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Proton induced spallation reactions investigated within the framework of BUU model

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The BUU model provides an equation for the phase space density f(**x,p**) of the nucleons, the constituents of the colliding nuclei. Having the phase-space density one can subsequently work out all interesting observables that can later be compared to experimental data. Particles can be scattered into another phase-space cell (leakage) or scattered into considered cell, respectively. The process can be described as process of diffusion of interacting particles-nucleons.

$$\left[\partial_t + (\vec{\nabla}_p U)\vec{\nabla}_x - (\vec{\nabla}_x U)\vec{\nabla}_p\right] f(\vec{x},\vec{p}) =$$

$$\frac{4}{(2\pi)^3}\int d^3p_1 d^3p' d\Omega \; v \; \frac{d\sigma}{d\Omega} \; \delta(\vec{p}+\vec{p}_1-\vec{p}'-\vec{p}_1')$$

$$[\; f(\vec{x},\vec{p}')f(\vec{x},\vec{p}_1')(1-f(\vec{x},\vec{p}))(1-f(\vec{x},\vec{p}_1))$$

$$-(f(\vec{x},\vec{p})f(\vec{x},\vec{p}_1)(1-f(\vec{x},\vec{p}'))(1-f(\vec{x},\vec{p}_1')) \;]$$

This equation depicts the time evolution of the phase space density f(**x, p**) in the presence of the mean field potential U and two-particle collisions. The term on the right side is called collision term. dσ/dΩ is the corresponding two-particle collision cross section. The factors of the form (1-f) in the collision term take the Pauli principle into account - two particles cannot occupy the same phase-space cell. f is normalized so that f=1 means that phase-space is fully occupied. Through these Pauli factors quantum mechanics explicitly is taken into account. Is it sufficient? quantum mechanical interference is not considered in BUU ! Next, a question appears how reliably the Pauli factors are calculated; this will be considered soon.

It is important to point out the different constituents of the BUU-equation. These are:

- The basic structure of the BUU equation is on "responsibility" of statistical physics that acts as a background theory, i.e. the exact form of the BUU equation was obtained using statistical physics method (as discussed e.g. in book of Kerson Huang, "Statistical mechanics", the same equation with collision term set to zero gives for gas in equilibrium Maxwell-Boltzmann distribution,
  $f(v) = n\,(m/(2\pi kT))^{3/2}\ \exp(-mv^2/(2kT))$

- The collision term is modelled in order to respect features of quantum mechanics: Pauli blocking.
- Two components of the BUU model are delivered in by other models:

- 1)Mean-field potential U of the form

$$U(\vec{x},\vec{p}) = A\left(\frac{\rho(\vec{x})}{\rho_0}\right) + B\left(\frac{\rho(\vec{x})}{\rho_0}\right)^{\sigma}$$

is taken from physics of nuclear structure. The parameters A, B, σ describe static properties of the nuclei. One can conclude that it is necessary, indispensable to have good description of the relevant nuclei (i.e. targets in case of proton-induced reactions) at rest. If not, the time evolution of the nuclear system would be invalid.

2) The elementary cross section $d\sigma/d\Omega$ is typically taken from experimental data.

The BUU equation describes the full dynamics of the model system. The equations can not be solved analytically. In order to solve them, one approximates the continuous phase-space density $f(\mathbf{x}, \mathbf{p})$ by a phase space density of a large number of test-particles. Every real nucleon is substituted by N such testparticles; every of the test-particles imitates 1/N of real nucleon.

Nuclear density and phase-space density f(**x,p**) can be calculated using following formulas:

$$\rho(\mathbf{r}_g) = \frac{1}{N}\sum_{i=1}^{NA}\frac{1}{(2\pi\Delta^2)^{3/2}}exp(\frac{-(\mathbf{r}_g-\mathbf{r}_i)^2}{2\Delta^2}$$

$$f(\mathbf{r},\mathbf{p},t)) = \frac{6}{N}\frac{1}{N}\sum_{\mathbf{r}_g}\int d^3p\sum_{j=1}^{NA}\Theta(q_0-\mid \mathbf{p}_j-\mathbf{p}\mid)(2\pi\Delta^2)^{-3/2}exp(\frac{-(\mathbf{r}_g-\mathbf{r}_j)^2}{2\Delta^2})$$

Please notice that here some quantum mechanical effect is simulated – every test particle contributes to nuclear density and to phase-space density in some area rather then in specific point (“Gaussian smearing of test particle”).

The test particles move between collisions according to classical Hamilton equations of motion.

$$\dot{\mathbf{p}}_i = -\nabla_r U(r_i) \quad , \qquad \qquad \dot{\mathbf{r}}_i = \mathbf{p}_i / \sqrt{m^2 + p^2}$$

Please consider that sufficiently large number of test particles is required in order to reproduce e.g. nuclear density (and further Pauli blocking factors).