The Abdus Salam
International Centre for Theoretical Physics

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Educational, Scientific and
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IAEA
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College of Soil Physics

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Soil water retention and water flow equations

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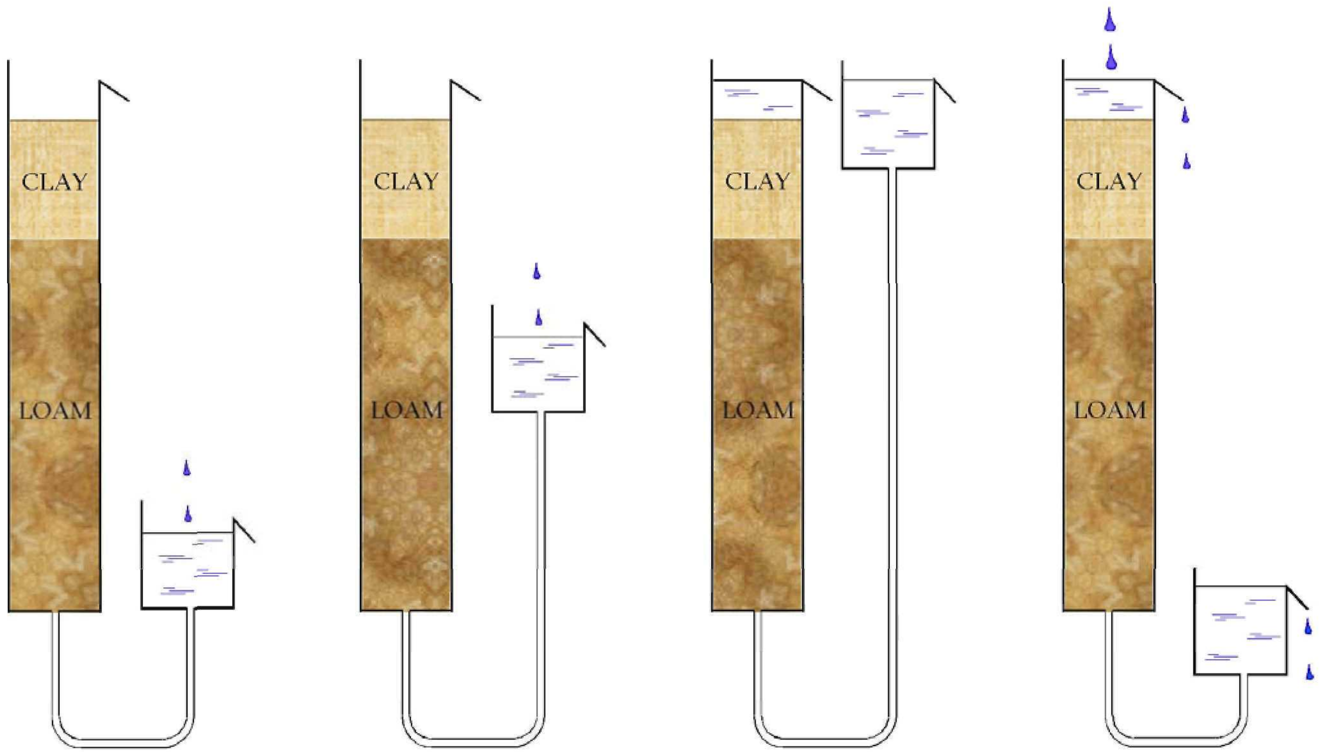
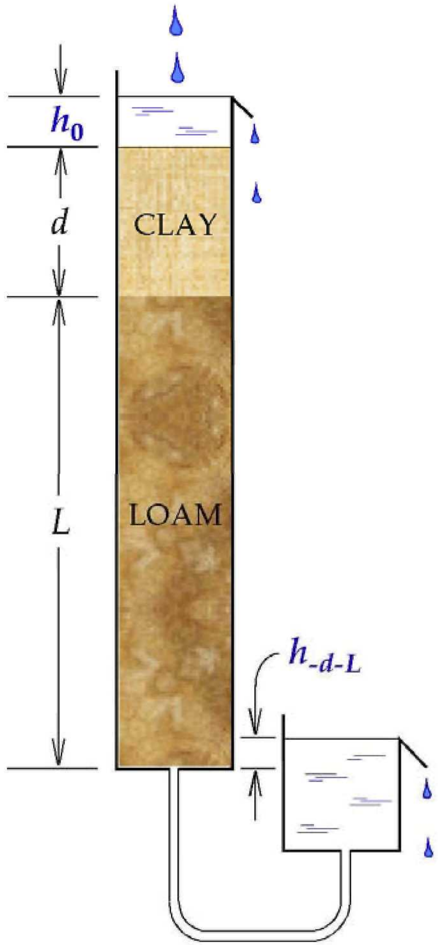


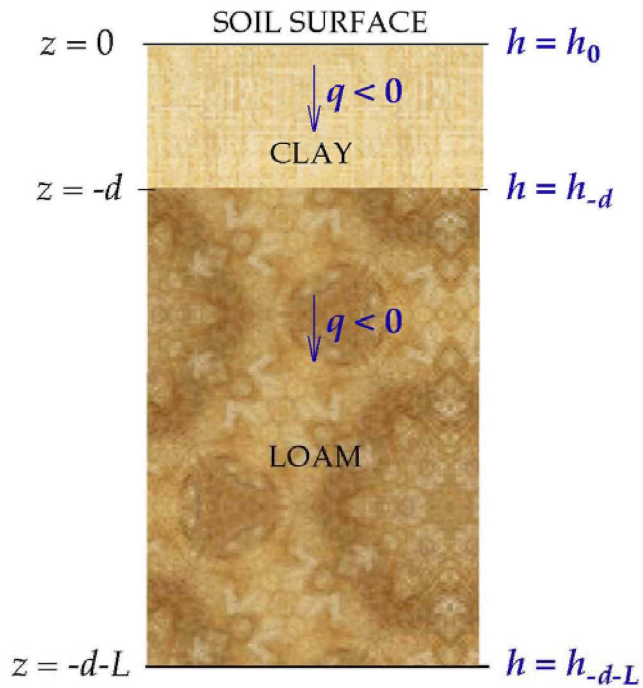
FILLING FROM THE BOTTOM
Slowly Raising Water Supply

WATER-FILLED SOIL
Water Not Flowing

WATER FLOWING
Downwards

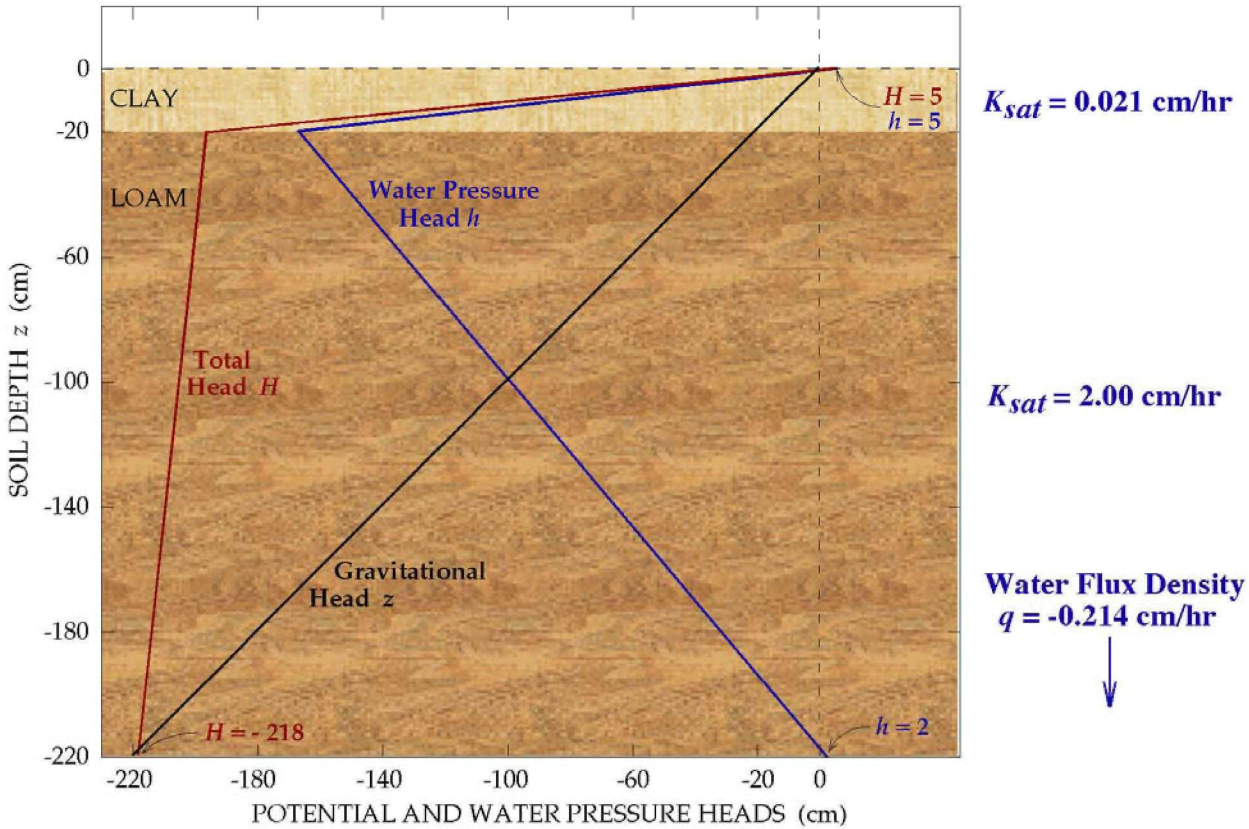


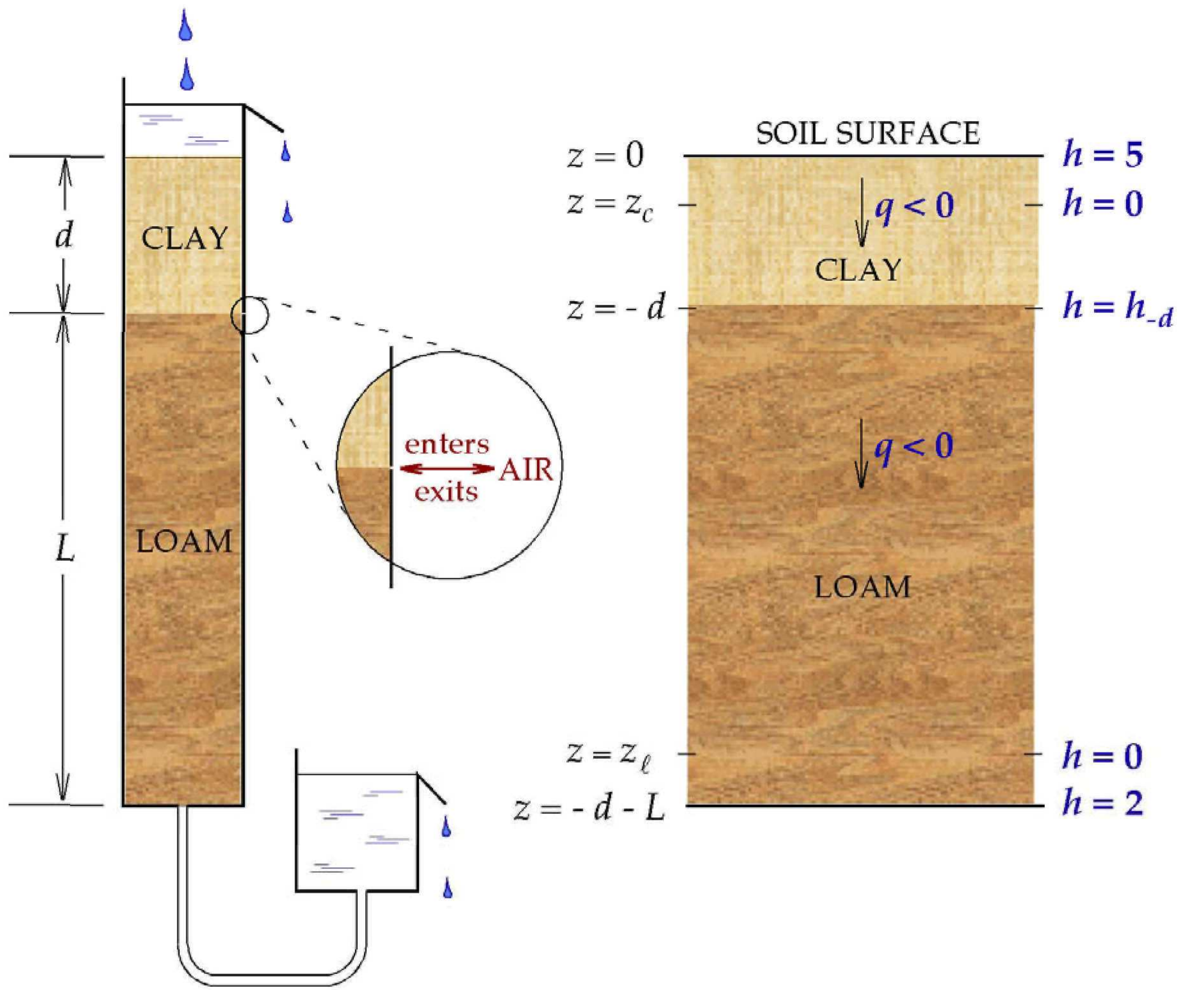
h_0
d
CLAY
L
LOAM
h_{-d-L}



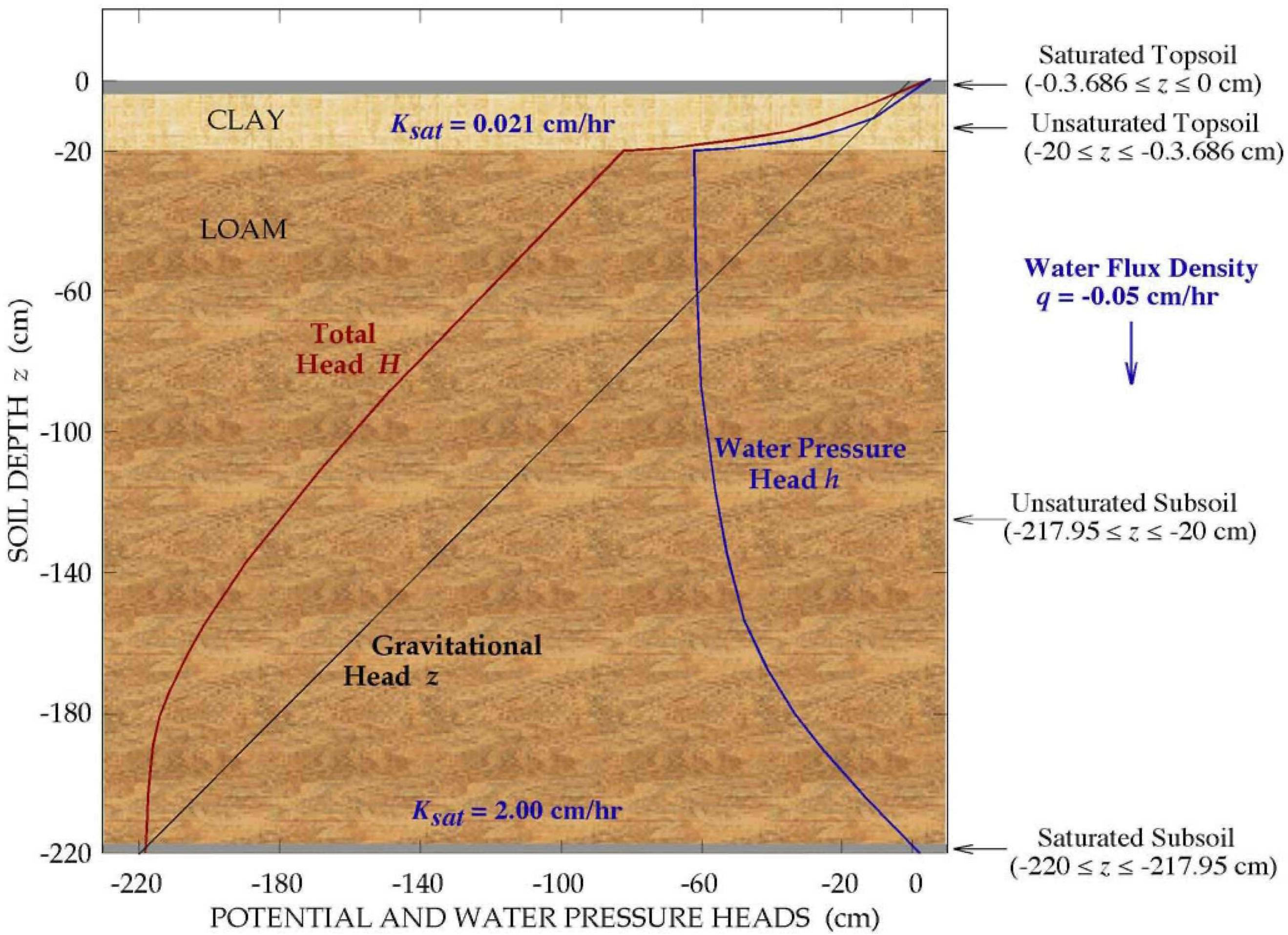
SOIL SURFACE
$z = 0$
$h = h_0$
$q < 0$
CLAY
$z = -d$
$h = h_{-d}$
$q < 0$
LOAM
$z = -d-L$
$h = h_{-d-L}$

"CLOSED" SYSTEM




d
CLAY
L
LOAM
enters
AIR
exits
SOIL SURFACE
z = 0
h = 5
z = z_c
q < 0
h = 0
CLAY
z = - d
h = h_-d
q < 0
LOAM
z = z_ℓ
h = 0
z = - d - L
h = 2

"OPEN" SYSTEM



CLAY
K_{sat} = 0.021 cm/hr
LOAM
Total Head H
Water Pressure Head h
Gravitational Head z
K_{sat} = 2.00 cm/hr
SOIL DEPTH z (cm)
0
-20
-60
-100
-140
-180
-220
-220
-180
-140
-100
-60
-20
0
POTENTIAL AND WATER PRESSURE HEADS (cm)
Saturated Topsoil
(-0.3.686 ≤ z ≤ 0 cm)
Unsaturated Topsoil
(-20 ≤ z ≤ -0.3.686 cm)
Water Flux Density
q = -0.05 cm/hr
Unsaturated Subsoil
(-217.95 ≤ z ≤ -20 cm)
Saturated Subsoil
(-220 ≤ z ≤ -217.95 cm)

TWO EQUATIONS, TWO UNKNOWNS

$$q = -K_{c_{sat}}\left(\frac{h_0 - h_{-d} + d}{d}\right)$$ TOPSOIL

$$q = -K_{\ell_{sat}}\left(\frac{h_{-d} - h_{-d-L} + L}{L}\right)$$ SUBSOIL

$$h_{-d} = \frac{d \cdot K_{\ell_{sat}}(h_{-d-L} - L) + L \cdot K_{c_{sat}}(h_0 + d)}{d \cdot K_{\ell_{sat}} + L \cdot K_{c_{sat}}}$$

TWO UNKNOWNS IN TOPSOIL

SATURATED TOPSOIL

$$\int_{z_c}^{0} dz = -\int_{0}^{5} \frac{dh}{q/K_{c\,sat} + 1}$$

$$z_c = \frac{5}{q/a_c\left(b_c + a_c/q\right)}$$

UNSATURATED TOPSOIL

$$\int_{-d}^{z_c} dz = -\int_{h_{-d}}^{0} \frac{a_c\,dh}{q(b_c + a_c/q + h^2)}$$

$$z_c = -d + \frac{a_c}{q(b_c + a_c/q)^{1/2}} \tan^{-1}\left(\frac{h_{-d}}{(b_c + a_c/q)^{1/2}}\right)$$

$$\frac{5 + d + db_c q/a_c}{\left(b_c + a_c/q\right)^{1/2}} = \tan^{-1}\left(\frac{h_{-d}}{(b_c + a_c/q)^{1/2}}\right)$$

TWO UNKNOWNS IN SUBSOIL

SATURATED SUBSOIL

$$\int_{-220}^{z_\ell} dz = -\int_2^0 \frac{dh}{(q/K_\ell + 1)}$$

$$z_\ell = -220 + \frac{2}{q/a_\ell(b_\ell + a_\ell/q)}$$

UNSATURATED SUBSOIL

$$\int_{z_\ell}^{-d} dz = -\frac{a_\ell}{q}\int_0^{h_{-d}} \frac{dh}{[(b_\ell + a_\ell/q)^{1/2}]^2 + h^2}$$

$$z_\ell = -d - \frac{a_\ell}{q}\frac{1}{2(-b_\ell - a_\ell/q)^{1/2}}\ln\left(\frac{(-b_\ell - a_\ell/q)^{1/2} + h_{-d}}{(-b_\ell - a_\ell/q)^{1/2} - h_{-d}}\right)$$

$$\frac{[(-440 + 2d)(b_\ell q/a_\ell + 1) + 4]}{(-b_\ell - a_\ell/q)^{1/2}} = \ln\left(\frac{(-b_\ell - a_\ell/q)^{1/2} + h_{-d}}{(-b_\ell - a_\ell/q)^{1/2} - h_{-d}}\right)$$

SOLVING FOR THE TWO UNKNOWNS

TOPSOIL

$$\frac{5 + d \; + d b_c \boldsymbol{q} / a_c}{\left(b_c + a_c / \boldsymbol{q}\right)^{1/2}} = \tan^{-1}\left(\frac{\boldsymbol{h_{-d}}}{\left(b_c + a_c / \boldsymbol{q}\right)^{1/2}}\right)$$

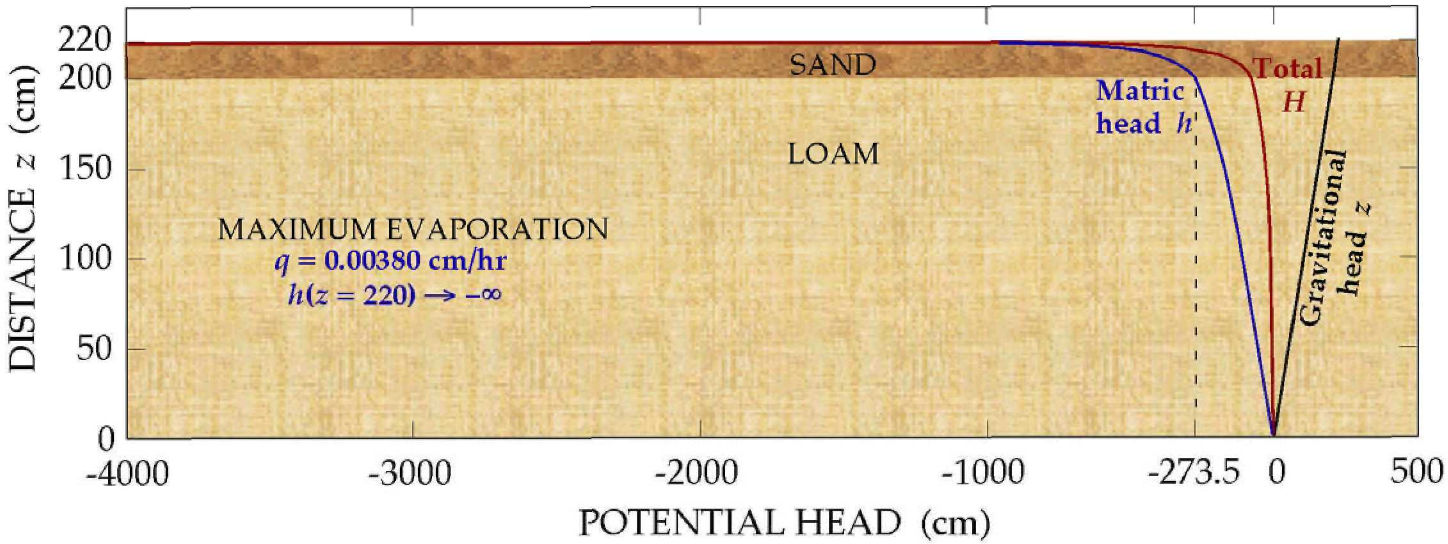
SUBSOIL

$$\frac{[(-440 + 2d)(b_\ell \boldsymbol{q} / a_\ell + 1) + 4]}{(-b_\ell - a_\ell / \boldsymbol{q})^{1/2}} = \ln\left(\frac{(-b_\ell - a_\ell / \boldsymbol{q})^{1/2} + \boldsymbol{h_{-d}}}{(-b_\ell - a_\ell / \boldsymbol{q})^{1/2} - \boldsymbol{h_{-d}}}\right)$$

q = - 0.05 cm/hr

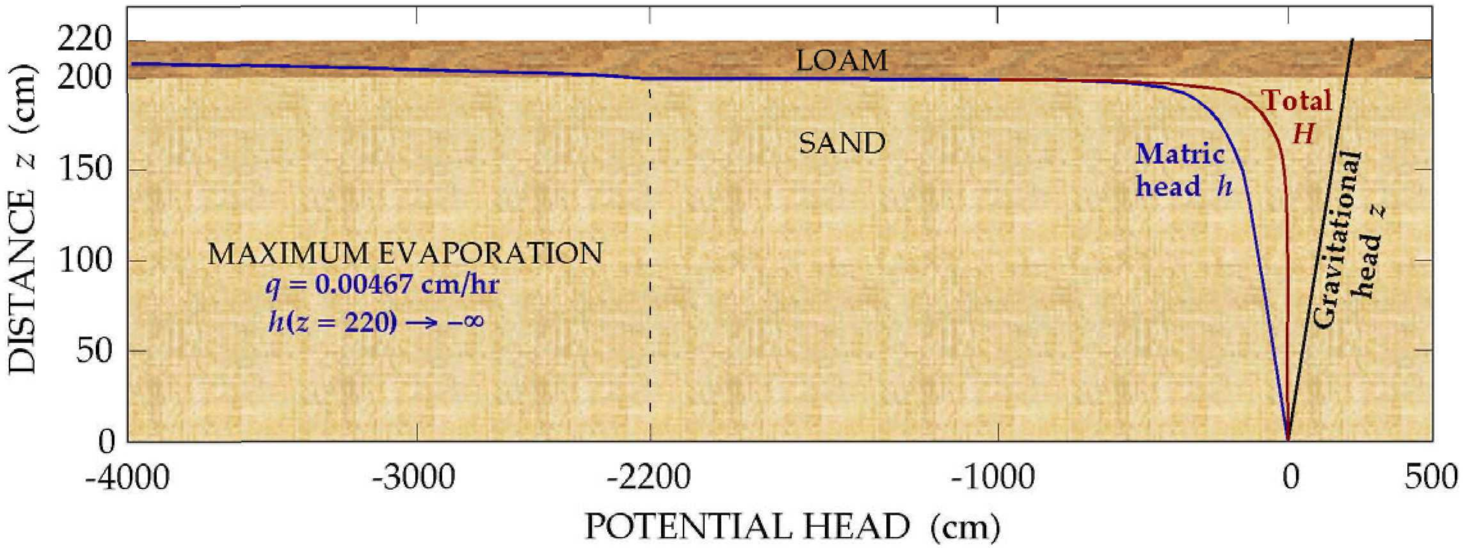
h_{-d} = - 62.21 cm

MAXIMUM STEADY STATE EVAPORATION



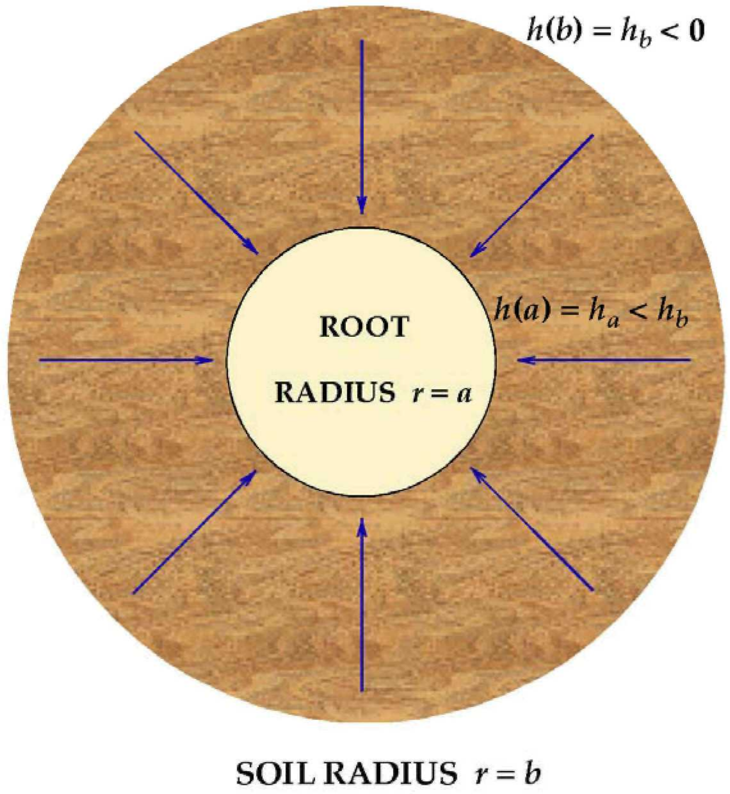
IF SOIL ENTIRELY A SAND, q_{max} **= 0.00323 cm/hr**

MAXIMUM STEADY STATE EVAPORATION



IF SOIL ENTIRELY A LOAM, q_{max} = 0.0101 cm/hr

SOIL WATER FLOW TO PLANT ROOTS



SOIL WATER FLOW TO PLANT ROOTS

RICHARDS EQUATION IN CYLINDRICAL COORDINATES

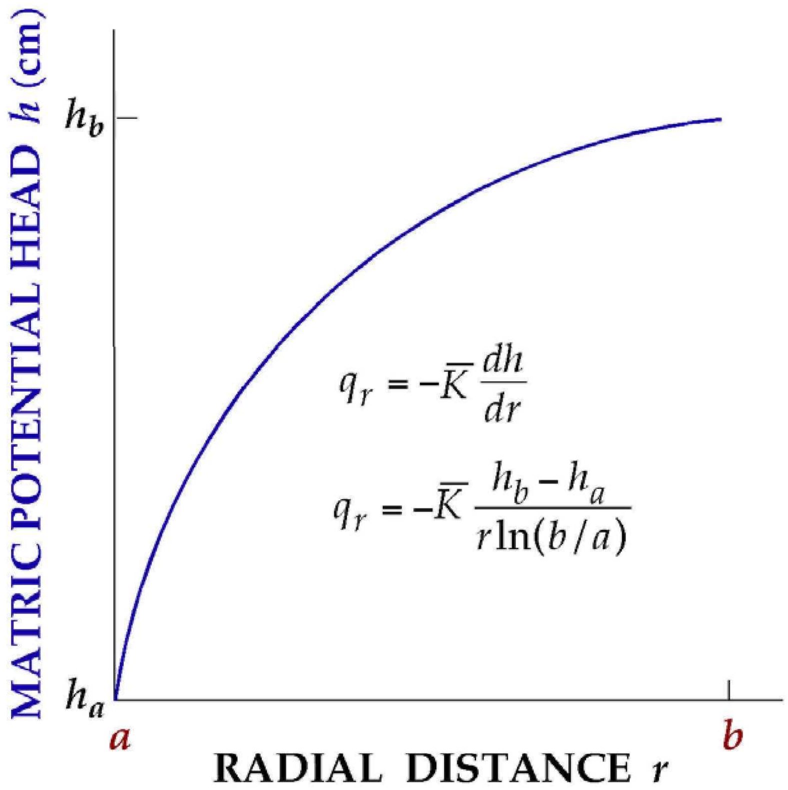
$$\frac{\partial \vartheta}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(rK\frac{\partial H}{\partial r}\right) + \frac{1}{r}\frac{\partial}{\partial \theta}\left(K\frac{1}{r}\frac{\partial H}{\partial \theta}\right) + \frac{\partial}{\partial z}\left(K\frac{\partial H}{\partial z}\right)$$

STEADY-STATE RADIAL FLOW ONLY

$$\frac{d}{dr}\left(rK\frac{dh}{dr}\right) = 0 \qquad \overline{K} = \frac{1}{h_a - h_b}\int_{h_a}^{h_b} K(h)dh$$

$$\frac{d}{dr}\left(r\frac{dh}{dr}\right) = 0 \qquad h = A\ln r + B$$

$$\frac{h - h_a}{h_b - h_a} = \frac{\ln(r/a)}{\ln(b/a)}$$



$$\frac{J_r}{(2\pi r\cdot 1)} = -\overline{K}\,\frac{(h_b - h_a)}{r\ln(b/a)} \qquad J_r = -\frac{2\pi\overline{K}(h_b - h_a)}{\ln(b/a)}$$

ABSORPTION OF SOIL WATER BY PLANT ROOTS

AN ABUNDANT NUMBER OF ROOTS

Wheat: 5 to 40 (km root length)/(m^2 soil surface)

HUGE AREA FOR ABSORBING WATER

Tens of m^2 of root surface area

WATER MOVES VERY SHORT DISTANCES IN SOIL

Only a few mm's