



**The Abdus Salam  
International Centre for Theoretical Physics**



**1867-35**

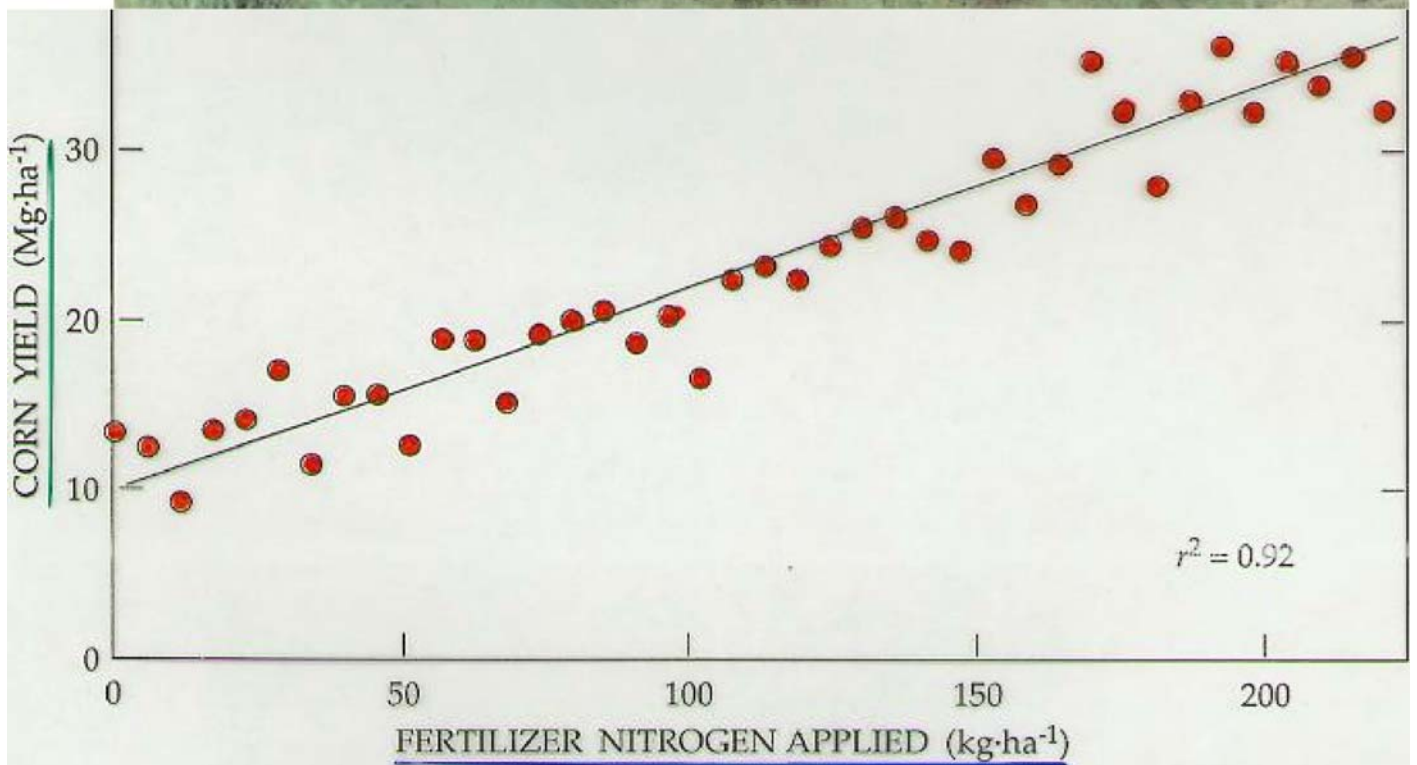
**College of Soil Physics**

*22 October - 9 November, 2007*

**Sampling soils and their vegetation**

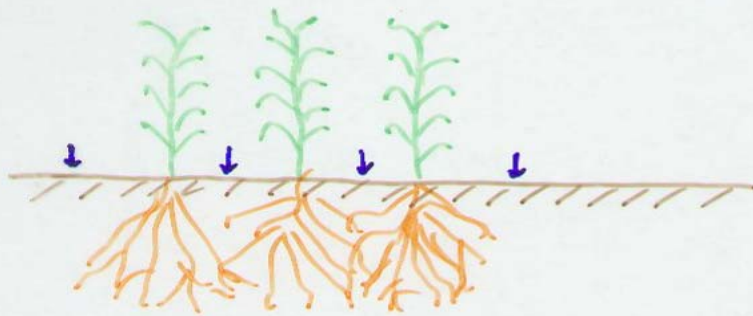
Donald Nielsen  
*University of California, Davies  
USA*

# Corn versus N



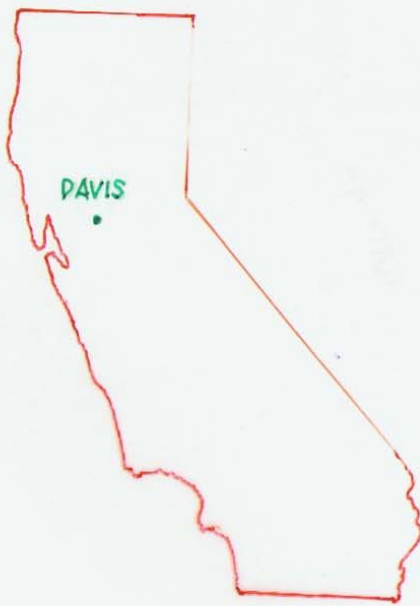


## LEACHING BELOW AGRICULTURAL FIELDS



$$\downarrow J_w \times C_{NO_3^-} = J_{NO_3^-} \downarrow$$

$^{15}\text{N}$ -FERTILIZER TO TRACE NITROGEN IN ENVIRONMENT



# Irrigation-fertility expt

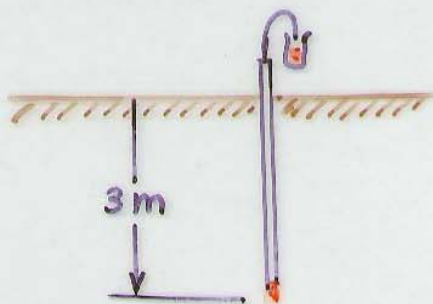
5-YEAR IRRIGATION-FERTILITY EXPERIMENT FOR MAIZE

IRRIGATION       $\frac{1}{3}ET$  ,  $\frac{3}{3}ET$  ,  $\frac{5}{3}ET$   
 $^{14}NH_4SO_4$       0 , 90 , 180 , 360 kg N/ha

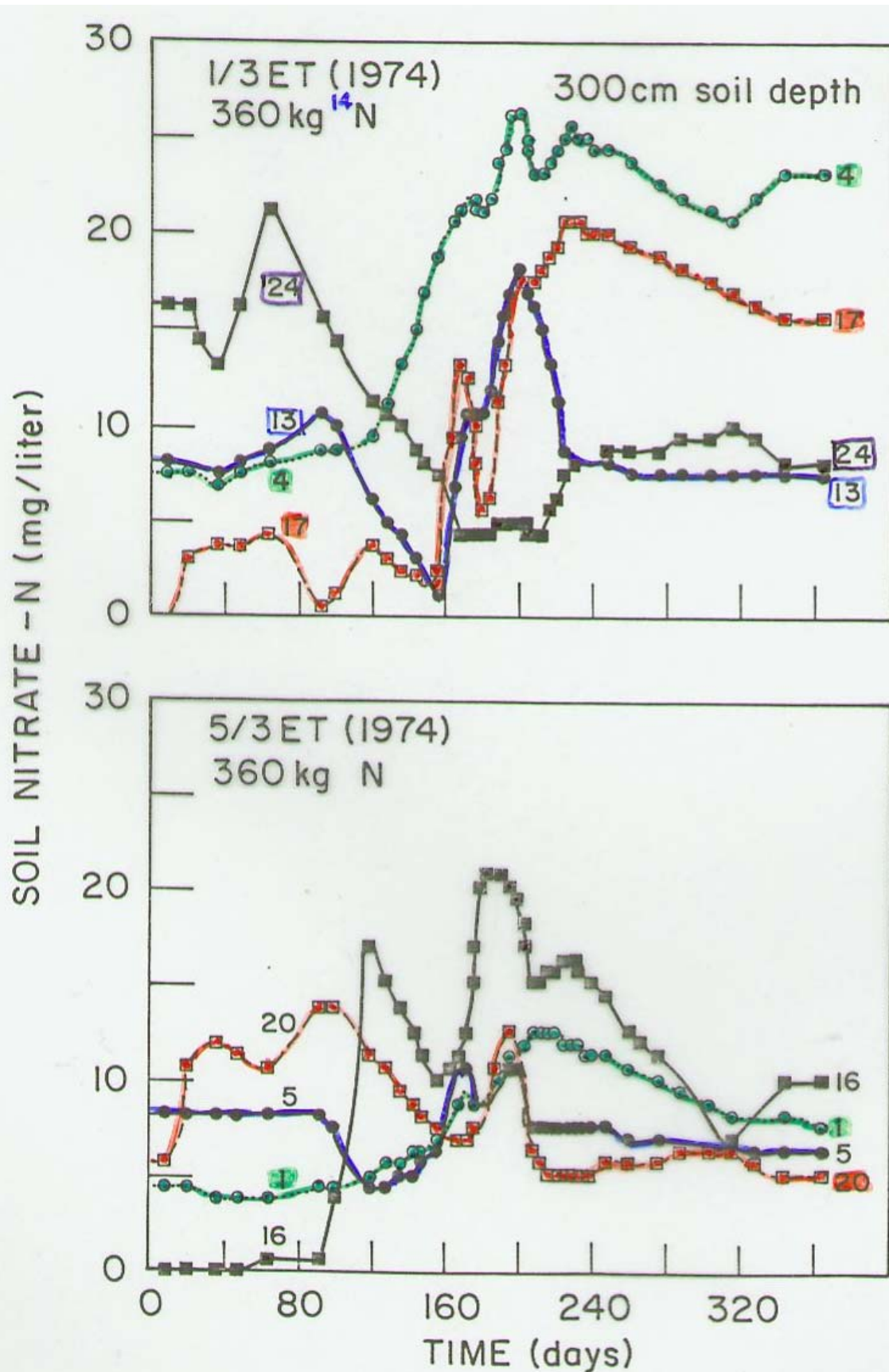
I	II
III	IV

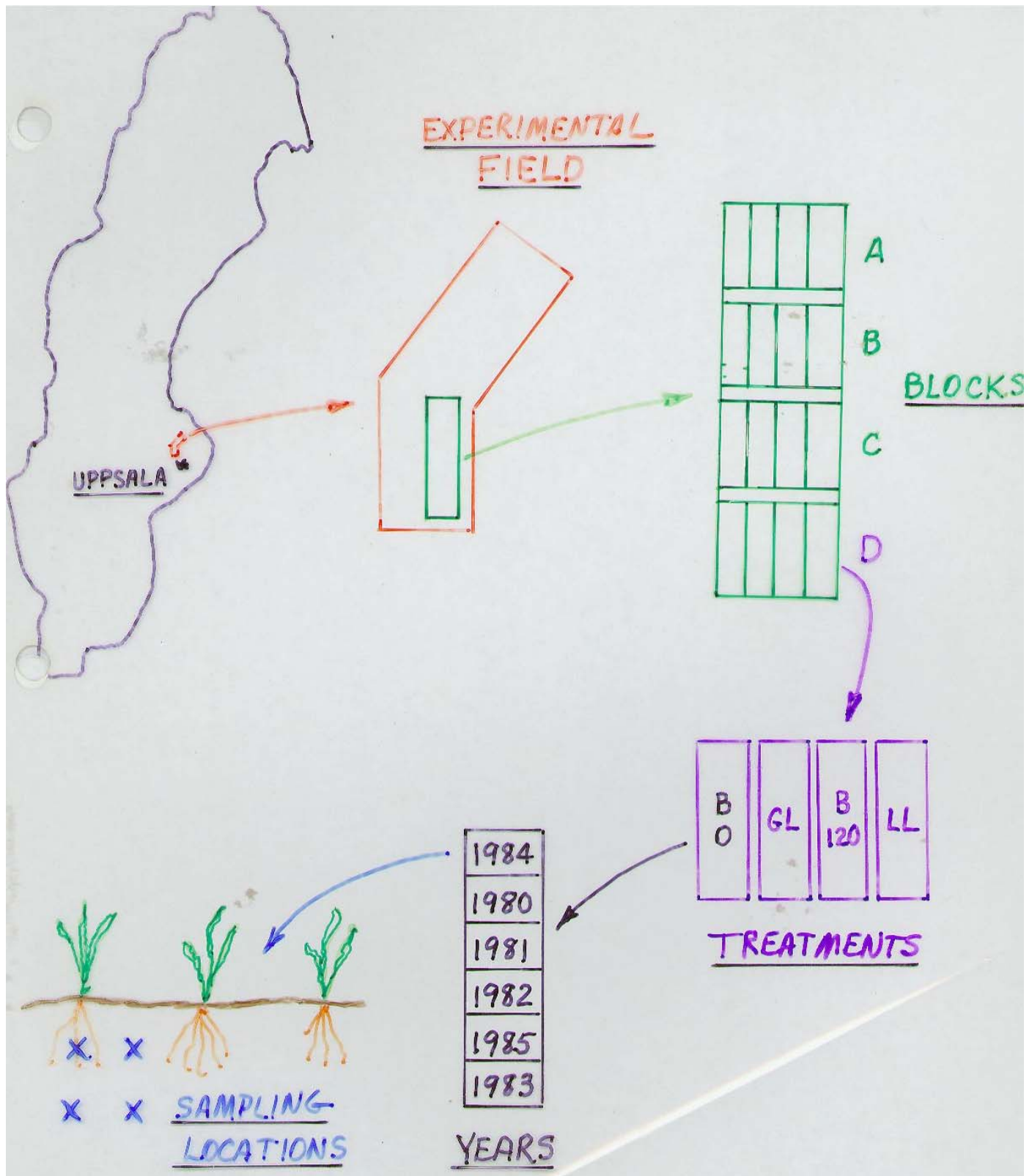

$\frac{1}{3}ET$   
360 kg

SOIL SOLUTION



# Soil nitrate conc







# We talk

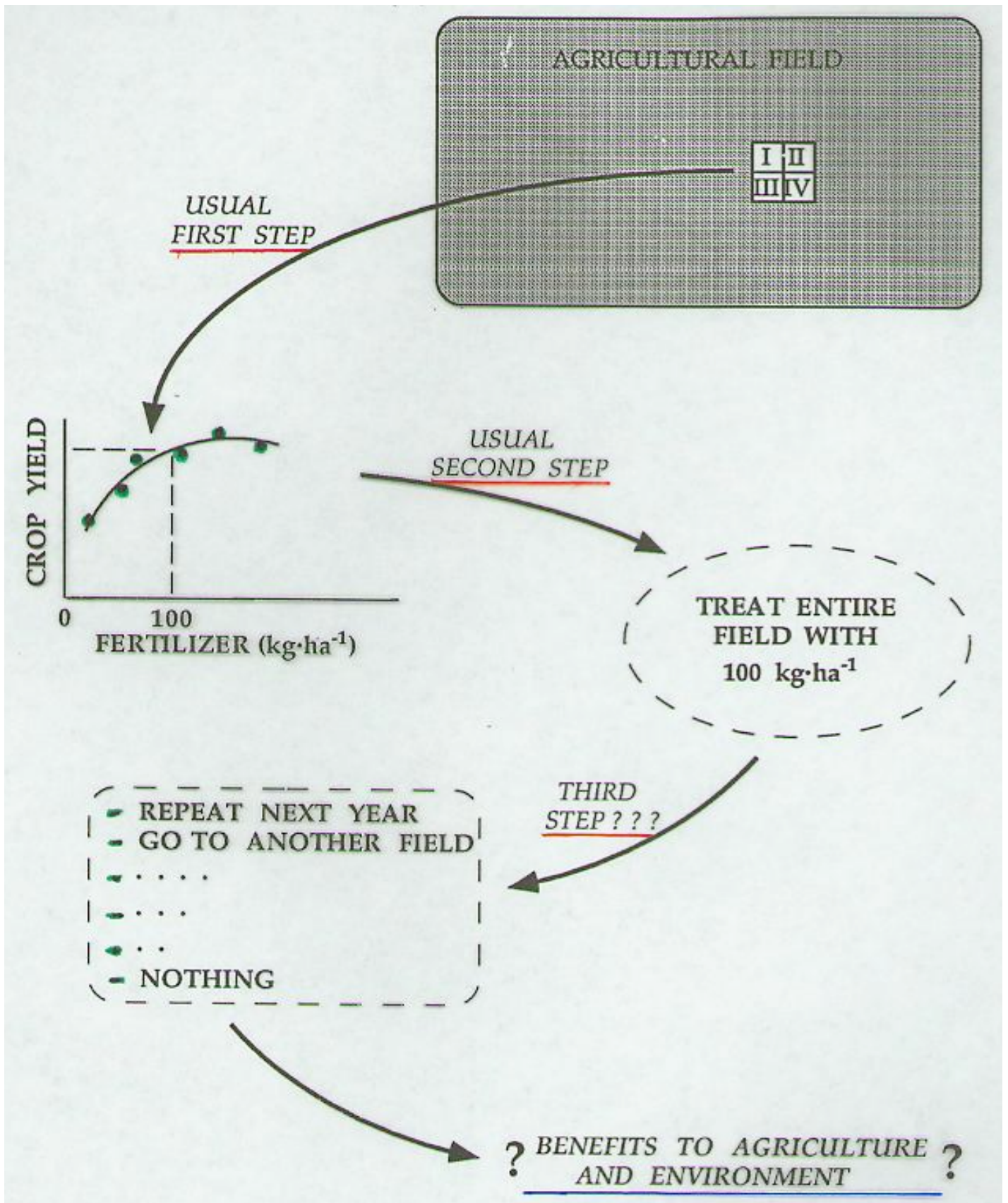
WE TALK ABOUT IMPROVING THE ENVIRONMENT, INCREASING AGRICULTURAL PRODUCTION AND SOLVING ENVIRONMENTAL PROBLEMS, BUT WE STILL USE SMALL PLOTS ON SELECTED SITES TO CONDUCT OUR RESEARCH! WE DO NOT KNOW HOW TO QUANTITATIVELY MAKE ASSESSMENTS ACROSS THE LANDSCAPE!

WE RESTRICT OUR RESEARCH BY IMPOSING TREATMENTS WITHIN GLASSHOUSES, ON SMALL PLOTS AND ON LYSIMETERS.

CROP PRODUCTION	- FERTILIZERS
PEST MANAGEMENT	- PESTICIDES
EROSION CONTROL	- SOIL SURFACE GEOMETRIES
TRANSPIRATION	- IRRIGATION SCHEDULES

WE STILL USE THE SAME STATISTICAL METHODS DESIGNED 75 YEARS AGO BY FISHER WHEN ONLY A PAPER & PENCIL, SLIDE RULE AND ABACUS WERE AVAILABLE FOR MATHEMATICAL CALCULATIONS.

# Present-day research

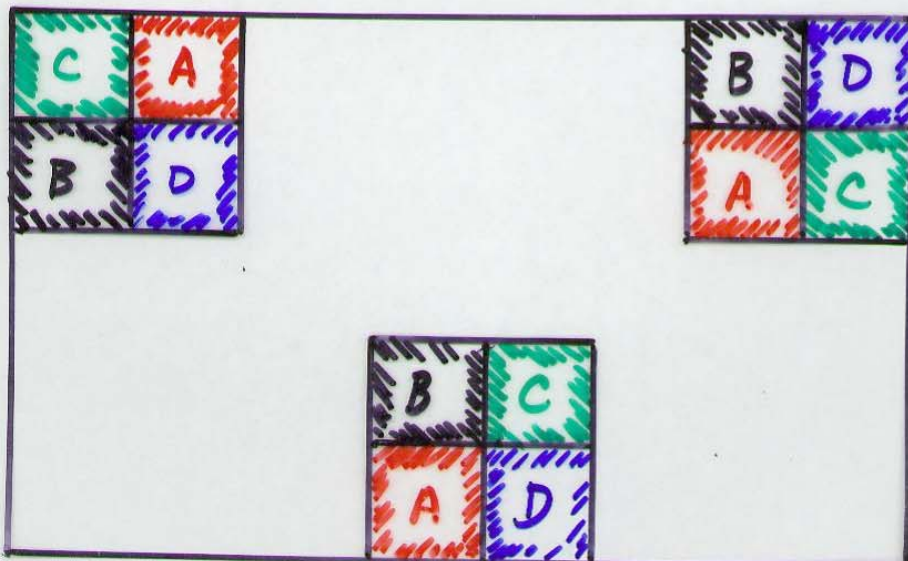
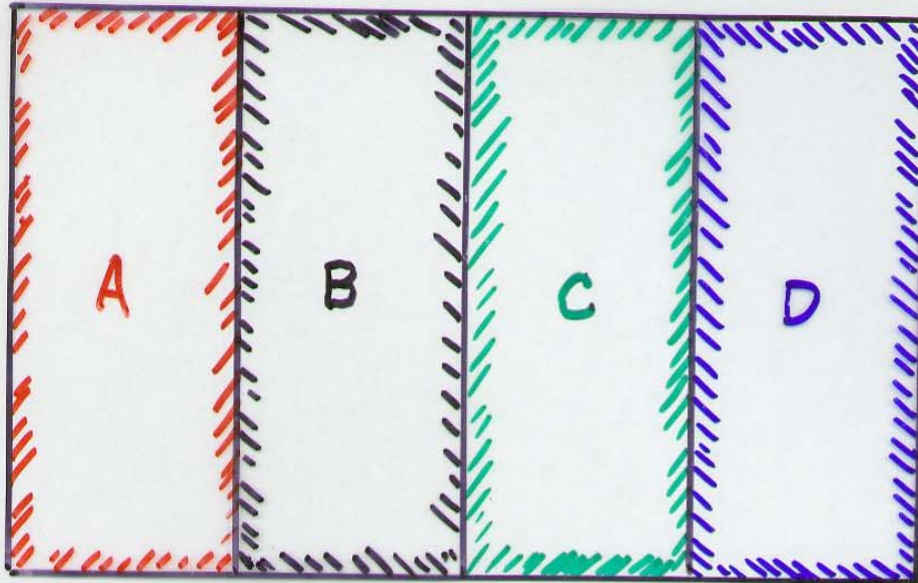


# Fisher blocks

FIELD EXPERIMENTS

BLOCKS

R.A. FISHER



# Classical Eq.

## MEAN

$$\bar{z} = \frac{1}{n} \sum_{i=1}^n z_i$$

## VARIANCE

$$s^2 = \frac{1}{n} \sum_{i=1}^n (z_i - \bar{z})^2$$

## NORMAL DISTRIBUTION

$$N = \frac{n\Delta z}{s\sqrt{2\pi}} \exp\left[-\frac{(z-m)^2}{2s^2}\right]$$

## LOG-NORMAL DISTRIBUTION

$$N = \frac{n\Delta z}{zs_{\ln}\sqrt{2\pi}} \exp\left[-\frac{(\ln z - m_{\ln})^2}{2s_{\ln}^2}\right]$$

## COVARIANCE

$$cov_{yz} = \frac{1}{n-1} \sum_{i=1}^n (y_i - \bar{y})(z_i - \bar{z}).$$

## CORRELATION COEFFICIENT

$$r_{yz} = \frac{cov_{yz}}{s_y s_z}$$

## LINEAR REGRESSION

$$\hat{y} = az + b$$

$$\begin{vmatrix} z & \hat{y} & 1 \\ \sum z_i & \sum y_i & n \\ \sum z_i^2 & \sum z_i y_i & \sum z_i \end{vmatrix} = 0$$

# Questions to ask

FOR A SMALL PLOT, A FIELD, AN ENSEMBLE OF FARMS  
OR A GEOGRAPHICAL REGION, WE SHOULD KNOW  
THE ANSWERS TO THESE QUESTIONS:

- HOW MANY SAMPLES?
- WHEN TO TAKE SAMPLES?
- WHERE TO TAKE SAMPLES?
- WHAT SIZE OF A DOMAIN DOES A SAMPLE REPRESENT?
- UTILITY OF A SAMPLE IN RELATION TO A CONCEPTUAL MODEL?
- UTILITY OF A CONCEPTUAL MODEL IN RELATION TO A SAMPLE?

# Optimal Freq&tim

AN OPTIMAL FREQUENCY IN TIME AND SPACE FOR  
MAKING OBSERVATIONS  
OF A SOIL ATTRIBUTE  
REMAINS AN ENIGMA FOR ALL SOIL SCIENTISTS

# The Enigma

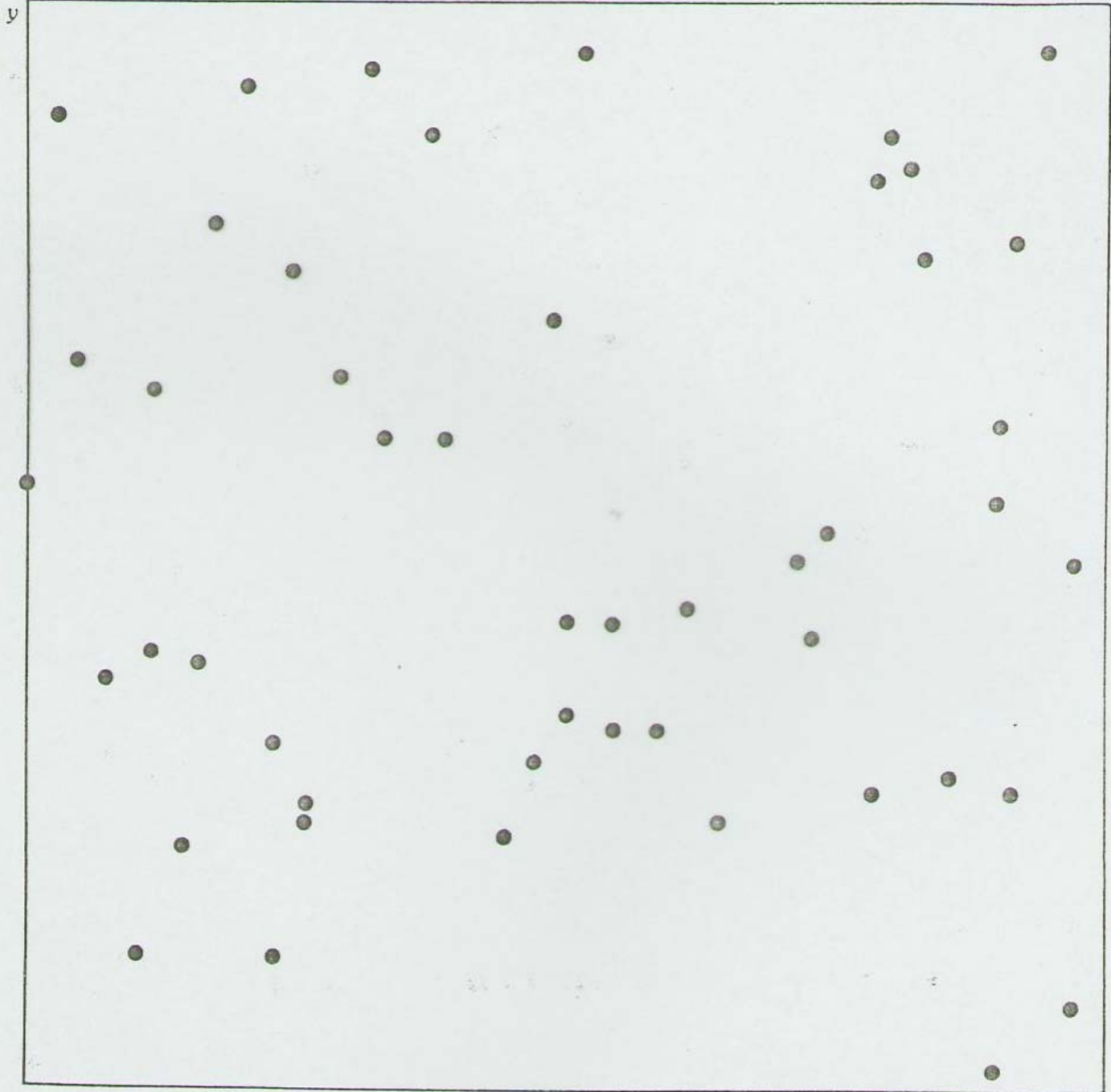
## THE ENIGMA

Proven concepts and strategies to interpret the cause of spatial crop patterns do not exist, and we are unable to reliably predict and achieve yield patterns.

The enigma persists owing to the propensity of agronomists limiting their research to classical randomized block field experiments having different treatments thought to have an impact on crop growth.

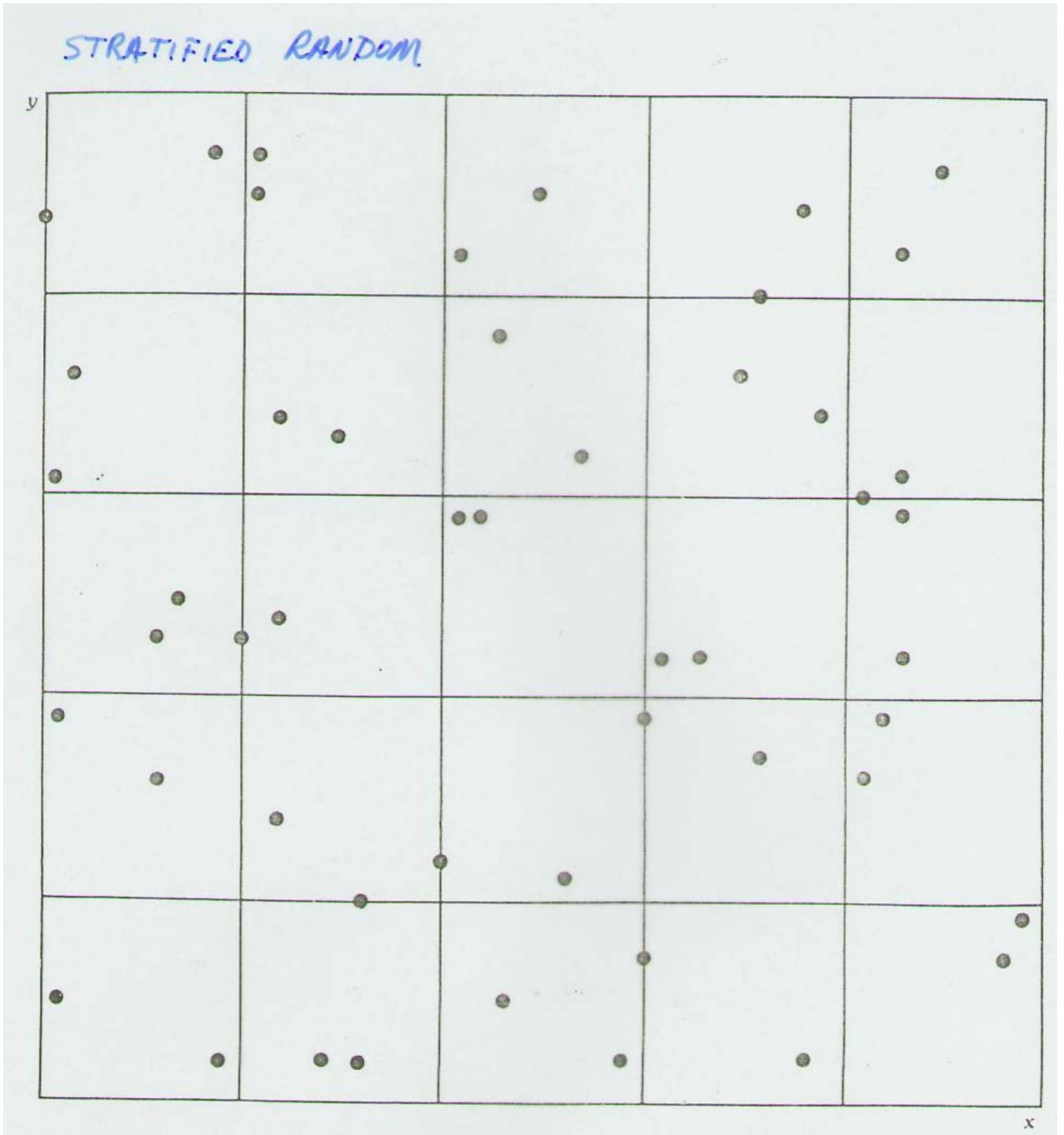
And the enigma will not readily be solved by merely importing tremendous amounts of field data into geographic information systems to produce spatial distribution maps that remain open to question because underlying assumptions are neither met nor sufficiently examined.

SIMPLE RANDOM

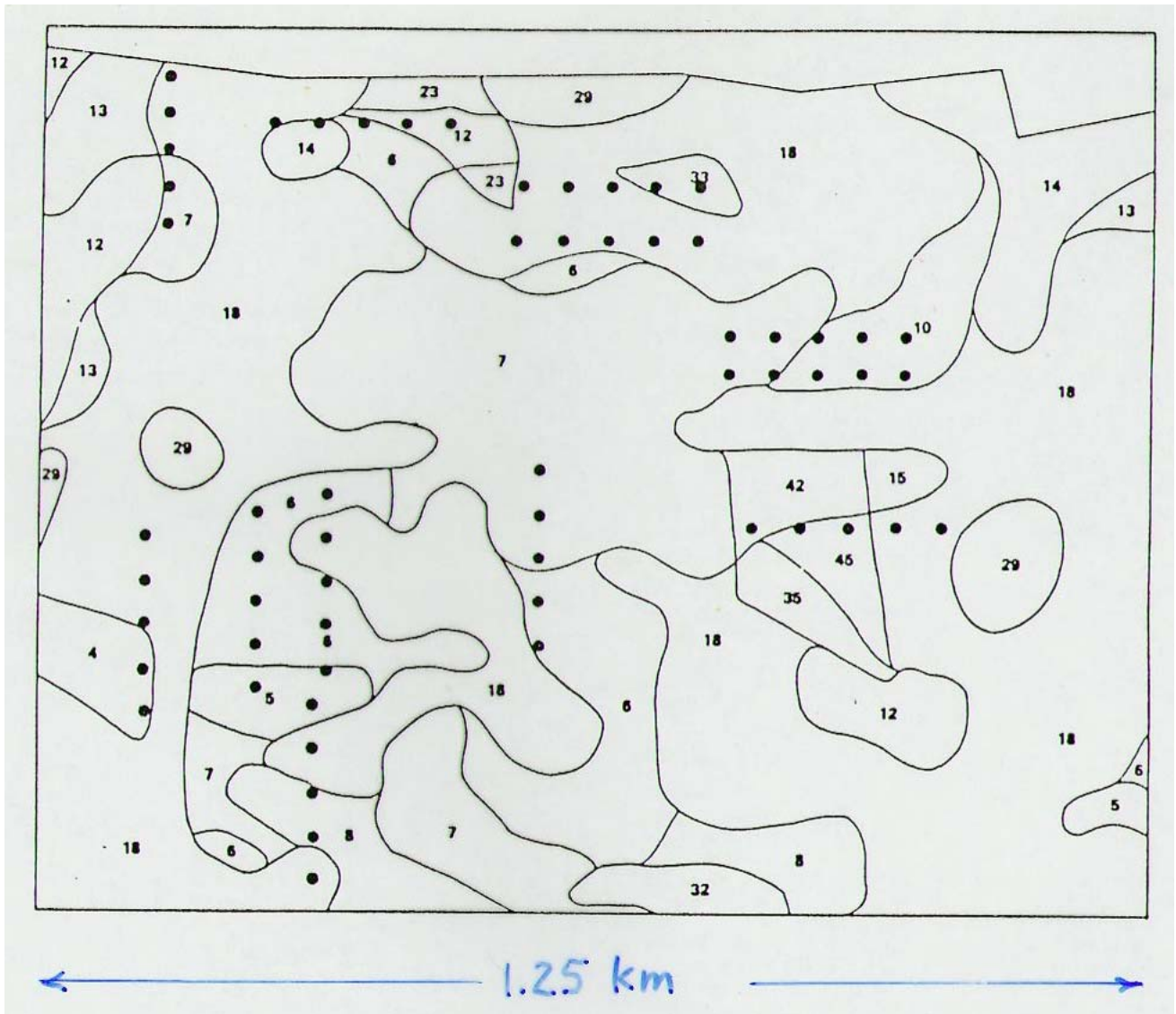




# Stratified random



# Dutch sampling

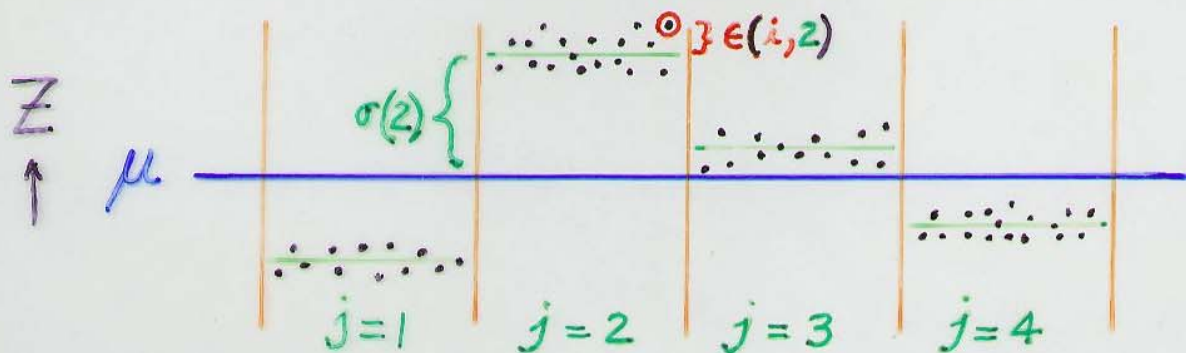


# ANOVA

## ANALYSIS OF VARIANCE

$$Z(i,j) = \mu + \sigma(j) + \epsilon(i,j)$$

observation      mean      class deviation      error



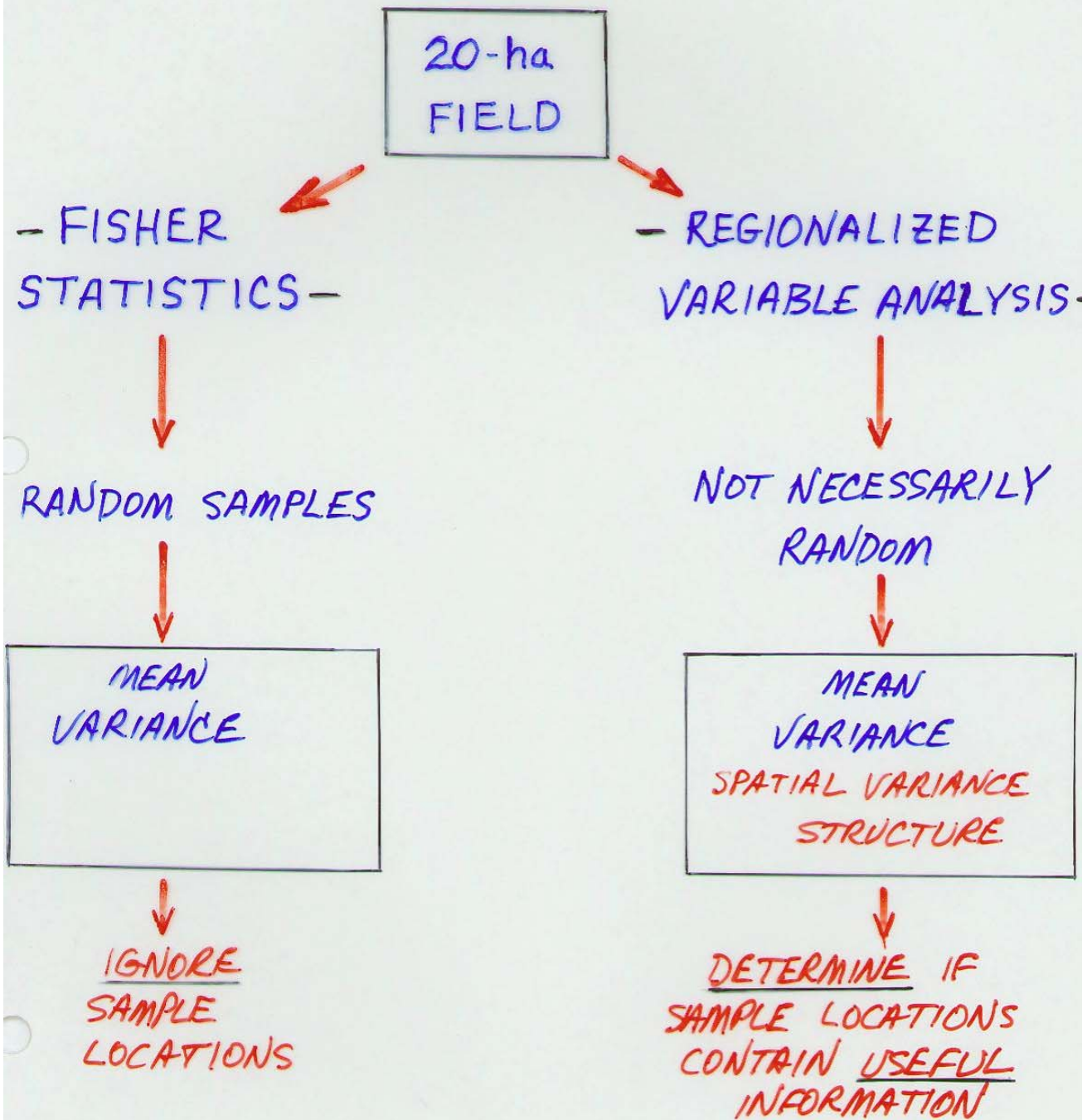
GEOGRAPHIC LOCATIONS OF OBSERVATIONS ARE IGNORED!

ALTHOUGH 50 YEARS OF USING ANOVA HAS BEEN HIGHLY BENEFICIAL, IT HAS UNFORTUNATELY LEAD TO:

- UNNECESSARILY IMPOSING "TREATMENTS" TO MOST EXPERIMENTS
- SELECTING "TYPICAL" OR "REPRESENTATIVE" SITES
- TAKING "RANDOM" SAMPLES TO "AVOID BIAS"
- AVOIDING SPATIALLY VARIABLE LOCATIONS
- ASSUMING VARIABILITY IS "BAD"
- ASSUMING VARIABILITY IS "ERROR"
- NOT TRUSTING ONLY ONE SAMPLING
- NOT TRUSTING ONLY ONE YEAR'S RESULTS

# Aggie vs regionalized

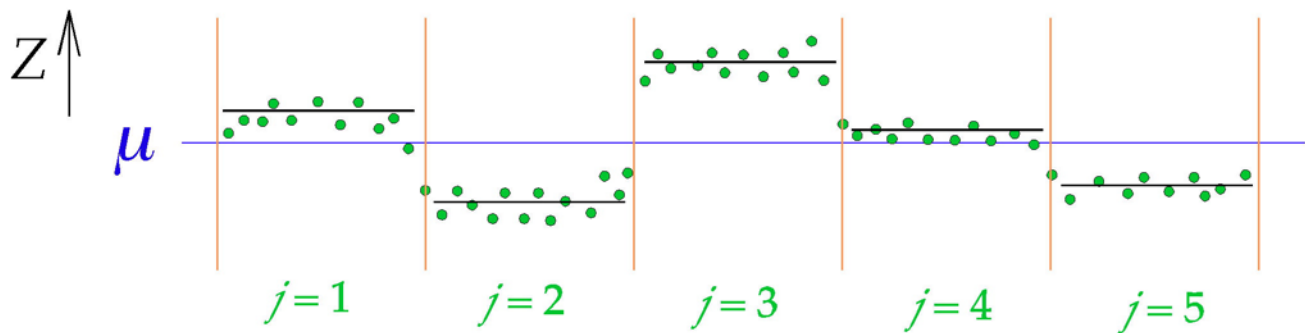
## FISHER or "AGGIE" STATISTICS VERSUS REGIONALIZED VARIABLE ANALYSIS



# Fisher ANOVA

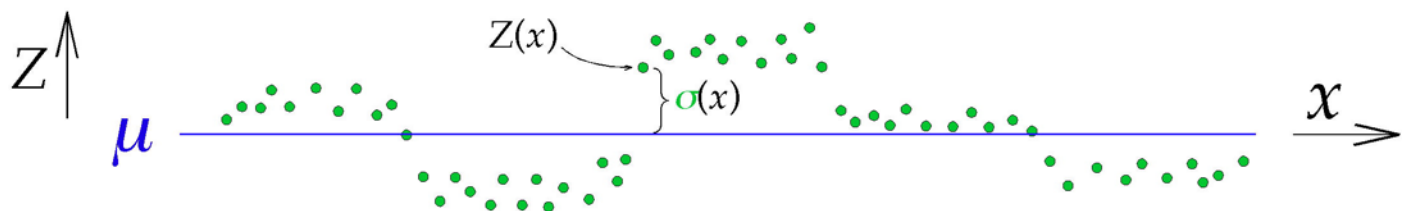
## FISHER ANALYSIS OF VARIANCE

$$Z(\mathbf{i}, j) = \mu + \sigma(j) + \varepsilon(\mathbf{i}, j)$$



## REGIONALIZED VARIABLE ANALYSIS

$$Z(x) = \mu + \sigma(x)$$



# Opportunities

## OPPORTUNITIES

- CORRELOGRAMS
- VARIOGRAMS
- KRIGING
- SPECTRAL ANALYSIS
- COHERENCY
- STATE-SPACE ANALYSIS
- MONTE CARLO SIMULATIONS
- KINEMATIC WAVE EQUATIONS
- NONLINEAR KALMAN FILTERING
- FRACTAL DISTRIBUTIONS
- AUTOREG. MOVING AVERAGES
- 
- 
- 
- 
- 
- 
- CROSS-CORRELOGRAMS
- CO-VARIOGRAMS
- CO-KRIGING
- CO-SPECTRAL ANALYSIS
- SPLIT MOVING WINDOWS
- MARKOV PROCESSES
- TRANSFER FUNCTIONS
- FUZZY SETS
- STOCHASTIC EQUATIONS
- SCALING CONCEPTS
- 
- 
- 
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- 
-

# Lecture content

## - INTRODUCTION

- Review of agricultural research experiments with small field plots
- Disadvantages of present-day experiments and statistical analyses
- Need for improved technology applicable to a farmer's entire field
- Need for improved technology applicable to an ensemble of fields

## - CONCEPT OF A REGIONALIZED VARIABLE

- Autocorrelation
- Crosscorrelation
- Examples

## - GEOSTATISTICAL METHODS

- Variograms and covariograms
- Kriging and cokriging
- Examples

## - APPLIED TIME SERIES ANALYSIS

- Spectral and cospectral analyses
- Coherency
- Examples

## - STATE-SPACE ANALYSIS

- Autoregressive functions
- Nonlinear Kalman filtering
- Examples

## - NEAREST NEIGHBOR ANALYSIS

- Example

## - FUTURE RESEARCH OPPORTUNITIES

# Objectives

- IDENTIFY ANALYTICAL METHODS NOT NORMALLY USED IN FIELD STUDIES OF SOILS AND AGROECOLOGY
- PROVIDE EXAMPLES OF A POTENTIAL NEW FIELD TECHNOLOGY FOR DESCRIBING, ANALYZING AND MANAGING OUR SOIL RESOURCES



# Taking advantage of variability

TAKING ADVANTAGE OF  
SPATIAL VARIABILITY  
RATHER THAN IGNORING IT  
WHEN  
ANALYZING FIELD SOILS

# Title of autocorrelation

SPATIAL OR TEMPORAL  
AUTOCORRELATION

# Questions for autocorrel

## QUESTIONS FOR AUTOCORRELATION

- HOW FAR APART SHOULD I TAKE SAMPLES?
- WHAT SIZE SAMPLE SHOULD I TAKE?
- HOW OFTEN SHOULD I TAKE SAMPLES?
- WHAT IS THE AREA REPRESENTED BY A SAMPLE?
- WHAT IS THE LENGTH OF TIME REPRESENTED BY A SAMPLE?

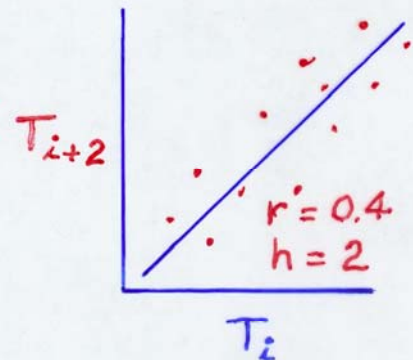
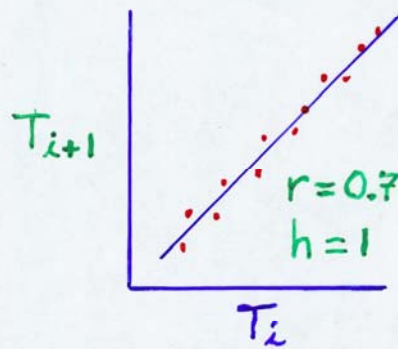
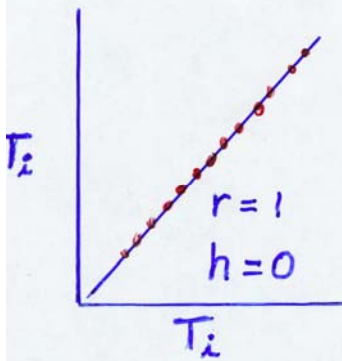
"CORRELATION LENGTH" VERSUS SAMPLE DIMENSION

# Concept of autocorrel

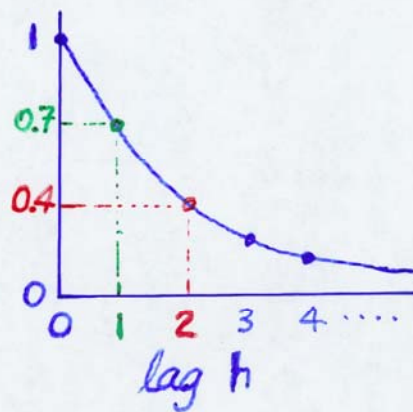
## CONCEPT OF AUTOCORRELATION

$T_1$   $T_2$   $T_3$   $T_4$   $\dots$   $T_i$   $\dots$   $T_{n-1}$   $T_n$   
 $x_1$   $x_2$   $x_3$   $x_4$   $\dots$   $x_i$   $\dots$   $x_{n-1}$   $x_n$

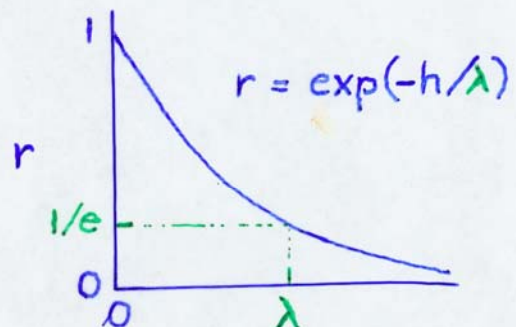
$$r(h) = \frac{\text{cov}(T_i, T_{i+h})}{\text{var}(T_i)}$$



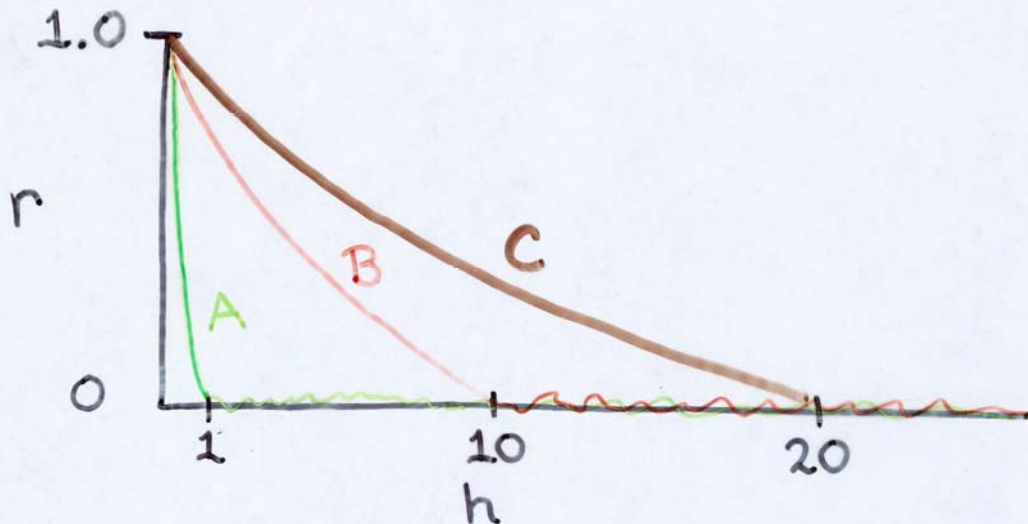
## CORRELOGRAM



## CORRELATION LENGTH $\lambda$



# ACF fields a,b,c

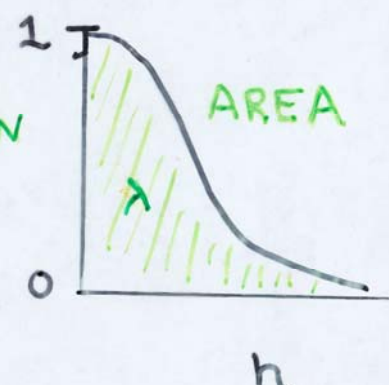
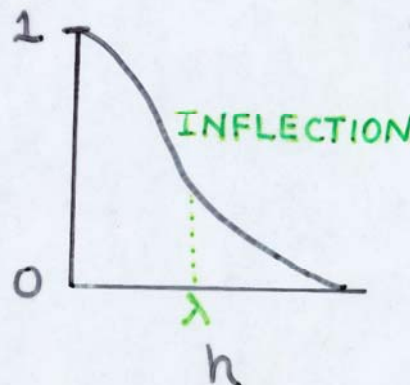
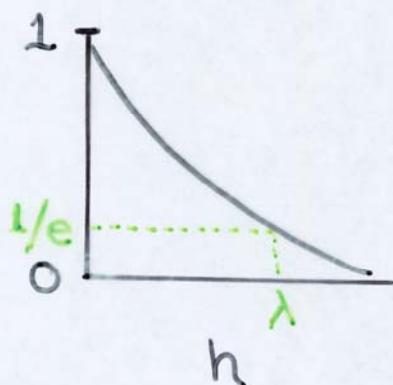


IF FOR  $h=1$ ,  $r \rightarrow 0$  : FISHER STATISTICS

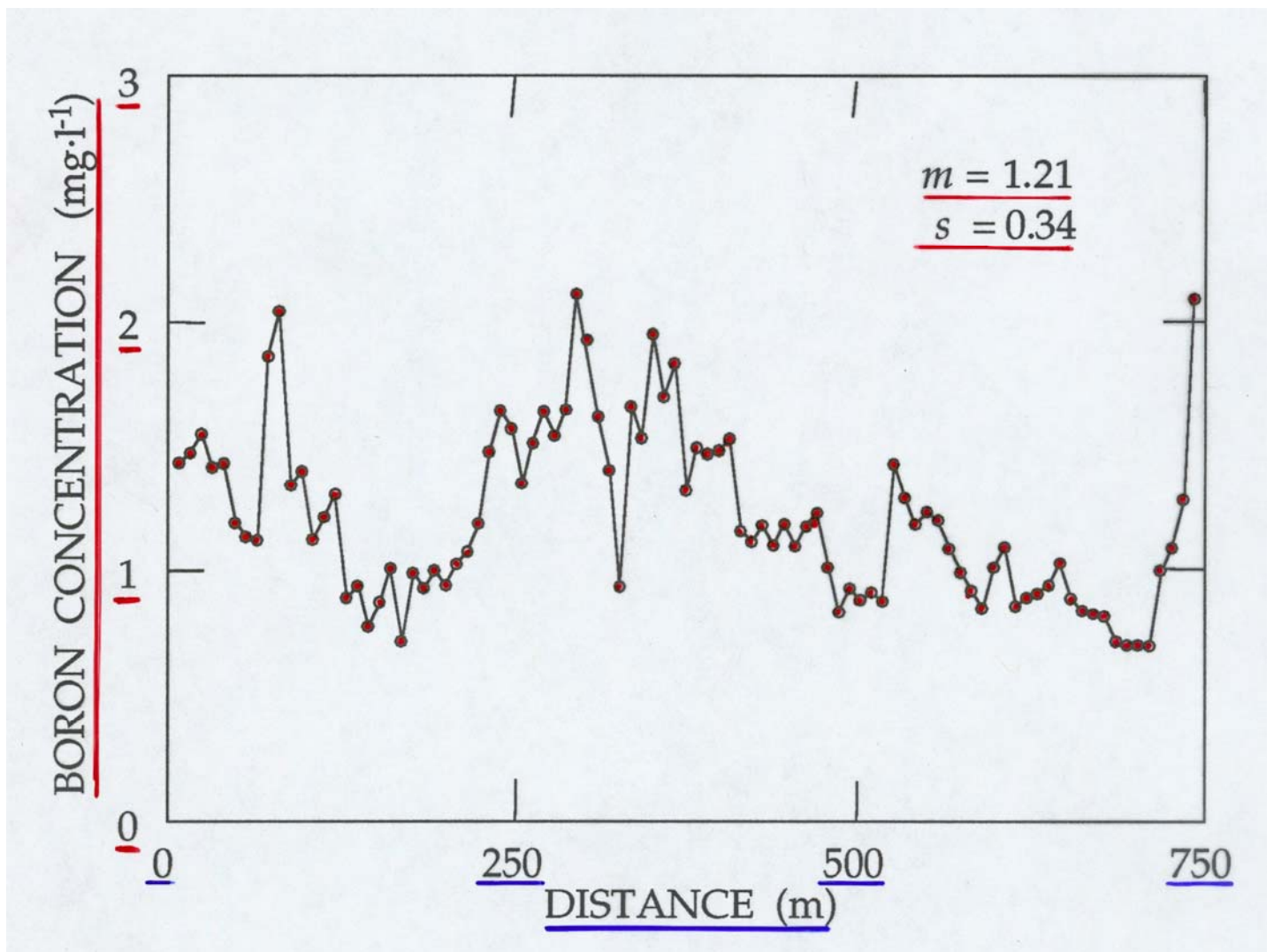
IF FOR  $h > 1$ ,  $r \neq 0$  : GEOSTATISTICS

## AUTOCORRELATION LENGTH

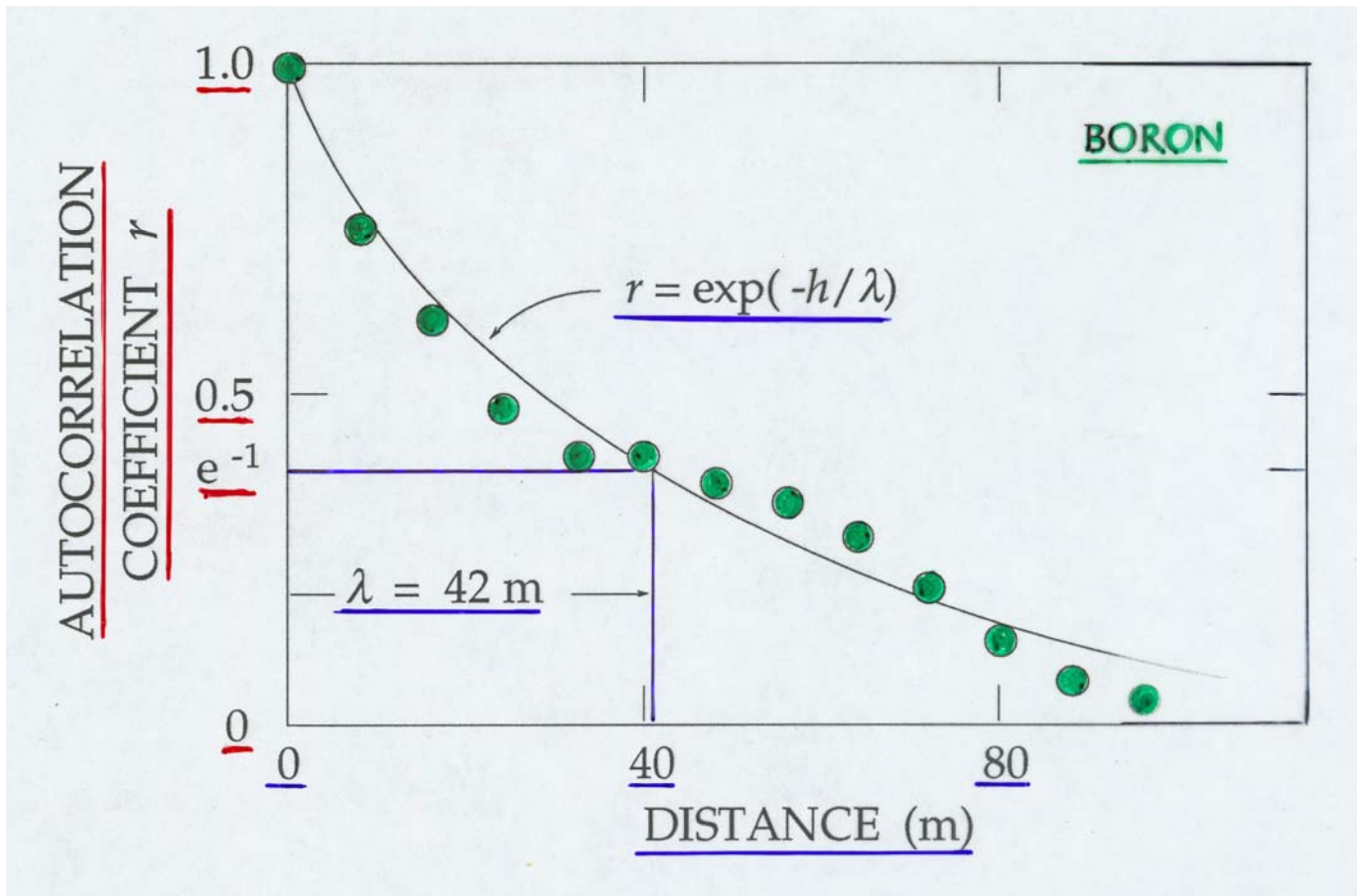
- DISTANCE ACROSS THE LANDSCAPE CHARACTERIZED BY A SINGLE OBSERVATION WITHIN THE FIELD



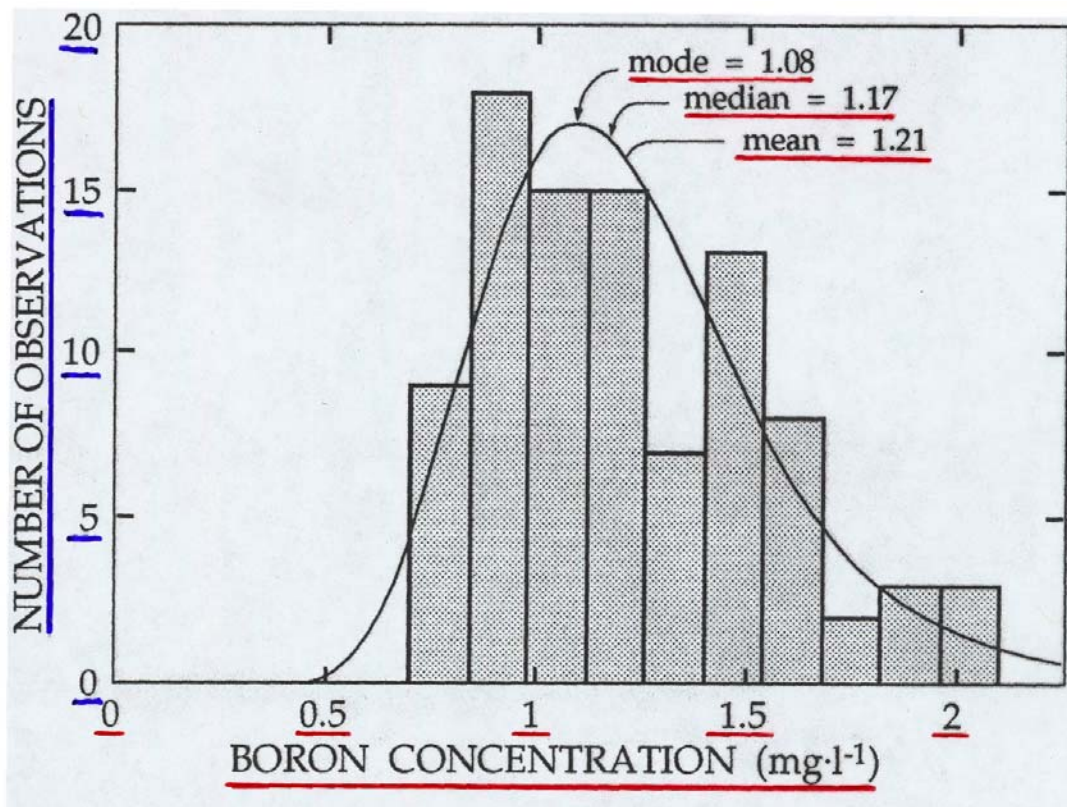
# Boron versus distance



# ACF Boron

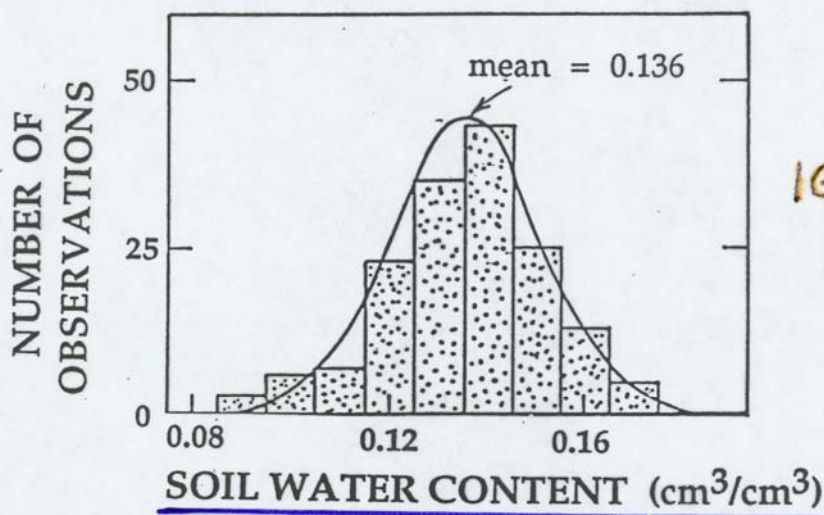
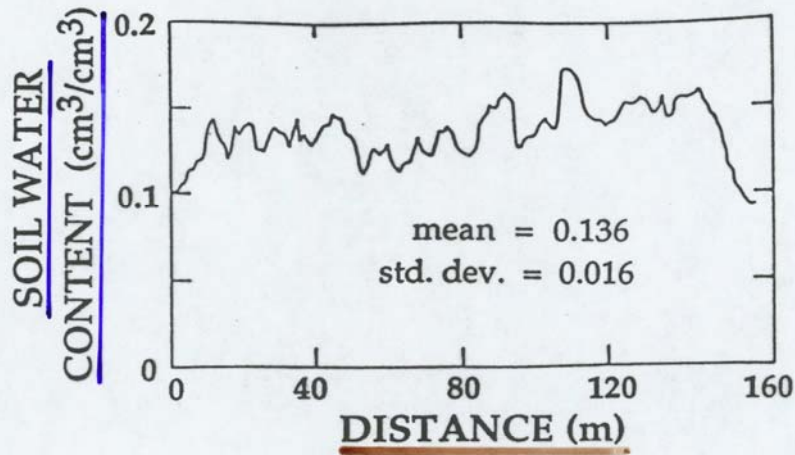


# Boron

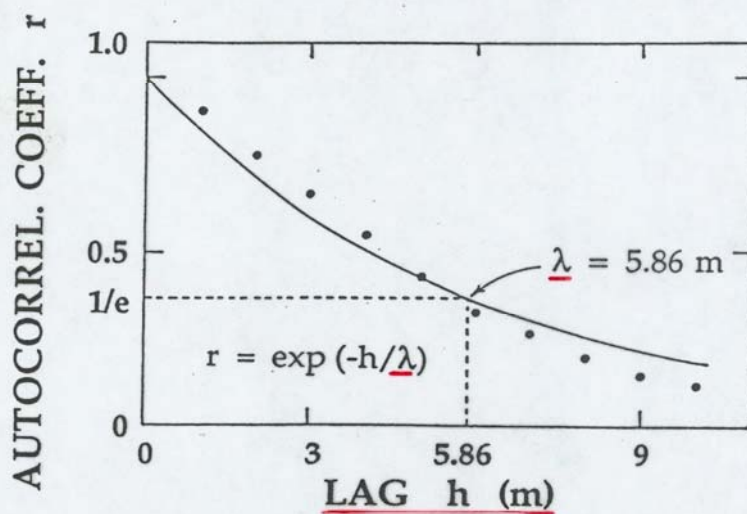




# Soil water content

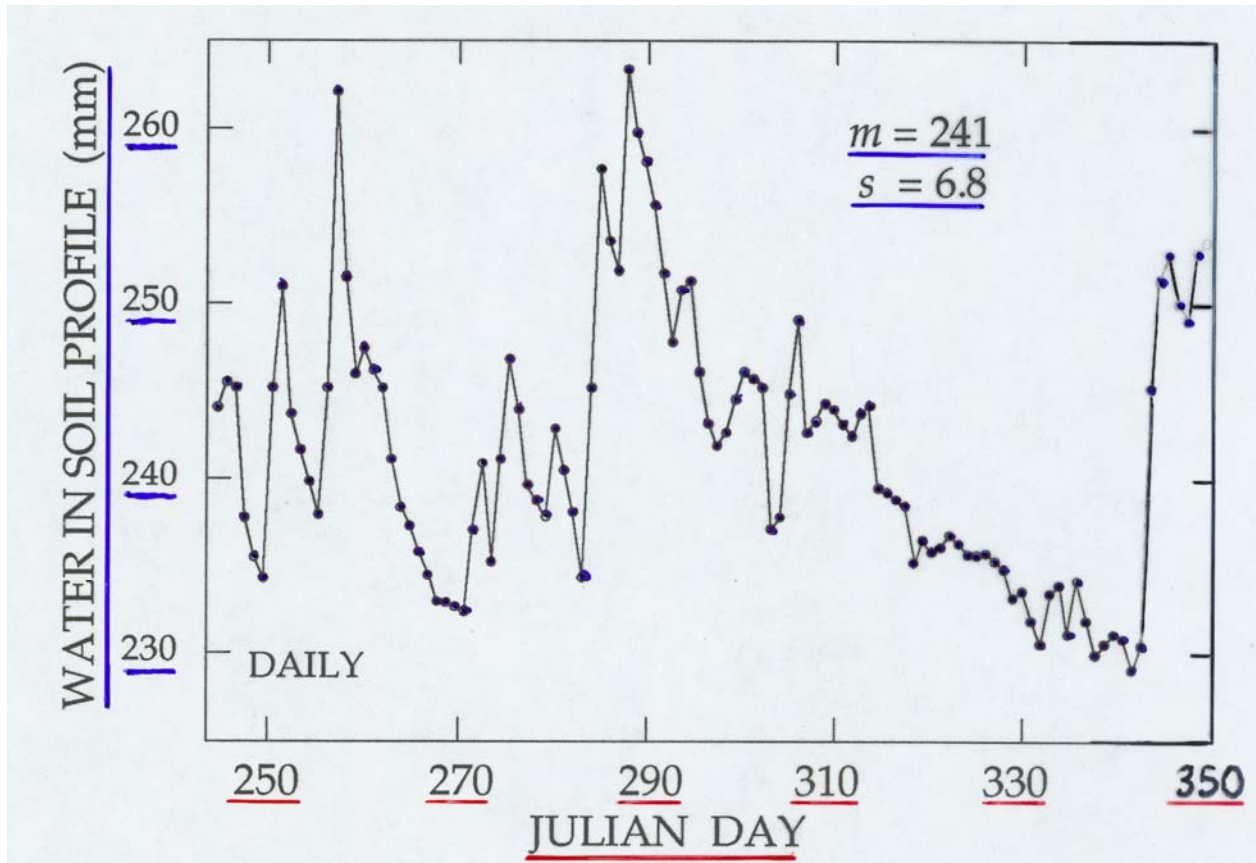


IGNORE LOCATIONS

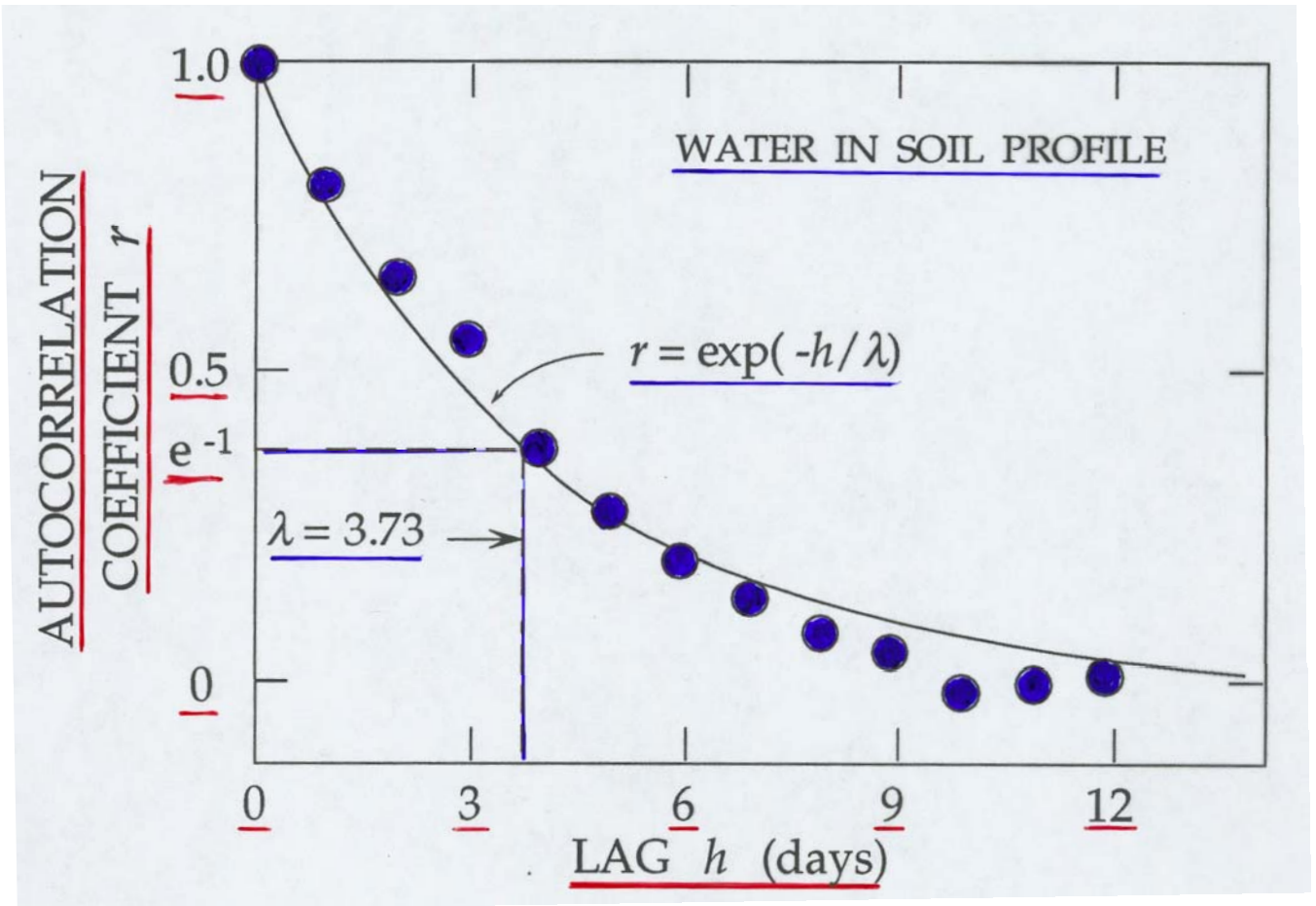


CONSIDER LOCATIONS

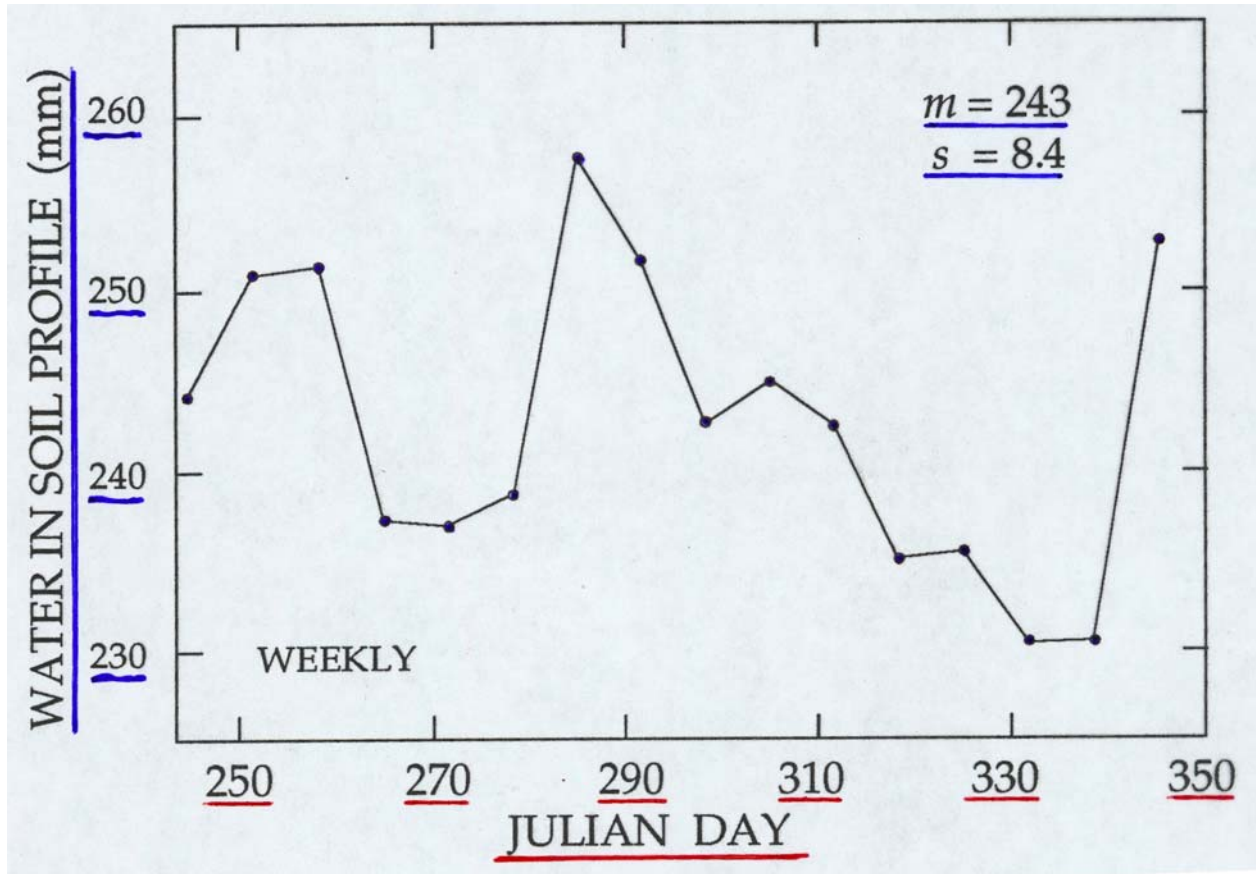
# Daily soil water



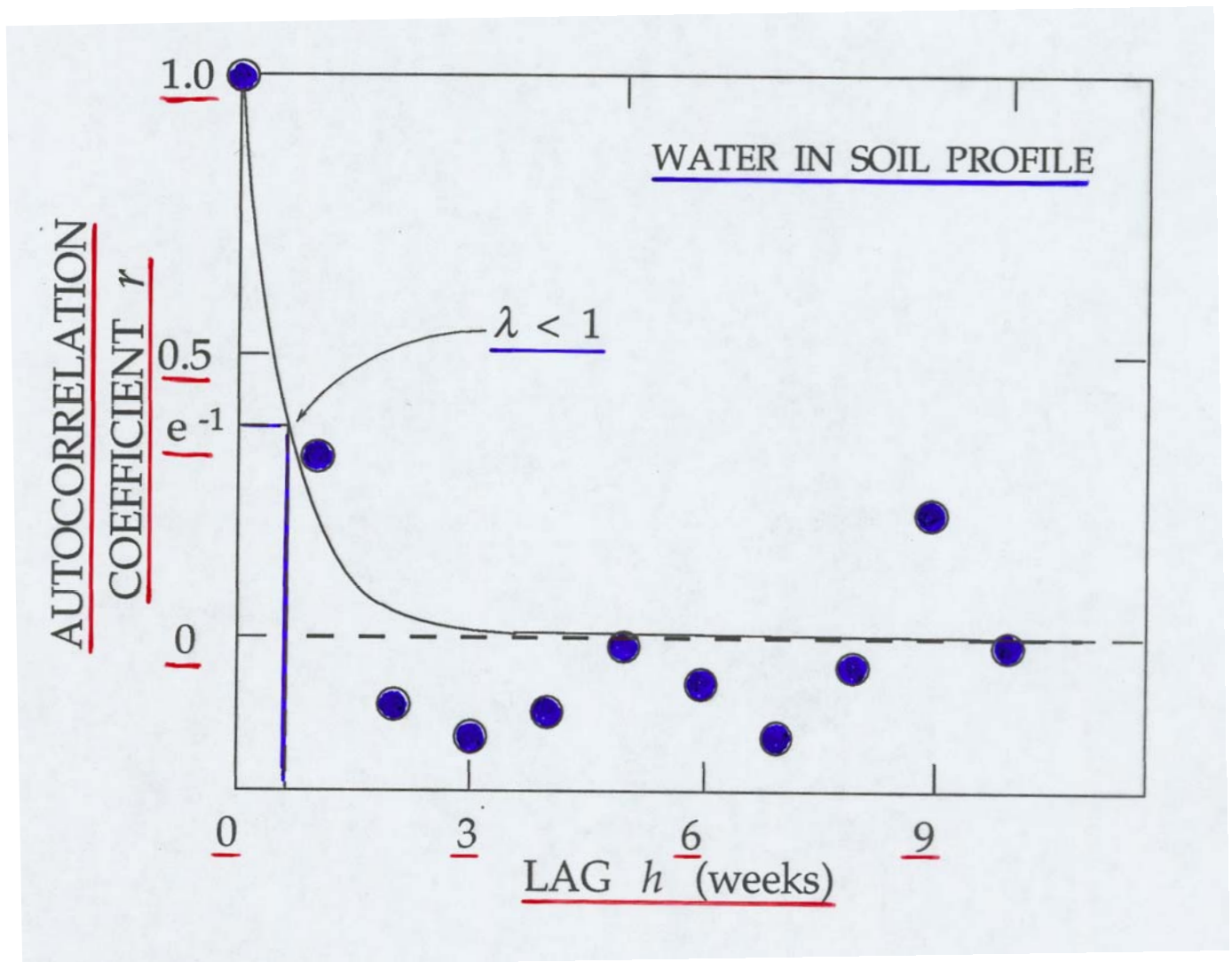
# ACF Daily water



# Weekly soil water

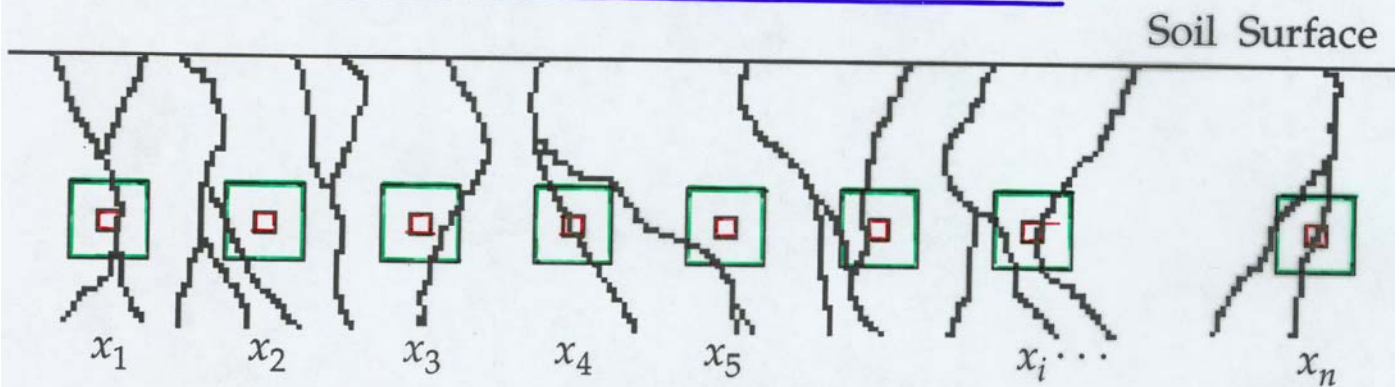


# ACF Weekly soil water

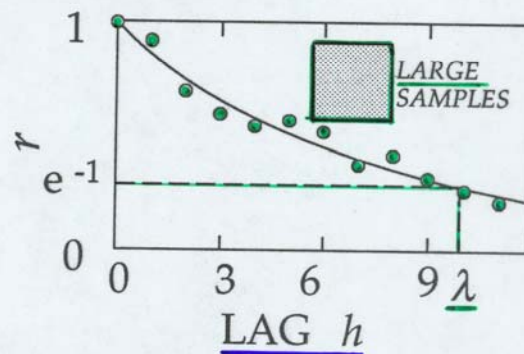
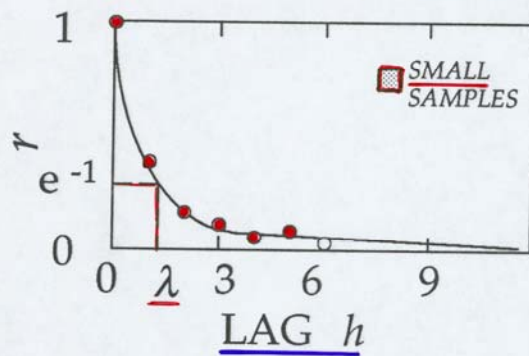


# Sampling preferential flow

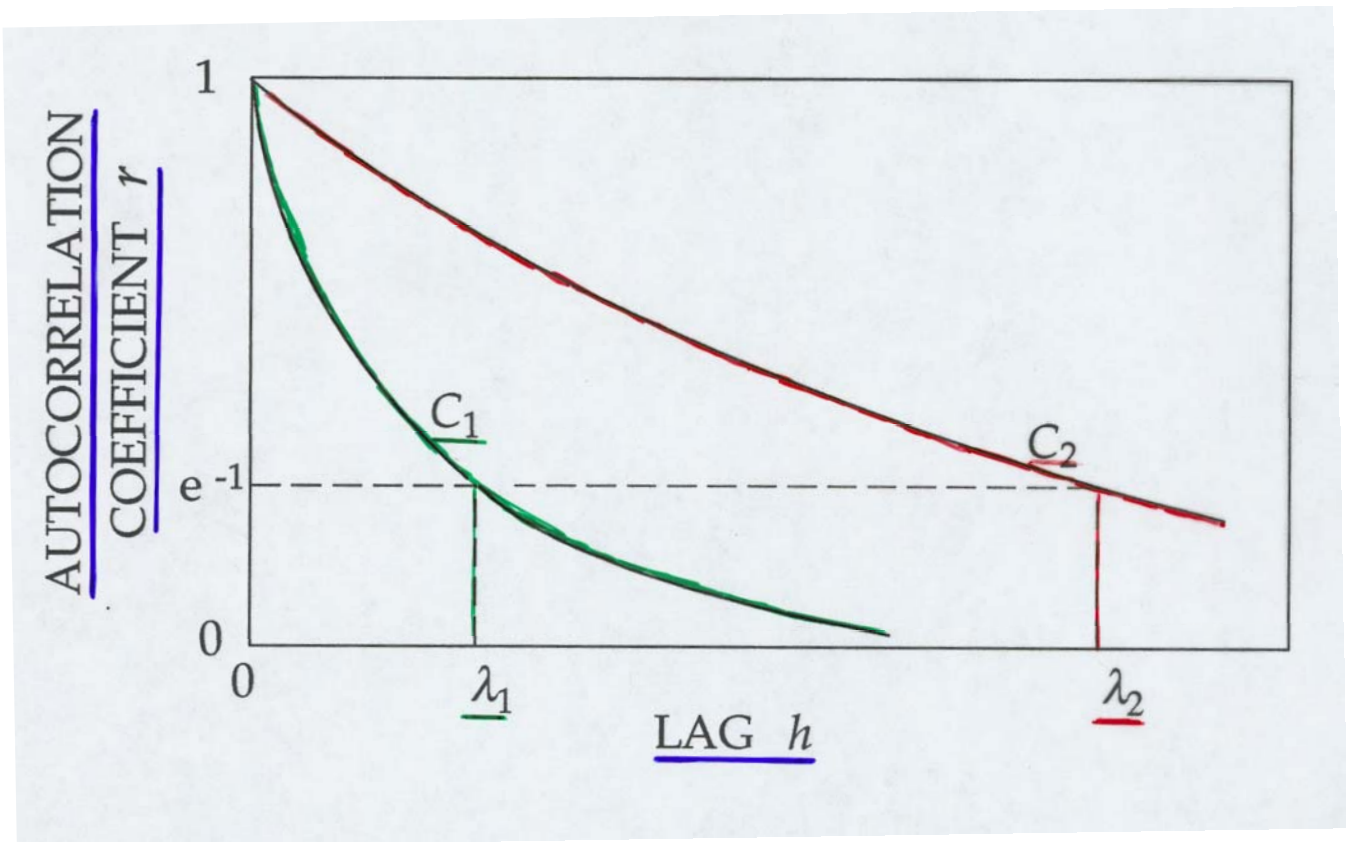
## SAMPLING SOIL SOLUTION WITH PREFERENTIAL FLOW PATHS



## AUTOCORRELOGRAMS

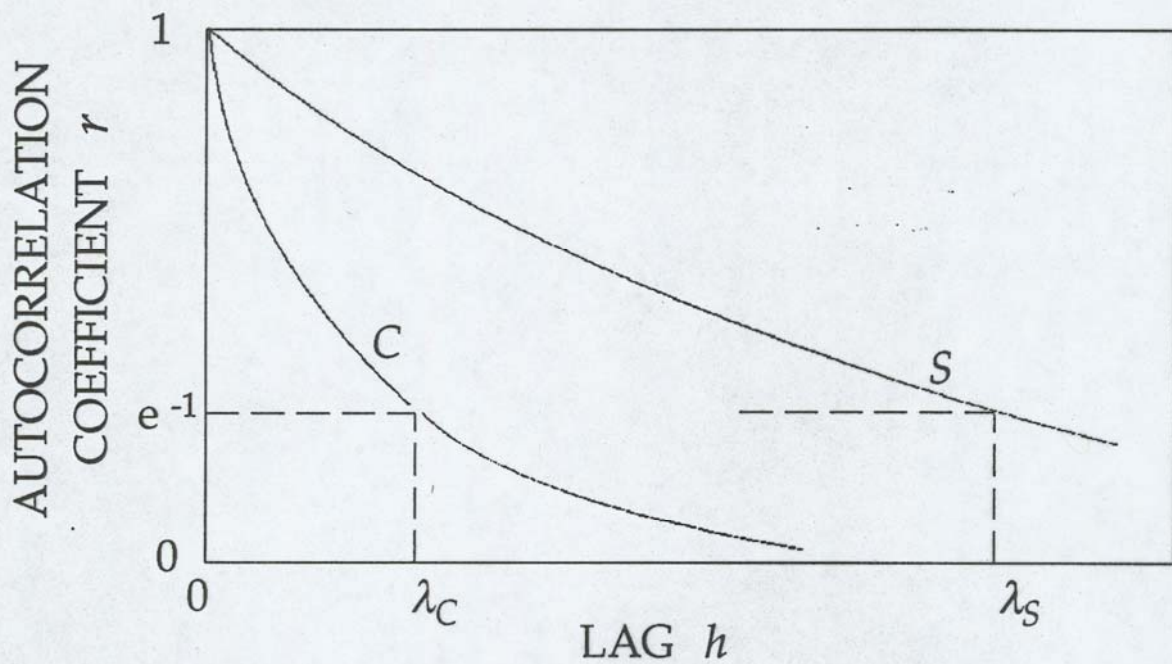


# ACF C1 and C2



# ACF C&S NH4

IF  $\lambda_C = \lambda_S$   $S = RC$  or  $f(C)$   
 $\lambda_C \neq \lambda_S$   $S \stackrel{?}{=} RC$  or  $f(C)$  ?



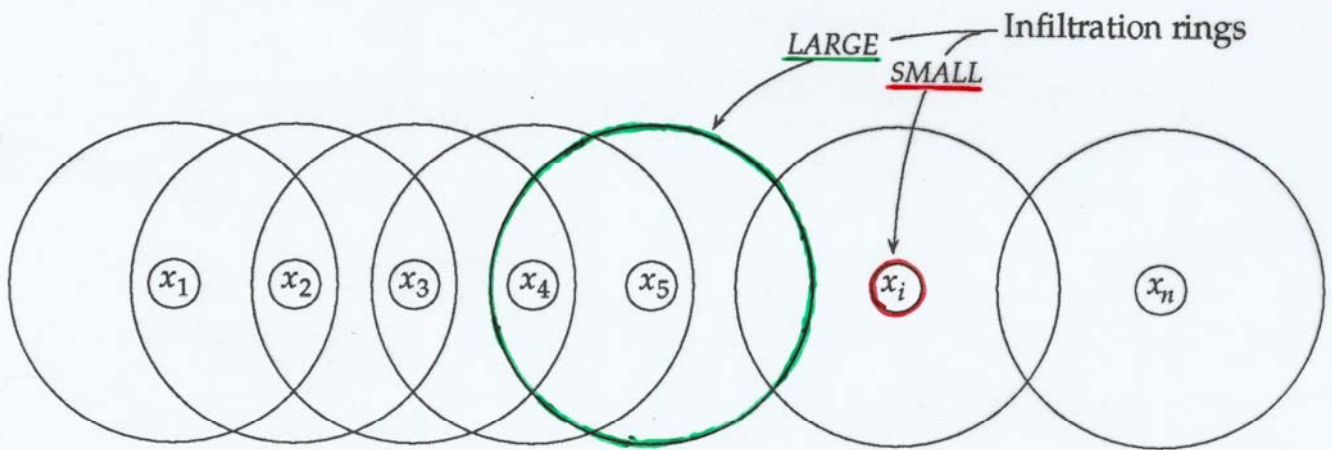
$$q_{\text{solute}} \stackrel{?}{=} -D \nabla C$$

$$q_{\text{water}} \stackrel{?}{=} -K \nabla H$$

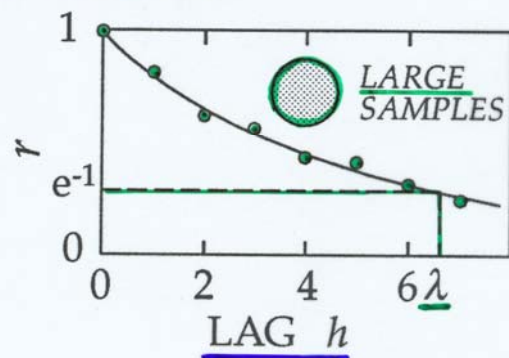
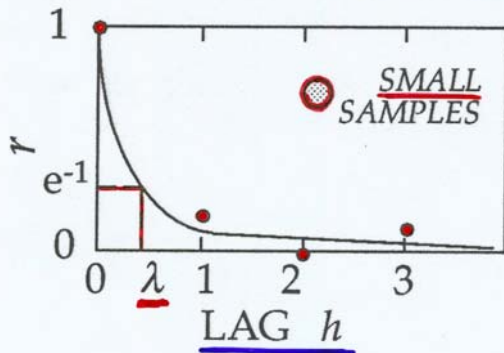


# Sampling infiltration rings

## SAMPLING INFILTRATION INTO THE SOIL SURFACE

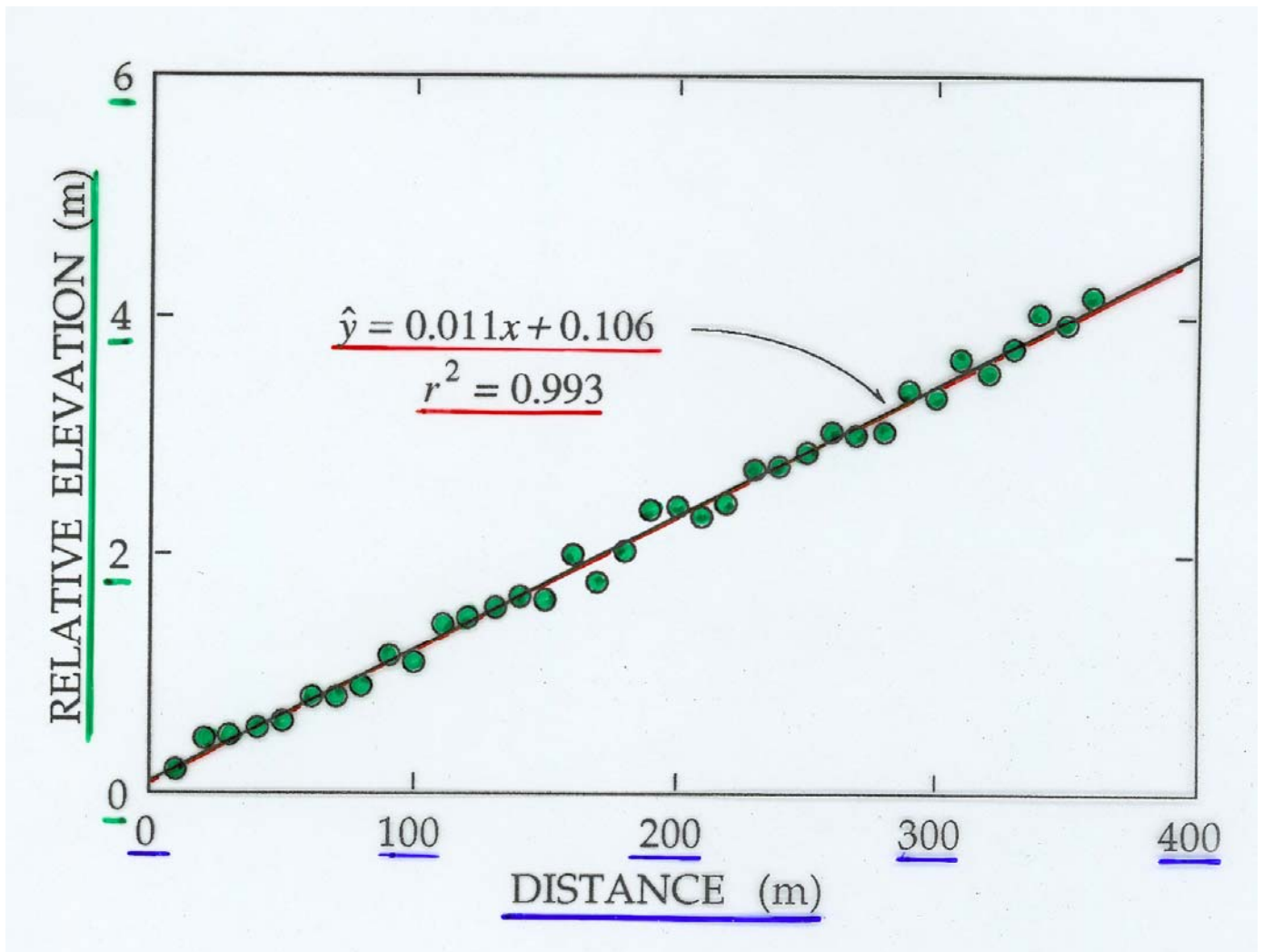


## AUTOCORRELOGRAMS

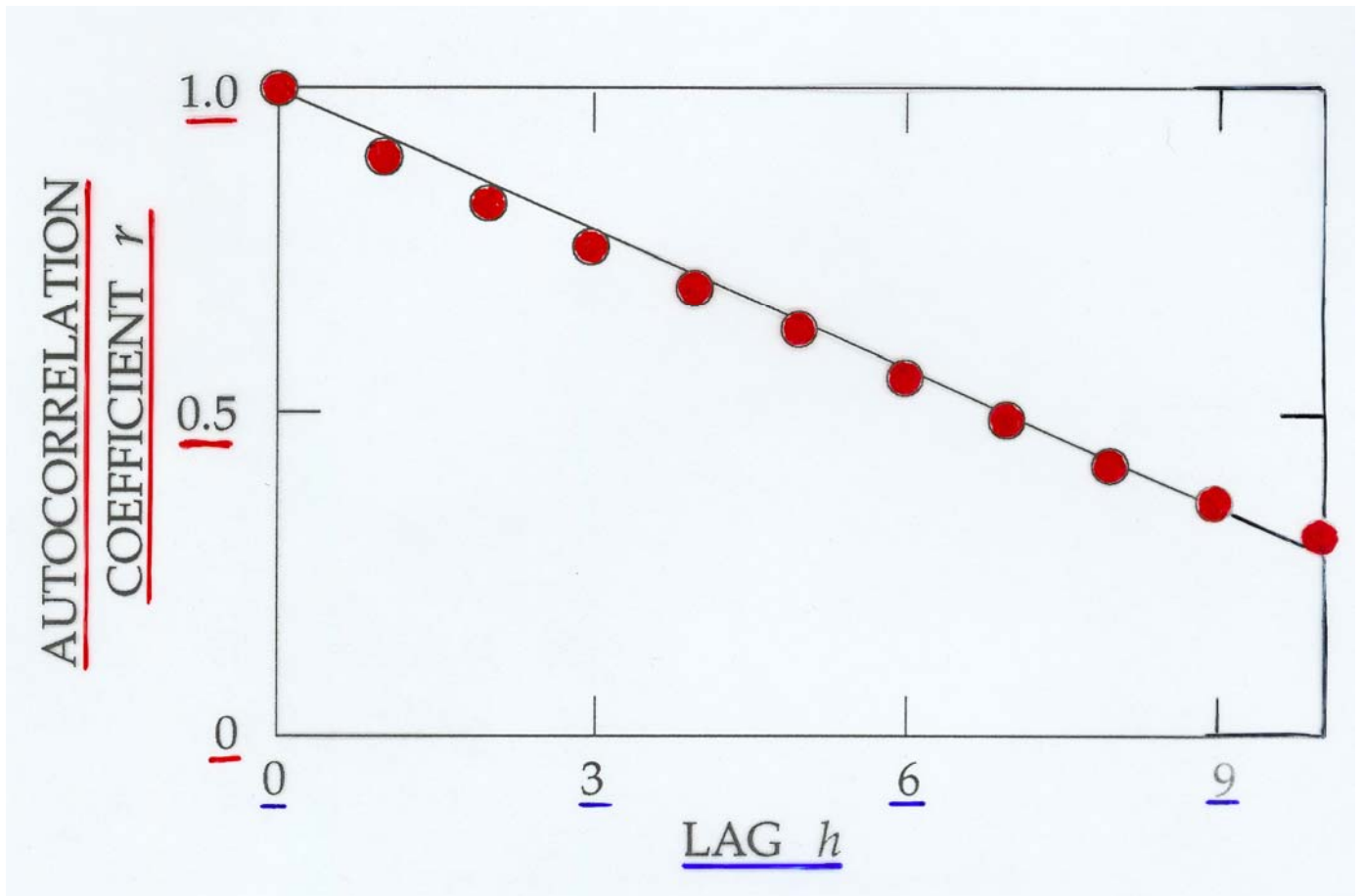


# IMPACT OF DETERMINISTIC TREND

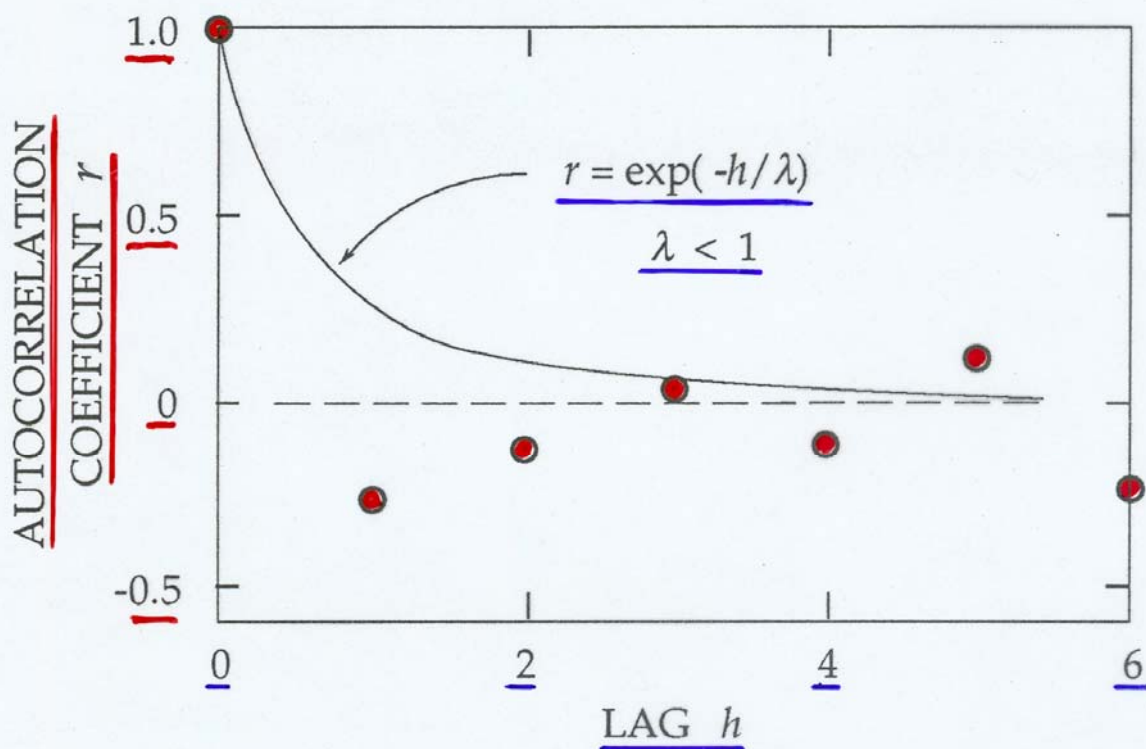
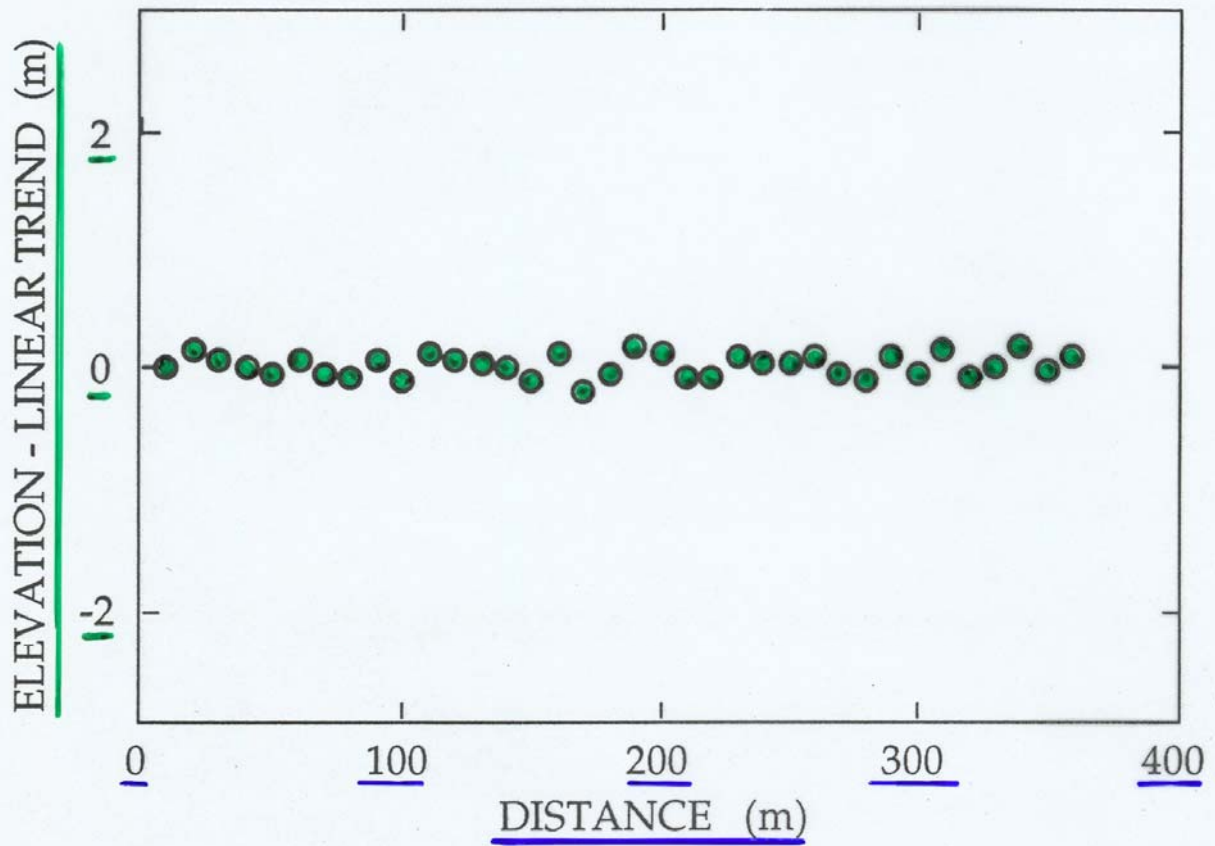
# Linear elevation vs distance



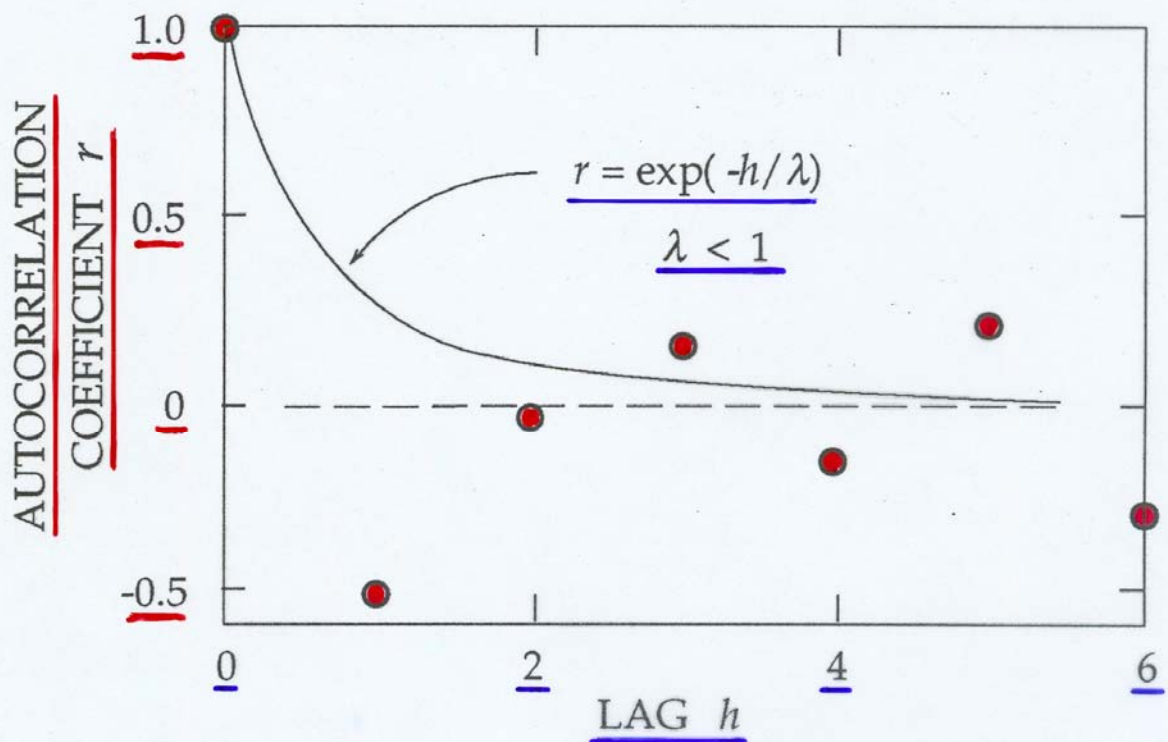
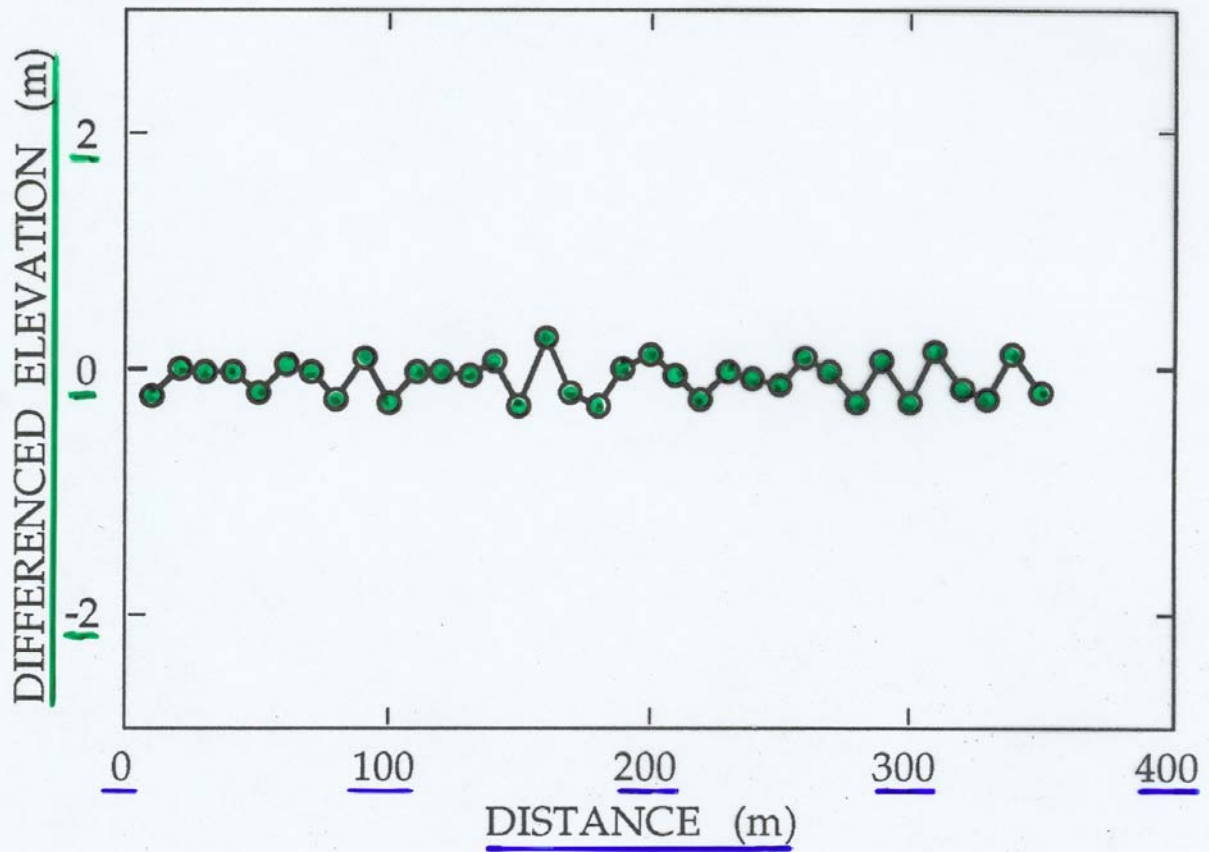
# ACF linear elevation



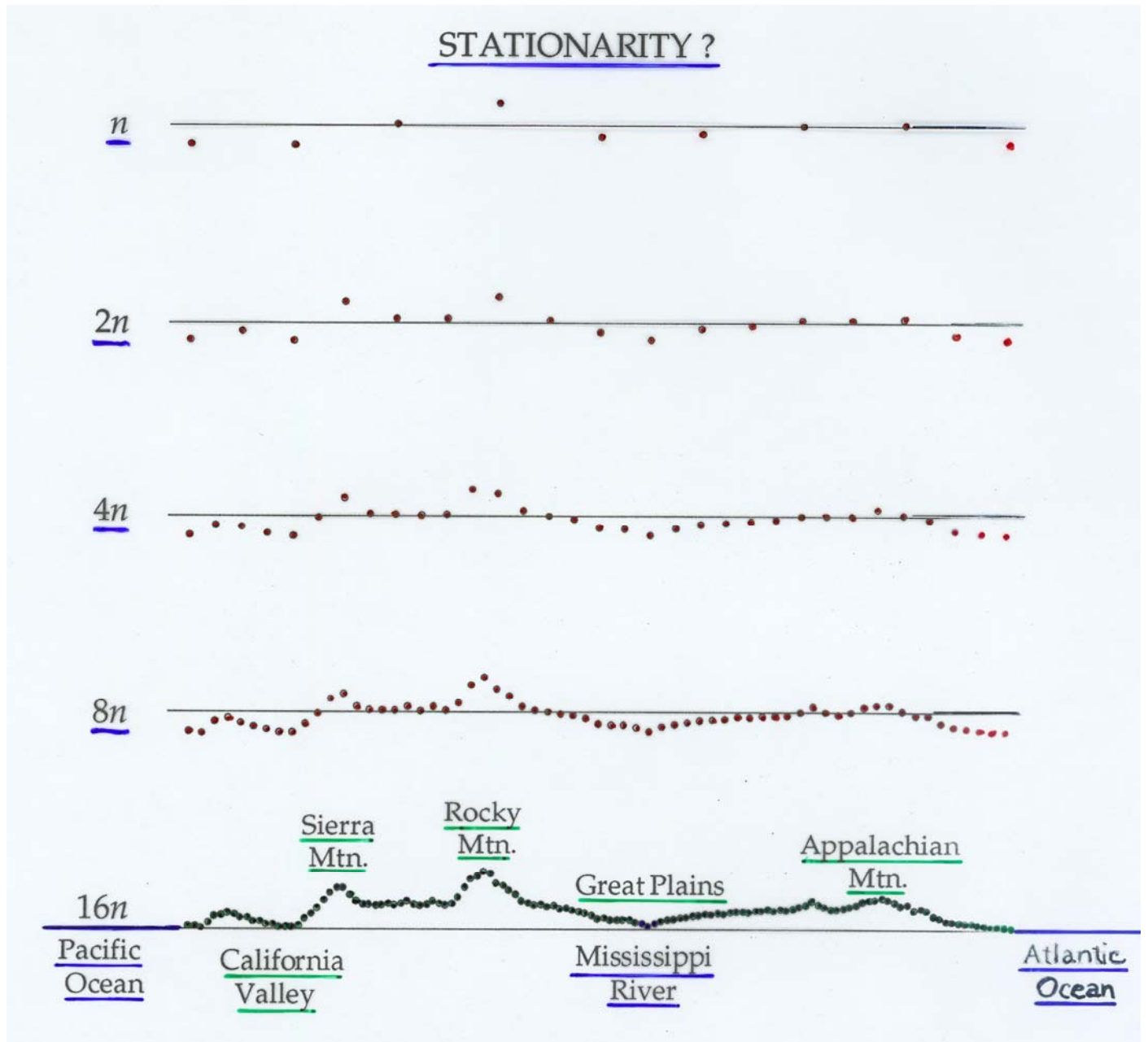
# Elevation minus linear



# Differenced elevation



# Stationarity?

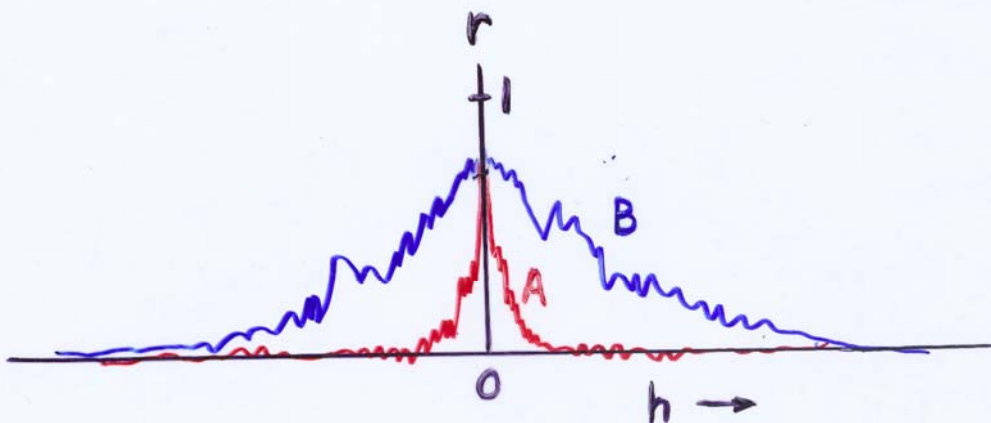
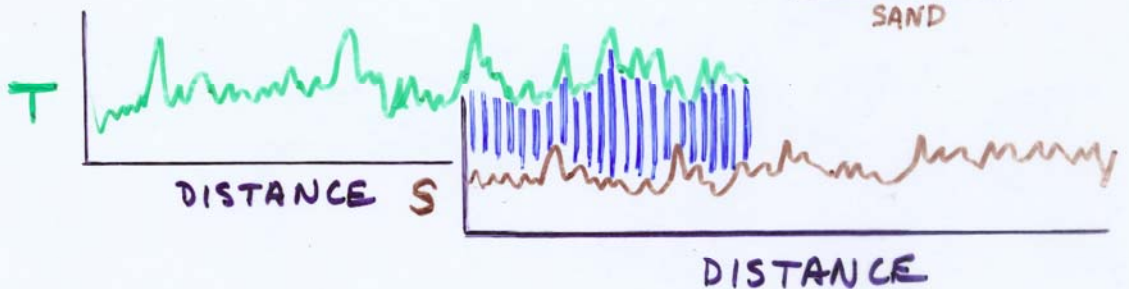
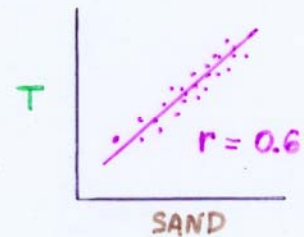
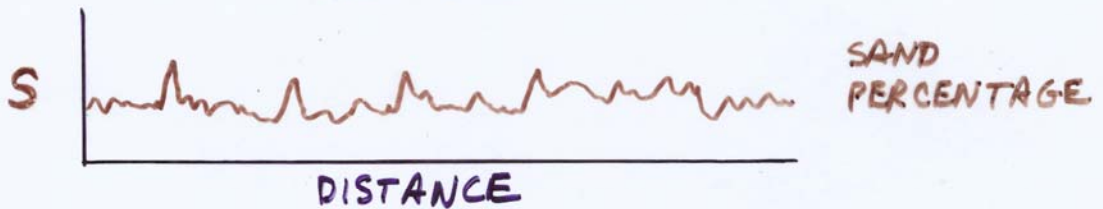
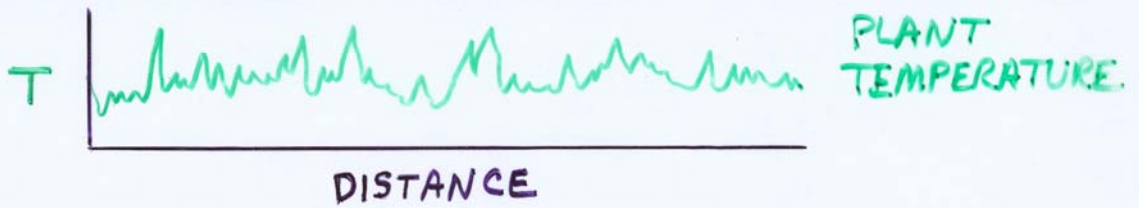


# SPATIAL OR TEMPORAL CROSS CORRELATION



# Crosscorrelation diagram

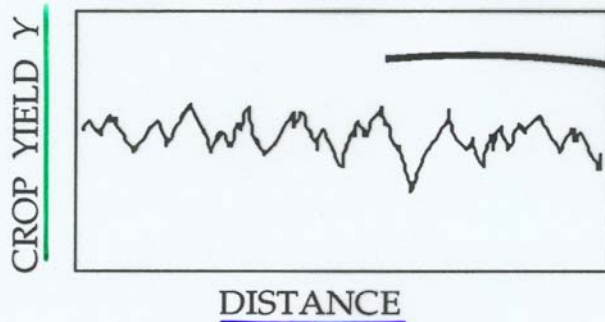
## SPATIAL OR TEMPORAL CROSS-CORRELOGRAMS



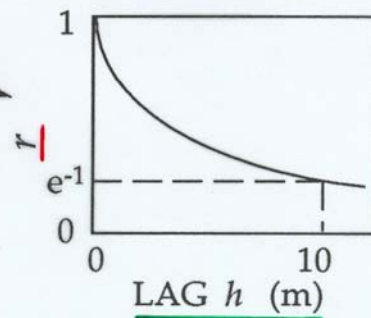
# Yield & fertility CCF

## CROP YIELD AND SOIL FERTILITY TEST

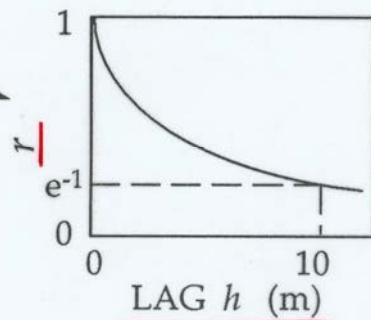
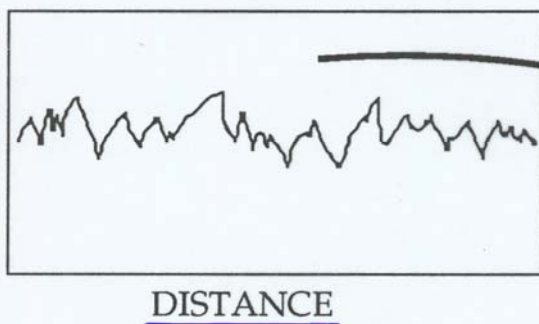
FIELD SAMPLES



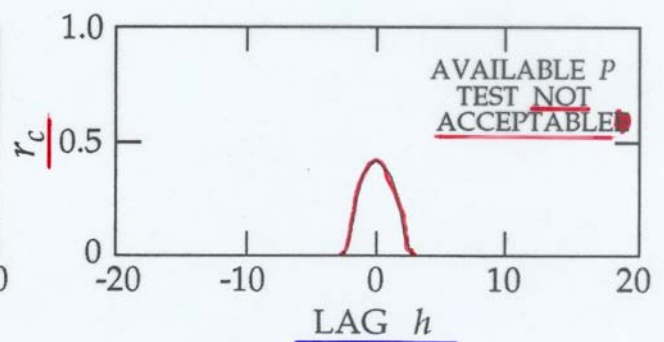
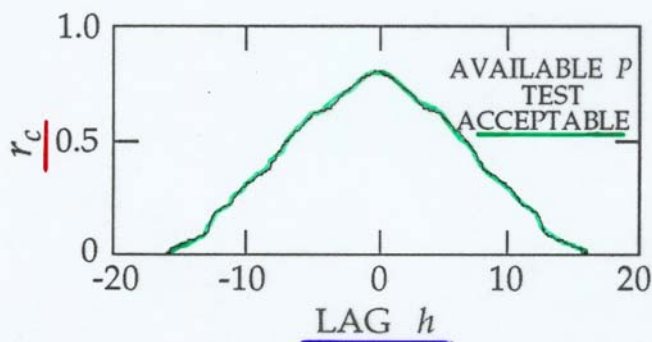
AUTOCORRELOGRAMS



AVAILABLE SOIL  
PHOSPHORUS P

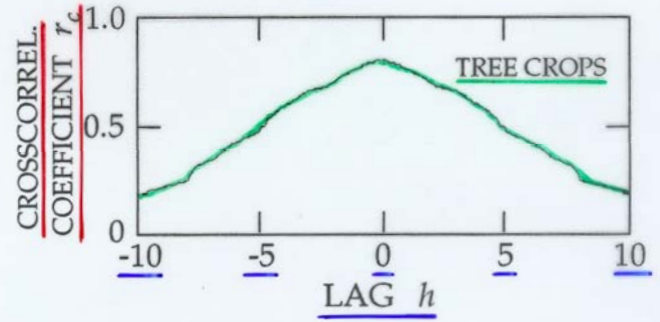
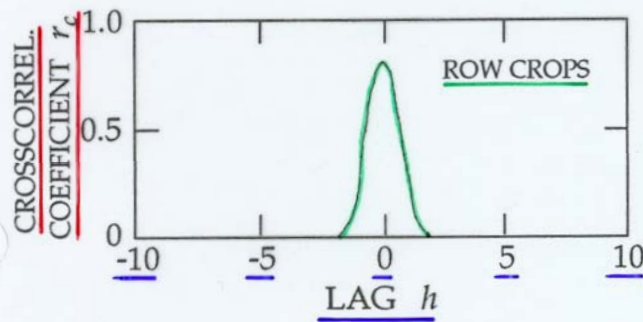
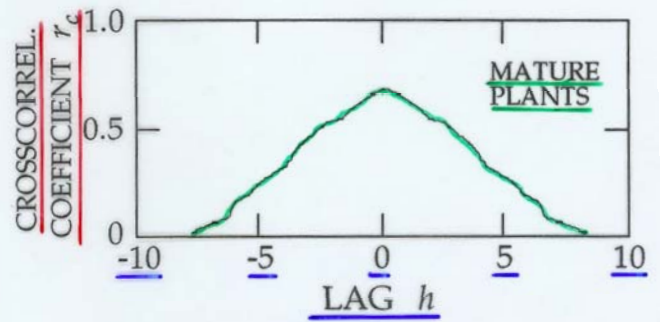
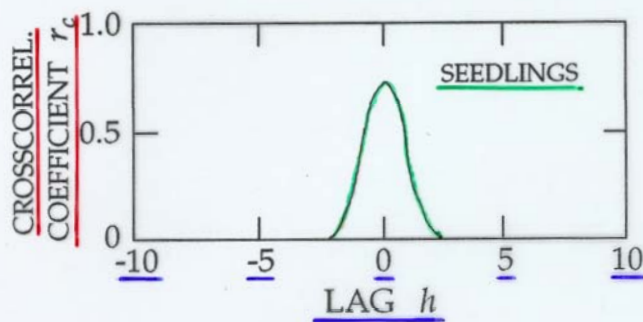


CROSSCORELOGRAMS



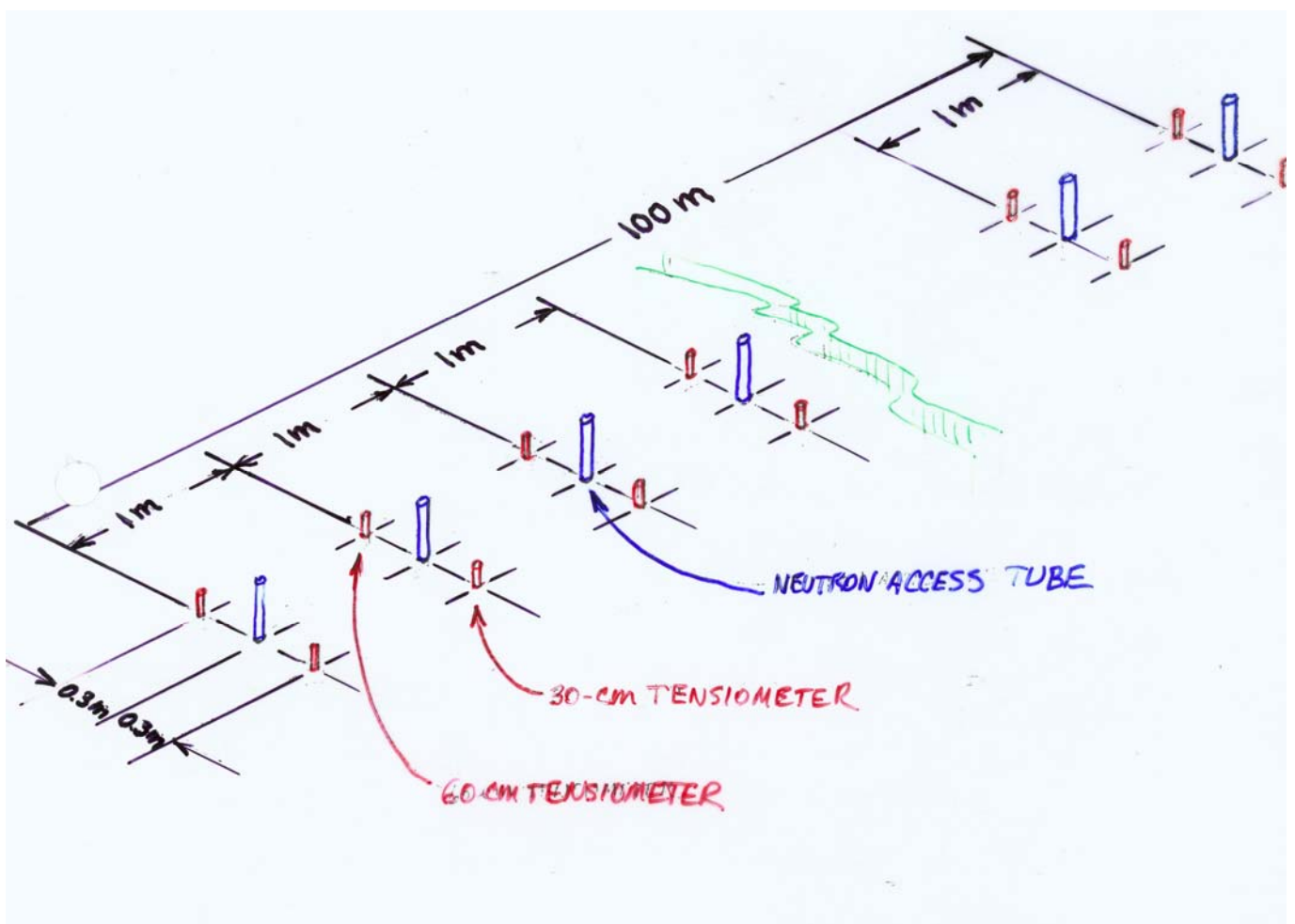
# Plant uptake & nutrients

PLANT UPTAKE VERSUS NUTRIENTS IN SOIL

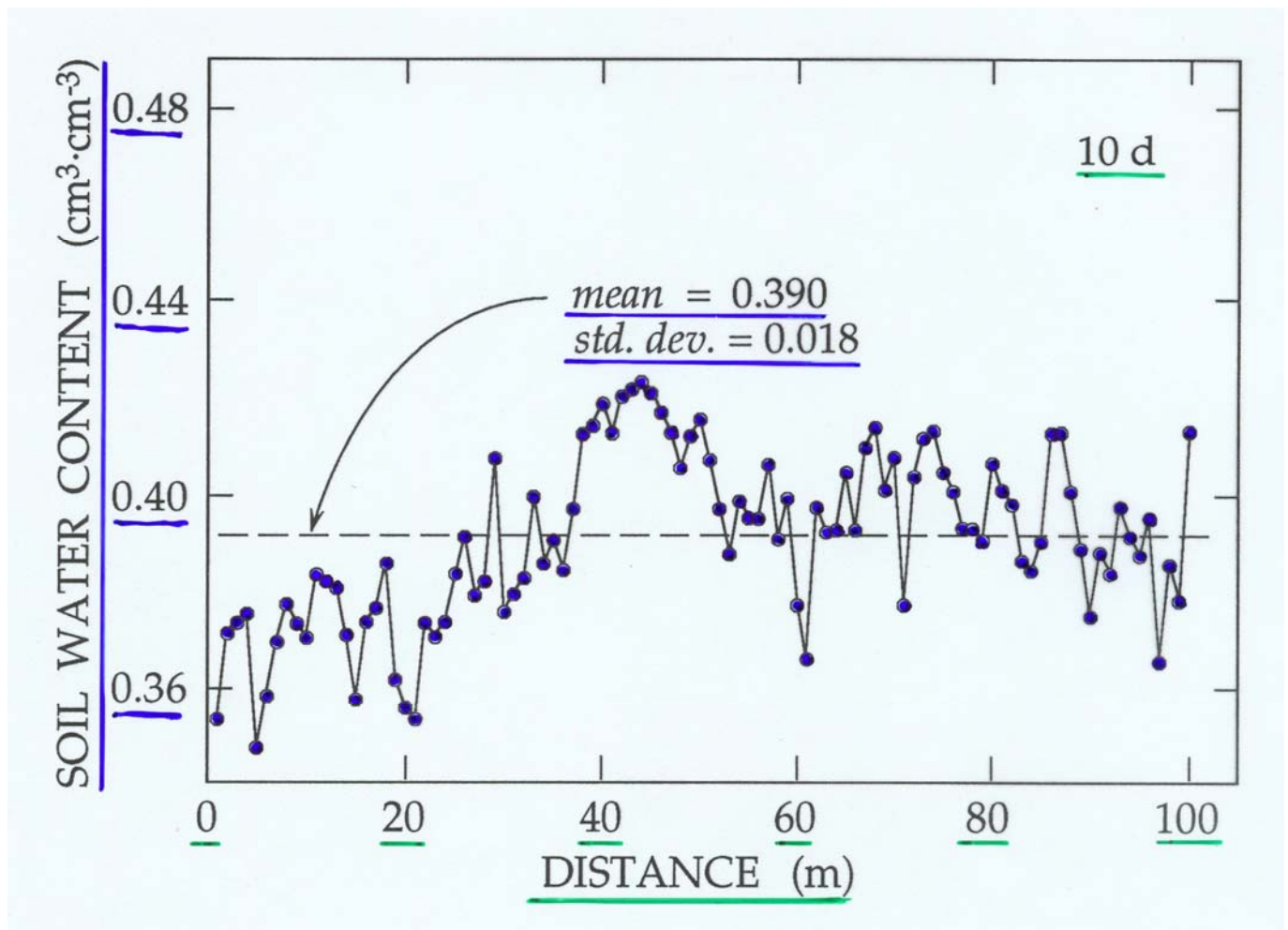


TWO SETS OF  
PHYSICALLY RELATED  
MEASUREMENTS

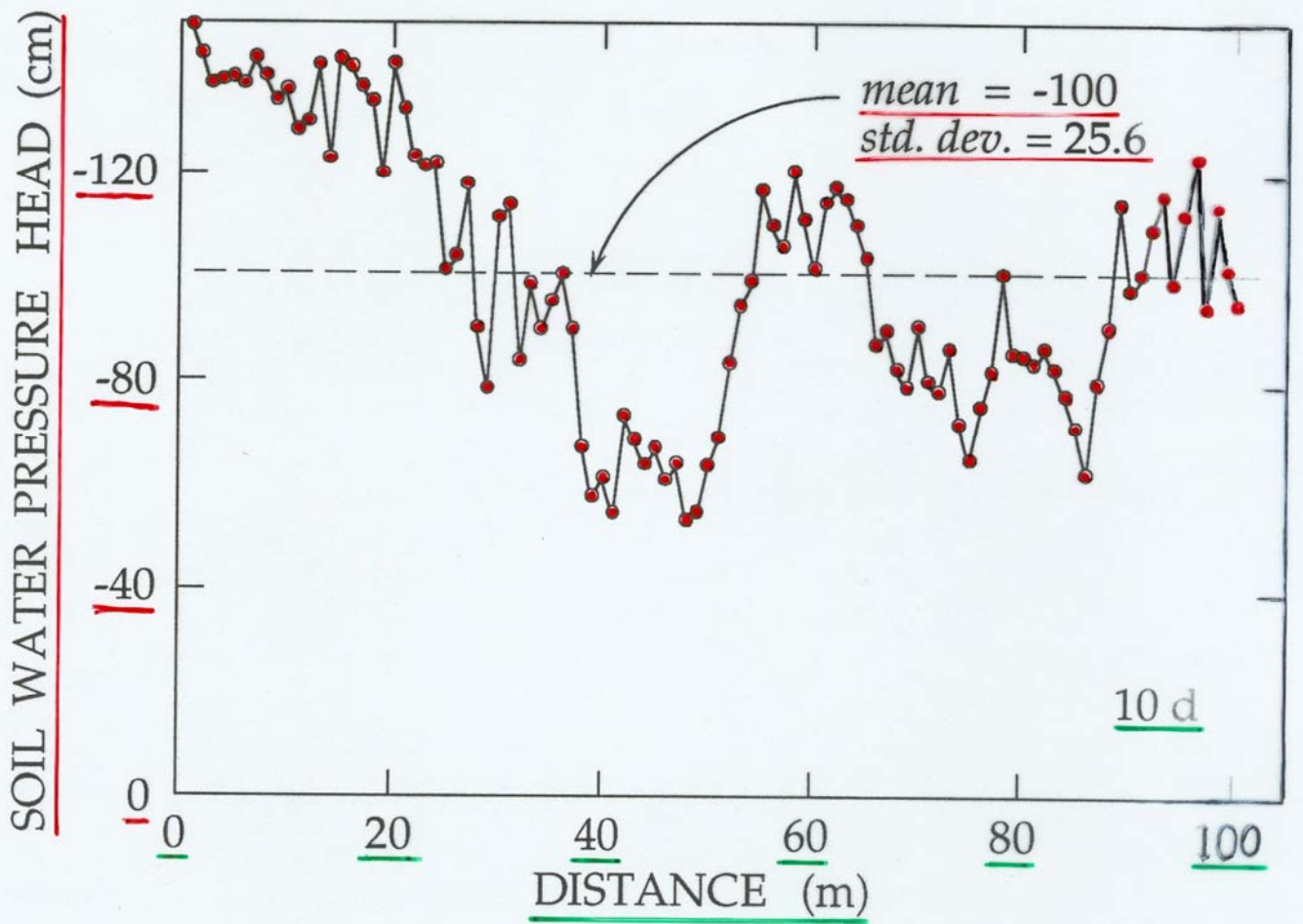
# Tension-neutron placement



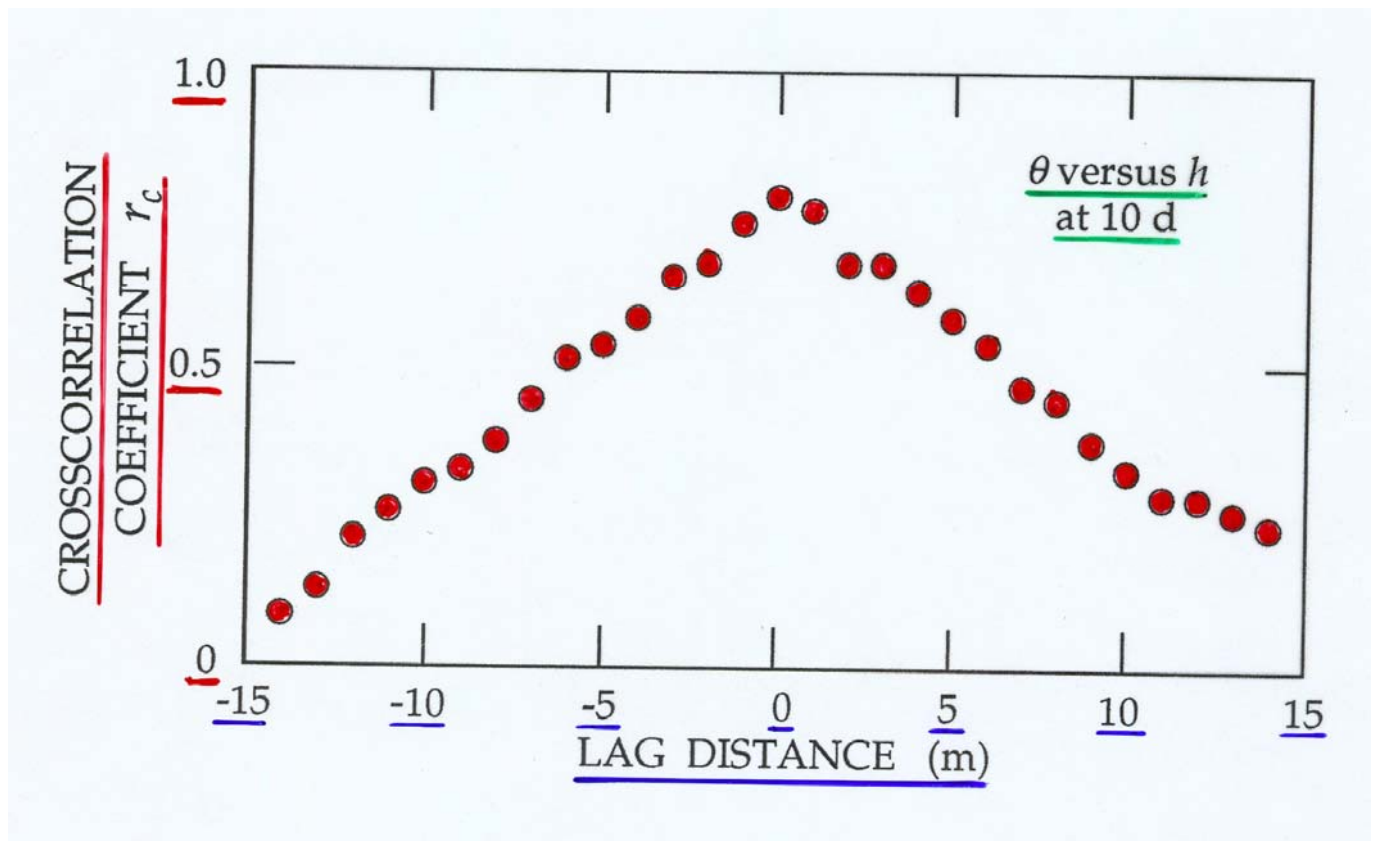
# Theta vs distance



# h vs distance

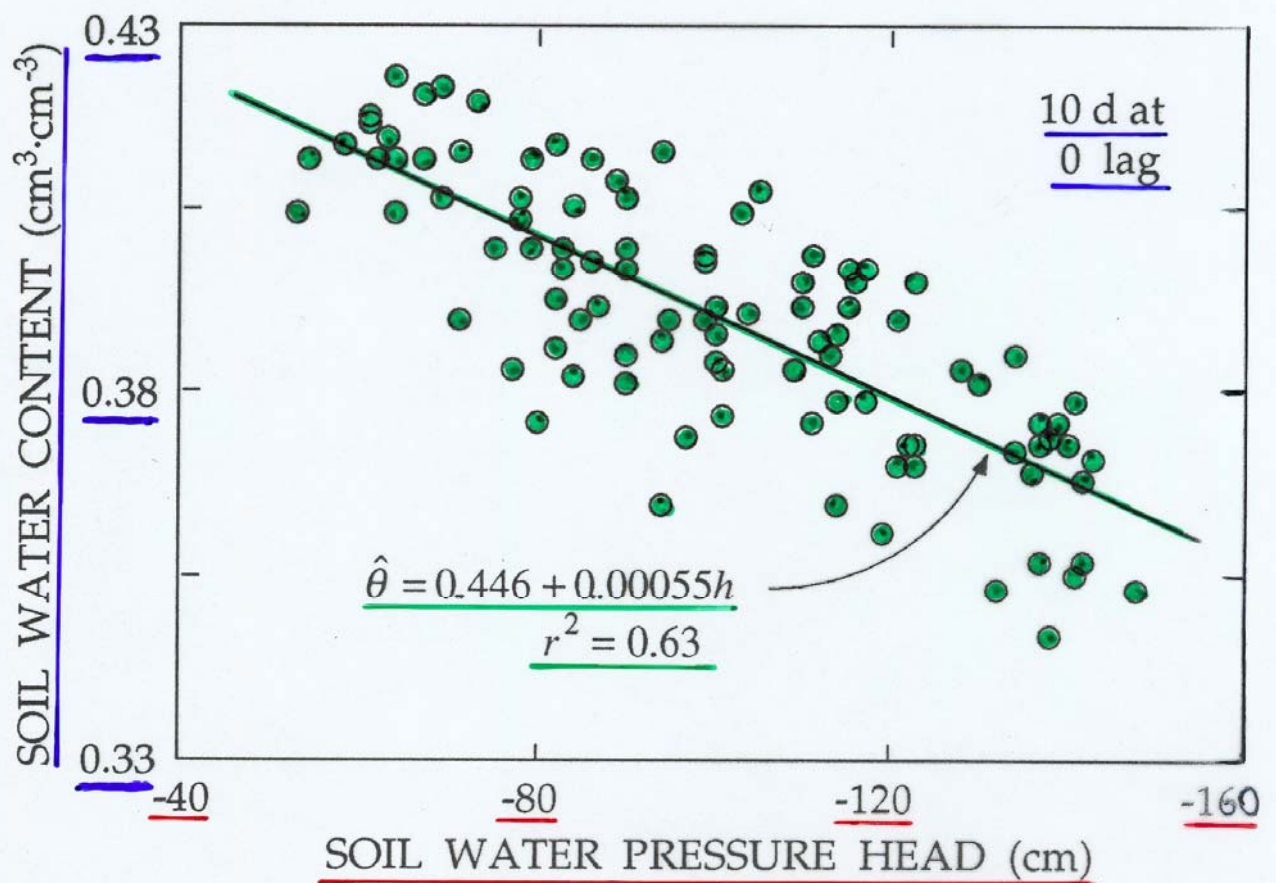


# CCF theta-h at 10 days

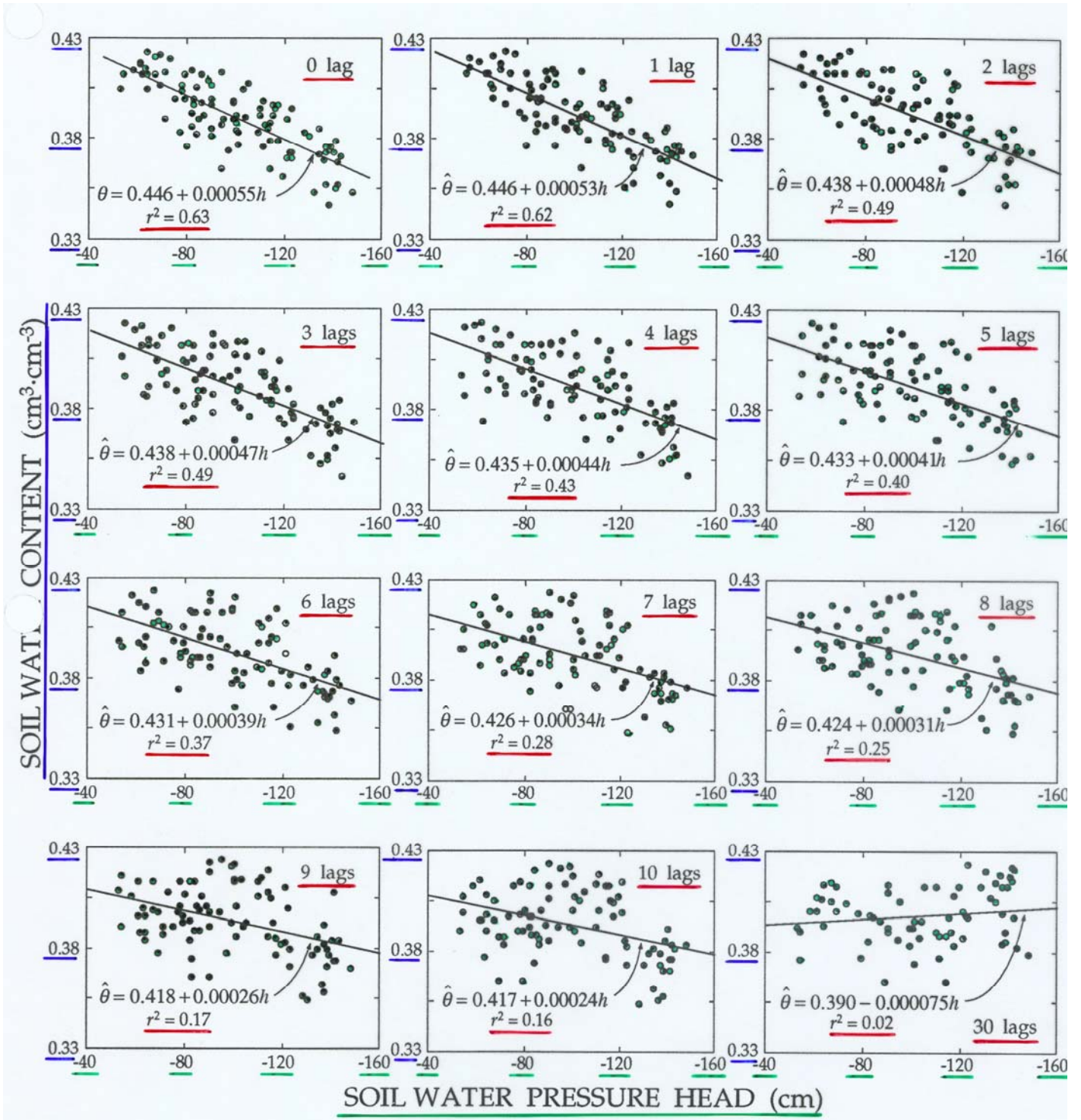




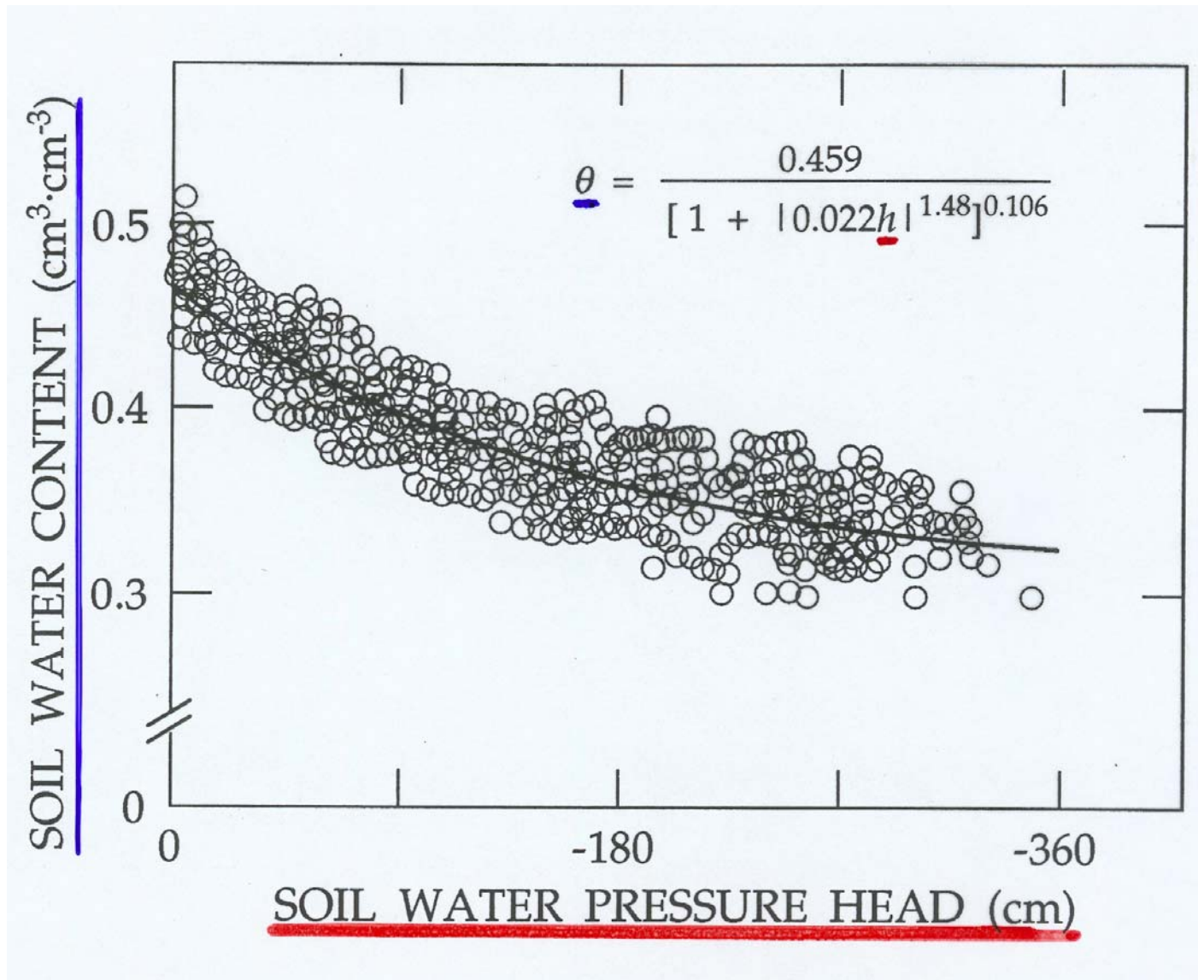
# Theta vs h at 10 days 0 lag



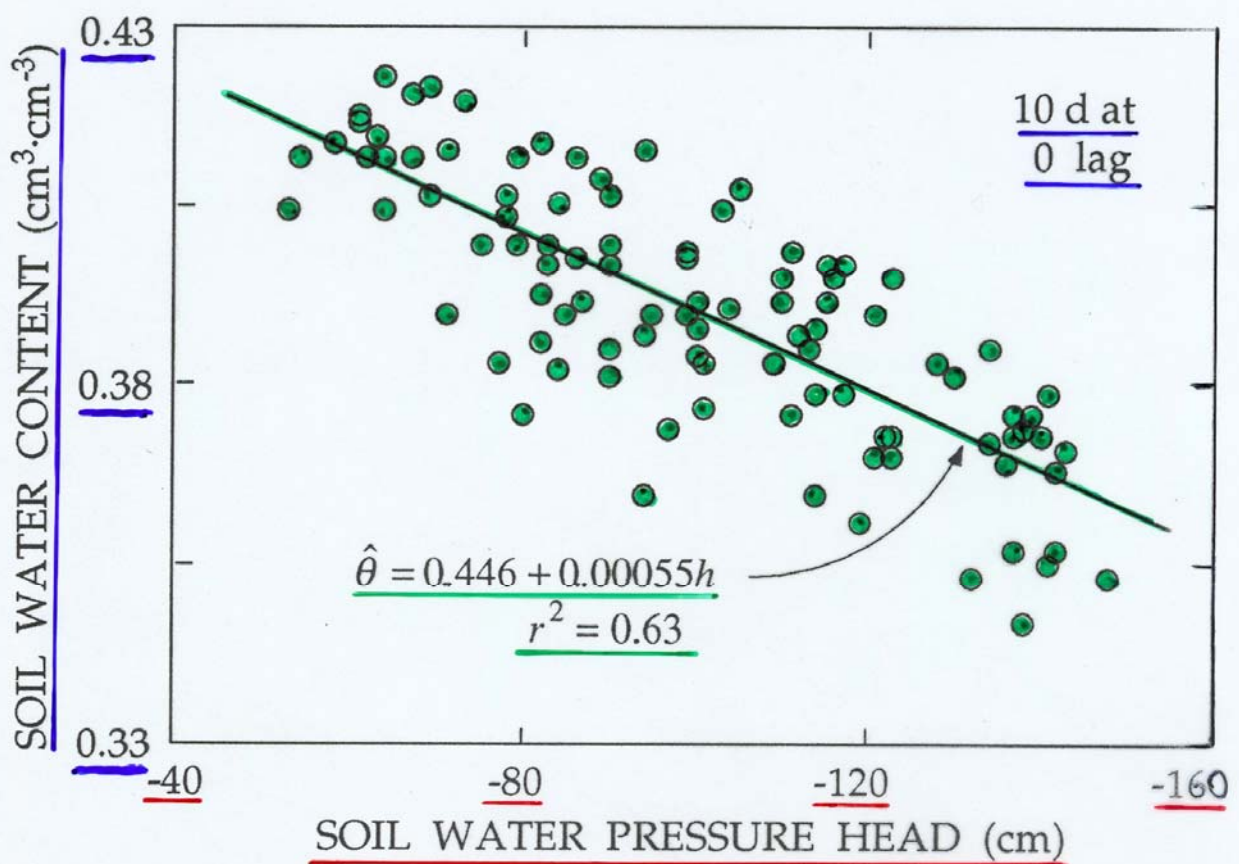
# $\theta$ vs $h$ 0-30 lags



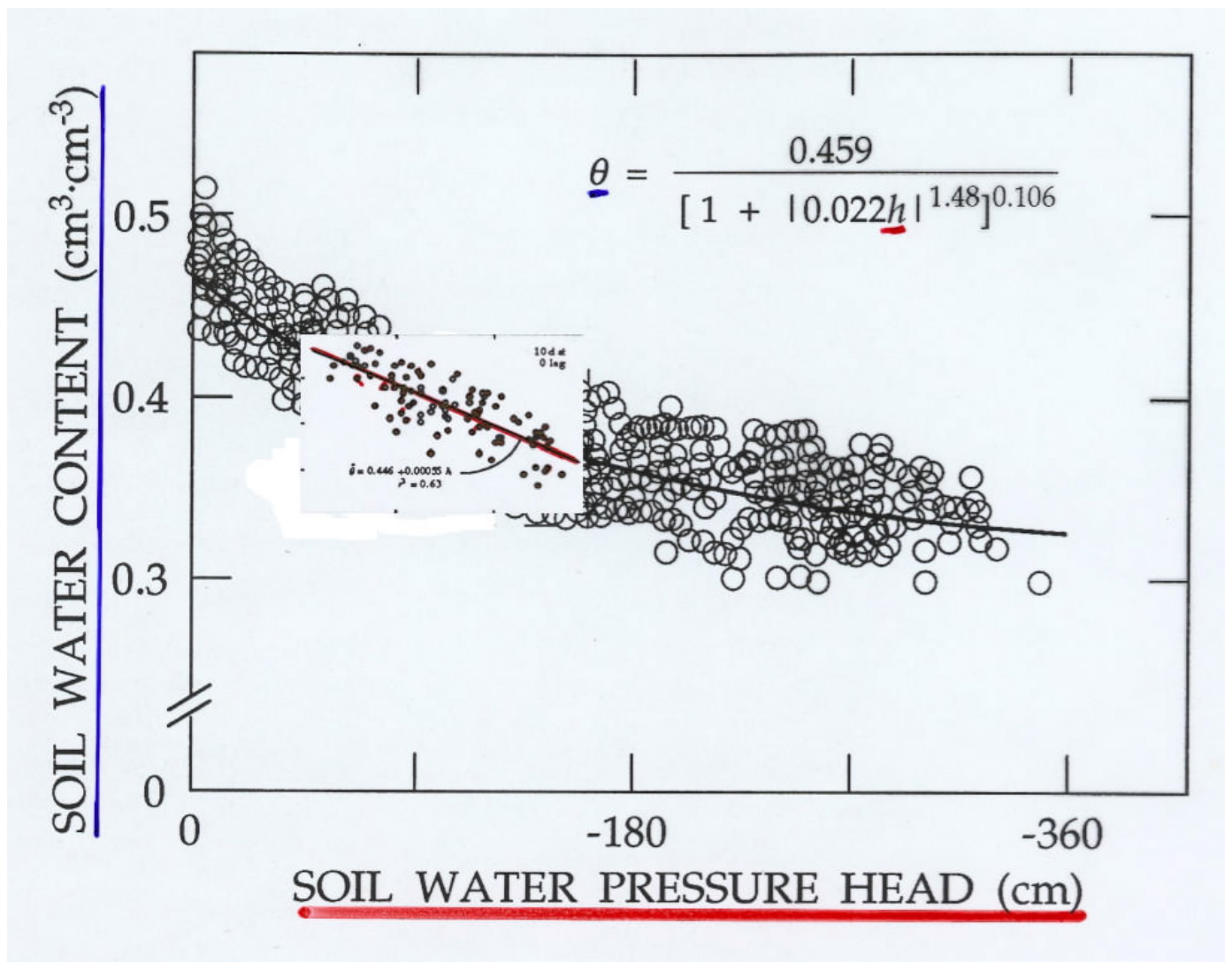
# $\theta$ vs $h$ all data



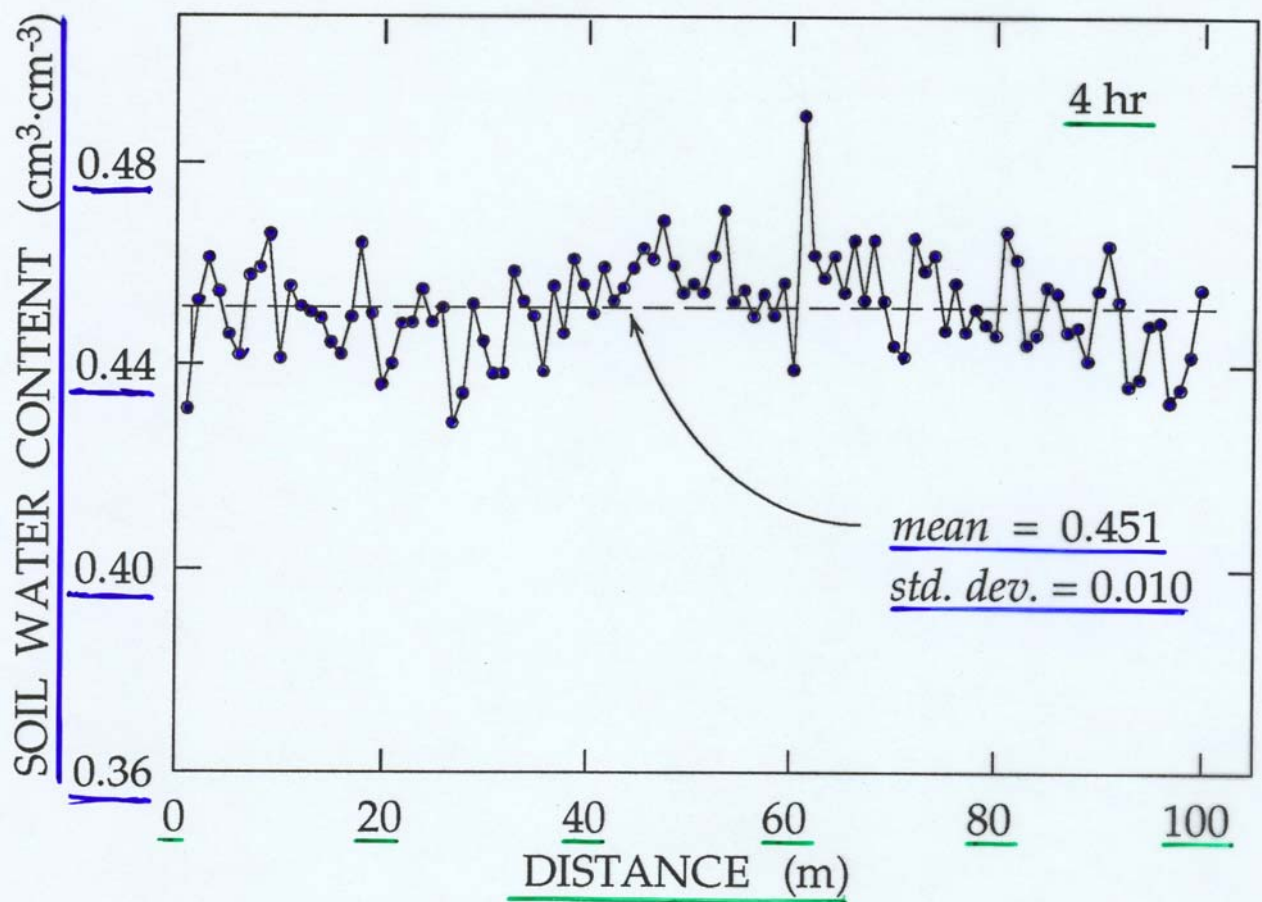
# $\theta$ vs $h$ 10 d 0 lag



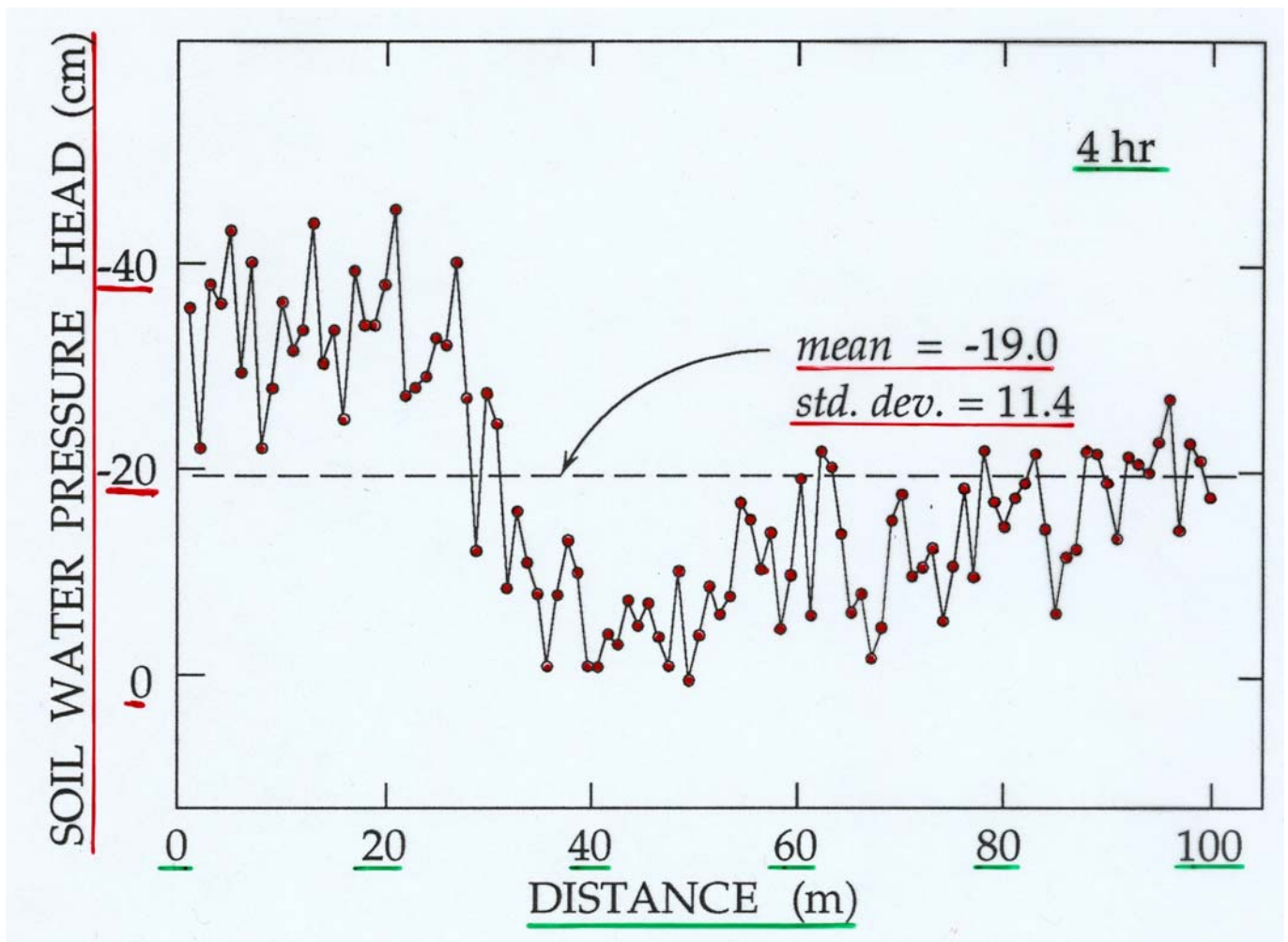
# q vs h all data + 0 lag



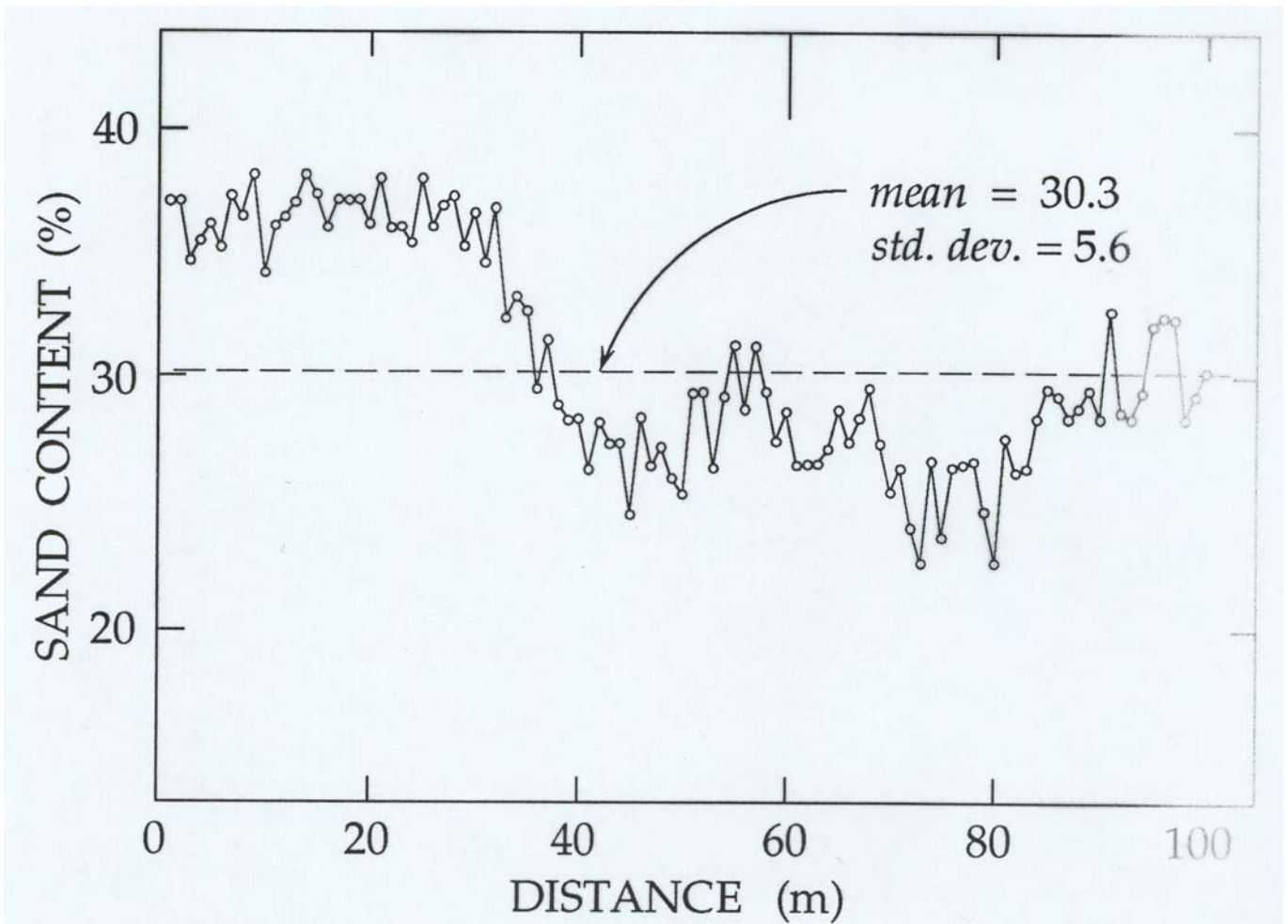
# q vs distance 4 hr



# h vs distance 4 hr

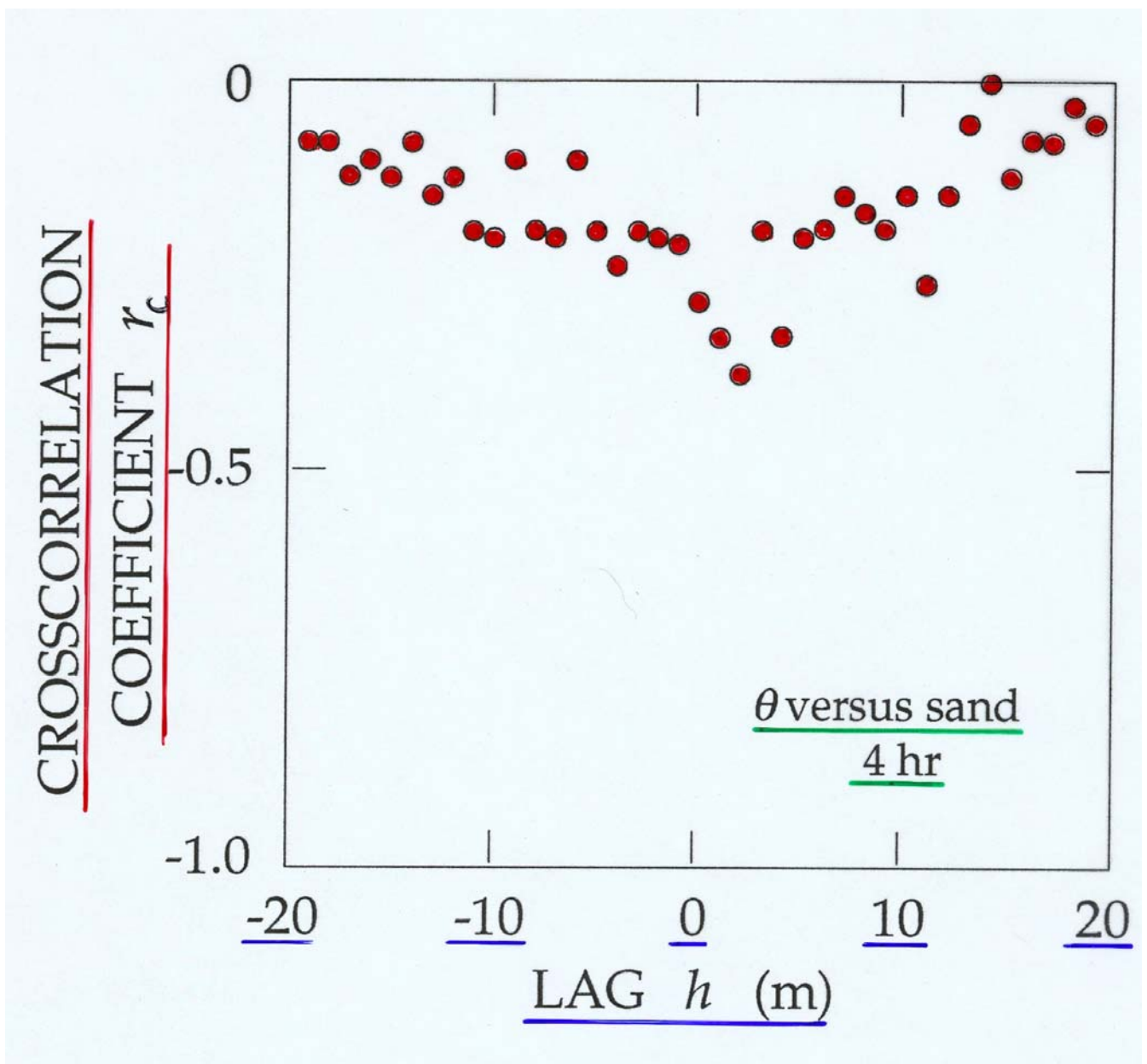


# sand vs distance

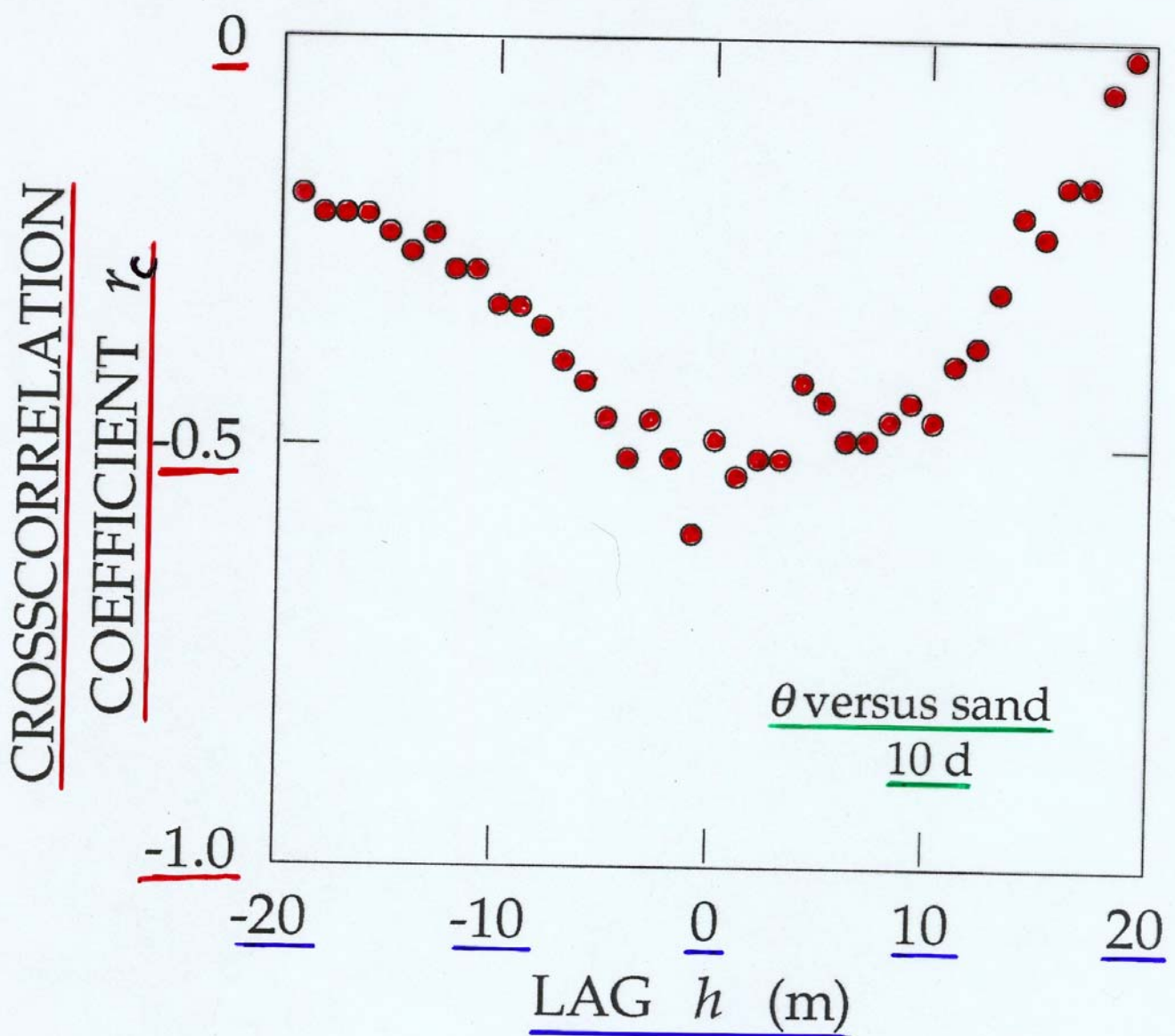




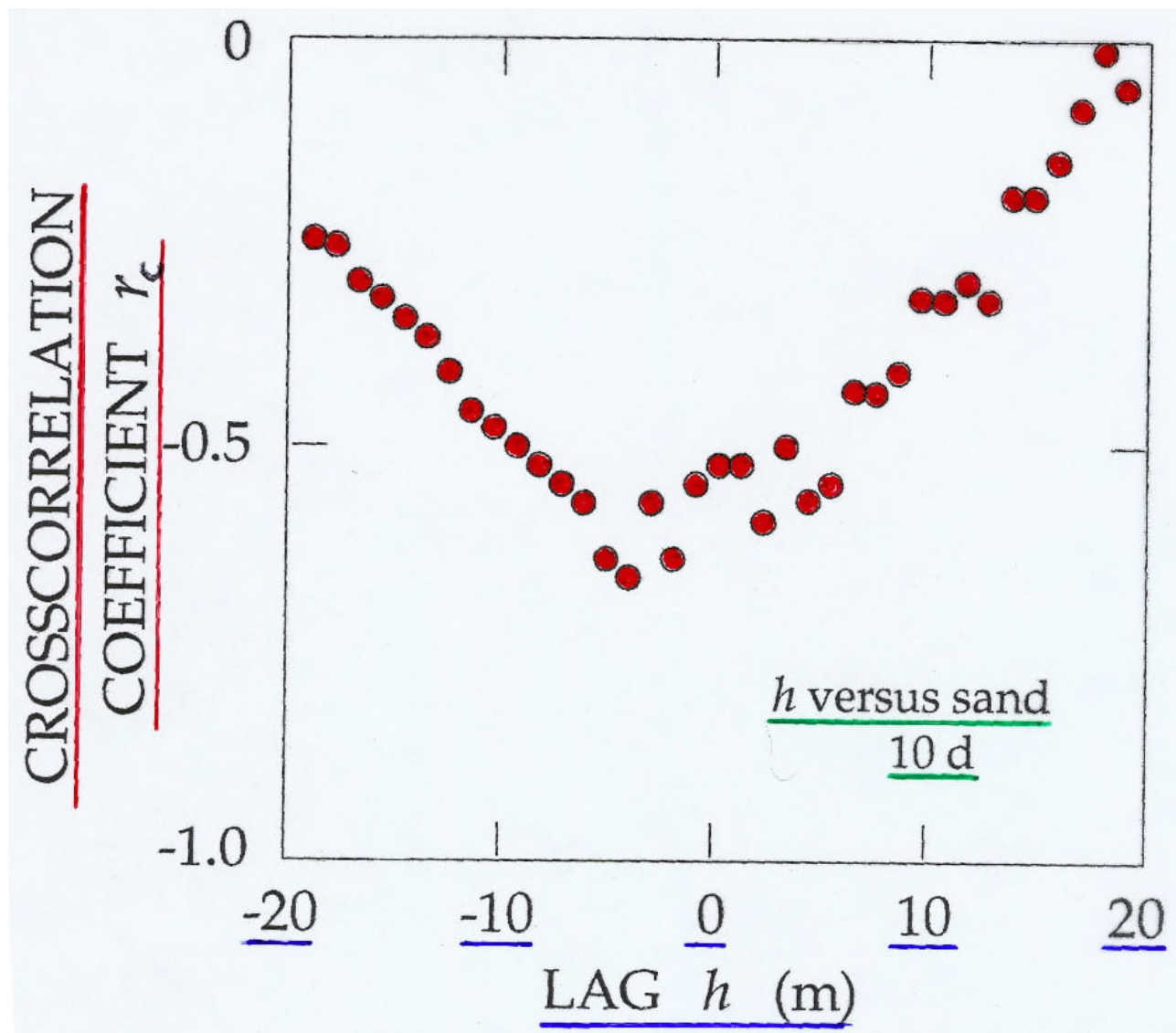
# CCF theta vs sand 4 hr



# CCF theta vs sand 10 days



# CCF h vs sand 10 days



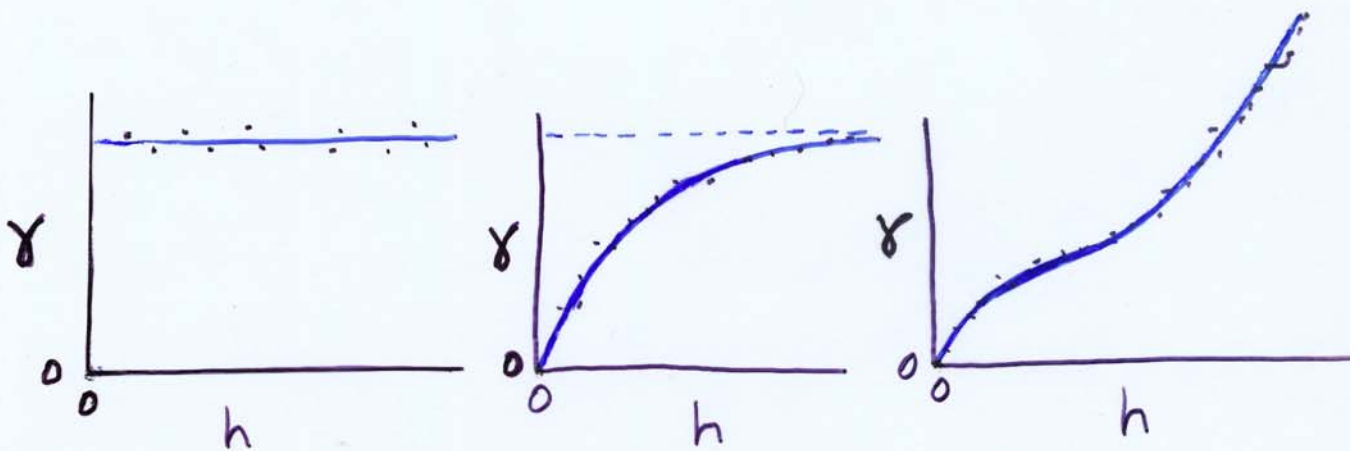
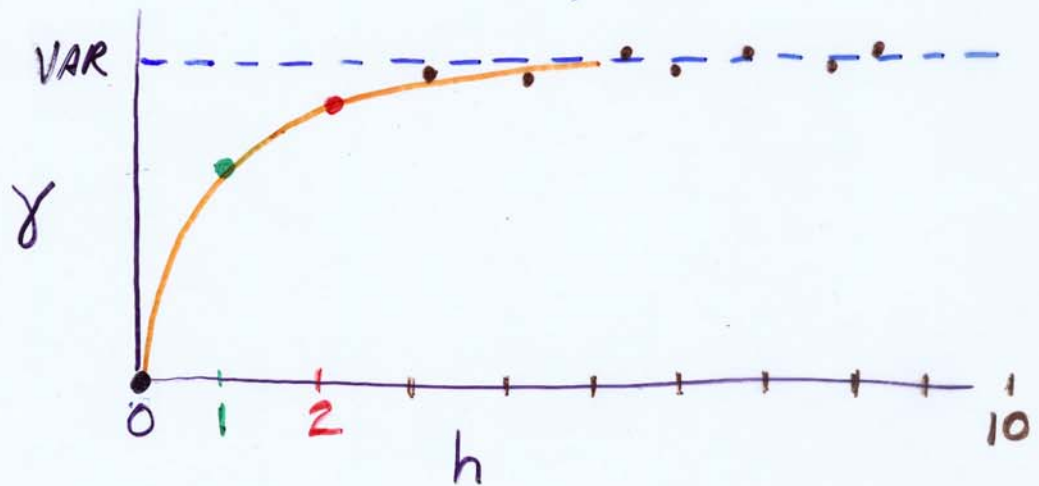
# SEMI-VARIOGRAMS

# Semivariogram calc

## SEMI-VARIOGRAMS

$T_1 \ T_2 \ T_3 \ T_4 \ \dots \ T_i \ T_{i+1}$

$$\gamma(h) = \frac{1}{2n} \sum_{i=1}^{n+1} (T_{i+h} - T_i)^2$$



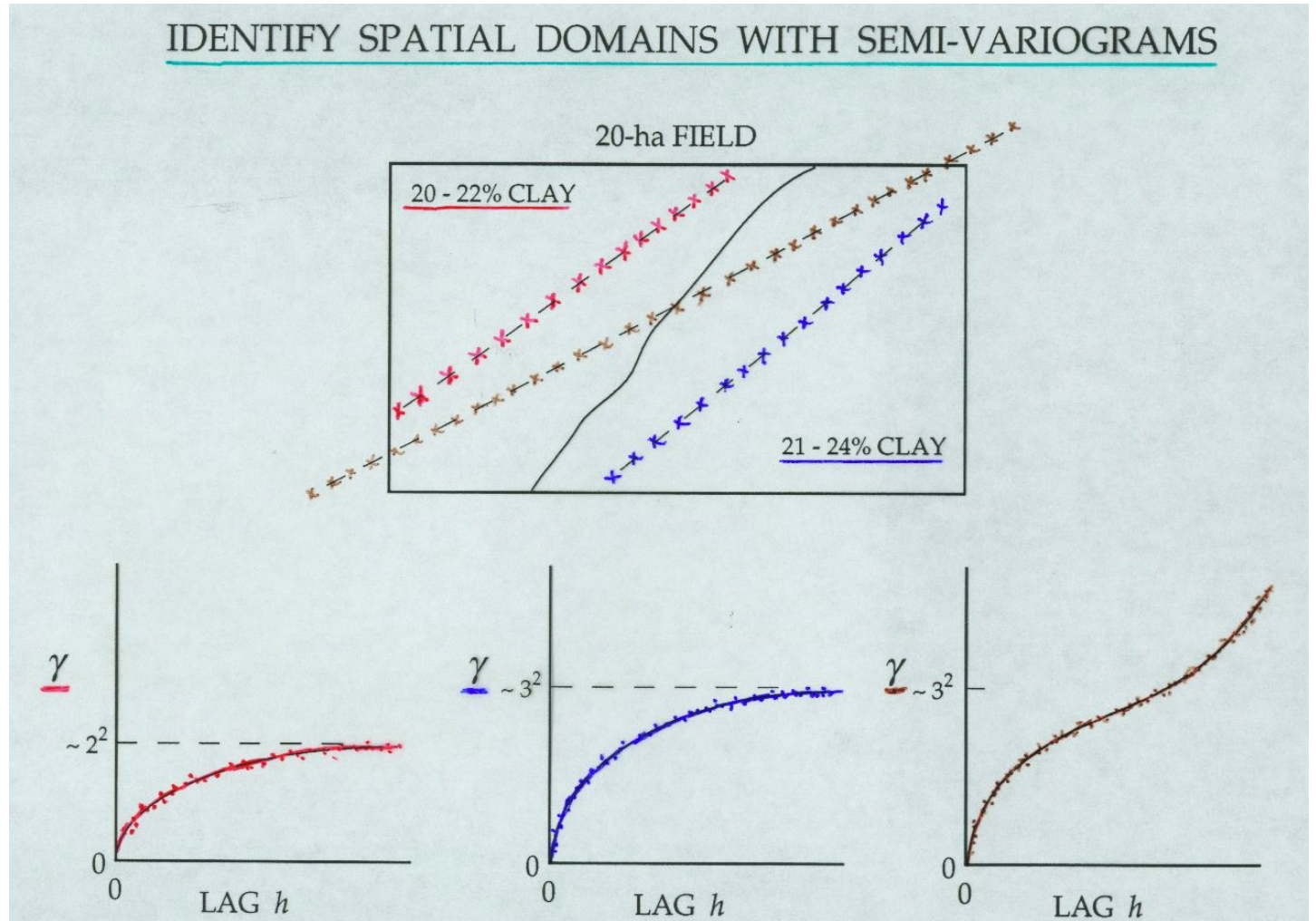
SPATIAL INDEPENDENT  
IN A DOMAIN

SPATIAL DEPENDENT  
IN A DOMAIN

CHANGING-DOMAIN

# Identifying domains

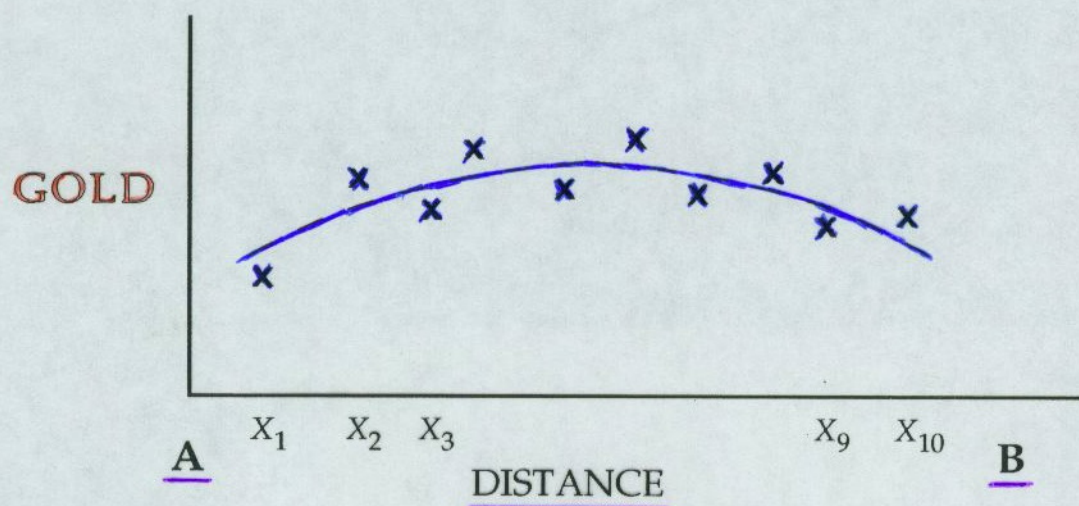
## IDENTIFY SPATIAL DOMAINS WITH SEMI-VARIOGRAMS



# KRIGING

# Interpolating, Kriging

## INTERPOLATE WITH SEMI-VARIOGRAMS - KRIGING

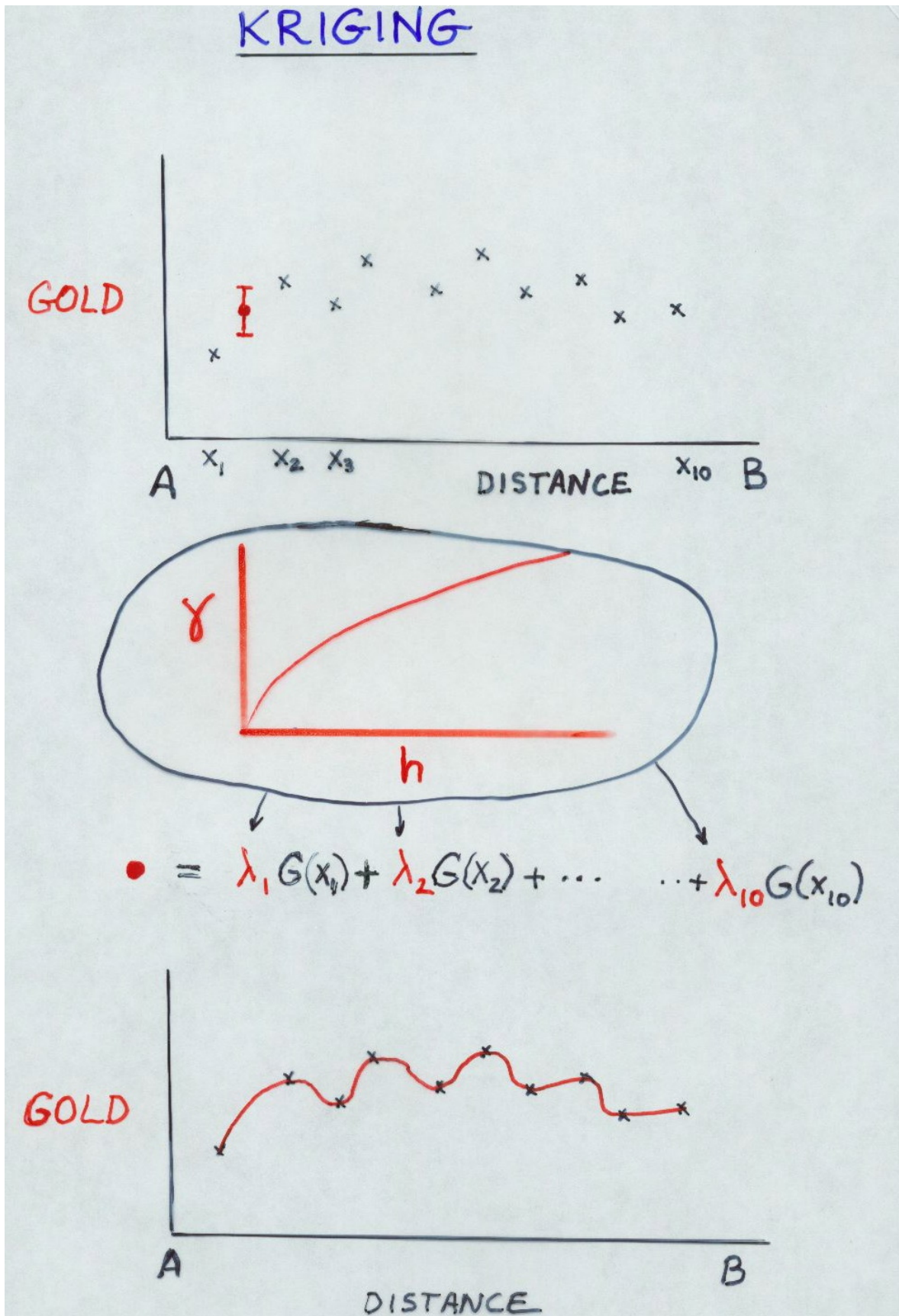


### CLASSICAL REGRESSION

<b>GOLD</b>	<u><math>= a_0</math></u>	<u><math>r = 0.5</math></u>
	<u><math>= a_0 + a_1x</math></u>	<u><math>r = 0.7</math></u>
	<u><math>= a_0 + a_1x + a_2x^2</math></u>	<u><math>r = 0.9</math></u>
	<u><math>= a_0 + a_1x + a_2x^2 + \dots + a_9x^9</math></u>	<u><math>r = 1</math></u>



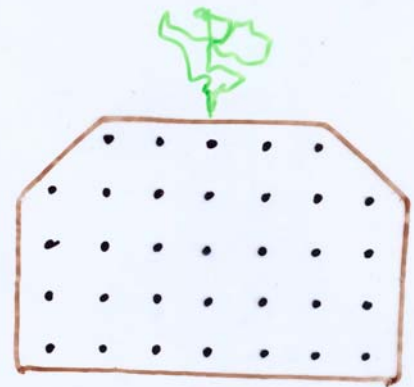
# Kriging gold



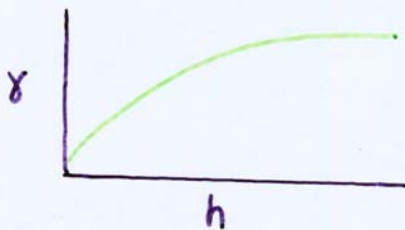
# Where to sample in and between rows

WHERE TO SAMPLE?

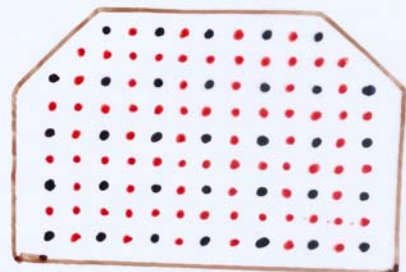
IN THE ROW, BETWEEN ROWS,  
HOW DEEP, HOW OFTEN ??



VARIOGRAM



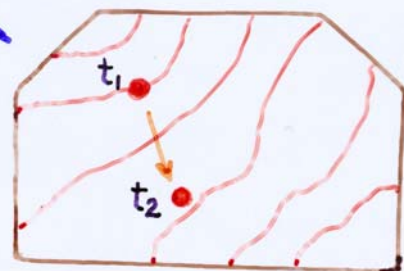
KRIGE



CONTOUR  
AT TIME 1

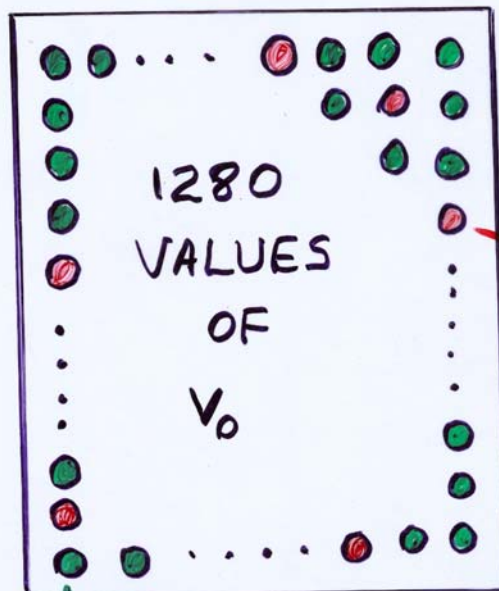


COMPARE  
AT TIME 2

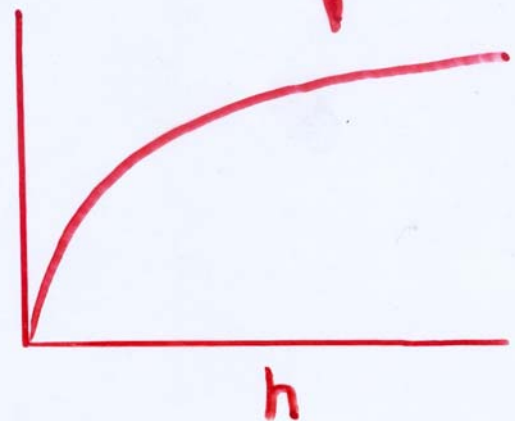


# 1280 infiltration measurements

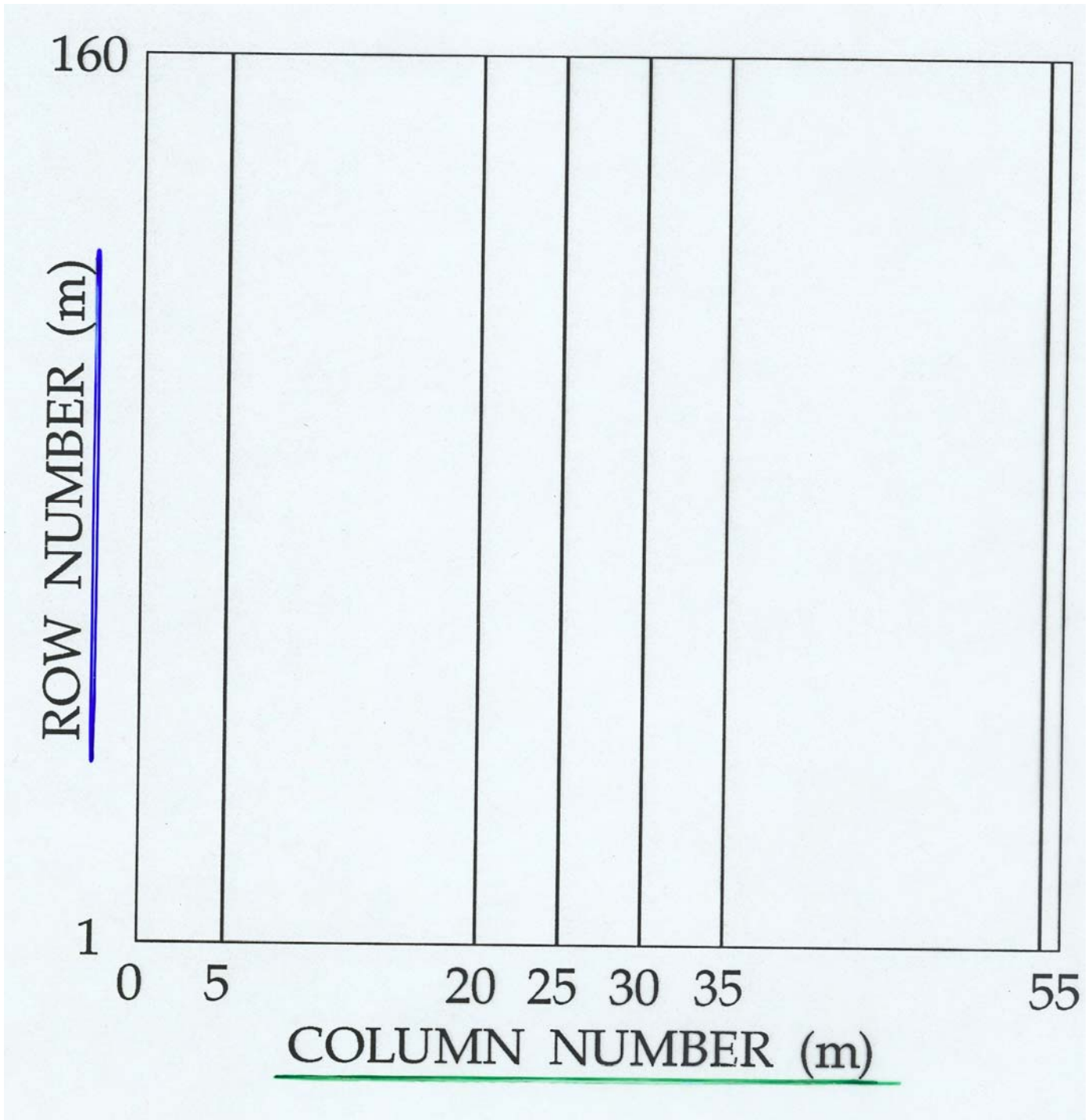
KRIGING INFILTRATION MEASUREMENTS IN  
A 1-HA. FIELD



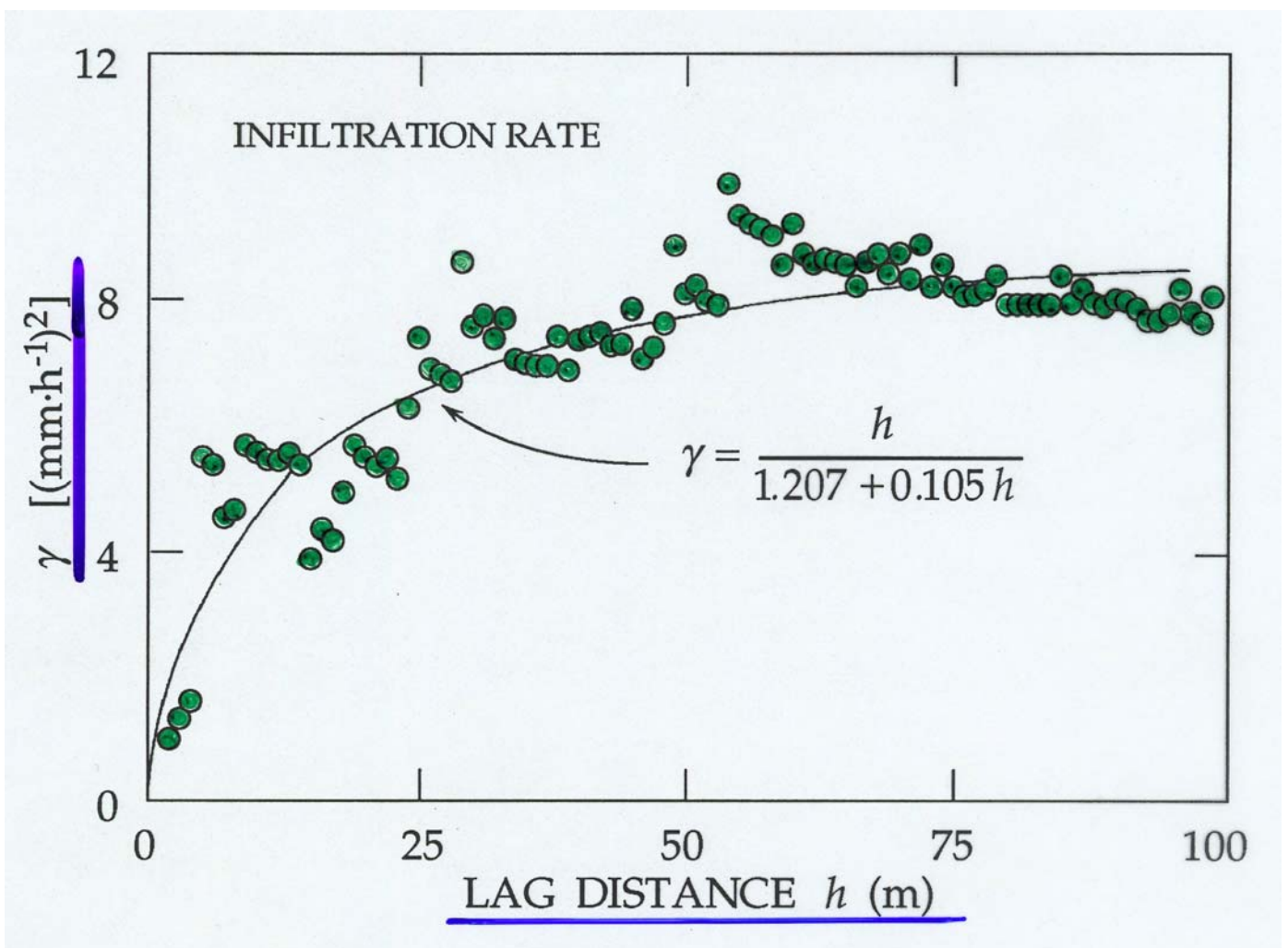
$\gamma$



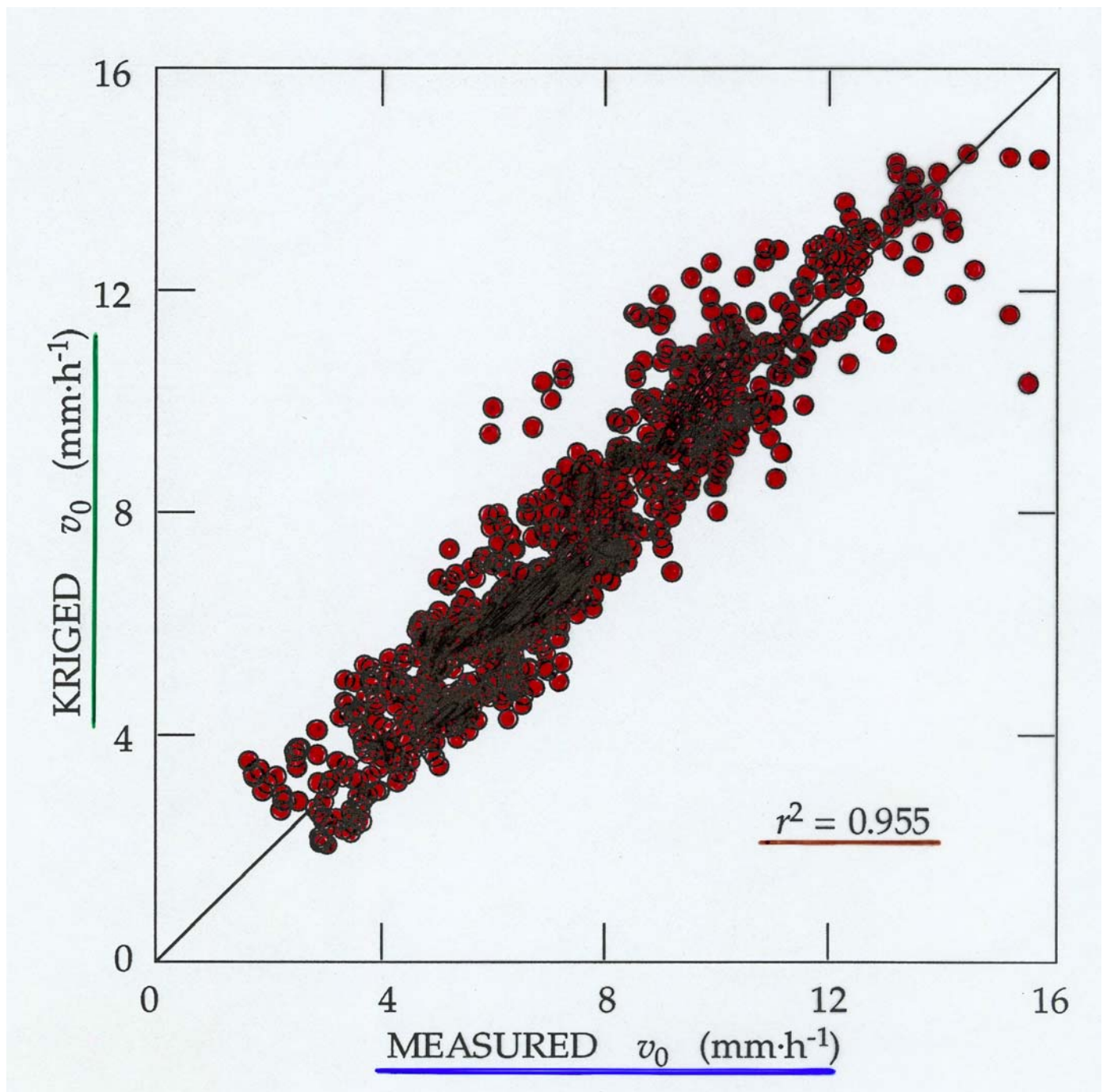
# Row & column number



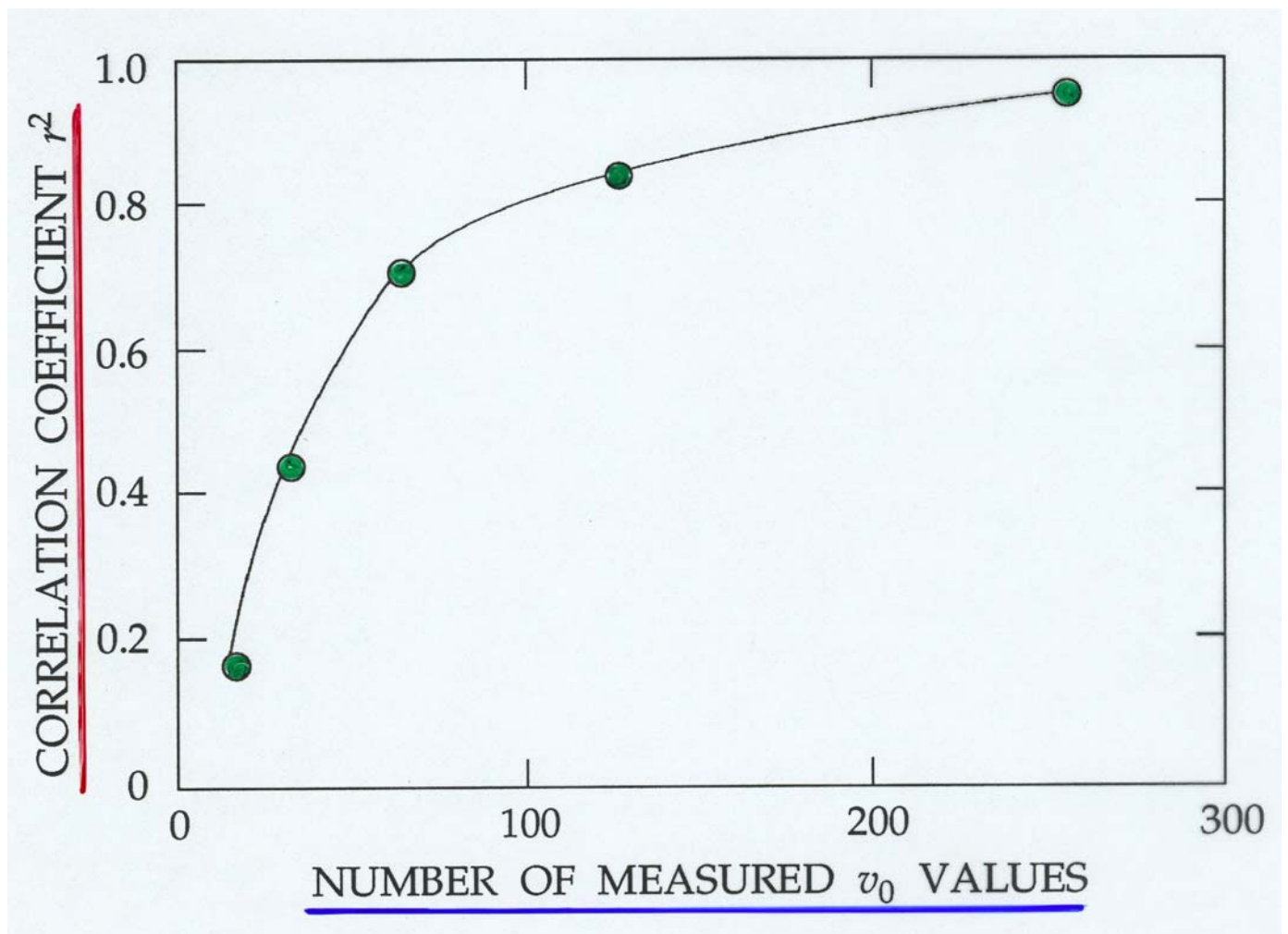
# Infilt. rate semivariogram



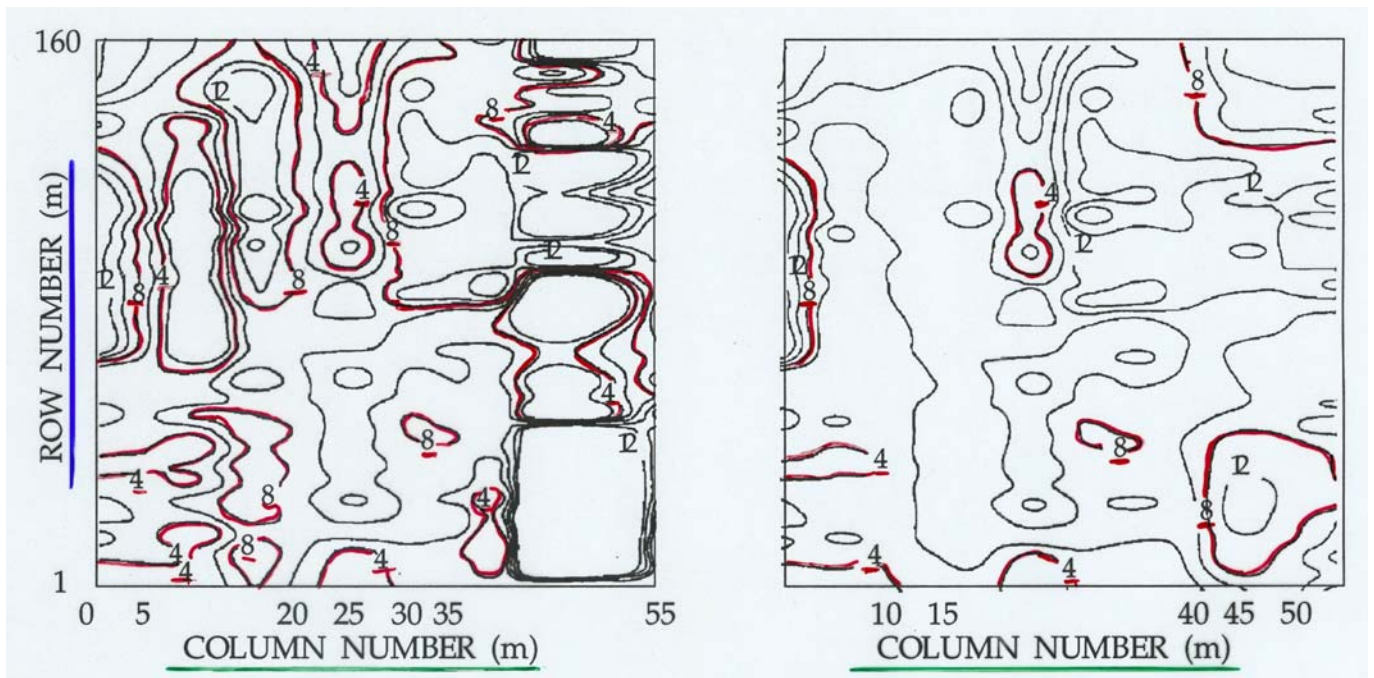
# Kriging vs measured $v_0$



# Correl coeff per no. of measurements

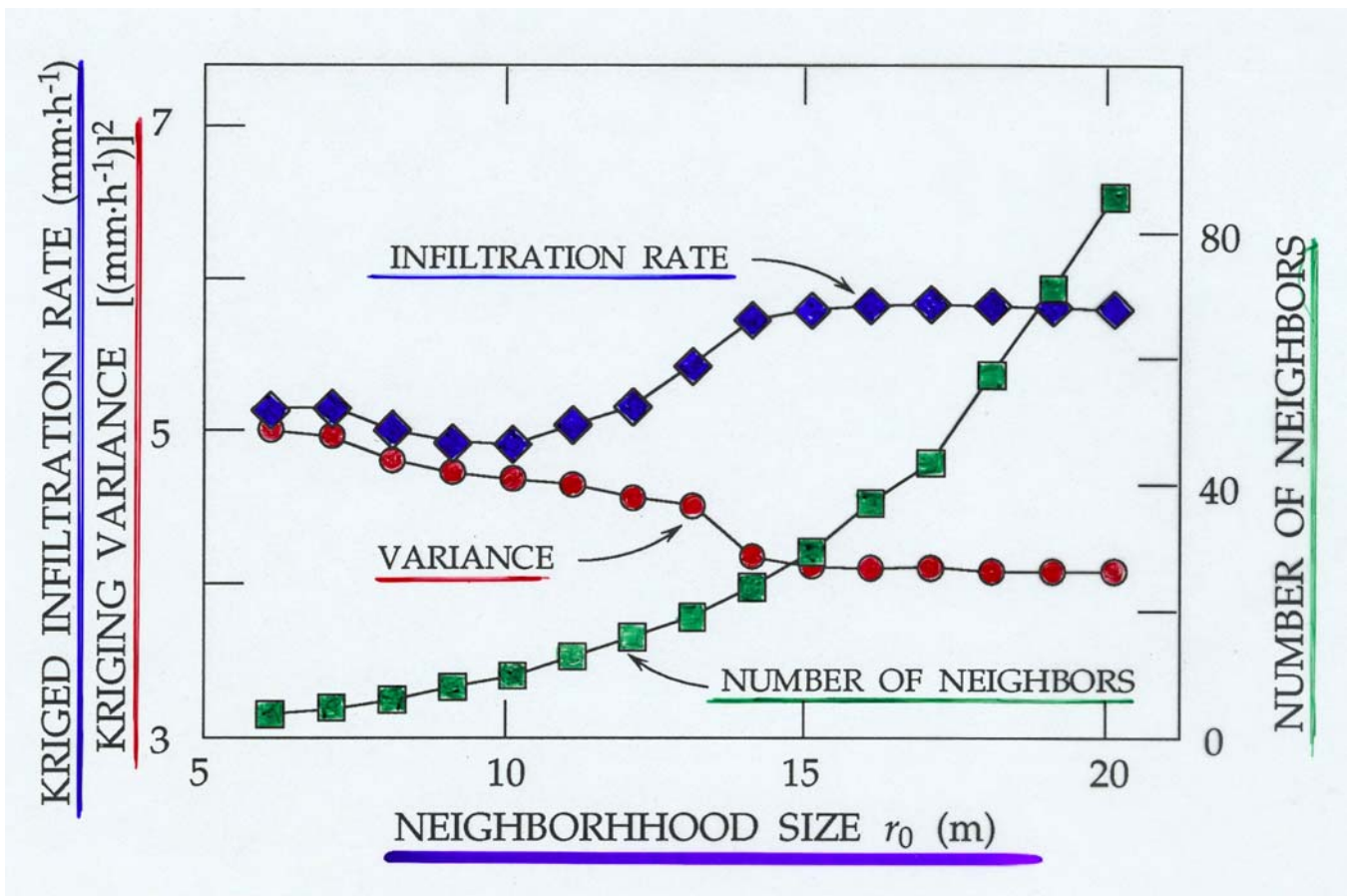


# Without and with 800 kriged values using same degree polynomial



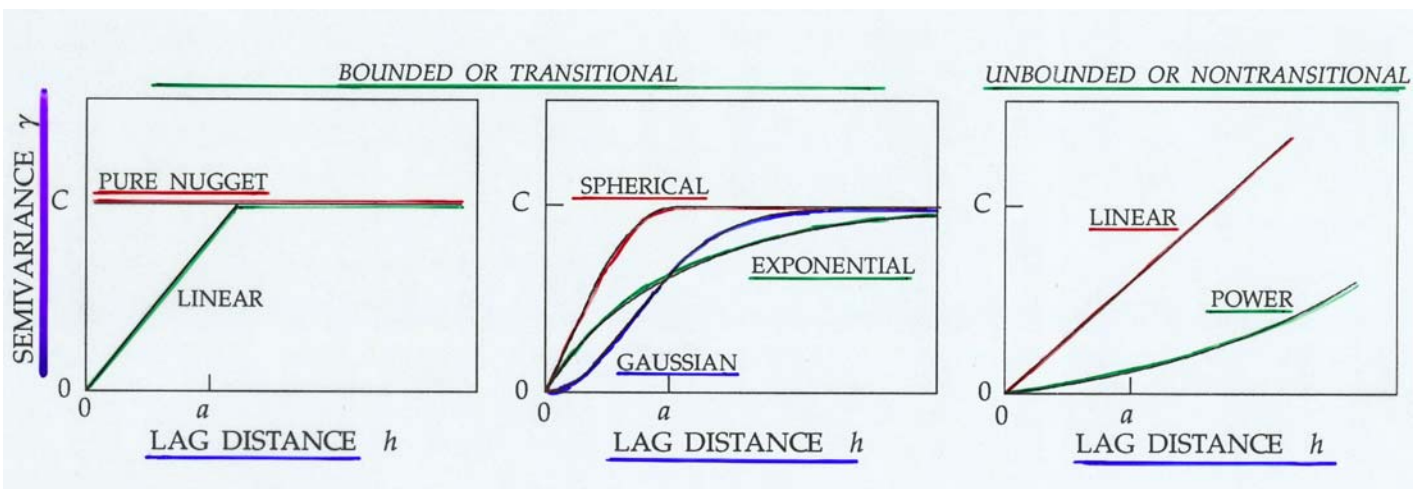


# infiltr rate-neighborhood size



# SEMI-VARIOGRAMS

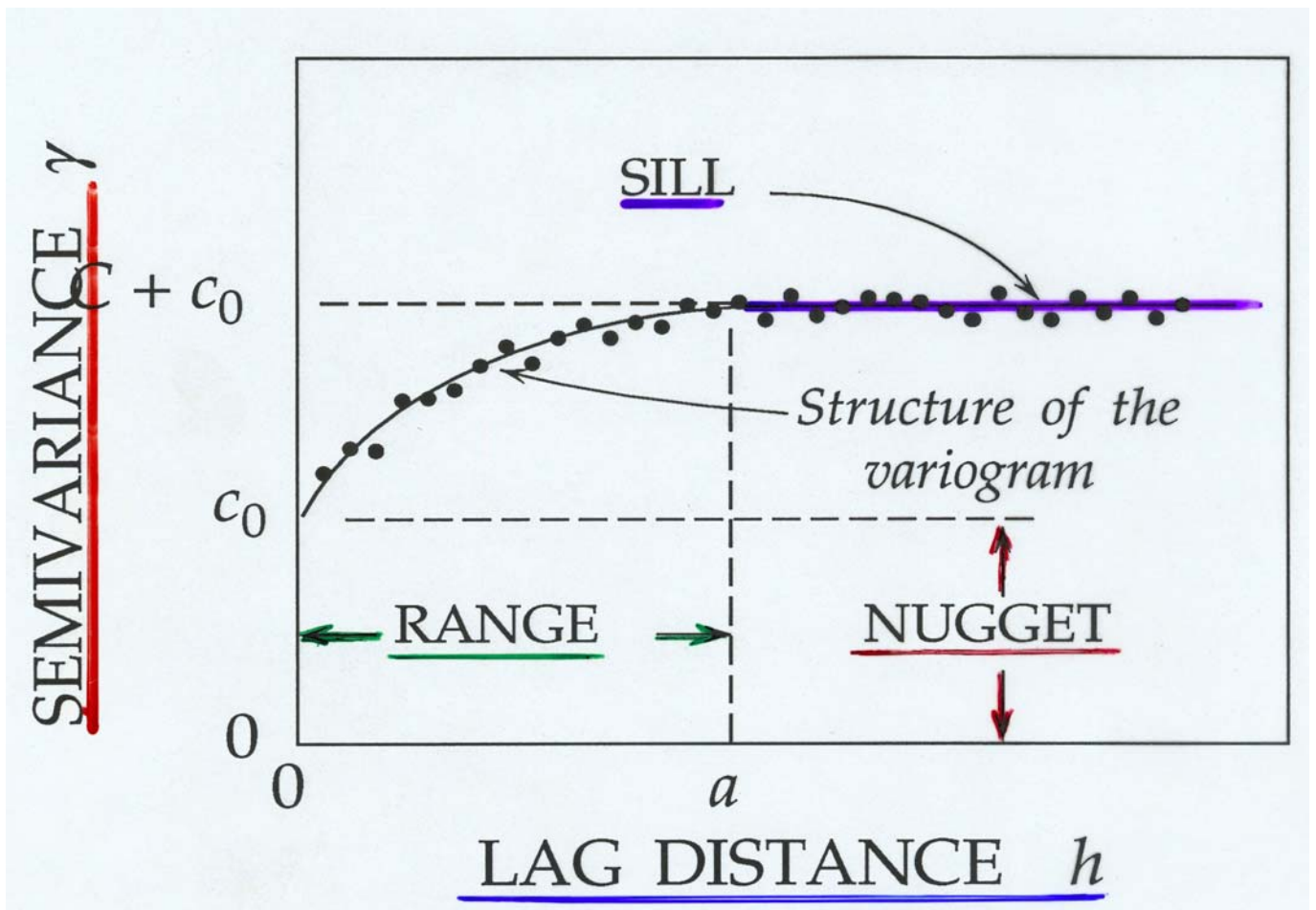
# Bounded & unbounded semivariograms



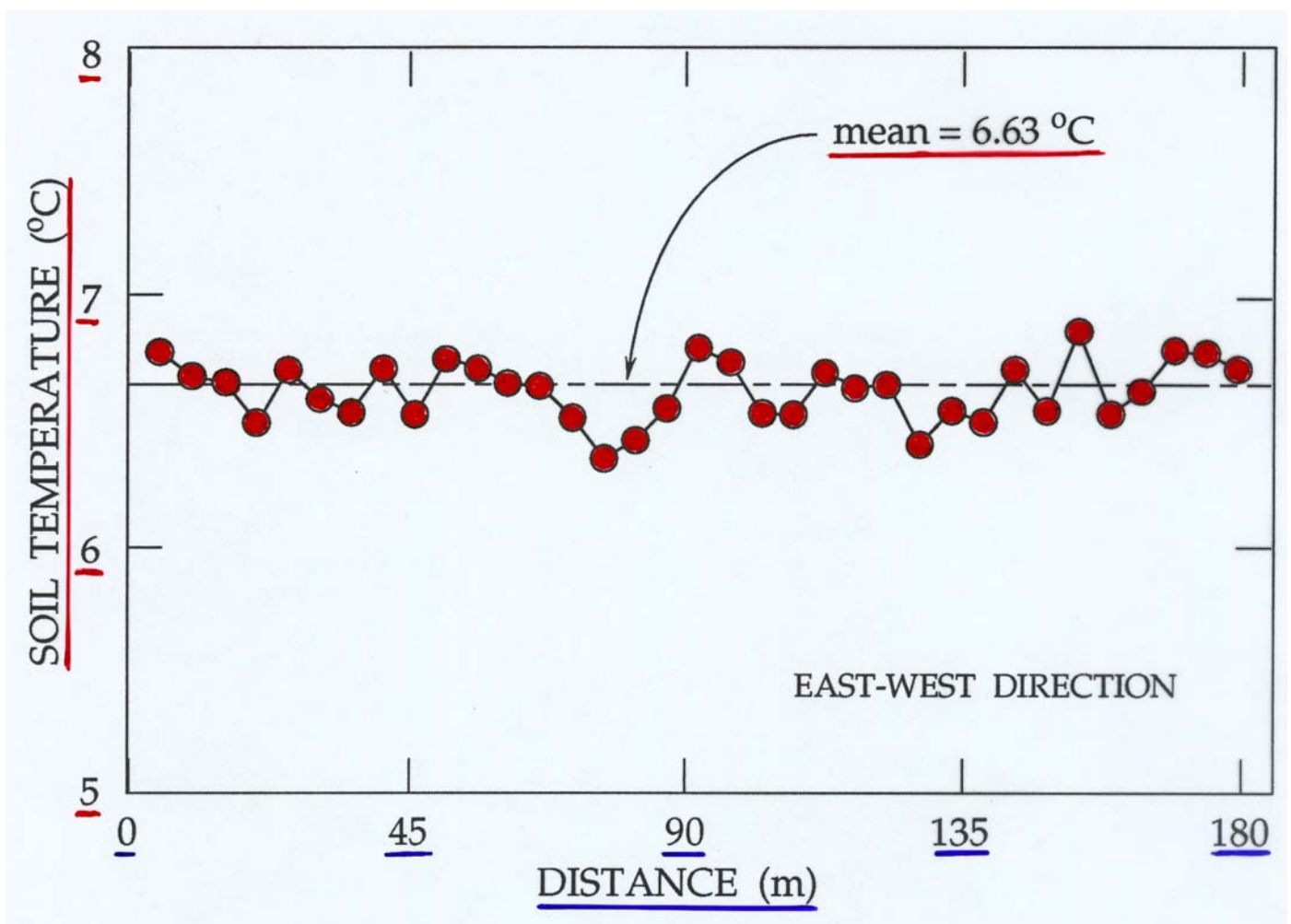
# Variogram equations

Model	Equation	Consideration
	<u>Bounded or Transitional</u>	
<u>Pure nugget</u>	$\gamma(h) = \begin{cases} 0 \\ C \end{cases}$	$h = 0$ $h \geq 1$
<u>Linear</u>	$\gamma(h) = \begin{cases} Ch/a \\ C \end{cases}$	$0 \leq h \leq a$ $h > a$
<u>Spherical</u>	$\gamma(h) = \begin{cases} C \left[ \frac{3h}{2a} - \frac{1}{2} \left( \frac{h}{a} \right)^3 \right] \\ C \end{cases}$	$0 \leq h \leq a$ $h > a$
<u>Exponential</u>	$\gamma(h) = C[1 - \exp(-h/a)]$	$h \geq 0$
<u>Gaussian</u>	$\gamma(h) = C \left\{ 1 - \exp \left[ -\left( \frac{h}{a} \right)^2 \right] \right\}$	$h \geq 0$
	<u>Unbounded or Nontransitional</u>	
<u>Linear</u>	$\gamma(h) = mh$	$h \geq 0$
<u>Power</u>	$\gamma(h) = mh^\beta$	$h \geq 0; 1 < \beta < 2$

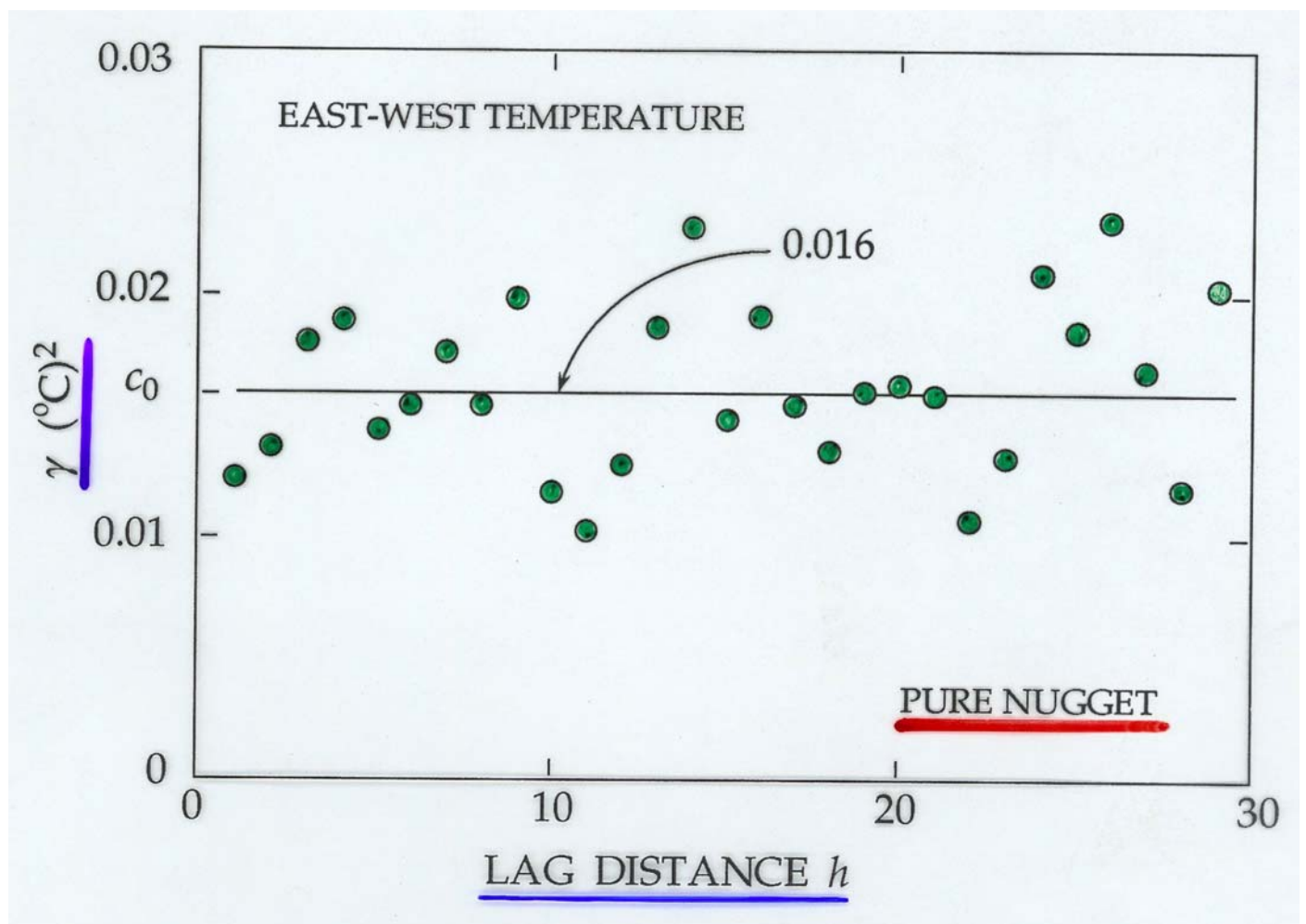
# Variogram structure



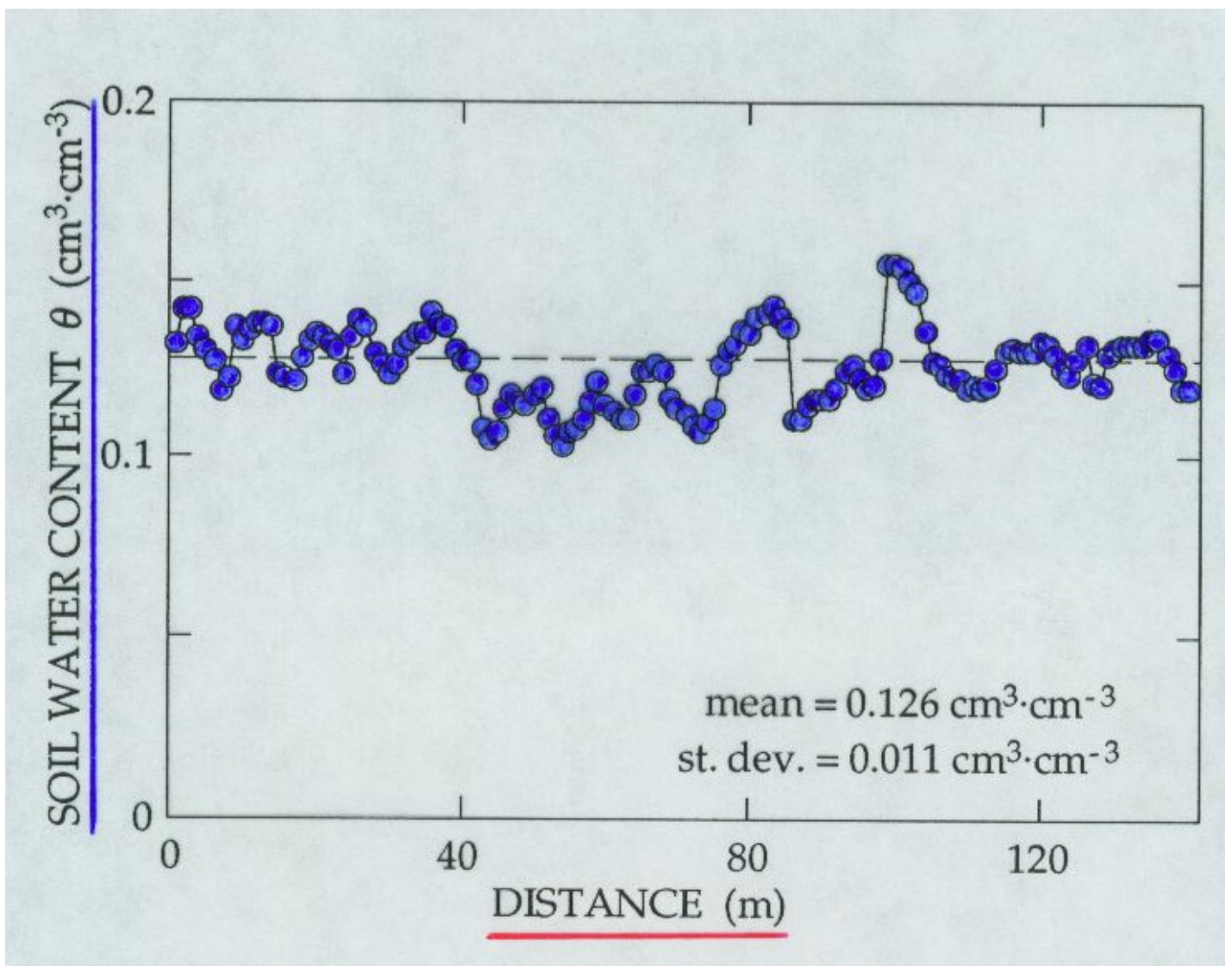
# Soil temp vs E-W distance



# Nugget variogram of E-W temp

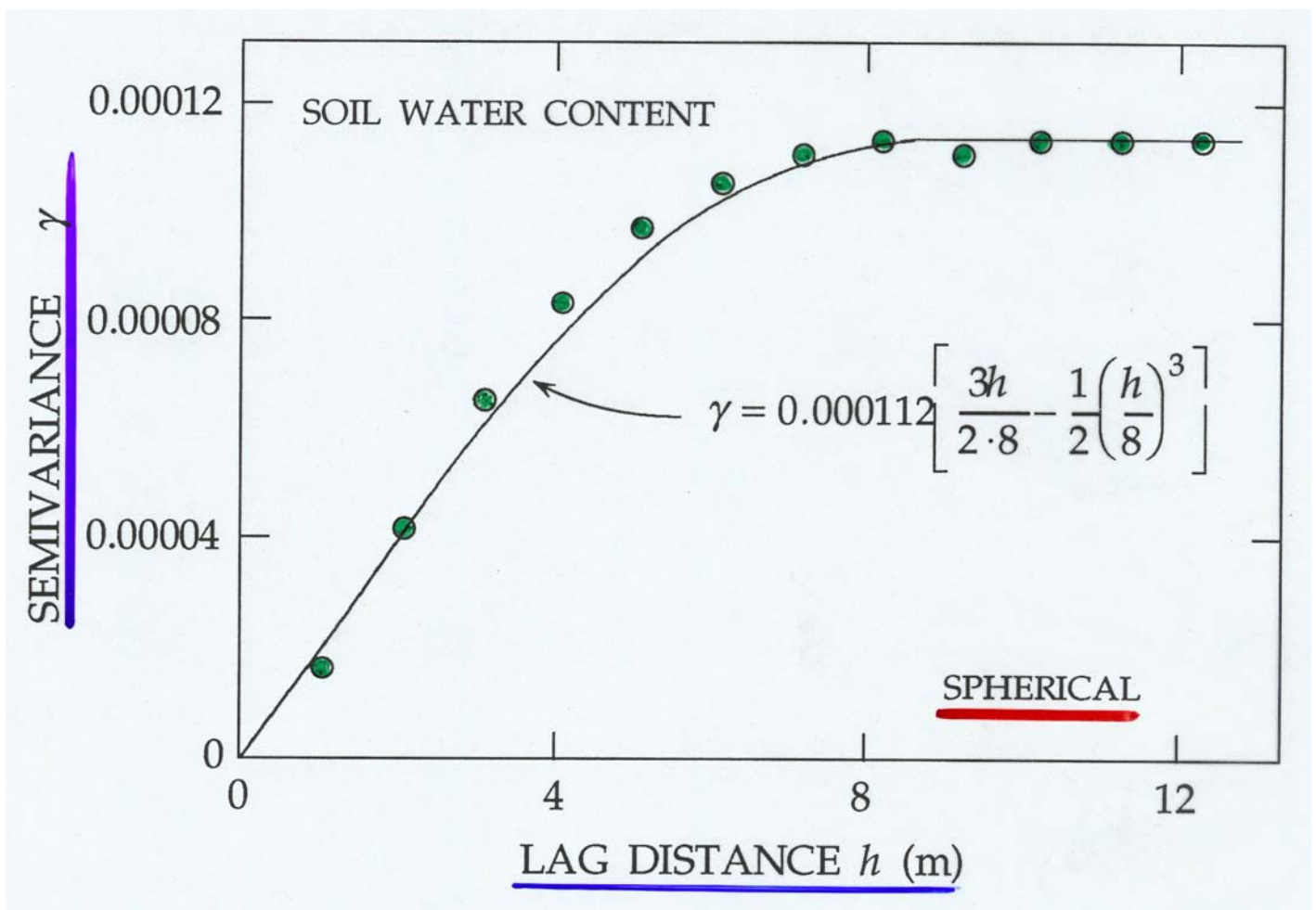


# Soil water vs distance

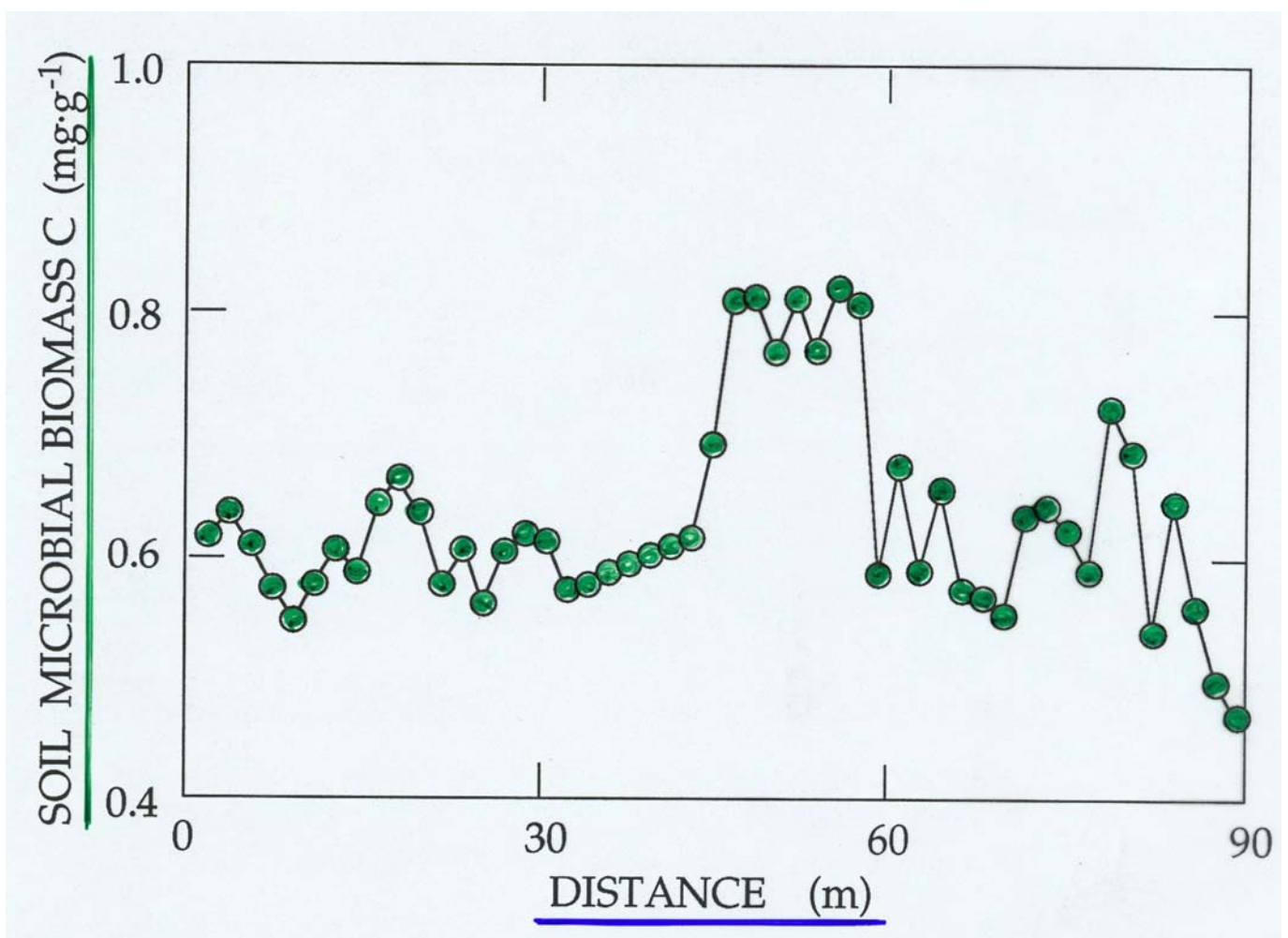




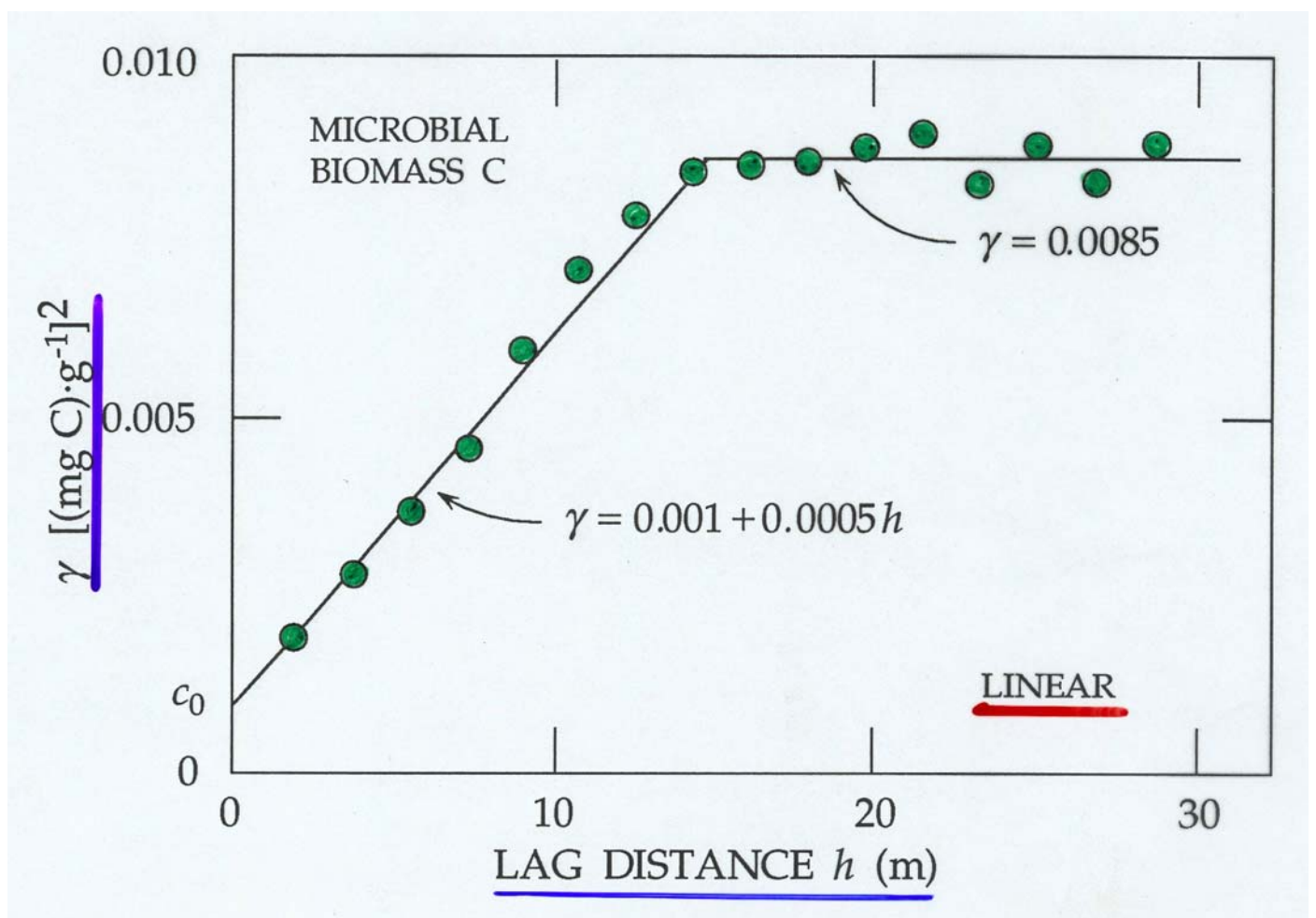
# Spherical variogram soil water



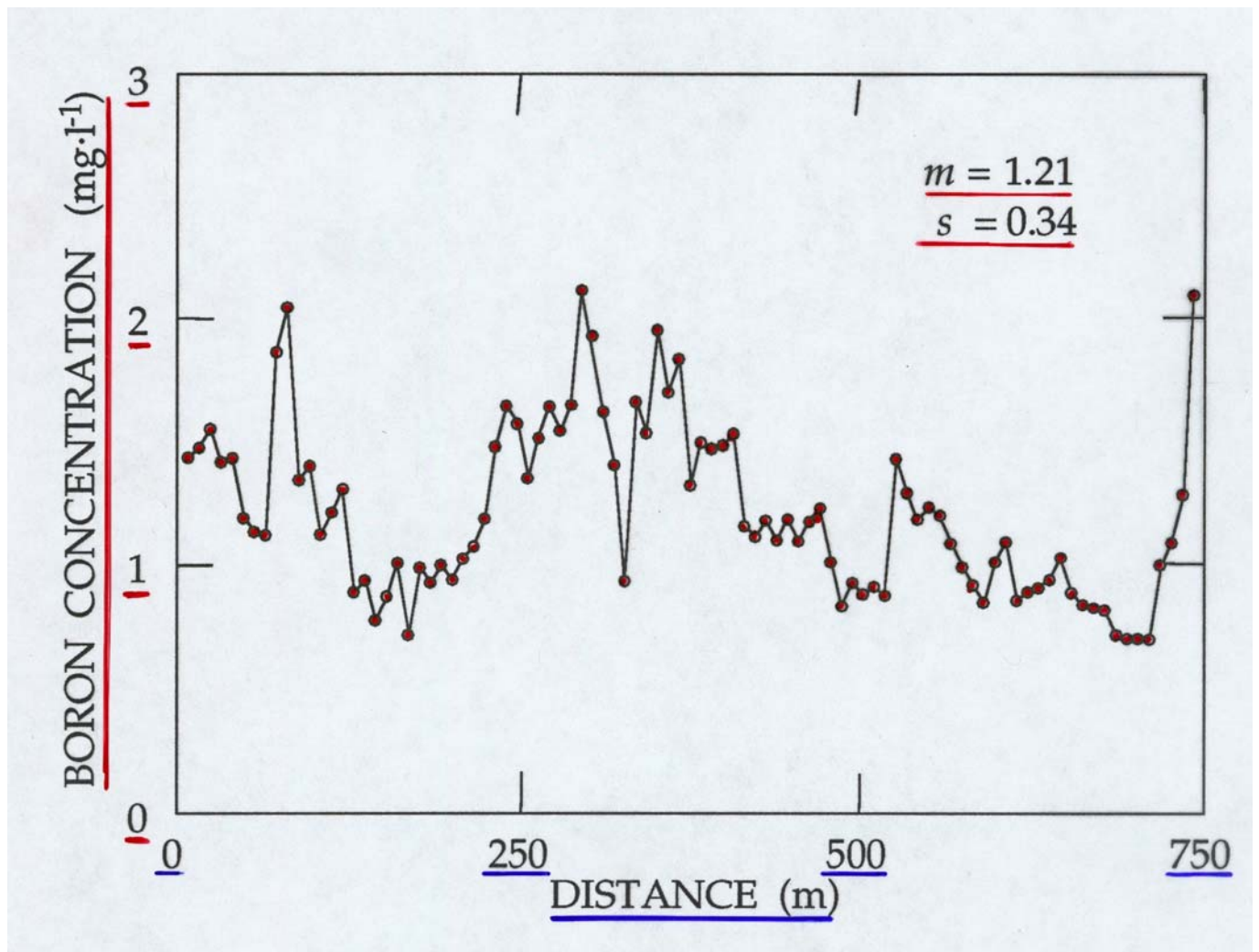
# Microbial biomass C vs distance



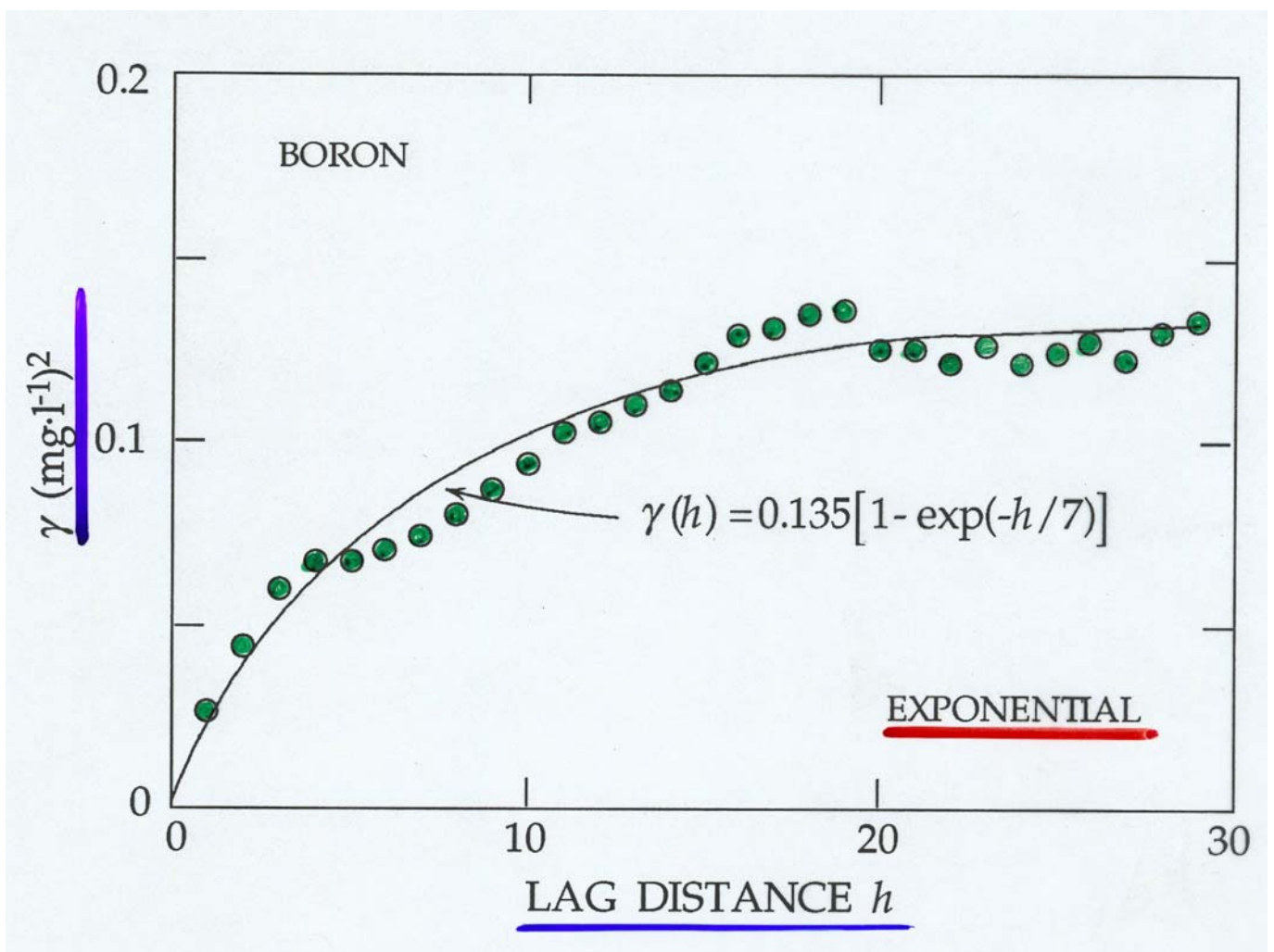
# Microbial biomass C variogram



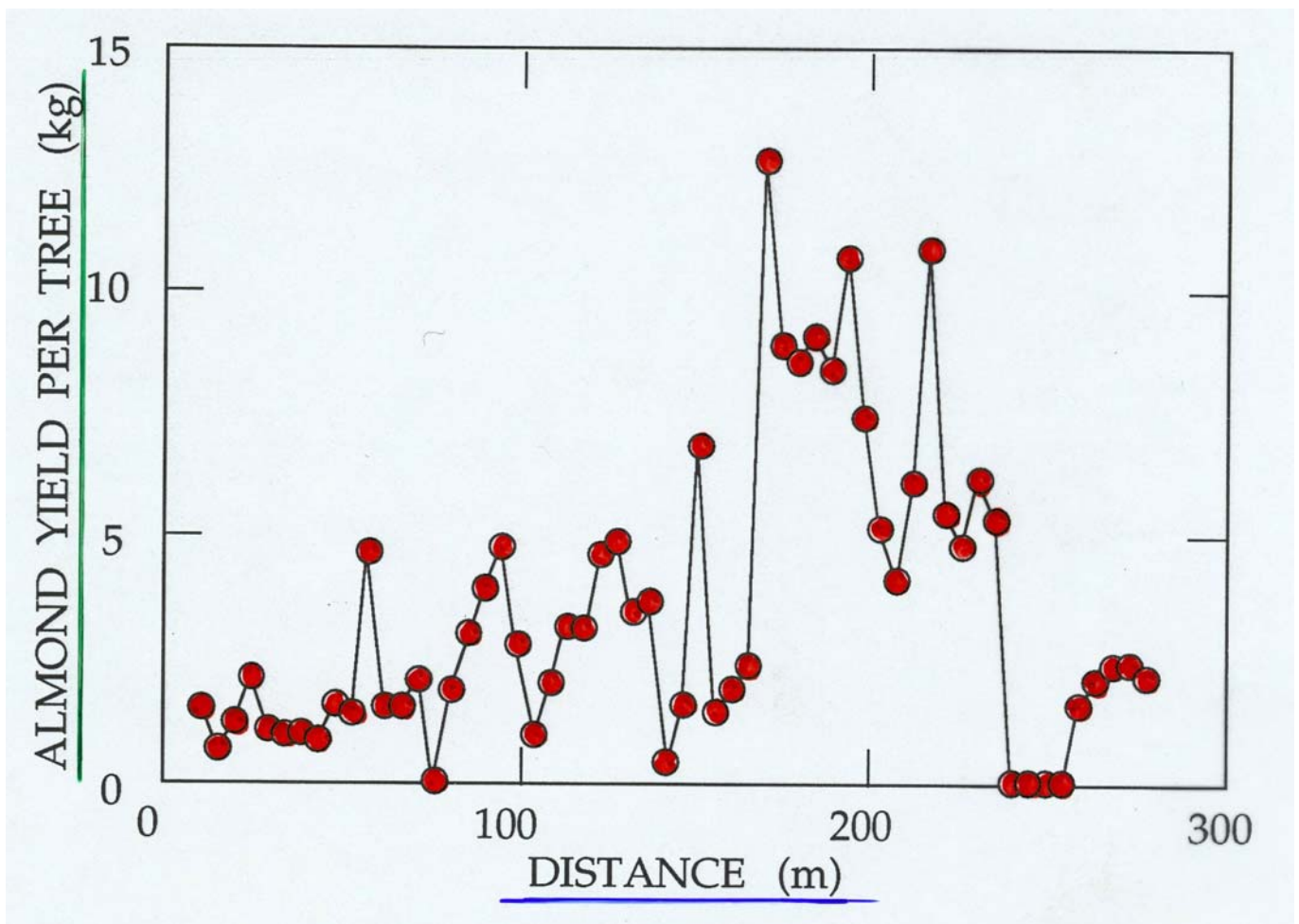
# Boron vs distance



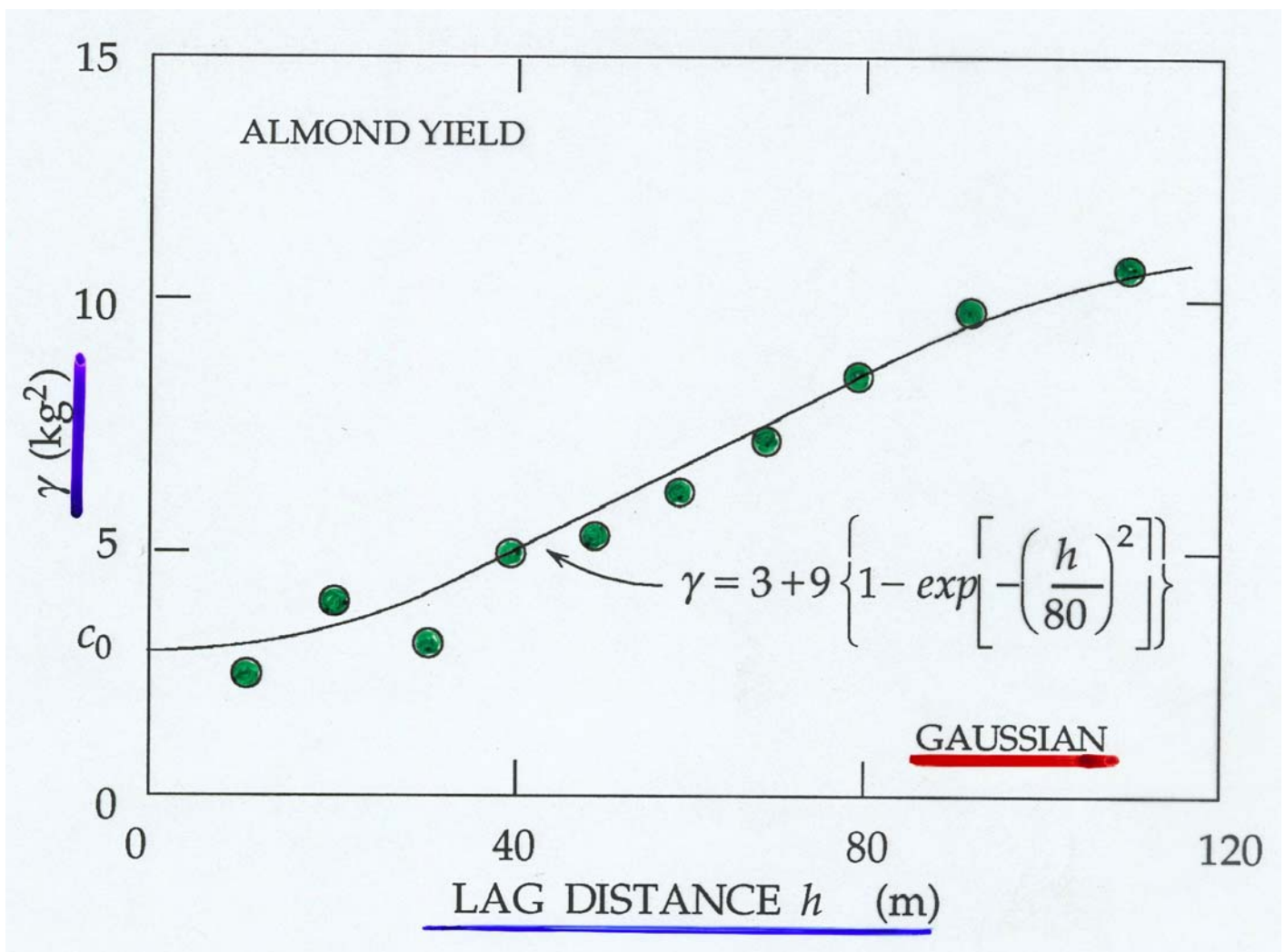
# Boron exponential variogram



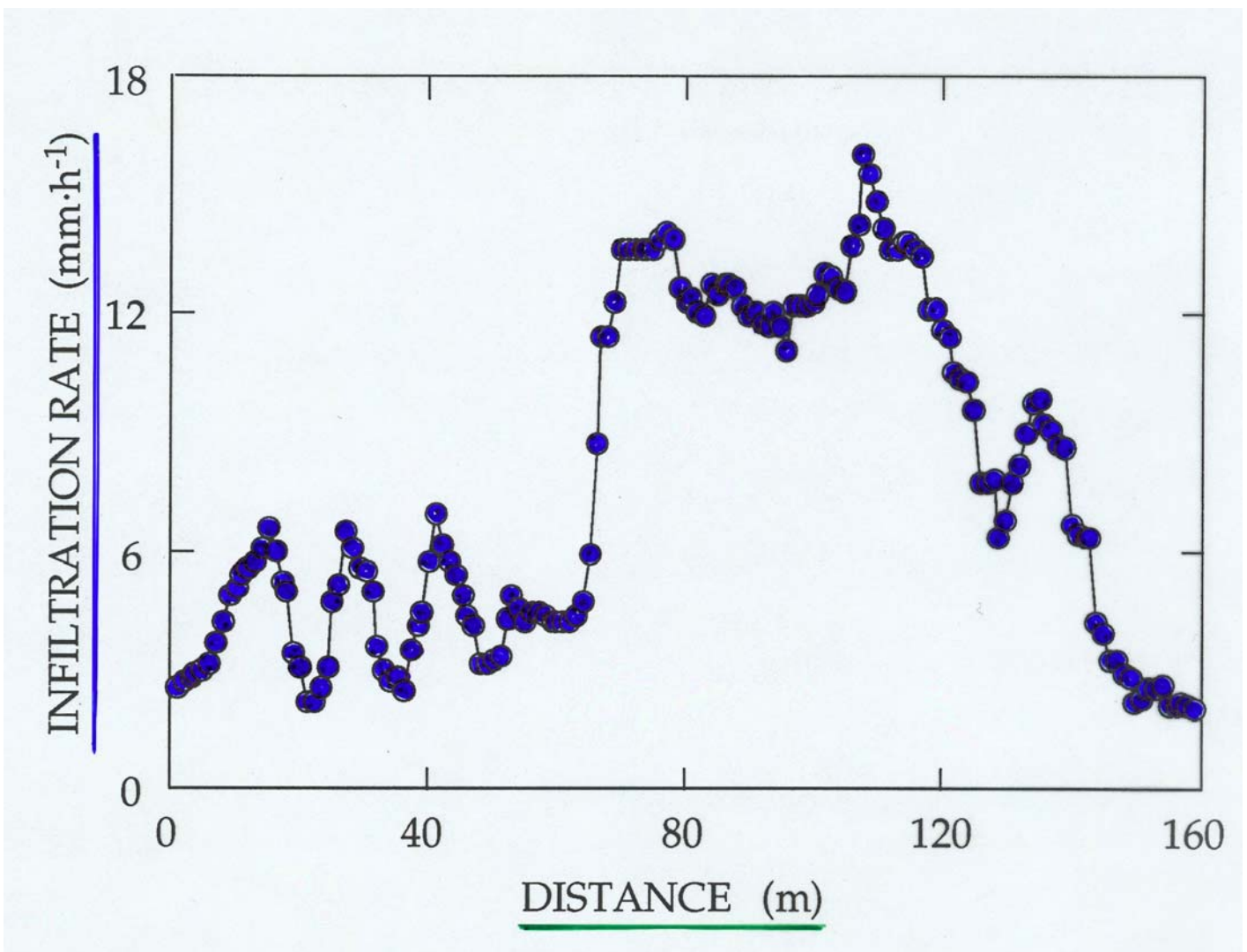
# Almond yield per tree vs distance



# Almond yield Gaussian variogram

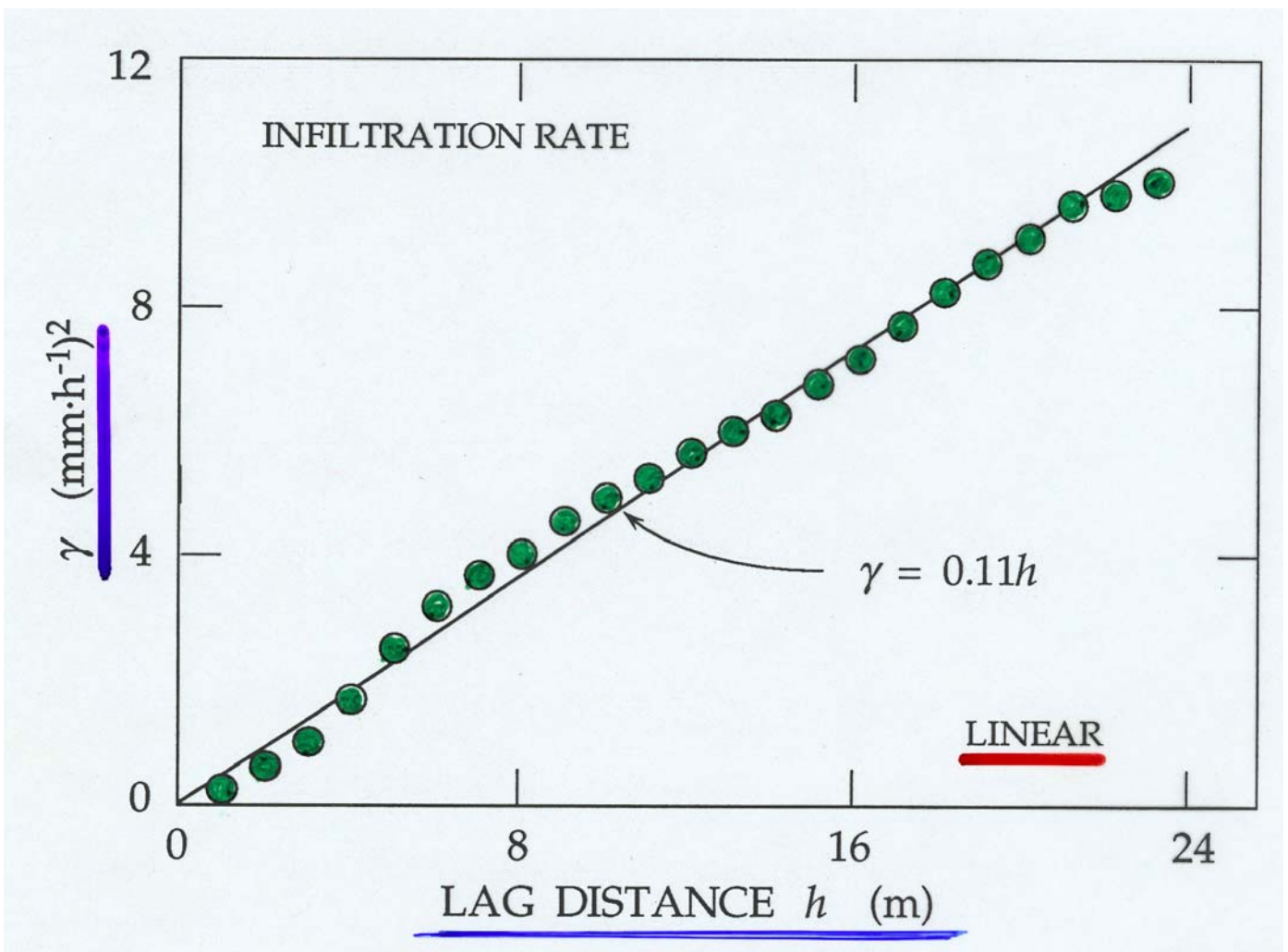


# infiltration rate vs distance

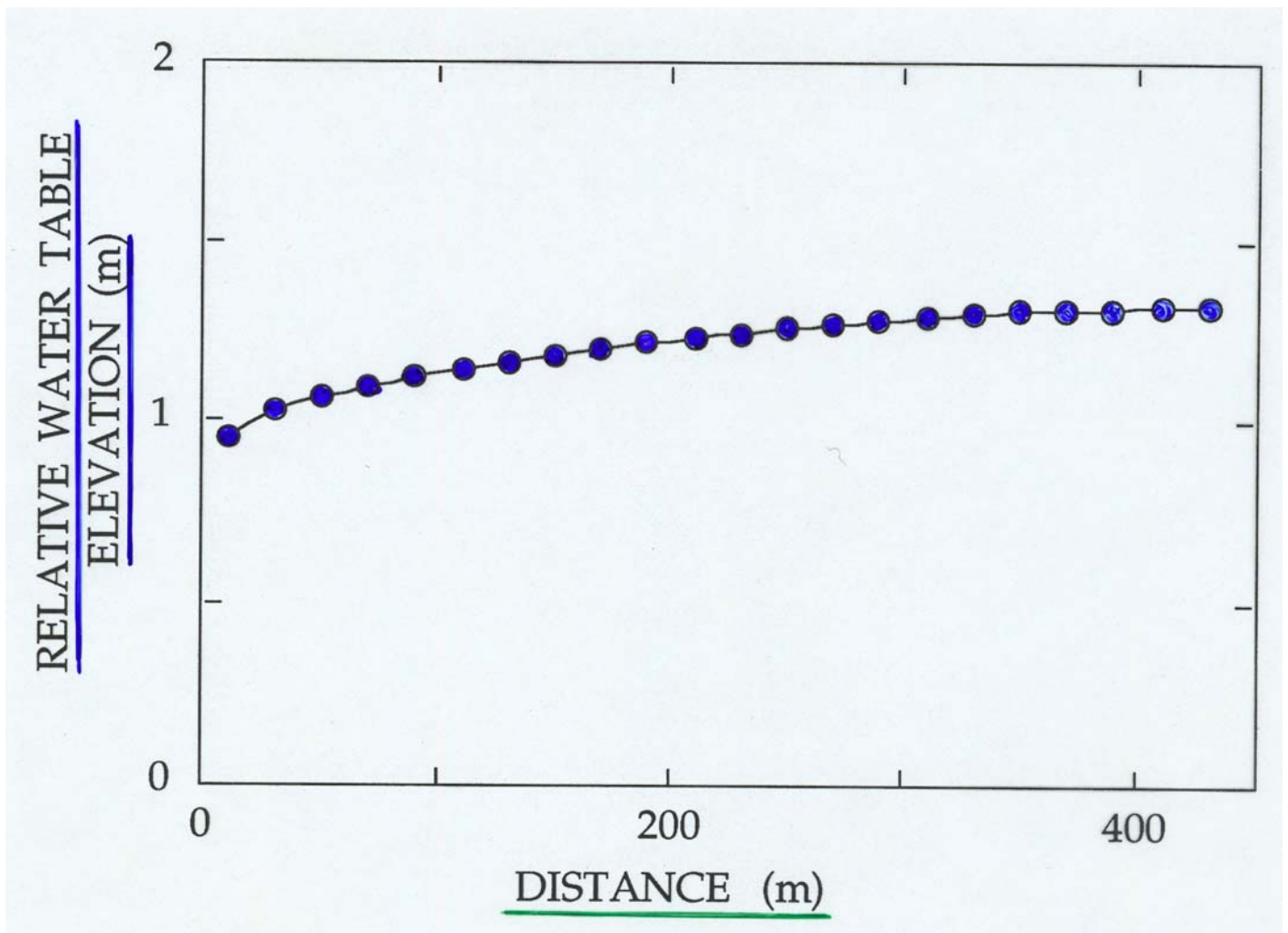




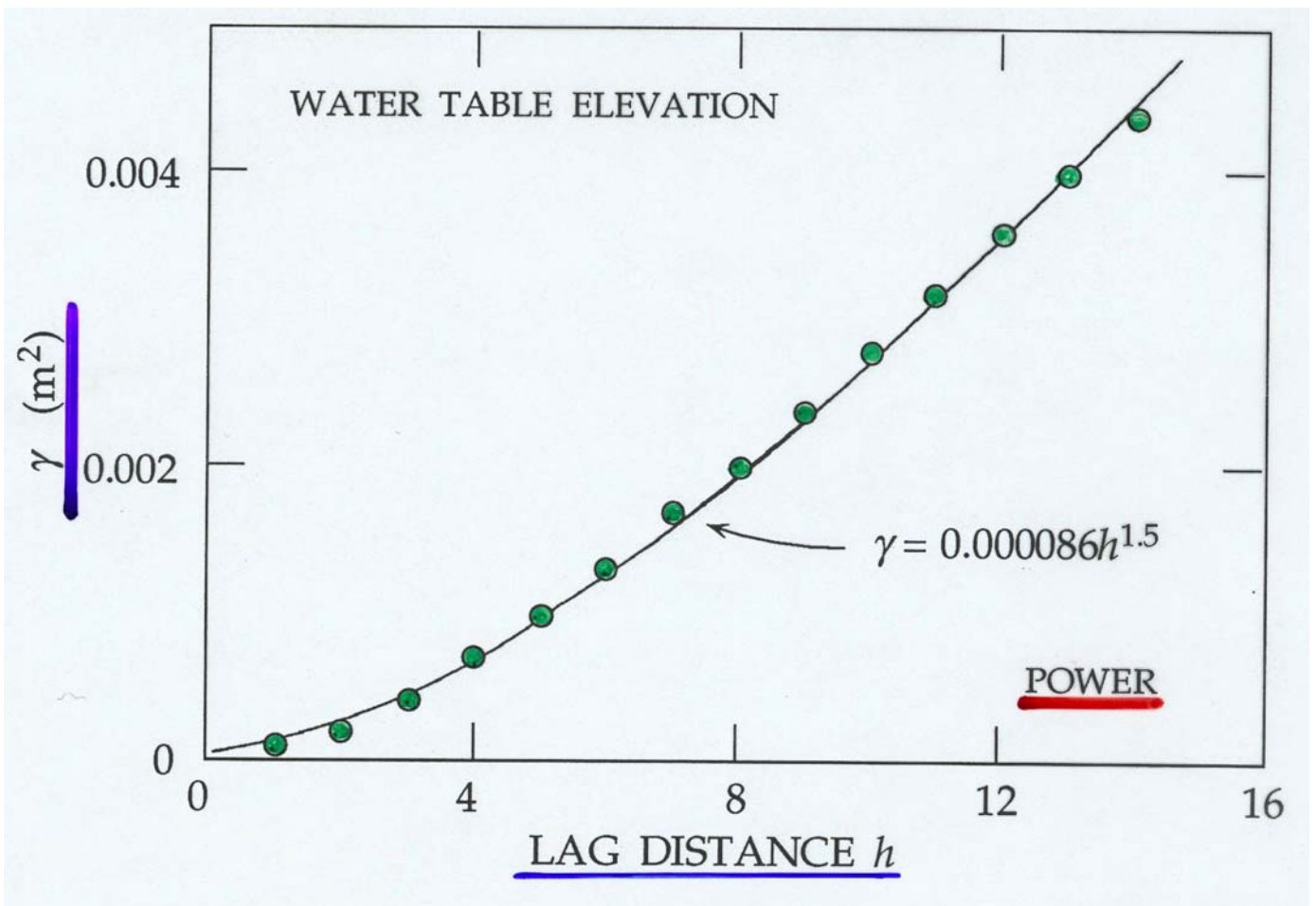
# infiltration rate linear variogram



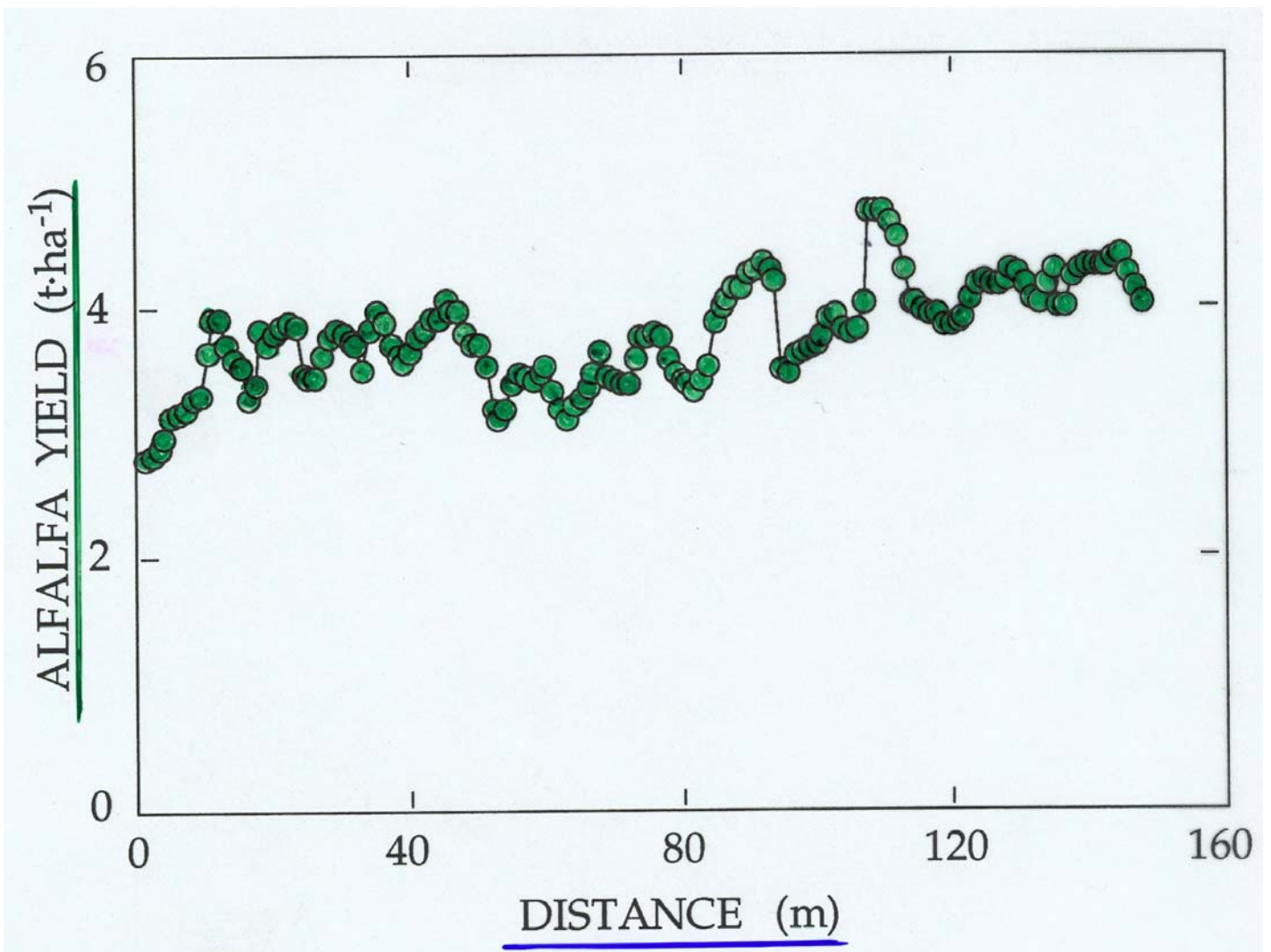
# water table elevation vs distance



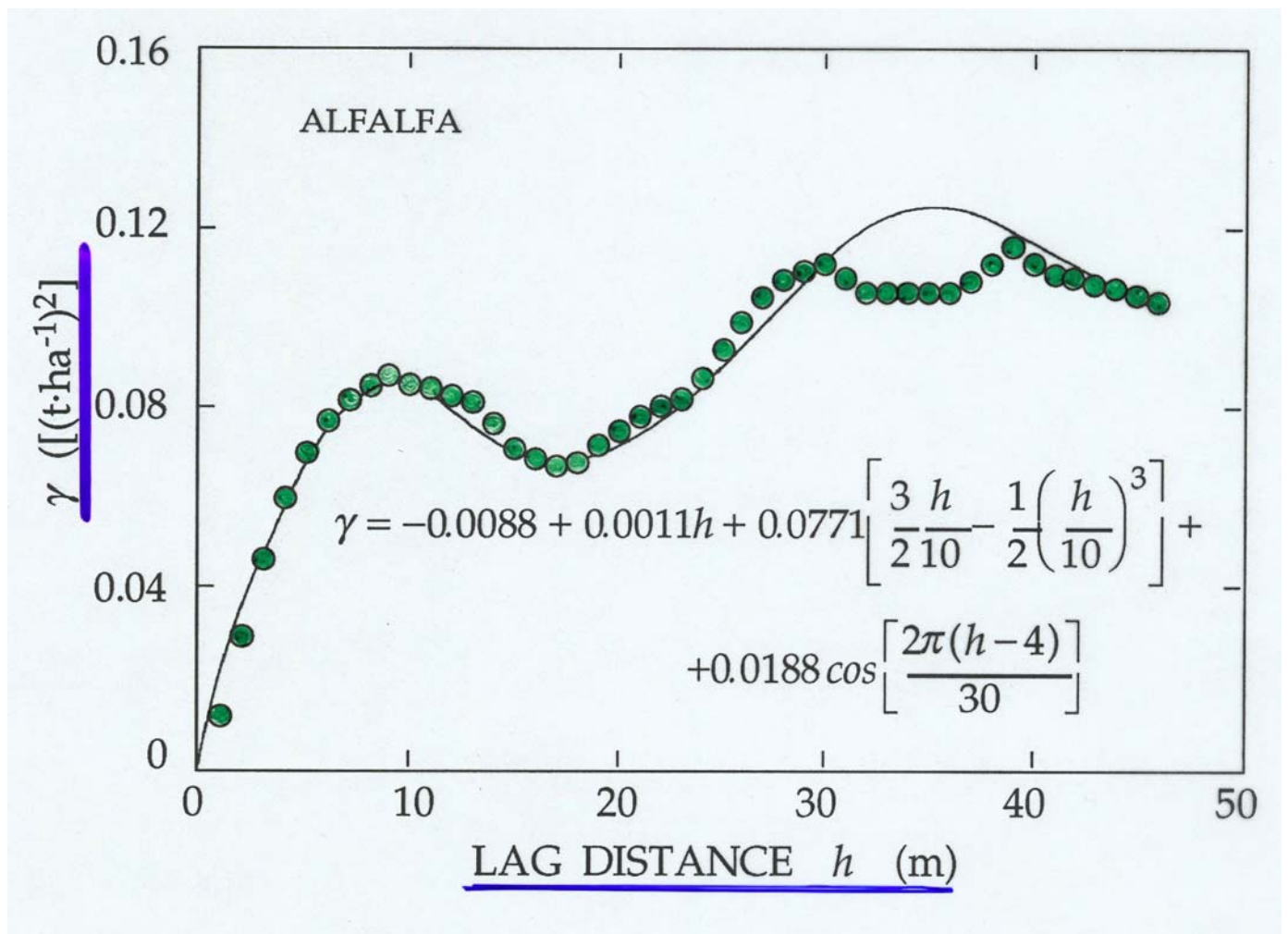
# water table elevation power variogram



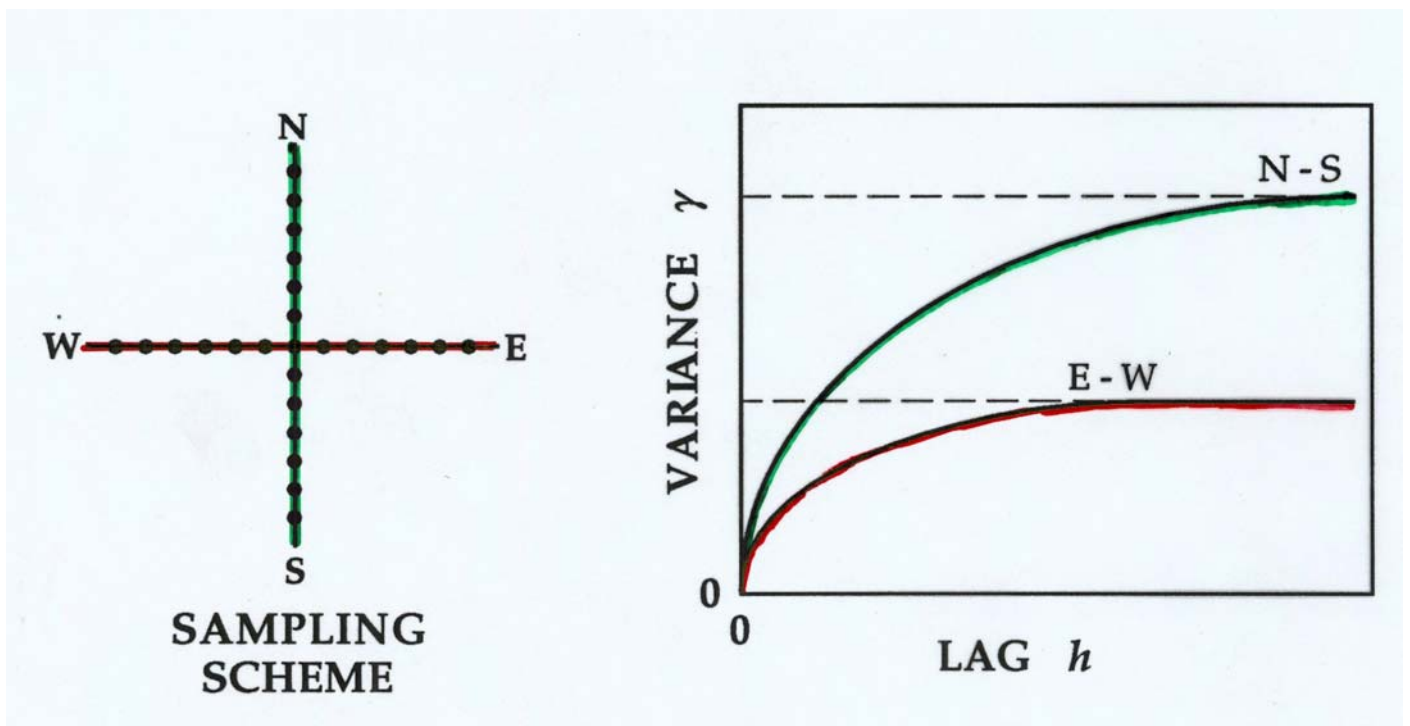
# alfalfa yield vs distance



# alfalfa comb. variogram

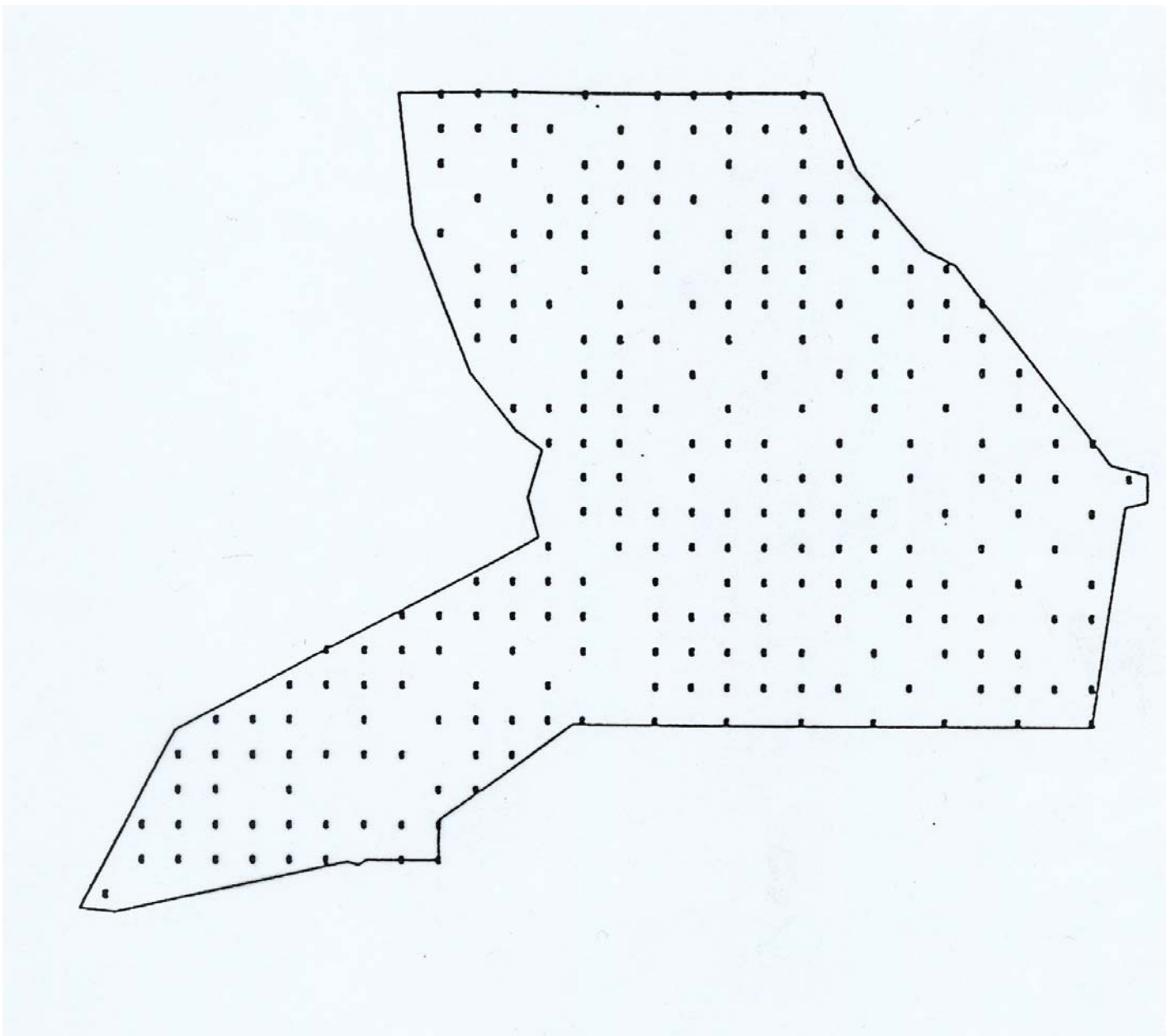


# directional variograms



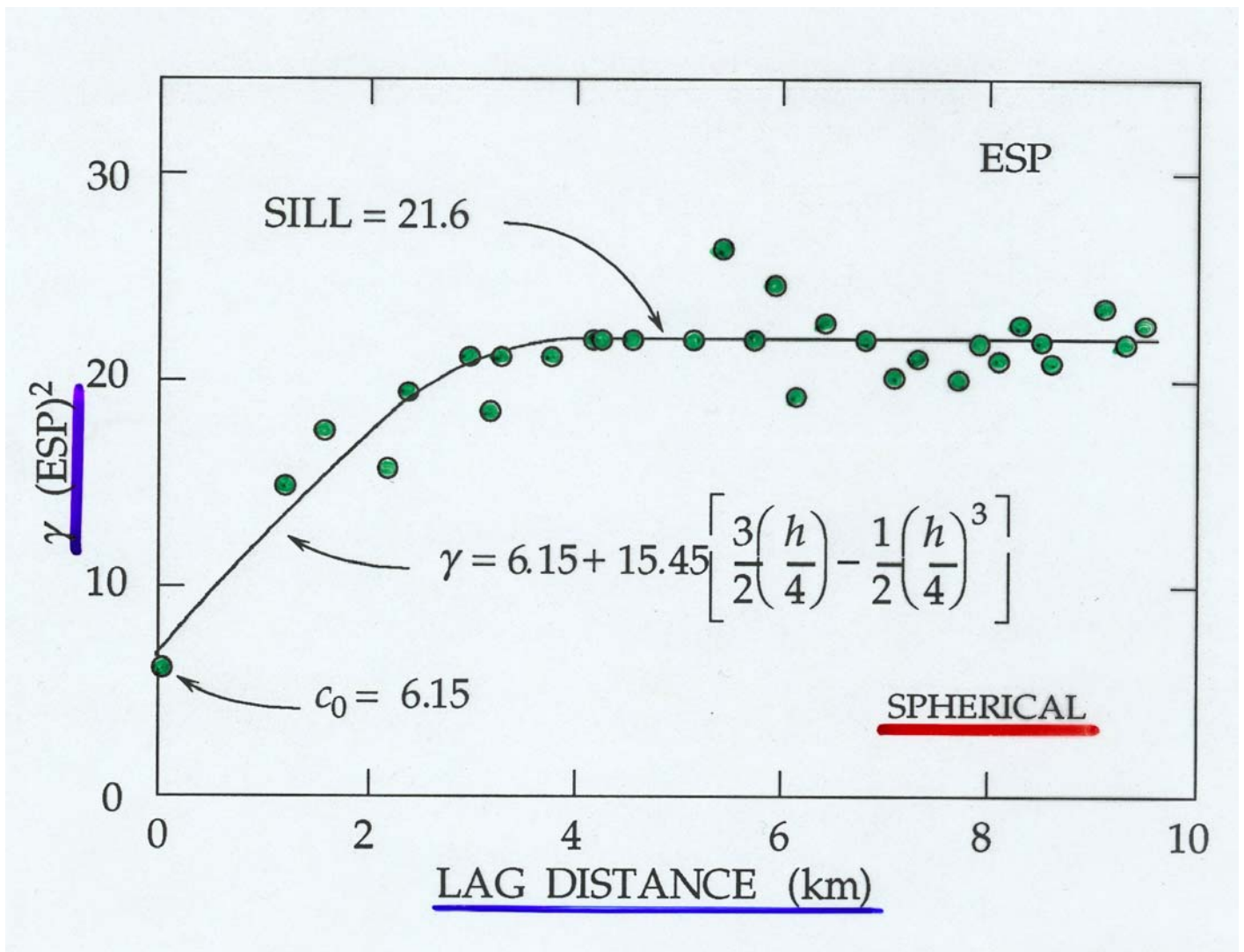
# EVALUATING THE “NUGGET”

# Sampling scheme in Sudan



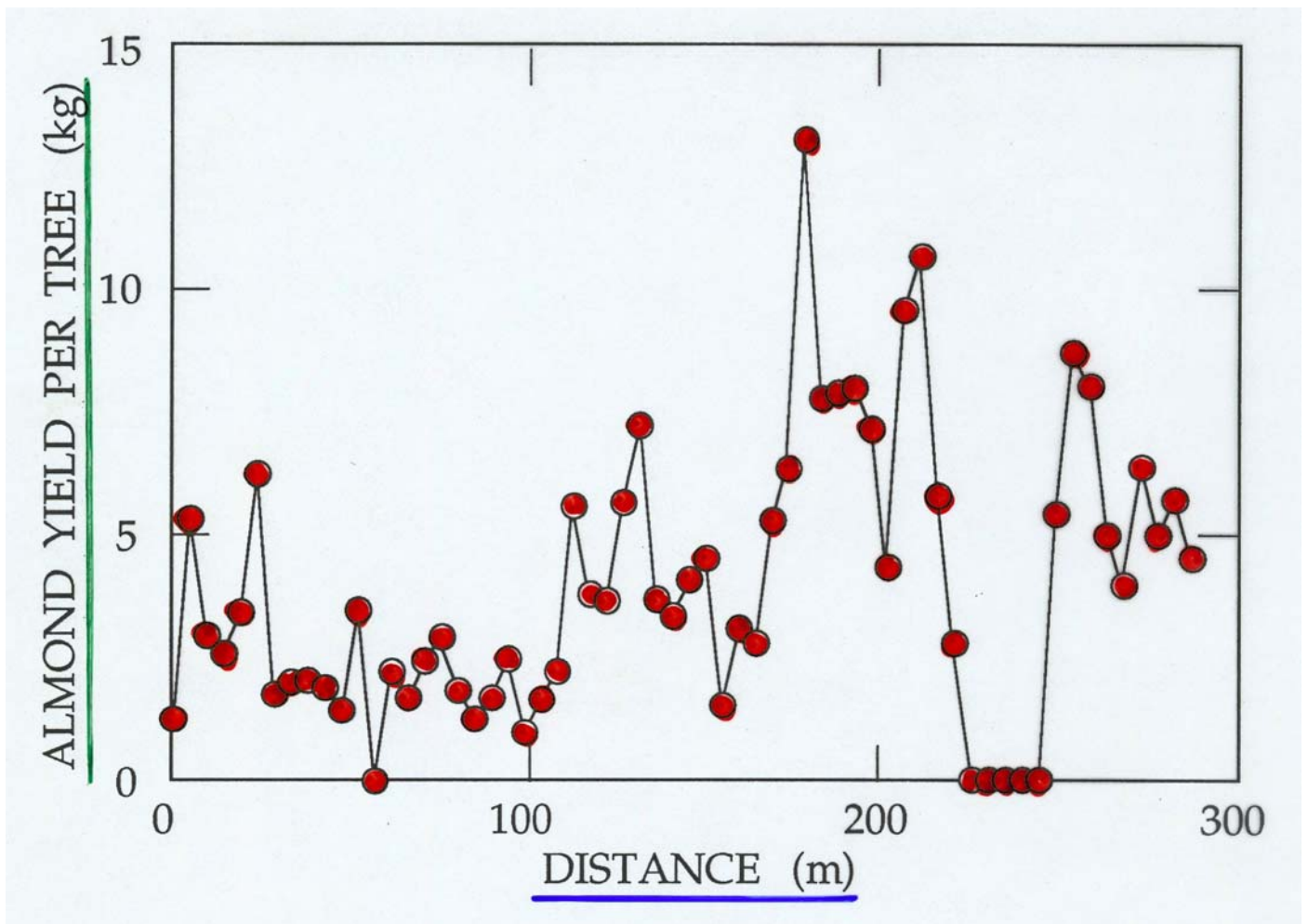


# ESP variogram Sudan

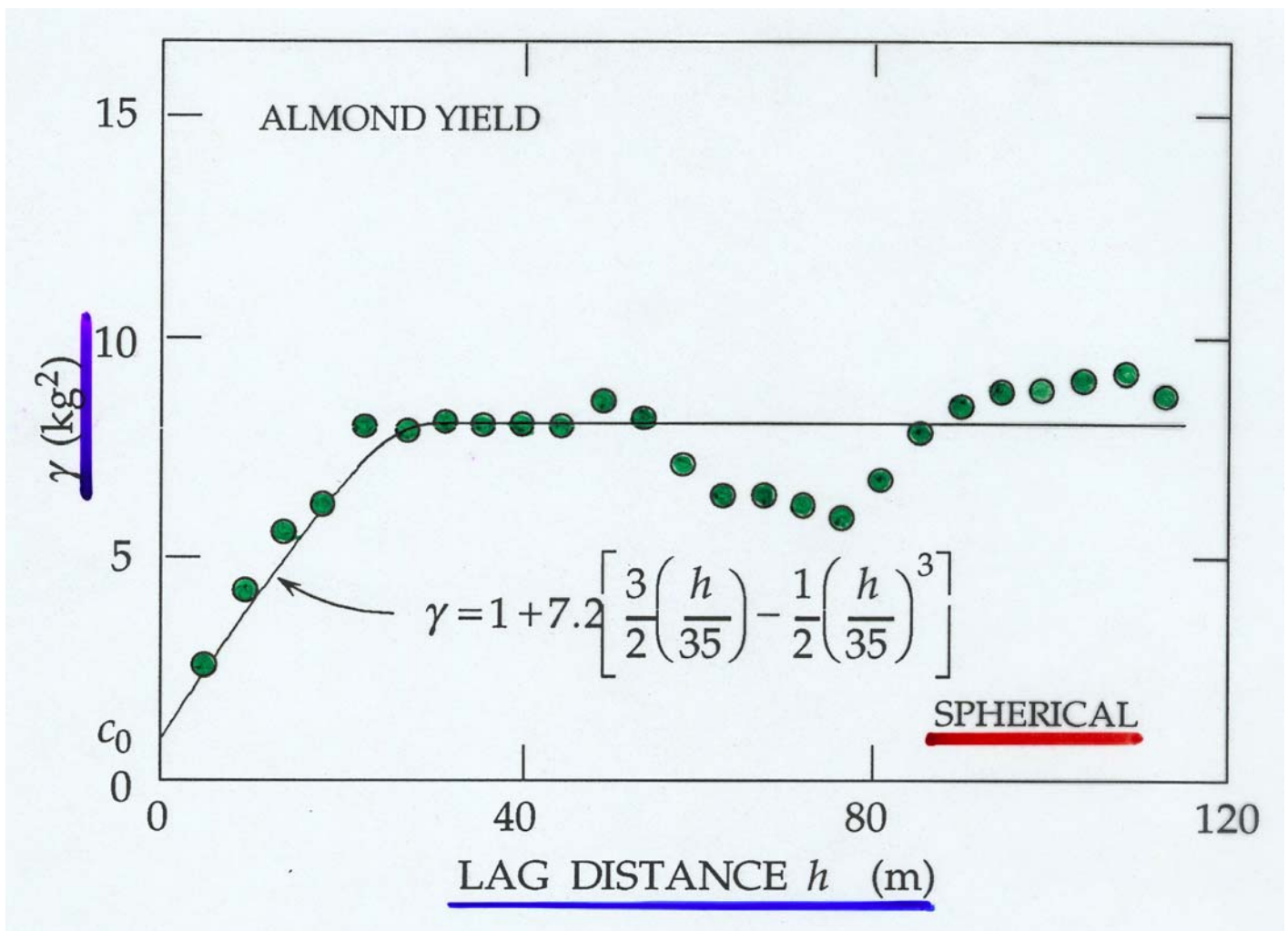


# KRIGING ALMOND NUT YIELDS

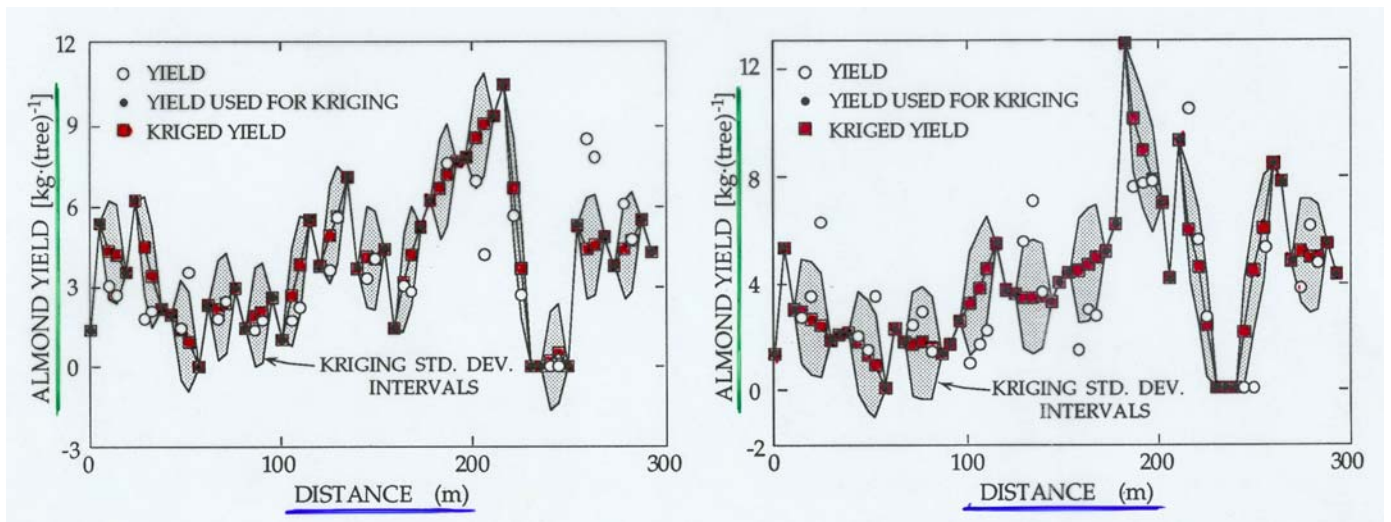
# almond yield per tree vs distance



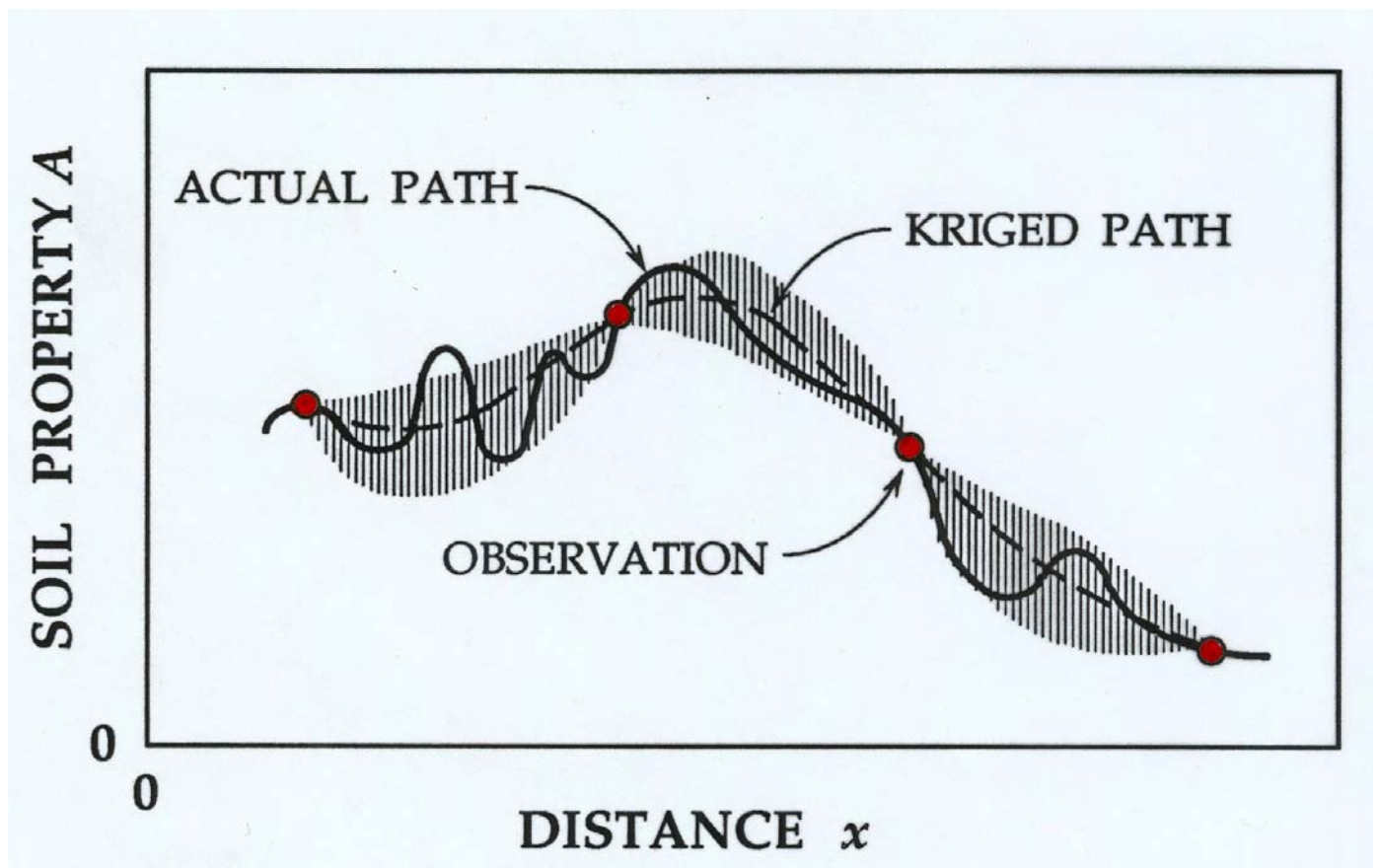
# almond yield spherical variogram



# kriging almond yield vs distance

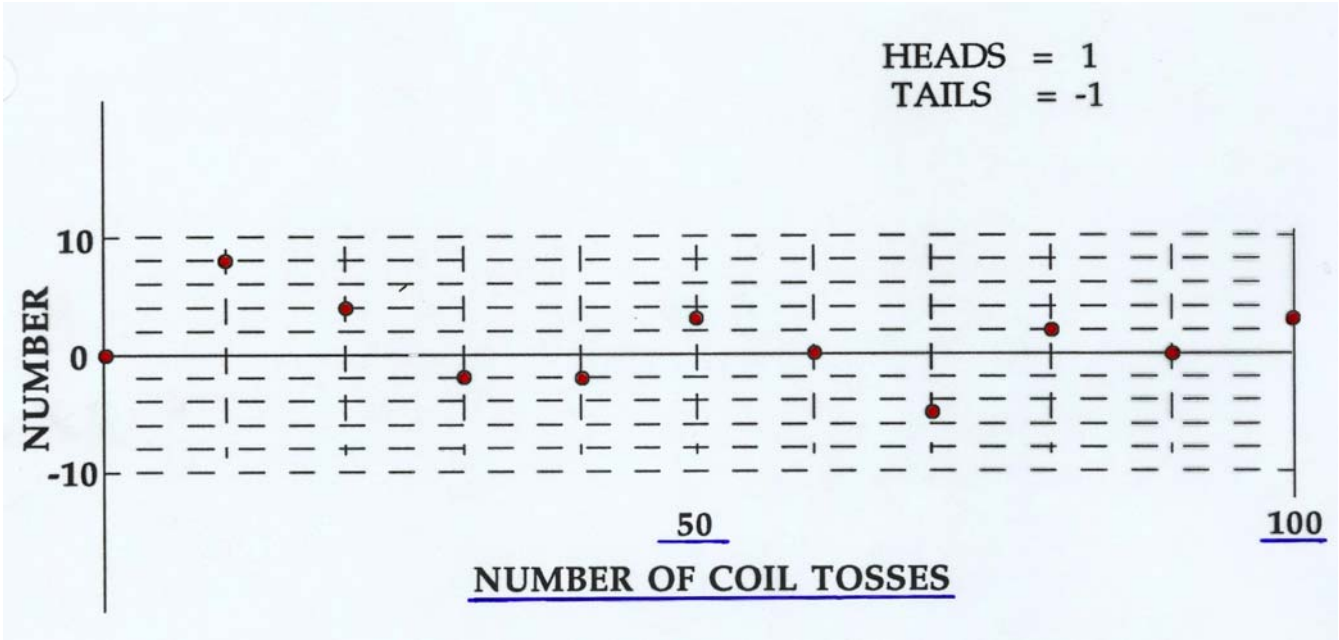


# kriging & real path



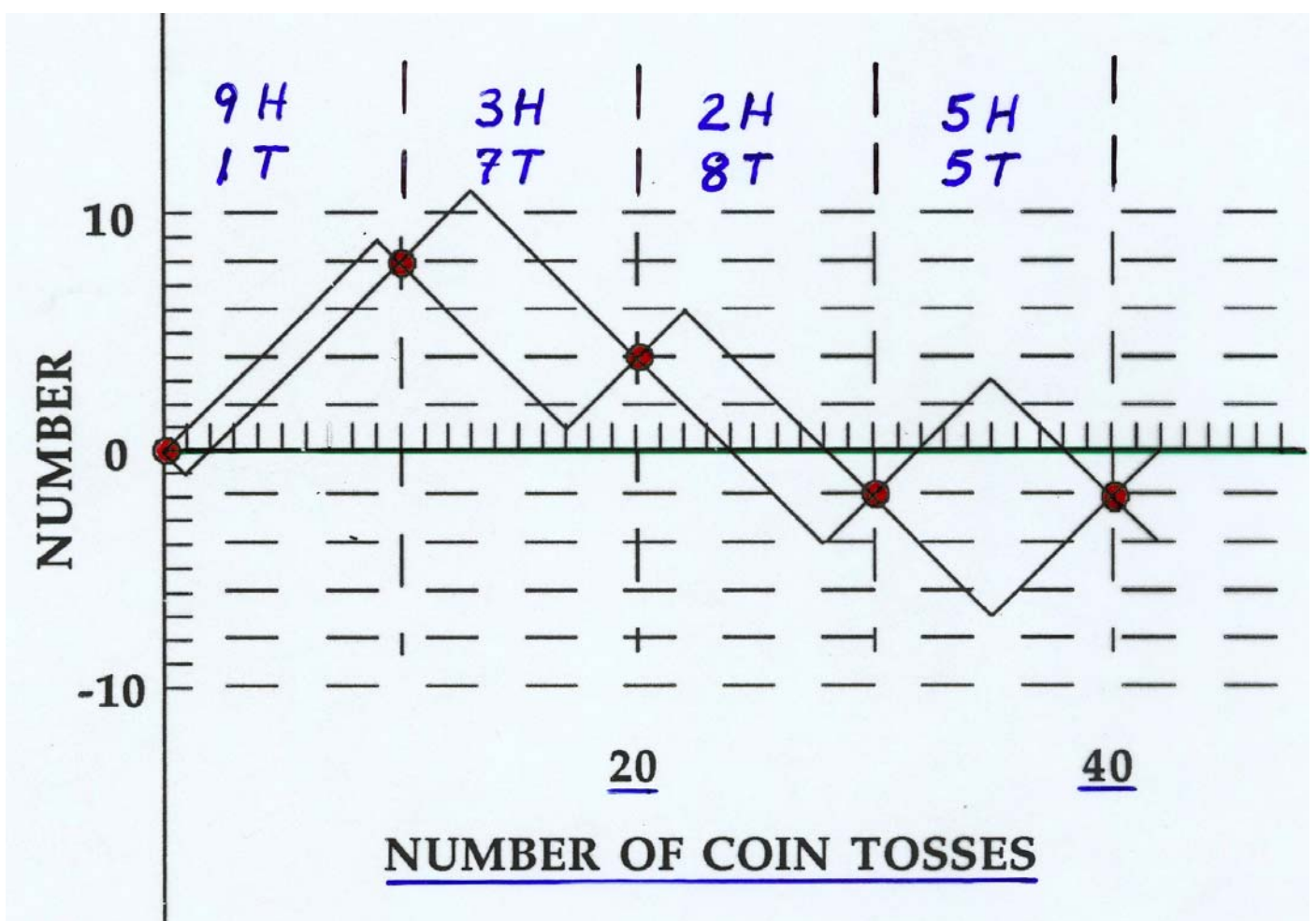
AN INSIGHT INTO THE KRIGING  
INTERPOLATION

# number of coin tosses (100)





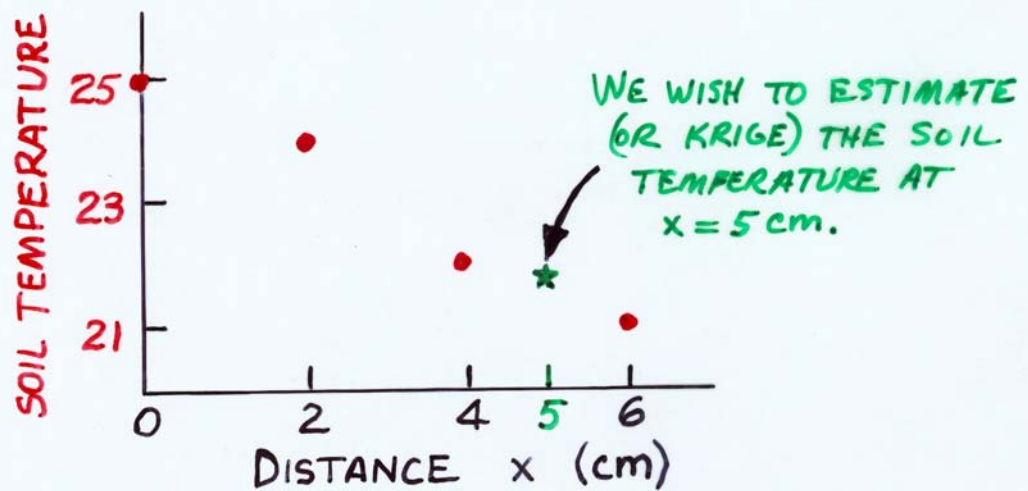
# number of coin tosses (40)



# simple example of kriging

## SIMPLE EXAMPLE OF KRIGING

### SOIL TEMPERATURE MEASURED AT 4 LOCATIONS



COORDINATE	0	2	4	6
LOCATION NUMBER	1	2	3	4
MEASURED VALUE	25	24	22	21

### COMPUTATION OF THE VARIOGRAM

$$\gamma(h) = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} [T(x_i+h) - T(x_i)]^2 \quad [1]$$

WHERE  $N(h)$  IS NUMBER OF COUPLES OF LOCATIONS SEPARATED BY A DISTANCE  $h$ .

# kriging example continued

FOR  $h=2$ ,  $N(2) = 3$

$$\gamma(2) = \frac{1}{2(3)} [(24-25)^2 + (22-24)^2 + (21-22)^2] = 1$$

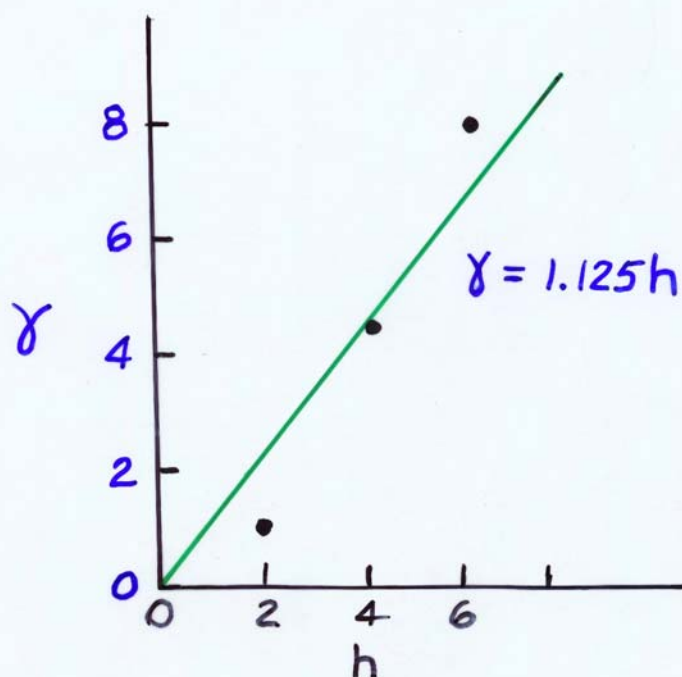
FOR  $h=4$ ,  $N(4) = 2$

$$\gamma(4) = \frac{1}{2(2)} [(22-25)^2 + (21-24)^2] = 4.5$$

FOR  $h=6$ ,  $N(6) = 1$

$$\gamma(6) = \frac{1}{2(1)} [(21-25)^2] = 8$$

CALCULATE SMOOTH FUNCTION TO DESCRIBE THE  
RAW VARIOGRAM



# kriging the value of T

USING THE SMOOTH VARIOGRAM ( $\gamma = 1.125h$ ), WE  
CALCULATE VALUES OF  $\gamma$  AT EACH SEPARATION  $h$ :

$$\gamma(0) = 0$$

$$\gamma(4) = 4.5$$

$$\gamma(2) = 2.25$$

$$\gamma(6) = 6.75$$

KRIGING THE VALUE OF T AT  $x = 5$  CM

$$T^*(x_0) = \lambda_1 T(x_1) + \lambda_2 T(x_2) + \lambda_3 T(x_3) + \lambda_4 T(x_4)$$

$$T^*(5) = \lambda_1 \cdot 25 + \lambda_2 \cdot 24 + \lambda_3 \cdot 22 + \lambda_4 \cdot 21$$

THE WEIGHTS  $\lambda_i$  ARE CALCULATED ON THE BASIS THAT:

$$E[T^*(x_0) - T(x_0)] = 0$$

AND

$\text{VAR}[T^*(x_0) - T(x_0)]$  IS A MINIMUM

WHERE  $T^*(x_0)$  IS THE ESTIMATED VALUE AT  $x_0$

$T(x_0)$  IS THE TRUE VALUE AT  $x_0$ .

THESE ASSUMPTIONS LEAD TO 5 EQUATIONS  
AND 5 UNKNOWNNS.

# five equations to be solved

FIVE EQUATIONS TO BE SOLVED:

$$\lambda_1 \gamma_{11} + \lambda_2 \gamma_{12} + \lambda_3 \gamma_{13} + \lambda_4 \gamma_{14} + \mu = \gamma_{10}$$

$$\lambda_1 \gamma_{21} + \lambda_2 \gamma_{22} + \lambda_3 \gamma_{23} + \lambda_4 \gamma_{24} + \mu = \gamma_{20}$$

$$\lambda_1 \gamma_{31} + \lambda_2 \gamma_{32} + \lambda_3 \gamma_{33} + \lambda_4 \gamma_{34} + \mu = \gamma_{30}$$

$$\lambda_1 \gamma_{41} + \lambda_2 \gamma_{42} + \lambda_3 \gamma_{43} + \lambda_4 \gamma_{44} + \mu = \gamma_{40}$$

$$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 1$$

FROM THE SMOOTH VARIOGRAM, WE CALCULATE  $\gamma_{ij}$

$$h = 2 \quad \gamma_{12} = \gamma_{21} = 2.25$$

$$h = 4 \quad \gamma_{13} = \gamma_{31} = 4.50$$

$$h = 6 \quad \gamma_{14} = \gamma_{41} = 6.75$$

$$h = 2 \quad \gamma_{23} = \gamma_{32} = 2.25$$

$$h = 4 \quad \gamma_{24} = \gamma_{42} = 4.50$$

$$h = 2 \quad \gamma_{34} = \gamma_{43} = 2.25$$

$$h = 0 \quad \gamma_{11} = \gamma_{22} = \gamma_{33} = \gamma_{44} = 0$$

$$h = 5 \quad \gamma_{10} = 5.625$$

$$h = 3 \quad \gamma_{20} = 3.375$$

$$h = 1 \quad \gamma_{30} = 1.125$$

$$h = 1 \quad \gamma_{40} = 1.125$$

SOLUTION OF FIVE EQUATIONS YIELD:

$$\lambda_1 = 0 \quad \lambda_3 = 0.5 \quad \mu = 0$$

$$\lambda_2 = 0 \quad \lambda_4 = 0.5$$

# kriged value of temp and its variance

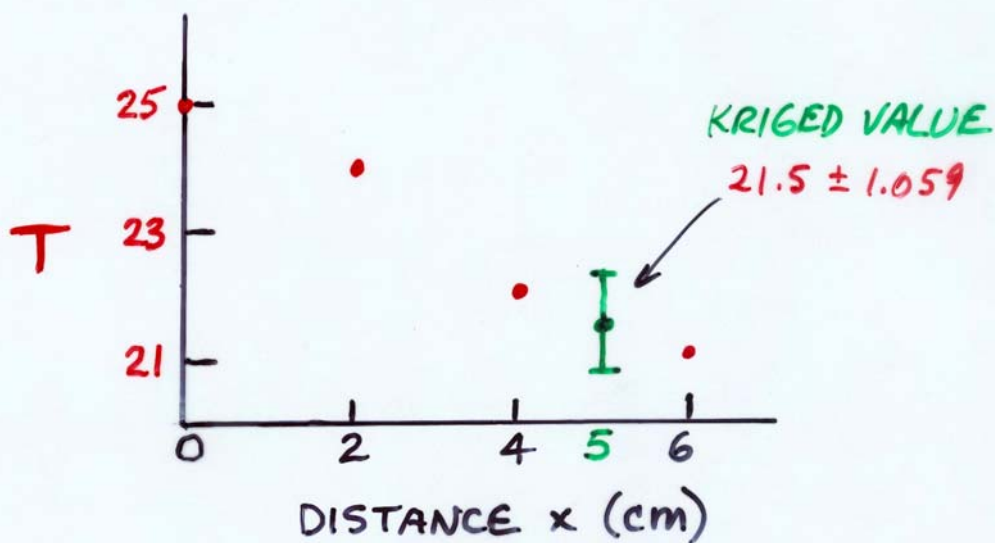
KRIGED VALUE OF TEMPERATURE & ITS VARIANCE ( $x=5$ ):

$$T^*(x_0) = \lambda_1 T(x_1) + \lambda_2 T(x_2) + \lambda_3 T(x_3) + \lambda_4 T(x_4)$$

$$\begin{aligned} T^*(5) &= \lambda_1 \cdot 25 + \lambda_2 \cdot 24 + \lambda_3 \cdot 22 + \lambda_4 \cdot 21 \\ &= 0 \cdot 25 + 0 \cdot 24 + 0.5 \cdot 22 + 0.5 \cdot 21 = 21.5 \end{aligned}$$

$$\sigma^2(x_0) = \lambda_1 \gamma_{10} + \lambda_2 \gamma_{20} + \lambda_3 \gamma_{30} + \lambda_4 \gamma_{40} + \mu$$

$$\begin{aligned} \sigma^2(5) &= 0 \cdot 5.625 + 0 \cdot 3.375 + 0.5 \cdot 1.125 + 0.5 \cdot 1.125 + 0 \\ &= 1.125 \end{aligned}$$



# CROSS VARIOGRAMS

# cross variogram calc. A & B

## CROSS VARIOGRAM

### OBSERVATION A

$A_1$	$A_2$	$A_3$	$A_4$	$A_5$	$A_6$	$A_7 \dots$	$A_i \dots$	$A_n$
○	○	○	○	○	○	○	○	○
$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	$x_7 \dots$	$x_i \dots$	$x_n$

### OBSERVATION B

$B_1$	$B_2$	$B_3$	$B_4$	$B_5$	$B_6$	$B_7 \dots$	$B_i \dots$	$B_n$
○	○	○	○	○	○	○	○	○
$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	$x_7 \dots$	$x_i \dots$	$x_n$

$$\Gamma(h) = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} [A(x_i) - A(x_i + h)][B(x_i) - B(x_i + h)]$$

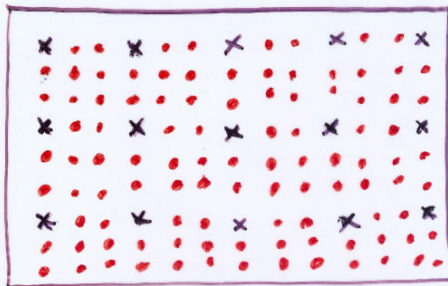
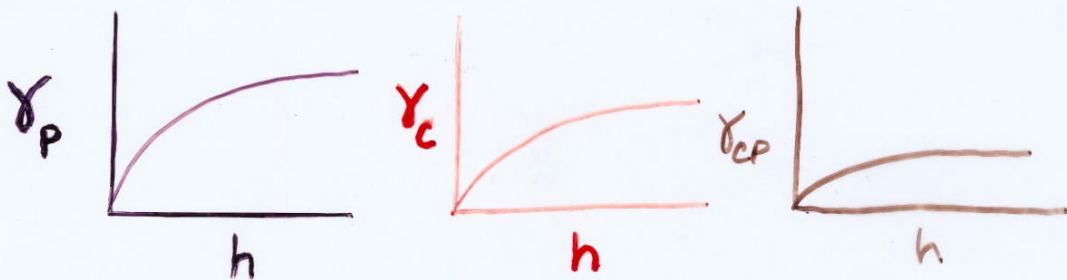


# cross variogram calculation scheme

## CROSS VARIOGRAMS

TAKE ADVANTAGE OF ONE EASILY MEASURED PARAMETER TO PREDICT ANOTHER MORE DIFFICULT OR MORE EXPENSIVELY MEASURED PARAMETER

1. AVAILABLE P AND CLAY
2. PESTICIDE LEACHING AND CLAY
3. SOIL TEMPERATURE AND SOIL WATER CONTENT



$$x^* = \lambda_1(x_1) + \lambda_2 x_2 \dots + \lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 \dots$$



# CROP YIELD AND SOIL EROSION

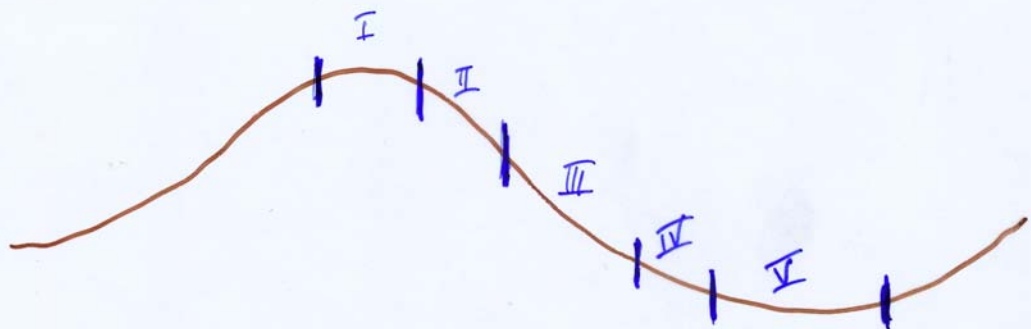
# erosion study

## EROSION STUDY

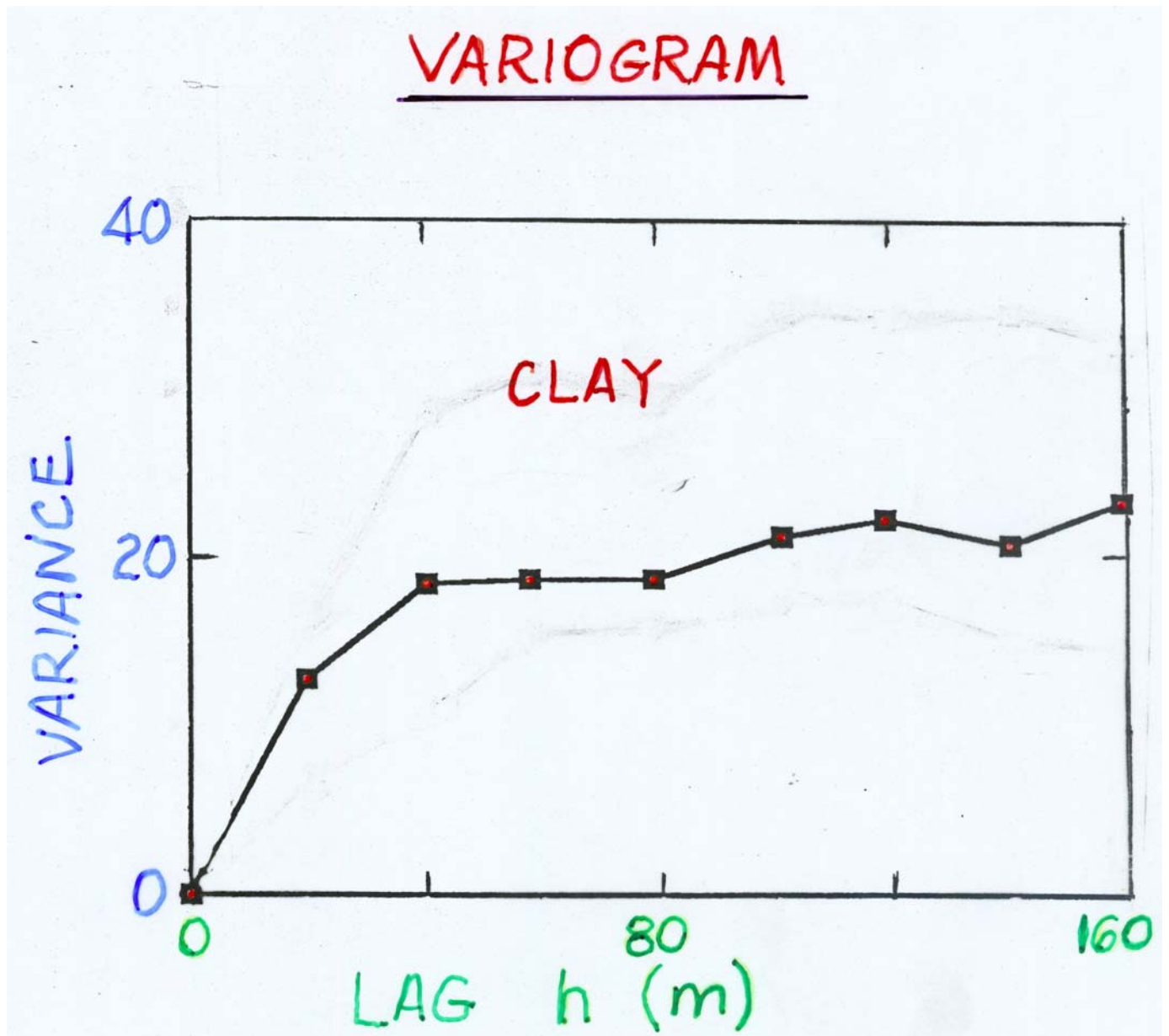
### SEVERAL GEOLOGIC CYCLES OF GENTLE UPLIFTING & EROSION

- HILLY & ROLLING RELIEF SEEN TODAY
- WHEAT BIOMASS INCREASES FROM KNOLLS TO SWALES
- CLAY & OTHER PROPERTIES VARY BY LANDSCAPE POSITION

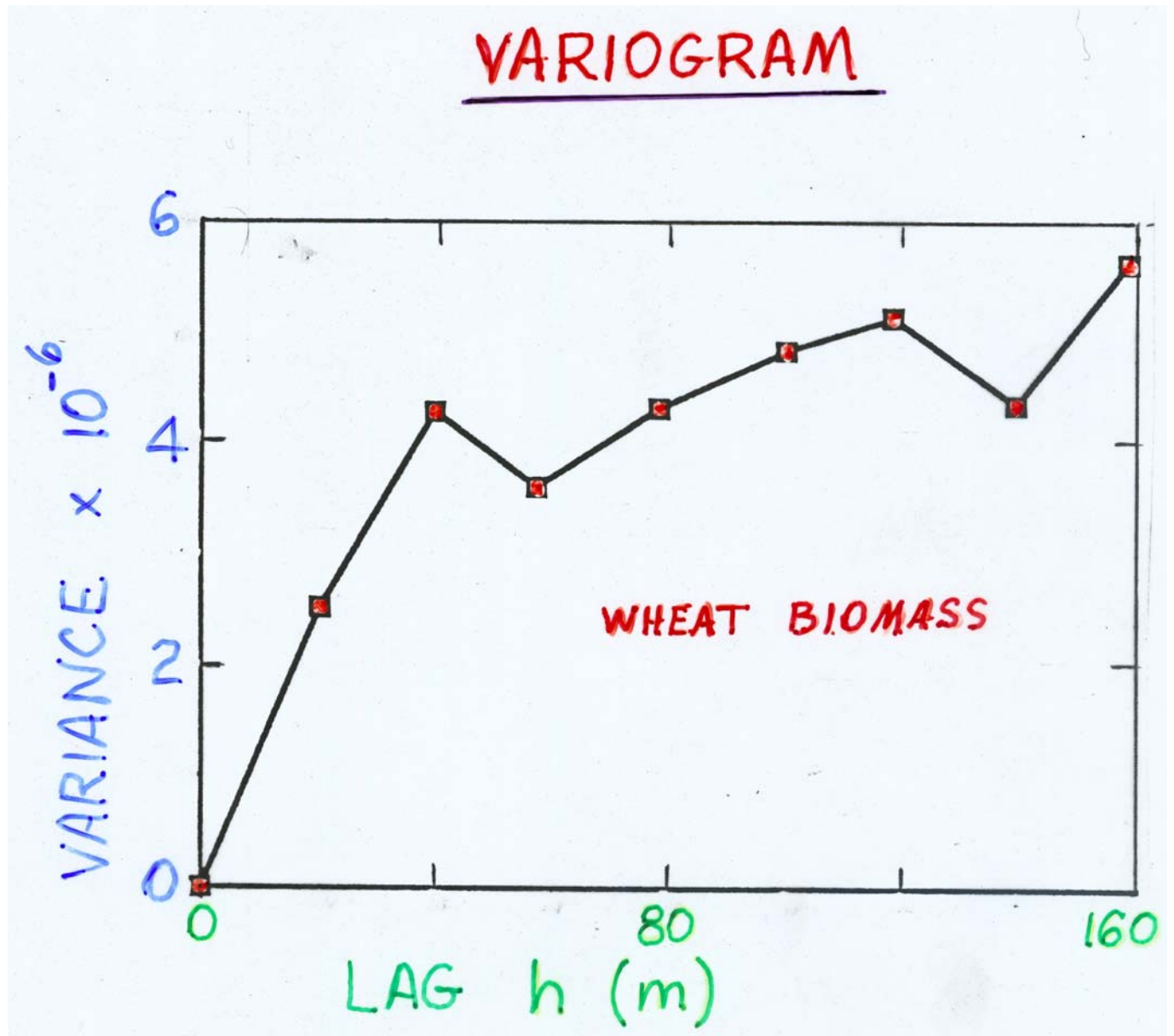
### SAMPLED WHEAT & CLAY CONTENT ON A 20-M GRID



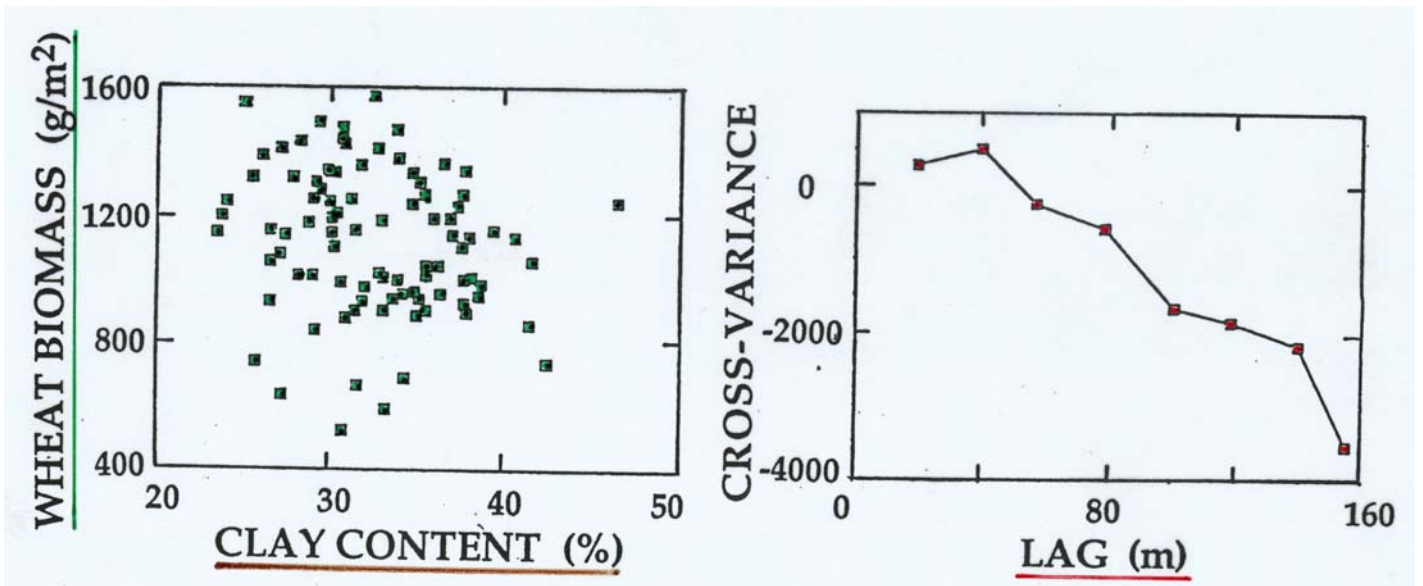
# clay variogram



# wheat biomass variogram

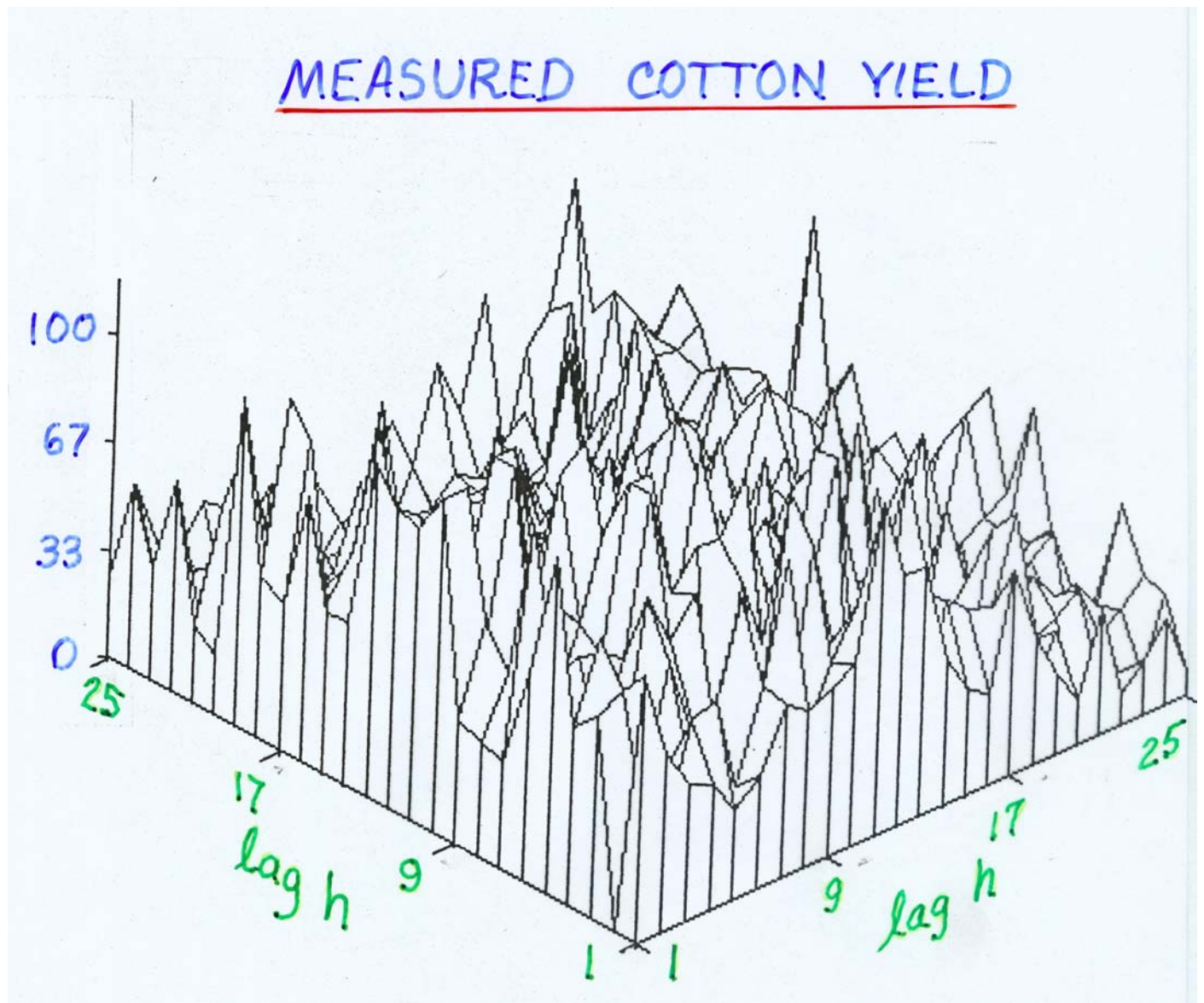


# cross variogram wheat biomass vs clay



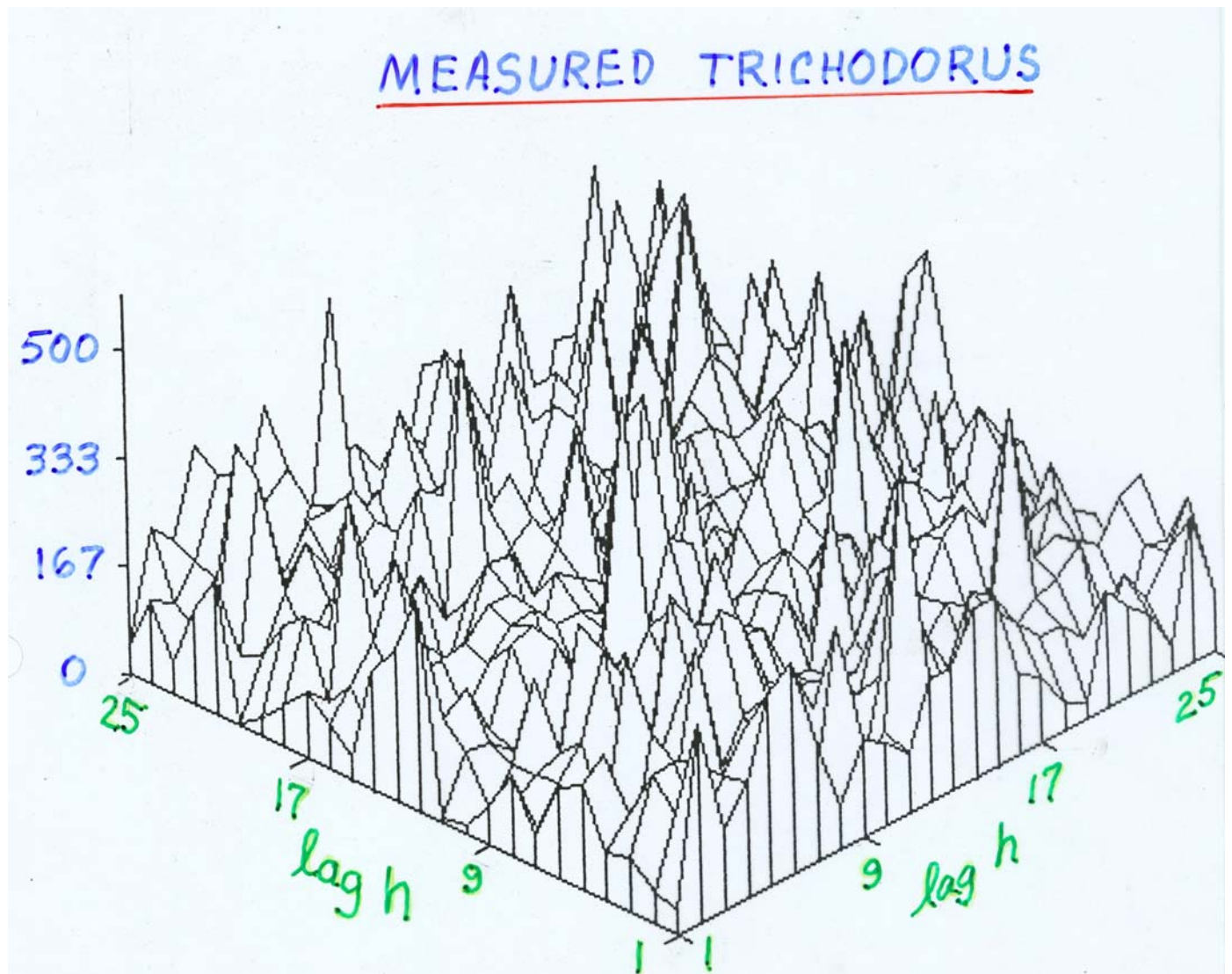
# CROP YIELD AND NEMATODES

# cotton yield (nematodes)

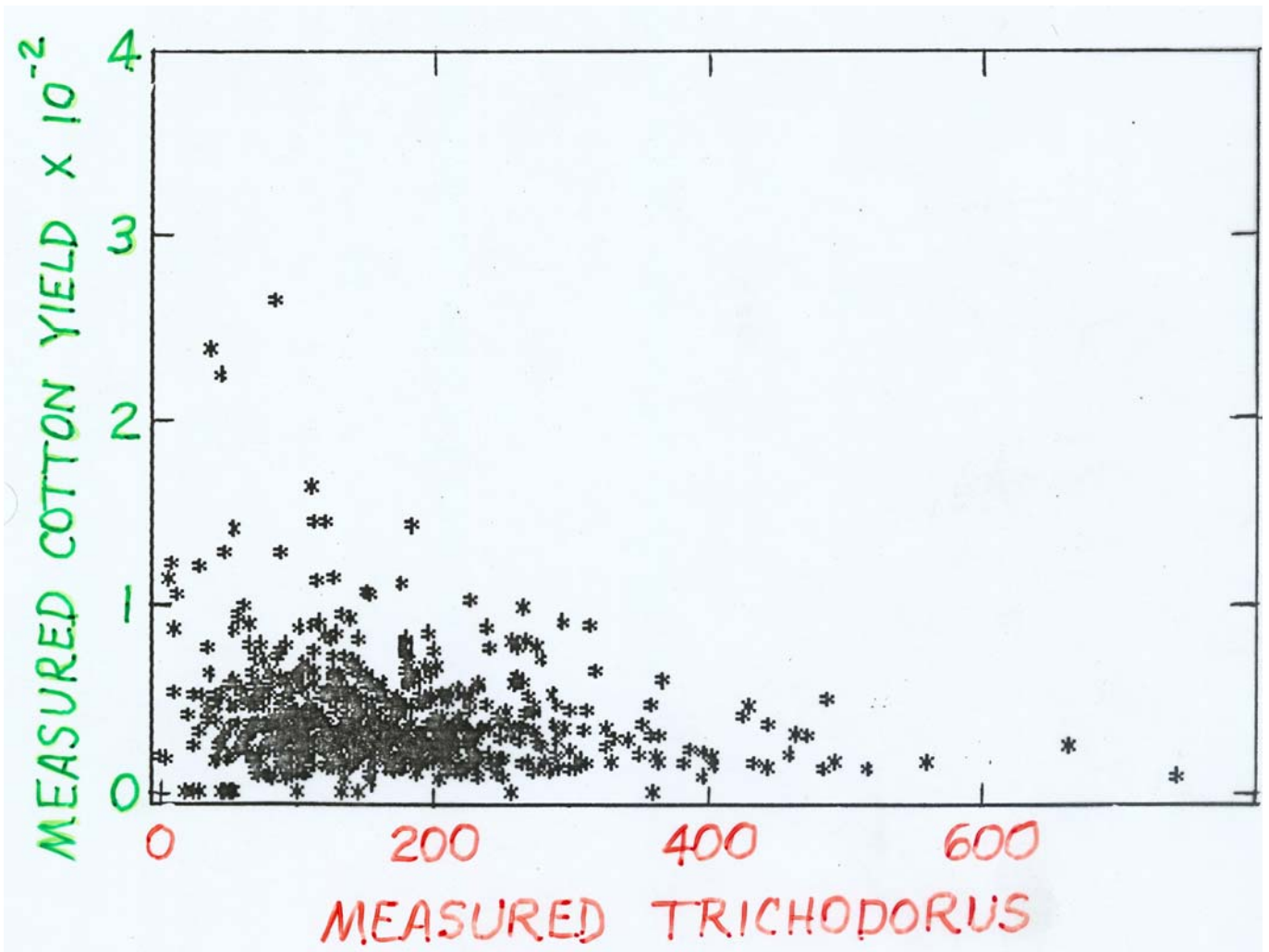




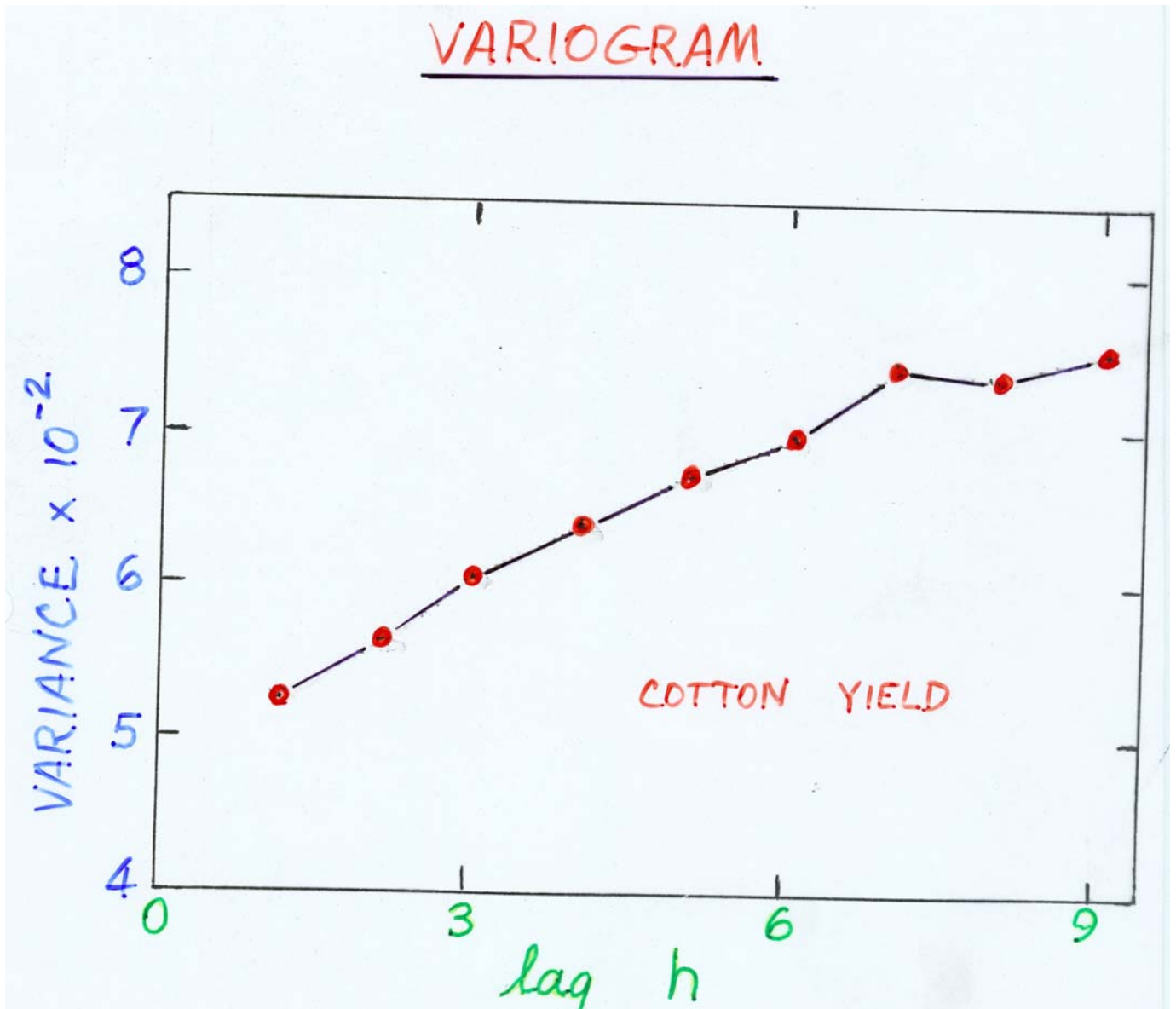
# trichodorus (cotton)



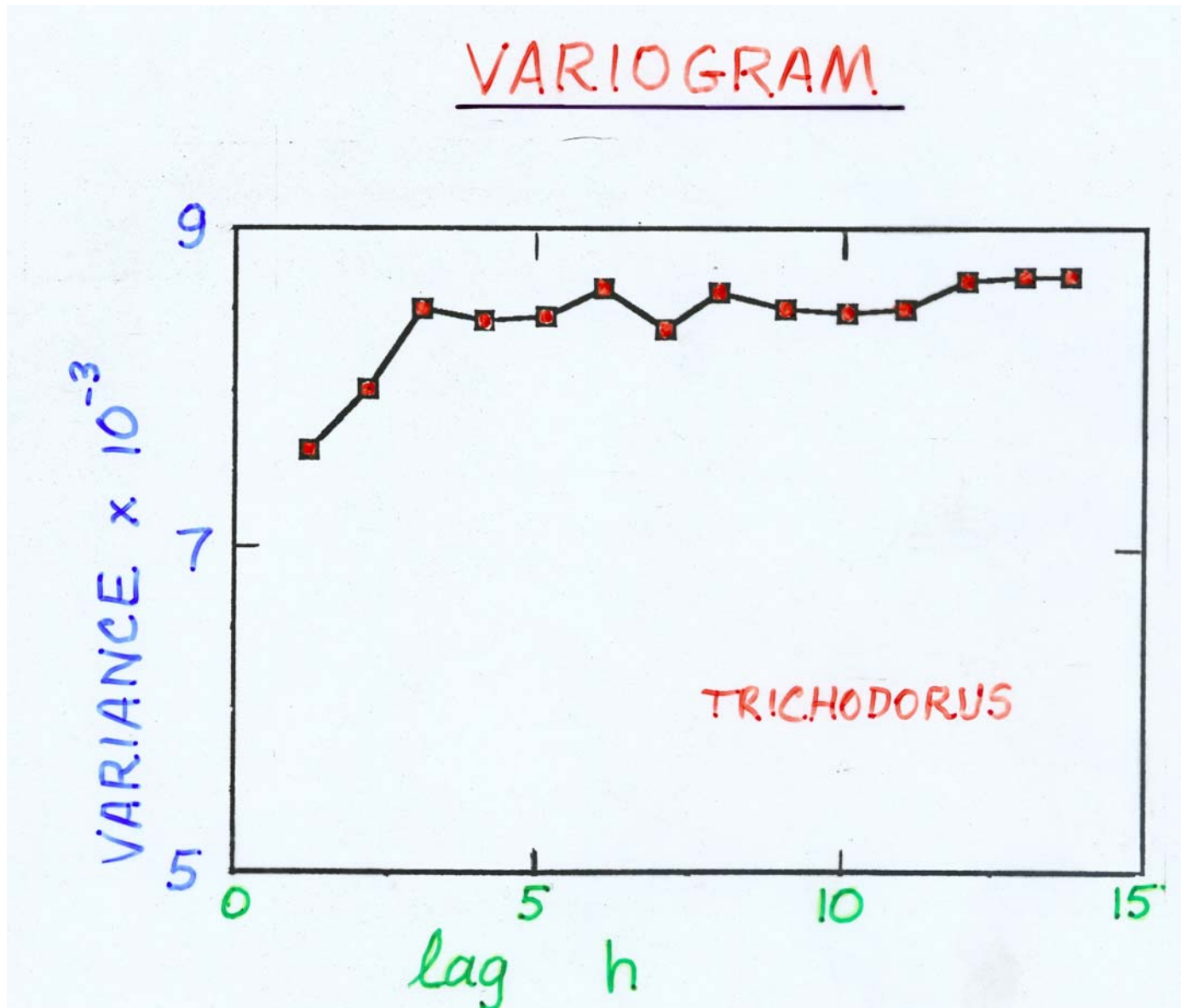
# cotton yield vs trichodorus



# variogram cotton yld (nematodes)

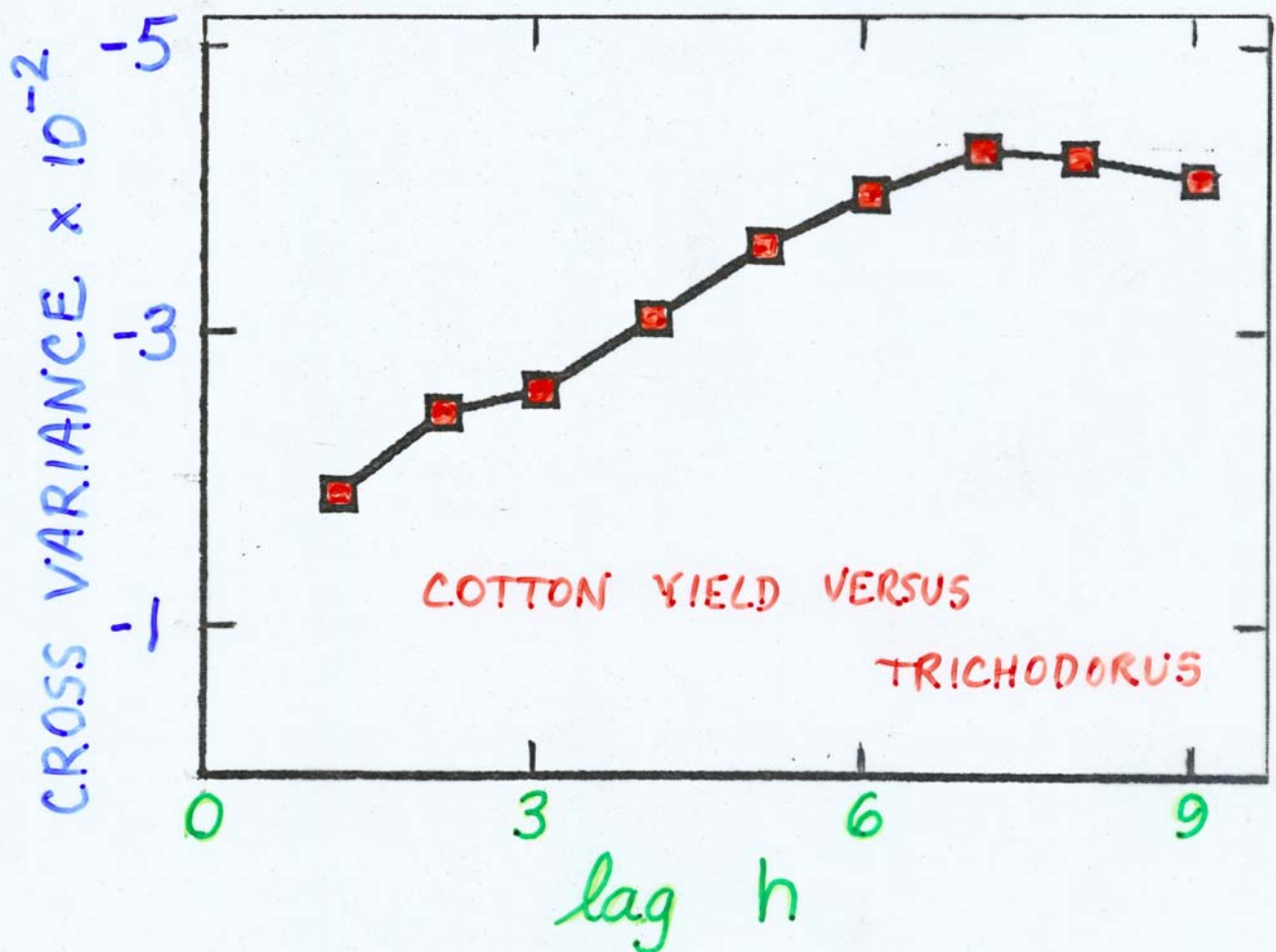


# variogram nematodes (cotton yld)

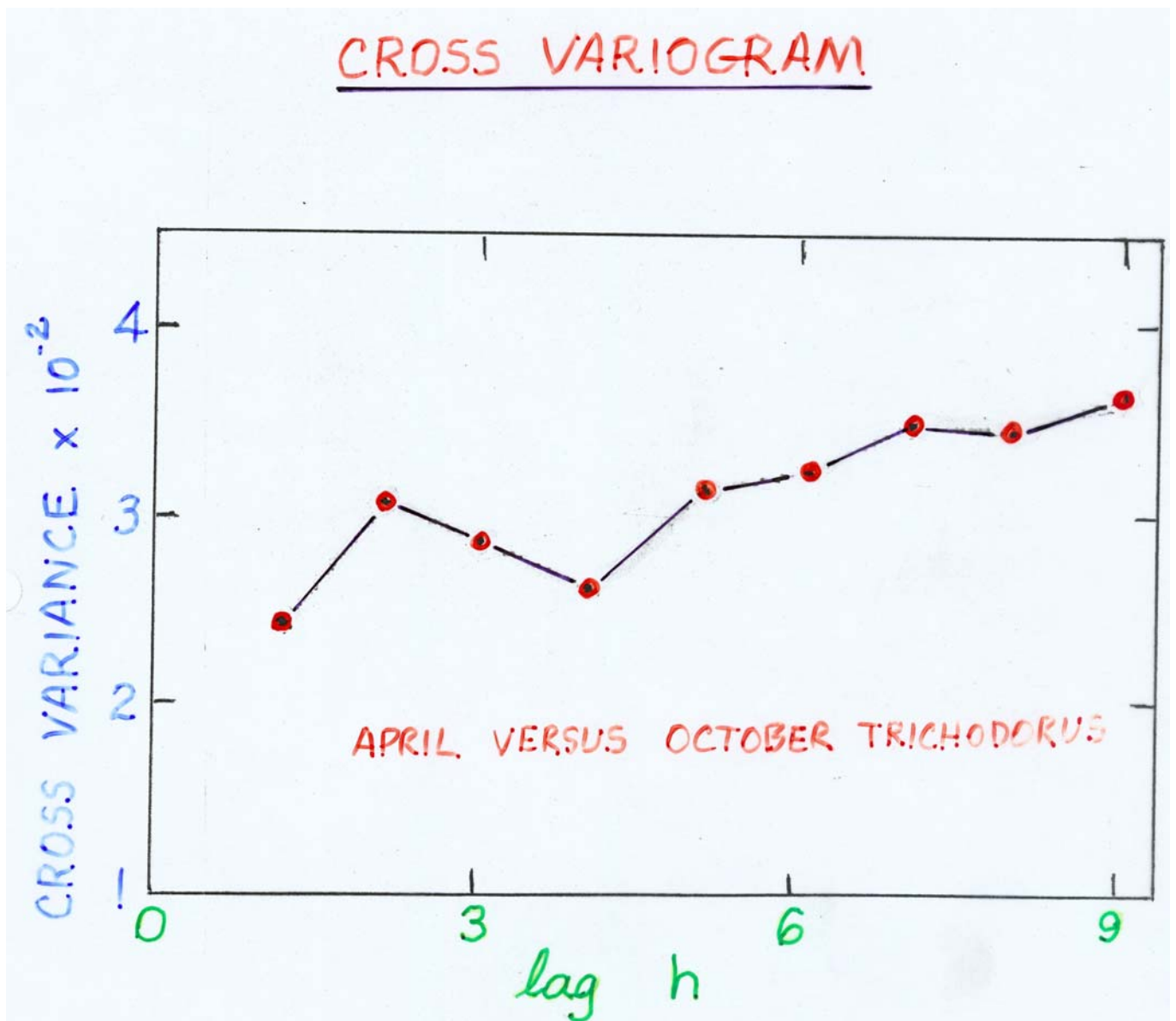


# cross variogram cotton vs nematodes

## CROSS VARIOGRAM!

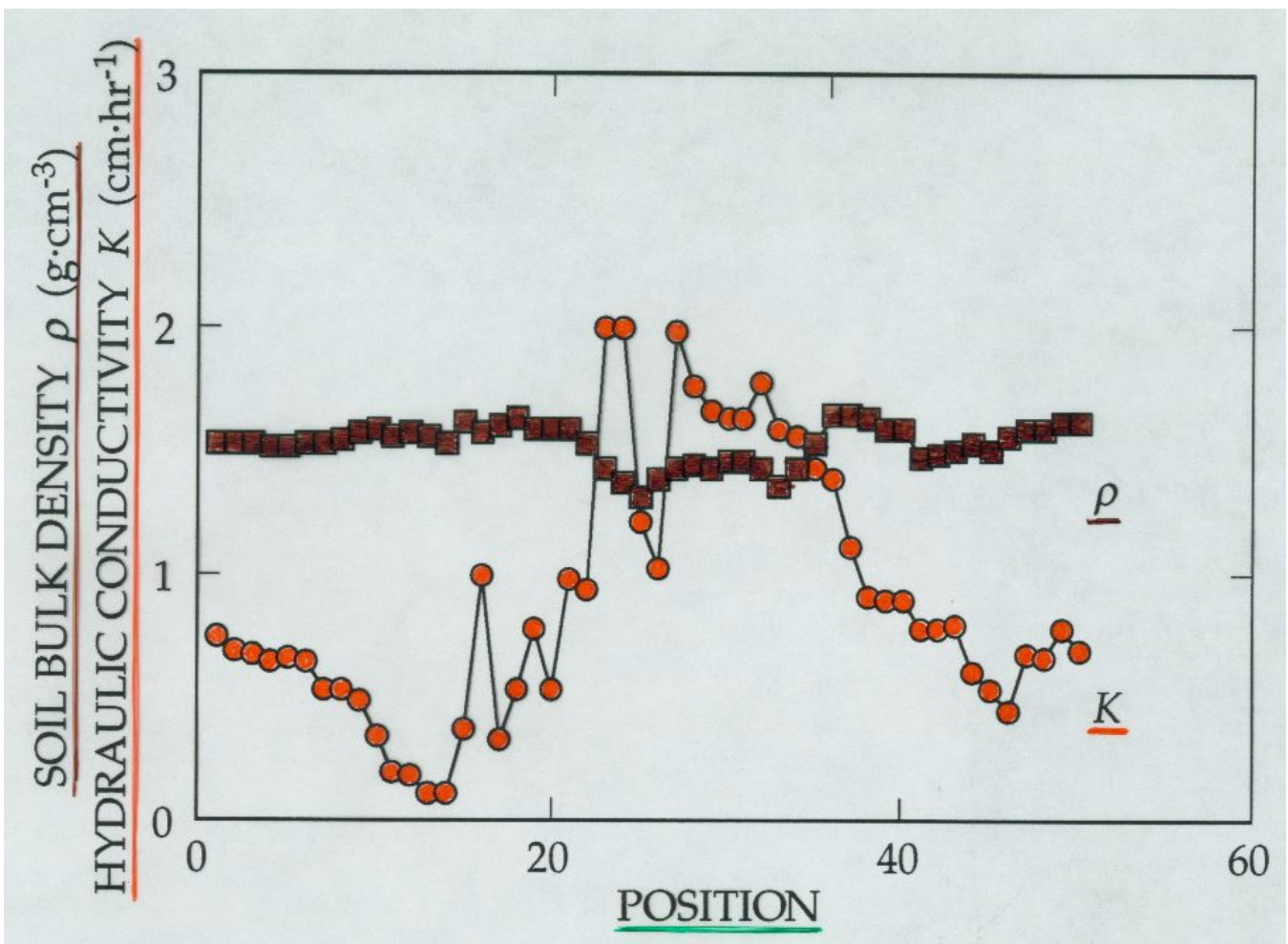


# cross variogram April vs Oct nematodes



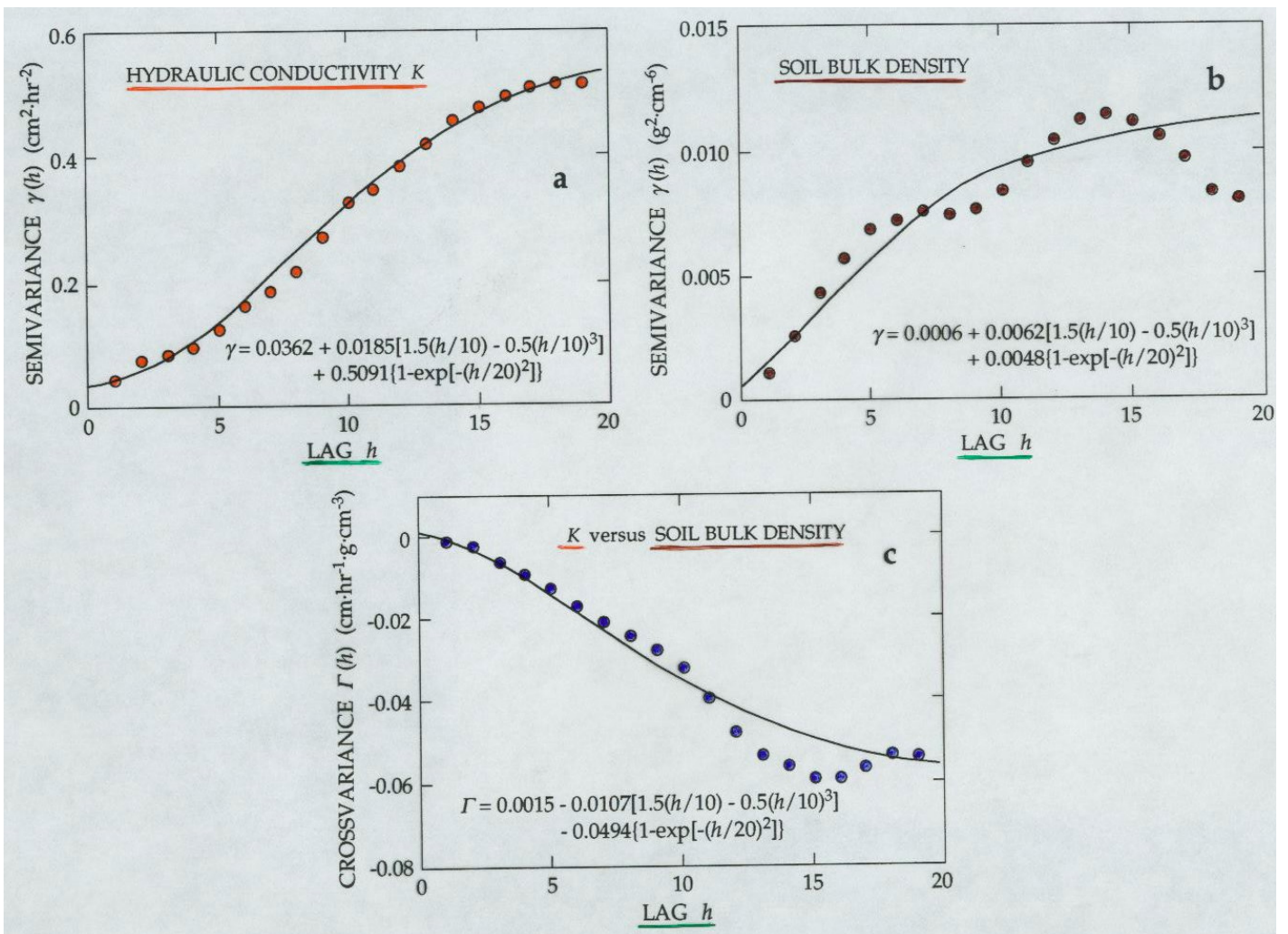
COKRIGING SOIL HYDRAULIC CONDUCTIVITY  
WITH SOIL BULK DENSITY

# K and BD versus distance

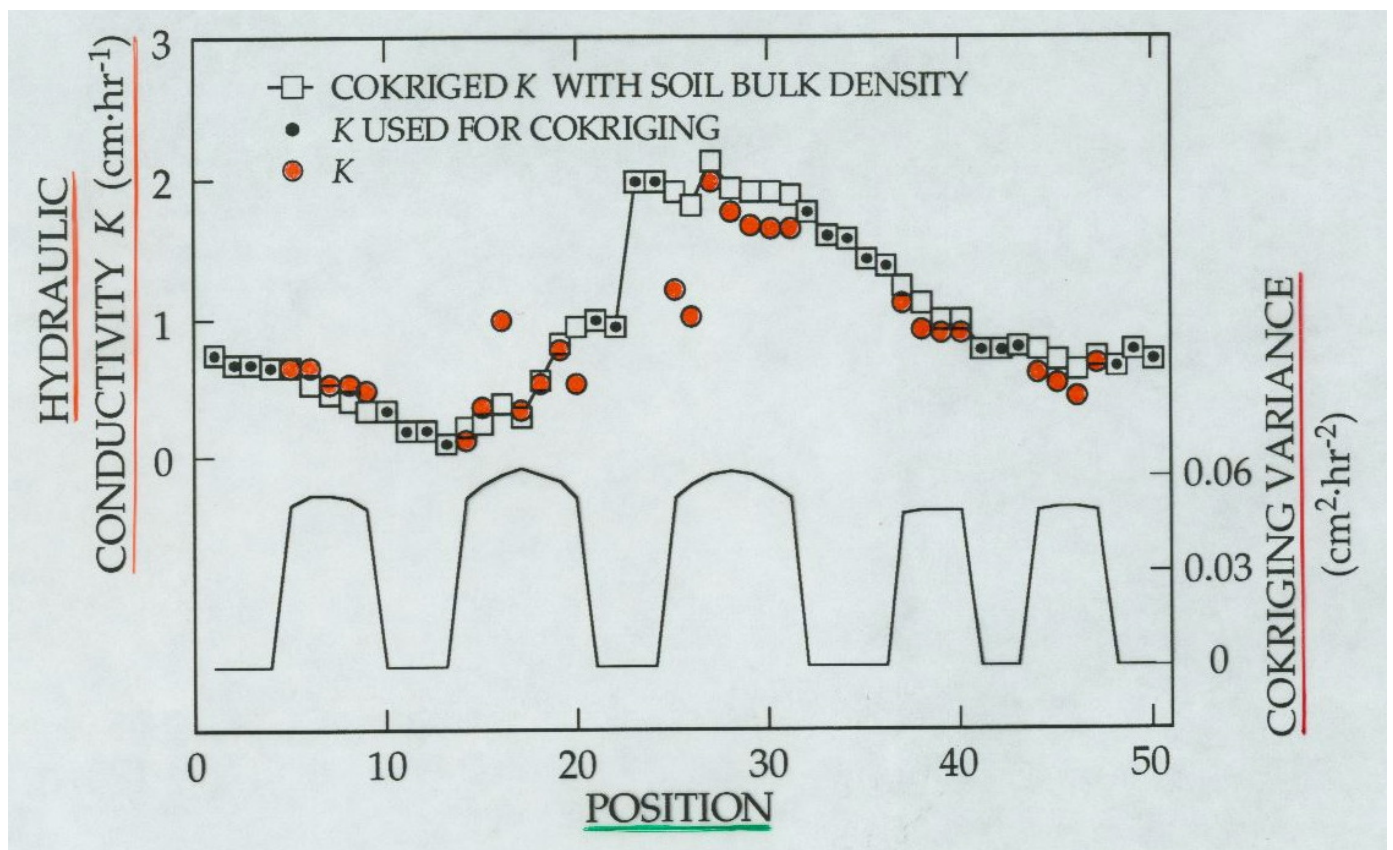




# variograms&covar K&BD

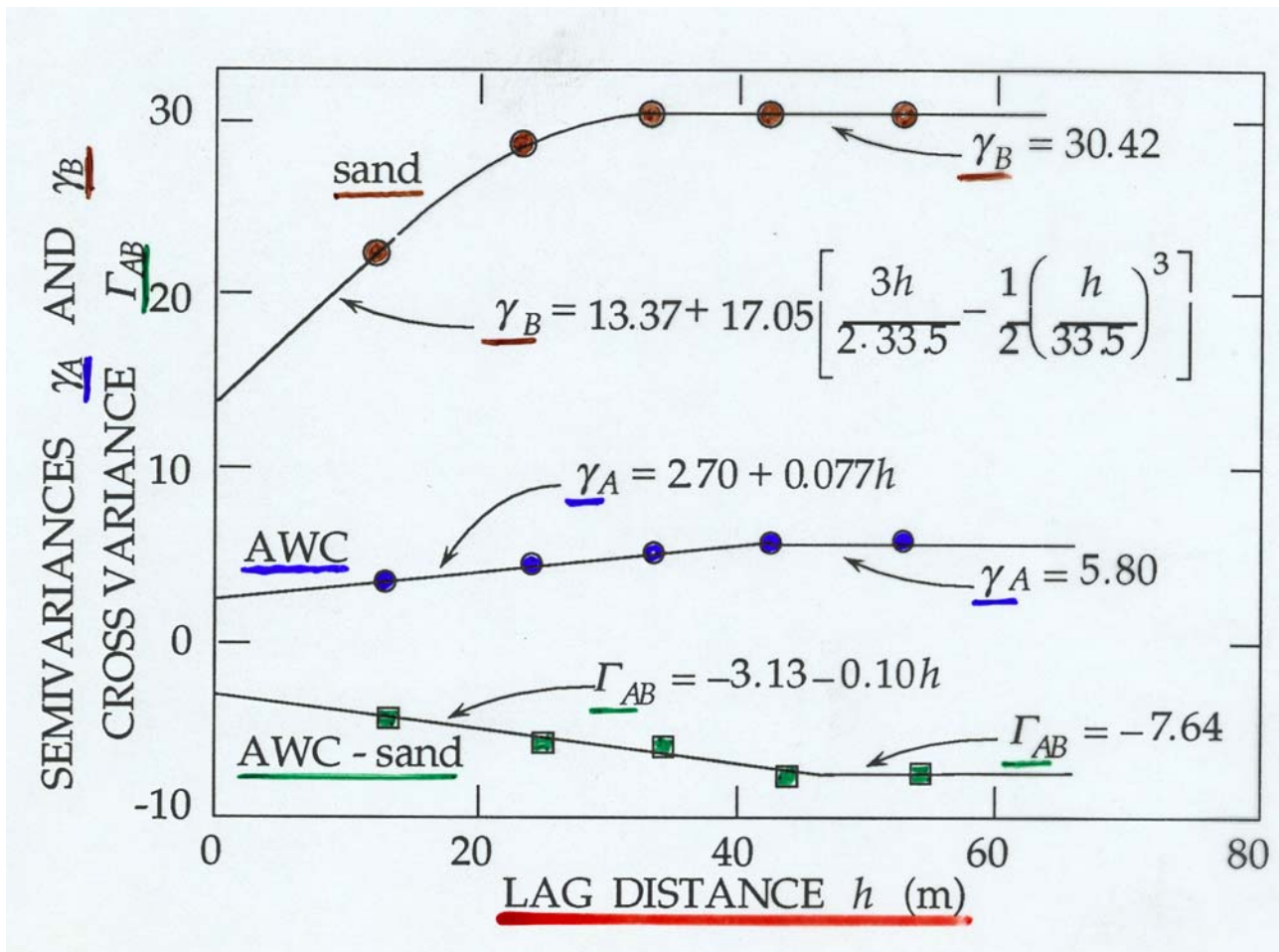


# cokriged K with BD



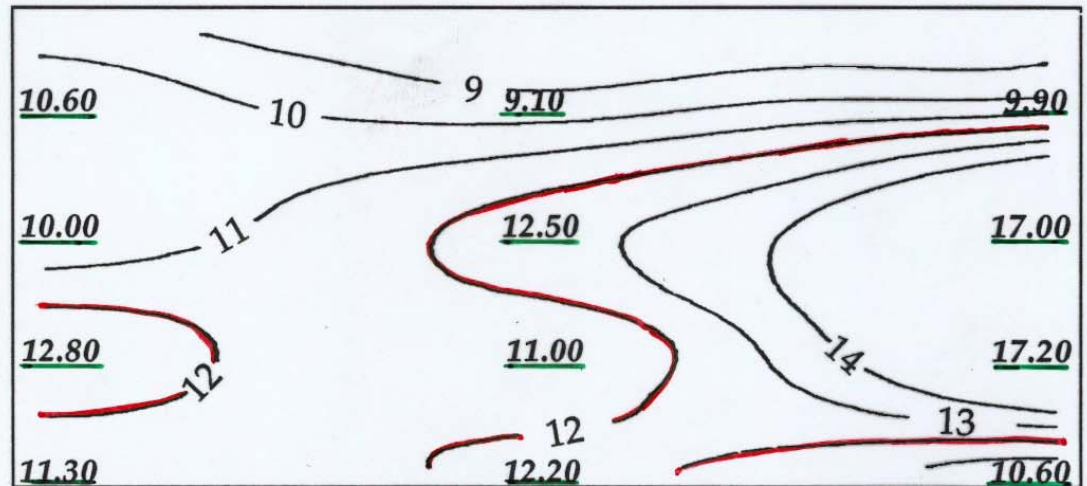
COKRIGING AVAILABLE WATER CONTENT  
WITH SAND CONTENT

# variograms & covariogram of AWC & sand

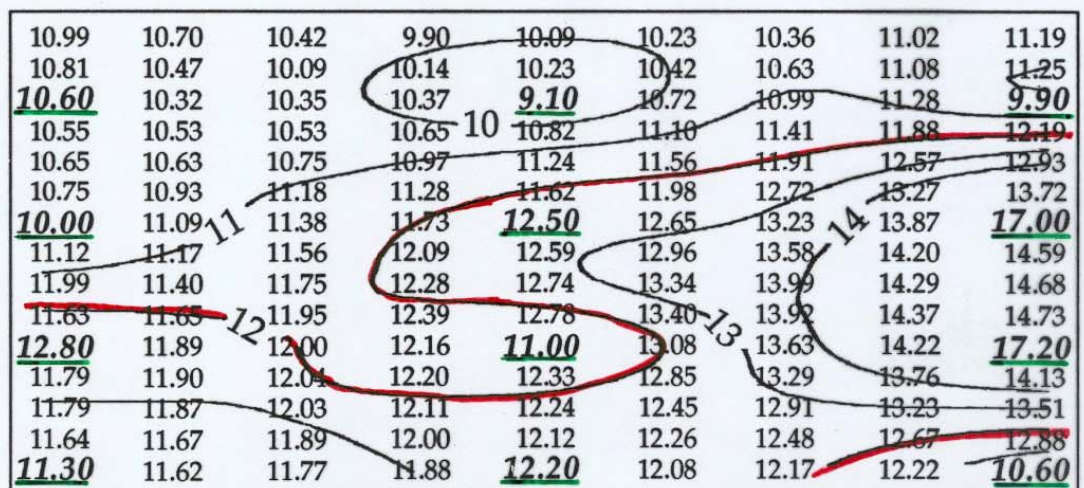


# original data, kriged & cokrige

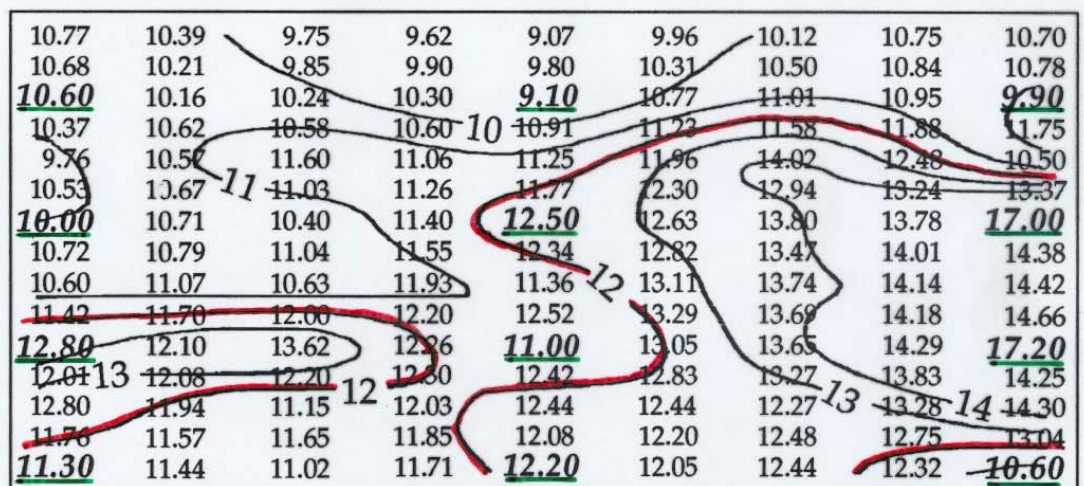
ORIGINAL  
DATA



KRIGED



COKRIGED

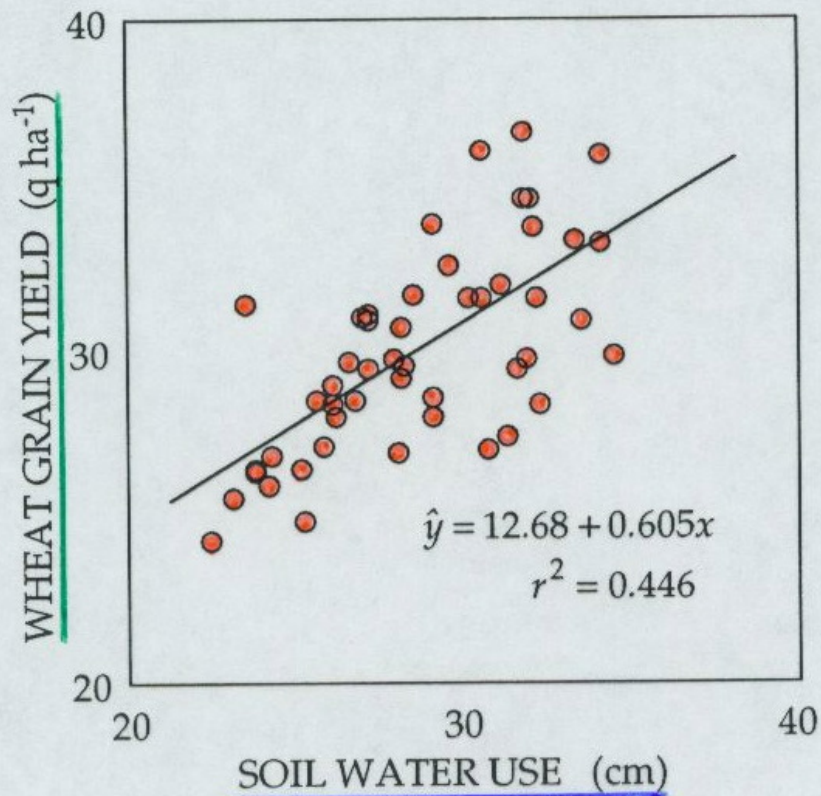
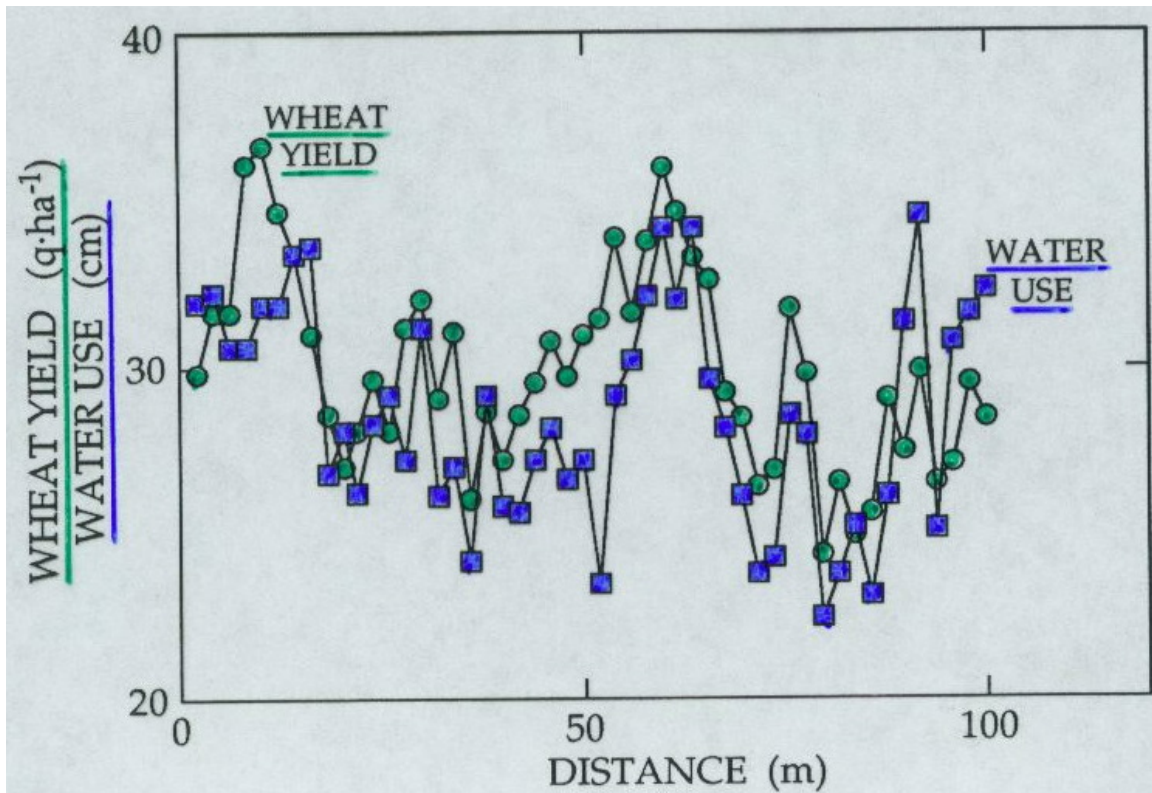


WHEAT YIELD

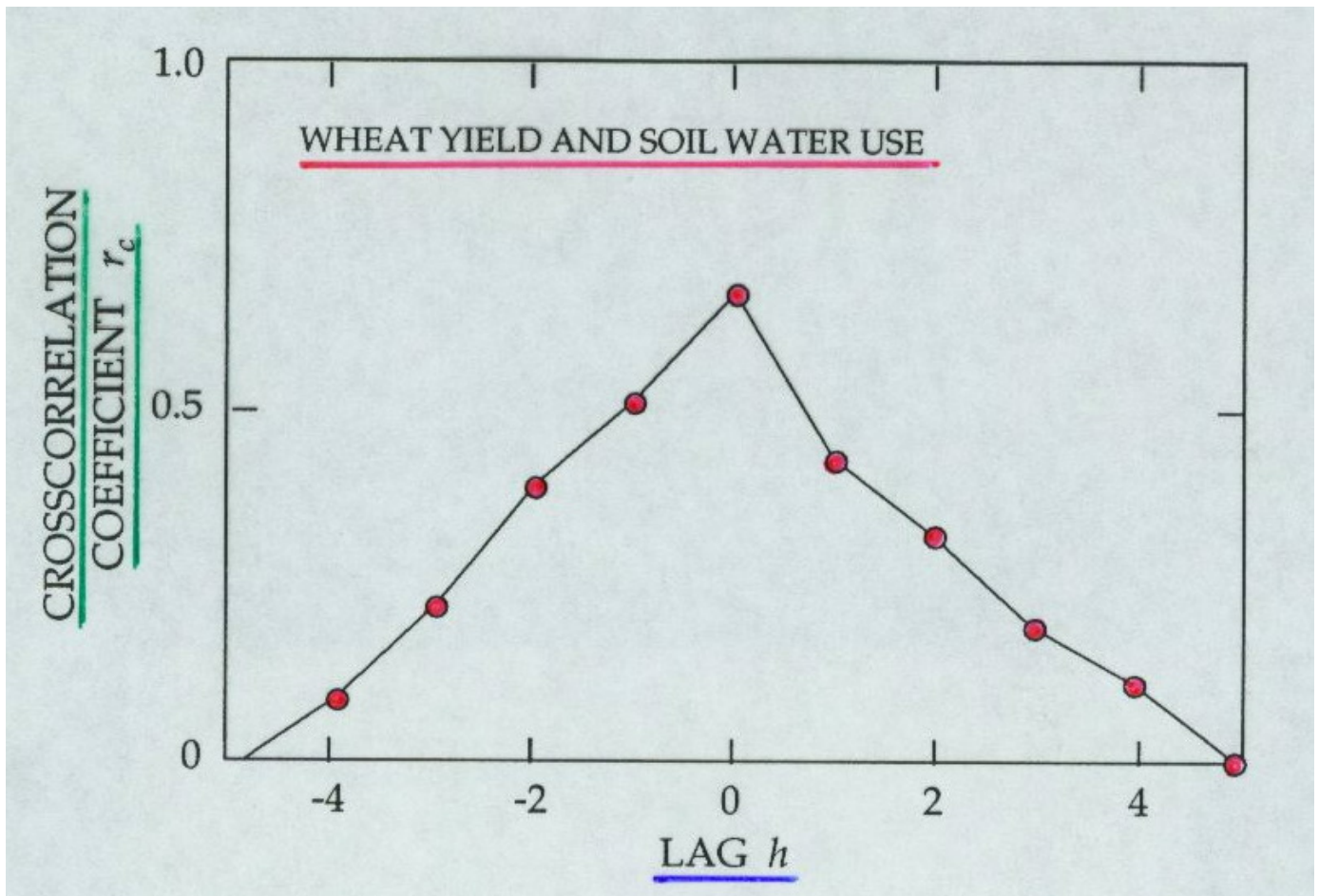
AND

WATER USE

# wheat versus water use

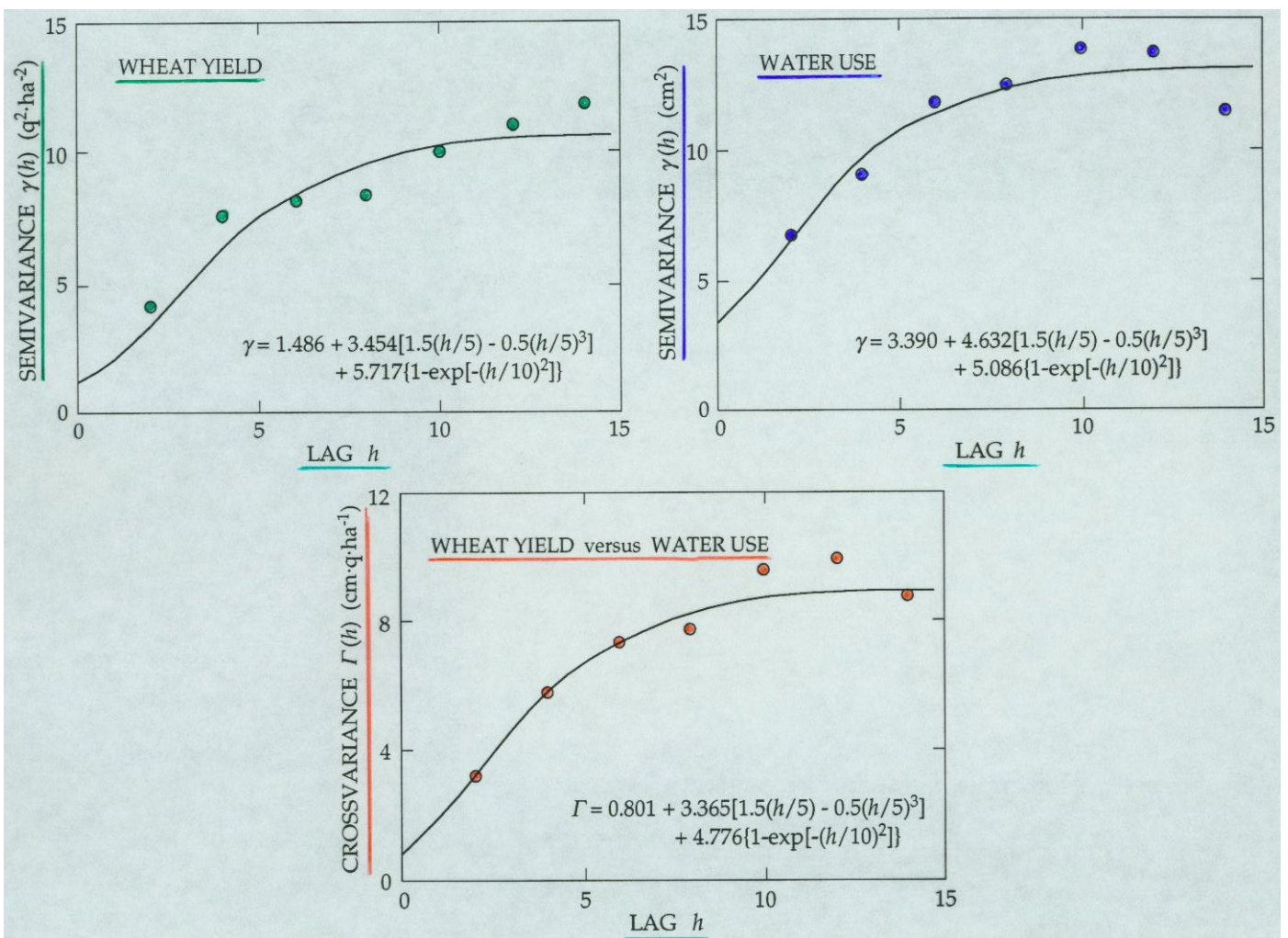


# CCF wheat&water use

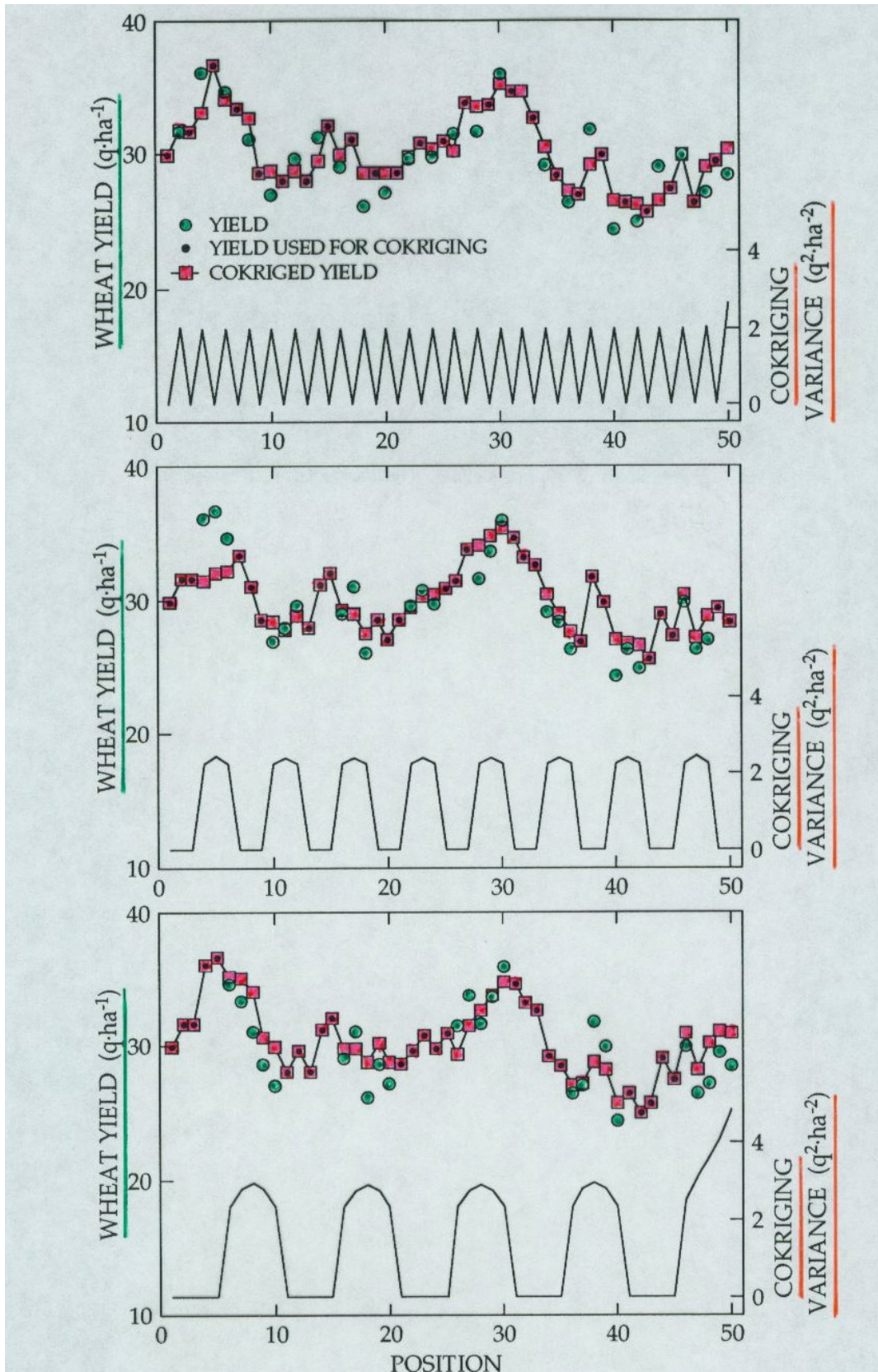




# wheat & water use vario & covariogram



# wheat & water use cokriging



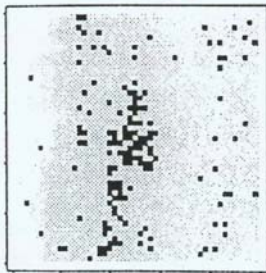
## SPATIO-TEMPORAL ANALYSIS OF DOWNY MILDEW IN A FIELD OF CABBAGE

- MODELING AND PREDICTING THE SPATIAL PATTERN OF THE DISEASE AT ANY TIME
- DEVELOPING SAMPLING SCHEMES FOR FUTURE ASSESSMENT
- DETERMINING THE INITIAL LOCATION AND SPREADING RATE OF DISEASE

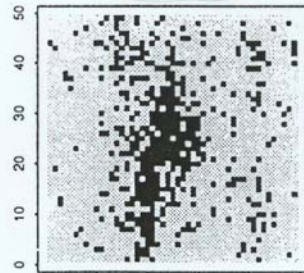
# downy mildew in cabbage

## DOWNY MILDEW IN CABBAGE

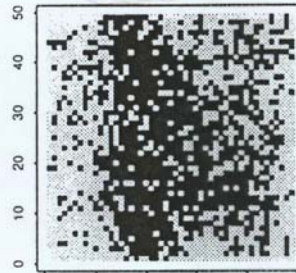
5 June



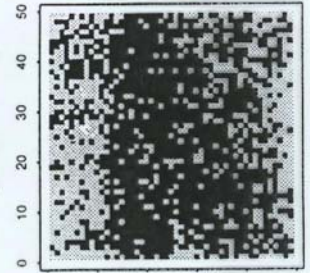
10 June



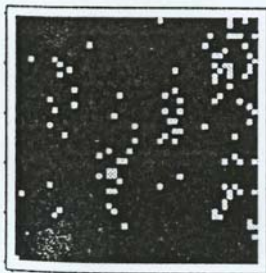
18 June



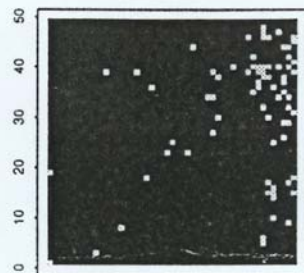
25 June



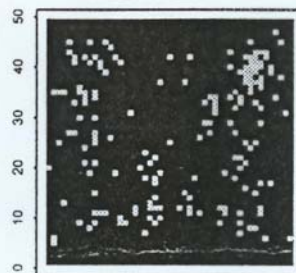
1 July



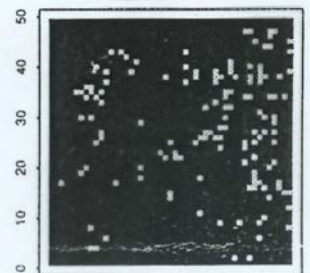
8 July



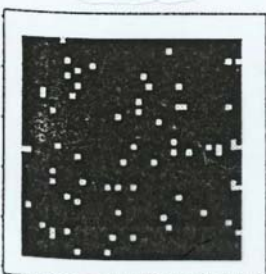
15 July



22 July



29 July



5 August

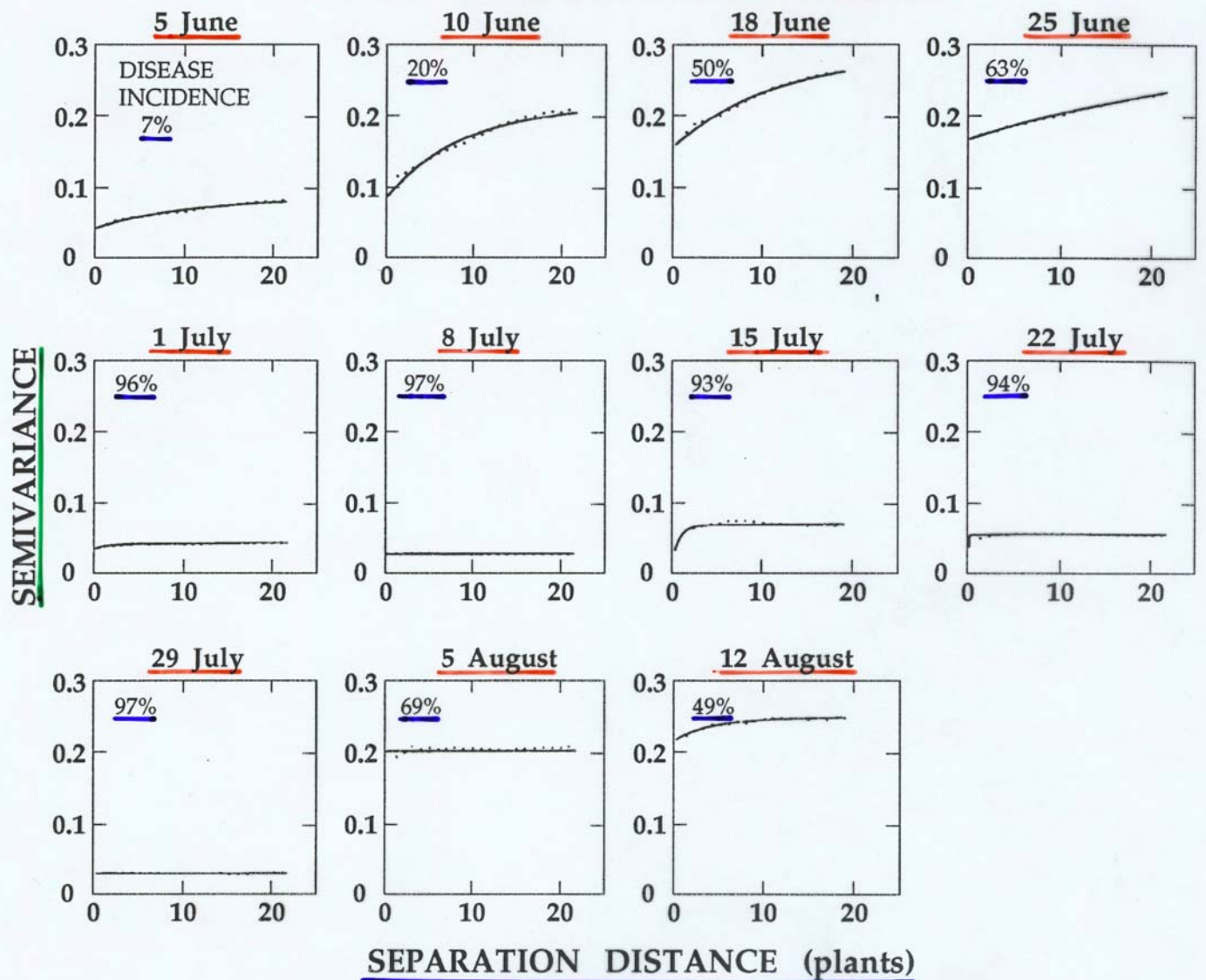


12 August



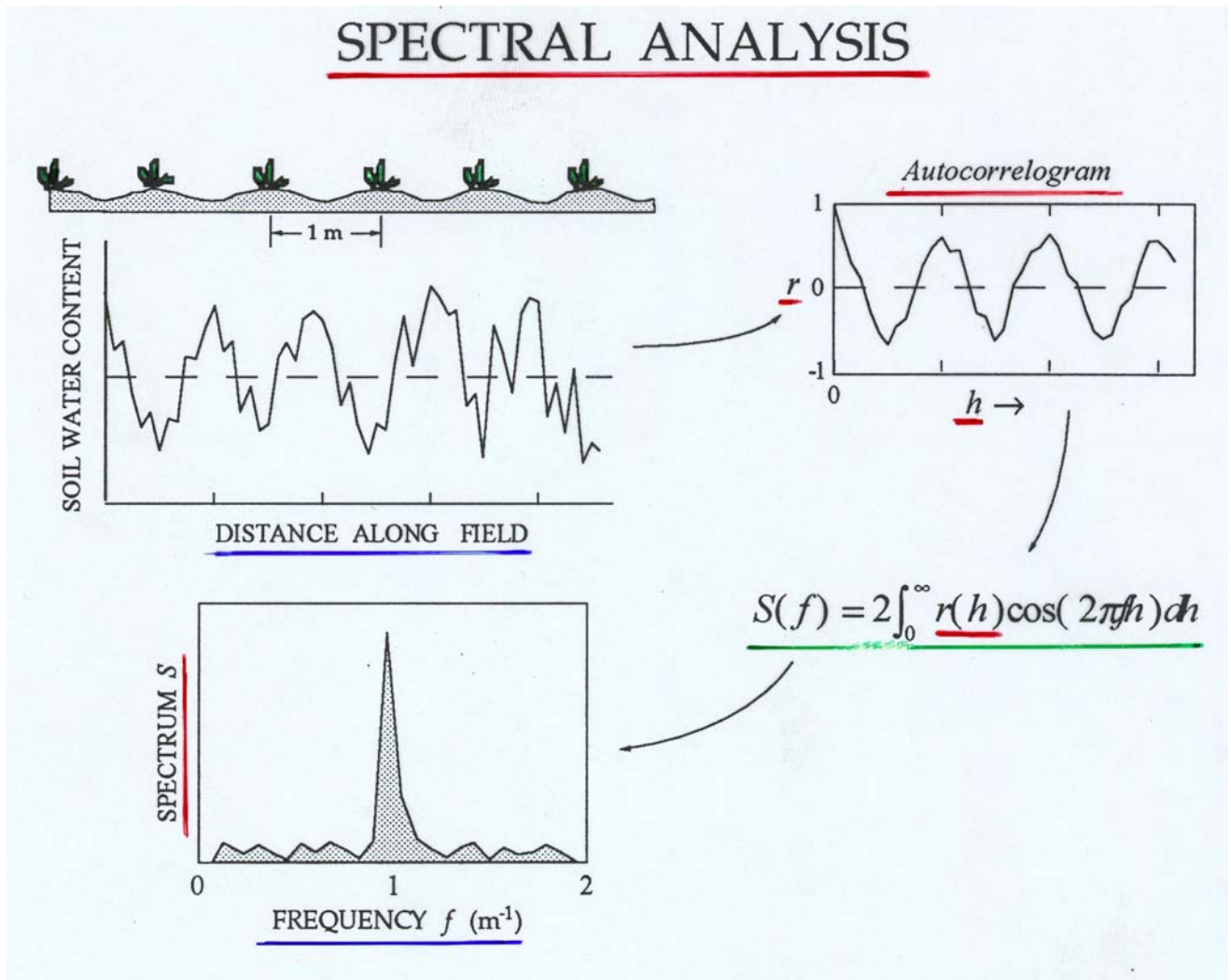
# variograms downy mildew

## DOWNY MILDEW IN CABBAGE

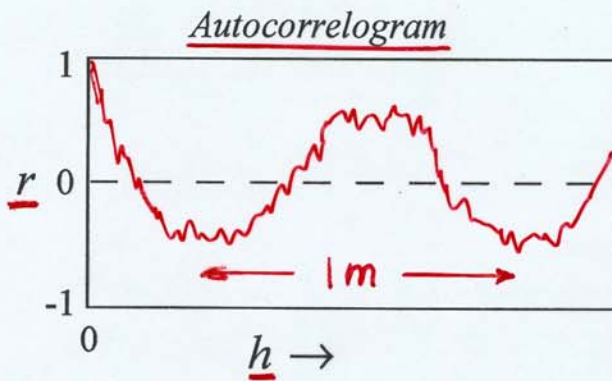
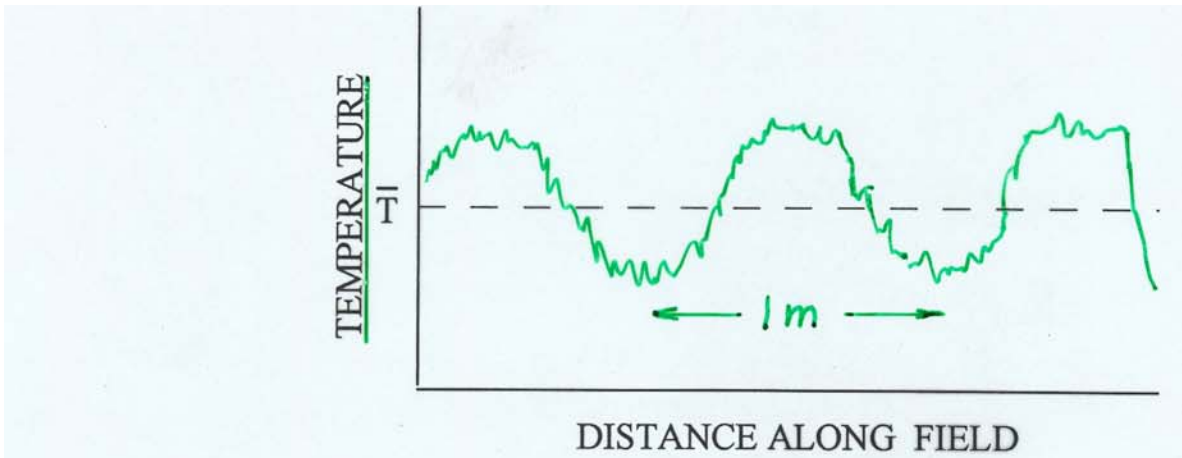


# SPECTRAL ANALYSIS

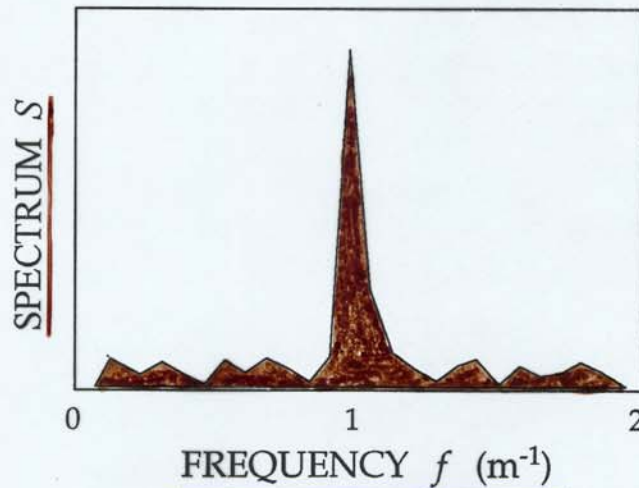
# spectral analysis illustration



# spectral analysis concept calc

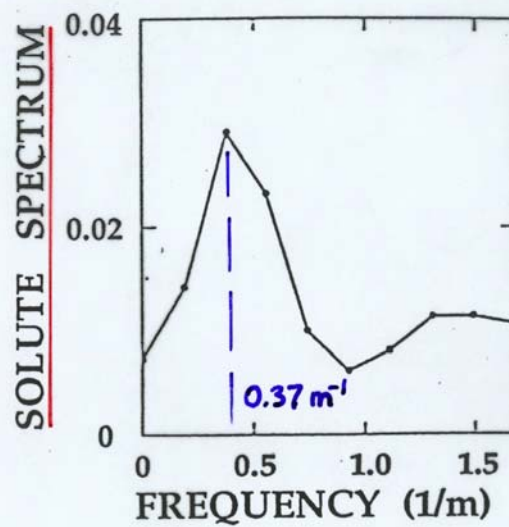
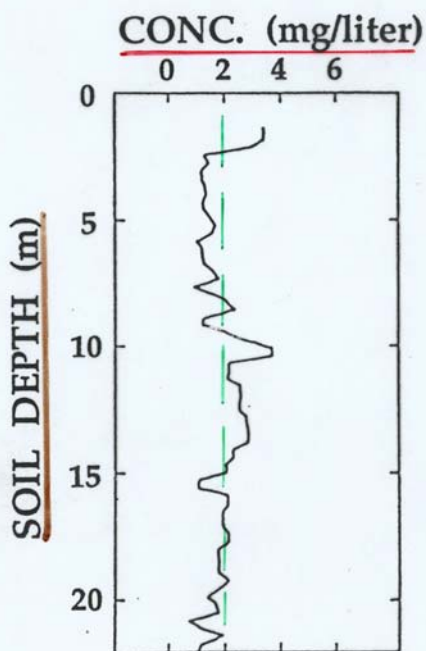


$$S(f) = 2 \int_0^{\infty} r(h) \cos(2\pi fh) dh$$





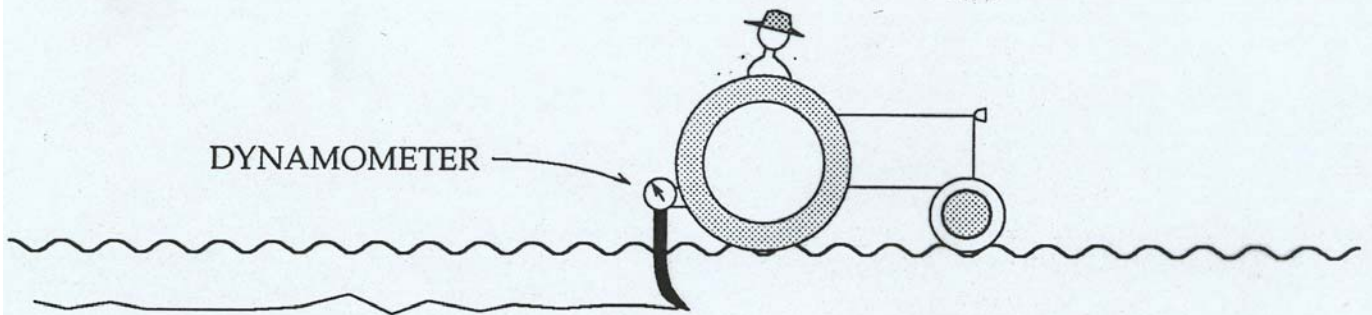
# soil depth solute spectrum NM



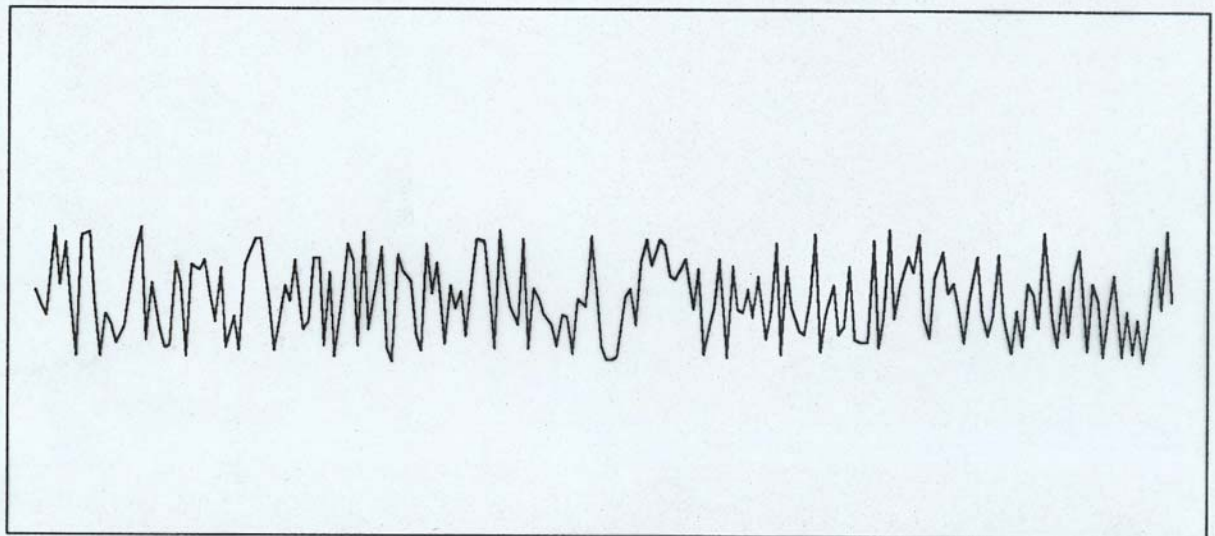
$0.37\text{m}^{-1} \sim 2.7\text{m}$  DISTANCE TRAVEL  
DEEP PERCOLATION  
IRRIGATION SEASON  
IN DESERT ENVIRONMENT

# soil compaction dynamometer

## SOIL COMPACTION



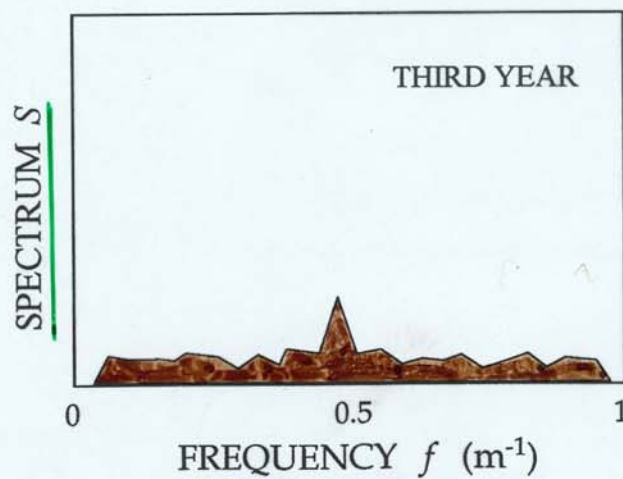
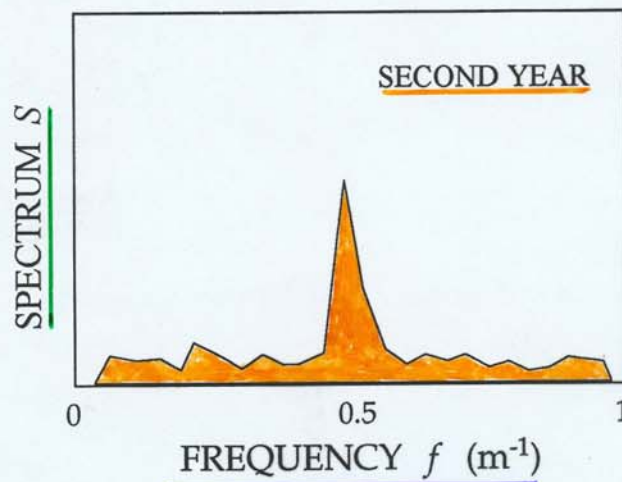
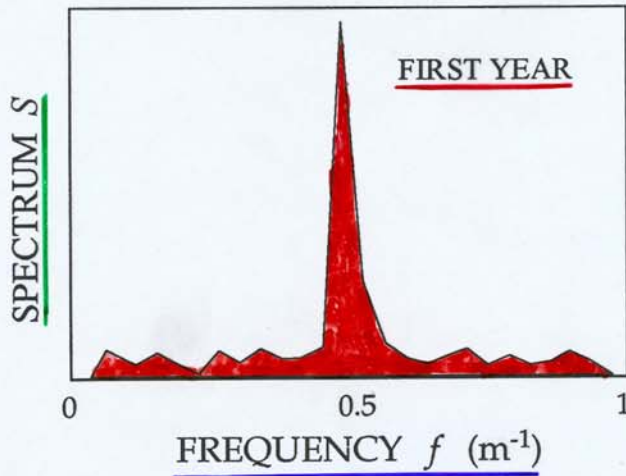
DRAWBAR PULL (newtons)



DISTANCE

# drawbar pull spectra 3 yrs

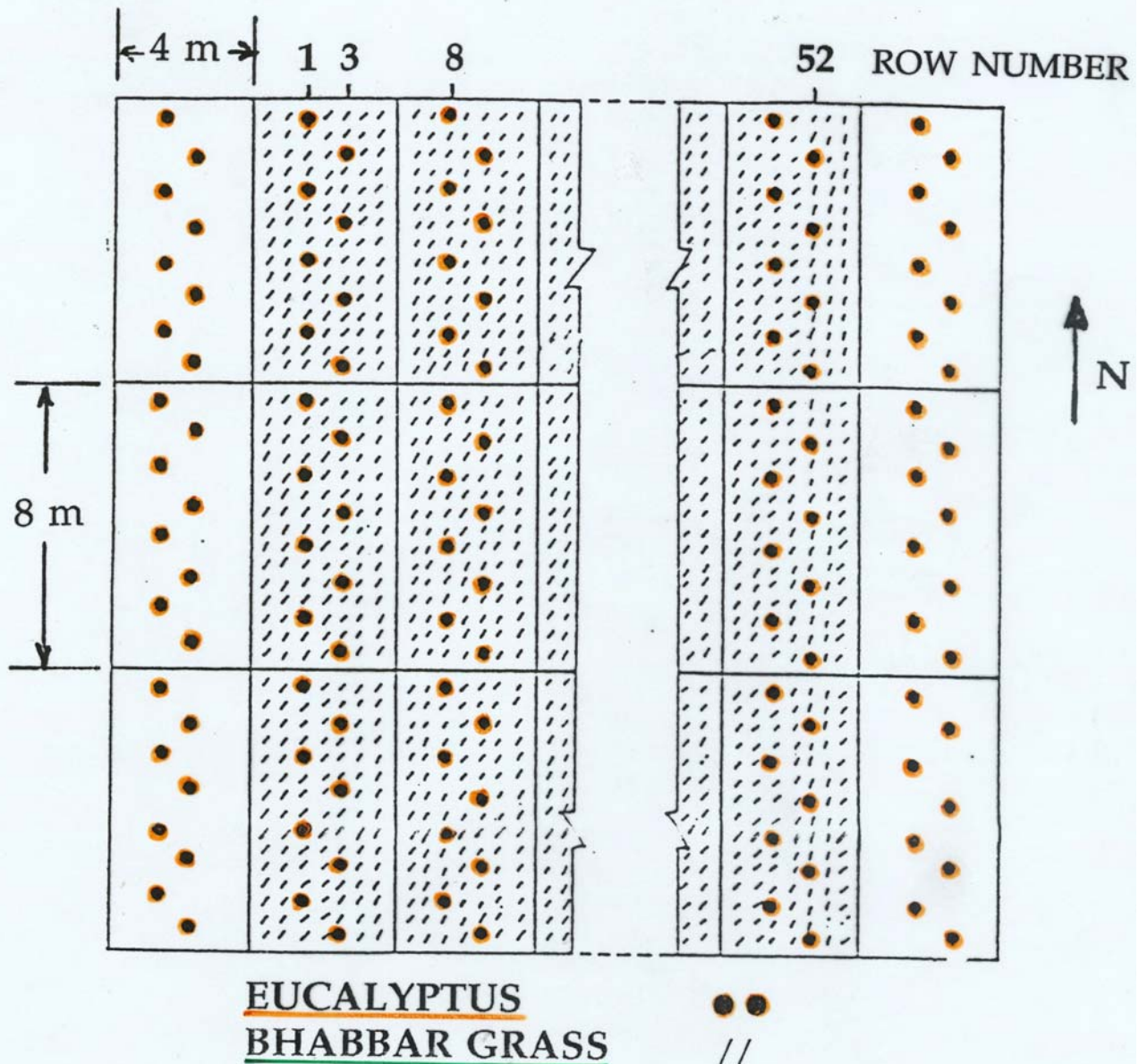
## DRAWBAR PULL



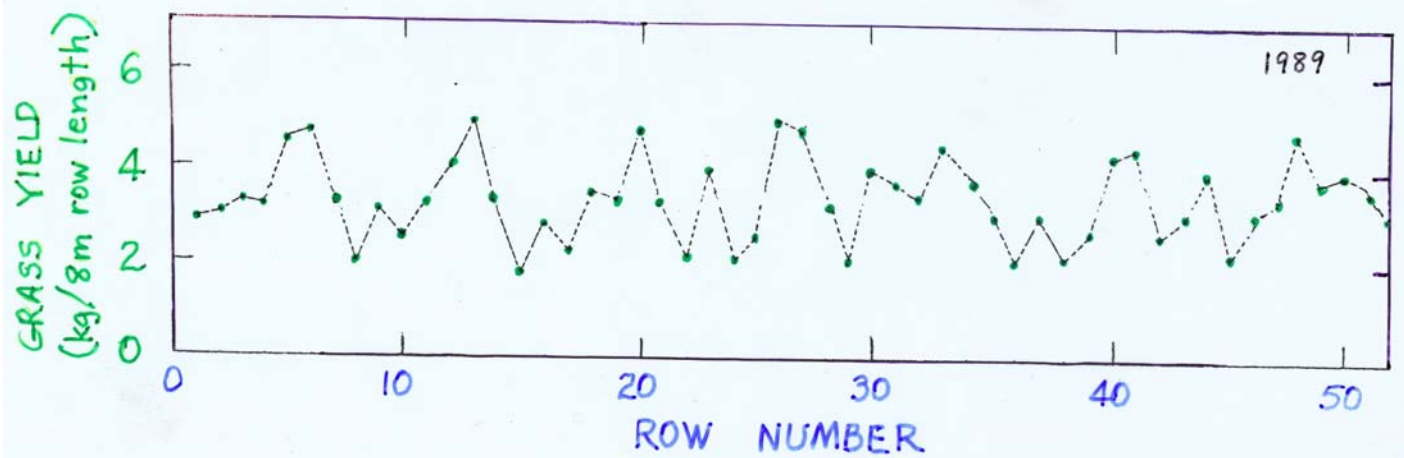
# spectral analysis bhabbar grass

## SPECTRAL ANALYSIS

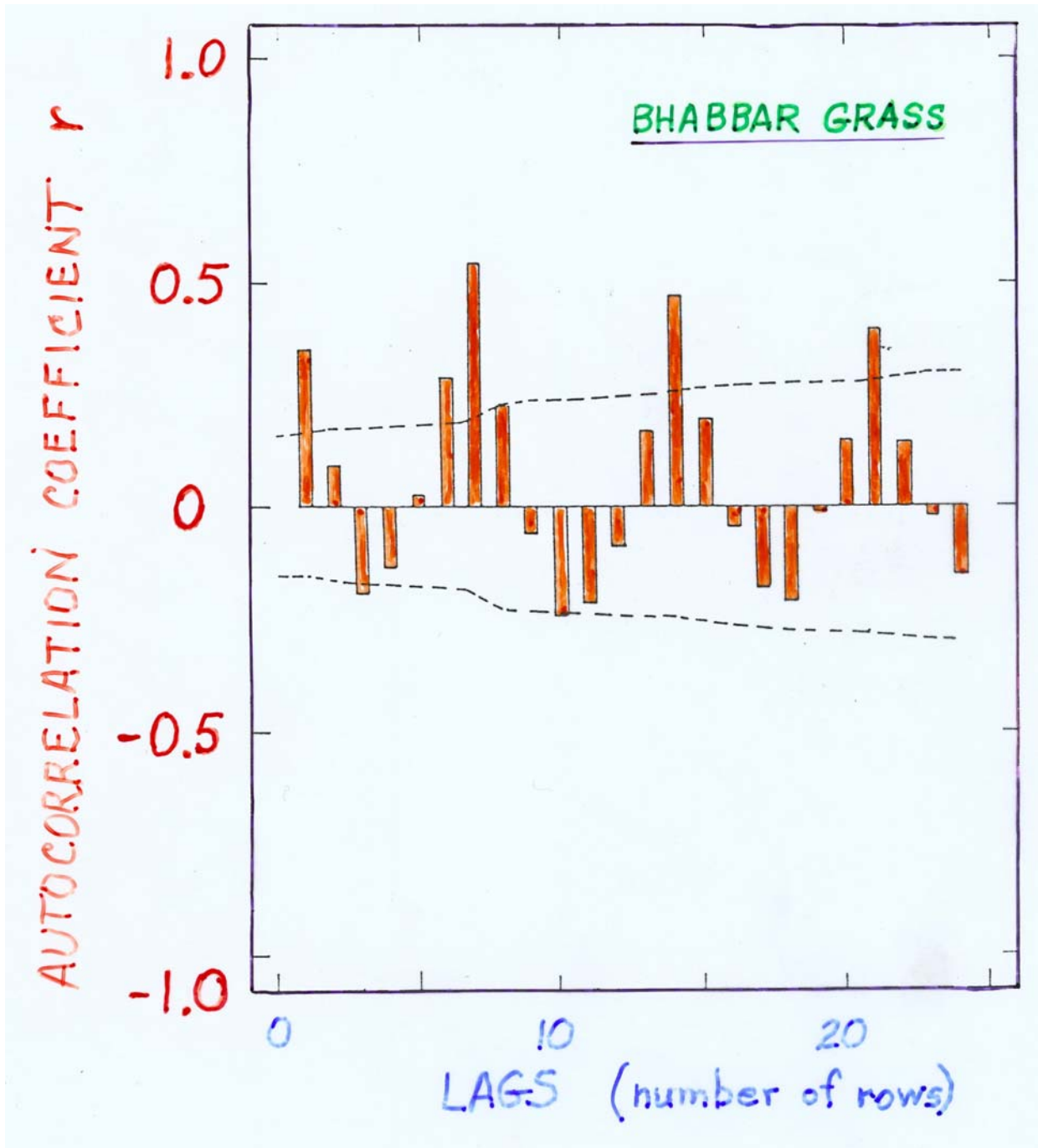
### INTER-CROPPING COMPETITION



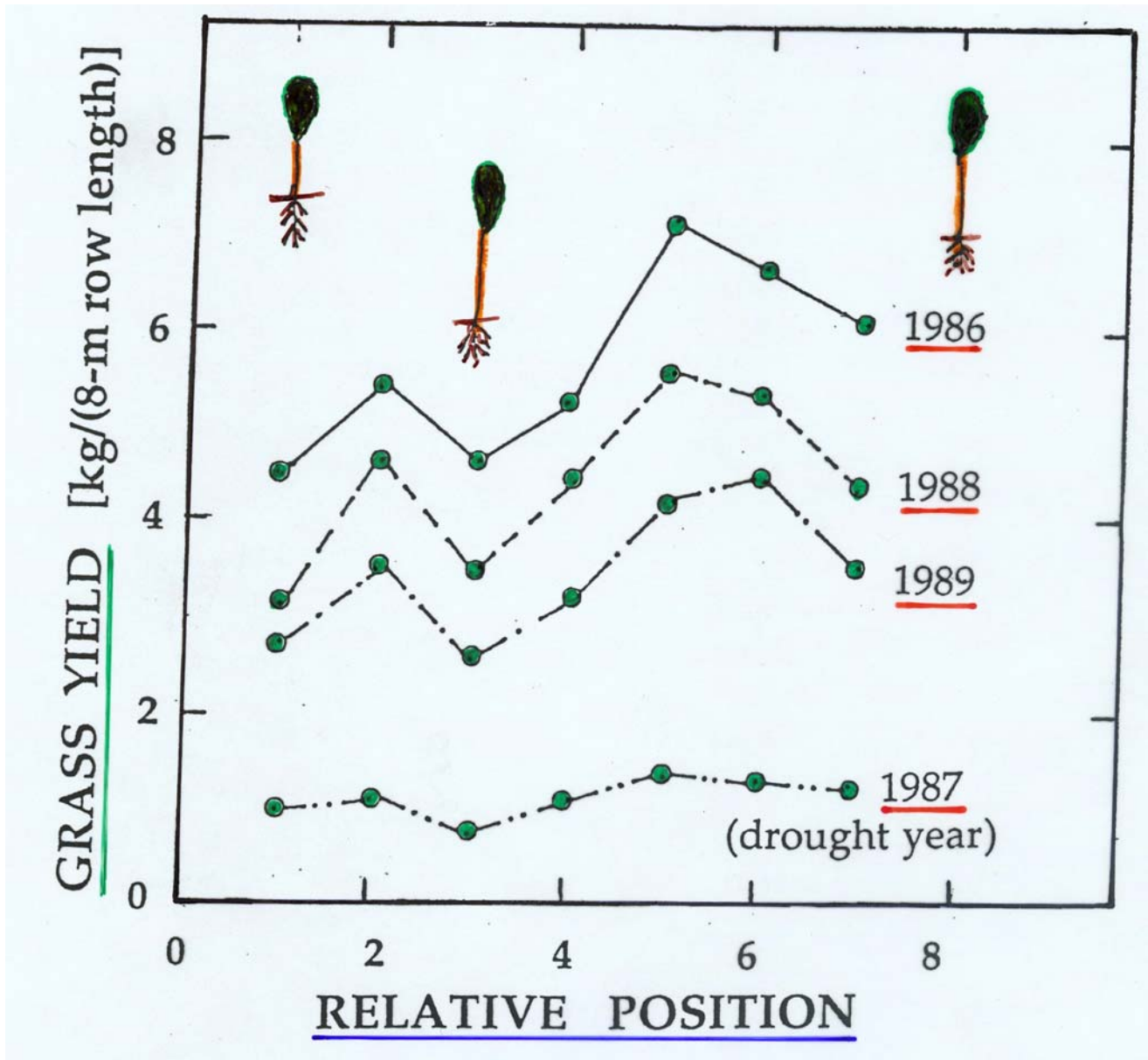
# bhabbar grass yield vs location



# ACF bhabbar grass



# bhabbar grass vs location



# bhabbar grass&eucalyptus conclusions

## CONCLUSIONS

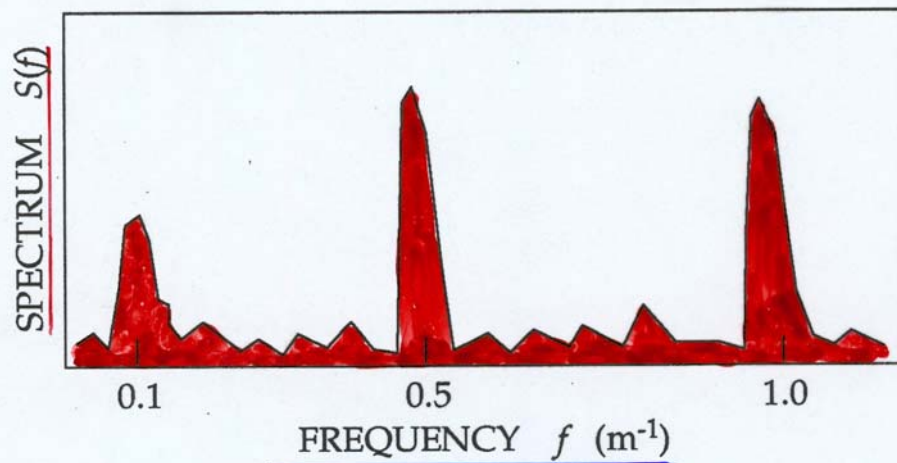
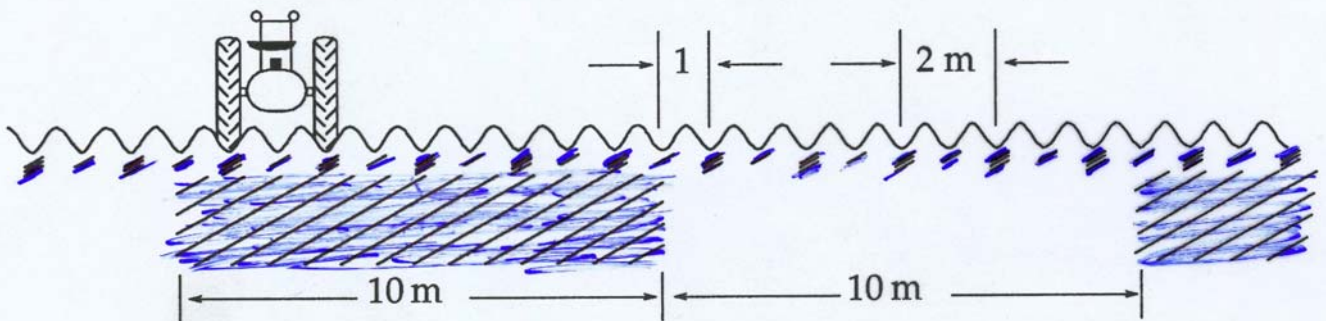
- VARIATION IN GRASS PRODUCTION AT A FREQUENCY CORRESPONDING TO 3.5m IS RELATED TO EUCALYPTUS TREES DURING THE FIRST YEAR.
- ADDITIONAL VARIATION IN GRASS PRODUCTION AT A FREQUENCIES CORRESPONDING TO 1.75 AND 1.17m DURING OTHER YEARS AS TREES GREW TALLER.



# CO-SPECTRAL ANALYSIS

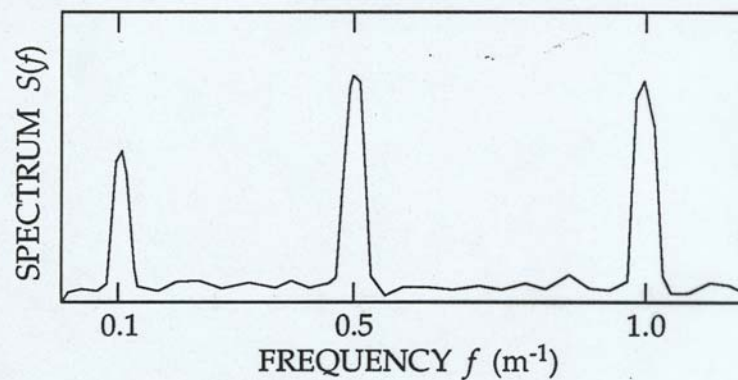
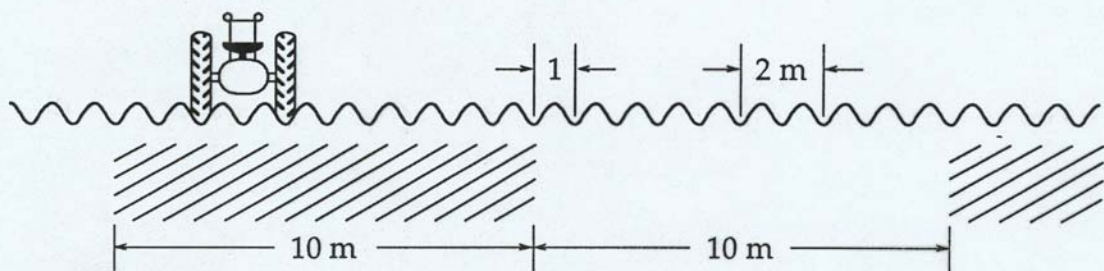
# variation in water soil content

## VARIATION IN SOIL WATER CONTENT

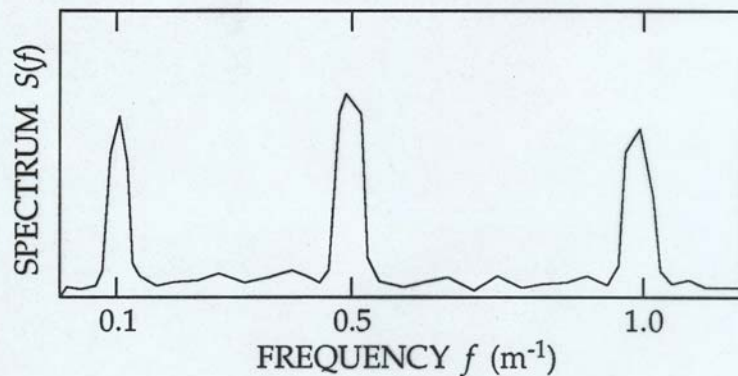


# 161 variation in soil water content & crop temp

## VARIATION IN SOIL WATER CONTENT

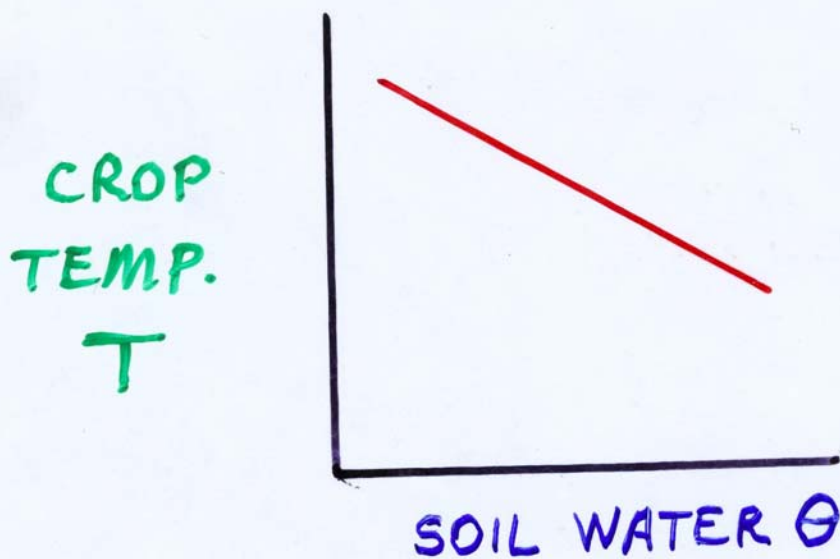


## VARIATION IN CROP CANOPY TEMPERATURE



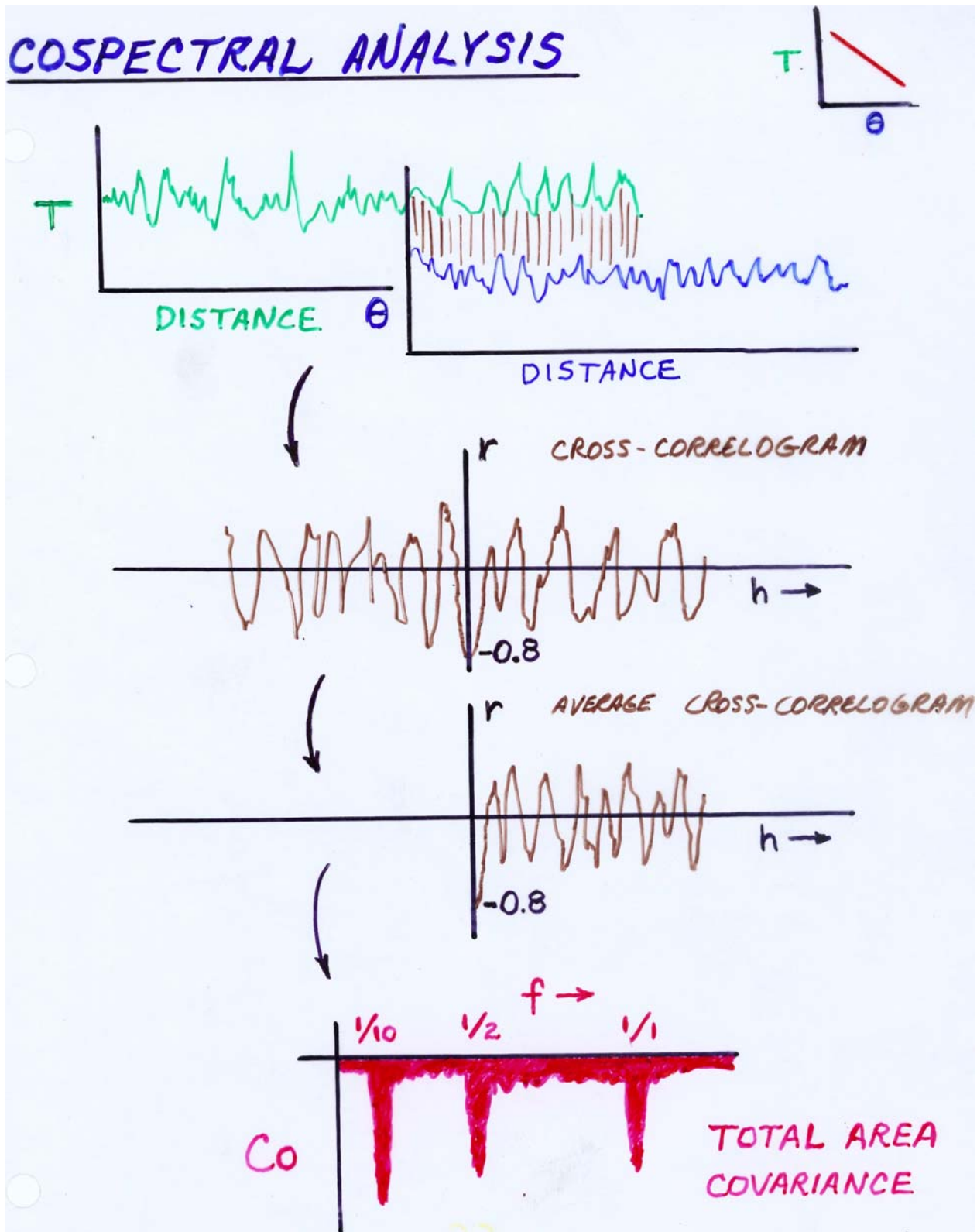
if crop  $T$  inversely related  
to  $\theta$

IF:



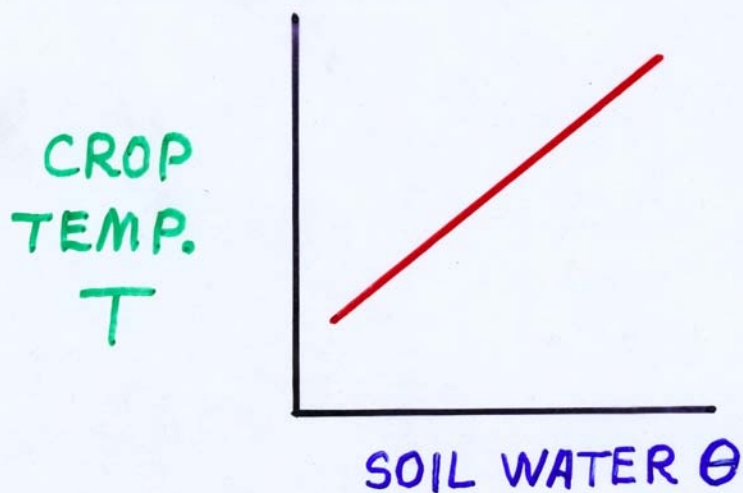
- LOWER WATER CONTENT  $\theta$
- LESS CROP AVAILABLE WATER
  - REDUCED TRANSPIRATION
  - GREATER CROP TEMPERATURE  $T$

# cospectral analysis diag no compaction



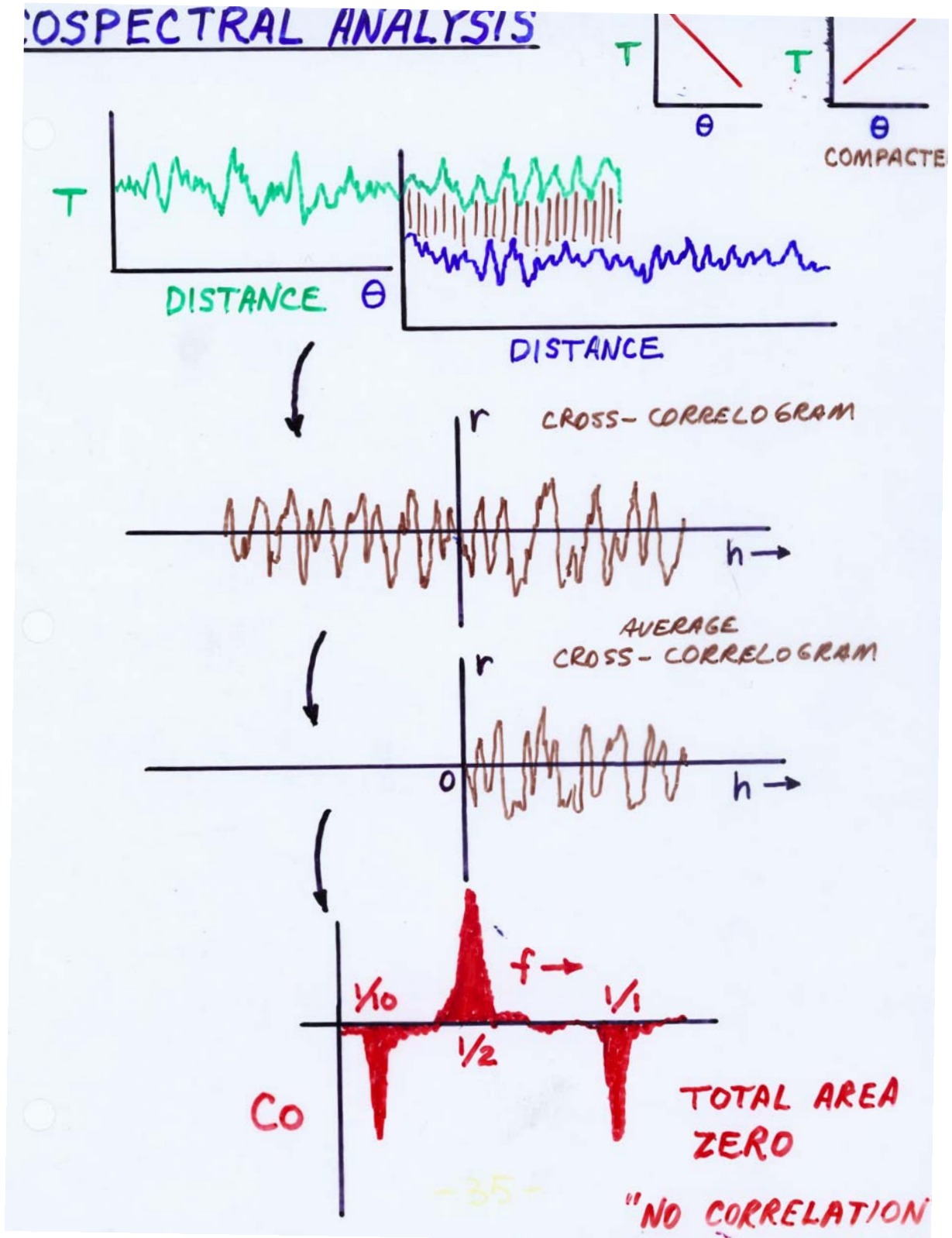
# if compaction in tractor wheel furrow

IF IN TRACTOR WHEEL FURROW:



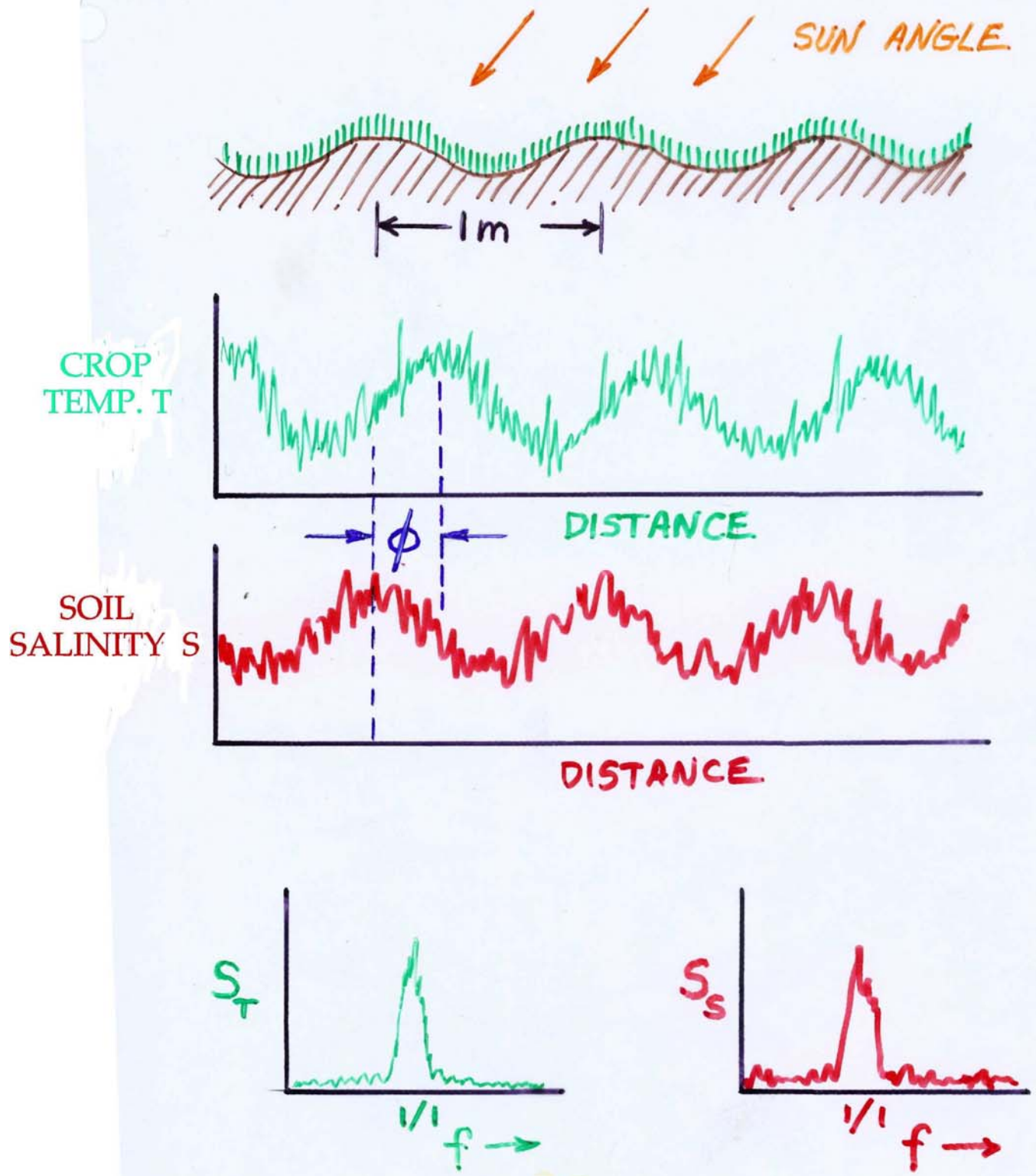
- SOIL IS COMPACTED
- GREATER SOIL WATER CONTENT  $\theta$
  - FUNGAL ROOT ROT DISEASE
  - POOR WATER EXTRACTION
  - REDUCED TRANSPIRATION
  - GREATER CROP TEMPERATURE T

# cospectral analysis diag compaction



# cospectral phase angle

## COSPECTRAL PHASE ANGLE $\phi$

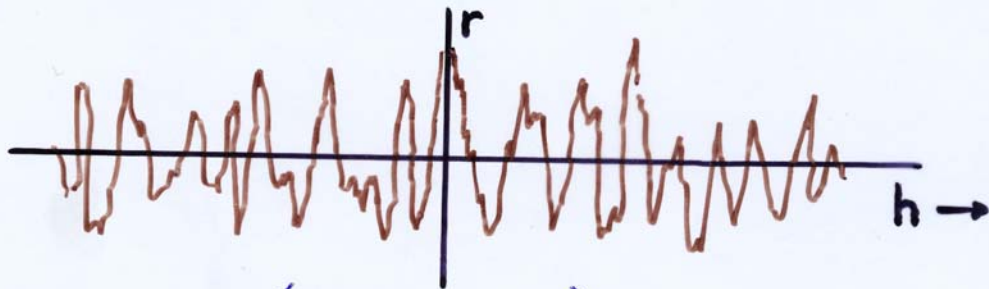




# phase angle illustration

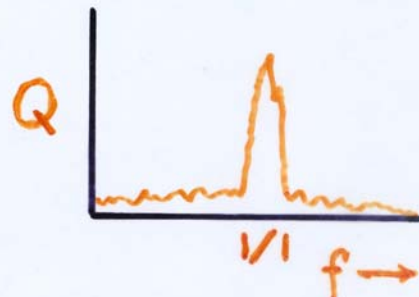
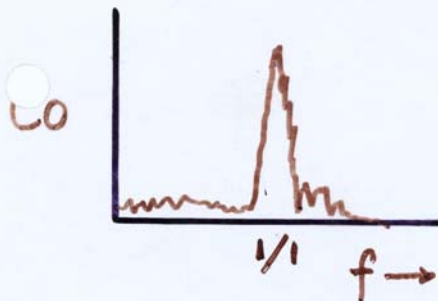
## PHASE ANGLE $\phi$

CROSS-CORRELOGRAM (CROP TEMP.  $\neq$  SOIL SALINITY)



$$\frac{r(h>0) + r(h<0)}{2}$$

$$\frac{r(h>0) - r(h<0)}{2}$$



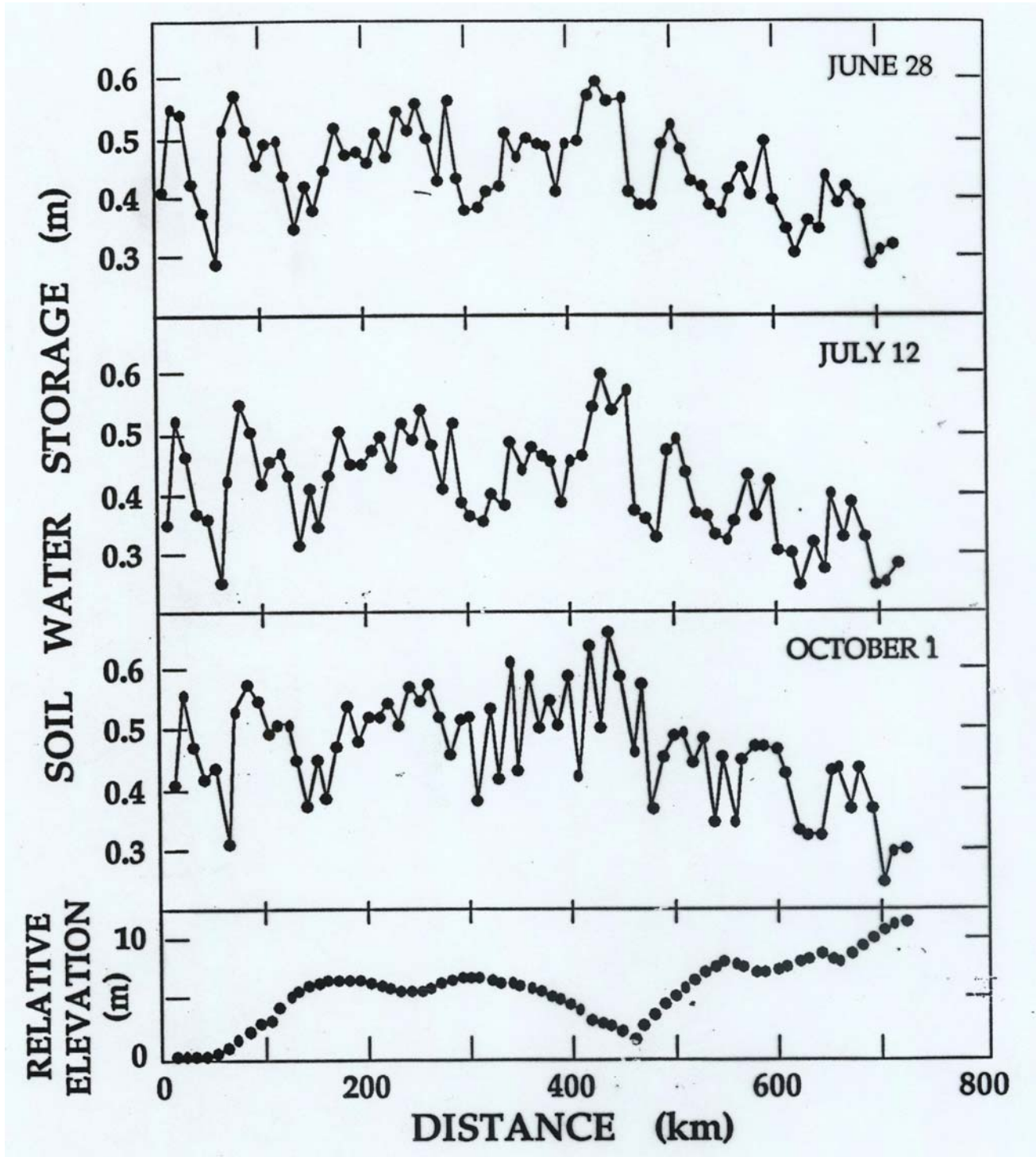
$$\phi = \frac{1}{2\pi f} \tan^{-1} \left[ \frac{Q(f)}{Co(f)} \right]$$

$$Coh(f) = \frac{Q^2(f) + Co^2(f)}{S_T(f) S_S(f)}$$

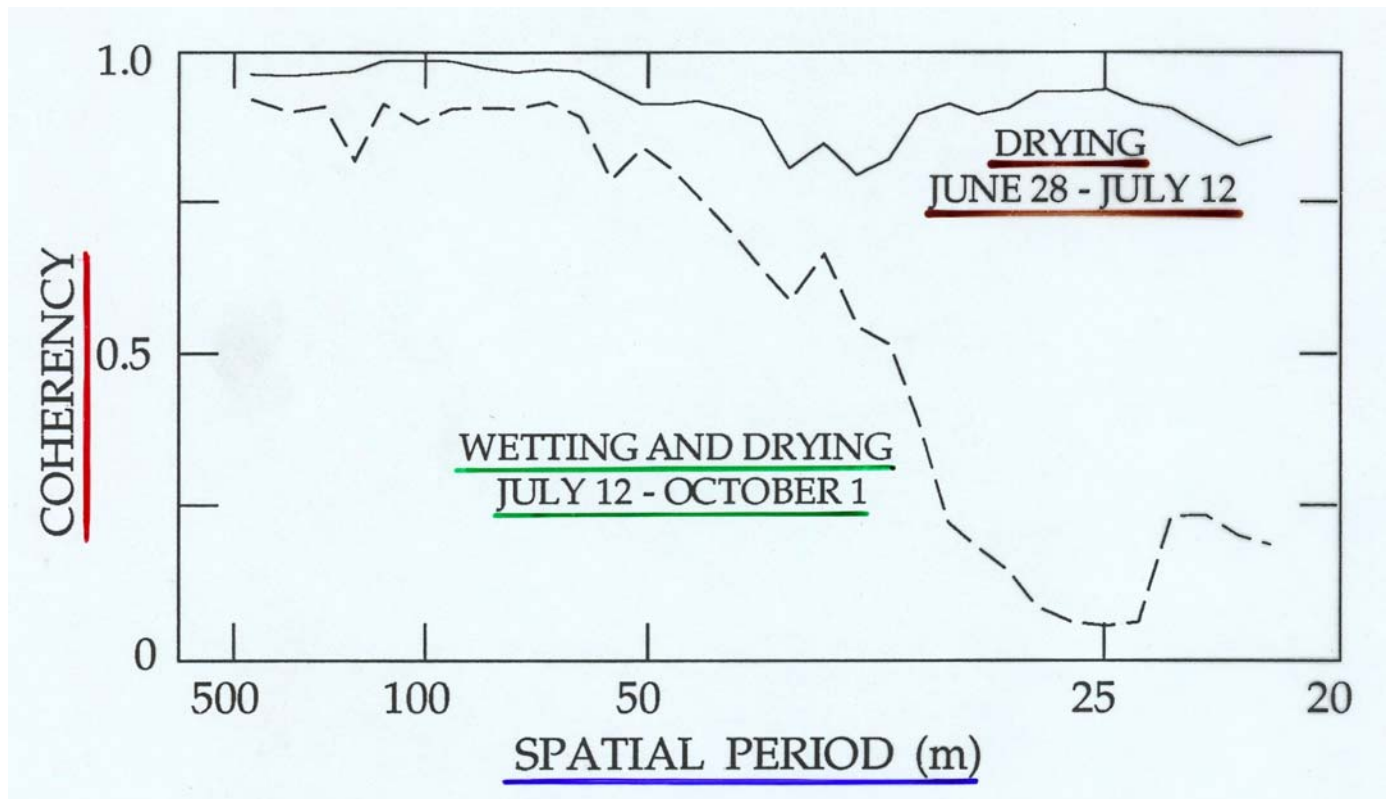
$$0 < Coh(f) < 1$$

**COHERENCY OF SOIL WATER STORAGE  
AT DIFFERENT TIMES**

# soil water storage vs distance

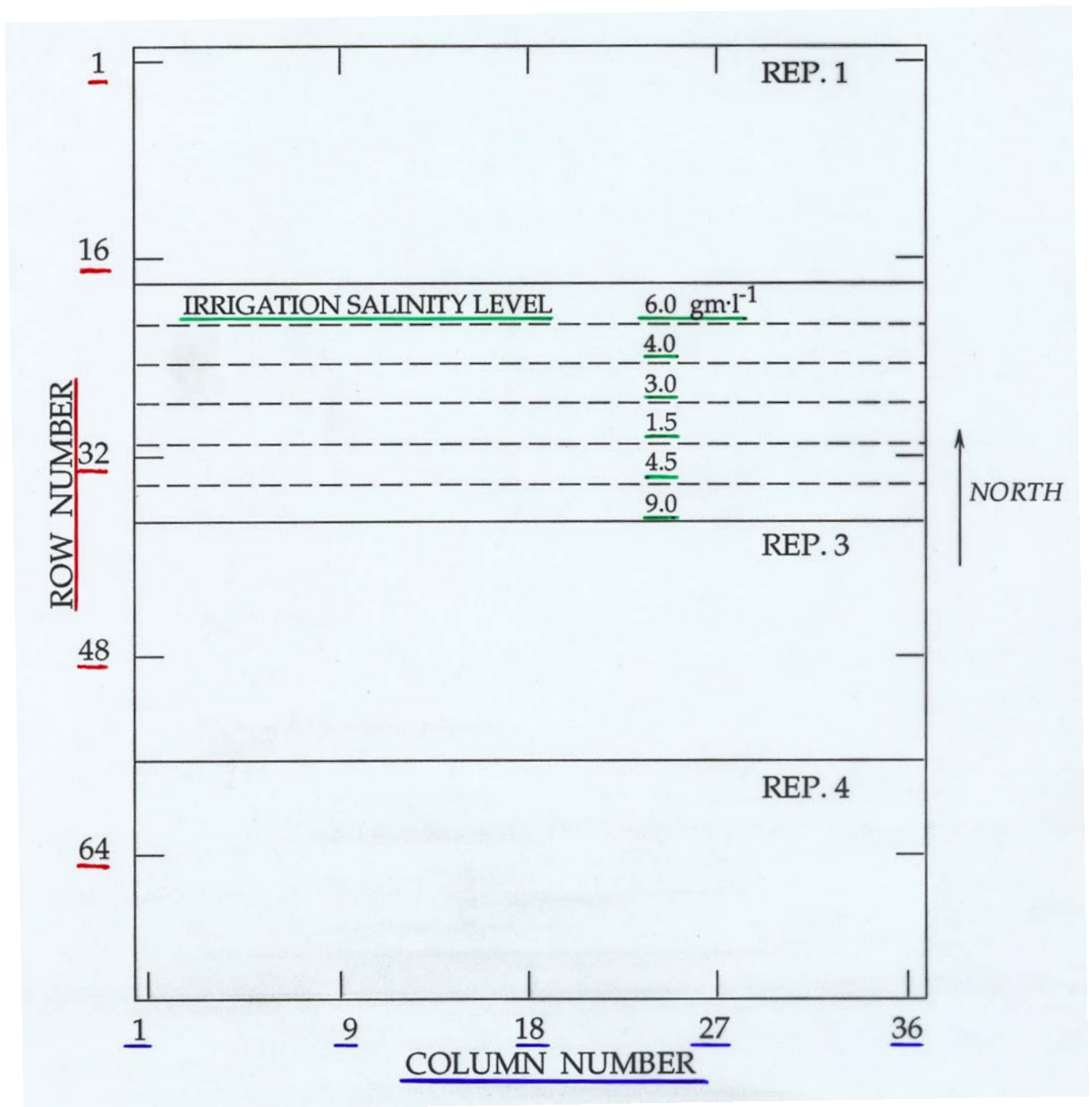


# coherency vs spatial period

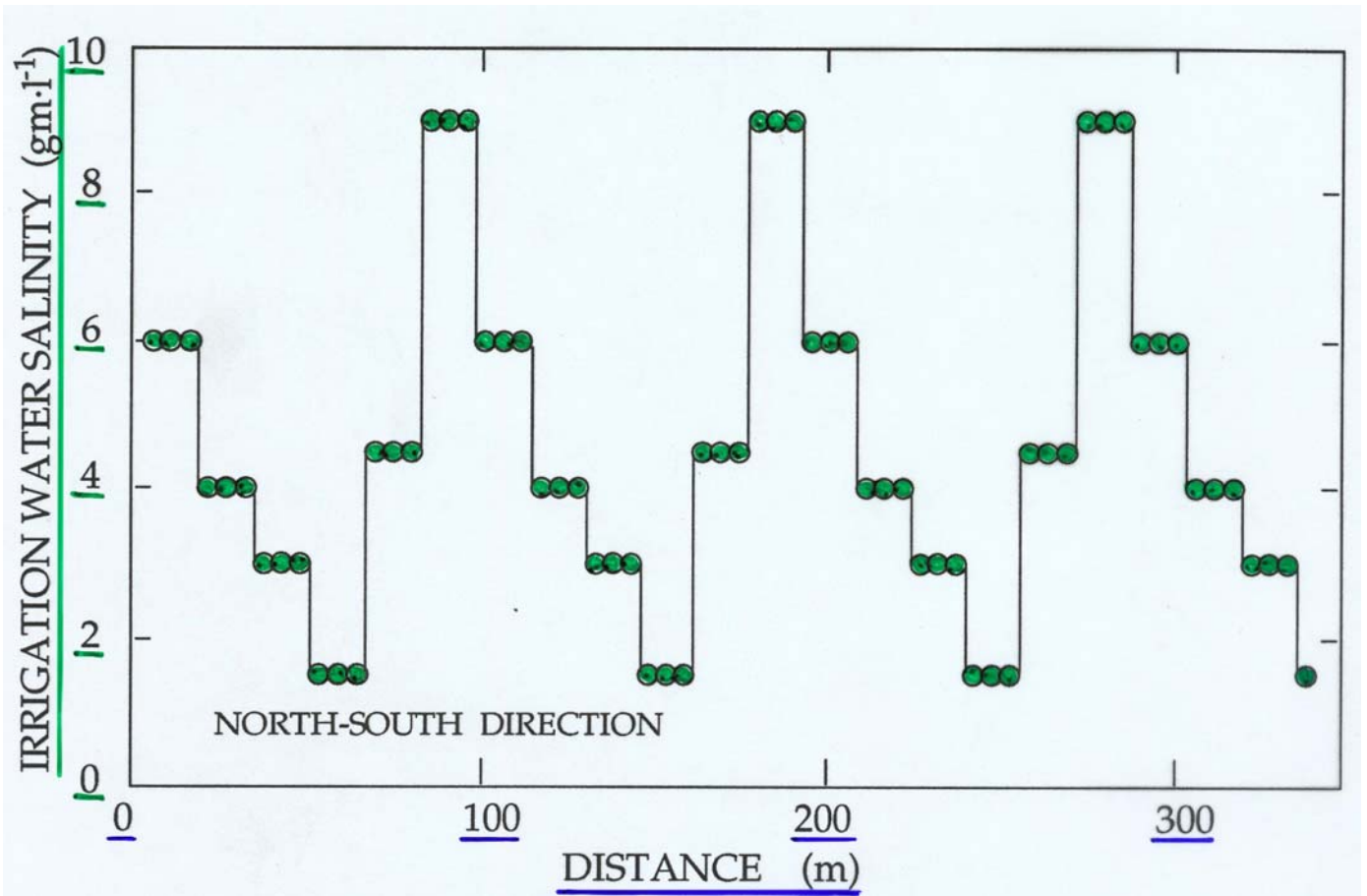


QUALITY OF IRRIGATION WATER  
RELATED TO SOIL TEMPERATURE

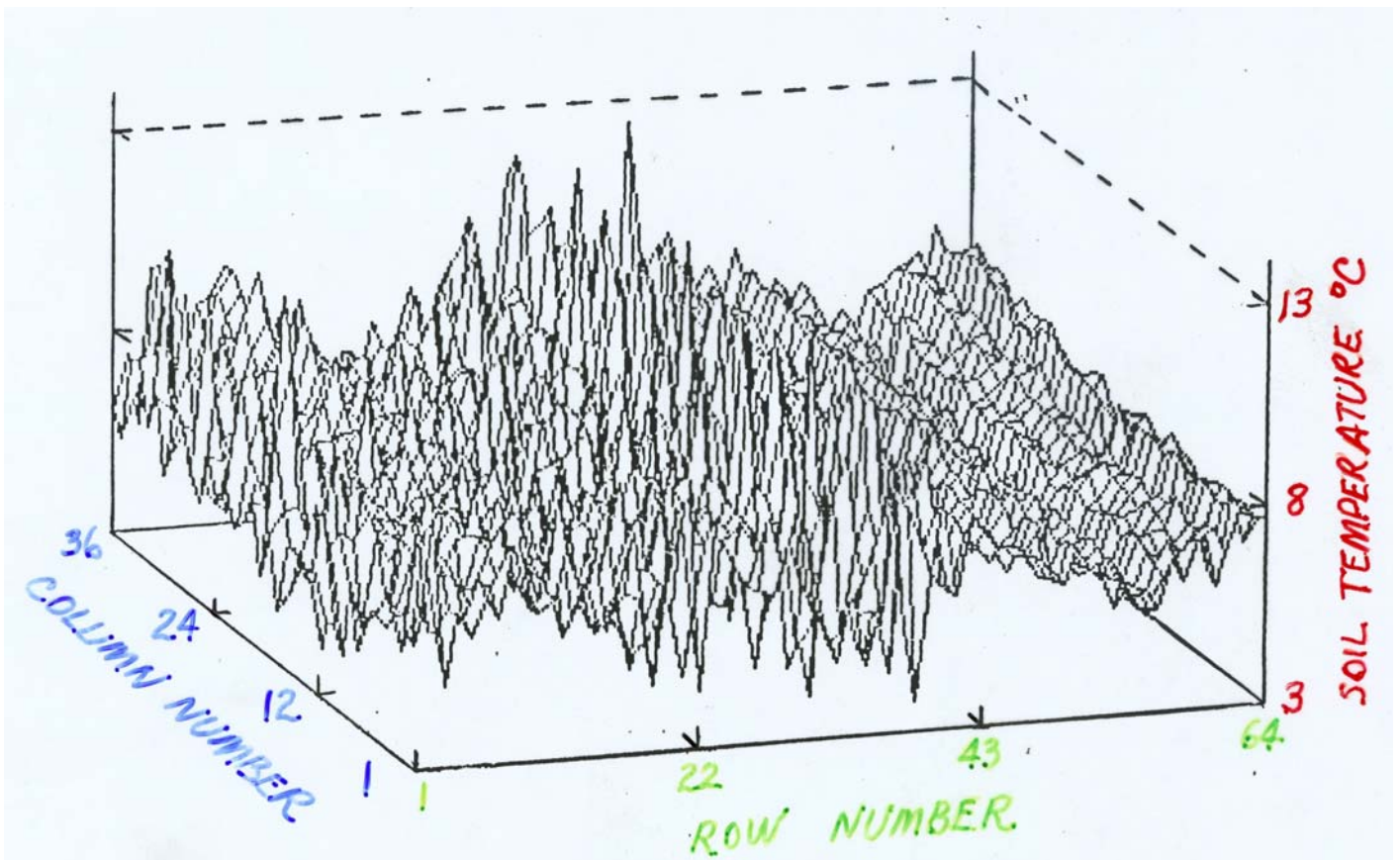
# row and column number irrig&salinity



# irrig water salinity vs distance

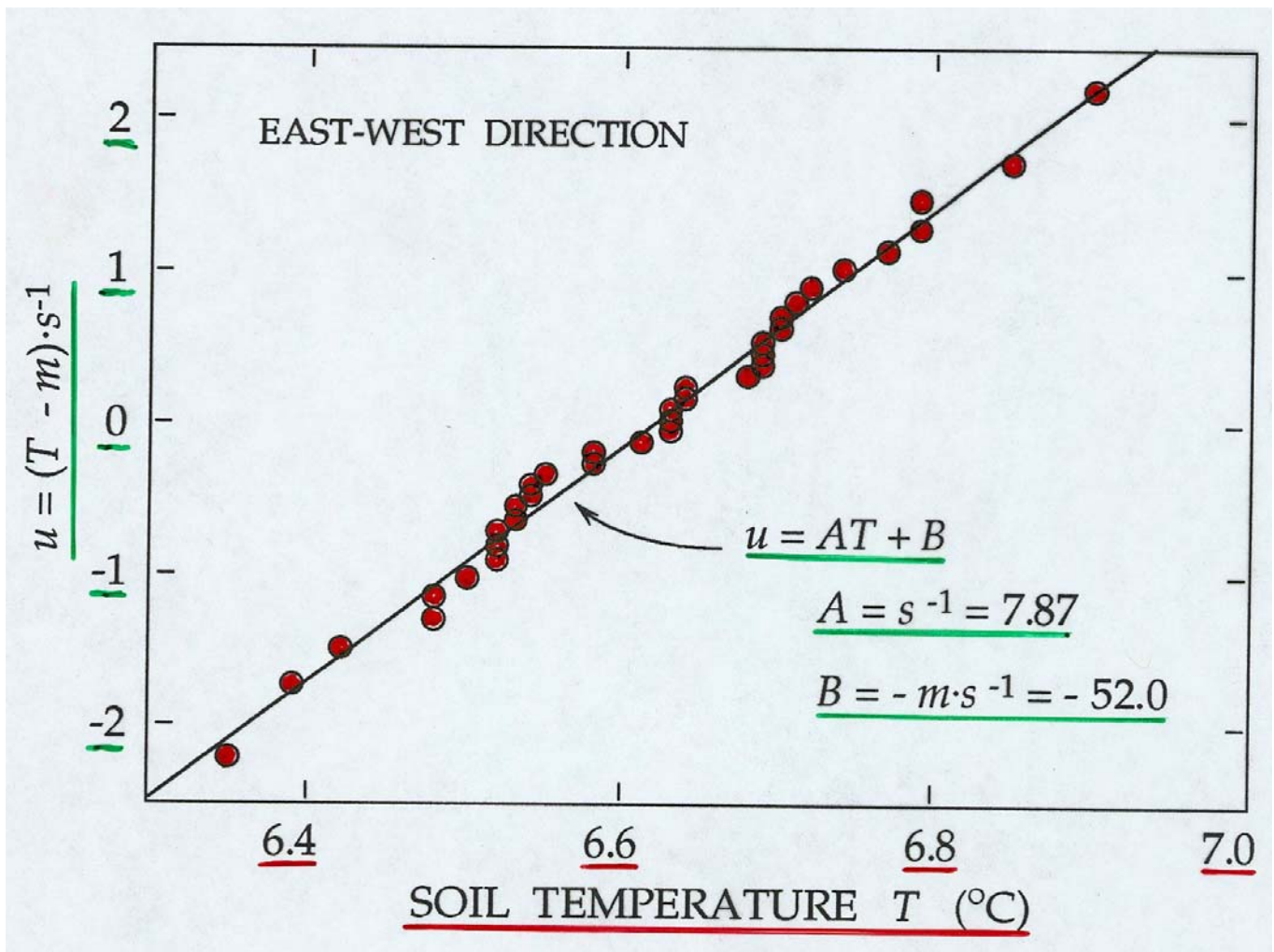


# soil temp vs row & column numbers

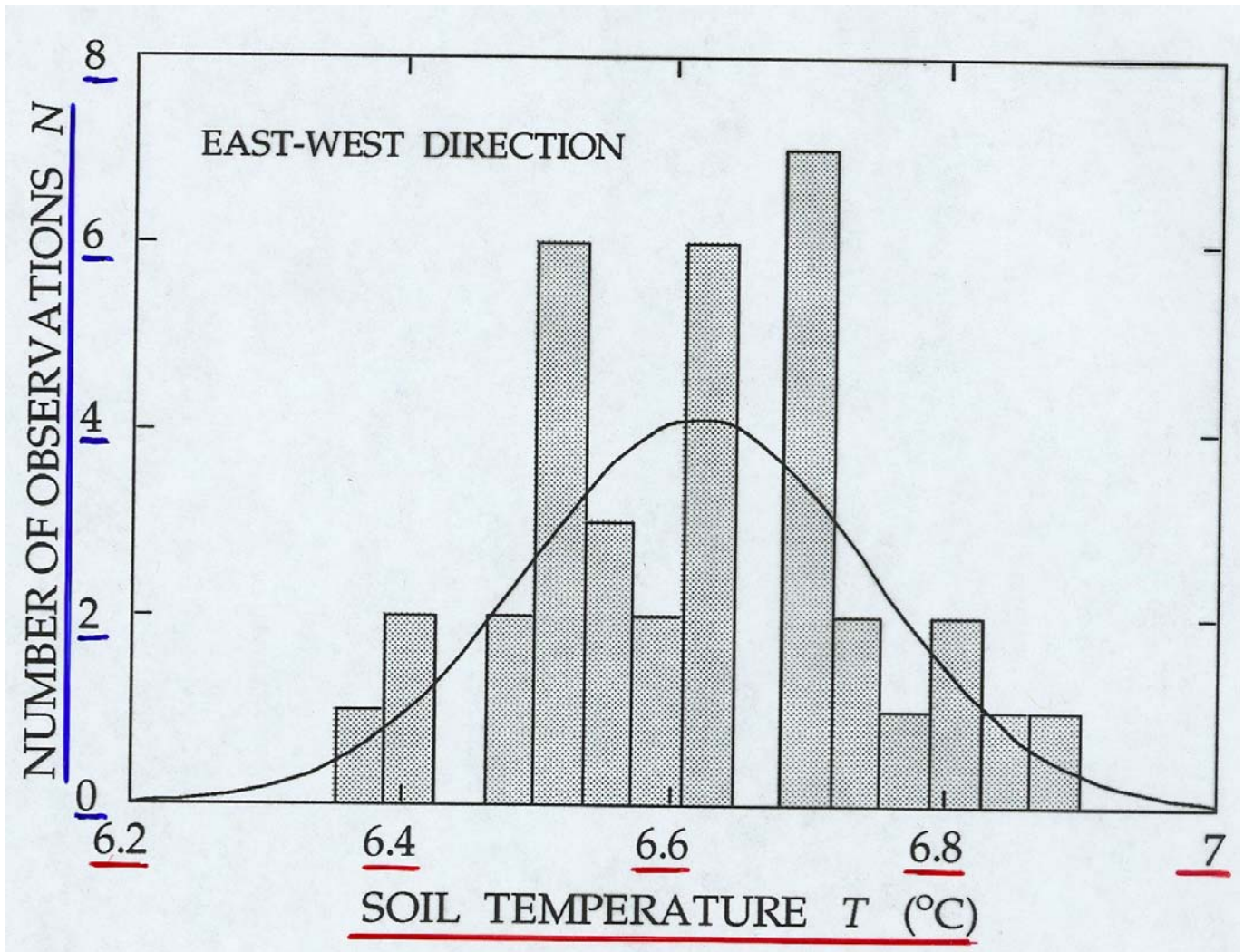




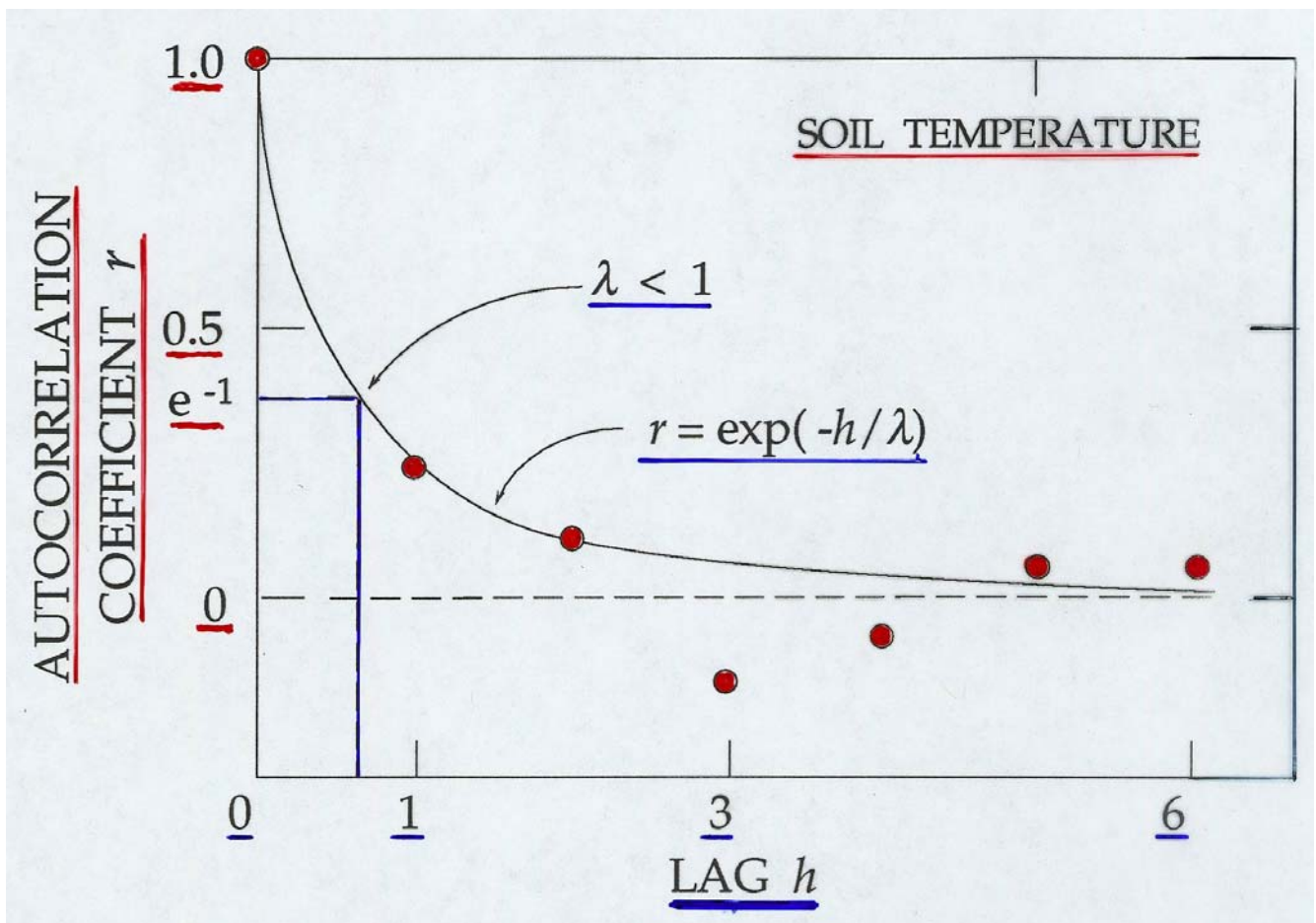
# fractile diagram soil temp (E-W)



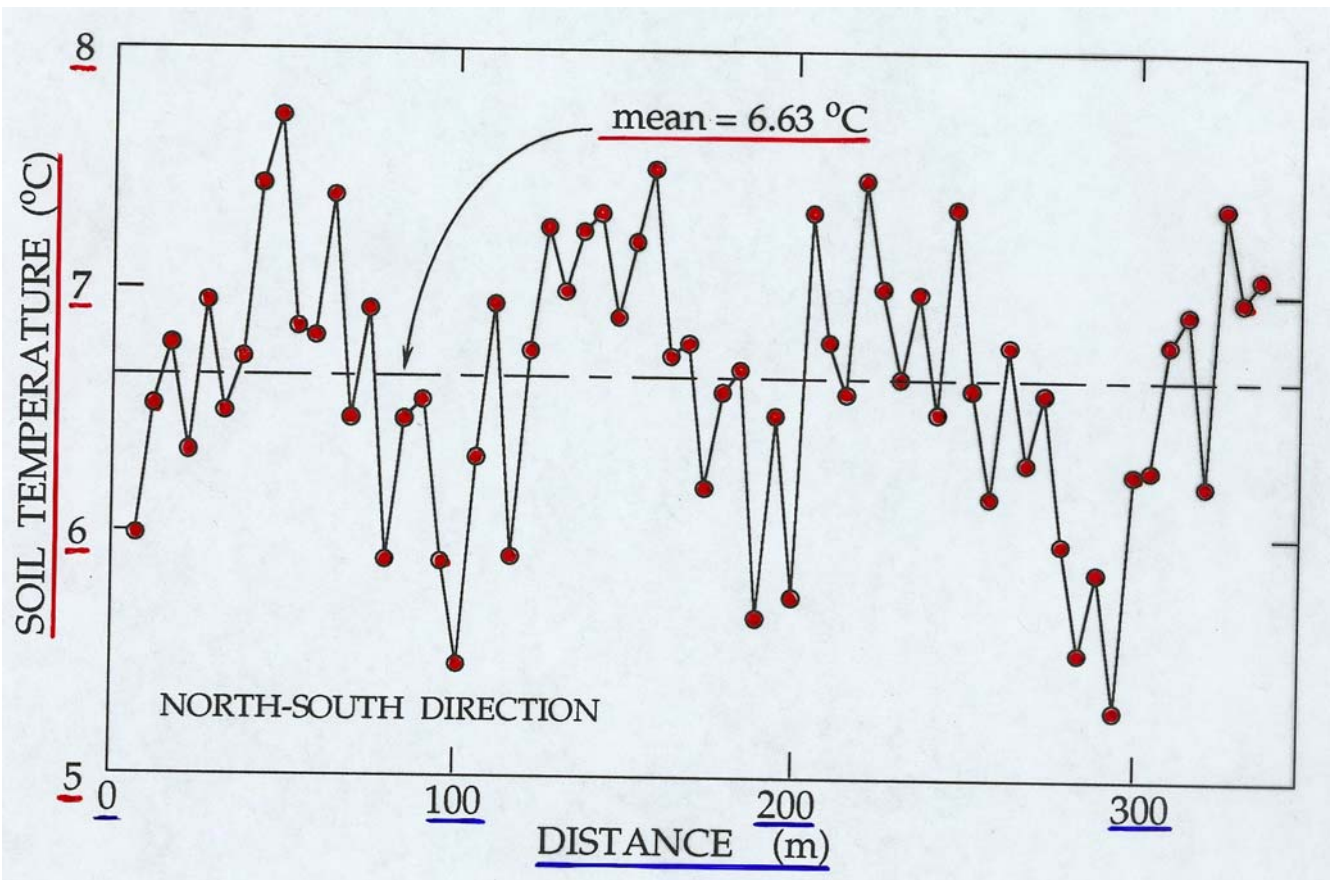
# pdf soil temperature (E-W)



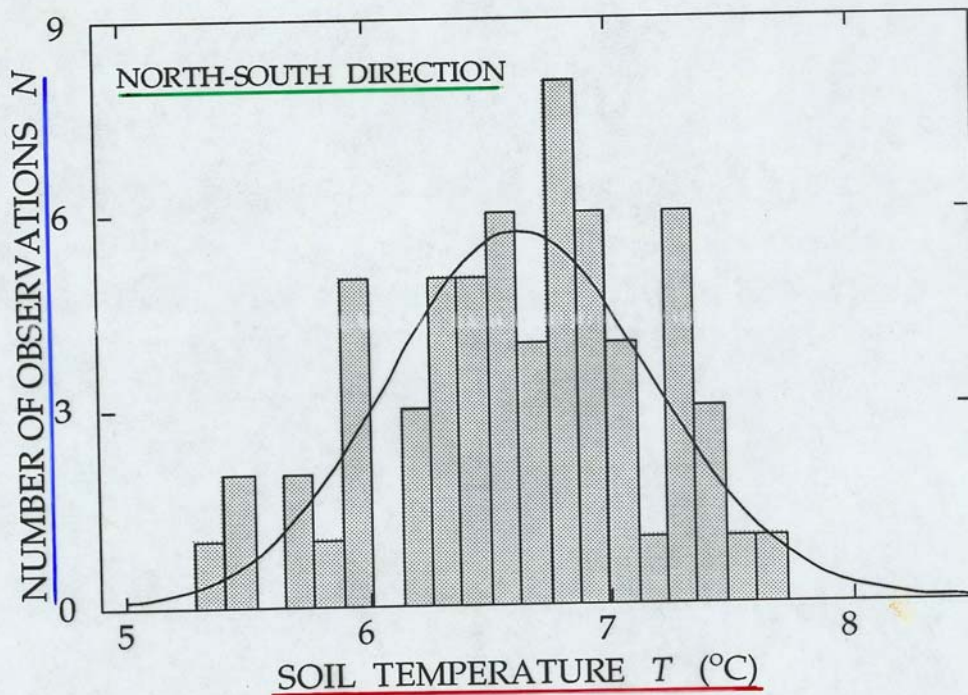
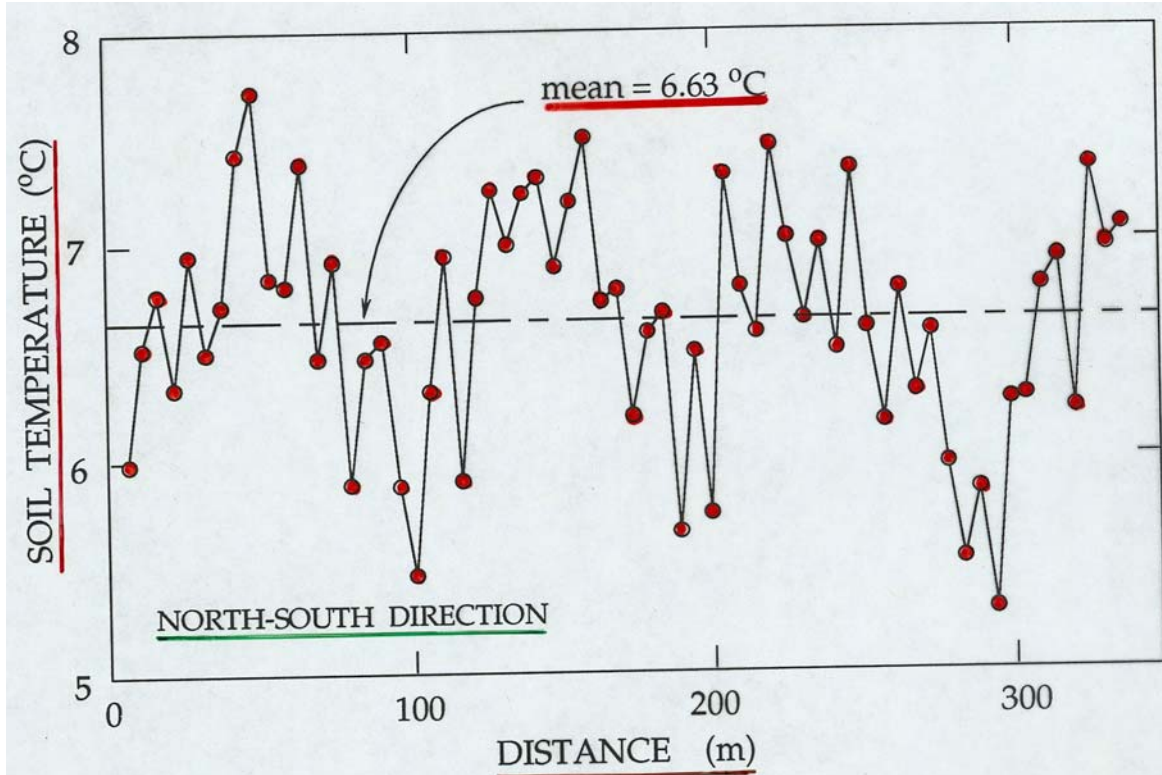
# autocorrelogram E-W soil temp.



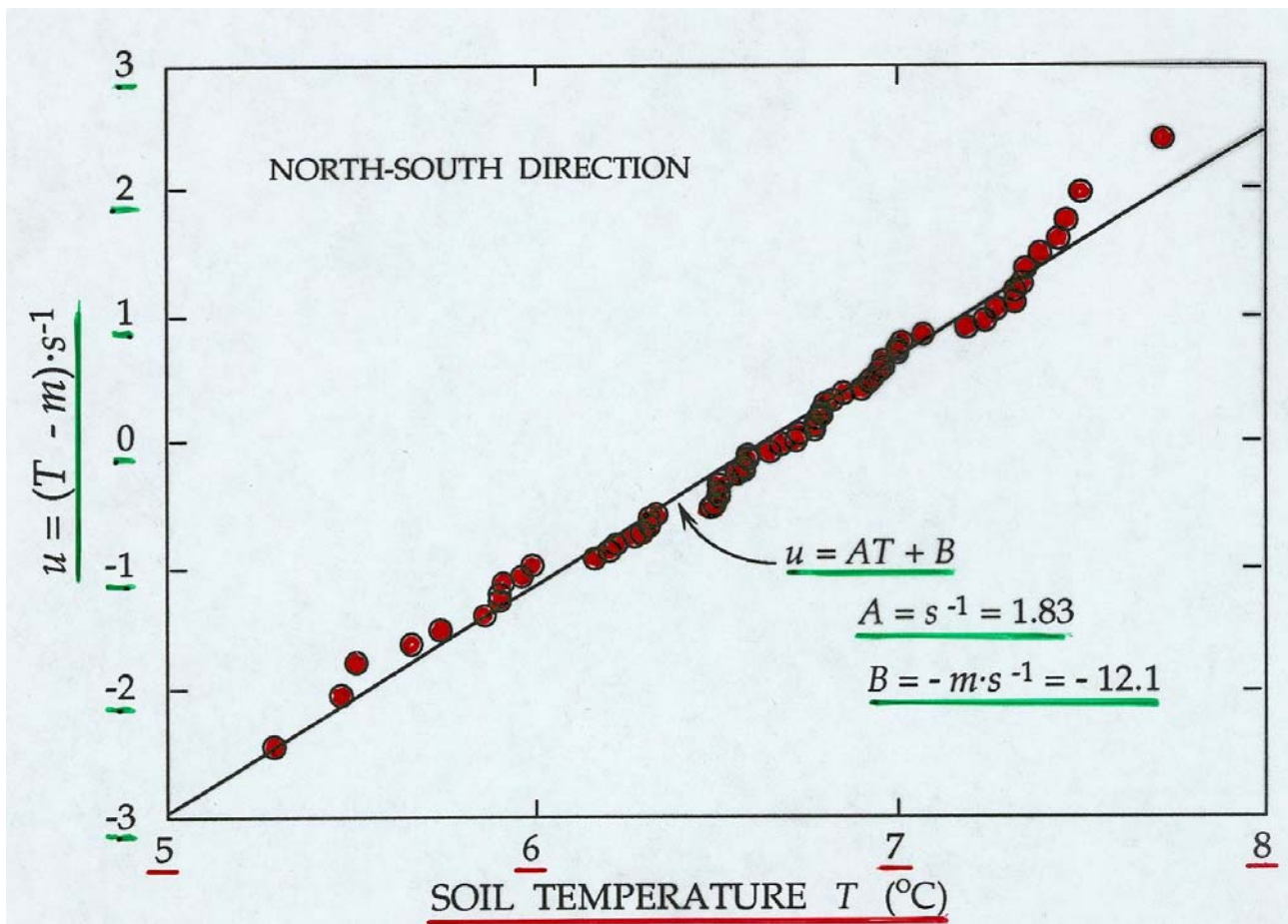
# soil temperature vs distance (N-S)



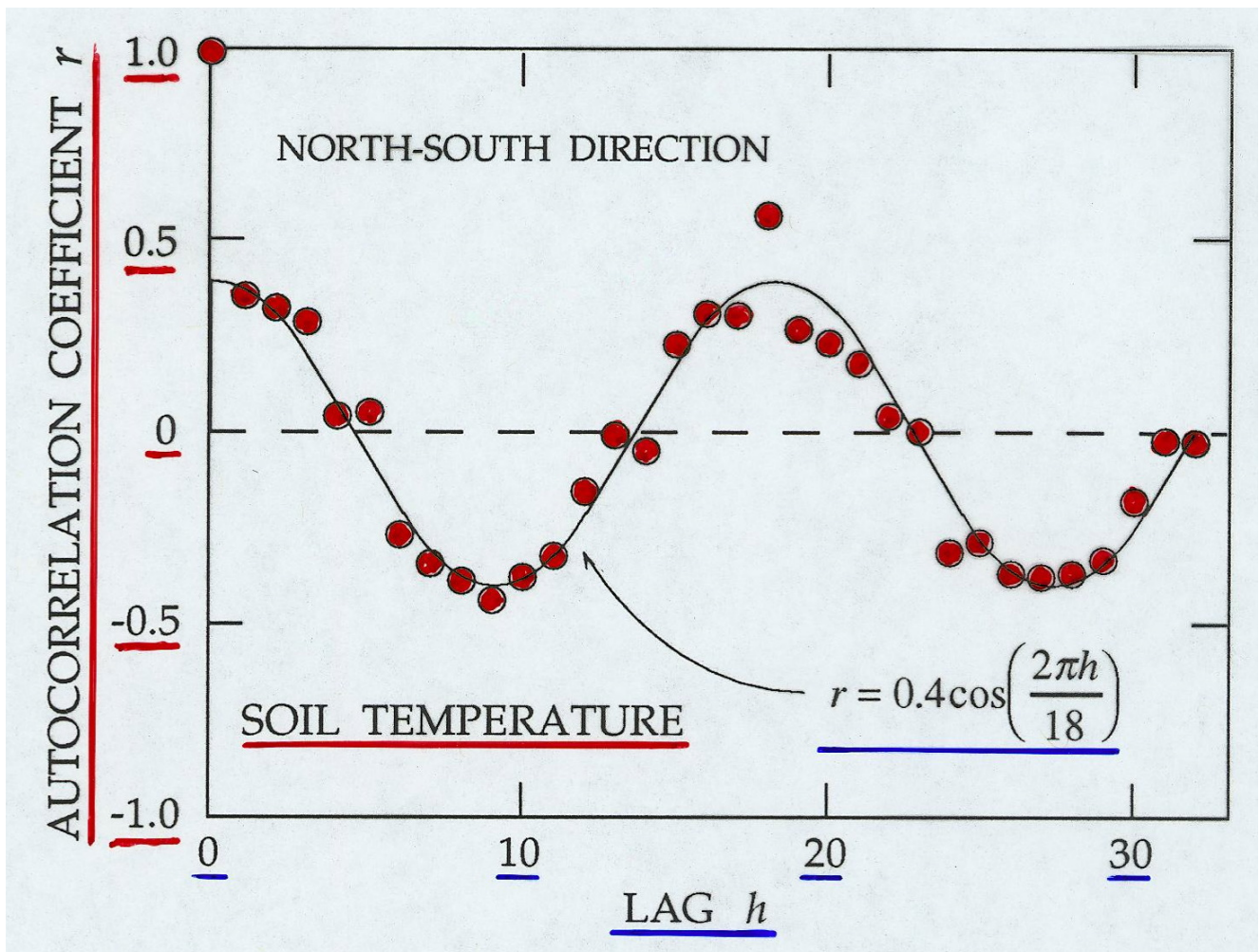
# soil temperature vs distance & pdf (N-S)



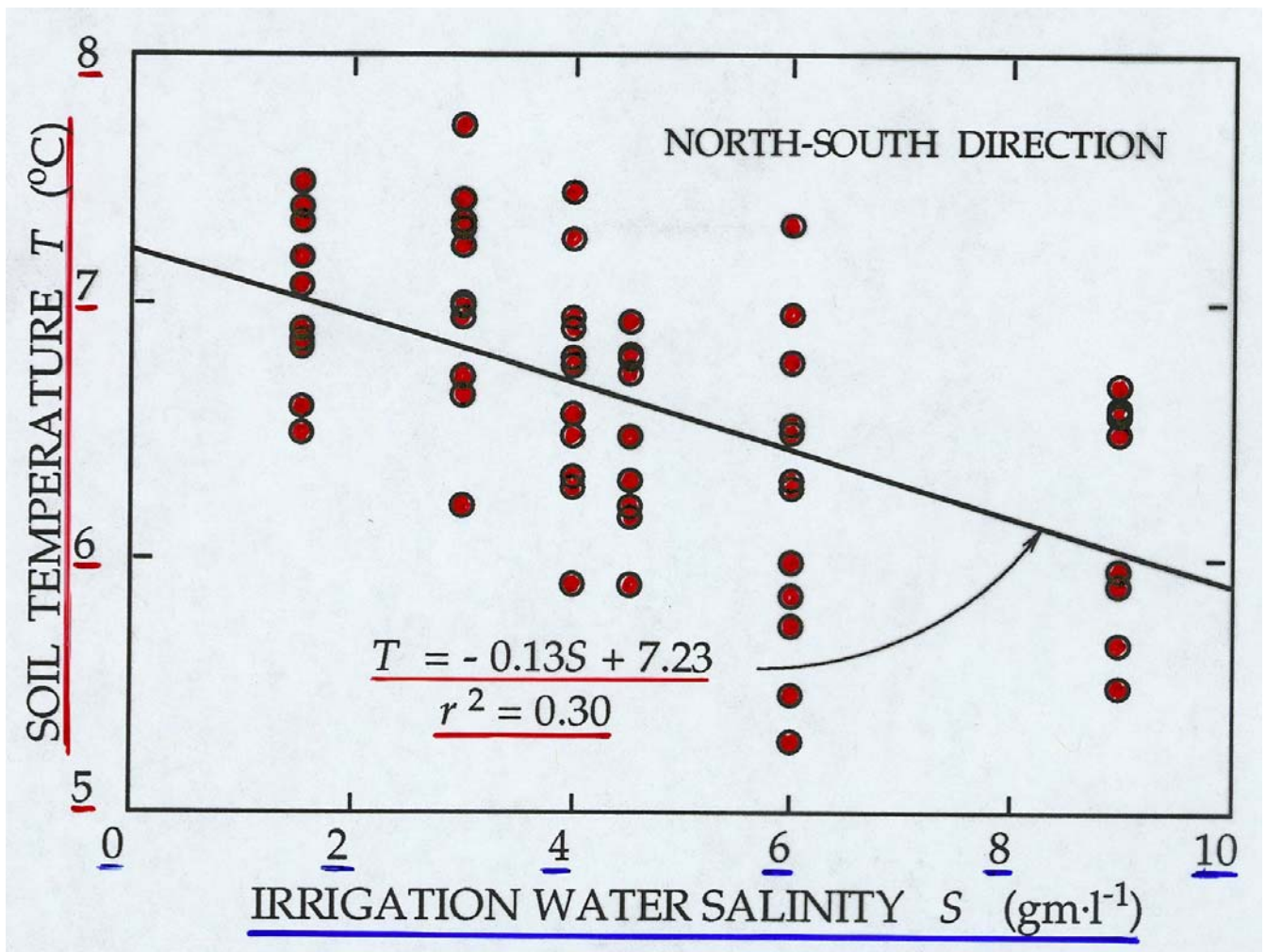
# fractile diagram soil temperature (N-S)



# ACF N-S soil temp

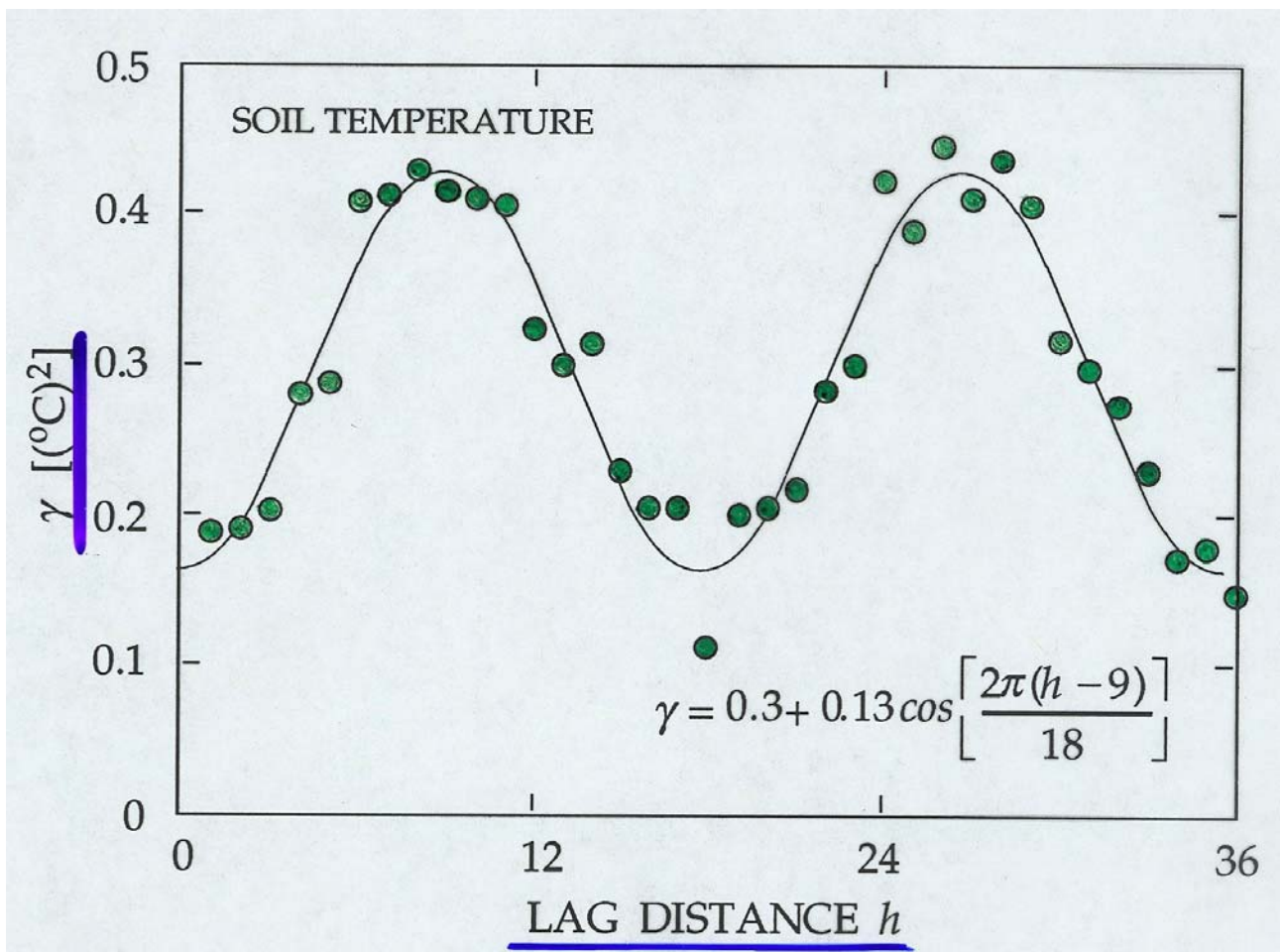


# linear regress soil temp vs irrig salinity

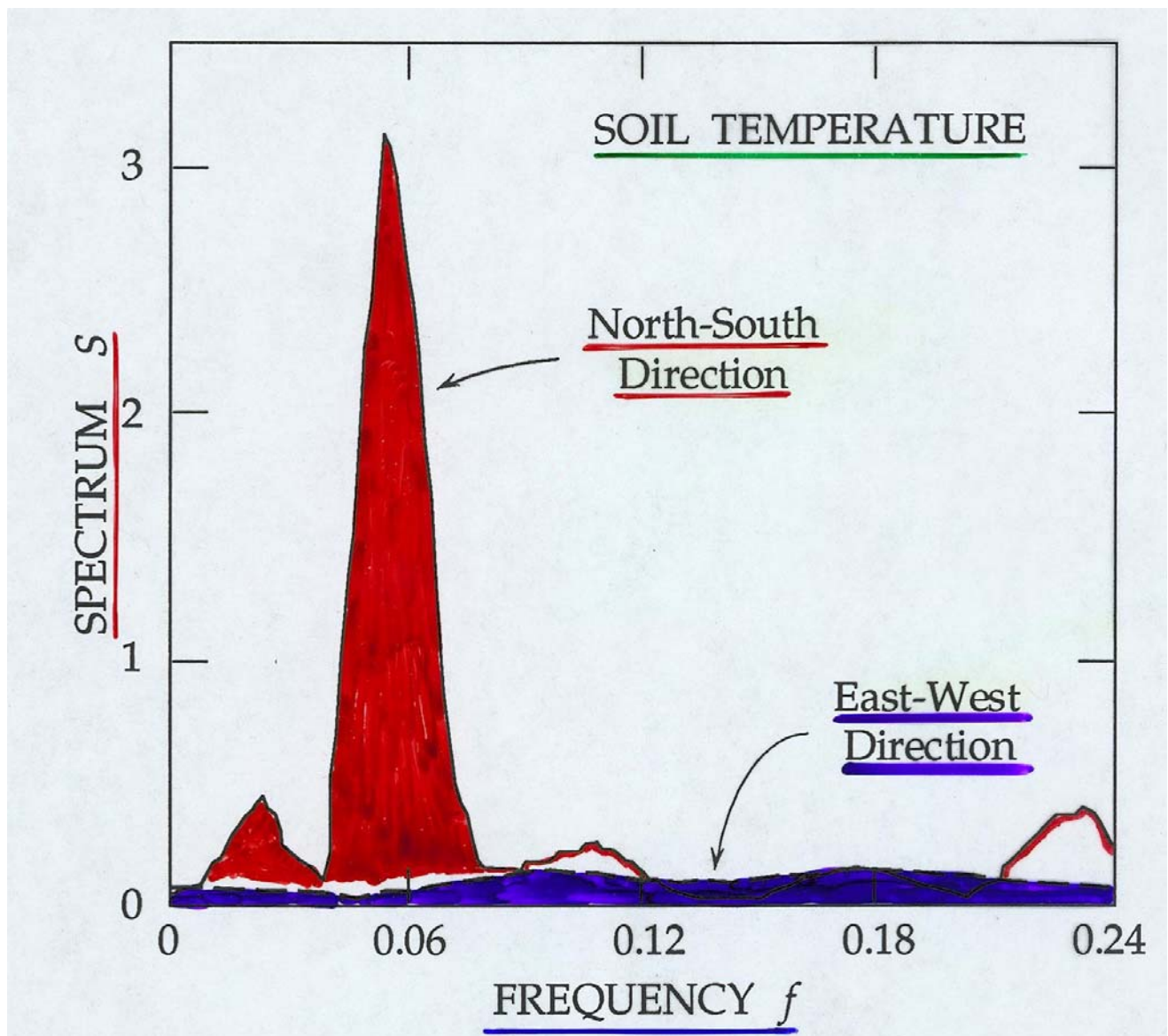




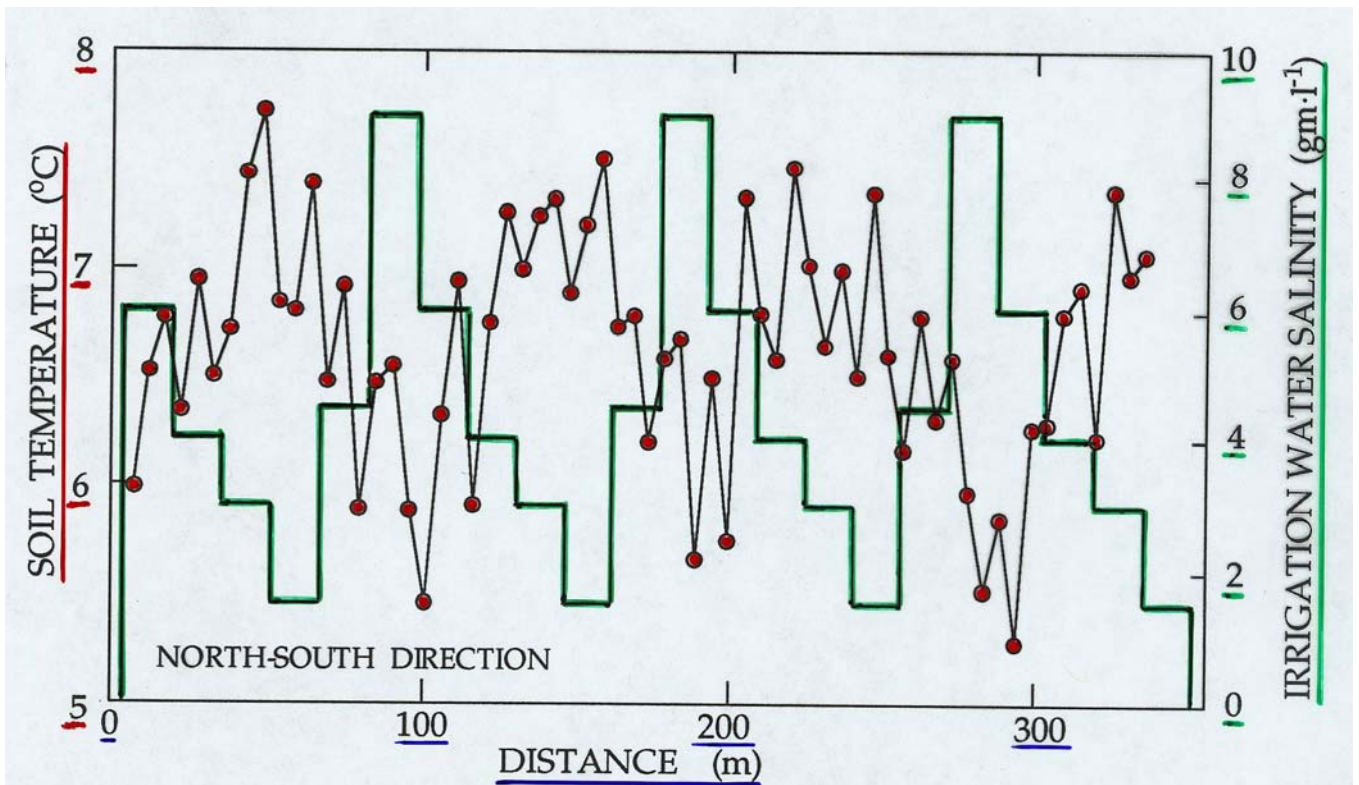
# variogram soil temperature (cosine)



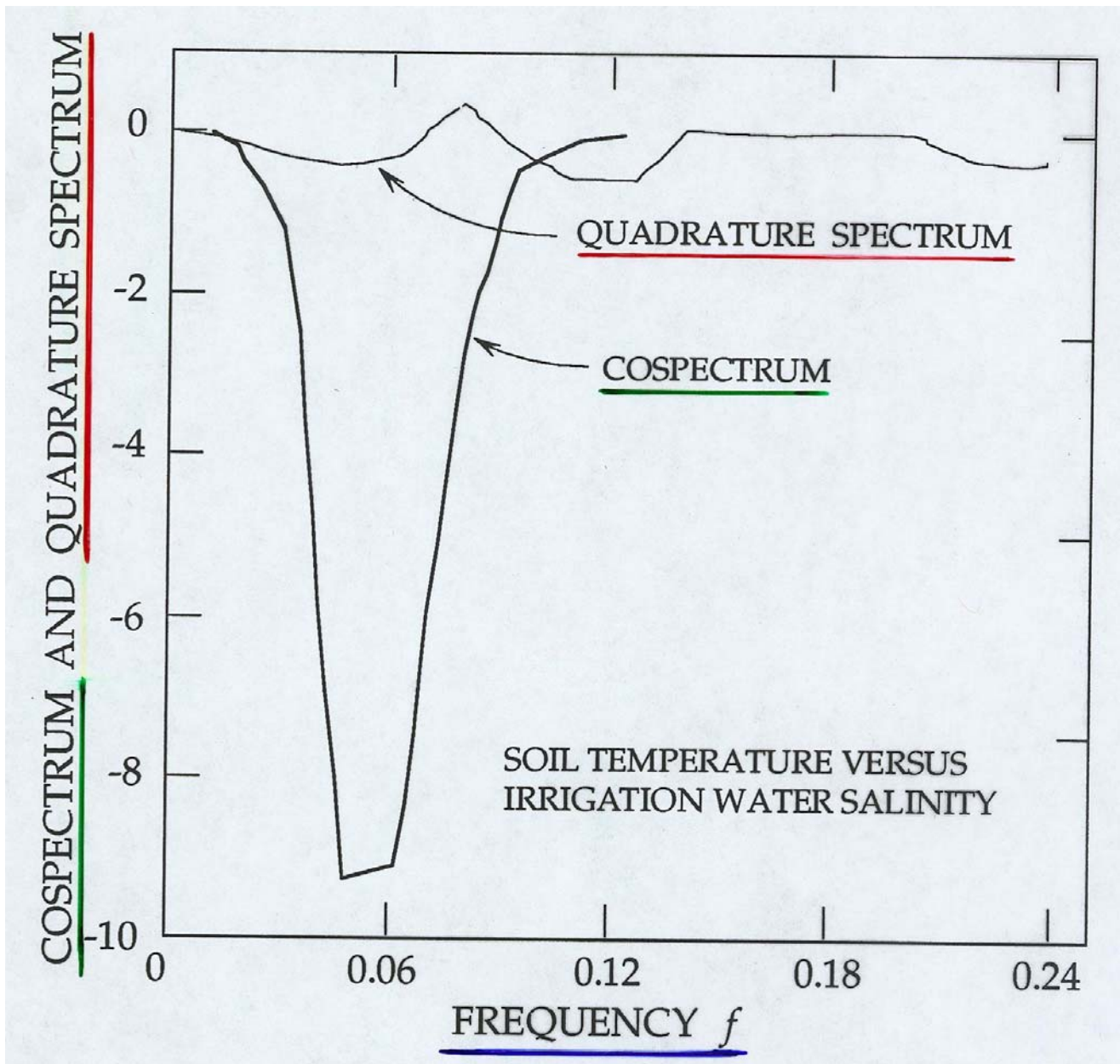
# spectra soil temp (N-S & E-W)



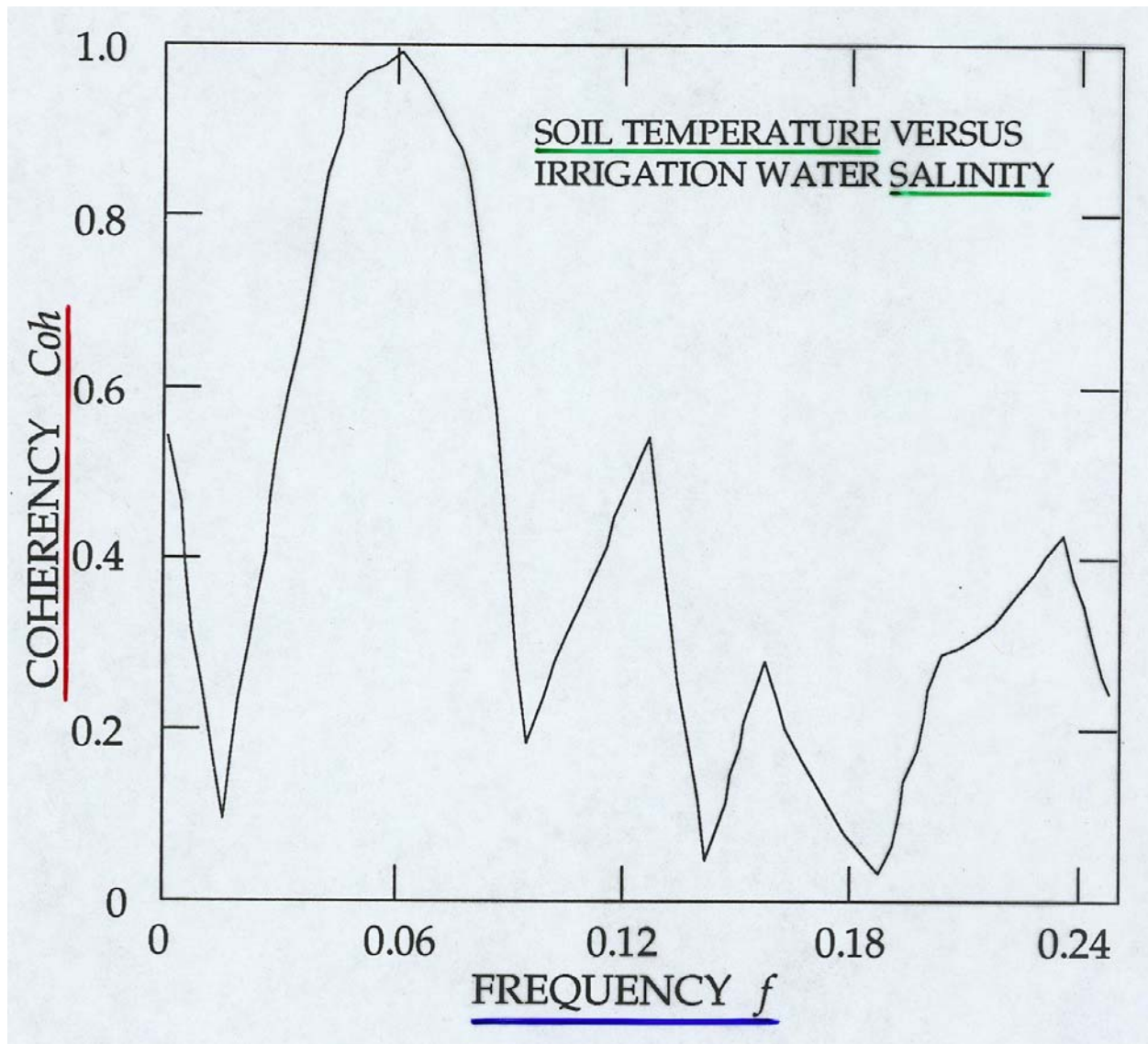
# soil temp & irrigation salinity vs distance



# co-& quad-spectra temp vs water salinity



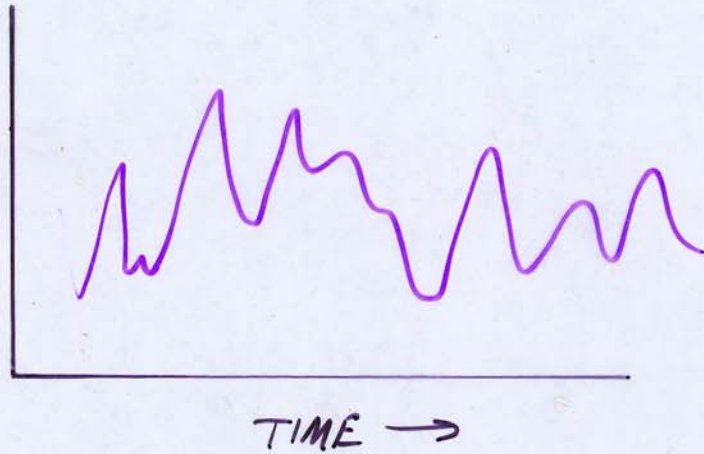
# coherency soil temp vs irrig water salinity



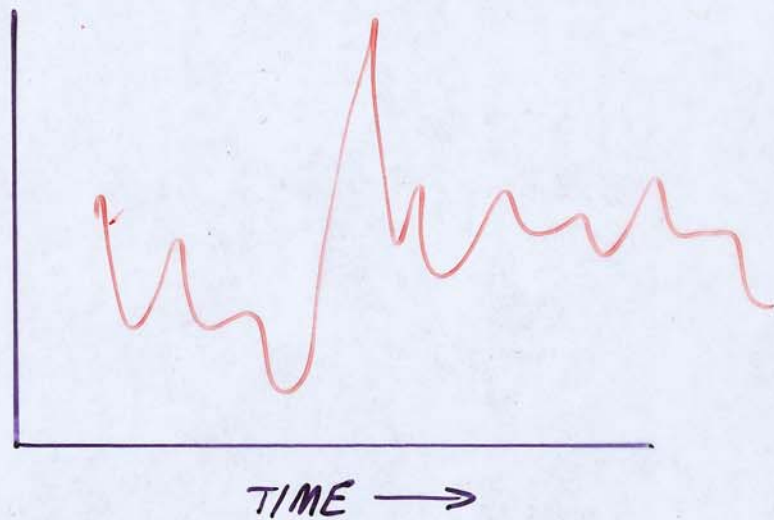
conclusions

# protozoa vs time, etc.

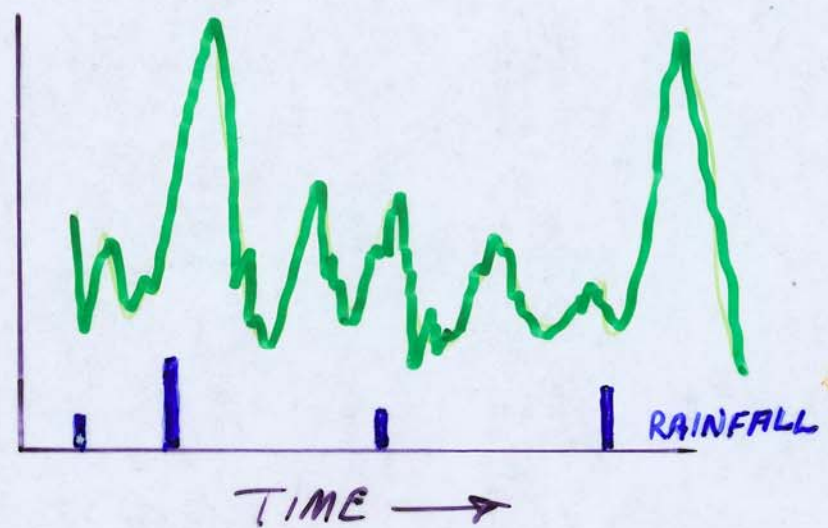
PROTOZOA



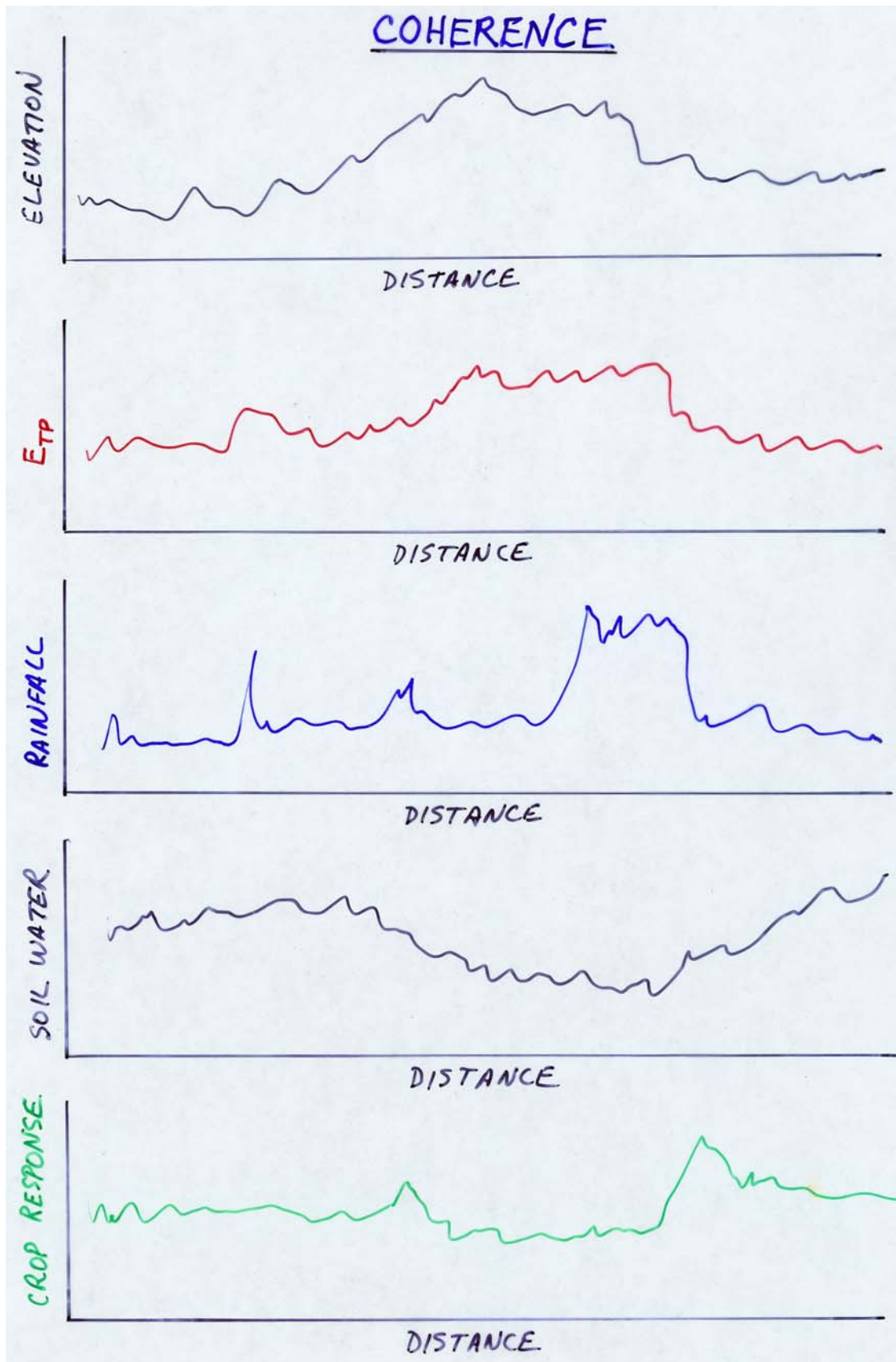
BACTERIA



PLANT  
UPTAKE  
OF N



# coherence across landscape



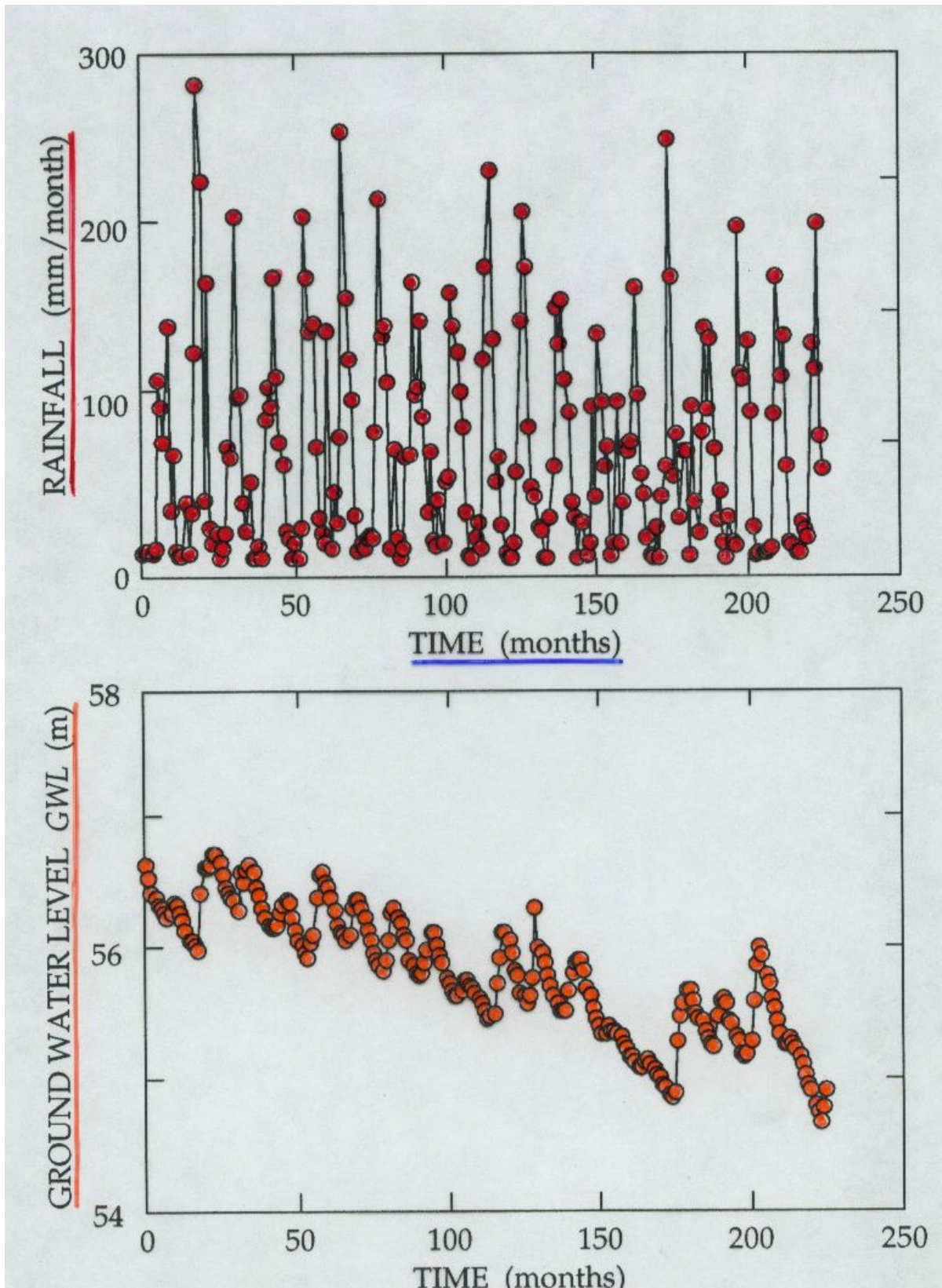


TRAVEL TIME

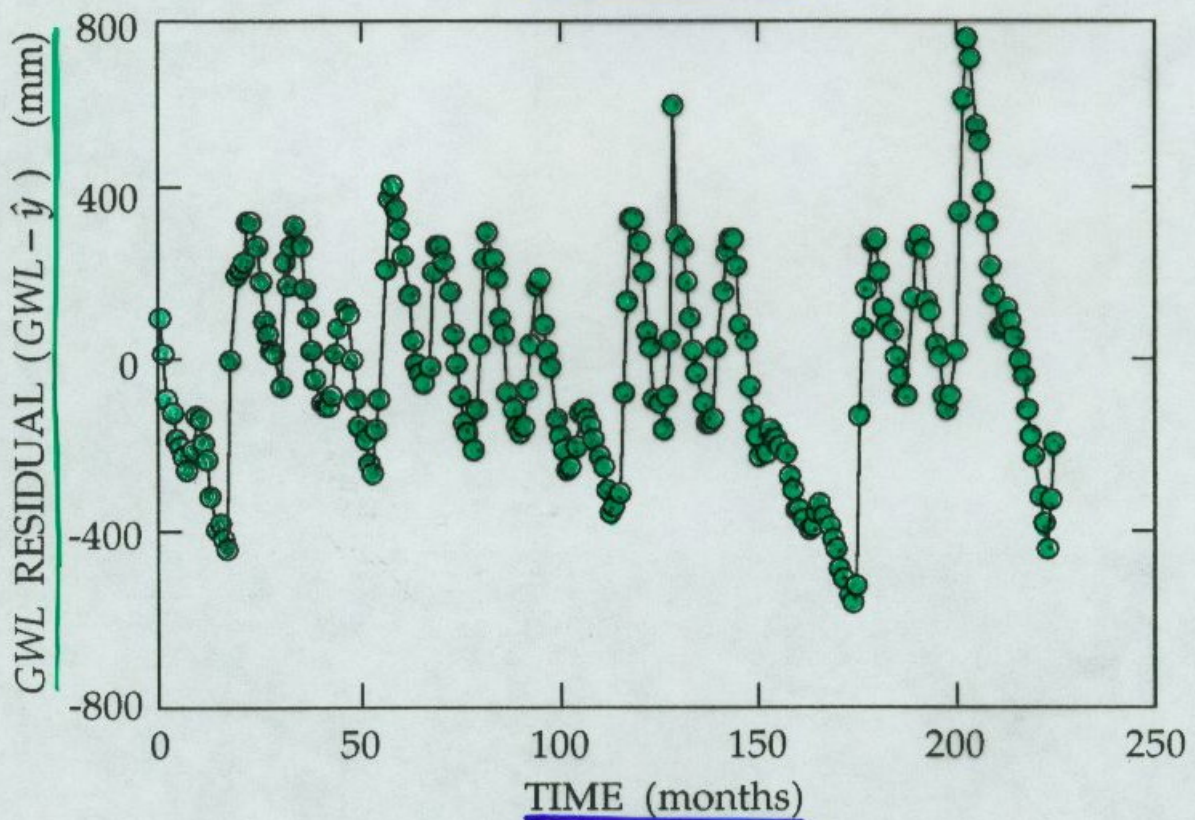
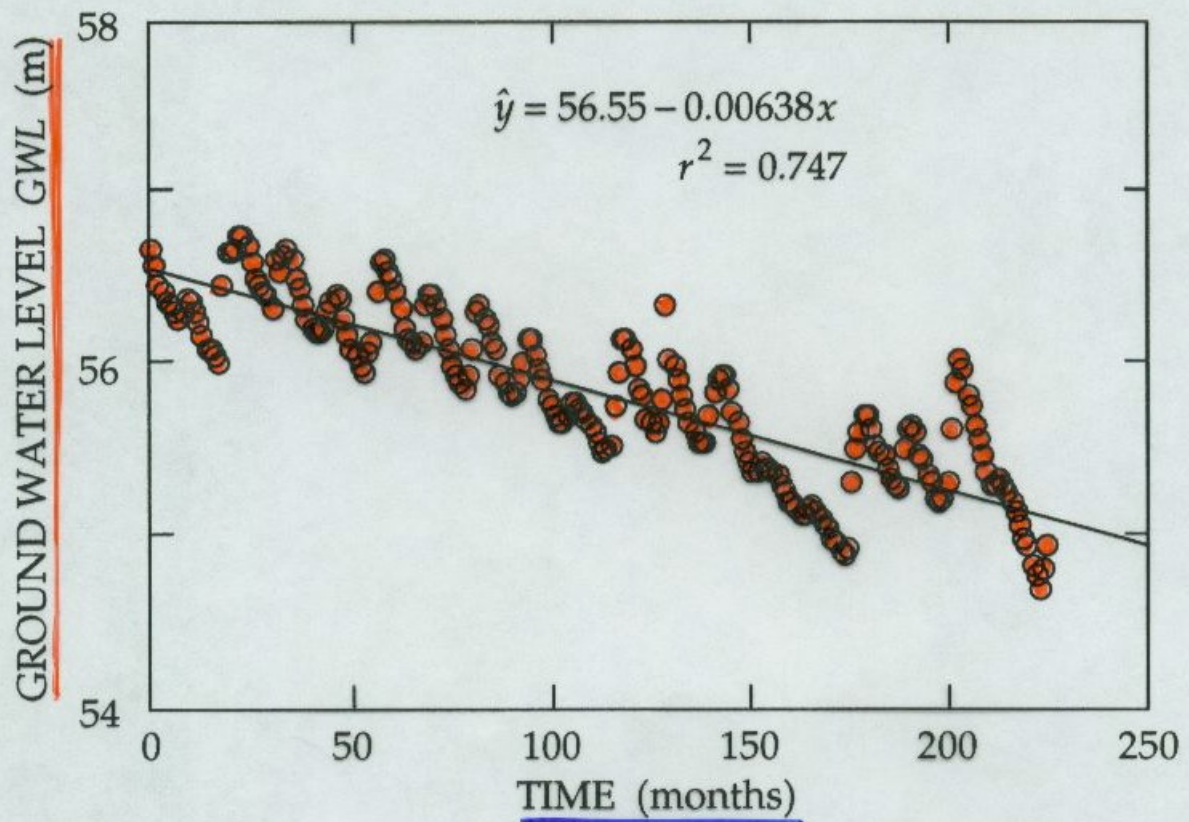
FOR WATER INFILTRATING THE SOIL SURFACE

TO REACH THE WATER TABLE

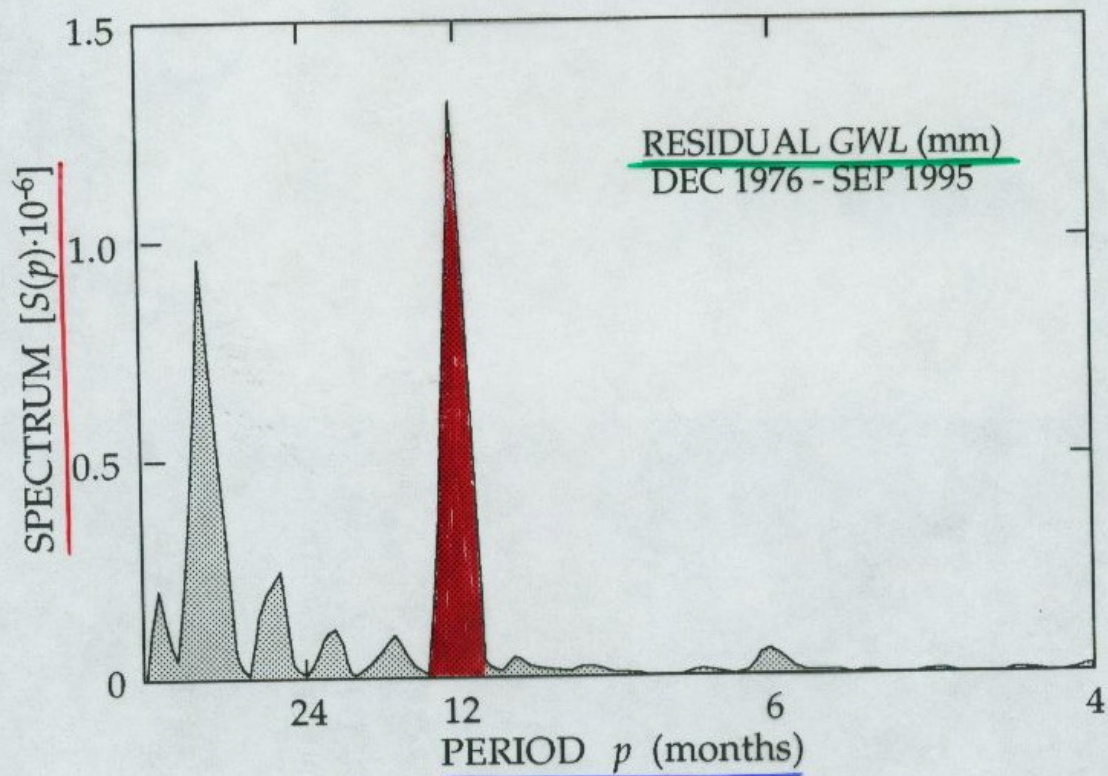
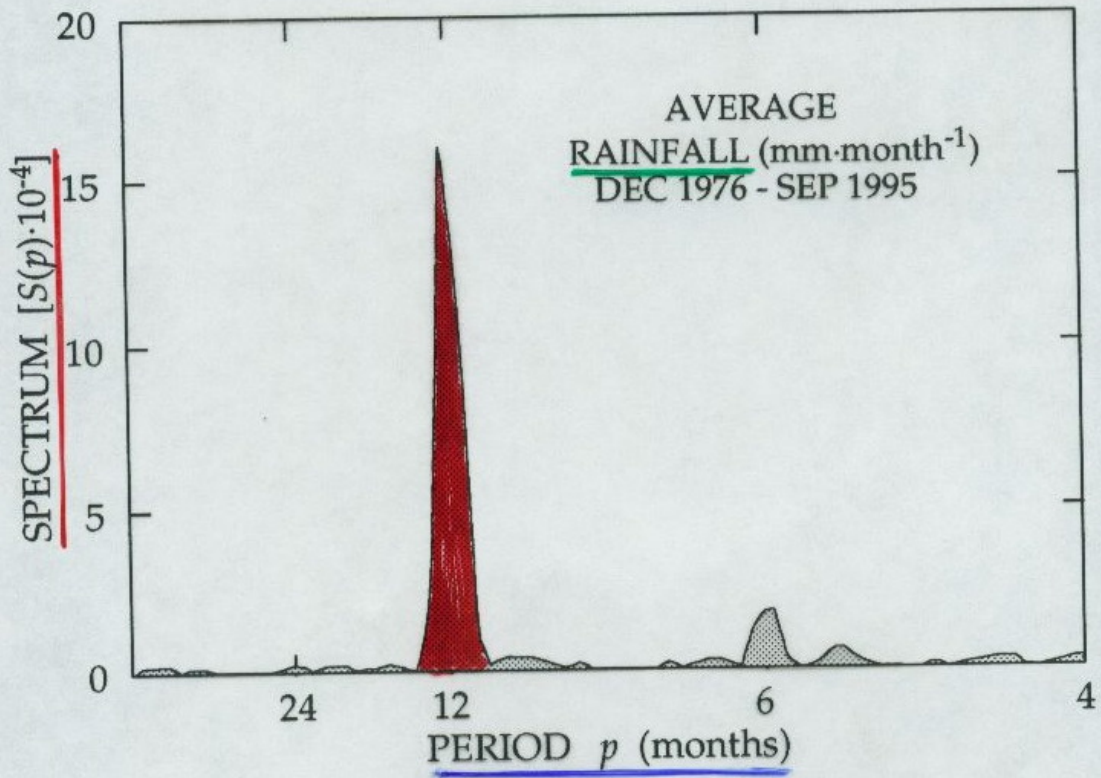
# rainfall and GWL



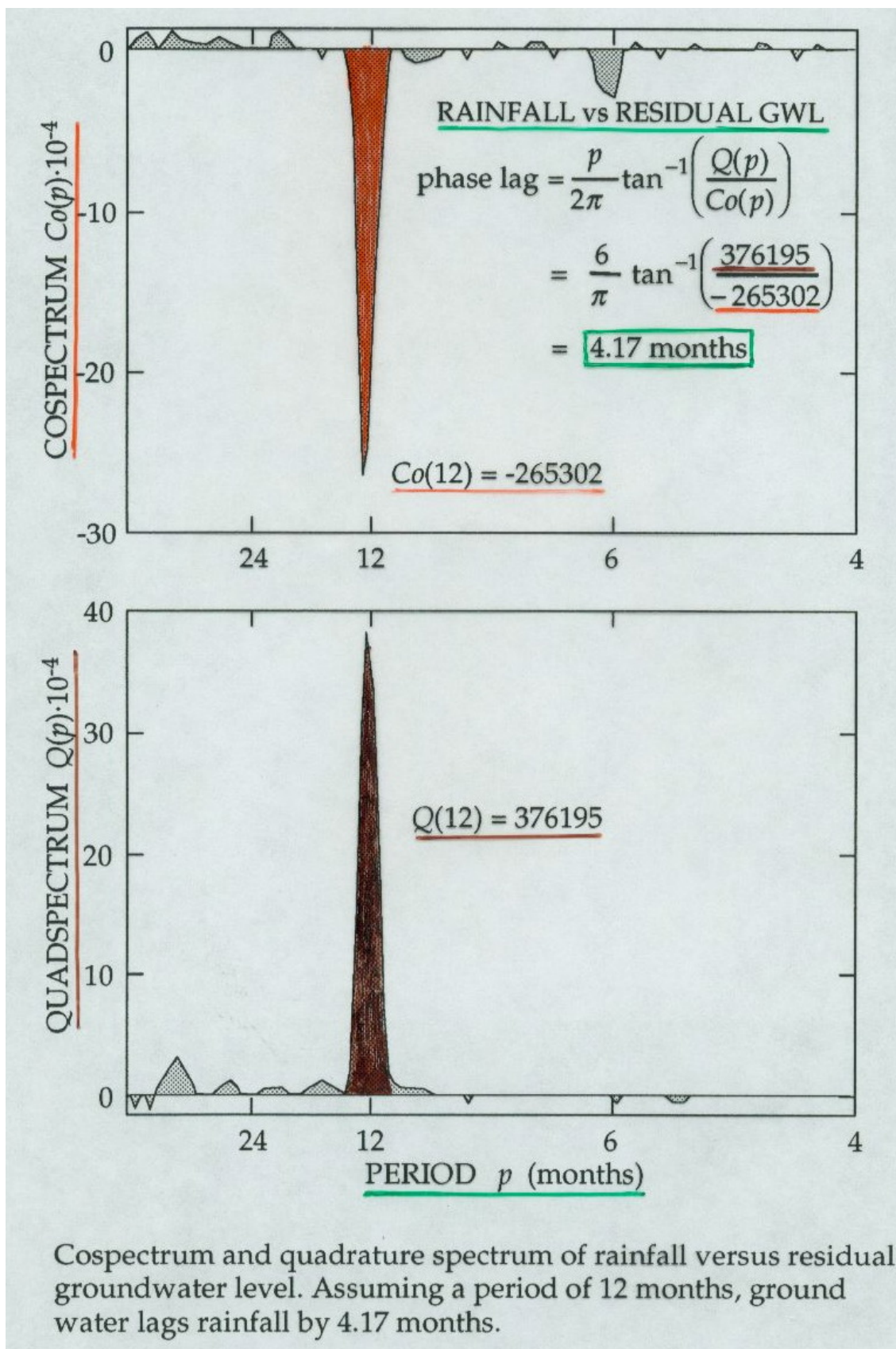
# groundwater fluctuations



# spectra rain & GWL



# crosspectra rain & GWL

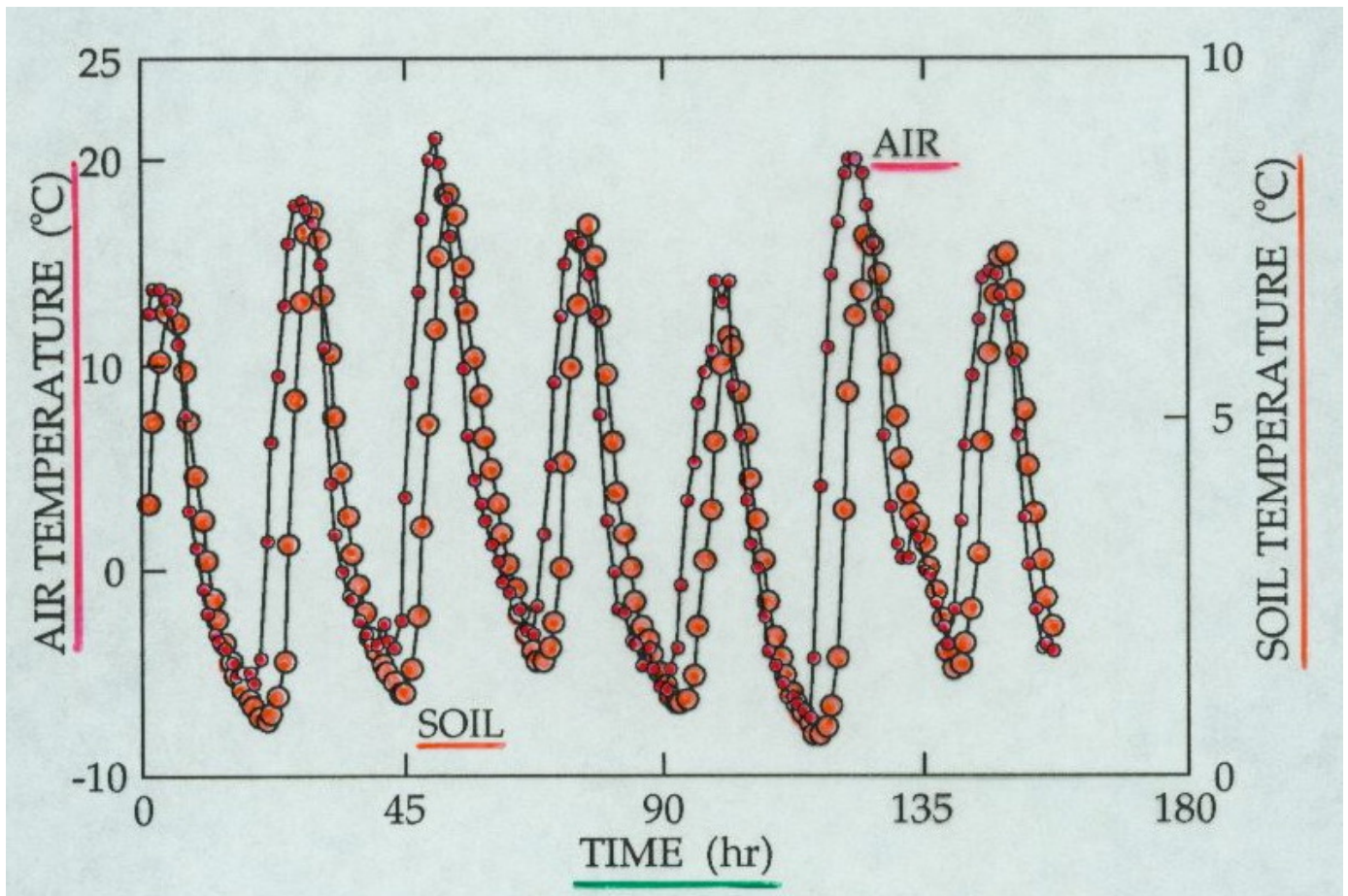


AIR TEMPERATURE

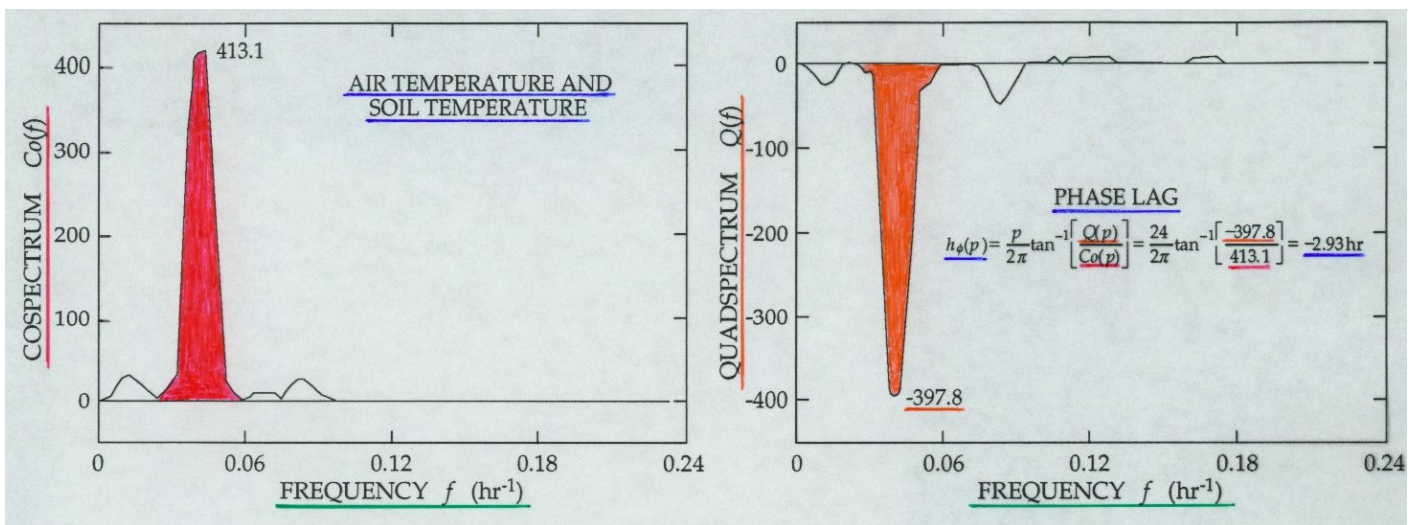
AND

SOIL TEMPERATURE

# air&soilT versus time

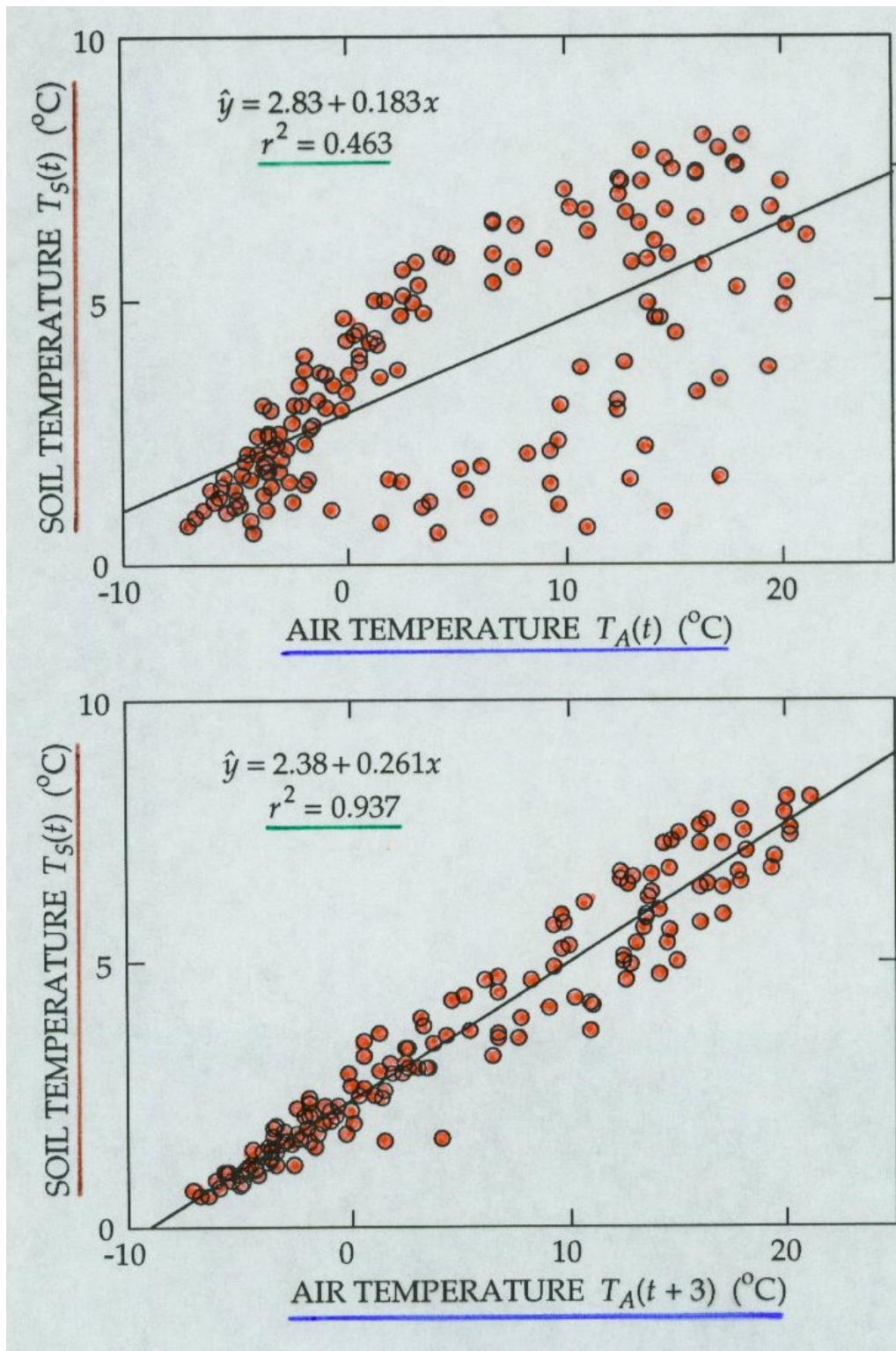


# Air & soil temperature cospect





regressions same time and  
lag of 3 hr

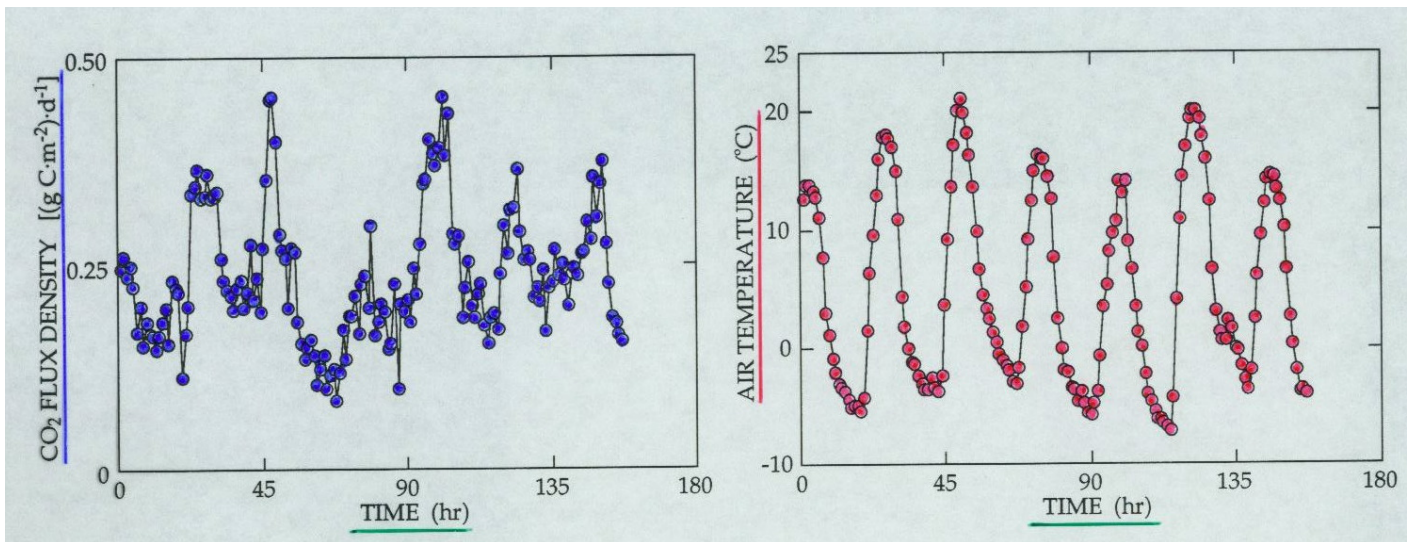


RESPIRATION

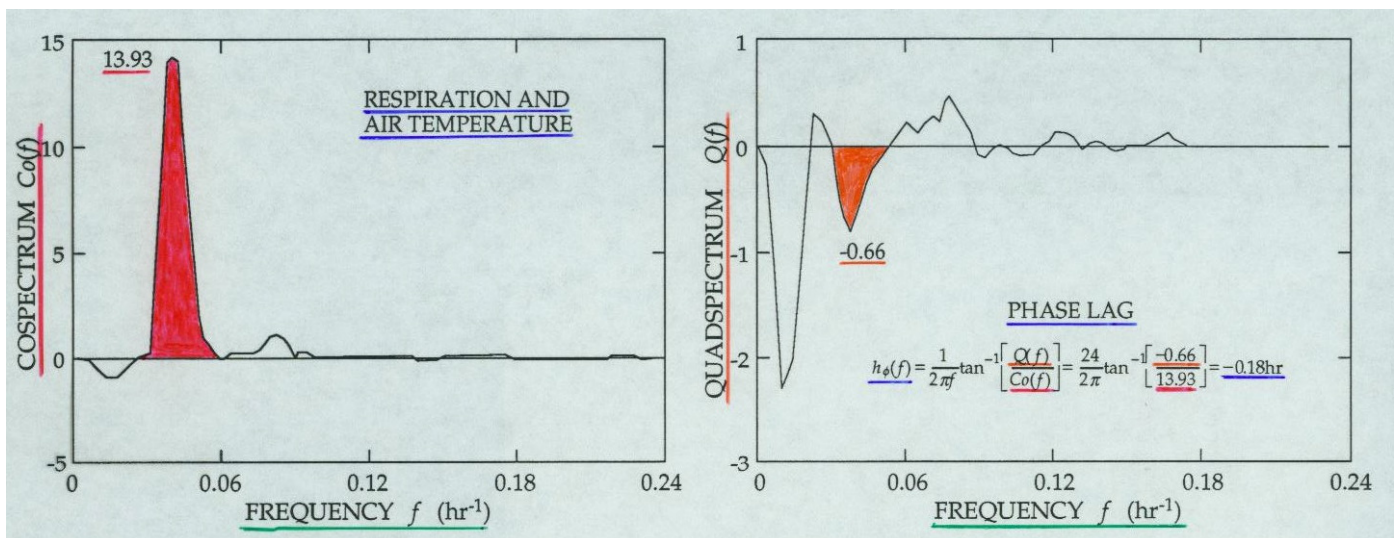
AND

AIR TEMPERATURE

# resp & air T versus time



# Resp & air T cospectrum

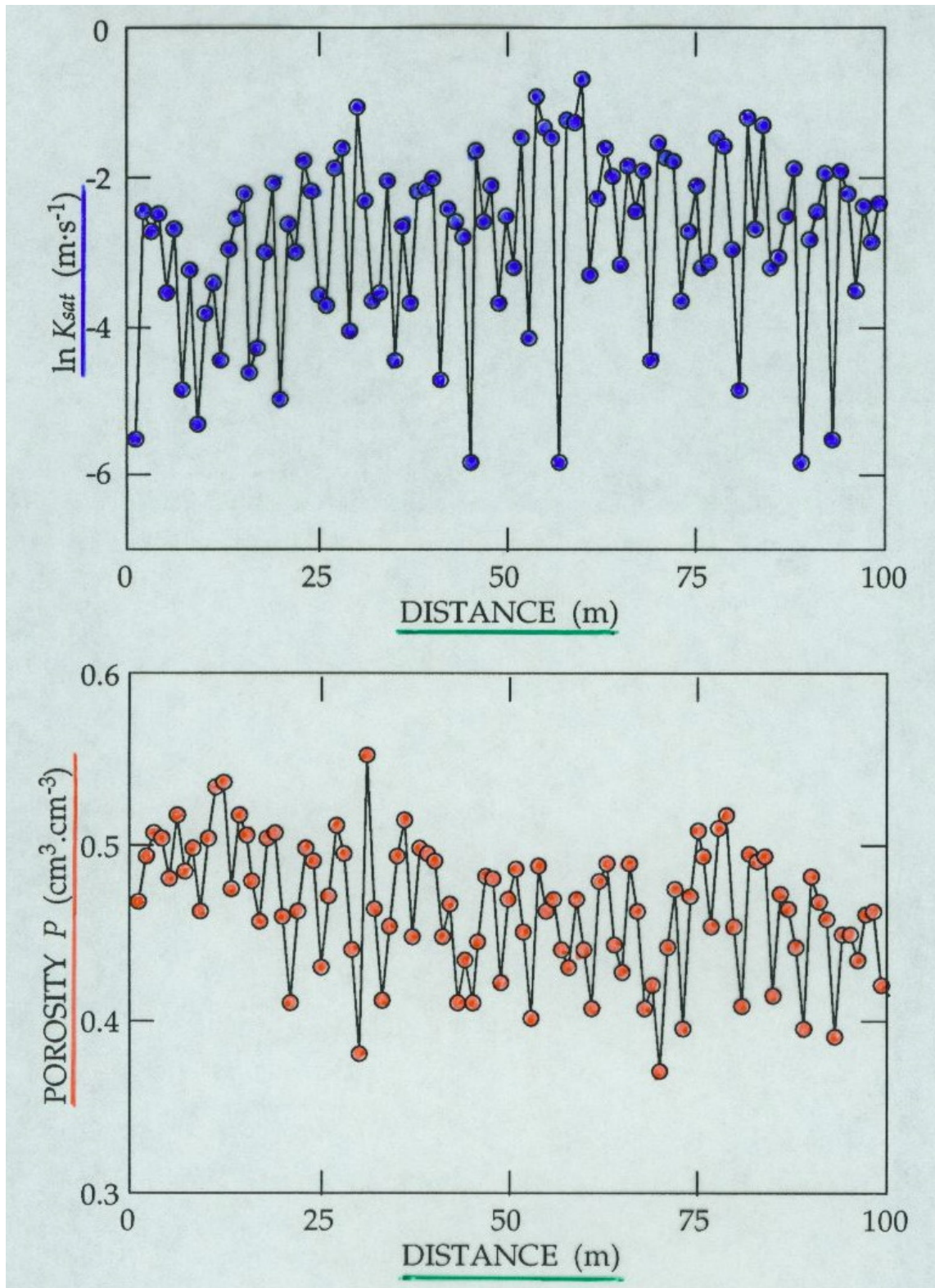


SOIL HYDRAULIC CONDUCTIVITY

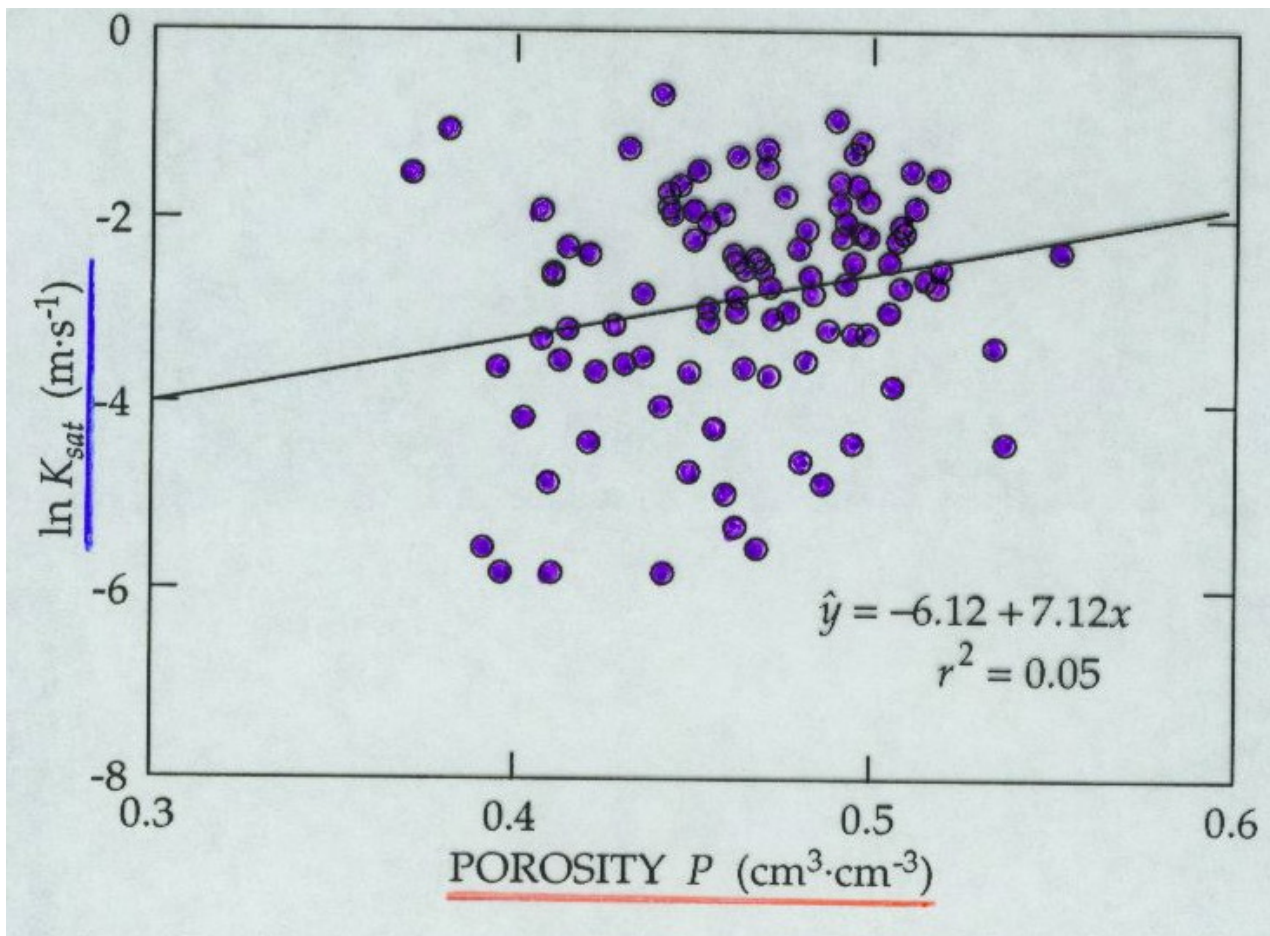
AND

SOIL POROSITY

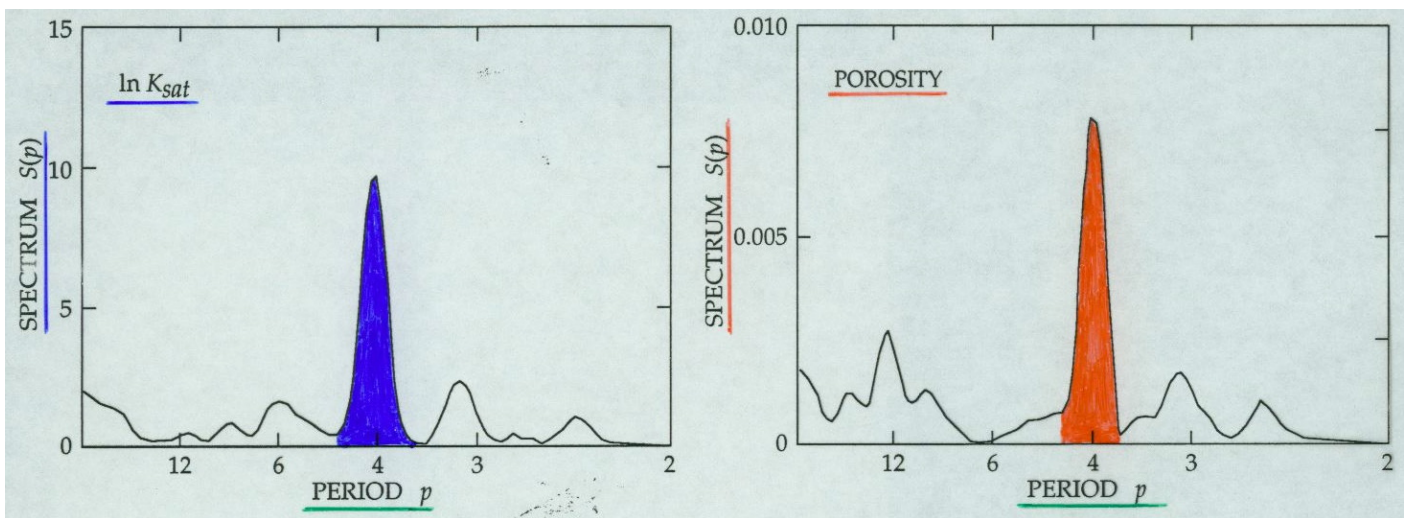
# K & porosity versus distance



# K versus porosity regres

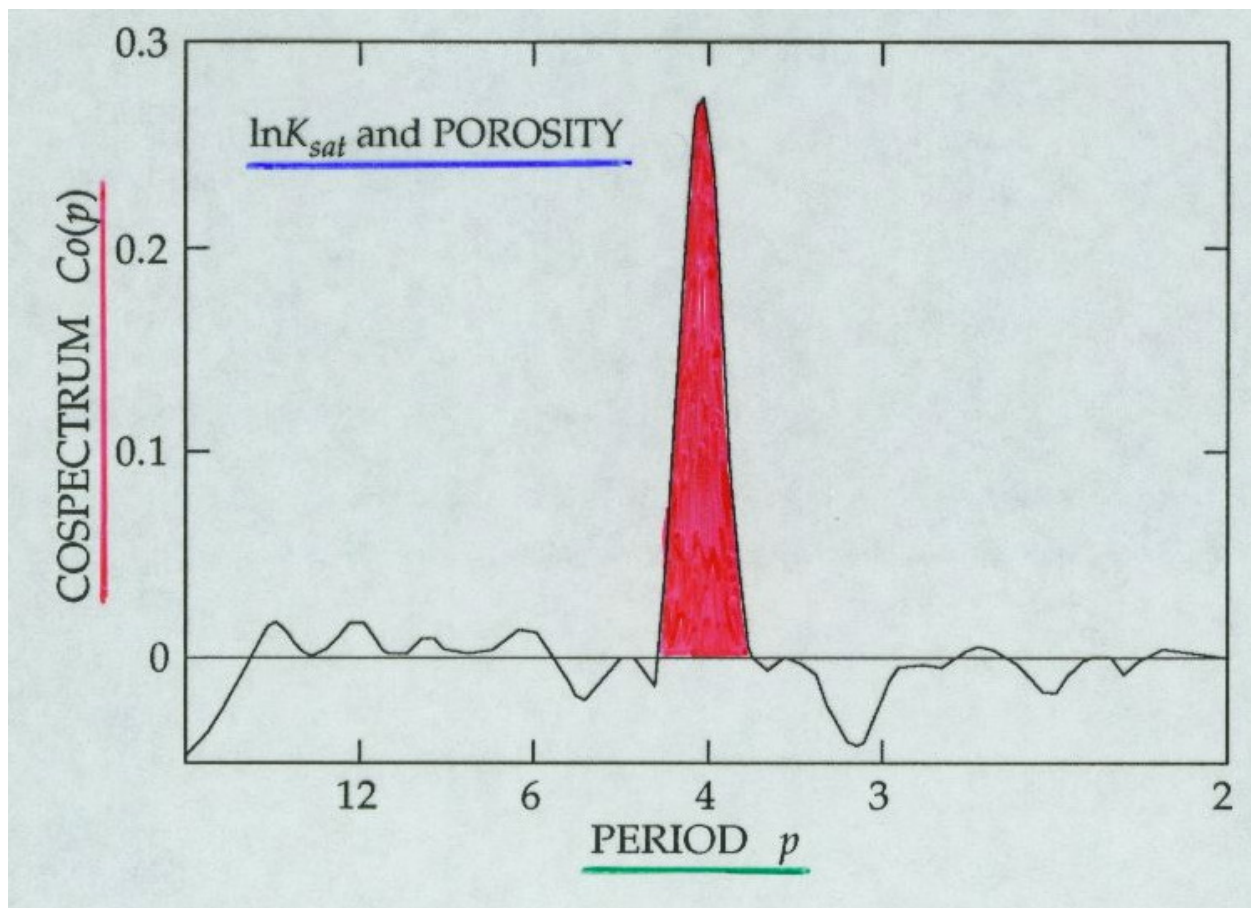


# K versus porosity spectrum

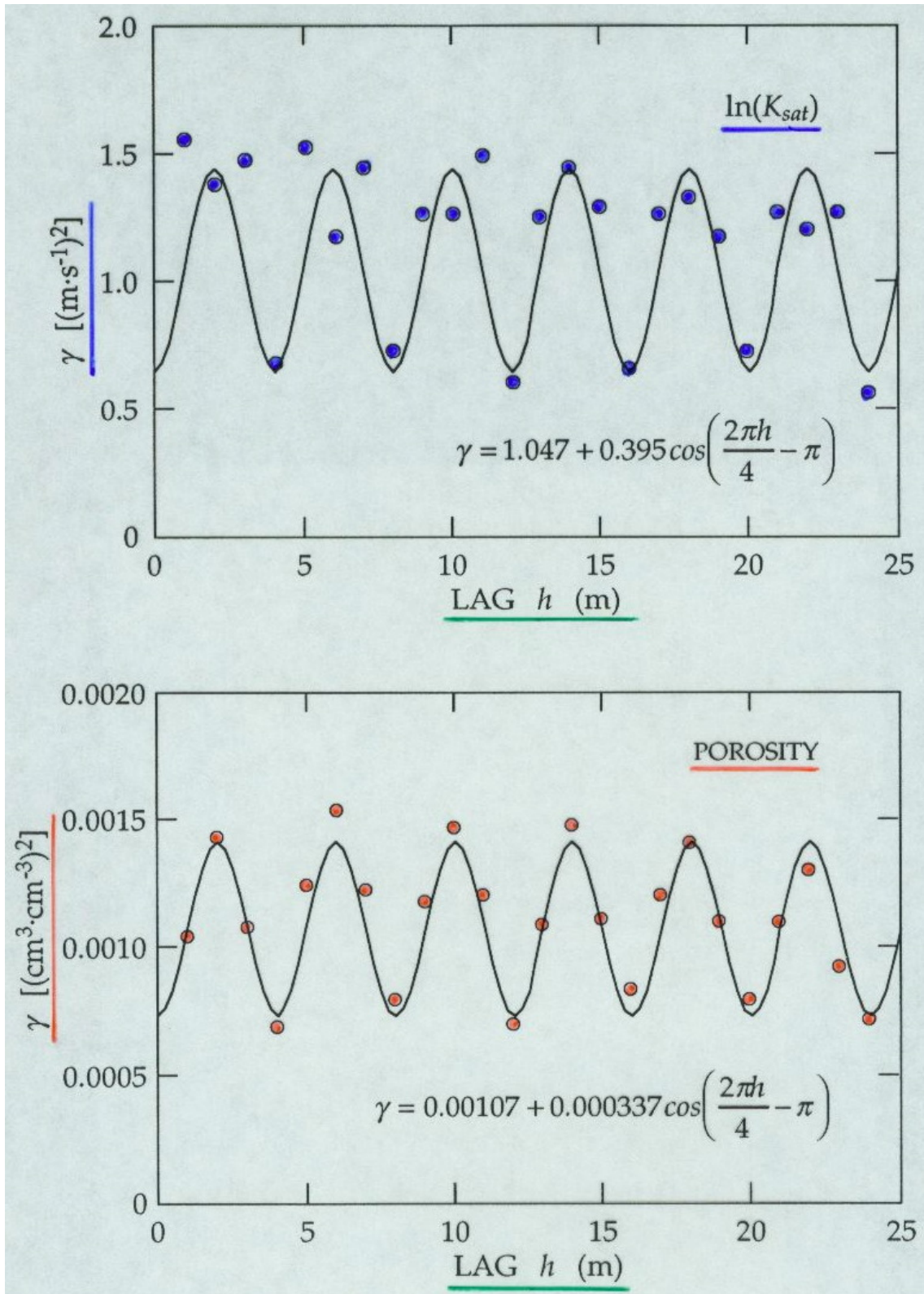




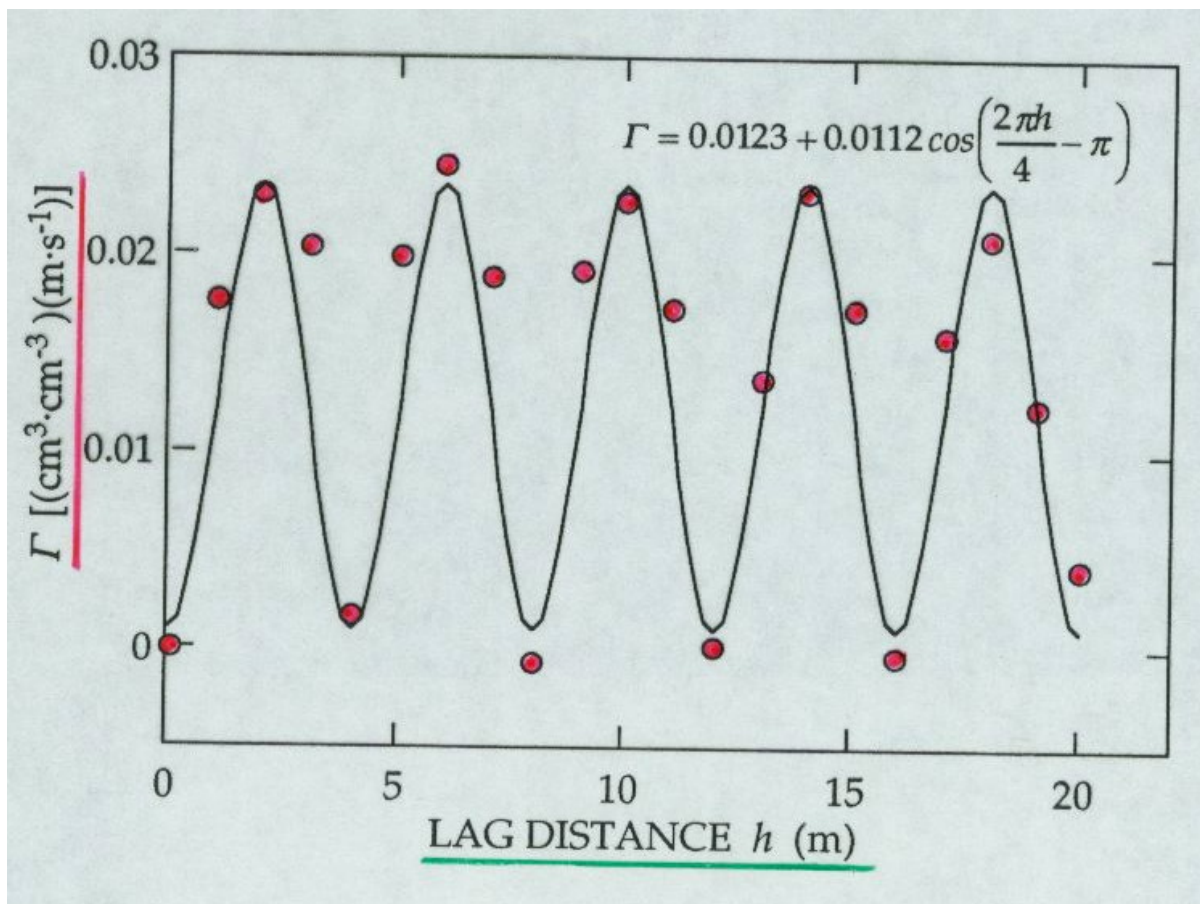
# K versus porosity cospectrum



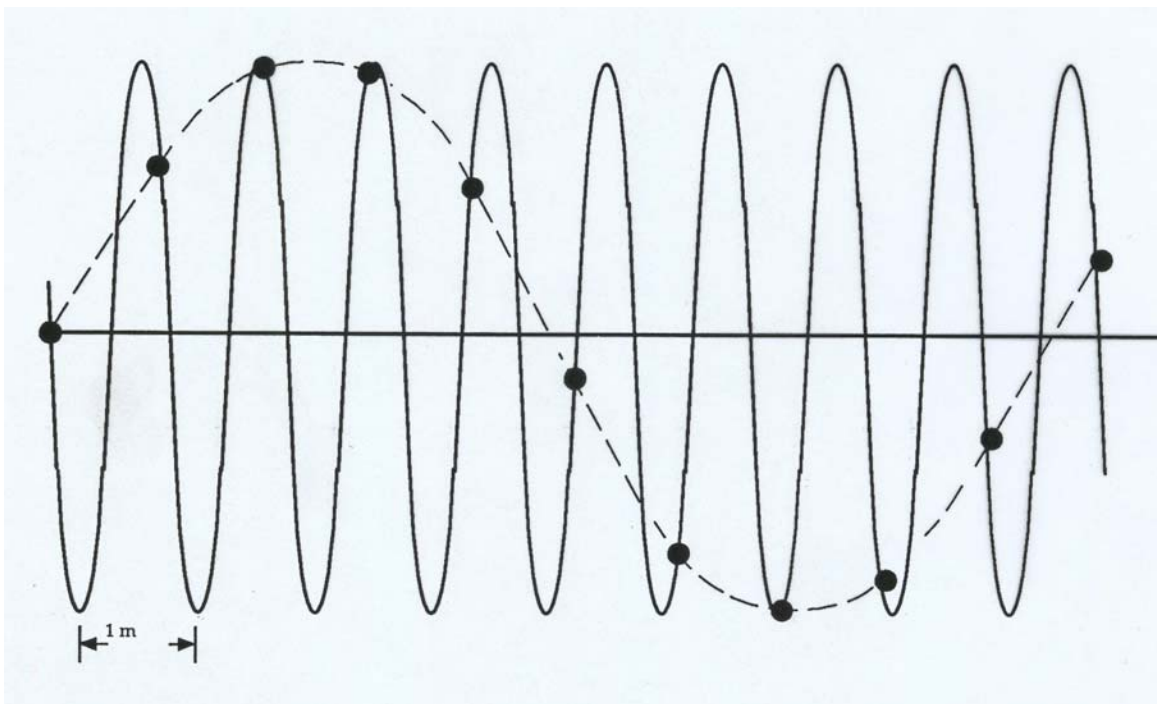
# K and porosity variogram



# K and porosity covariogram



# minimal sampling spectral analysis



# why spectral analysis?

## ¿ WHY SPECTRAL ANALYSIS ?

Fields are traversed in cyclic patterns

Impact and persistence of cyclic patterns should be known

Link observations with different physical and chemical phenomena

Size of sensor should depend upon period of oscillation

Cyclic pattern may be more important than average behavior

Over what distances should observations be made to obtain a "meaningful average"?

Filter out trends across a field to look at local variations, or vice versa

Analyze entire field or group of fields rather than small plots

AUTOREGRESSIVE AND  
MOVING AVERAGE FUNCTIONS

# AR and MA equations

## Random Walk Model

$$A_i - A_{i-1} = \omega_i$$

$$A_i = A_{i-1} + \omega_i$$

## Autoregressive Model AR(p)

$$A_i = \phi_1 A_{i-1} + \phi_2 A_{i-2} + \dots + \phi_p A_{i-p} + \omega_i$$

### Partial Autocorrelation Function

$$\phi_{i+1,i+1} = \frac{r(i+1) - \sum_{j=1}^i \phi_{p,j} \cdot r(i+1-j)}{1 - \sum_{j=1}^i \phi_{i,j} \cdot r(j)}$$

where

$$\phi_{i+1,j} = \phi_{i,j} - \phi_{i+1,i+1} \cdot \phi_{i,i-j+1}$$

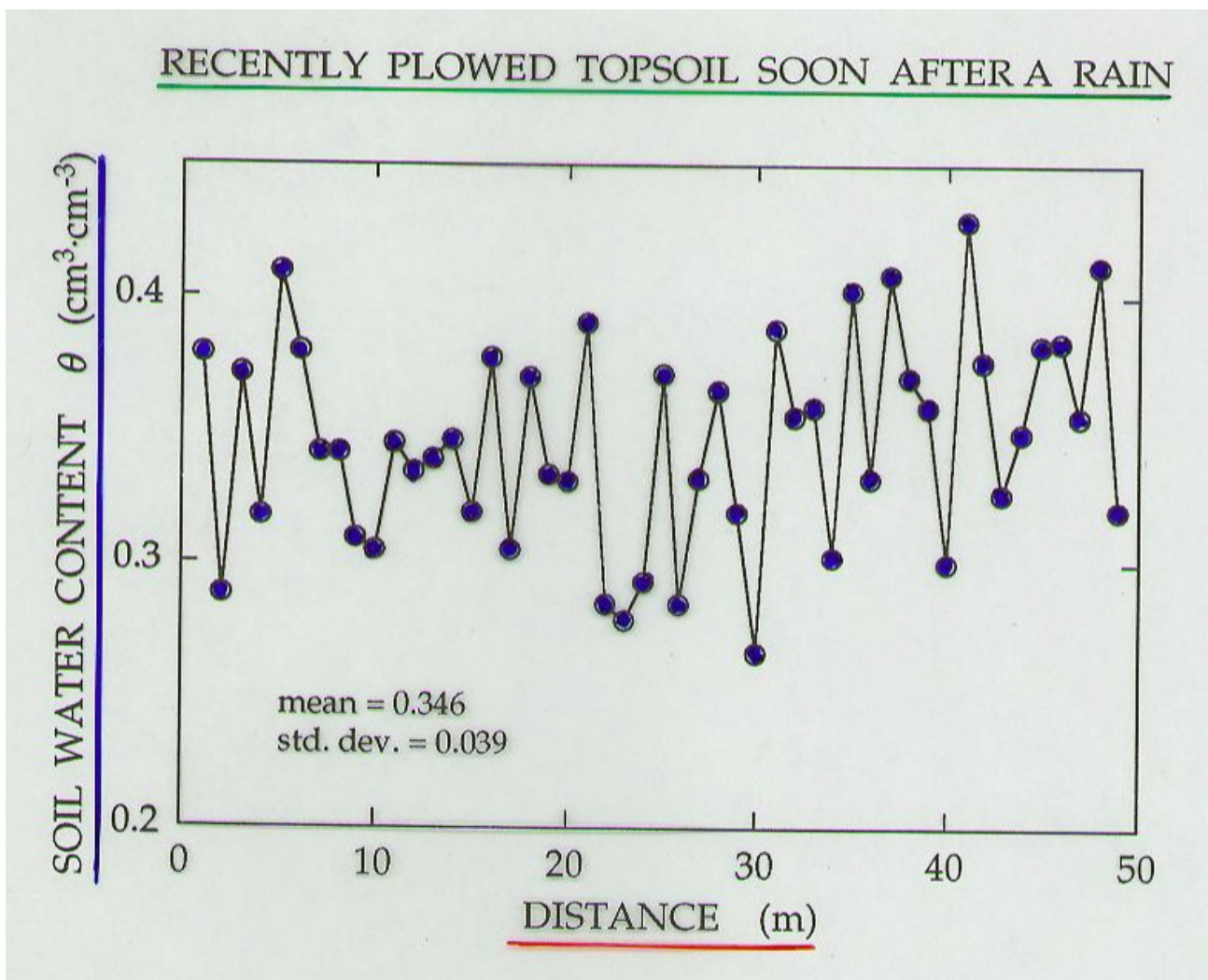
## Moving Average Model MA(q)

$$A_i = \omega_i - \theta_1 \omega_{i-1} - \theta_2 \omega_{i-2} - \dots - \theta_q \omega_{i-q}$$

## Autoregressive Moving Average Model ARMA(p,q)

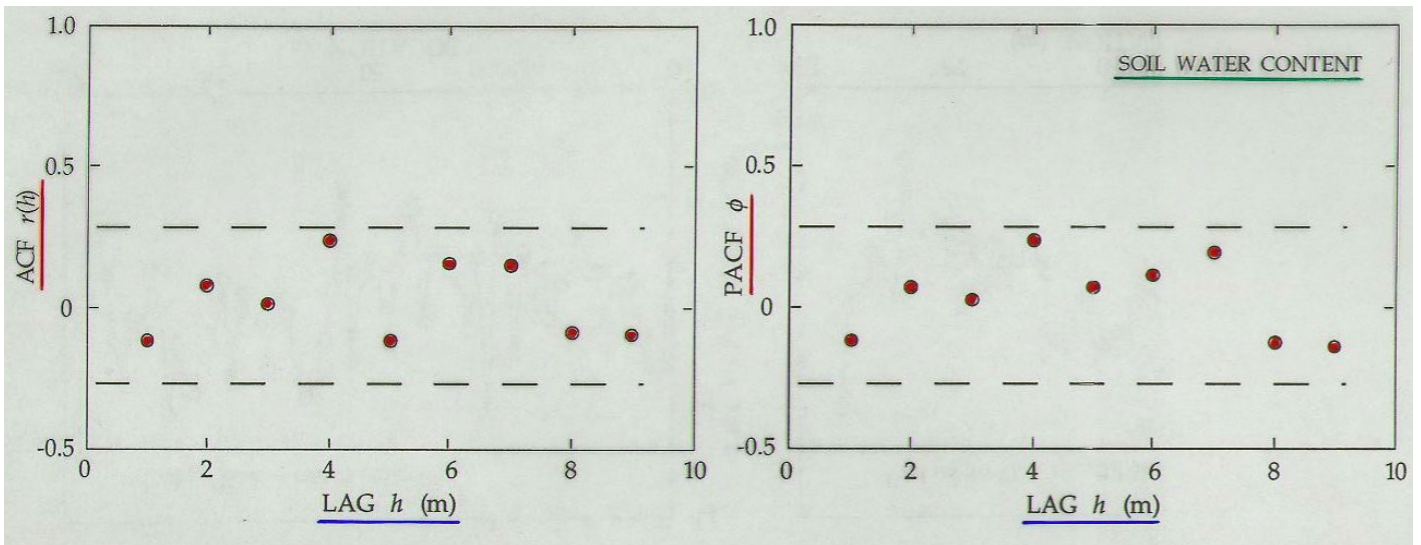
$$A_i = \phi_1 A_{i-1} + \phi_2 A_{i-2} + \dots + \phi_p A_{i-p} + \omega_i - \theta_1 \omega_{i-1} - \theta_2 \omega_{i-2} - \dots - \theta_q \omega_{i-q}$$

# soil water after rain vs distance

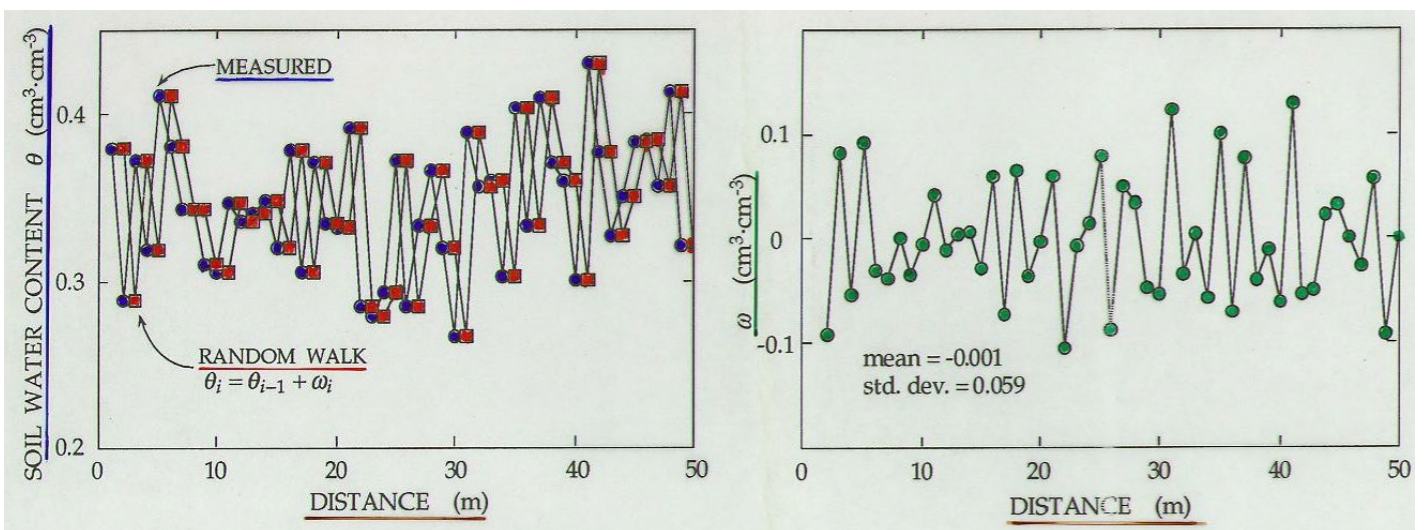




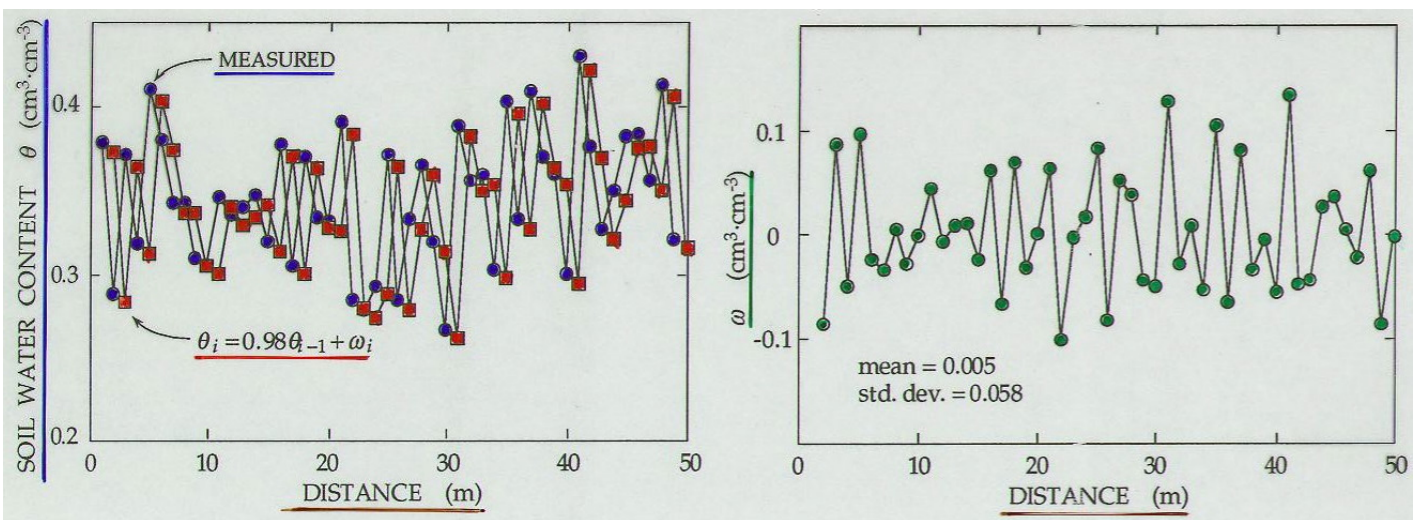
# ACF & PACF of soil water



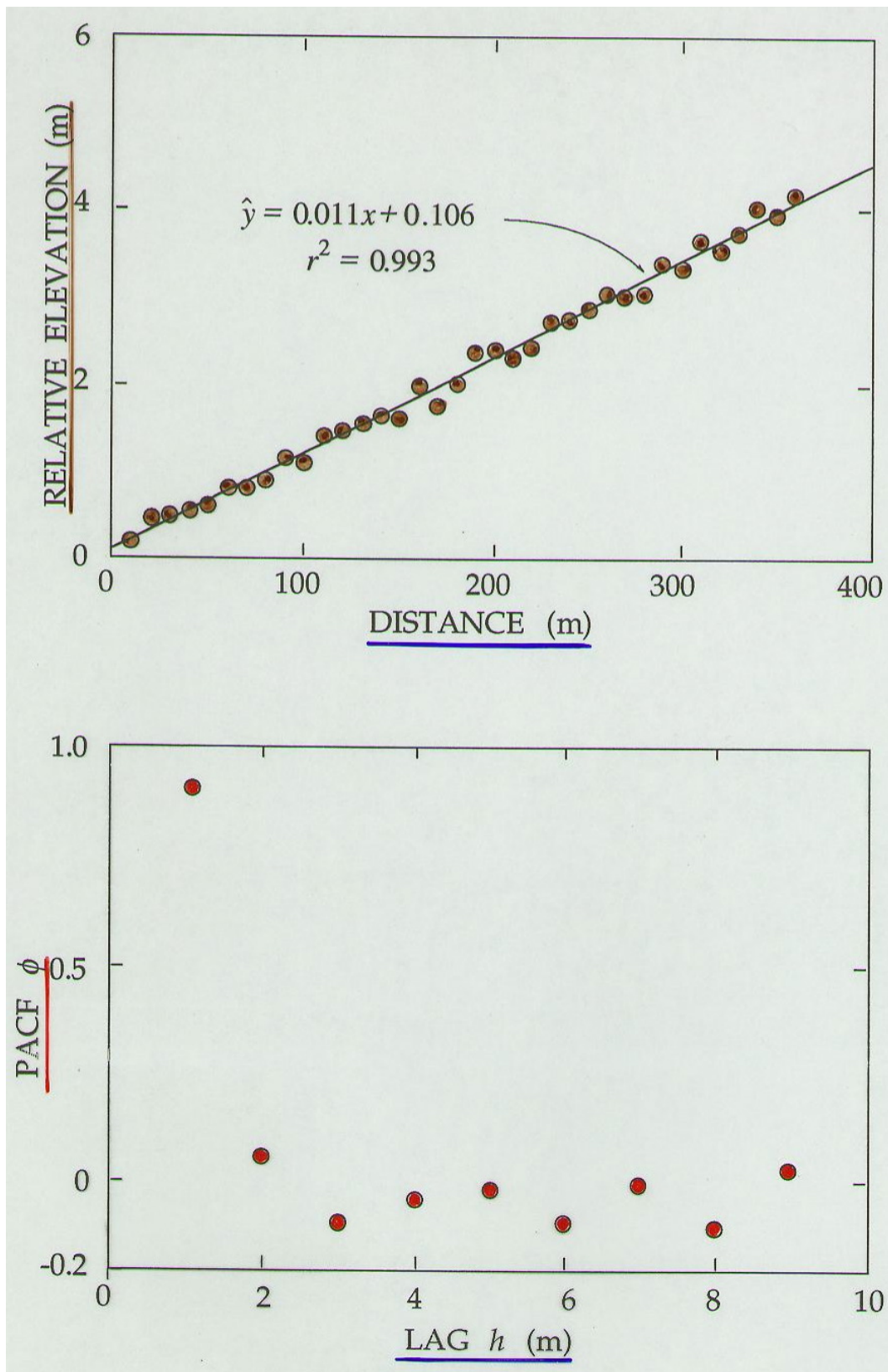
# random walk soil water



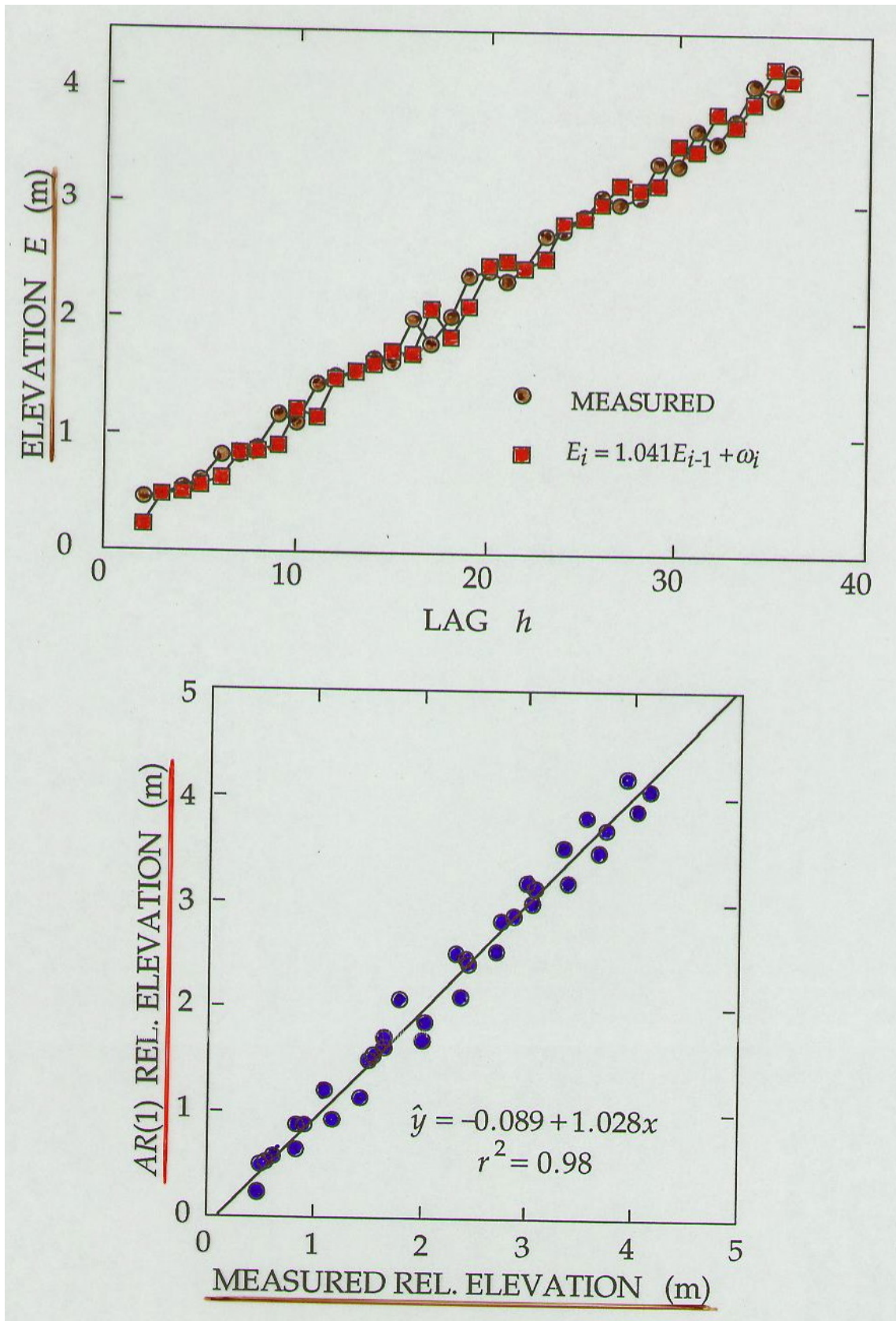
# AR(1) soil water



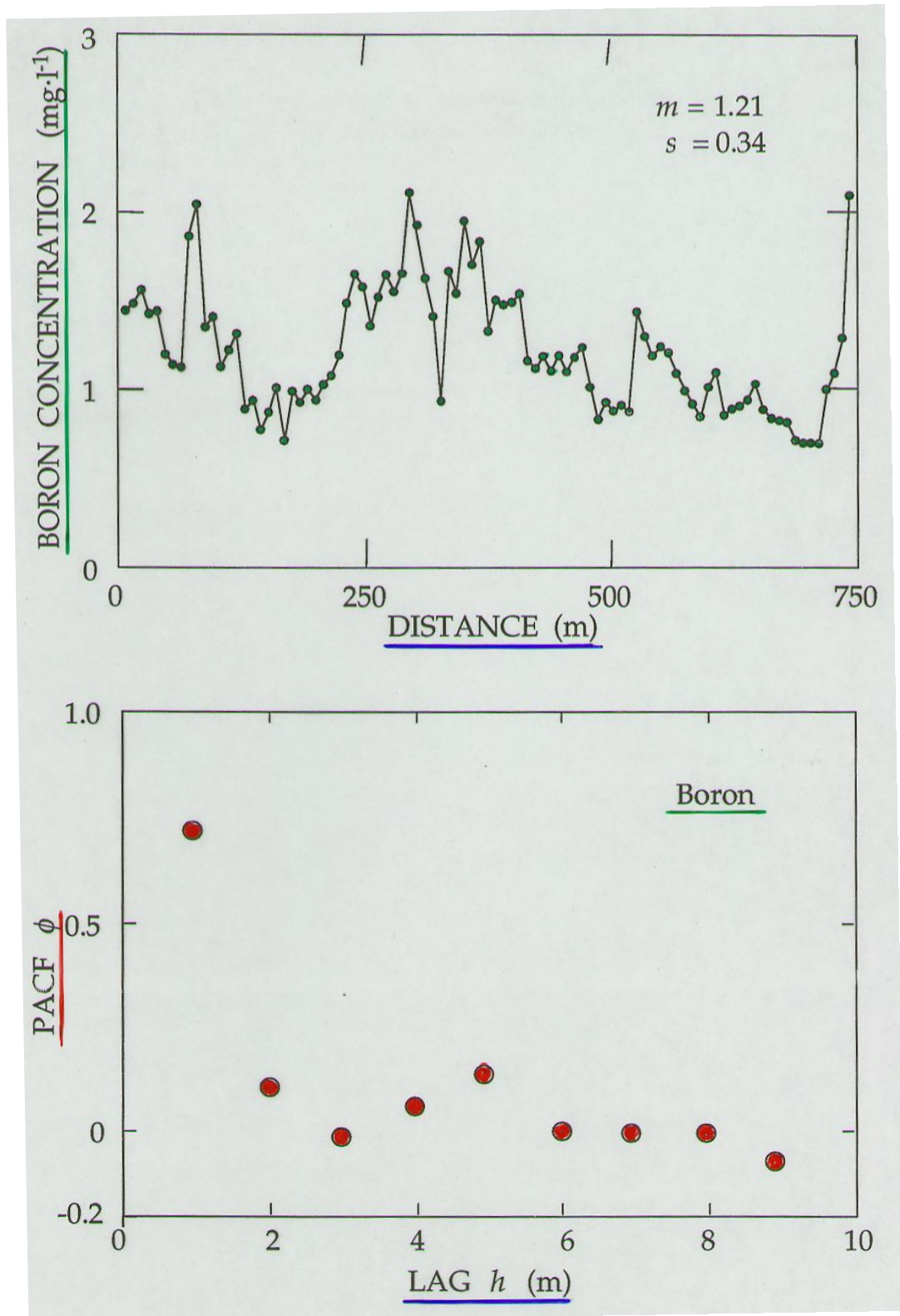
# linear soil elevation and ACF



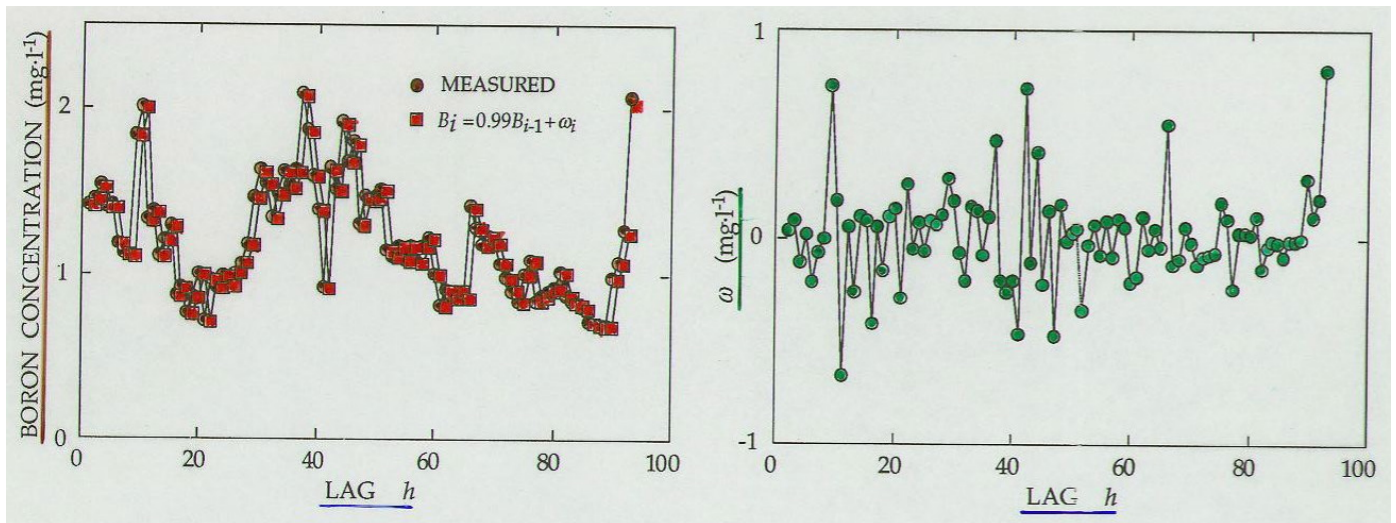
# AR(1) of soil elevation



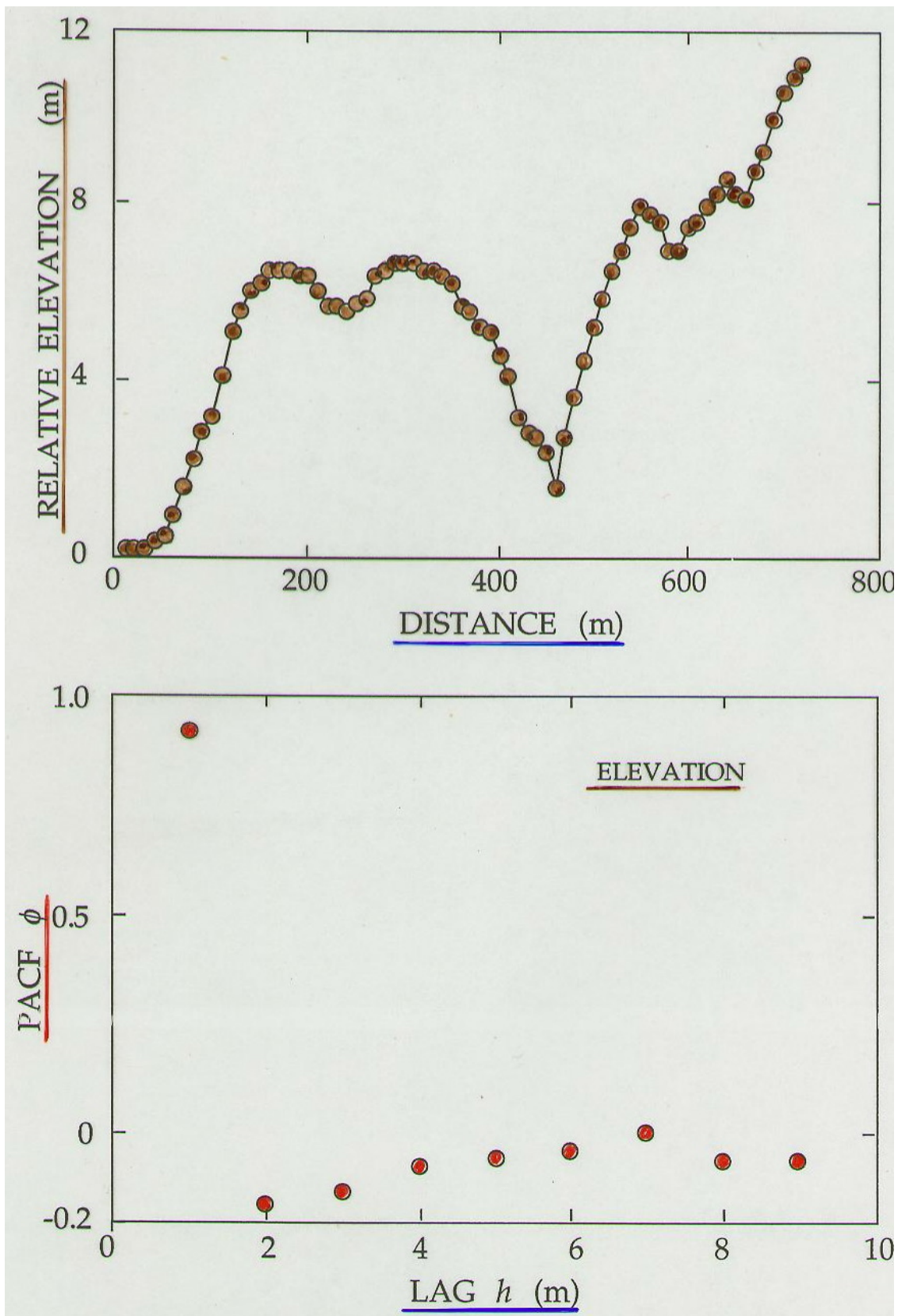
# boron conc vs distance & PACF (NM)



# AR(1) of boron NM

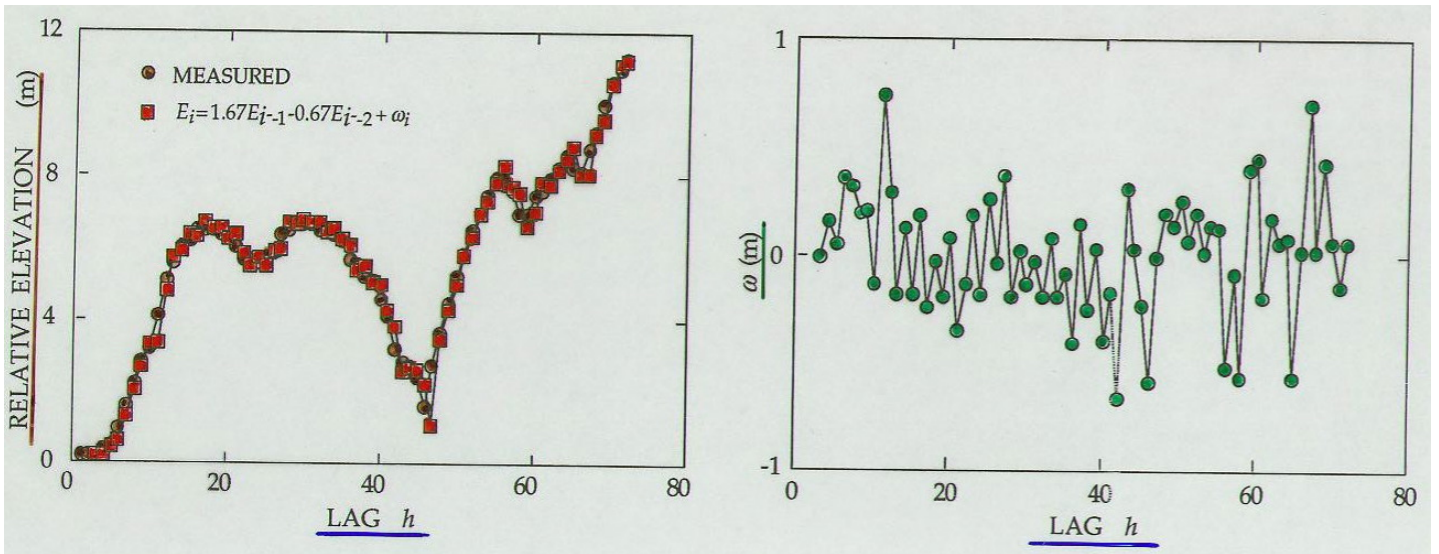


# soil elevation vs distance

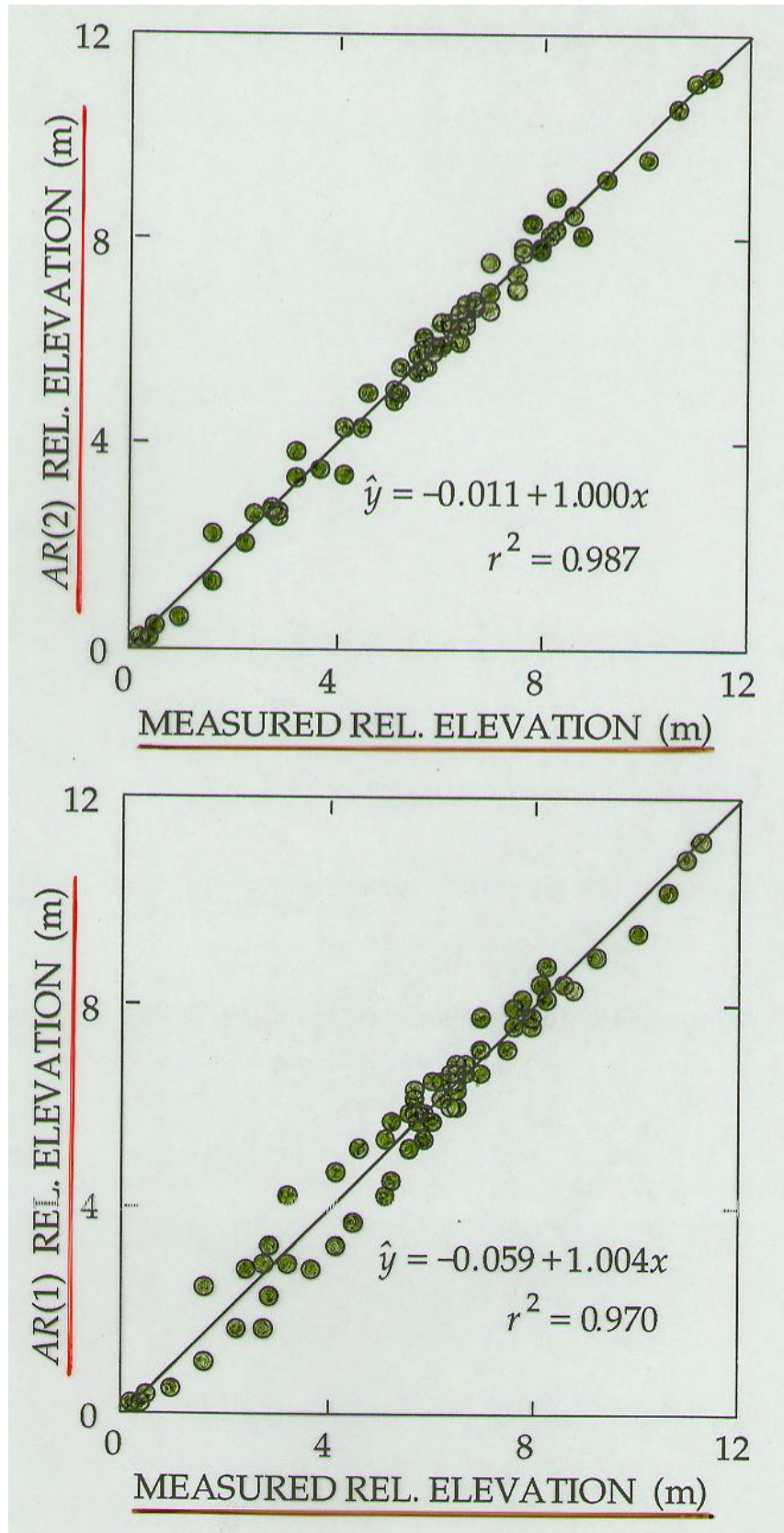




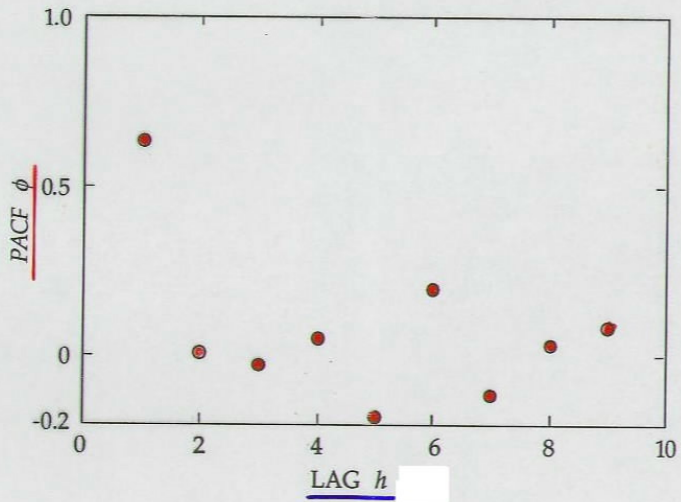
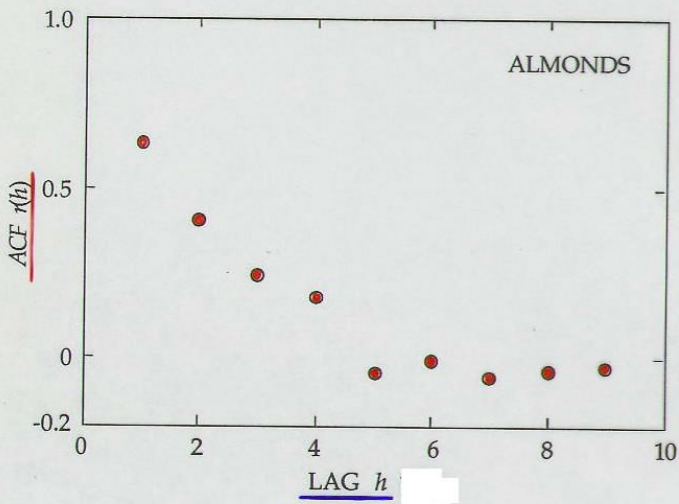
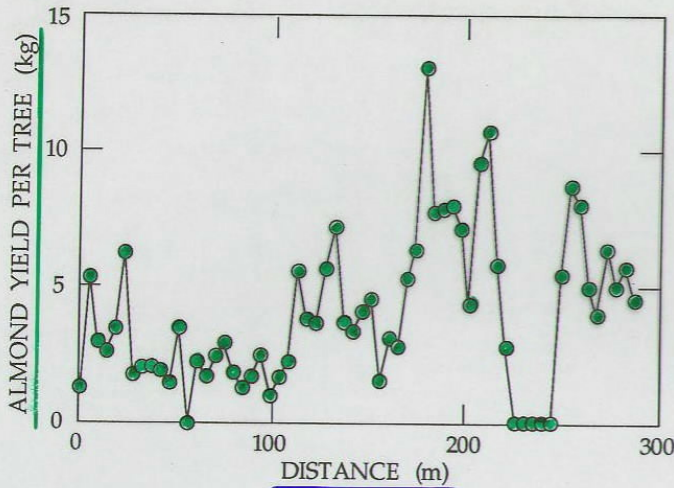
# AR(1) and residual elevation



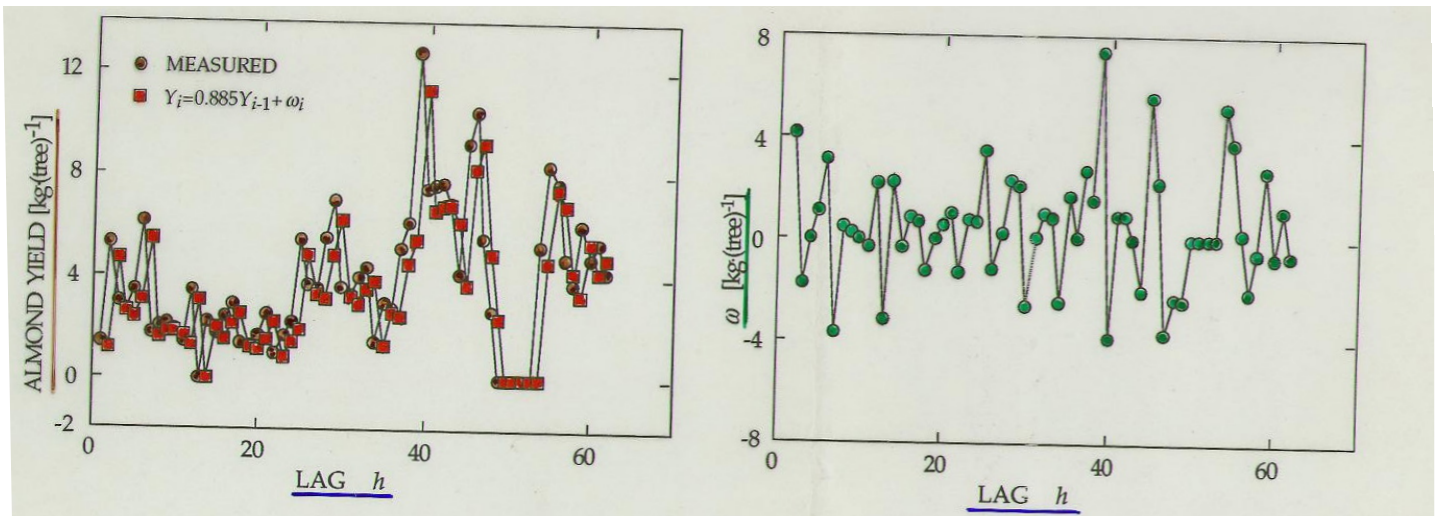
# AR(1) & AR(2) elevation



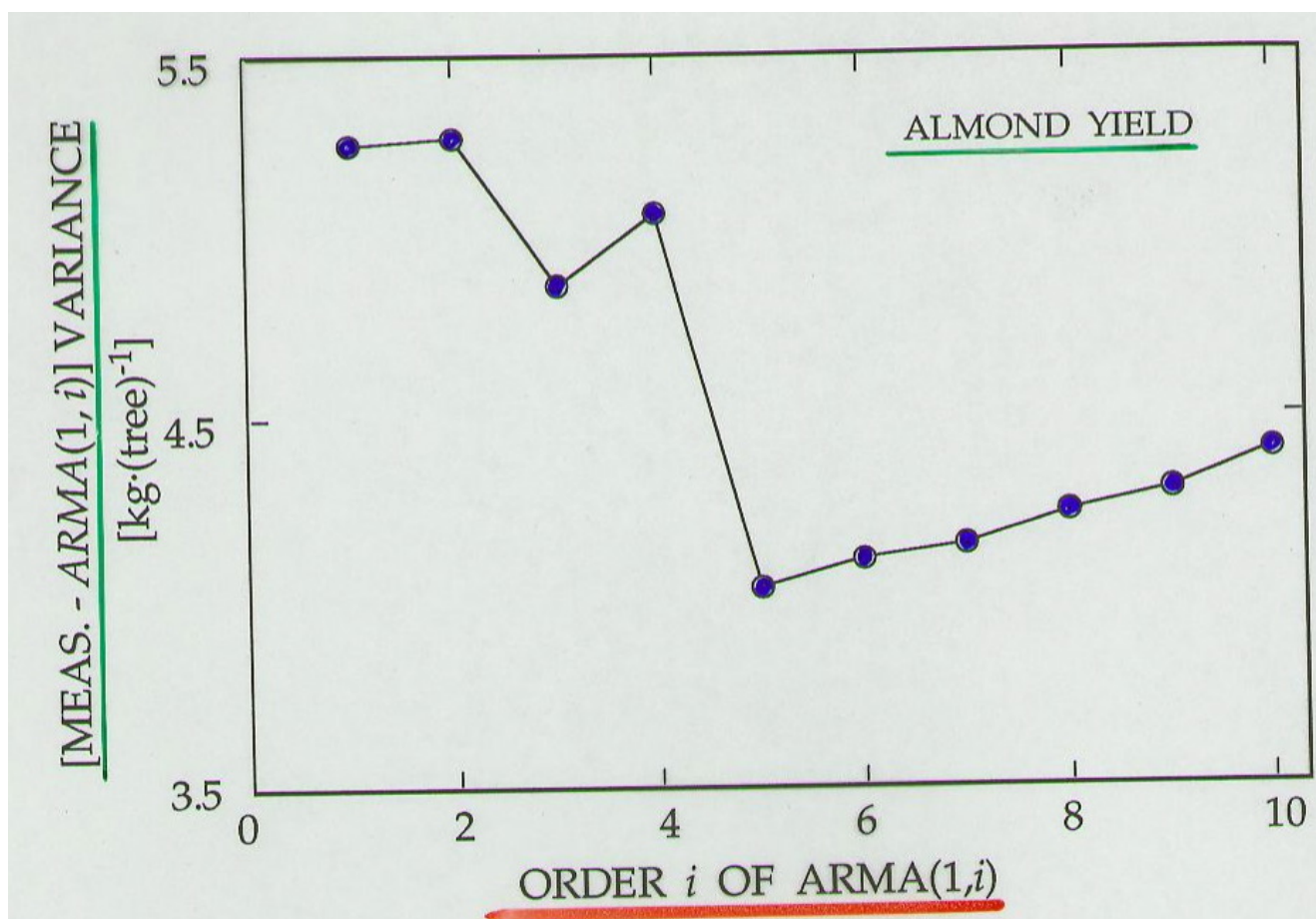
# almond yield per tree vs distance



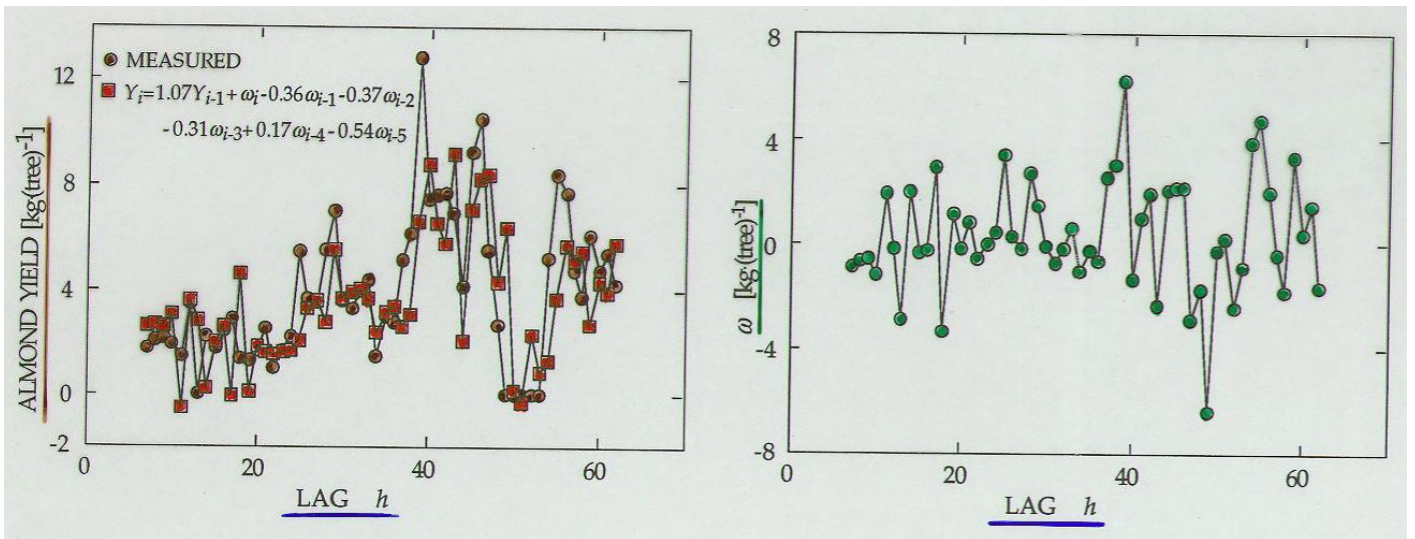
# AR(1) of almond yield per tree



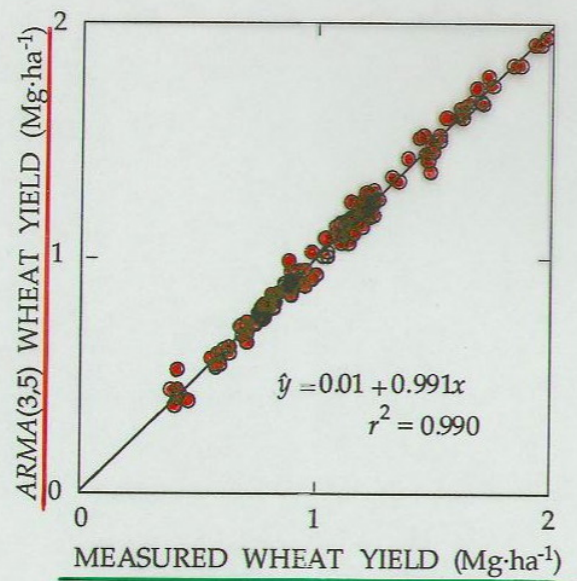
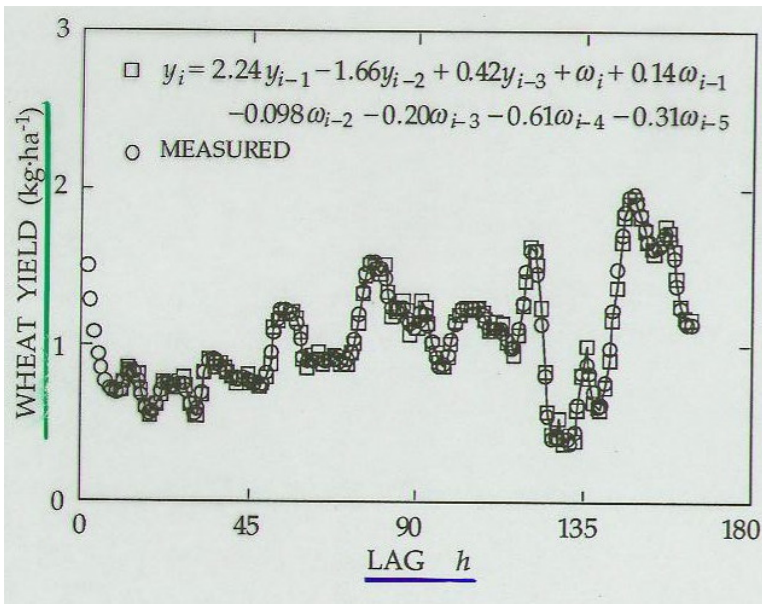
# almond yield variance vs AR order



# AR(1) and moving ave almond



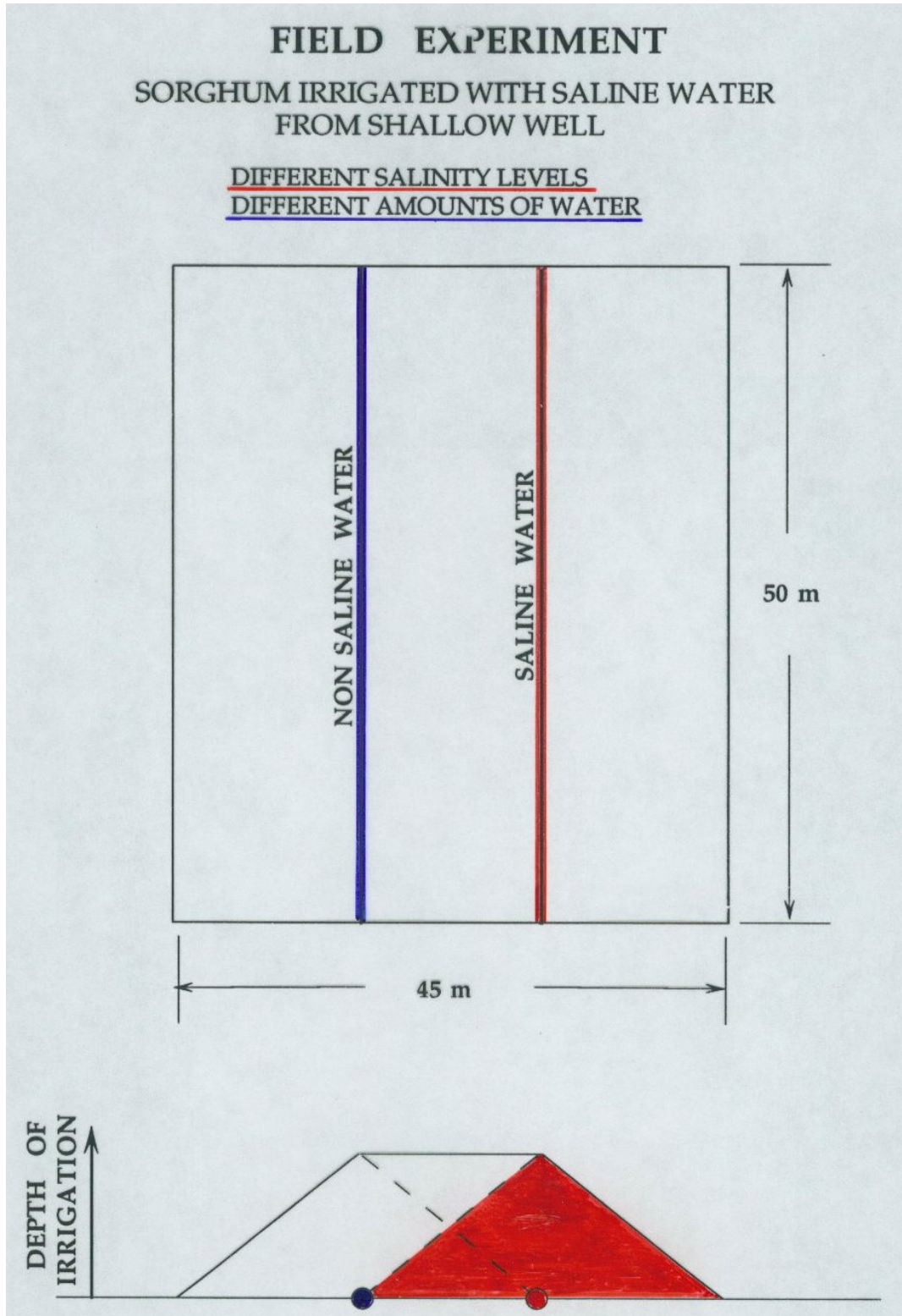
# AR(3) and moving ave. wheat



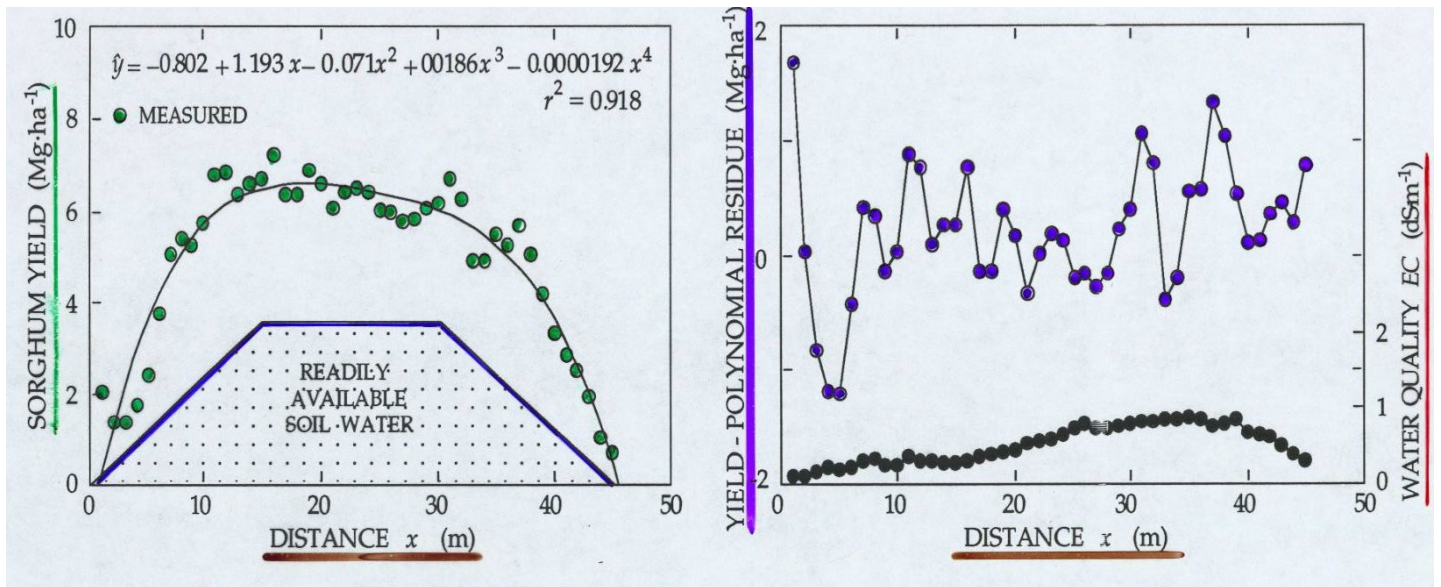
SORGHUM YIELD, SOIL WATER CONTENT  
AND SOIL SALINITY



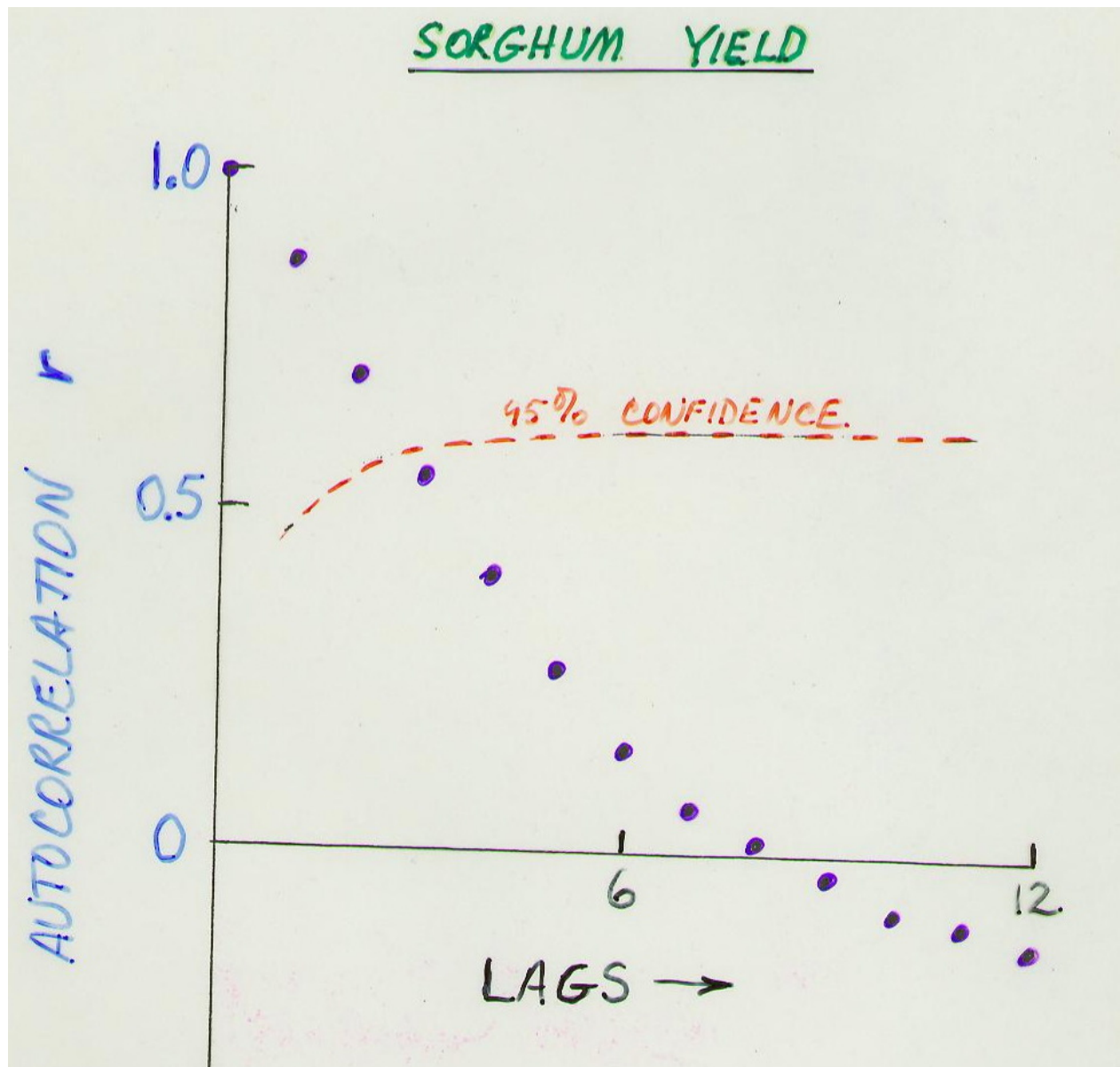
# sorghum irrigation expt design



# sorghum yld (4th order & residuals)

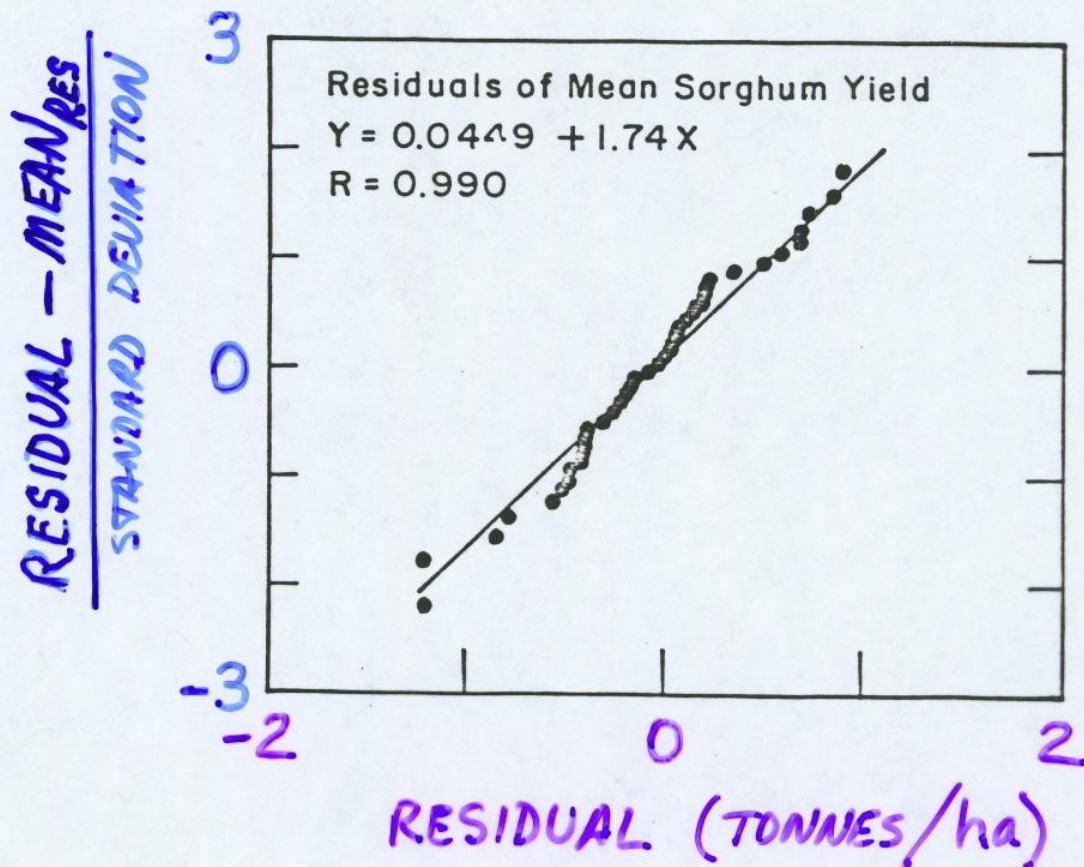


# ACF sorghum yield

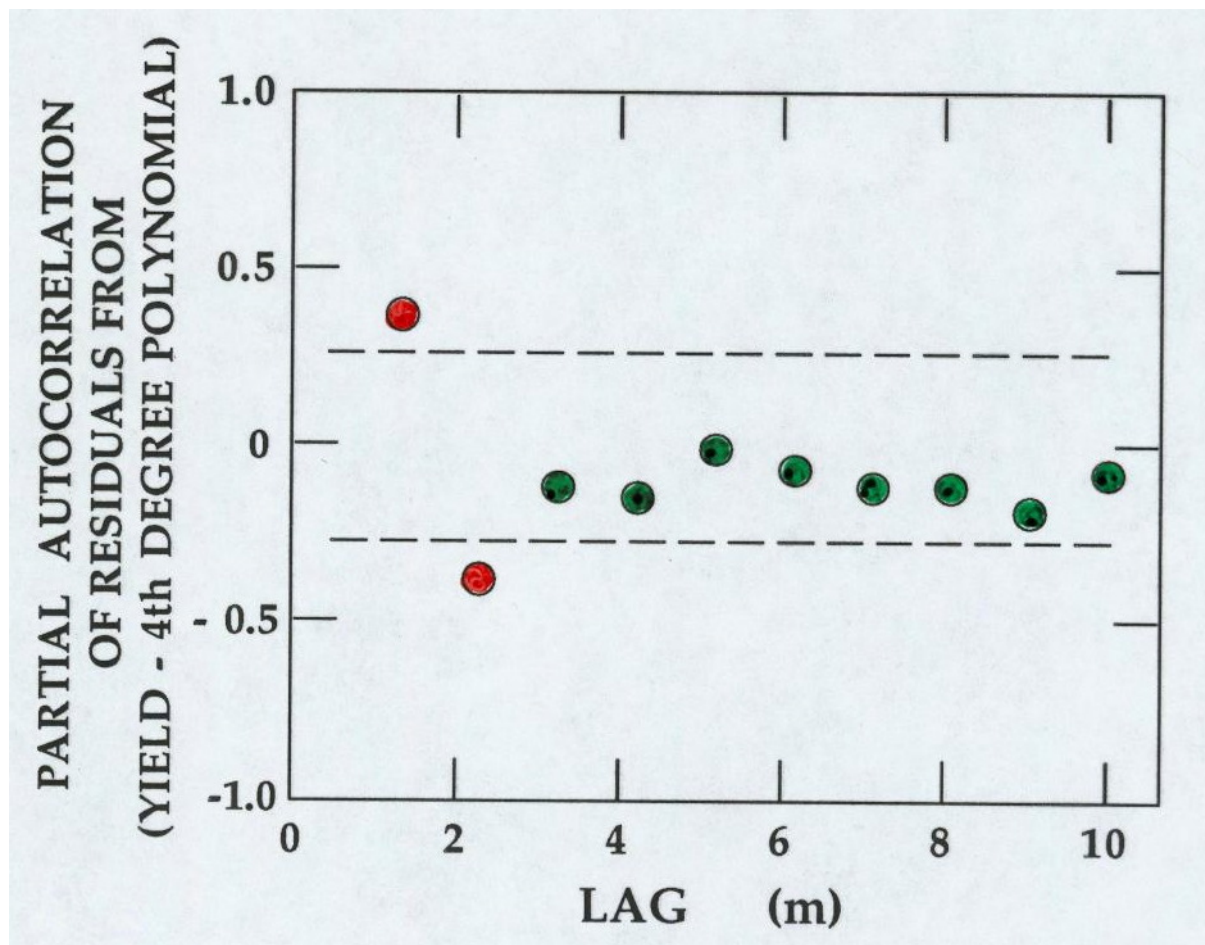


# sorghum residuals fractile diagram

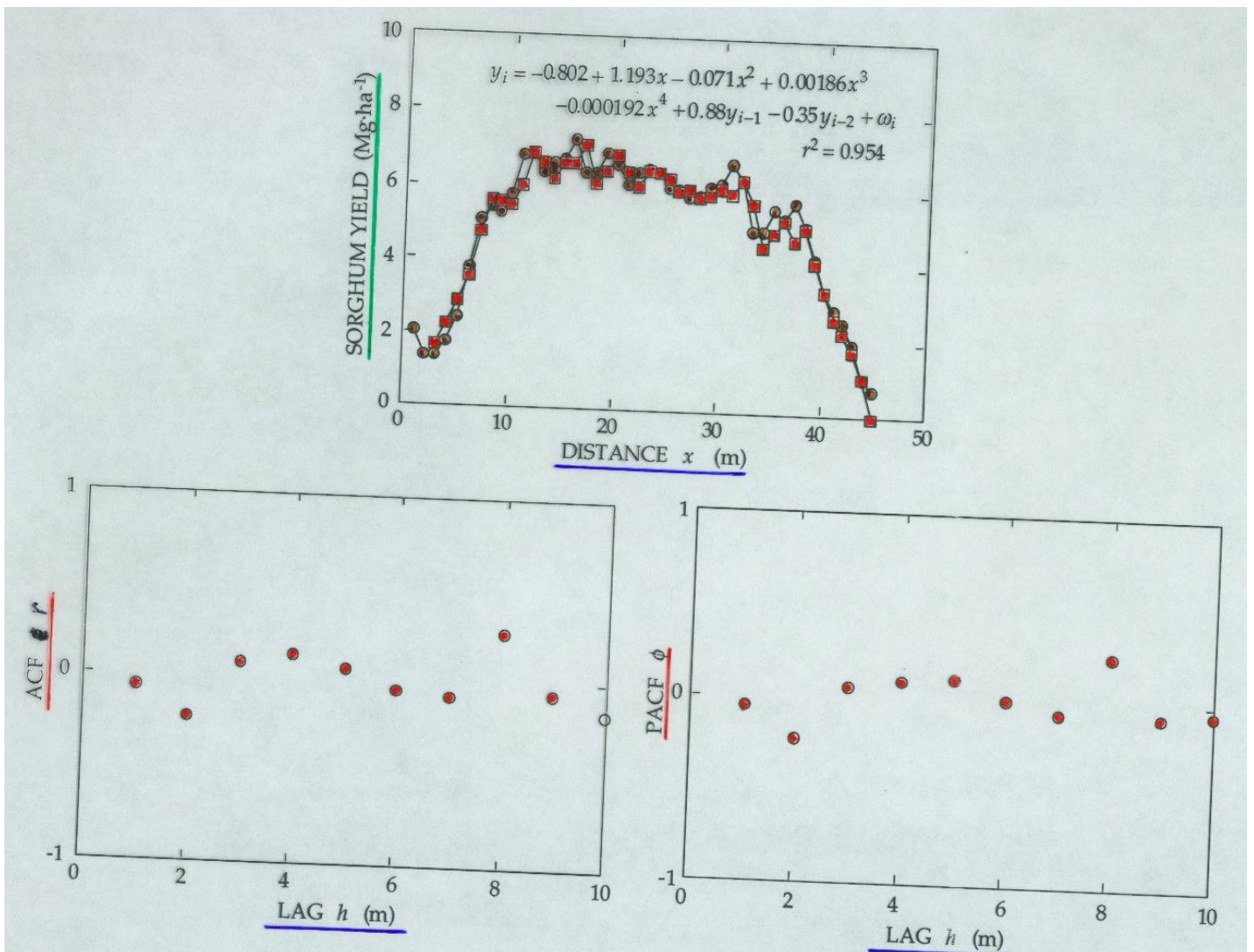
RESIDUAL = YIELD - 4<sup>th</sup> ORDER POLY.



# PACF (sorghum yield - 4th order)



# 4th order polynomial plus AR(2) sorghum

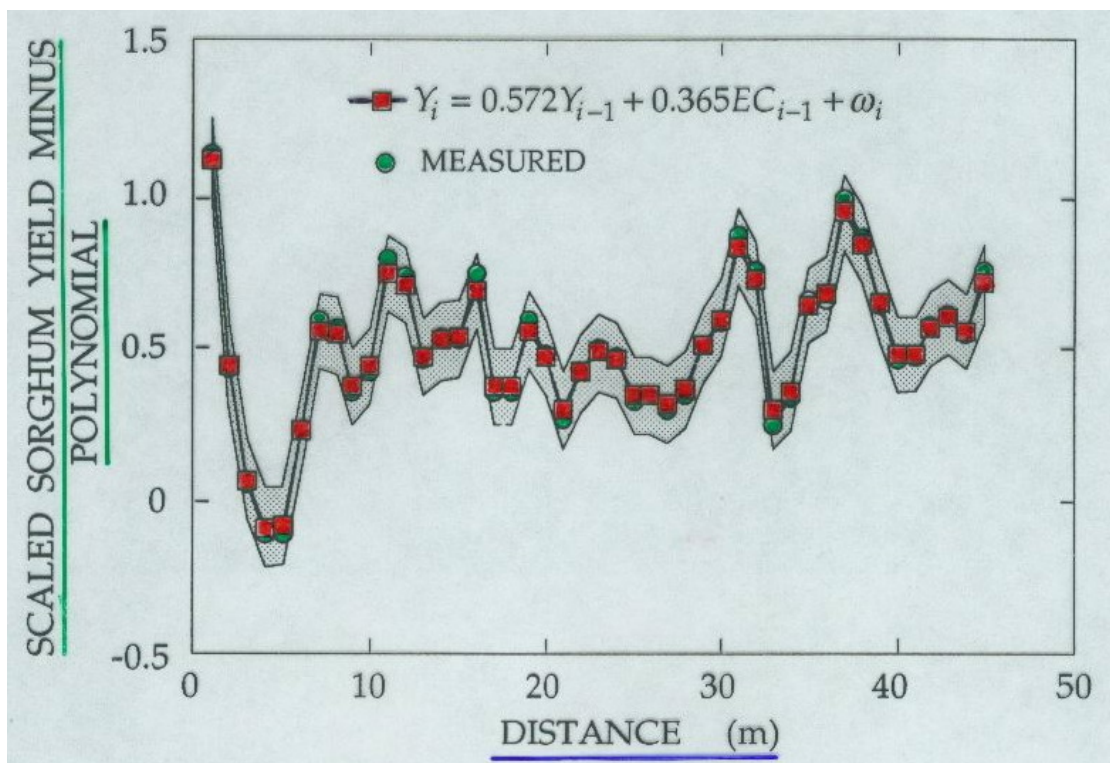


# 4th order polynomial plus AR(2) equation

$$\text{YIELD} = a_0 + a_1 X^1 + a_2 X^2 + a_3 X^3 + a_4 X^4 + \text{AR}(2)$$

SOIL WATER                      SALINITY

# State-space sorghum minus 4th polynomial

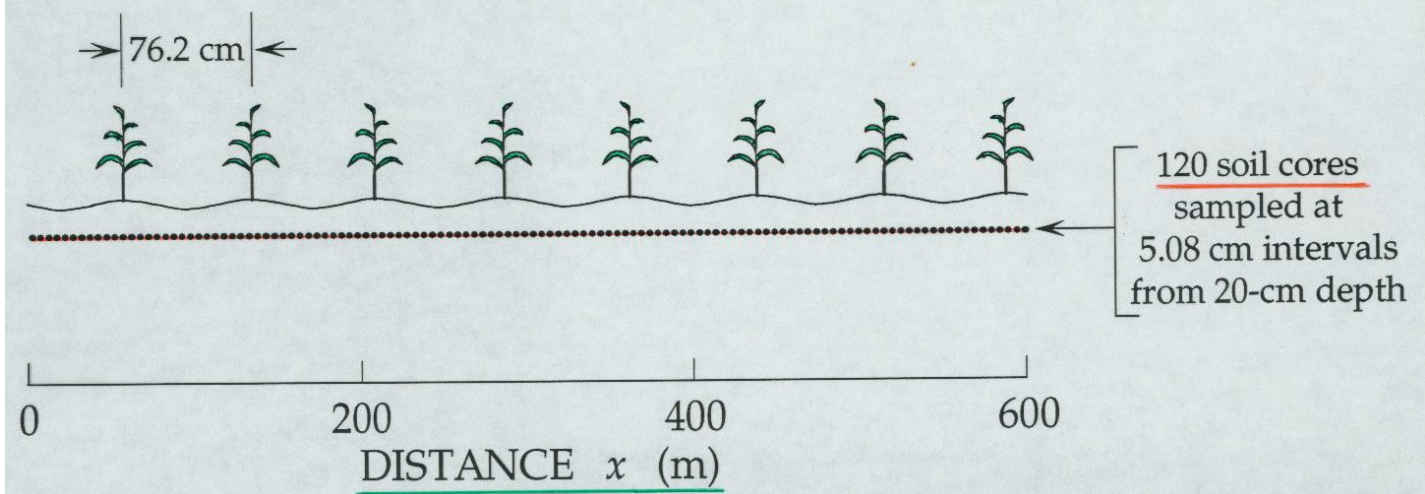




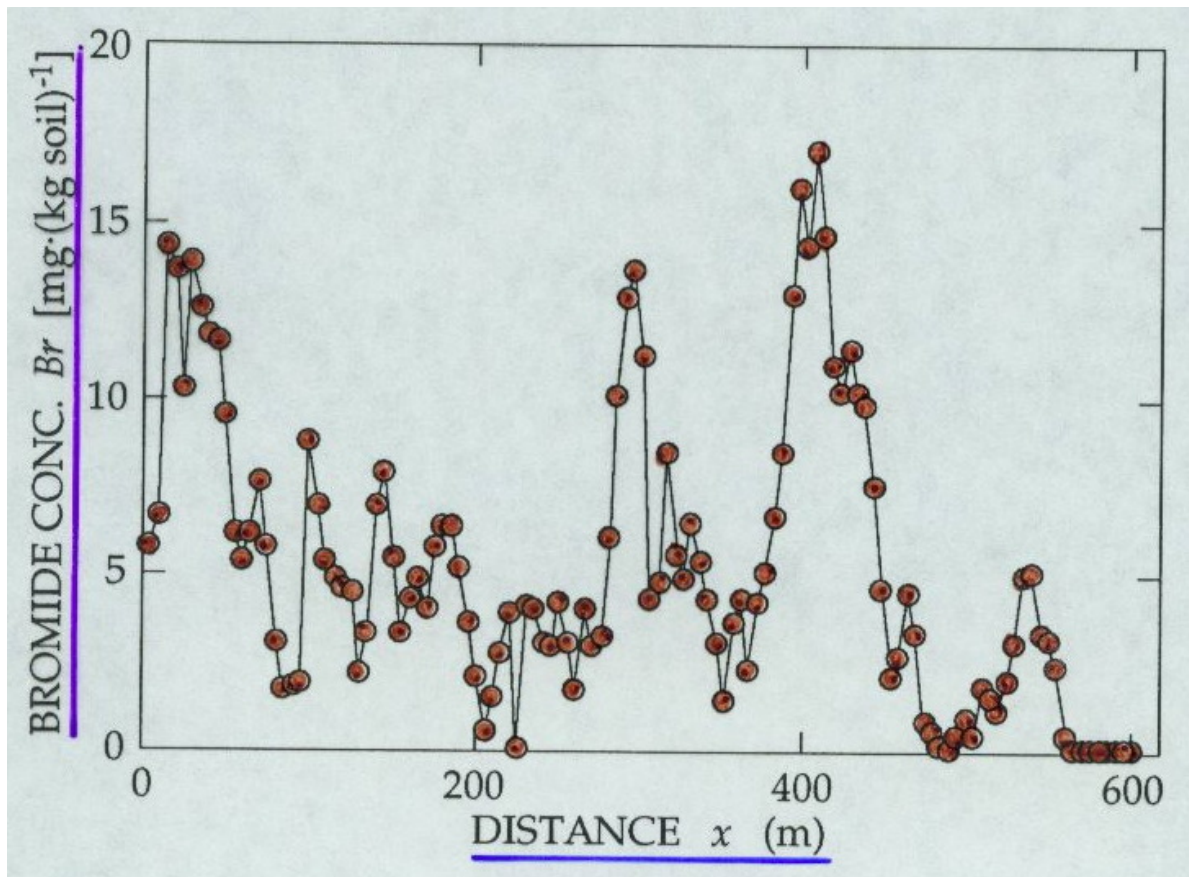
# Bromide application expt design

KBr added uniformly to soil surface of field planted to corn.

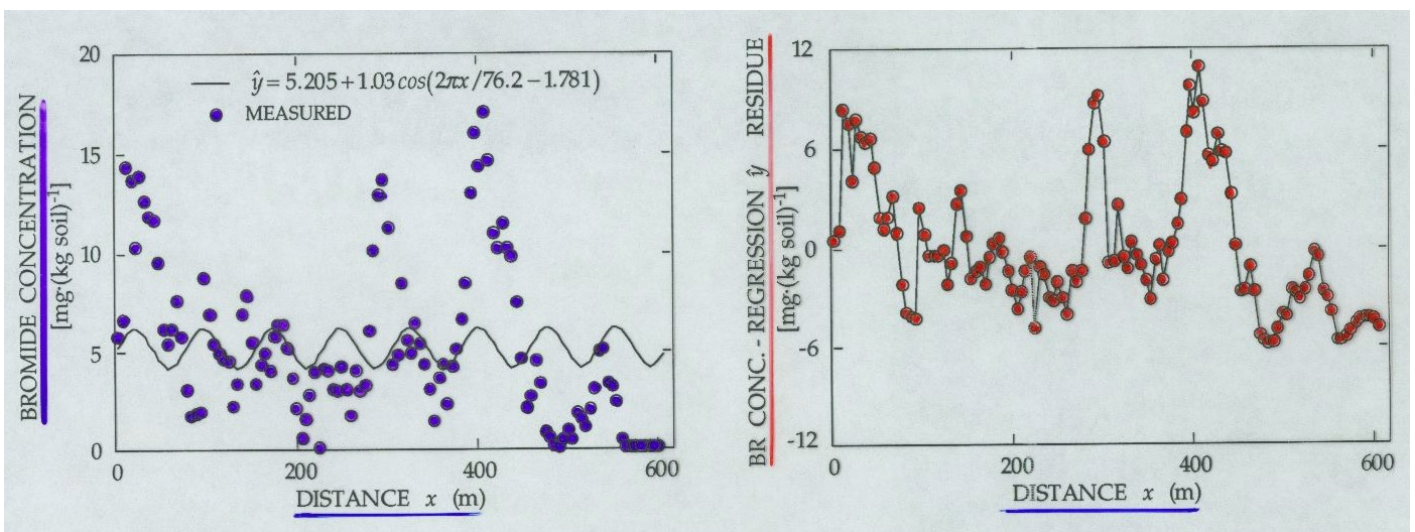
One month later after receiving 14.7 cm rainfall and irrigation, a transect of soil cores were analyzed for Br



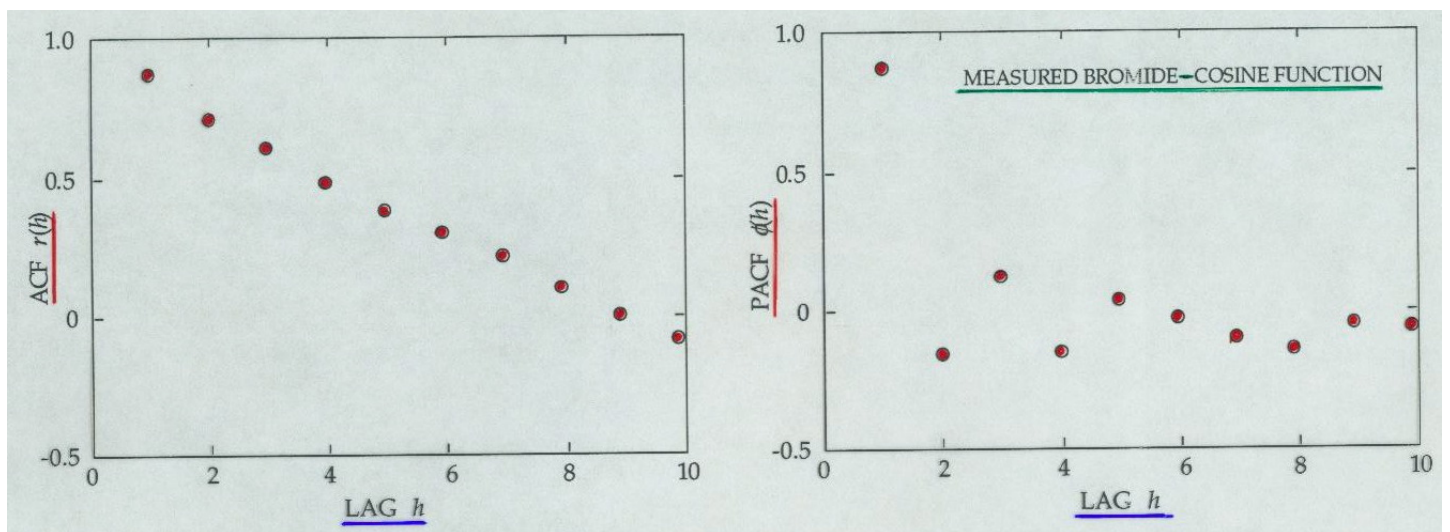
# bromide measurements vs distance



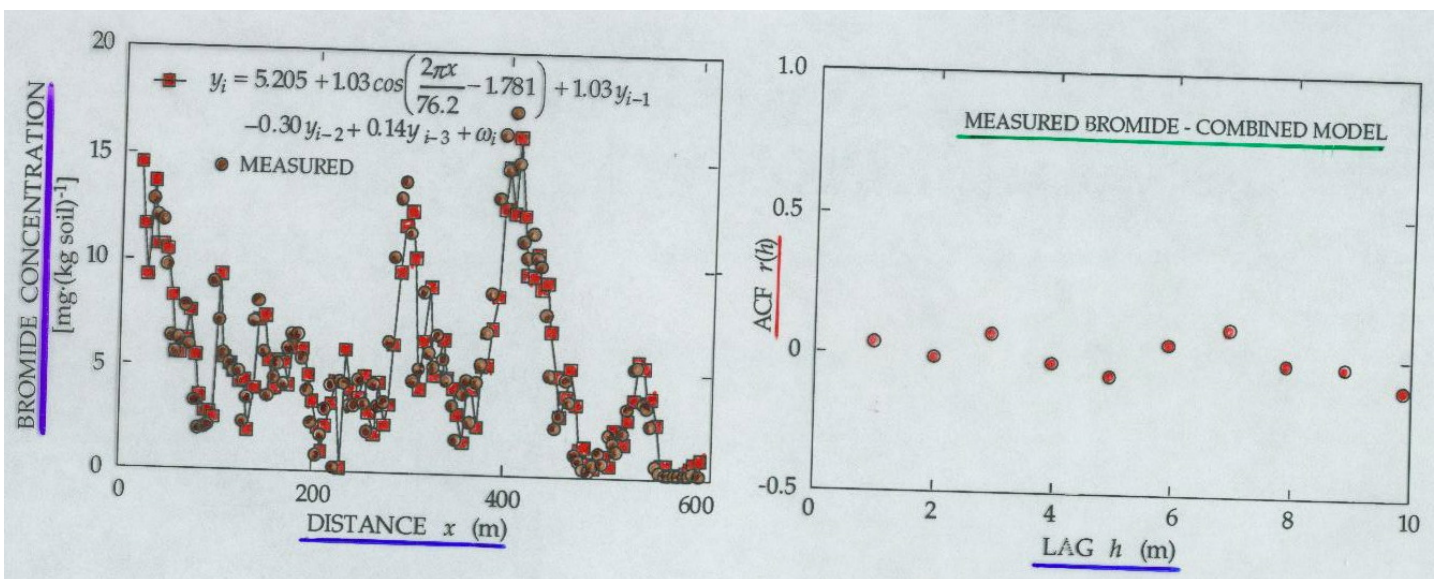
# measured bromide & cosine



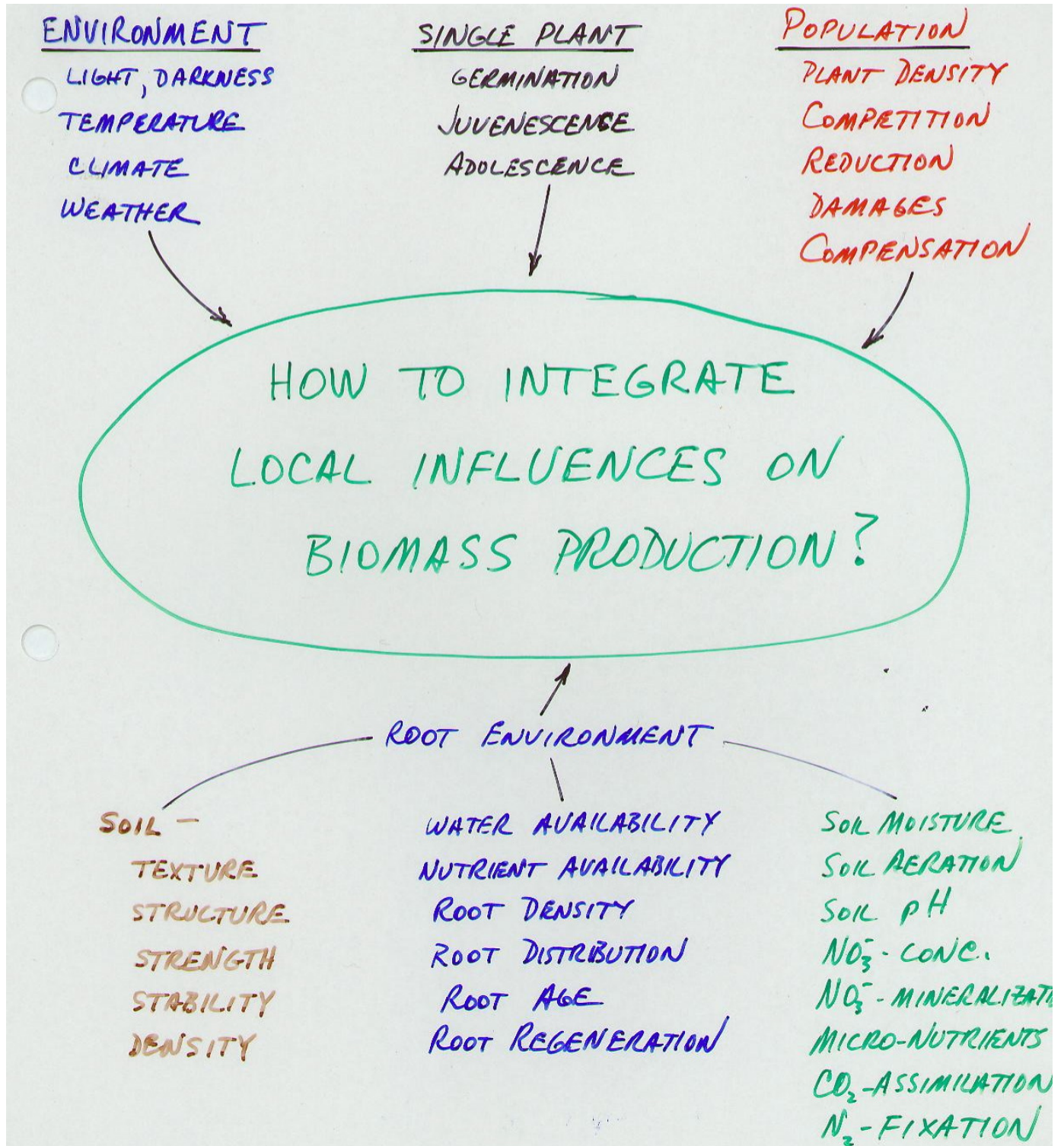
# ACF & PACF (Br minus cos)



# Cosine + AR(3) br



# how to integrate local influence



# STATE-SPACE ANALYSIS

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# state space analysis logic

## STATE-SPACE ANALYSIS

- THE LOGIC BEHIND STATE-SPACE ANALYSIS IS SIMILAR TO THAT OF A FARMER WALKING ACROSS A CULTIVATED FIELD COMPARING THE HEALTH AND STATE OF A CROP AND ITS UNDERLYING SOIL CONDITION AT FIRST ONE LOCATION, THEN AT A SECOND LOCATION IN CLOSE PROXIMITY TO THE FIRST, AND THEN AT A THIRD LOCATION AND SO ON UNTIL THE FARMER HAS TRAVERSED THE ENTIRE FIELD.
- THE FARMER DOES NOT HAVE THE KNOWLEDGE TO INTEGRATE THE COUNTLESS REACTIONS AND PROCESSES OCCURRING WITHIN THE FIELD TO PREDICT THE EXACT HEALTH AND STATE OF A CROP AT ANY LOCATION.
- BUT THE FARMER CAN READILY OBSERVE DIFFERENCES IN CROP BEHAVIOR AT DIFFERENT LOCATIONS WITHIN THE FIELD AT ANY ONE TIME OR DURING A SERIES OF TIMES.
- THE FARMER ALSO INTUITIVELY KNOWS THAT CROP DIFFERENCES OCCURRING OVER RELATIVELY SHORT DISTANCES IN ONE REGION ARE NOT NECESSARILY CAUSED BY THE SAME REACTIONS AND PROCESSES OCCURRING IN OTHER REGIONS OF THE FIELD.
- THE FARMER'S OBJECTIVE AND THAT OF STATE-SPACE ANALYSIS ARE IDENTICAL – TO UNDERSTAND OR DIAGNOSE WHAT ARE THE CAUSES OF THE DIFFERENT CROP BEHAVIOR IN ORDER TO IMPROVE CONDITIONS AT ALL LOCATIONS FOR BETTER CROP PRODUCTION.



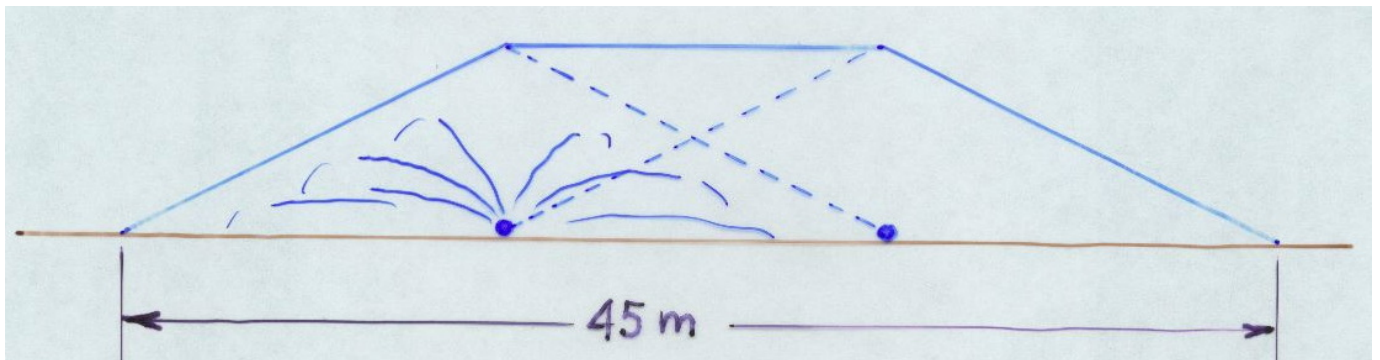
# state space questions

## RELEVANT QUESTIONS

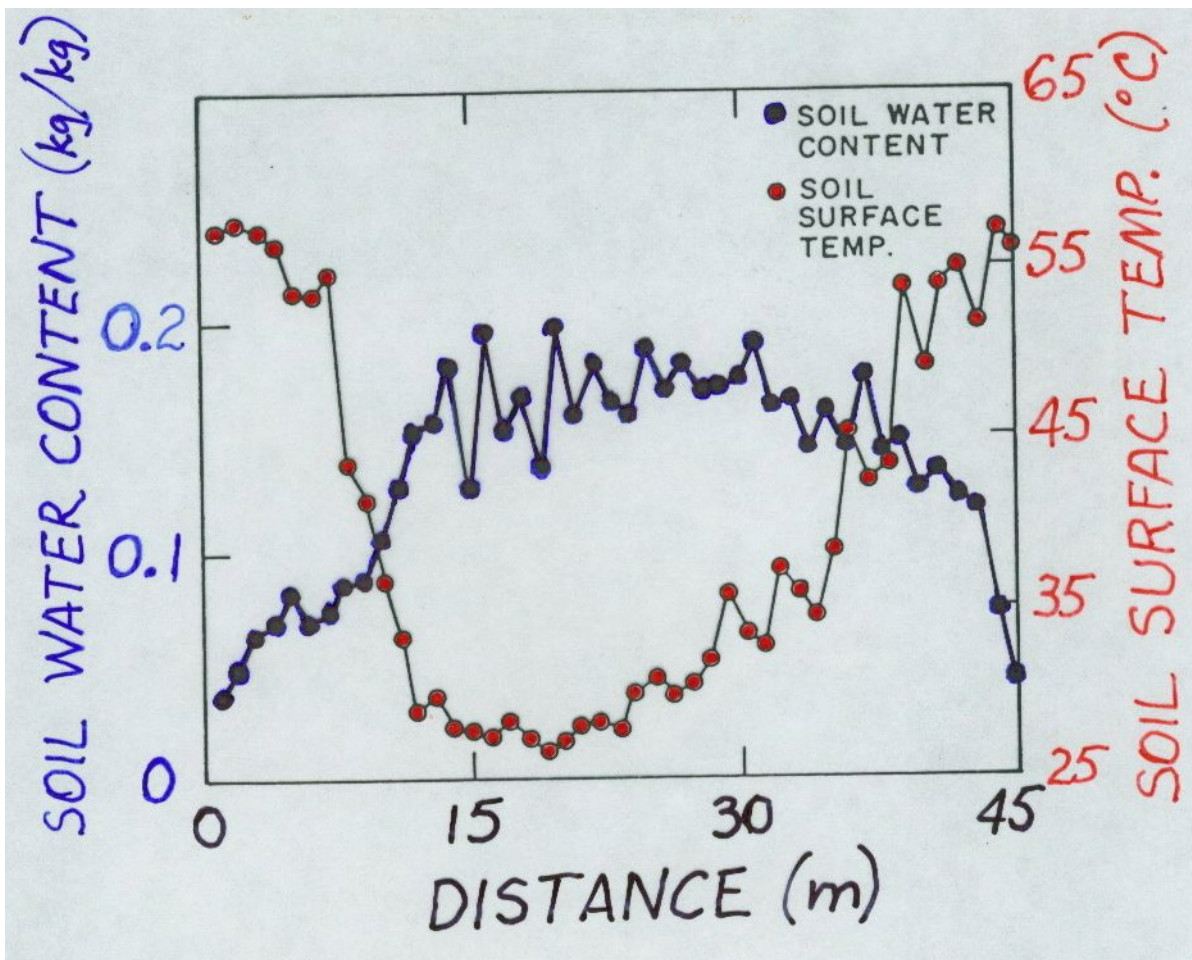
- WHAT PROCESSES AND CONDITIONS ARE RESPONSIBLE OR RELATED TO SPATIAL VARIATIONS OF CROP OR SOIL ATTRIBUTES ACROSS THE LANDSCAPE?
- WHAT STATE VARIABLES SHOULD BE OBSERVED TO IDENTIFY IMPORTANT PHYSICAL, CHEMICAL AND BIOLOGICAL PROCESSES?
- WHAT SHOULD BE THE SAMPLE SPACING FOR OBSERVING STATE VARIABLES?
- WHICH OBSERVATIONS REVEAL THE GREATEST INFORMATION ABOUT THE STATE OF A SOIL OR CROP?

**SURFACE SOIL TEMPERATURE  
AND SOIL WATER CONTENT**

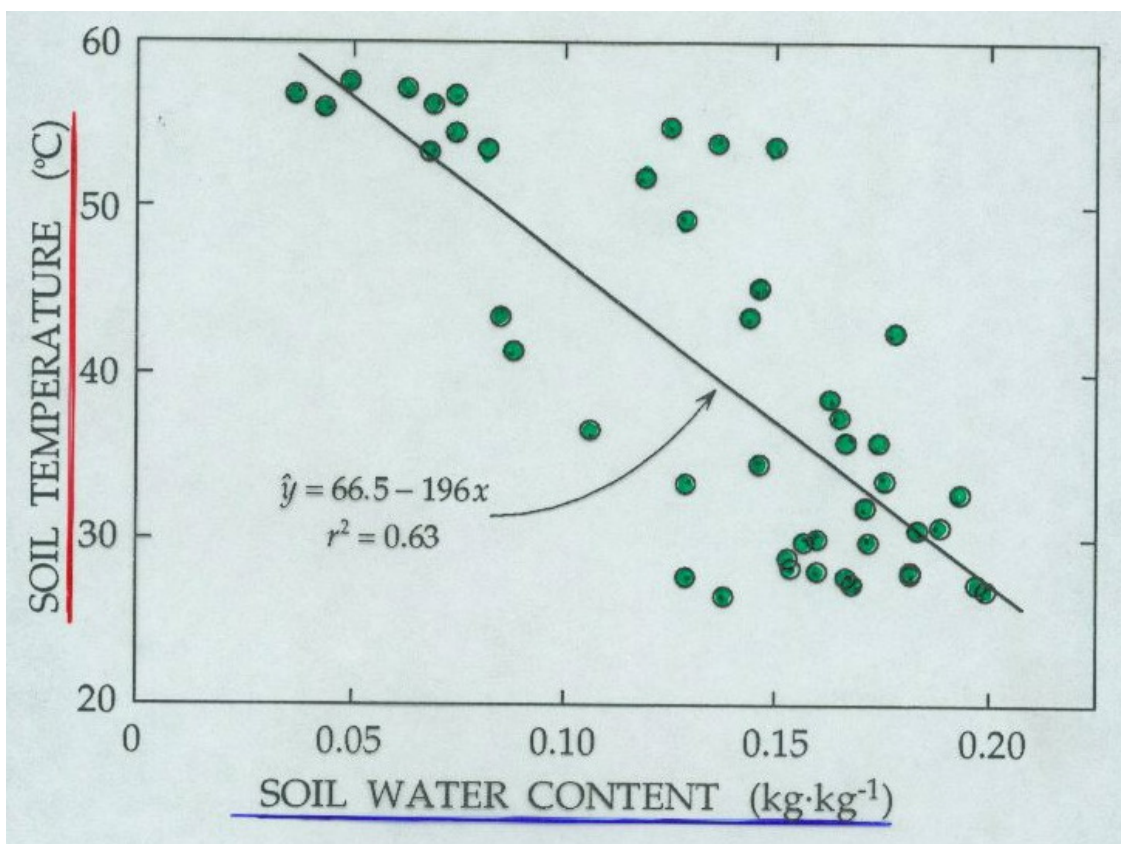
# Sorghum irrigation line source



measured T and theta



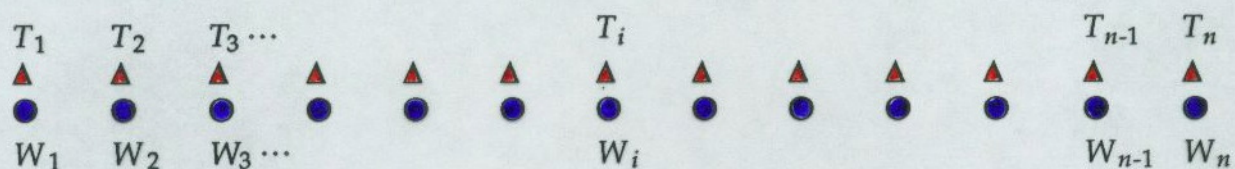
# T vs theta regression



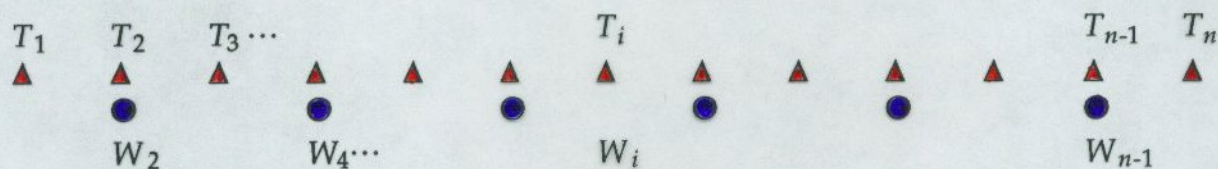
# st space equation illustration

## STATE-SPACE ANALYSIS

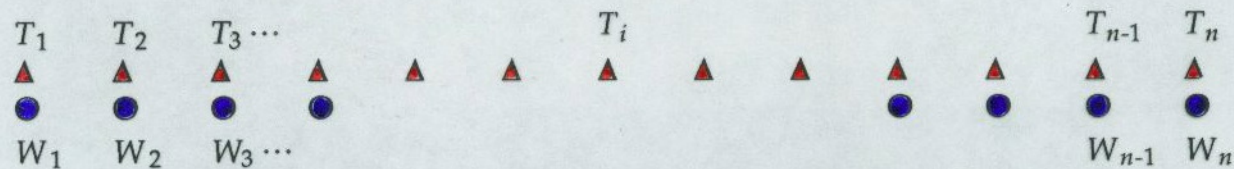
OBSERVATIONS  $T_i, W_i$



COMPLETE SET



$W_{odd}$  NOT OBSERVED



OBSERVATIONS OF  $W$  MISSING AT CONSECUTIVE LOCATIONS

# st space equations

## STATE-SPACE ANALYSIS

$$\underline{Z_i = \Phi Z_{i-1} + \omega_i}$$

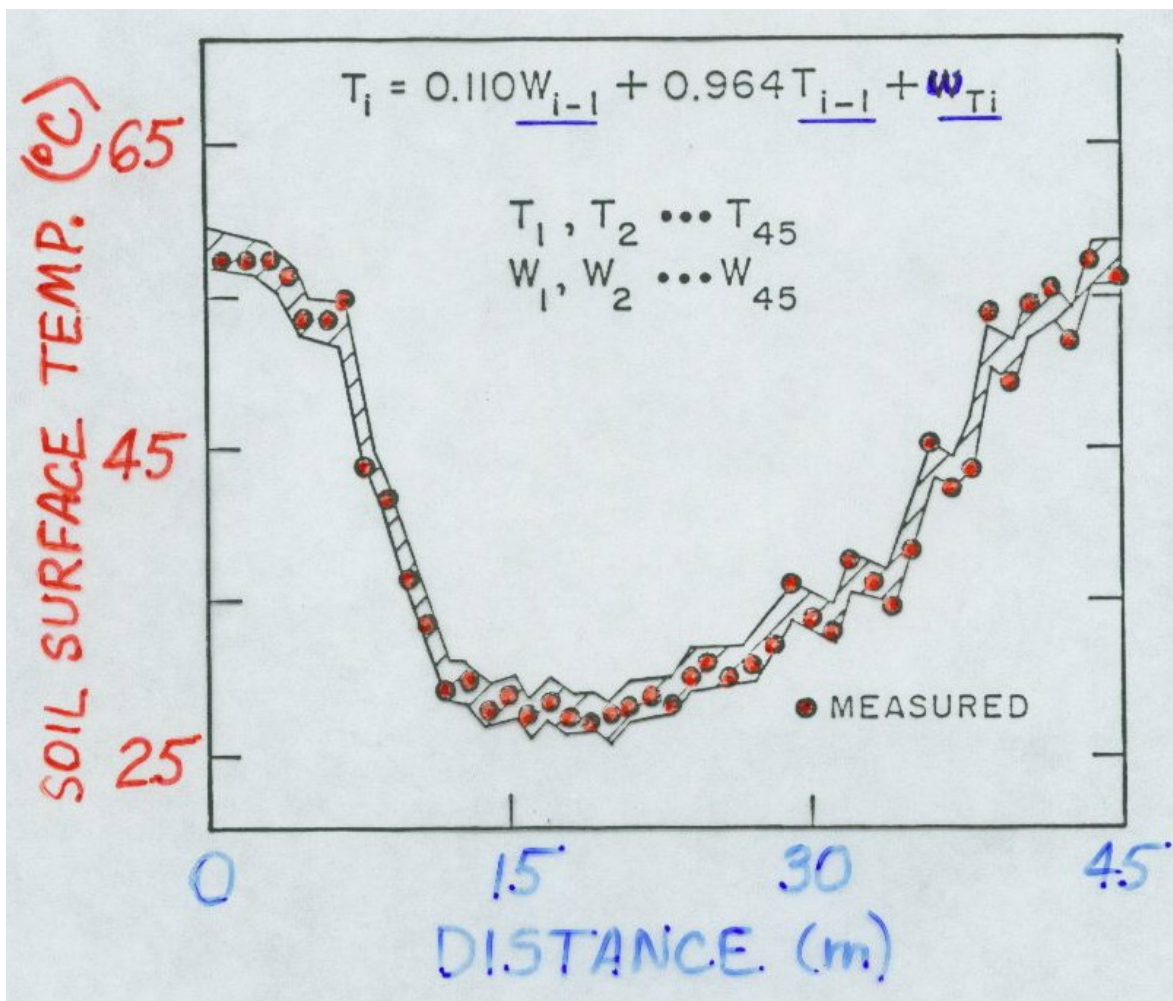
- $Z_i$  state vector (a set of  $p$  variables at location  $i$ )
- $\Phi$   $p \times p$  matrix of state coefficients  $\phi$  indicating spatial regression
- $\omega_i$  uncorrelated zero mean model error

$$\underline{Y_i = M_i Z_i + v_i}$$

- $Y_i$  observed vector
- $M_i$  observation matrix
- $Z_i$  state vector
- $v_i$  uncorrelated, zero mean observation error

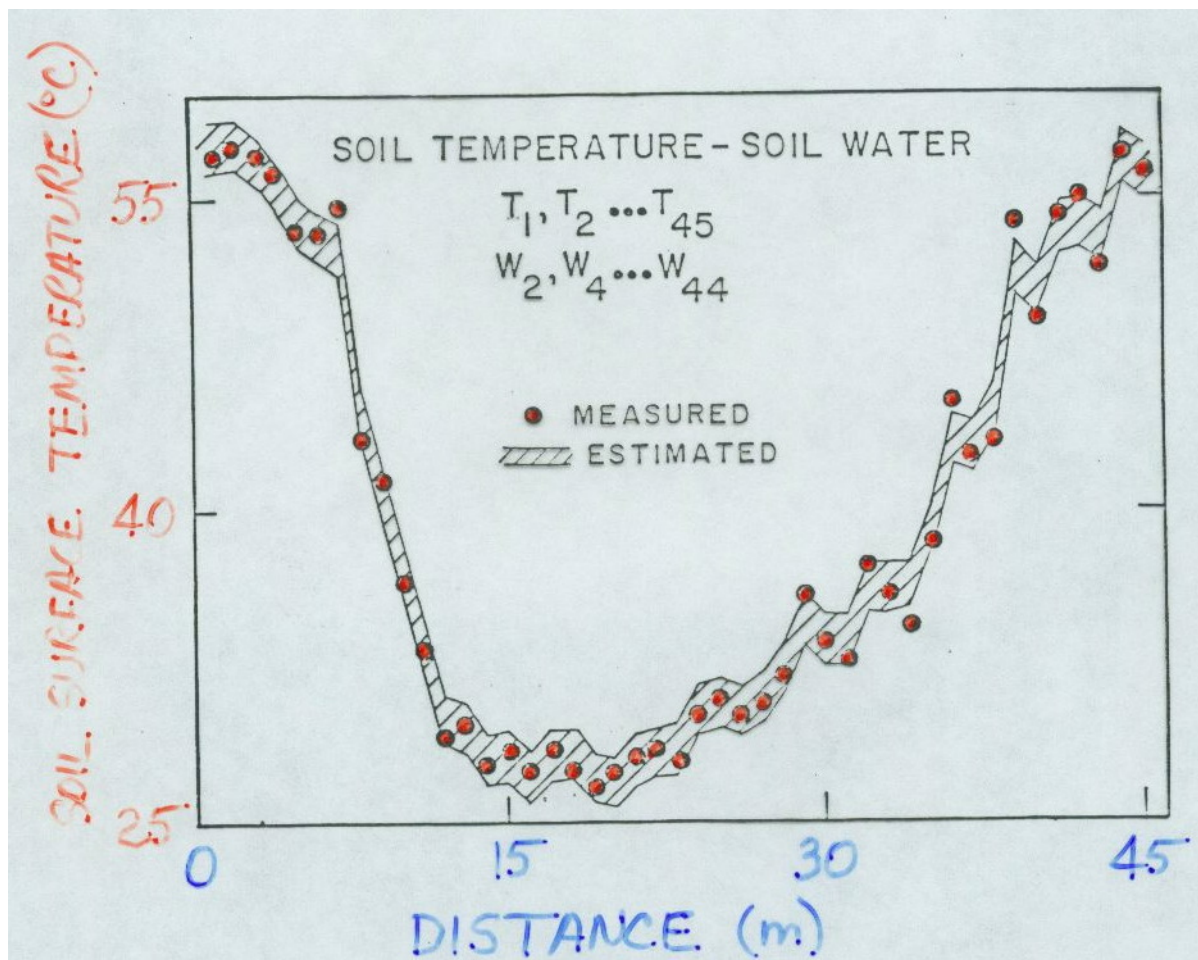
- Observations contain measurement and calibration errors.
- The state coefficient and covariance matrices are optimized with Kalman filtering.

# st space estimation T

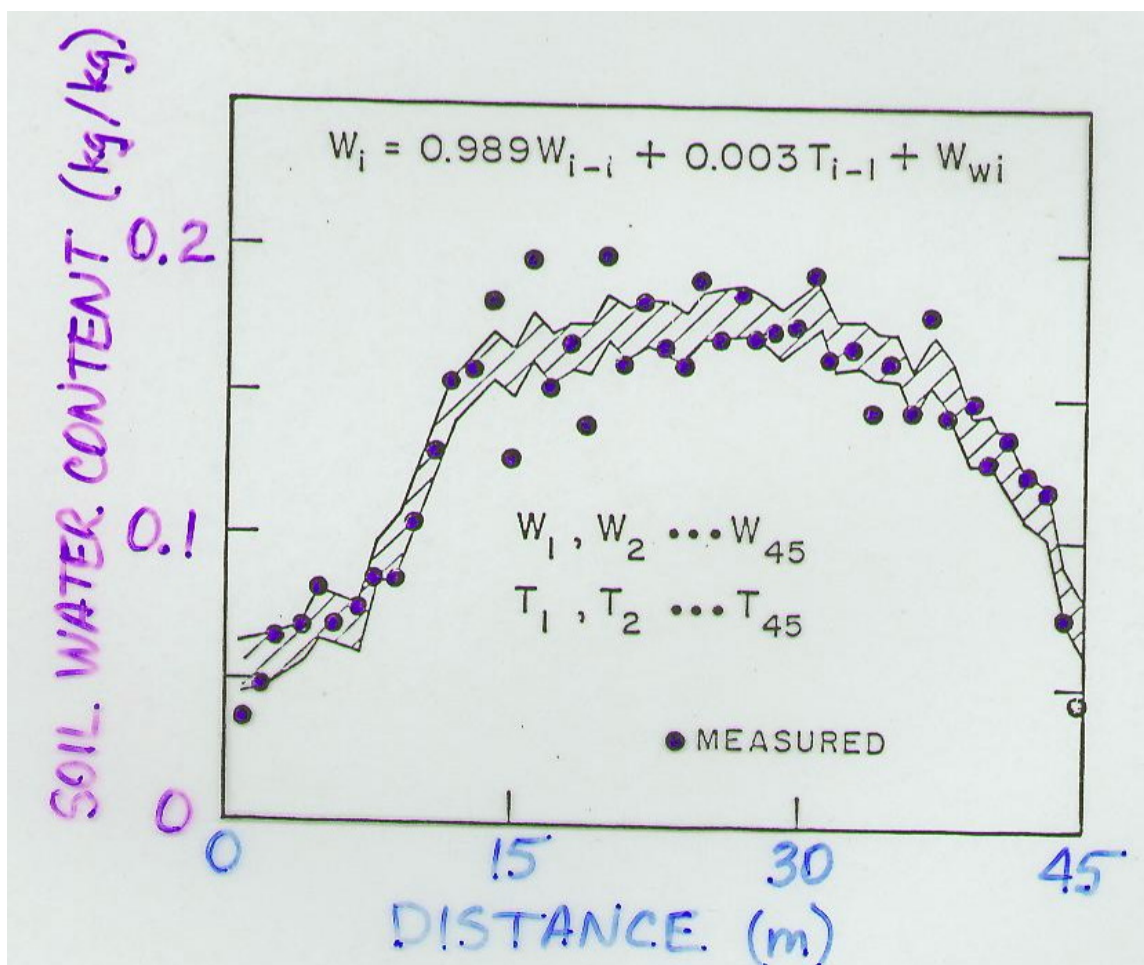




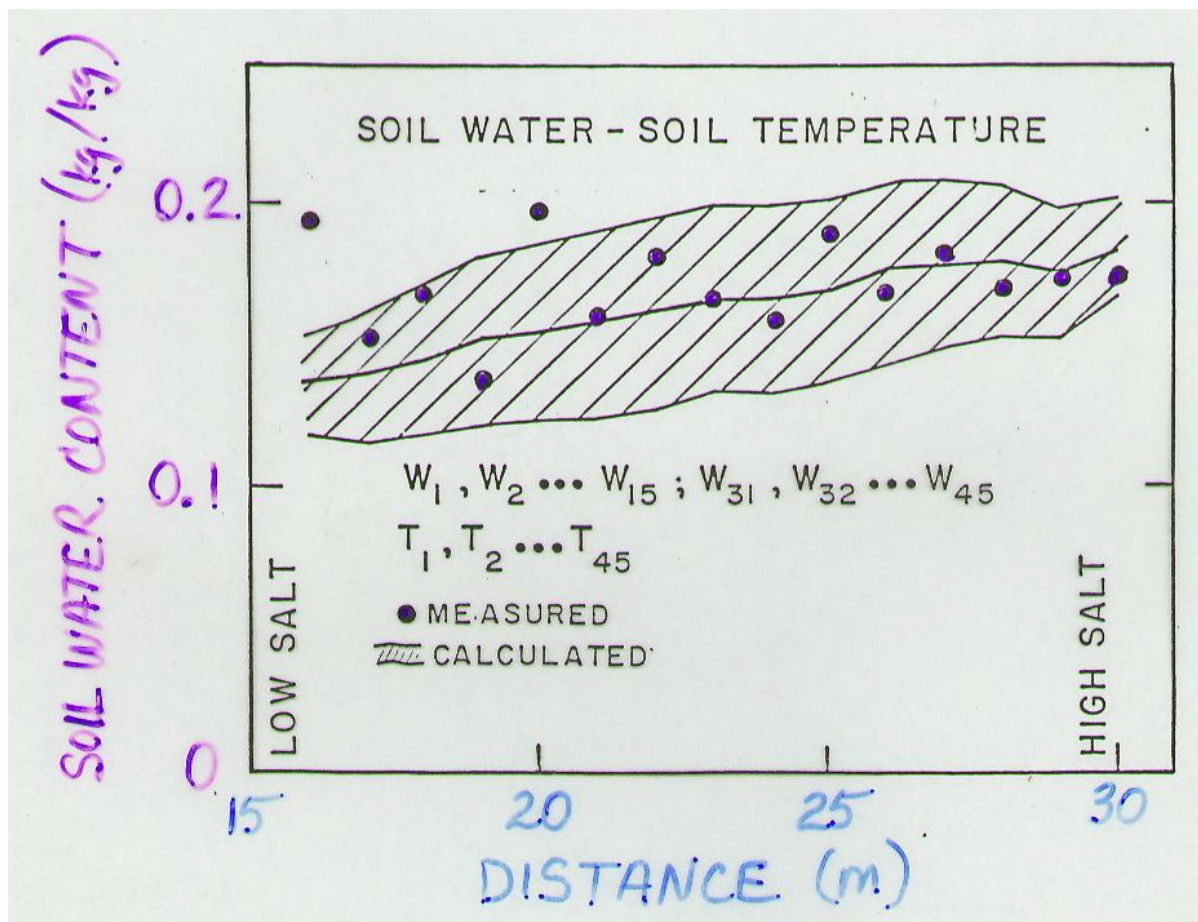
st space estimation T (even q)



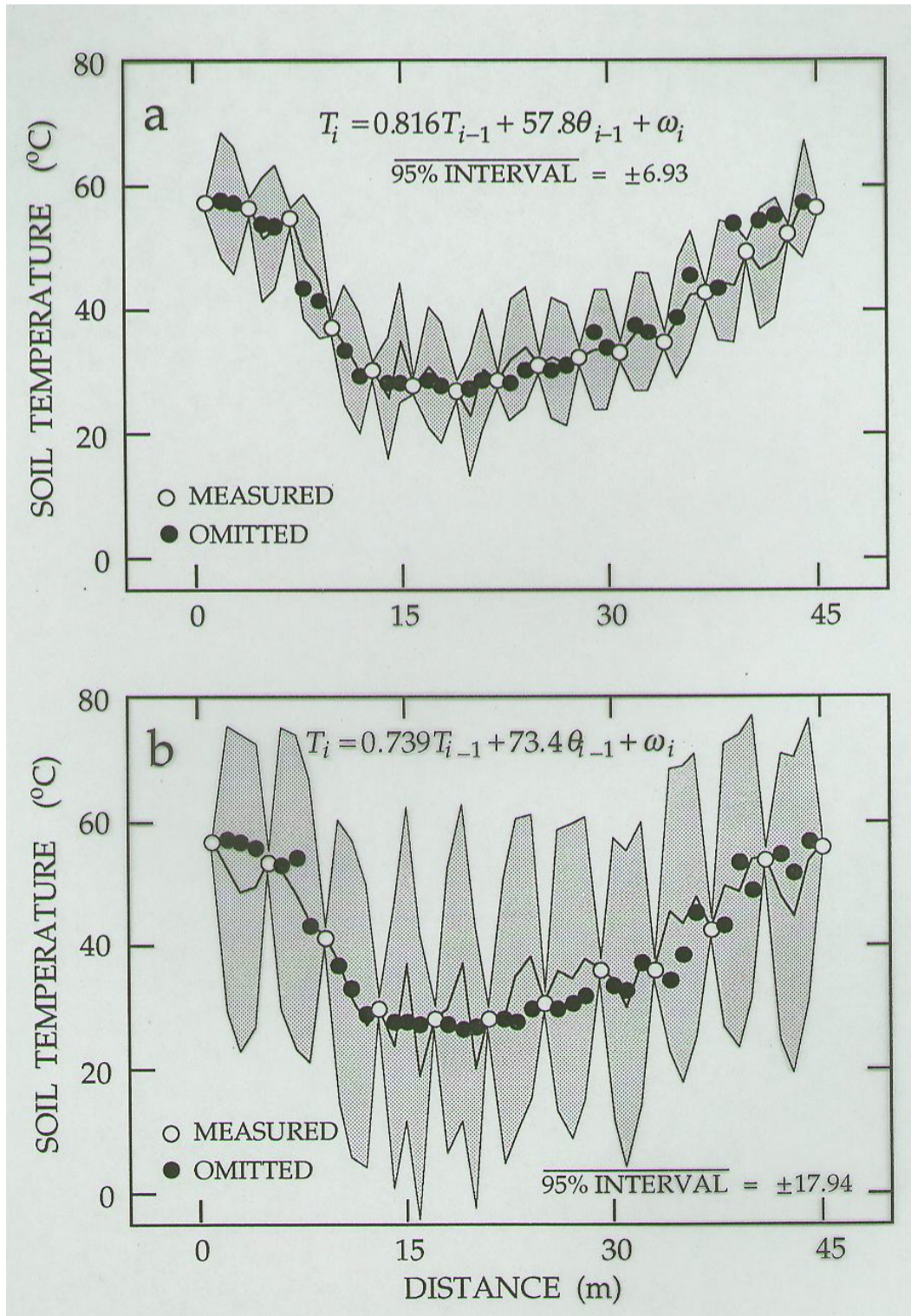
# st sp soil water content (temp)



# st sp soil water calculated (15-30m)



# kriged soil temp (omitting 2&4 values)

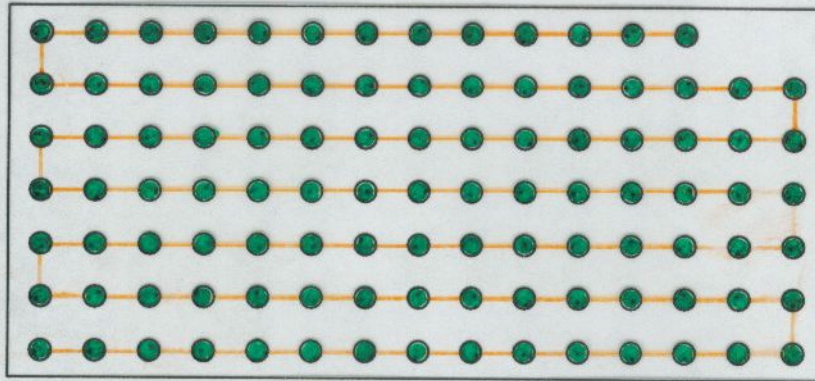


**CORN YIELD  
AND SOIL NUTRIENTS**

# Michigan corn field samples

## MICHIGAN FARM

Sampled 103 locations  
on a 30.5-m (100-ft) grid



## AT EACH LOCATION

- Corn yield
- Plant population
- Ca (0-5 and 5-15 cm)
- Mg (0-5 and 5-15 cm)
- K (0-5 and 5-15 cm)



# st space eqns corn

## STATE-SPACE EQUATIONS OF CORN YIELD, Ca, Mg, K AND PLANT POPULATION FOR A MICHIGAN FARM

$$X_{i,1} = \phi_{11} X_{i-1,1} + \phi_{12} X_{i-1,2} + \phi_{13} X_{i-1,3} + \phi_{14} X_{i-1,4} + \omega_{i1}$$

$$X_{i,2} = \phi_{21} X_{i-1,1} + \phi_{22} X_{i-1,2} + \phi_{23} X_{i-1,3} + \phi_{24} X_{i-1,4} + \omega_{i2}$$

$$X_{i,3} = \phi_{31} X_{i-1,1} + \phi_{32} X_{i-1,2} + \phi_{33} X_{i-1,3} + \phi_{34} X_{i-1,4} + \omega_{i3}$$

$$X_{i,4} = \phi_{41} X_{i-1,1} + \phi_{42} X_{i-1,2} + \phi_{43} X_{i-1,3} + \phi_{44} X_{i-1,4} + \omega_{i4}$$

where  $X_{i,1} = \frac{Y_i}{Y_{max}}$      $X_{i,2} = \frac{\ln Mg_i}{\ln Mg_{max}}$      $X_{i,3} = \frac{\ln Ca/K_i}{\ln Ca/K_{max}}$      $X_{i,4} = \frac{Pl_i}{Pl_{max}}$

with  $Y_i = \text{Corn Yield}$

$$\ln Mg_i = \left[ \ln \left( \frac{5 Mg_{(0-5cm)} + 15 Mg_{(5-20cm)}}{20} \right) \right]_i$$

$$\ln Ca/K_i = \left[ \ln \left( \frac{5 Ca_{(0-5cm)} + 15 Ca_{(5-20cm)}}{5 K_{(0-5cm)} + 15 K_{(5-20cm)}} \right) \right]_i$$

$Pl_i = \text{Plant Population}$

Subscripts  $i$  and  $max$  indicate sampling location and maximum value observed in the field, respectively.

$$\frac{Y_i}{Y_{max}} = 0.520 X_{i-1,1} + 0.051 X_{i-1,2} - 0.044 X_{i-1,3} - 0.007 X_{i-1,4} + \omega_i$$

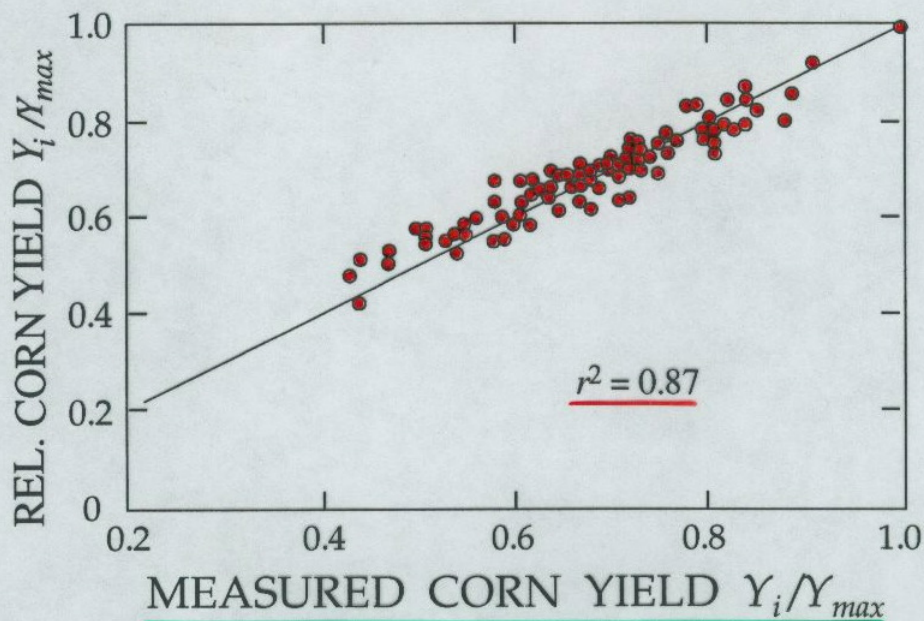
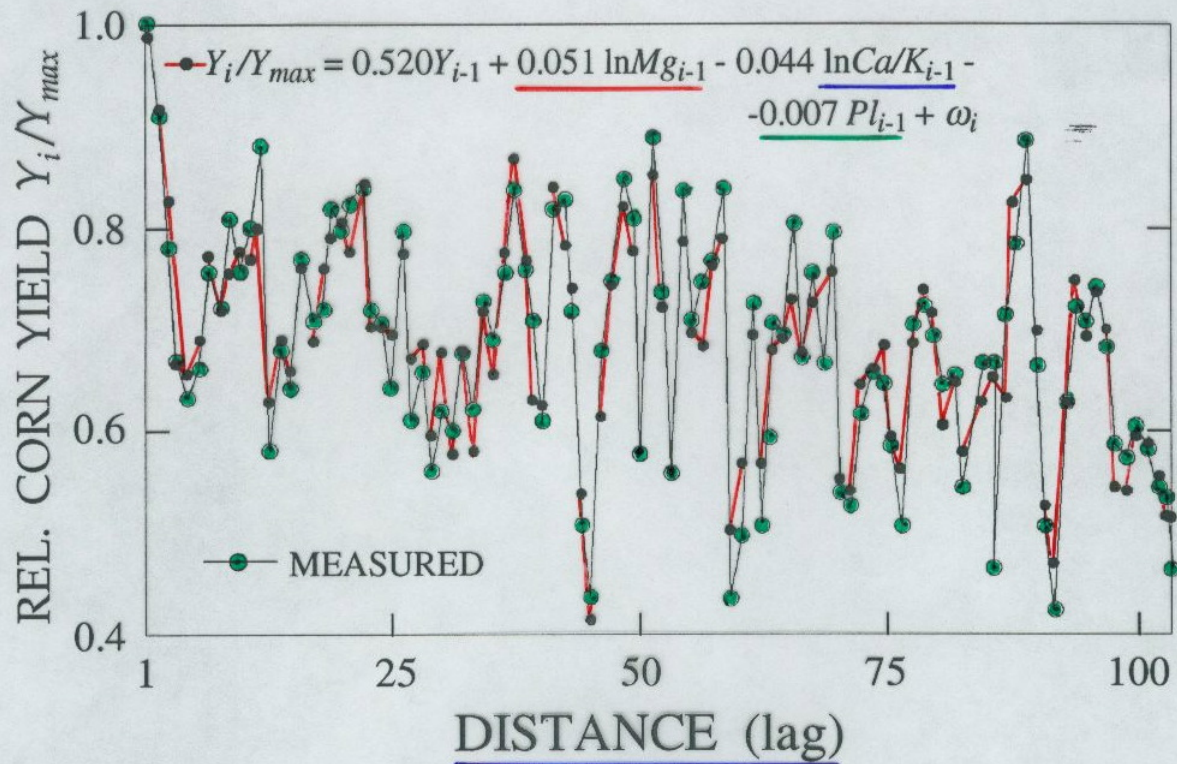
## MULTIPLE REGRESSION EQUATION FOR CORN YIELD

$$\frac{\hat{Y}_i}{Y_{max}} = 1.458 X_{i,2} + 3.911 X_{i,3} + 0.0034 X_{i,4} - 1.593$$

# st space est. corn

## STATE-SPACE ANALYSIS

(locations included)

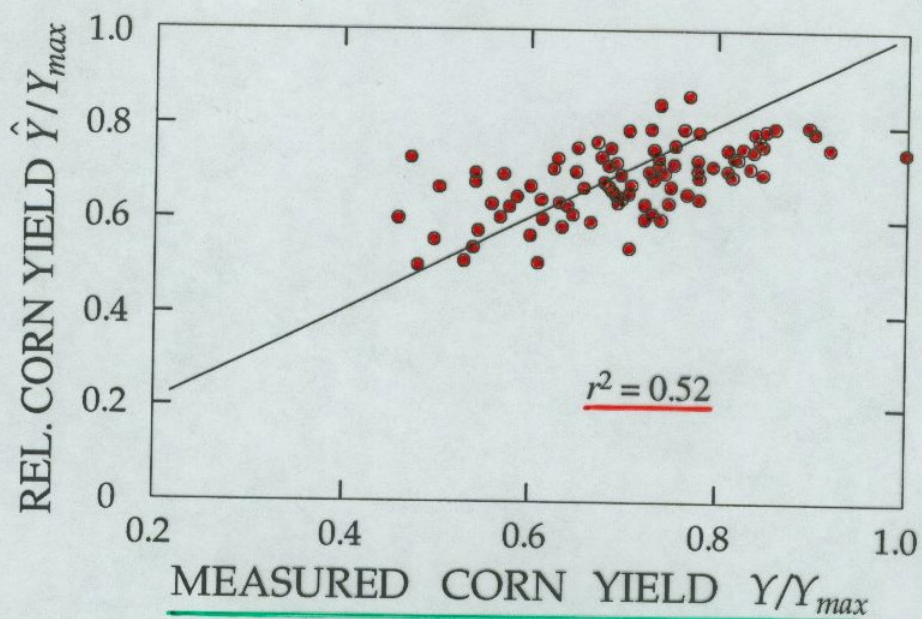
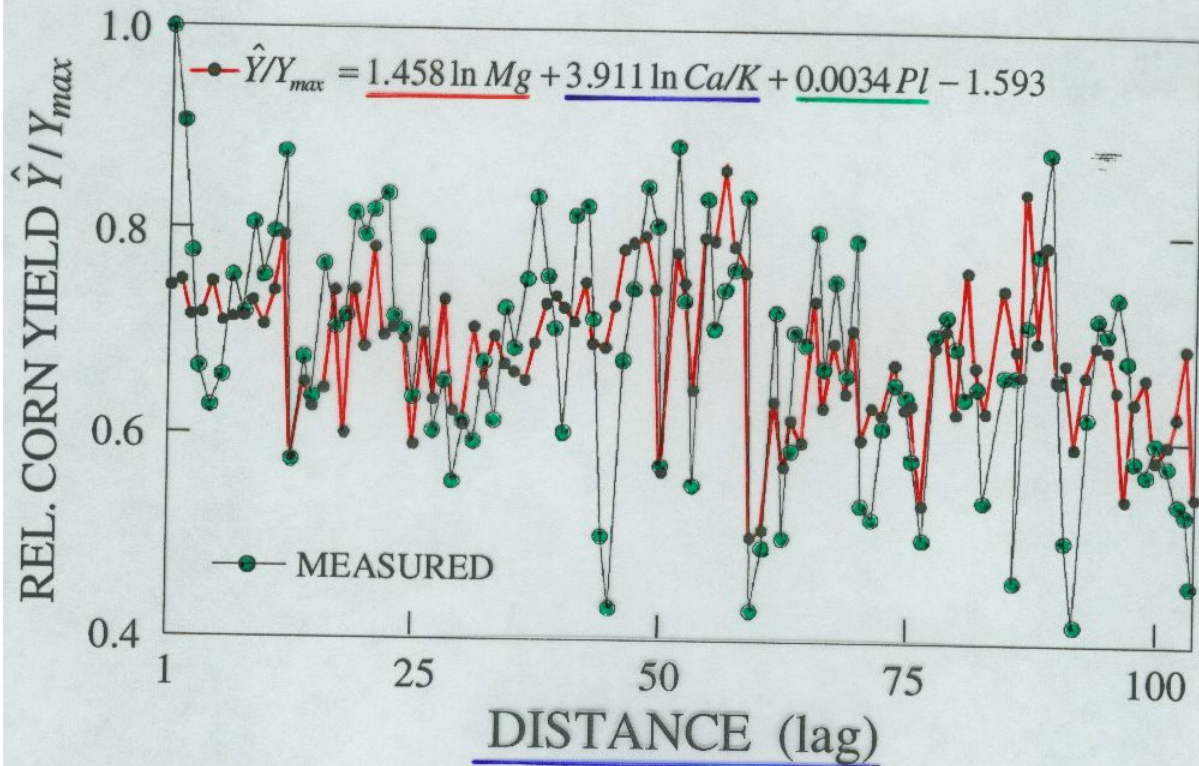




# mult. regr est. corn

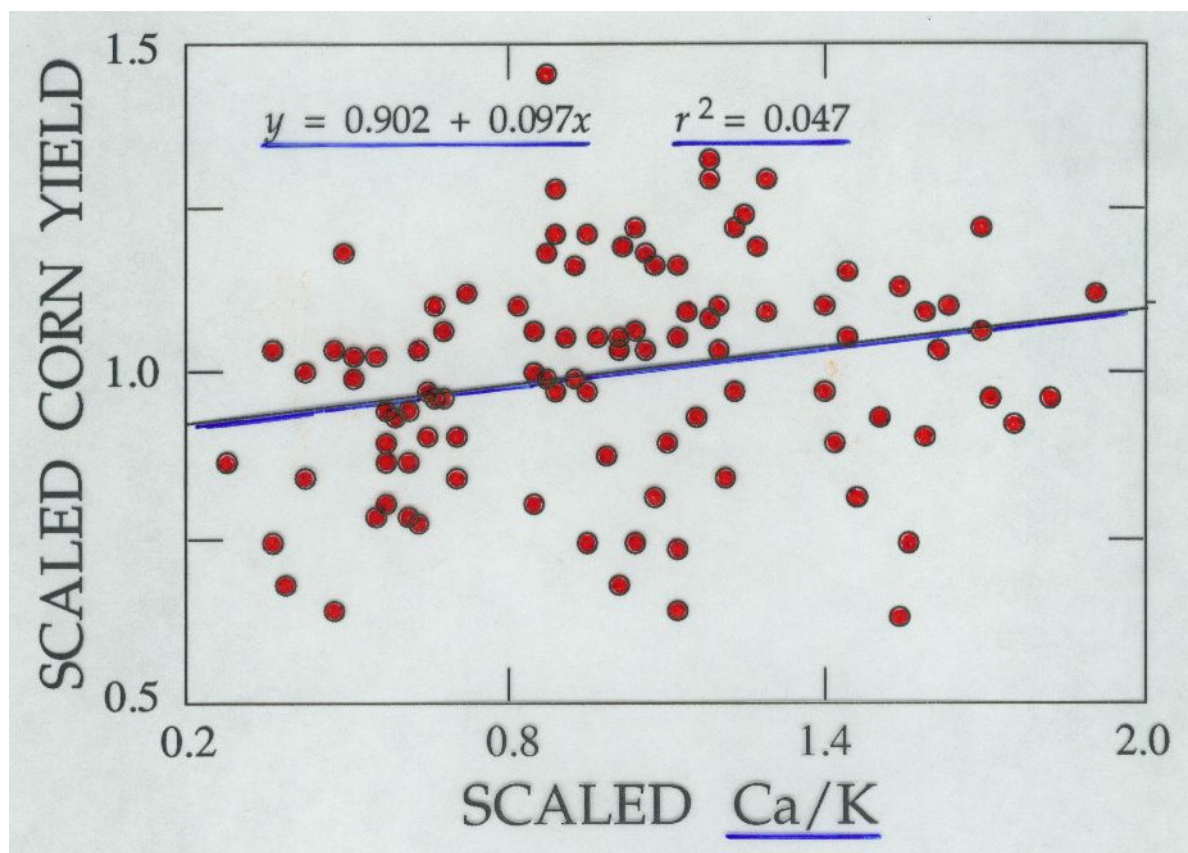
## MULTIPLE REGRESSION

(locations ignored)

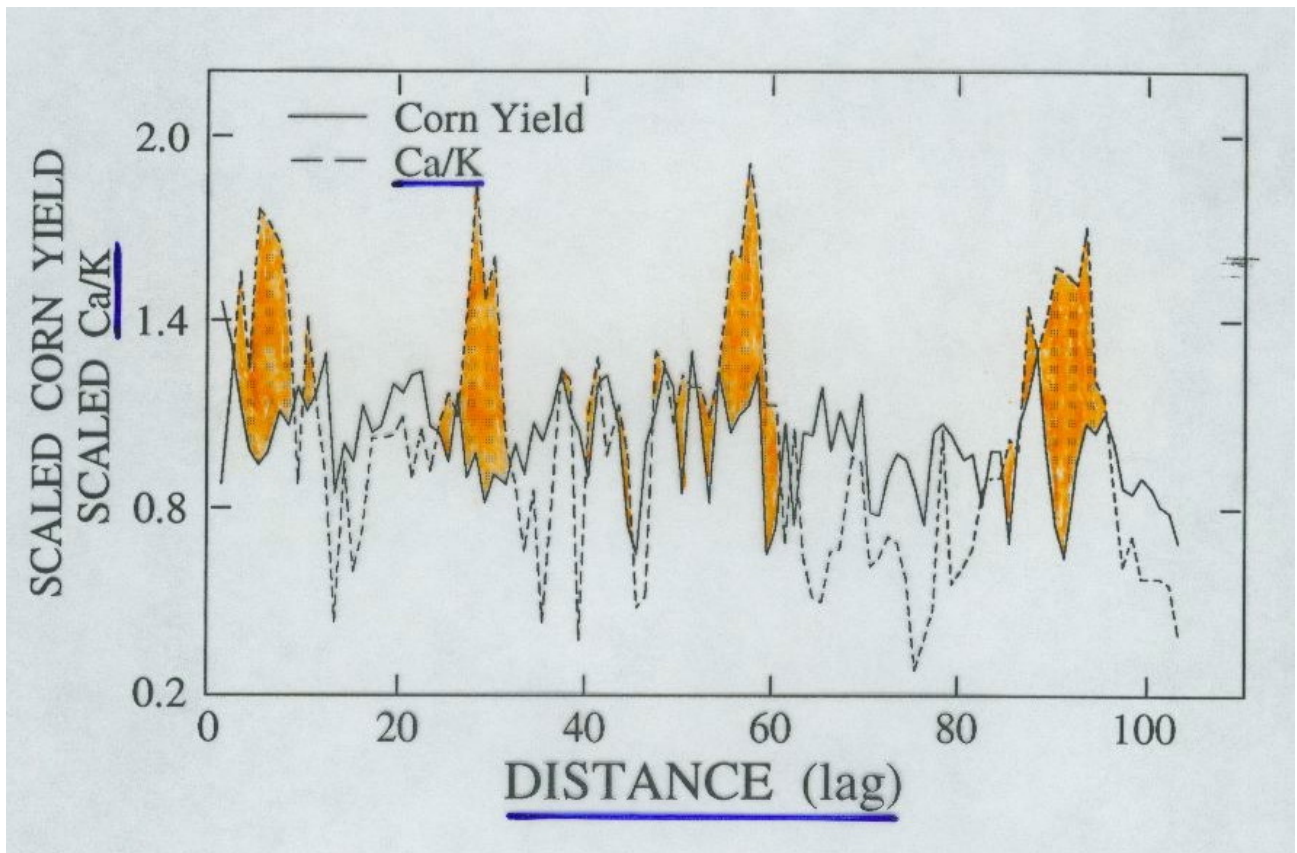


LOCAL, FUNCTIONALLY SIMILAR DOMAINS  
WITHIN A FARMER'S FIELD

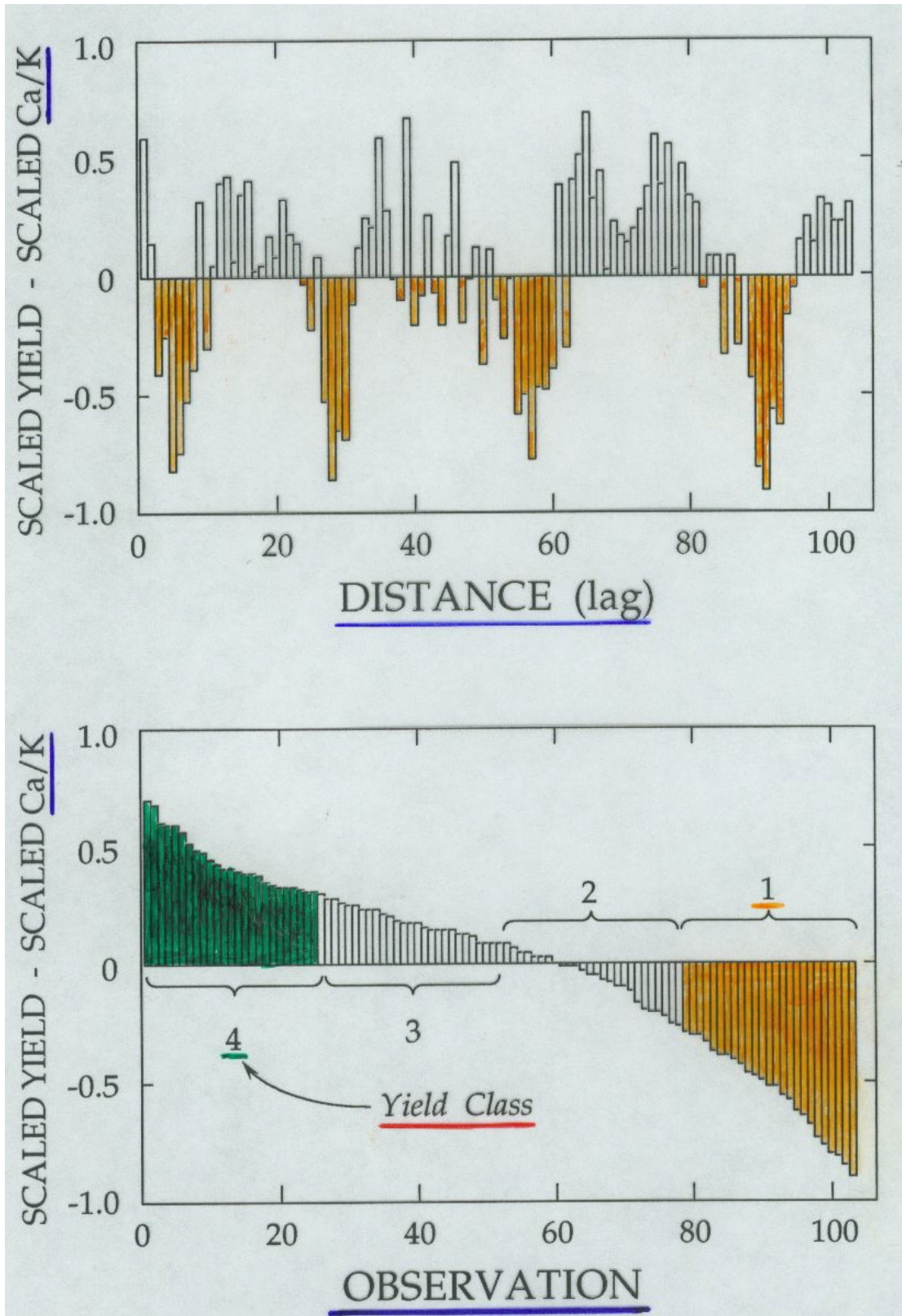
# corn yld vs Ca/K



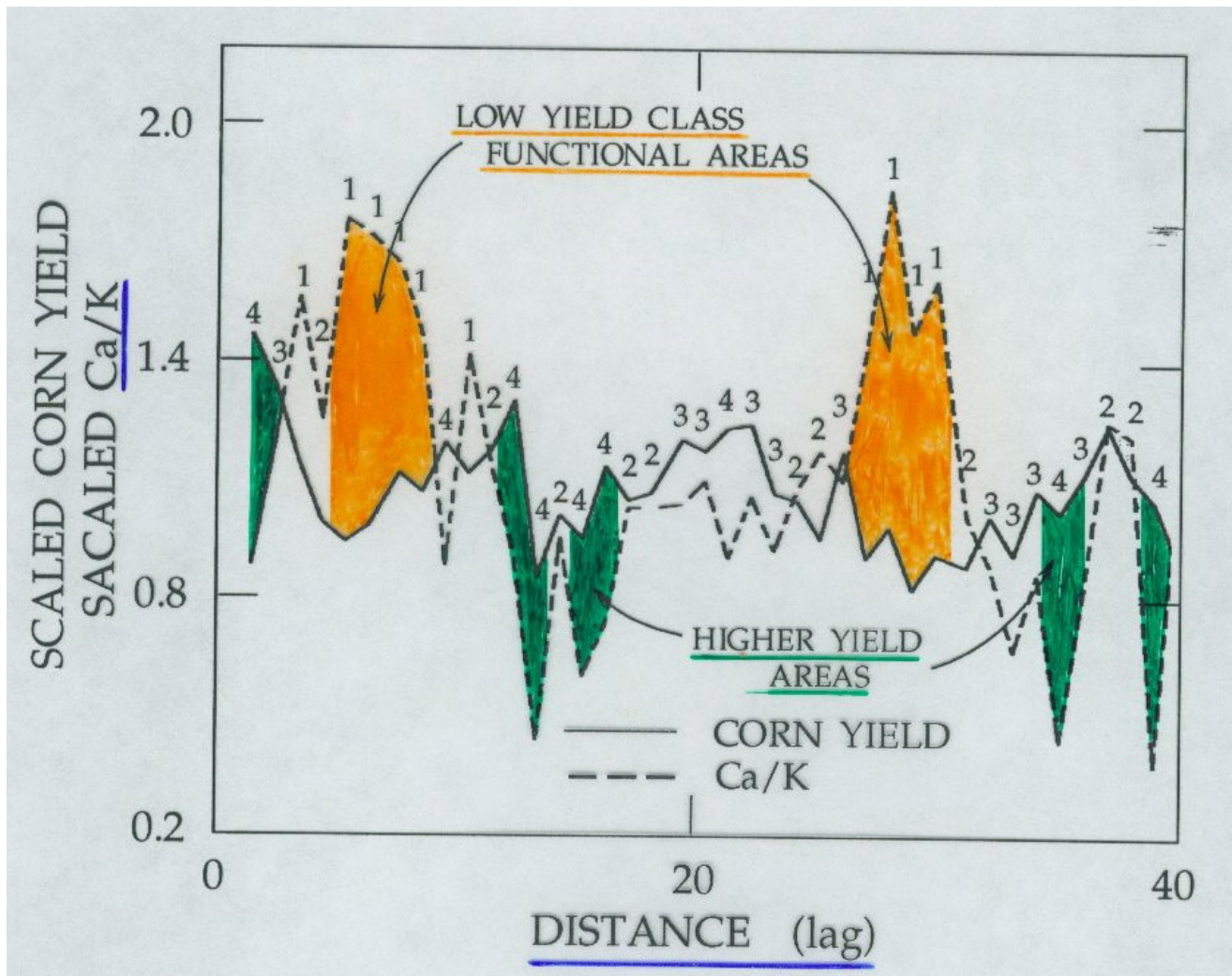
# corn yld&Ca/K vs distanc



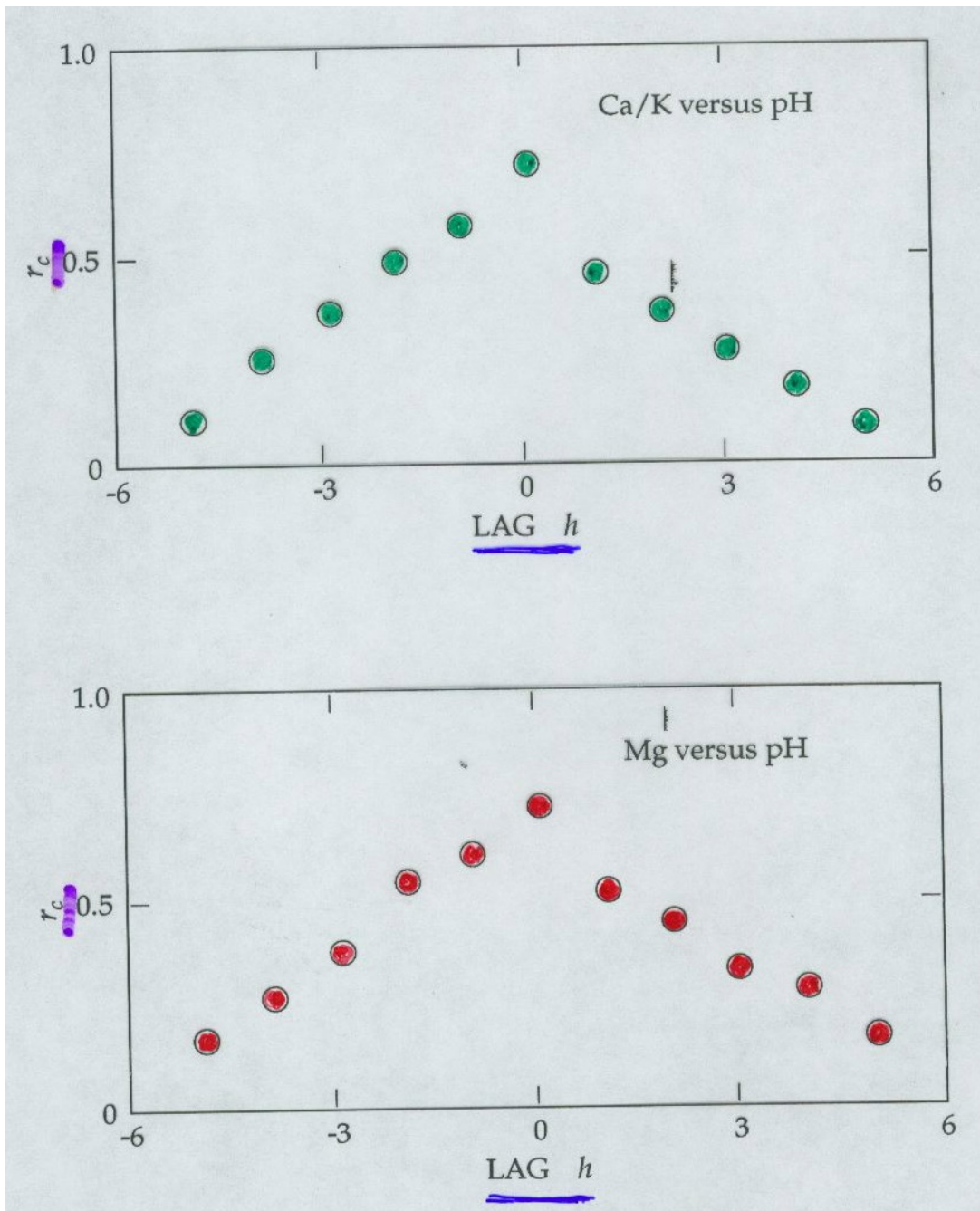
# corn yield classes



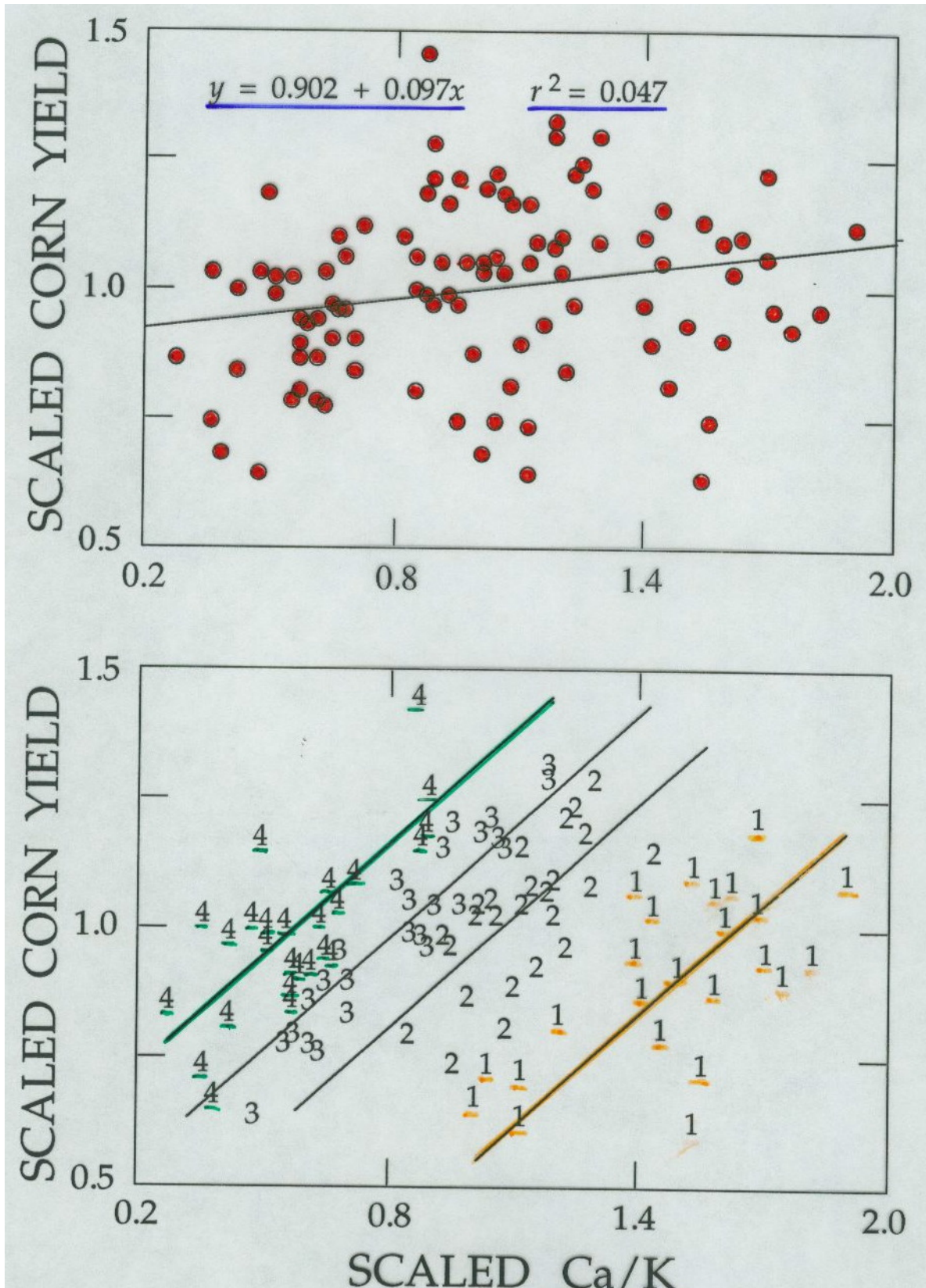
# corn yield class vs distance



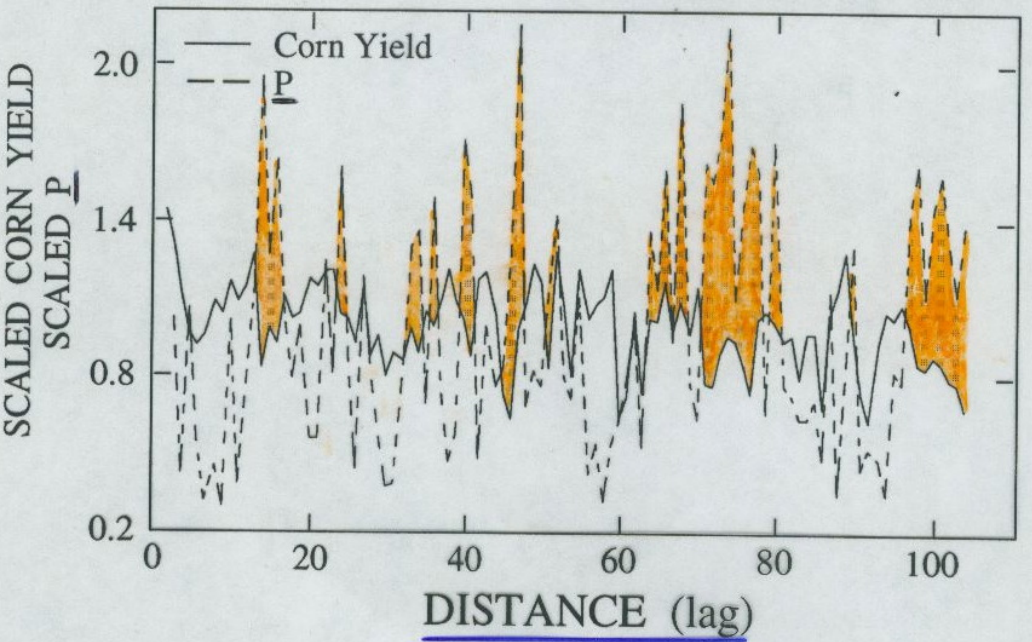
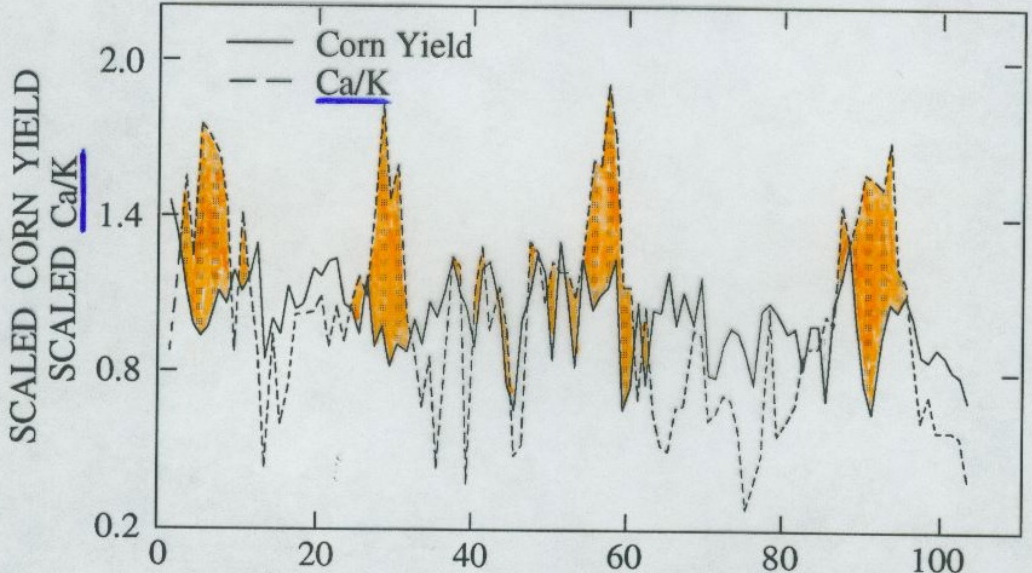
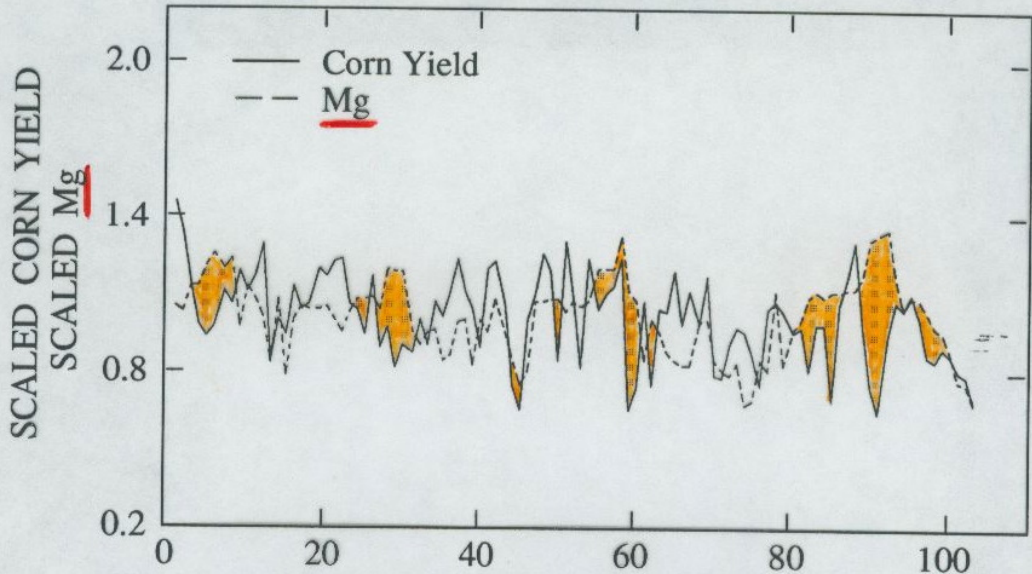
# CCF Ca vs pH



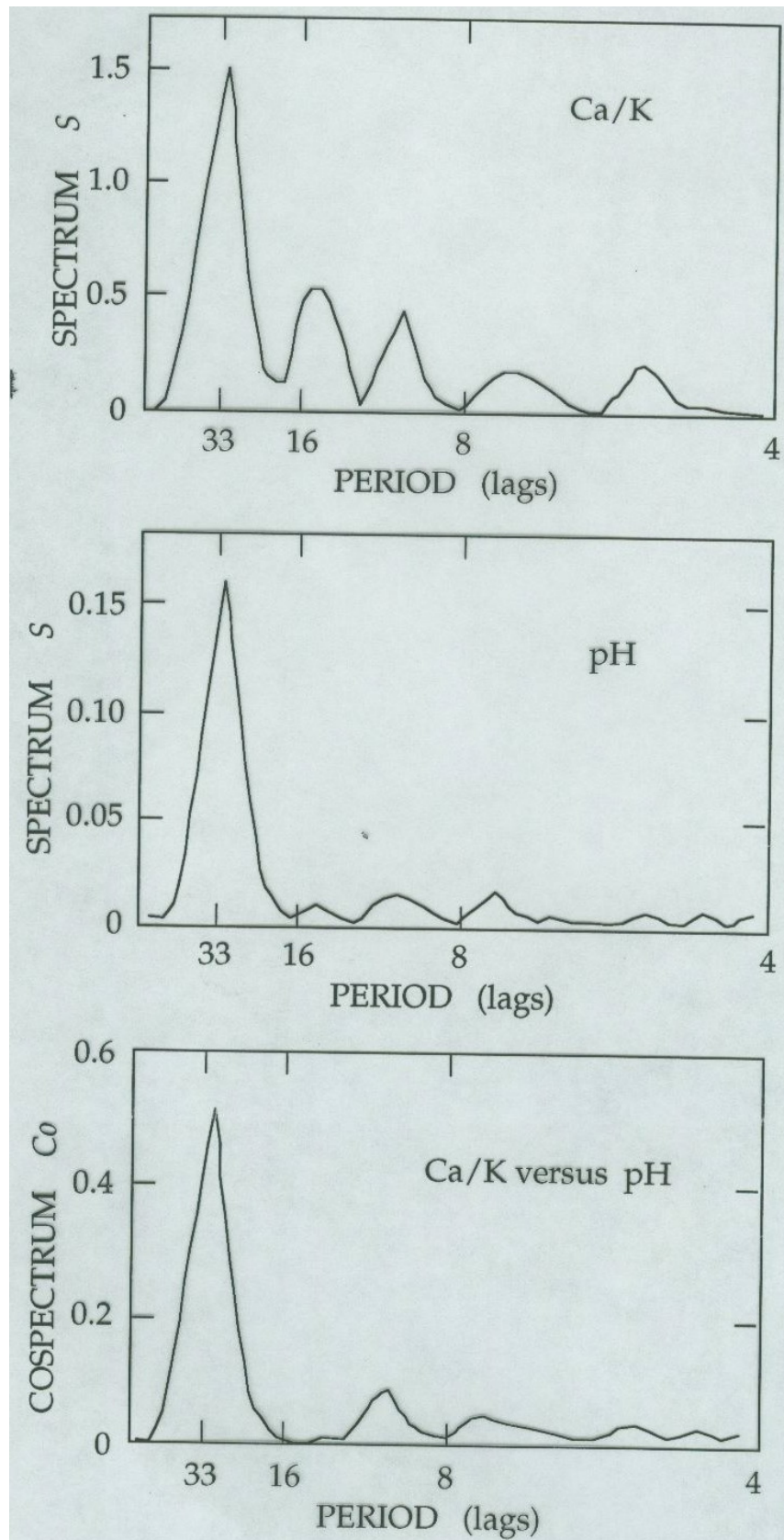
# corn yield vs Ca by class



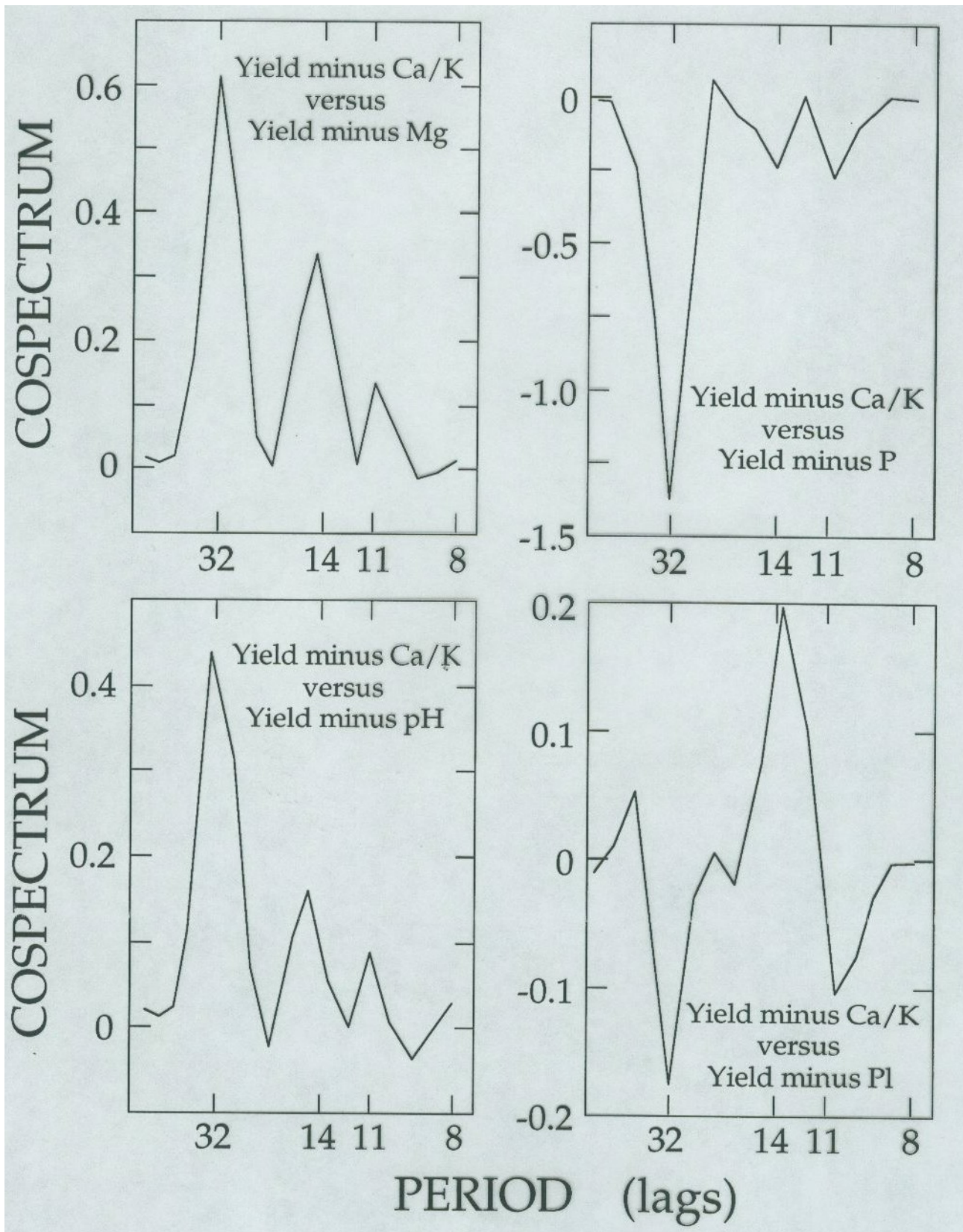




# spectra Ca, Mg,K



# Cospectra Ca vs pH



# Conclusions & future

## CONCLUSION

State-space analysis provides an alternative for exploring on-farm crop yield processes.

Local, functionally distinct areas within a field were identified with applied time series analysis.

## THE FUTURE

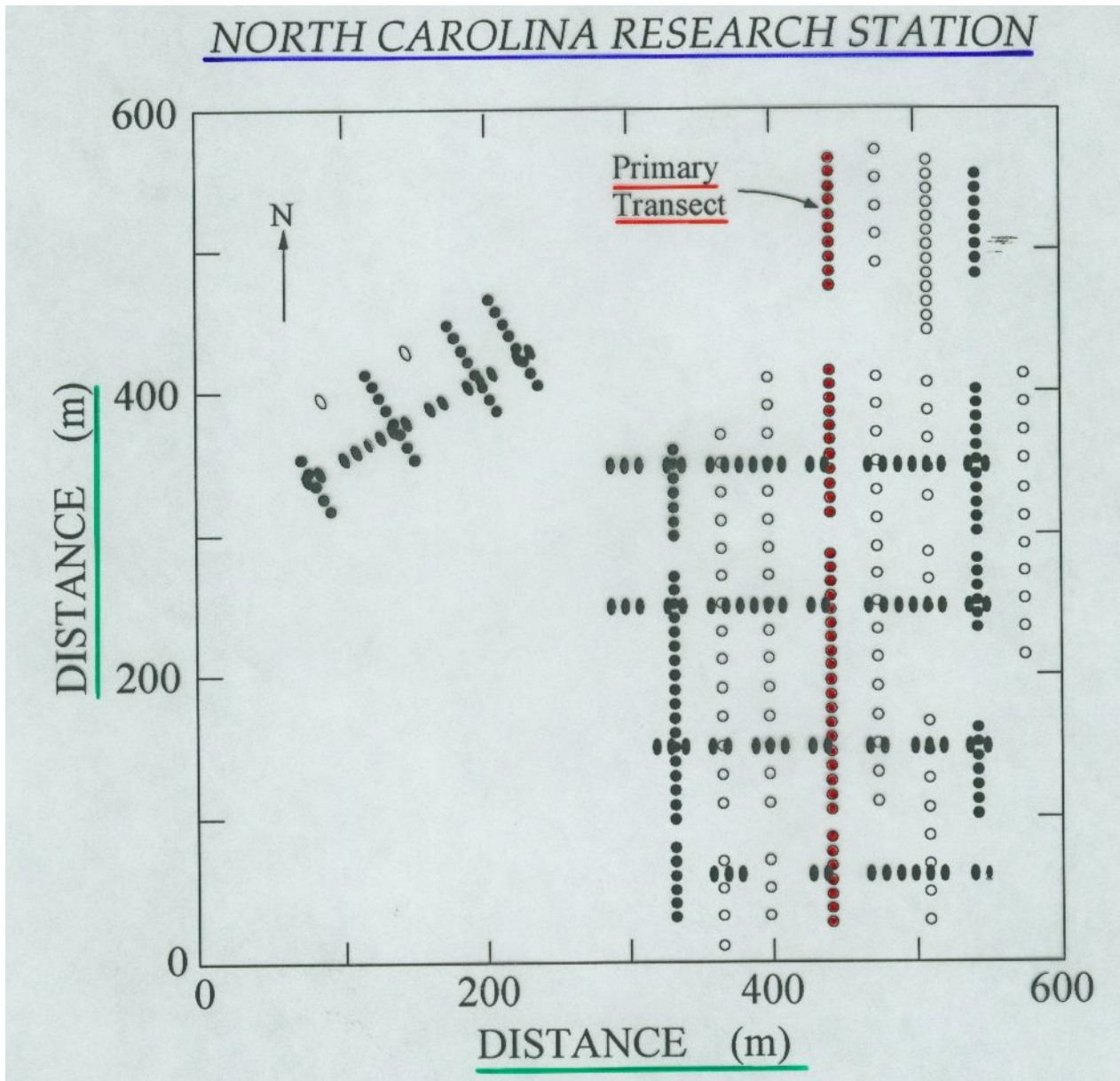
With the inclusion of physically based equations into state-space analysis, we anticipate the enigma of crop yield patterns will be solved.

WHEAT YIELD AND LANDSCAPE  
ATTRIBUTES

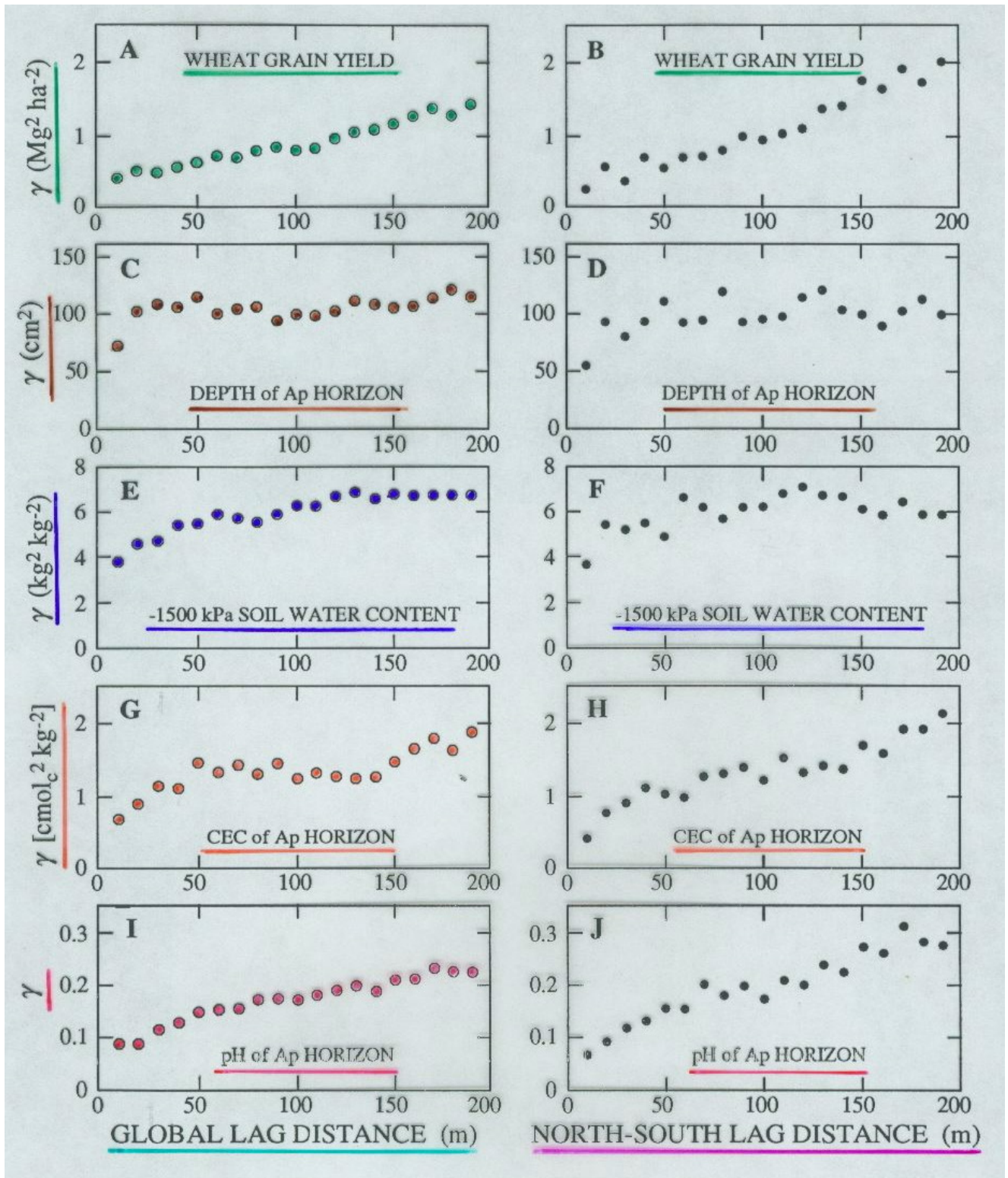
# wheat yield across NCSU field station



# Field station transects

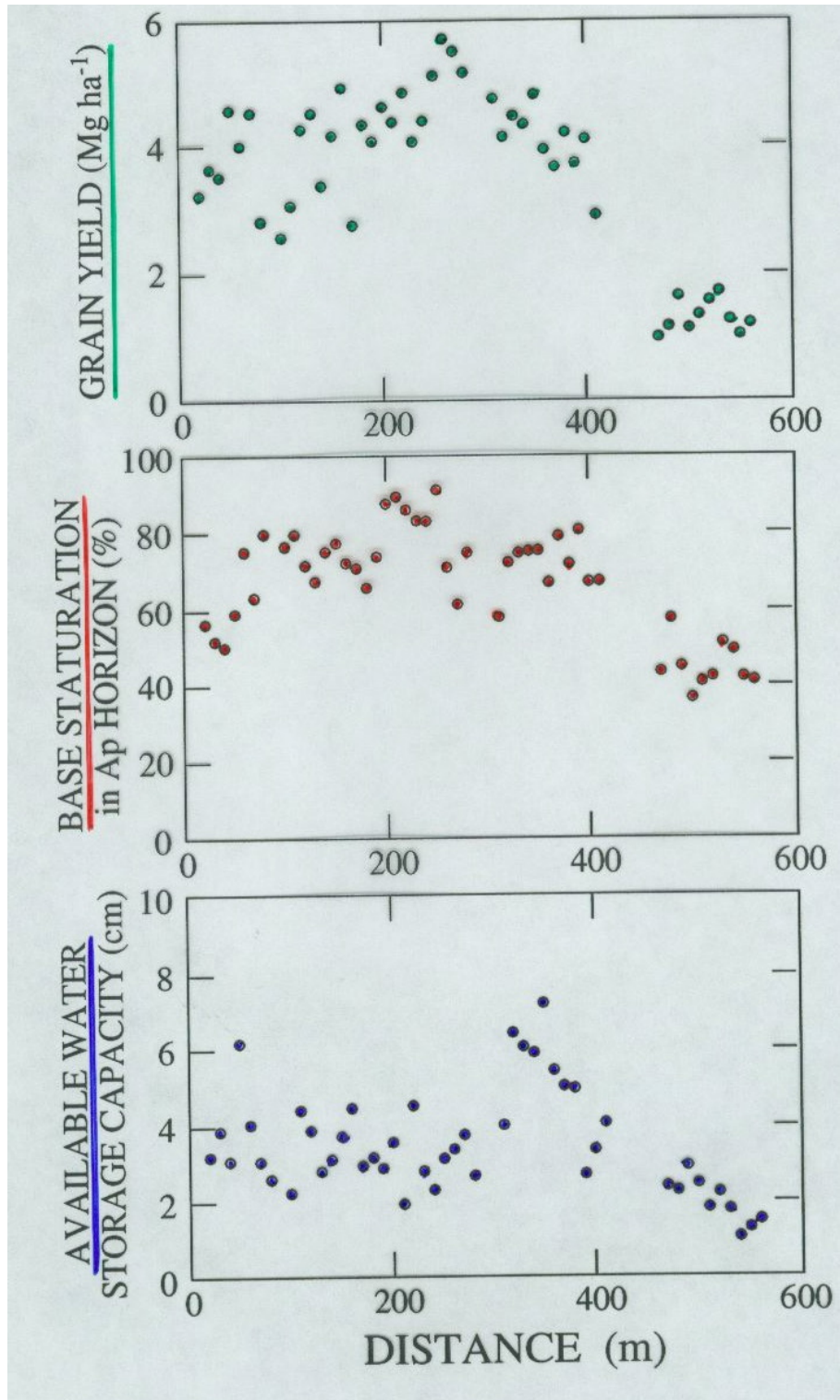


# variograms field

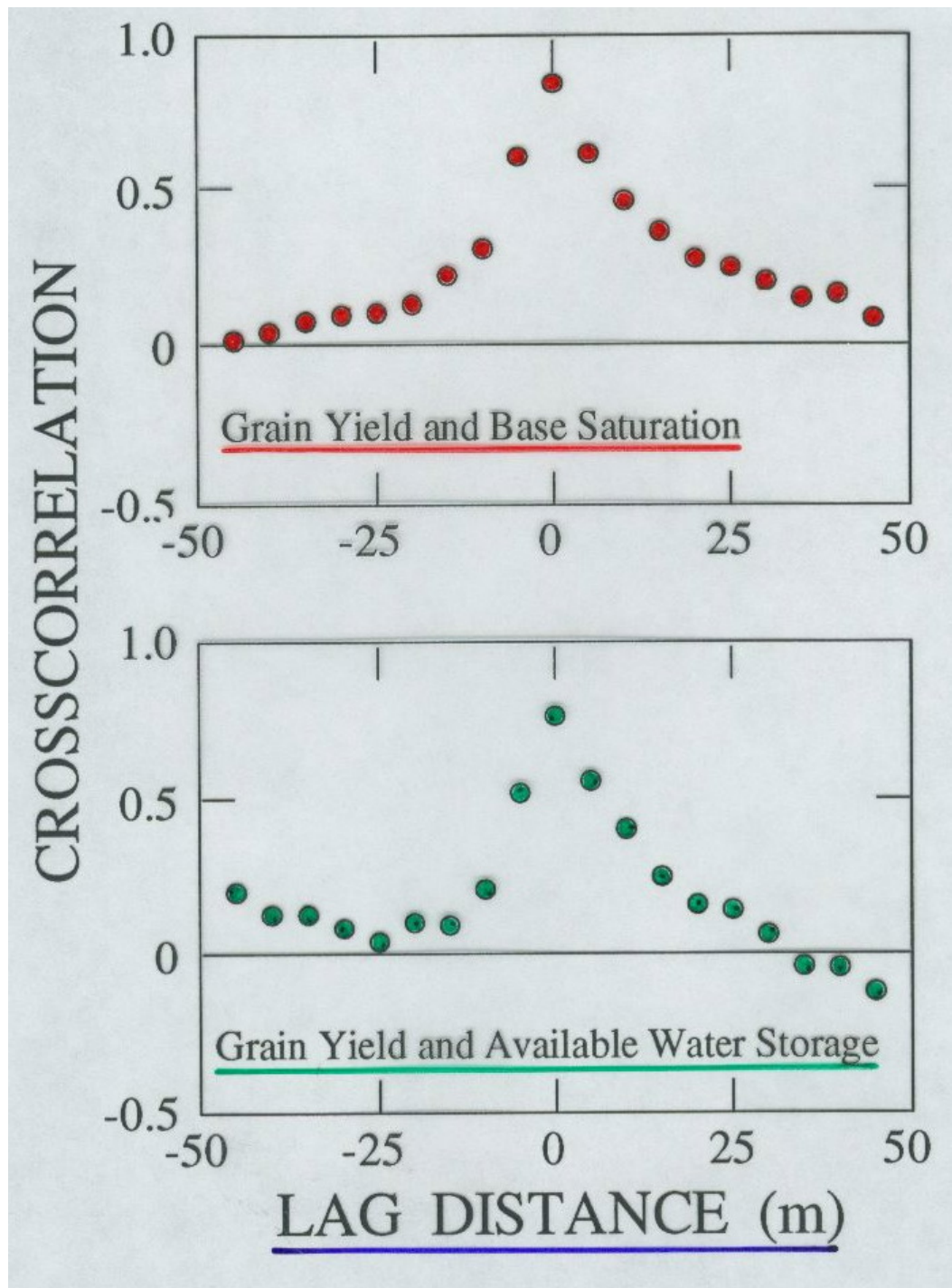




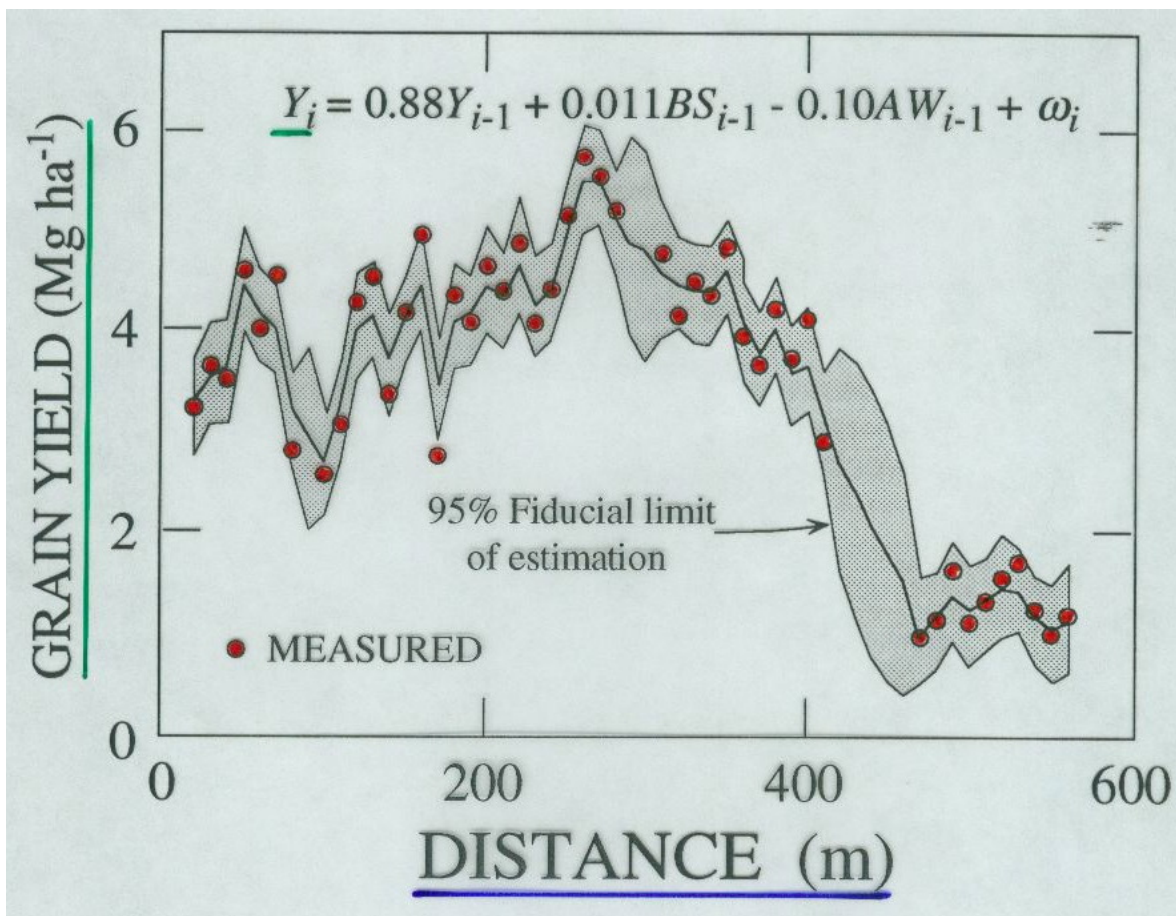
# soil properties across transects



# CCF soil properties



# state-space wheat NCS



# biological basis for time series

## BIOLOGICAL BASIS FOR A TIME SERIES MODEL OF CROP YIELD

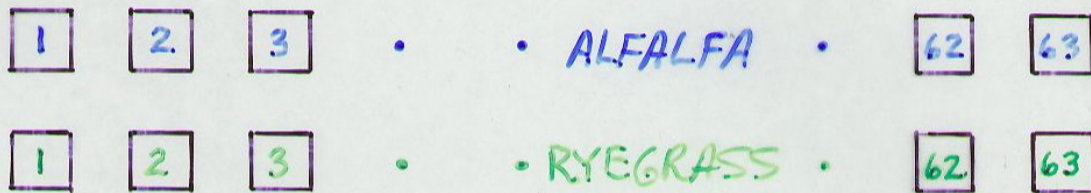
OBJECTIVE: UNDERSTAND N-FIXATION DISTRIBUTION ACROSS A CULTIVATED FIELD

WHEREVER LACK OF INFORMATION AND LIMITED KNOWLEDGE ABOUT EFFECTS OF AND INTERACTIONS BETWEEN VARIOUS PARAMETERS MAKE A DETERMINISTIC EXPLANATION OF THE FIELD SITUATION IMPOSSIBLE, PROCESSES IN FIELDS CAN BE DESCRIBED USING STOCHASTIC APPROACHES EXAMINING HOW OBSERVATIONS CHANGE IN SPACE.

# NITROGEN FIXATION

# state space analysis N fixation

## STATE-SPACE ANALYSIS



20 kg <sup>15</sup>N-UREA

←————— 96m —————→

## % NITROGEN FIXED BY ALFALFA

### DIFFERENCE METHOD

$$\% \text{ N FIXED} = \left( \frac{\text{N IN SHOOTS OF ALF.} - \text{N IN SHOOTS OF RYEGR.}}{\text{N IN SHOOTS OF ALFALFA}} \right) 100$$

18.8%

### ISOTOPE DILUTION METHOD

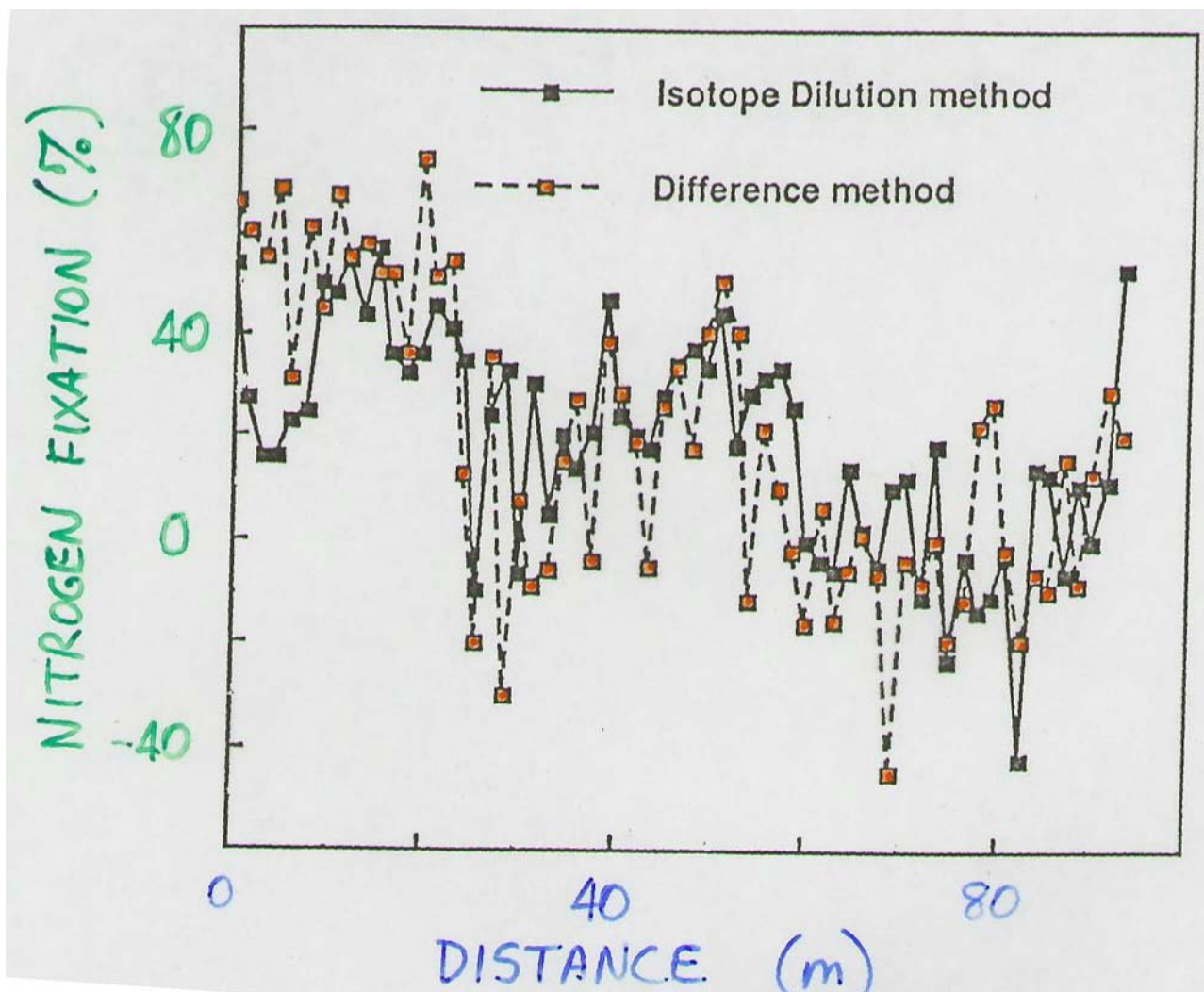
$$\% \text{ N FIXED} = \left( \frac{\%^{15}\text{N IN RYEGRASS} - \%^{15}\text{N IN ALFALFA}}{\%^{15}\text{N IN RYEGRASS}} \right) 100$$

19.6%

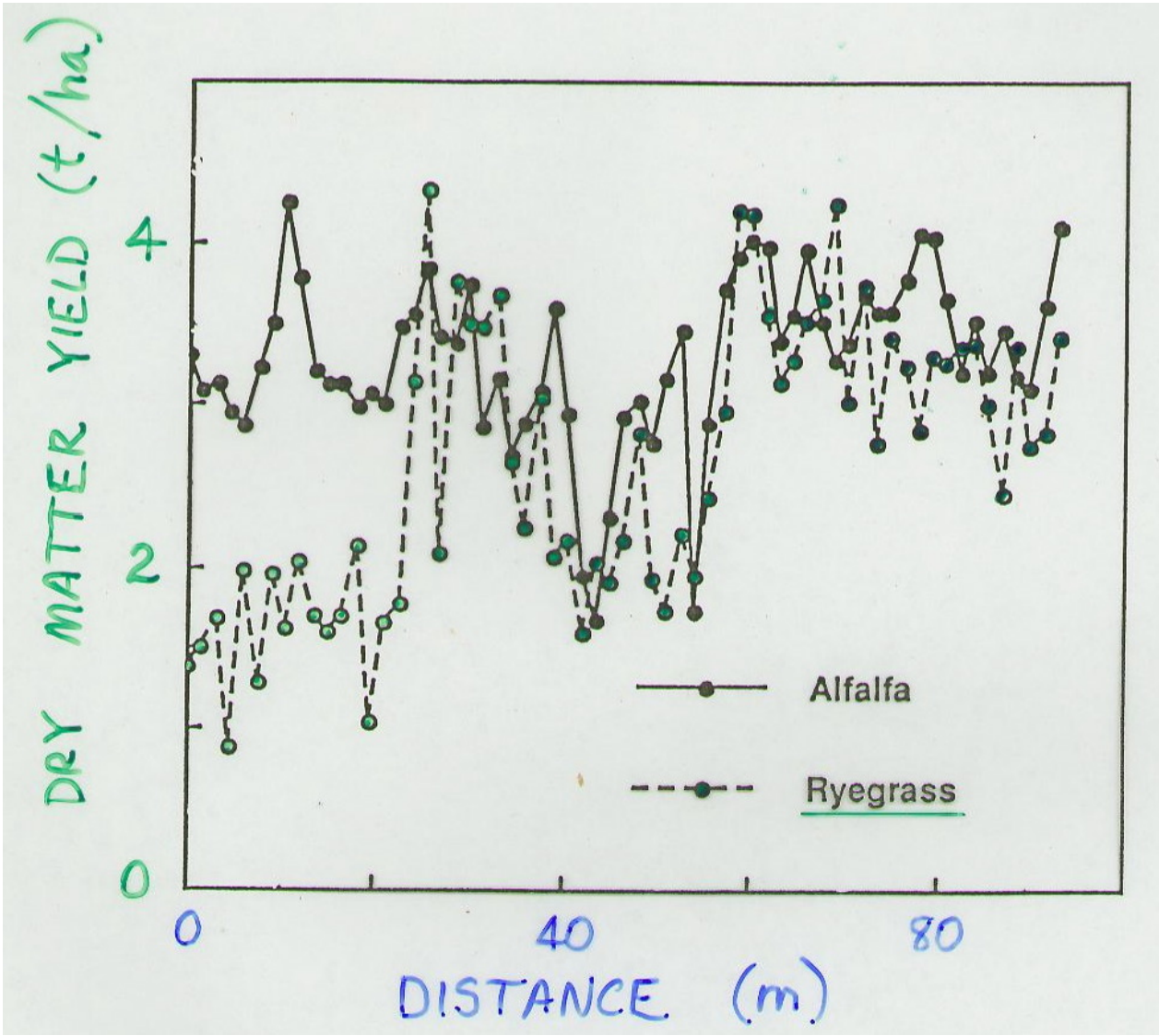
### MEASURED

DRY MATTER, TOTAL CROP N CONTENT, <sup>15</sup>N CONTENT OF CROP,  
STONE CONTENT, TOTAL SOIL N CONTENT

# N fixation vs distance (2 methods)

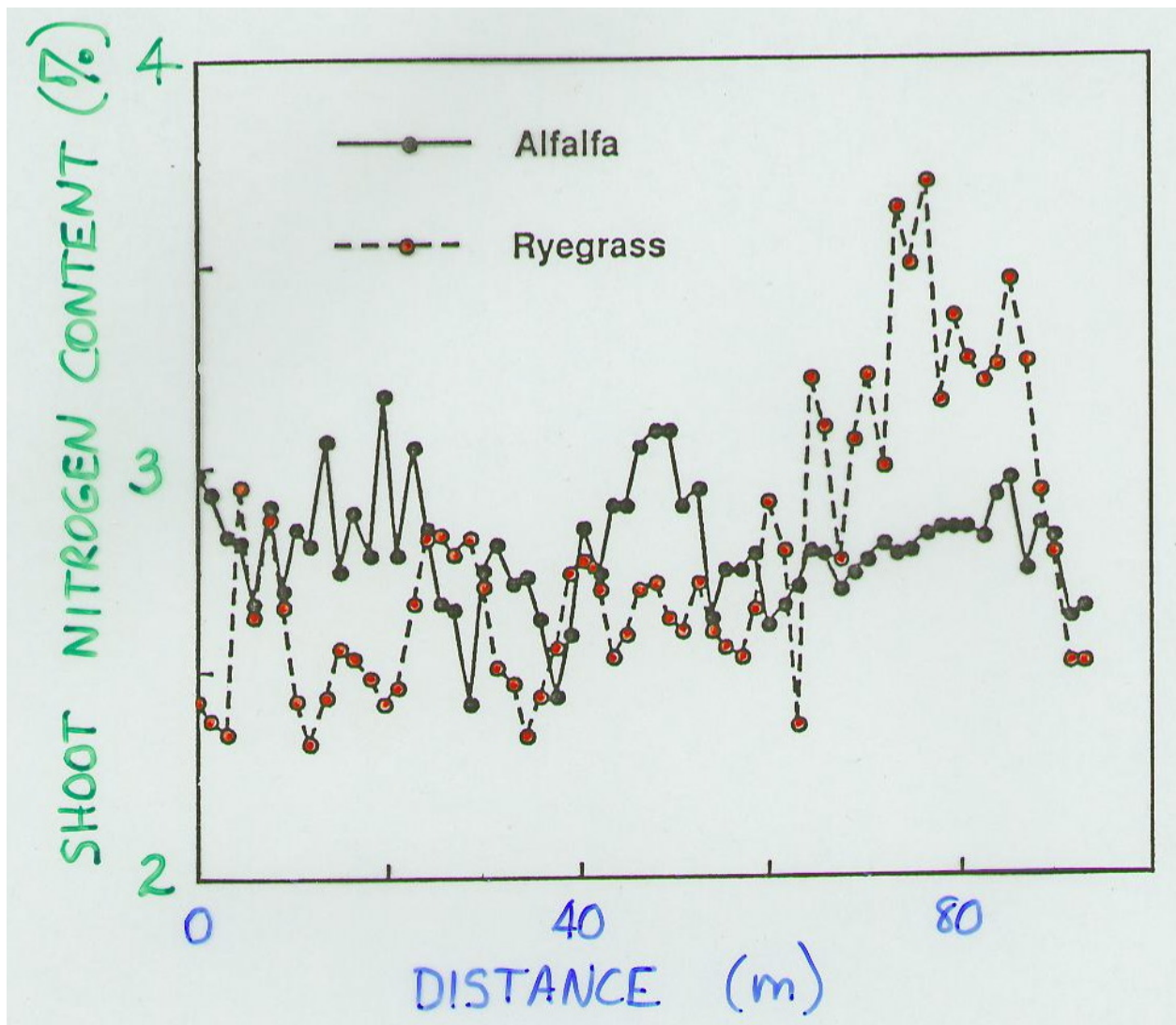


# alfalfa & rye grass yield

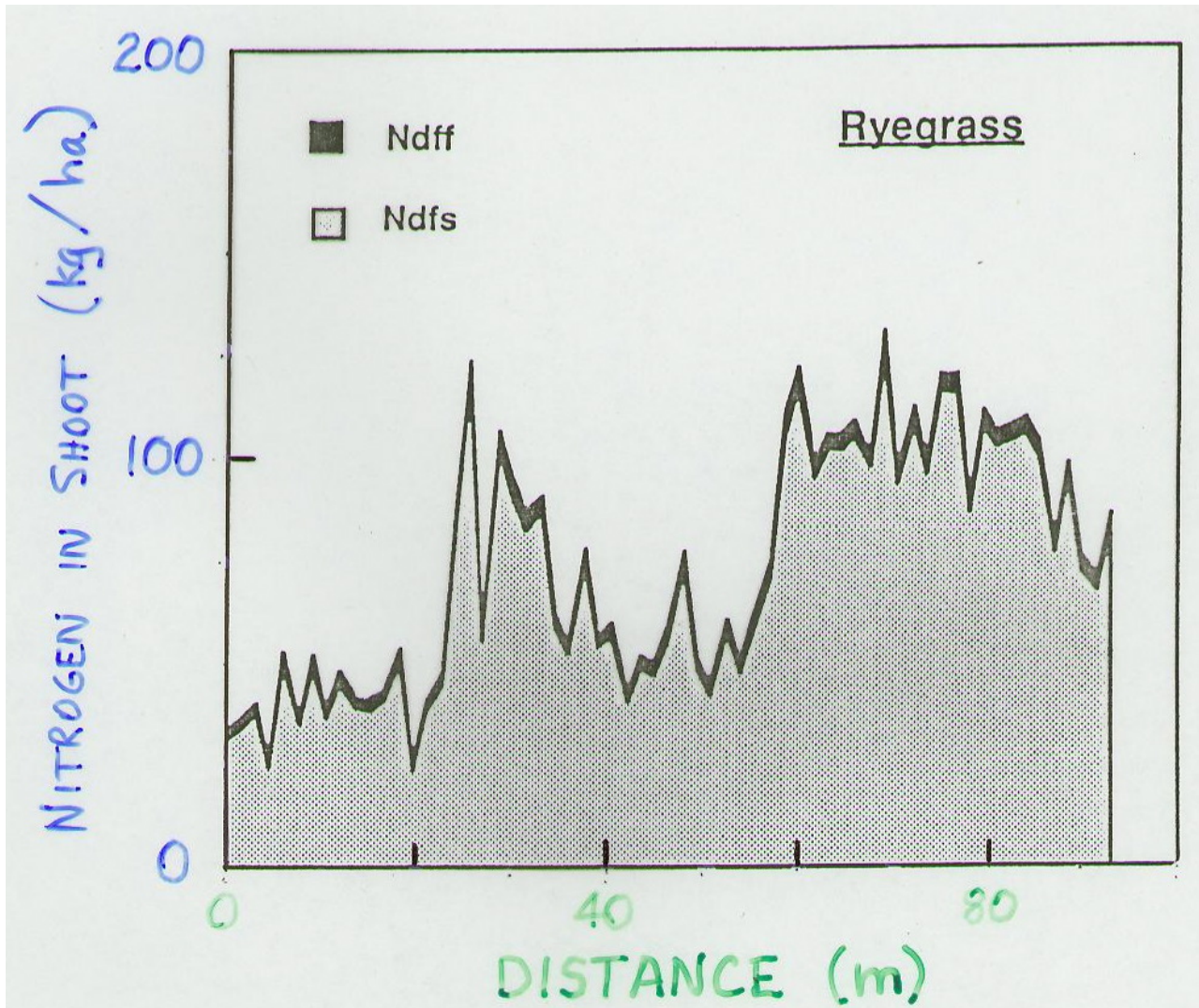




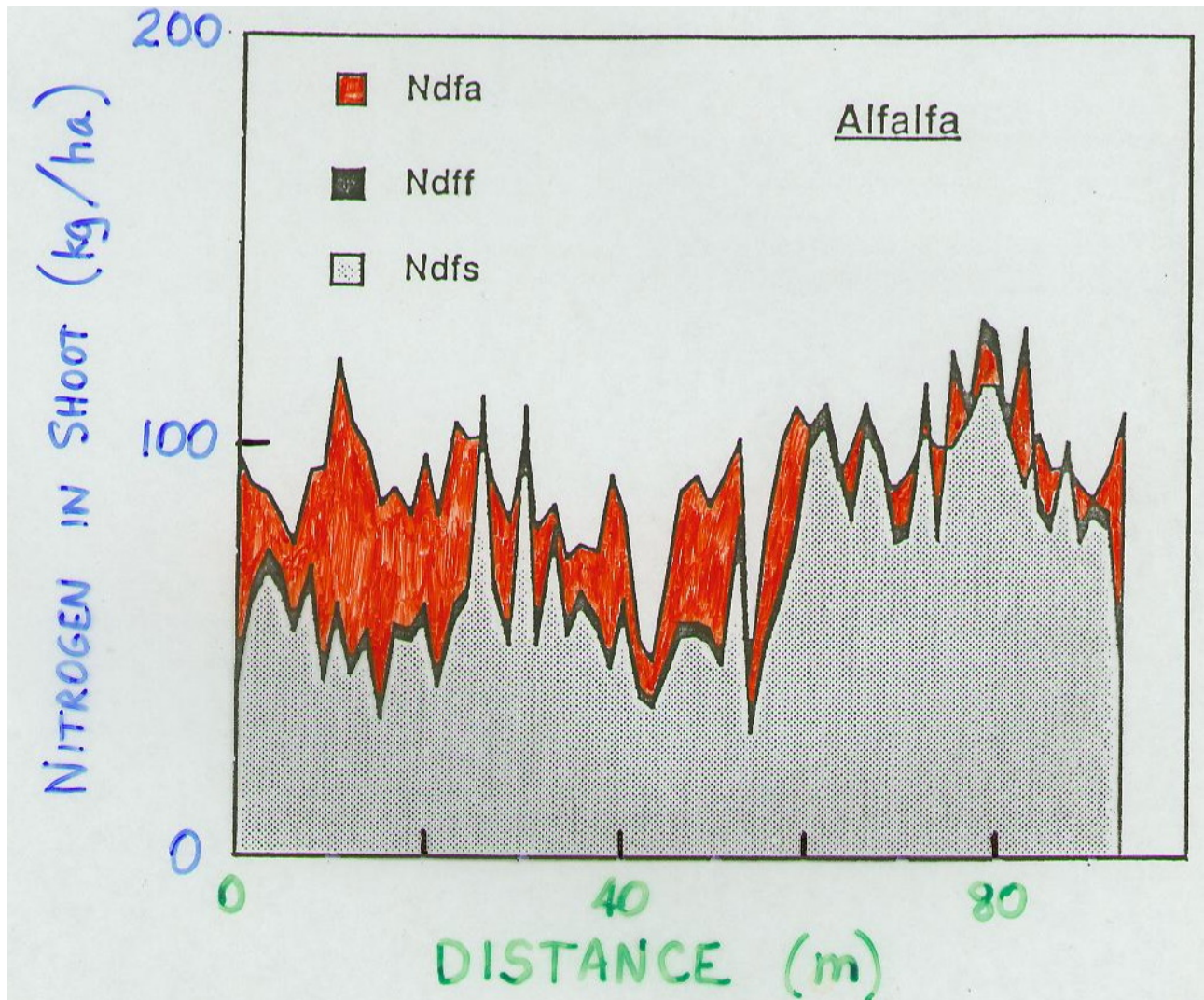
# shoot N in rye grass & alfalfa



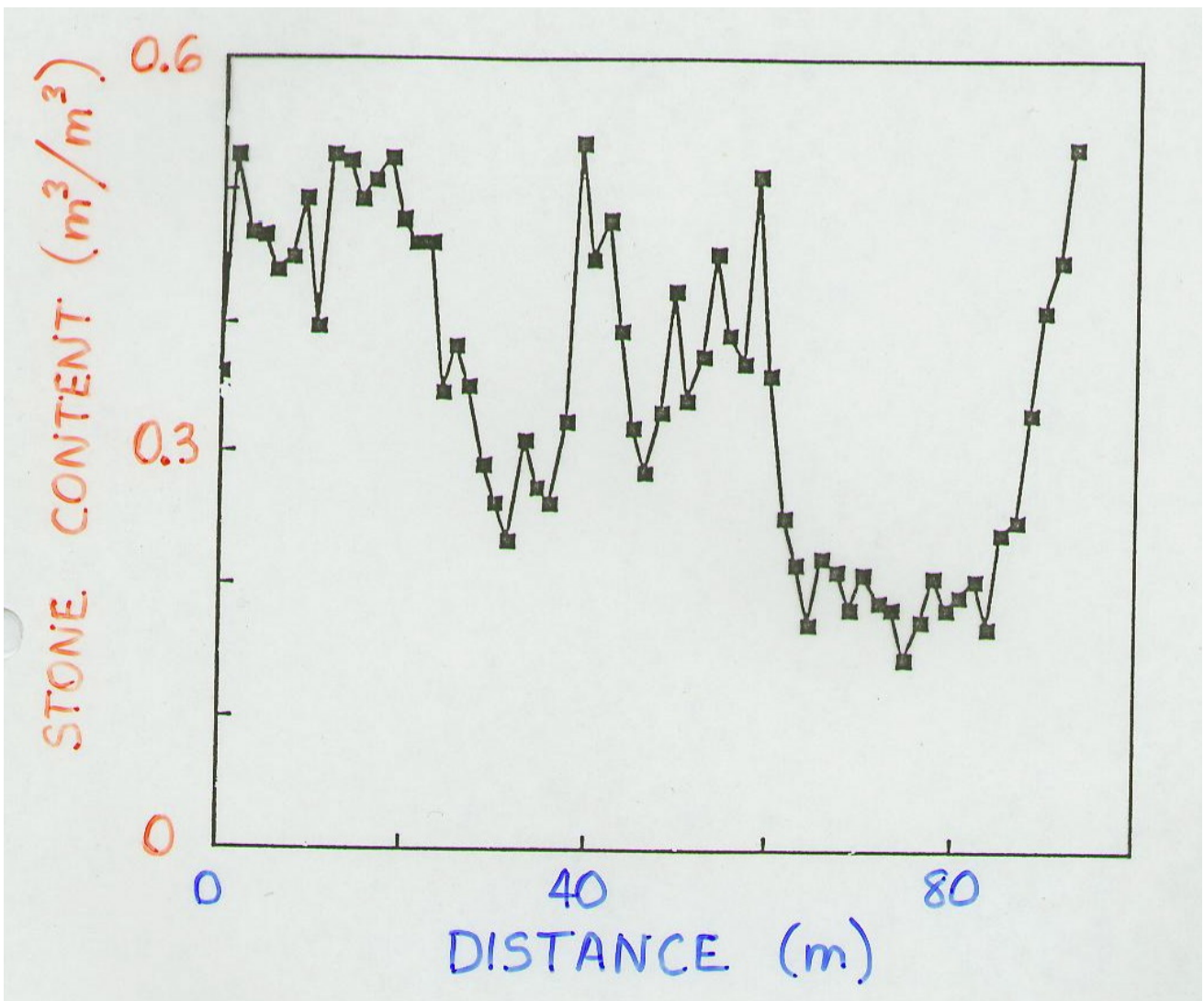
# N in ryegrass shoot



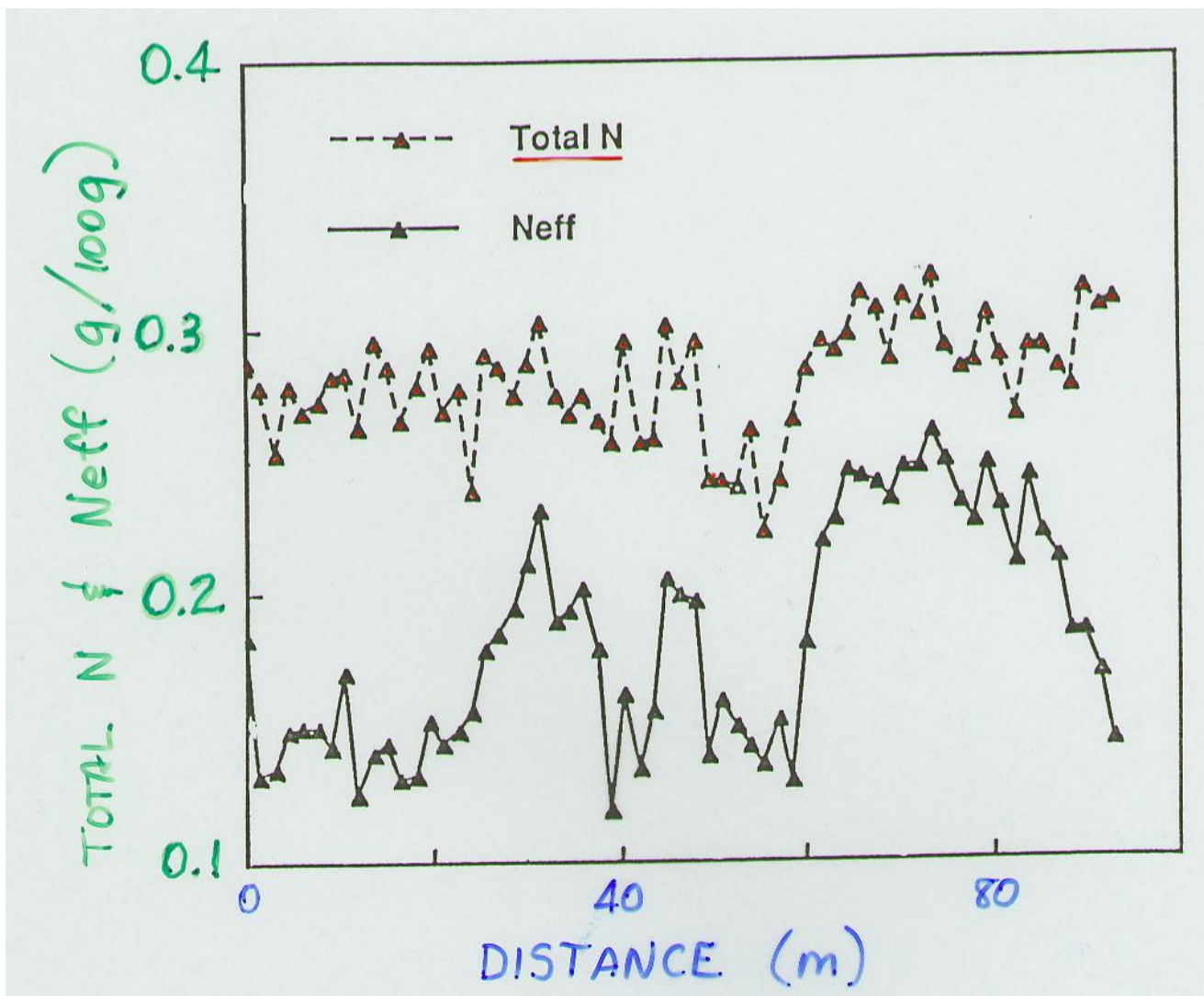
# N in alfalfa shoot



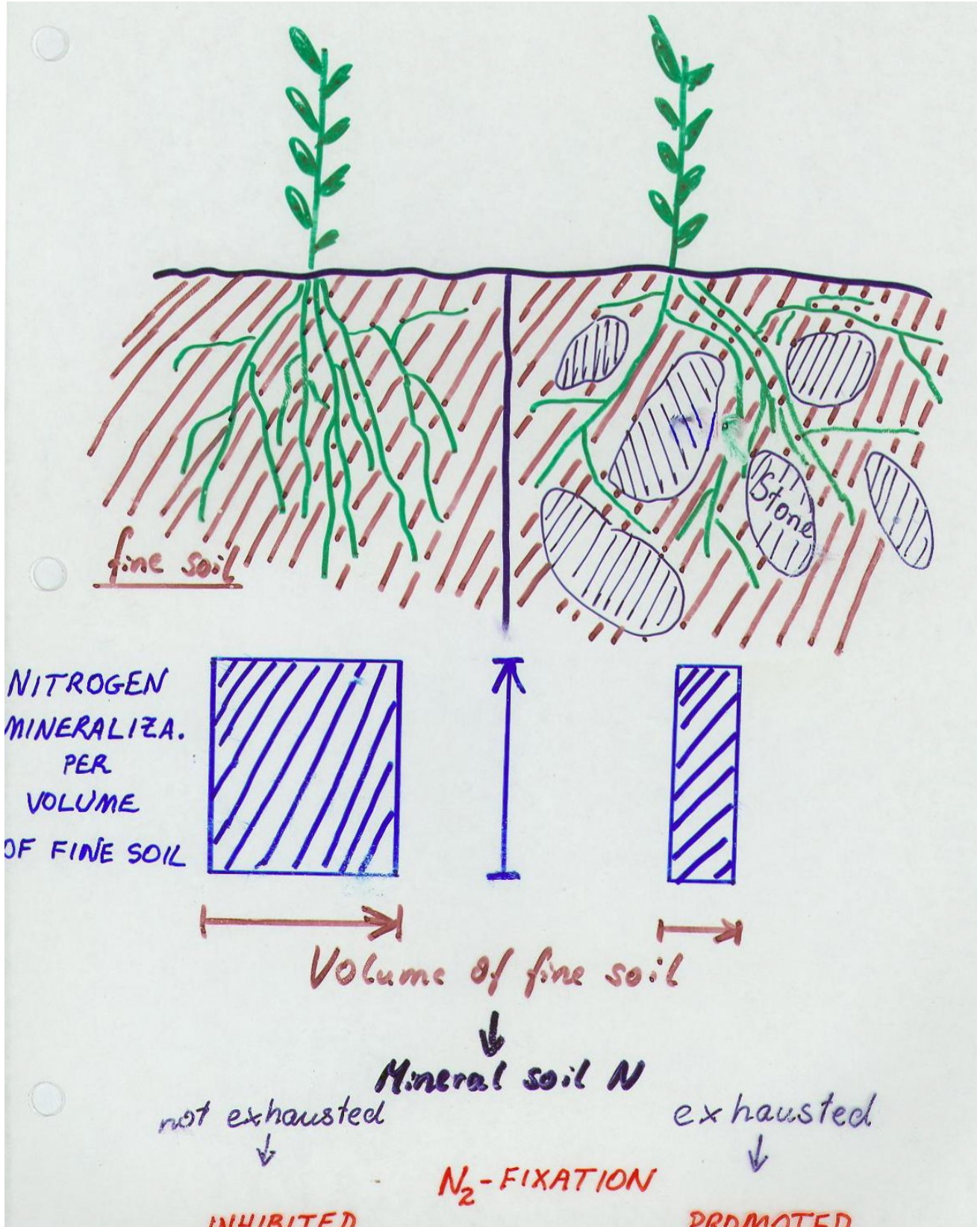
# stone content



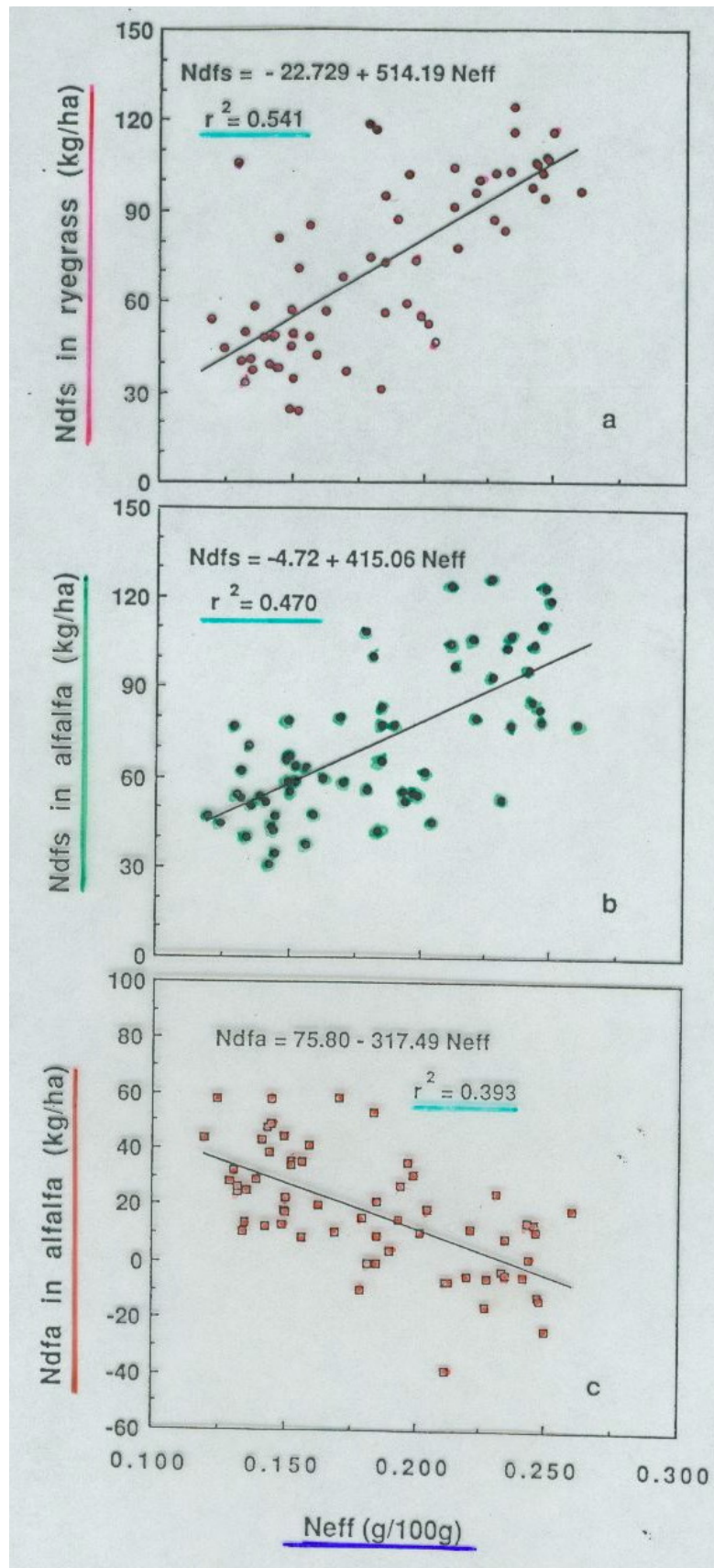
# total N & Neff versus distance



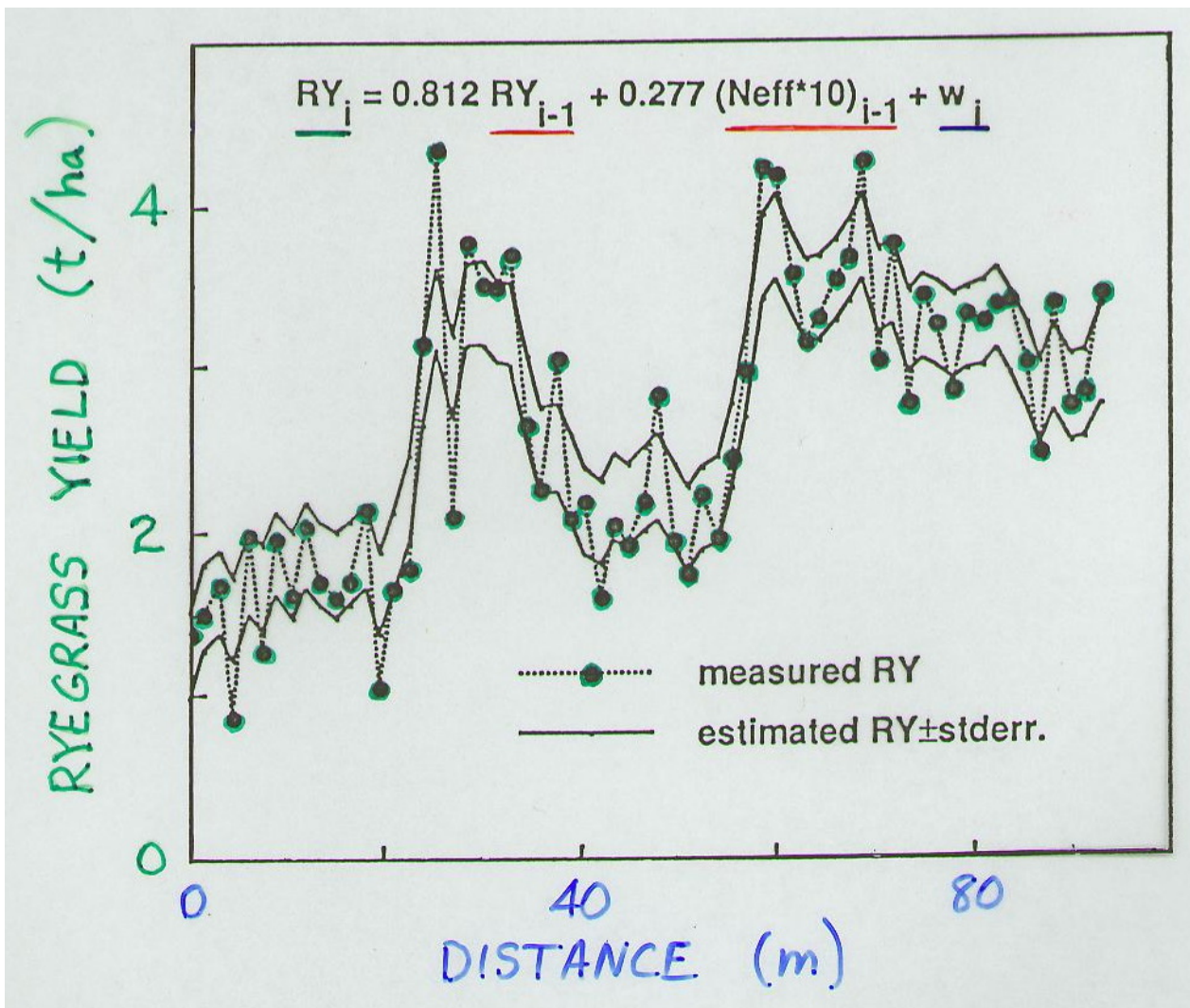
# N<sub>2</sub> fixation (stones in soil)



# Ndfs versus Neff

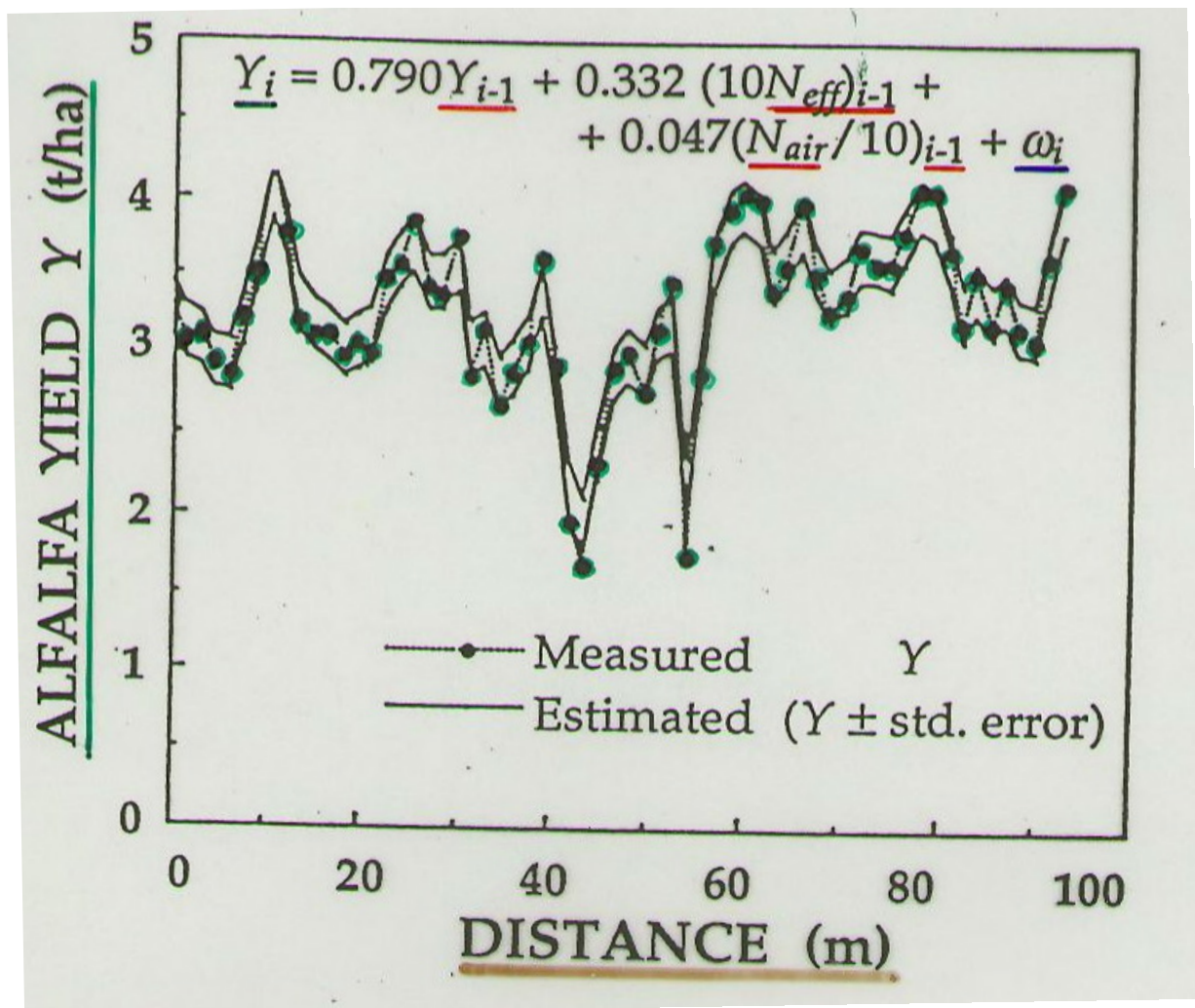


# state-space ryegrass yield

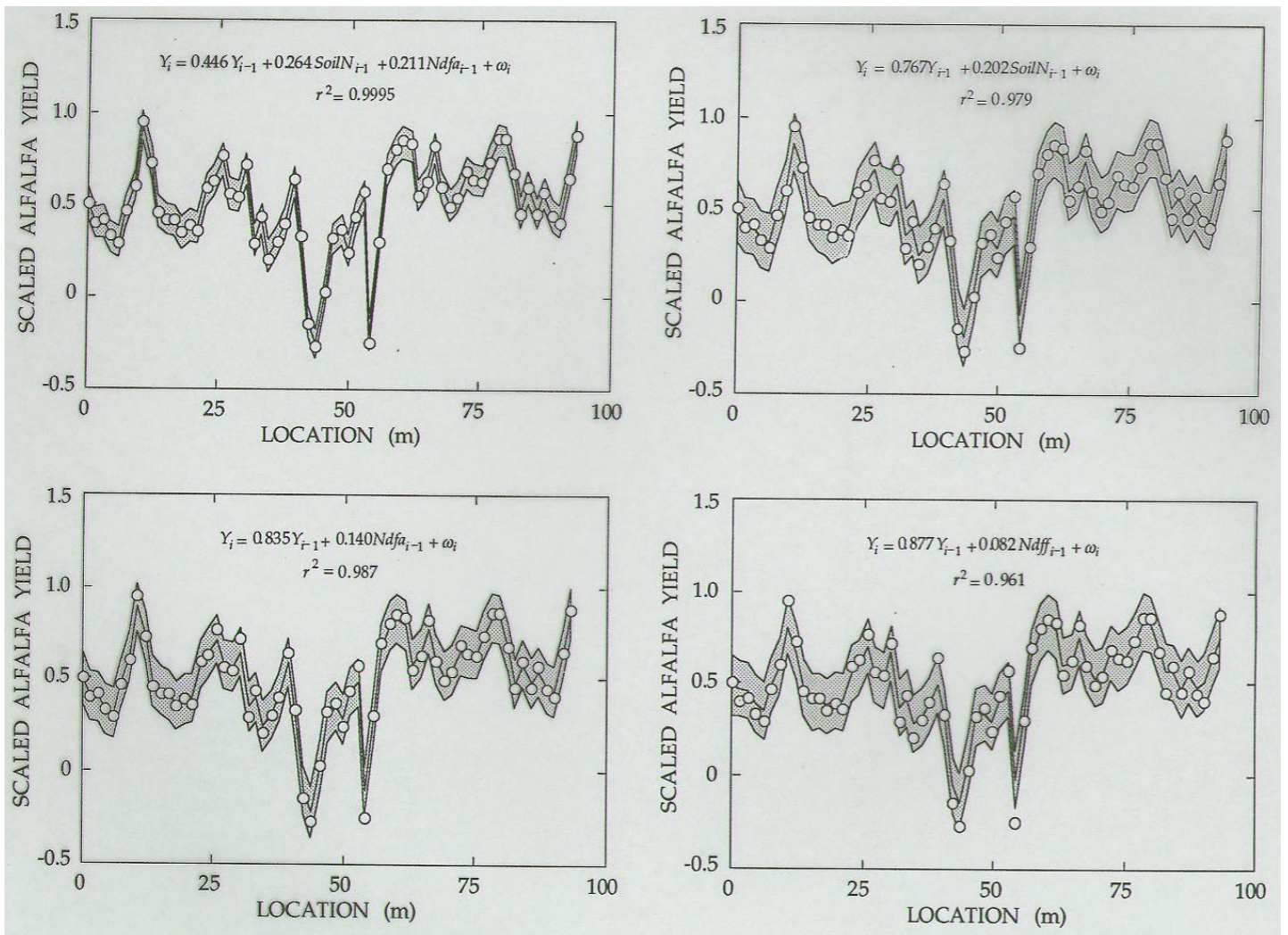




# state-space alfalfa yield



# state-space est. alfalfa



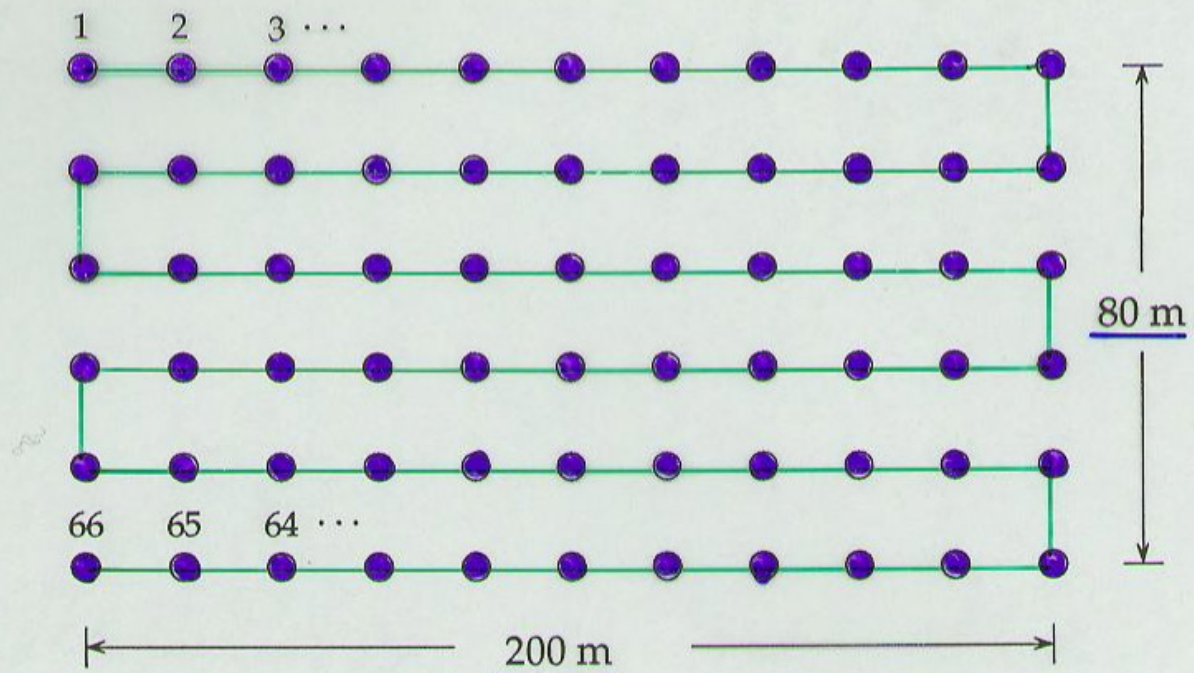
# N<sub>2</sub> fixation conclusions

## CONCLUSIONS

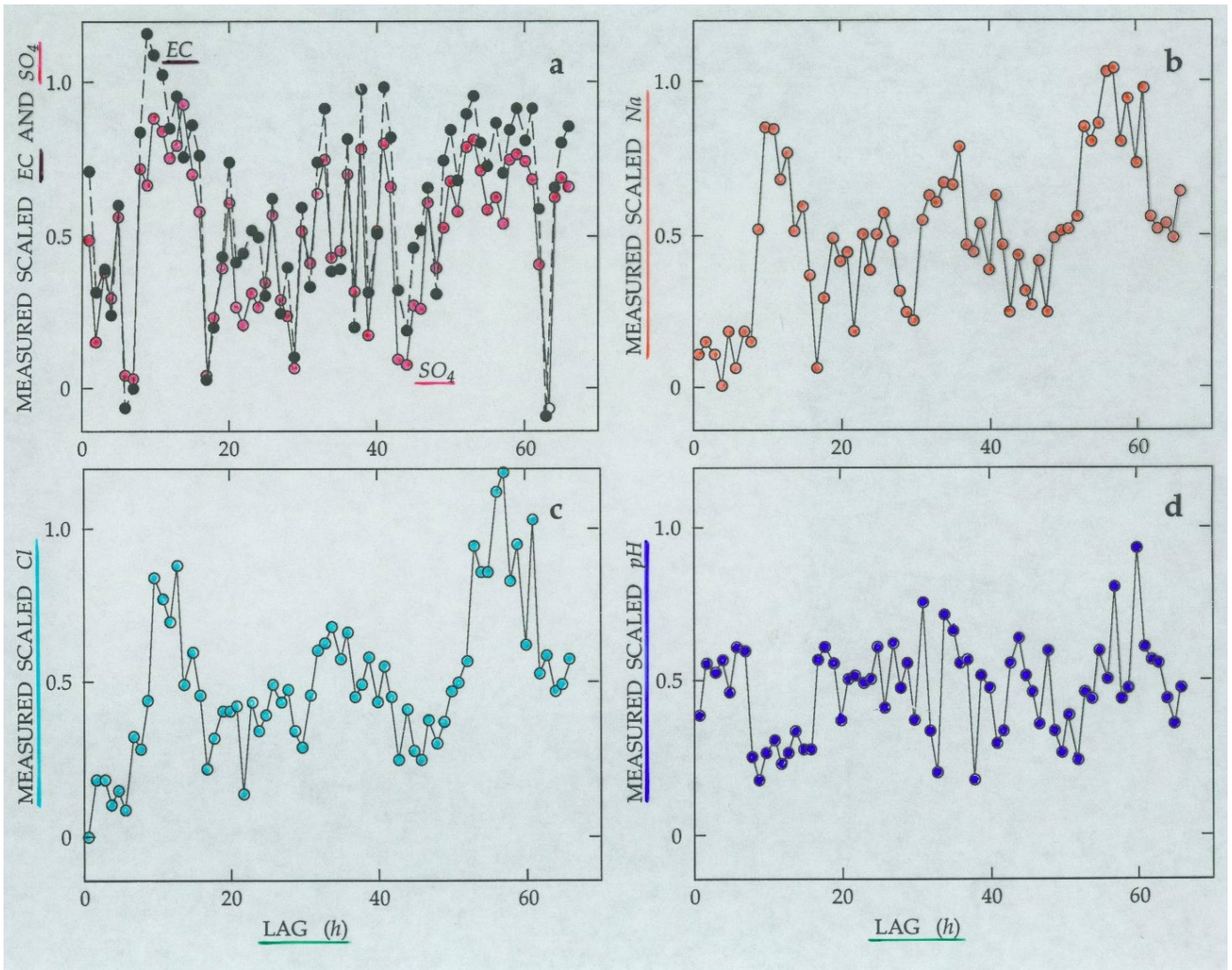
- MEANINGFUL FIELD EXPERIMENTS DO NOT HAVE TO BE CONDUCTED ON UNIFORM AREAS OF A FIELD.
- DIFFERENT TREATMENTS WERE NOT IMPOSED NOR NECESSARY.
- NITROGEN FIXATION, FERTILIZER UTILIZATION AND CROP RESPONSE ARE BETTER UNDERSTOOD BY SAMPLING ACROSS ENTIRE FIELDS TO CONSIDER THE SPATIAL VARIANCE AND CO-VARIANCE STRUCTURES.

# sampling a sea-botton

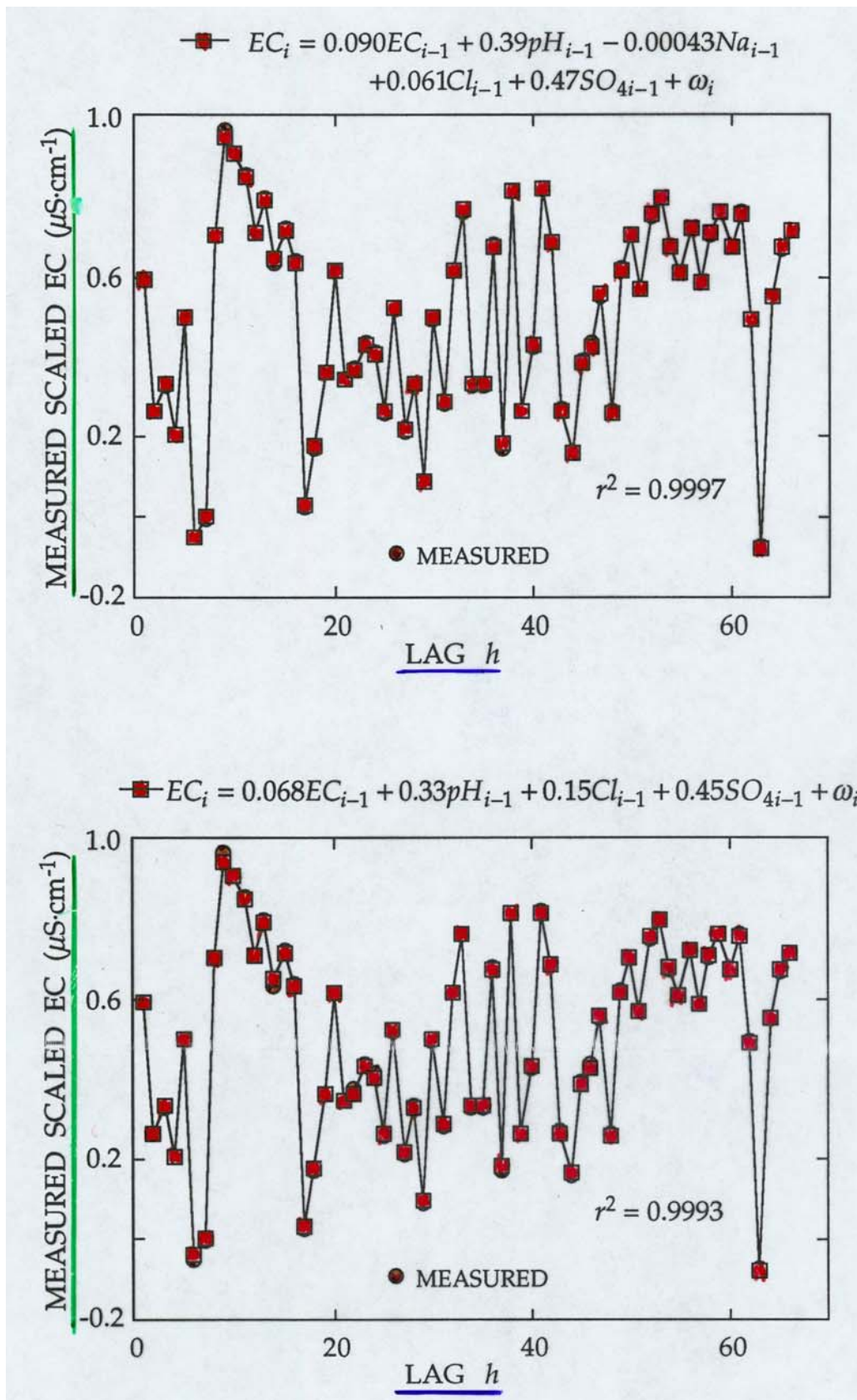
## SAMPLING A SEA-BOTTOM RECLAMATION FIELD



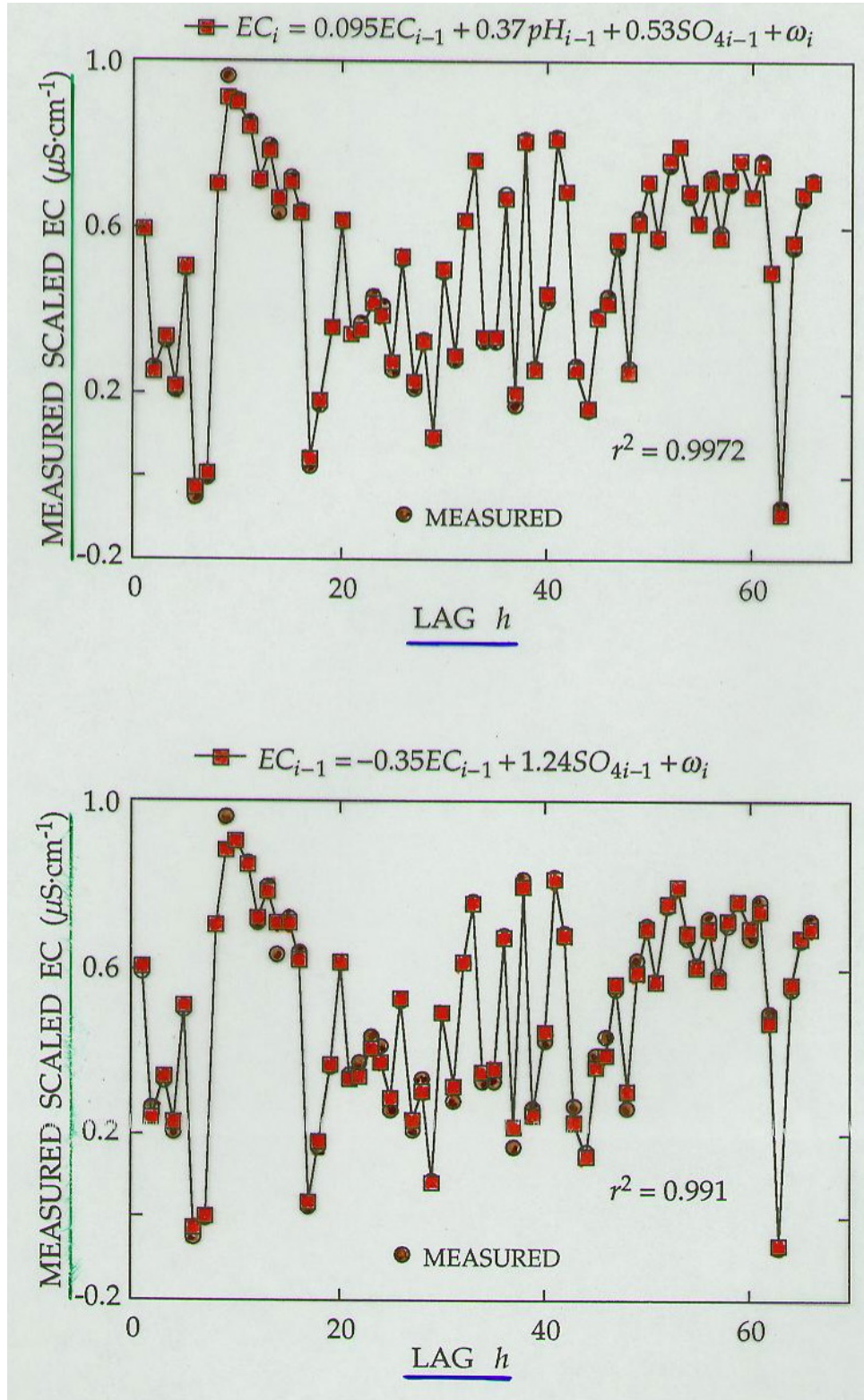
# scaled sea data



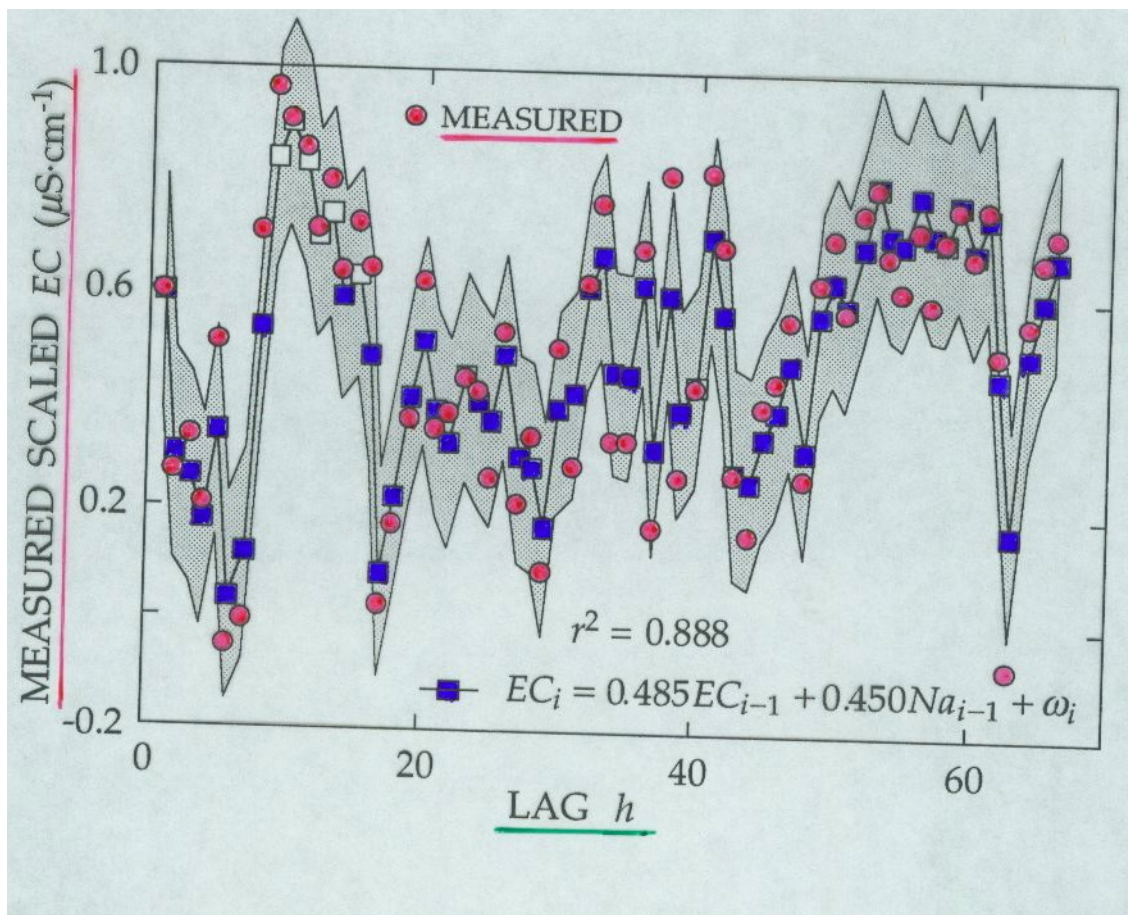
# state-sp EC(all variables)



# state-sp EC less variables



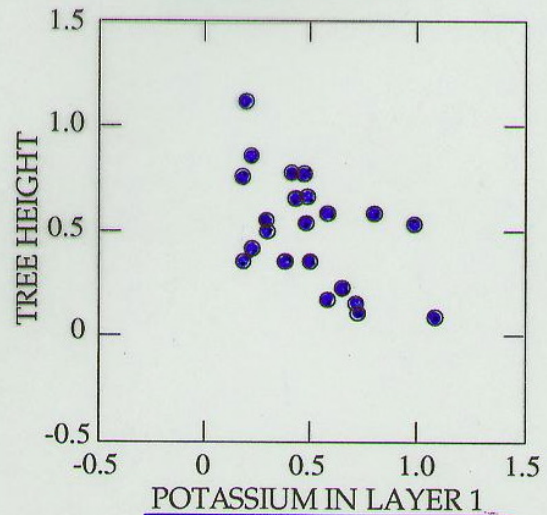
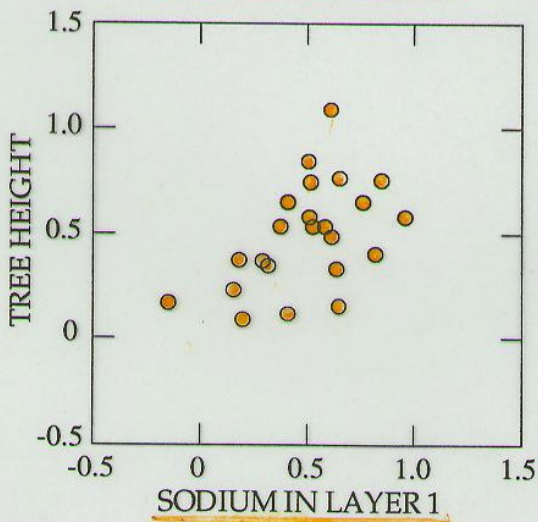
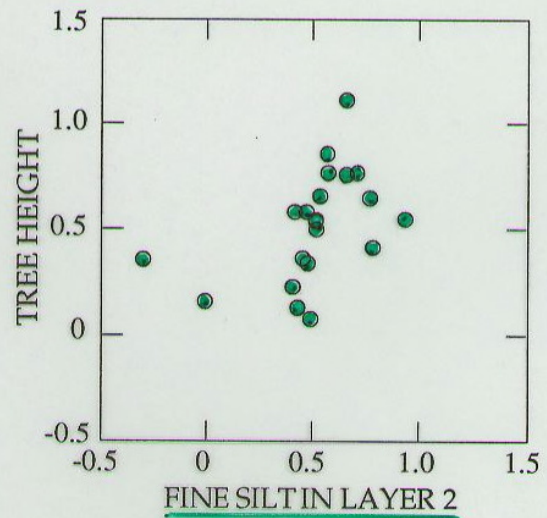
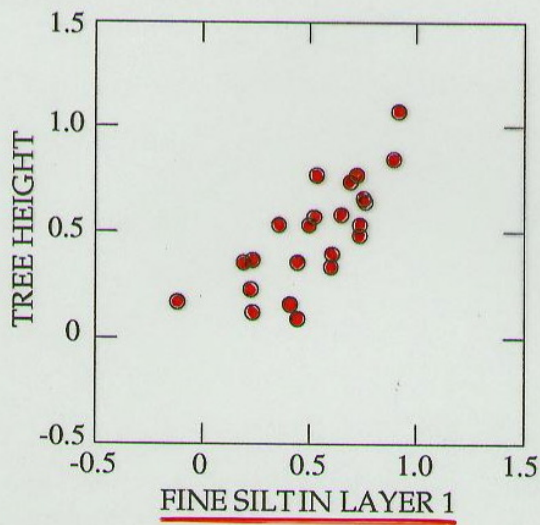
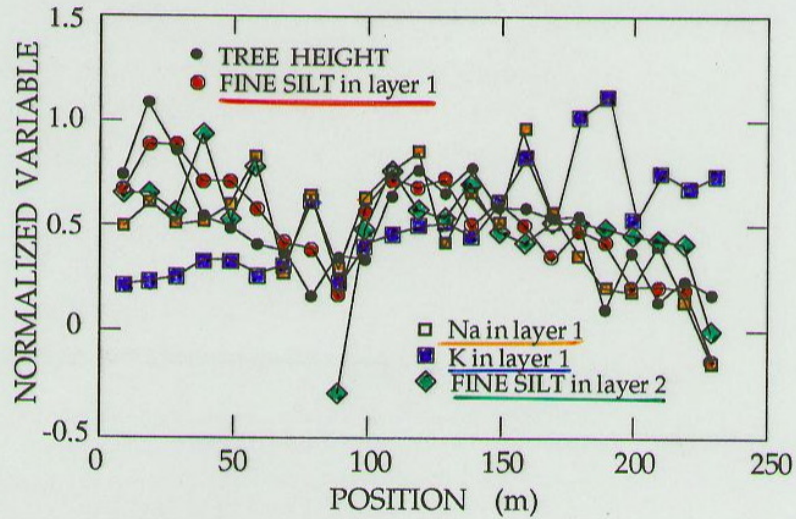
# state-sp EC vs SO4



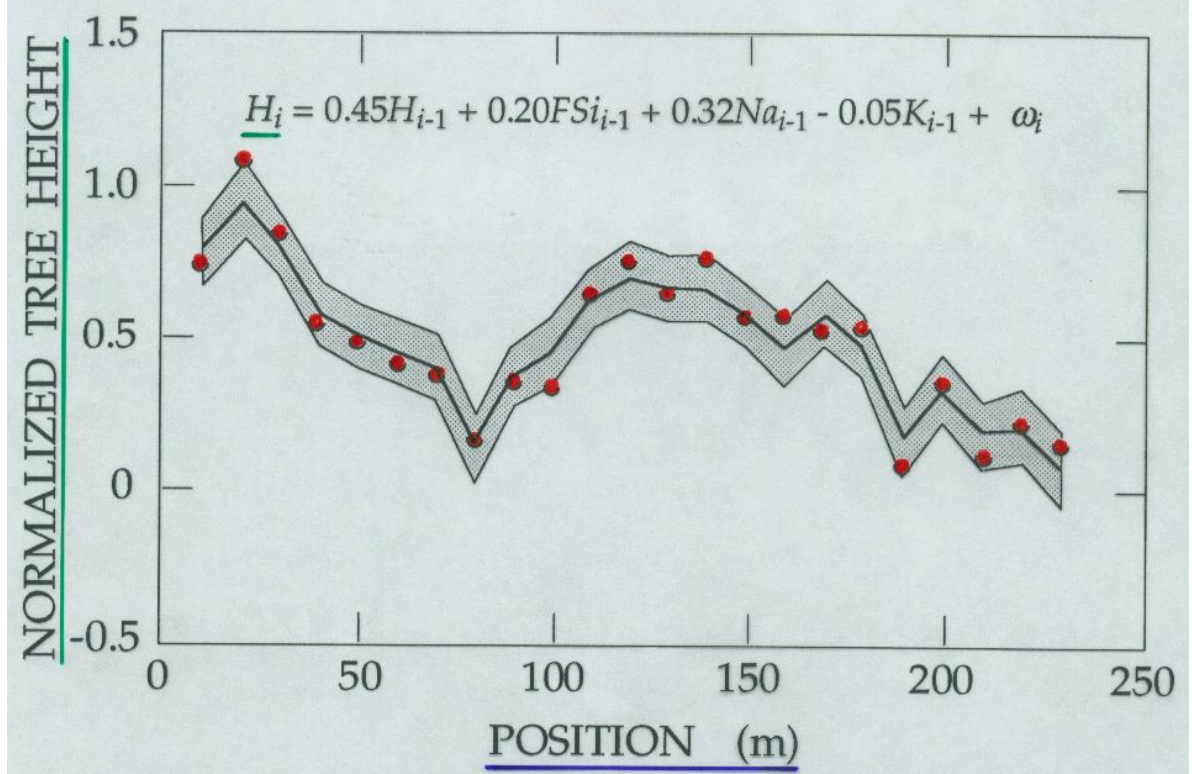
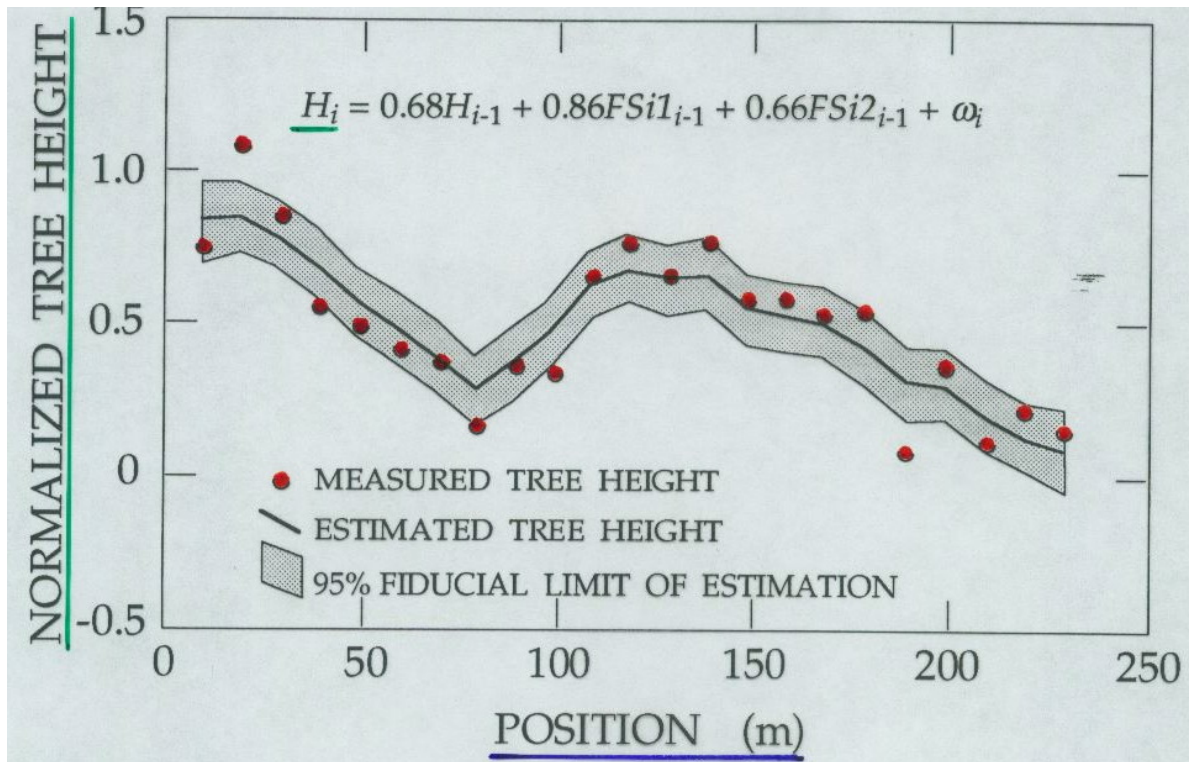


# red oak growth

## OAK TREE GROWTH IN FRANCE

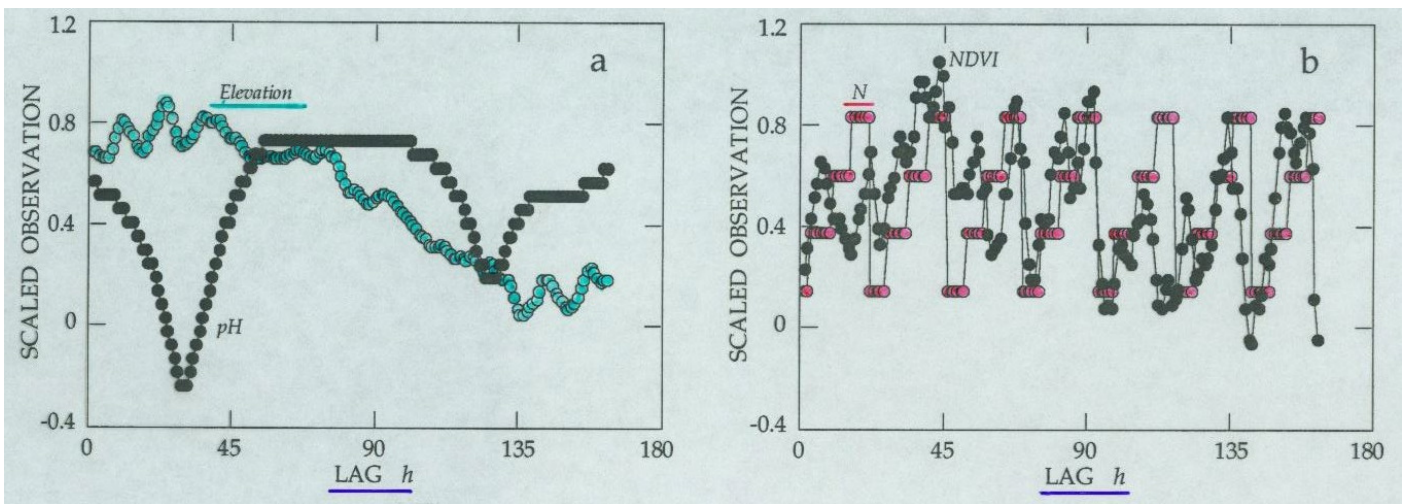


# state-space oak tree

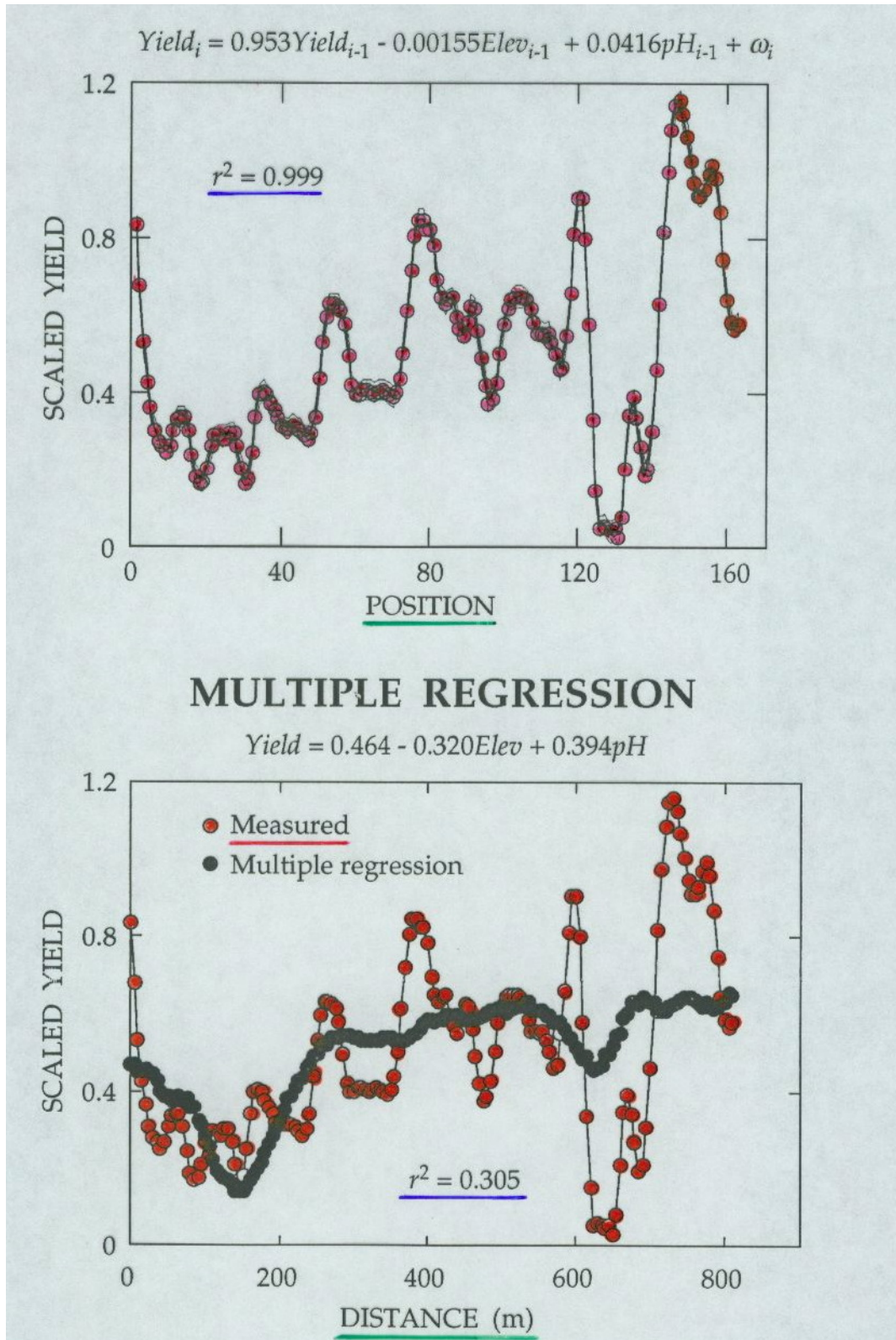


WHEAT YIELD  
AND SOIL PROPERTIES

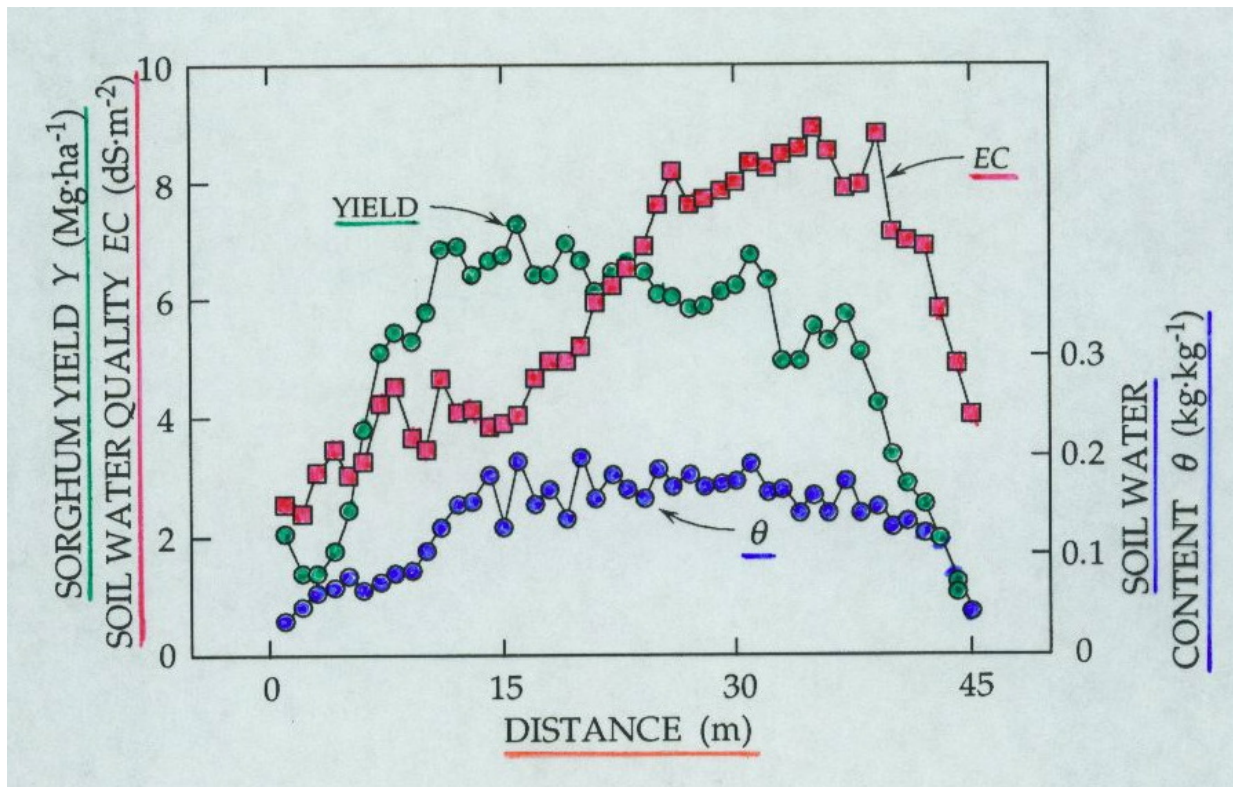
# scaled wheat & soil properties



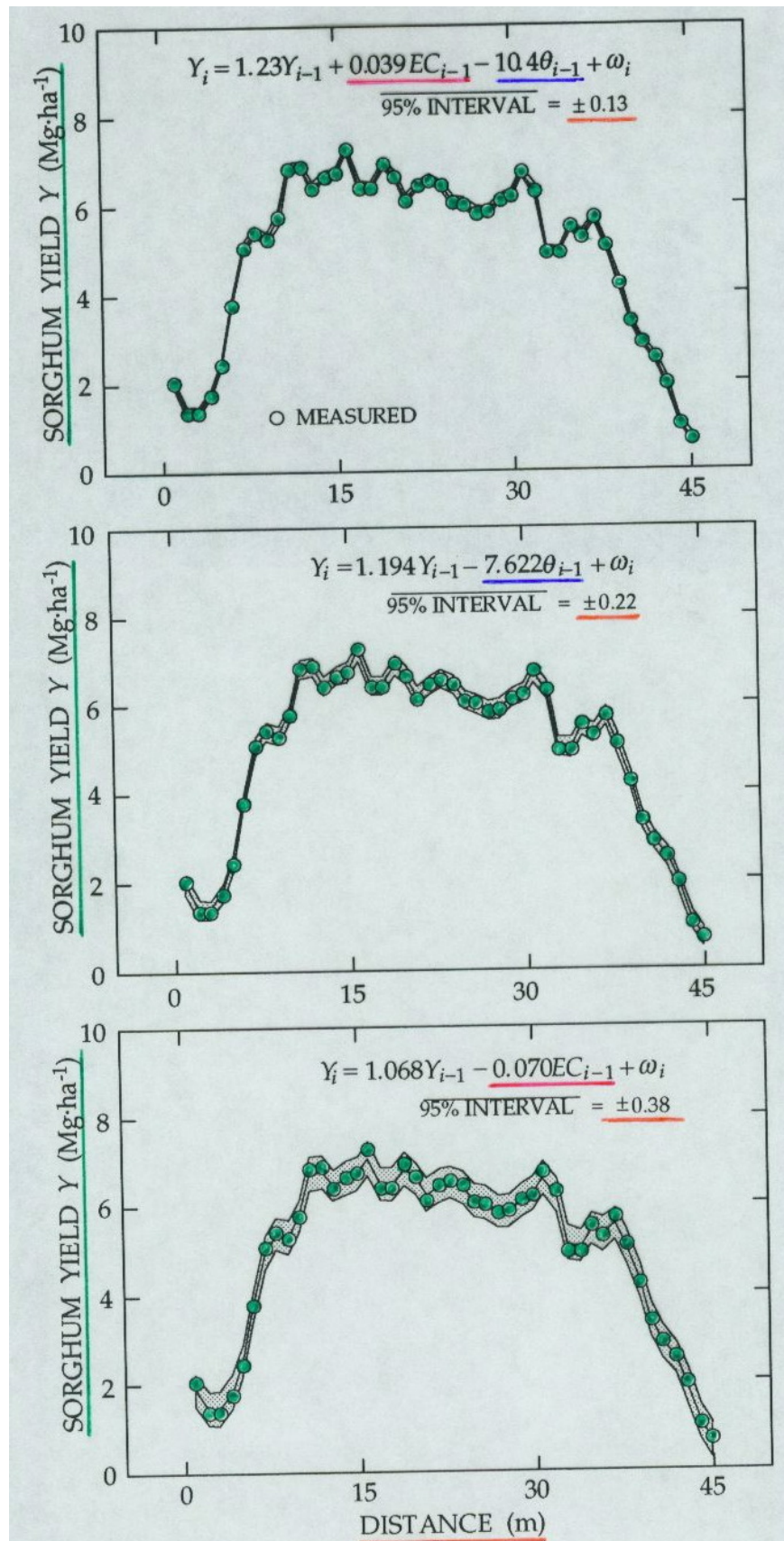
# state-space & MR wheat



# sorghum yield, EC and water



# state-space yield, EC. water



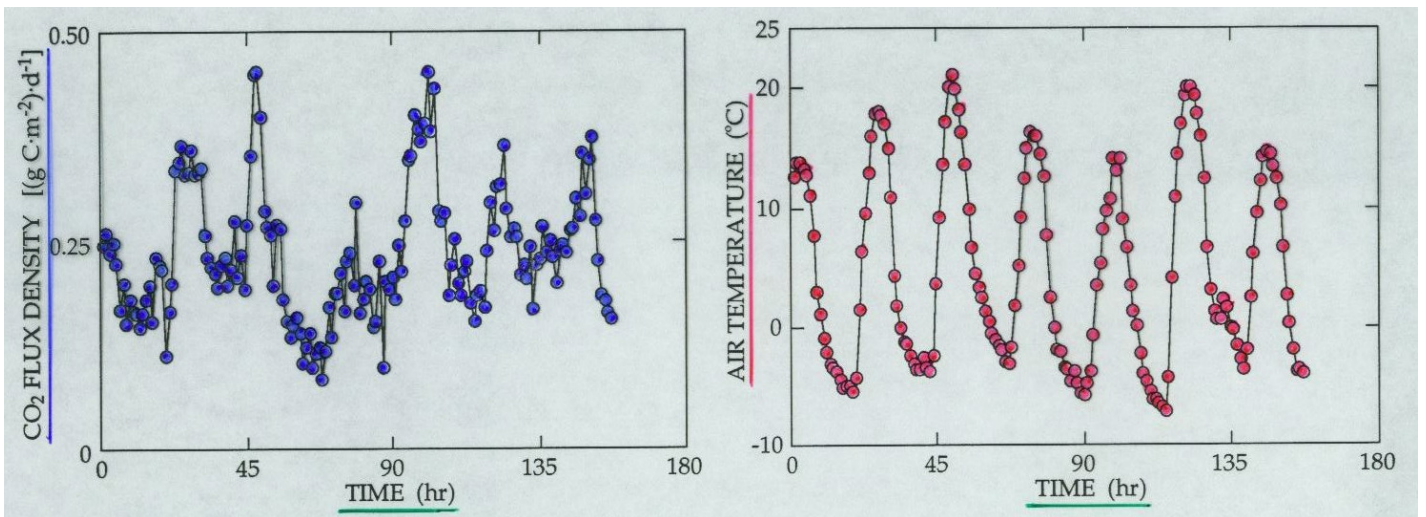
RESPIRATION

AND

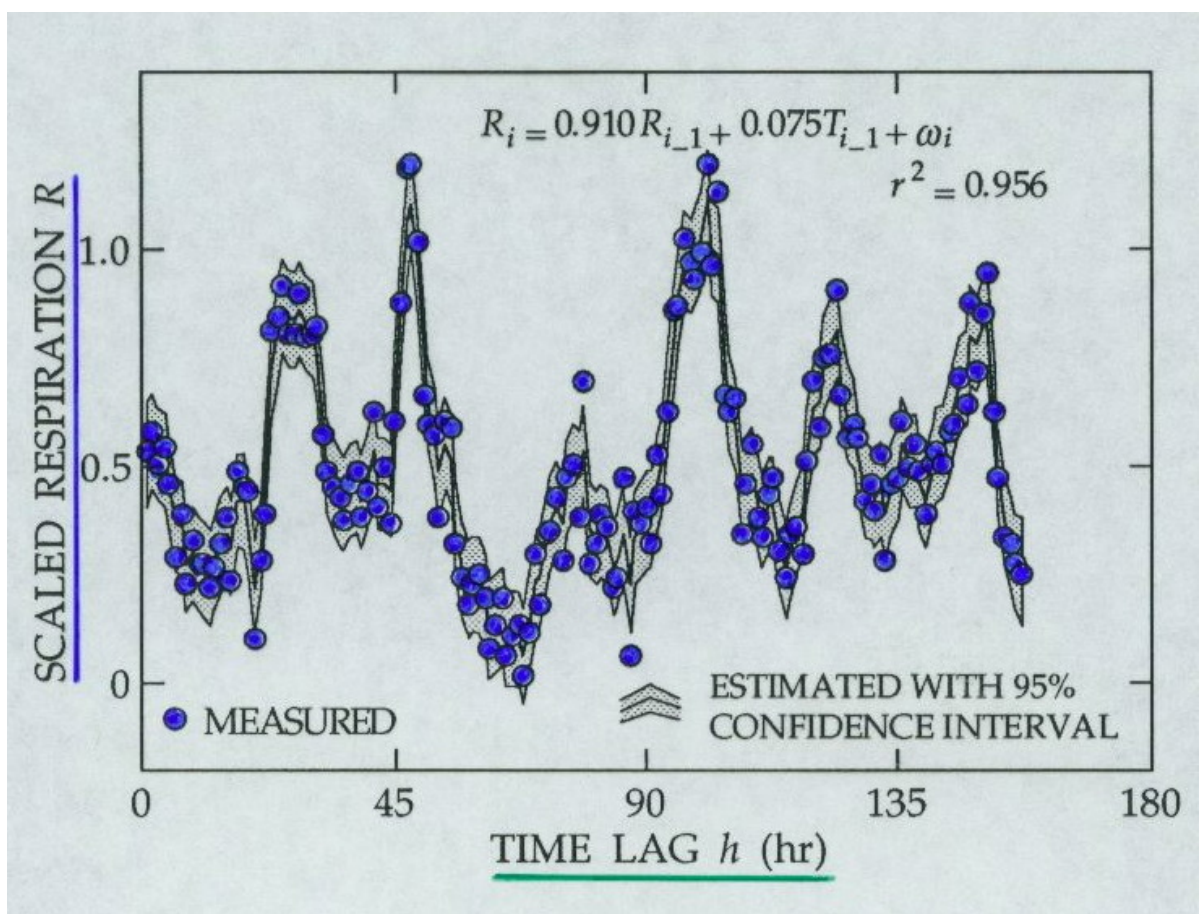
AIR TEMPERATURE



# Respiration and air temp data

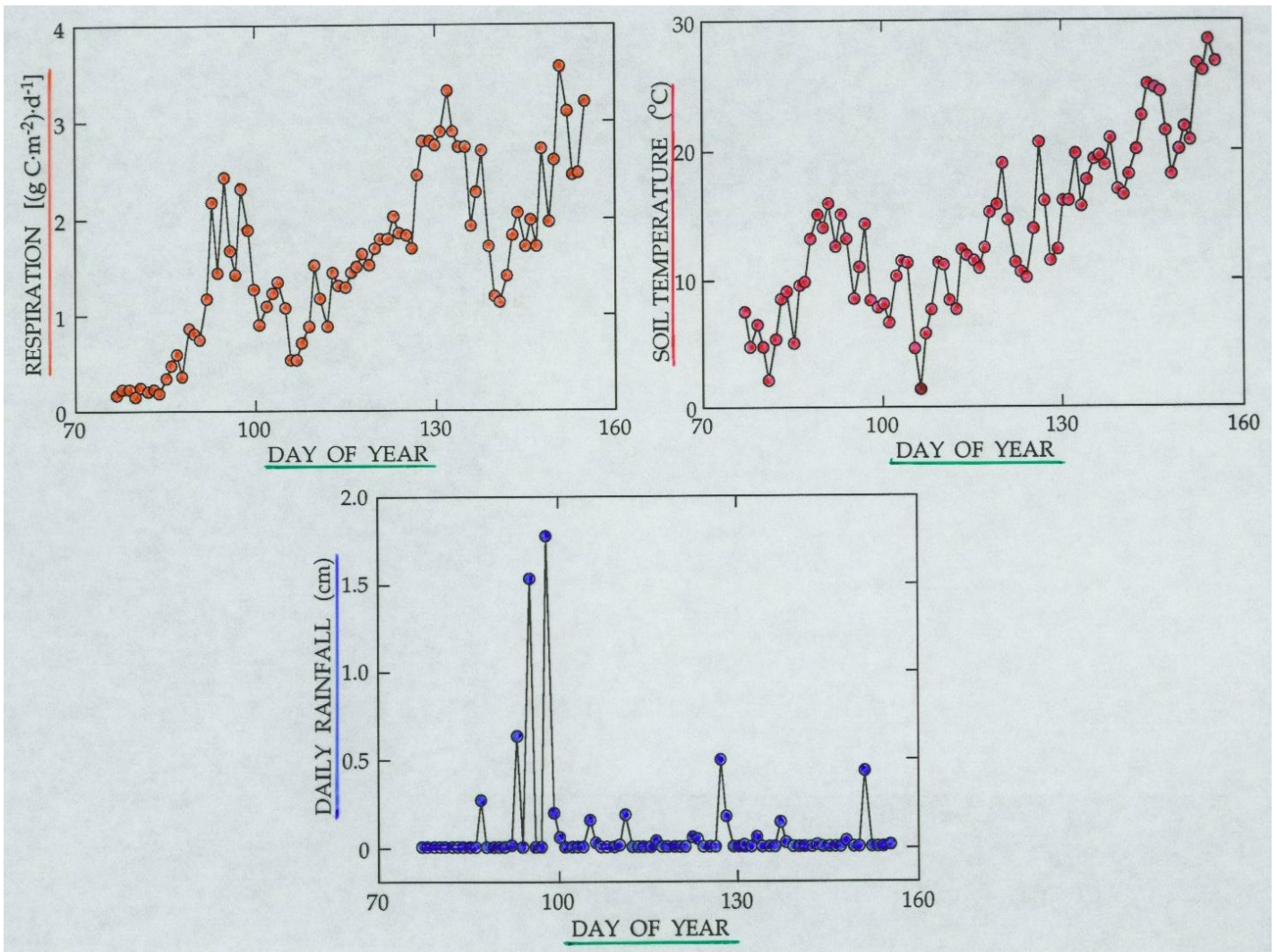


# state-space respir & airT

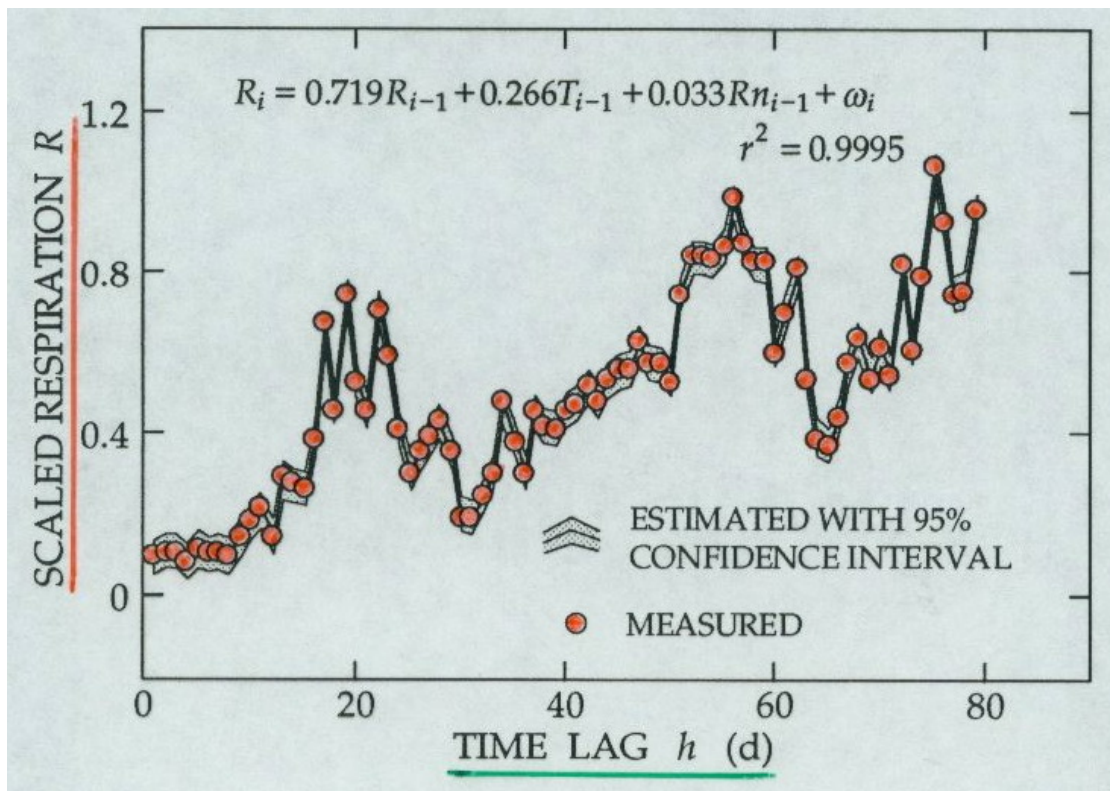


## RESPIRATION, SOIL TEMPERATURE AND RAINFALL

# respiration, temperature & rain data



# state-space respiration, temperature & rain



# Thus Far, next step

## THUS FAR

- STATE VARIABLES SELECTED UPON AGRONOMIC EDUCATION, EXPERIENCE AND INTUITION
- EXPECTED STATE VARIABLES TO BE IMPORTANT AND CORRELATED
- HAD CORRELATIONS NOT BEEN FOUND, DIFFERENT VARIABLES SELECTED BY TRIAL AND ERROR

## NEXT STEPS

- INVOKE PHYSICALLY BASED EQUATIONS INTO THE STATE-SPACE ANALYSIS
- BE GUIDED BY PROVEN EQUATIONS EXPRESSING PROCESSES OF KNOWN CHEMICAL, PHYSICAL AND BIOLOGICAL IMPORTANCE.
- TRANSFORM EQUATIONS DESCRIBING PROCESSES IN THE TOPSOIL OF A FARMER'S FIELD INTO STATE-SPACE FORMULATIONS.
- SIMULTANEOUSLY EXAMINE EQUATIONS AND THEIR PARAMETERS FOR BETTER UNDERSTANDING AND IDENTIFICATION OF LOCAL, FUNCTIONALLY SIMILAR AREAS IN A FARMER'S FIELD

**COMBINATION OF PHYSICAL EQUATION  
AND OBSERVATIONAL EQUATION**

# state-space outline

## A PHYSICALLY DERIVED STATE-SPACE EQUATION SOLVED IN COMBINATION WITH AN OBSERVATION EQUATION

- A THEORETICAL EQUATION WITH AN ERROR TERM EXPLICITLY  
EMBRACING ASSUMPTIONS MADE IN ITS DERIVATION
- AN OBSERVATION EQUATION WITH AN ERROR TERM EXPLICITLY  
EMBRACING INSTRUMENTATION CALIBRATION AND SOIL  
HETEROGENEITY
- AN EXAMINATION OF THE FREQUENCY OF OBSERVATIONS MADE  
IN SPACE AND TIME RELATIVE TO THE VARIANCES OF THE  
ESTIMATED PARAMETERS OF THE THEORETICAL EQUATION



# state-space eqn derivation

Let  $Y(z, t)$  be a state variable (e.g. soil water content, temperature, pH, solute concentration)

$$\frac{\partial Y(z, t)}{\partial t} = - \frac{\partial q_Y(z, t)}{\partial z}$$

$q_Y$  is the mass (or energy) flux density of the state variable and  $z$  and  $t$  are soil depth and time

Integrating from the soil surface to depth  $Z$ ,

$$- \frac{\partial}{\partial t} \left[ \int_0^Z Y(z, t) dz \right] = q_Y(Z, t) - q_Y(0, t)$$

Defining

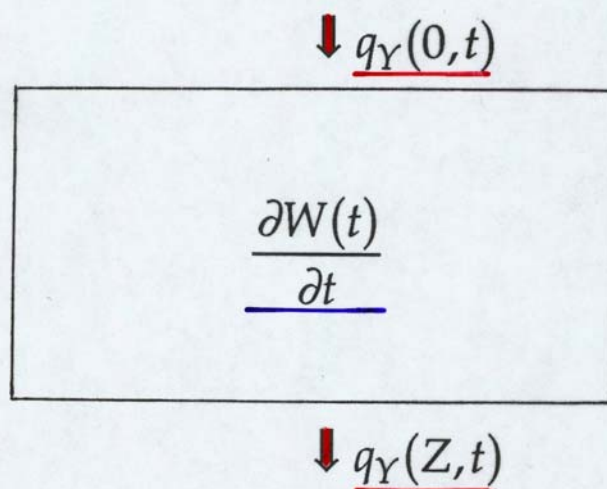
$$W(t) = \int_0^Z Y(z, t) dz$$

we have:

$$\underline{\frac{\partial W(t)}{\partial t}} = \underline{q_Y(0, t) - q_Y(Z, t)}$$

# schematic physical eqn to state-space

Schematically,



- We assume that a relation between the state variable  $Y(z, t)$  at depth  $Z$  and  $W(t)$  is known

Hence,

$$\underline{\frac{dW(t)}{dt}} = f[W(t)] + \underline{\varepsilon(t)}$$

where  $\varepsilon$  is noise signifying the equation is not exact

# state space formulation

## STATE-SPACE FORMULATION

$$\frac{dX(t)}{dt} = f[X(t)] + \varepsilon(t)$$

$$Z_m(t_k) = X(t_k) + v(t_k) \quad k=0,1,2,3,\dots,N$$

where  $Z_m(t_k)$  is the measured value of the state variable  $X$  (that represents  $W$ ) at a discrete time  $t_k$  and  $v$  is the measurement error

# state-space $K(\theta)$ single location

Example -  $K(\theta)$  from observations within a single profile.

Simplifying Richards' equation in terms of stored water  $W$  between the soil surface and depth  $b$  for redistribution of soil water after infiltration with no evaporation at the soil surface

$$\frac{\partial W}{\partial t} = -K(W) \left( \frac{\partial H}{\partial z} \right)_{z=b}$$

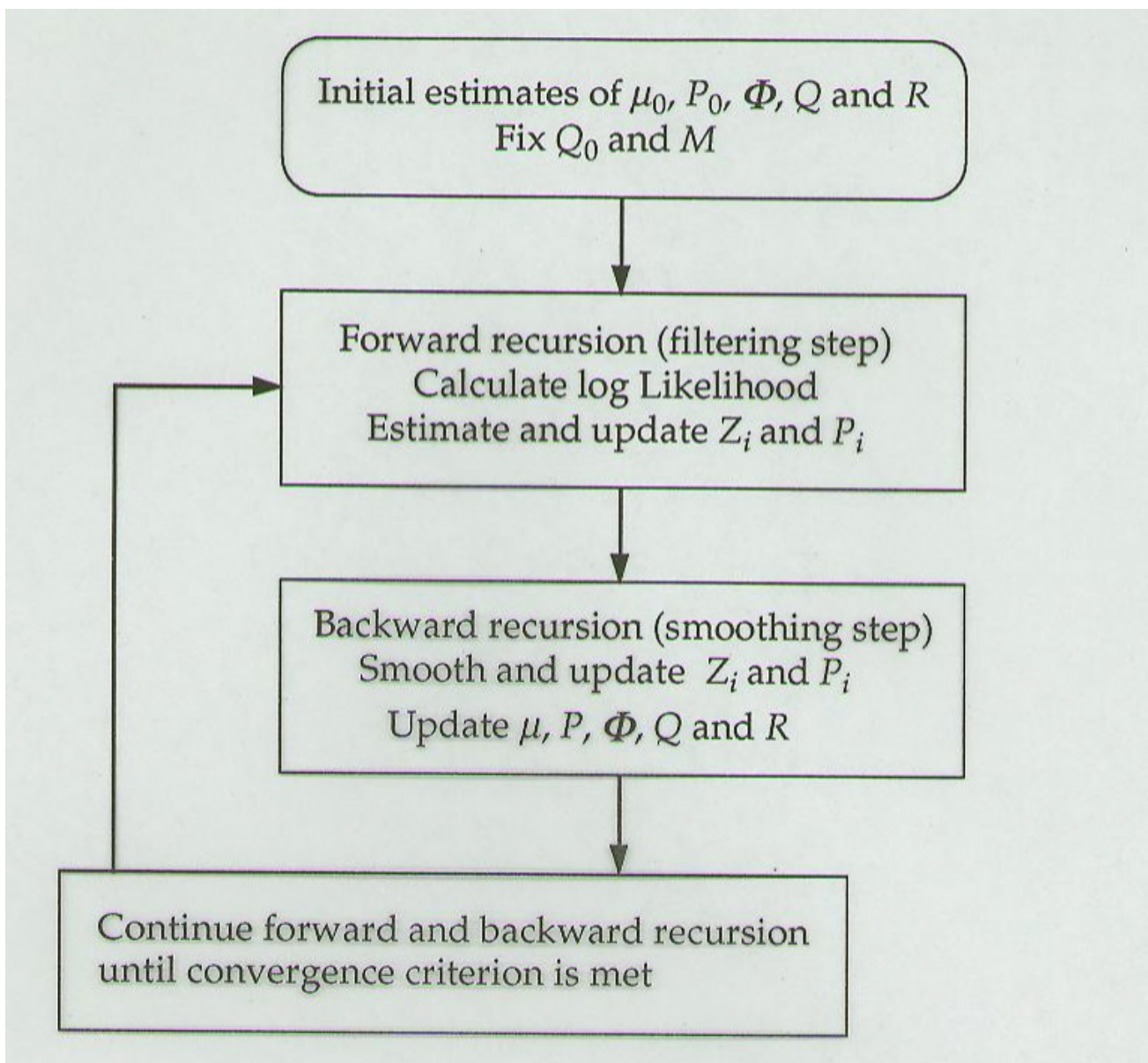
## STATE-SPACE FORMULATION

$$X(t) = -A \exp[B X(t)] \left( \frac{\partial H}{\partial z} \right) + \epsilon_s(t)$$

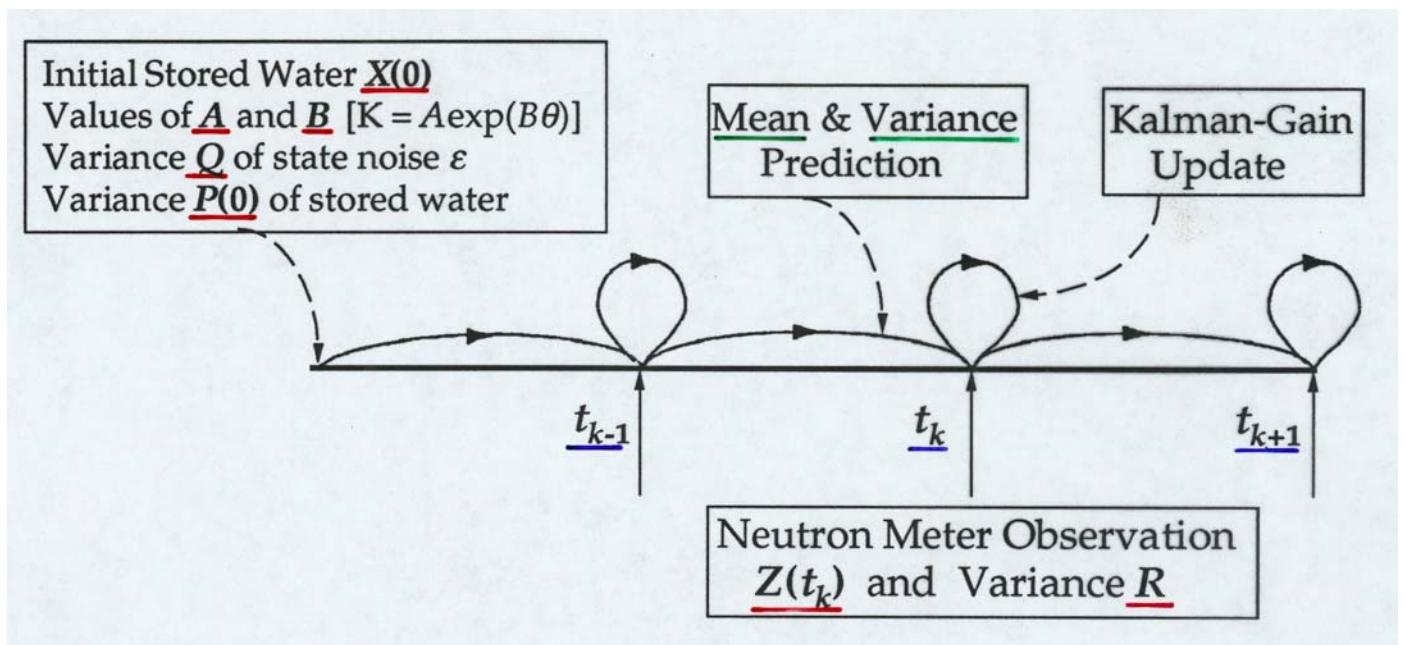
$$Z_m(t_k) = X(t_k) + v_m(t_k)$$

where  $K(W)$  is of the form  $A \exp(B\theta)$

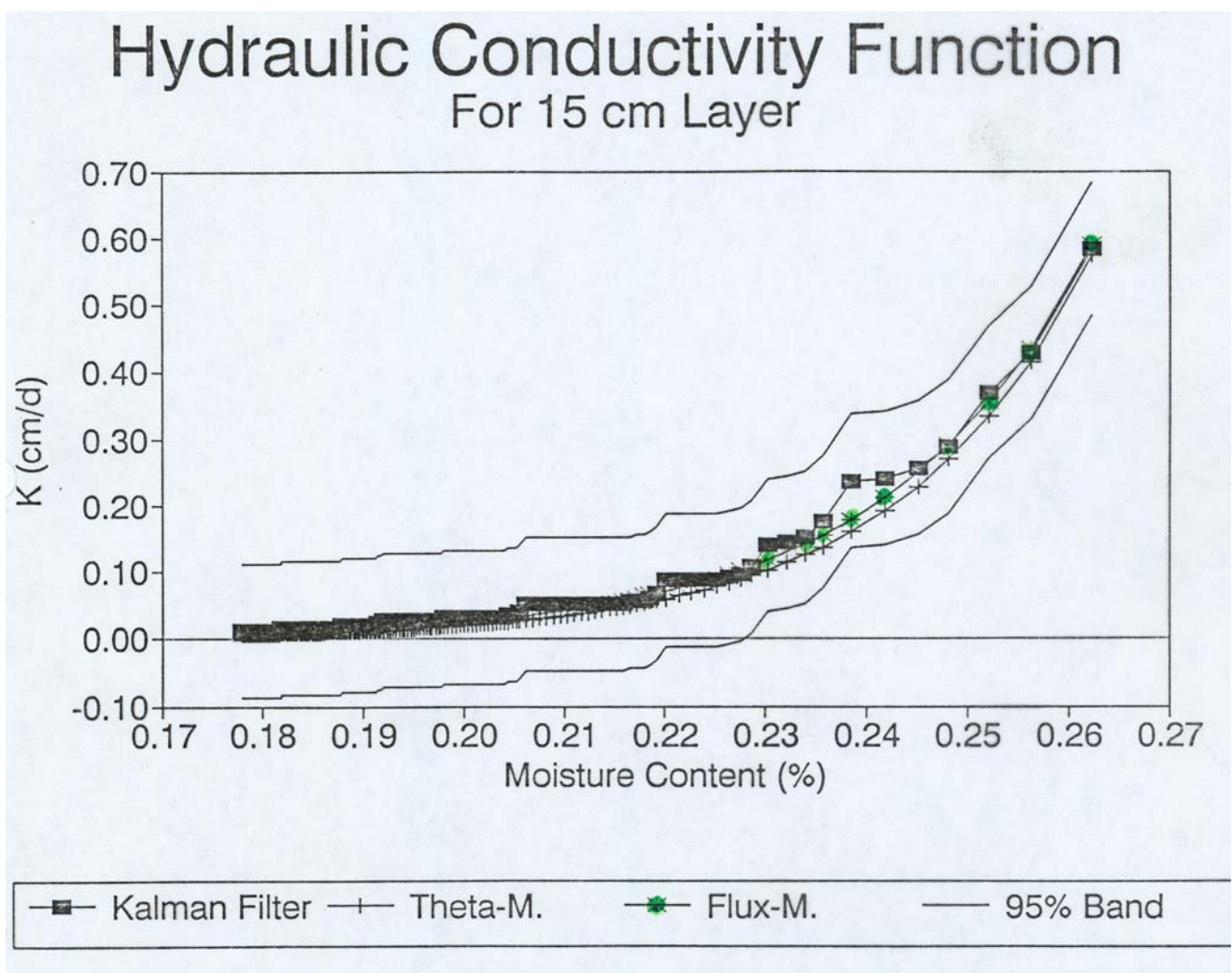
# Kalman filter scheme



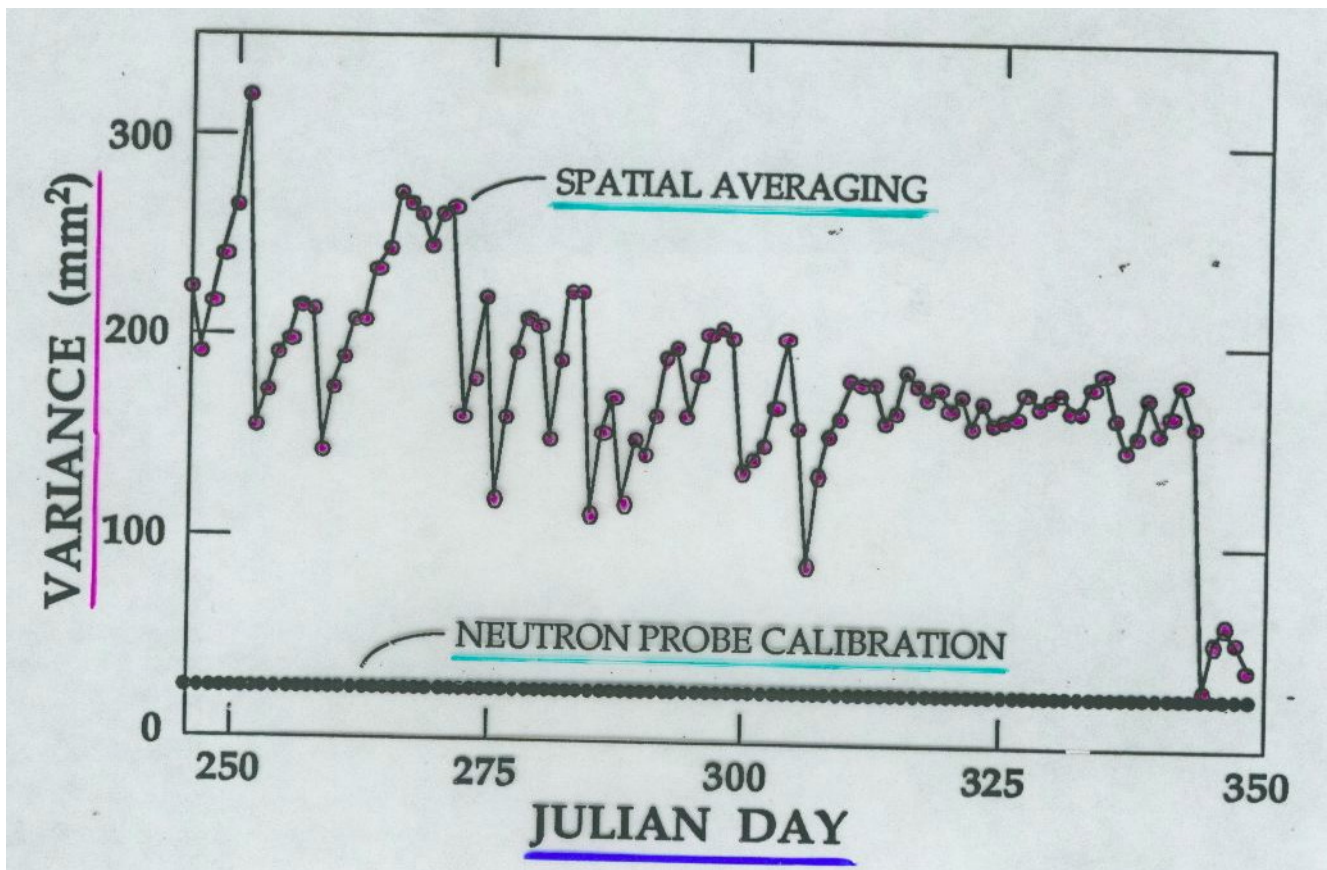
# soil water kalman filter scheme



# K(theta) with fiducial limits



# variances of neutron meter & equation





# irrigating slowly permeable soils

## IRRIGATING SLOWLY PERMEABLE FIELD SOILS

### ASSUME:

- SALT IS NOT TAKEN UP BY THE PLANT OR ABSORBED BY THE SOIL
- LEACHING ONLY OCCURS WHEN THE SOIL IS NEARLY WATER-SATURATED (A FINE-TEXTURED CLAY SOIL WITHOUT STRUCTURE)
- NO SOLUTES ARE IN THE RAIN WATER – ONLY IN THE IRRIGATION WATER

### LET:

$I$  = RATE OF IRRIGATION ( $\text{cm}\cdot\text{yr}^{-1}$ )

$P$  = Rainfall

$E$  = EVAPOTRANSPIRATION

$L$  = LEACHING FLUX DENSITY

AT DEPTH  $z$  BELOW ROOT ZONE, THE WATER BALANCE IS:

$$I + P - E = L$$

# Long term salt

Example - Long-term solute dynamics in an irrigated soil (Rose et al., 1979).

Rate of change of salinity averaged over a soil profile of depth  $b$  is

$$b\bar{\theta}_s \frac{d\bar{C}}{dt} = q_0 C_0 - q_a C_a$$

where  $\bar{\theta}_s$  and  $\bar{C}$  are the saturated soil water content and soil solute concentration,  $q_0$  the average water flux density at the soil surface caused by irrigation,  $C_0$  the average solute concentration of the irrigation water,  $q_a$  the soil water flux density at the bottom of the profile and  $C_a$  the soil solute concentration at depth  $b$ .

Assume that values of  $b$ ,  $\bar{\theta}_s$ ,  $q_0$  and  $C_0$  are deterministically known and reliably measured.

Values of  $q_a C_a$  and  $\bar{C}$  are not reliably known.

# state-space formulation slowly permeable

- Assuming  $q_a C_a$  is some function of  $\bar{C}$ ,

$$b\bar{\theta}_s \frac{d\bar{C}}{dt} = q_0 C_0 - A[1 - \exp(B\bar{C})]$$

- We seek values of  $\bar{C}$  from our field measurements

## STATE-SPACE FORMULATION

$$X_t = \frac{q_0 C_0 - A[1 - \exp(BX_t)]}{b\bar{\theta}_s} + v_s$$

Solving for A and B, the rate at which solute leaves the profile is known

# state-space thermal diffusivity

Example - The thermal diffusivity  $\lambda$  from observations within a soil profile.

Starting with

$$\frac{\partial(cT)}{\partial t} = \frac{\partial}{\partial z} \left[ \lambda \frac{\partial T}{\partial z} \right]$$

$$\int_0^b [\partial(cT) / \partial t] dz = - \left( \lambda \frac{\partial T}{\partial z} \right) \Big|_{z=0}$$

$$b\bar{c} \frac{d\bar{T}}{dt} = A \exp(B\bar{T})$$

## STATE-SPACE FORMULATION

$$X_t = \frac{A \exp(BX_t)}{b\bar{c}} + v_s$$

$$Z_m(t_k) = X(t_k) + v(t_k) \quad k=0,1,2,3,\dots,N$$

Here, the stochastic state variable  $X_t$  represents the average soil temperature  $\bar{T}$ , and  $Z_m$  is the average soil temperature from thermocouple measurements

# state-space $D(\theta)$ several locations

## Example - $D(\theta)$ from observations within several profiles.

Water stored  $S(t)$  within a soil profile from depths 0 to  $b$  was measured daily with a neutron meter during evaporation from a bare soil during a period 100 days at 5 locations along a 75-m transect. Fifteen small irrigations (each  $< 20$  mm) were applied between 4 Sept. to 12 Dec.

With the gravitational force neglected

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left[ D(\theta) \frac{\partial \theta}{\partial z} \right]$$

$$E_t = \frac{\pi^2 S D(S/b)}{4b^2} \quad \text{for } \frac{Dt}{b^2} > 0.3$$

$$\frac{dS}{dt} = P_t - \left( \frac{\pi}{2b} \right)^2 S D(S)$$

# state-space formulation of diffusivity

## STATE-SPACE FORMULATION

For an exponential diffusivity function  
 $D(S / b) = A \exp(BS / b),$

$$X_t = P_t - \left( \frac{\pi}{2b} \right)^2 X_t A \exp(BX_t / b) + v_S$$

$$Z_m(t_k) = X_{t_k} + v_m(t_k)$$

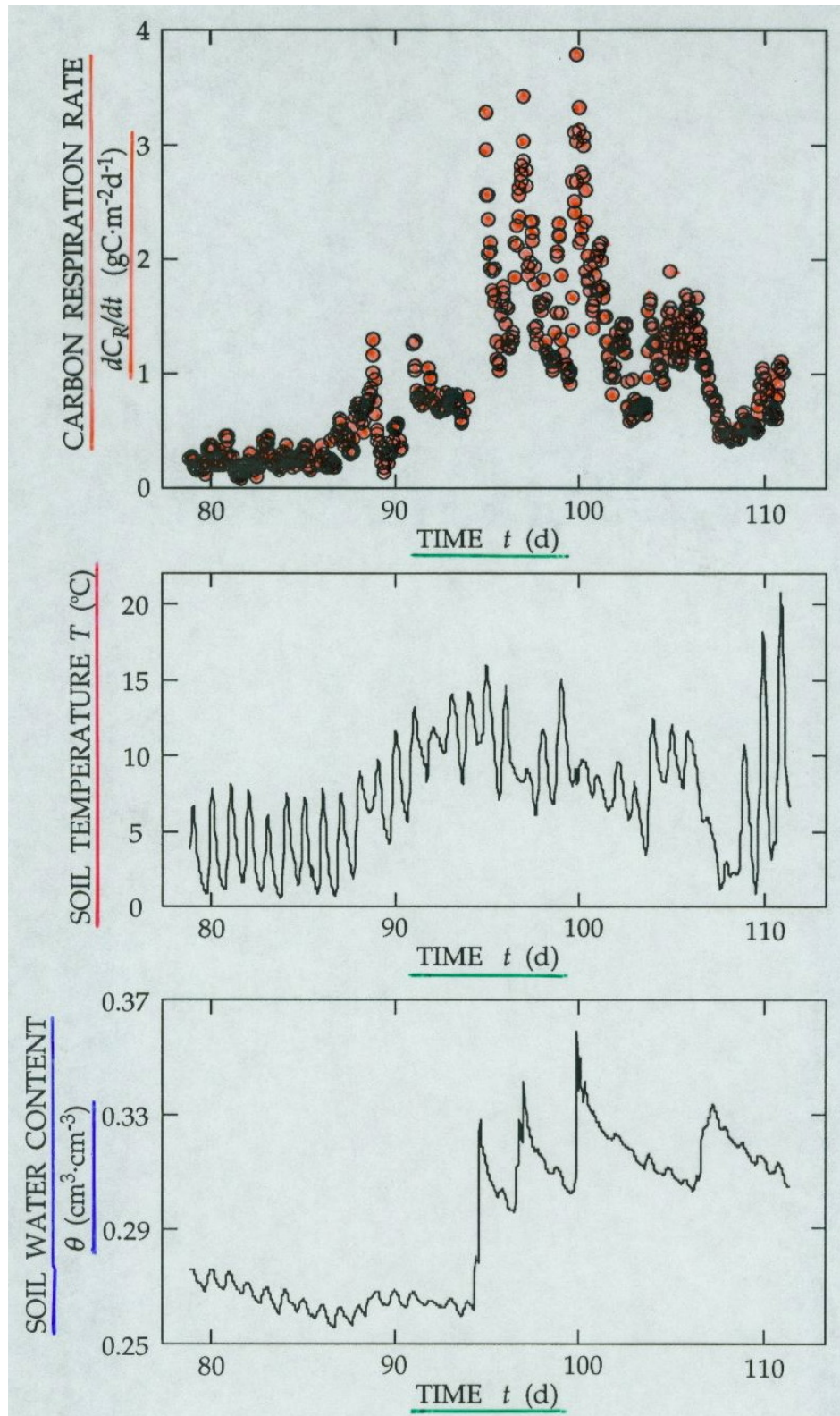
RESPIRATION

SOIL TEMPERATURE

AND

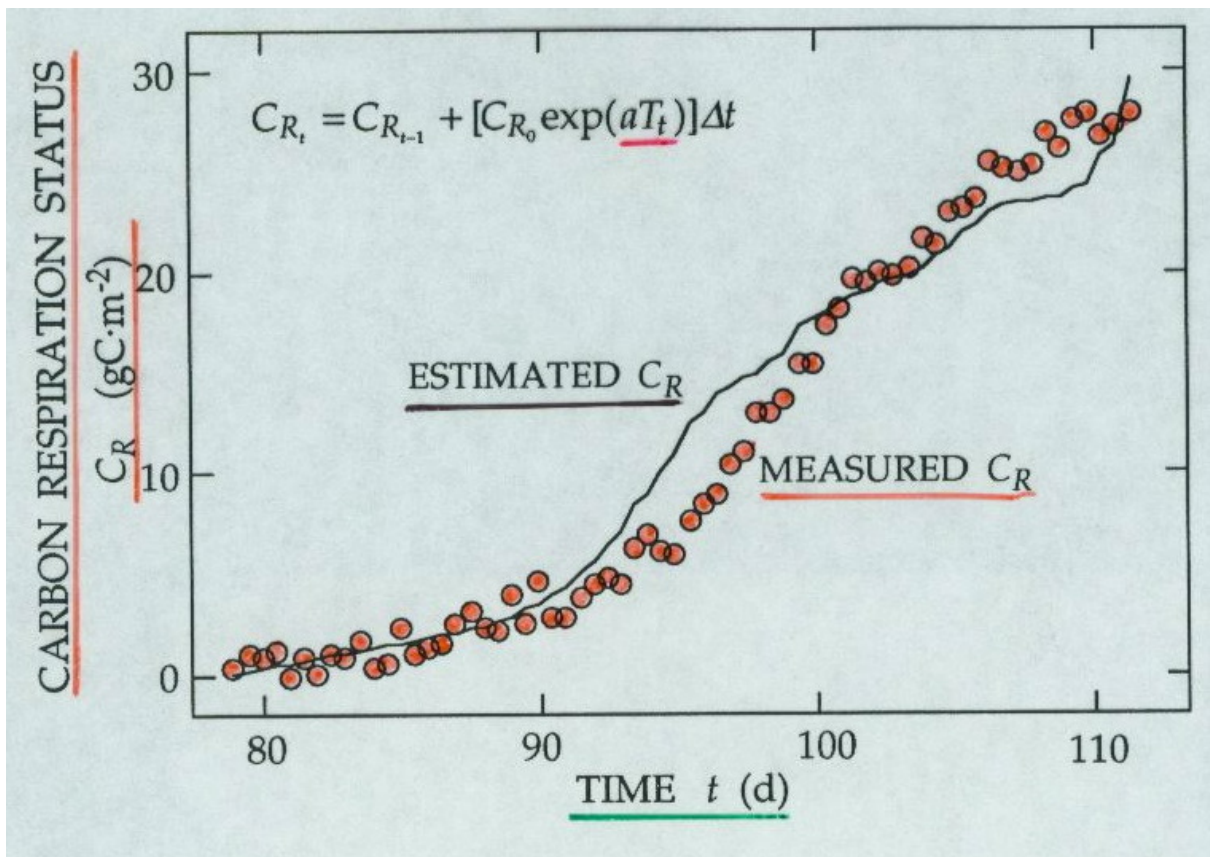
SOIL WATER CONTENT

# respiration, Temp & theta data

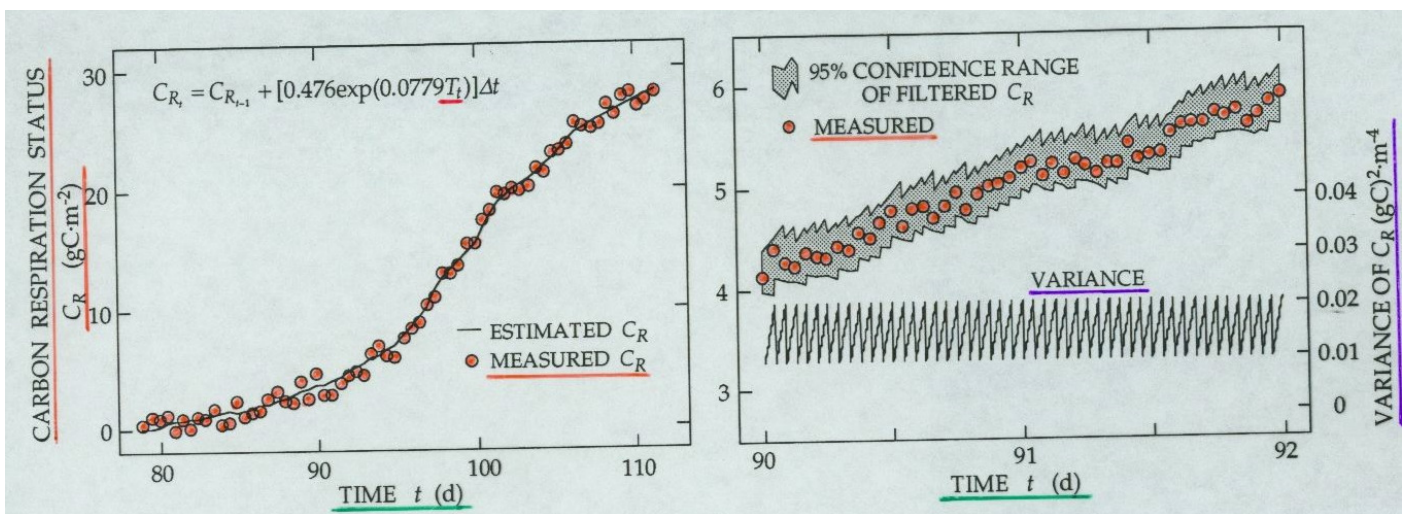




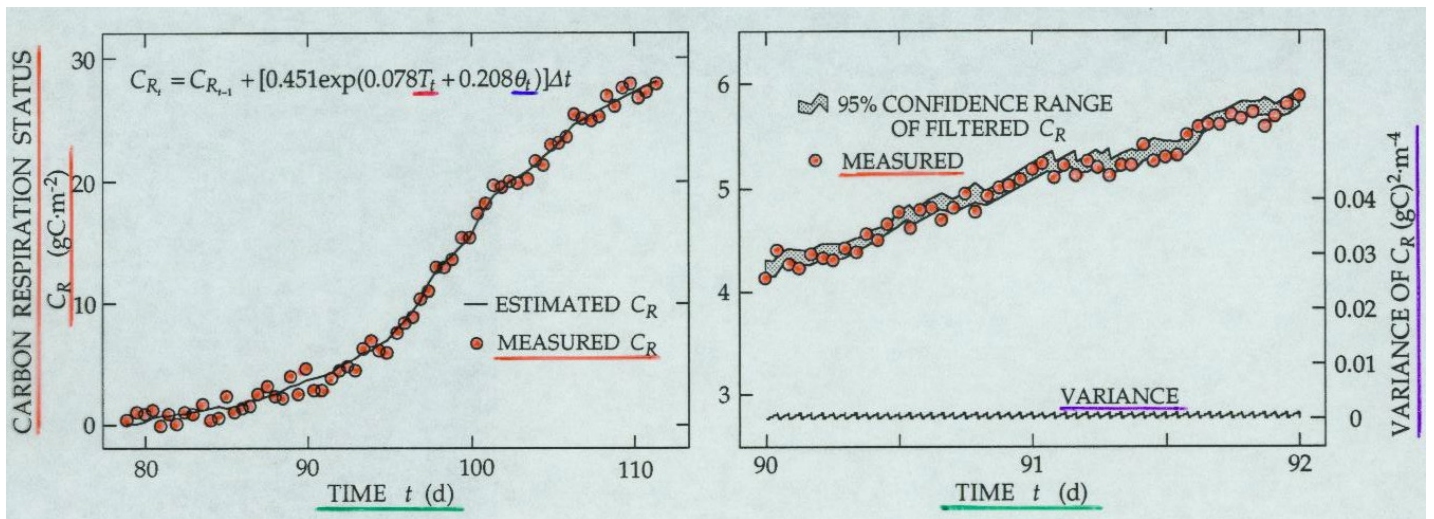
# respiration versus Temp



# state space respiration versus Temp



# state space respir vs T, q

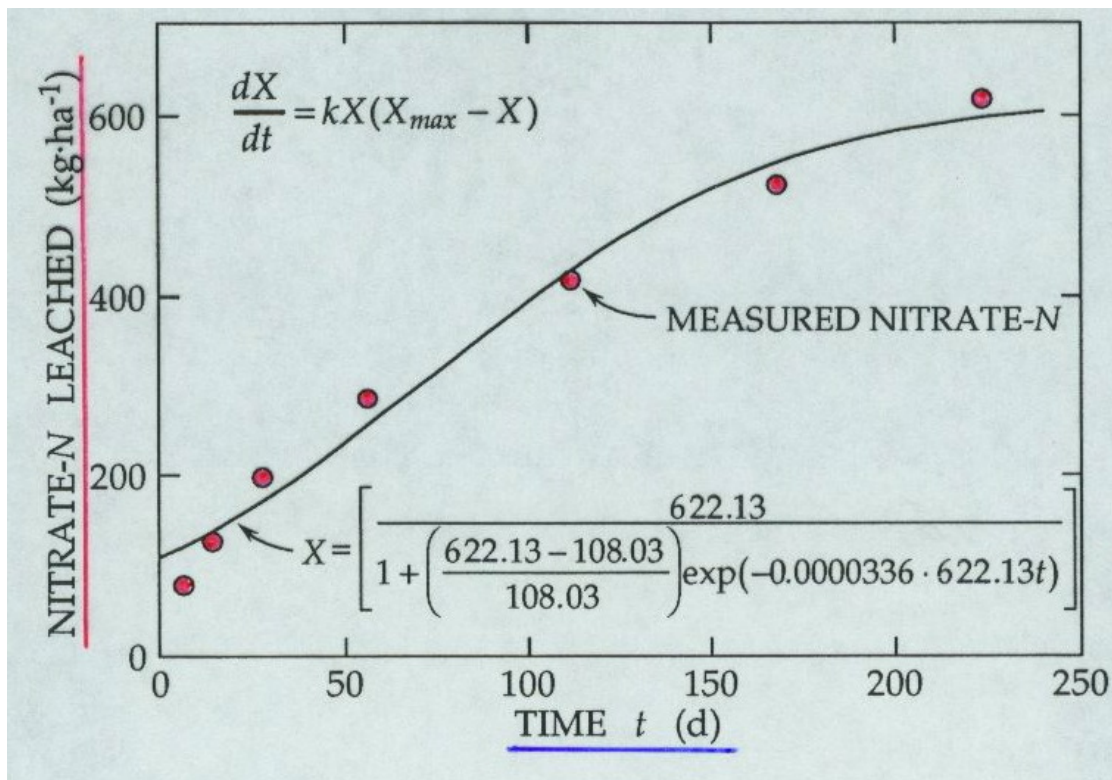


TIME SERIES

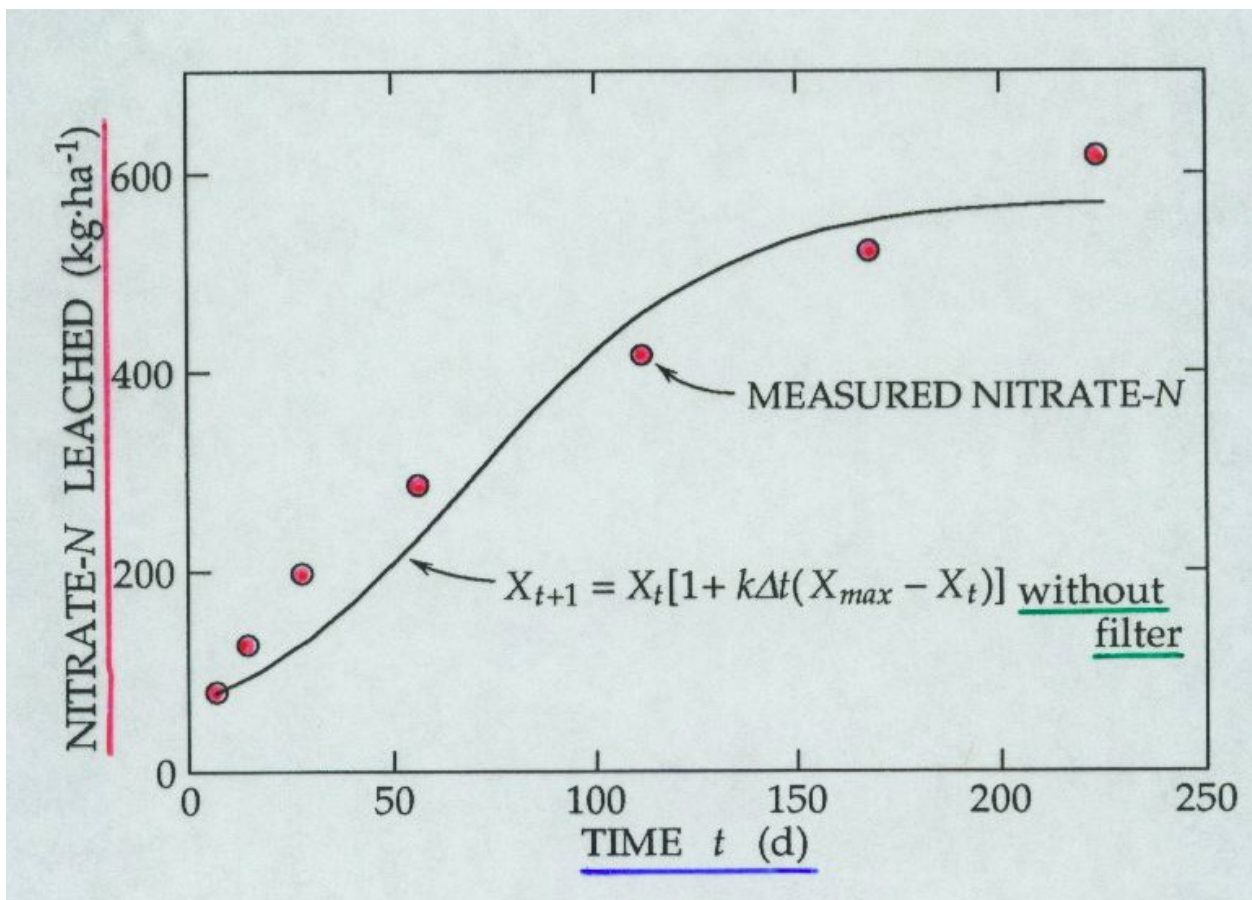
OF

SOIL NITROGEN MINERALIZATION

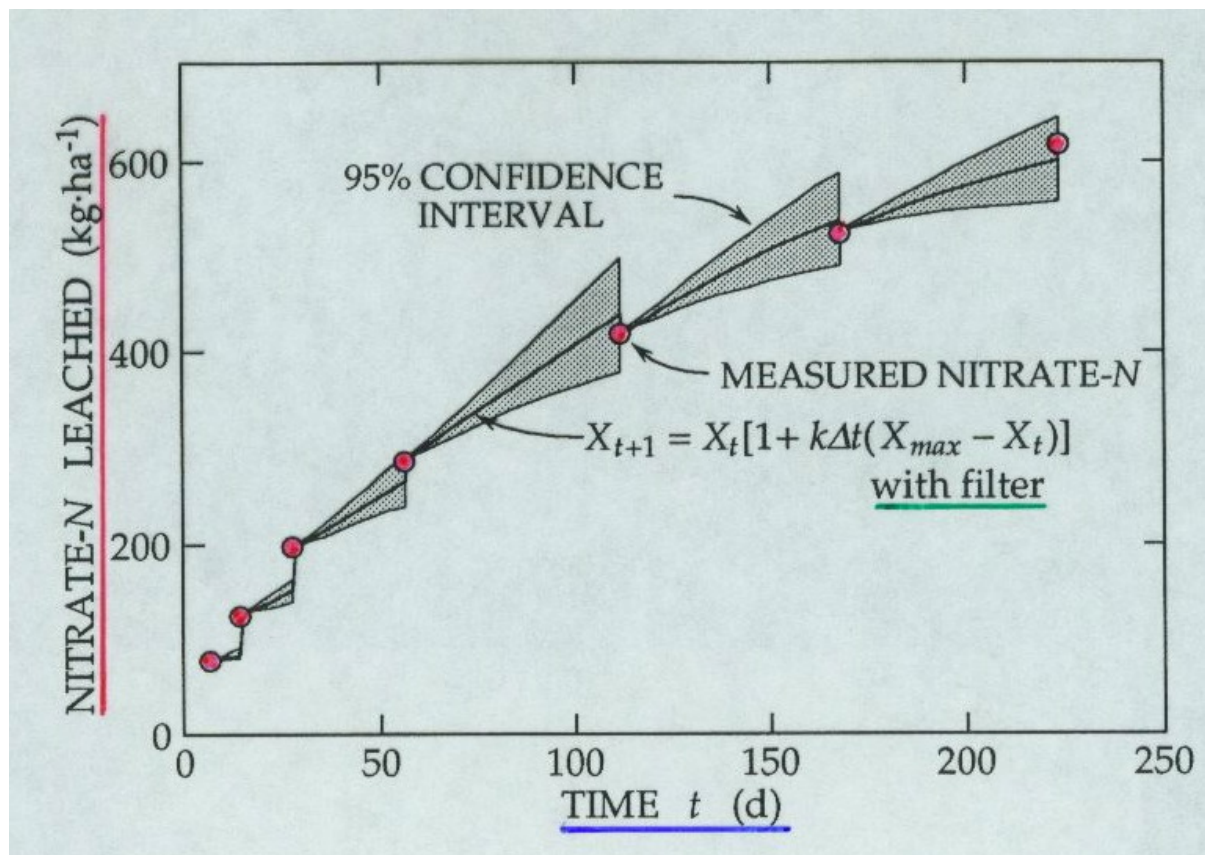
# N leaching analytic equation



# N leaching no filter



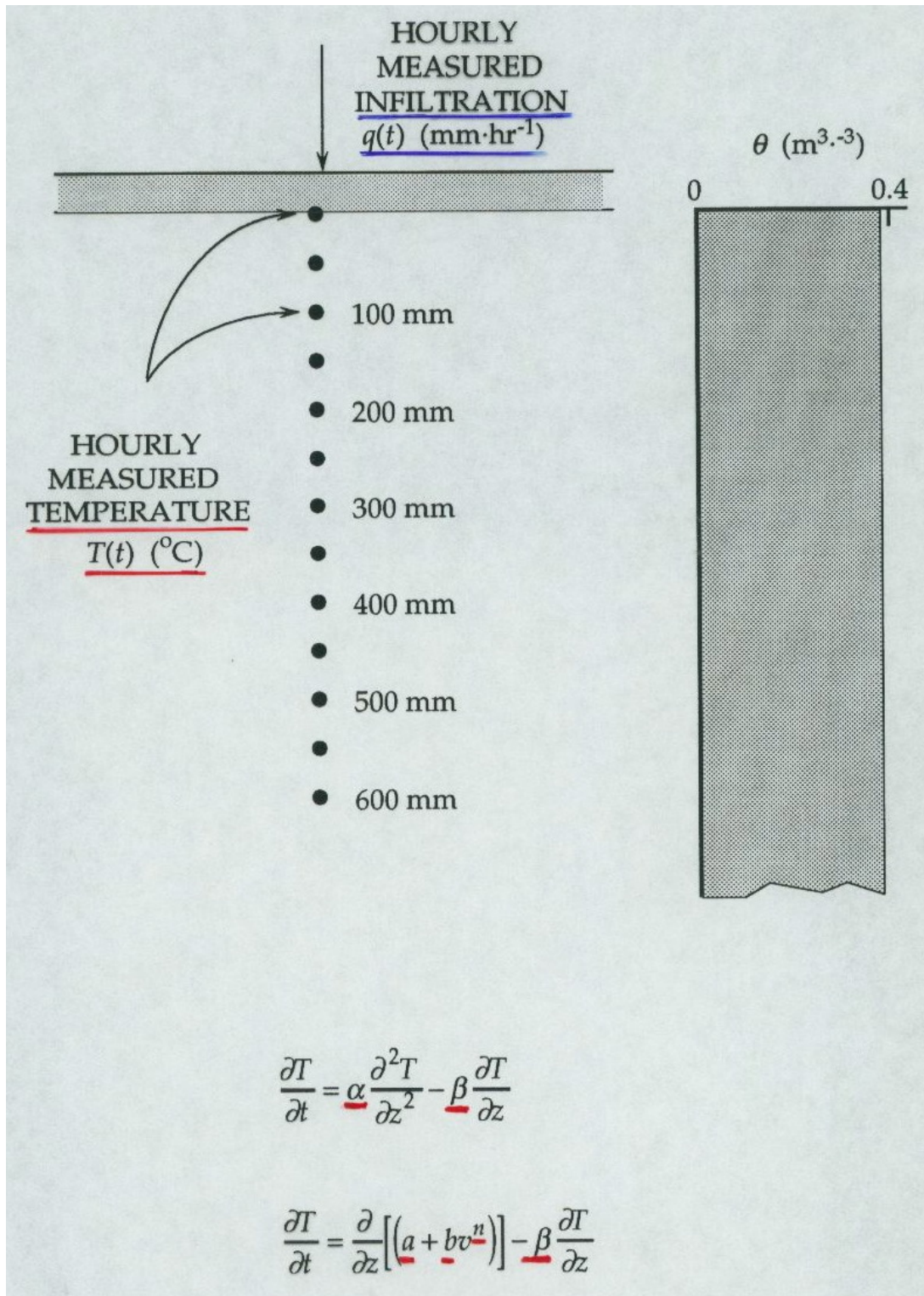
# N leaching with filter



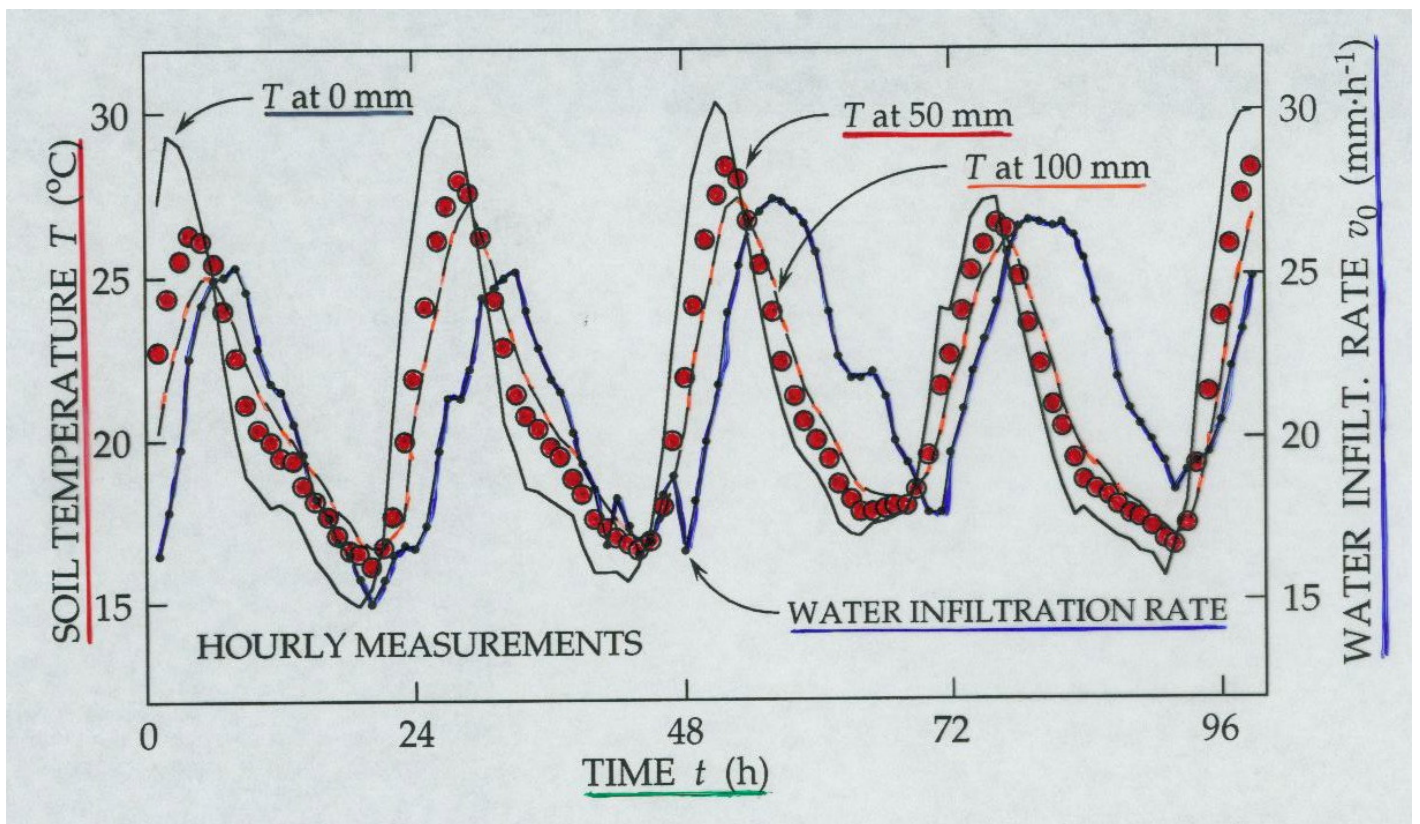
WATER WITH  
DAILY TEMPERATURE FLUCTUATIONS  
INFILTRATING  
THE  
SOIL SURFACE



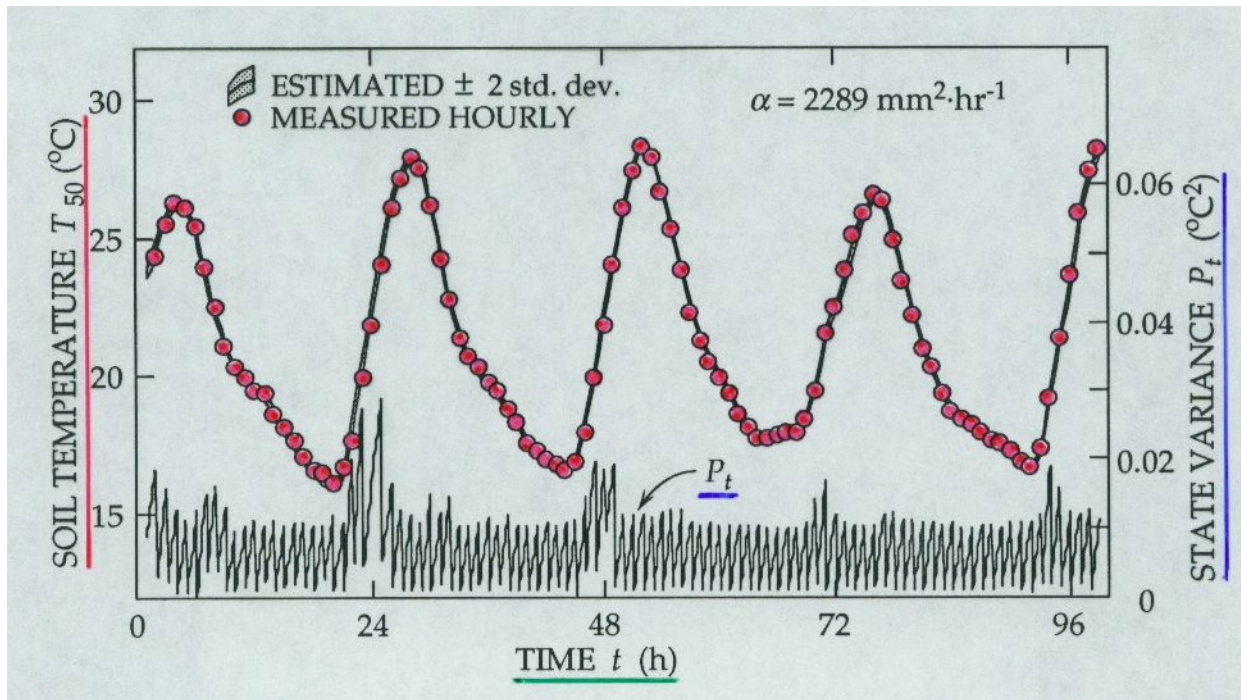
# infiltrating water field expt



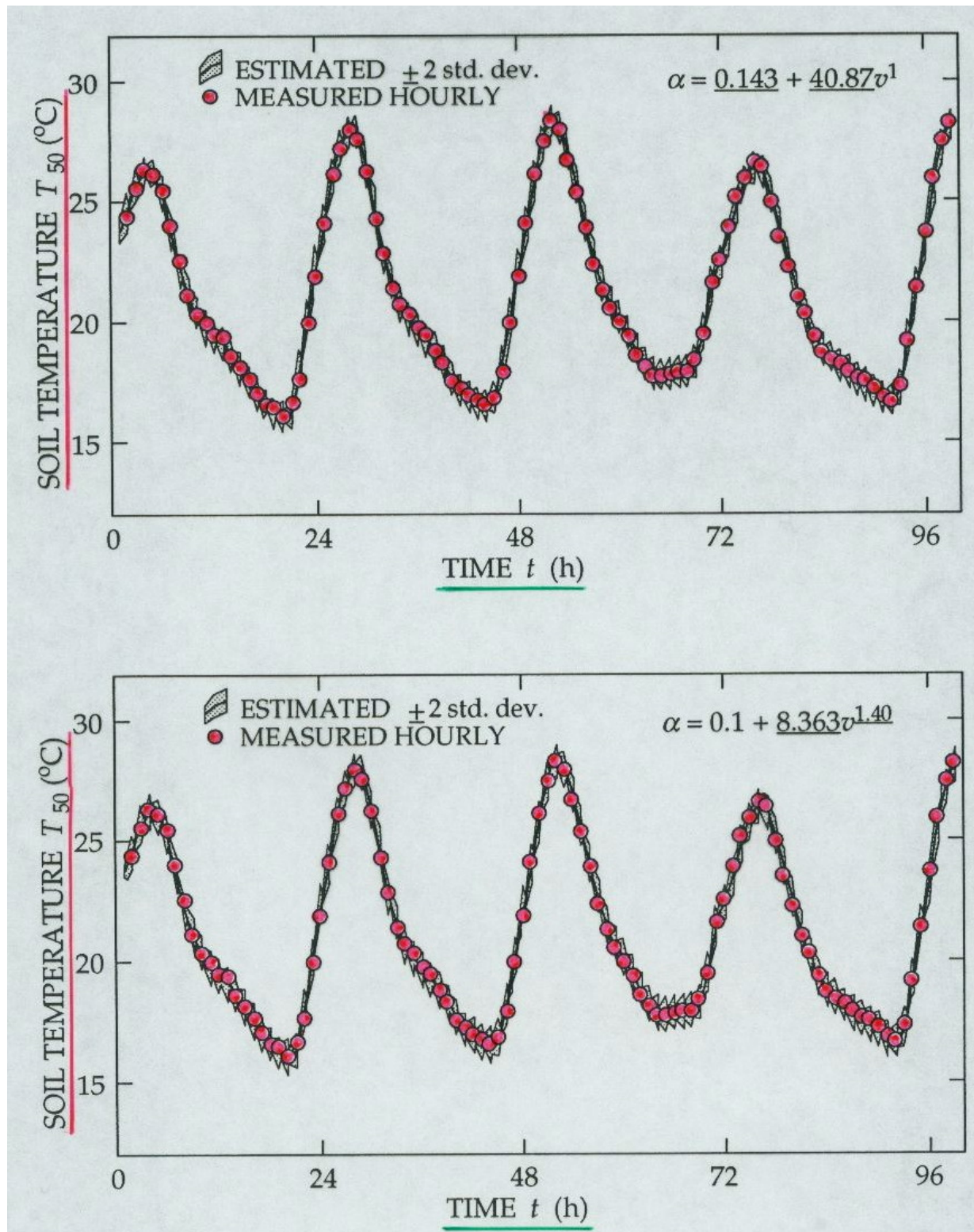
# infiltrating water data



# infiltrating constant diffusivity alpha



# infiltrating variable alpha with a,b,n



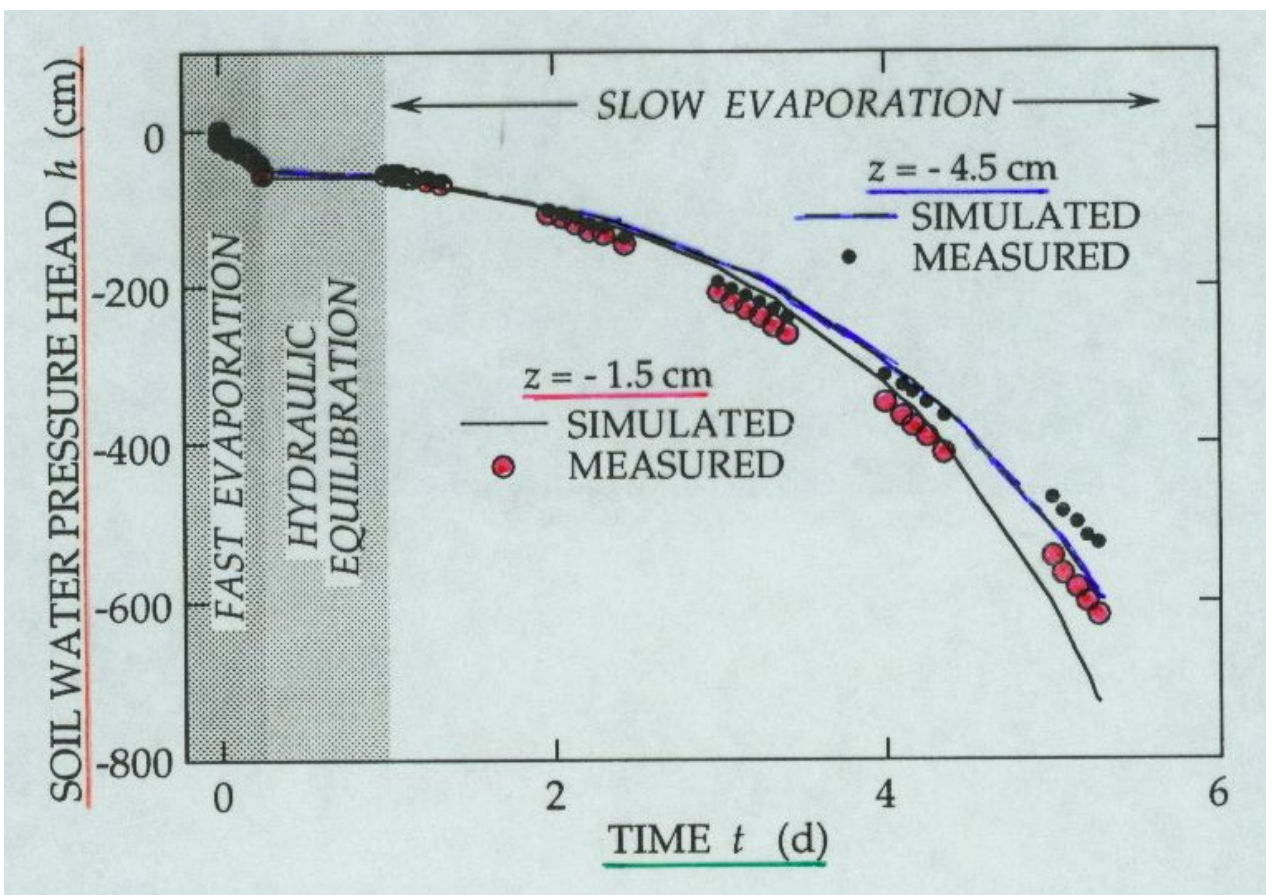
# temp table

Optimization results for the three different scenarios of soil temperature and water infiltration data (Jaynes,1990).

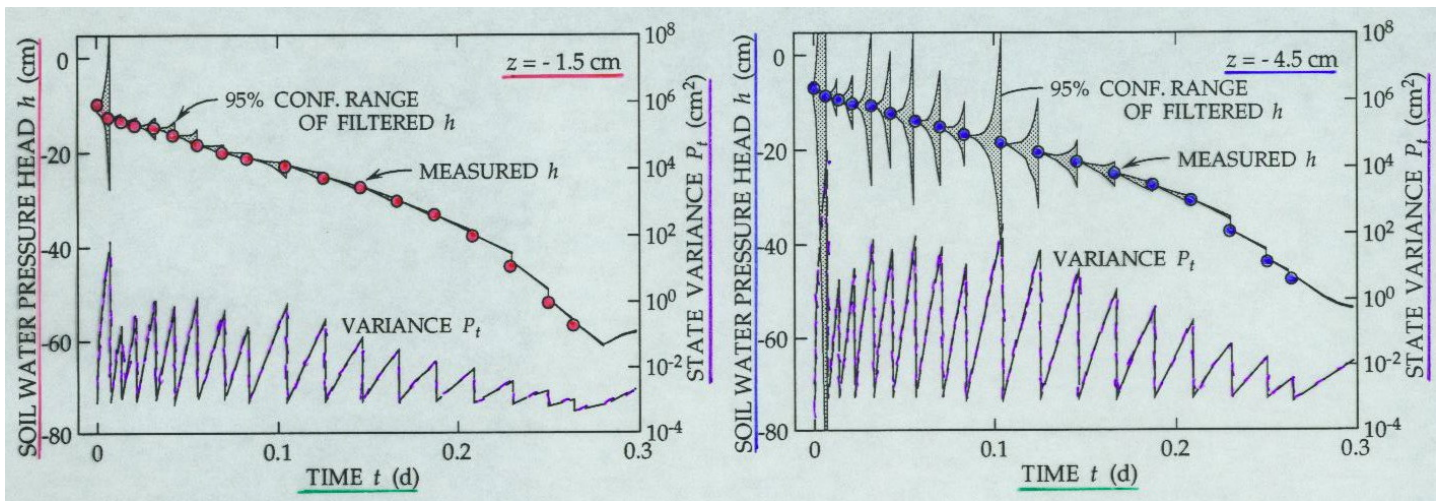
Scenario	$\alpha$ (mm <sup>2</sup> ·hr <sup>-1</sup> )	OF (°C <sup>2</sup> )				
<u>constant <math>\alpha</math></u>	2289	<u>7.174</u>				
<u><math>\alpha = a + bv</math></u>	<table border="1"> <thead> <tr> <th><math>a</math></th> <th><math>b</math></th> </tr> </thead> <tbody> <tr> <td><u>0.143</u></td> <td><u>40.87</u></td> </tr> </tbody> </table>	$a$	$b$	<u>0.143</u>	<u>40.87</u>	<u>5.146</u>
$a$	$b$					
<u>0.143</u>	<u>40.87</u>					
<u><math>\alpha = 0.1 + bv^n</math></u>	<table border="1"> <thead> <tr> <th><math>b</math></th> <th><math>n</math></th> </tr> </thead> <tbody> <tr> <td><u>8.363</u></td> <td><u>1.40</u></td> </tr> </tbody> </table>	$b$	$n$	<u>8.363</u>	<u>1.40</u>	<u>4.643</u>
$b$	$n$					
<u>8.363</u>	<u>1.40</u>					

ESTIMATING UNSATURATED  
SOIL HYDRAULIC CONDUCTIVITY  
USING THE EVAPORATION METHOD  
ON SOIL CORES  
IN THE LABORATORY

# inverse simulation pressure

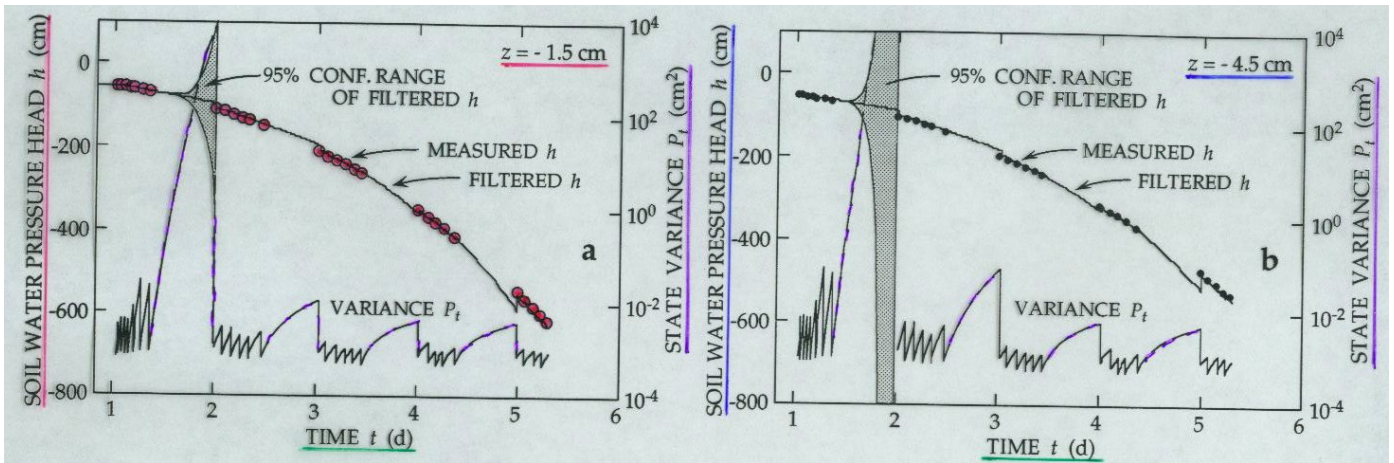


# state-space press hrs

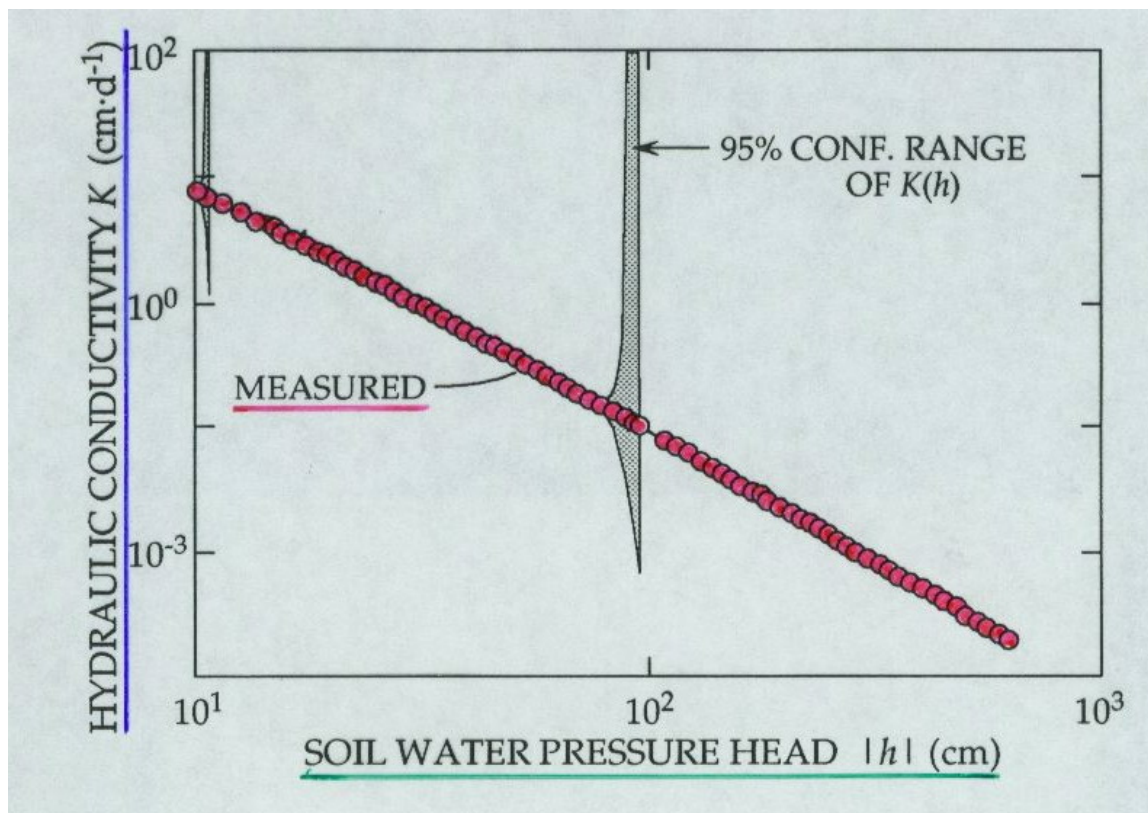




# state-space press days



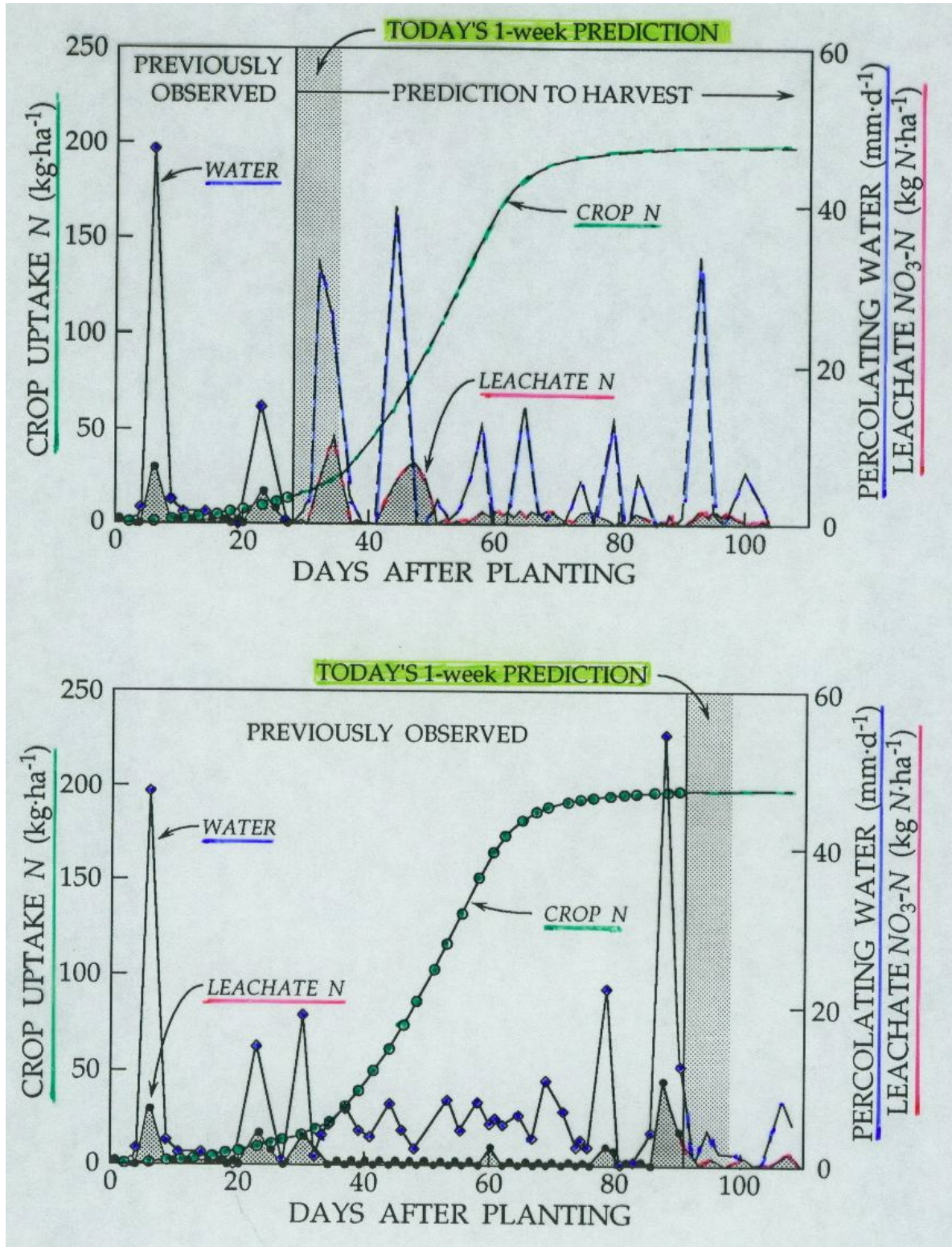
# K vs theta



MANAGEMENT-ORIENTED

MODELING

# Manage-Oriented-Modeling



# advantages of st space approach

## ADVANTAGES OF STATE-SPACE APPROACH

Regardless of the instrument or field sampling technique used to observe a soil physical, chemical or biological process, most samples represent some sort of depth-averaged value.

- Hence, it is not only convenient but more correct to use equations that describe depth-averaged phenomena.
- In the future we envision state-space equations to be used in other disciplines of soil science. The development of soil horizons, growth and water extraction of plant roots, and soil depths over which microbiologically-induced and chemical reactions predominantly occur are some examples.
- We also expect that progress could be made using time-averaged equations to examine critical periods during which soil processes occur.

# advantages of st.space

## ADVANTAGES OF STATE-SPACE APPROACH

- Explicit acknowledgment that the equation is definitely approximate and contains a model error.
- Explicitly solving for the model variance, and through its examination, ascertain the impact of simplifications, and identify improvements for a more realistic equation.
- The inclusion of an observation error. It can be treated as a known and measured quantity or alternatively, treated as an unknown.
  - A known observation error allows a reconsideration of the state variable in the equation or an improvement in instrumentation or calibration.
  - Treating the observation error as an unknown, its behavior in space and time can be related to spatial and temporal correlation lengths.

# future agr. Research dependence

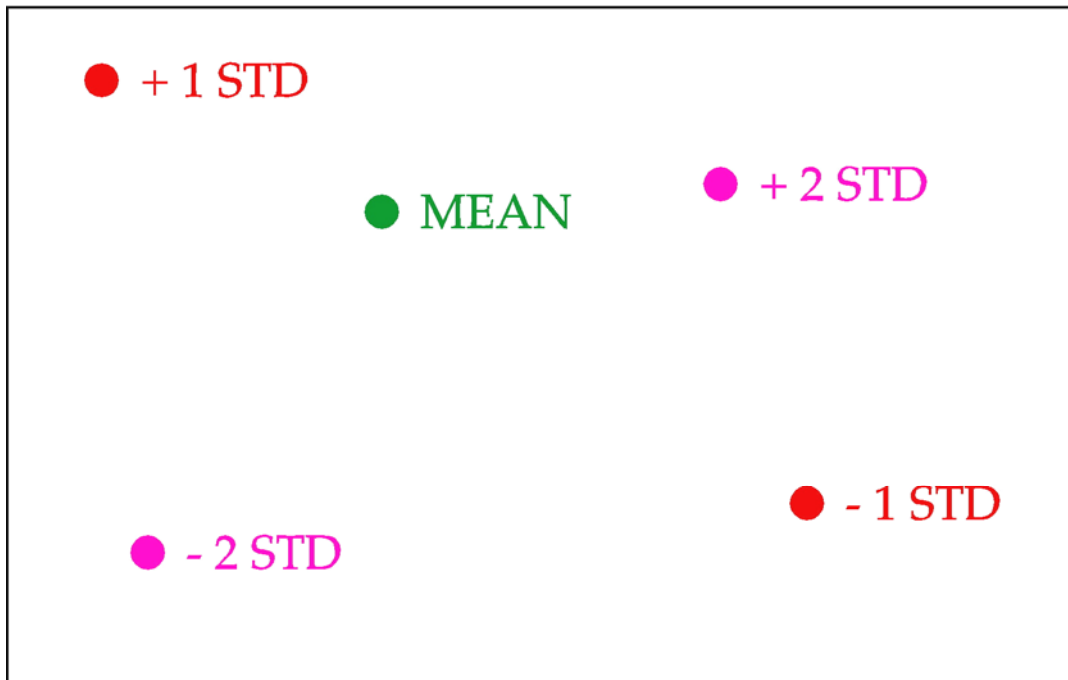
## FUTURE AGRICULTURAL & ECOLOGICAL RESEARCH

- WILL NOT DEPEND UPON SMALL PLOTS  
TREATED DIFFERENTLY.
- WILL TAKE ADVANTAGE OF SPATIAL  
AND TEMPORAL VARIABILITY OF  
SOILS RATHER THAN IGNORING IT.

# Time invariance

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## TIME INVARIANCE OF FIELD SPATIAL VARIABILITY



ANALYZE THE ENTIRE FIELD  
WITH ONLY 3 TO 5 OBSERVATIONS



# future

## Future work

- Characterize soil properties in relation to soil mapping units
- Ascertain spatial and temporal variance structures within soil mapping units
- Examine spatial and temporal covariances between soil parameters and agroecological parameters
- Develop ability to translate from one space or time scale to another
- Shift from deterministic to stochastic methods to improve our technology to manage natural resources

Develop field technology to answer:

- What sample size?
- How many samples?
- How far apart?
- How often?

# achievements

## WHAT DO WE HOPE TO ACHIEVE ?

- TODAY SUSTAINABLE AGRICULTURE AND QUALITY ENVIRONMENT
- YESTERDAY BIOTECHNOLOGY AND CULTIVAR DEVELOPMENT
- LAST WEEK FOOD PRODUCTION
- LAST CENTURY FOOD & HEALTH FOR CIVILIZATION
- LAST MILLENIUM ?
- LAST 300 MILLENIA ACHIEVMENTS OF CIVILIZATION  
V. GORDON CHILDE (1882 – 1957)

1. IRRIGATION
2. THE PLOW
3. HARNESSING ANIMAL POWER
4. SAILING BOATS
5. WHEELED VEHICLES
6. ORCHARD HUSBANDRY
7. FERMENTATION
8. PRODUCTION & USE OF COPPER
9. BRICKS
10. THE ARCH
11. GLAZING
12. CONFIDENTIAL SEAL OF A LETTER
13. SOLAR CALENDAR
14. ALPHABET
15. WRITING
16. NUMERICAL NOTATION
17. PRODUCTION OF BRONZE
18. SMELTING IRON
19. AQUEDUCTS FOR CITY WATER
20. ???

# 20th Achievement

20<sup>TH</sup> ACHIEVEMENT OF CIVILIZATION

A GLOBAL POPULATION SUFFICIENTLY EDUCATED

TO

MANAGE ITS CONTINENTAL RESOURCES

WITHOUT SOIL EXHAUSTION

AND

WITH SUSTAINED WATER QUALITY

# doggie

“FUTURE SOIL PHYSICIST”



NEW INSTRUMENTS  
& LABORATORY WORK

LANDSCAPE  
OBSERVATIONS

NEW THEORIES  
& STATISTICS

EDUCATIONAL  
REFORMS