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Gamma and X ray attenuation and introduction to computed tomography 2

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ELETROMAGNETIC WAVE ATTENUATION IN SOIL PHYSICS

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1.INTRODUCTION

Electromagnetic waves of high energy, like gamma-rays and X-rays, have the property of penetrating into relatively dense materials and are, therefore, very useful for “inside” views. The attenuation of a beam of this kind of radiation is a function of the density of the material, and this fact opens the possibility to study several materials, including soils. We will here give more emphasis to the measurement of soil water contents and bulk densities, but also extend the technique to soil mechanical analysis.

2.GAMMA AND X RAY PROPERTIES

Gamma and X rays are electromagnetic waves like sun or radio waves, which propagate in the vacuum with the speed of light c , each having a characteristic wavelength λ (or frequency f) and, therefore, a characteristic energy E :

$$E = hf \quad ; \quad c = \lambda f = \text{constant}$$

h being Plank´s constant.

Radiation type	Wave length λ (μm)
Gamma	$4 \times 10^{-8} - 1 \times 10^{-4}$
X	$1 \times 10^{-5} - 0.01$
ultra violet	0.01 – 0.38
visible light	0.38 – 0.78
infrared	0.78 – 1.000

Gamma rays are originated from unstable nuclei, while X rays are the consequence of electron energy loss during target bombardment or due to electron jumps between different atomic energy levels.

Table 1 lists radioisotopes mostly used as gamma-radiation sources. From these, the most commonly used in soil science are Americium, Cesium and Cobalt. X rays are produced by high voltage

Table 1. Radioisotopes suitable for gamma attenuation experiments in soils

Radioisotopes	Half-life (years)	Main energy peaks (KeV)
²⁴¹ Am (Americium)	458	60
¹⁰⁹ Cd (Cadium)	1.24	88
¹⁴⁴ Ce (Cerium)	0.78	134
¹³⁴ Cs (Cesium)	2.50	570
¹³⁷ Cs (Cesium)	30	662
⁶⁰ Co (Cobalt)	5.30	1173
¹⁹² Ir (Iridium)	0.20	296
²² Na (Sodium)	2.60	511

KeV= kiloelectron-volt, an energy unit: 1eV= 1,602x10⁻¹⁹ J

When gamma or X radiation interacts with the atoms of the matter they are passing through, mainly three processes may occur, which are responsible for the attenuation of the beam. For low energy radiation the photo-electric process is very probable (Figure 1).

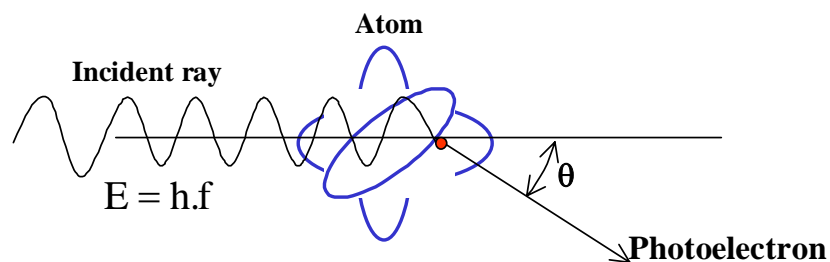


Figure 1: Photo-electric effect.

By this process, the photon (or gamma, X ray) collides with an inner shell electron, is completely absorbed, and as a consequence the electron is ejected from the atom.

For medium energy photons the Compton-effect is the most probable (Figure 2).

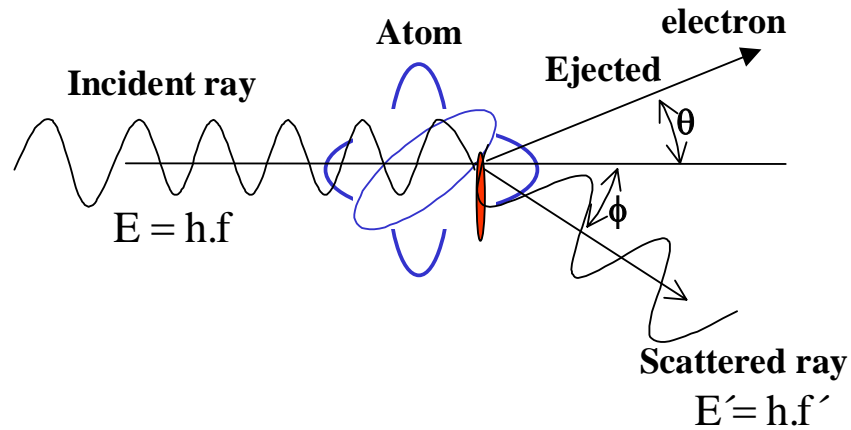


Figure 2: Compton effect.

Here a photon also collides with an electron, but there is only partial energy loss and the ray is deviated from its original trajectory. Through this process gamma or X radiation is scattered and changes its energy and wavelength.

Only for energies higher than 1.02 MeV, photons may interact with target nuclei and become transformed in an electron and a positron. This process is called pair-production (Figure 3).

$$E = h.f = 2.m_0.c^2 = 1.02 \text{ MeV}$$

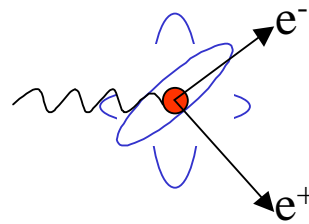


Figure 3. Pair-production effect.

Due to these and other less probable processes, a gamma or X-ray beam of a given intensity I becomes attenuated when passing through matter. The attenuation process depends on

the energy of the photons, on the nature and density of the target matter and on the length of the travel path of the radiation through this matter. For a mono-energetic radiation beam, Beer's law is valid:

$$I = I_0 \exp(-\mu \rho x) \quad (1)$$

where I_0 is the incident beam intensity [number of photons per unit area (cm^2) per unit time (s), or counts per s (cps), or counts per minute (cpm)]; I the transmitted beam intensity; μ the mass attenuation coefficient (cm^2/g); ρ the density of the absorbing material (g/cm^3); x the absorption length (cm). Figure 4 illustrates the process.

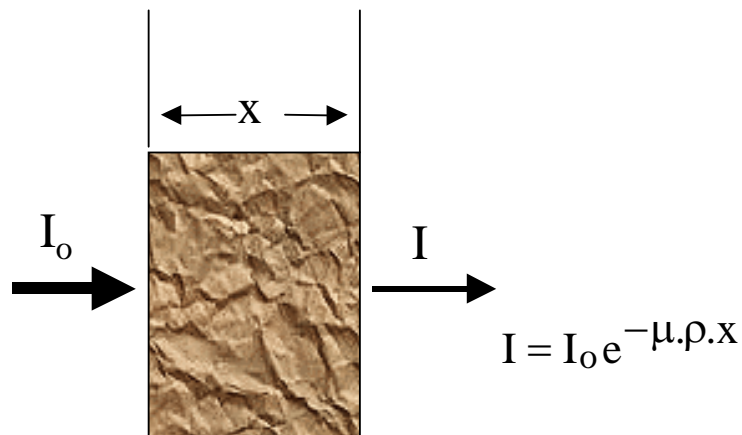


Figure 4. Schematic diagram of the attenuation process of a monoenergetic radiation beam by a homogeneous material of thickness x , density ρ and attenuation coefficient μ .

The absorption coefficient μ is a function of the absorbing material and of the energy of the gamma or X rays. Knowing μ and measuring I_0 and I , the attenuation process can be used to measure ρ if x is known, or to measure x if ρ is known, using equation (1). The coefficient μ is more correctly called mass absorption coefficient. Since the product $\mu \rho x$ has to be dimensionless, μ is given in cm^2/g , when the density is given in g/cm^3 and x in cm. For a given radiation and a given material (aluminum, acrylic plastic, glass, water, etc) they are constant, and are determined in the laboratory. This is the principle of the process.

Very important details, which will not be treated here, are i. source intensity; ii. beam collimation; iii. counting equipment; iv. peak definition; v. mass absorption coefficient measurement; etc. Radiation safety has also to be mentioned. In general, to collimate radiation beams, gamma sources or X-ray tubes, are involved in lead (Pb) shields, of calculated thickness to protect the operator. Radiation is only allowed to pass through a collimation whole, which defines the cross section of the beam (circular, rectangular, generally with less than 1 cm²). Along the beam direction, radiation levels are high and care should be taken in order not to expose hands and other parts of the body to this radiation. When manipulating samples within the beam path, the collimation whole should be closed with a lead shield (Figure 5).

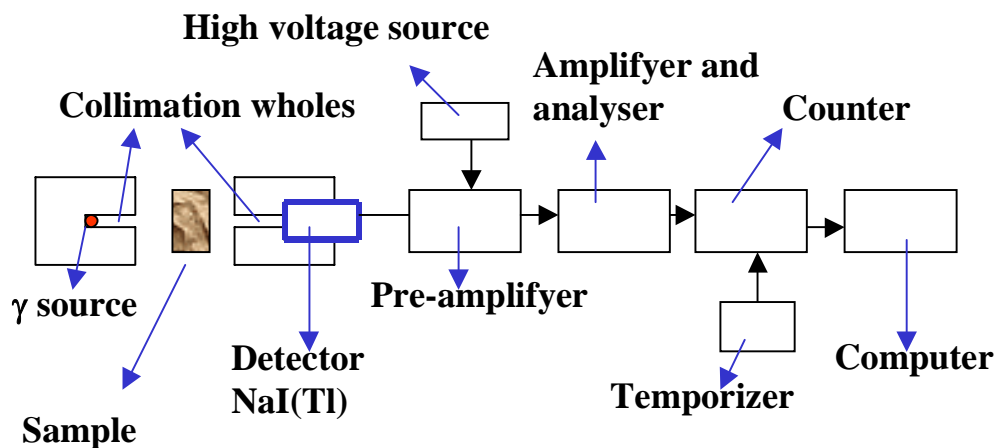


Figure 5: Schematic view of a gamma-attenuation system.

3. ATTENUATION IN SOILS

Soils are not homogeneous and equation (1) must be extended for heterogeneous materials. We will assume that the solid fraction of one given soil is homogeneous and so a moist soil sample consists of soil, water and air, and its thickness x can be represented by:

$$x = x_s + x_w + x_a \quad (2)$$

where $x_s + x_w + x_a$ are the equivalent thickness of solids, water and air, within x .

Since a soil sample generally comes in a container, and the radiation source is located at a “fair” distance from the radiation detector the total radiation absorbing distance x from source to the detector will be:

$$x_t = x_{a1} + 2x_c + x_s + x_w + x_a + x_{a2} \quad (3)$$

Figure 6 illustrates schematically these distances, indicating air and container thickness (x_{a1} , x_{a2} , x_c). Considering the attenuation process as additive, equation (1) for the system described in Figure 6, is extended to:

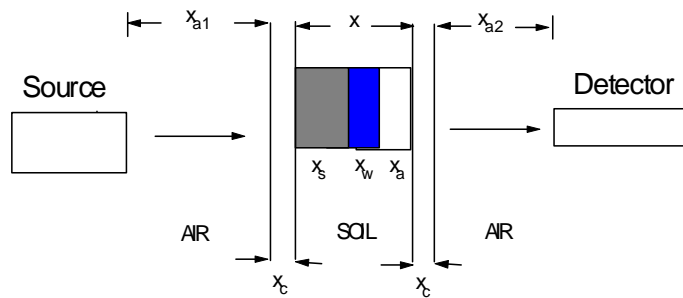


Figure 6: Schematic diagram of attenuation distances for a soil sample packed in a container.

$$I = I_0 \exp \{ -[\mu_a \rho_a (x_{a1} + x_a + x_{a2}) + 2\mu_c \rho_c x_c + \mu_s \rho_s x_s + \mu_w \rho_w x_w] \} \quad (4)$$

If I_0 is measured with the empty container, the constant attenuation of the air outside the soil and container is already taken care of, and recognizing that:

$$\rho_s x_s = d_b x \quad \text{and} \quad \rho_w x_w = \theta x$$

where: ρ_s = density of soil particles, (g.cm^{-3});

d_b = soil bulk density, (g.cm^{-3});

ρ_w = density of water, (g.cm^{-3});

θ = soil water content, ($\text{cm}^3.\text{cm}^{-3}$).

equation (4) reduces to:

$$I = I_0 \exp \{-x(\mu_s d_b + \mu_w \theta)\} \quad (5)$$

Using carefully measured values of I_0 , I , x , μ_s and μ_w , soil bulk density d_b and soil water content θ , can be estimated within the volume of the path of the radiation beam. Rearranging equation (5) we have:

$$d_b = \frac{1}{\mu_s} \left[\ln \left(\frac{I_0}{I} \right) + \mu_w \theta \right] \quad (6)$$

and

$$\theta = \frac{1}{\mu_w} \left[\ln \left(\frac{I_0}{I} \right) + \mu_s d_b \right] \quad (7)$$

The great difficulty in using equations (6) and (7) is that to measure d_b one needs to know θ and to measure θ one needs know d_b . For monoenergetic gamma or X-ray beams, the only possibilities are the measurement of d_b in a dry sample ($\theta = 0$) and the measurement of θ in the same soil sample considering d_b invariant in time and in θ , with previous measurement of d_b .

Since μ_s and μ_w are functions of the energy of the radiation, if a convenient choice of a double-energy (E_1 and E_2) radiation beam is made, which determines significantly different values of μ_s and μ_w , soil bulk density d_b and soil water content θ can be measured simultaneously solving the set of equations:

$$\text{For } E_1: I_1 = I_{01} \exp \{-x(\mu_{s1} d_b + \mu_{w1} \theta)\} \quad (5a)$$

$$\text{For } E_2: I_2 = I_{02} \exp \{-x(\mu_{s2} d_b + \mu_{w2} \theta)\} \quad (5b)$$

The solution is:

$$d_b = \frac{\left[\mu_{w1} \ln\left(\frac{I_2}{I_{o2}}\right) - \mu_{w2} \ln\left(\frac{I_1}{I_{o1}}\right) \right]}{x(\mu_{s1}\mu_{w2} - \mu_{s2}\mu_{w1})} \quad (8)$$

$$\theta = \frac{-\left[\mu_{s1} \ln\left(\frac{I_2}{I_{o2}}\right) - \mu_{s2} \ln\left(\frac{I_1}{I_{o1}}\right) \right]}{x(\mu_{s1}\mu_{w2} - \mu_{s2}\mu_{w1})} \quad (9)$$

The use of equations (6), (7), (8) and (9) implies in the knowledge of the attenuation coefficients μ_i . Ferraz and Mansel (1979) present values for several soils and for water, for several radiation energies. Some of them are reproduced in Table 2. As it can be seen from the μ_i values of soils, for Americium and for Cesium, these two sources are a very good choice for a double energy beam. Since μ_i values vary from soil to soil, they have to be determined for each soil. This is easily done through equation (1), using an artificially packed dry soil sample of known bulk density d_b , as it will be seen in Example 1.

Table 2. Soil and other absorber materials mass attenuation coefficients μ_i for 60 keV (^{241}Am) and 662 (^{137}Cs) keV gamma photons

Material	Clay (%)	Silt (%)	Sand (%)	μ_i ($\text{cm}^2 \cdot \text{g}^{-1}$)	
				60 keV	662 keV
Dark red latosol	48	31	21	0.31647	0.07424
Yellow red latosol	17	10	73	0.27501	0.07834
Red yellow podsol	8	10	82	0.26411	0.07755
Alluvial soil	33	43	24	0.30440	0.07837
Regosol	16	9	75	0.25518	0.07724
Washed sand	-	-	100	0.25008	0.07666
Water (distilled)	-	-	-	0.20015	0.08535

Example 1: To measure the mass absorption coefficient of a soil for the gamma radiation of ^{137}Cs (662 keV), an oven dry soil sample was used, packed in a rectangular acrylic container of absorption thickness $x = 5.7$ cm and a bulk density ρ of 1.473 g.cm^{-3} . The measured gamma intensities were $I_0 = 102,525$ cpm (container without soil) and $I = 53,575$ cpm (container with homogeneously packet dry soil). In this case:

$$53575 = 102525 \exp(-\mu_s \times 1.473 \times 5.7)$$

and

$$\mu_s = 0.0773 \text{ cm}^2.\text{g}^{-1}$$

Example 2: Using the same container filled with distilled water, the attenuated gamma intensity changed to $I = 63156$ cpm. Therefore:

$$63156 = 102525 \exp(-\mu_w * 1.000 * 5.7)$$

and

$$\mu_w = 0.0850 \text{ cm}^2.\text{g}^{-1}$$

Example 3: A soil sample of thickness 6.62 cm is submitted to a double gamma ray beam and the following data was obtained:

Radiation 1:

$$I_{01} = 253,428 \text{ cpm}$$

$$I_1 = 4,776 \text{ cpm}$$

$$\mu_{s1} = 0.40139 \text{ cm}^2.\text{g}^{-1}$$

$$\mu_{w1} = 0.20015 \text{ cm}^2.\text{g}^{-1}$$

Radiation 2:

$$I_{02} = 116,438 \text{ cpm}$$

$$I_2 = 48,574 \text{ cpm}$$

$$\mu_{s2} = 0.07881 \text{ cm}^2.\text{g}^{-1}$$

$$\mu_{w2} = 0.08535 \text{ cm}^2.\text{g}^{-1}$$

Using equations (5a) and (5b) we have:

$$4,776 = 253,428 \exp[-6.62 (0.40139d_b + 0.200150)]$$

$$48,574 = 116,438 \exp [-6.62 (0.07881d_b + 0.08535\theta)]$$

and solving this set of equations we obtain:

$$d_b = 1.340 \text{ g.cm}^{-3} \quad \text{and} \quad \theta = 0.310 \text{ cm}^3.\text{cm}^{-3}$$

4.EXPERIMENTAL ERRORS ASSOCIATED IN d_b AND θ MEASUREMENTS

4.1.Sample Thickness x

Sample thickness x is critical and has to be measured carefully, with minimal errors. In example 3 (above) if x would be 6.52 instead of 6.62 cm, i.e., with an error of 1.5%, the values of d_b and θ would be 1.361 and 0.314, respectively.

Since the radiation attenuation process is exponential, the reduction of I_0 is very high, and directly related to the sample thickness x . In Example (3) we observe a reduction of I_0 of 98% for radiation 1 (low energy) and of 58% for radiation 2 (high energy). If x is increased excessively the values of I for the low energy gamma become too small, compromising counting statistics. Ferraz and Mansel (1979) show that there is an optimum thickness x^* in terms of measurement statistics, which depends on the type of radiation and of the values of d_b and θ . Too thin samples or too large samples introduce great errors in the measurements. They show that x^* is given by:

$$x^* = \frac{2}{\mu_s d_b + \mu_w \theta} \quad (10)$$

For the data of example 3 we have:

$$\text{Radiation 1: } x_1^* = \frac{2}{0.40139 \times 1.34 + 0.20015 \times 0.31} = 3.3 \text{ cm}$$

$$\text{Radiation 2: } x_2^* = \frac{2}{0.07881 \times 1.34 + 0.08535 \times 0.31} = 15.1 \text{ cm}$$

Since x is more critical for the low energy, when using double beams, x has to be closer to x^* for the low energy. For the above example, $x = 6.62$ is a fairly good choice. More details for the choice of x are found in Ferraz and Mansel (1979).

4.2.Errors in d_b and θ Measurements

Ferraz and Mansel (1979) show that the minimum resolvable changes σ of d_b and θ , when using a monoenergetic beam, are:

$$\sigma_{d_b} = \frac{1}{x\mu_s\sqrt{I_0}} \exp\left[\frac{x}{2}(\mu_s d_b + \mu_w \theta)\right] \quad (11)$$

$$\sigma_{\theta} = \frac{1}{x\mu_w\sqrt{I_0}} \exp\left[\frac{x}{2}(\mu_s d_b + \mu_w \theta)\right] \quad (12)$$

As it can be seen, the minimum resolvable changes σ depend on all parameters and measurements of the attenuation process: I_0 , x , μ_s , μ_w , d_b and θ . For example 3 analyzing separately the case of each radiation, we have,

Radiation 1:

$$\sigma_{d_{b1}} = \frac{1}{6.62 \times 0.40139 (253428)^{1/2}} \exp\left[\frac{6.62}{2} (0.40139 \times 1.43 + 0.20015 \times 0.31)\right]$$

$$\sigma_{\theta_1} = \frac{1}{6.62 \times 0.20015 (253428)^{1/2}} \exp\left[\frac{6.62}{2} (0.40139 \times 1.43 + 0.20015 \times 0.31)\right]$$

and

$$\sigma_{d_{b1}} = 0.006 \text{ g.cm}^{-3} \quad ; \quad \sigma_{\theta_1} = 0.012 \text{ cm}^3.\text{cm}^{-3}$$

Radiation 2:

$$\sigma_{db2} = \frac{1}{6.62 \times 0.07881 (116438)^{1/2}} \exp \left[\frac{6.62}{2} (0.07881 \times 1.43 + 0.08535 \times 0.31) \right]$$

$$\sigma_{\theta 2} = \frac{1}{6.62 \times 0.08535 (116438)^{1/2}} \exp \left[\frac{6.62}{2} (0.07881 \times 1.43 + 0.08535 \times 0.31) \right]$$

and

$$\sigma_{db2} = 0.009 \text{ g.cm}^{-3} \quad ; \quad \sigma_{\theta 2} = 0.008 \text{ cm}^3.\text{cm}^{-3}$$

indicating errors of about 0.5% for bulk density and 3.2% for water content measurements.

When using the double beam, a system of equations is solved and parameters of both radiations interfere in the measurements of d_b and θ . For this case:

$$\sigma_{db} = \frac{\left[\frac{(\mu_{w1})^2}{I_2} + \frac{(\mu_{w2})^2}{I_1} \right]^{1/2}}{x(\mu_{s1}\mu_{w2} - \mu_{s2}\mu_{w1})} \quad (13)$$

$$\sigma_{\theta} = \frac{\left[\frac{(\mu_{s1})^2}{I_2} + \frac{(\mu_{s2})^2}{I_1} \right]^{1/2}}{x(\mu_{s1}\mu_{w2} - \mu_{s2}\mu_{w1})} \quad (14)$$

For the data of example 3, using the double beam, we have:

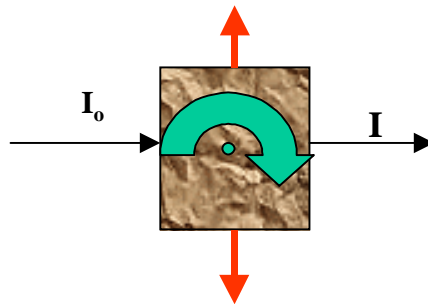
$$\sigma_{db} = \frac{\left[\frac{(0.20015)^2}{48574} + \frac{(0.08535)^2}{4776} \right]^{1/2}}{6.61(0.40139 \times 0.08535 - 0.20015 \times 0.07881)} = 0.012 \text{ g.cm}^{-3}$$

$$\sigma_o = \frac{\left[\frac{(0.40139)^2}{48574} + \frac{(0.07881)^2}{4776} \right]^{1/2}}{6.61(0.40139 \times 0.08535 - 0.20015 \times 0.07881)} = 0.018 \text{ g.cm}^{-3}$$

indicating errors of 0.8% and 5.8% for d_b and θ , respectively. As it can be seen, although the double gamma technique is an improvement, the measurements have greater errors as compared to the mono gamma technique.

5.FURTHER IMPROVEMENTS AND APPLICATIONS OF THE TECHNIQUE

One shortcoming of the gamma or X-ray attenuation technique is the measurement of x , which is critical for the estimation of d_b and θ , and difficult to be measured accurately for odd shaped samples like a soil clod. It is only easy to be measured for cases of soil samples packed in rectangular or cylindrical acrylic containers, precisely manufactured. In other cases, like plants growing in commercial soil plots or even soil clods, it is very difficult to measure x , which varies for each measurement point. One solution to this problem could be the use a triple energy beam, leaving x also as an unknown? This is not possible because x multiplies d_b and θ in equation (5) and the three resulting simultaneous equations will not be independent.



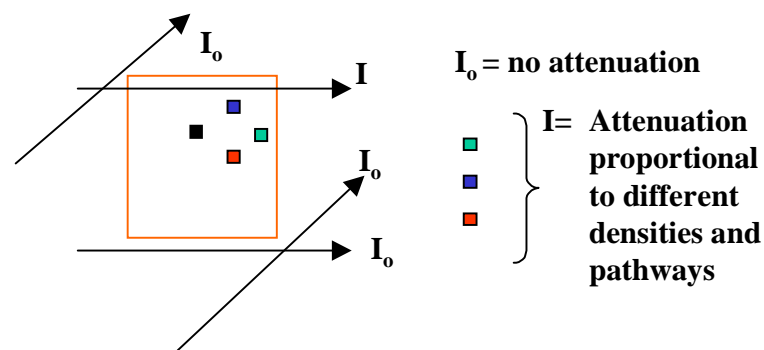


Figure 7: Schematic principle of tomography

So, as things stand today, x has to be measured as precisely as possible for mono and double beam attenuation measurements. One improvement has, however, been made through the computed tomography. This technique, first introduced into Soil Science by Crestana et al (1985) gives d_b and θ distributions in irregularly shaped soil samples, without the need of measuring x . When taking a tomography, the sample rotates around an axis and a very high number of attenuation measurements are made within the rotation plane, which involve different beam paths, each having its x , d_b and θ . Solving all these unknowns through computation one obtains the d_b or θ distribution over the rotation plane, i.e., a cross-section “picture” is obtained, indicating the d_b or θ distribution, with a resolution (pixel) that can go down to 1 mm^2 . Vaz et al (1992) gives more details of the technique.

The last improvement, not yet published, is the two media measurement, by which the thickness x is measured indirectly by the attenuation process.

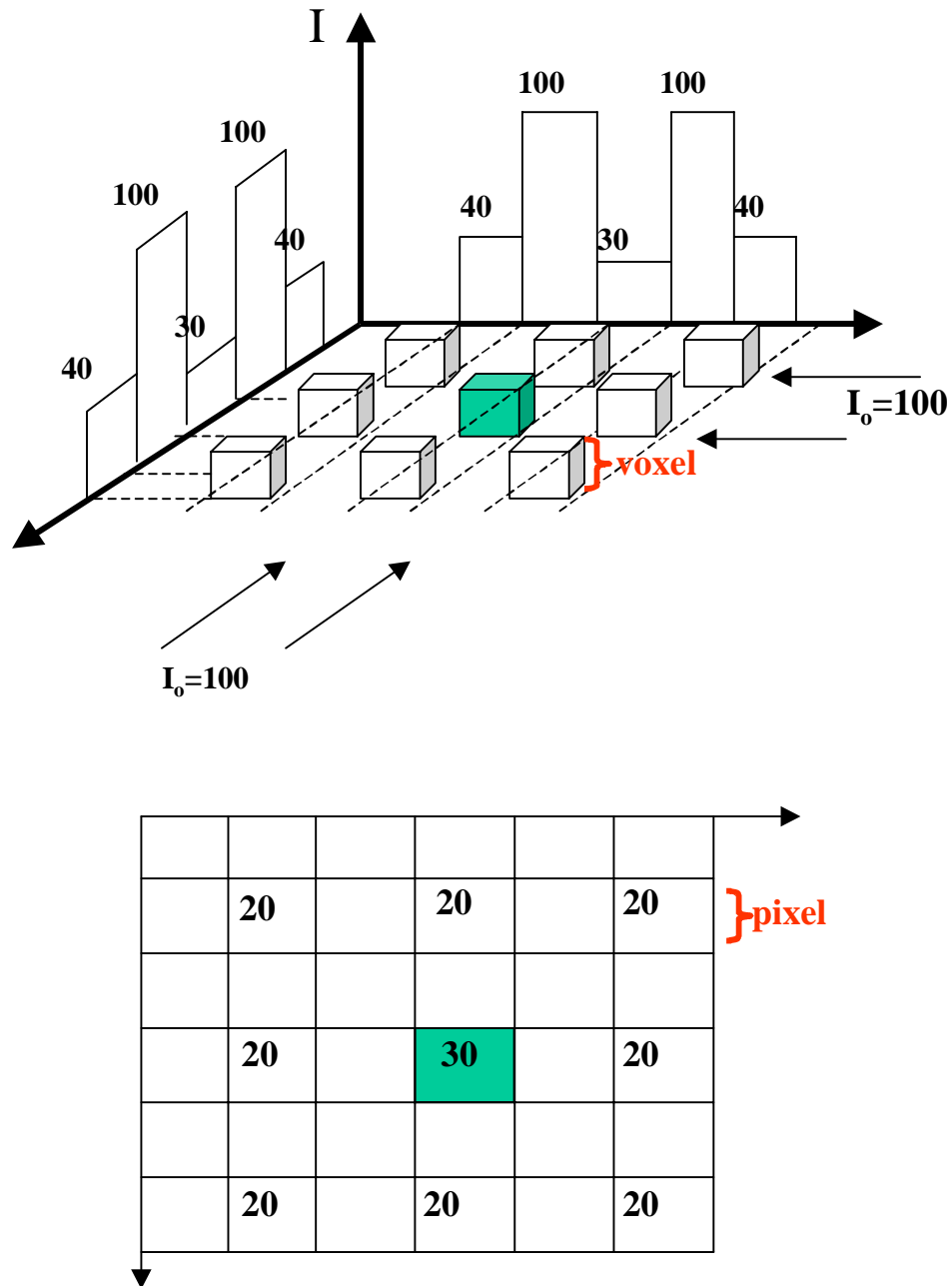


Figure 8: Principle of image formation in tomography.

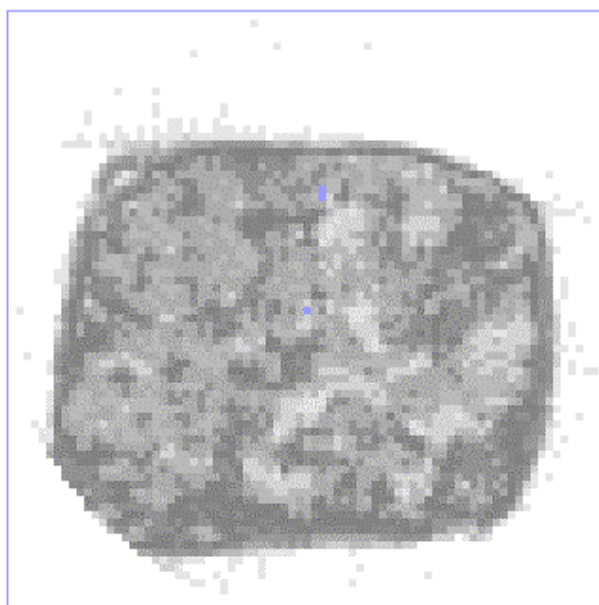
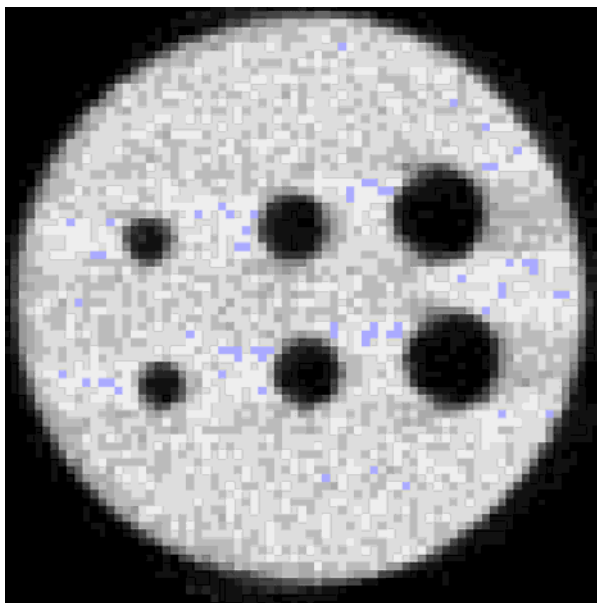


Figure 9: Examples of tomographic images obtained at CENA/USP, Piracicaba, Brazil

6.APPLICATION IN SOIL PHYSICS

6.1.Infiltration tests in homogeneous soils

The gamma-attenuation technique is very suitable for laboratory studies that involve water movement in soils. The main advantage of the methodology is its non-destructive character. As water moves thorough the soil, the changing water content can be monitored at different positions and times, with measurement times of less than 1 minute per point. Infiltration tests are examples for which gamma-attenuation has contributed significantly. These tests are normally performed on homogeneous soil columns. Columns are packet carefully with dry soil and before submitting to water infiltration are tested for homogeneity through bulk-density distributions. This can be performed by gamma-attenuation and, when the colimation beam is of the order of mm, d_b can be measured mm by mm. Columns presenting undesired d_b discontinuities can be descarted and repacked (Davidson et al, 1963 and Reichardt et al, 1972).

6.2.Soil Mechanical analysis

The intensity of a gamma beam passing through a soil suspension at a given depth is related to the concentration of the suspension as it varies with time. From the changes in the attenuation of the beam intensity it is possible to calculate soil particle fractions (Vaz et al, 1992 and Oliveira et al, 1997).

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