The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific and
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IAEA
International Atomic Energy Agency

1867-30

College of Soil Physics

22 October - 9 November, 2007

Soil water hydrostatics-Soil water potential

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FORCES THAT HOLD WATER IN SOIL

or

WHY IS IT NECESSARY TO APPLY A FORCE TO REMOVE WATER FROM SOIL?

ANSWER: TO OVERCOME VARIOUS FORCES ACTING IN THE SOIL:

- "SURFACE TENSION" FORCES
- "ADHESIVE" AND "COHESIVE" FORCES
- "SWELLING" AND "SHRINKING" FORCES
- "INTERFACIAL" FORCES BETWEEN SOIL PARTICLES AND WATER

ORIGIN OF FORCE	FORCE	EXAMPLE
ION-ION	$1/r^2$	Ionic Crystals
ION-DIPOLE	$1/r^3$	Water around Ion
DIPOLE-DIPOLE	$1/r^4$	Water to water
ION-INDUCED DIPOLE	$1/r^5$	Water to Non-Polar Fluid
DIPOLE- INDUCED DIPOLE	$1/r^7$	Adsorption of Non-Polar Molecule to Clay

WE NEVER MEASURE THE FORCES DIRECTLY

WE MEASURE THE WORK
AND EXPRESS THAT WORK AS A POTENTIAL ENERGY USUALLY AS

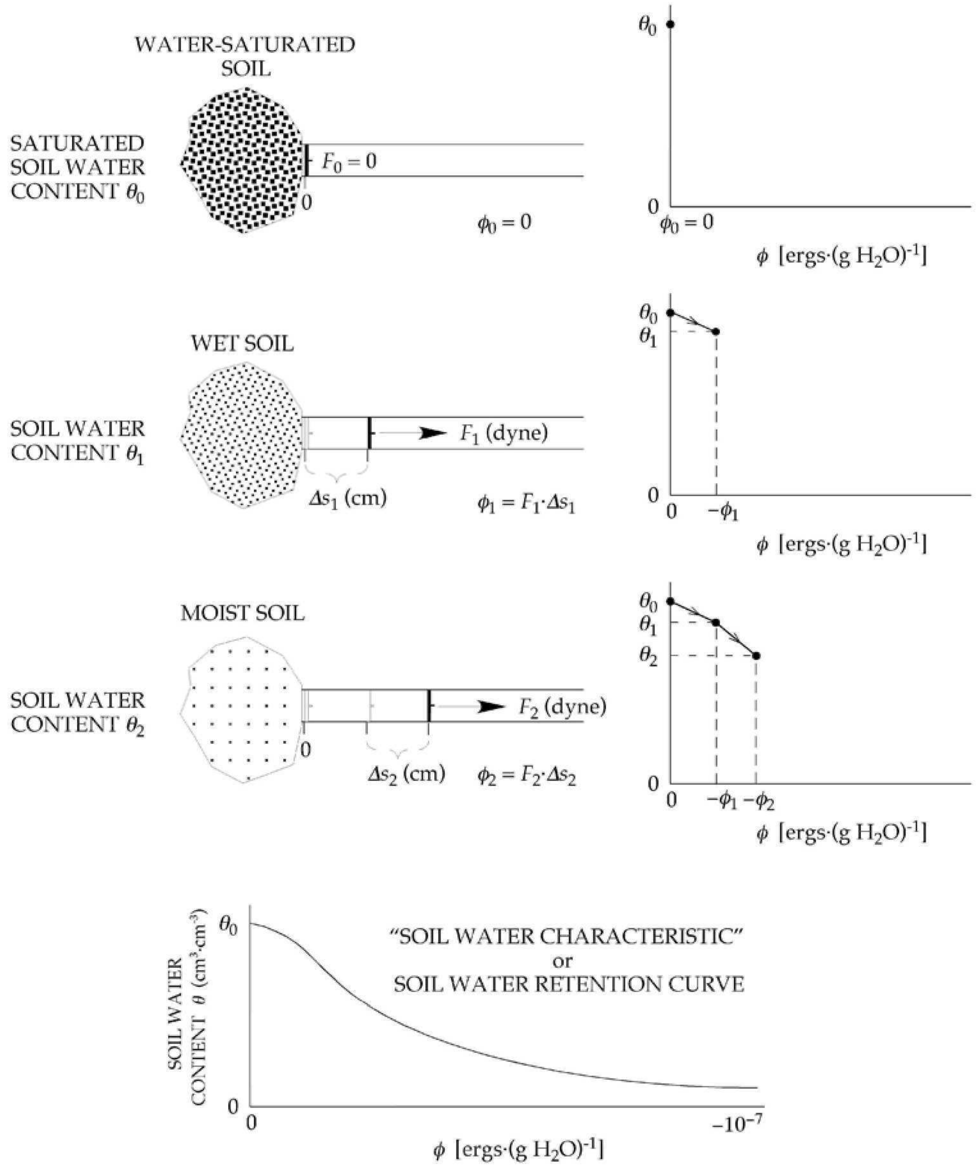
Soil Water Potential Head	h	(cm)
Soil Water Potential (mass basis)	$\phi_m = gh$	(ergs/gm)
Soil Water Potential (volume basis)	$\phi_v = \rho gh$	(ergs/cm^3)

For an unsaturated soil, the value of $h < 0$ (less than atmospheric pressure).

Soil water potential on a volume basis is the same as pressure = ρgh (dynes/cm^2)

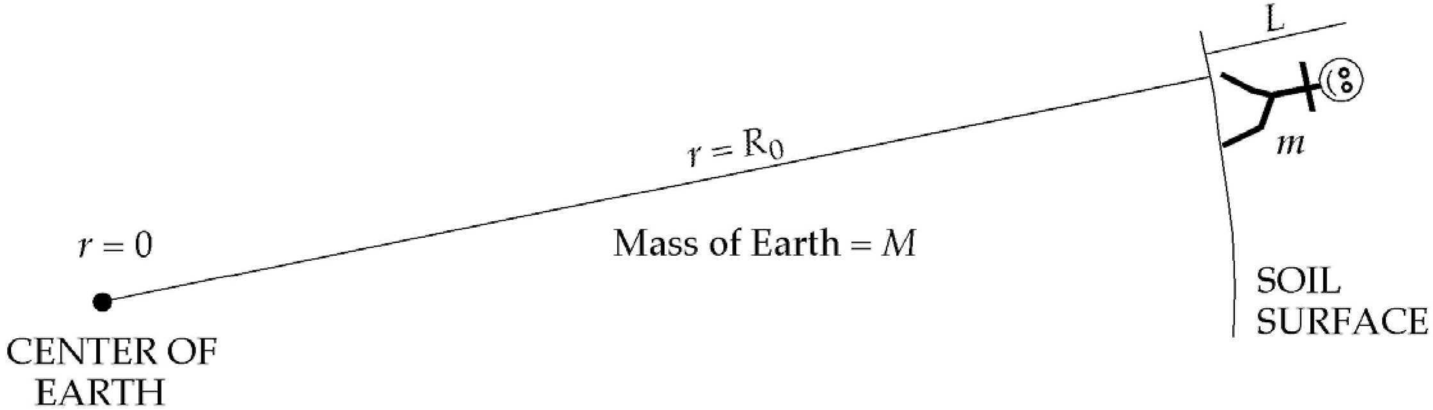
CONCEPT OF
SOIL WATER POTENTIAL OR MATRIC POTENTIAL ϕ

Work (ergs) = **Force·Distance** (dyne·cm) = **Pressure·Volume** (dyne·cm^{-2})·(cm^3)



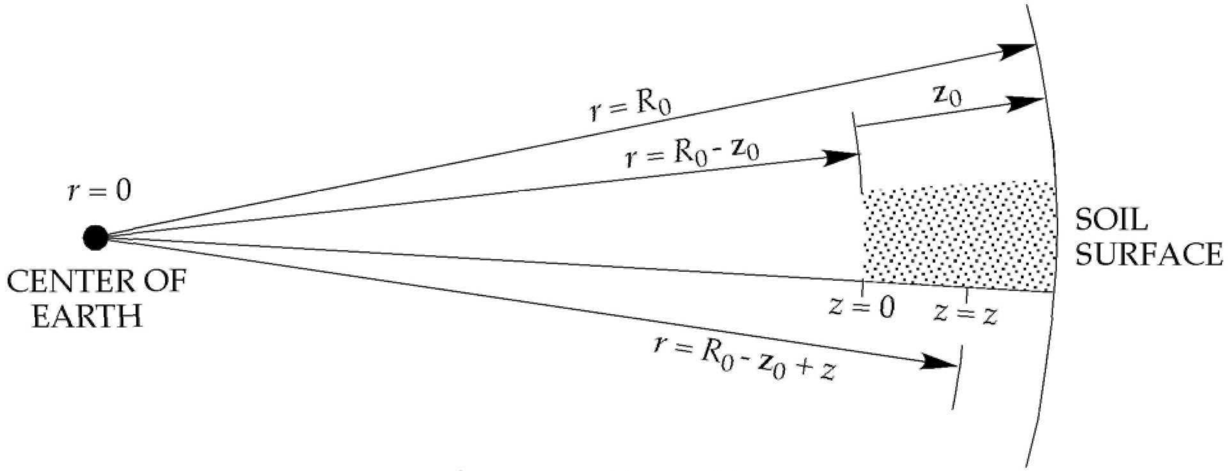
NEWTON'S LAW OF UNIVERSAL GRAVITATION

$$F = -G\frac{mM}{s^2}$$



$$\frac{F}{m} = -G\frac{M}{s^2}\bigg|_{s=R_0+L/2} = -G\frac{M}{(R_0+L/2)^2} \approx -G\frac{M}{R_0^2} = -g$$

GRAVITATIONAL POTENTIAL ENERGY FOR SOIL WATER ψ



$$F = -\frac{d\psi}{dr} \qquad \frac{F}{m} = -G\frac{M}{r^2}$$

$$-\int_{R_0 - z_0}^{R_0 - z_0 + z} G\frac{M}{r^2}dr = -\int_0^{\psi} d\psi$$

$$-\psi = G\frac{M}{r}\bigg|_{R_0 - z_0}^{R_0 - z_0 + z}$$

$$\psi = -GM\left(\frac{1}{R_0 - z_0 + z} - \frac{1}{R_0 - z_0}\right) = -GM\left[\frac{-z}{(R_0 - z_0 + z)(R_0 - z_0)}\right] \approx \left(\frac{GM}{R_0^2}\right)z$$

$$\psi = gz$$

TOTAL SOIL WATER POTENTIAL Φ

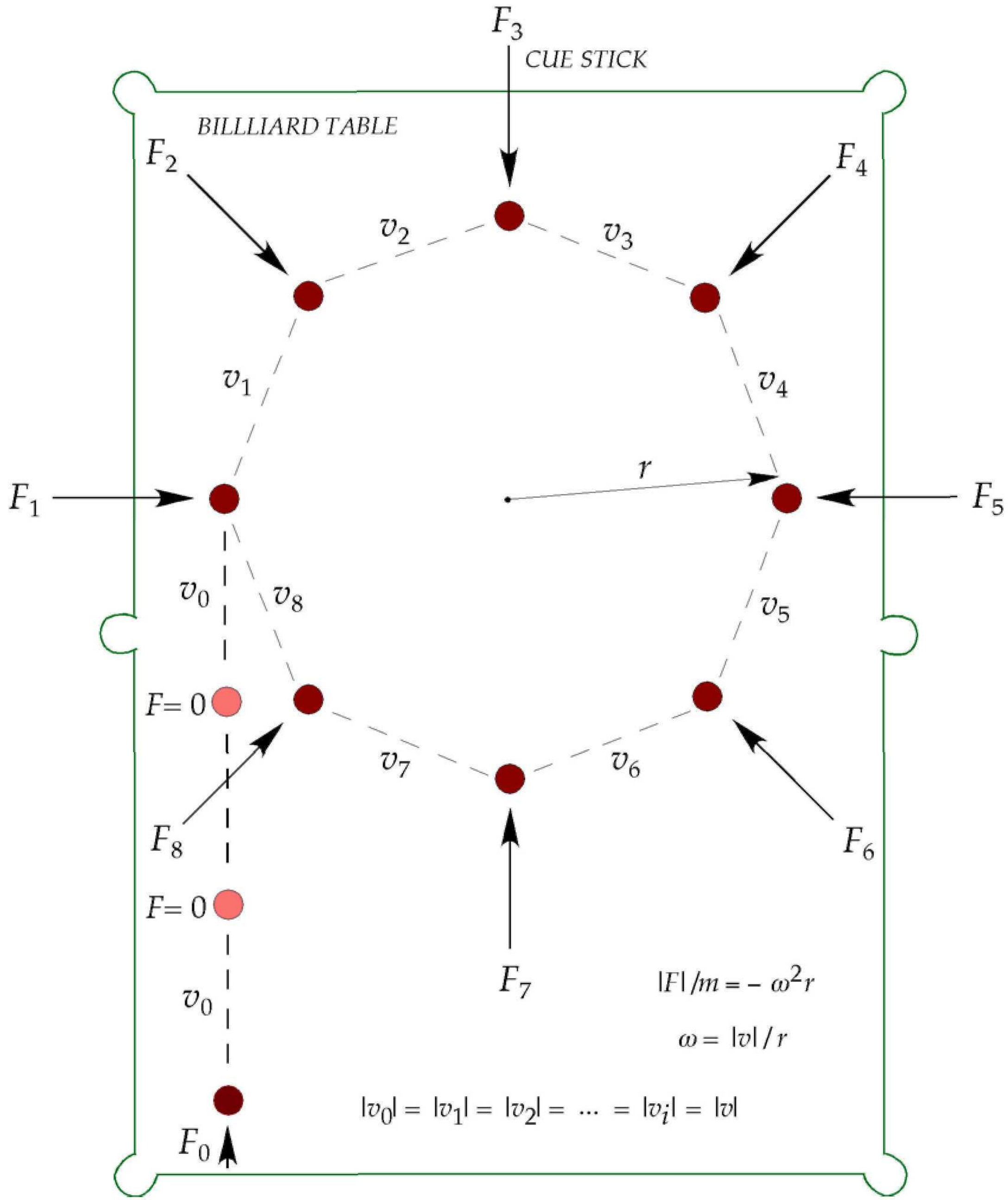
$$\Phi = \phi + \psi = gh + gz$$

$$H = h + z$$

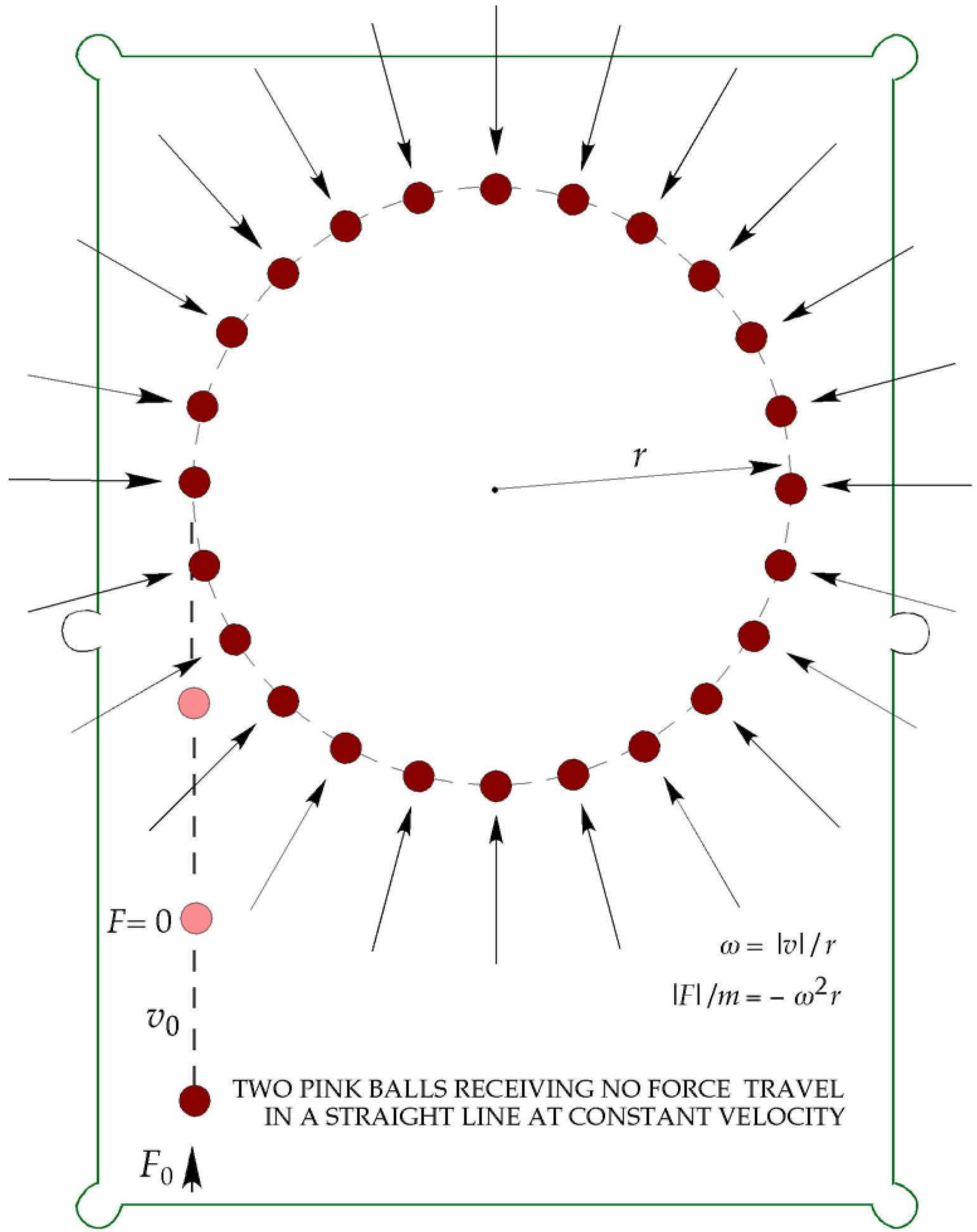
SUM OF FORCES *F* ACTING ON SOIL WATER

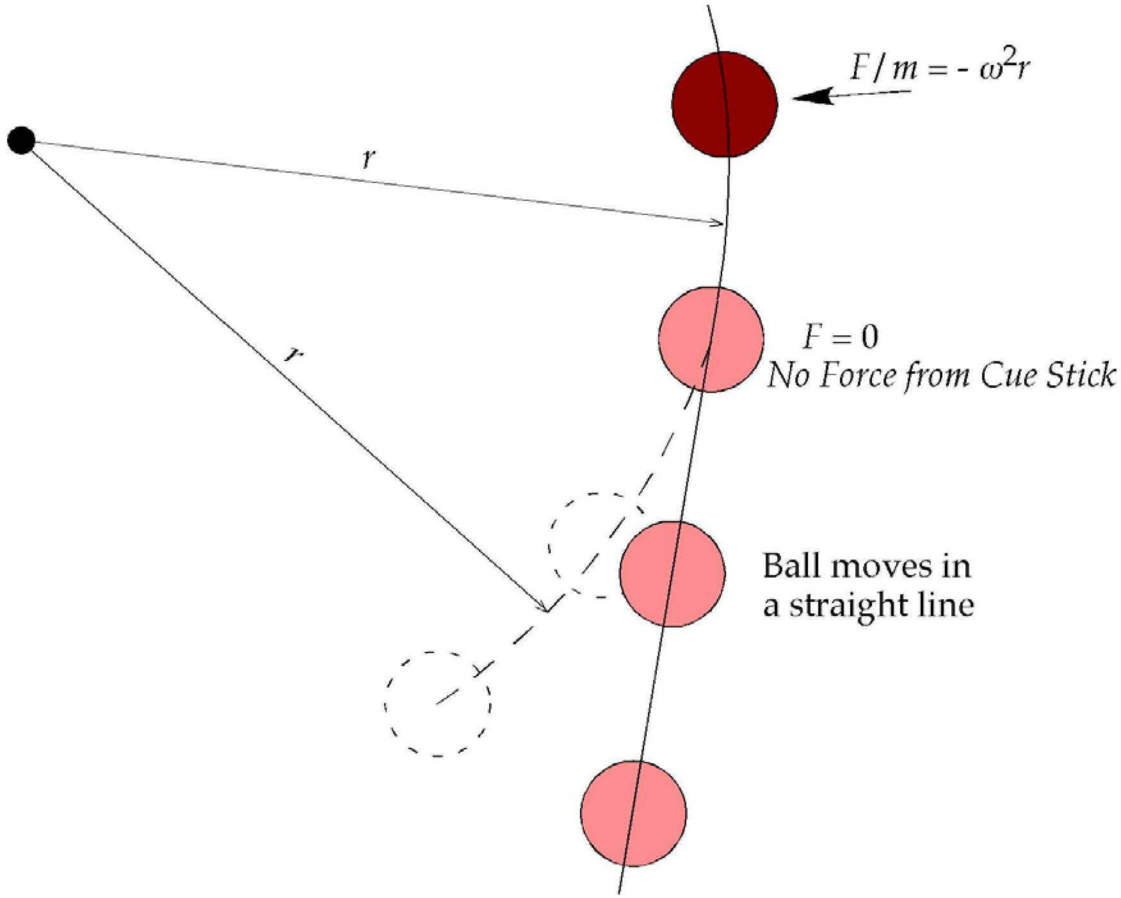
$$F = -\frac{d\Phi}{dz} = -\frac{d(\phi + \psi)}{dz} \quad \text{or} \quad = -g\left(\frac{dh}{dz} + \frac{dz}{dz}\right)$$

$$F = -\frac{d\phi}{dz} - \frac{d\psi}{dz} \quad \text{or} \quad = -g\left(\frac{dh}{dz} + 1\right)$$



F_3
CUE STICK
BILLIARD TABLE
F_2
F_4
v_2
v_3
v_1
v_4
r
F_1
F_5
v_0
v_8
v_5
$F = 0$
v_7
v_6
F_8
F_6
$F = 0$
F_7
$|F|/m = -\omega^2 r$
$\omega = |v|/r$
v_0
$|v_0| = |v_1| = |v_2| = \dots = |v_i| = |v|$
F_0


r
ω = |v| / r
|F| / m = − ω²r
F= 0
v0
F0
TWO PINK BALLS RECEIVING NO FORCE TRAVEL
IN A STRAIGHT LINE AT CONSTANT VELOCITY

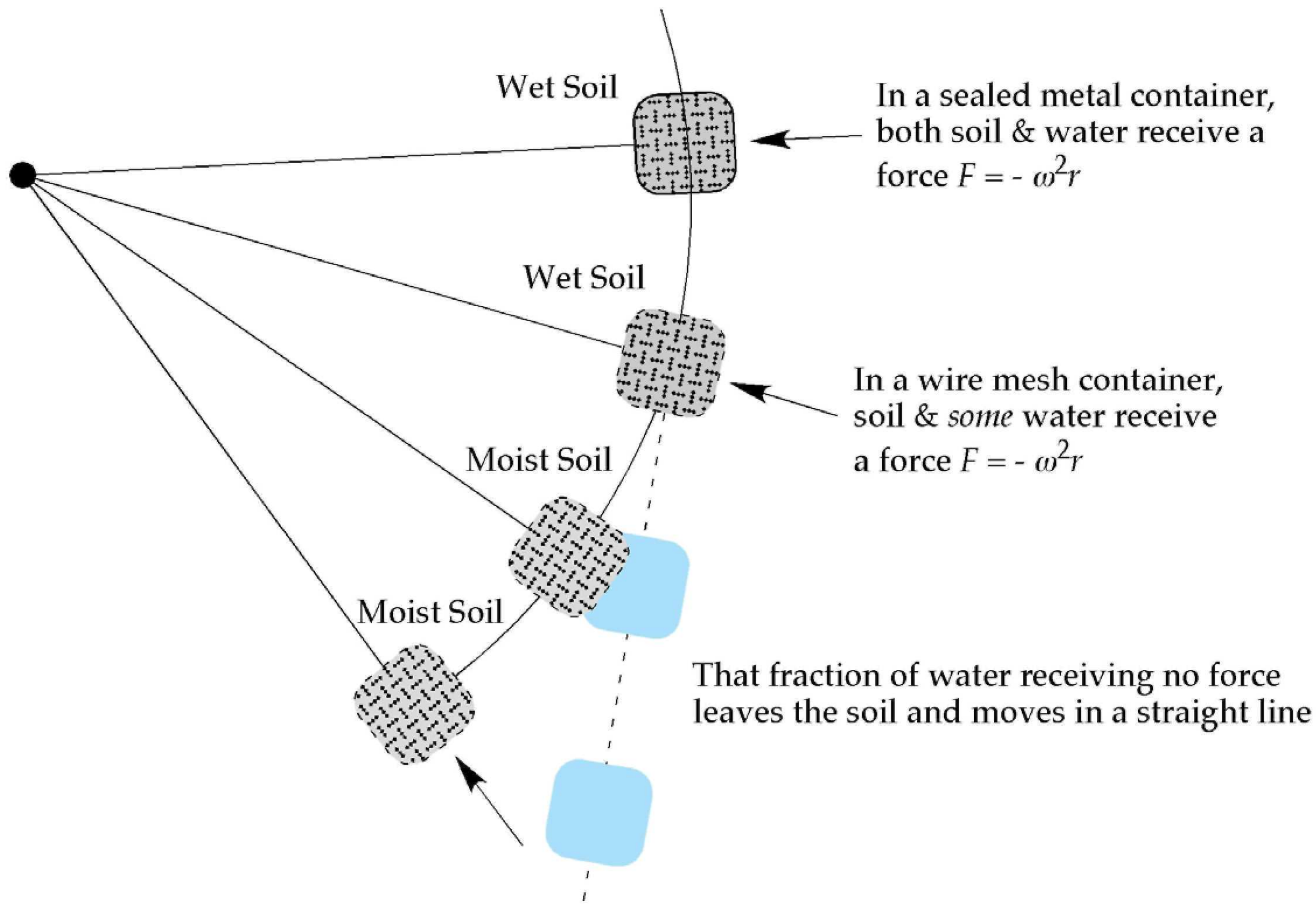


$F / m = - \omega^2 r$
r
r
$F = 0$
No Force from Cue Stick
Ball moves in
a straight line

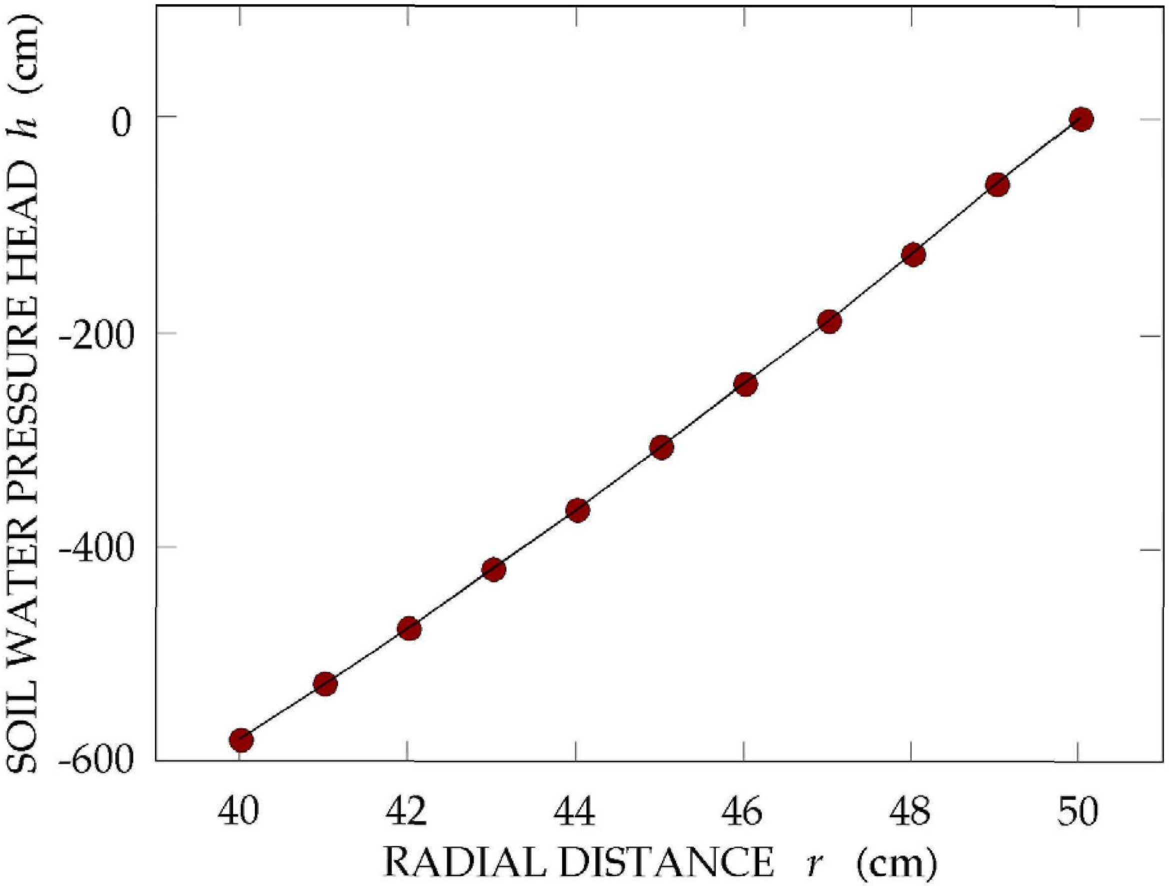
$$-\frac{d\phi}{dr} = \frac{F(r)}{m} = -\omega^2 r$$

$$\int_{gh(r)}^{gh_2} d\phi = \int_{r}^{r_2} \omega^2 r dr$$

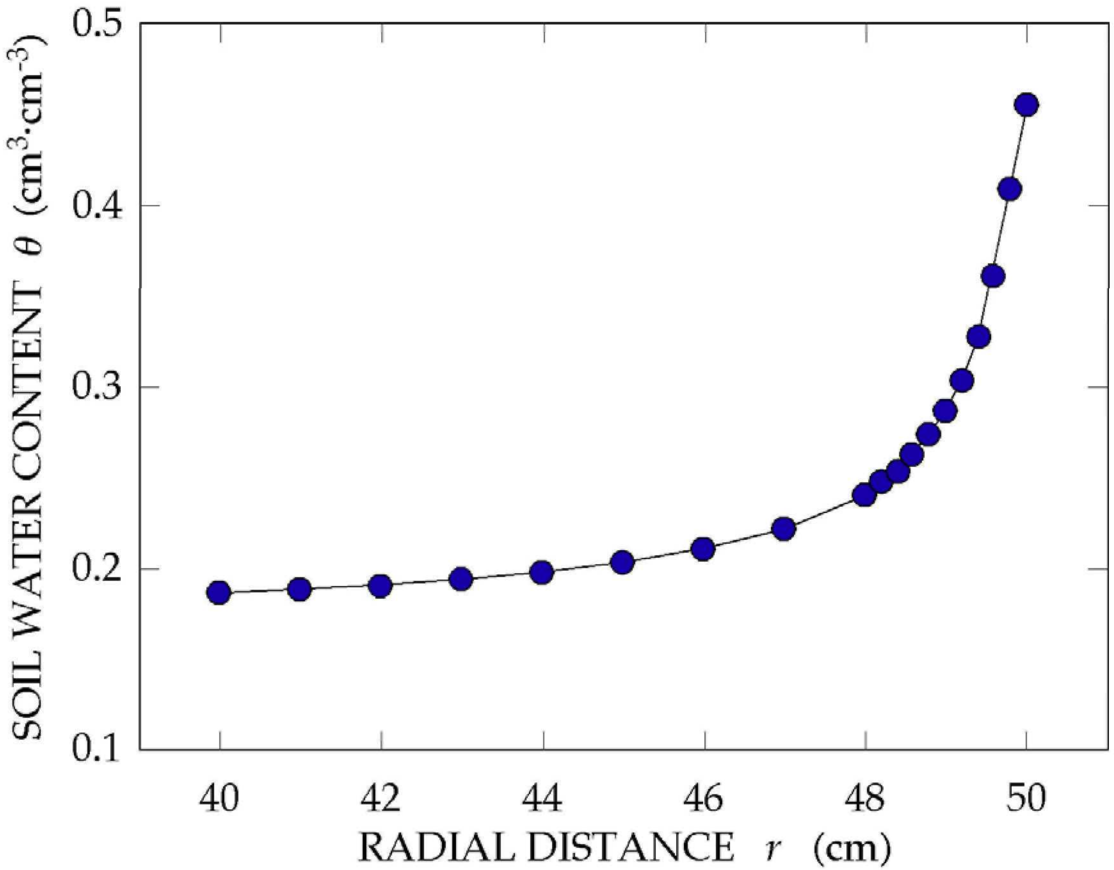
$$h(r) = -\frac{\omega^2}{2g}\left(r_2^2 - r^2\right) + h_2$$



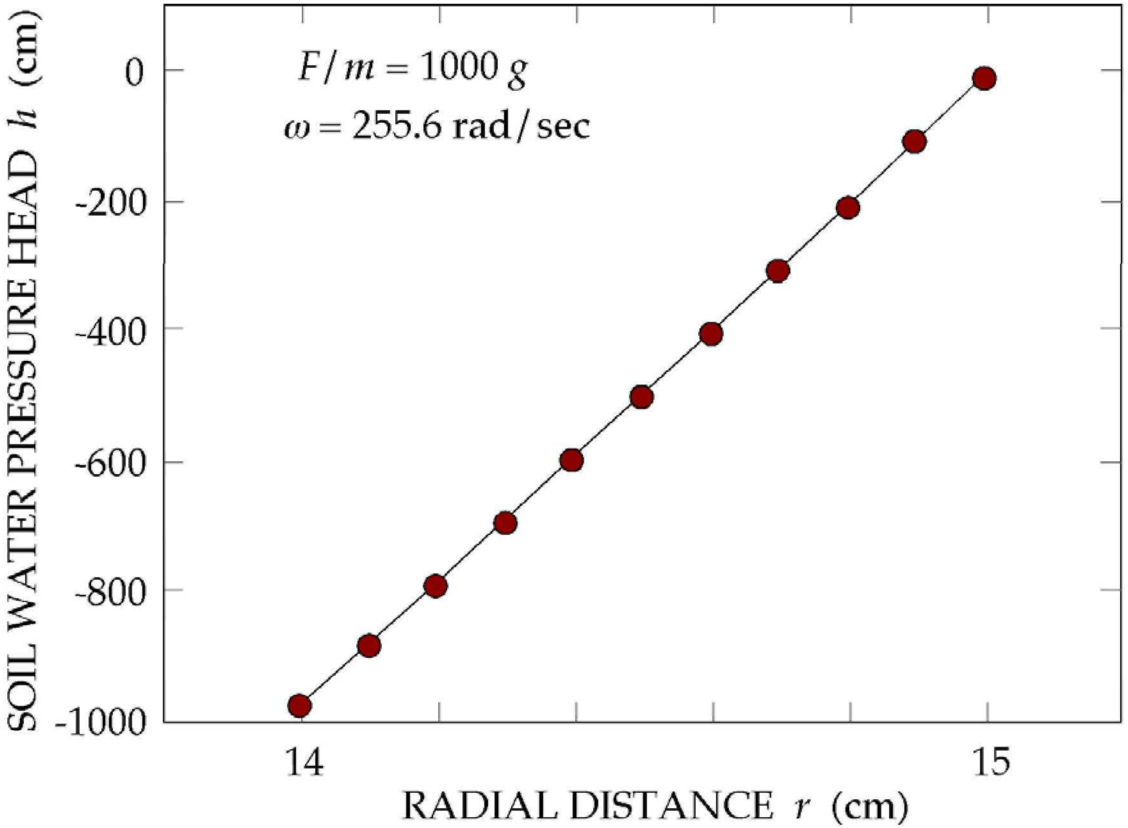
Wet Soil
In a sealed metal container, both soil & water receive a force $F = -\omega^2 r$
Wet Soil
In a wire mesh container, soil & *some* water receive a force $F = -\omega^2 r$
Moist Soil
Moist Soil
That fraction of water receiving no force leaves the soil and moves in a straight line



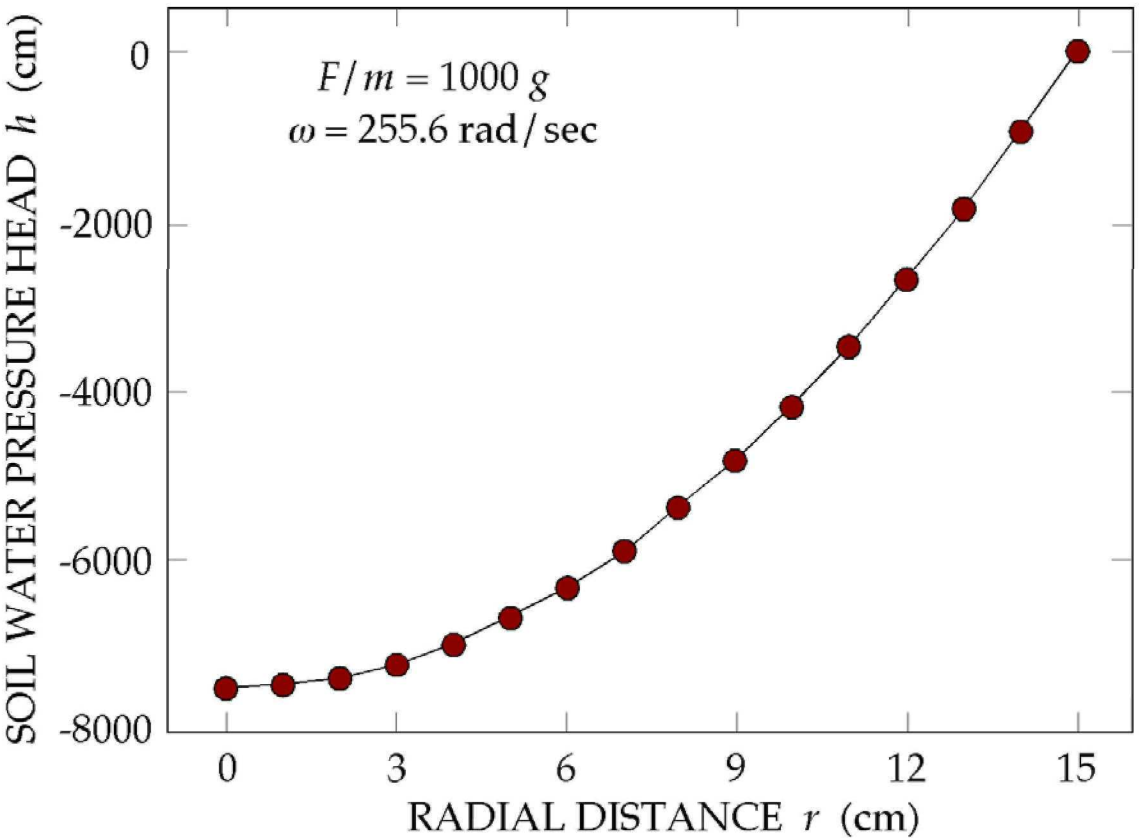
SOIL WATER PRESSURE HEAD h (cm)
0
-200
-400
-600
40
42
44
46
48
50
RADIAL DISTANCE r (cm)



0.5
0.4
0.3
0.2
0.1
40
42
44
46
48
50
SOIL WATER CONTENT θ (cm³·cm⁻³)
RADIAL DISTANCE r (cm)

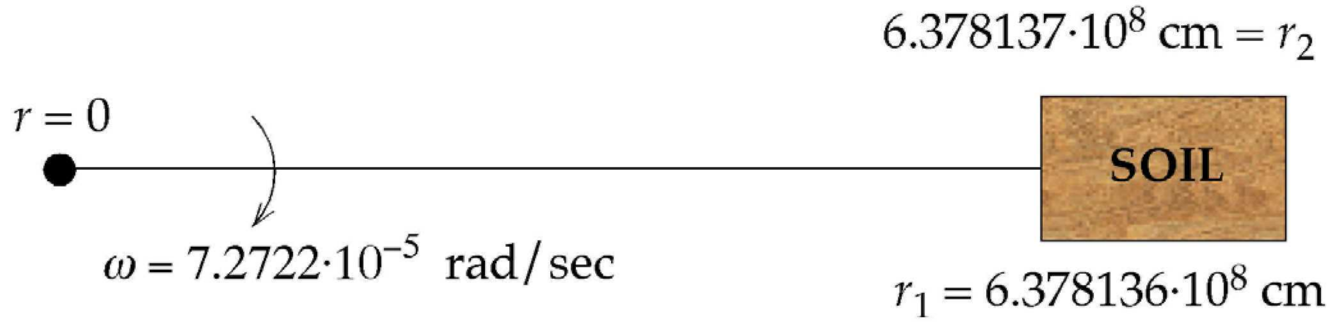


$F/m = 1000\ g$
$\omega = 255.6$ rad/sec
SOIL WATER PRESSURE HEAD h (cm)
0
-200
-400
-600
-800
-1000
14
15
RADIAL DISTANCE r (cm)



$F/m = 1000\ g$
$\omega = 255.6$ rad/sec
SOIL WATER PRESSURE HEAD h (cm)
0
-2000
-4000
-6000
-8000
0
3
6
9
12
15
RADIAL DISTANCE r (cm)

100-cm SOIL PROFILE AT THE RADIUS OF THE EARTH



THE SOIL IS INITIALLY WATER-SATURATED

THE SAMPLE ROTATES ONCE EVERY DAY

WHAT IS THE MATRIC POTENTIAL HEAD AT r_1 ?

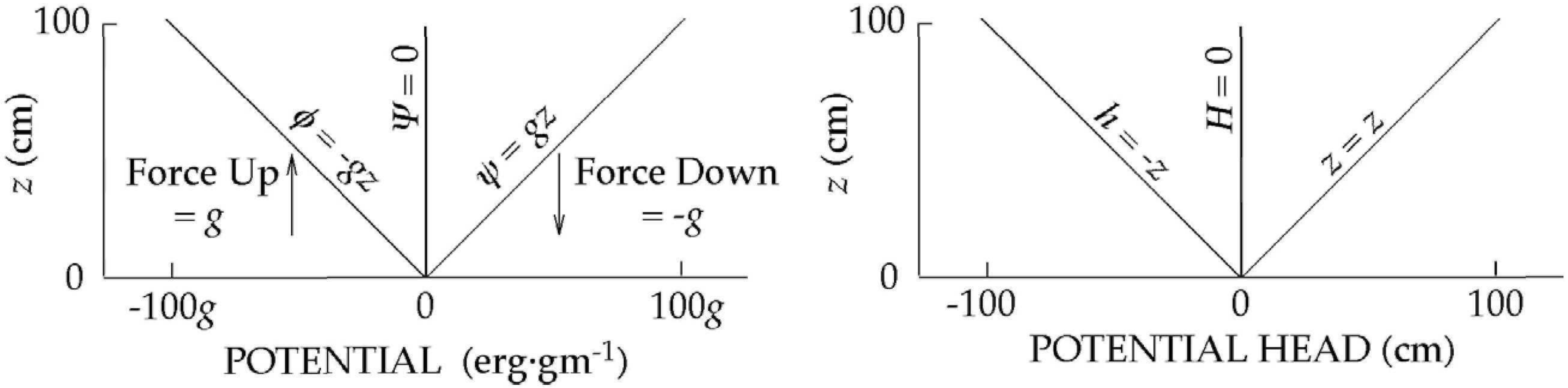
WHEN DOES SOIL WATER NOT MOVE??

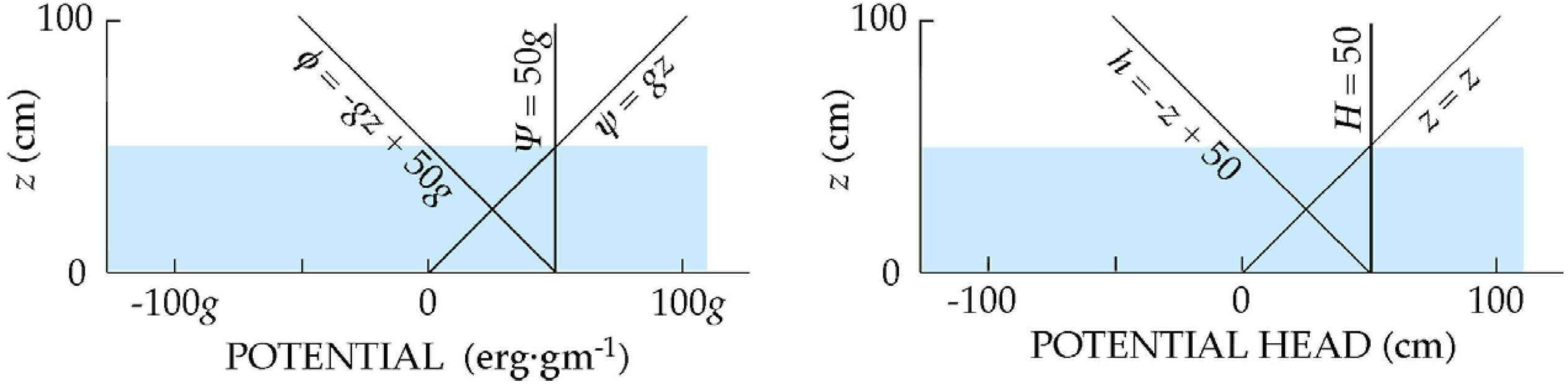
WHEN SUM OF FORCES *F* ACTING ON SOIL WATER ARE ZERO

$$F = -\frac{d\phi}{dz} - \frac{d\psi}{dz} = 0$$

$$\frac{d\phi}{dz} = -\frac{d\psi}{dz}$$

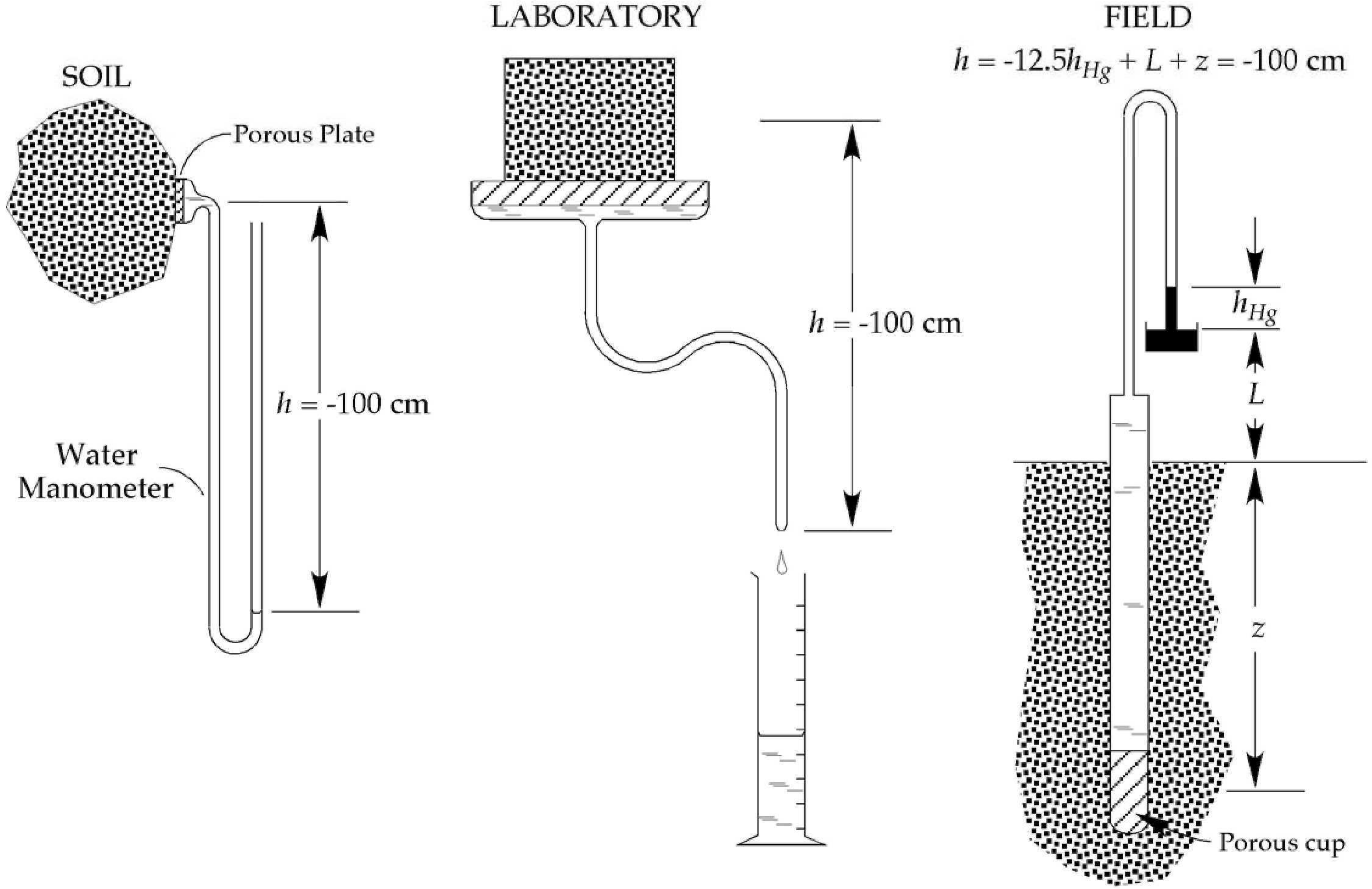
$$\frac{dh}{dz} = -1$$


100
0
z (cm)
Ψ = 0
ϕ = -gz
ψ = gz
Force Up
= g
Force Down
= -g
-100g
0
100g
POTENTIAL (erg·gm-1)
100
0
z (cm)
H = 0
h = -z
z = z
-100
0
100
POTENTIAL HEAD (cm)



100
z (cm)
0
-100g
0
100g
POTENTIAL (erg·gm-1)
φ = -gz + 50g
Ψ = 50g
ψ = gz
100
z (cm)
0
-100
0
100
POTENTIAL HEAD (cm)
h = -z + 50
H = 50
z = z

MEASURING THE SOIL WATER MATRIC POTENTIAL



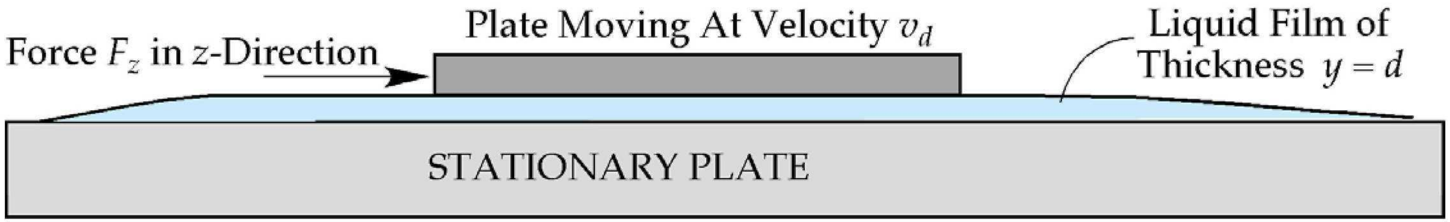
WHAT HAPPENS IF THE SUM OF FORCES F ACTING ON SOIL WATER ARE NOT ZERO?

THE SOIL WATER MOVES !

HOW FAST? HOW MUCH? HOW LONG?

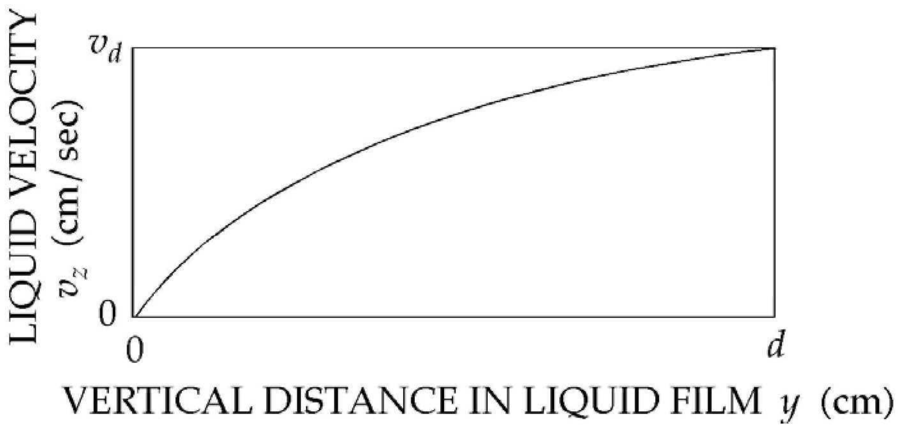
VISCOSITY IS PART OF THE ANSWER

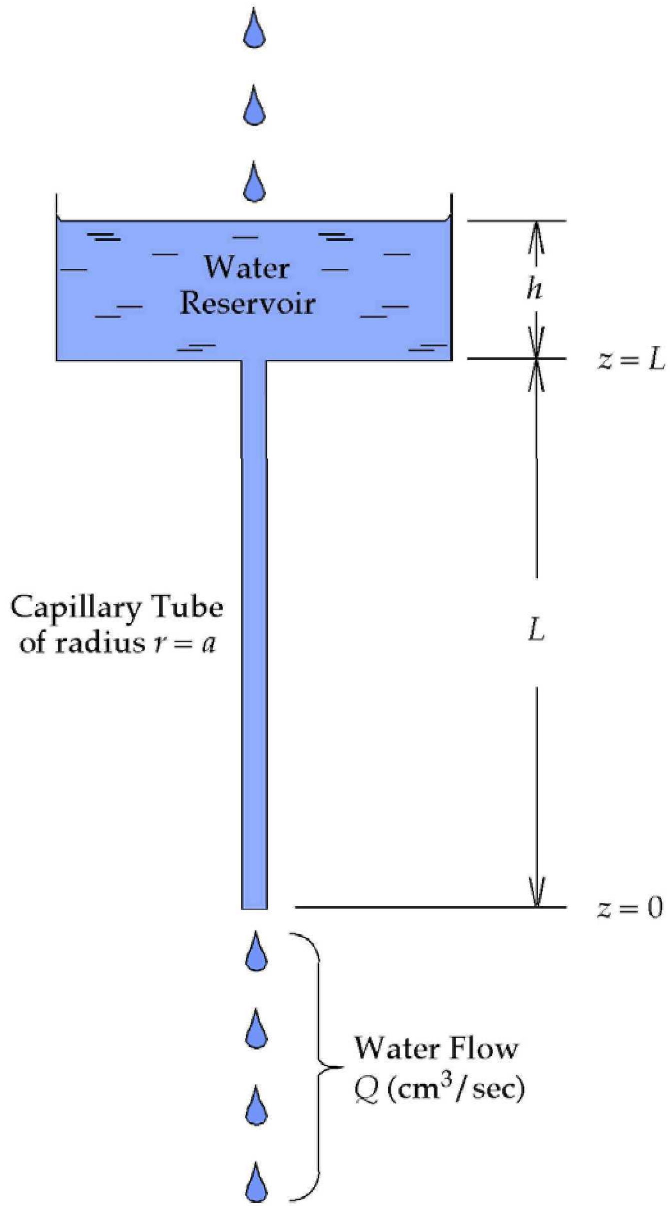
Viscosity η determines the amount of friction between water molecules, which in turn determines the energy absorbed by the water movement



$$F_z = \eta \frac{dv_z}{dy} \sim \eta \frac{v_d - 0}{d - 0}$$

$$v_d = \frac{d}{\eta} F_z$$





Water
Reservoir
h
z = L
L
Capillary Tube
of radius r = a
z = 0
Water Flow
Q (cm3/sec)

The water velocity in the capillary is in equilibrium between
gradient of the total water potential
and
shearing force of the water

$$\text{area} \cdot \left(-\frac{\partial \Phi}{\partial z} \right) + \text{circumference} \cdot \left(-\eta \frac{\partial v}{\partial r} \right) = 0$$

$$\pi r^2 \left(\rho g \frac{\partial (h+z)}{\partial z} \right) + 2\pi r L \eta \frac{\partial v}{\partial r} = 0$$

$$\pi r^2 \left(\rho g \frac{[(h+L)-(0+0)]}{(L-0)} \right) + 2\pi r L \eta \frac{\partial v}{\partial r} = 0$$

$$\frac{dv}{dr} = -\frac{r[\rho g(h+L)]}{2L\eta}$$

$$\int_{v(r)}^{0} dv = -\frac{[\rho g(h+L)]}{2L\eta} \int_{r}^{a} r\,dr$$

$$v(r) = 2v_0 \left(1 - \frac{r^2}{a^2} \right) \quad \text{where} \quad v_0 = \frac{a^2[\rho g(h+L)]}{8L\eta}$$

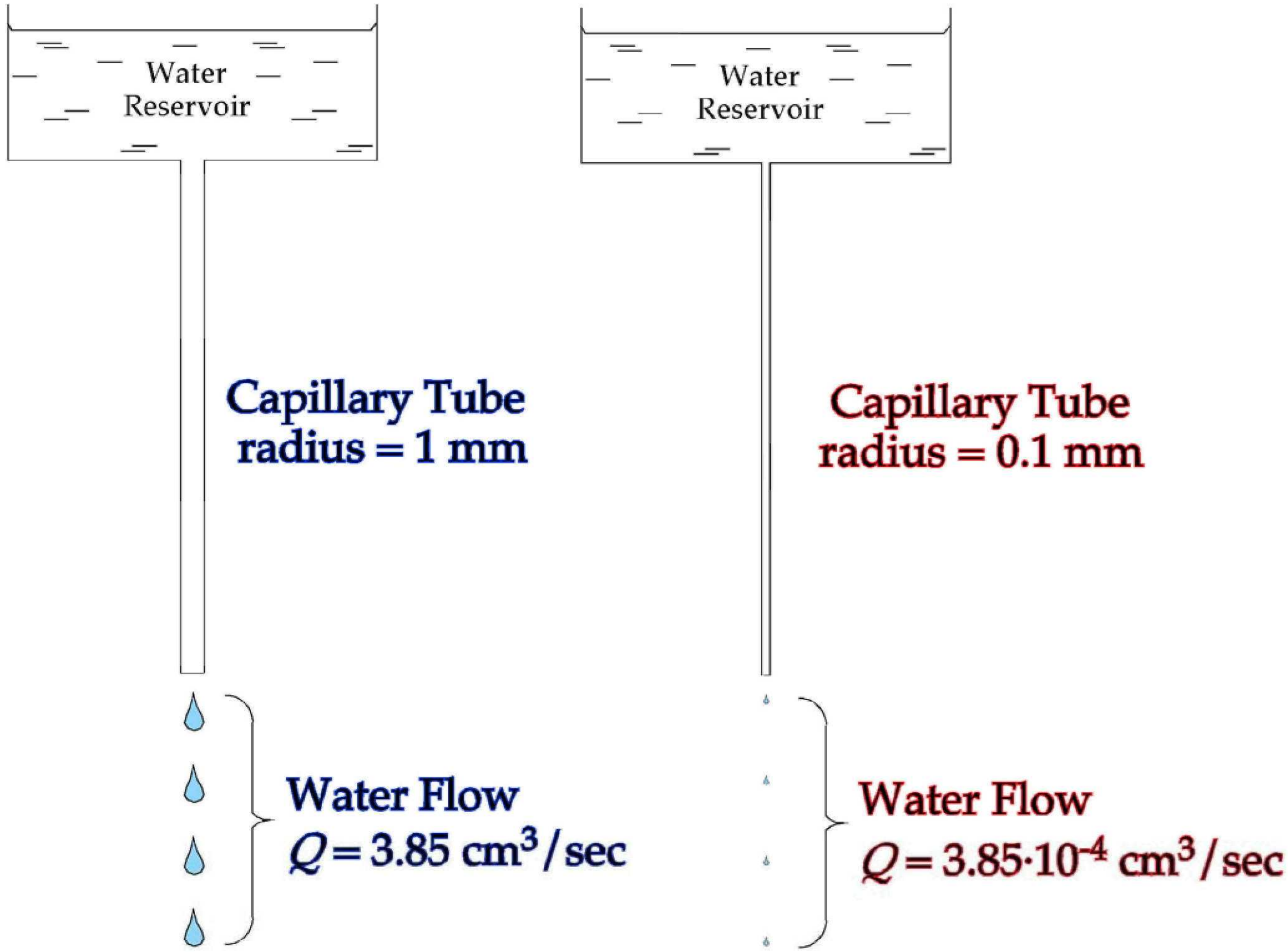
The volumetric flow through the capillary Q is obtained by integrating the velocity $v(r)$ with its cross-sectional area

$$Q = \int_0^a 2\pi r v(r) dr = 4\pi v_0 \int_0^a \left(r - \frac{r^3}{a^2} \right) dr$$

$$Q = a^2 \pi v_0 = \frac{a^4 \pi \rho g (h + L)}{8 L \eta}$$

When the depth of water at the top of the capillary is maintained at zero, we obtain

$$Q = \frac{\pi \rho g a^4}{8 \eta}$$



1 Capillary Tube = 10,000 Capillary Tubes

HOW FAST DOES WATER MOVE THROUGH SOIL?

IT MOVES AT A RATE PROPORTIONAL TO THE FORCE ACTING ON IT

$$q = \frac{\text{volume of water}}{(\text{area of surface}) \cdot (\text{interval of time})}$$

$$q \;\alpha\; F \qquad \text{and since} \qquad F = -\frac{d\Phi_v}{dz},$$

$$q \;\alpha\; \left[-\frac{d(\phi_v + \psi_v)}{dz}\right]$$

$$q \;\alpha\; \left[-\rho g\left(\frac{dh}{dz} + \frac{dz}{dz}\right)\right]$$

IF THE PROPORTIONALITY CONSTANT $\alpha = k/\eta$

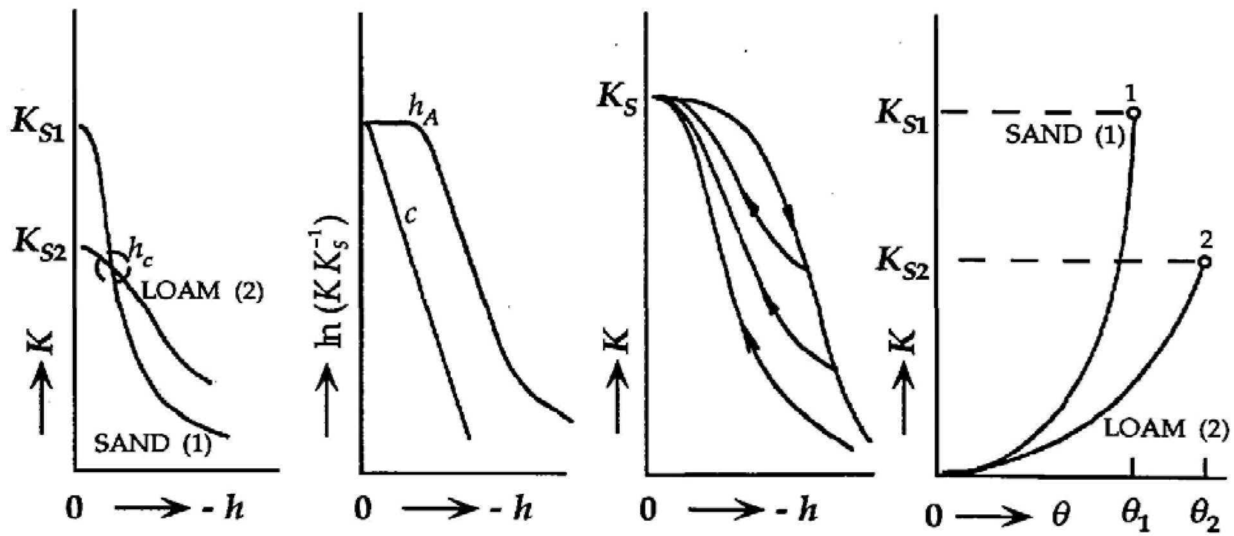
$$q = -\frac{\rho g k(h)}{\eta}\left(\frac{dh}{dz} + 1\right)$$

$$q = -K(h)\left(\frac{dh}{dz} + 1\right)$$

Note that ρ/η is a property of the water, g the gravitational acceleration and $k(h)$ is a measure of the soil pore geometry.

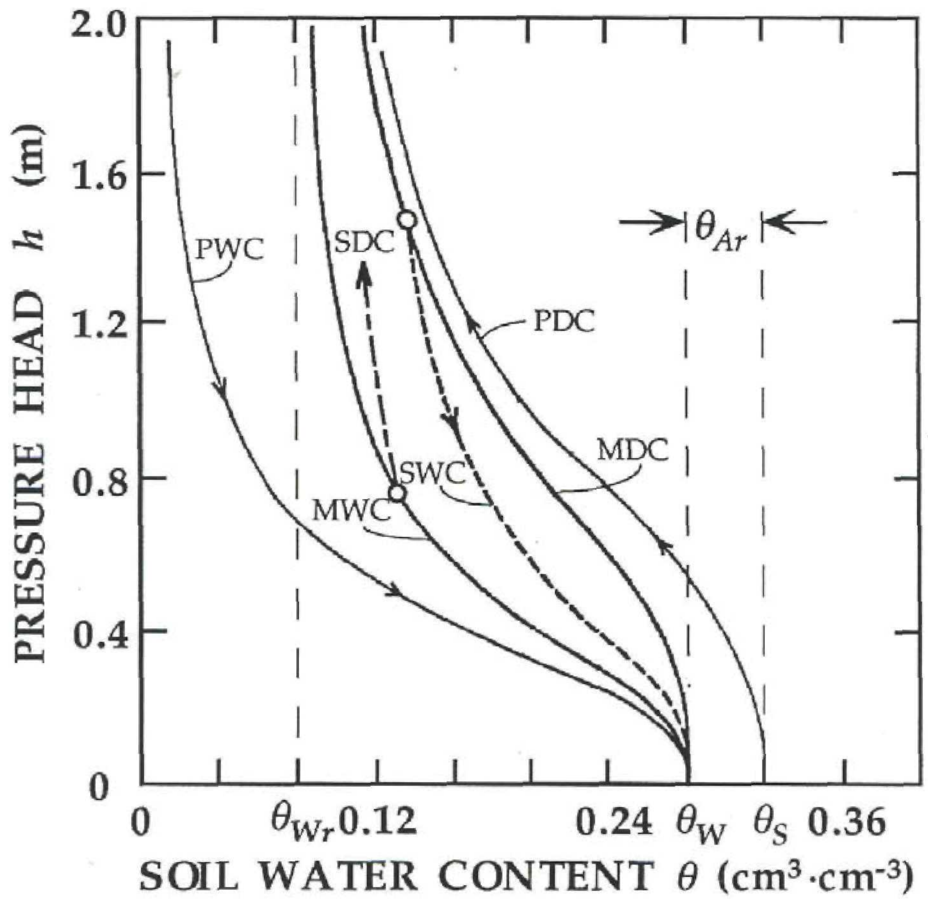
For h>0, Darcy's equation for saturated flow.

For h<0, Buckingham-Darcy equation for unsaturated flow.

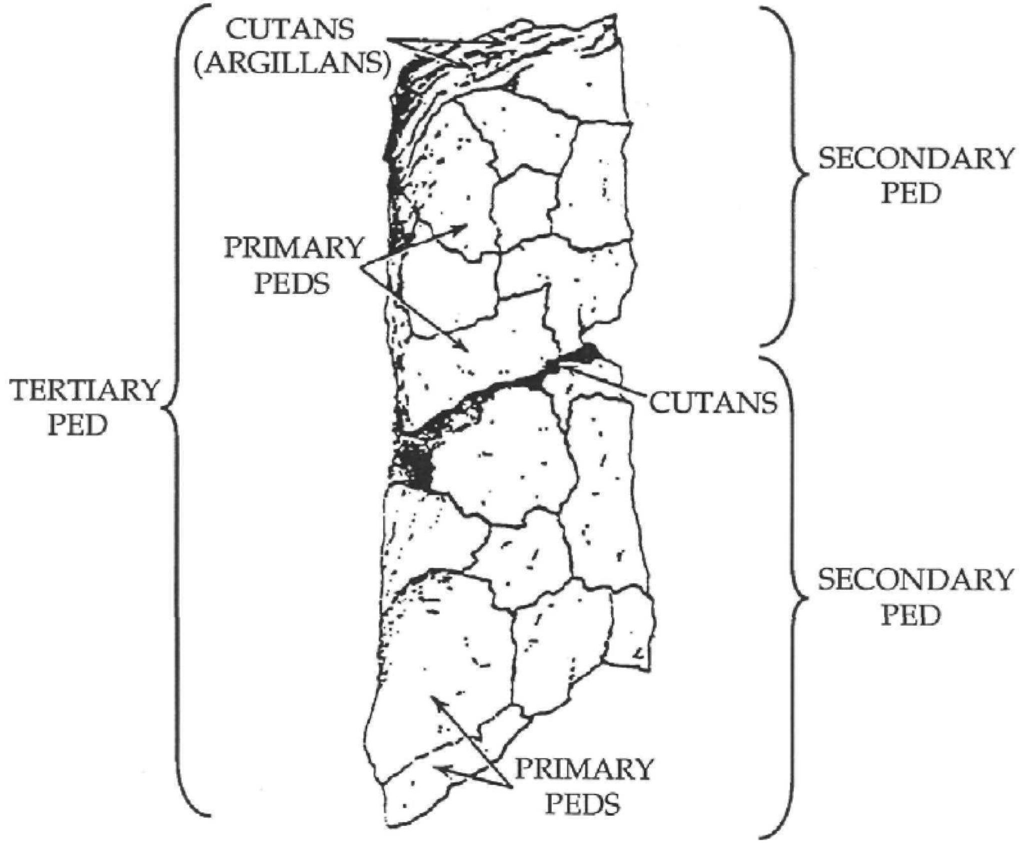


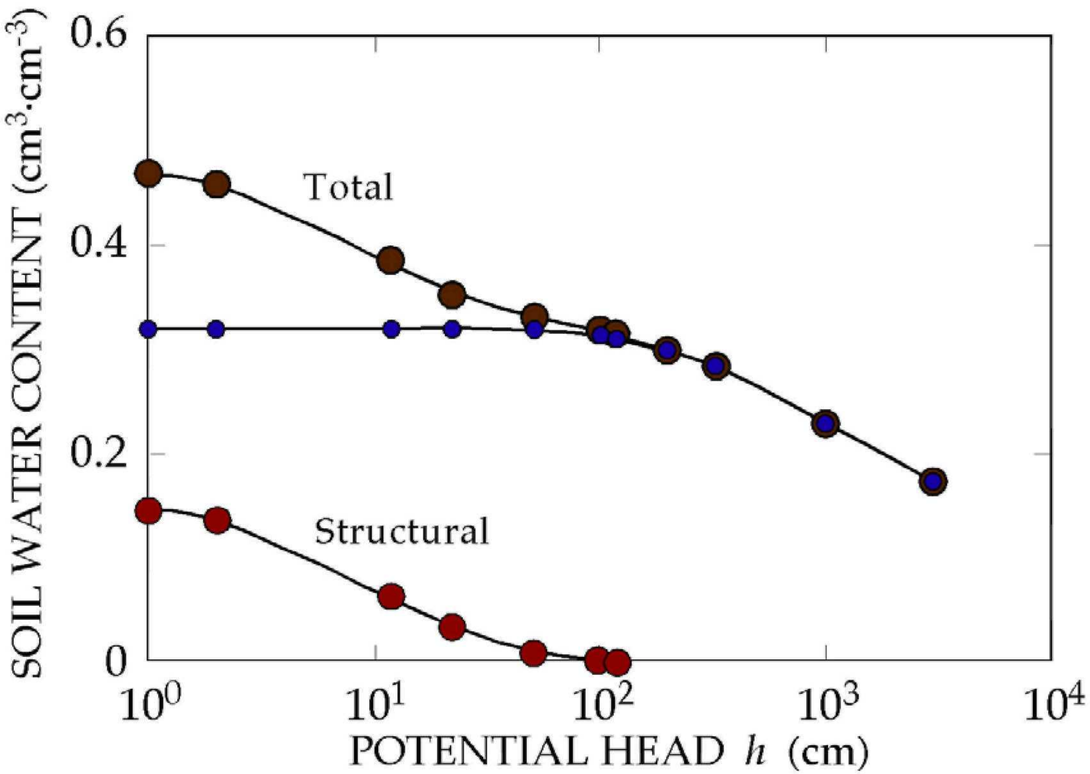
K_{S1}
K_{S2}
h_c
LOAM (2)
SAND (1)
K
0
- h
h_A
c
$\ln(K K_S^{-1})$
0
- h
K_S
K
0
- h
K_{S1}
1
SAND (1)
K_{S2}
2
K
LOAM (2)
0
θ
θ_1
θ_2

HYSTERESIS

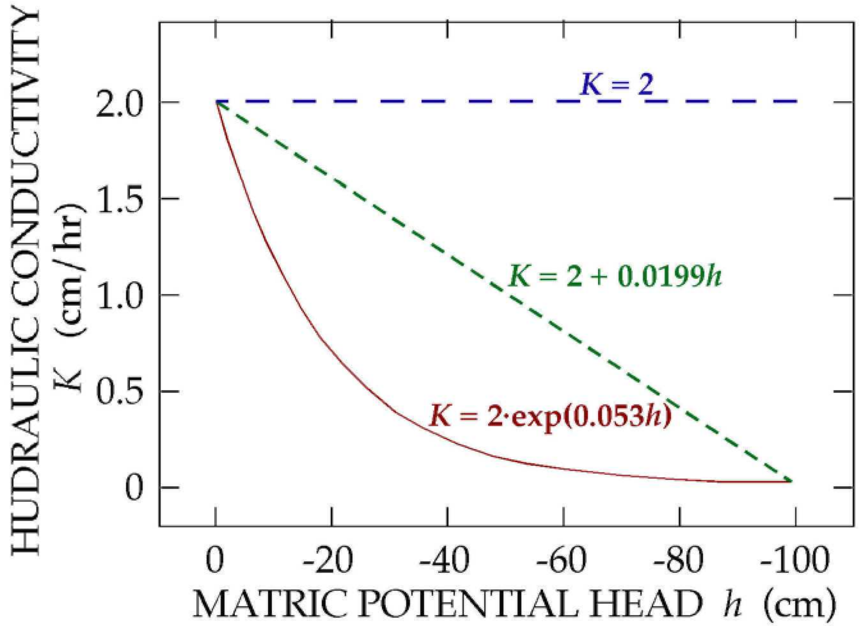


2.0
1.6
1.2
0.8
0.4
0
PRESSURE HEAD h (m)
PWC
SDC
SWC
MWC
PDC
MDC
θAr
0
θWr 0.12
0.24 θW θS 0.36
SOIL WATER CONTENT θ (cm3·cm-3)

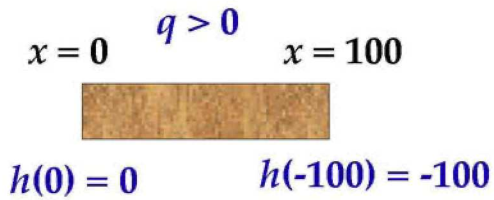

CUTANS
(ARGILLANS)
PRIMARY
PEDS
TERTIARY
PED
SECONDARY
PED
CUTANS
SECONDARY
PED
PRIMARY
PEDS



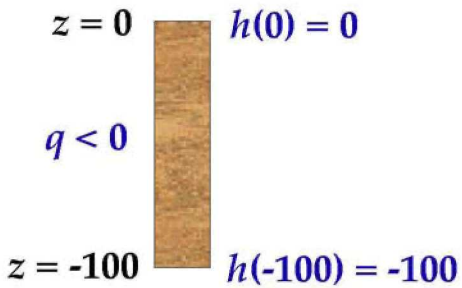
SOIL WATER CONTENT (cm3·cm-3)
0.6
0.4
0.2
0
Total
Structural
10^0
10^1
10^2
10^3
10^4
POTENTIAL HEAD h (cm)



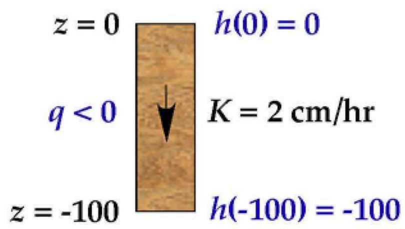
HUDRAULIC CONDUCTIVITY
K (cm/hr)
2.0
1.5
1.0
0.5
0
0
-20
-40
-60
-80
-100
MATRIC POTENTIAL HEAD h (cm)
K = 2
K = 2 + 0.0199h
K = 2·exp(0.053h)



x = 0
q > 0
x = 100
h(0) = 0
h(-100) = -100



z = 0
h(0) = 0
q < 0
z = -100
h(-100) = -100



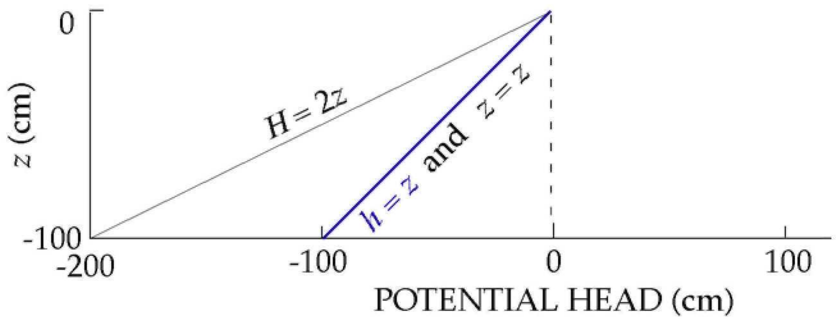
$$q = -K(h)\left(\frac{dh}{dz} + 1\right)$$

$$\int_{-100}^{0} \left(\frac{q}{K} + 1\right) dz = -\int_{-100}^{0} dh \quad \text{or} \quad \left(\frac{q}{K} + 1\right) z \Big|_{-100}^{0} = -h\Big|_{-100}^{0}$$

$$\left(\frac{q}{K} + 1\right) 100 = -100 \quad \text{or} \quad q = -2K = -4$$

$$\int_{-100}^{z} \left(\frac{q}{K} + 1\right) dz = -\int_{-100}^{h} dh \quad \text{or} \quad \left(\frac{q}{K} + 1\right) z \Big|_{-100}^{z} = -h\Big|_{-100}^{r}$$

$$h = z$$



$z = 0$ $\quad h(0) = 0$

$q < 0$ $\quad K = 2 + 0.0199h$ cm/hr

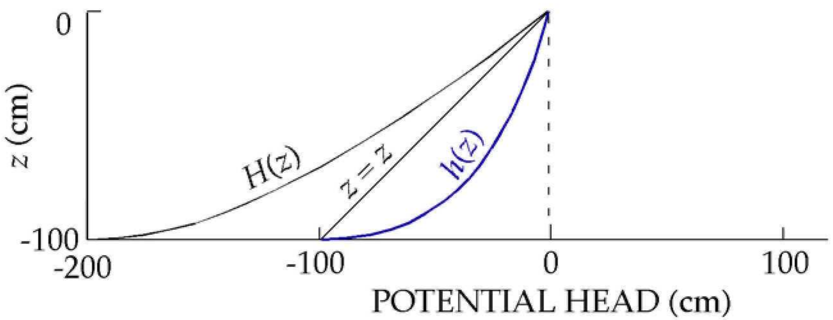
$z = -100$ $\quad h(-100) = -100$

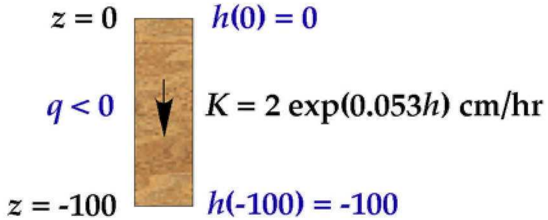
$$\int_{-100}^{0} dz = -\int_{-100}^{0} \frac{dh}{(q/K+1)} = -\int_{-100}^{0} \frac{dh}{\left[q/(ah+b)+1\right]}$$

$$\int_{-100}^{0} dz = -\int_{-100}^{0} \frac{(ah+b)dh}{(ah+b+q)} \qquad \frac{200a}{q} = \ln\left(\frac{b+q}{-100a+b+q}\right)$$

$$\frac{3.98}{q} = \ln\left(\frac{2+q}{0.01+q}\right) \qquad q = -2.154$$

$$\int_{-100}^{z} dz = -\int_{-100}^{h(z)} \frac{dh}{q/(ah+b)+1} \qquad z = -200 - h(z) - 126.3\ln\left(\frac{0.514-0.0199h(z)}{2.504}\right)$$



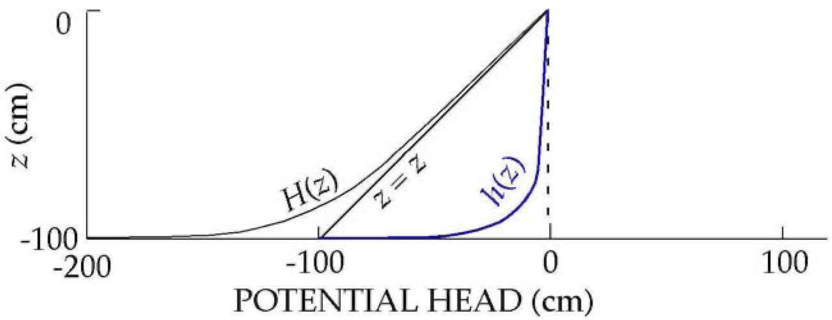


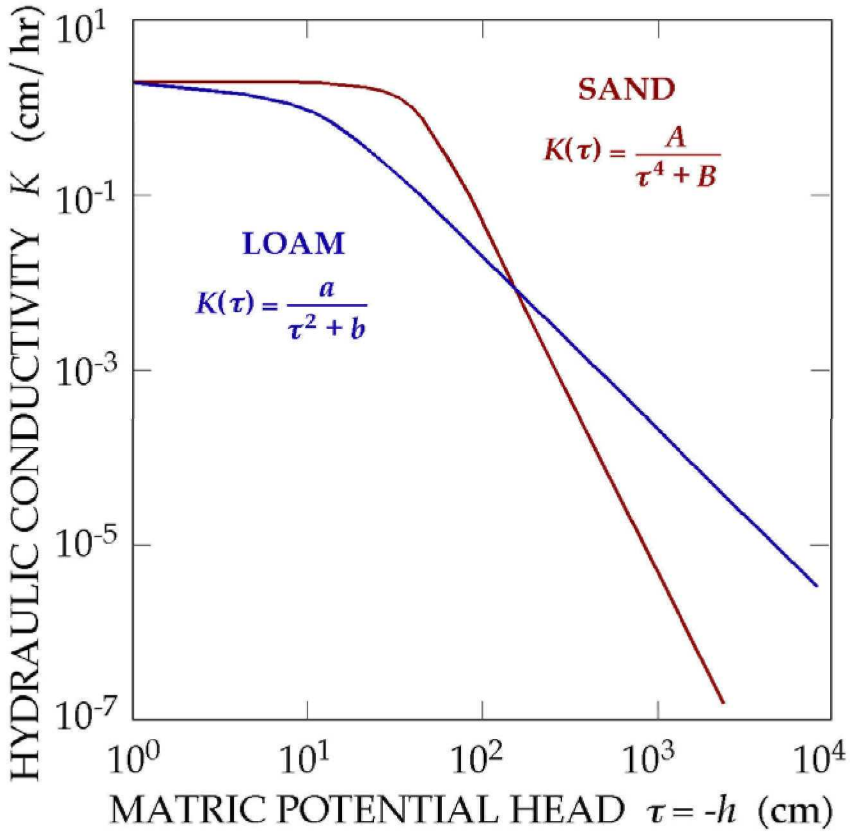
$$\int_{-100}^{0} dz = -\int_{-100}^{0} \frac{dh}{q/K+1} = -\int_{-100}^{0} \frac{Kdh}{K+q} = -\int_{-100}^{0} \frac{adh}{\left(a+q\exp(-bh)\right)}$$

$$z\Big|_{-100}^{0} = -h\Big|_{-100}^{0} - \frac{1}{b}\ln\left(a+q\exp(-bh)\right)\Big|_{-100}^{0}$$

$$100 = -100 - \frac{1}{b}\ln\left(\frac{a+q}{a+q\exp(100b)}\right) \qquad q = -2.01$$

$$z = -200 - h(z) - \frac{1}{0.053}\ln\left\{\frac{2-2.01\exp[-0.053h(z)]}{-400}\right\}$$





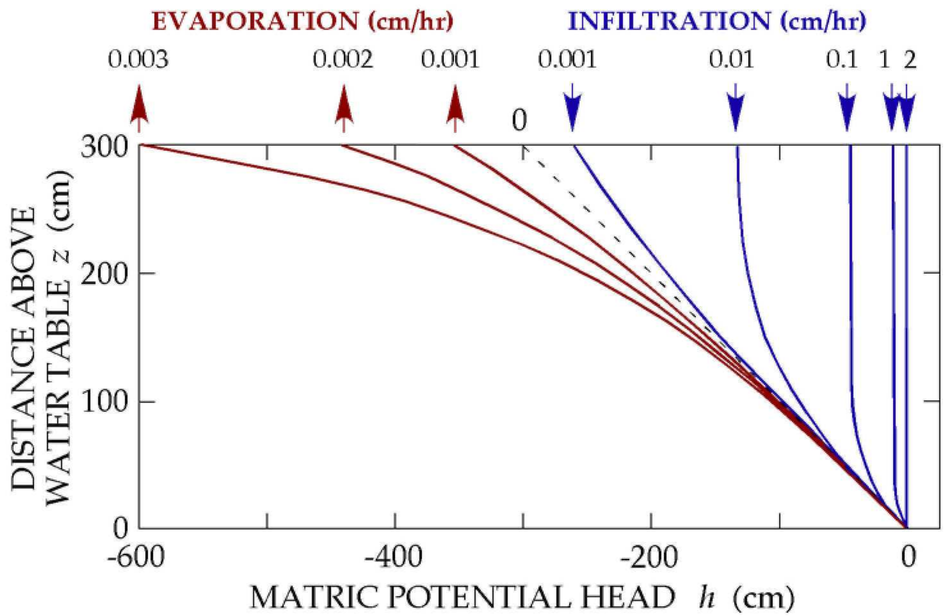
SAND
$K(\tau) = \frac{A}{\tau^4 + B}$
LOAM
$K(\tau) = \frac{a}{\tau^2 + b}$
HYDRAULIC CONDUCTIVITY K (cm/hr)
10^{1}
10^{-1}
10^{-3}
10^{-5}
10^{-7}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
MATRIC POTENTIAL HEAD $\tau = -h$ (cm)

STEADY STATE EVAPORATION RATE FOR THE LOAM

$$h = -(b + a/q)^{1/2} \tan\left[\frac{z(b + a/q)^{1/2}}{a/q}\right]$$

STEADY STATE INFILTRATION RATE FOR THE LOAM

$$z = \frac{a}{q}\frac{1}{2(-b - a/q)^{1/2}} \ln\left(\frac{(-b - a/q)^{1/2} + h}{(-b - a/q)^{1/2} - h}\right)$$

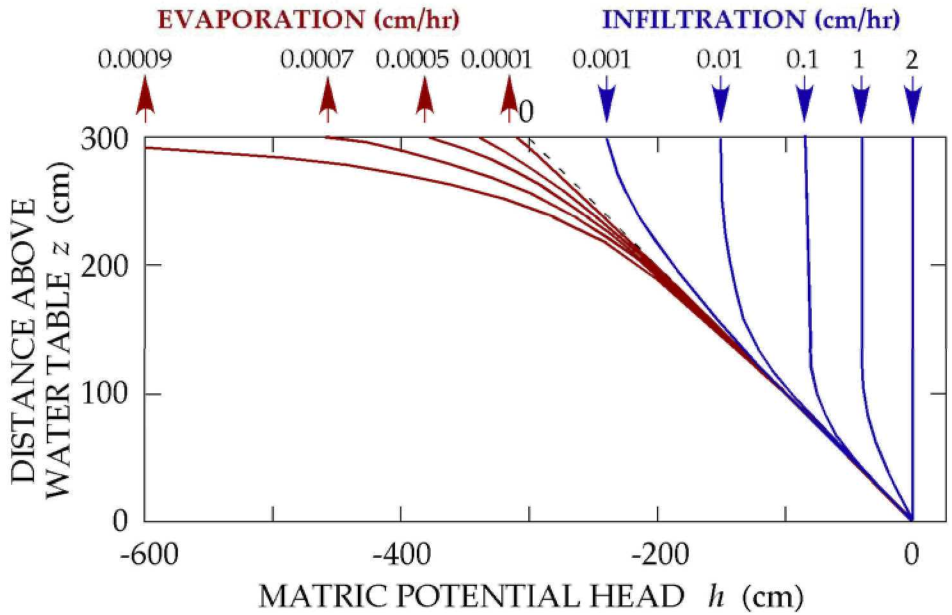


STEADY STATE EVAPORATION RATE FOR THE SAND

$$z\frac{q}{A}=\frac{1}{2\sqrt{2}(B+A/q)^{3/4}}\left\{\frac{1}{2}\ln\left[\frac{\tau^2+\sqrt{2}(B+A/q)^{1/4}\tau+(B+A/q)^{1/2}}{\tau^2-\sqrt{2}(B+A/q)^{1/4}\tau+(B+A/q)^{1/2}}\right]+\tan^{-1}\left[\frac{\sqrt{2}(B+A/q)^{1/4}\tau}{(B+A/q)^{1/2}-\tau^2}\right]\right\}$$

STEADY STATE INFILTRATION RATE FOR THE SAND

$$z=\frac{A}{2q(-A/q-B)^{3/4}}\left[\frac{1}{2}\ln\left|\frac{(-A/q-B)^{1/4}+h}{(-A/q-B)^{1/4}-h}\right|+\tan^{-1}\left(\frac{h}{(-A/q-B)^{1/4}}\right)\right]$$



GENERALITIES

- SOIL IS ABLE TO INFILTRATE VIRTUALLY ALL WATER COMING AT RAINFALL INTENSITIES ≤ K_{sat}

- AT SLOW INFILTRATION RATES, THE MATRIC POTENTIAL IN THE TOPSOIL IS VIRTUALLY CONSTANT. HENCE, A UNIT HYDRAULIC GRADIENT EXISTS.

- WITH A WATER TABLE AT 3-m DEPTH, THE AMOUNT OF WATER EVAPORATING IS GENERALLY < 1 mm/day.

- SOIL IS AN EXCELLENT RESERVOIR FOR STORING WATER AGAINST LOSSES BY EVAPORATION AND DEEP DRAINAGE. WITHOUT PLANTS TO ABSORB WATER, MOST OF THE WATER REMAINS IN THE PROFILE FOR A LONG TIME.

- TO PREVENT SOIL SALINIZATION, WATER TABLES SHOULD BE AT LEAST 3m DEEP TO REDUCE EVAPORATION.