The Abdus Salam
International Centre for Theoretical Physics

United Nations
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1867-34

College of Soil Physics

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Transient temperature and water vapor in soils

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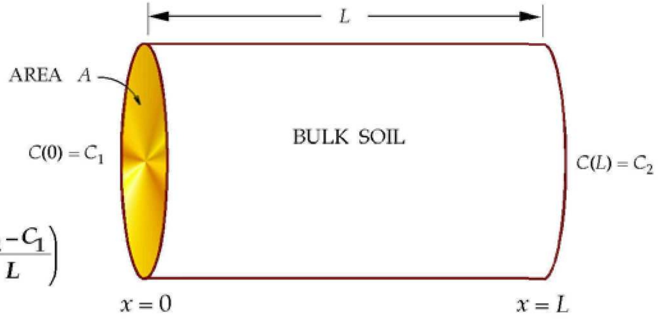
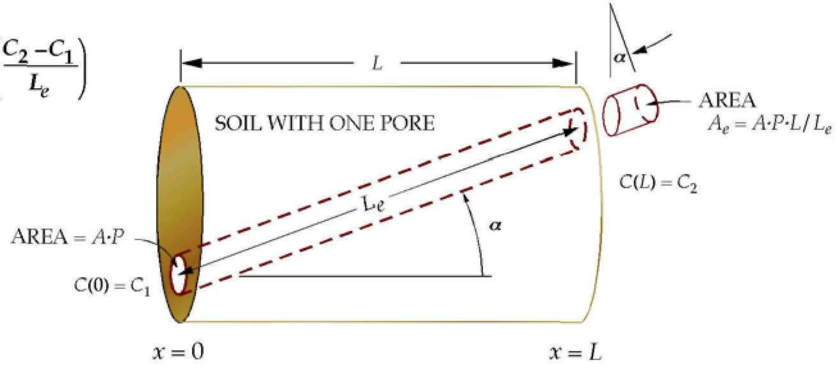
ISOTHERMAL GASEOUS DIFFUSION



$$Q = -D_m A\left(\frac{C_2 - C_1}{L}\right)$$

$$D_m = D_o P\left(L/\,L_e\right)^2$$

$$Q = -D_o A_e\left(\frac{C_2 - C_1}{L_e}\right)$$



$$D_m = D_o P\left(L/L_e\right)^2 \quad \text{(Buckingham, 1904)}$$

D_m = DIFFUSION COEFFICIENT IN BULK SOIL

D_o = DIFFUSION COEFFICIENT IN AIR

P = AIR-FILLED POROSITY

$\left(L/L_e\right)^2$ = TORTUOSITY

$= \cos^2 a \approx 0.66$ (Penman, 1940)

Marshall (1958) used $P\left(L/L_e\right)^2 = P^{3/2}$

M & Q (1959) used $P\left(L/L_e\right)^2 = P^{4/3}$

A GASEOUS OXYGEN DIFFUSION EXAMPLE

$$q = -D_m \frac{dc}{dx} \qquad D_m = 6000\ \text{cm}^2\cdot\text{day}^{-1}$$

$$q = -6000\,\frac{(0-2.8\cdot 10^{-4})}{100-0} = \mathbf{0.0168\ gm\cdot(cm^2\cdot day)^{-1}}$$

Note that the oxygen in 60 cm^3 of air can diffuse daily through a 1-cm^2 soil column having a length of 1 m

ISOTHERMAL WATER VAPOR DIFFUSION IN SOILS

$$q_v = -D_m \frac{d\rho_v}{dx}$$

Let us assume that $p_v V = nRT$

$$q_v = -D_m \frac{1}{R_v T} \frac{dp_v}{dx}$$

Know that

$$p_v = p_0 \exp\left(\frac{\phi_w}{RT}\right)$$

p_0 = saturated water vapor pressure
ϕ_w = matric potential + osmotic potential

Neglecting gravity

$$q_v = -D_m \frac{p_v}{(RT)^2} \frac{d\phi_w}{dx}$$

A robust equation for water vapor diffusion in soil ignoring temperature gradients.

$$q_v = -D_m \frac{p_v}{(RT)^2} \frac{d\phi_w}{dx}$$

WHEN SOIL WATER CONTENT IS NOT VERY DRY
(ABOVE PERMANENT WILTING PERCENTAGE)

WATER VAPOR PRESSURE $p_v \rightarrow p_0$

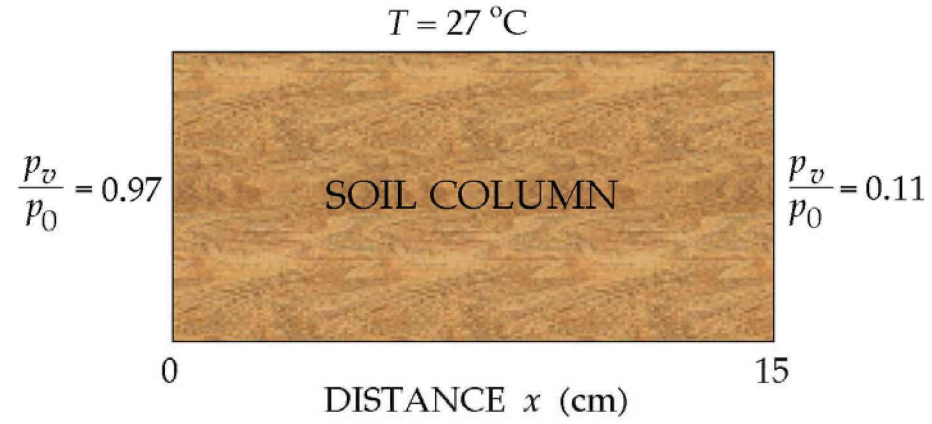
RELATIVE HUMIDITY OF SOIL AIR $\rightarrow 1$

THE AMOUNT OF VAPOR THAT DIFFUSES UNDER ISOTHERMAL CONDITIONS IS VERY SMALL!

WATER VAPOR MOVES MORE READILY THROUGH SOILS

1. **IN PRESENCE OF LARGE VAPOR PRESSURE GRADIENTS**
2. **UNDER A THERMAL GRADIENT IN MOIST SOILS**

STEADY STATE WATER VAPOR DIFFUSION



$T = 27\ ^{\circ}\mathrm{C}$

$\frac{p_v}{p_0} = 0.97$

SOIL COLUMN

$\frac{p_v}{p_0} = 0.11$

0

15

DISTANCE x (cm)

GOAL: MEASURE VAPOR DIFFUSION COEFFICIENT

LIQUID

$$q_\theta = -K\frac{dh}{dx}$$

$$q_\theta = -K\frac{dh}{d\theta}\frac{d\theta}{dx} = -D_\theta\frac{d\theta}{dx}$$

VAPOR

$$q_v = -D_m\frac{d\rho_v}{dx}$$

$$q_v = -D_v\frac{\partial\rho_v}{\partial\theta}\frac{d\theta}{dx}$$

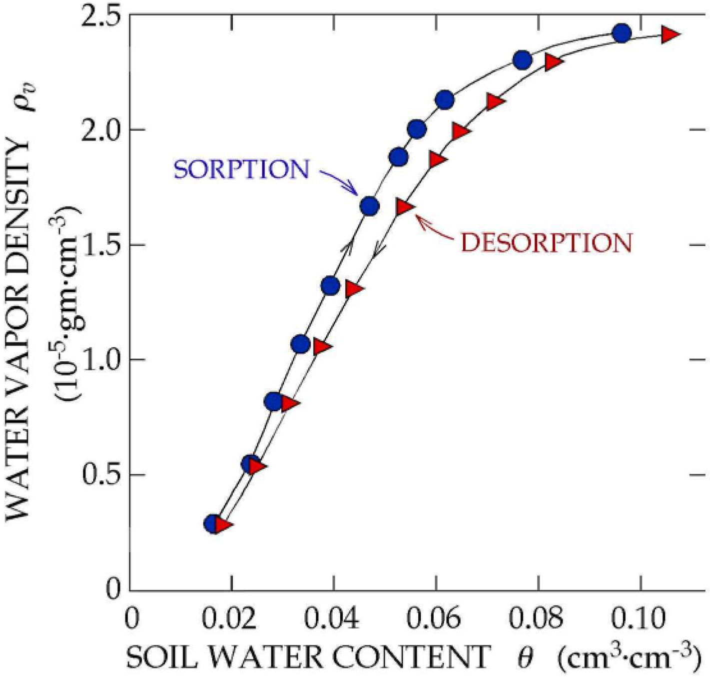
LIQUID & VAPOR

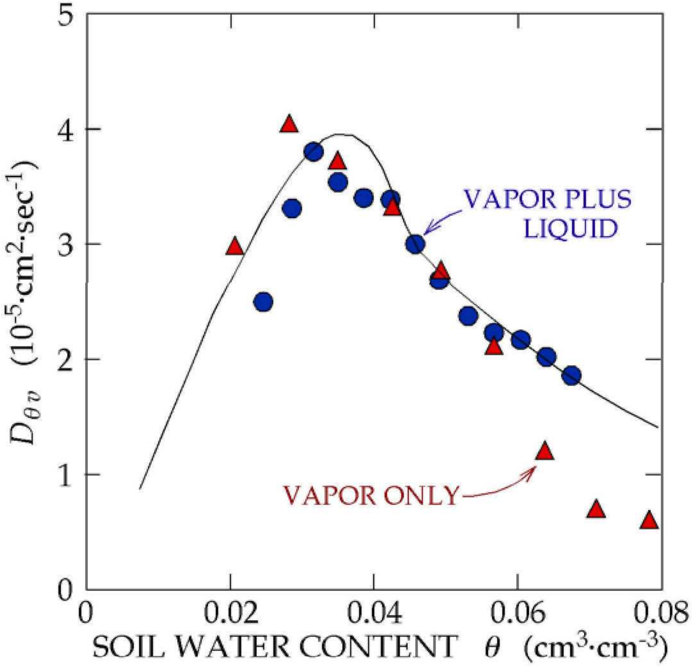
$$q = q_\theta + q_v = -\left(D_\theta + D_v\frac{d\rho_v}{d\theta}\right)\frac{d\theta}{dx}$$

$$q = -D_{\theta v}\frac{d\theta}{dx}$$

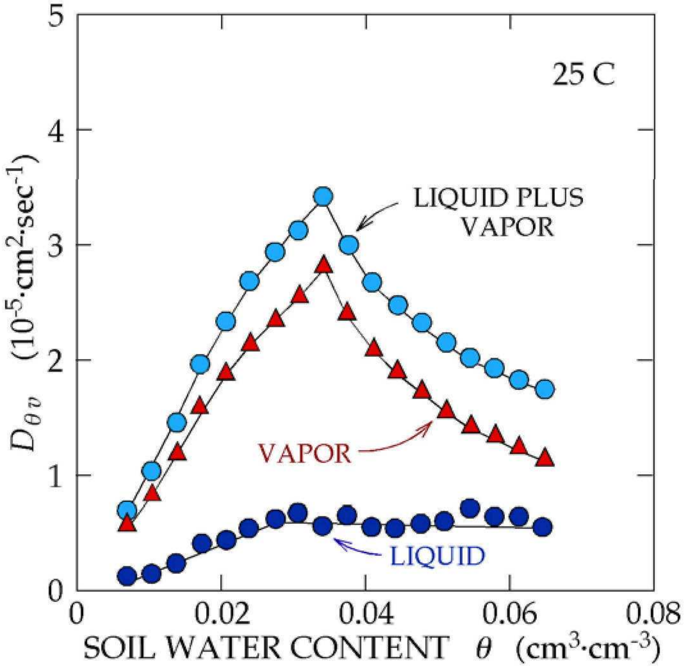
where $D_{\theta v} = D_\theta + D_v\dfrac{d\rho_v}{d\theta}$

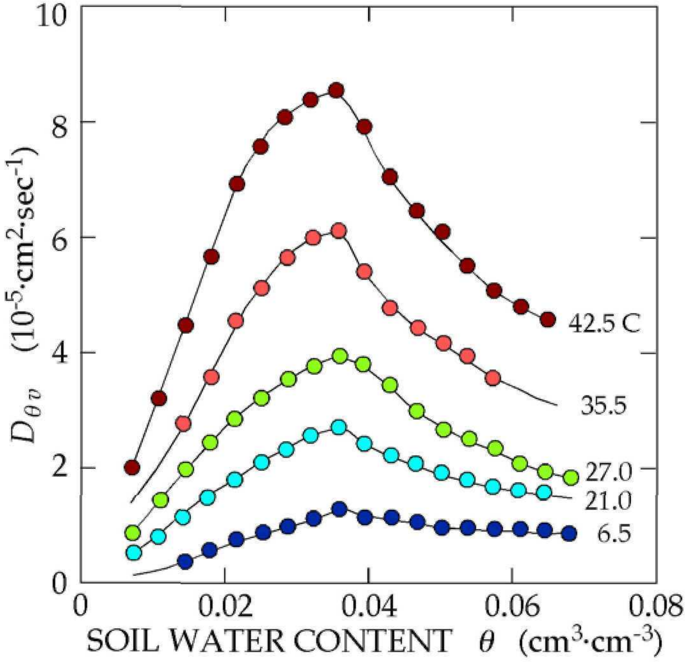
ISOTHERMS

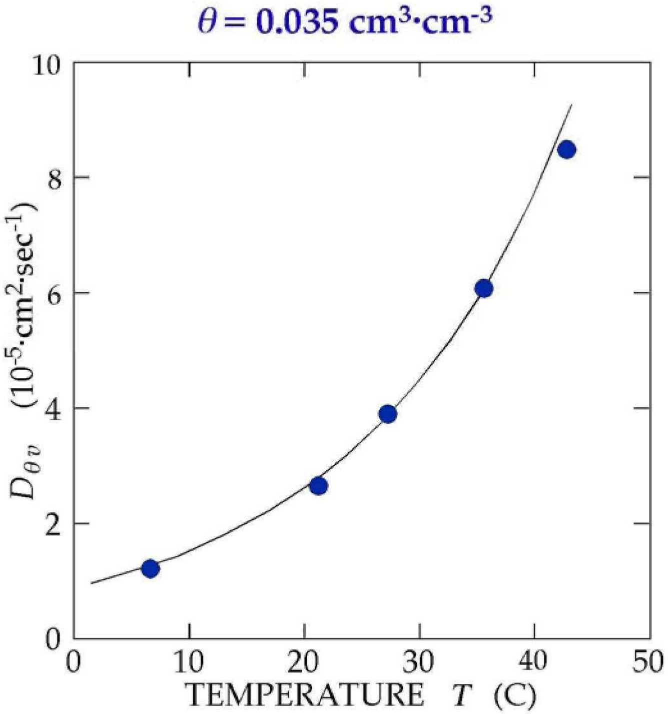

2.5
2.0
1.5
1.0
0.5
0
WATER VAPOR DENSITY ρ_v
(10^{-5}·gm·cm^{-3})
SORPTION
DESORPTION
0
0.02
0.04
0.06
0.08
0.10
SOIL WATER CONTENT θ (cm^3·cm^{-3})



$$D_v = D_0 (L / L_e)^2 (P - \theta)$$
$$= \mathbf{0.0818 - 0.176}\,\theta$$


25 C
LIQUID PLUS VAPOR
VAPOR
LIQUID
$D_{\theta v}$ ($10^{-5}\cdot cm^2\cdot sec^{-1}$)
SOIL WATER CONTENT θ ($cm^3\cdot cm^{-3}$)
0
1
2
3
4
5
0
0.02
0.04
0.06
0.08


10
8
6
4
2
0
$D_{\theta v}$ $(10^{-5}\cdot cm^2\cdot sec^{-1})$
0
0.02
0.04
0.06
0.08
SOIL WATER CONTENT θ $(cm^3\cdot cm^{-3})$
42.5 C
35.5
27.0
21.0
6.5


$\theta = 0.035$ cm^3·cm^{-3}
$D_{\theta v}$ (10^{-5}·cm^2·sec^{-1})
0
2
4
6
8
10
0
10
20
30
40
50
TEMPERATURE T (C)

$$\frac{\partial \rho_v}{\partial t} = \frac{\partial}{\partial x}\left(\frac{\mathrm{D}_v}{\varepsilon}\frac{\partial \rho_v}{\partial x}\right) - \frac{1}{\varepsilon}\frac{\partial \theta}{\partial t}$$

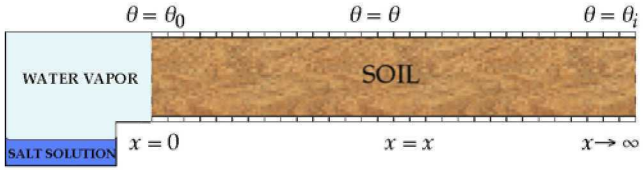
Assume $\frac{\partial \theta}{\partial t} >> \varepsilon \frac{\partial \rho_v}{\partial t}$

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left(D_v \frac{\partial \rho_v}{\partial x}\right) \qquad \frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left(D_v \frac{\partial \rho_v}{\partial \theta}\frac{\partial \theta}{\partial x}\right)$$

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left(D_{vap} \frac{\partial \theta}{\partial x}\right) \qquad q_\theta = -K \frac{dh}{d\theta}\frac{d\theta}{dx} = -D_\theta \frac{d\theta}{dx}$$

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left(\left[D_\theta + D_{vap}\right] \frac{\partial \theta}{\partial x}\right)$$

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left(D_{\theta v} \frac{\partial \theta}{\partial x}\right)$$

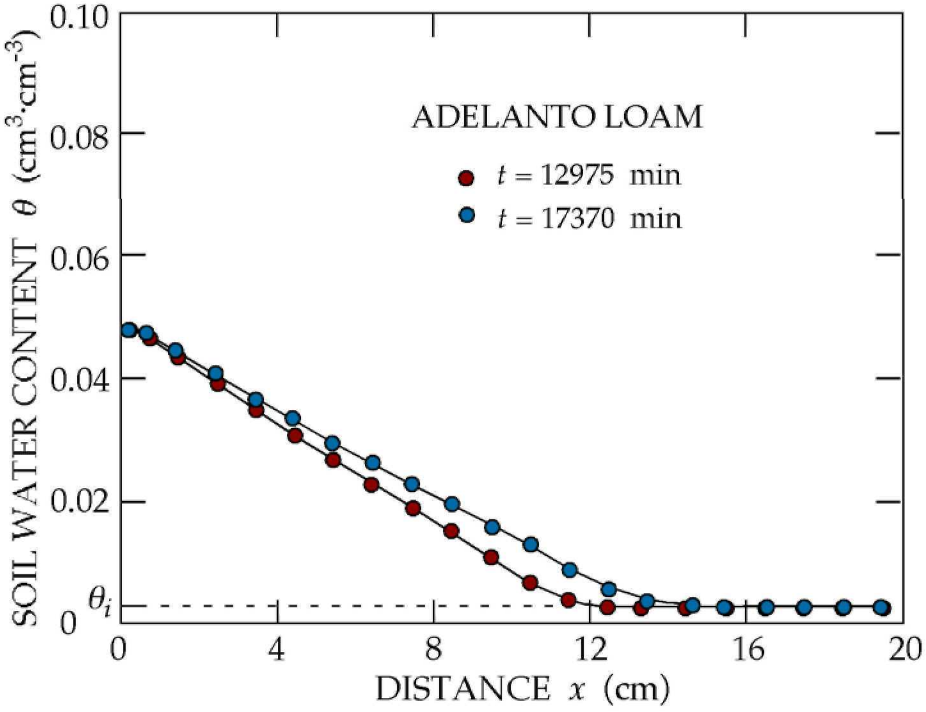


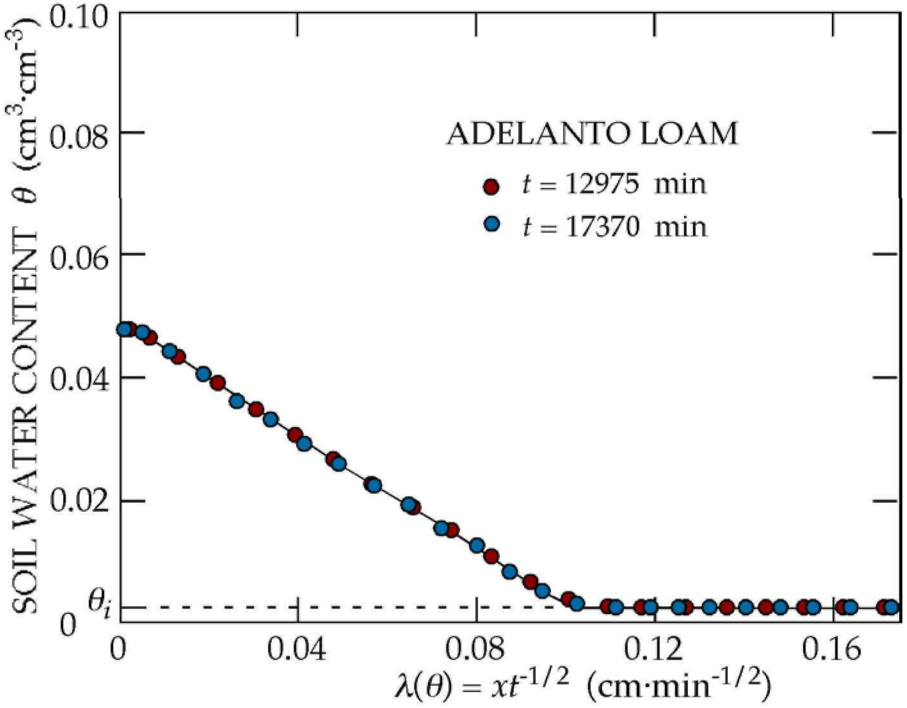
$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left(D_{\theta v} \frac{\partial \theta}{\partial x}\right)$$

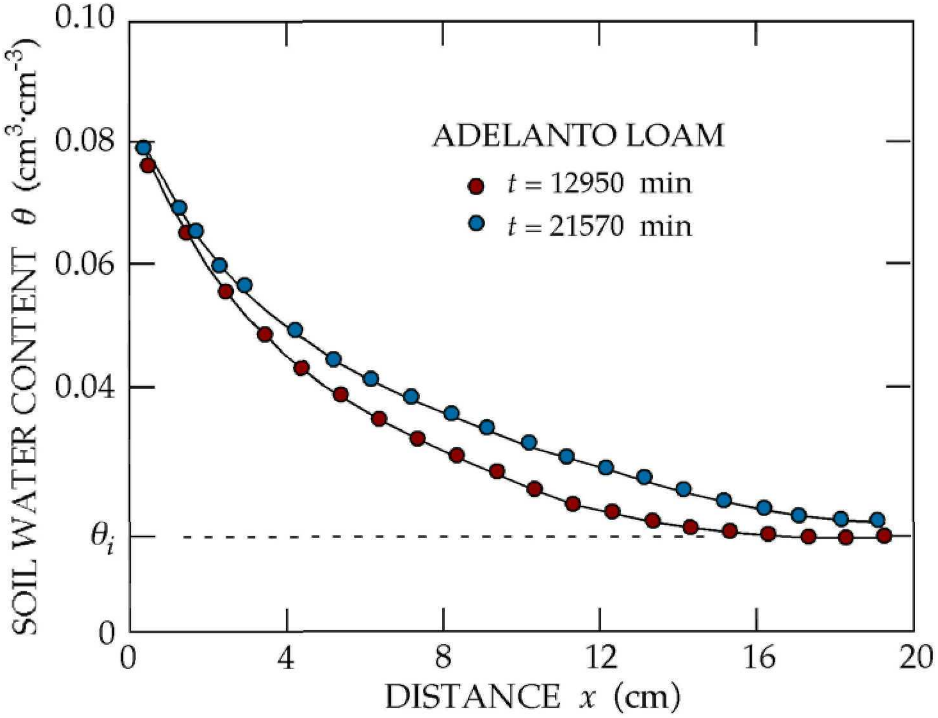
$$\begin{array}{lll} \theta = \theta_i & x > 0 & t = 0 \\ \theta = \theta_0 & x = 0 & t > 0 \end{array}$$

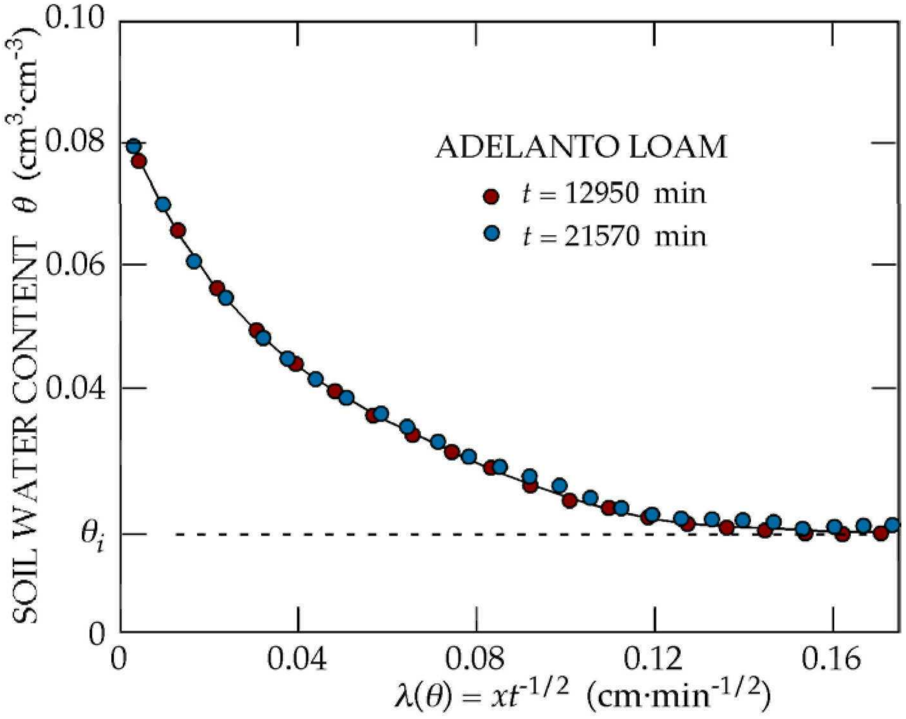
$$\lambda(\theta) = x t^{-1/2}$$

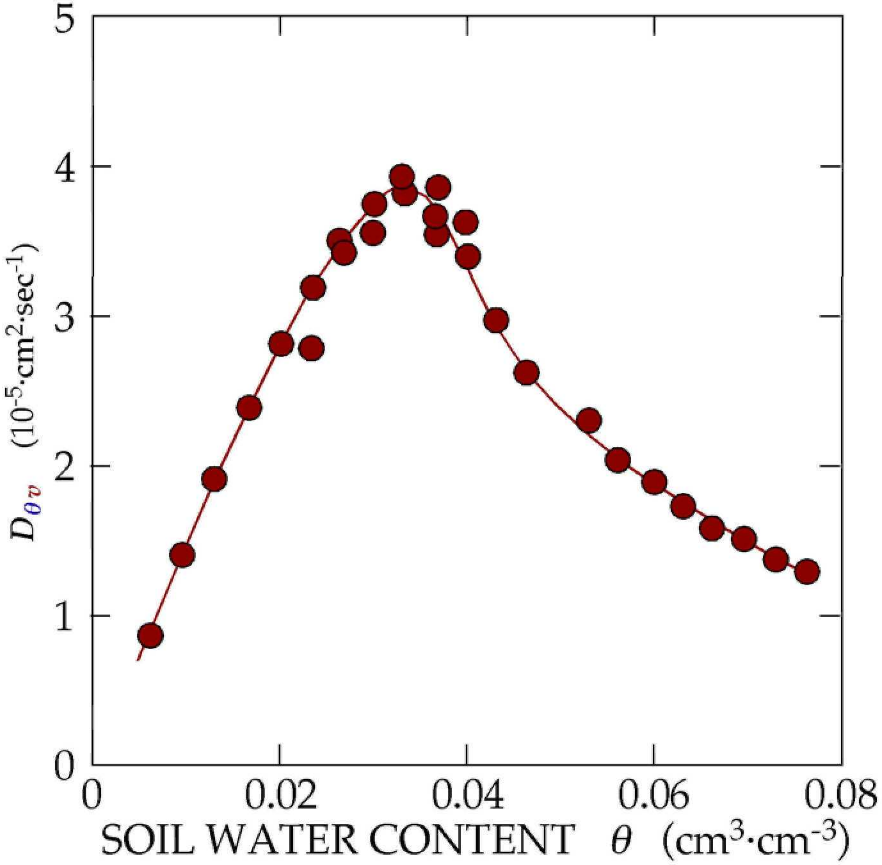
$$D_{\theta v}\left[\theta(x, t_1)\right] = -\frac{1}{2t_1} \frac{dx}{d\theta} \int_{\theta_i}^{\theta} x \, d\theta$$

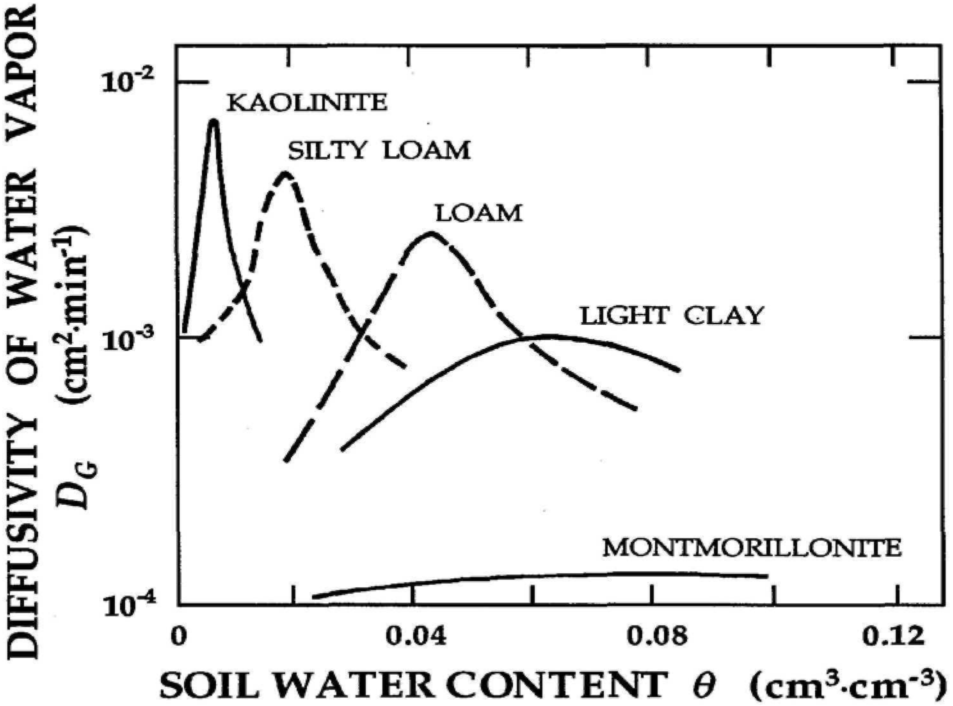

ADELANTO LOAM
● $t = 12975$ min
● $t = 17370$ min
SOIL WATER CONTENT θ (cm^3·cm^{-3})
DISTANCE x (cm)
0.10
0.08
0.06
0.04
0.02
θ_i
0
0
4
8
12
16
20


ADELANTO LOAM
● t = 12975 min
● t = 17370 min
SOIL WATER CONTENT θ (cm^3·cm^{-3})
0.10
0.08
0.06
0.04
0.02
θ_i
0
0
0.04
0.08
0.12
0.16
$\lambda(\theta) = xt^{-1/2}$ (cm·min$^{-1/2}$)


ADELANTO LOAM
● t = 12950 min
● t = 21570 min
SOIL WATER CONTENT θ (cm^3·cm^{-3})
0.10
0.08
0.06
0.04
θ_i
0
DISTANCE x (cm)
0
4
8
12
16
20


0.10
0.08
0.06
0.04
θ_i
0
SOIL WATER CONTENT θ (cm^3·cm^{-3})
ADELANTO LOAM
● $t = 12950$ min
● $t = 21570$ min
0
0.04
0.08
0.12
0.16
$\lambda(\theta) = xt^{-1/2}$ (cm·min$^{-1/2}$)


5
4
3
2
1
0
0
0.02
0.04
0.06
0.08
$D_{\theta v}$ $(10^{-5}\cdot cm^2\cdot sec^{-1})$
SOIL WATER CONTENT θ $(cm^3\cdot cm^{-3})$


DIFFUSIVITY OF WATER VAPOR
D_G (cm²·min⁻¹)
10⁻²
10⁻³
10⁻⁴
KAOLINITE
SILTY LOAM
LOAM
LIGHT CLAY
MONTMORILLONITE
0
0.04
0.08
0.12
SOIL WATER CONTENT θ (cm³·cm⁻³)

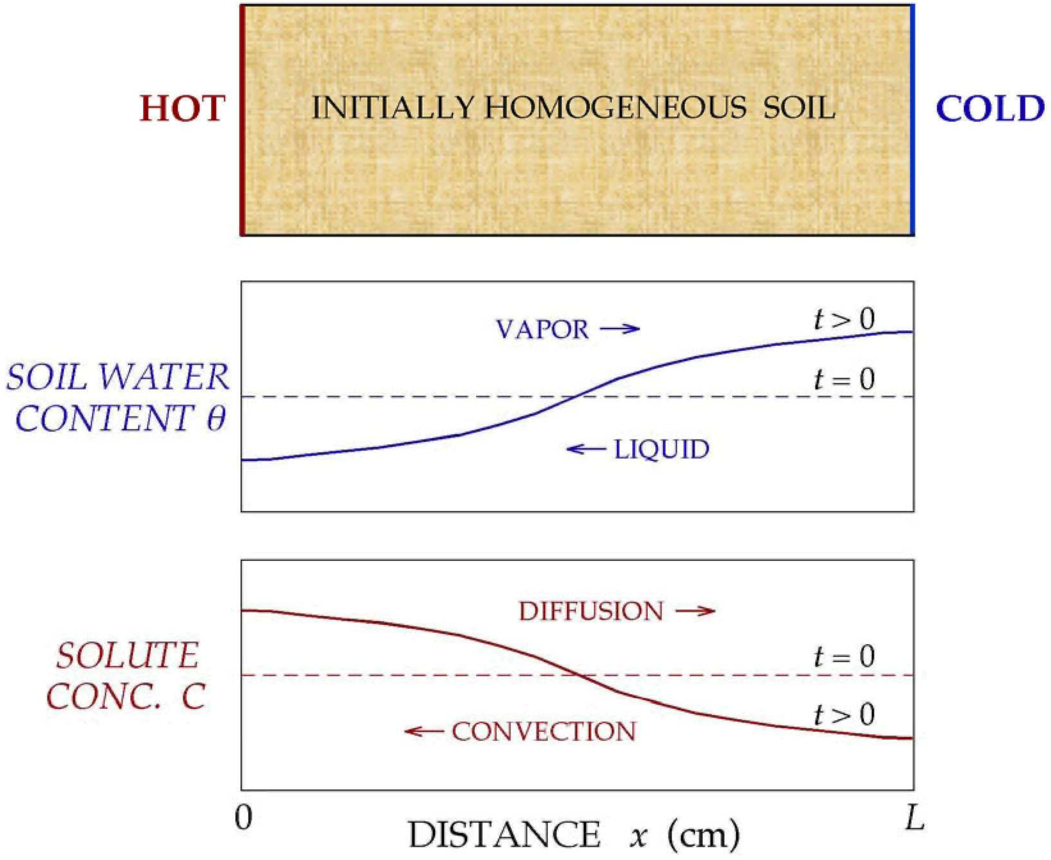
VALUES $D_{\theta v}$ COULD NOT BE SEPARATED INTO THEIR INDIVIDUAL COMPONENTS OF VAPOR AND LIQUID TRANSPORT

WE LEARNED THAT

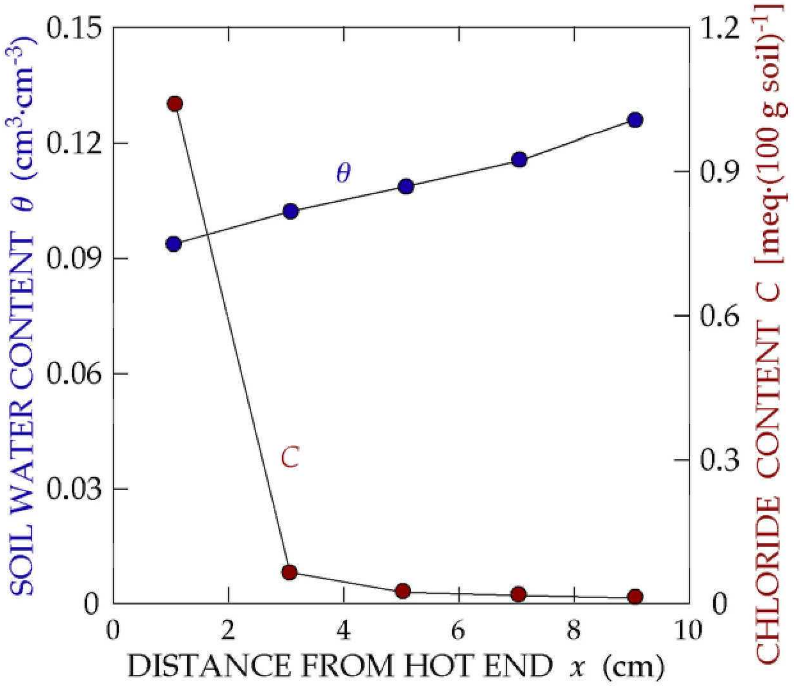
IN DRY SOILS UNDER ISOTHERMAL CONDITIONS.
LARGE GRADIENTS OF VAPOR DENSITY CAUSE
SMALL CHANGES IN LIQUID WATER CONTENT
OVER SEVERAL DAYS AND WEEKS

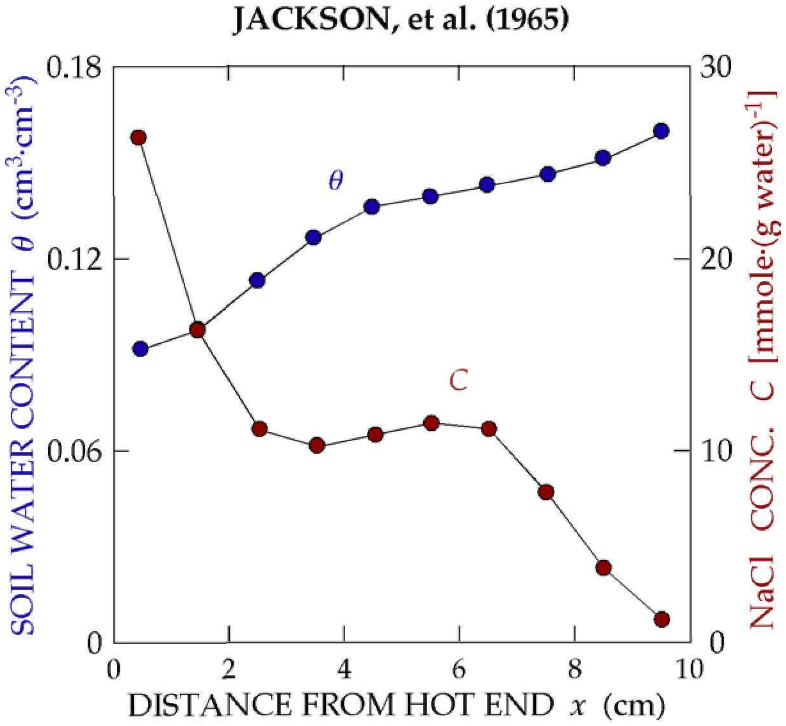
BUT, AS THE LIQUID SOIL WATER CONTENT INCREASES
VAPOR DENSITY GRADIENTS DRASTICALLY DECREASE
WITH MOST WATER MOVING AS A LIQUID

NONISOTHERMAL TRANSPORT PROCESSES

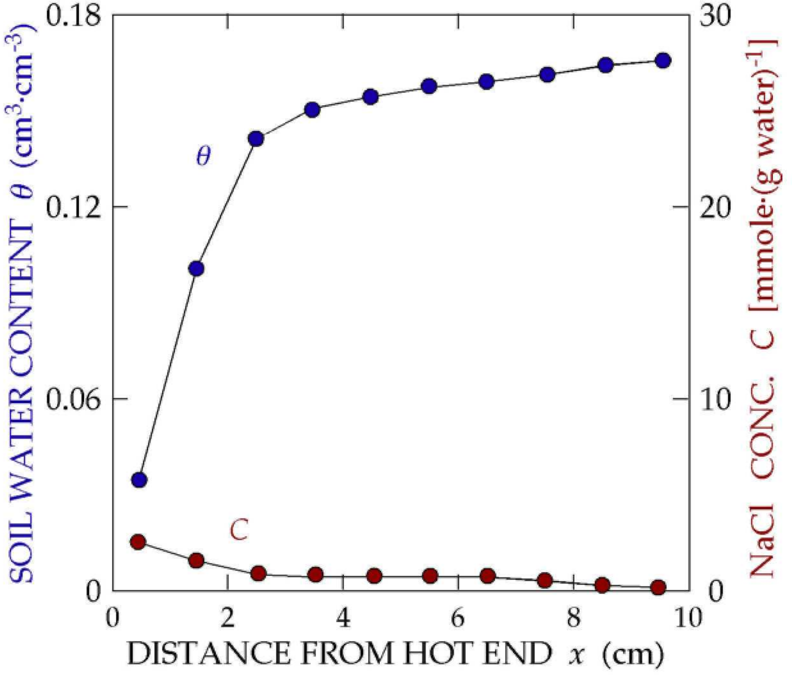


GURR, MARSHALL & HUTTON (1952)

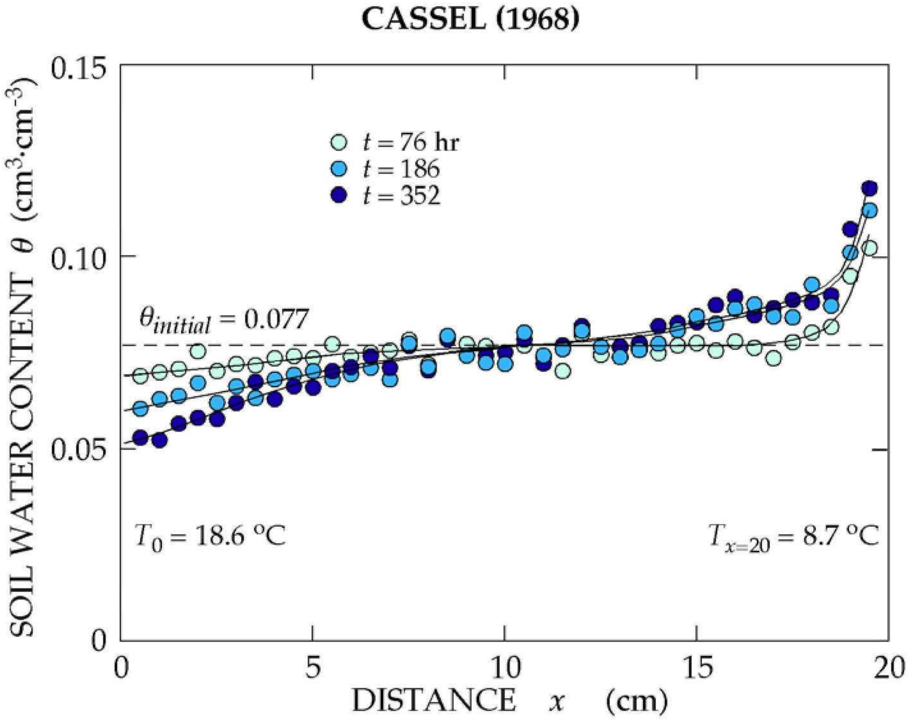




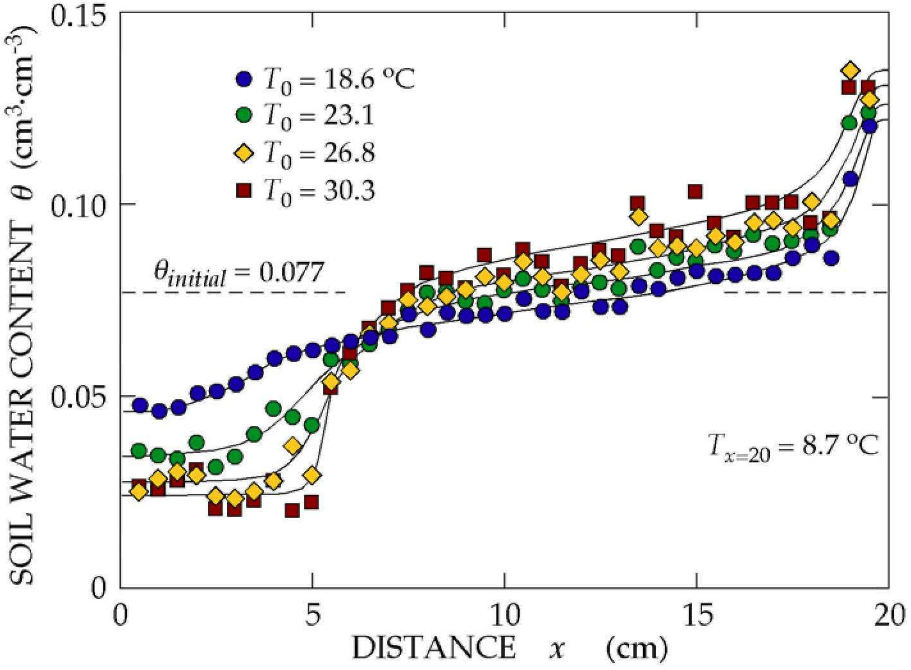
JACKSON, et al. (1965)

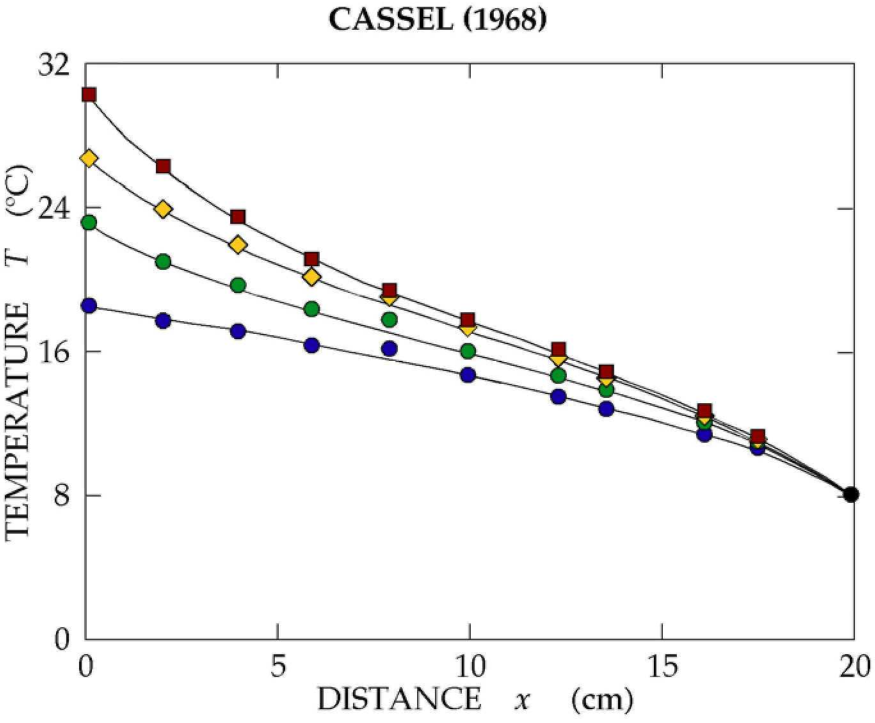


JACKSON, et al. (1965)

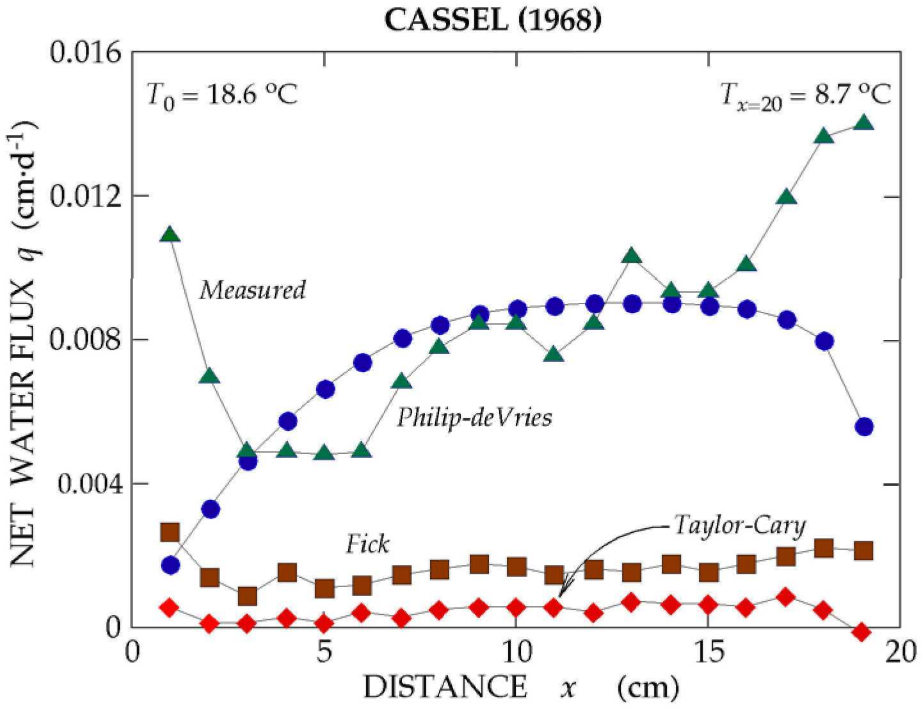

CASSEL (1968)
SOIL WATER CONTENT θ (cm^3·cm^{-3})
0.15
0.10
0.05
0
t = 76 hr
t = 186
t = 352
$\theta_{initial}$ = 0.077
T_0 = 18.6 °C
$T_{x=20}$ = 8.7 °C
0
5
10
15
20
DISTANCE x (cm)

CASSEL (1968)


SOIL WATER CONTENT θ (cm³·cm⁻³)
0.15
0.10
0.05
0
0
5
10
15
20
DISTANCE x (cm)
T0 = 18.6 °C
T0 = 23.1
T0 = 26.8
T0 = 30.3
θinitial = 0.077
Tx=20 = 8.7 °C



CASSEL (1968)



CASSEL (1968)

MORE RECENT, SIGNIFICANT RESEARCH ON SOIL WATER DIFFUSION WAS PUBLISHED BY NASSAR AND HORTON (1989).

DAILY FLUCTUATIONS OF WATER AND SOLUTES IN A TOPSOIL WITHOUT VEGETATION

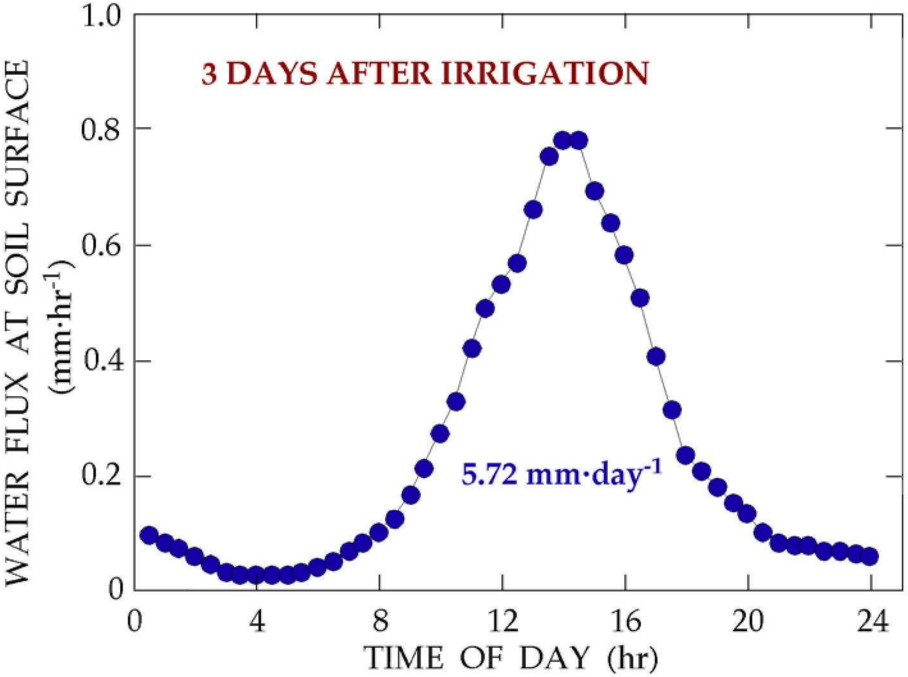
Jackson, Nakayama, Kimball and Reginato (1971)

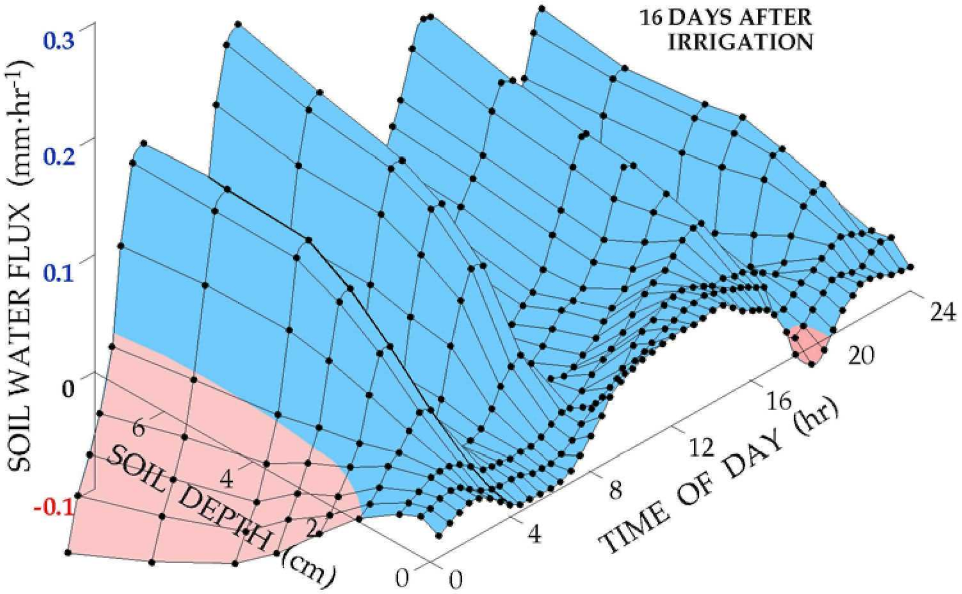
THOUSANDS OF GRAVIMETRIC MEASUREMENTS
AT 8 SOIL-DEPTH INTERVALS (0 TO 9 cm) 48 TIMES EACH DAY
DURING 16 SELECTED DAYS FOLLOWING A 10-cm IRRIGATION
CONTAINING 12 meq Cl per L.

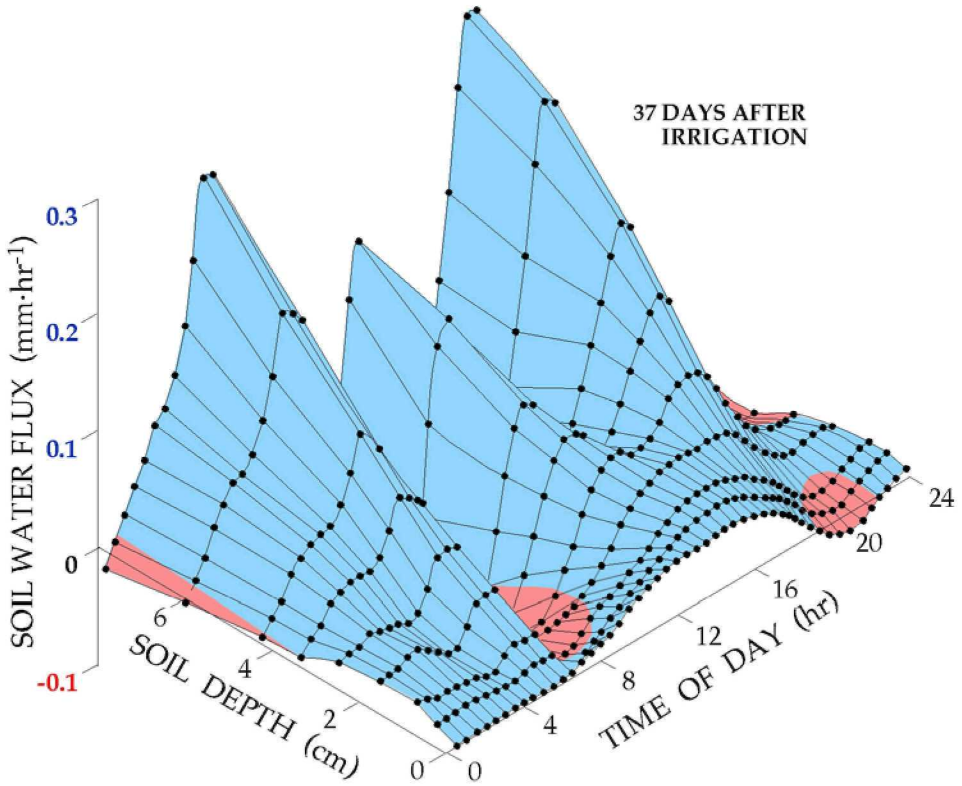
WATER EVAPORATION DATA WERE OBTAINED WITH LYSIMETERS

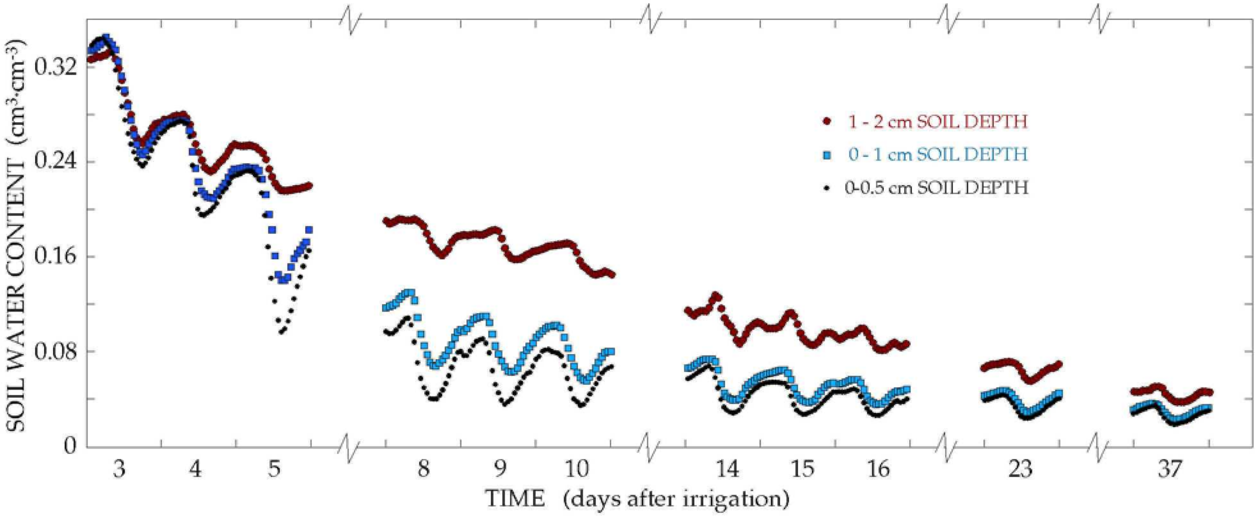
THE EXPERIMENTAL ERROR FOR SOIL WATER CONTENT (± 0.001 $cm^3 \cdot cm^{-3}$)

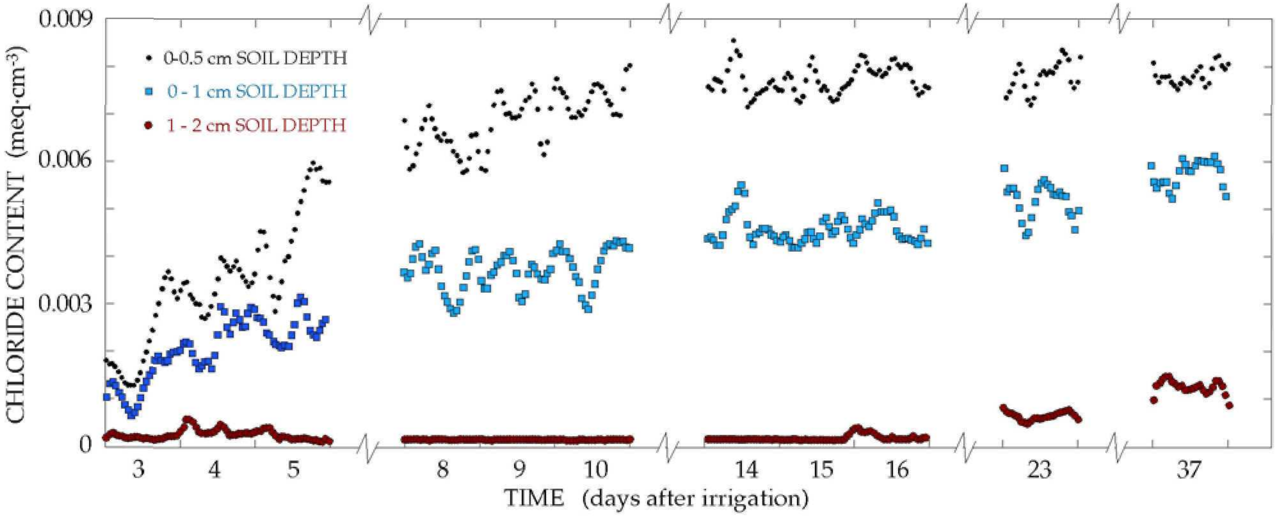
THE EXPERIMENTAL ERROR FOR WATER FLUX DENSITY (± 0.04 $mm \cdot hr^{-1}$)

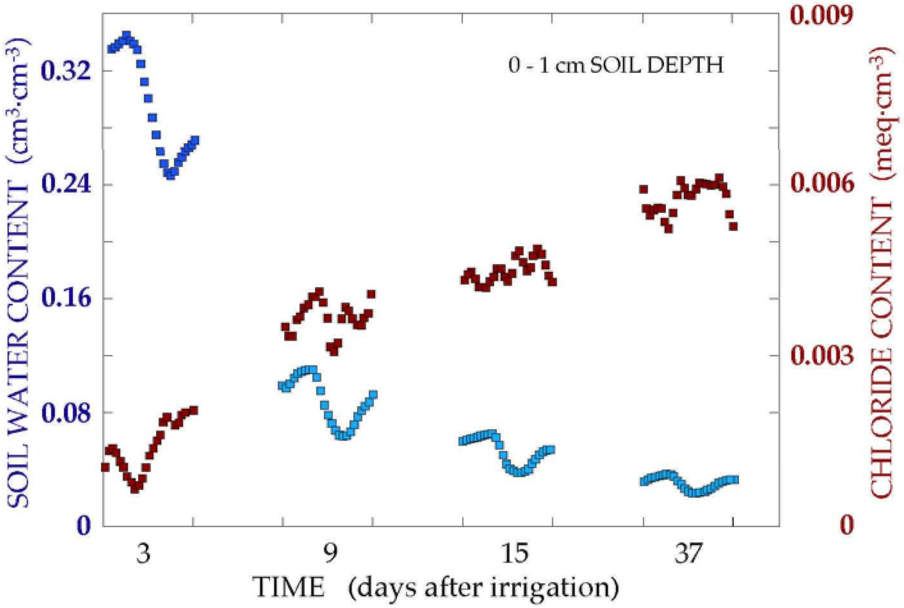

3 DAYS AFTER IRRIGATION
5.72 mm·day-1
WATER FLUX AT SOIL SURFACE (mm·hr-1)
TIME OF DAY (hr)
1.0
0.8
0.6
0.4
0.2
0
0
4
8
12
16
20
24

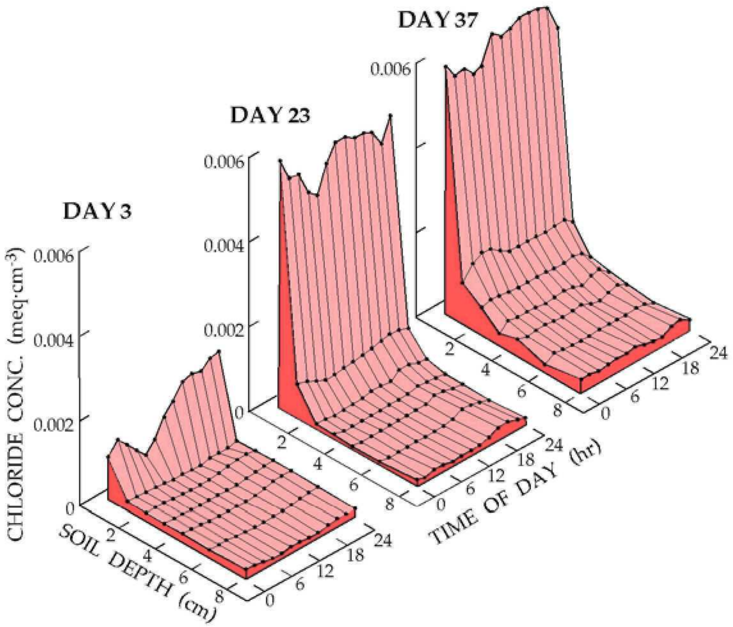

16 DAYS AFTER
IRRIGATION
SOIL WATER FLUX (mm·hr^{-1})
0.3
0.2
0.1
0
-0.1
SOIL DEPTH (cm)
6
4
2
0
TIME OF DAY (hr)
0
4
8
12
16
20
24


37 DAYS AFTER
IRRIGATION
SOIL WATER FLUX (mm·hr-1)
0.3
0.2
0.1
0
-0.1
SOIL DEPTH (cm)
6
4
2
0
TIME OF DAY (hr)
0
4
8
12
16
20
24


SOIL WATER CONTENT (cm³·cm⁻³)
0.32
0.24
0.16
0.08
0
● 1 - 2 cm SOIL DEPTH
■ 0 - 1 cm SOIL DEPTH
• 0-0.5 cm SOIL DEPTH
3
4
5
8
9
10
14
15
16
23
37
TIME (days after irrigation)


0.009
0.006
0.003
0
CHLORIDE CONTENT (meq·cm⁻³)
● 0-0.5 cm SOIL DEPTH
■ 0 - 1 cm SOIL DEPTH
● 1 - 2 cm SOIL DEPTH
3
4
5
8
9
10
14
15
16
23
37
TIME (days after irrigation)


0 - 1 cm SOIL DEPTH
SOIL WATER CONTENT (cm³·cm⁻³)
0.32
0.24
0.16
0.08
0
CHLORIDE CONTENT (meq·cm⁻³)
0.009
0.006
0.003
0
3
9
15
37
TIME (days after irrigation)



CHLORIDE CONC. INCREASES GREATLY ONLY IN TOP 1 cm

CHLORIDE STOPPED INCREASING AS θ DECREASED TO ABOUT 0.04 $cm^3 \cdot cm^{-3}$

For $\theta < 0.04$ $cm^3 \cdot cm^{-3}$, liquid water flow is drastically reduced

Cl ions are repelled from negatively charged soil particle surfaces

For $\theta < 0.04$ $cm^3 \cdot cm^{-3}$, Cl diffusion coefficients approach a value of zero

HENCE, SMALL MAGNITUDES OF DOWNWARD Cl DIFFUSION ARE MATCHED BY SMALL QUANTITIES OF UPWARD Cl CONVECTION

Below 1 cm, water vapor and liquid water (with its dissolved salts) move upward toward the soil surface

Water leaves the soil surface by evaporation

THE AMOUNT OF SALT LEACHED WITH EACH IRRIGATION OR RAINFALL AND THE AMOUNT OF SALT ACCUMULATED NEAR A SOIL SURFACE IS A DYNAMIC PROCESS

THE FREQUENCY & AMOUNTS OF IRRIGATION TO SALINIZE OR DE-SALINIZE A SOIL DEPEND UPON

- **The physical properties of the soil**
- **The quality of the applied water**
- **The nature and extent of the boundary conditions**
  - **at its surface**
  - **at particular depths within the soil profile**
  - **in the vadose zone below**