Simulation of Particle Beam Transport in High Energy Accelerators

Particle Beam Transport Codes

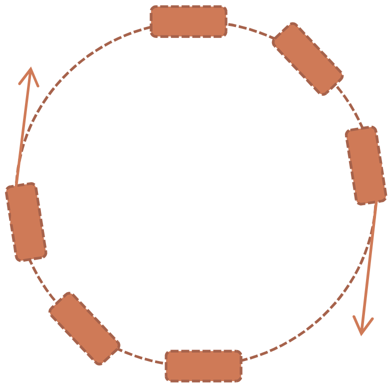
Optics code (*MAD*)

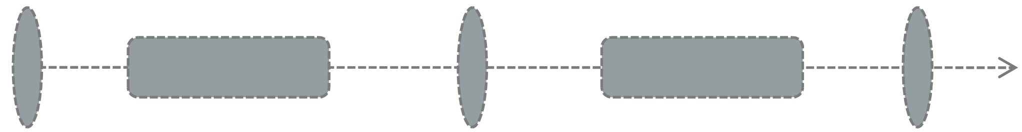
- *Global parameters of the Accelerator:*
  - synchrotron
  - FEL linac
  - collider,…
- *Transverse and sometime longitudinal Single Particle dynamics:*
  - transverse focusing
  - dynamic aperture
  - synchrotron radiation integrals,…

Particle Tracking code (*Elegant*)

- *Detailed study of the Accelerator performance:*
  - brightness degradation,…
- *Transverse and longitudinal Collective Effects:*
  - interaction with vacuum chamber
  - coherent synchrotron radiation,…

How to construct an input command file





properties

type of element
(keyword)

arbitrary
name

Element definitions:

```
MQ1:  quadrupole,  L=1.0, K1=1
MQ2:  quadrupole,  L=1.0, K1=-1
D:    drift,       L=2.0
MB:   sbend,       L=2.0, ANGLE=0.1
RFC:  rfcavity,    L=0.5, FREQ=2997e6, VOLT=250e3
```

ordered sequence
of elements

Line definition:

```
TOTAL: line=(MQ1, D, MB, D, MQ2, D, MB, D, MQ1)
```

or

```
HALF: line=(MQ1, D, MB, D)
TOTAL: line=(HALF, MQ2, -HALF)
```

Combination of sub-lines

Conventions for the Lattice

Conventions

- Elements are placed along the curvilinear reference orbit/trajectory
- Local coordinate system moves along the *s*-coordinate
- Elements coordinate can be referred to the entry, centre or exit

Dipole (bending) magnet:

ANGLE>0, clockwise

$$\rho\theta = l$$

$$p[GeV/c] = eB\rho = 0.3B[T]\rho[m]$$

Quadrupole magnet:

K1>0, x-focusing & y-defocusing

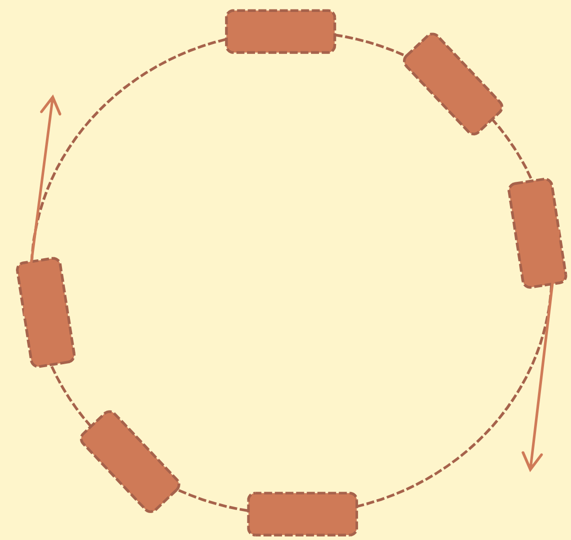
$$K1 = \frac{e}{p}\left(\frac{\partial B_y}{\partial x}\right)_0 = \frac{eg}{p} = \frac{0.3g[T/m]}{p[GeV/c]} \equiv \frac{1}{l \cdot f}$$

Thin/Thick Lens model:

Thin → zero length, symplectic, integral field
→ path lengths not correct, fringe field not correct

Thick → finite length, fringe field and path length correct
→ not symplectic

Exercise 1



Elettra storage ring
24 identical dipoles
L=1.44m

→What is the dipole magnetic field B at 2.4 GeV?

Elettra quadrupoles:
QF, l=0.41m, K1=2.0
QD, l=0.26m, K1=1.4
Q2, l=0.50m, K1=2.2

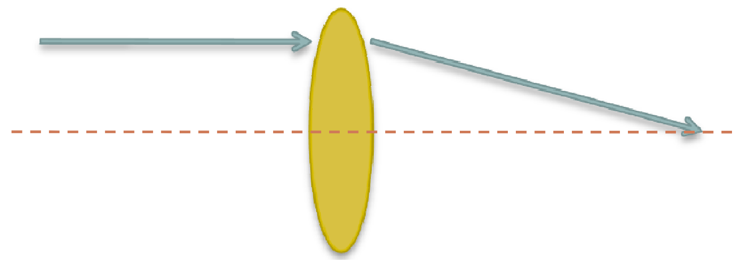
→ What is the maximum integrated gradient $g \cdot l$ at 2.4GeV?

Example of Optics code: MAD

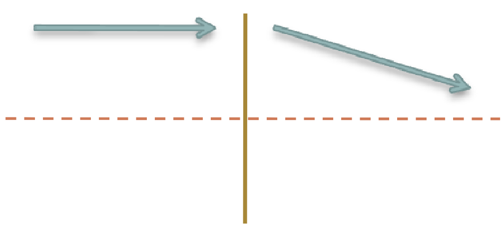
- ❑ http://frs.home.cern.ch/frs/Xdoc/mad-X_day.html

- ❑ The particle coordinates (6-D phase space) are transported through the lattice by matrix formalism.

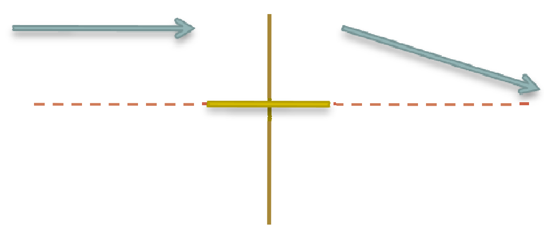
$$[Q] = \begin{pmatrix} 1 & 0 \\ -\frac{l}{f} & 1 \end{pmatrix} \qquad \begin{pmatrix} x_f \\ x'_f \end{pmatrix} = [Q] \begin{pmatrix} x_i \\ x'_i \end{pmatrix}$$



Thins lens model



Thick lens model

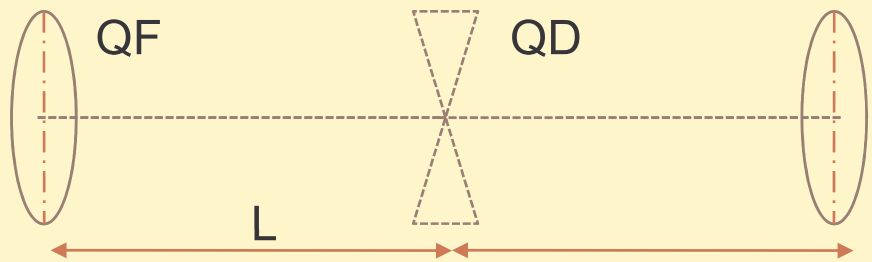


- ❑ Twiss parameters (betatron functions, dispersion) are propagated from known/periodical initial conditions through matrix formalism.

- ❑ Particle tracking is possible but it needs specification of particle-by-particle initial coordinates.

Exercise 2

Symmetric FODO lattice



$$M_{FODO} = [QF][D][QD][D][QF] =$$

$$= \begin{pmatrix} 1 - 2\frac{L^2}{f^2} & 2L\left(1 + \frac{L}{f}\right) \\ 2\left(1 - \frac{L}{f}\right)\left(\frac{L}{f^2}\right) & 1 - 2\frac{L^2}{f^2} \end{pmatrix}$$

$$\hat{\beta} = f \cdot \frac{1 + \sin(\phi/2)}{\cos(\phi/2)} \equiv \beta_x(QF) = \beta_y(QD)$$

$$\breve{\beta} = f \cdot \frac{1 - \sin(\phi/2)}{\cos(\phi/2)} \equiv \beta_y(QF) = \beta_x(QD)$$

with $f = \dfrac{L}{\sin(\phi/2)}$

→ What is the phase advance that minimizes the beam envelope ~ $\beta_x + \beta_y$ in both planes?
tip: take β_x and β_y as function of ϕ at a known location. Remind that f is also function of ϕ

Useful Commands

Particle Beam Definition

BEAM → total energy
mass
charge
transverse and longitudinal emittance (beam sizes and energy spread)
initial Twiss parameters (optics) for open line

Actions

TWISS → Lattice optics that define the beam transport line

TUNES → usually for storage ring only: betatron tunes to check resonances

PRINT → Layout file with sequence of elements, positions and optics

MATCH → match desired Twiss parameters at given location or some global beam parameters (tunes, dynamic aperture, etc…)

PLOT → graphical output of lattice optics, dynamic aperture, tune diagram, phase space, orbit/trajectory, etc…

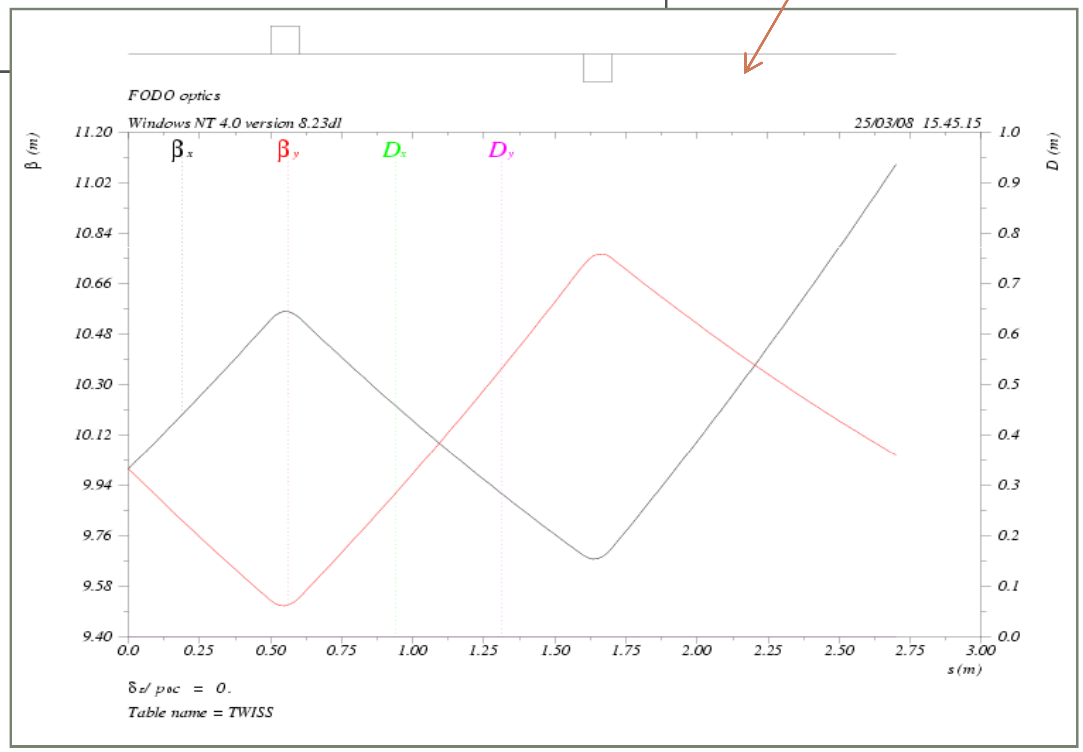
```
BEAM, particle=electron, energy=0.1

CALL, FILENAME="test.xsif"  !! which lattice file should be used?

USE, test                   !! which lattice line should be used?

PRINT, #s/#e

TWISS, save, betx=10.0, bety=10.0, &
             alfx=-0.5, alfy=0.5   !! explicit intial conditions for open line

PLOT, TITLE='FODO optics',&
table=twiss, haxis=s, vaxis1=betx,bety,vaxis2=dx,dy, &
colour=100, spline

STOP
```

Command file

Lattice file

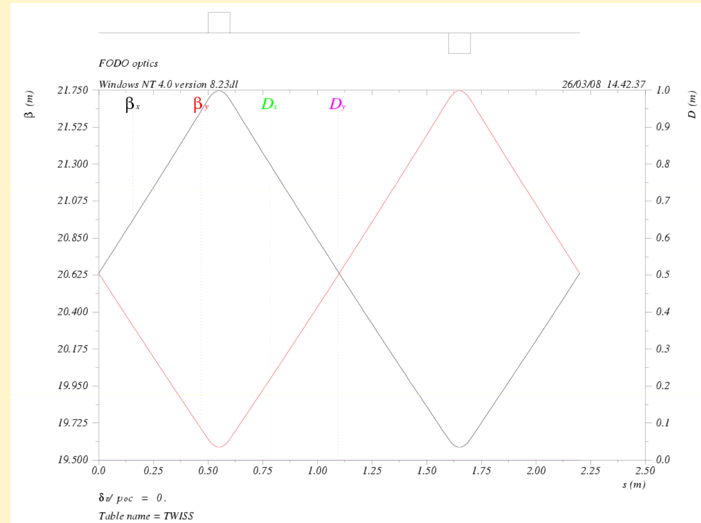
```
TITLE='example of FODO lattice'

! here the element definitions start

D1: DRIFT, L=0.5
D2: DRIFT, L=1.0

QF: QUADRUPOLE, L=0.1, K1=1.0
QD: QUADRUPOLE, L=0.1, K1=-1.0


! here the line definition starts

TEST: LINE=( D1, QF, D2, QD, D2 )
```

Plot of lattice optics

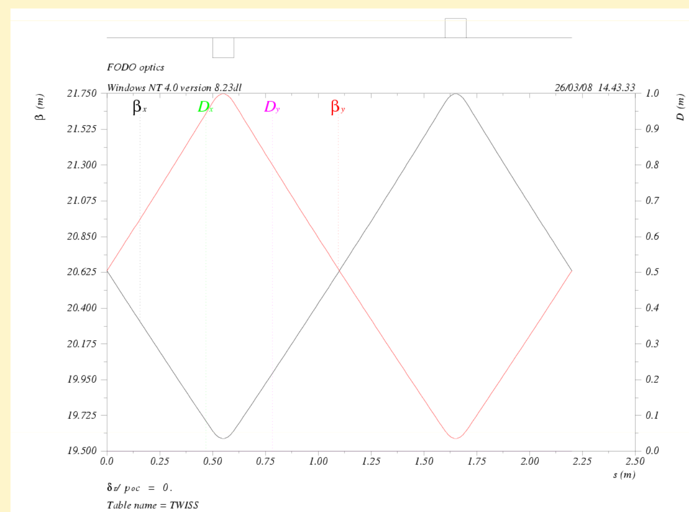


Exercise 3

→ Run "test_1" to reproduce the plot in the previous slide



→ By changing the layout, make the optics periodic
tip: you need to change both the command and the lattice file

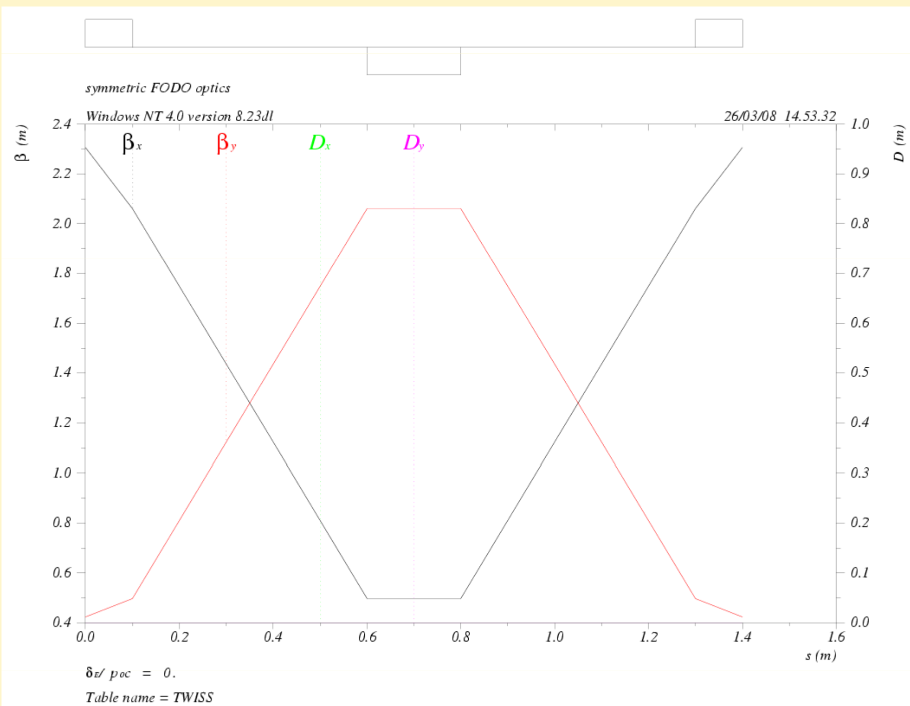


→ Now by changing the lattice file, invert the x- and y- betatron functions

Exercise 4

→ Run "test_2". Which kind of lattice is described?

→ Use the MATCH command to find the quadrupole strengths that minimizes the beam envelope in both the transverse planes.
tip: the betatron phase advances in Mad are in [1/2π] unit



output "print" file

```
Qx            =      0.250000
Qx'           =     -0.317755
betax(max)    =      2.307401
Dx(max)       =      0.000000
Dx(r.m.s.)    =      0.000000
xco(max)      =      0.000000
xco(r.m.s.)   =      0.000000
```

```
Qy            =      0.250000
Qy'           =     -0.317755
betay(max)    =      2.061859
Dy(max)       =      0.000000
Dy(r.m.s.)    =      0.000000
yco(max)      =      0.000000
yco(r.m.s.)   =      0.000000
```

Betatron Resonances

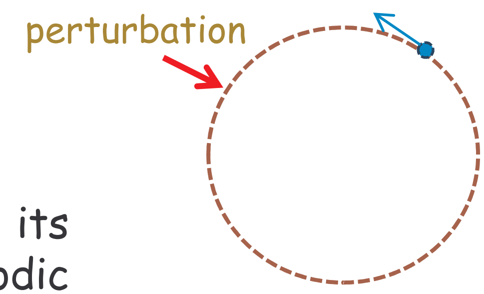
- The particle betatron motion is quasi-periodic oscillation along the transport line:

$$x = \sqrt{a\beta(s)}\cos(\Delta\phi(s))$$

$$x' = \frac{p_x}{p_s} = \frac{dx(s)}{ds}$$

The representation of an oscillator in the phase space (x, x') is an ellipse (closed orbit means periodic motion)

- Perturbations in the accelerator (field errors, misalignments,...) are represented by error kick angle (Δx') that forces the particle to unstable motion.



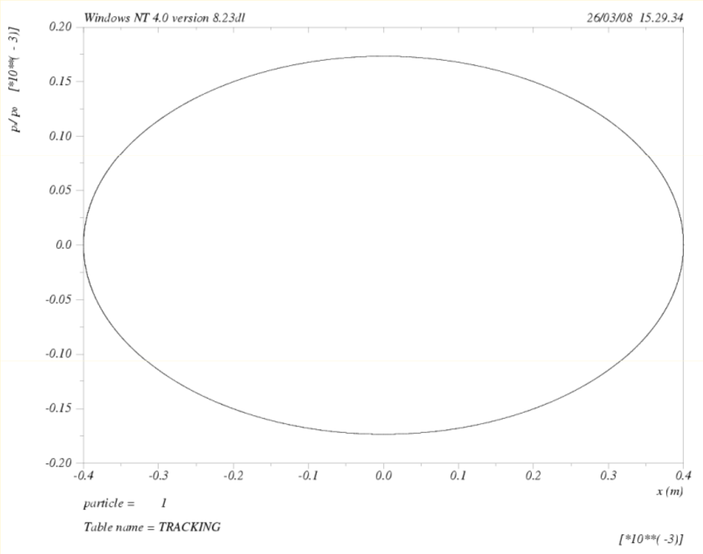
- The perturbation is critical if the particle returns to the its location always with the *same position* and *velocity* (exact periodic motion).
In this case, the phase advance in the lattice satisfies the following resonance condition:

$$\boxed{p \cdot Q = m}$$

with p,m integer and Q is the *betatron tune* = *phase advance/2π*

Exercise 5

→ Run "test_3" and plot the particle horizontal phase space over many turns (ideally, the FODO lattice is repeated infinitely)



→ Run "test_3" by changing the quadrupole strength step-by-step closer to the value found in exercise 4 (see previous slide).

→ How the phase space is changing ? After how many turns the particle returns to the same (x, x') point ? Which "resonance" are you approaching?

Example of Particle Tracking code: Elegant

Particle tracking codes have usually the same capabilities of the Optics code but also something more.....

❑ http://www.aps.anl.gov/Accelerator_Systems_Division/Operations_Analysis/software.shtml

❑ Particle beam can be an external file (6 data columns per particle) or a self-generated particle distribution (Gaussian, Uniform).

❑ Beam transport by integration of the equation of motion performed for each single particle ⇒ optics can be calculated starting from the particle distribution

⇓

Be sure the beam Twiss parameters are matched to the lattice parameters !!!

❑ Collective effects are simulated by implementing analytical models or by calculating the Coulomb force between the particles (intra-beam effects) or between each particle and the external source (beam-beam effect, image charge,...)

Input command and lattice have the same structure of Mad code but they accept more options…output files in SDDS format (they need the proper toolkit to be managed)