



*The Abdus Salam
International Centre for Theoretical Physics*



1866-4

School on Pulsed Neutrons: Characterization of Materials

15 - 26 October 2007

Cage Structures and Clathrates

Werner Press
*Christian-Albrechts-Universität Kiel
Institut für Experimentelle und Angewandte Physik
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24098 Kiel
Germany*

Cage-systems

Werner Press (Kiel)

H_2O plus Xe or CH_4 (seafloor-gashydrates)

O_2Si plus CH_4 or CO_2 (volcanoes - clathrasils)

Si/Ge plus Na, K, Rb...

Skutterudites XRu_4Se_{12}

Inclusion compounds

Form structures stable in surprising p, T-regions

not: H in metals or systems with interconn. channels

Cage-systems: properties

Framework & (inert) host

Polyhedra

Marine research

Store energy

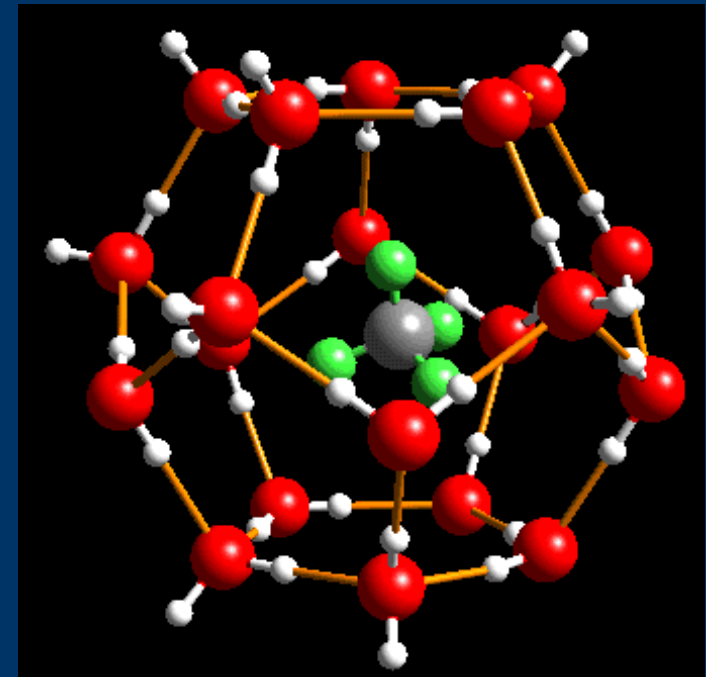
Electrochemical systems

Rattling modes

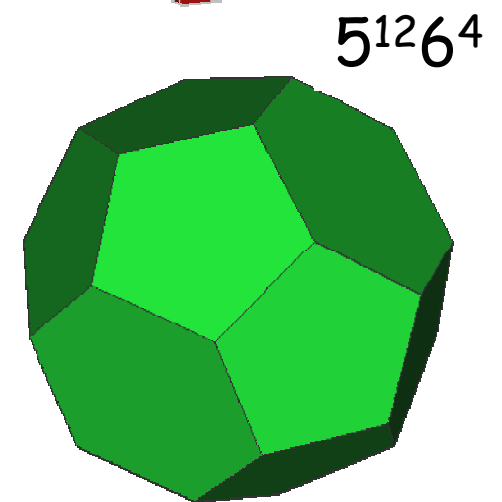
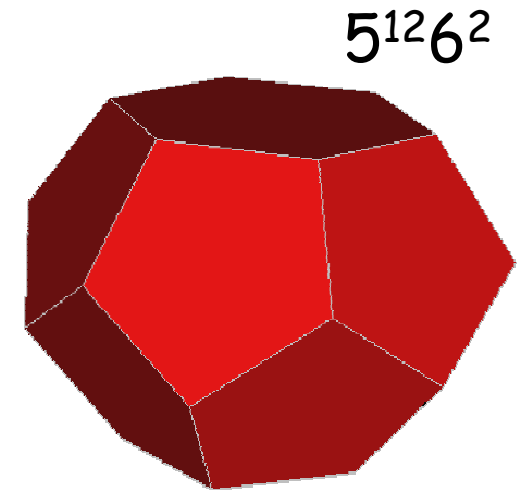
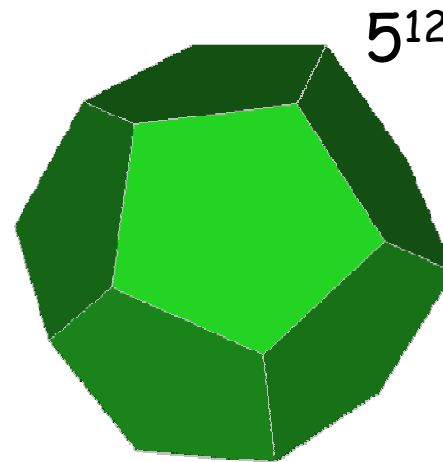
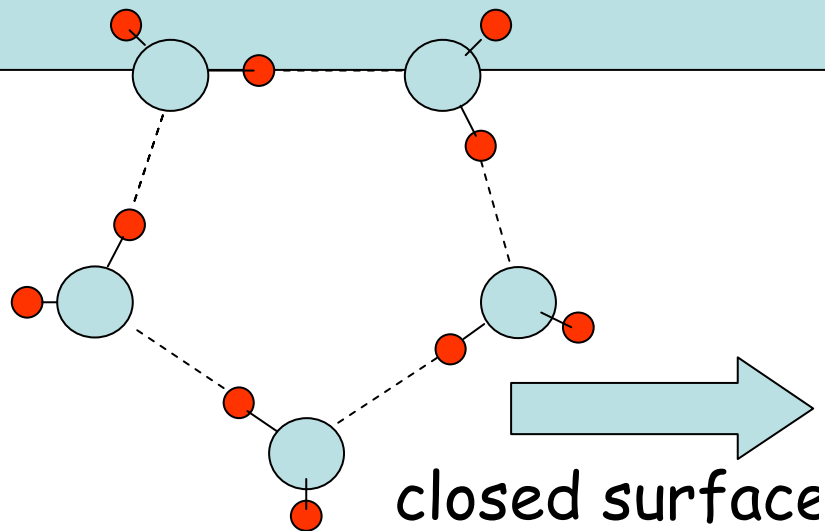
Einstein oscillationors

glass-like thermal conductivity

Neutron moderators ?



motif Cage-formation



Cageology

- 1) Polyeder is instable without enclosed guest
- 2) There are not enough bonds in the surface

1

vertex (= oxygen)

3

edges/vertex (= 1.5 hydrogens)

4th

edge/vertex needed

out-of-surface bonds, 3rd dimension

Outline of talk

Introduction

Gashydrates

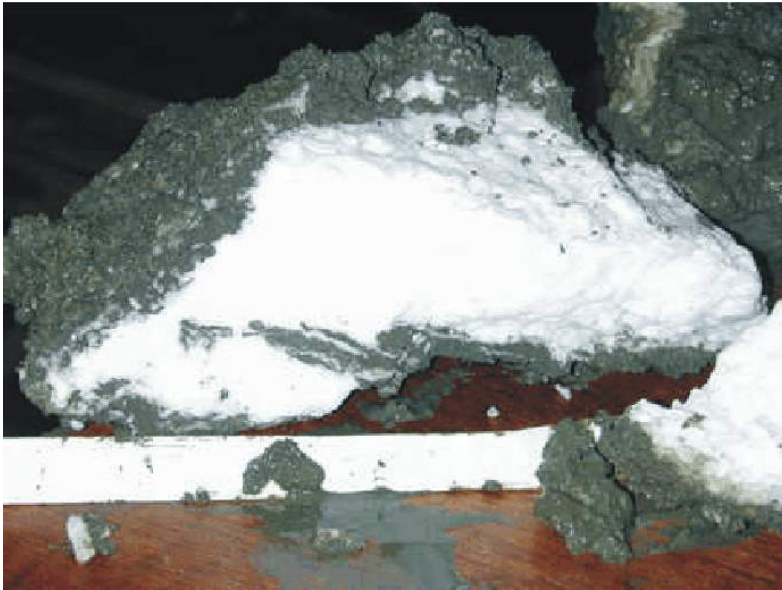
Skutterudites

Alkali-silicon system

Summary/outlook

Methane hydrate (burning ice)

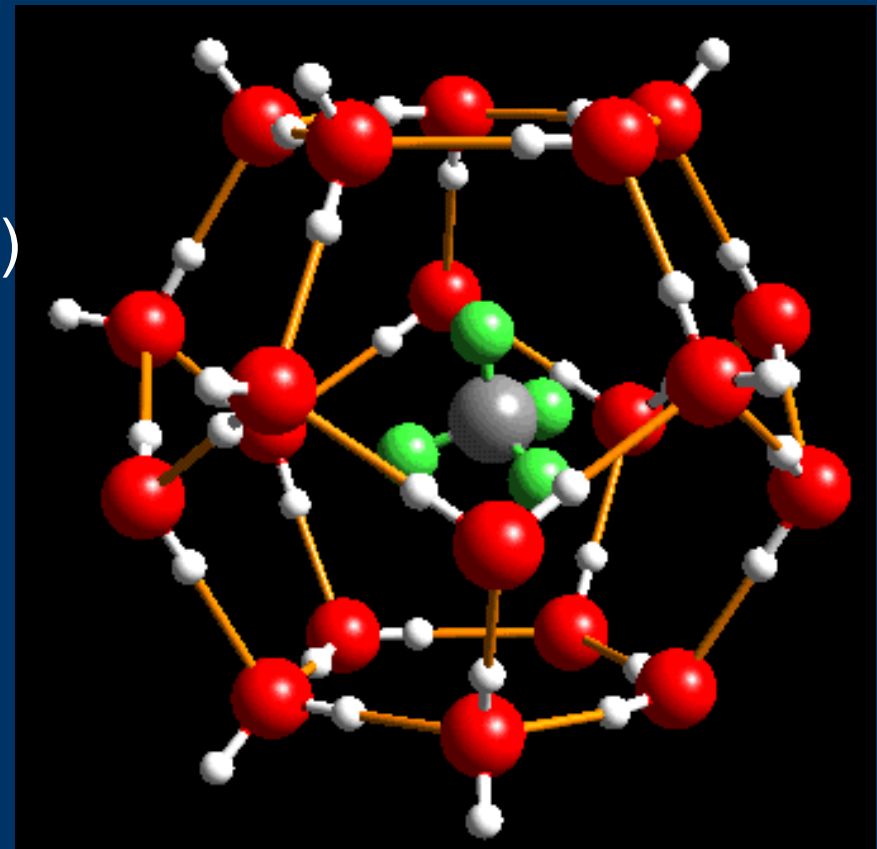
Werner Press (Univ. Kiel)



Macroscopic
(from Pacific)



Microscopic
(hydrate
cage)



Methane hydrate (burning ice)

Study with neutrons & X-rays

1997 - 2003

2006 - 2007

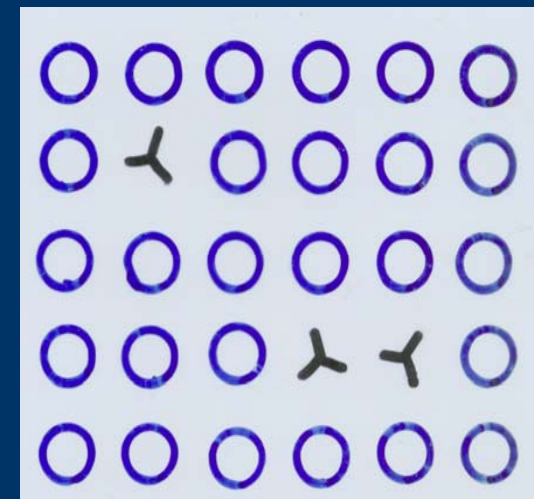
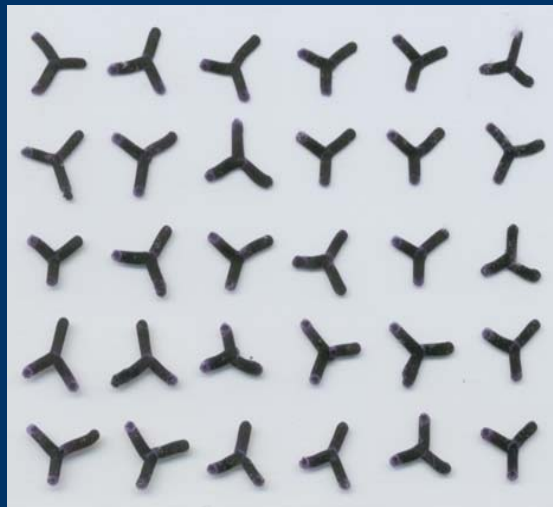
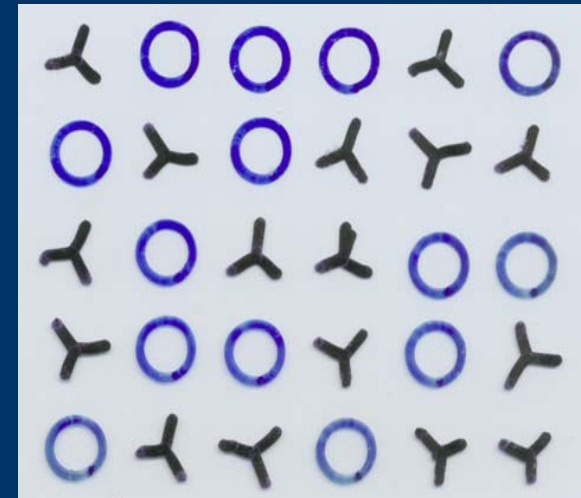
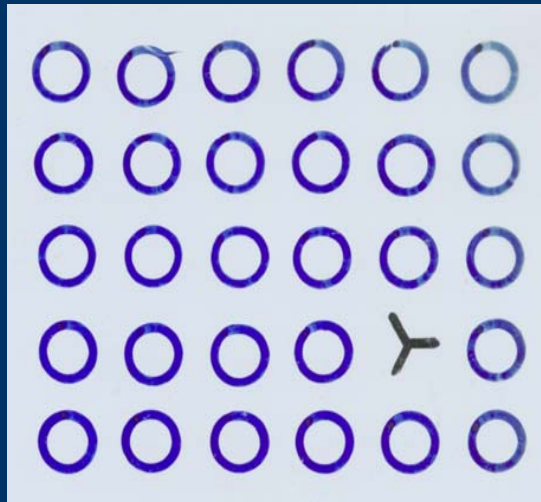
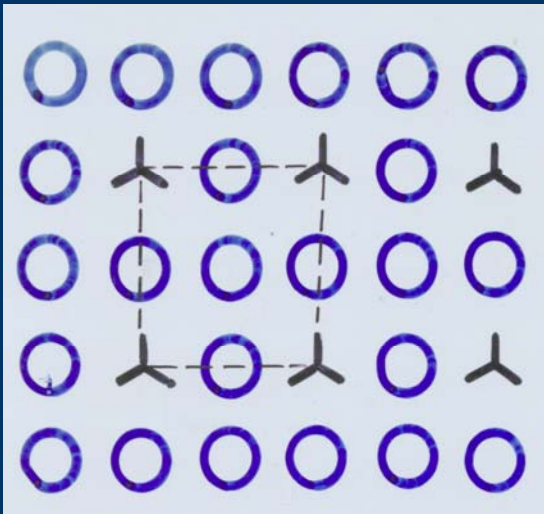
Julian Baumert †
Christian Gutt (DESY Hamburg)

Alfred Hüller (Uni Erlangen)
John Tse (Saskatchewan, Canada)
Mark Johnson (ILL)
Sasha Krivchikov (Charkow, Ukraine)



Molecular Solids:

Position R Orientation φ, Ω, ω



Long personal history of research with XH_y

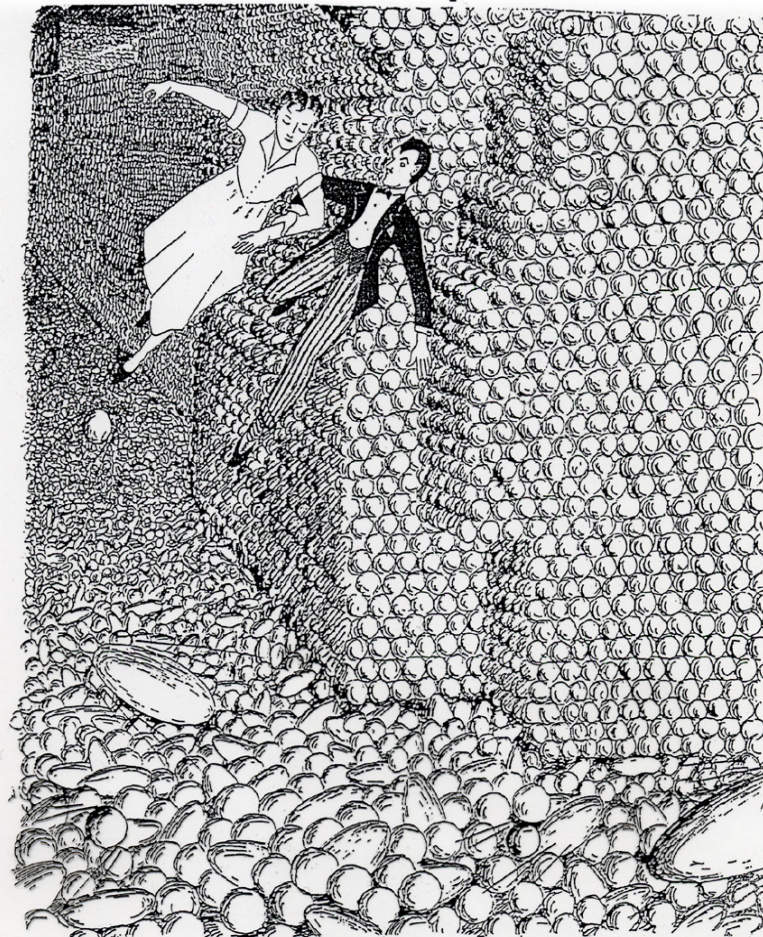


$$\sigma_{inc}(H) = 80 \text{ barn} \quad B = \hbar^2/2\Theta$$

- fundamental research
- energy storage
- geosciences
- planetary environments

Phase III
Structure
Tunneling (CH_4 , CD_4)

??????
meaning



Sieht etwa so die Hölle aus?

Gamov

H_2O
Ice: many phases
close: hexagonal Ice I_h
Dipole moment

Gas Hydrates

Combine the two:



More stable than ice
(presence of pressure
> ~50 bar)

Preparation:

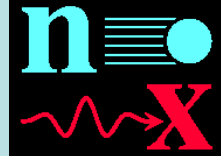
need pressure
Biomaterials(CH_4)
 H_2O

nature
laboratory



Research Ship "Sonne"

Universität Kiel
Institut für Experimentelle
und Angewandte Physik



Research Ship "Sonne"



Storage &
transport



History of Gas Hydrate Research

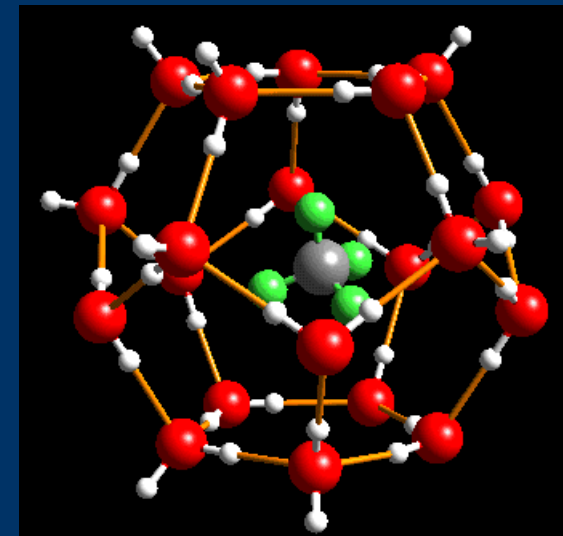
- ~ 1810 Davy , Faraday
- 1928 Review : W .Schroeder , "Gashydrate"
- ~ 1935 Pipelines blocked
- ~ 1950 Structural comprehension :
Stackelberg
- ~ 1970 small quantities of natural samples
(Soviet Union & USA/Canada)
- 1984 Type I Structure
- 1997 E. Sloan " Clathrate Hydrates of
Natural Gases "
- etc

How did it all start ?

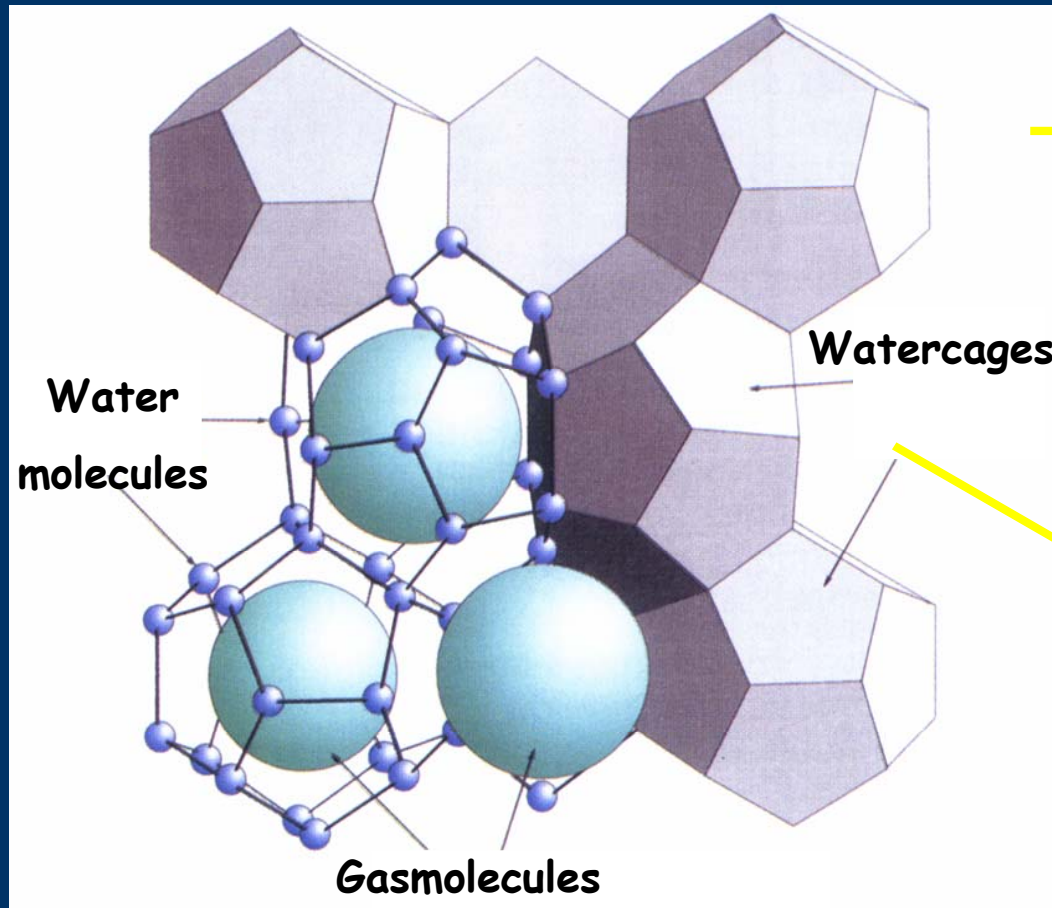
phone call ~1997
article in "Die Zeit"
burning ice

IFM-Geomar, Kiel
NRC Ottawa

Small
cubic
cage 5^{12}

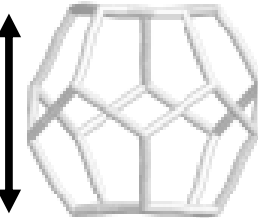


Gas Hydrates



2x

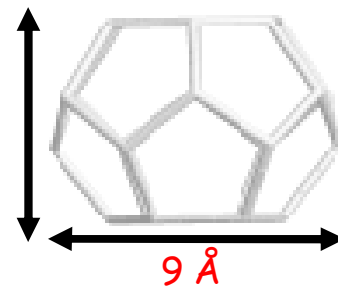
5 Å



5¹² spherical

6x

5.9 Å



5¹² 6² elliptical

Structure I : cubic $Pm\bar{3}n$ (CH_4 , Xe, ...)
8 $CH_4 \cdot 46 H_2O$ Lattice Parameter $a_0 \approx 11.81 \text{ \AA}$

Structure of gas hydrates: Cages !

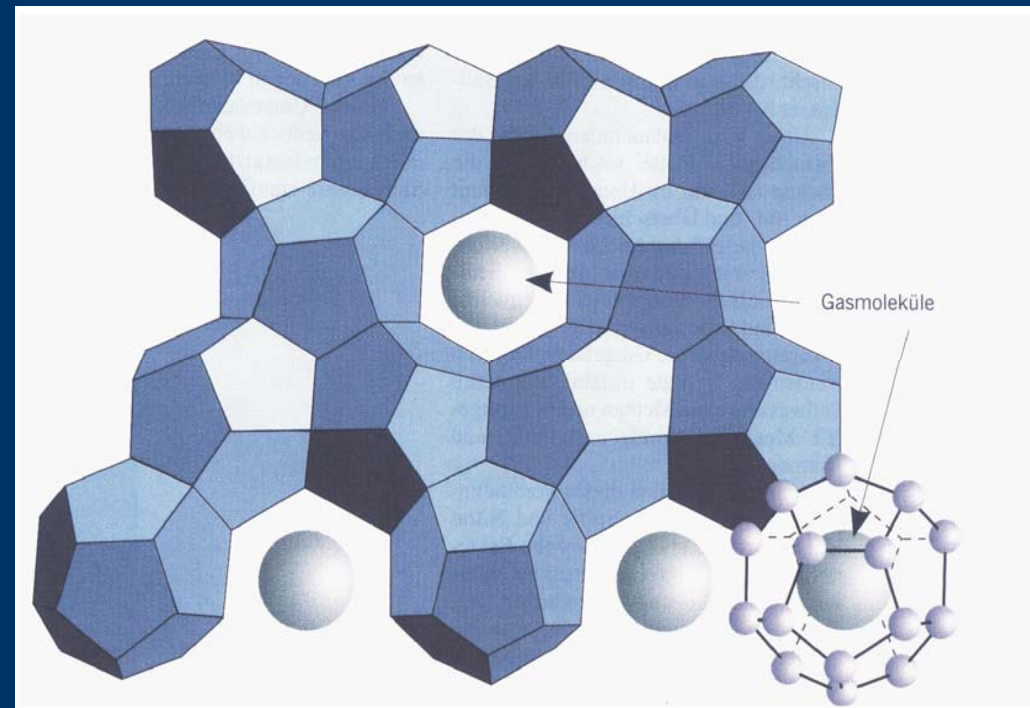
host: ice framework
guest: CH_4 , Xe
NB alkanes are hydrophobic

(1) low temperature

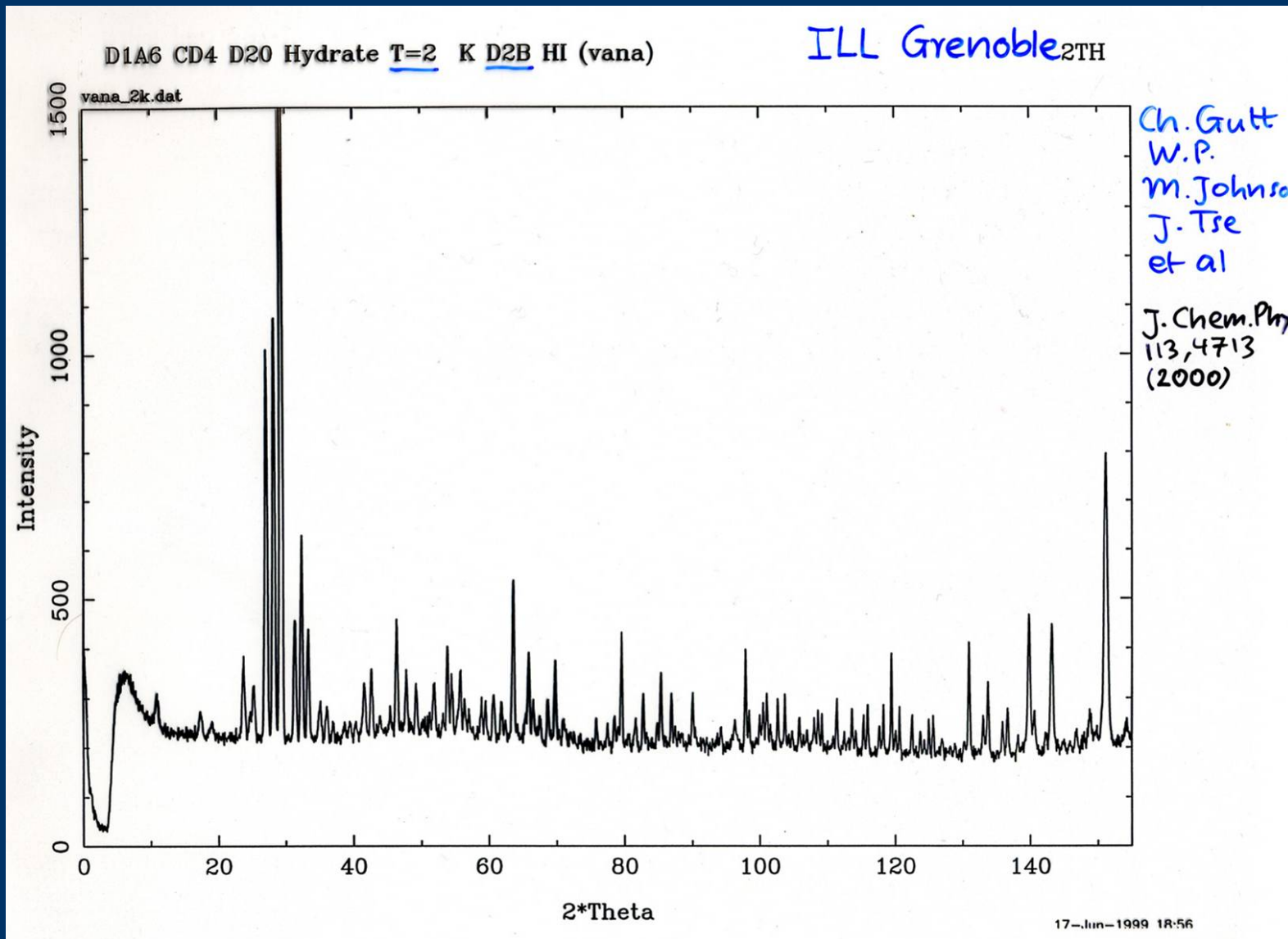
(2) natural conditions

Powder diffraction
ILL D2B
deuterated compound

T = 2K, 20K etc
Normal pressure



Structure : Diffraction at ILL/D2B (old) $\lambda = 1.59\text{\AA}$



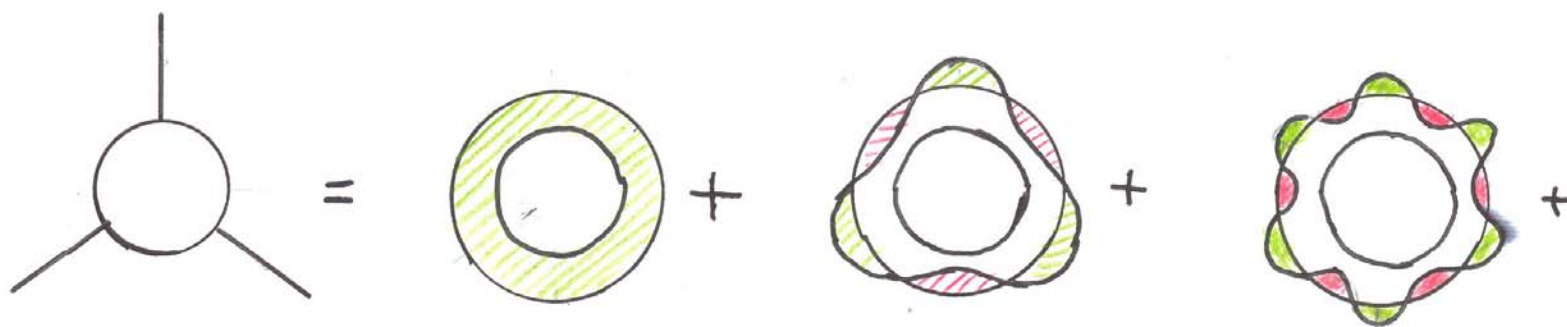
Density on spherical surface: expansion into symmetry-adapted harmonics

$$\rho(r) = \int \rho_T(R) \rho_{Rot}(r - R) dR$$

$$F(Q) = \exp(iQR_0) \exp[-W(Q)] F_{Rot}(Q)$$

$$F_{Rot}(Q) = 4\pi \sum_{l=0}^{\infty} \sum_{m=1}^{2l+1} i^l j_l(Q\rho) c_{lm} K_{lm}(\Omega_Q)$$

Guests:
 Site-occupation
 Centered?



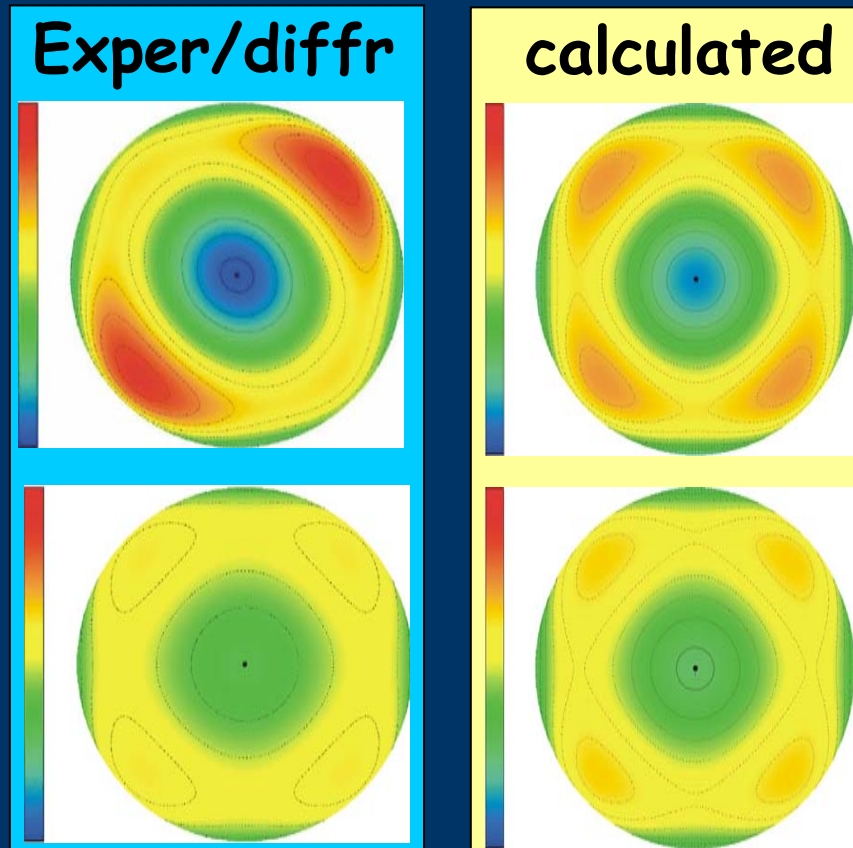
modulation

Fourier TF(3D) $j_0(Q\rho) - b_{31} i j_3(Q\rho) K_{31}(\Omega_Q) + \dots$

Scattering Length Density in CH₄ Hydrate

elliptical $\bar{4}m2$
cages

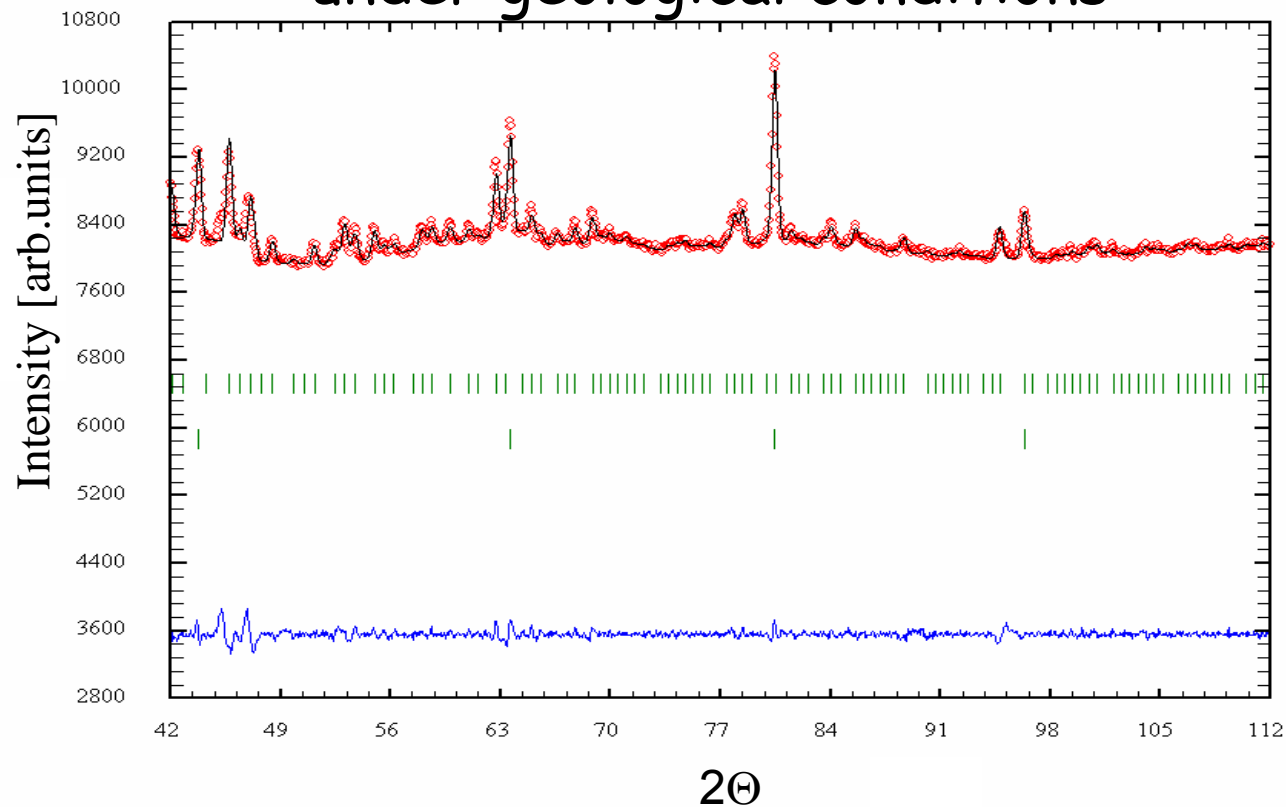
spherical $m3$
cages



C. Gutt, W. Press, A. Huller, J.S. Tse and H. Casalta, *J. Chem. Phys.*, 114, 4160 (2001)

Structure of Methane Hydrate

Diffraction pattern of $\text{CD}_4\text{-D}_2\text{O}$ hydrate
under geological conditions



$T=280\text{K}$, $p=100\text{bar}$



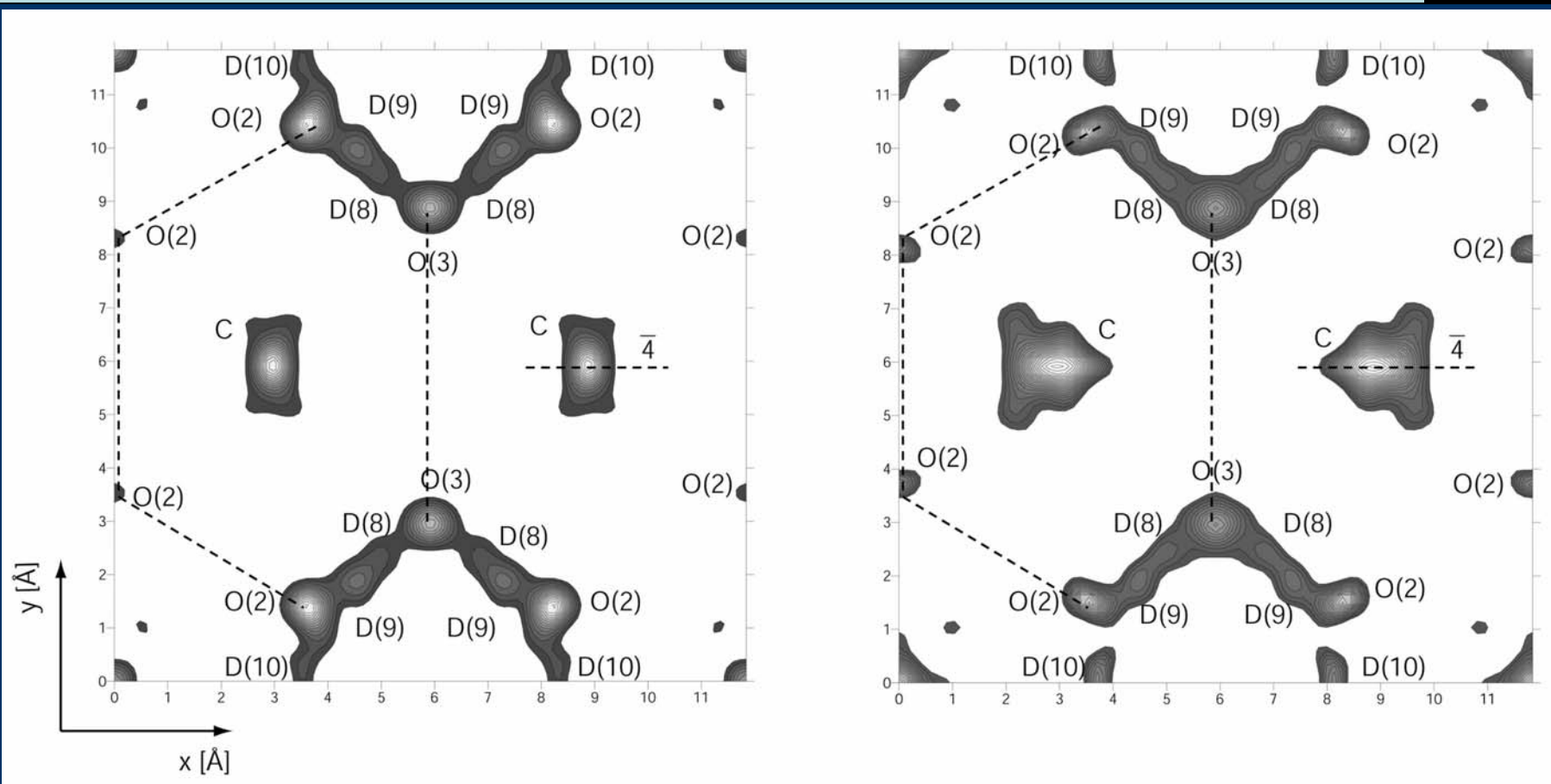
$T=4^\circ\text{C}$, 1000m water depth

Julian Baumert

Scattering Length Density

Low temp.
T=2K, ambient pressure

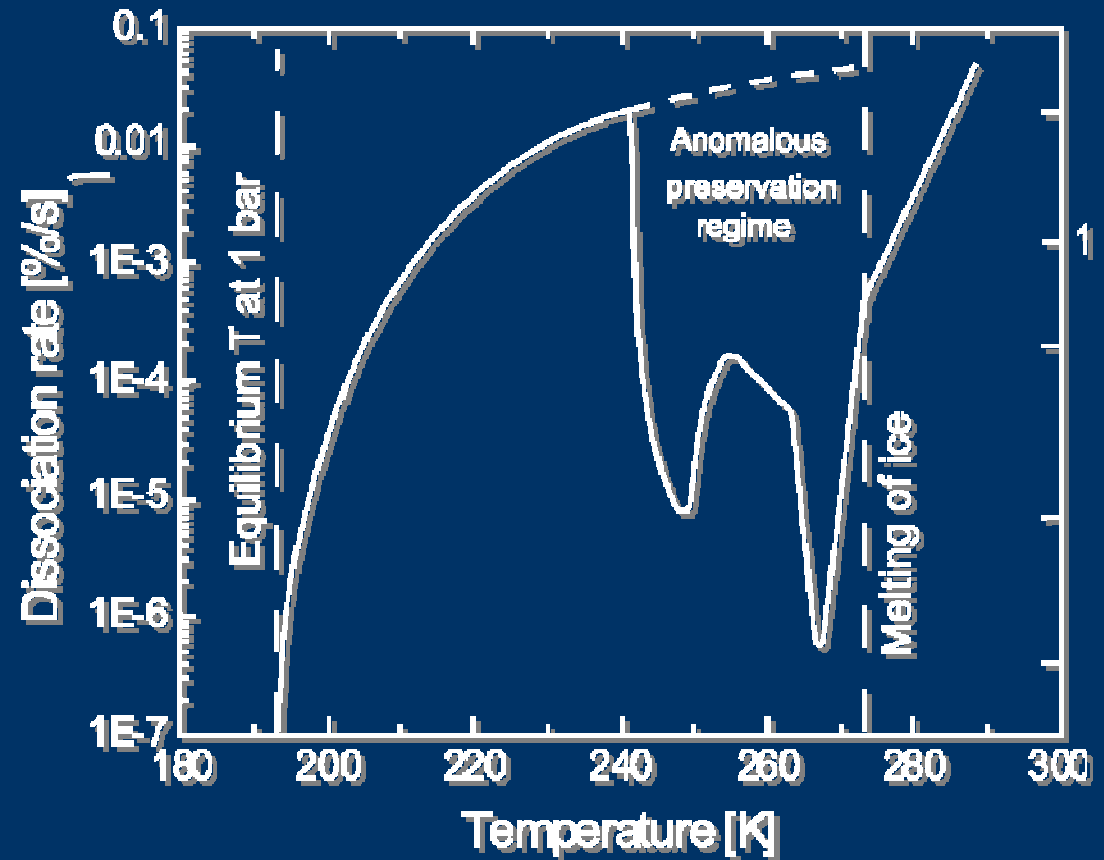
High temp.
T=285K, p=100bar



Scattering length density map ([barn/Å³]) of the (001) plane
obtained from MEM analysis

Decomposition kinetics

Dissociation rate
of methane
hydrate
Stern et al
J.Phys.Chem.
B105 , 1756 (2001)

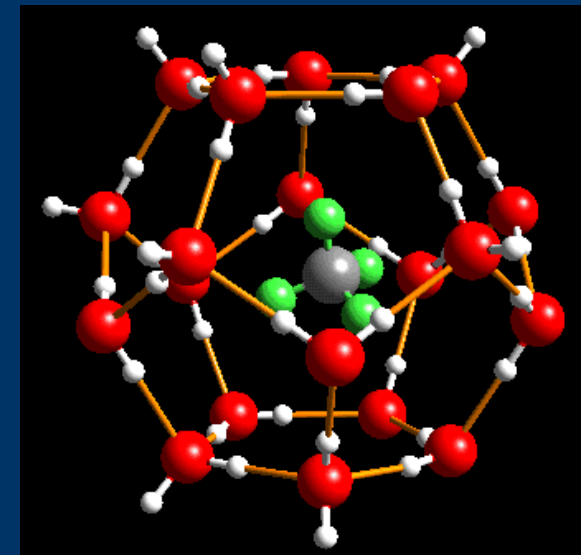


Anomalous preservation of gas hydrates

Diffraction study on D2O/ILL
ice I_h has many stacking faults
methane diffusion fast

annealed ice I_h stops diffusion

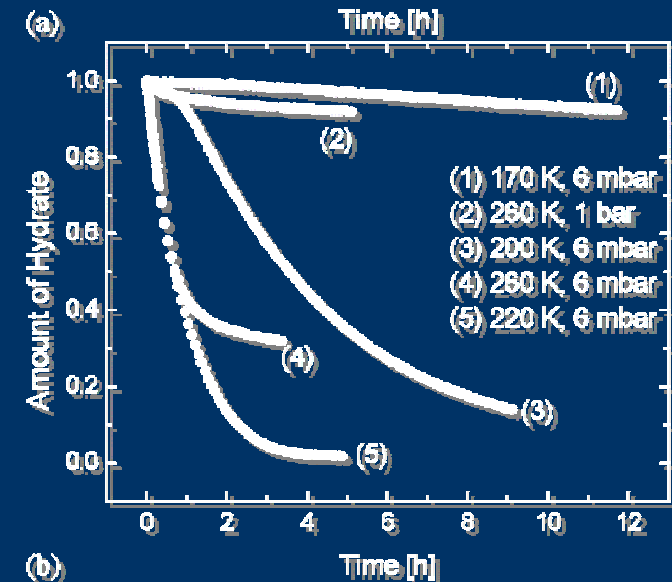
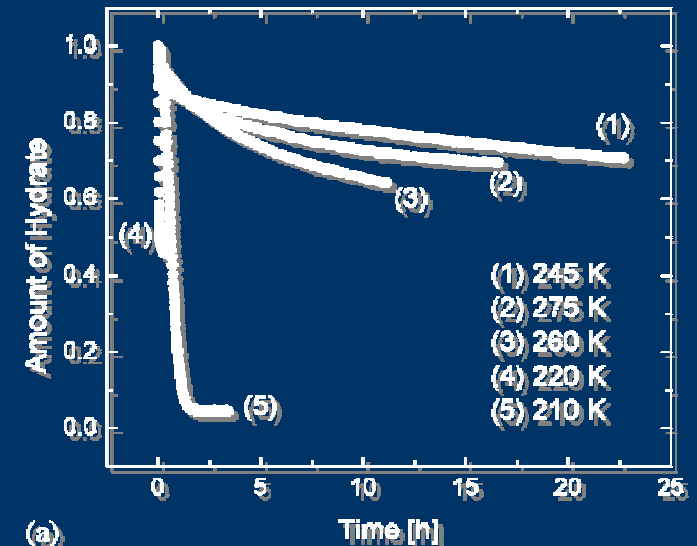
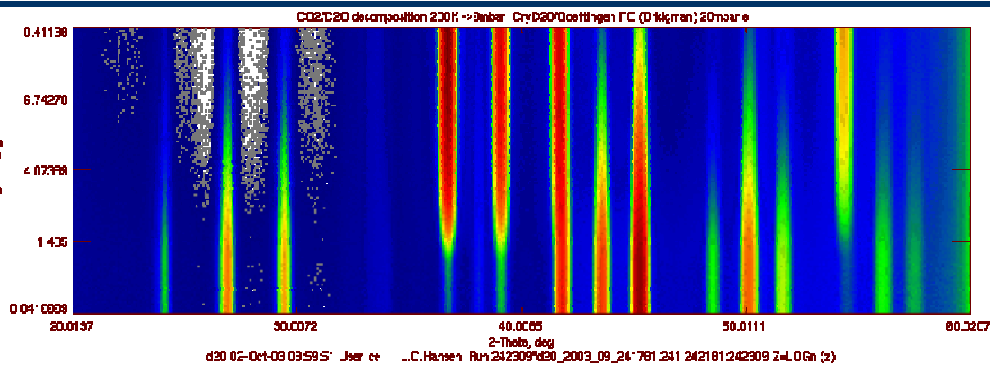
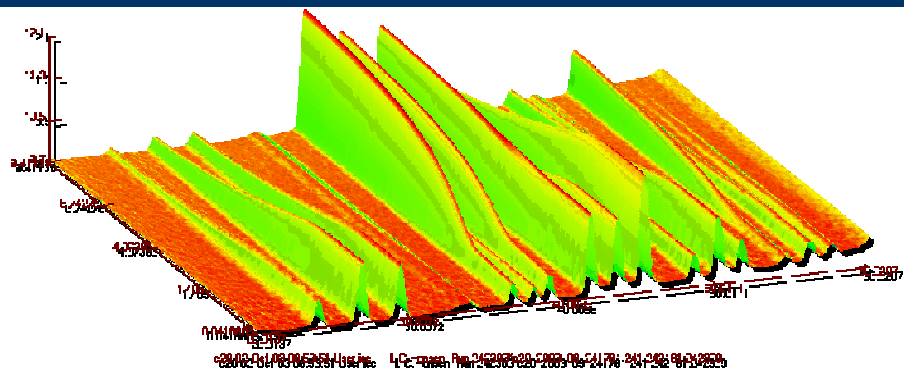
(Kuks et al: with I_c decomp. & CO_2)
CPPC 6 (2004)



hydrate decomposition : time dependence

D2O in situ experiment (W.Kuhs, T.Hansen et al)

- Decomposition of $\text{CH}_4^{(a)}$ & $\text{CO}_2^{(b)}$ hydrates at different T and p
 - Here: CO_2 at 200 K, 6 mbar
 - Covering 10 hours



interlude: methane hydrate

Research

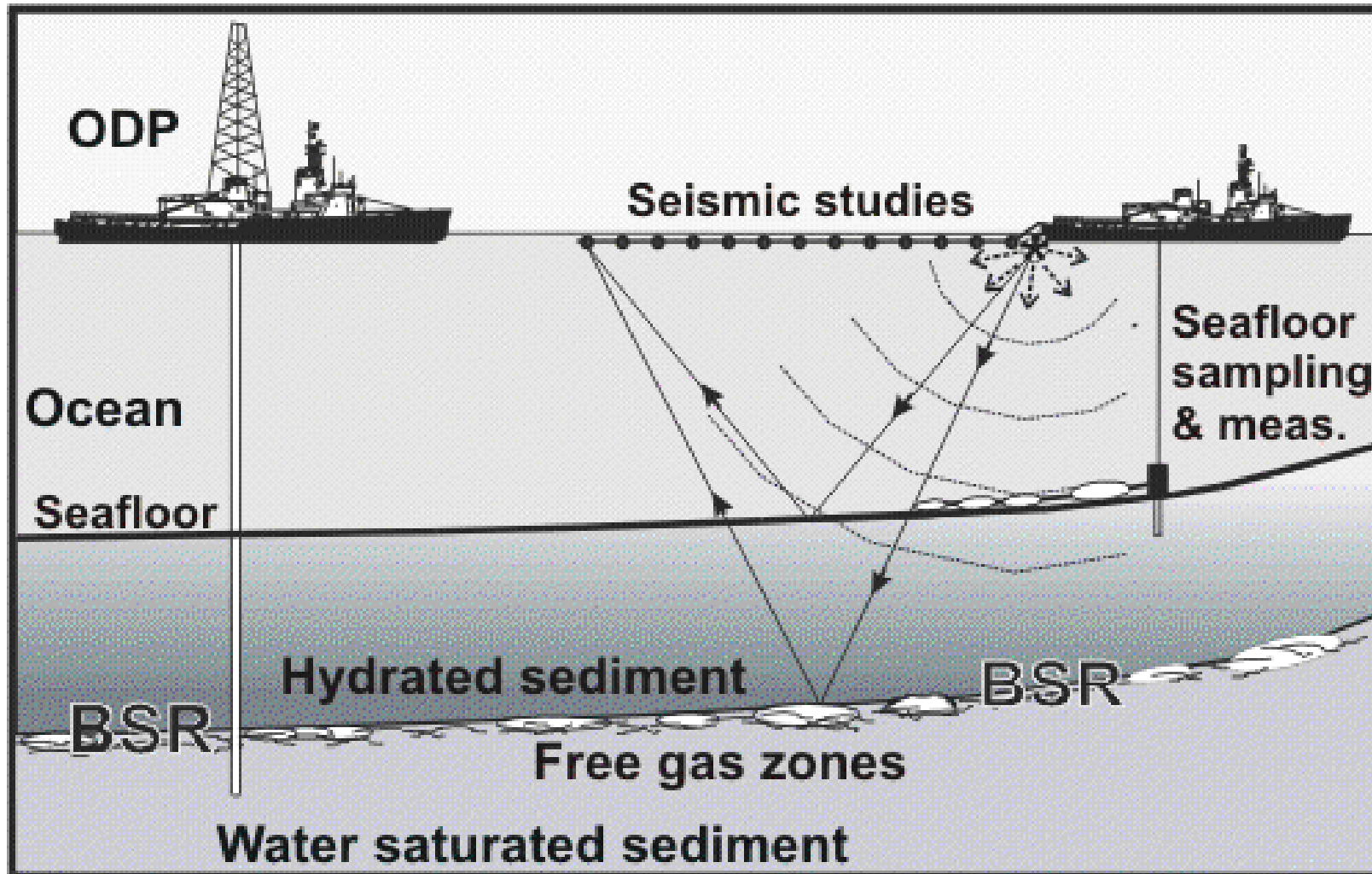
of deep sea : little done

only area ~ a dozen football fields
investigated

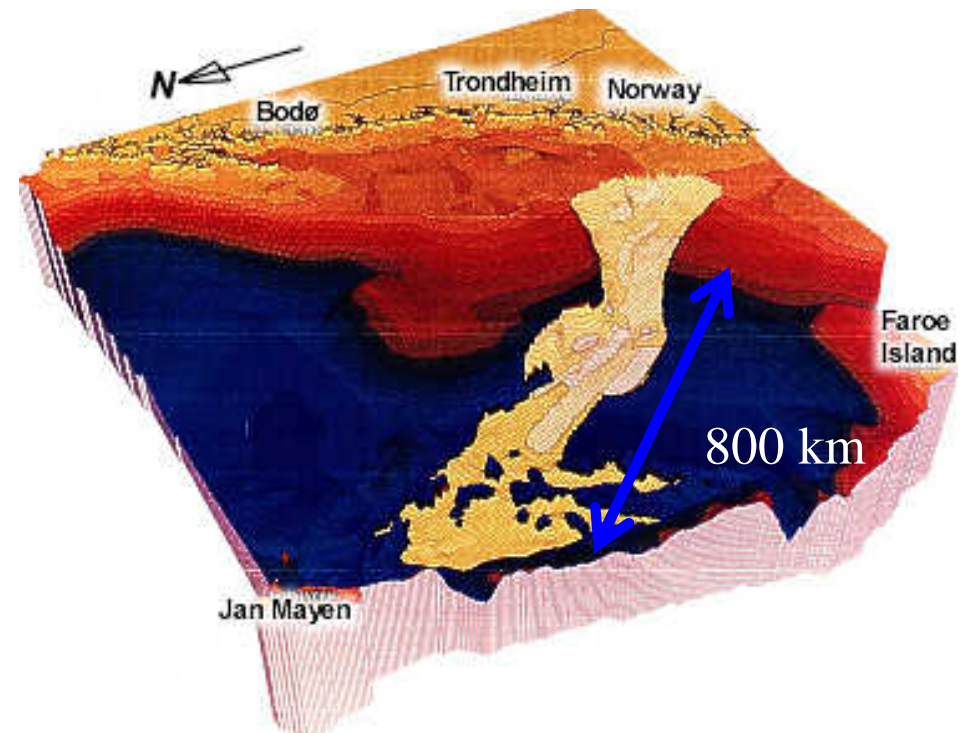
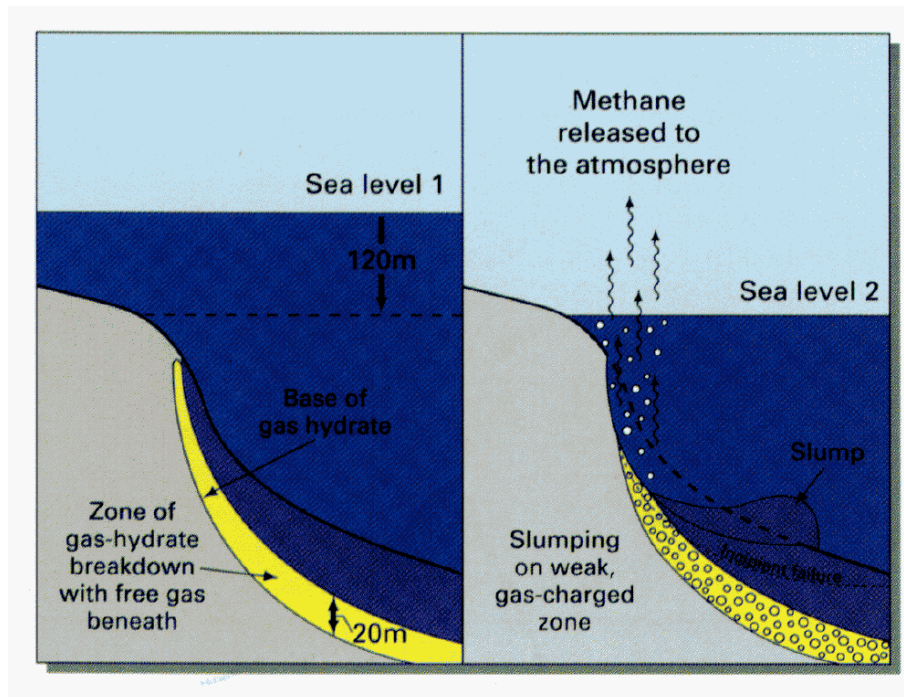
much remains to be done !!!

yet fragile

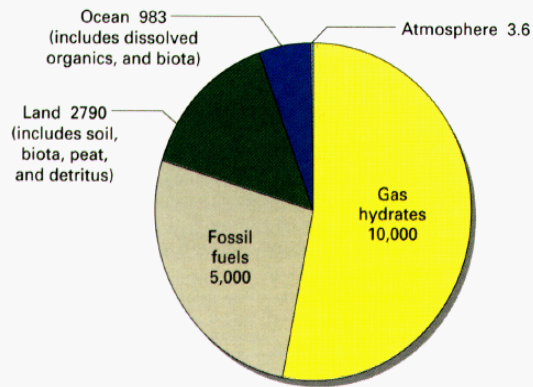
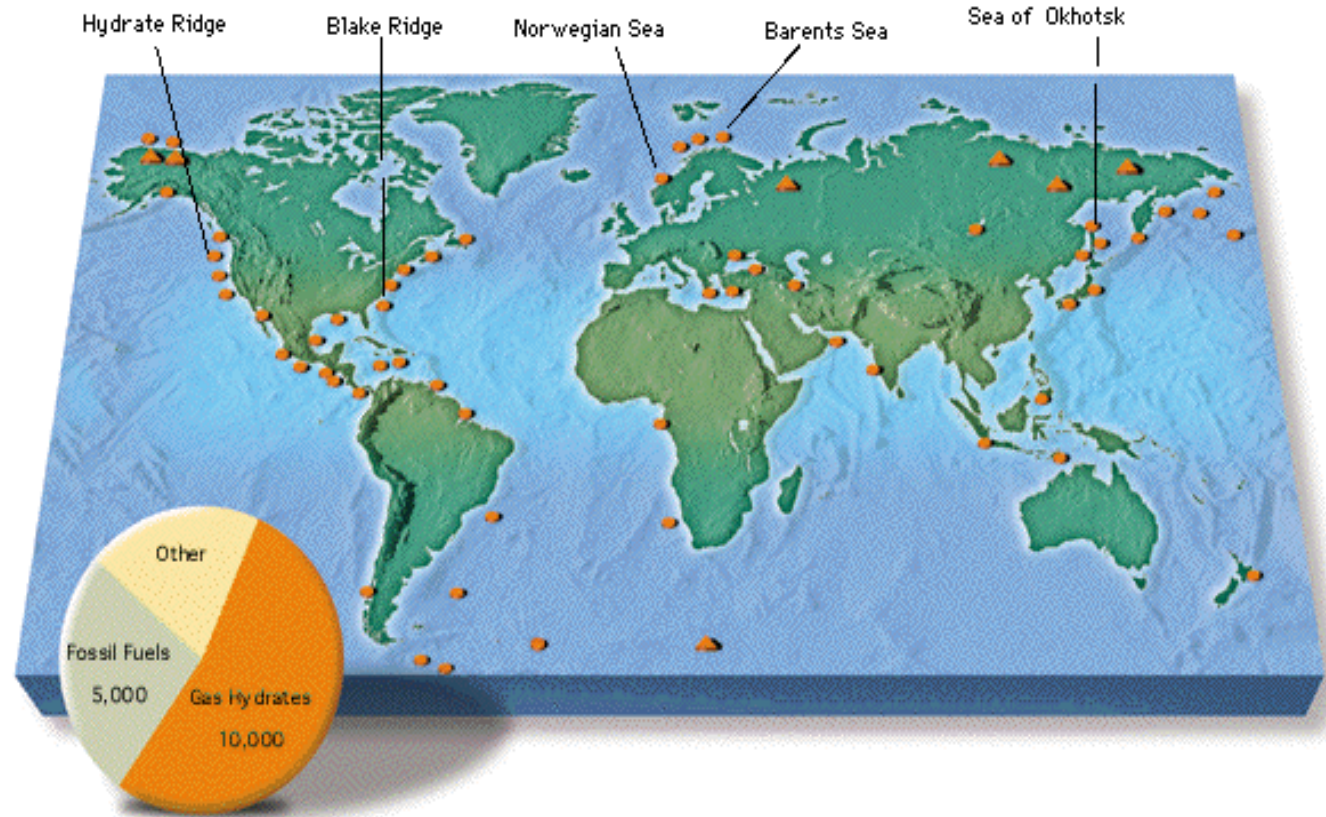
Hydrate Seismic Detection



Hang-Slides: Hydrate Decomposition



Hydrate Deposits

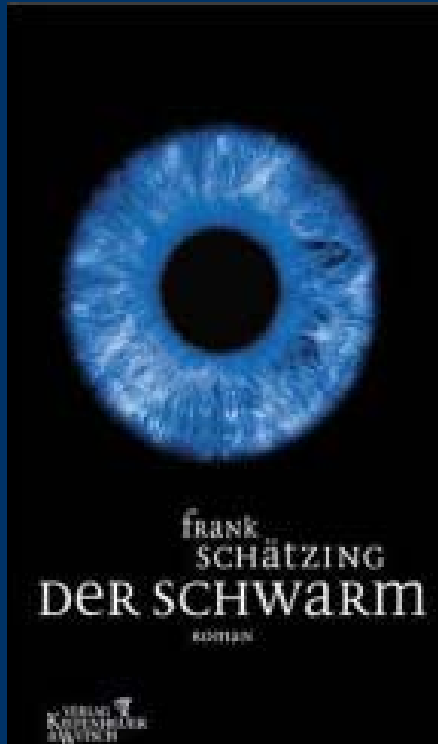


Distribution of organic carbon in Earth reservoirs (excluding dispersed carbon in rocks and sediments, which equals nearly 1,000 times this total amount). Numbers in gigatons (10⁹ tons) of carbon.

Life in CH₄-deposits (discovered 1997) & Schätzing: The Swarm (Der Schwarm)



Public interest



Global warming

DIE  ZEIT

Why neutrons?

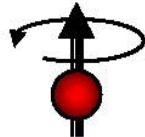
„commercial“



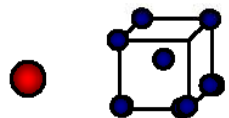
Neutrons are neutral: high penetration power, non-destructive



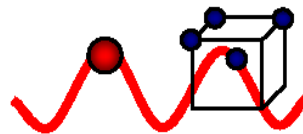
Neutrons have a magnetic moment: magnetic structures and excitations



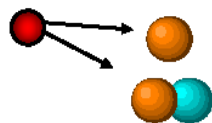
Neutrons have a spin: polarized neutrons, coherent and incoherent scattering, nuclear magnetism **rotations**



Neutrons have thermal energies: excitation of elementary modes, phonons, magnons, librations, rotons, tunneling, etc.



Neutrons have wavelengths similar to atomic spacings: structural information, short and long range order, pore and grain sizes, cavities, etc.



Neutrons see nuclei: sensitive to light atoms, exploiting isotopic substitution, contrast variation with isotopes



Incoherent & coherent scattering

Isotopes with different b_j
Nuclei with b_+ and b_- (statistical ! nucl.spin)

σ_{inc} no interference
local structure and dynamics

H

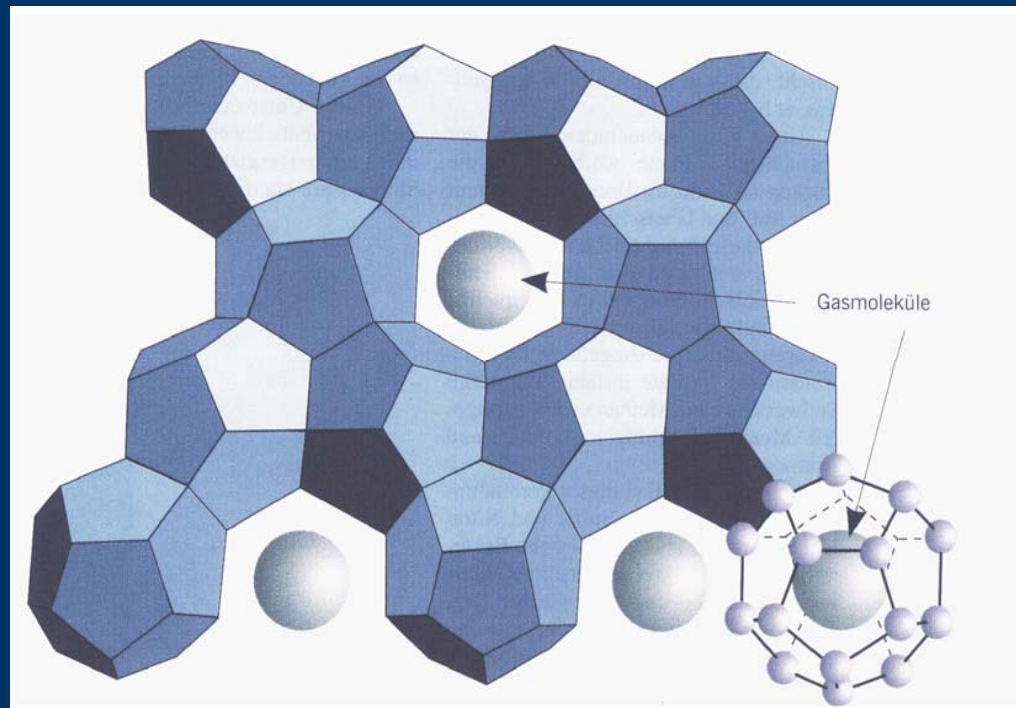
σ_{coh} interference
long range order & collective dynamics

D, C

Quantum rotation of methane hydrate

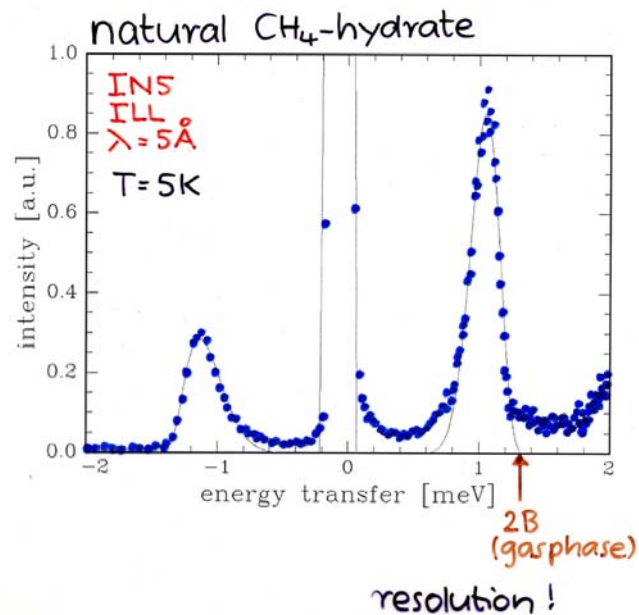
(1) low temperature

(2) natural conditions



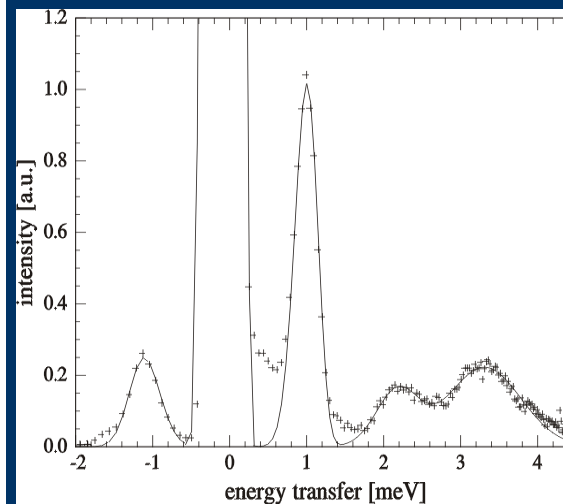
Methan hydrate : natural sample (quantum rotation)

Ch. Gutt et al
Europhys. Lett. 48, 269 (1999)



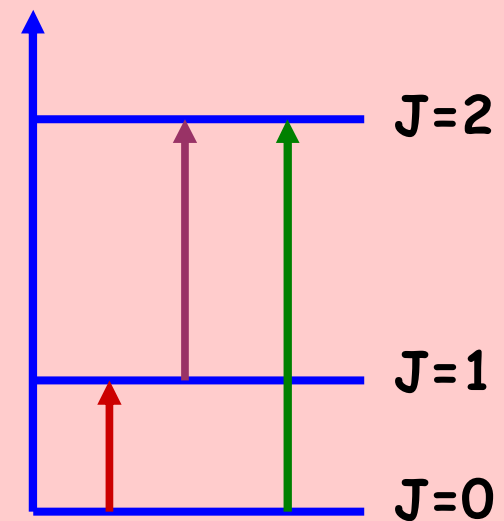
laboratory-made sample:
J.S. Tse, C. Ratcliffe, Y.P. Handa et al
J. Phys. Chem. A101, 4491 (1997)

T=5K



$$\hbar\omega = 1.08 \text{ meV}$$

$$\Delta E/E_{\text{free}} = 20 \%$$



C. Gutt, J. Baumert, W. P. et al. *Europhys. Lett.* 48, 269 (1999) FOCUS / PSI

Mathematical description (2D-Rot.)

Hamiltonian (Operator $J = \frac{\hbar}{2\pi} \frac{d}{d\varphi}$)

$$\begin{aligned} H &= \frac{1}{2} \Theta J^2 + V \\ &= -\frac{\hbar^2}{2\Theta} \frac{d^2}{d\varphi^2} + V \\ &= -B \frac{d^2}{d\varphi^2} + V \end{aligned}$$

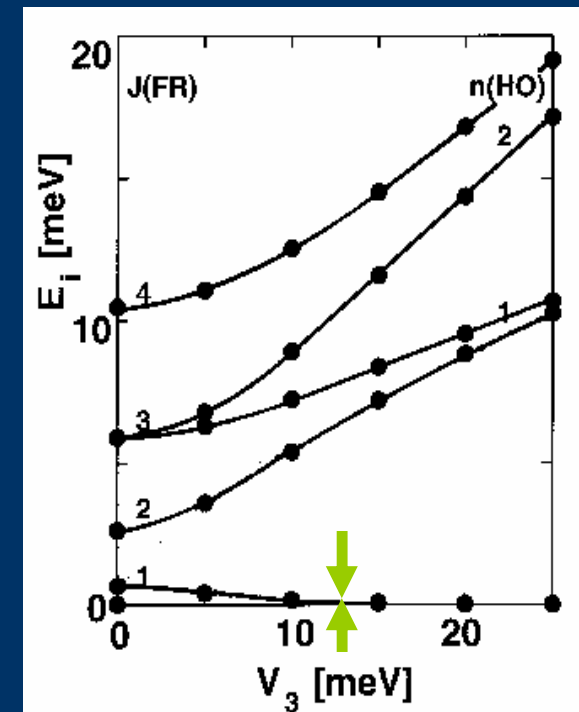
$$V(\varphi) = \sum_{n=1}^N V_{3n} (1 - \cos(3n\varphi))$$

Solutions of the Schrödinger equation

$$\frac{H}{B} \Psi = \frac{E}{B} \Psi$$

Eigen functions:

$$\begin{aligned} \Psi &= \Phi \xi \\ \Phi &= \left(\sum_{m=-M}^M a_m \exp(im\varphi) \right) \\ \xi &: \text{spin-eigenfunction} \end{aligned}$$



F-Rot

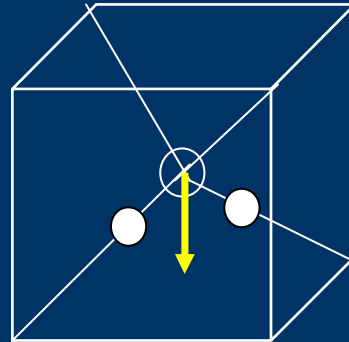
H-Osc

sensitivity: $\hbar\omega_t \sim e^{-\alpha V(\varphi)^n}$

$$\Psi = \Phi_{\text{rot}} \chi_{\text{spin}}$$

Linewidth explained with dipole-octopole interaction & disorder (2 types of cages)

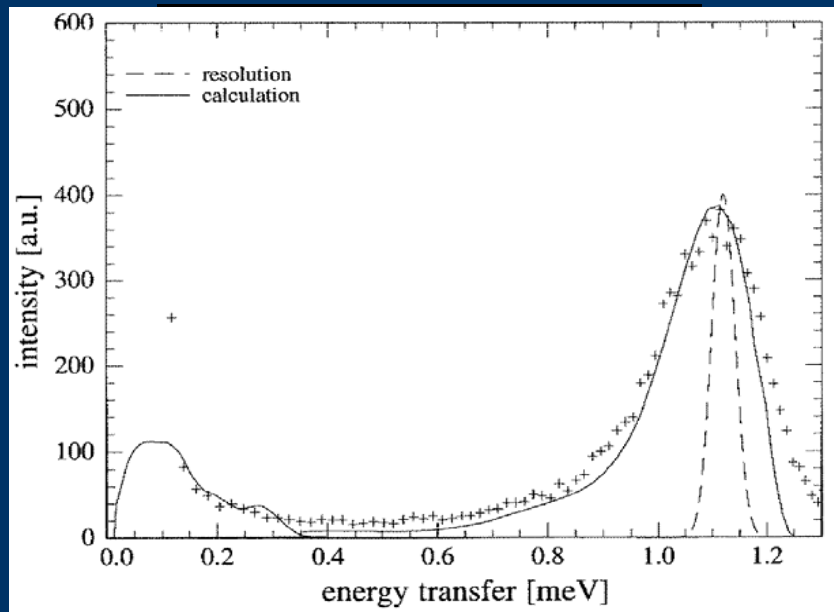
H₂O dipoles
 6 orientations
 (ice rules)



O is tetrahedrally
 coordinated

Electrostatic model
 $q = 0.13e$ on H of CH₄
 $q = 0.8 e$ on H of H₂O

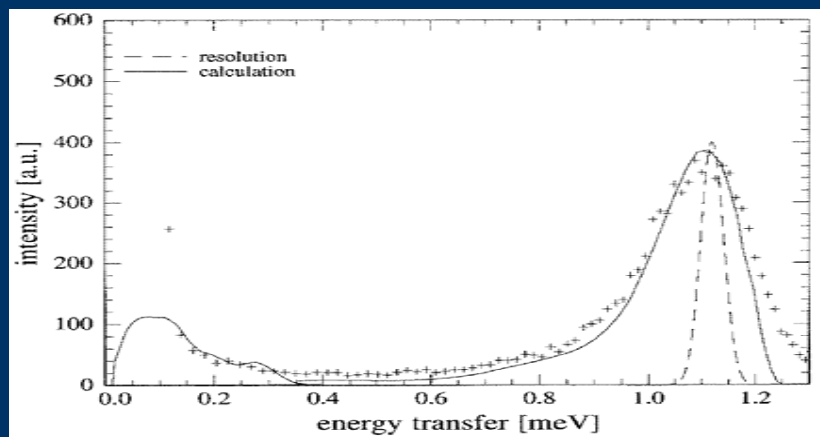
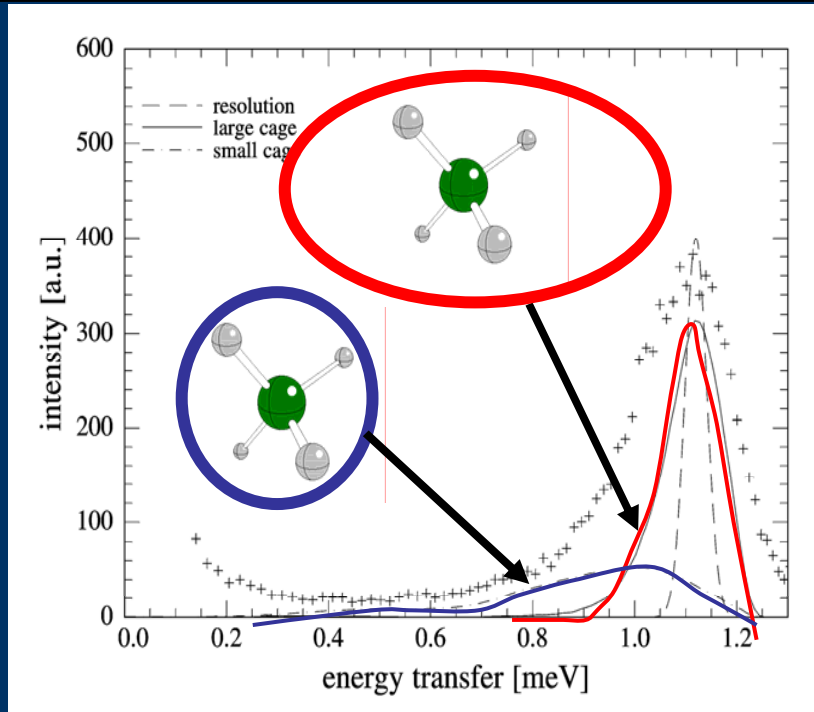
$$W = \iint d\vec{r}_0 d\vec{r} \frac{\rho_{\text{cage}}(\vec{r}_0) \rho_{\text{CH}_4}(\vec{r})}{|\vec{r} - \vec{r}_0|}$$



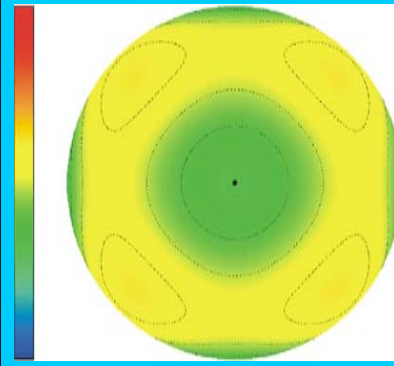
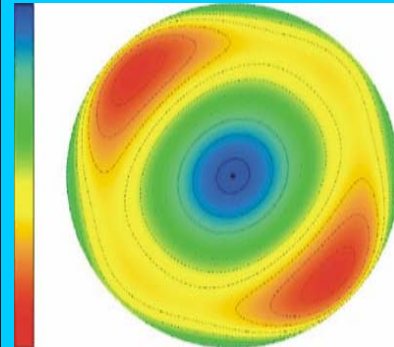
Cubic harmonics $K_{lm}(\theta, \phi)$
 Quaternions τ , $H_{\mu\nu}^{(l)}(\tau)$

Rotational States ($J = 0 \longrightarrow 1$)
 from 2nd order perturbation
 theory
 ~ 3000 configurations

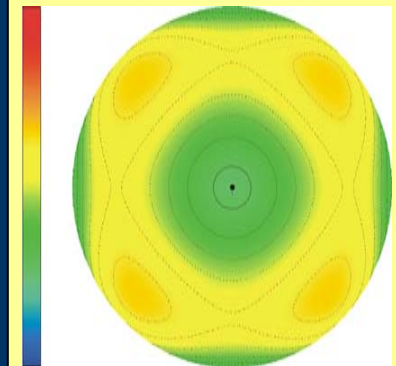
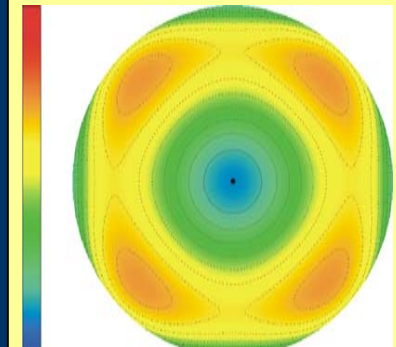
Scattering Length Density in CH₄ Hydrate



Exper/diffr



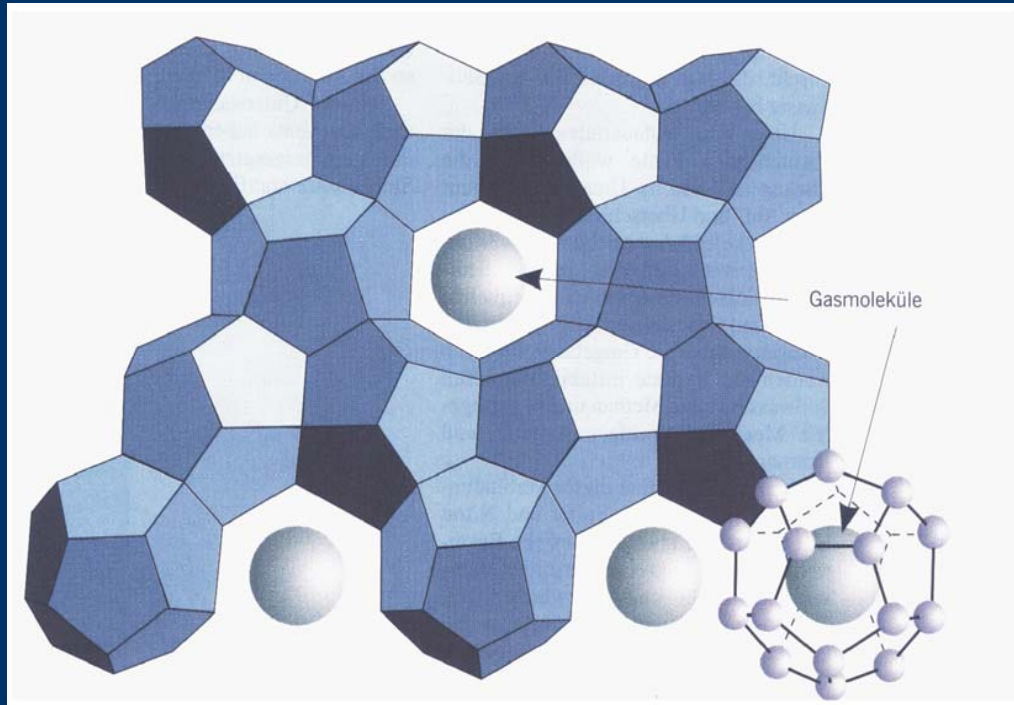
calculated



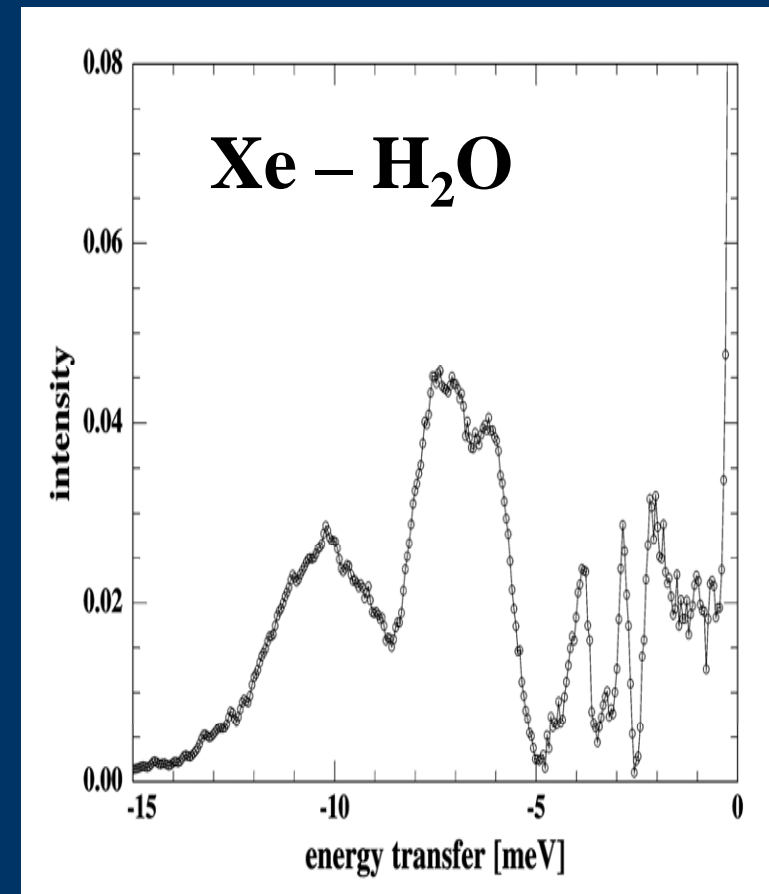
C. Gutt, W. Press, A. Huller, J.S. Tse and H. Casalta, *J. Chem. Phys.*, 114, 4160 (2001)



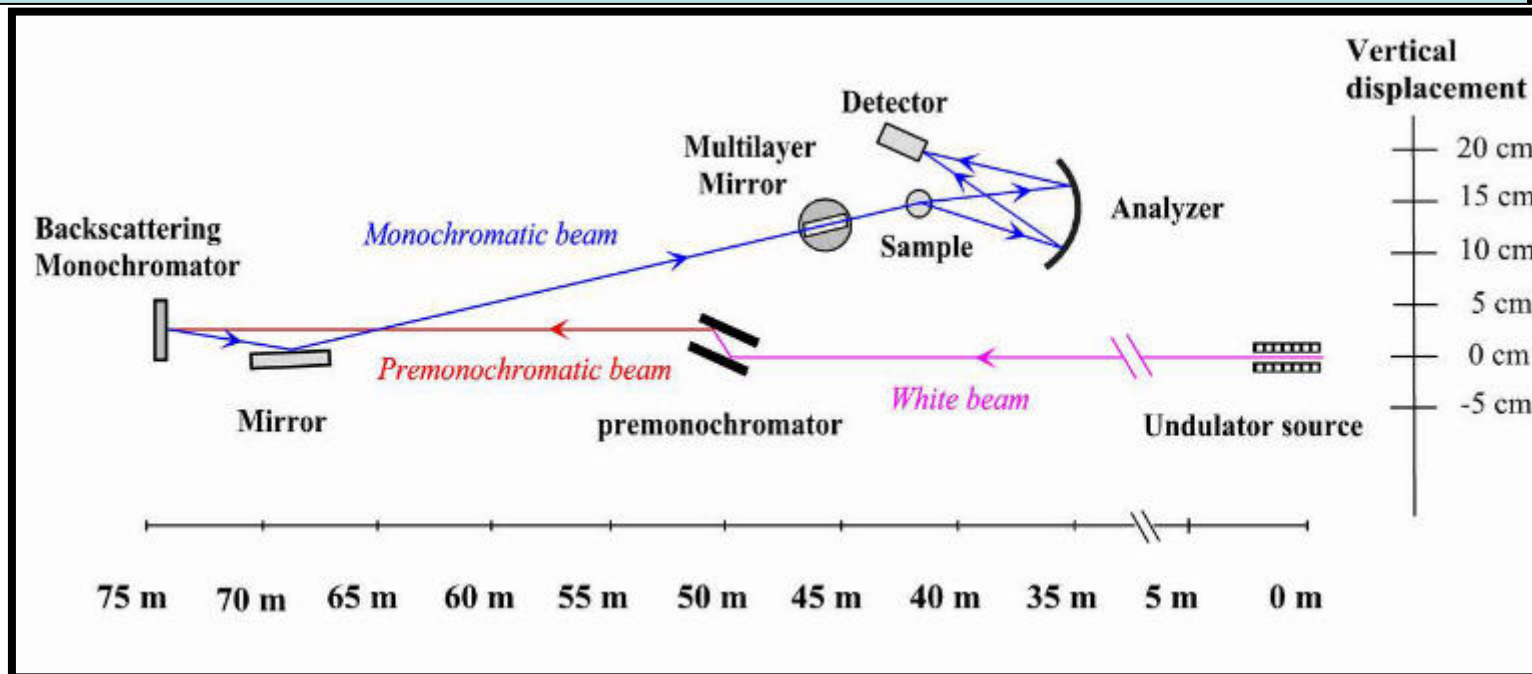
Quantum rotations + translational "rattling" motion



Neutrons
X-ray & nucl. resonance
Simulation
Context: glassy thermal cond.



Inelastic X-ray Spectrometer ID28

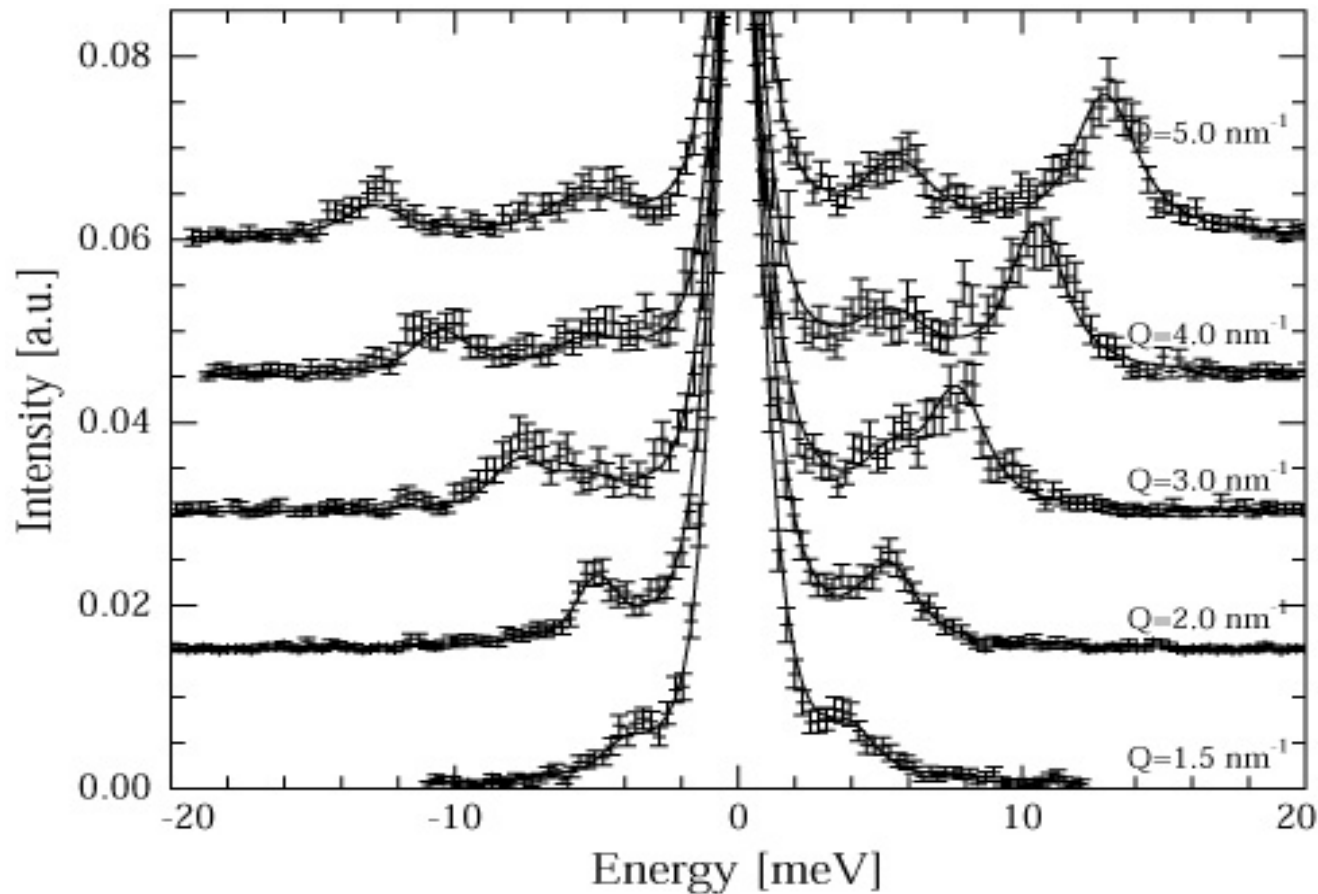


mono- chromator	photon- energy	wavelength	instr. energy- resolution	max. Q-range	aQ betw. analyzers	Q-reso- lution	flux at 200mA (*)
Si(hkl)	[eV]	[Å]	[meV]	[nm ⁻¹]	[nm ⁻¹]	[nm ⁻¹]	[10 ⁹ phot./s]
8 8 8	15817	0.7839	5.5	67.75	2.10	0.245	22.67
9 9 9	17794	0.6968	2.7	76.22	2.38	0.276	5.42
11 11 11	21747	0.5701	1.5	93.16	2.91	0.337	1.81
13 13 13	25703	0.4824	0.9	110.1	3.43	0.399	??

Brillouin Scattering! $I \sim \vec{Q} \cdot \vec{u}$

Lattice Dynamics of Methane Hydrate

Inelastic x-ray spectra $\text{CH}_4 - \text{H}_2\text{O}$ @ 100K



ID28, ESRF

J. Baumert, C. Gutt, V.P. Shpakov, J.S. Tse, M. Krisch, M. Müller, H. Requardt,
D.D. Klug, S. Janssen, and W. Press, *Phys. Rev. B* **68**, 174301 (2003)

backscattering

Some math : difference between
particles (neutrons) and
radiation (X-rays)

$$2d \sin\Theta = \lambda$$

$$2d\Delta\cos\Theta = \Delta\lambda$$

$$\Delta\lambda / \lambda = 2d \cot\Theta \Delta\Theta$$

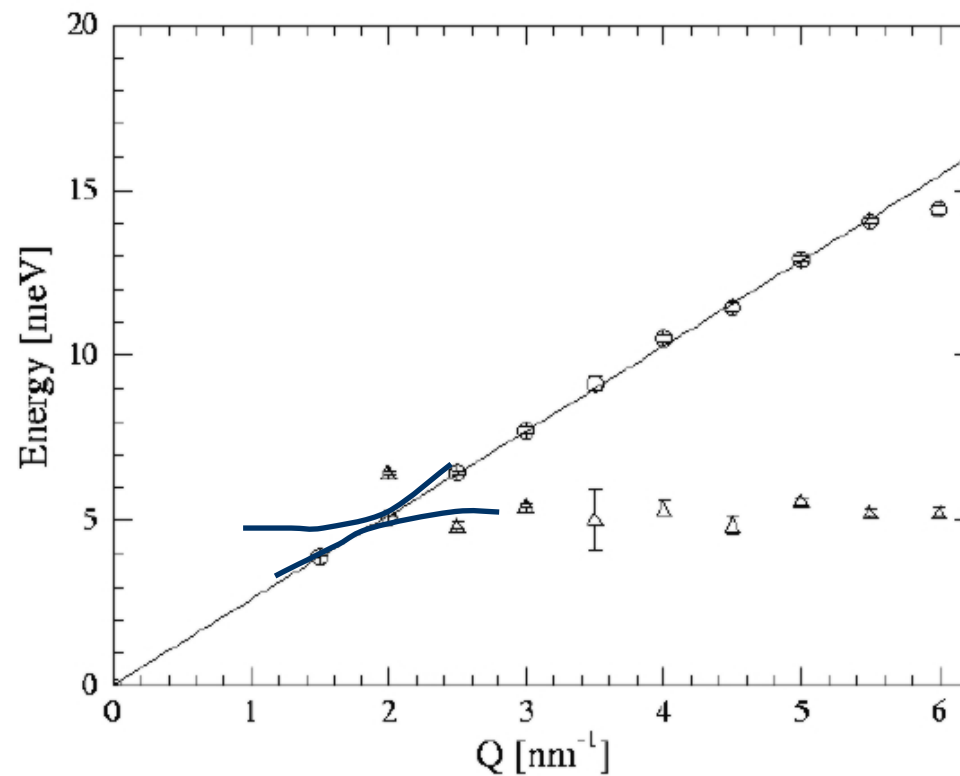
$$\Theta \sim 90^\circ$$

$$E_{\text{ph}} \sim k \sim 1/\lambda$$

$$E_{\text{neutr}} \sim k^2$$

Phonon Dispersion

$\text{CH}_4 - \text{H}_2\text{O} @ 100\text{K}$

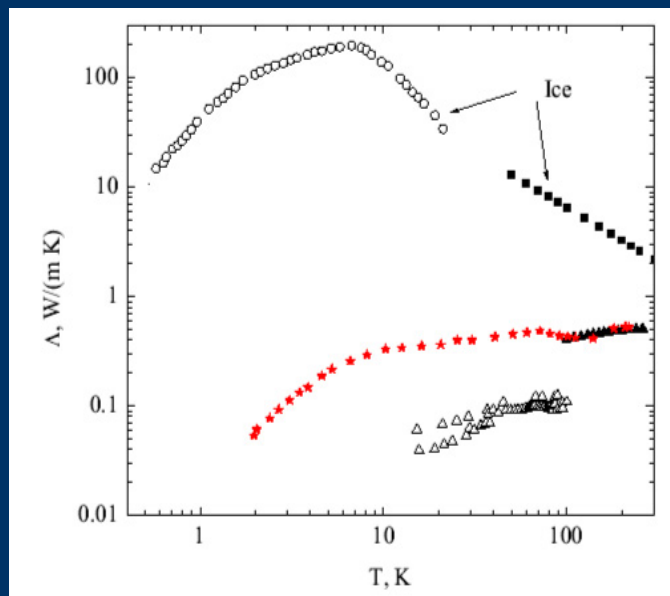


Velocity of Sound: 3900 ± 50 m/s

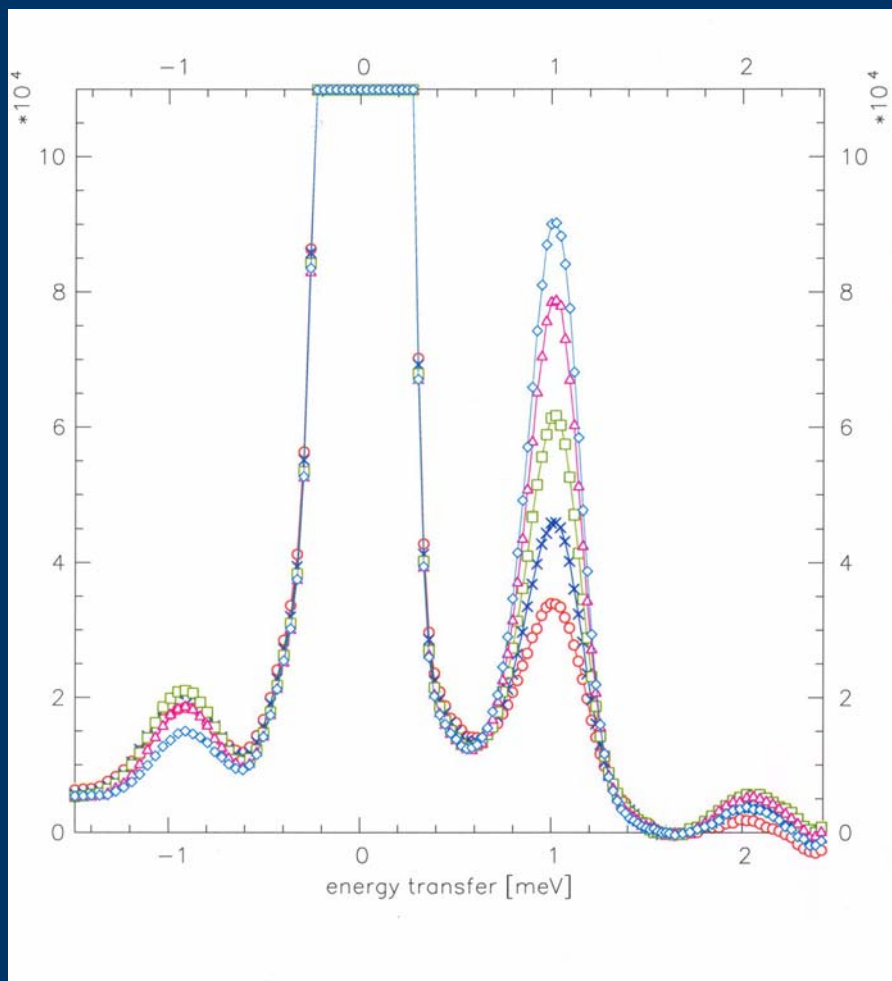
Other related work:

Thermal conductivity: **A. Krivchikov** et al
JCP 2005

^{83}Kr nuclear resonant scattering: J.Tse et al
Nature Mat. Dec 2005



„RT-coupling“ rotation $J = 0 - 1$

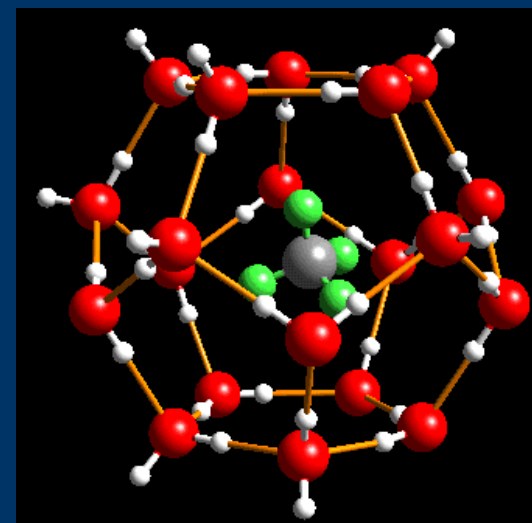


Nuclear spin conversion

$$\Psi = \Phi_{\text{rot}} \chi_{\text{spin}}$$

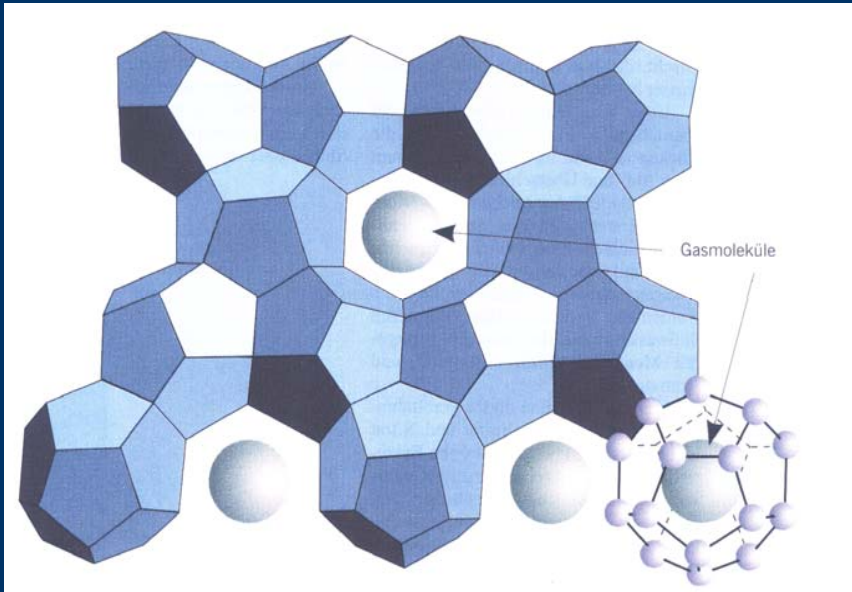
from rotat. thermometer

$$4\text{K} < T_{\text{rot}} < 14\text{K}$$

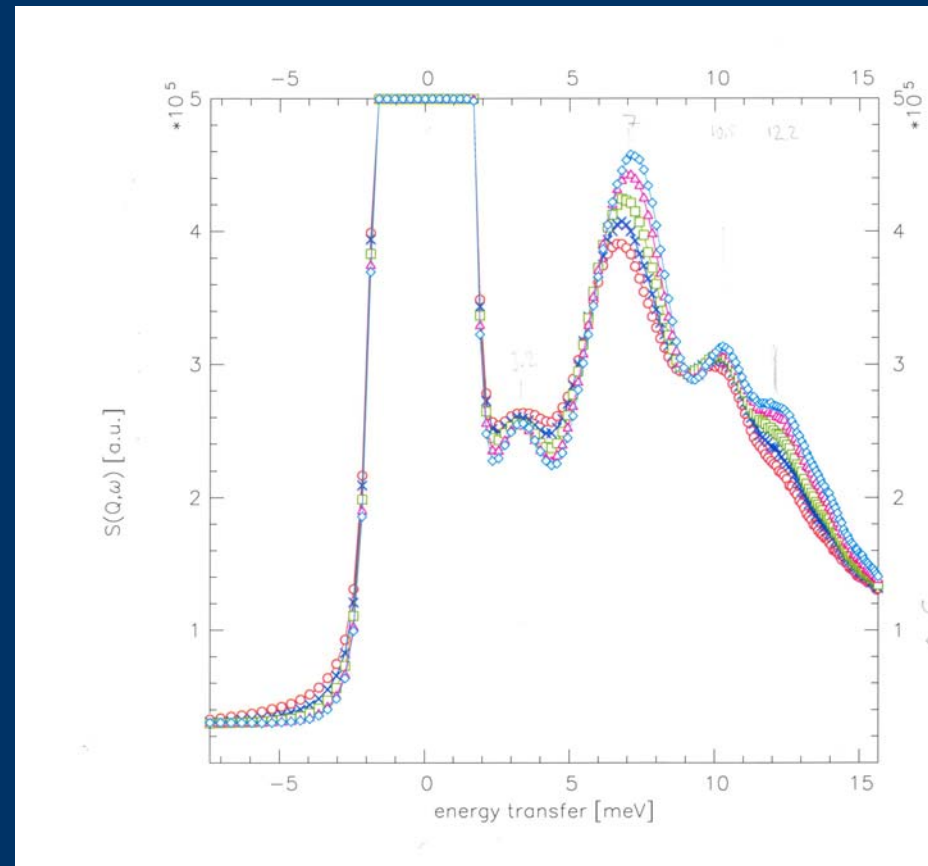
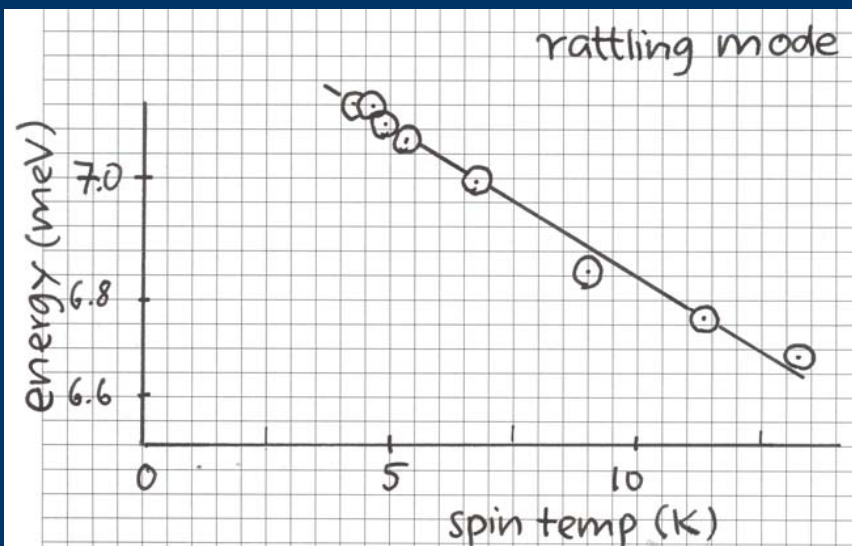


TOF SV29 FRJ2 Juelich

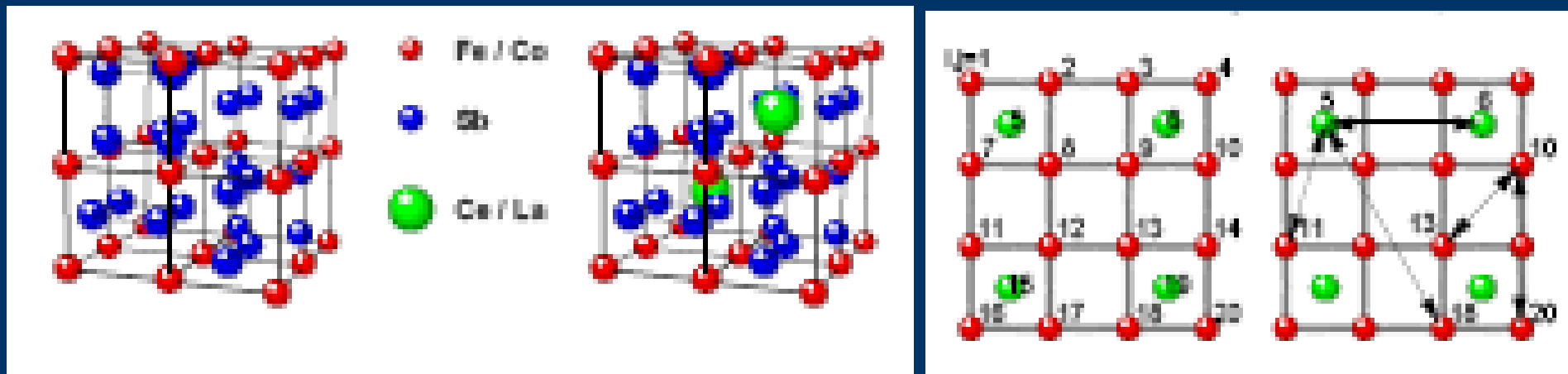
Quantum rotations + translational "rattling" motion



Frequency of rattling
depends on rotational
state



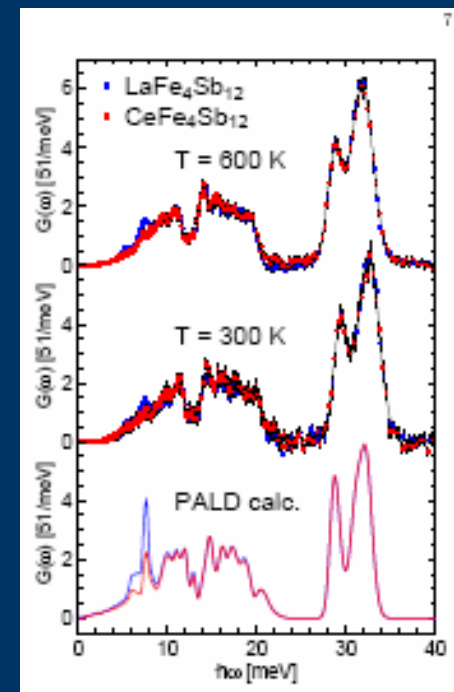
Skutterudites XFe_4Sb_{12}



Spacegroup $Im\bar{3}$ ~ bcc cages $a_0 \sim 10\text{\AA}$
 Filled skutterudites

What is new? electrical conductivity!

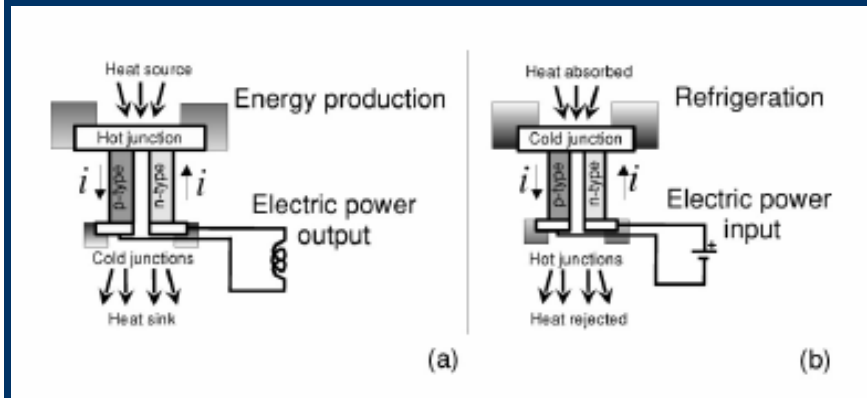
Thanks to
 Marek Koza
 Raphael Herrman



Skutterudites & Si,Ge - clathrates

$$\vec{j} = L (\nabla T)$$

Thermoelectrical effect
 $\text{grad}T, \text{grad}U = E$

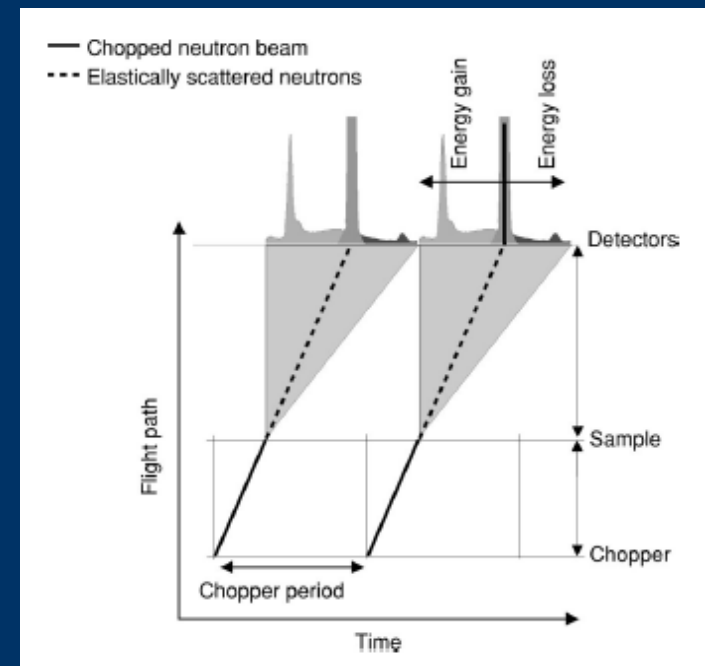
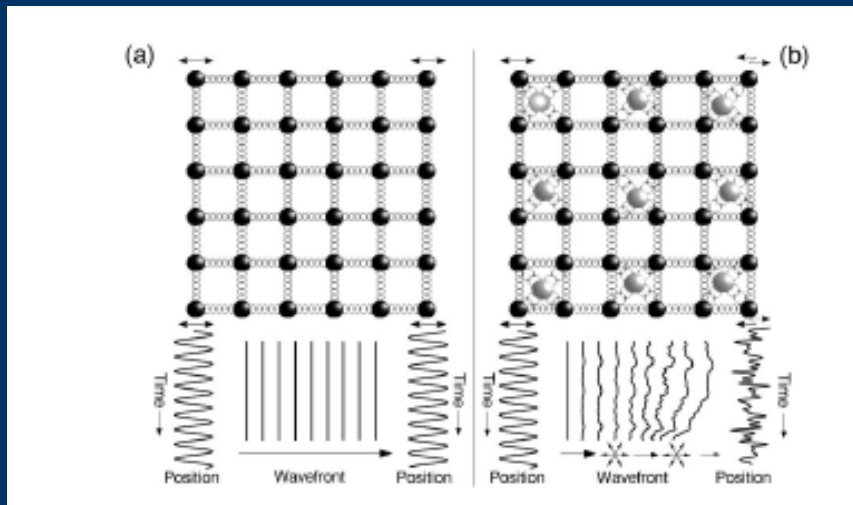


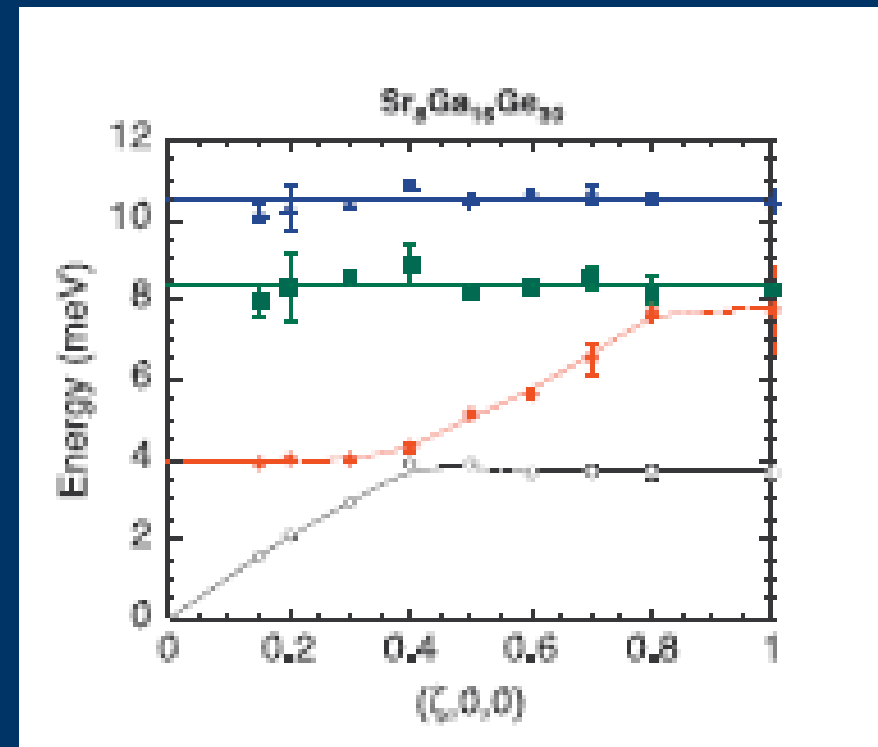
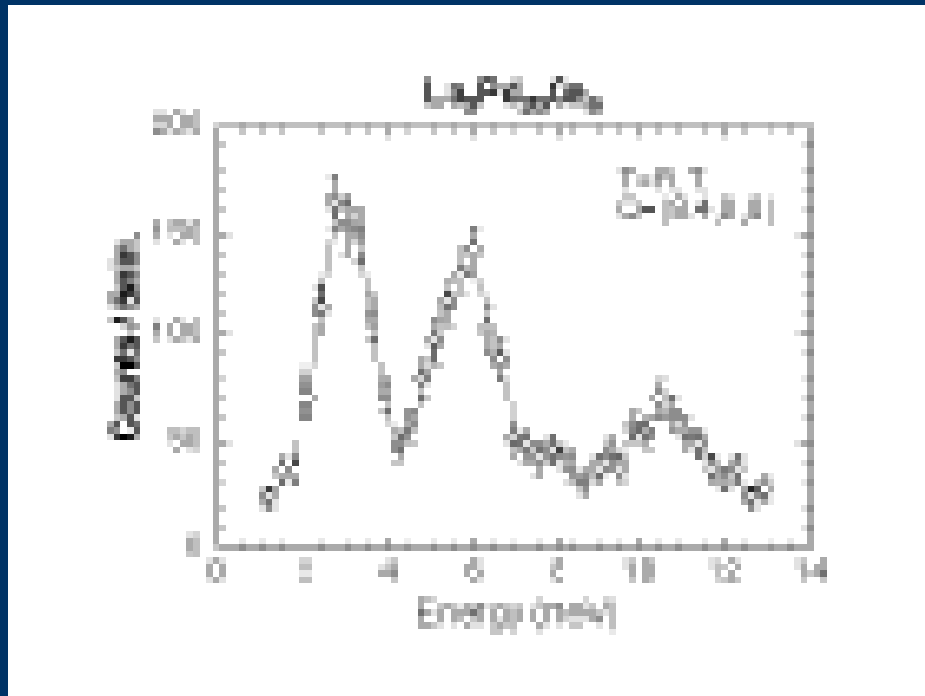
Seebeck (conversion of therm. to electric)
Peltier

No dissipation: good electrical conductivity σ
Large thermal gradient: κ_{th} low
„electron crystal & phonon glass“

Recommended reference for further reading:
R.P.Herrman et al Am. J. Phys. 73, 110-118 (2005)

Einstein-type motion (low-lying optical mode, not Debye-behaviour !) by itself fascinating ...





Rattling modes =
 Einstein modes of guest atoms

~ single particle translational oscillations
 weak coupling (anticrossing, $I_{inel} \neq Q^{*2}$),
 anharmonicity

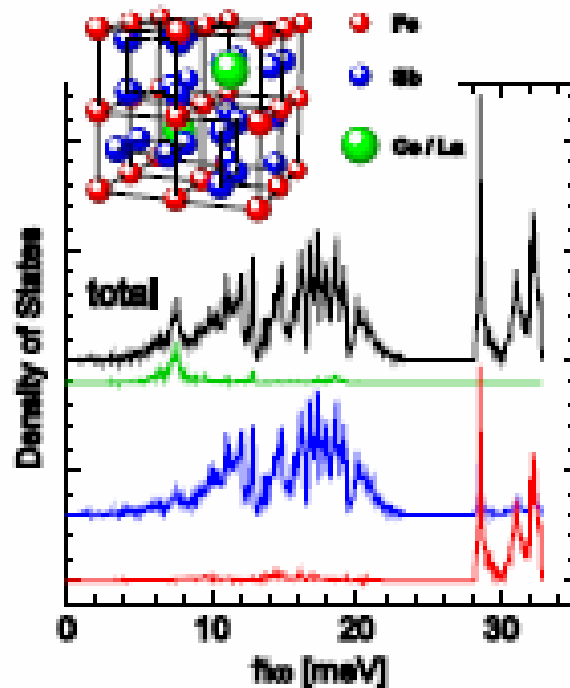


Figure 5: Total and partial density of states as obtained from the *ab initio* lattice dynamics calculation.

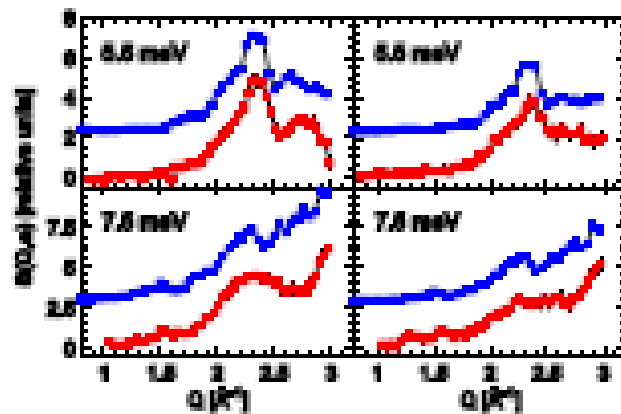


Figure 6: Constant energy slices of the calculated (blue) and measured (red) inelastic intensity of $\text{LaFe}_3\text{Sb}_{12}$ (left) and $\text{CeFe}_3\text{Sb}_{12}$ (right) samples. Calculated data are shifted for clarity.

$I(Q) \sim Q^2$ signals
 single particle behaviour

Departures !

Data from time-of-flight
 IN6, IN4 at ILL
 M. Koza et al, 2006

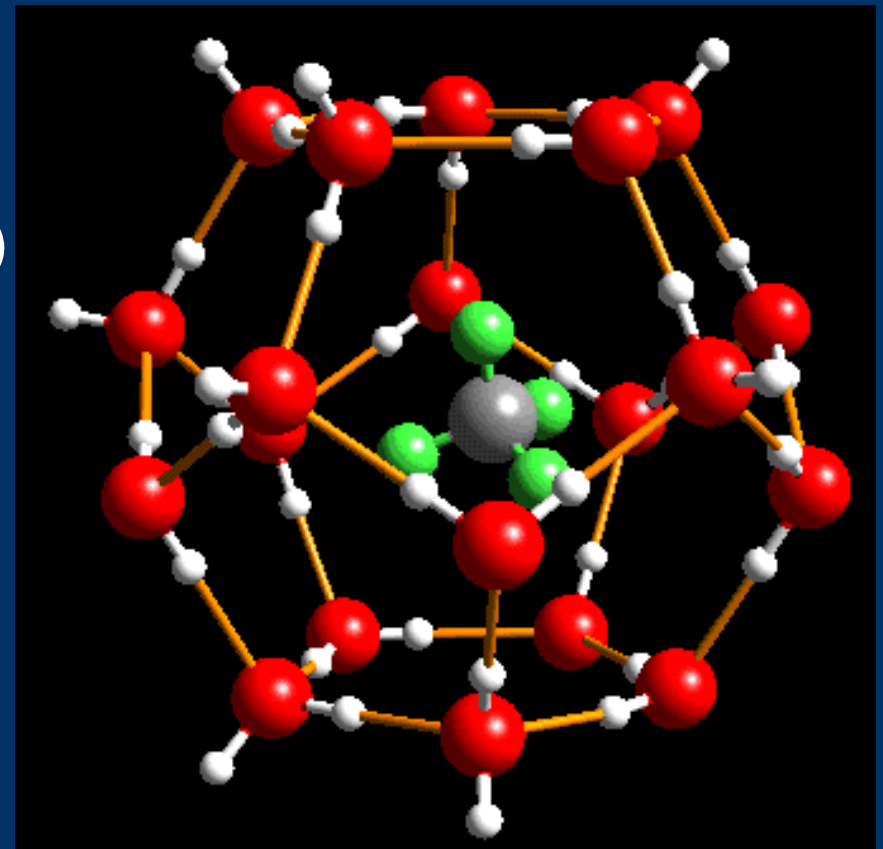
Methane hydrate (burning ice)



Macroscopic
(from Pacific)



Microscopic
(hydrate
cage)



Open

Marine research needs macroscopic features

Thermal conductivity not fully understood

$$H = H_{\text{Trans}} + H_{\text{Rot}} + H_{\text{RT}}$$

(even H_{Trans})

Yet many surprising features
and even some model character

Fin

CH4 II Rotational Excitations

