The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA International Atomic Energy Agency

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Preparatory School to the Winter College on Micro and Nano Photonics for Life Sciences

4 - 8 February 2008

Photonic crystal basics.

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Photonic Crystals Basics

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Outline

Introduction:	Photonic Crystals: definition and History; 1D, 2D,3D Photonic Crystals: examples;
Propagation of e.m waves in periodic media:	Bloch's Theorem, Band Diagrams; Group velocity, band edge effects;
Analysis of 1D Ph. C.	Calculation of photonic bands; Off axis propagation, isofrequency curves; Finite size structures;
2D and 3D structures:	Reciprocal lattice; Brillouin zone, irreducible Brillouin zone; Optical properties of Bulk Ph. C.
Photonic Crystals with Intentional defects:	High Q cavities; Control of spontaneous emission: Bragg waveguides; Photonic crystal waveguides; Photonic crystal fibers

Photonic Crystals

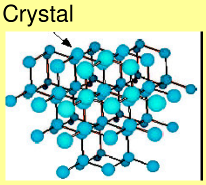
Periodic dielectric structures that can interact resonantly with radiation with wavelengths comparable to the periodicity length of the dielectric lattice.

WHY ARE THEY CALLED PHOTONIC CRYSTALS?

Can affect the properties of photons in much the same way that ordinary semiconductor and conductor crystals affect the properties of electrons.
Properties of electrons in ordinary crystals are affected by parameters like lattice size and defects, for example. Crystals are made of periodic arrays of atoms at atomic length scales.

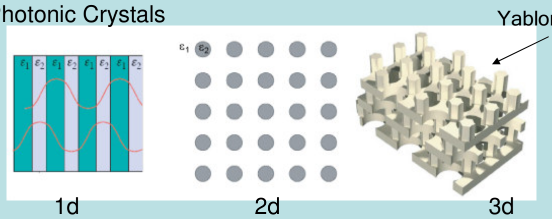
HOW DO THEY LOOK?

Crystal



Photonic Crystals

Yablonovite



1d 2d 3d

From Bragg Gratings to Photonic Crystals in 5 Steps

1785: The first man-made diffraction grating was made by David Rittenhouse, who strung hairs between two finely threaded screws.

1913: Bragg formulation of X-ray diffraction by cristalline solids

1928: Bloch's Theorem describe the conduction of electrons in crystalline solids. (Math developed by Floquet in 1D case in 1883)

1976: A.Yariv and P. Yeh, study of dielectric multilayer stacks, waveguides and bragg fibers.

1987: <u>Prediction of photonic crystals</u>

S. John, Phys. Rev. Lett. 58,2486 (1987), "*Strong localization of photons in certain dielectric superlattices*"
E. Yablonovitch, Phys. Rev. Lett. 58 2059 (1987), "*Inhibited spontaneous emission in solid state physics and electronics*"

Suggested readings

Websites:

http://www.pbglink.com **for Extensive web listing of PBG information**
http://ab-initio.mit.edu/mpb **MIT photonic- band package**
http://homepages.udayton.edu/~sarangan **Dayton Univ. Lectures on Photonic crystals**

Books:

Photonic Crystals: Towards Nanoscale Photonic Devices, J-M Lourtioz et al. Springer (2003)

Roadmap on Photonic Crystals by Susumu Noda (Editor), Toshihiko Baba (Editor), Kluwer Academic Publishers; (2003).

Nonlinear Photonic Crystals by R. E. Slusher and B. J. Eggleton Eds., (2003).

Photonic Crystals: The Road from Theory to Practice: S. G. Johnson and J. D.Joannopoulos, Kluwer (2002).

Optical Properties of Photonic Crystals K. Sakoda, Springer (2001).

Quantum Electronics 3° edition A. Yariv, Wiley (1989).

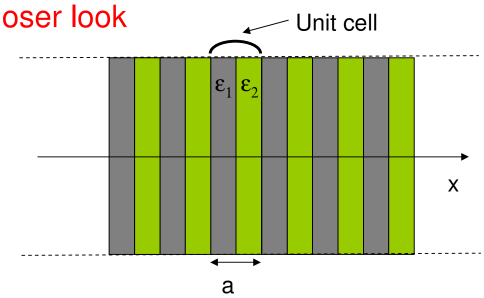
Optical waves in Layered media Pochi Yeh, Wiley (1988).

1-D Photonic Crystals

a closer look

The dielectric constant is periodically modulated in one dimension:

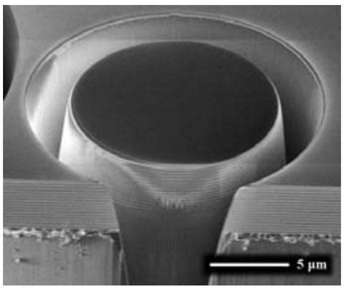
$$\varepsilon(\vec{r}) = \varepsilon(x) = \varepsilon(x + na);$$
$$n = 0, \pm 1, \pm 2, \ldots$$



From cartoons to pictures….

A scanning electron microscopy image of a tiny dielectric mirror being cut from a larger piece of material.

The alternating layers of Ta_2O_5 and SiO_2 which form the mirror are clearly visible.

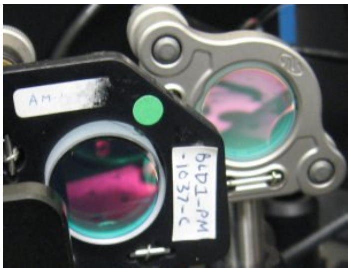


1-D Photonic Crystals

a closer look

According to their definition, photonic crystals can interact resonantly with radiation with wavelengths comparable to the periodicity length of the dielectric lattice.

Broadband dielectric mirrors used in optics experiments. The characteristic color of the mirrors is caused by their wavelength dependent reflectivity.



2-D Photonic Crystals

a closer look

The dielectric constant is periodically modulated in two dimensions:

$$\varepsilon(\vec{r}) = \varepsilon(x, y) = \varepsilon(x + na_x, y + ma_y);$$
$$n, m = 0, \pm 1, \pm 2, \ldots$$

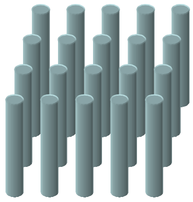
A cylinder is put at every lattice point.
Any **Bravais** 2-D lattice is defined by 2 primitive vectors, for the square lattice they are:

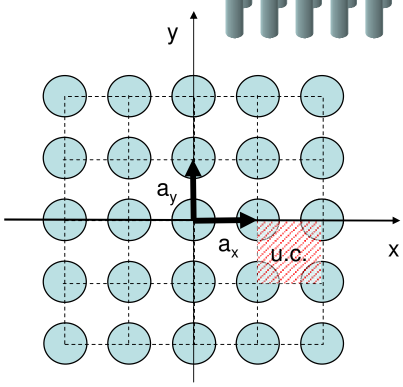
$$\vec{a}_1 = a_x \hat{x};$$
$$\vec{a}_2 = a_y \hat{y};$$

Thus every lattice point can be written as:

$$\vec{R} = n\vec{a}_1 + m\vec{a}_2;$$
$$n, m = 0, \pm 1, \pm 2, \ldots$$

Square lattice of cylindrical rods





2-D Photonic Crystals

a closer look

Primitive unit cell: Unit Cell: a volume (if 3D) space that, when translated through all the vectors of a Bravais lattice, just fills all of space without either overlapping itself or leaving voids.

There is no unique way of choosing a primitive cell for a given Bravais lattice.

Obvious primitive cell can be defined starting from the primitive vectors:

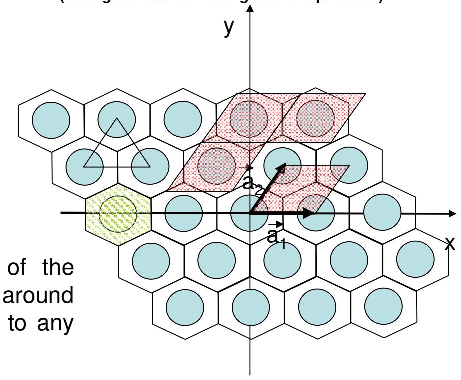
$$\vec{a}_1; \quad \varepsilon(\vec{r}) = \varepsilon(\vec{r} + n\vec{a}_1 + m\vec{a}_2);$$
$$\vec{a}_2; \quad n, m = 0, \pm 1, \pm 2, \ldots$$

With $\vec{r}$ belonging to the primitive cell

Wigner-Seitz primitive cell

Primitive cell with the full symmetry of the Bravais lattice. It is the region of space around a lattice point, closer to that point than to any other lattice point

Rhombic lattice or isosceles triangular lattice
(triangular lattice if triangles are equilateral)



2-D Photonic Crystals

a closer look

Living example of a 2D Photonic Crystal, the “Sea Mouse” spine.

Fig. 1: A sea mouse or *Aphrodita* (note the iridescent felt on the edge of its body). From Sue Daly, *Marine Life of the Channel Islands*, 1998 (with permission).

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Structural colours through photonic crystals

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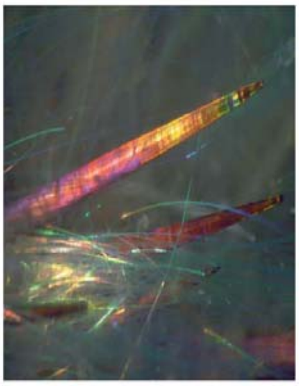
2-D Photonic Crystals

a closer look



- Electronmicrographs of a spine and a felt hair. Both contain close packed voids of water in chitin (refractive index 1.52). The spine has 88 layers of holes with a spacing of 0.51 μm.

3-D Photonic Crystals

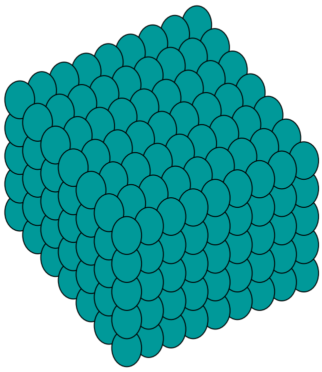
a closer look

The dielectric constant is periodically modulated in three dimensions.
Lattice is defined by 3 primitive vectors:

$$\vec{a}_1;$$
$$\vec{a}_2;$$
$$\vec{a}_3; \qquad \vec{R} = n\vec{a}_1 + m\vec{a}_2 + l\vec{a}_3;$$
$$n, m, l = 0, \pm 1, \pm 2, \ldots$$

$$\varepsilon(\vec{r}) = \varepsilon(\vec{r} + n\vec{a}_1 + m\vec{a}_2 + l\vec{a}_3);$$
$$n, m, l = 0, \pm 1, \pm 2, \ldots$$

With $\vec{r}$ belonging to the primitive cell

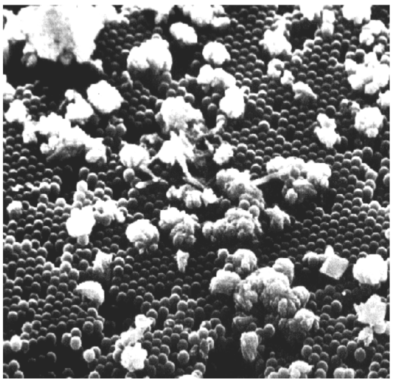


Reference: Ashcroft & Mermin, Solid State Physics, Saunders College Publishing for details on 3D Bravais lattices and crytal symmetries

3-D Photonic Crystals

a closer look

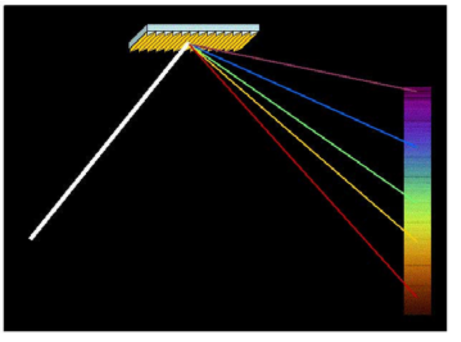
At the micro scale precious opal is composed of silica(SiO_2) spheres some 150 to 300 nm in diameter in a hexagonal or cubic closed packed lattice. These ordered silica spheres produce the internal colors by causing the interference and diffraction of light passing through the microstructure of opal



Propagation of e.m waves in periodic media:

Scattering of light in periodically patterned media

Periodicity comparable to incident light wavelengths can be responsible for constructive interferences among scattered light by every lattice point. The phenomenon is exactly the same as it happens in diffraction gratings.



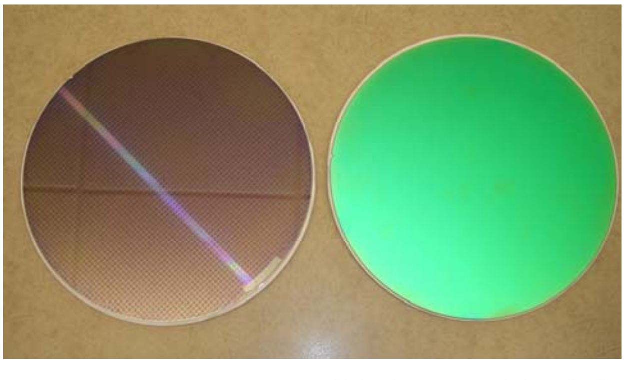
The tracks of a compact disc act as a diffraction grating, producing a separation of the colors of white light. The nominal track separation on a CD is 1.6 micrometers, corresponding to about 625 tracks per millimeter. This is in the range of ordinary laboratory diffraction gratings. For red light of wavelength 600 nm, this would give a first order diffraction maximum at about 22°.

Periodically patterned media
Vs.
Random media

http://my.fit.edu/~phsu/

Si wafer top side with devices - patterned roughness.

wafer back side with random roughness



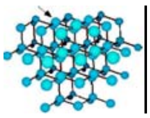
Resonant reflectivity at given incident angle and wavelengths

Light is scattered in all directions, there is no evidence of frequency or angle dependent response induced by random roughness

Introduction to Bloch's Theorem

waves in a periodic medium can propagate without scattering

Electrons



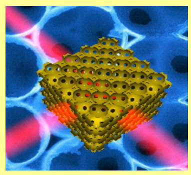
If the atoms were randomly placed the free electrons would experience strong scattering with the lattice atoms and a short mean free path is expected.

This is in contrast with the high value of measured conductivity for some crystals.

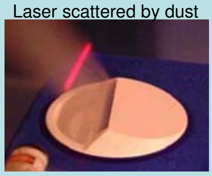
But crystal lattice are periodic with typical sizes of a few Å and electrons can be treated as waves (quantum mechanics) with energies corresponding to a typical wavelength comparable to lattice size.

Photons

Light scattered from a random media: The effect can be weak or strong depending on density of scatterers. Size and shape of the scatterers produce wavelength and angular dependence on the efficiency of scattering



Laser scattered by dust



Photonic crystals are characterized by primitive cells of a size comparable to the wavelength of the photon. Resonant scattering can occur as a function of frequency and wavevector.

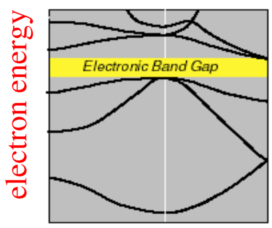
Introduction to Bloch's Theorem

Electrons

As a results electrons **can propagate in a periodic lattice** without experiencing scattering with a proper **dispersive relation** (energy as a function of the wave vector)

Band diagrams can be calculated using Bloch theorem

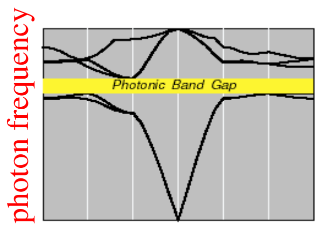
NOTE forbidden region called GAPs. Electrons with forbidden energies cannot propagate inside the crystals.



Photons

Periodic modulation of the dielectric constant can affect the properties of photons in much the same way that ordinary semiconductor and conductor crystals affect the properties of electrons.

For certain geometries it exists a range of frequencies called GAP for which light is forbidden to exist inside the crystal



Bloch's Theorem

for e.m waves in photonic crystals

Starting point: Maxwell equations and constitutive relations.

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t},$$
$$\nabla\times\vec{H} = \frac{\partial\vec{D}}{\partial t} + \cancel{\vec{j}},$$
$$\nabla\cdot\vec{D} = \cancel{\rho},$$
$$\nabla\cdot\vec{B} = 0.$$

If the e.m field is time-harmonic and if the bodies are at rest or in very slow motion relative to each other:

$$\vec{j} = \sigma\vec{E}, \quad \vec{D} = \varepsilon_0\hat{\varepsilon}_r\vec{E}, \quad \vec{B} = \mu_0\hat{\mu}_r\vec{H}.$$

Neutral Insulators or dielectrics: ρ=0
σ**(conductivity) is negligibly small;** non magnetically active μ_r**=1;**

If we seek solutions of the form: $\vec{E}(\vec{r},t) = \vec{E}(\vec{r})e^{-i\omega t}; \quad \vec{H}(\vec{r},t) = \vec{H}(\vec{r})e^{-i\omega t};$

We have….

Ashcroft & Mermin, Solid State Physics, Saunders College Publishing for a proof of the theorem

Bloch's Theorem

for e.m waves in photonic crystals

…the following eigenvalue equations:

$$\hat{\Gamma}_E\vec{E}(\vec{r}) = \frac{1}{\varepsilon_r(\vec{r})}\vec{\nabla}\times\left\{\vec{\nabla}\times\vec{E}(\vec{r})\right\} = \frac{\omega^2}{c^2}\vec{E}(\vec{r}),$$
$$\hat{\Gamma}_H\vec{H}(\vec{r}) = \vec{\nabla}\times\left\{\frac{1}{\varepsilon_r(\vec{r})}\vec{\nabla}\times\vec{H}(\vec{r})\right\} = \frac{\omega^2}{c^2}\vec{H}(\vec{r}),$$

If ε_r is a periodic function of the spatial coordinate Bloch's Theorem states that fields solutions are characterized by a **Bloch wave vector K**, a band index n and have the form:

$$\vec{E}(\vec{r}) = \vec{E}_{\vec{K}n}(\vec{r})e^{i\vec{K}\cdot\vec{r}}; \qquad \vec{H}(\vec{r}) = \vec{H}_{\vec{K}n}(\vec{r})e^{i\vec{K}\cdot\vec{r}};$$

with:

$$\varepsilon_r(\vec{r}) = \varepsilon_r(\vec{r}+\vec{a}_i);$$
$$\vec{E}_{\vec{K}n}(\vec{r}) = \vec{E}_{\vec{K}n}(\vec{r}+\vec{a}_i); \qquad i=1,2,3.$$
$$\vec{H}_{\vec{K}n}(\vec{r}) = \vec{H}_{\vec{K}n}(\vec{r}+\vec{a}_i);$$

$\vec{a}_1;$ $\vec{a}_2;$ $\vec{a}_3;$ Primitive vectors of the periodic lattice

Bloch's Theorem

for e.m waves in photonic crystals

Substituting Bloch's solution into eigenvalue equations it is possible to calculate:

Eigen-angular frequencies: $\omega_{\vec{K}n}$;

Bloch modes (eigenvectors): $\vec{E}_{\vec{K}n}(\vec{r})$; $\vec{H}_{\vec{K}n}(\vec{r})$;

If we consider infinite periodic structures, real values of ε (i.e lossless case), Γ_E and Γ_H are Hermitian eigen-operators:

$$\omega_{\vec{K}n} \in R$$

$\vec{E}_{\vec{K}n}(\vec{r})$; $\vec{H}_{\vec{K}n}(\vec{r})$; Are a complete set of orthogonal eigen-functions.

Solving equations for several values of K and w it is possible to calculate and plot band diagrams (dispersion relations)

Band Diagrams

Some Computational tools:

Plane Wave Expansion (PWE) Method -CPU time demanding and poor convergence

Koringa-Kohn-Rostker (KKR) Method - spherical waves expansion

Transfer matrix method - Developed by Pendry's group at Imperial College. It also provides amplitude and phase information.
- Rigorous Coupled-wave analysis RCWA

Dispersive properties:

Group velocity=Energy velocity (proof: Yariv's Optical waves in crystals)

$$\vec{v}_g = \vec{\nabla}\omega(\vec{K})\Big|_{\vec{K}=\vec{K}_0}$$

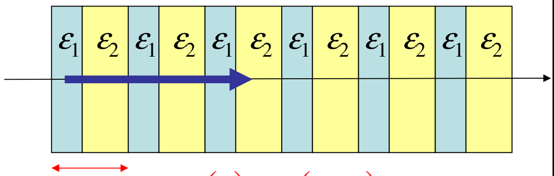
Phase velocity CANNOT be defined appropriately in photonic crystals because eigenfunctions are superposition of plane waves and equiphase surfaces cannot be defined properly. Nevertheless, in the effective medium approximation, long wavelengths with respect to lattice periodicity the effective phase velocity can be defined as:

$$\vec{v}_\varphi = \frac{\omega(\vec{K}_0)}{|K_0|}\hat{K}_0$$

Analysis of 1D photonic crystals:

Analysis of 1D Photonic crystals

Also called Bragg gratings since 1887



$$\varepsilon(x) = \varepsilon(x+a)$$

For 1D systems we can write the field as:

$$\vec{E}(x) = \boxed{\vec{E}_K(x)} e^{iKx} \qquad \text{with} \quad \vec{E}_K(x) = \vec{E}_K(x+ma); \qquad m = 0, \pm 1, \pm 2, \ldots$$

But what happens if we consider the Bloch mode with a wavevector K'=K+2*π/a?

$$\vec{E}(x) = \boxed{\vec{E}_{K'}(x) e^{i\frac{2\pi x}{a}}} e^{iKx}$$

Periodic function satisfying the same conditions as

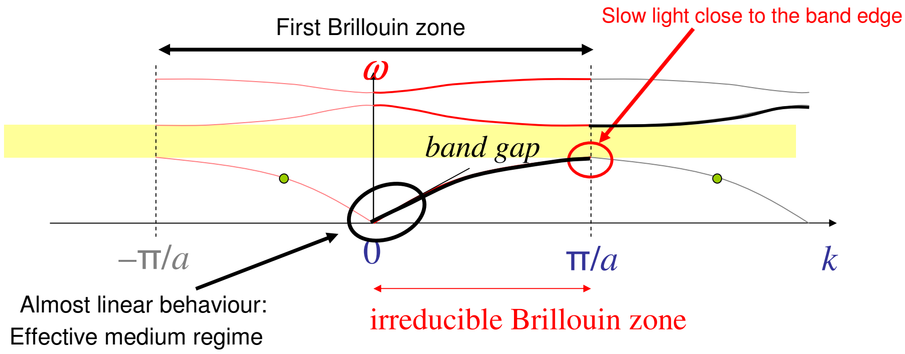
$$\vec{E}_K(x) = \vec{E}_K(x+ma);$$

Analysis of 1D Photonic crystals

k is periodic:

$k + 2\pi/a$ equivalent to k

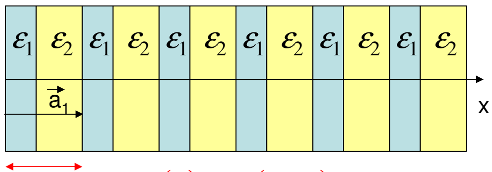
"quasi-phase-matching"



Analysis of 1D Photonic crystals

1D lattice has only one primitive vector

$\vec{a}_1 = a\ \hat{x};$



$\varepsilon(x) = \varepsilon(x+a)$

Being a periodic function, the dielectric constant can be expanded as a Fourier serie

$$\varepsilon(x) = \sum_{G=n2\pi/a} e^{iGx}\tilde{\varepsilon}(G), \quad n = 0, \pm 1, \pm 2, ...$$

The set of values of G are called the **reciprocal lattice** vectors:

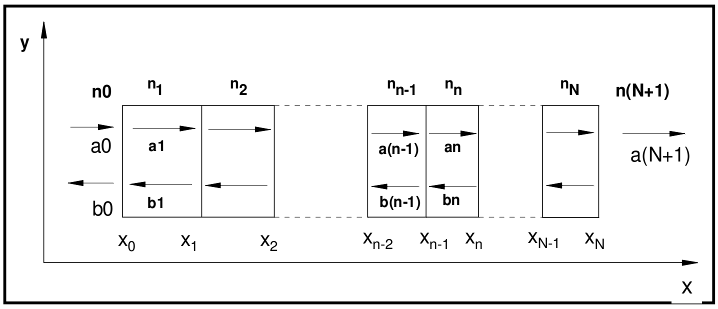
NOTE: the primitive cell of the reciprocal lattice is $g=2\pi/a$

Dispersion curves are periodic with a unit cell = $2\pi/a$ called Brillouin zone

The Brillouin zone is a primitive cell of the reciprocal lattice

Analysis of 1D Photonic crystals

Calculating photonic bands



For simplicity we consider only TE polarized waves.
In the *l*-th layer the field at angular frequency w can be written as:

$$E_z(x,y,t) = \left[a_l e^{ik_x^l(x-x_l)} + b_l e^{-ik_x^l(x-x_l)}\right] e^{ik_y y} e^{-i\omega t}$$

Where k_y is a conserved quantity in every layer and $k^l = \frac{\omega}{c} n_l; \;\; k_x^l = \sqrt{(k^l)^2 - k_y^2};$

Analysis of 1D Photonic crystals

Calculating photonic bands

Thus the field in x_l can be described by the following vector:

$$\begin{pmatrix} a_l \\ b_l \end{pmatrix} \qquad l = 0,1,\ldots,N$$

With this field's representation and imposing continuity of E_z and H_y at every interface it can be shown that: **(see Optical waves in layered media by Pochi Yeh edited by Wiley & Sons)**

$$\begin{pmatrix} a_{l-1} \\ b_{l-1} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 + k_x^l/k_x^{l-1} & 1 - k_x^l/k_x^{l-1} \\ 1 - k_x^l/k_x^{l-1} & 1 + k_x^l/k_x^{l-1} \end{pmatrix} \begin{pmatrix} e^{-ik_x^l d_l} & 0 \\ 0 & e^{ik_x^l d_l} \end{pmatrix} \begin{pmatrix} a_l \\ b_l \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} a_l \\ b_l \end{pmatrix}$$

Back propagation through the l-layer

Jump at the interface from *l* to *l*-1

Analysis of 1D Photonic crystals

Calculating photonic bands

For periodic structures with a unit cell composed of 2 layers, the transfer matrix of a unit cell, relating the fields at the beginning of a unit cell with the field at the next cell is:

$$\begin{pmatrix} a_{n-1} \\ b_{n-1} \end{pmatrix} = \begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix} \begin{pmatrix} A_2 & B_2 \\ C_2 & D_2 \end{pmatrix} \begin{pmatrix} a_n \\ b_n \end{pmatrix} = \hat{M} \begin{pmatrix} a_n \\ b_n \end{pmatrix}$$

* The index n refers to the unit cell number

Transfer matrix of the unit cell

Transfer matrix of the layer composed by material 2

Transfer matrix of the layer composed by material 1

Next step: Use Bloch's theorem to calculate bands!

Analysis of 1D Photonic crystals

Calculating photonic bands

According to Bloch's Theorem the field in the (*n*)-th unit cell can be written as:

$$\vec{E}(x+(n-1)a) = \vec{E}_K(x)e^{iKx}e^{iK(n-1)a} \qquad 0 \le x < a$$

The field in the (*n-1*)-th unit cell can be written as:

$$\vec{E}(x+(n-2)a) = \vec{E}_K(x)e^{iKx}e^{iK(n-1)a}e^{-iKa} = \vec{E}(x+(n-1)a)e^{-iKa}$$

The field at the first layer of the (n)-th unit cell is related to the field at the first layer of the (n-1)-th unit cell by the relation:

$$\begin{pmatrix} a_{n-1} \\ b_{n-1} \end{pmatrix} = e^{-iKa} \begin{pmatrix} a_n \\ b_n \end{pmatrix}$$

But also:

$$\begin{pmatrix} a_{n-1} \\ b_{n-1} \end{pmatrix} = \hat{M} \begin{pmatrix} a_n \\ b_n \end{pmatrix}$$

Analysis of 1D Photonic crystals

Calculating photonic bands

Thus we have an eigenvalue problem:

$$\begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}\begin{pmatrix} a_n \\ b_n \end{pmatrix} = e^{-iKa}\begin{pmatrix} a_n \\ b_n \end{pmatrix},$$

So we can calculate eigen values and eigenvectors:

$$\begin{pmatrix} a_n \\ b_n \end{pmatrix} = e^{inKa}\begin{pmatrix} M_{12} \\ e^{-iKa} - M_{11} \end{pmatrix}$$

and

$$K(\mathrm{k}_y,\omega) = \frac{1}{a}\arccos\left[\frac{1}{2}(M_{11}+M_{22})\right]$$

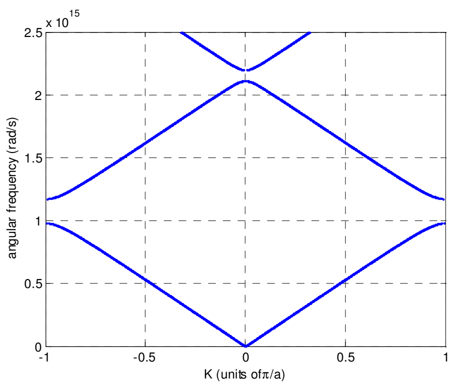
If:

$|(M_{11}+M_{22})|<2$; $K \in R$ $\rightarrow$ propagating Bloch wave

$|(M_{11}+M_{22})|>2$; $K = m\pi/a + iK_i$ $\rightarrow$ evanescent Bloch wave

Analysis of 1D Photonic crystals

Calculating photonic bands



Onedimsimple.m

Matlab code developed using the model previously described. Codes are available.

Analysis of 1D Photonic crystals

Calculating photonic bands

Off axis propagation in one dimensional photonic crystals

Consider the plane x,y. and e.m. wave propagating with an angle with respect to the direction normal to interfaces.

K_y component is conserved because it is parallel to the interfaces.

$$K_y = \frac{\omega}{c} n_1 \sin(\theta_1) = \frac{\omega}{c} n_2 \sin(\theta_2);$$

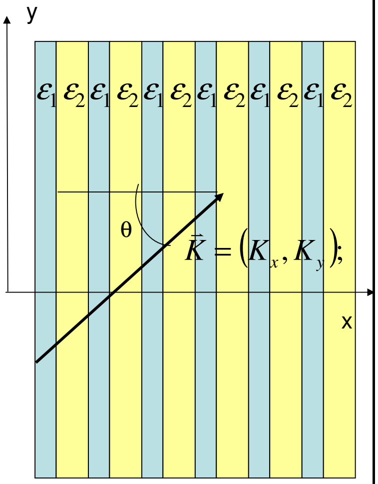
Electric field can be written as:

$$\vec{E}(x,y) = \vec{E}_{K_x}(x) e^{iK_x x} e^{iK_y y}$$

Bloch mode for 1D PhC

Bloch wavevector

$$\vec{K} = (K_x, K_y);$$



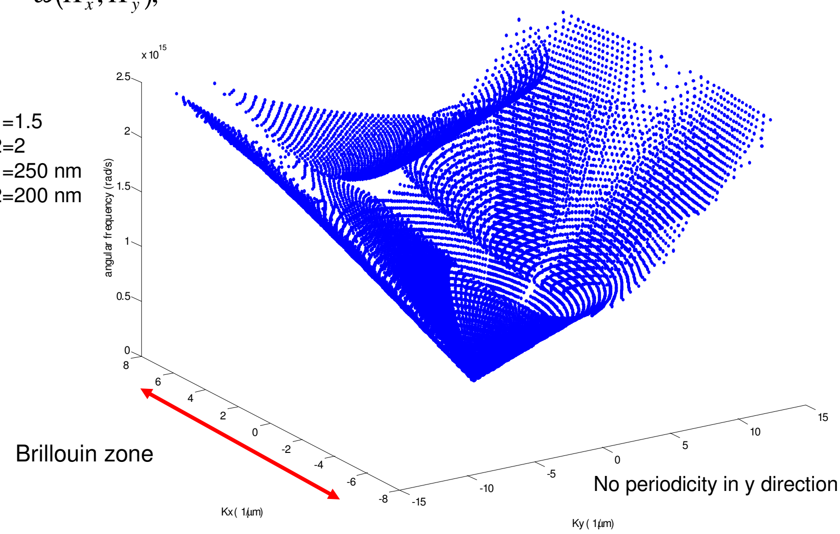
a $\varepsilon(x) = \varepsilon(x+a)$

Analysis of 1D Photonic crystals

Calculating photonic bands

$$\omega = \omega(K_x, K_y);$$

n1=1.5
n2=2
d1=250 nm
d2=200 nm

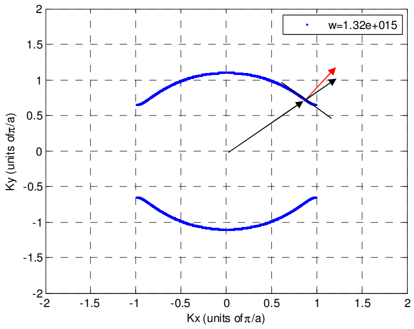


Analysis of 1D Photonic crystals

Calculating photonic bands

Isofrequency curves: slices of $\omega(K_x,K_y)$=cost.

The group velocity (or energy velocity) is orthogonal to the isofrequency curves



$$\vec{v}_g = \vec{\nabla}\omega(\vec{K})\Big|_{\vec{K}=\vec{K}_0}$$

$\boxed{\vec{v}_g}$ and $\boxed{\vec{K}}$ are not parallel

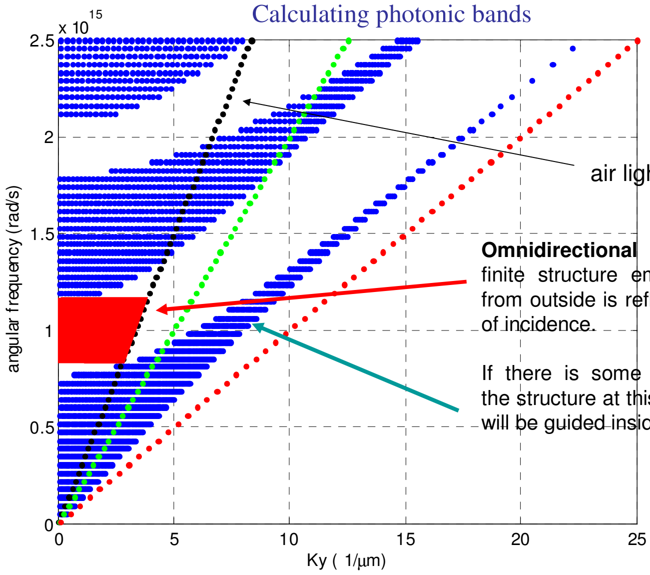
Anisotropy induced by stratification in one direction. For 2D cases strong curvatures of dispersion curves leads to **superprism effects** and **negative refraction.**

Onedimband.m

Matlab code developed using the model previously described. Codes are available.

Analysis of 1D Photonic crystals

Calculating photonic bands



n1=1.5
n2=3
d1=250 nm
d2=200 nm

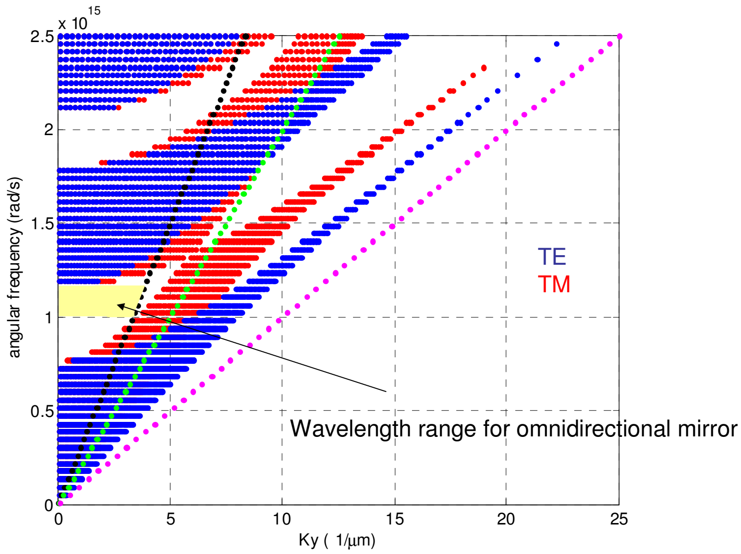
Omnidirectional mirror: Consider a finite structure embedded in air,light from outside is reflected at any angles of incidence.

If there is some fluorescence inside the structure at this frequency range, it will be guided inside the layers

NOTE: a True omnidirectional mirror should work for both TE and TM polarization. Dispersion curves for TM polarized field should be calculated and overlapped to the dispersion curves for TE pol.

Analysis of 1D Photonic crystals

Calculating photonic bands



Onedimguide.m

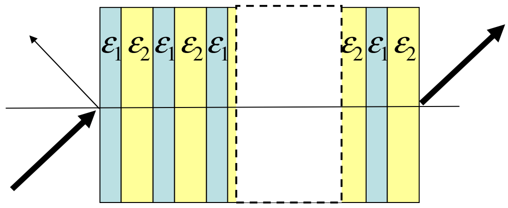
Finite Size 1D Photonic Crystals

Consider a structure made of N periods embedded in air:

For a a given incident plane monochromatic wave we want to calculate transmitted field and reflected field.

WE use the transfer matrix method:

$$\begin{pmatrix} a_{n-1} \\ b_{n-1} \end{pmatrix} = \hat{M} \begin{pmatrix} a_n \\ b_n \end{pmatrix}$$



a $\varepsilon(x) = \varepsilon(x+a)$

In this case we cannot invoke Bloch's Theorem because the translational symmetry is broken. Nevertheless we can take advantage of the following relation:

$$\begin{pmatrix} a_N \\ b_N \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 + k_x^{out} / k_x^{\varepsilon_2} & 1 - k_x^{out} / k_x^{\varepsilon_2} \\ 1 - k_x^{out} / k_x^{\varepsilon_2} & 1 + k_x^{out} / k_x^{\varepsilon_2} \end{pmatrix} \begin{pmatrix} \boxed{a_{out}} \\ 0 \end{pmatrix}$$

At the end of the structure there is only the transmitted field

Finite Size 1D Photonic Crystals

Going back to the input:

Unitary incident field

$$\begin{pmatrix}1\\ r\end{pmatrix}=\frac{1}{2}\begin{pmatrix}1+k_x^{\varepsilon_1}/k_x^{in} & 1-k_x^{\varepsilon_1}/k_x^{in}\\ 1-k_x^{\varepsilon_1}/k_x^{in} & 1+k_x^{\varepsilon_1}/k_x^{in}\end{pmatrix}\hat{P}_1\hat{D}_{21}\hat{P}_2\hat{M}^{N-1}\frac{1}{2}\begin{pmatrix}1+k_x^{out}/k_x^{\varepsilon_2} & 1-k_x^{out}/k_x^{\varepsilon_2}\\ 1-k_x^{out}/k_x^{\varepsilon_2} & 1+k_x^{out}/k_x^{\varepsilon_2}\end{pmatrix}\begin{pmatrix}t\\ 0\end{pmatrix}$$

Reflected field

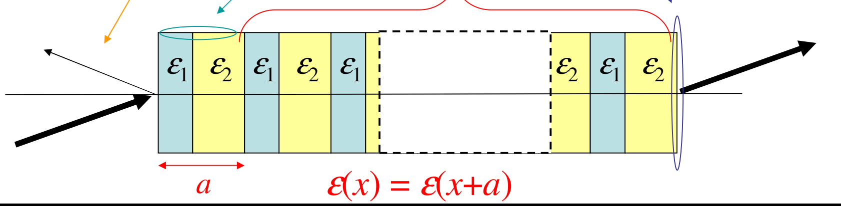
n1 material→input

First unit cell

Transfer matrix of the unit cell applied N-1 times

out→n2 material

Transmitted field



Finite Size 1D Photonic Crystals

Gap for the infinite structure
Transmittance for N=5 and N=10 periods
Normal incidence

n1=1.5;
n2=2.5;
d1=250 nm
d2=400 nm

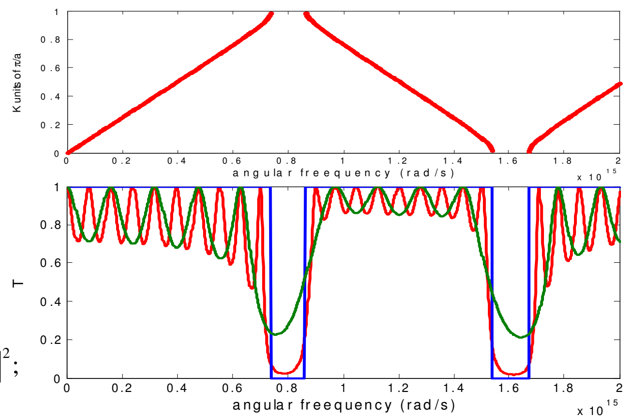
nin=1;
nout=1;

$$\begin{pmatrix}1\\ r\end{pmatrix}=\begin{pmatrix}A & B\\ C & D\end{pmatrix}\begin{pmatrix}t\\ 0\end{pmatrix}$$

$$t=\frac{1}{A};\quad r=\frac{C}{A};$$

$$T=\frac{I_{out}}{I_{in}};$$

$$T=\frac{n_{out}\cos(\theta_{out})}{n_{in}\cos(\theta_{in})}|t|^2;$$



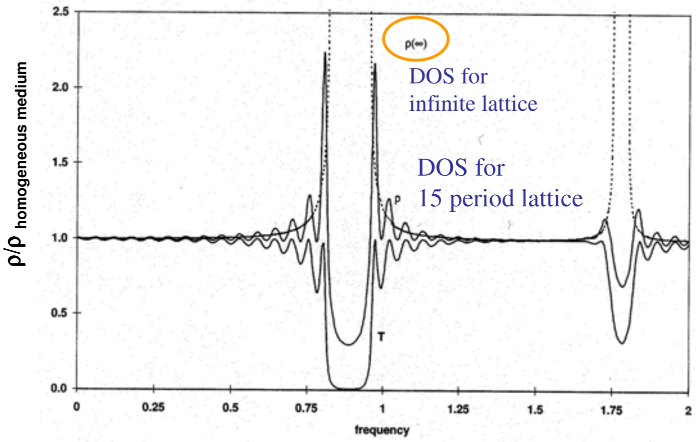
Onedimfinite.m

Matlab code developed using the model previously described. Codes are available.

Finite Size 1D Photonic Crystals

- Density of States $\rho = \left|\frac{dk}{d\omega}\right|$

Calculation of DOS for 2D and 3D requires a complex procedure. Nevertheless a s ageneral property of photonic crystals, DOS is maximum at band edges



Finite Size 1D Photonic Crystals

Fields inside the structure can be calculated starting from the value of transmitted field and applying the transfer matrix backward, layer by layer:

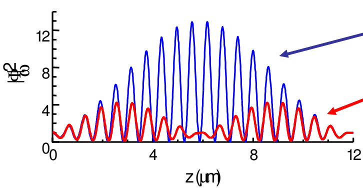
$$\begin{pmatrix} a_{2N} \\ b_{2N} \end{pmatrix} = \hat{D}_{out \to 2} \begin{pmatrix} t \\ 0 \end{pmatrix}$$

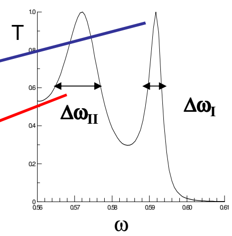
Interface from output to n2 material

$$\begin{pmatrix} a_{2N-1} \\ b_{2N-1} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 + k_x^{2N} / k_x^{2N-1} & 1 - k_x^{2N} / k_x^{2N-1} \\ 1 - k_x^{2N} / k_x^{2N-1} & 1 + k_x^{l} / k_x^{2N-1} \end{pmatrix} \begin{pmatrix} e^{-ik_x^{2N} d_{2N}} & 0 \\ 0 & e^{ik_x^{2N} d_{2N}} \end{pmatrix} \begin{pmatrix} a_{2N} \\ b_{2N} \end{pmatrix}$$

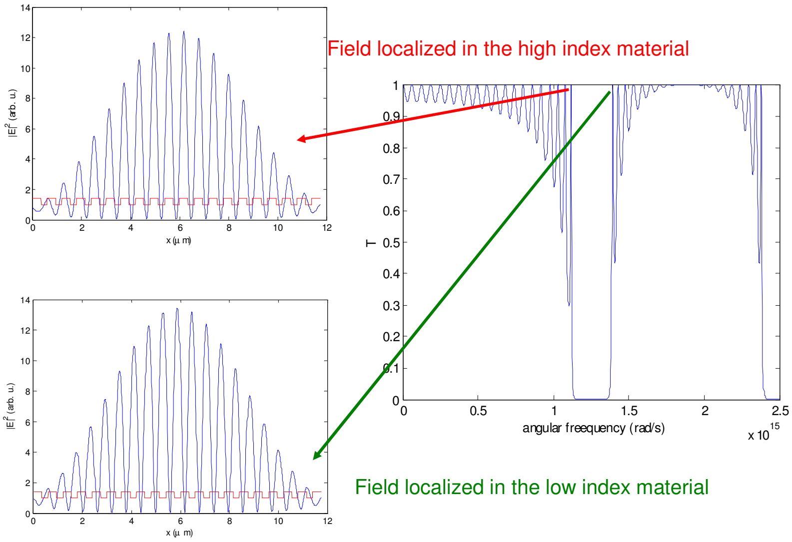
And so on….

Localization of field is related to the bandwidth of the resonance. Maximum field localization is achieved at the band edge.





Finite Size 1D Photonic Crystals



Finite Size 1D Photonic Crystals

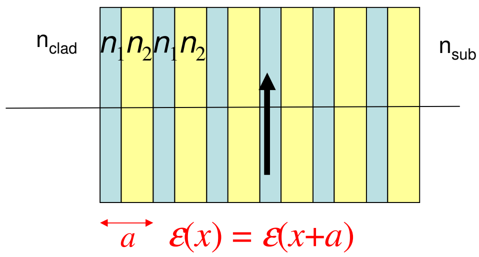
Guided modes

Condition for guided modes can be set using the transfer matrix method in the following way:

Guided mode must fulfill the condition to be evanescent waves out of the guiding structure, thus:

$$k_y > \frac{\omega}{c} n_{clad};$$

$$k_y > \frac{\omega}{c} n_{sub};$$



Moreover, if A is the matrix that relates the fields in the substrate to the field in the cladding, the following relation must be fulfilled

$$\begin{pmatrix} 0 \\ b_0 \end{pmatrix} = \hat{A} \begin{pmatrix} a_{N+1} \\ 0 \end{pmatrix} \Longrightarrow A_{11}=0$$

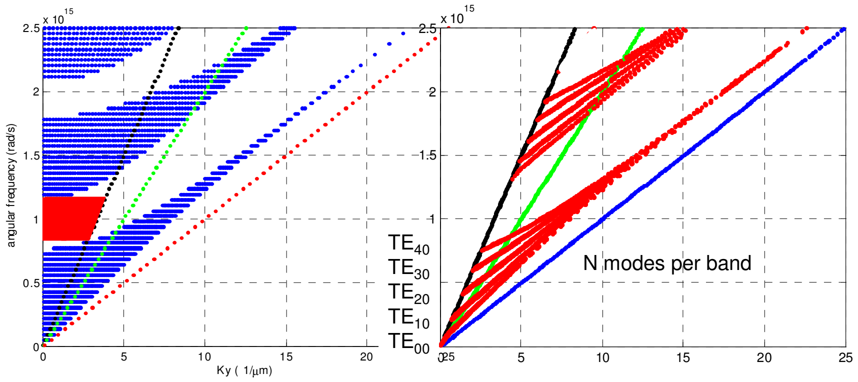
Values of K_y fulfilling the condition $A_{11}=0$ are the propagation constants of guided modes

Finite Size 1D Photonic Crystals

Guided modes

Infinite structure bands

N=5 period structure guided modes



Fields profile of guided modes can be calculated the same way as before using transfer matrix.

Analysis of 2D and 3D structures:

Analysis of 2D and 3D Photonic crystals

2D and 3D structures have many properties in common with 1D structures but they offer the opportunity to tailor localization properties in 3D.

WE start by giving a general definition of the **reciprocal lattice**, already introduced for the one dimensional case.

A function of the 3d space with a given periodicity can be written as a function of a vector **r** belonging to the primitive cell:

$$\underbrace{\vec{r} = \alpha\vec{a}_1 + \beta\vec{a}_2 + \gamma\vec{a}_3}_{\text{Primitive vectors}} \qquad 0 \le \alpha, \beta, \gamma \le 1$$

$$f(\vec{r} + n\vec{a}_1 + m\vec{a}_2 + l\vec{a}_3) = f(\vec{r}) \qquad n, m, l = 0, \pm 1, \pm 2, \ldots; \vec{r} \in u.c.$$

A periodic function can be expanded according to Fourier series:

$$f(\vec{r}) = \sum_{\vec{G}} e^{i\vec{G}\cdot\vec{r}} F(\vec{G});$$

Analysis of 2D and 3D Photonic crystals

Ther set of G vectors are selected in order to have a periodic oscillating function over the unit cell. This means that in a lattice point identified by vector **R**:

$$e^{i\vec{G}\cdot\vec{R}} = 1$$

Equivalent to:

$$\vec{G}\cdot\vec{R} = 2\pi p; \qquad p = 0, \pm 1, \pm 2, \ldots$$

Reminding that every lattice point can be obtained by linear combination of primitive vectors:

$$\vec{R} = n\vec{a}_1 + m\vec{a}_2 + l\vec{a}_3;$$
$$n, m, l = 0, \pm 1, \pm 2, \ldots$$

The reciprocal lattice is defined as the set of vectors G generated by its three reciprocal primitive vectors, through the formula :

$$\vec{g}_1 = 2\pi \frac{\vec{a}_2 \times \vec{a}_3}{\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)};$$

$$\vec{g}_2 = 2\pi \frac{\vec{a}_3 \times \vec{a}_1}{\vec{a}_2 \cdot (\vec{a}_3 \times \vec{a}_1)};$$

$$\vec{g}_3 = 2\pi \frac{\vec{a}_1 \times \vec{a}_2}{\vec{a}_3 \cdot (\vec{a}_1 \times \vec{a}_2)};$$

Any Bravais lattice has a reciprocal lattice

Analysis of 2D and 3D Photonic crystals

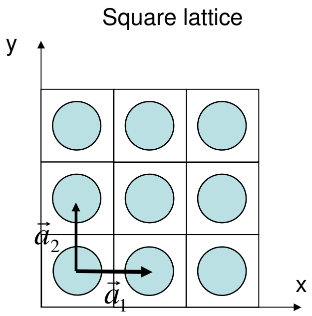
Examples in 2D

Exercise: Verify for 1D the reciprocal lattice primitive cell corresponds to the Brillouin zone.

This statement is always true and it can be used as a definition:

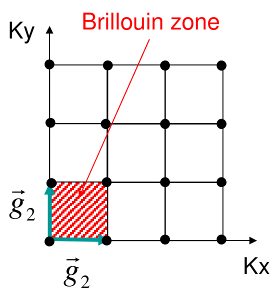
The Brillouin zone for a Bravais lattice is the primitive unit cell of the reciprocal lattice.
As a consequence, for a given lattice, band diagrams are periodic functions over its reciprocal lattice.

Example



$$\vec{g}_1 = \frac{2\pi}{a_1}\hat{a}_1;$$

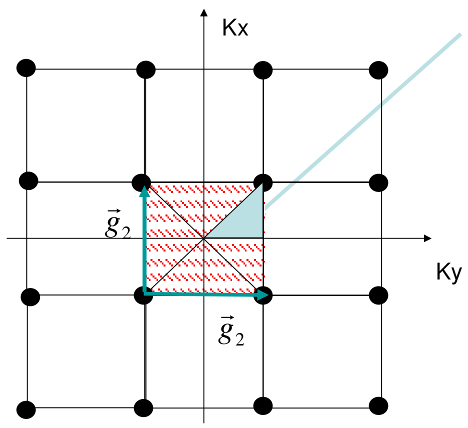
$$\vec{g}_2 = \frac{2\pi}{a_2}\hat{a}_2;$$



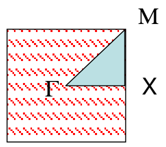
a_3 would have infinite modulus and orthogonal to a_1 and a_2

Analysis of 2D and 3D Photonic crystals

Examples in 2D



Irreducible Brillouin Zone for the square lattice: first Brillouin zone reduced by all thhe symmetries in the point group of the lattice.



Γ Center of the Brillouin zone

M Center of an edge

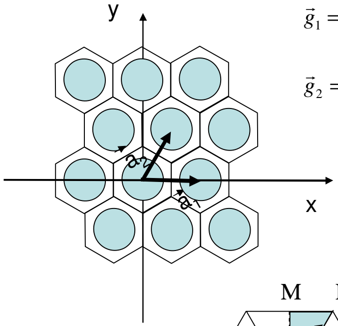
X Center of a face

It is enough to calculate band diagrams in the irreducible Brillouin zone and then use the symmetry of the lattice to extend the diagrams to the first Brillouin zone.

Analysis of 2D and 3D Photonic crystals

Examples in 2D

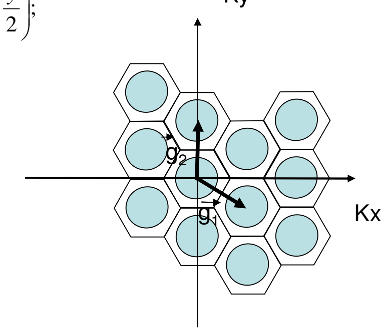
Triangular lattice



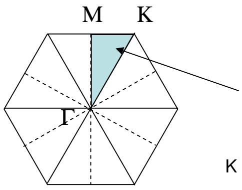
$$\vec{g}_1 = \frac{4\pi}{a\sqrt{3}}\left(\frac{\hat{x}\sqrt{3}}{2} - \frac{\hat{y}}{2}\right);$$

$$\vec{g}_2 = \frac{4\pi}{a\sqrt{3}}\hat{y};$$

Reciprocal lattice



Brillouin zone:

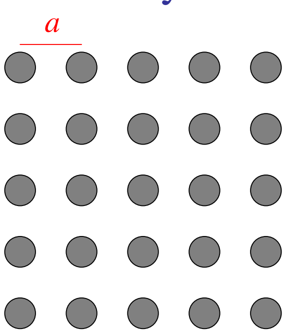


Irreducible Brillouin zone

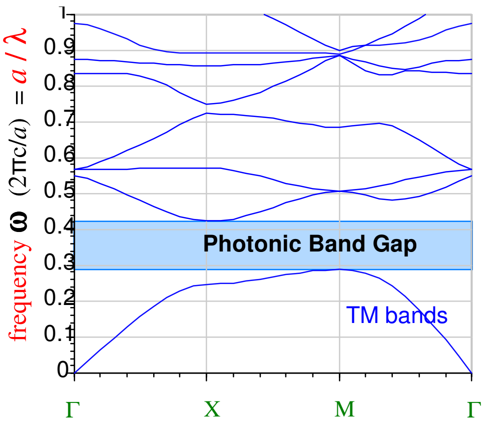
K Middle of an edge joining two hexagonal faces

Analysis of 2D and 3D Photonic crystals

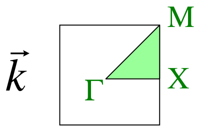
Examples in 2D



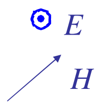
ε=12:1



irreducible Brillouin zone



TM

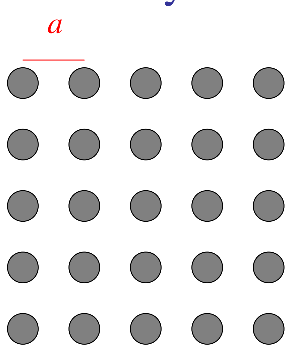


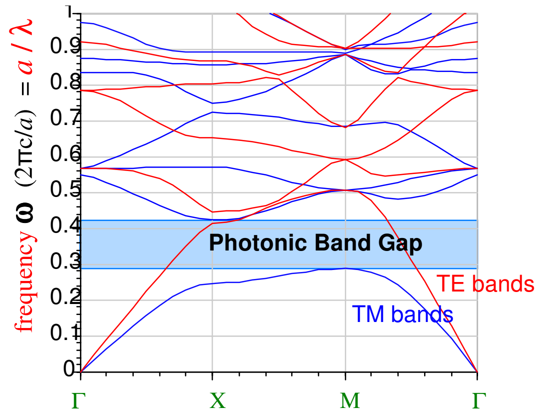
gap for
$n > \sim 1.75:1$

http://ab-initio.mit.edu/photons/tutorial/

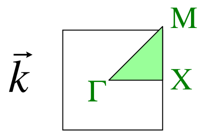
Analysis of 2D and 3D Photonic crystals

Examples in 2D





irreducible Brillouin zone



http://ab-initio.mit.edu/photons/tutorial/

Analysis of 2D and 3D Photonic crystals

Examples in 2D

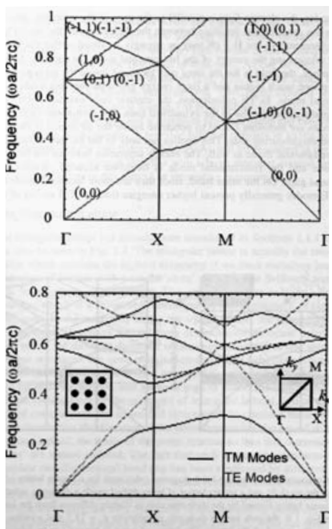
Comparison between 2D free space dispersion diagram and 2D Photonic crystal dispersion diagram.

$$\omega = \frac{c}{\sqrt{\varepsilon_b}}\sqrt{k_x^2 + k_y^2};$$

$\Gamma \rightarrow X$; $k_y=0$;

$X \rightarrow M$; k_x=cost

$M \rightarrow \Gamma$; $k_x=k_y$



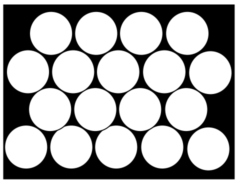
Analysis of 2D and 3D Photonic crystals

Examples in 2D

Triangular lattice

TM band gaps tend to occur in lattices formed by isolated high-permittivity regions; TE gaps in connected lattices.

High degree of compactness of triangular lattice make the connected lattice keep some of the properties of the isolated lattice.



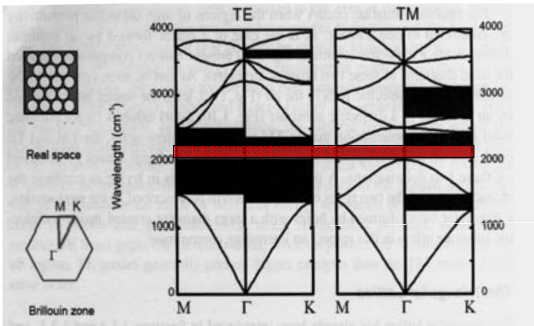


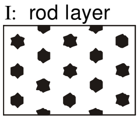
Fig. 1.11. *Left.* Triangular lattice formed by pores created in a dielectric substrate and corresponding Brillouin zone. *Right.* Band diagram calculated for a dielectric substrate with a permittivity $\varepsilon = 11.7$ and a period $a = 2\mu m$, while the radius of the pores is $r/a = 0.45$. The band gaps are indicated by grey bands

Analysis of 2D and 3D Photonic crystals

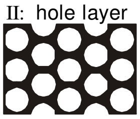
3D example

Yablonovite

I: rod layer



II: hole layer



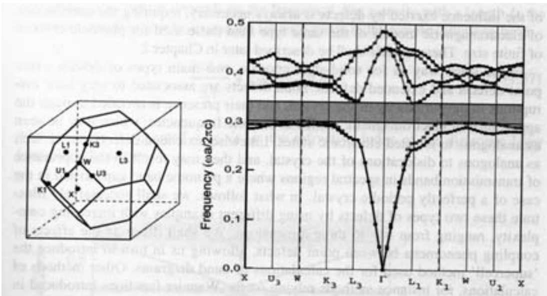
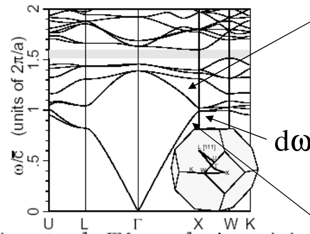


Fig. 1.14. Brillouin zone of the fcc lattice and band diagram of the Yablonovite structure calculated for the first six bands. The permittivity of the dielectric is $\varepsilon = 13$, while the ratio between the radius of the holes and the period of the crystal is such that the air filling factor is equal to 57% after the holes have been drilled (David 2003)

Review on Bulk Photonic crystals properties



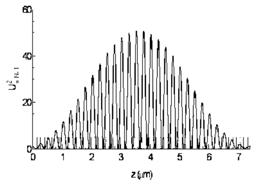
Complex band structure

backwards slope:
negative refraction

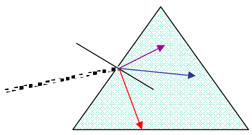
$d\omega/dk \to 0$: slow light
(e.g. DFB lasers)

strong curvature:
super-prisms, …

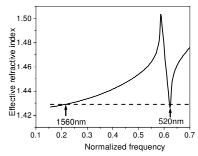
(+ negative refraction)



Internal field enhancement
- Low threshold lasing
- Enhanced nonlinear optical effects



Spatial dispersion (Superprism effect)
- Negative refraction
- Large angle deflection 500x
- Self-collimation



Dispersive refractive index dispersion
- Control of light propagation
- Phase-matching for harmonic generation

Photonic crystals with intentional defects

Control of Electromagnetic Waves

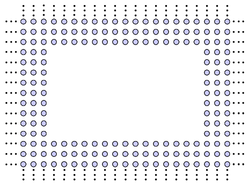
Using Photonic Crystals with intentional defects

Photonic GAPs can be used to confine light in a region of space

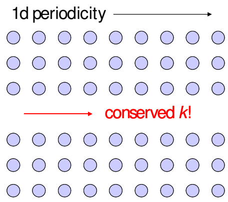
RESONANT CAVITIES

Light cannot propagate inside the crystal and it is confined in a finite region of space.

Mode of electromagnetic waves inside the cavity must fulfill proper boundary conditions
(discrete eigenfrequency spectrum)



WAVEGUIDES

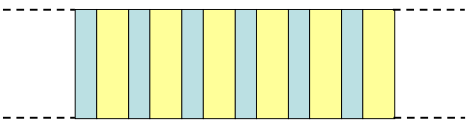


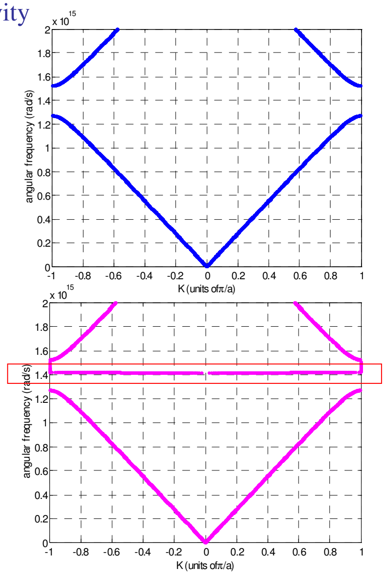
Light tuned in the gap cannot exit form the line defect which acts as a waveguide. Propagation constants of guided modes will appear in the gap of the perfectly periodic crystal's band diagram.

Control of Electromagnetic Waves

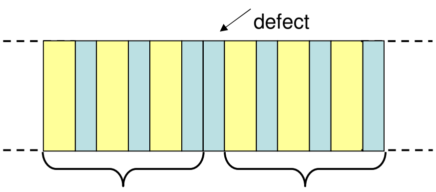
1D Micro-cavity

Consider an infinite stack:
n1=1.5; d1=250 nm;
n2=2.0; d2=150 nm;





We introduce a defect, by adding a layer of material 2. The structure is no longer periodic. We can calculate the new dispersion relation defining a **supercell.**
NOTE: in order to get good results distance between periodic defects in the supercell must be large with respect to wavelength (weak coupling)



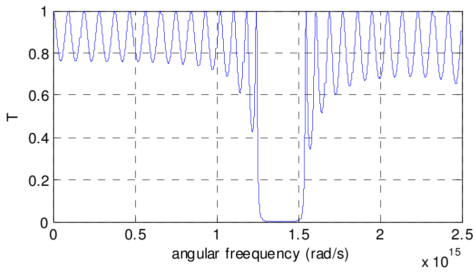
s.c=8periods-defect-8periods

Control of Electromagnetic Waves

1D Micro-cavity

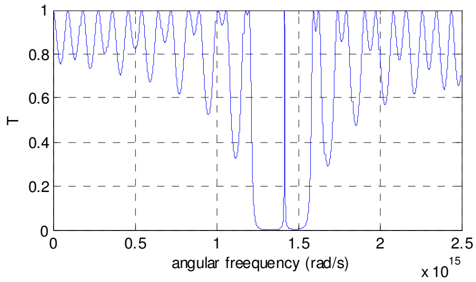
In we consider finite structures:

Symmetric structure,
N=14 periods (HL)+H



Structure with central defect:

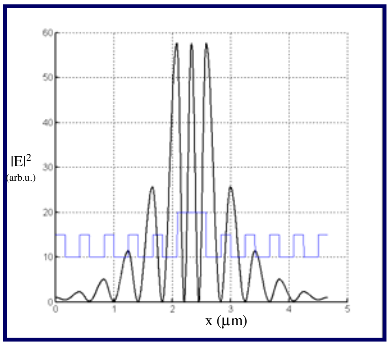
7 periods (HL)+H+ 7periods (LH)

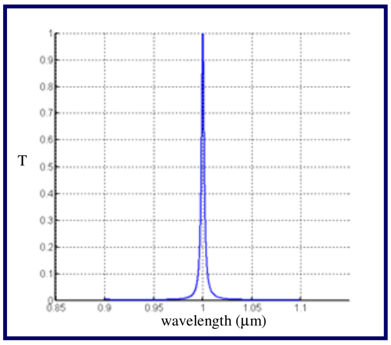


Control of Electromagnetic Waves

1D Micro-cavity

Micro-cavity sandwiched between two Bragg mirrors.
High Q factors, high fields intensity enhancement inside the structure.

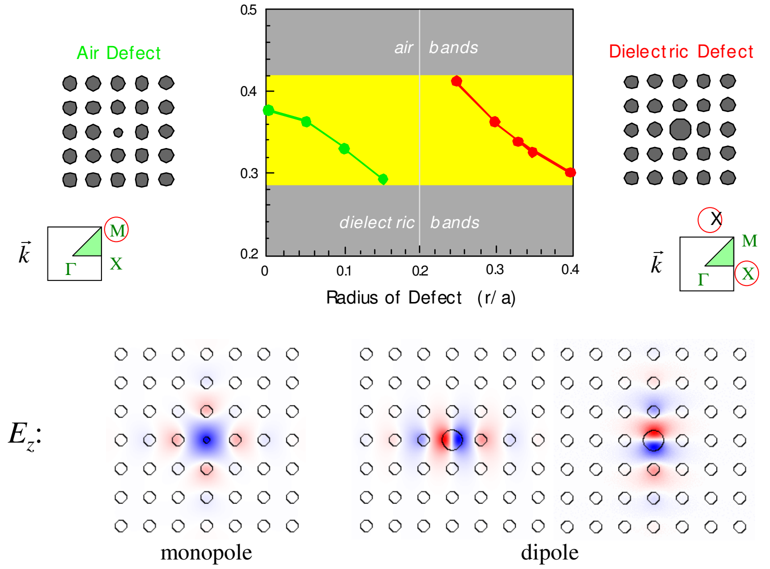




$$Q = 2\pi \frac{Stored\ Energy}{Energy\ lost\ per\ cicle} = \omega_0 \frac{\langle Energy \rangle}{\langle Power\ loss \rangle} \approx \frac{\omega_0}{\Delta\omega_0}$$

Control of Electromagnetic Waves

2D single mode Micro Cavity

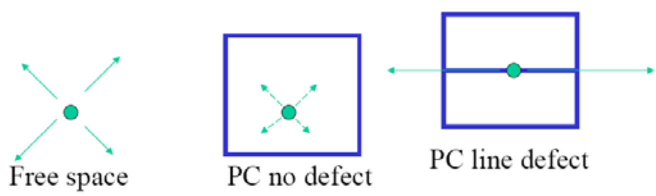


Control of Electromagnetic Waves

Spontaneous emission suppression

E. Yablonovitch, PRL (1987)

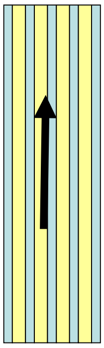
- Excited atoms in a Photonic crystal will not radiate; i.e. no radiation mode is available.
- Add a defect that introduces a single mode in the band gap. Now when an atom radiates its first photon it has no other choice except into the defect mode. Thereafter the atoms radiate by stimulated emission. Zero threshold laser.



Control of Electromagnetic Waves

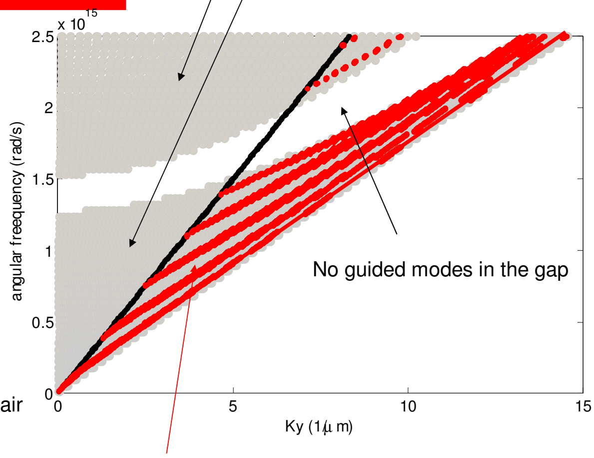
Photonic Crystal waveguides

Consider a finite stack as in the picture:
n1=1.5; d1=250 nm;
n2=2.0; d2=150 nm;



N=4 (HL)+1H; embedded in air

Bands of the infinite structure



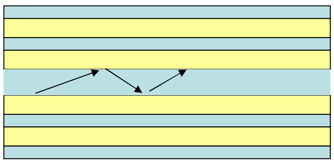
No guided modes in the gap

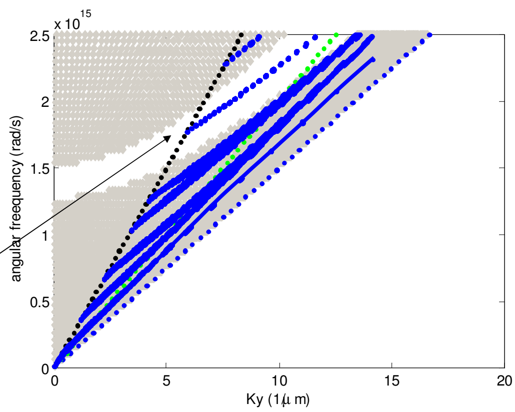
Guided modes for the finite structure

Control of Electromagnetic Waves

Photonic Crystal waveguides

We add a defect:





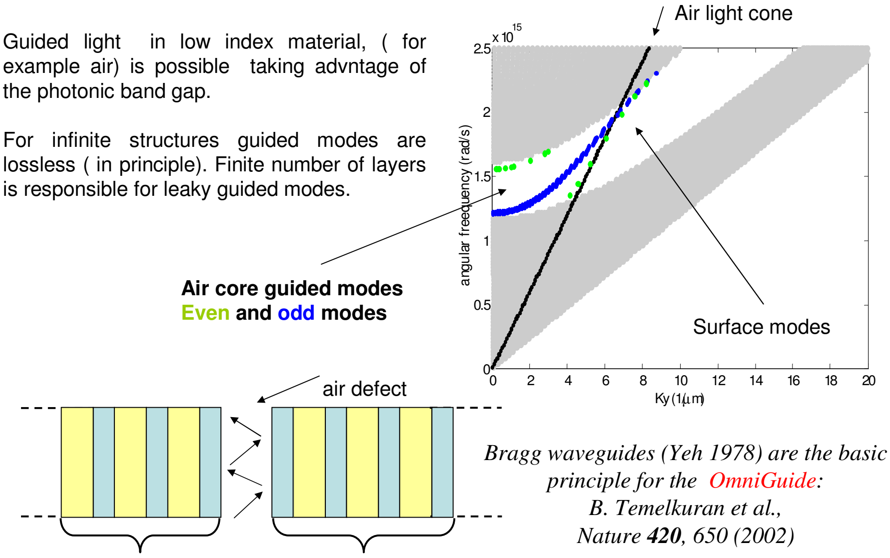
guided mode in the gap
Combination of index guiding and
photonic band gap guiding

Control of Electromagnetic Waves

Hollow Bragg waveguides

Guided light in low index material, (for example air) is possible taking advntage of the photonic band gap.

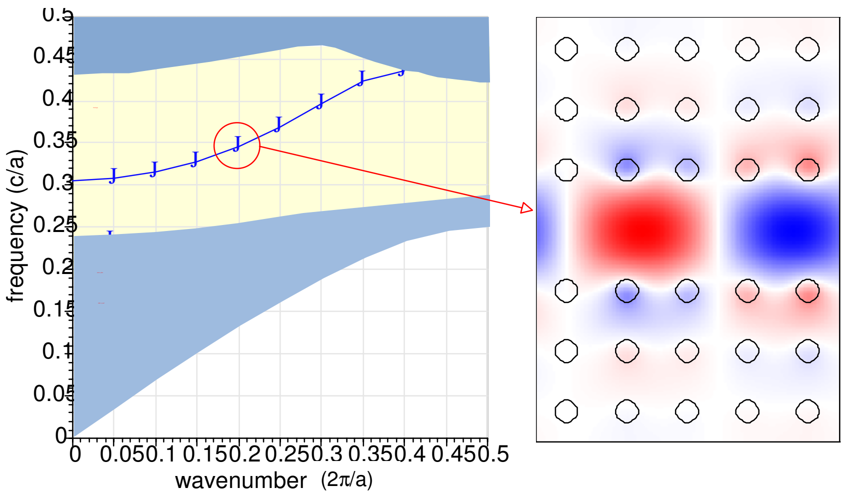
For infinite structures guided modes are lossless (in principle). Finite number of layers is responsible for leaky guided modes.



*Bragg waveguides (Yeh 1978) are the basic principle for the OmniGuide: B. Temelkuran et al., Nature **420**, 650 (2002)*

Control of Electromagnetic Waves

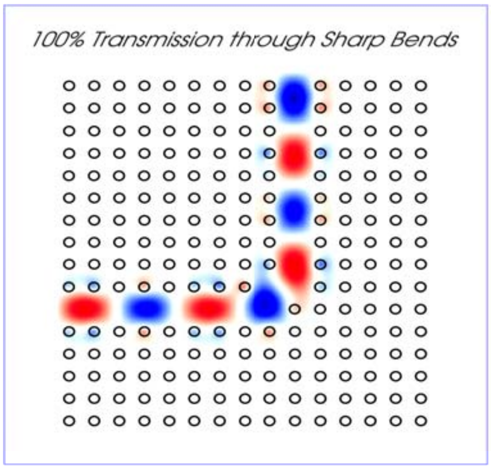
2D waveguides



any state in the gap cannot couple to bulk crystal –> localized

Control of Electromagnetic Waves

2D waveguides



Lossless Bends

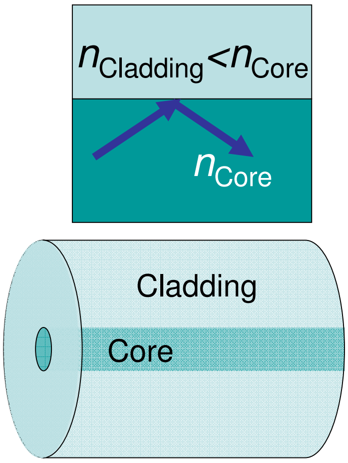
[A. Mekis *et al.*, *Phys. Rev. Lett.* **77**, 3787 (1996)]

symmetry + single-mode + “1d” = resonances of 100% transmission

Photonic Crystal Fibers

Guiding light: Conventional Optical Fibres

- Total Internal Reflection (TIR)
  - $n_{Cladding} < n_{Core}$
  - Core must be dielectric material
  - Interaction between light and matter unavoidable
    - Non-linearity
    - Material Dispersion
    - Losses

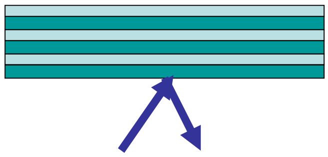


$n_{Core} > n_{Cladding}$

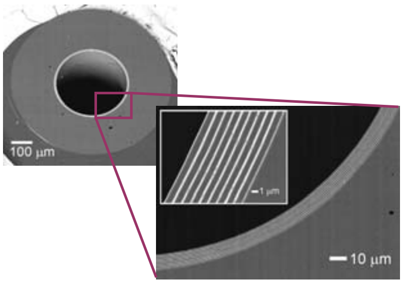
http://www.physics.usyd.edu.au/cudos/

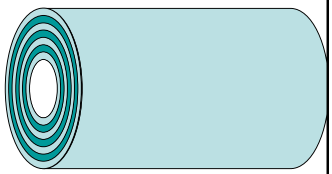
Photonic Crystal Fibers

- Bragg reflection
  - Very low losses
  - Bandwidth ?
  - Angle of incidence ?
  - Index contrast ?
  - Fabrication ?



- Bragg fibres, “OmniGuide” fibres

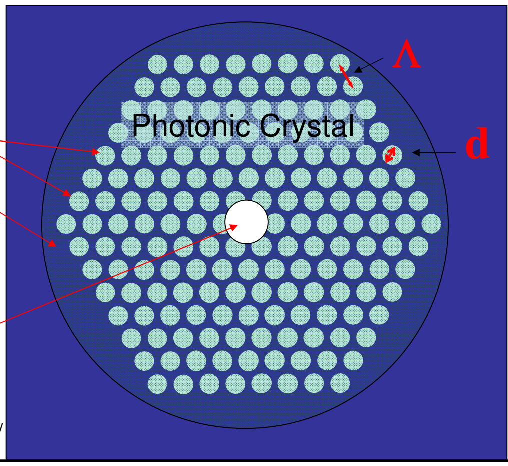




Burak Temelkuran *et al*
Nature <u>420</u>, 650-653, December 2002.

Photonic Crystal Fibers

Birks, Roberts, Russel, Atkin, Shepherd, Electron.
Lett. **31**, 1941-1942 (1995)

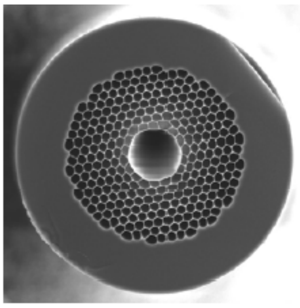


http://www.physics.usyd.edu.au/cudos/

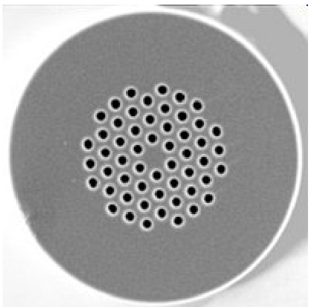
Photonic Crystal Fibers

Holey Silica Cladding

- Hollow core and solid core PCFs



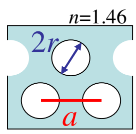
Mangan *et al,* OFC 2004
(1.7dB/km Loss@1550nm)



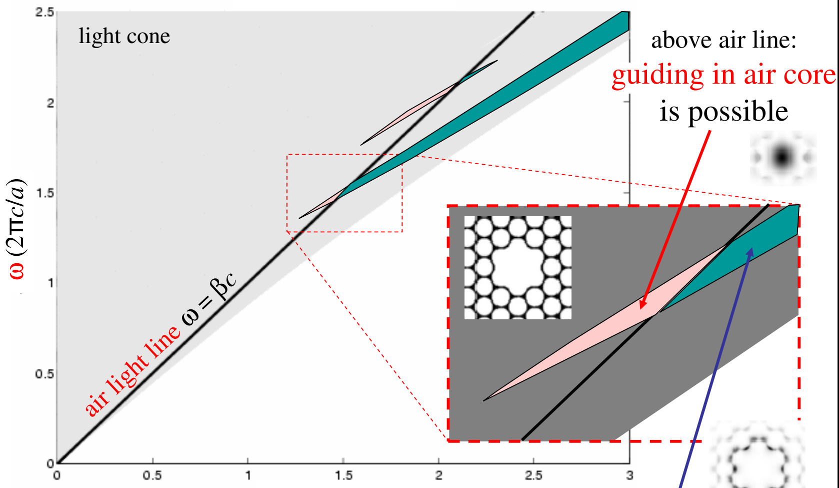
blazephotonics

Photonic Crystal Fibers

Hollow Core and Holey Silica Cladding



$r = 0.45a$



[figs: West *et al*,
Opt. Express **12** (8), 1485 (2004)]

Photonic Crystal Fibers

Effective index vs PBG guidance

Mechanism	TIR "averaged index"	coherent scattering
Periodicity	Not necessary	Crucial
Bandwidth	Unlimited	$\Delta\lambda/\lambda$~10%
Core	Solid	Hollow or solid
Hole/pitch	Small or large	Large (typ. d/Λ>0.9

Winter college lectures on Photonic Crystals

Photonic Crystals (1 hour)

- Natural Photonic crystal
- Fabrication technologies
- Properties of bulk photonic crystals
- super prism, super lens
- enhancement of nonlinear effects

Photonic Crystals- applications (1hour)

- Photonic crystal cavities – sensors and detectors
- Photonic crystal waveguides – data processing
- Photonic crystal fibers – high intensity laser delivery
  - supercontinuum generation for spectroscopy