The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific and
Cultural Organization

IAEA
International Atomic Energy Agency

1932-5

Winter College on Micro and Nano Photonics for Life Sciences

11 - 22 February 2008

Supplement to plasmons

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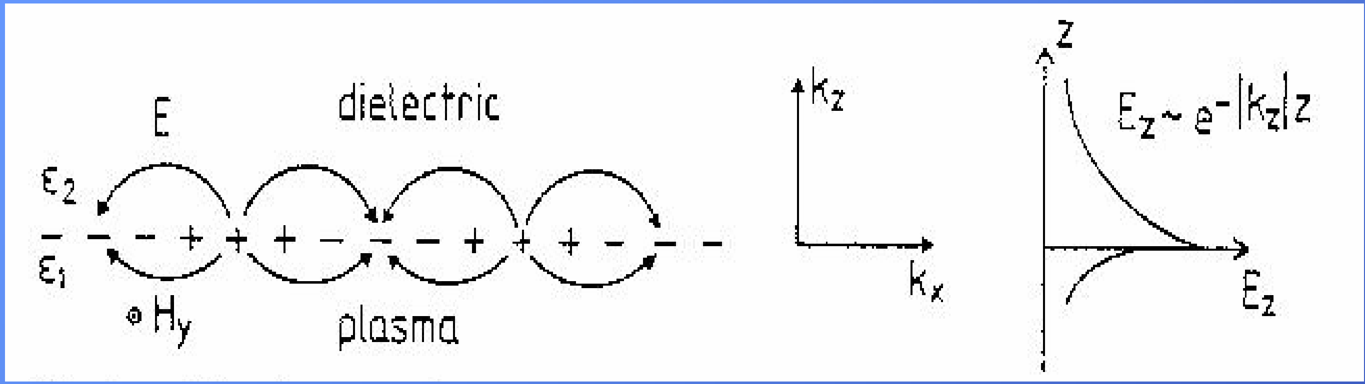


The wave propagates in the x-direction. The problem does not depend on y. The field is

$$z>0 \qquad H_2=(0,H_{2y},0)\exp i\left(k_{2x}x+k_{2z}z-\omega t\right)$$

$$E_2=\left(E_{2x},0,E_{2z}\right)\exp i\left(k_{2x}x+k_{2z}z-\omega t\right) \qquad (1)$$

$$z<0 \qquad H_1=(0,H_{1y},0)\exp i\left(k_{1x}x-k_{1z}z-\omega t\right)$$

$$E_1=\left(E_{1x},0,E_{1z}\right)\exp i\left(k_{1x}x-k_{1z}z-\omega t\right) \qquad (2)$$

The fields obey to Maxwell equations

$$rot\, H_i = \mu_i \varepsilon_i \frac{\partial E_i}{\partial t} \quad \therefore \quad rot\, H_i = \varepsilon_i \frac{\partial E_i}{\partial t} \qquad (3)$$

$$rot\, E_i = -\frac{\partial B_i}{\partial t} \quad \therefore \quad rot\, E_i = -\mu \frac{\partial H_i}{\partial t} \qquad (4)$$

$$div\, \varepsilon_i E_i = 0 \qquad (5)$$

$$div\, H_i = 0 \qquad (6)$$

with the continuity conditions

$$B = \mu H = \frac{E}{c}$$

$$E_{1x} = E_{2x} \qquad (7)$$

$$H_{1y} = H_{2y} \quad \rightarrow \quad \frac{E_{1z}}{v_1 \mu} = \frac{E_{2z}}{\mu v_2} \qquad (8)$$

$$\sqrt{\varepsilon_1} E_{1z} = \sqrt{\varepsilon_2} E_{2z} \qquad (9)$$

$$\mu v_1 = \frac{\mu}{\sqrt{\varepsilon \mu}} = \sqrt{\frac{\mu}{\varepsilon}}$$

Taking (1) and (2) at z = 0

$$z > 0 \qquad H_2 = (0, H_{2y}, 0)\exp i\left(k_{2x}x + k_{2z}z - \omega t\right)$$

$$E_2 = \left(E_{2x}, 0, E_{2z}\right)\exp i\left(k_{2x}x + k_{2z}z - \omega t\right) \qquad (1)$$

$$z < 0 \qquad H_1 = (0, H_{1y}, 0)\exp i\left(k_{1x}x - k_{1z}z - \omega t\right)$$

$$E_1 = \left(E_{1x}, 0, E_{1z}\right)\exp i\left(k_{1x}x - k_{1z}z - \omega t\right) \qquad (2)$$

$$H_2 = H_{2y}\exp i\left\{k_{2x}x - \omega t\right\} \qquad (10a)$$

$$E_2 = E_{2x}\exp i\left\{k_{2x}x - \omega t\right\} \qquad (10b)$$

$$= E_{2z}\exp i\left\{k_{2x}x - \omega t\right\} \qquad (10c)$$

$$H_1 = H_{1y}\exp i\left\{k_{1x}x - \omega t\right\} \qquad (10d)$$

$$E_1 = E_{1x}\exp i\left\{k_{1x}x - \omega t\right\} \qquad (10e)$$

$$= E_{2z}\exp i\left\{k_{2x}x - \omega t\right\} \qquad (10f)$$

Because $E_{1x} = E_{2x}$ it follows

$$E_{1x}\exp i(k_1x - \omega t) = E_{2x}\exp i\left\{k_2x - \omega t\right\}$$

therefore

$$k = k_{1x} = k_{2x} \qquad (11)$$

From $rot\, H = \varepsilon \frac{\partial E}{\partial t}$

$$rot\, H_i = \begin{vmatrix} \vec{i} & \vec{1} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = -\vec{i}\,\frac{\partial H_{yi}}{\partial z} = \varepsilon_i \frac{\partial E_{ix}}{\partial t}\vec{i}$$

or

$$\frac{\partial H_{yi}}{\partial z} = \omega\varepsilon_i E_{1x}$$

that is

$$k_{2z} H_{y2} = \omega\varepsilon_2 E_{x2}$$
$$-k_{1z} H_{y1} = \omega\varepsilon_1 E_{x1} \qquad (12)$$

Because $E_{x2} = E_{x1}$ it is

$$\frac{k_{2z}}{\varepsilon_2} H_{2y} = \omega E_{x2} \quad \text{e} \quad \frac{k_{1z}}{\varepsilon_1} H_{1y} = -\omega E_{x1} \qquad (13)$$

summing ($E_{x2} = E_{x1}$)

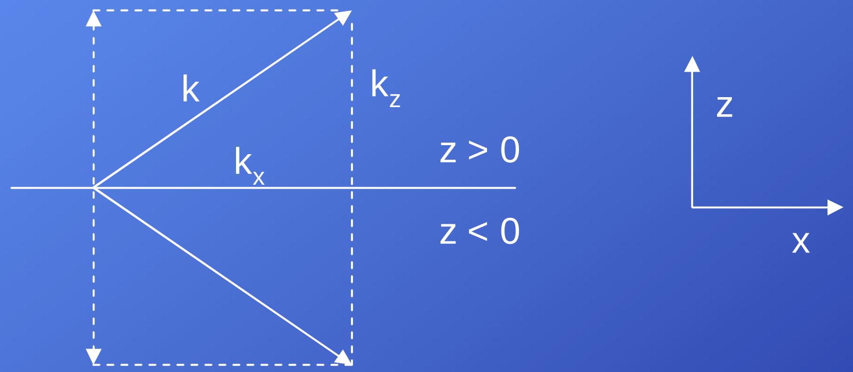
$$\frac{k_{1z} H_{1y}}{\varepsilon_1} + \frac{k_{2z} H_{2y}}{\varepsilon_2} = 0 \qquad (14)$$

The system (13) and (14) has solutions if

$$\begin{vmatrix} 1 & -1 \\ \frac{k_{1z}}{\varepsilon_1} & \frac{k_{2z}}{\varepsilon_2} \end{vmatrix} = 0 = \frac{k_{2z}}{\varepsilon_2} + \frac{k_{1z}}{\varepsilon_1} \qquad (15)$$

Moreover

$$k_x^2 + k_{z1}^2 = \varepsilon_i \left(\frac{\omega}{c} \right)^2 \qquad (16)$$



or

$$k_x^2 = \varepsilon_i \left(\frac{\omega}{c} \right)^2 - k_{zi}^2$$

$$k_x^2 = \varepsilon_1 \left(\frac{\omega}{c}\right)^2 - k_{z1}^2 \qquad \frac{k_{1z}}{\varepsilon_1} = -\frac{k_{2z}}{\varepsilon_2}$$

$$k_x^2 = \varepsilon_2 \left(\frac{\omega}{c}\right)^2 - k_{z2}^2$$

$$2k_x^2 = \left(\frac{\omega}{c}\right)^2 (\varepsilon_1 + \varepsilon_2) - k_{z1}^2 - \frac{k_{1z}^2 \varepsilon_2^2}{\varepsilon_1^2}$$

$$= \left(\frac{\omega}{c}\right)^2 (\varepsilon_1 + \varepsilon_2) - k_{z1}^2 \left(1 + \frac{\varepsilon_2^2}{\varepsilon_1^2}\right)$$

$$= \left(\frac{\omega}{c}\right)^2 (\varepsilon_1 + \varepsilon_2) - \left[k_x^2 + \varepsilon_1 \left(\frac{\omega}{c}\right)^2\right] \left(1 + \frac{\varepsilon_2^2}{\varepsilon_1^2}\right)$$

$$k_x^2 \left(1 - \frac{\varepsilon_2^2}{\varepsilon_1^2}\right) = \left(\frac{\omega}{c}\right)^2 (\varepsilon_1 + \varepsilon_2) - \left(\frac{\omega}{c}\right)^2 \varepsilon_1 \left(1 + \frac{\varepsilon_2^2}{\varepsilon_1^2}\right)$$

$$k_x^2 \frac{\varepsilon_1^2 - \varepsilon_2^2}{\varepsilon_1^2} = \left(\frac{\omega}{c}\right)^2 \left[\varepsilon_1 + \varepsilon_2 - \varepsilon_1 - \frac{\varepsilon_2^2}{\varepsilon_1}\right]$$

$$= \left(\frac{\omega}{c}\right)^2 \varepsilon_2 \left(1 - \frac{\varepsilon_2}{\varepsilon_1}\right) = \left(\frac{\omega}{c}\right)^2 \varepsilon_2 \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1}$$

$$k_x^2 \frac{(\varepsilon_1 - \varepsilon_2)(\varepsilon_1 + \varepsilon_2)}{\varepsilon_1} = \left(\frac{\omega}{c}\right)^2 \varepsilon_2 (\varepsilon_1 - \varepsilon_2)$$

$$k_x^2 = \left(\frac{\omega}{c}\right)^2 \left(\frac{\varepsilon_2 \varepsilon_1}{(\varepsilon_1 + \varepsilon_2)}\right)$$

$$k_x = \frac{\omega}{c}\left(\frac{\varepsilon_1 \varepsilon_2}{(\varepsilon_1 + \varepsilon_2)}\right)^{1/2} \qquad (17)$$

If we suppose $\varepsilon_2 = 1$ (air) and $\varepsilon_1 < 0$ (metal) with $|\varepsilon_1| < 1$ then $k_x > \frac{\omega}{c}$ and 51

$$k_{zi}^2 = \varepsilon_i \left(\frac{\omega}{c}\right)^2 - k_x^2$$

that in this case

$$k_{2z}^2 = \left(\frac{\omega}{c}\right)^2 - k_x^2 < 0$$

and k_{2z} is immaginary or complex.