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International Centre for Theoretical Physics*



1930-9

Joint ICTP-IAEA Advanced Workshop on Model Codes for Spallation Reactions

4 - 8 February 2008

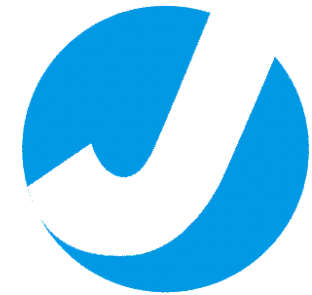
Proton induced spallation reactions investigated within the framework of BUU model

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Proton induced spallation reactions investigated within the framework of BUU model



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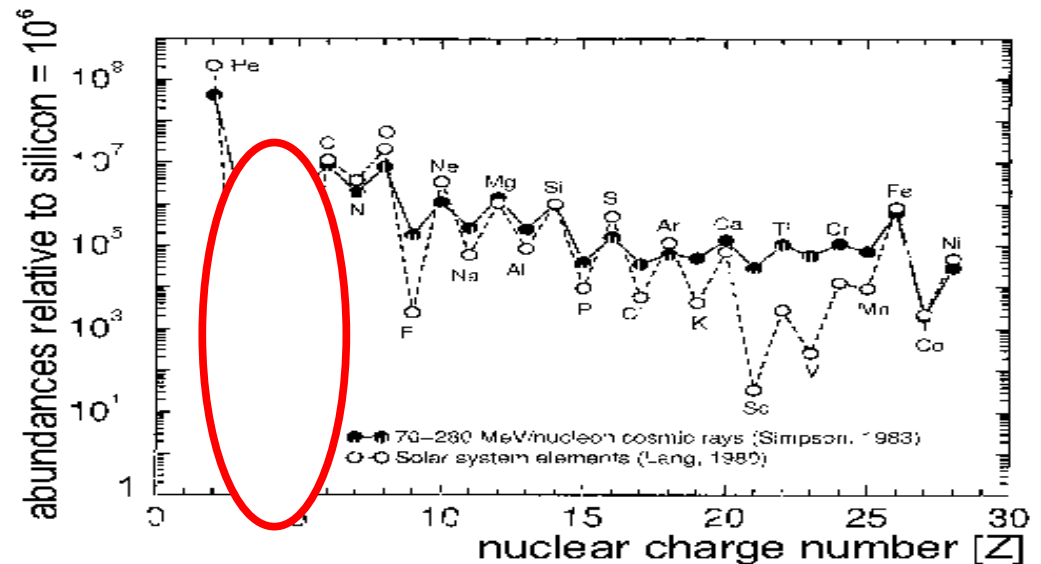
Outline:

- ***Motivation***
- ***Characteristics of the reactions***
- ***BUU model description***
- ***Model results - comparison with experimental data***
- ***Conclusions***



Why are proton induced spallation reactions of interest?

- **Knowledge of the reaction mechanism:**
- **mechanism of spallation reaction is not well known**
- **double differential cross sections of emitted particles in the reactions are necessary for testing, validation and developing of theoretical models**
- **Applications:**
- **astrophysical aspects:**
- **comparison of cosmic ray elements and the solar system abundances**
- **Li, Be and B in cosmic rays are enriched by more than 6 orders of magnitude**



S. G. Mashnik, *On Solar System and Cosmic Rays Nucleosynthesis and Spallation Processes*, LANL, Report LA-UR-00-3658, (2000)

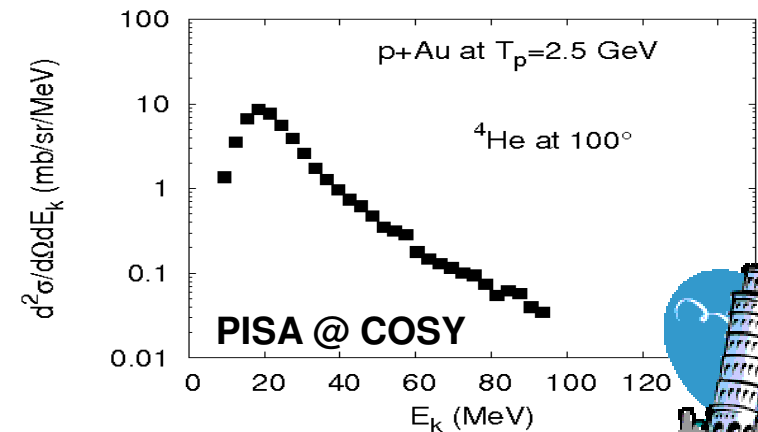
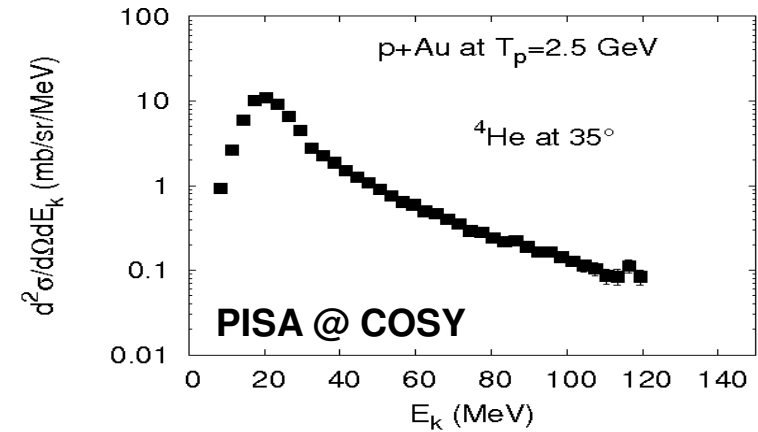


Experimental observations:

Study of the reactions possible due to development of accelerator technics
– the end of forties –
accelerators provide projectiles with energies higher than 100 MeV

- **2 – component spectra of emitted particles:**
- **high energy part – dominant in forward angles,**
- **isotropic low energy part**

Based on experimental observations –
general rules of spallation processes
are established



Historically:

▪ **Metropolis**

N. Metropolis et al., Phys. Rev. 110(1958)185

• **Dostrovsky**

I. Dostrovsky et al., Phys. Rev. 111(1958)1658

• – *using the idea of*

R. Serber, Phys. Rev. 72(1947)1114

• **Serber and Weisskopf –**

V. Weisskopf, Phys. Rev. 52(1937)295

▪ **suggested description of spallation as two step process:**

▪ **energy deposition**

▪ **subsequent evaporation**

Such treatment of spallation reactions is used from that time up to now !



The Spallation Process:

• **first (fast) stage: Microscopic models**

• **high energy proton causes an intra-NC on a time scale $\approx 10^{-22}$ s**

⊗ **highly non-equilibrated process, incoming proton deposits excitation energy and angular momentum**

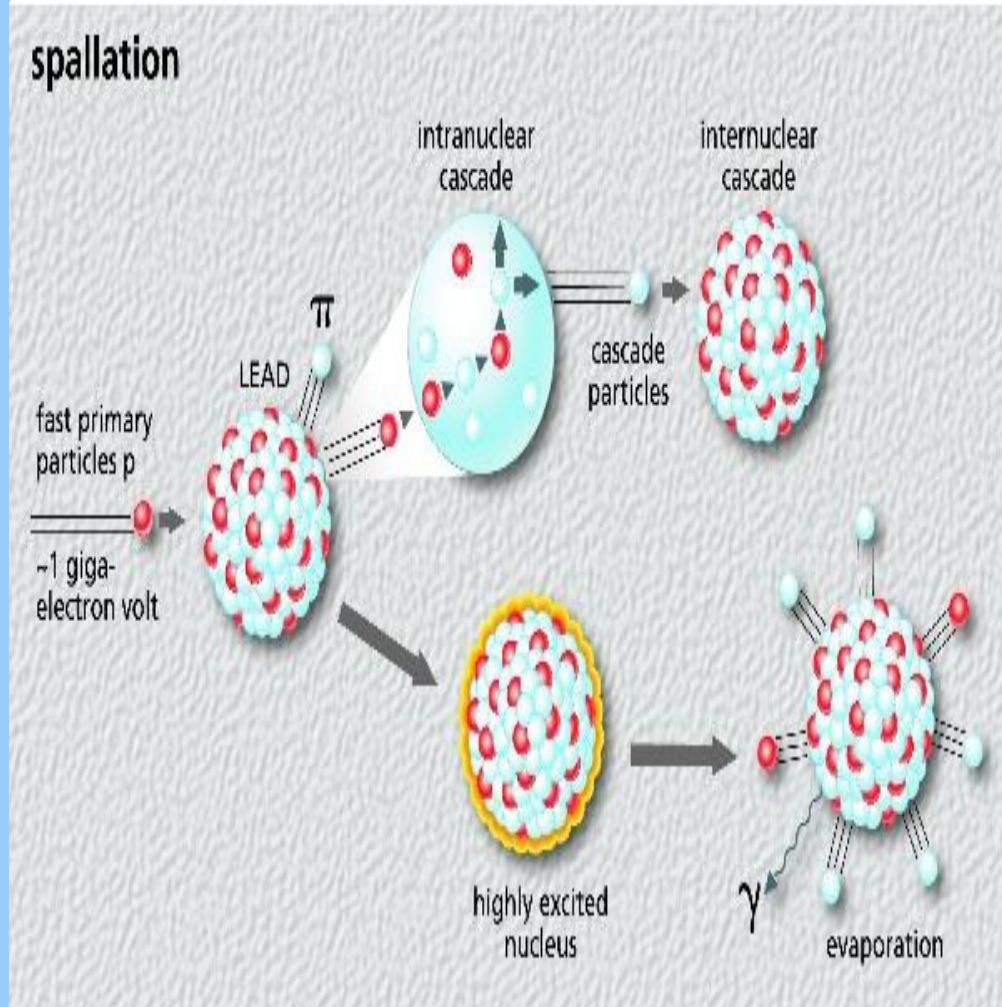
⊗ **high energy nucleons, pions and light ions are produced**

⊗ **the result: excited residual nucleus in thermodynamical equilibrium with a few MeV/N of excitation energy**

• **emitted particles (n, p, π) can cause an inter-NC placing individual nuclei into excited states**

⊗ **second (slow) stage: Statistical models**

• **de-excitation by evaporating $n, p, d, t, \alpha, \gamma \dots$ on a time scale $\approx 10^{-18} - 10^{-16}$ s**



Microscopic models:

- ***Intranuclear Cascade (INC)***
– *constant static potential*

Bertini

Cugnon

Boudard

- ***Boltzmann-Uehling-Uhlenbeck (BUU)***
– *dynamically changing mean field*

- ***Quantum Molecular Dynamics (QMD)***
– *two- and three- body potentials*

Aichelin

Niita



Boltzmann-Uehling-Uhlenbeck (BUU) model:

Based on transport equation

- *Originate in classical Boltzmann equation for one body phase – space distribution*
- *In 1933 – developed by **Uehling** and **Uhlenbeck**, by adding Pauli blocking factors*

A. E. Uehling and G. E. Uhlenbeck, Phys. Rev.43(1933)552

- *In 1984 – used first time to nuclear collision description, by **Bertsch***

G. F. Bertsch, et al., Phys. Rev. C, 29(1984)673



The transport equation:

$$\left\{ \frac{\partial}{\partial t} + \left(\frac{\vec{p}_1}{m_1} + \frac{\partial U(\vec{r}, \vec{p}_1, t)}{\partial \vec{p}_1} \right) \frac{\partial}{\partial \vec{r}} - \frac{\partial U(\vec{r}, \vec{p}_1, t)}{\partial \vec{r}} \frac{\partial}{\partial \vec{p}_1} \right\} f(\vec{r}, \vec{p}_1, t) = \frac{4}{(2\pi)^3} \int d^3 p_2 d^3 p_3 d\Omega v_{12} \frac{d\sigma_{12}}{d\Omega} \delta^3(\vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4) \cdot [f_3 f_4 \bar{f}_1 \bar{f}_2 - f_1 f_2 \bar{f}_3 \bar{f}_4]$$

- $f_i \equiv f(\vec{r}, \vec{p}_i, t)$ - one-body phase-space distribution
- $\bar{f}_i \equiv 1 - f(\vec{r}, \vec{p}_i, t)$ - Pauli blocking factors
- v_{12} - relative velocity of colliding particles 1 and 2
- Ω - angle between momenta of outgoing particles: \vec{p}_3 and \vec{p}_4
- $\frac{d\sigma_{12}}{d\Omega}$ - differential cross section of the reaction

- $U(\vec{r}, \vec{p}_1, t)$ - mean-field potential, dynamically changing, calculated as a function of local density:

$$U(\vec{r}) = \frac{3}{4} t_0 \rho(\vec{r}) + \frac{7}{8} t_3 \rho(\vec{r})^{4/3} + V_0 \int d^3 r' \frac{\exp(-\mu|\vec{r}-\vec{r}'|)}{\mu|\vec{r}-\vec{r}'|} \rho(\vec{r}') + V_{Coul}$$

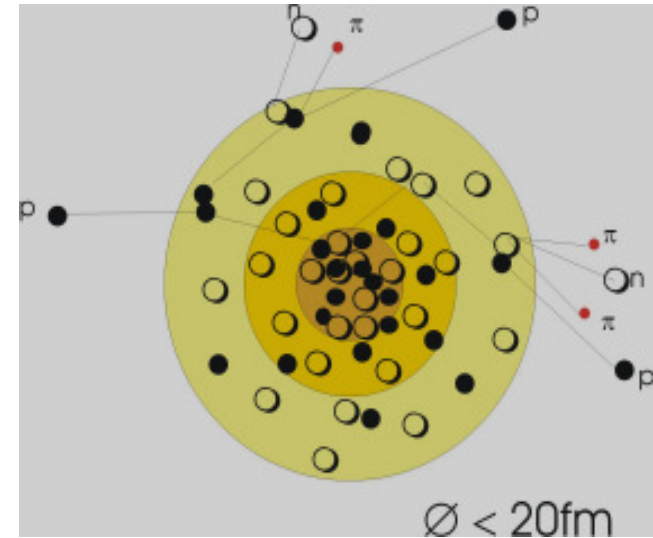
where:

$$t_0 = -1124 \text{ MeV} \cdot \text{fm}^3; t_3 = 2037 \text{ MeV} \cdot \text{fm}^4; V_0 = -378 \text{ MeV}; \mu = 2.175 \text{ fm}^{-1}.$$



Boltzmann-Uehling-Uhlenbeck (BUU) model:

- ***Classical Boltzmann transport equation complemented with Pauli blocking factors***
- ***$p + A$ collision is described as cascade of $N + N$ collisions***
- ***between collisions nucleons are moving in mean field being a function of nuclear density inside nucleus***
- ***the equation is solved using Monte Carlo method, generating positions and momentum of particles in successive time steps***



***K. Niita, W. Cassing and U. Mosel, Nucl. Phys. A 504(1989)391
G. F. Bertsch and S. Das Gupta, Phys. Rep. 160(1988)189***

- Interactions are based on elementary cross sections derived from empirical approximations of :

- $NN \rightarrow NN$ (elastic)

- $NN \rightarrow NR \rightarrow N\pi N$

$R = \Delta, N(1440), N(1535)$

- $NN \rightarrow RR \rightarrow N\pi N\pi$

- $\pi N \rightarrow \pi R \rightarrow \pi N\pi$

- $NR \rightarrow NN$ (delta absorption)

- $\pi N \rightarrow \pi N$ (elastic, charge exchange)

- production and propagation of other baryons ($\Lambda, \Sigma, \Sigma^*, \Xi, \Omega$), corresponding antibaryons and mesons ($K, \eta, \eta', \rho, \omega, \phi, K^*, a_1$)

- Low energy limit:

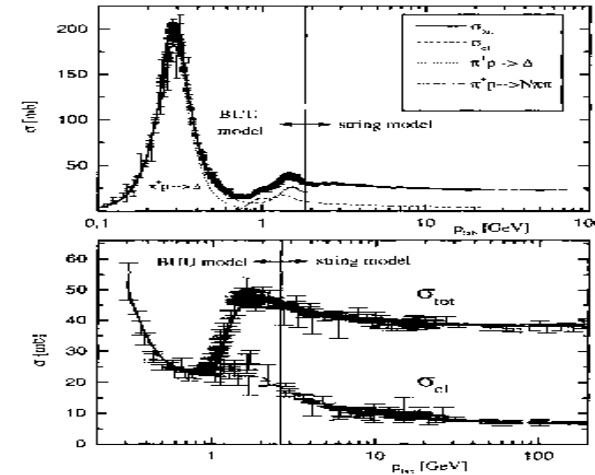
- De-Broglie-wavelength λ of cascade particles smaller than average distance of nucleons in nucleus ($\delta \approx 1.8 \text{ fm}$) and mean free path length ($L \approx 2 \text{ fm}$) in nuclear matter: $\lambda \ll \delta, \lambda \ll L$

→ few hundred MeV

- High energy limit:

- imposed by implemented processes

→ about 3.0 GeV



PDG, Phys. Rev. D 50(1994)1173

and other experimental informations !

J. Geiss, W. Cassing, C. Greiner, Nucl. Phys. A, 644(1998)107

$\lambda = 0.7 \text{ fm} \leftrightarrow 1000 \text{ MeV}$

$\lambda = 2.7 \text{ fm} \leftrightarrow 100 \text{ MeV}$

$\lambda = 9 \text{ fm} \leftrightarrow 10 \text{ MeV}$

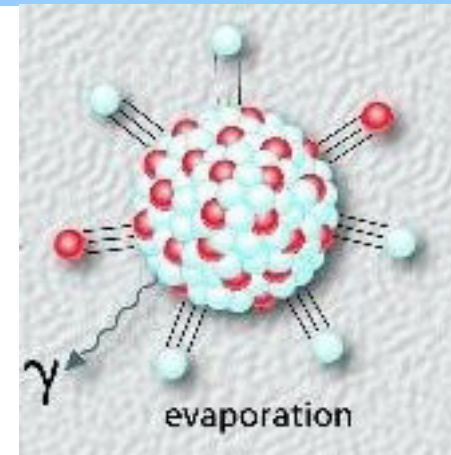


BUU + evaporation model:

Output of BUU model

defines

input for evaporation model (“afterburner”)



Output of BUU model:

Properties of residual nucleus:

A, Z, E^*, p, L

evaluate by exploring the conservation of total energy, mass number, momentum and angular momentum:

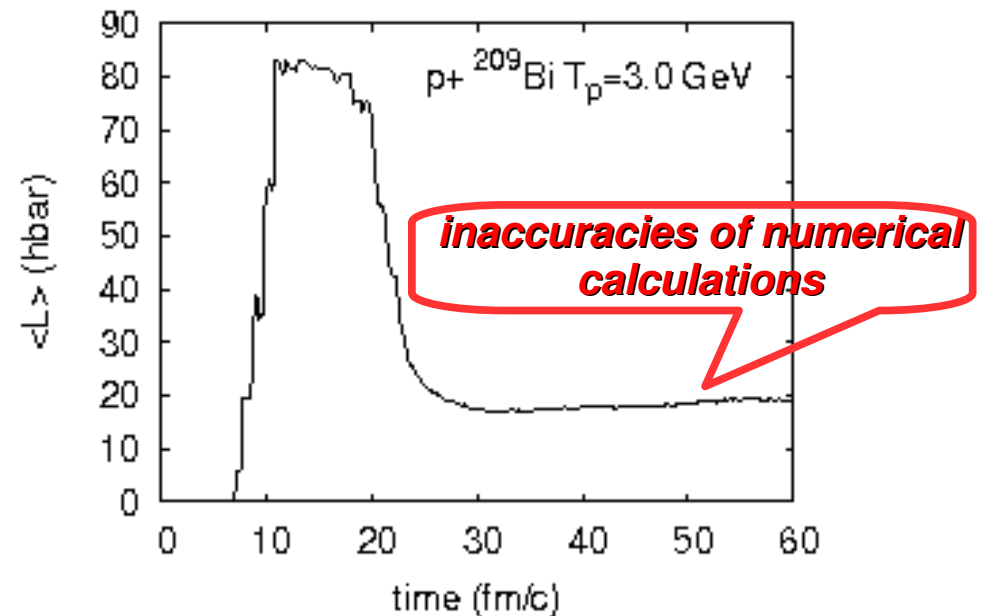
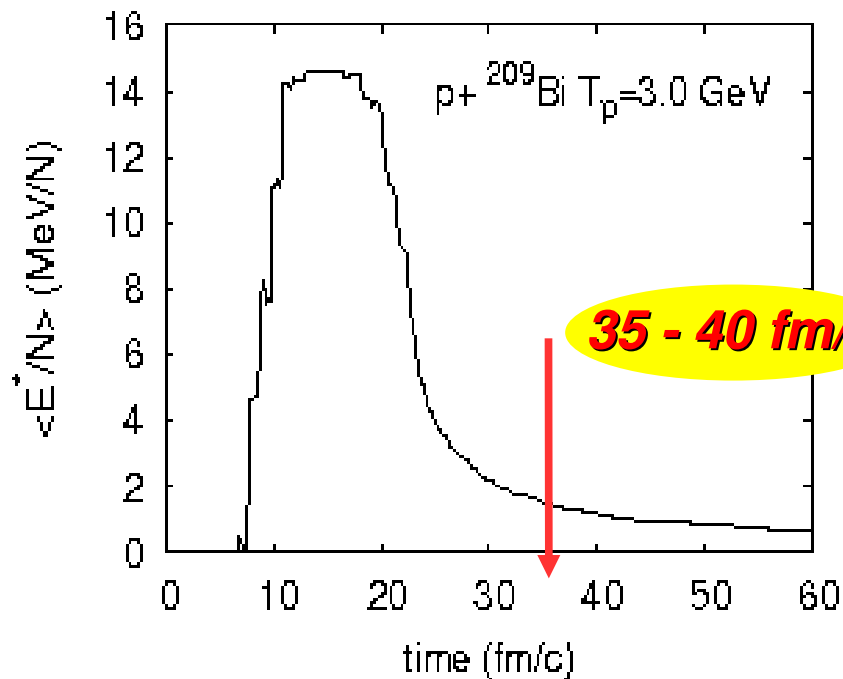
$$\begin{aligned}\langle E_R^* \rangle (t) &= E_{tot} - \sum_{i=1}^{N_p(t)} \sqrt{p_i^2 + m_i^2} - M_R - E_C \\ \langle A_R \rangle (t) &= A_T + A_P - N_p(t) \\ \langle \vec{p}_R \rangle (t) &= \vec{p}_{tot} - \sum_{i=1}^{N_p(t)} \vec{p}_i(t) \\ \langle L \rangle (t) &= L_{tot} - \sum_{i=1}^{N_p(t)} \vec{r}_i(t) \times \vec{p}_i(t)\end{aligned}$$

where: $N_p(t)$ denotes the number of escaped particles, M_R is the mass of the residual nucleus, A_T is a mass of original target, $A_P = 1$ stands for incoming proton and E_C is the energy of Coulomb interaction between the emitted particles and the residual nucleus.



Stopping time for the BUU model calculations:

- **Time evolution of the average values of properties of excited nucleus**
- **termination of the first stage indicated by stabilization of the values in time**

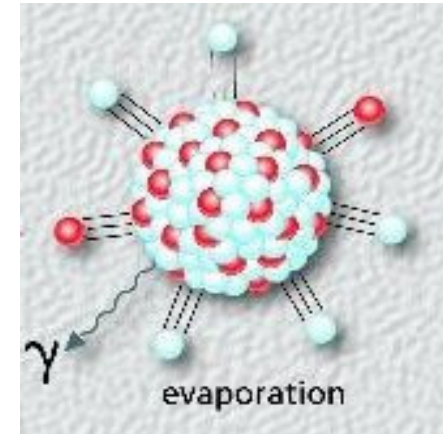


BUU + evaporation model:

Generalized Evaporation Model

GEM

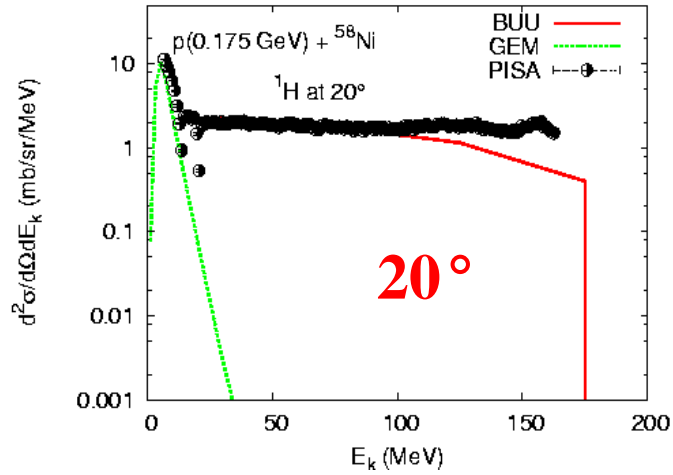
(evaporation in competition with fission)



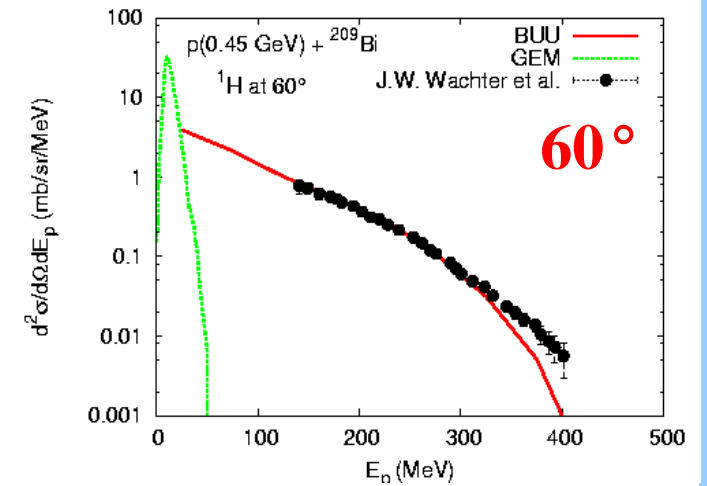
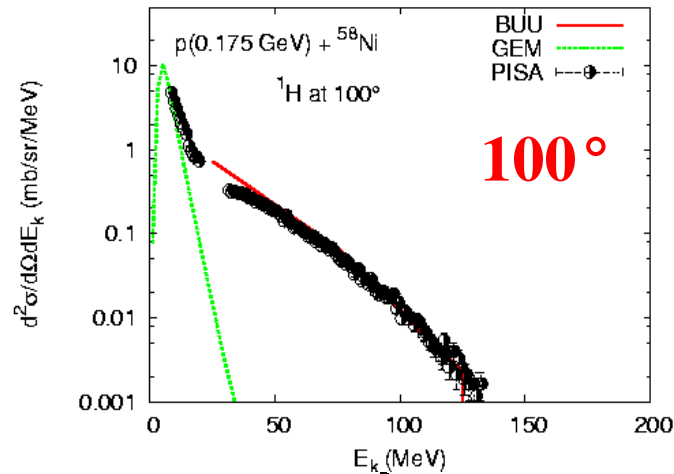
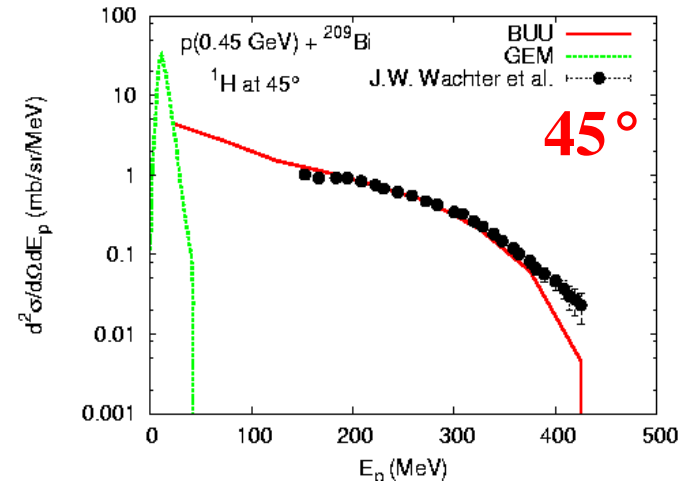
S. Furihata, Nucl. Inst. Meth. in Phys. Res. B 171(2000)251

Results: *proton spectra*

p + Ni @ 0.175 GeV



p + Bi @ 0.45 GeV



PISA @ COSY

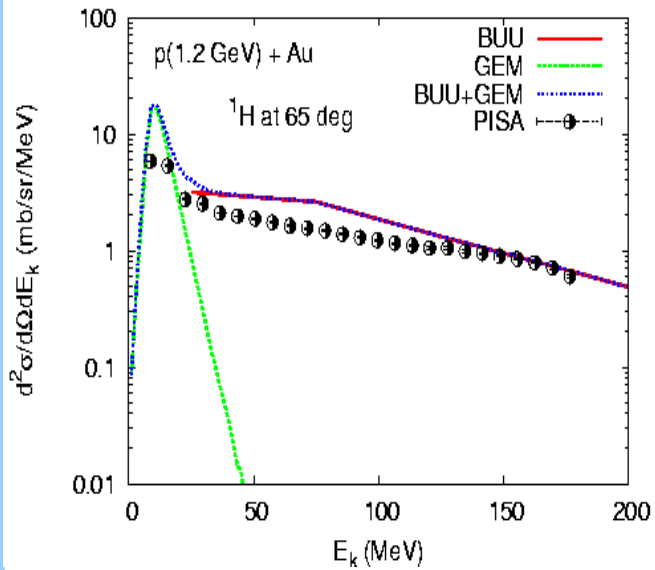


J.W. Wachter et al., PRC 6(1972)1496

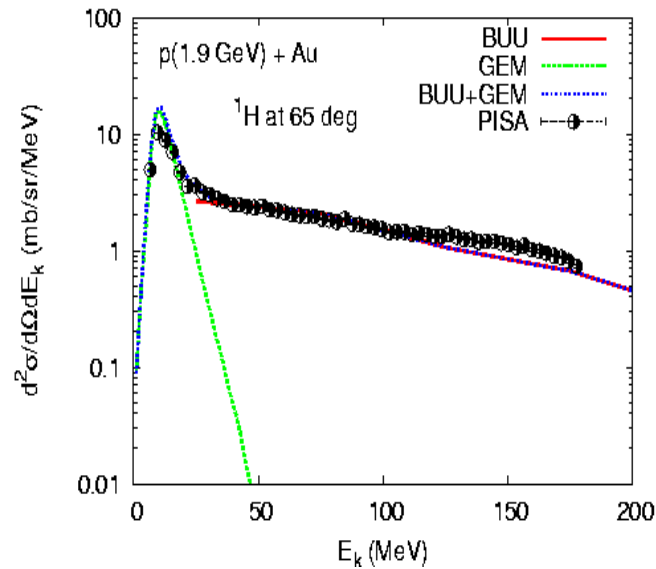


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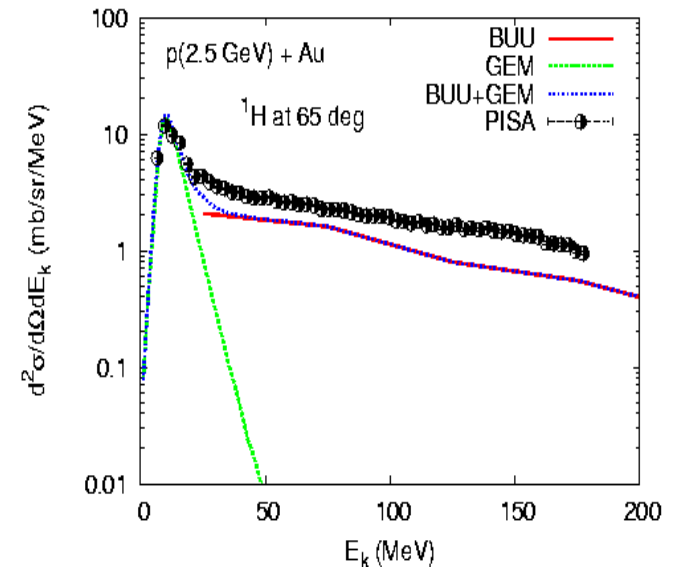
p + Au @ 1.2 GeV



p + Au @ 1.9 GeV



p + Au @ 2.5 GeV



65°

PISA @ COSY

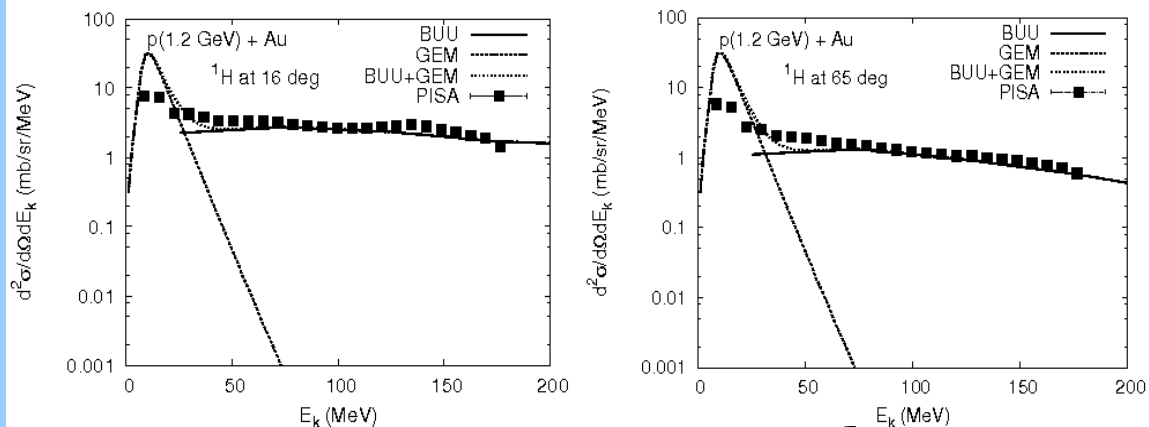


R. Barna et al., NIM A 519 (2004) 610
A. Bubak et al., PRC 76 (2007) 014618

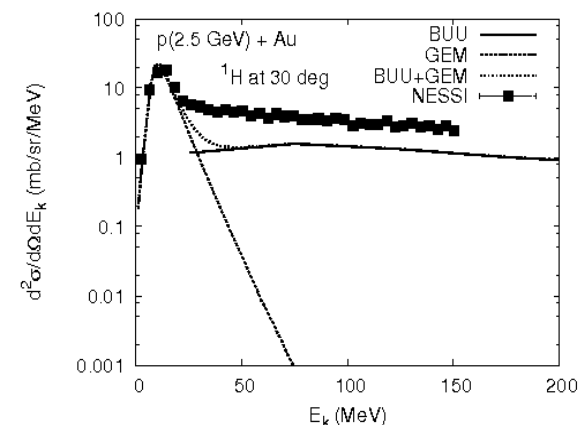


Results: proton spectra

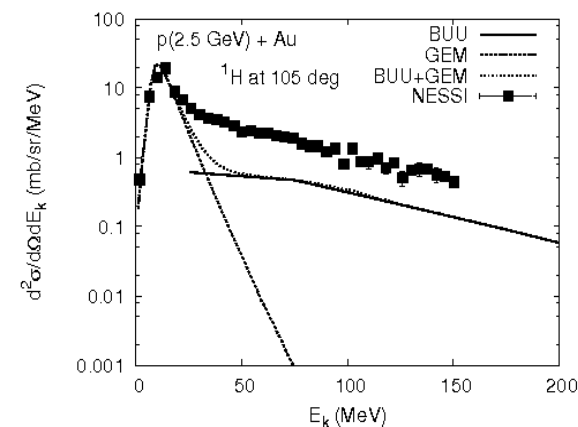
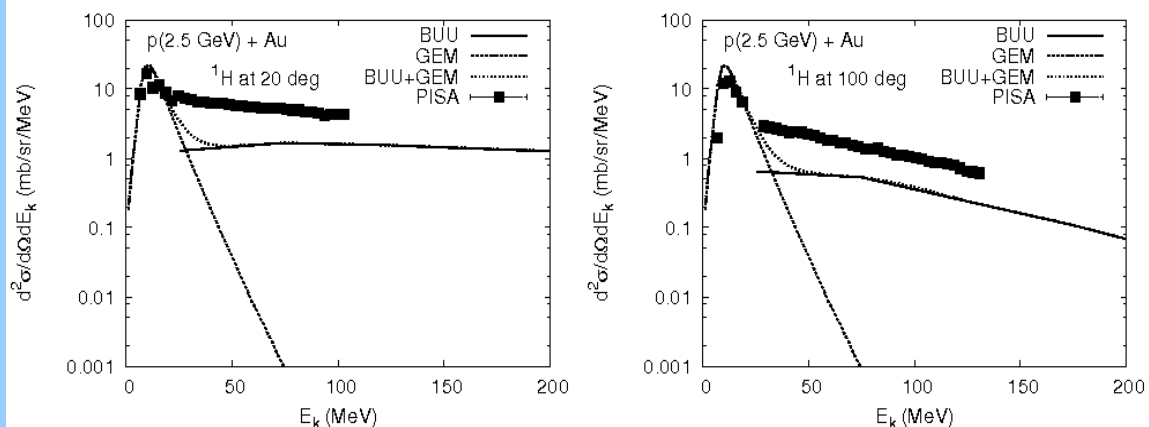
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PISA @ COSY

R. Barna et al., NIM A 519 (2004) 610



NESSI @ COSY

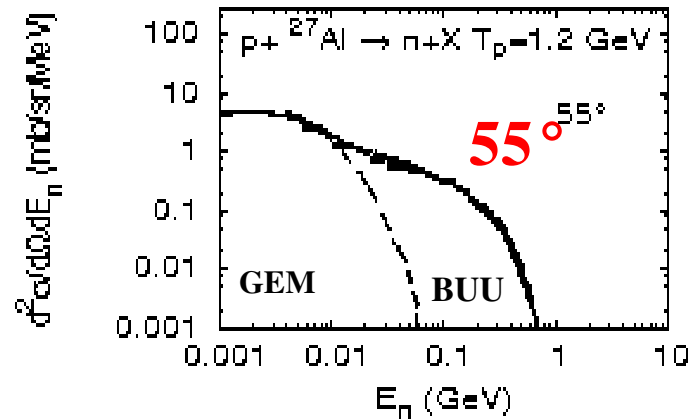
A. Letourneau et al., Nucl. Phys. A, 712(2002)133



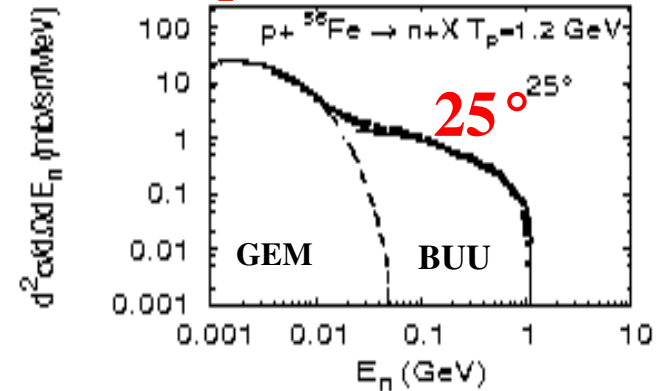
Results: neutron spectra

SATURNE (Saclay)

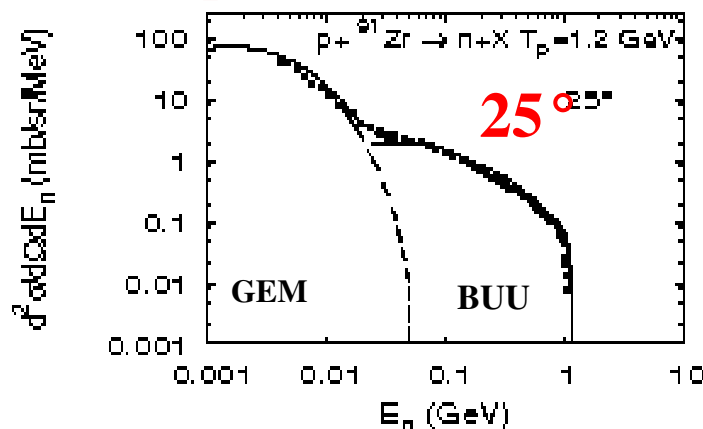
p + Al @ 1.2 GeV



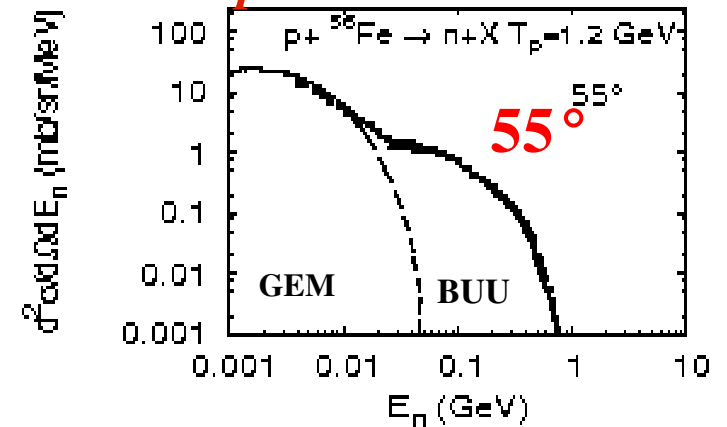
p + Fe @ 1.2 GeV



p + Zr @ 1.2 GeV



p + Fe @ 1.2 GeV

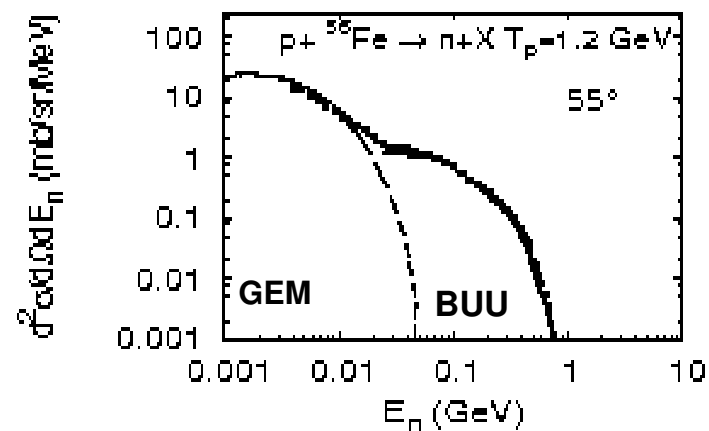
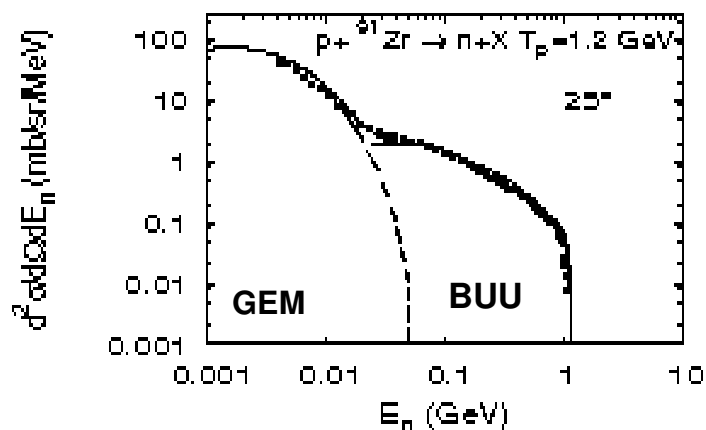
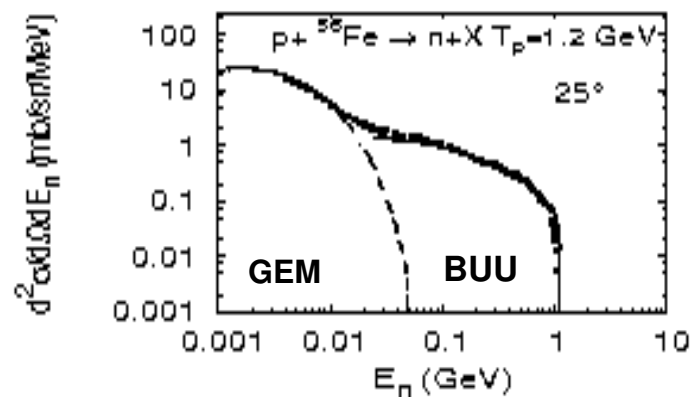
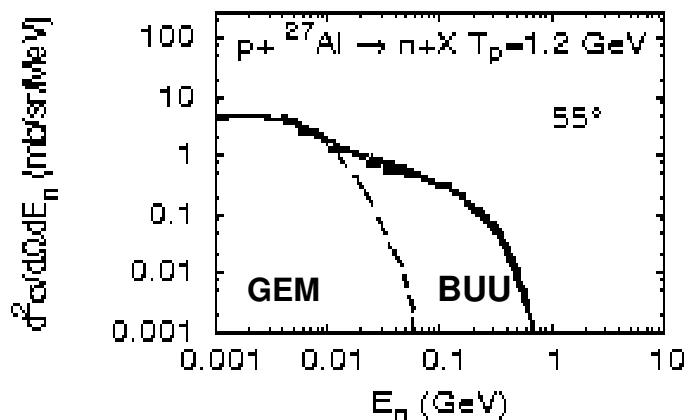


S. Leray et al., Phys. Rev. C, 65(2002)044621



Results: *neutron spectra*

SATURNE (Saclay)



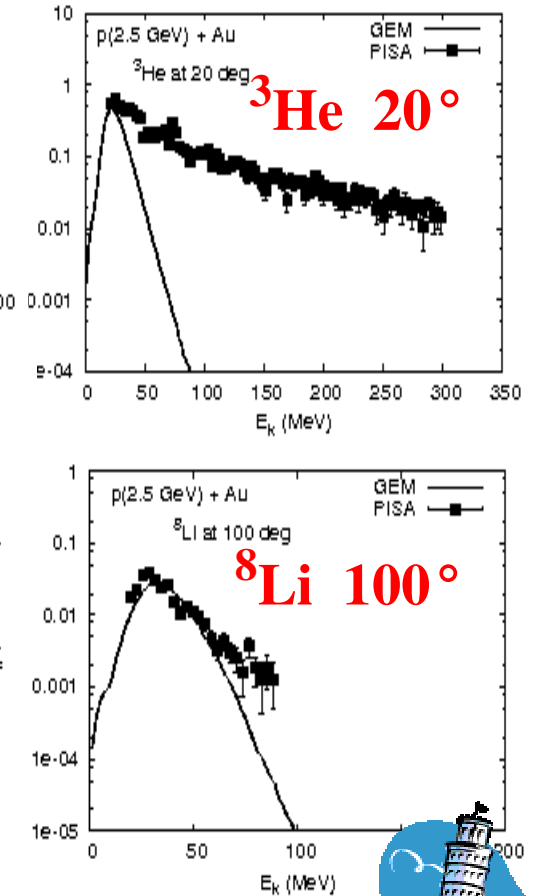
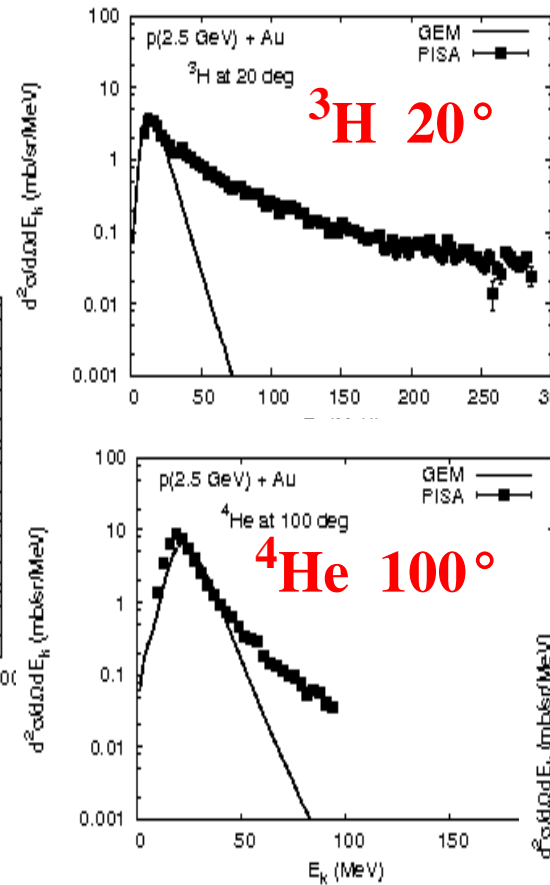
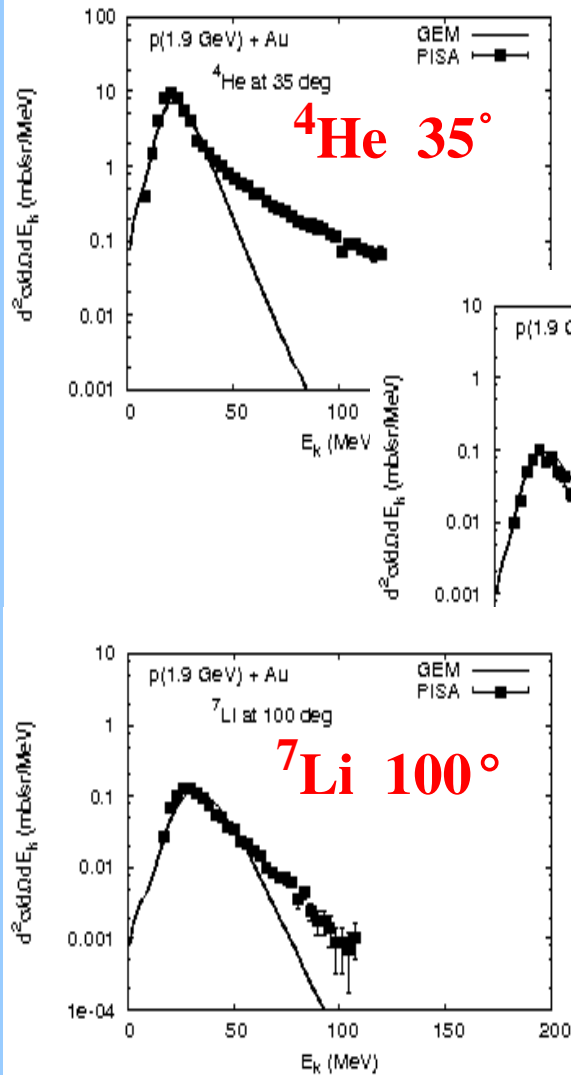
S. Leray et al., Phys. Rev. C, 65(2002)044621



Results: other ejectiles

p + Au @ 1.9 GeV

p + Au @ 2.5 GeV

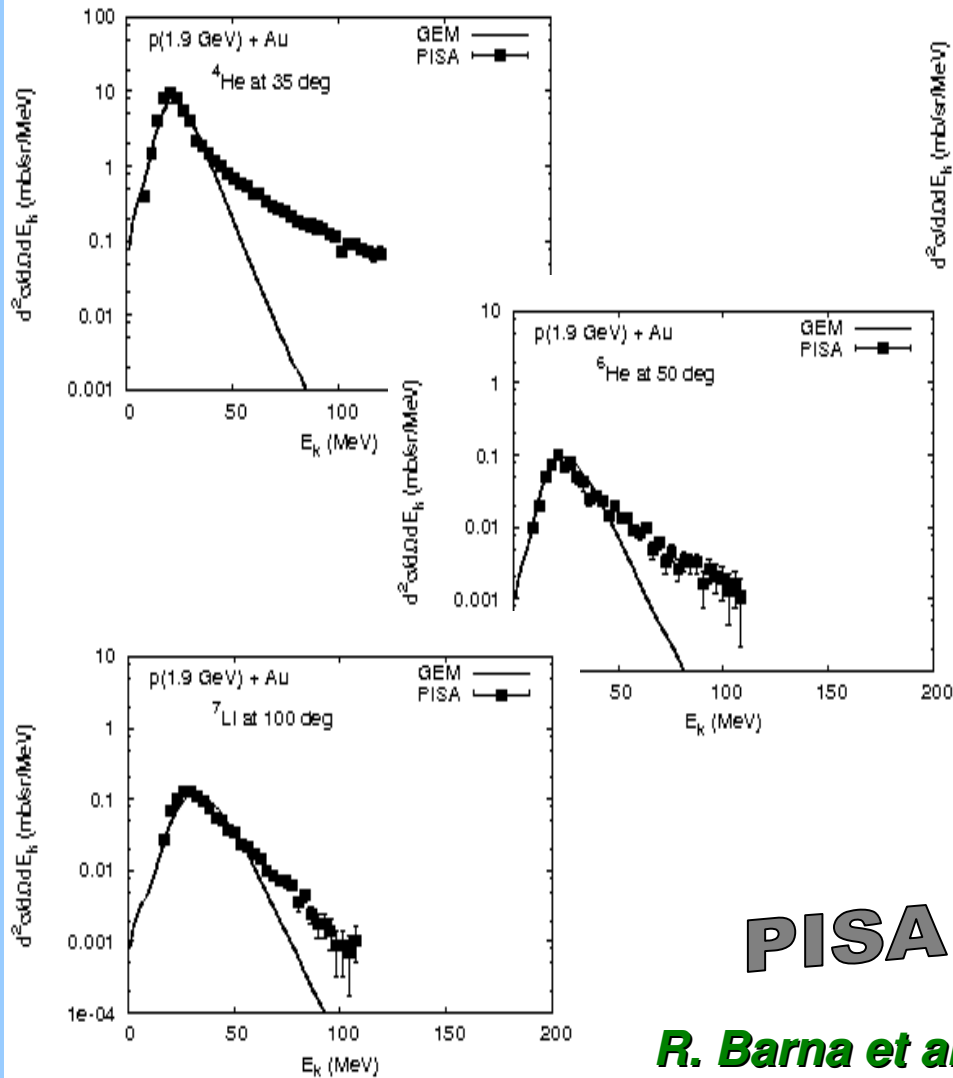


PISA @ COSY

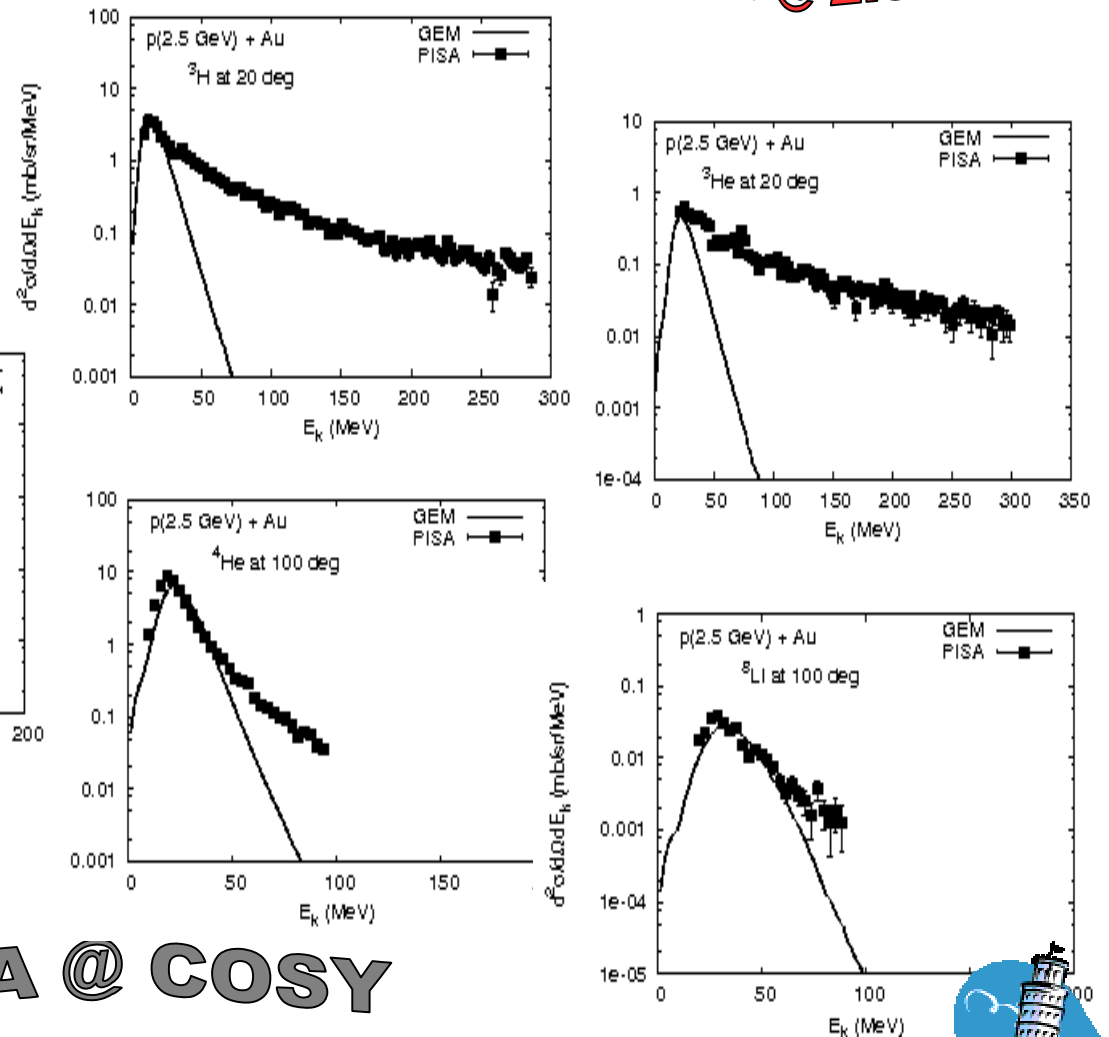


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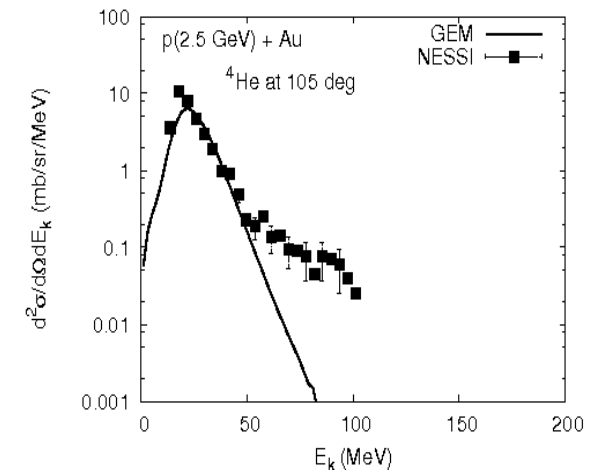
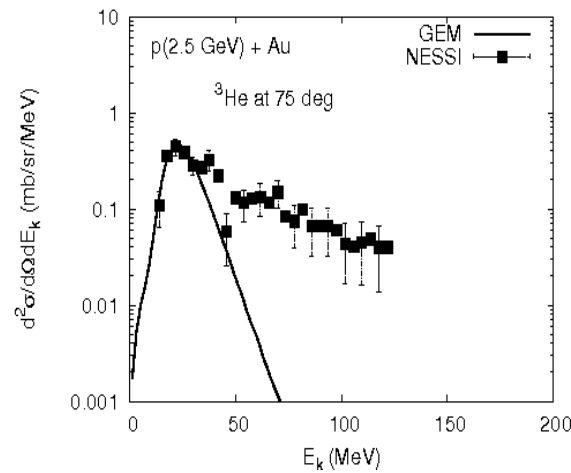
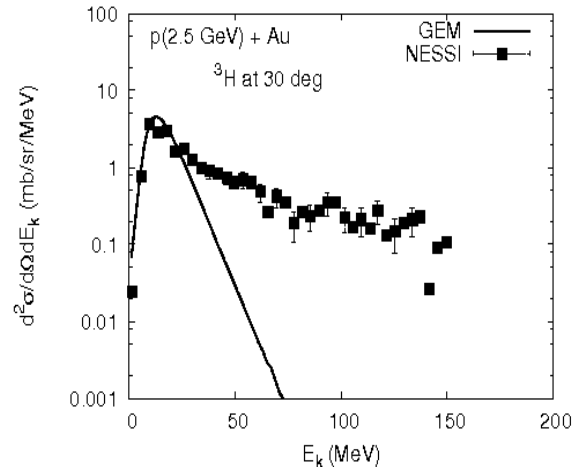
PISA @ COSY

R. Barna et al., NIM A 519 (2004) 610



Results: other ejectiles

p + Au @ 2.5 GeV



NESSI @ COSY

A. Letourneau et al., Nucl. Phys. A, 712(2002)133



SOLUTION: TEST PARTICLE METHOD

Represent the one-body phase-space distribution by discretized test particles:

$$f(\vec{r}, \vec{p}, t) = \frac{1}{N} \sum_{i=1}^{N \cdot A(t)} \delta^3(\vec{r} - \vec{r}_i(t)) \delta^3(\vec{p} - \vec{p}_i(t))$$

N – number of test particles

$A(t)$ – number of real particles at time t

The test particles propagate between collisions according to classical Hamilton equations of motion:

$$\dot{\vec{p}}_i = - \frac{\partial U(\vec{r}_i, \vec{p}_i, t)}{\partial \vec{r}_i}$$

$$\dot{\vec{r}}_i = \vec{p}_i / \sqrt{m^2 + p^2} + \frac{\partial U(\vec{r}_i, \vec{p}_i, t)}{\partial \vec{p}_i}$$

Literature:

K.Niita, W.Cassing, U.Mosel, Nucl.Phys.A 504(1989)391

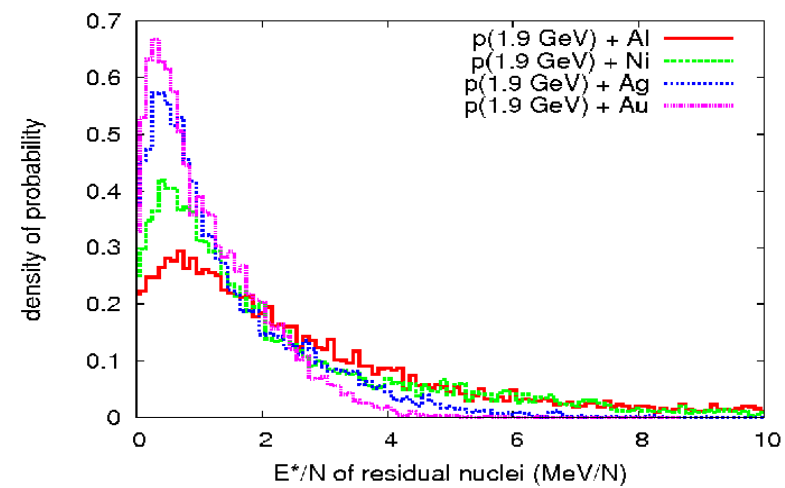
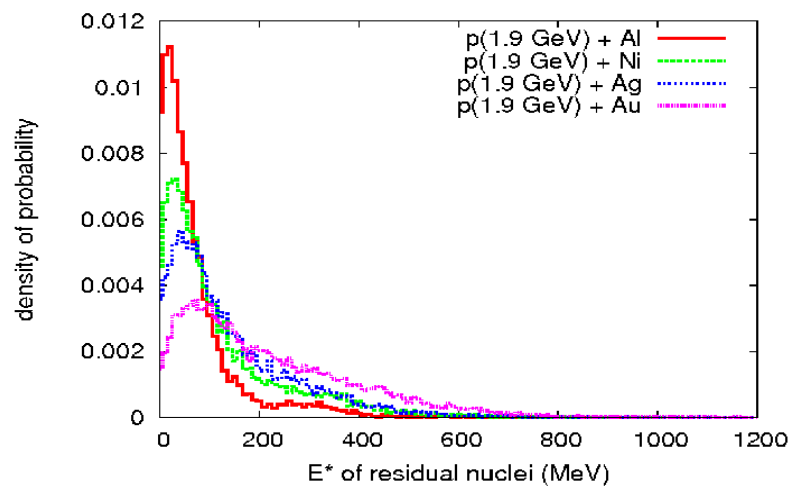
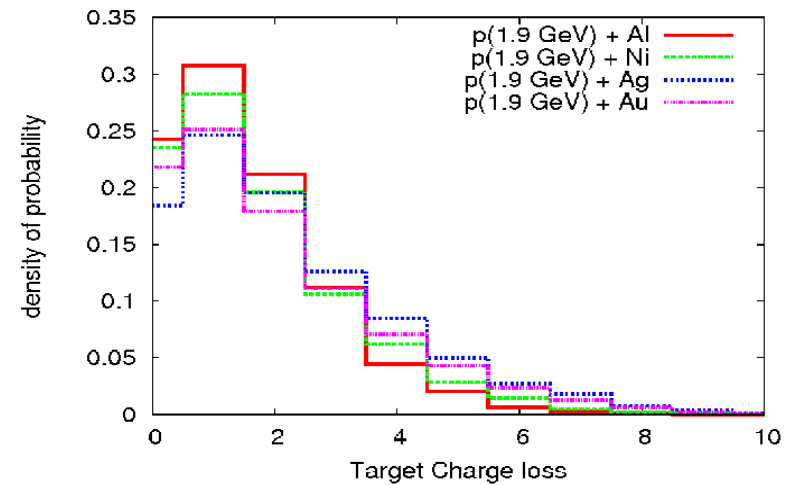
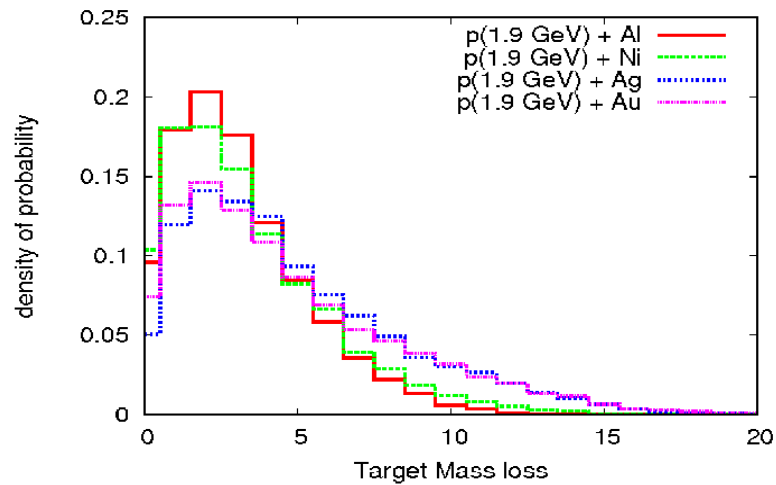
G.F.Bertsch,S.Das Gupta, Phys.Rep. 160(1988)189

J.Geiss, W.Cassing, C.Greiner, Nucl.Phys.A 644(1998)107

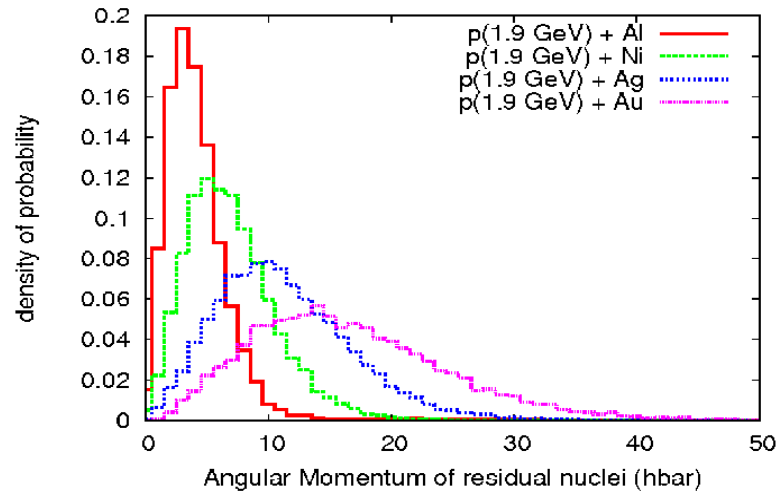
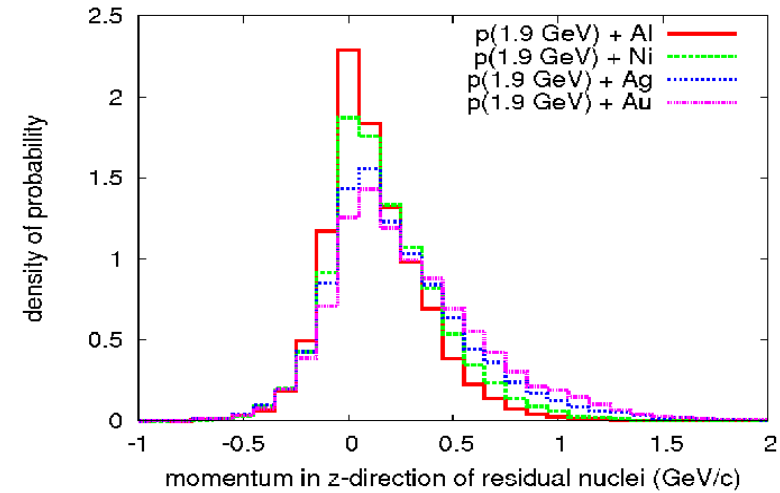
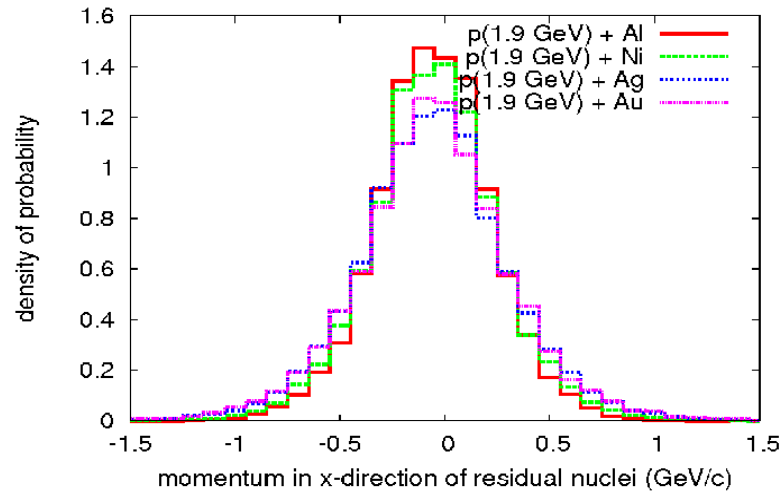
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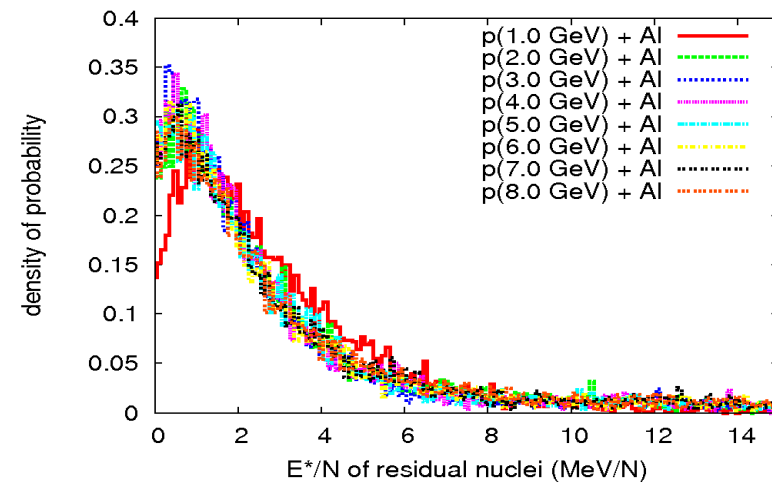
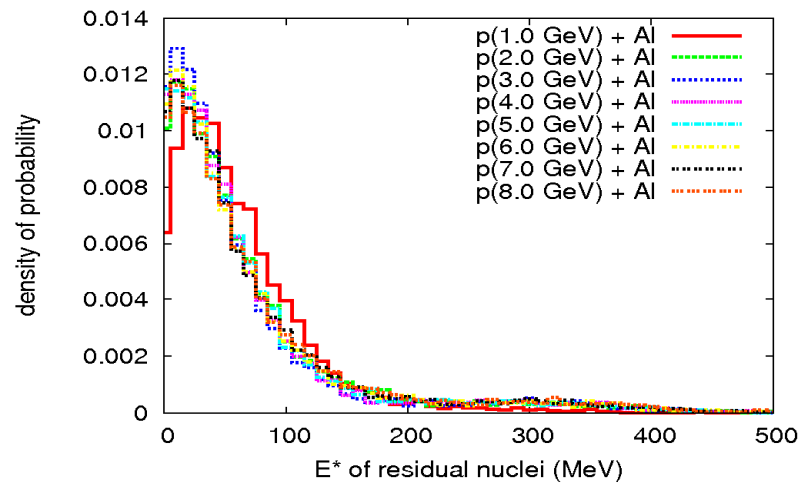
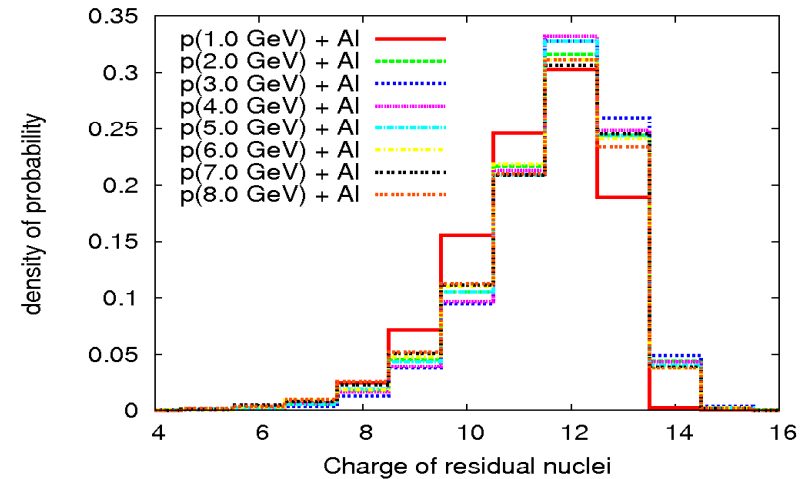
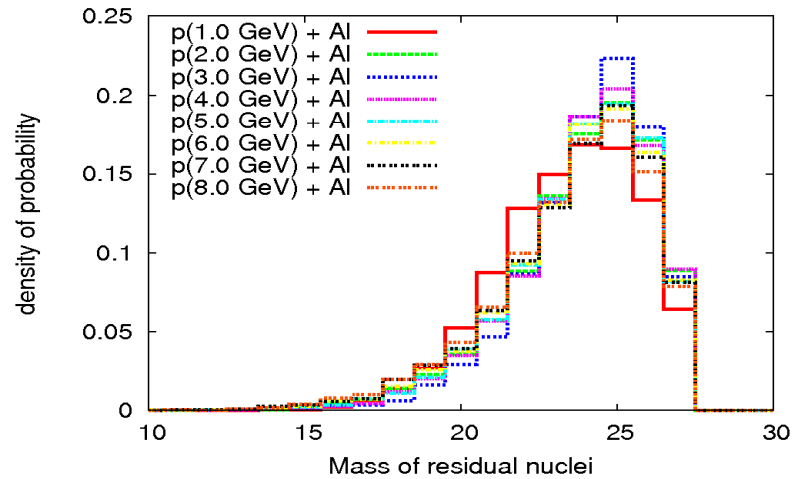
Properties of residual (hot) nuclei



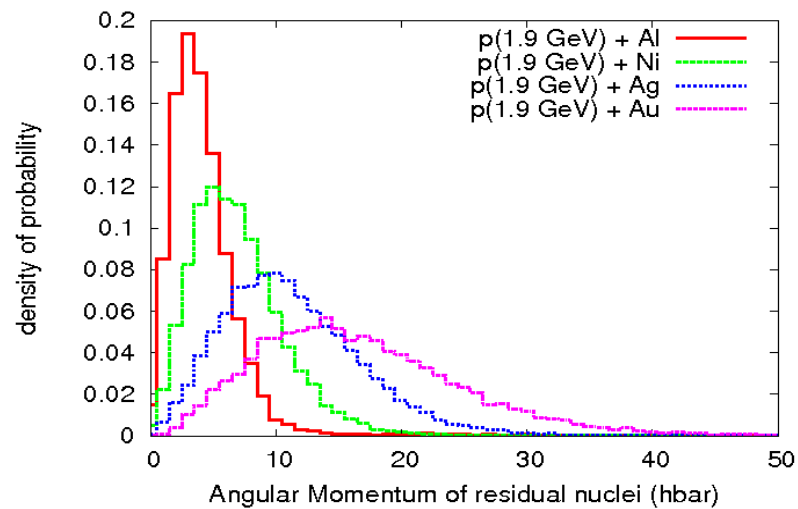
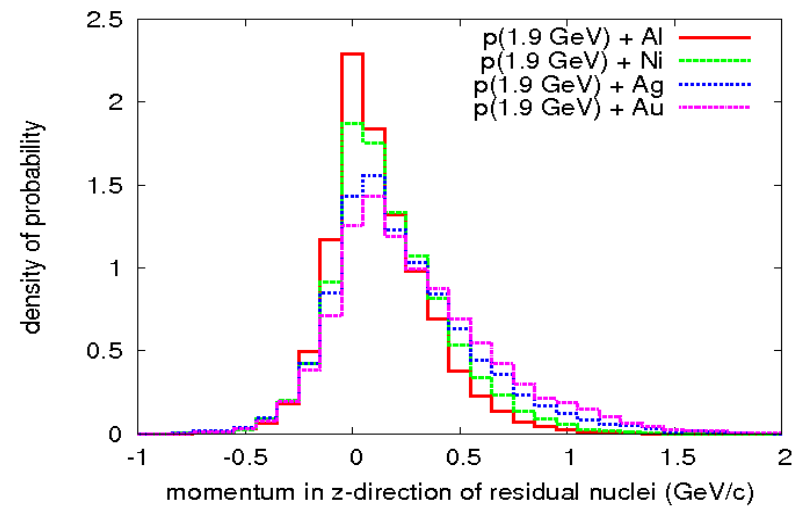
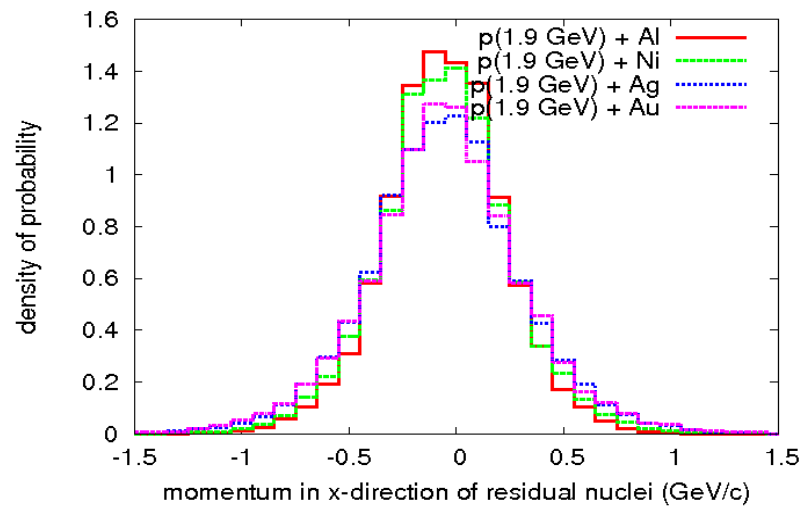
Properties of residual (hot) nuclei

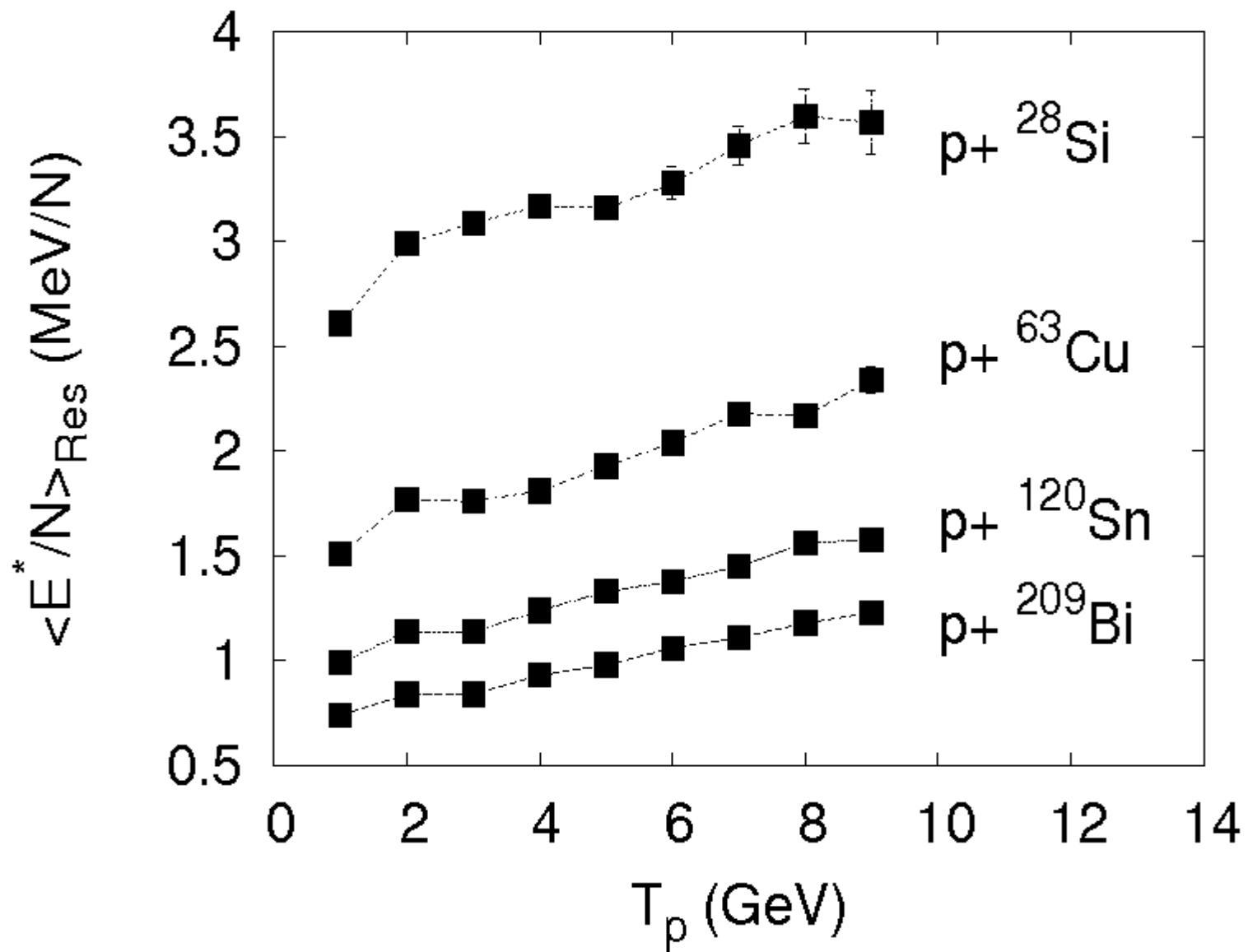


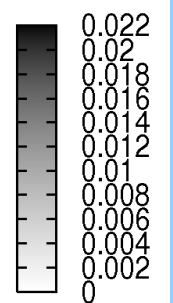
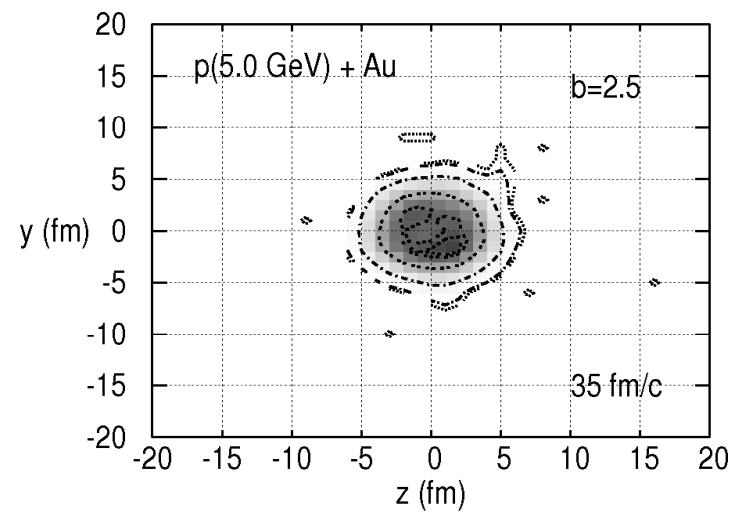
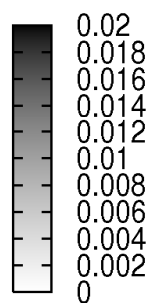
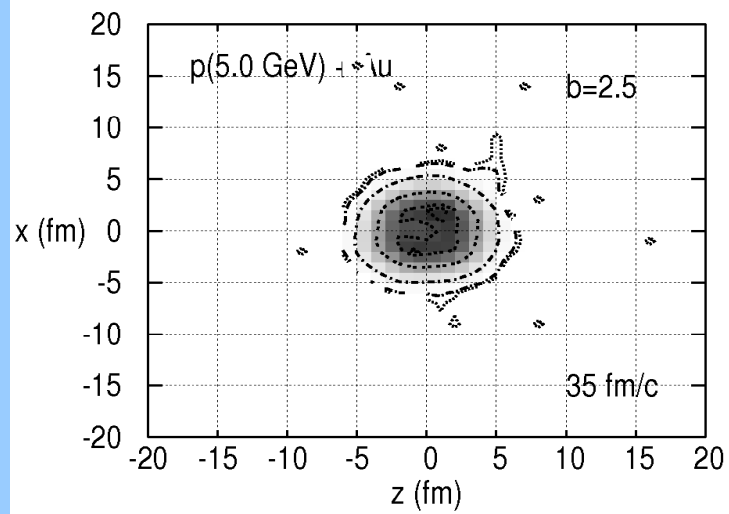
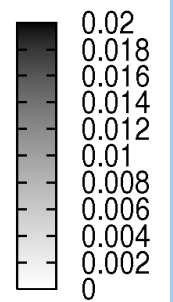
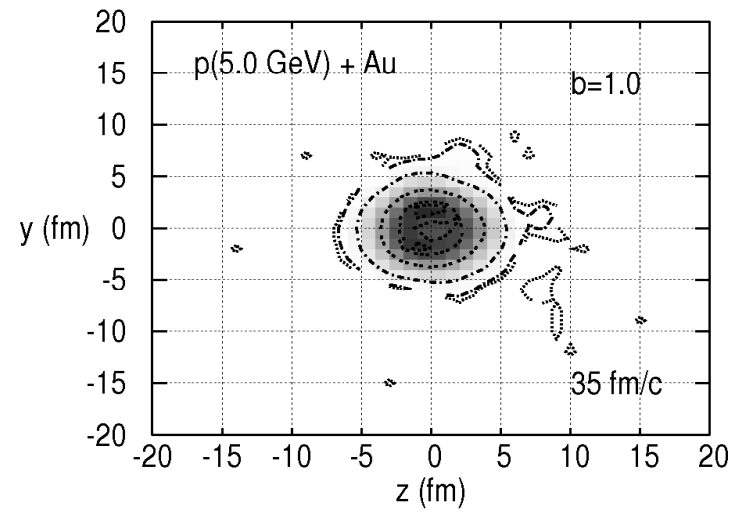
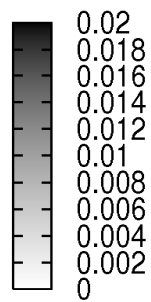
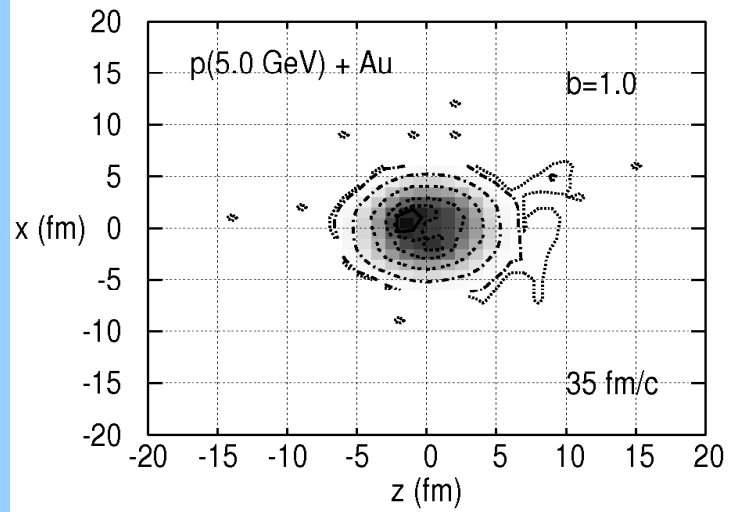
Properties of residual (hot) nuclei

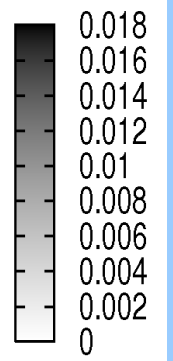
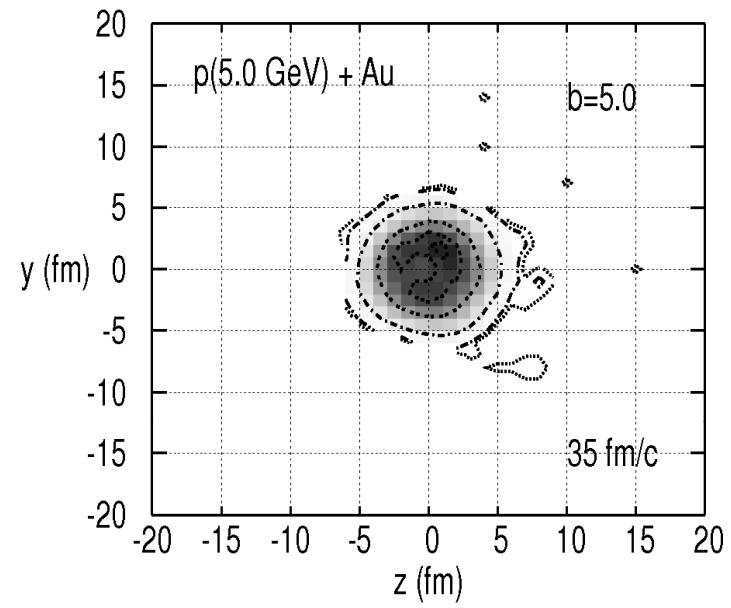
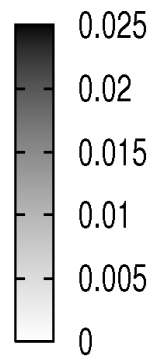
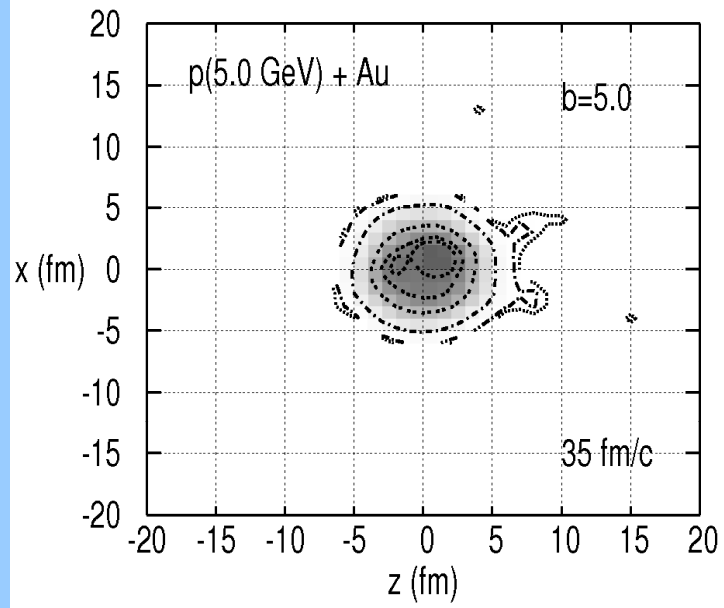


Properties of residual (hot) nuclei

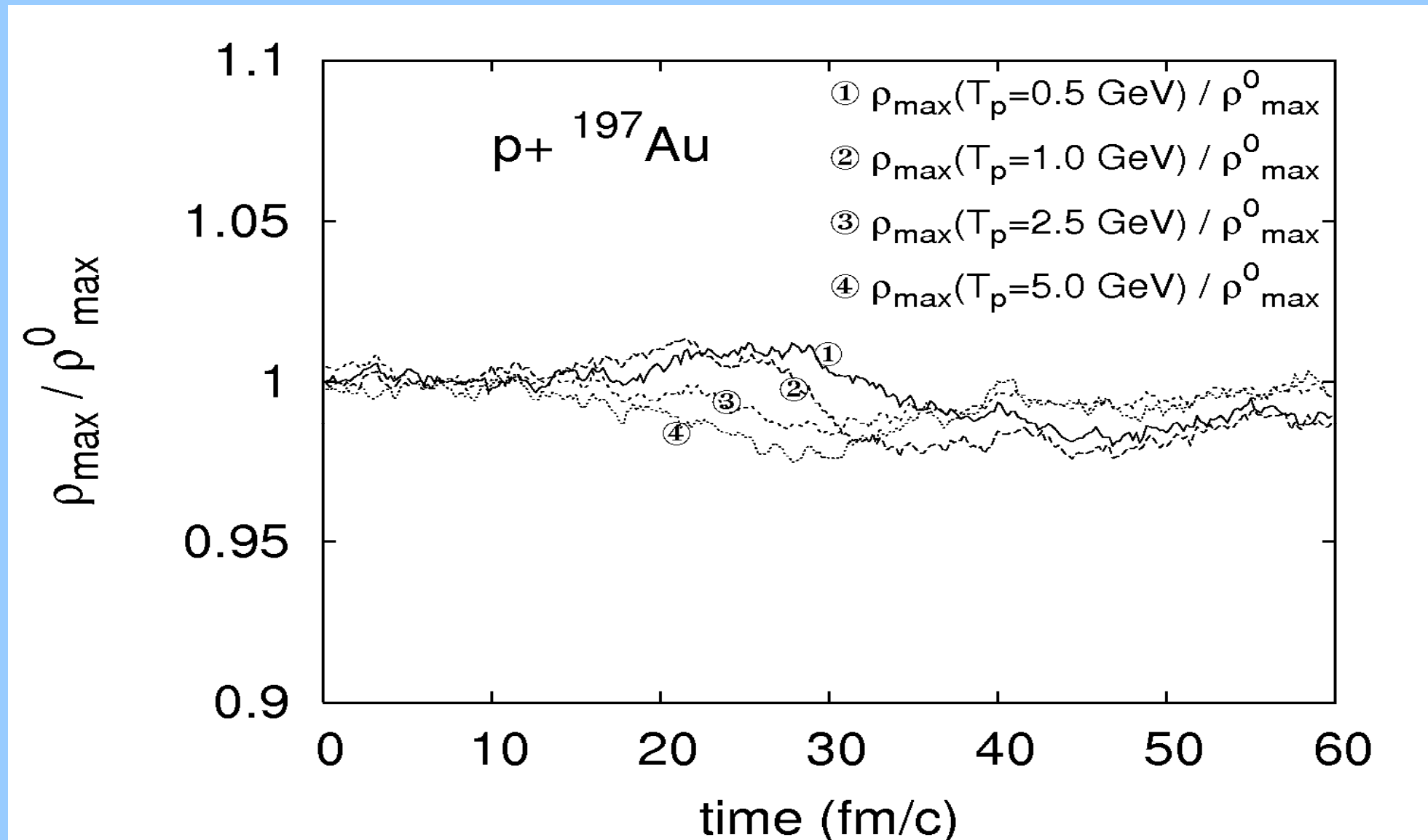




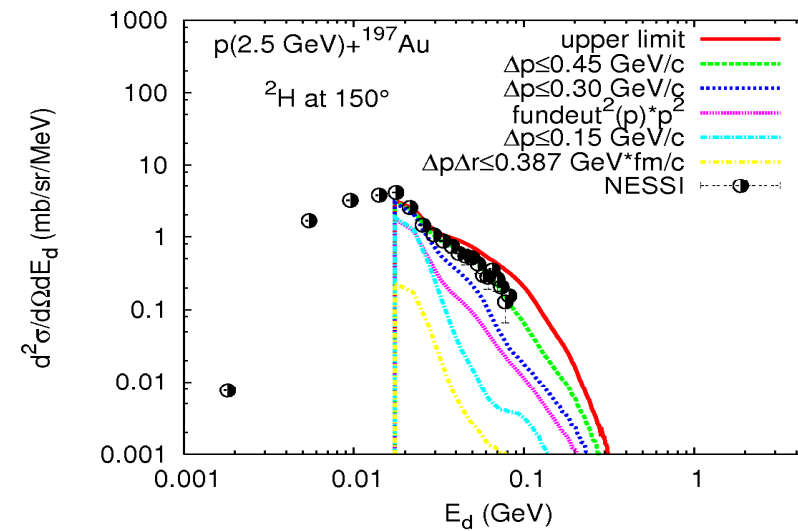
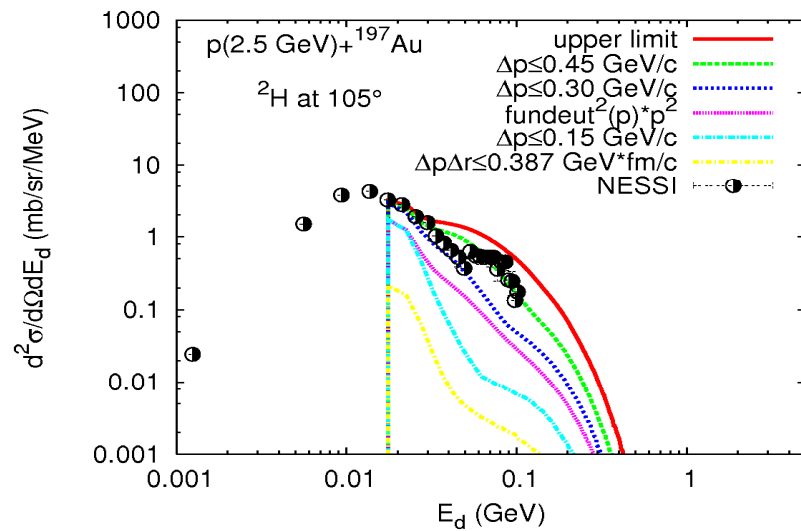
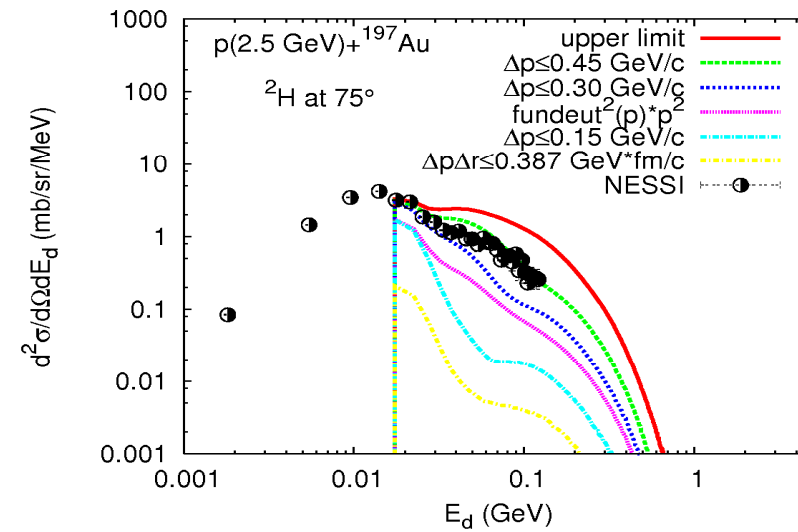
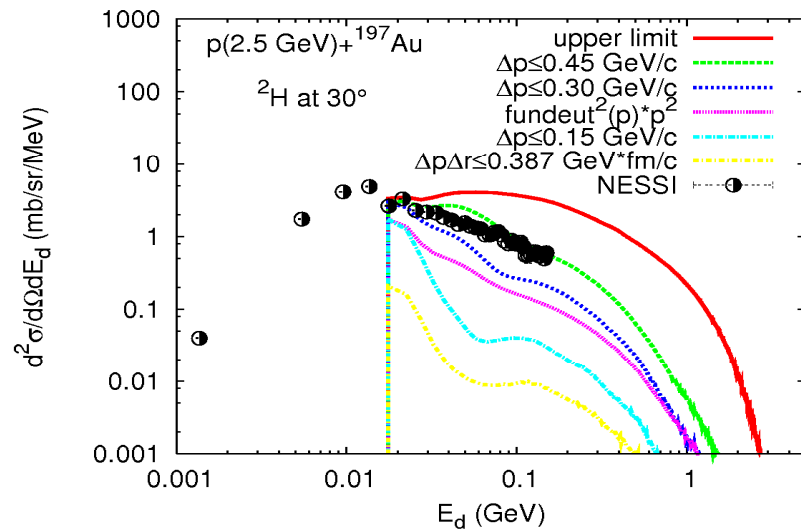




Properties of residual (hot) nuclei



"Coalescence"



BUU + evaporation

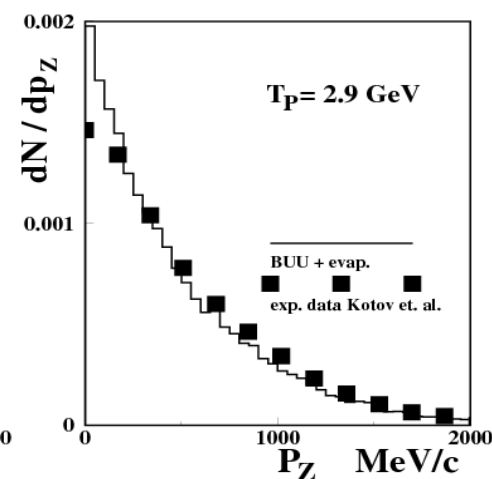
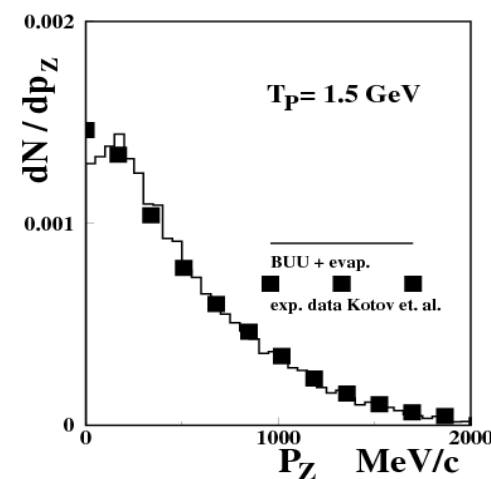
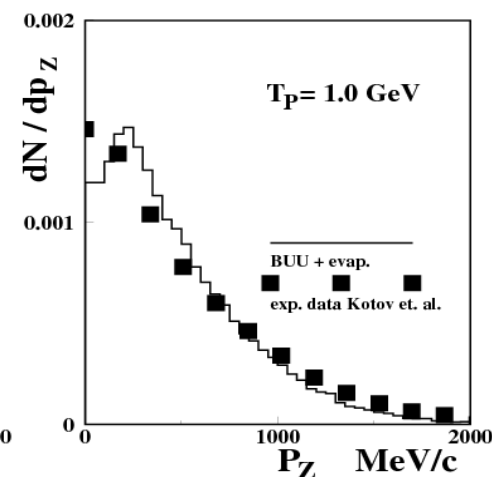
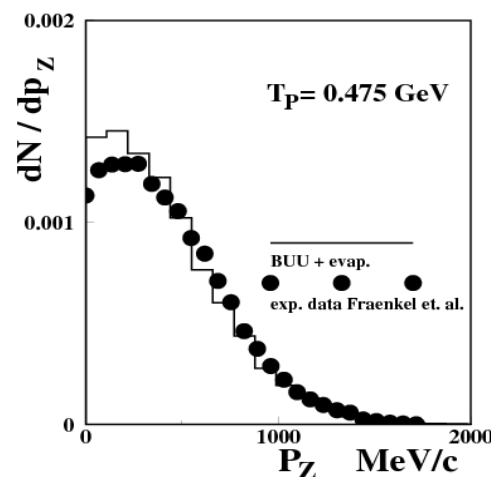
*Momentum in z direction
(i.e. beam direction)
of residual nuclei*

Fraenkel et al.

Phys. Rev. C41, 1050 (1990)

Kotov et al.

Sov. J. Nucl. Phys. 17, 498 (1974)



Conclusions:

- *Two – stage scenario of the proton induced spallation reactions (combined **BUU + GEM** model) gives good description of neutron and proton spectra*
- *In case of other ejectiles H, He, Li, ...*
 - *only low energy part of spectrum is described*
 - *high energy part description needs implementation of coalescence processes into the first stage models*
- *Proton induced spallation as rather non-invasive process*
- *Properties of residual nuclei depend weakly on proton impact energy, strongly on target mass*

