The Abdus Salam
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College of Soil Physics

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Unsaturated flow in homogeneous and heterogeneous porous media

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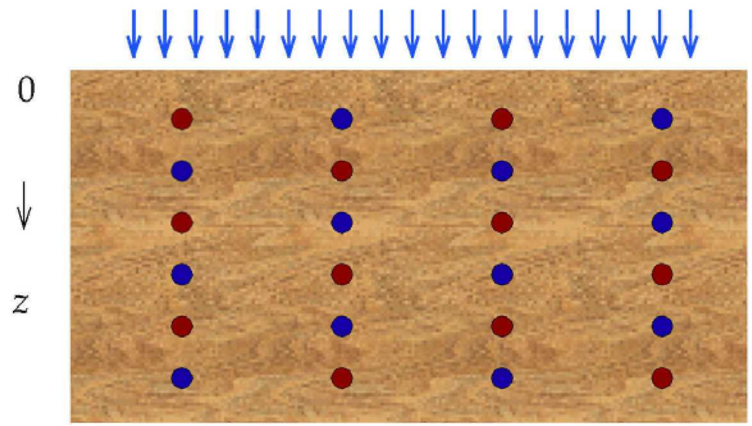
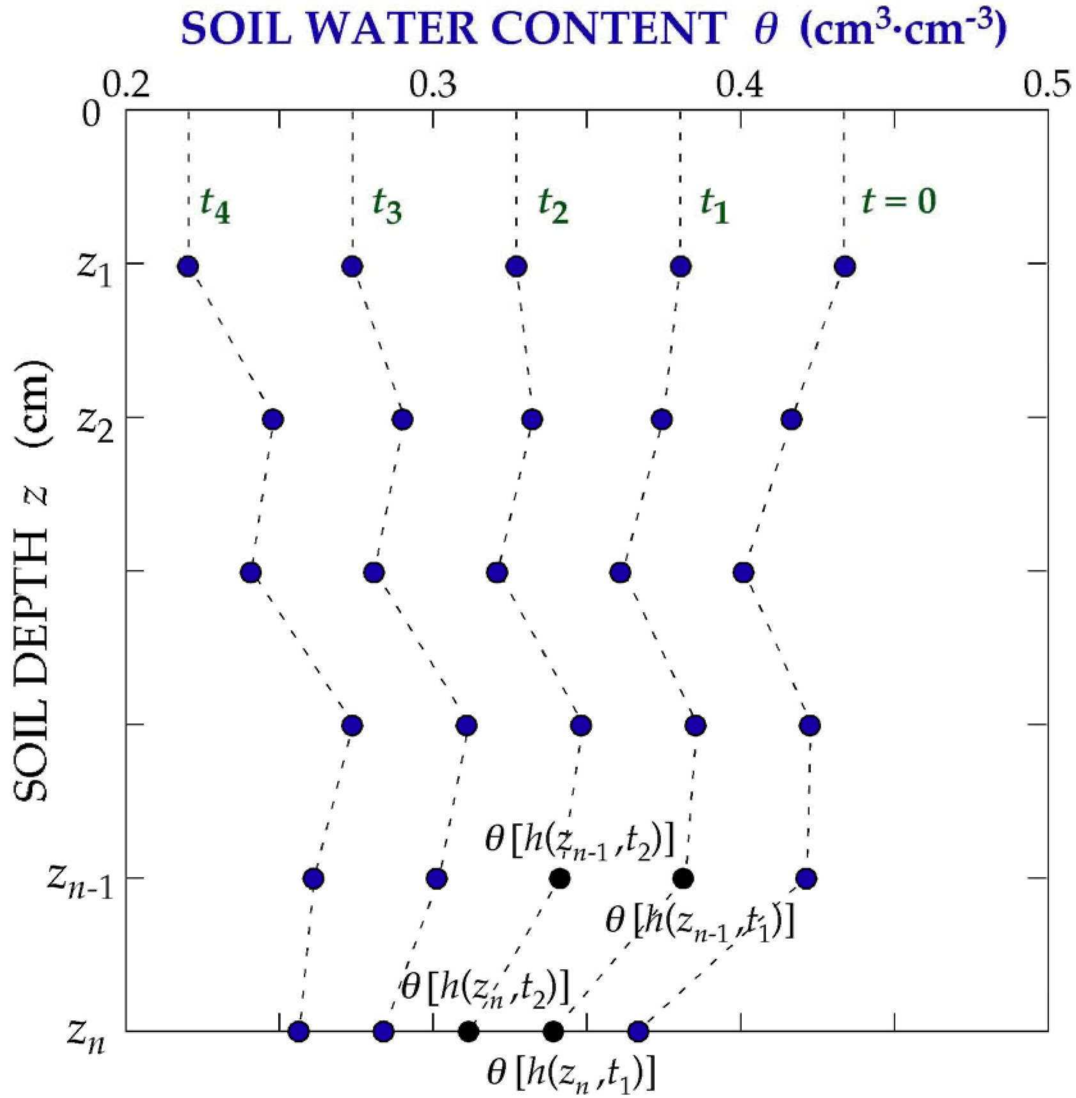
SIMPLE FIELD METHODS FOR MEASURING SOIL HYDRAULIC CONDUCTIVITY $K[\theta(h)]$

EACH METHOD IS BASED ON THE VALIDITY OF RICHARDS' EQUATION

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left(K[\theta(h)] \frac{\partial (h - z)}{\partial z} \right)$$





SOIL WATER CONTENT θ (cm³·cm⁻³)
0.2
0.3
0.4
0.5
0
SOIL DEPTH z (cm)
t_4
t_3
t_2
t_1
$t = 0$
z_1
z_2
z_{n-1}
z_n
$\theta\,[h(z_{n-1}, t_2)]$
$\theta\,[h(z_{n-1}, t_1)]$
$\theta\,[h(z_n, t_2)]$
$\theta\,[h(z_n, t_1)]$

STEADY STATE INFILTRATION

$$q = - K[\theta(h)] \frac{\partial(h - z)}{\partial z}$$

$$K(\bar{\theta}_{z_1, z_2}) = \frac{-q(z_2 - z_1)}{(\bar{h}_2 - \bar{h}_1) - (z_2 - z_1)}$$

INSTANTANEOUS PROFILE METHOD

$$\int_0^{z_n} \frac{\partial \theta}{\partial t} dz = \left[K(\theta) \frac{\partial H}{\partial z} \right]_{z=z_n} - \left[K(\theta) \frac{\partial H}{\partial z} \right]_{z=0}$$

$$\frac{\partial}{\partial t} \int_0^{z_n} \theta dz = \left[K(\theta) \frac{\partial H}{\partial z} \right]_{z=z_n}$$

$$\frac{\partial W(t)}{\partial t} = \left[K(\theta) \frac{\partial H}{\partial z} \right]_{z=z}$$

$$\frac{W(t_2) - W(t_1)}{t_2 - t_1} = K(\bar{\theta}) \left[\left(\overline{\frac{h_n - h_{n-1}}{z_n - z_{n-1}}} \right) - 1 \right]$$

ASSUME A UNIT HYDRAULIC GRADIENT EXISTS AT DEPTH *L*

$$\int_0^L \frac{\partial \theta}{\partial t} dz = \left[K(\theta)\frac{\partial H}{\partial z}\right]_{z=L} - \left[K(\theta)\frac{\partial H}{\partial z}\right]_{z=0}$$

$$\frac{\partial}{\partial t}\int_0^L \theta\, dz = \left[K(\theta)\frac{\partial H}{\partial z}\right]_{z=L}$$

$$\frac{\partial}{\partial t}\int_0^L \theta\, dz = -K(\theta)\Big|_{z=L}$$

The Flux Method

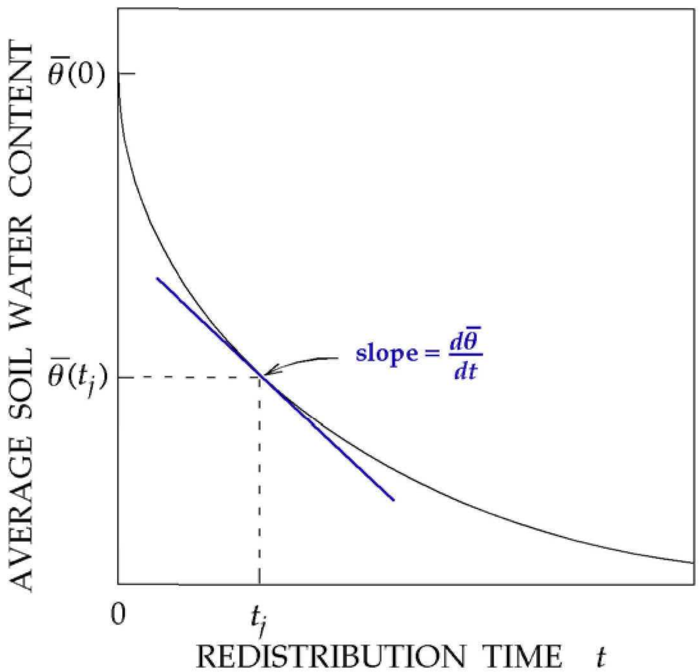
The Theta Method

The CGA Method

The Flux Method

$$\frac{\partial}{\partial t}\int_0^L \theta \, dz = -K(\theta)\Big|_{z=L}$$

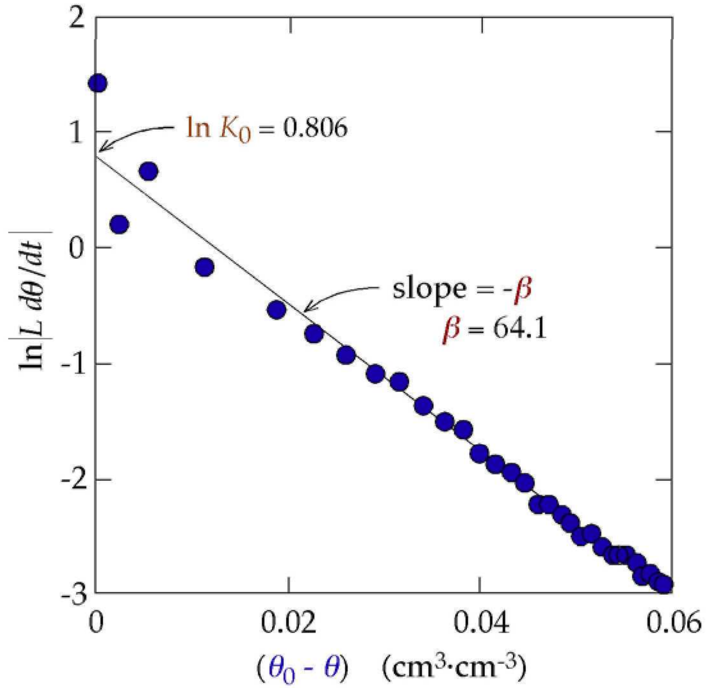
$$L\frac{d\bar{\theta}}{dt} = -K(\theta_L, t)$$



The Flux Method

$$L\frac{d\overline{\theta}}{dt} = -K(\theta_L, t) = -K_0 \exp[\beta(\theta - \theta_0)]$$

$$\ln\left|L\frac{d\overline{\theta}}{dt}\right| = -\beta(\theta_0 - \theta) + \ln K_0$$



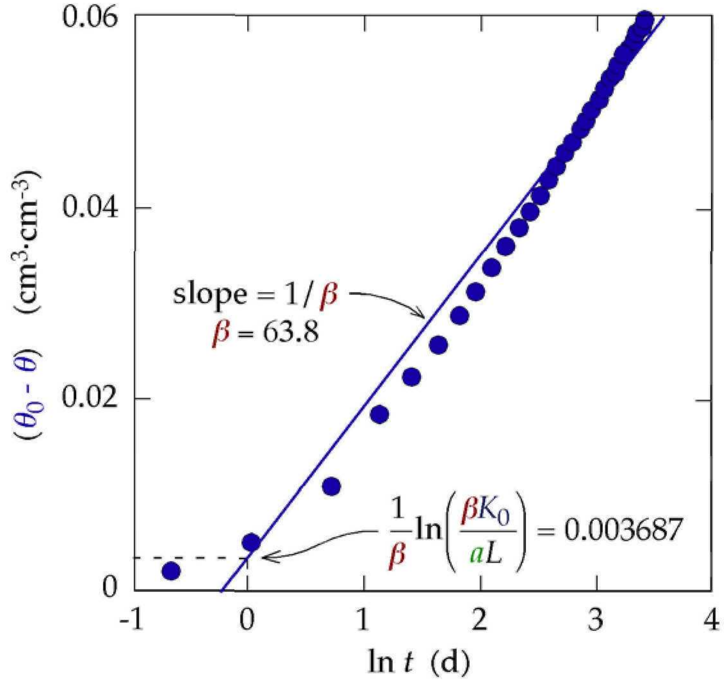
The Theta Method

$$L\frac{d\overline{\theta}}{dt} = -K(\theta_L, t) = -K_0 \exp[\beta(\theta - \theta_0)]$$

Assume $\overline{\theta} = a\theta + b$ $\qquad -aL\frac{d\theta}{dt} = K_0 \exp[\beta(\theta - \theta_0)]$

After integrating $\qquad \theta_0 - \theta = \frac{1}{\beta}\ln\left(1 + \frac{\beta K_0 t}{aL}\right)$

If $\frac{\beta K_0 t}{aL} >> 1$ $\qquad \theta_0 - \theta = \frac{1}{\beta}\ln t + \frac{1}{\beta}\ln\left(\frac{\beta K_0}{aL}\right)$

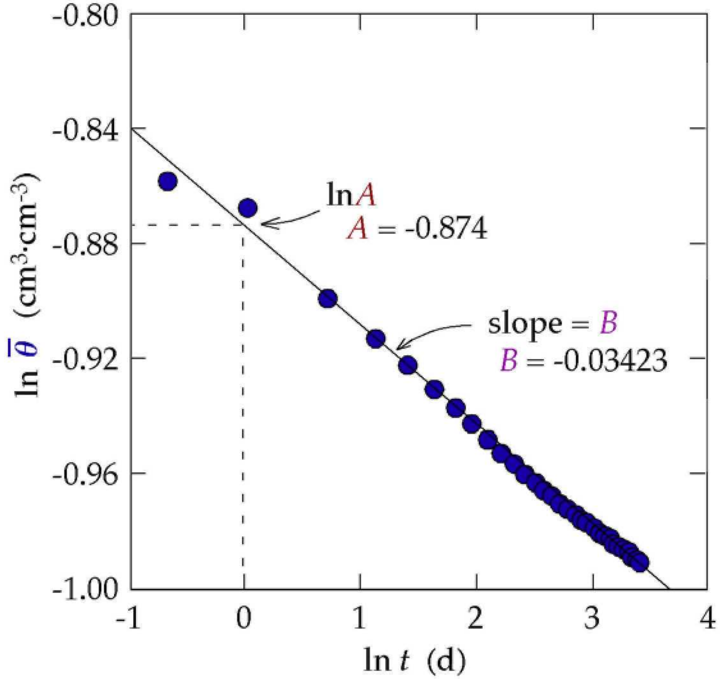


The CGA Method

$$L\frac{d\overline{\theta}}{dt} = -K(\theta_L, t)$$

Assume $\overline{\theta} = At^B$

$$K(\theta_L) = -A^{1/B} B L \overline{\theta}^{(B-1)/B}$$



COMPARISON OF SIMPLIFIED FIELD METHODS WITH INSTANTANEOUS PROFILE METHOD TO MEASURE HYDRAULIC CONDUCTIVITY

Parameter	Inst. P. Method	Flux Method	Theta Method	CGA Method
$K_0\,(\mathrm{cm\cdot d^{-1}})$	2.99	2.24	2.72	9.71
β	54.5	64.1	63.8	63.1

ARE SIMPLIFIED FIELD METHODS ADEQUATE ?

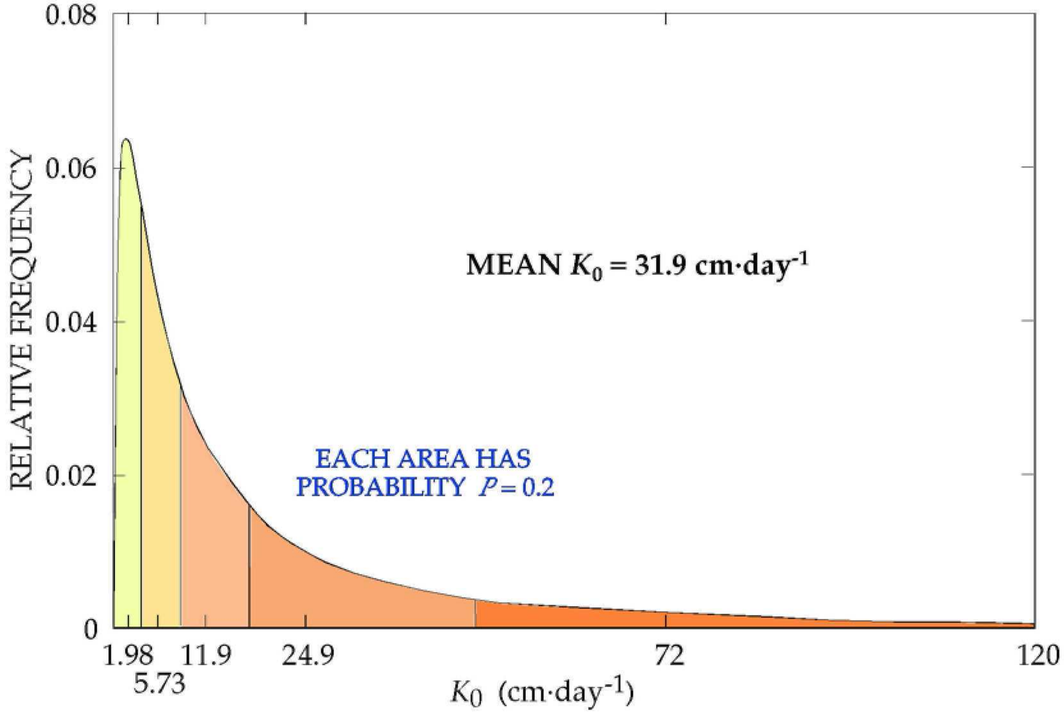
ADEQUATE FOR WHAT PURPOSE ?

FOR ESTIMATING THE WATER DRAINING J_L FROM A SOIL AT DEPTH L AFTER A HEAVY IRRIGATION OR RAINFALL

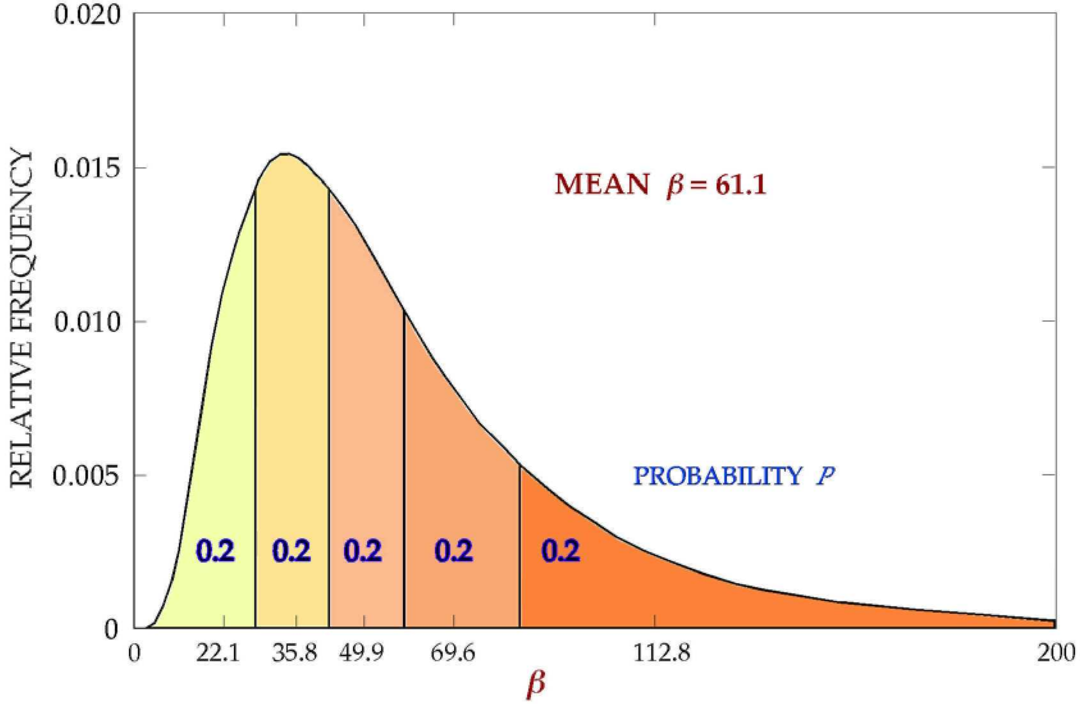
$$J_L(t) = \frac{K_0}{1 + \beta K_0 t / L}$$

FIELD DATA FOR 150 ha

Soil Depth	θ_0 Mean	θ_0 Std. Dev.	K_0 Mean	K_0 Std. Dev.	β Mean	β Std. Dev.
(cm)	($cm^3 \cdot cm^{-3}$)		($cm \cdot day^{-1}$)			
30	**0.397**	**0.037**	**14.7**	**17.7**	**151.0**	**126.9**
60	**0.409**	**0.042**	**15.1**	**16.2**	**70.5**	**28.6**
90	**0.411**	**0.060**	**26.8**	**35.5**	**57.4**	**27.9**
120	**0.446**	**0.042**	**18.3**	**18.7**	**60.0**	**25.3**
150	**0.446**	**0.059**	**22.6**	**23.6**	**51.9**	**22.9**
180	**0.458**	**0.061**	**26.2**	**30.8**	**61.1**	**41.7**



0.08
0.06
0.04
0.02
0
RELATIVE FREQUENCY
MEAN K_0 = 31.9 cm·day^{-1}
EACH AREA HAS
PROBABILITY $P = 0.2$
1.98
5.73
11.9
24.9
72
120
K_0 (cm·day^{-1})



0.020
0.015
0.010
0.005
0
RELATIVE FREQUENCY
MEAN $\beta = 61.1$
PROBABILITY P
0.2
0.2
0.2
0.2
0.2
0
22.1
35.8
49.9
69.6
112.8
200
β

DETERMINISTIC ESTIMATE OF SOIL WATER FLUX AT DEPTH L DURING REDISTRIBUTION AFTER 1 day

$$J_L(t) = \frac{E(K_0)}{1 + [E(\beta)E(K_0)t]/L}$$

$$J_{180}(1) = \frac{31.9}{1 + [61.1(31.9)1]/180} = 2.70$$

PROBABILISTIC ESTIMATE OF SOIL WATER FLUX AT DEPTH L DURING REDISTRIBUTION AFTER 1 day

$$\bar{J}_L(t) = \int_0^\infty \int_0^\infty J_L(t) f(K_0, \beta)\, dK_0\, d\beta$$

$$\bar{J}_{180}(1) = \sum_{i,j} (J_L)_{i,j} P_{i,j}$$

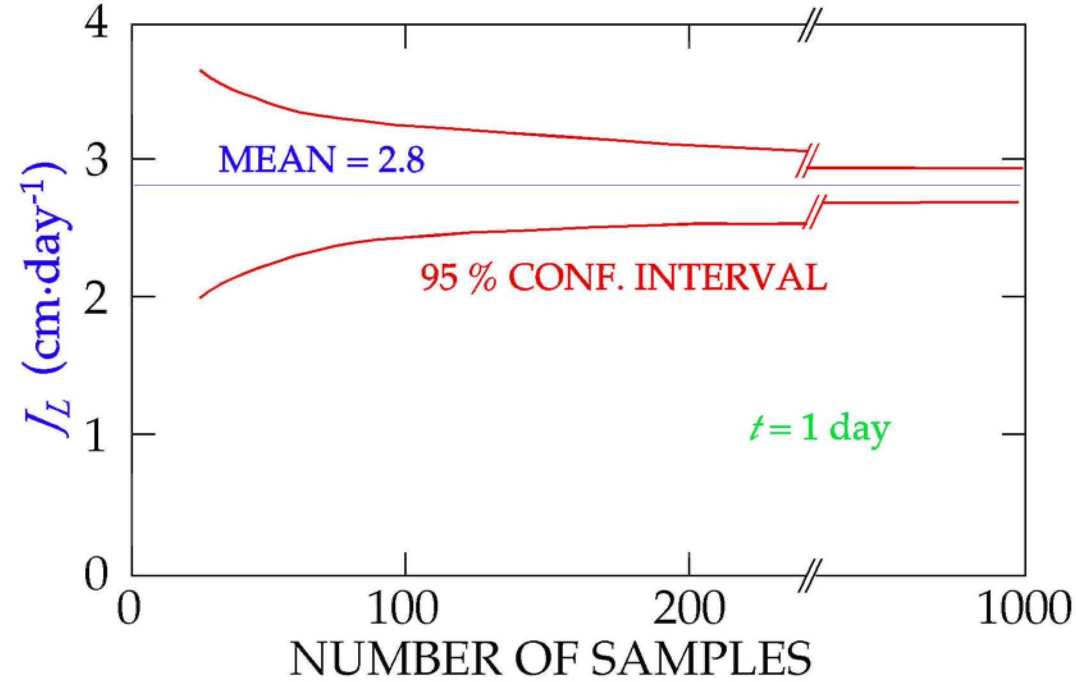
ESTIMATED MEAN VALUES OF SOIL WATER FLUX J_{180} IN 25 CELLS EACH WITH PROBABILITY 0.04

		20% Fractile j for K_0				
		1	2	3	4	5
20% Fractile i for β	β \ K_0	**1.98**	**5.73**	**11.9**	**24.9**	**72.0**
1	**22.1**	**1.59**	**3.36**	**4.84**	**6.14**	**7.32**
2	**35.8**	**1.42**	**2.68**	**3.54**	**4.19**	**4.71**
3	**49.9**	**1.28**	**2.21**	**2.77**	**3.15**	**3.44**
4	**69.6**	**1.12**	**1.78**	**2.12**	**2.34**	**2.50**
5	**112.8**	**0.88**	**1.25**	**1.41**	**1.50**	**1.56**

$$J_{180}(1) = \frac{1.98}{1+(22.1\cdot 1.98\cdot 1)/180} = \mathbf{1.59\ cm\cdot day^{-1}}$$

$$\bar{J}_L(t) = \sum_{i,j}^{5} 0.04 (J_L)_{i,j}$$

$$\bar{J}_{180}(1) = 0.04(1.59 + 3.36 + \cdots + 1.50 + 1.56) = 2.76$$



4
3
2
1
0
J_L (cm·day^{-1})
MEAN = 2.8
95 % CONF. INTERVAL
t = 1 day
0
100
200
1000
NUMBER OF SAMPLES