The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific and
Cultural Organization

IAEA
International Atomic Energy Agency

1867-33

College of Soil Physics

22 October - 9 November, 2007

Simple field methods for soil water properties

Donald Nielsen
University of California
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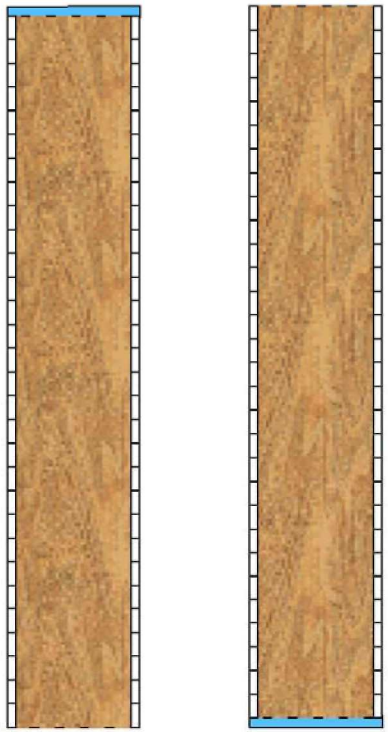
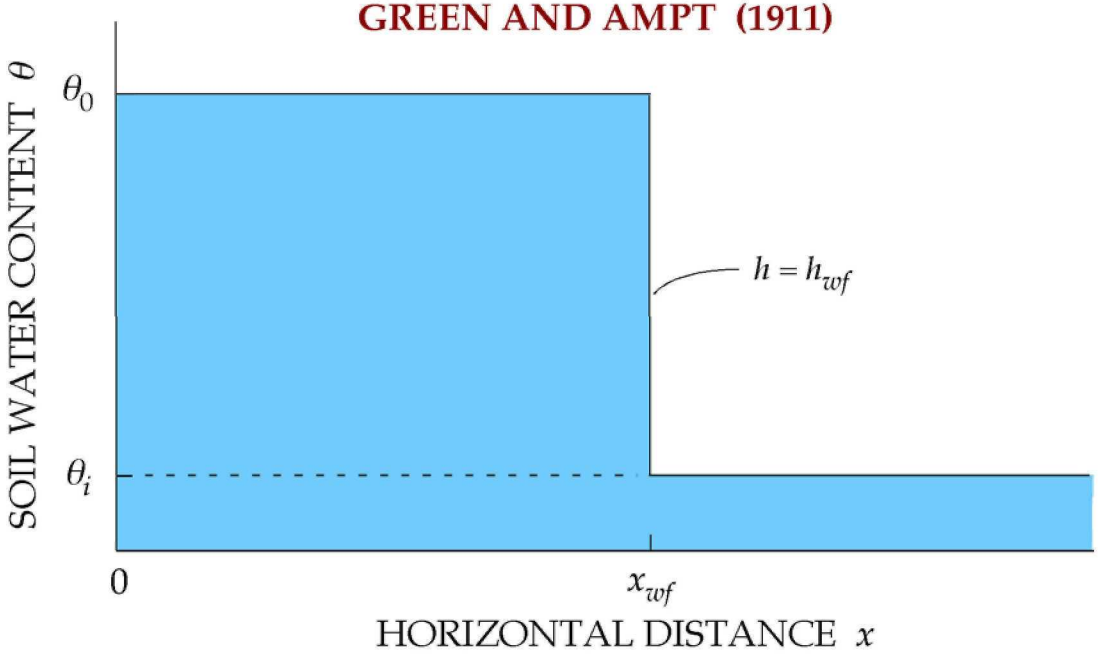
INFILTRATION



HORIZONTAL INFILTRATION

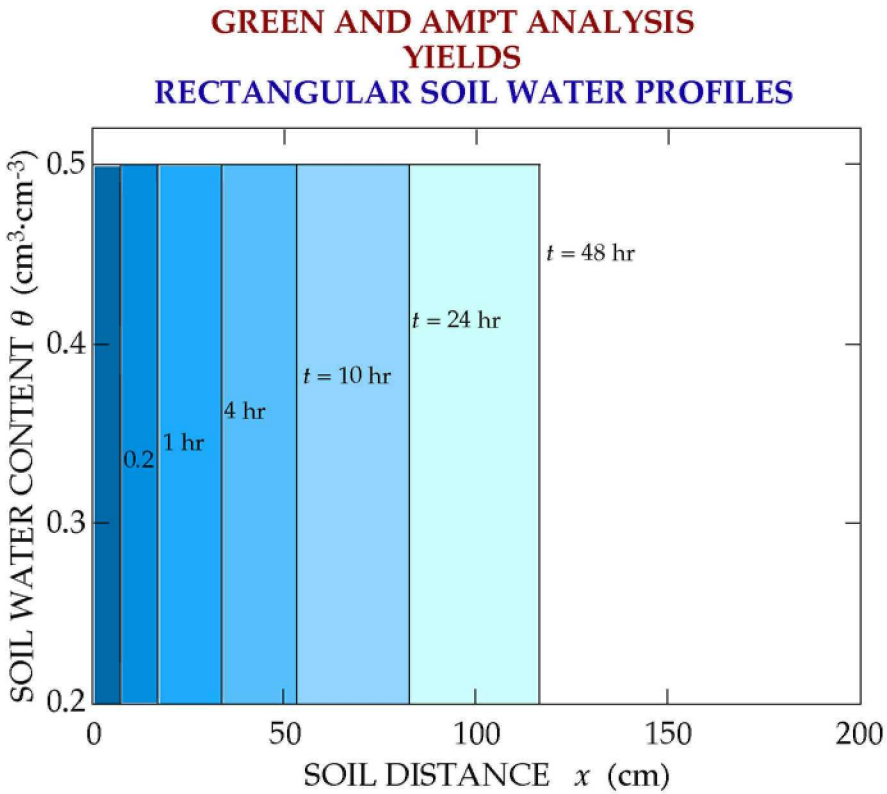
CAPILLARY RISE

GREEN AND AMPT (1911)

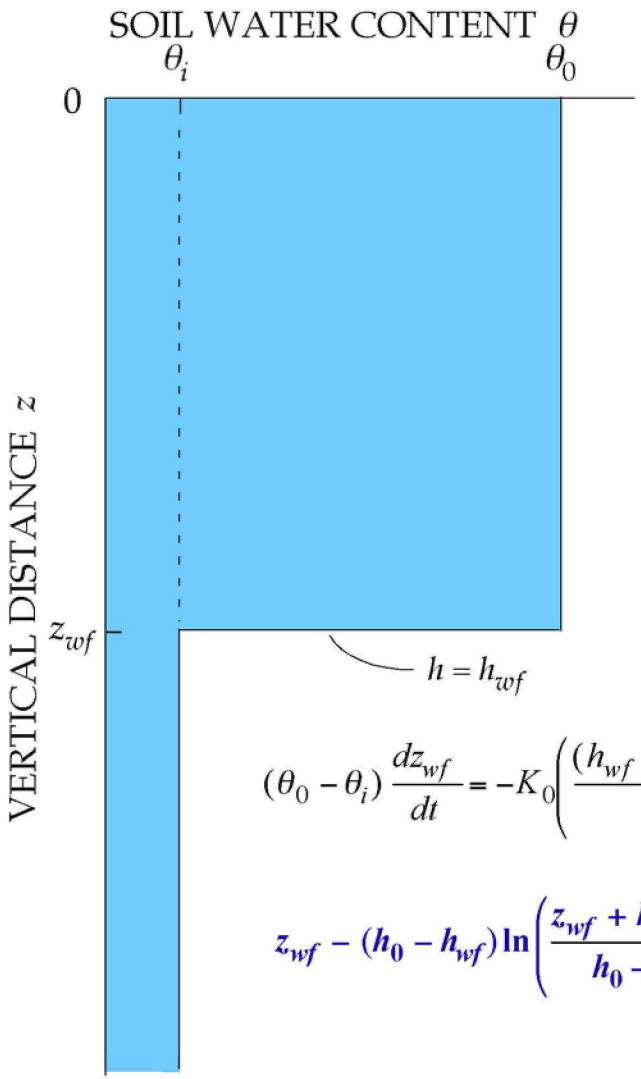


$$q = -K_0\left(\frac{(h_{wf} - h_0)}{x_{wf} - 0}\right) \qquad (\theta_0 - \theta_i)\frac{dx_{wf}}{dt} = K_0\left(\frac{h_0 - h_{wf}}{x_{wf}}\right)$$

$$\int_0^{x_{wf}} x_{wf}\,dx_{wf} = \frac{K_0(h_0 - h_{wf})}{(\theta_0 - \theta_i)}\int_0^t dt \qquad x_{wf} = \sqrt{\frac{2K_0(h_0 - h_{wf})}{(\theta_0 - \theta_i)}}\; t^{1/2}$$


GREEN AND AMPT ANALYSIS
YIELDS
RECTANGULAR SOIL WATER PROFILES
SOIL WATER CONTENT θ (cm³·cm⁻³)
0.5
0.4
0.3
0.2
0
50
100
150
200
SOIL DISTANCE x (cm)
0.2
1 hr
4 hr
t = 10 hr
t = 24 hr
t = 48 hr

GREEN AND AMPT 1911)



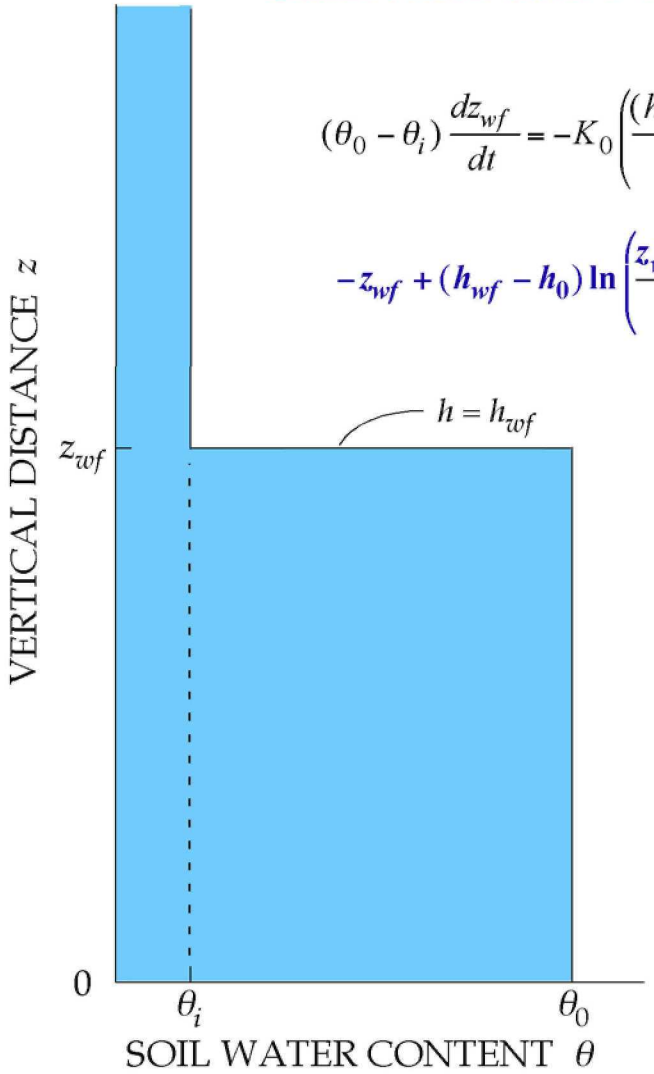
$$(\theta_0 - \theta_i)\frac{dz_{wf}}{dt} = -K_0\left(\frac{(h_{wf} - z_{wf}) - (h_0 - 0)}{z_{wf} - 0}\right)$$

$$z_{wf} - (h_0 - h_{wf})\ln\left(\frac{z_{wf} + h_0 - h_{wf}}{h_0 - h_{wf}}\right) = \frac{K_0 t}{\theta_0 - \theta_i}$$

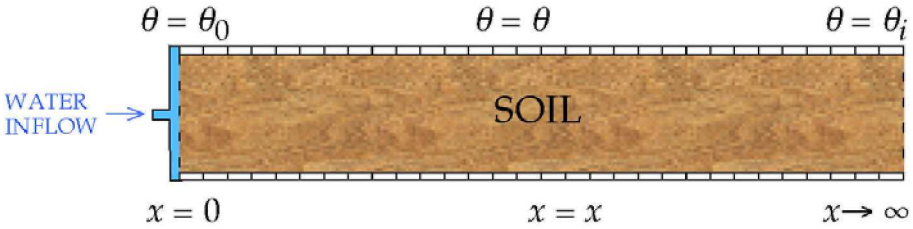
GREEN AND AMPT 1911)

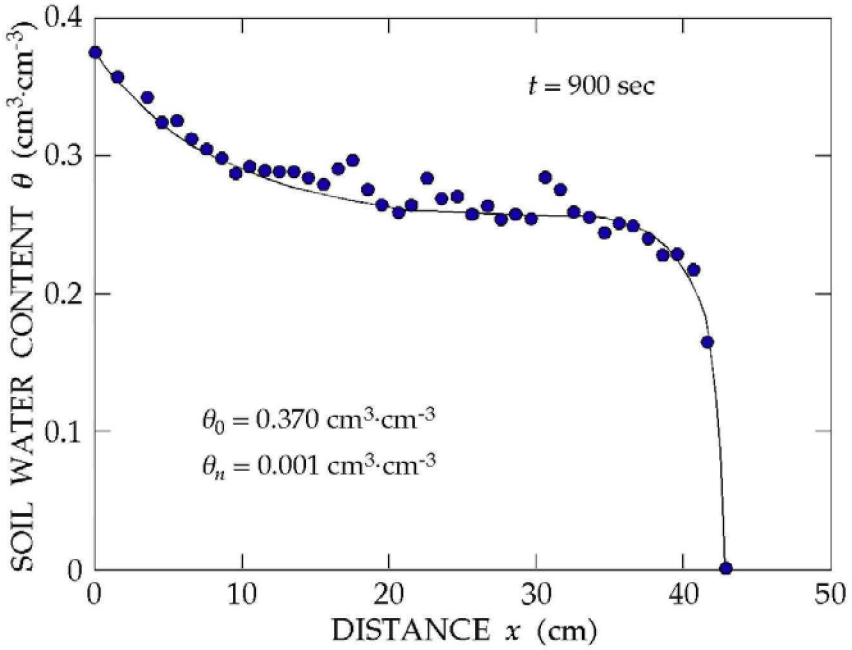
$$(\theta_0 - \theta_i)\frac{dz_{wf}}{dt} = -K_0\left(\frac{(h_{wf} + z_{wf}) - (h_0 - 0)}{z_{wf} - 0}\right)$$

$$-z_{wf} + (h_{wf} - h_0)\ln\left(\frac{z_{wf} + h_{wf} - h_0}{h_{wf} - h_0}\right) = \frac{K_0 t}{\theta_0 - \theta_i}$$



BRUCE AND KLUTE (1956)





BRUCE AND KLUTE (1956)

$$\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial x}\left(K(\theta)\frac{\partial h}{\partial x}\right) = \frac{\partial}{\partial x}\left(K(\theta)\frac{dh}{d\theta}\frac{\partial\theta}{\partial x}\right) \qquad \frac{\partial\theta}{\partial t} = \frac{\partial}{\partial x}\left(D(\theta)\frac{\partial\theta}{\partial x}\right)$$

$$\theta = \theta_i \qquad x \geq 0 \qquad t = 0$$

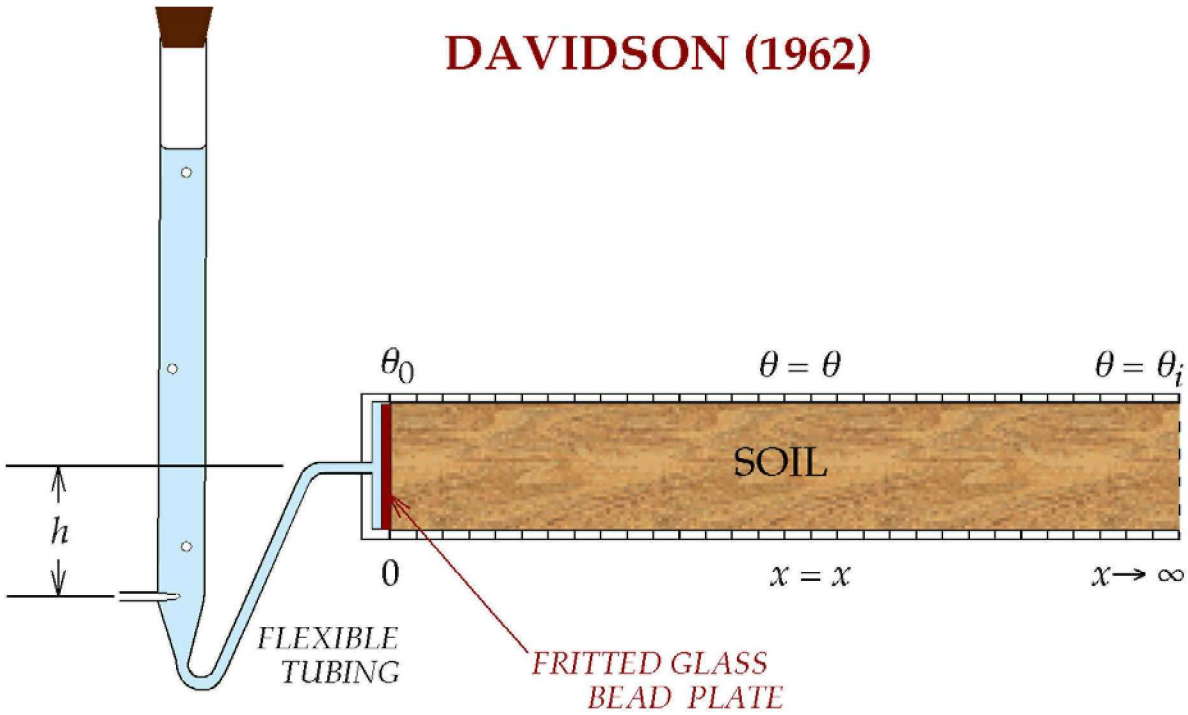
$$\theta = \theta_0 \qquad x = 0 \qquad t > 0$$

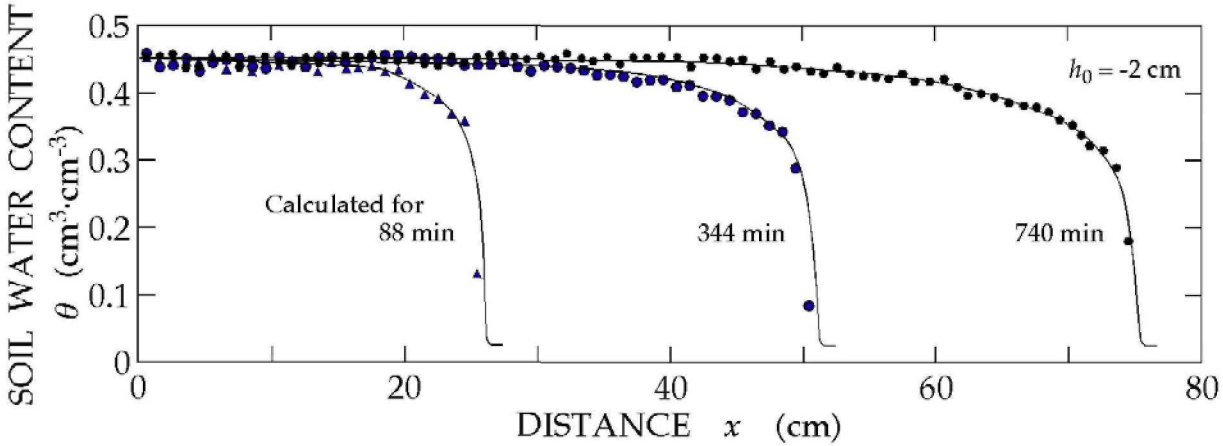
$$\theta = \theta_i \qquad x \to \infty \qquad t \geq 0$$

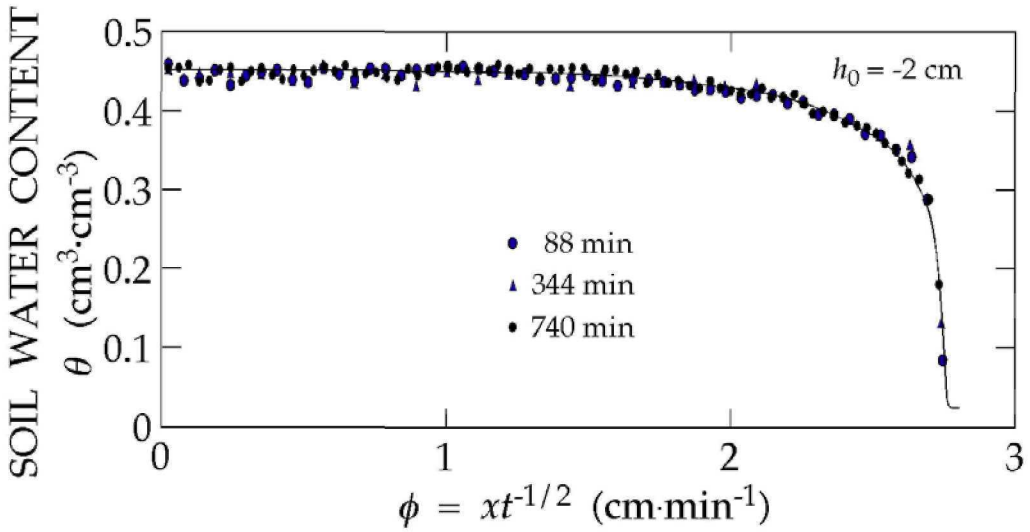
With the Boltzmann transform $\phi(\theta) = xt^{-1/2}$

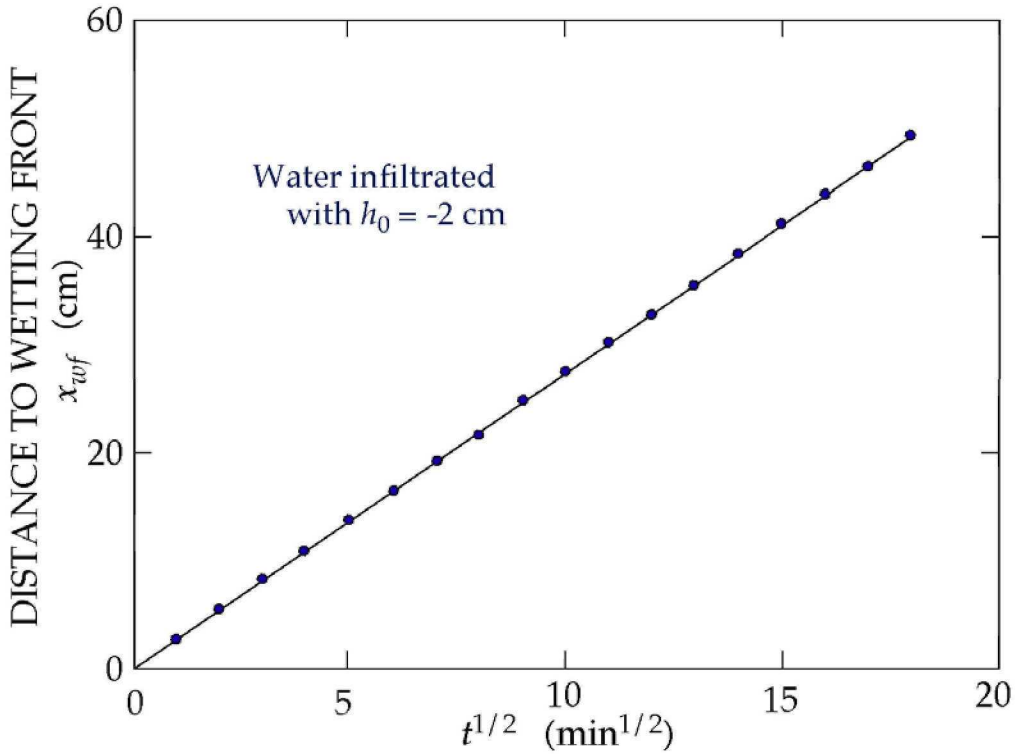
$$D(\theta) = -\frac{1}{2t_1}\left.\frac{dx}{d\theta}\right|_\theta \int_{\theta_i}^{\theta} x\,d\theta$$

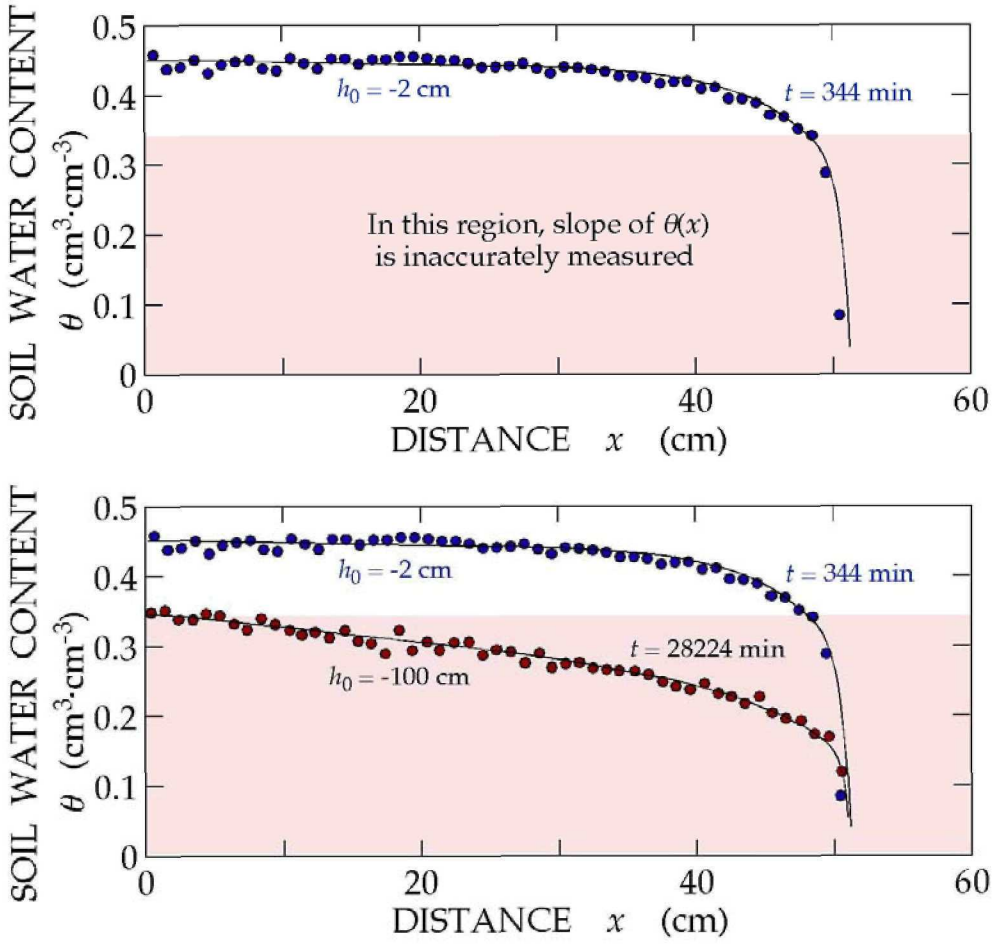
DAVIDSON (1962)

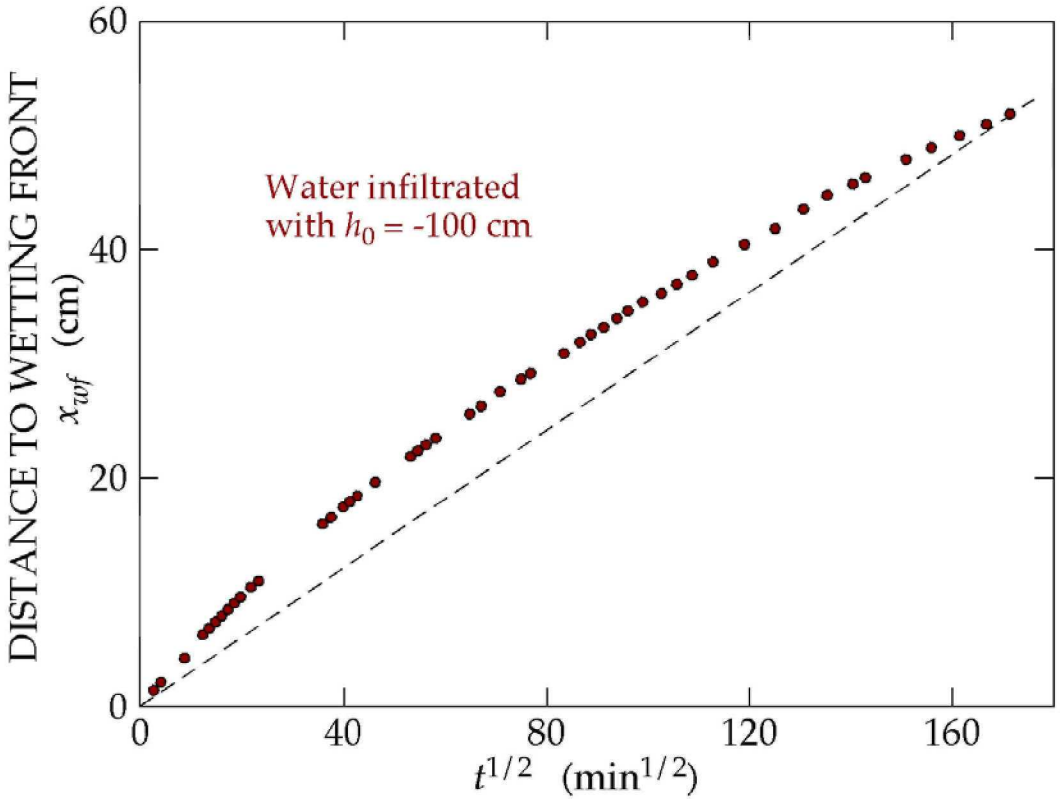



SOIL WATER CONTENT
θ (cm³.cm⁻³)
0.5
0.4
0.3
0.2
0.1
0
0
20
40
60
80
DISTANCE x (cm)
h0 = -2 cm
Calculated for
88 min
344 min
740 min

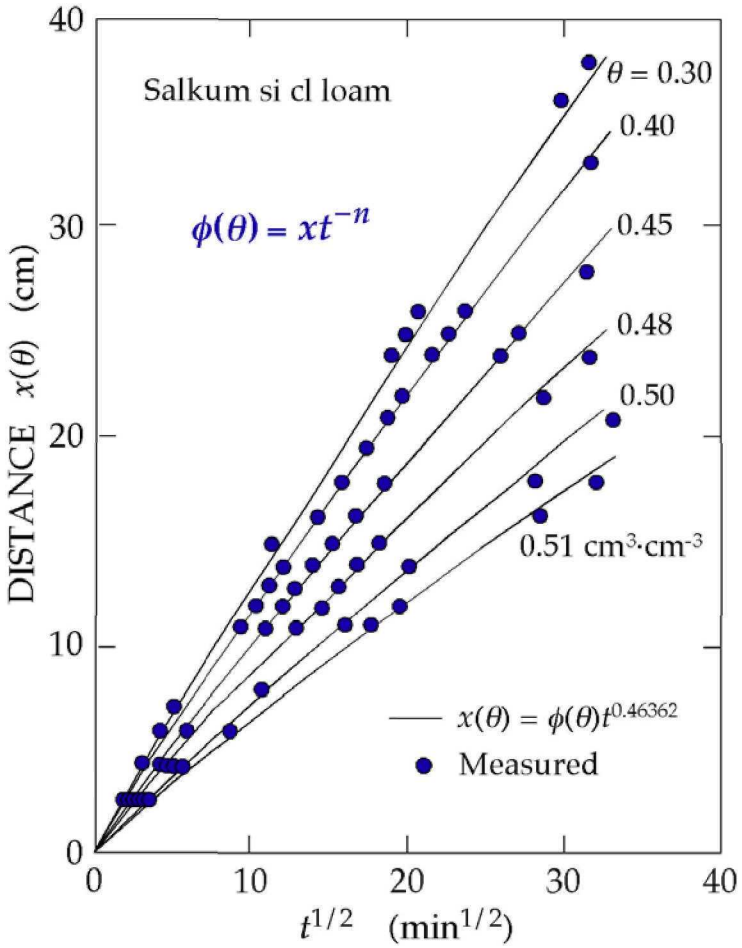

SOIL WATER CONTENT
θ (cm³·cm⁻³)
0.5
0.4
0.3
0.2
0.1
0
0
1
2
3
h0 = -2 cm
88 min
344 min
740 min
ϕ = xt-1/2 (cm·min-1)


DISTANCE TO WETTING FRONT
x_{wf} (cm)
Water infiltrated
with h_0 = -2 cm
60
40
20
0
0
5
10
15
20
$t^{1/2}$ (min$^{1/2}$)


SOIL WATER CONTENT
θ (cm³·cm⁻³)
0.5
0.4
0.3
0.2
0.1
0
h0 = -2 cm
t = 344 min
In this region, slope of θ(x)
is inaccurately measured
0
20
40
60
DISTANCE x (cm)
SOIL WATER CONTENT
θ (cm³·cm⁻³)
0.5
0.4
0.3
0.2
0.1
0
h0 = -2 cm
t = 344 min
t = 28224 min
h0 = -100 cm
0
20
40
60
DISTANCE x (cm)


DISTANCE TO WETTING FRONT
x_{wf} (cm)
Water infiltrated
with h_0 = -100 cm
0
20
40
60
0
40
80
120
160
$t^{1/2}$ (min$^{1/2}$)

GUERRINI AND SWARTZENDRUBER (1992)

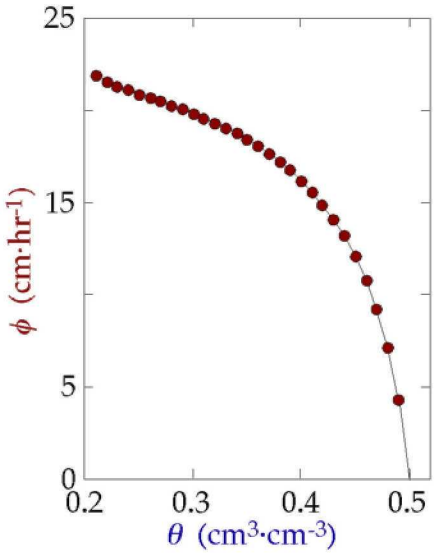


SOLUTION TO RICHARD'S EQUATION FOR INFILTRATION, PHILIP (1957)

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left(D(\theta) \frac{\partial \theta}{\partial z} \right) - \frac{\partial K(\theta)}{\partial z}$$

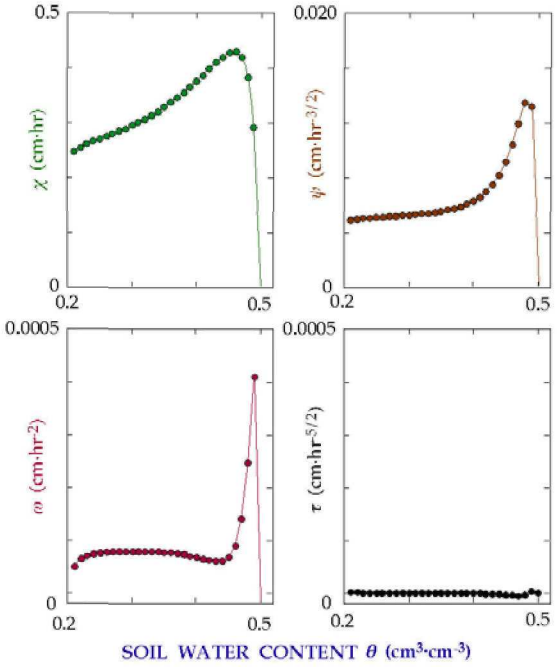
$\theta = \theta_n$	$z \geq 0$	$t = 0$
$\theta = \theta_0$	$z = 0$	$t > 0$
$\theta = \theta_n$	$z \rightarrow \infty$	$t \geq 0$

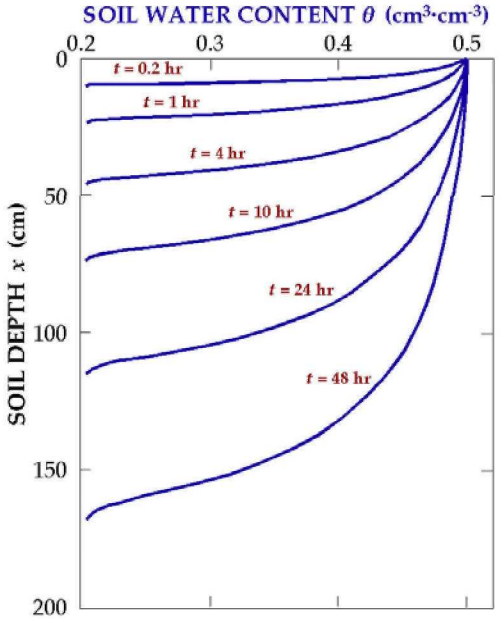
$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left(D(\theta) \frac{\partial \theta}{\partial z} \right) \qquad -\frac{1}{2}\int_{\theta_n}^{\theta} \phi \, d\theta = D(\theta) \frac{d\theta}{d\phi}$$

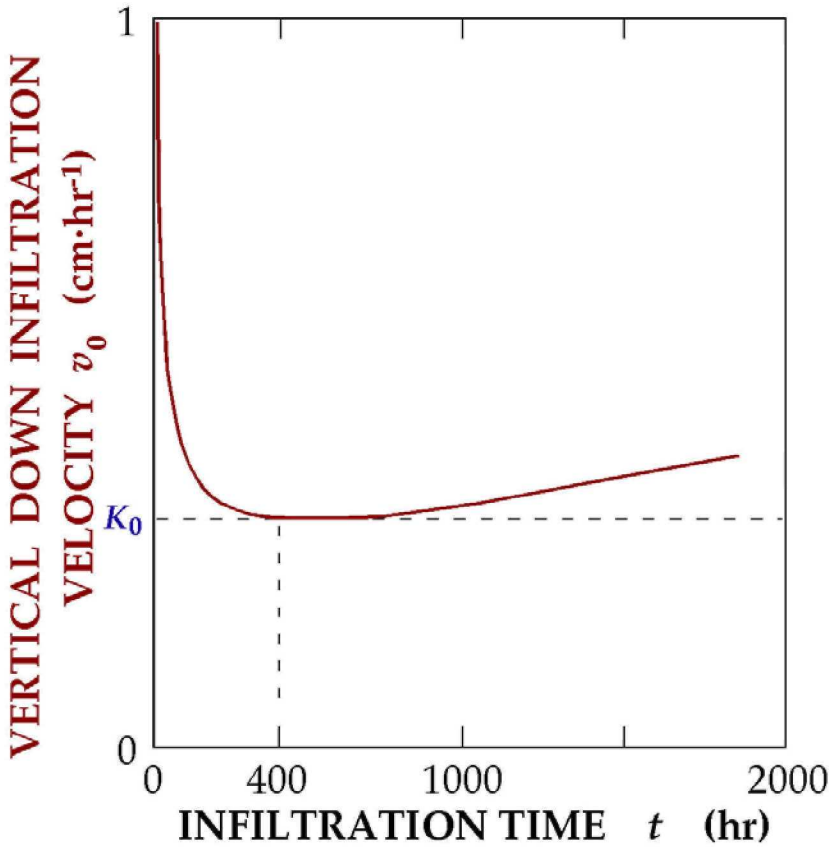


SOLUTION TO RICHARD'S EQUATION FOR INFILTRATION, PHILIP (1957)

$$z = \phi(\theta)t^{1/2} + \chi(\theta)t + \psi(\theta)t^{3/2} + \omega(\theta)t^{2} + \tau(\theta)t^{5/2} + \cdots$$

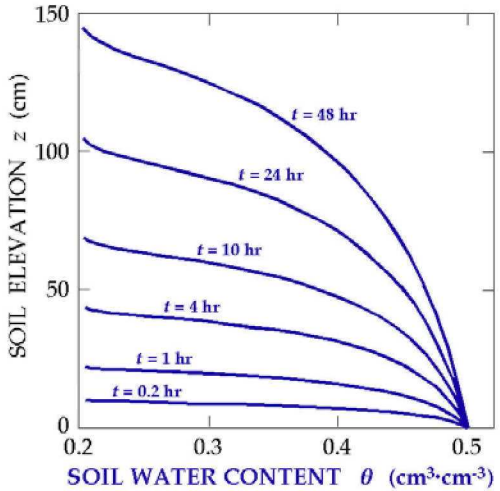


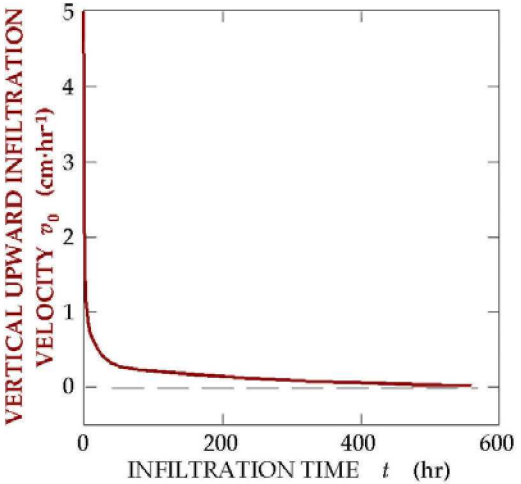


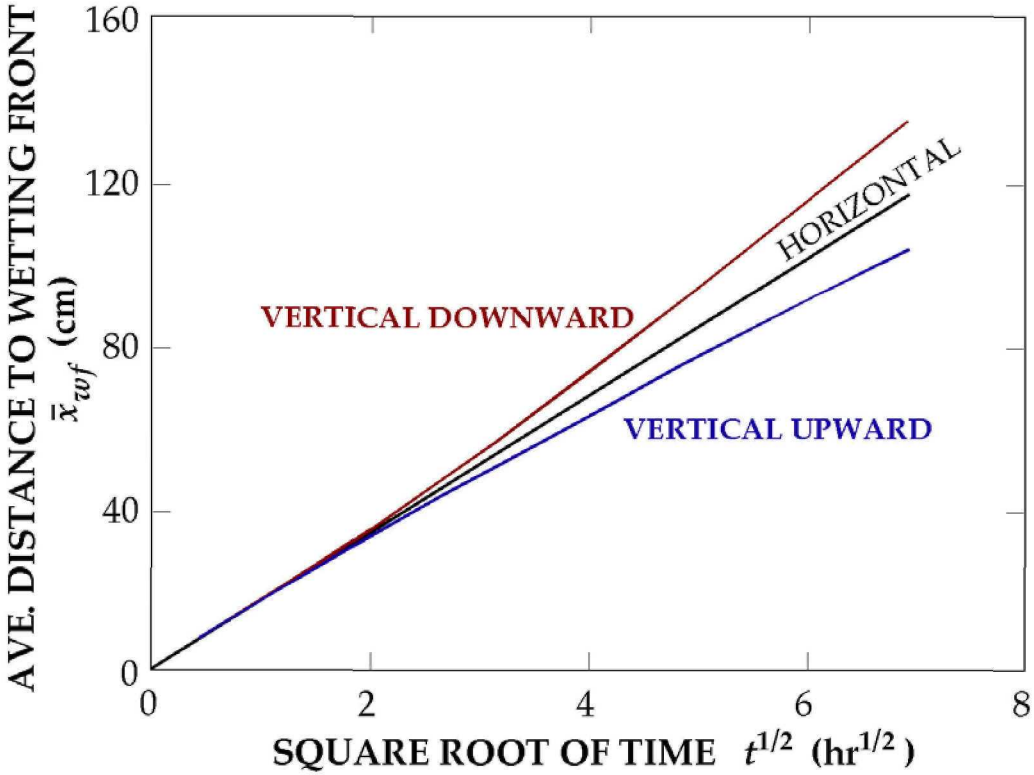

VERTICAL DOWN INFILTRATION VELOCITY v_0 (cm·hr^{-1})
1
K_0
0
0
400
1000
2000
INFILTRATION TIME t (hr)

SOLUTION TO RICHARD'S EQUATION FOR CAPILLARY RISE, PHILIP (1957)

$$z = \phi(\theta)t^{1/2} - \chi(\theta)t + \psi(\theta)t^{3/2} - \omega(\theta)t^{2} + \tau(\theta)t^{5/2} - \cdots$$






AVE. DISTANCE TO WETTING FRONT
$\bar{x}_{wf}$ (cm)
160
120
80
40
0
0
2
4
6
8
SQUARE ROOT OF TIME $t^{1/2}$ (hr$^{1/2}$)
VERTICAL DOWNWARD
HORIZONTAL
VERTICAL UPWARD