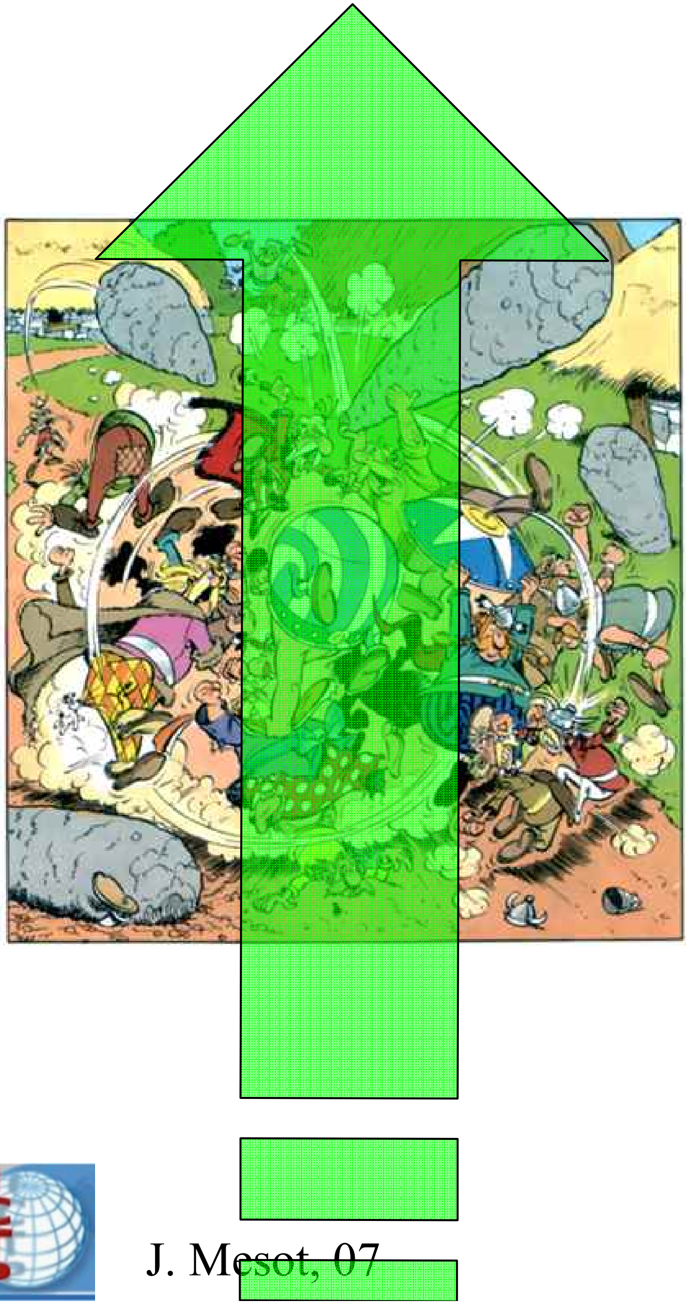
A complex and fascinating problem!

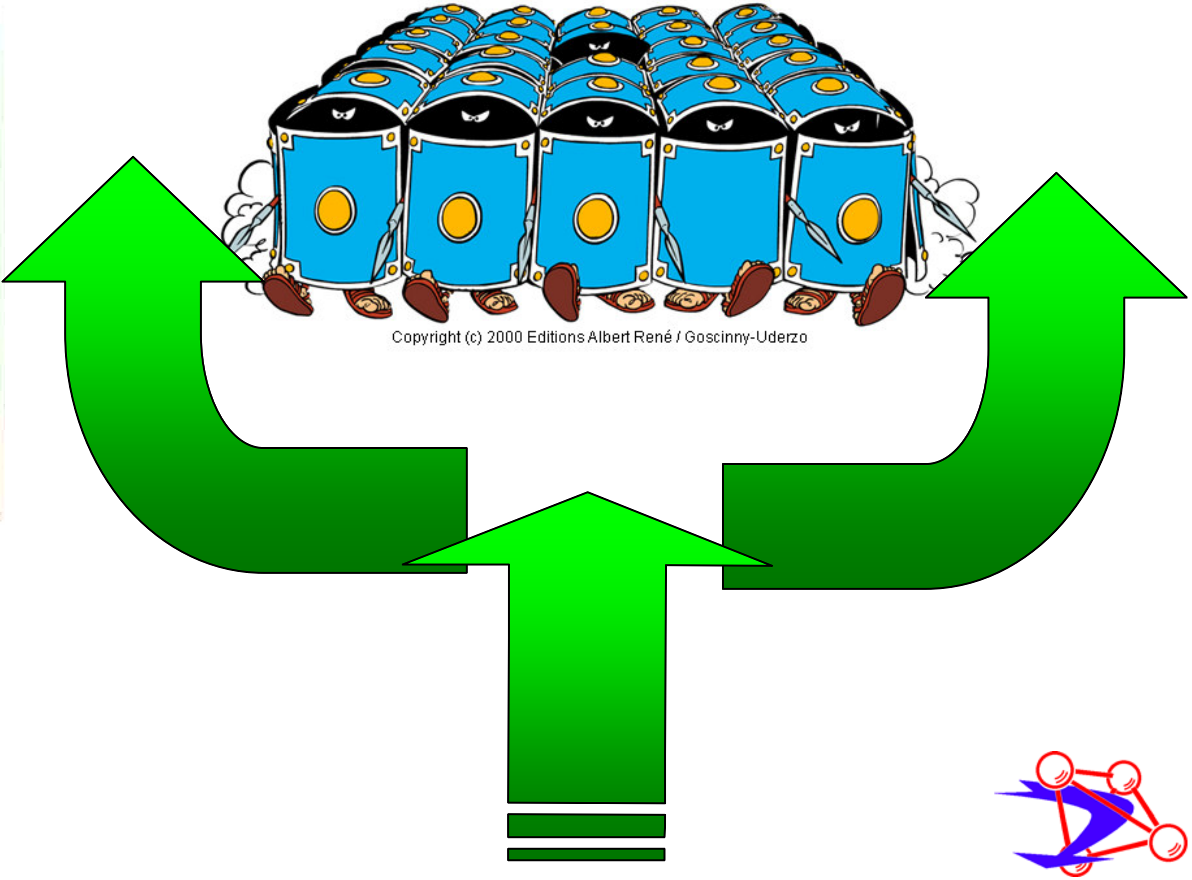


Magnetic Phase Diagram



Meissner effect (magnetic)





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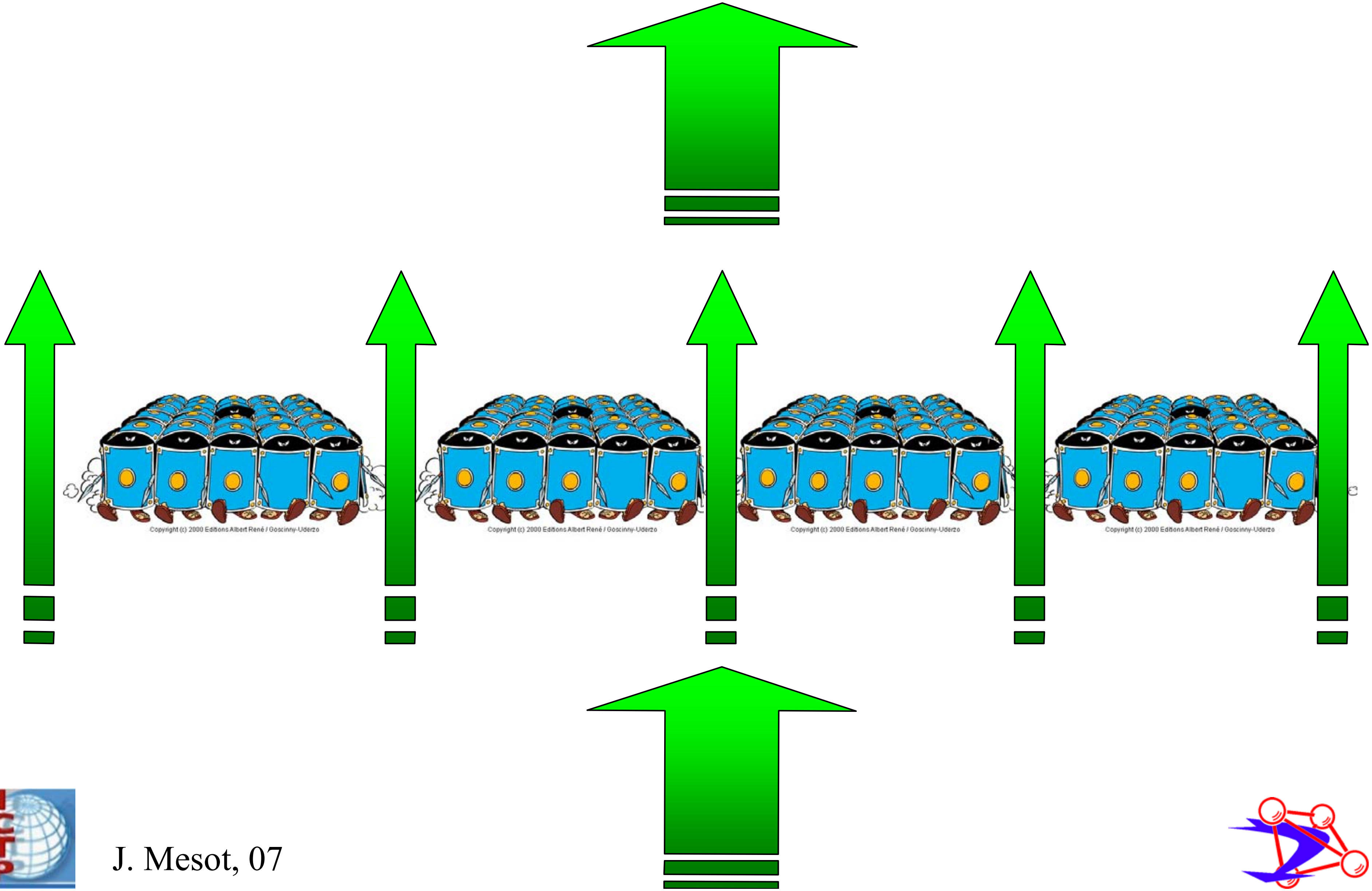
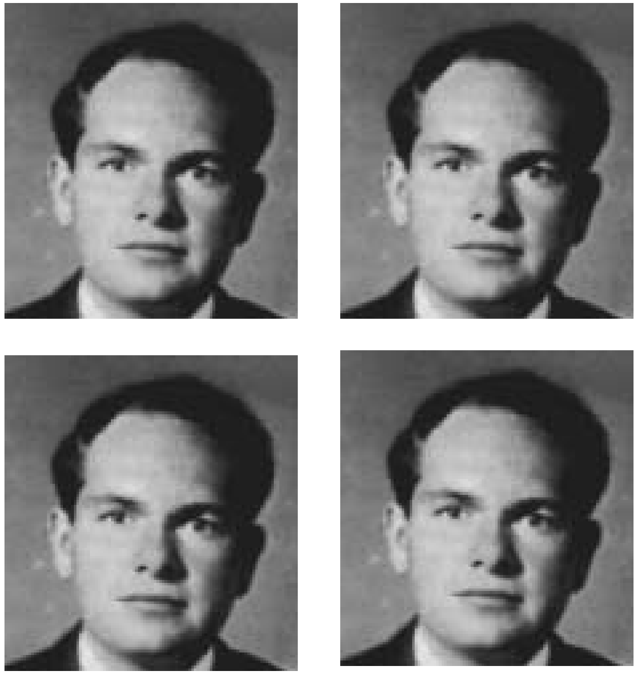
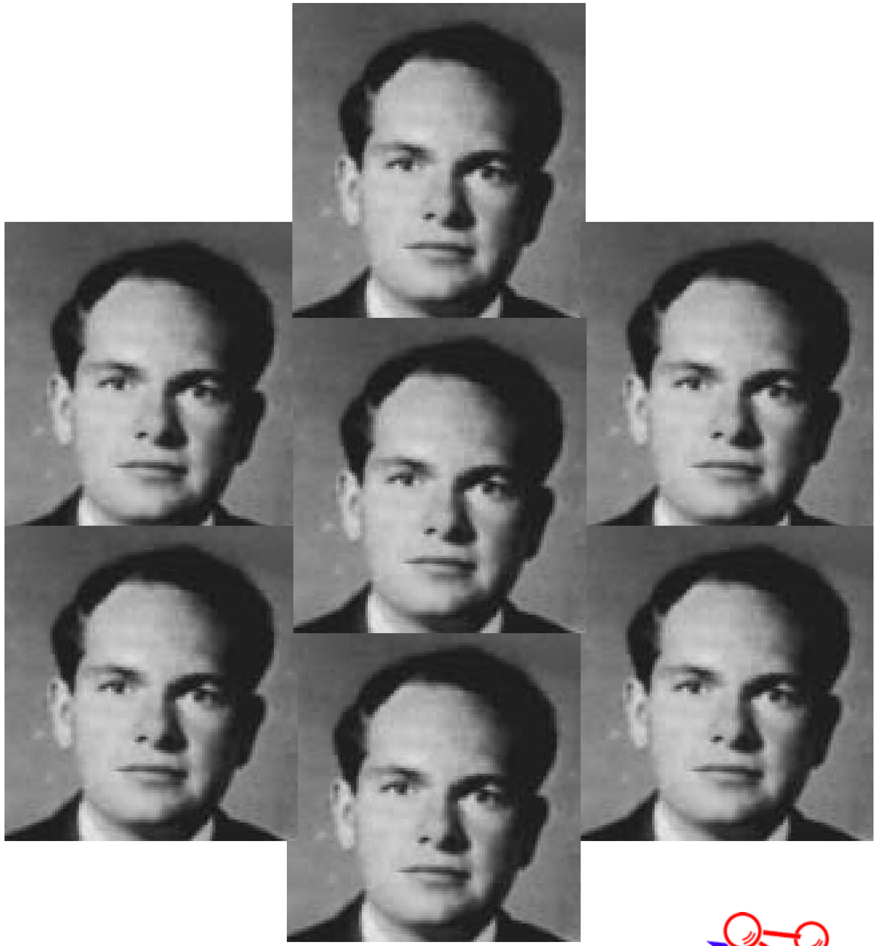
Flux lines (magnetic)



Abrikosov Lattice





Small Angle Neutron Scattering (SANS)

Diffraction of neutrons from flux lines ----> Bragg law: $\lambda=2d\sin(\Theta)$

$$d=\alpha\sqrt{\frac{\Phi_0}{B}}$$

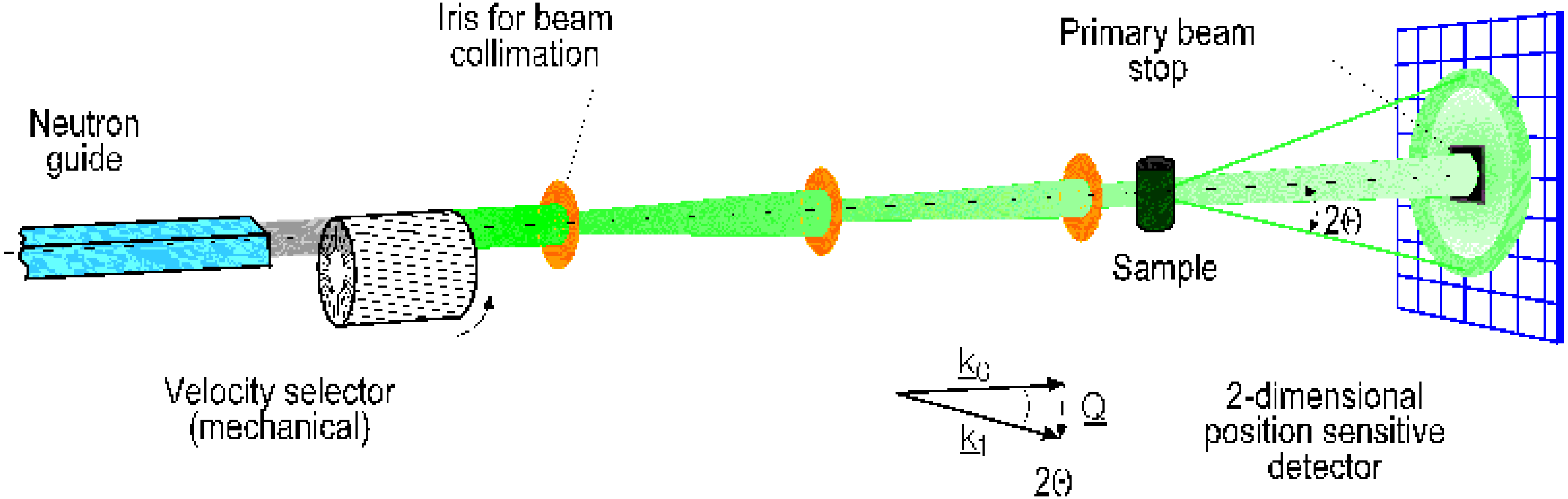
square: $\alpha=1.000$

hexagonal: $\alpha=1.075$

B=1 Tesla -----> d=455 Å

λ=10 Å -----> Θ=0.63 degrees

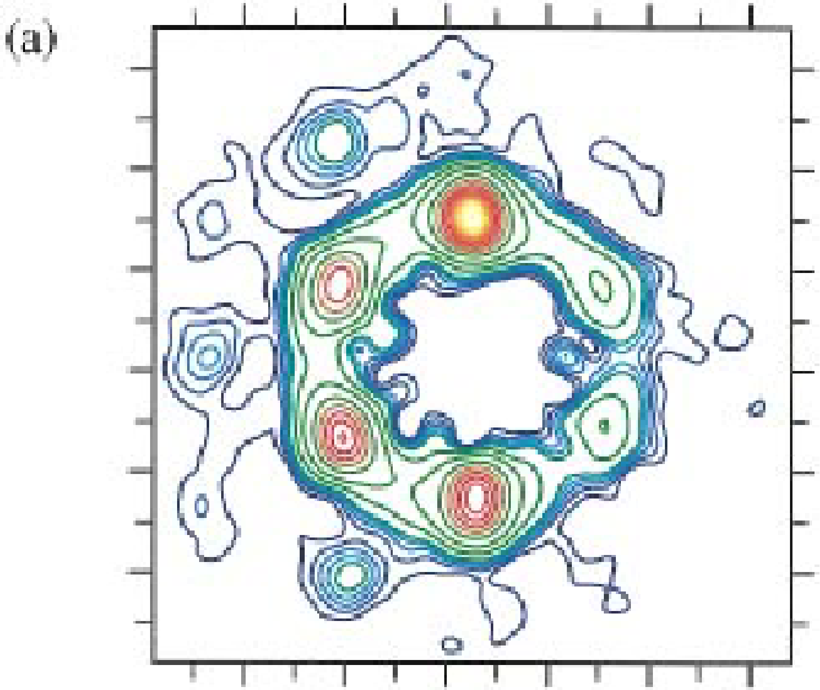
SMALL ANGLES!!!



11-Tesla Magnet

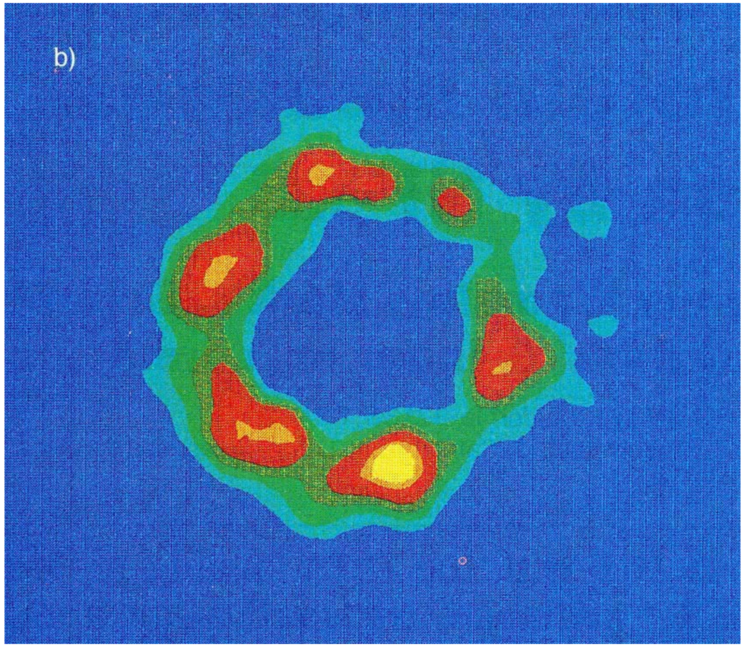
HTSC low fields

$YBa_2Cu_3O_7$ (B=0.2 T)



Johnson *et al*. PRL **82** (1999) 2792

$Bi_2Sr_2CaCu_2O_8$ (B=0.05 T)

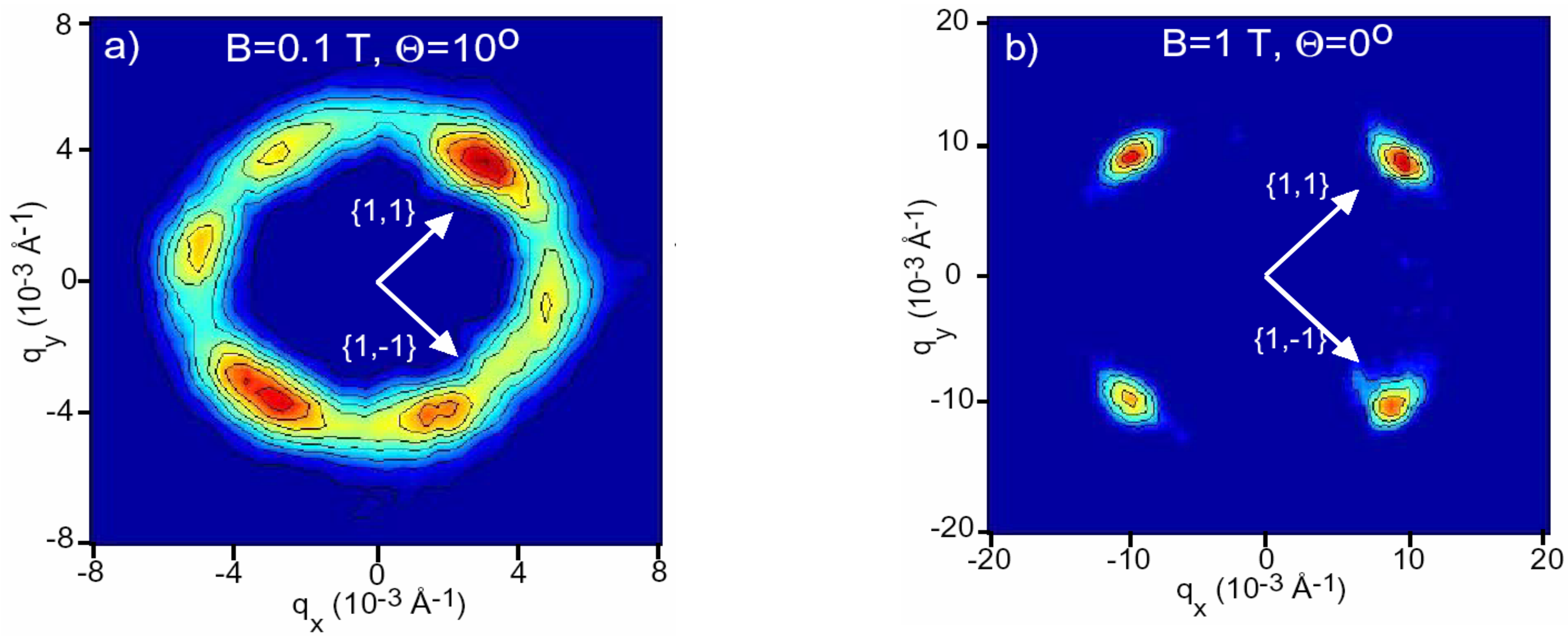


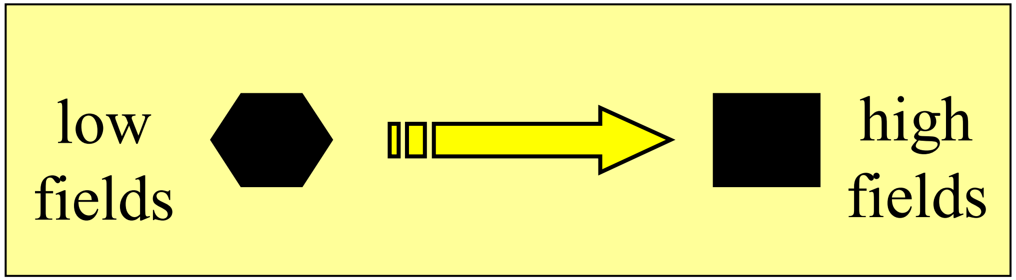
Cubitt *et al*., Nature **365** (1993) 407

J. Mesot, 07

$La_{2-x}Sr_xCuO_4$ (x=0.17)

{1,1}=(Cu-O-Cu)

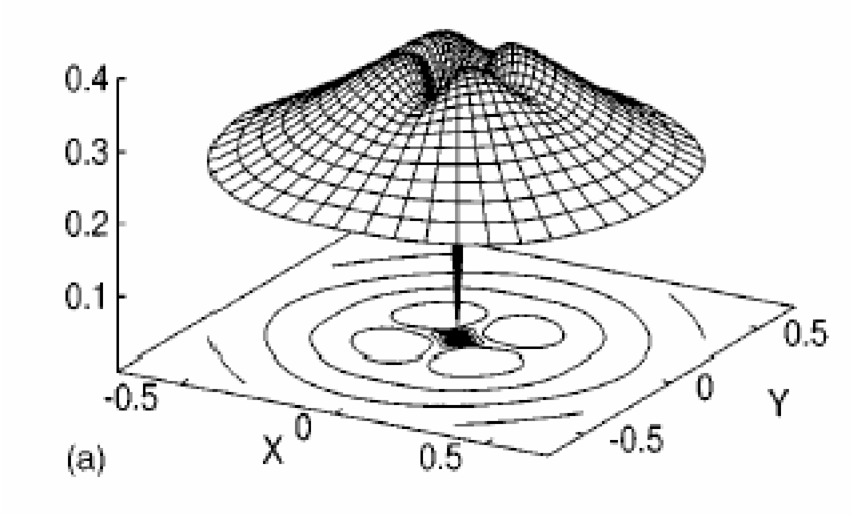


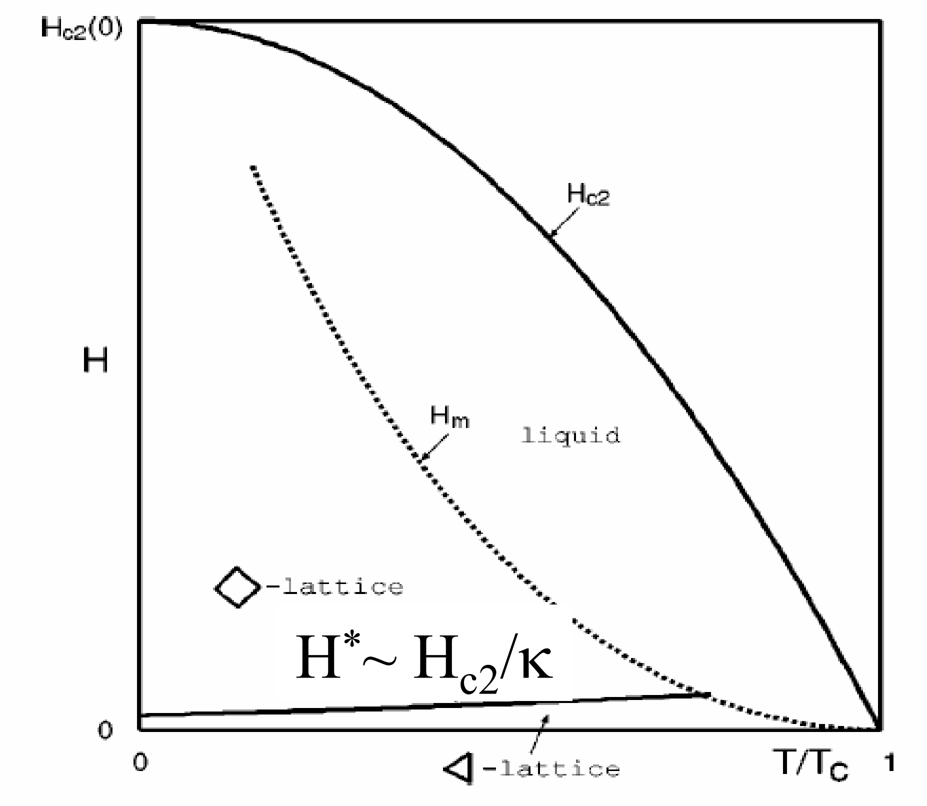

low fields → high fields

R. Gilardi *et al.*,
PRL **88** (2002) 217003

Vortex structure in d-wave superconductors

M. Ichioka et al., Phys. Rev. B **53** (1996) 15316



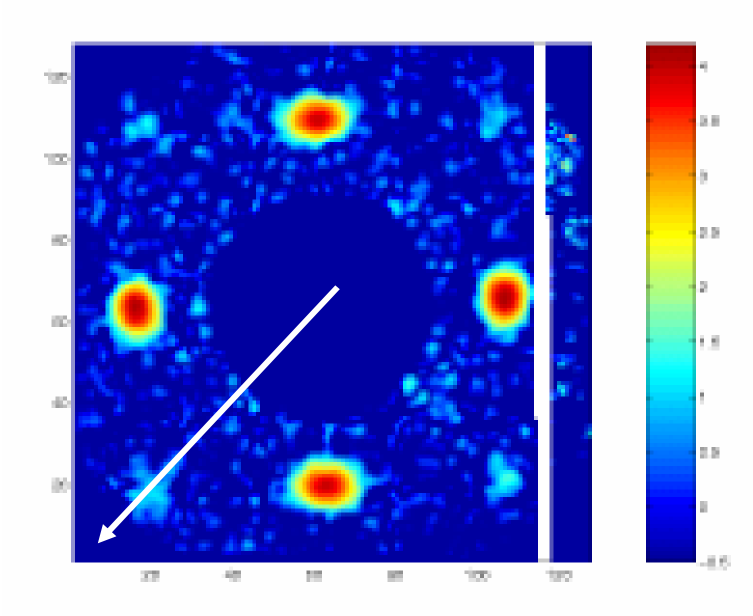


$H^{*}\sim H_{c2}/\kappa$

J. Shiraishi et al., PRB ***59*** *(1999) 4497*

Crossover hexagonal to square in YBCO at higher fields

B=9 Tesla



(Cu-O-Cu)

- S. Brown, T. Forgan, unpublished
- Keimer et al., PRL **75** (1994) 3459

$$\frac{H_{cross}(\mathrm{YBCO})}{H_{cross}(\mathrm{LSCO})} \approx 22.5$$

Lattice orientation

YBCO	**LSCO**
Nodal	**Anti-nodal**

J. Mesot, 07

Origin of the transition?

-d-wave: increased importance of vortex core anisotropy?

Predictions from theoretical works:

A.J. Berlinsky et al., Phys. Rev. Lett. ***73*** *(1995) 2200*

N. Schopohl and K. Maki, Phys. Rev. B ***52*** *(1995) 490*

M. Ichioka. N. Hayashi, N. Enomoto, and K. Machida, Phys. Rev. B ***53*** *(1996) 15316*

-Anisotropy of the Fermi velocity?

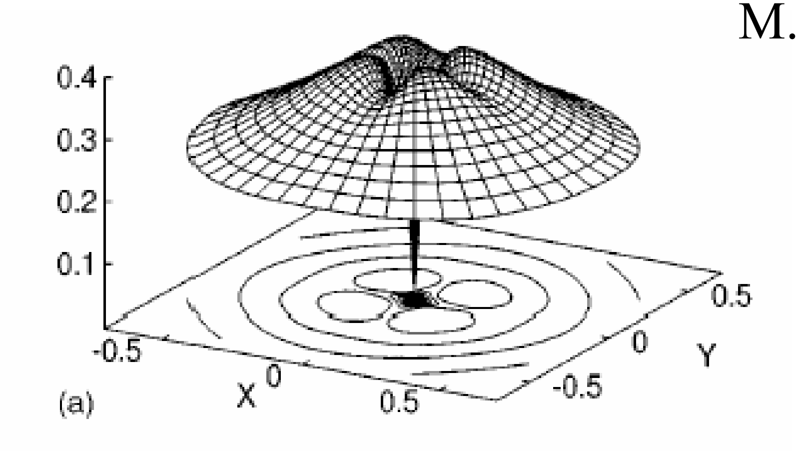
N. Nakai et al., PRL ***89*** *(2002) 237004*

-Presence of stripes?

Vortex structure in d-wave superconductors

Fourfold symmetry of current and magnetic field distribution around a vortex

M. Ichioka et al., Phys. Rev. B **53** (1996) 15316

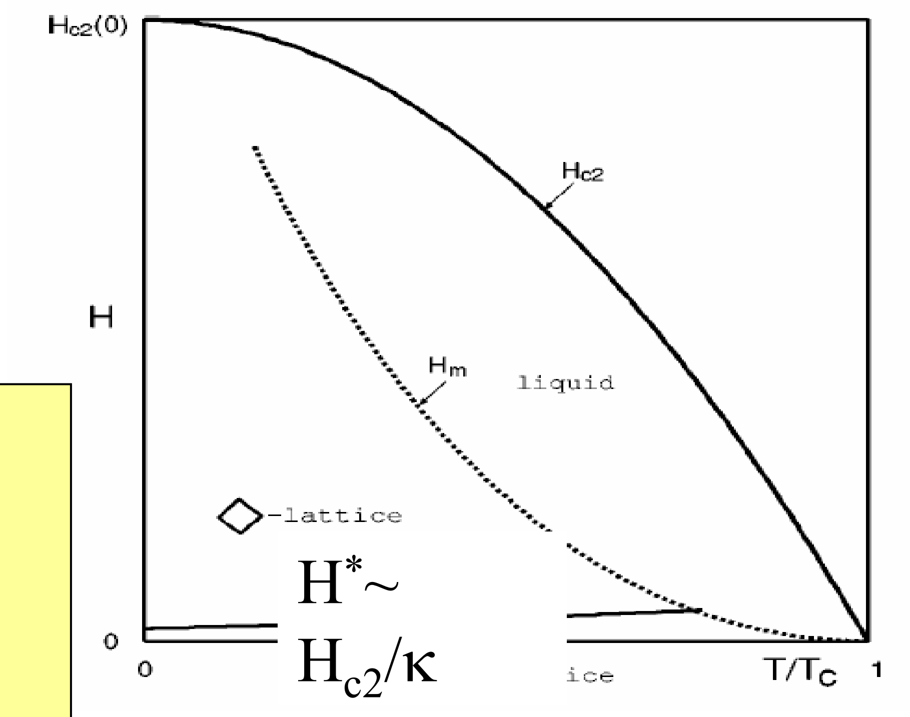


$\kappa \approx 100$

$H_{c2} \approx 60$ T

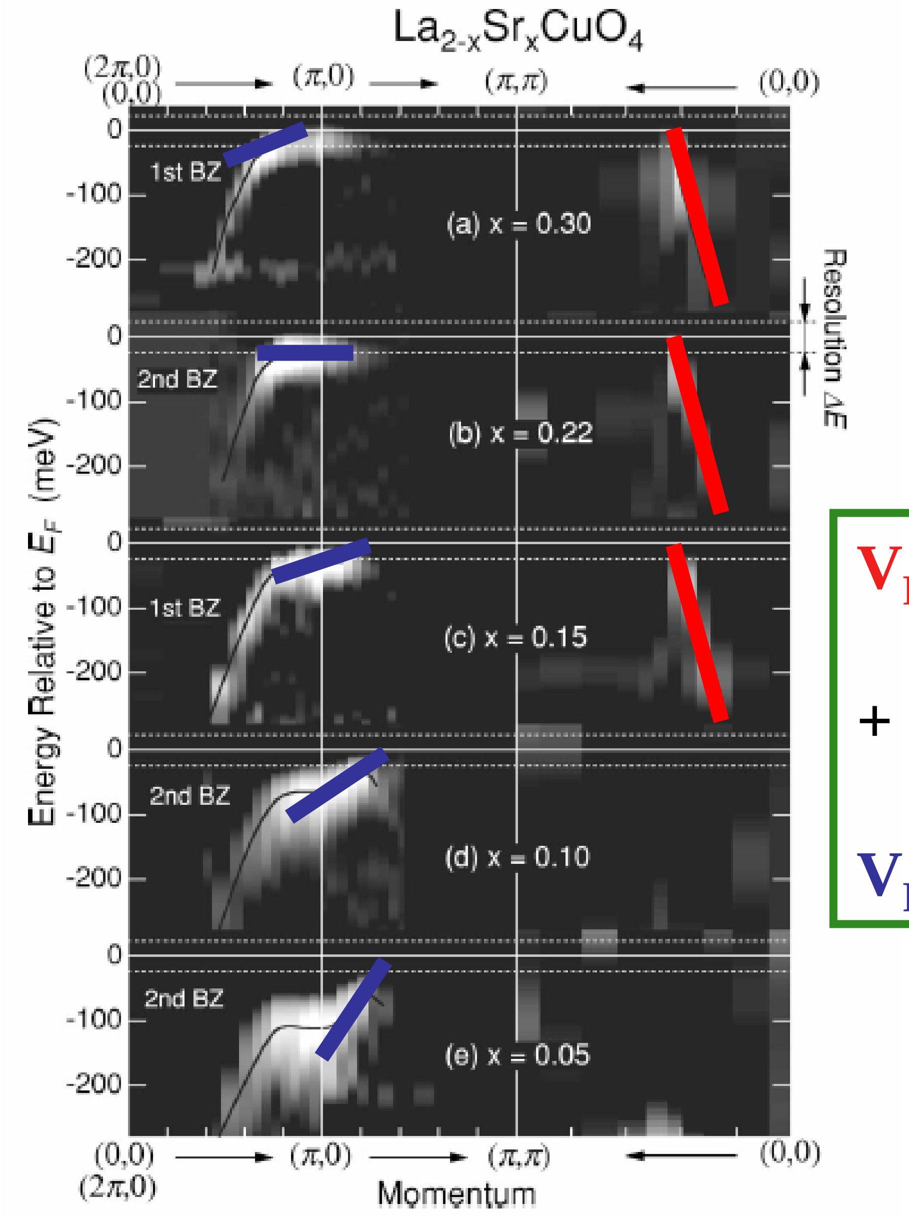
} H* ~ 0.6 Tesla ✓

But: VL expected to be rotated by 45°
(expected along nodal direction) ✗



$H^* \sim H_{c2}/\kappa$

*J. Shiraishi et al., PRB **59** (1999) 4497*



Ino et al., PRB
(2002) 094504

V_F(nodal) > V_F(anti-nodal)

+

V_F(anti-nodal) doping dependent !

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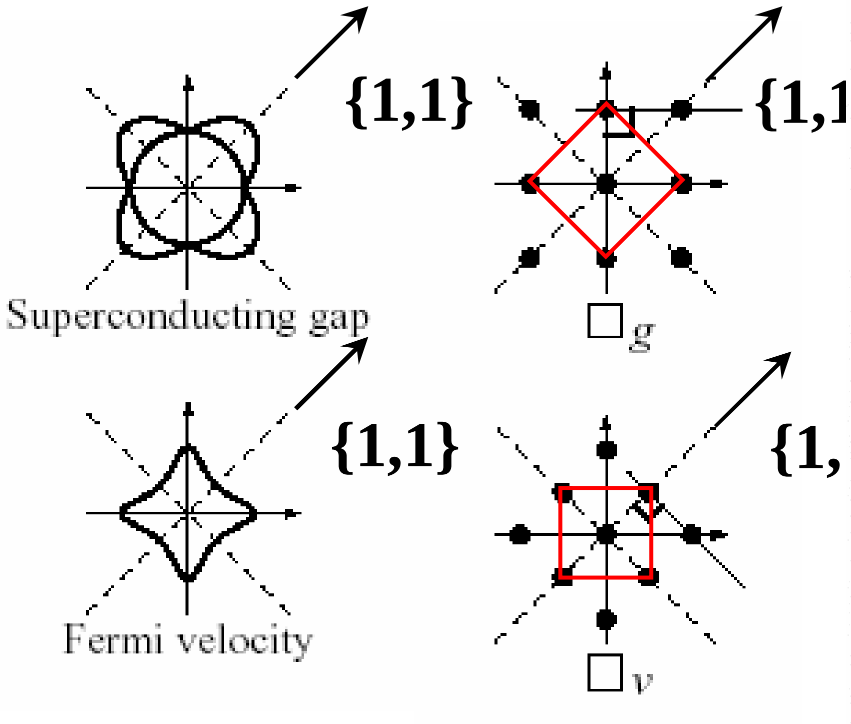
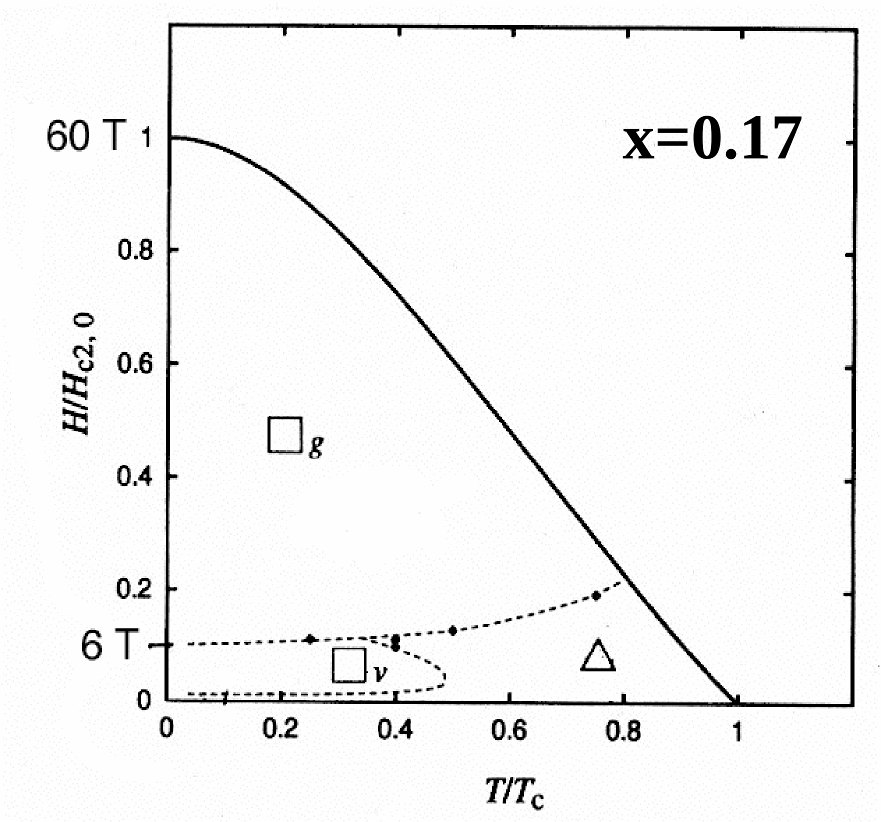
Interplay/Competition between Fermi velocity anisotropy & gap anisotropy

$La_{2-x}Sr_xCuO_4$





N. Nakai et al., PRL ***89*** *(2002) 237004*