



**The Abdus Salam  
International Centre for Theoretical Physics**



**1936-4**

**Advanced School on Synchrotron and Free Electron Laser Sources  
and their Multidisciplinary Applications**

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**Introduction to  
RIETVELD REFINEMENT  
&**

**Powder X-ray Diffraction  
Experimental setups**

Jasper Plaisier  
*Elettra, Trieste*



# Introduction to RIETVELD REFINEMENT

Jasper Plaisier  
MCX beamline –Elettra Trieste



# Outline

- Introduction
  - The method
  - The math
- Before you start refining
  - Indexing / unit cell, space group determination
  - Constructing a model
- Refinement
  - Important steps
  - Criteria of fit

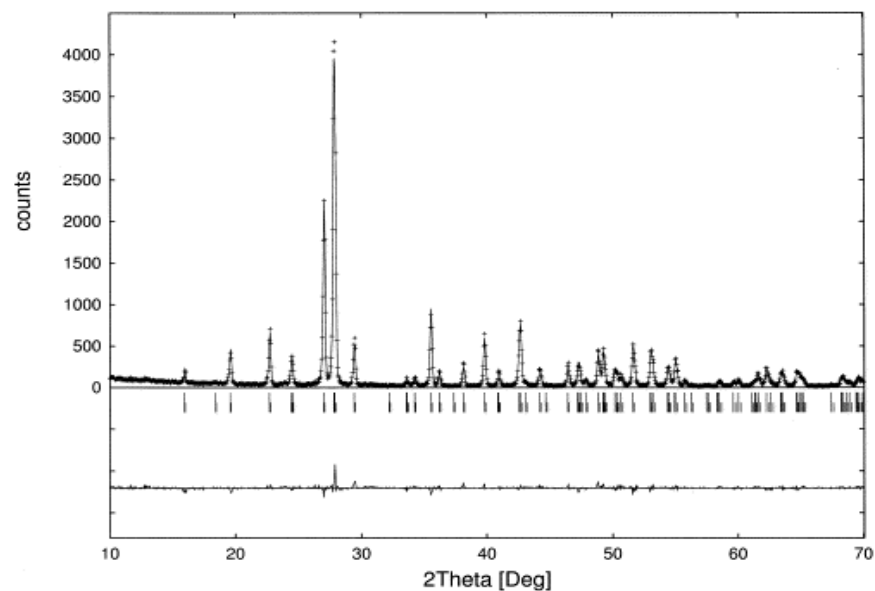
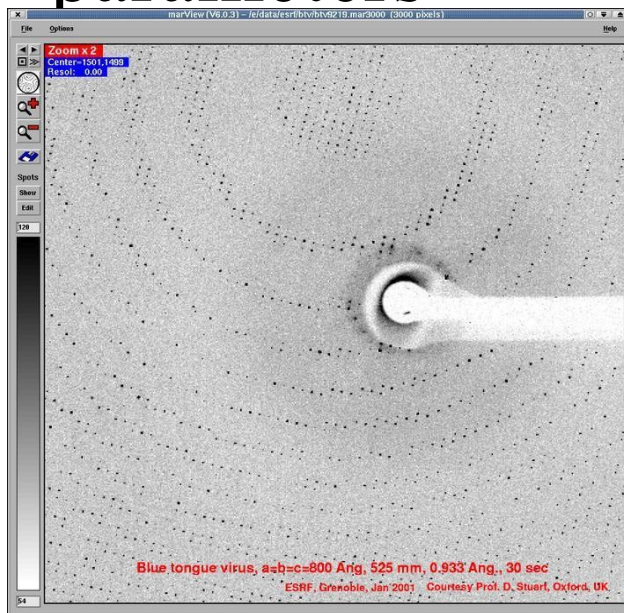
# Introduction

- Hugo M. Rietveld (Den Haag, 1932)
- Structure determination from neutron powder diffraction
- Original software was written in 1969
- First reports on the application in XRD: 1977



# Rietveld refinement

- Goal: get accurate structural parameters given a powder diffraction pattern.
- Peak intensities depend on structural parameters

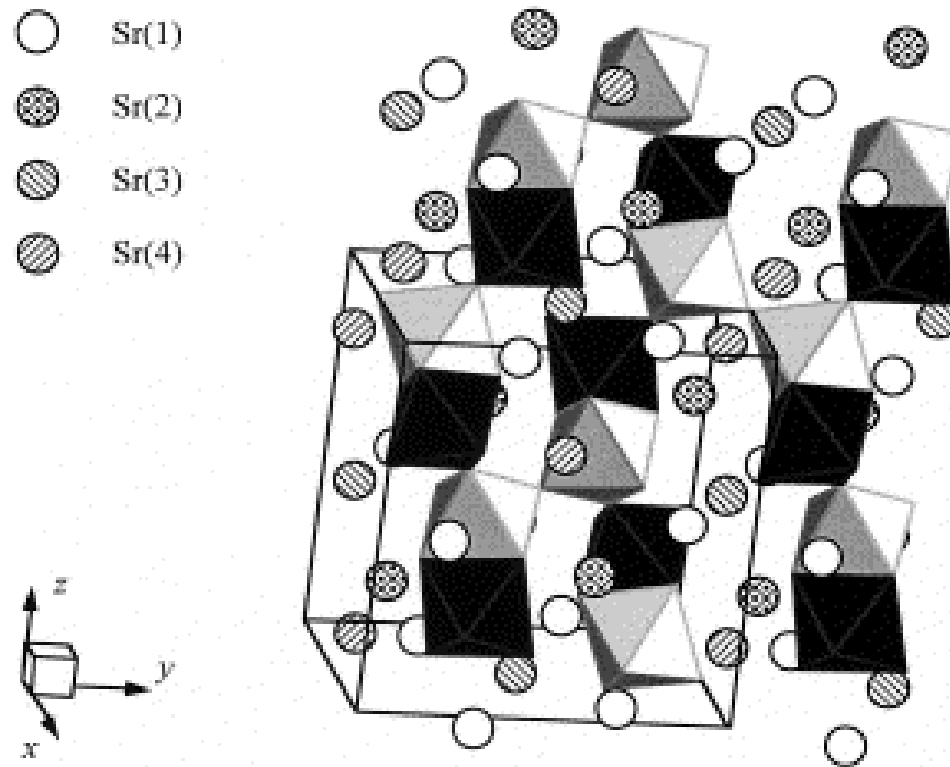




# Crystal Structure

- Unit cell: regular repeating pattern of atoms that forms the crystal (defined by lattice parameters  $a$ ,  $b$ ,  $c$ ,  $\alpha$ ,  $\beta$ ,  $\gamma$ )
- Space group: Describes the symmetry of the atomic structure (eg.  $P2_1/m$ )
- Atomic positions within unit cell ( $x, y, z$ )

# Crystal Structure





# Peak intensities

- Integrated area of peak gives intensity
- Peak height is not a good estimate, shape must be taken into account
- Factors affecting peak shape: temperature absorption, particle size, instrument parameters

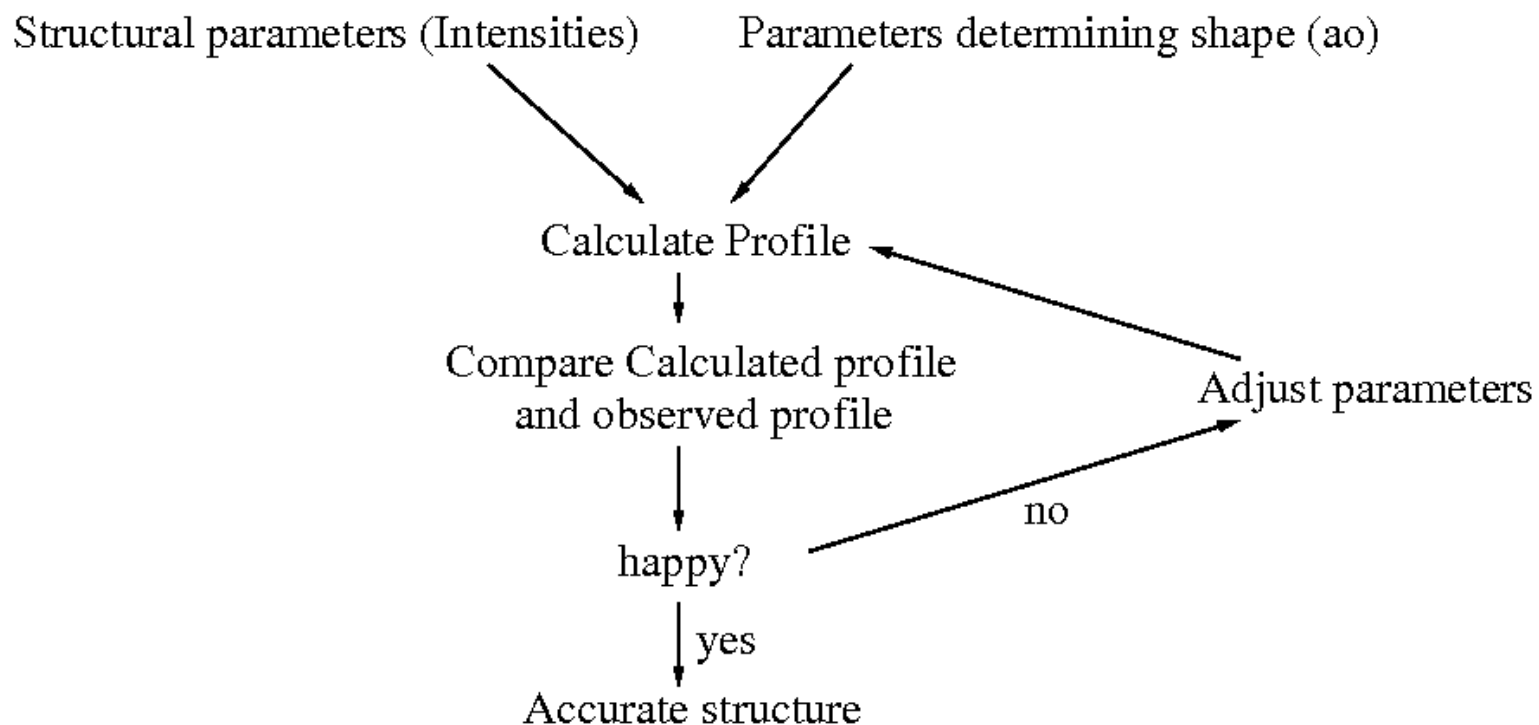




# Rietveld method

- In the Rietveld method the parameters are refined using the least squares method until the best fit between the calculated pattern and the observed pattern is obtained

# Rietveld method



# Rietveld math

- Minimization of residual  $S_y$

$$S_y = \sum_i w_i (y_i - y_{ci})^2$$

- $W_i = 1/y_i$
- $Y_i$  = Observed intensity at the  $i$ th step
- $Y_{ci}$  = Calculated intensity at the  $i$ th step

# Rietveld math

- Calculation of intensity  $y_{ic}$

$$y_i = s \sum_K L_K |F_K|^2 \varphi(2\Theta_i - 2\Theta_K) P_{KA} + y_{bi}$$

- $s_i$  = Scale factor
- $K$  represents Miller indices,  $hkl$ , for a Bragg reflection
- $L_K$  contains Lorentz, polarization and multiplicity factors
- $\varphi$  is the reflection profile function
- $P_K$  is the preferred orientation function
- $A$  is the absorption factor
- $F_K$  is the structure factor
- $Y_{bi}$  is the background intensity at the  $i$ th step



# Rietveld math

- Calculation of the structure factor

$$F_{hkl} = \sum_j N_j f_j \exp(2\pi i(hx_j + ky_j + lz_j)) \exp(-B_j \sin^2(\theta/\lambda^2))$$

- $hkl$  Miller indices
- $x_j, y_j, z_j$  are the atomic position parameters of the  $j$ th atom in the unit cell
- $B_j$  = Temperature factor (thermal vibrations) of the  $j$ th atom
- $N_j$  site occupancy for the  $j$ th atom site
- $f_j$  scattering factor of the  $j$ th atom



# Before we start refining

- Rietveld **refinement** requires a starting model that is close to the real model
- Starting values for instrument parameters can be determined using standard materials
- Necessary finding a reasonable estimate of
  - Unit cell parameters
  - Atomic positions
- Determine the correct space group



# Finding unit cell parameters

- Unit cell parameters are related to the peak positions by Bragg's law:

$$n \lambda = 2d \sin \theta$$

- In case of a cubic cell :  $a=b=c$ ,  $\alpha=\beta=\gamma=90^\circ$

$$d_{hkl}^2 = \frac{a^2}{h^2 + k^2 + l^2}$$



# Simple cubic example

- $a=4.0\text{\AA}$ , space group  $Pm\bar{3}m$

<u><math>2\Theta</math></u>	<u><math>d</math></u>	<u><math>1000/d^2</math></u>	<u><math>h\ k\ l</math></u>	<u><math>h^2+k^2+l^2</math></u>
22.21	4.00	62.5	1 0 0	1
31.61	2.83	125.0	1 1 0	2
38.97	2.31	187.5	1 1 1	3
45.31	2.00	250.0	2 0 0	4
51.01	1.79	312.5	2 1 0	5
56.29	1.63	375.0	2 1 1	6
66.00	1.41	437.5	2 2 0	8
70.58	1.33	500.0	3 0 0	9
			2 2 1	9



# Finding unit cell parameters

- More complex unit cells make manual indexing difficult
- Indexing is usually done with a computer program (Dicvol, Treor, Ito)
- Input:
  - 20-30 accurately determined peak positions
  - Wavelength
  - Maximum cell volume
  - Error in peak positions



# Finding space group

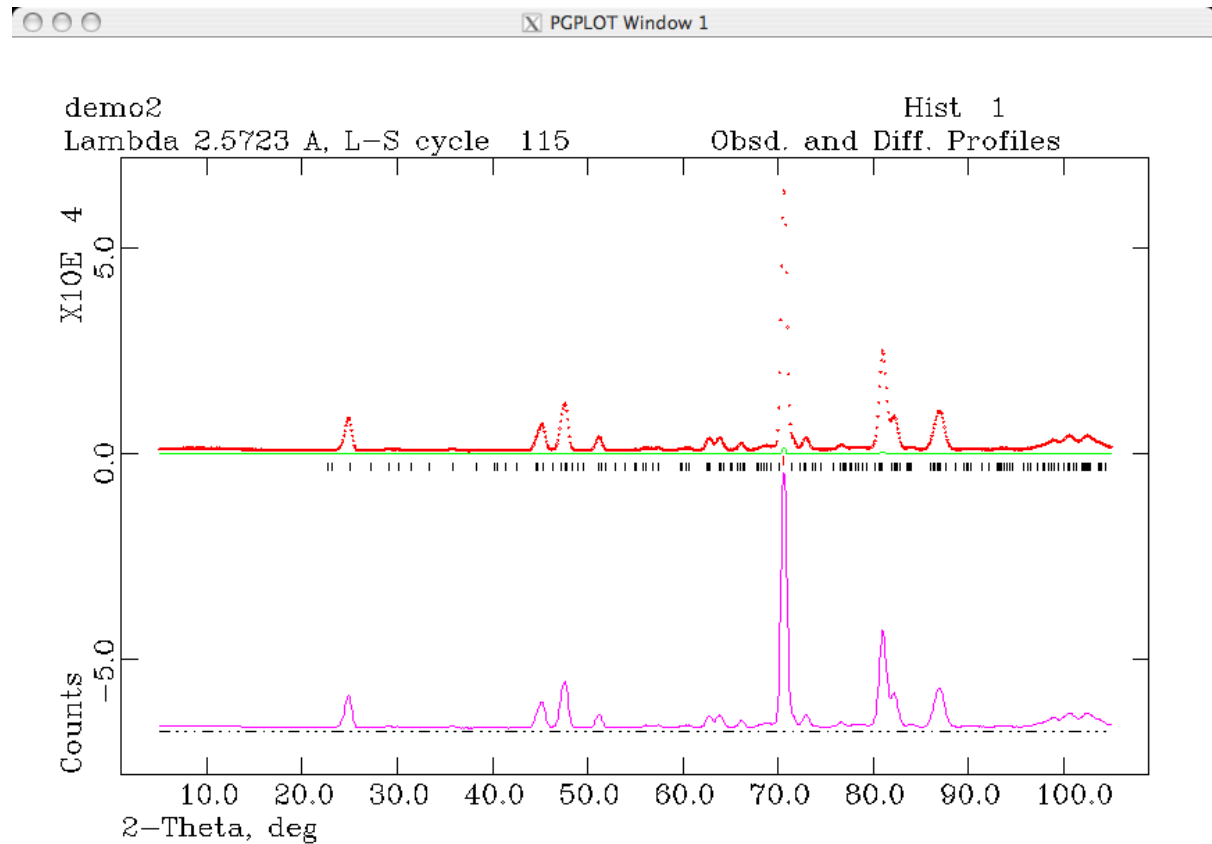
- Space group cannot be refined, must be right
- Some symmetry elements can be determined from systematic absences
- Often various refinements using different space groups are performed and their results compared

# Starting model: atomic parameters

- Find in the database a known structure that is similar
- Determine a structure ab-initio using direct methods or modeling
  - Needs accurately estimated peak intensities
  - Small peak widths of synchrotron powder diffraction helps out a lot



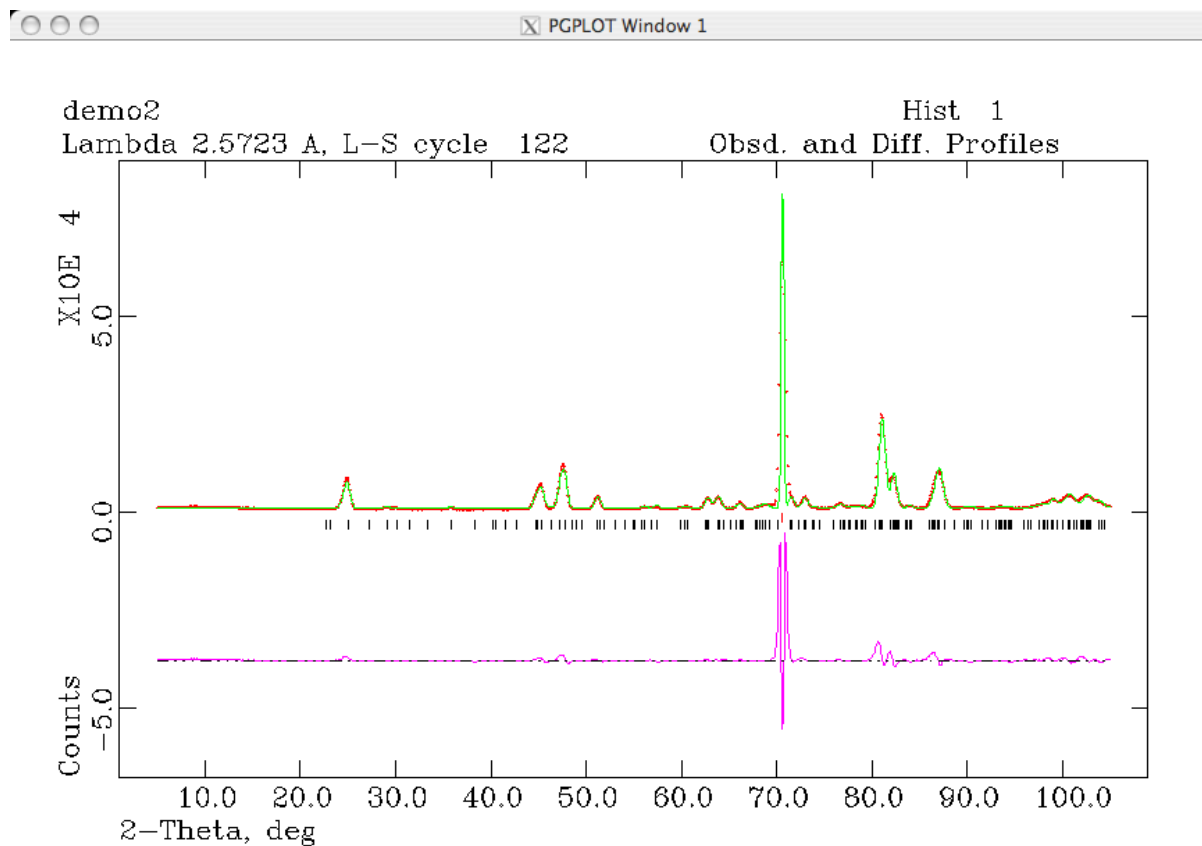
# Refinement steps



# Refinement steps

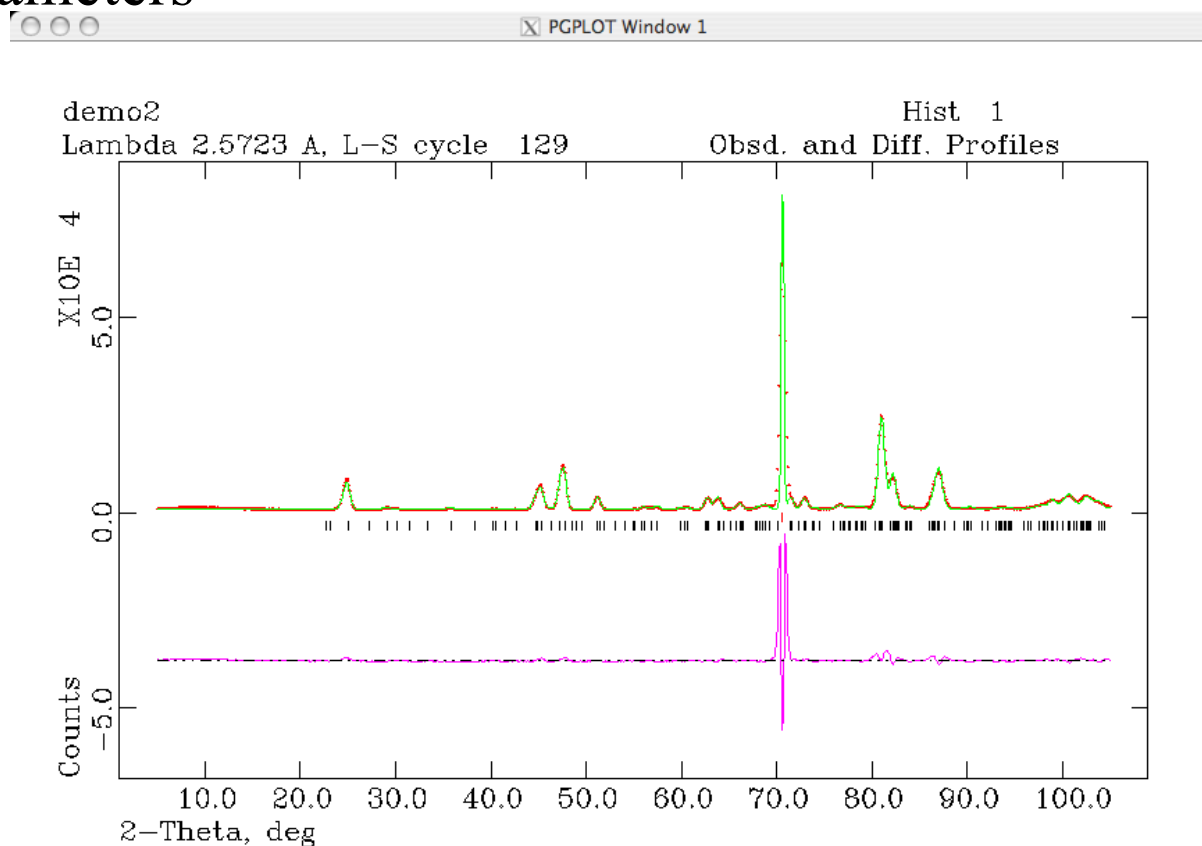
- Refining parameters to fit overall shape of diffraction pattern:
  - Scale factor
  - Background (function with 3-6 parameters)
  - Zero-point correction

# Refinement steps



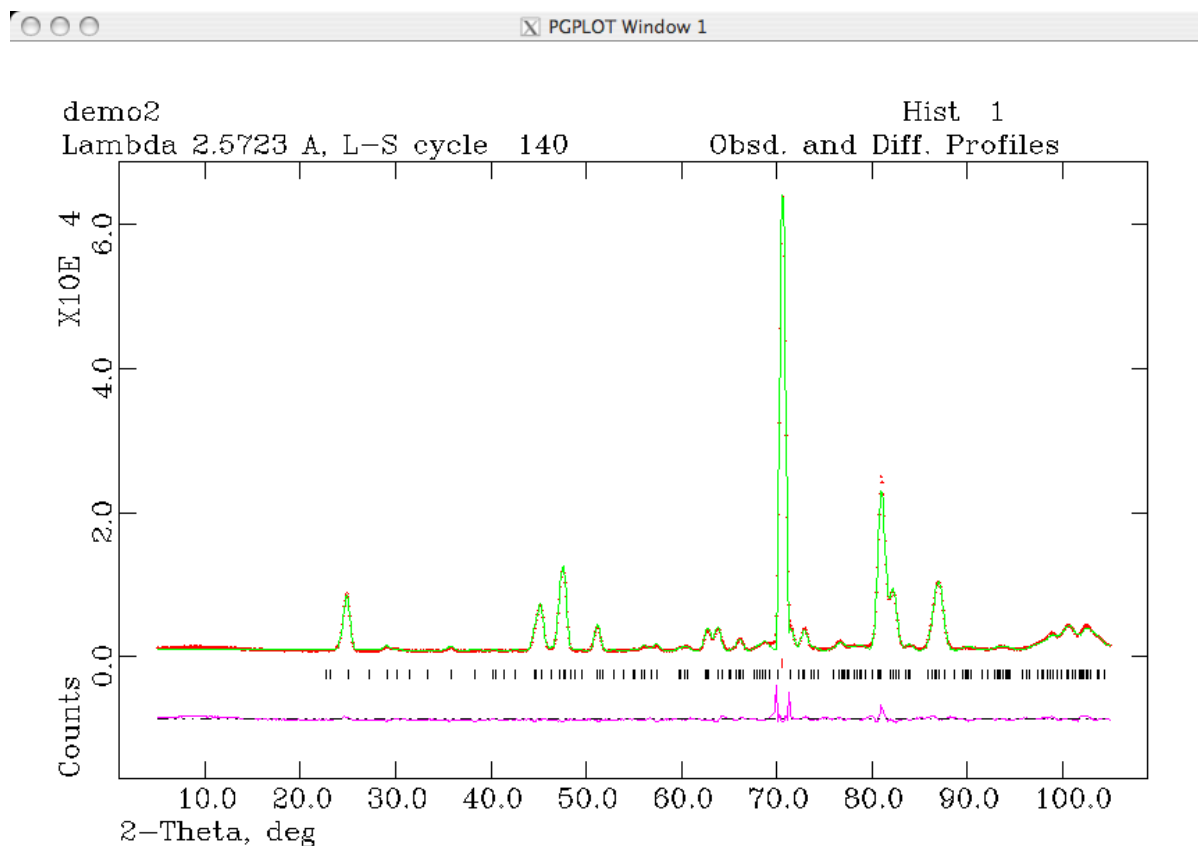
# Refinement steps

- Get accurate peak positions by refinement of the unit cell parameters



# Refinement steps

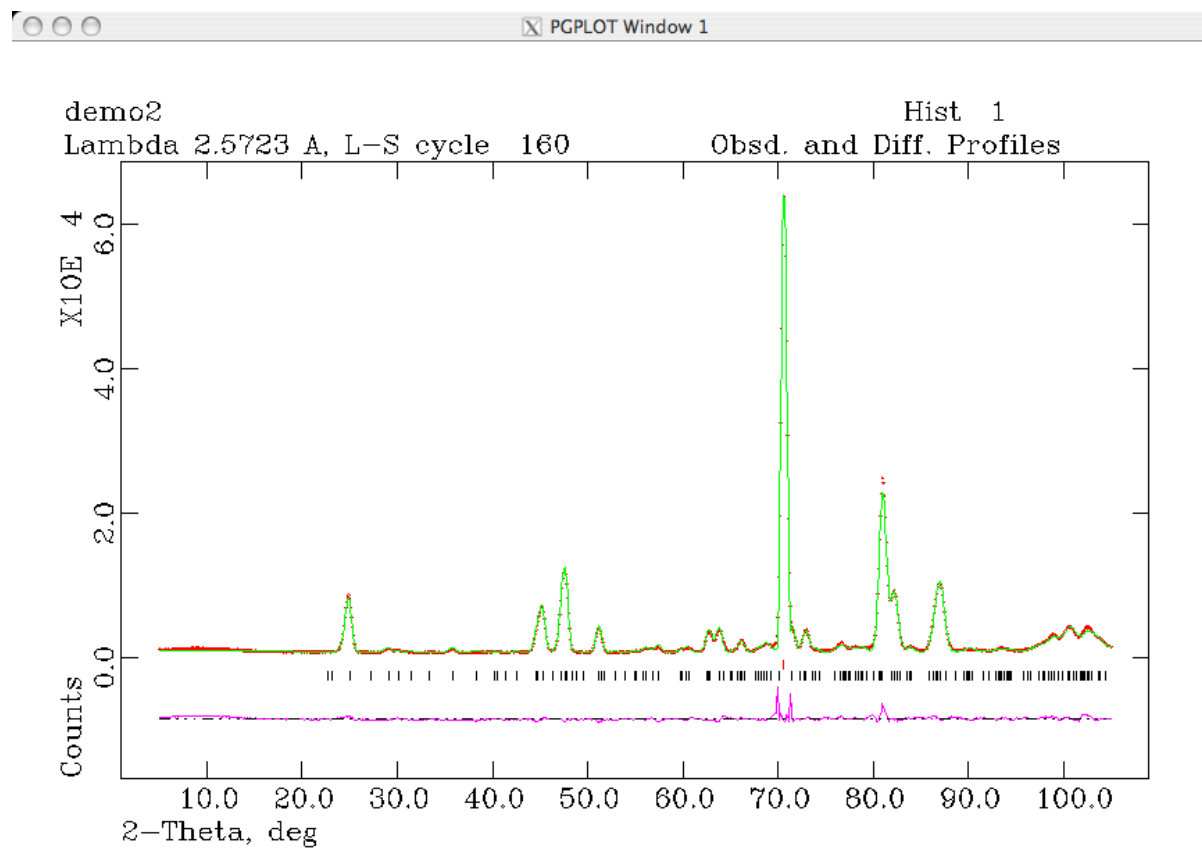
- Peak shape parameters





# Refinement steps

- Atomic parameters



# Refinement steps

- Sequence of the parameters to be included in the refinement depends on the quality of the starting model
- Looking at the difference between the calculated and observed profile helps to decide the order in which to include the parameters

# Criteria of fit

- Measure of the quality of the fit

- R-Bragg 
$$R_B = \frac{\sum |I_K(obs) - I_K(calc)|}{\sum I_k(obs)}$$

- R-pattern

$$R_p = \frac{\sum |y_i(obs) - y_i(calc)|}{\sum y_i(obs)}$$

- R-weighted pattern

$$R_{wp} = \frac{\sum w_i (y_i(obs) - y_i(calc))^2}{\sum w_i (y_i(obs))^2}$$

# Criteria of fit

- $I_K$  is never directly observed.
- R-bragg is biased towards the model
- $R_{wp}$  is directly related to the function that is minimized
- Don't forget to look at the patterns!!!



# Powder X-ray Diffraction Experimental setups

Jasper Plaisier

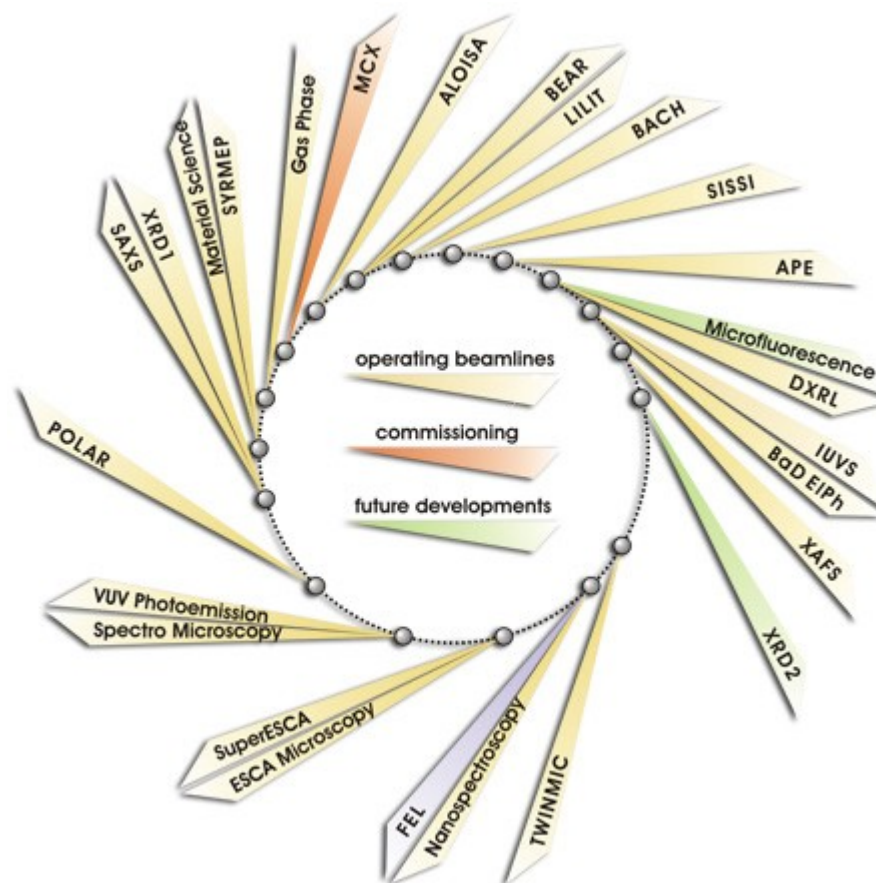
MCX beamline –Elettra Trieste

# Outline

- Introduction
- Experimental techniques at MCX
- MCX Experimental setup
  - Beam line optics
  - Experimental hutch
- Experimental stations
  - Flat plate - reflection
  - Capillary - transmission

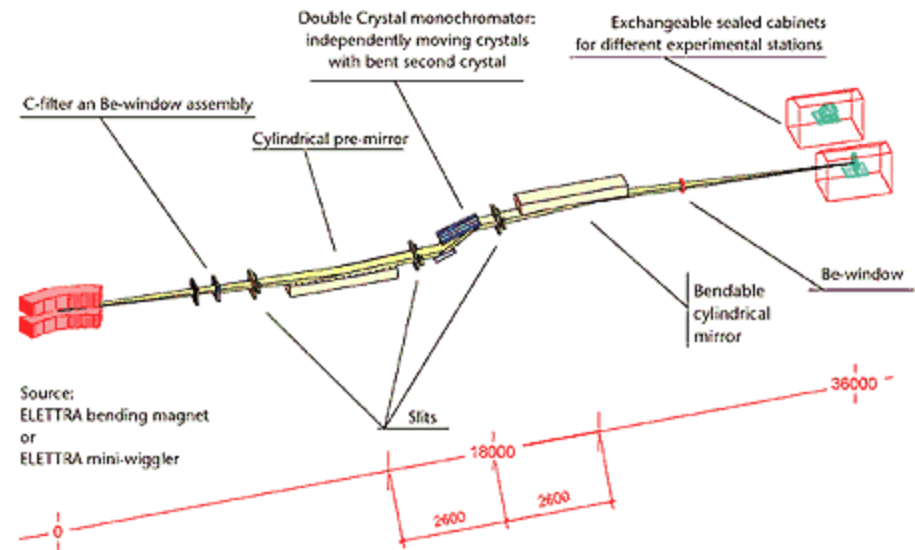
# Introduction

- MCX is under construction



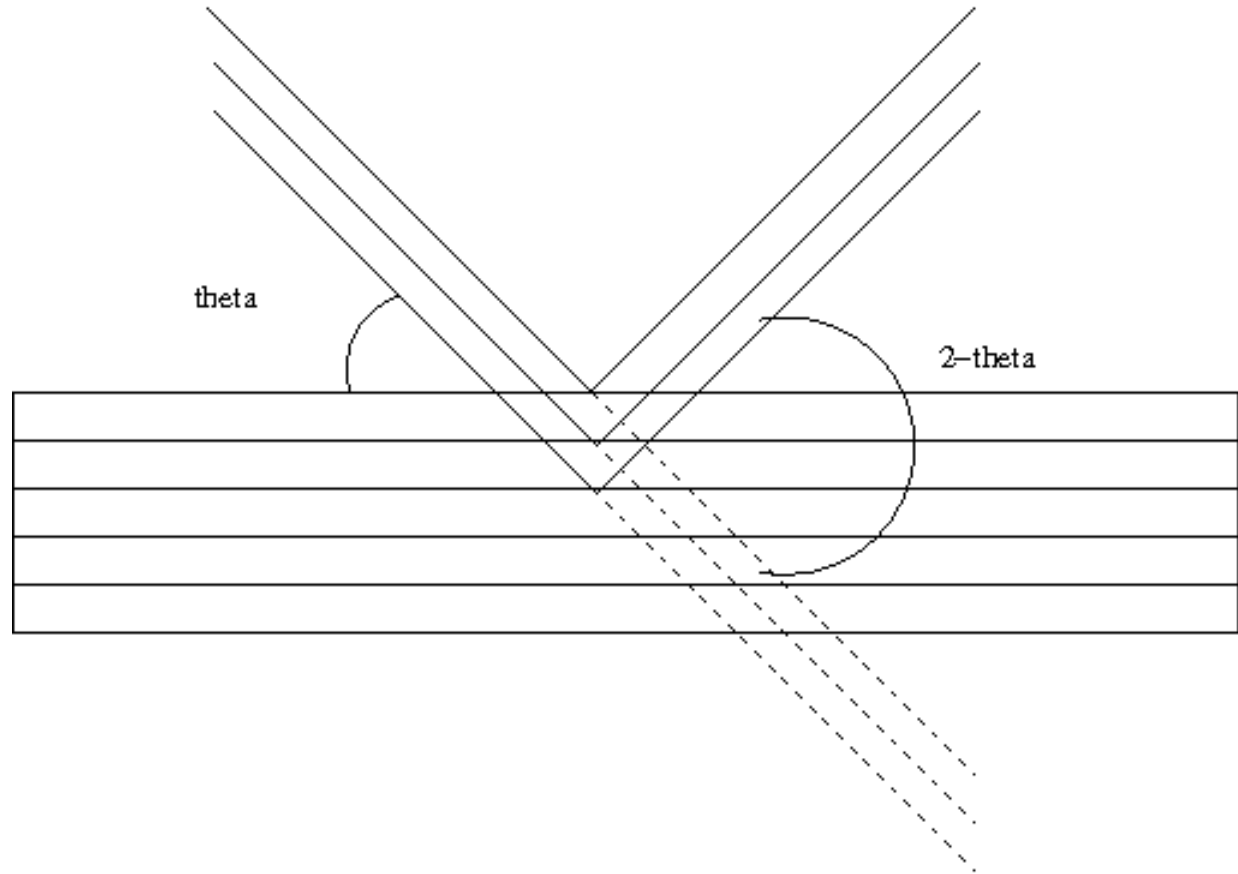
# Introduction

- Materials Characterization by X-rays

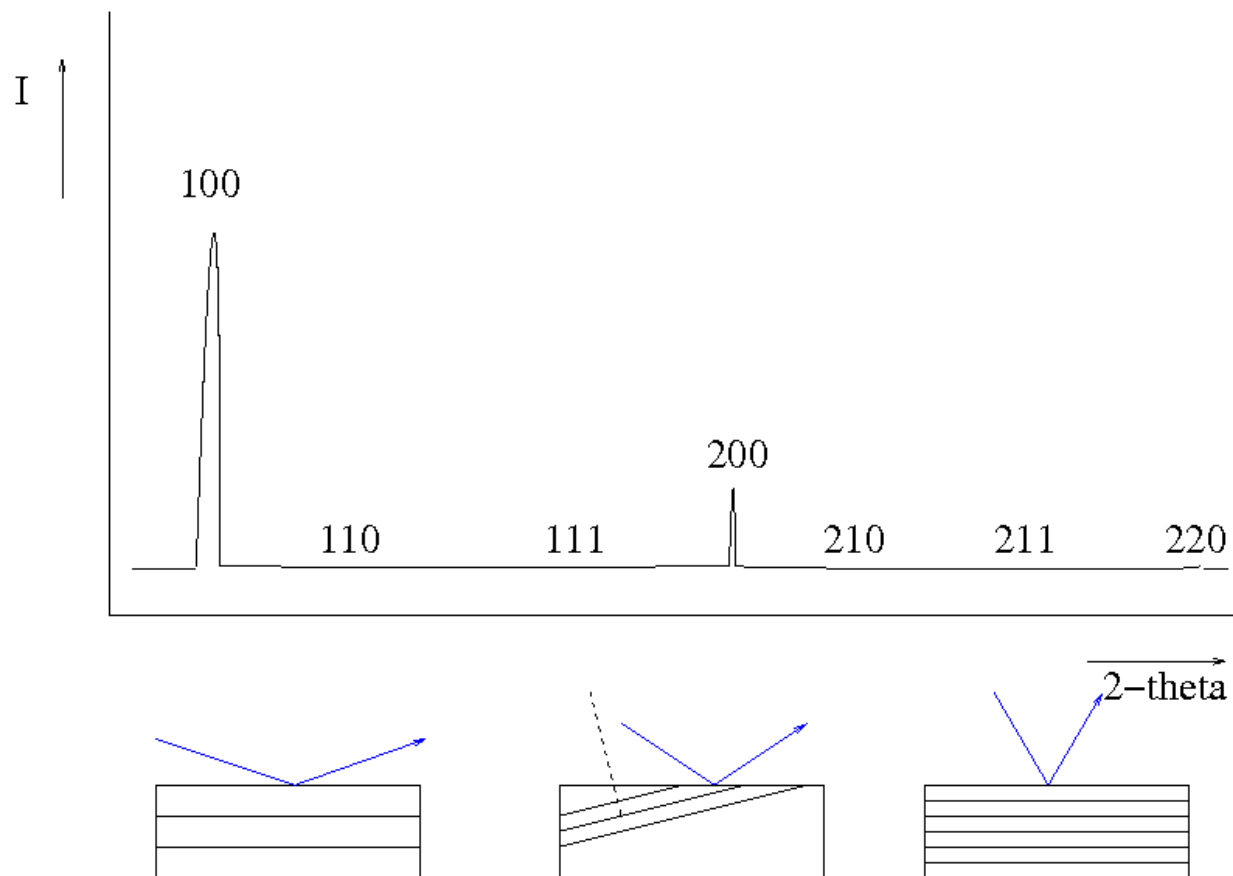




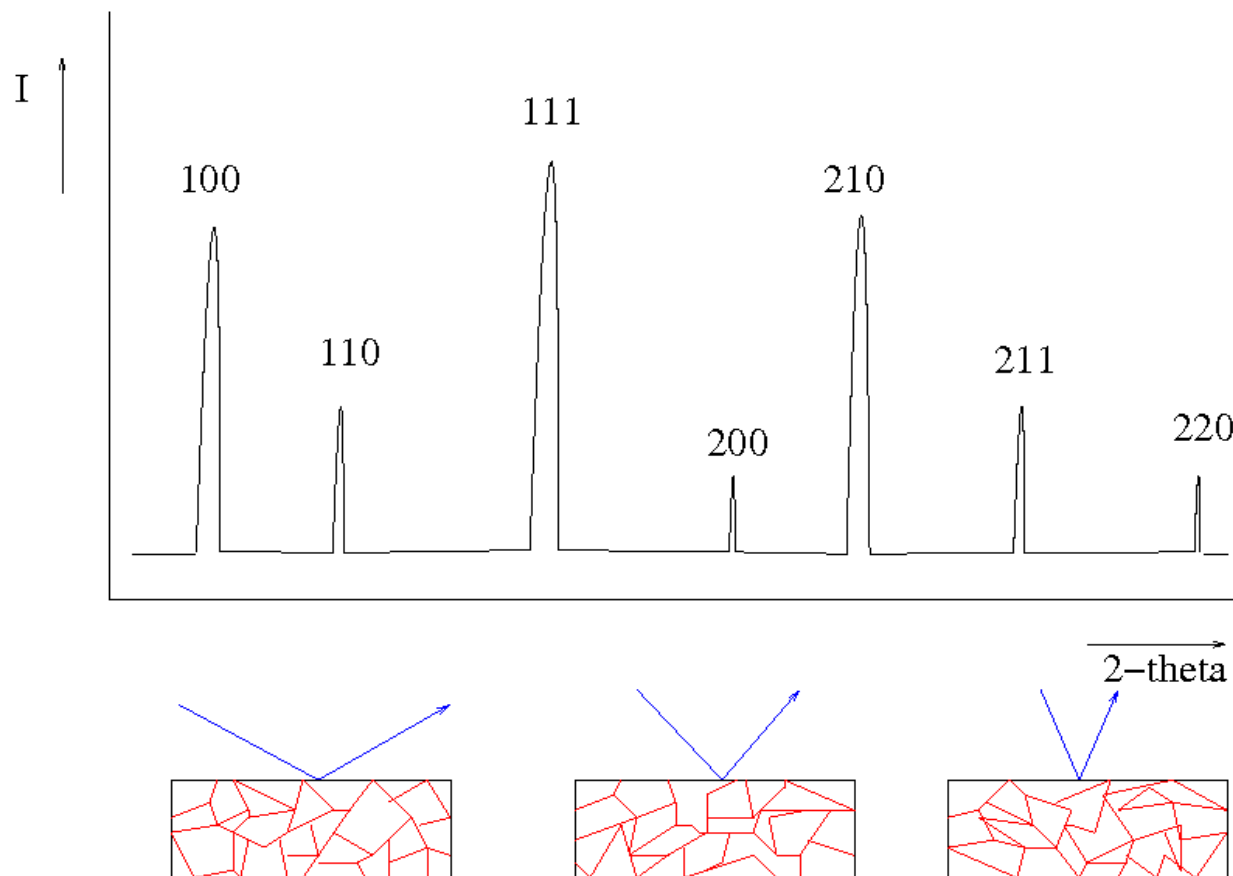
# Introduction



# Introduction



# Introduction





# Experimental techniques at MCX

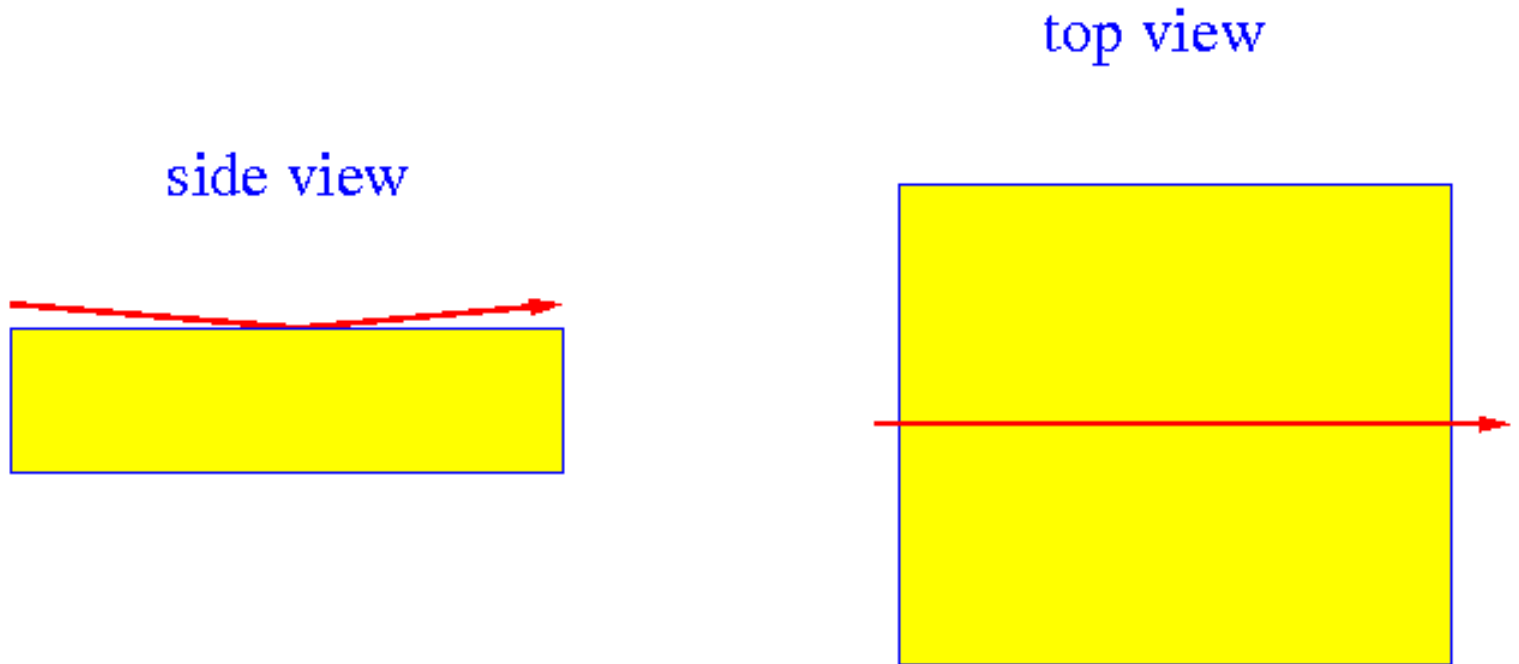
## Grazing angle: reflectivity

- Based on total reflection of X-ray beam
- Study of:
  - Surface morphology
    - Roughness
    - Waviness
  - Composition
  - Structure of multilayer systems
- Layer thickness .5-400 nm,
- Needs difference of electron density in layers

# Experimental techniques at MCX

## Grazing angle: reflectivity

- The measurement:



# Experimental techniques at MCX

## Grazing angle: diffraction

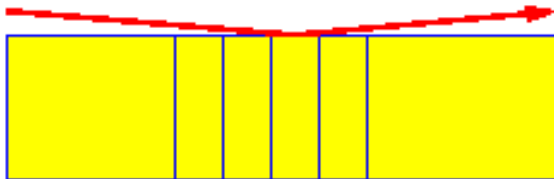
- Based on Bragg diffraction in thin layer
- X-rays enter sample with small incident angle to limit penetration depth
- Study of:
  - Identification and study of monolayers
  - Differences in crystallinity surface/bulk
  - Surface residual stress and texture
  - Depth profiling

# Experimental techniques at MCX

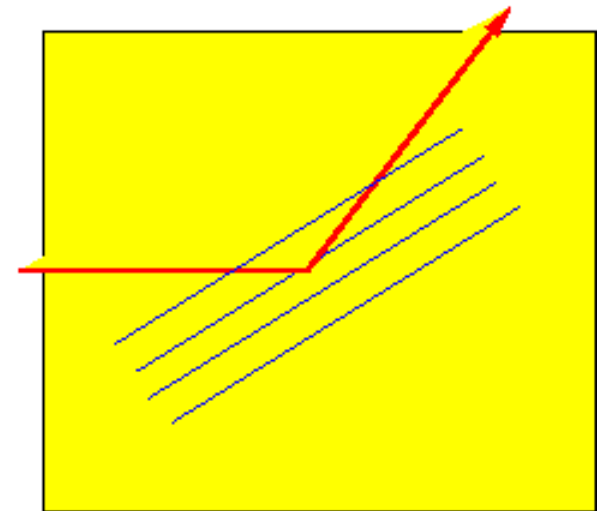
## Grazing angle: diffraction

- Symmetric configuration

side view



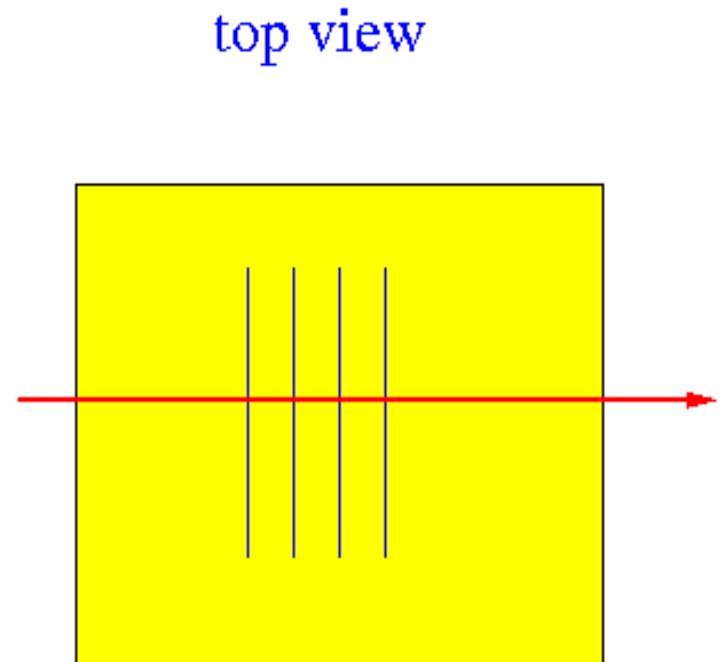
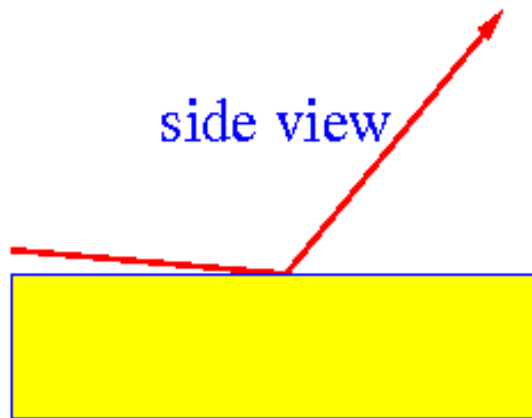
top view



# Experimental techniques at MCX

## Grazing angle: diffraction

- Asymmetric configuration







# Experimental techniques at MCX

## Stress and texture analysis

- Stress can be determined from precision lattice constant determination (diffraction peak shifts)
- Texture is the orientation distribution of the crystallites
- Texture is measured by rotating the sample while monitoring the intensity of a Bragg-reflection



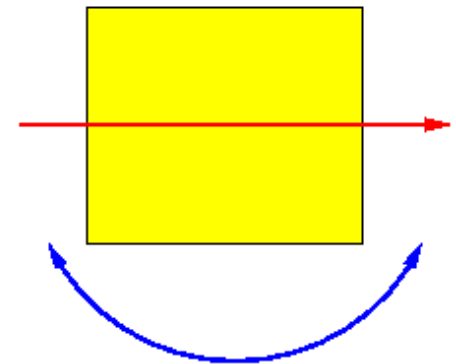
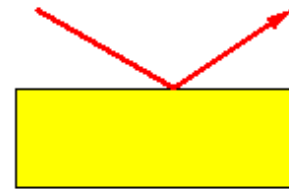
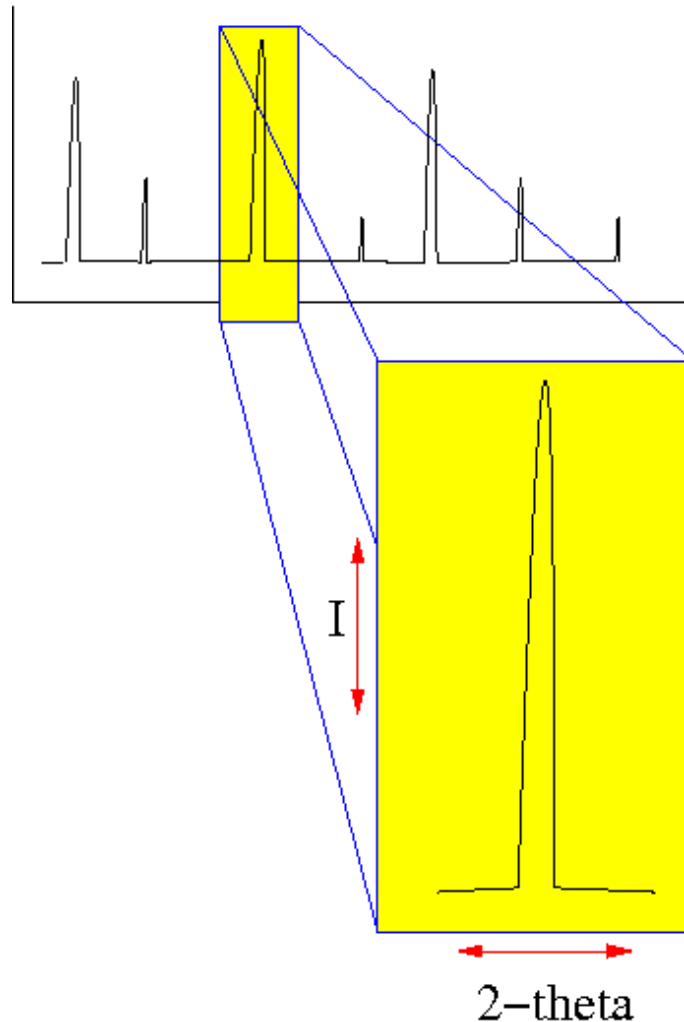
# Experimental techniques at MCX

## Elastic constants and mechanical properties

- Based on X-ray diffraction measurement while mechanical pressure is applied
- Elastic tensors can be determined by measuring different crystallographic directions

# Experimental techniques at MCX

## Elastic constants and mechanical properties





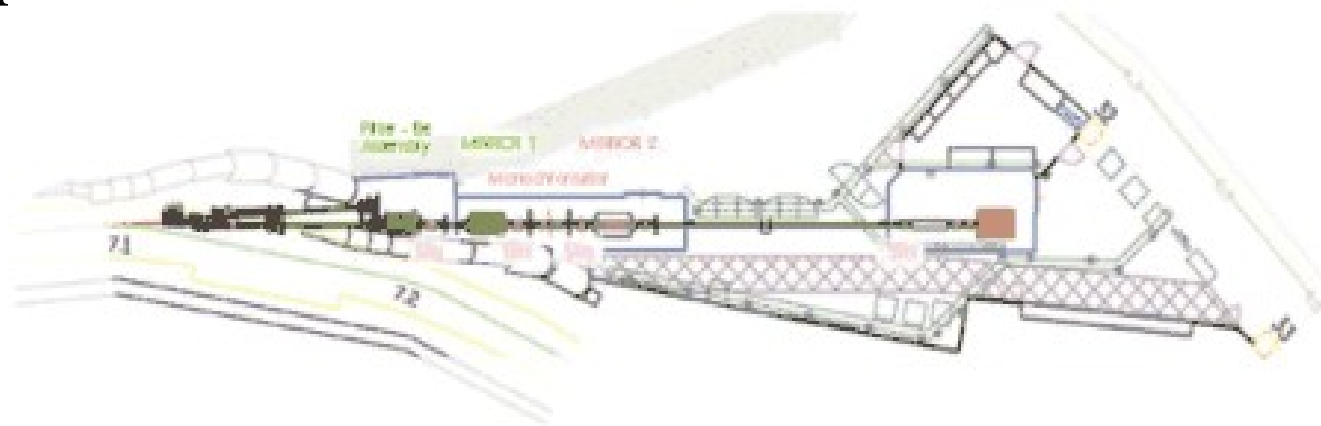
# Experimental techniques at MCX

## Phase identification and profile analysis

- Phase identification by XRD fingerprint
- Quantitative phase analysis
- Structure solution and refinement
- Size and shape distribution
- Study of lattice defects

# MCX Experimental setup

- Front end Hutch
- Beamline Optics Hutch
- Tube
- Experimental Hutch

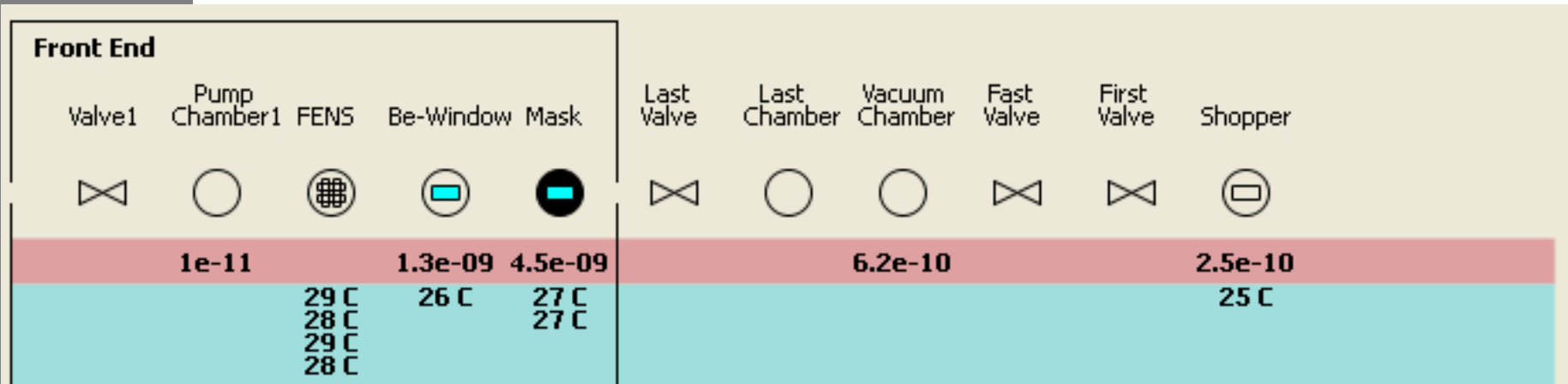




# MCX Experimental setup

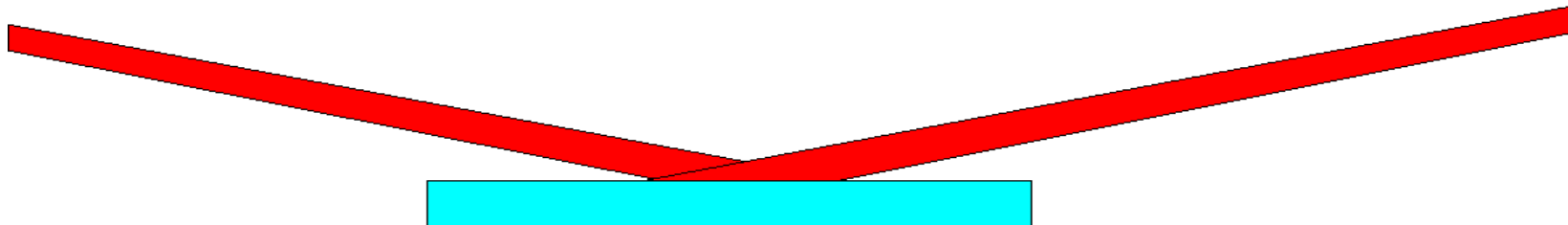
## Front End Hutch

- Shopper
- Mask
- Be window
- First Entrance Slits
- Several valves



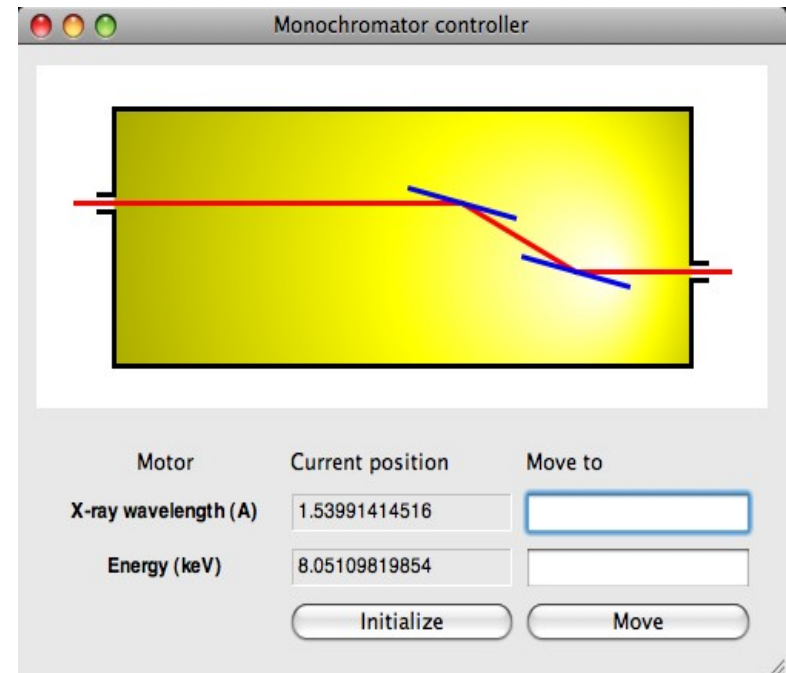
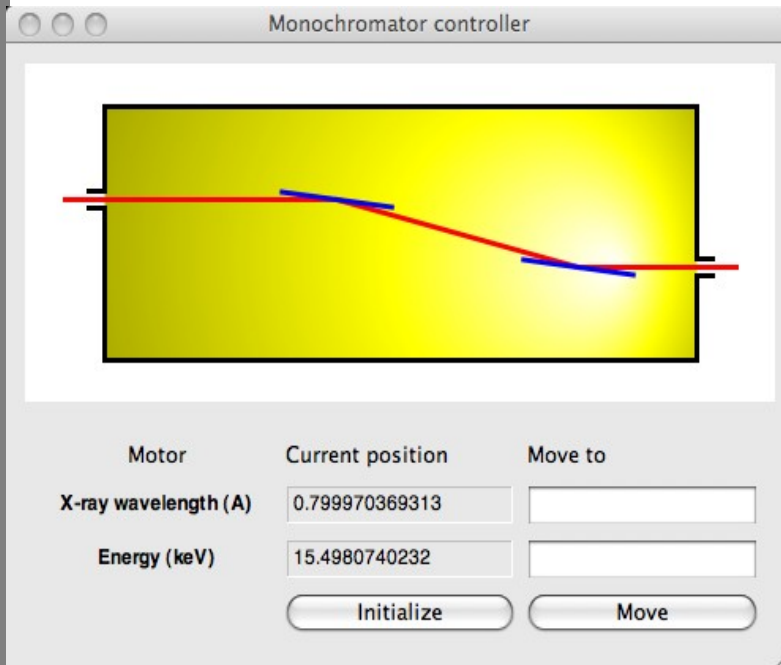
# MCX Experimental setup

## Optics – Mirror 1



# MCX Experimental setup

## Optics – Monochromator

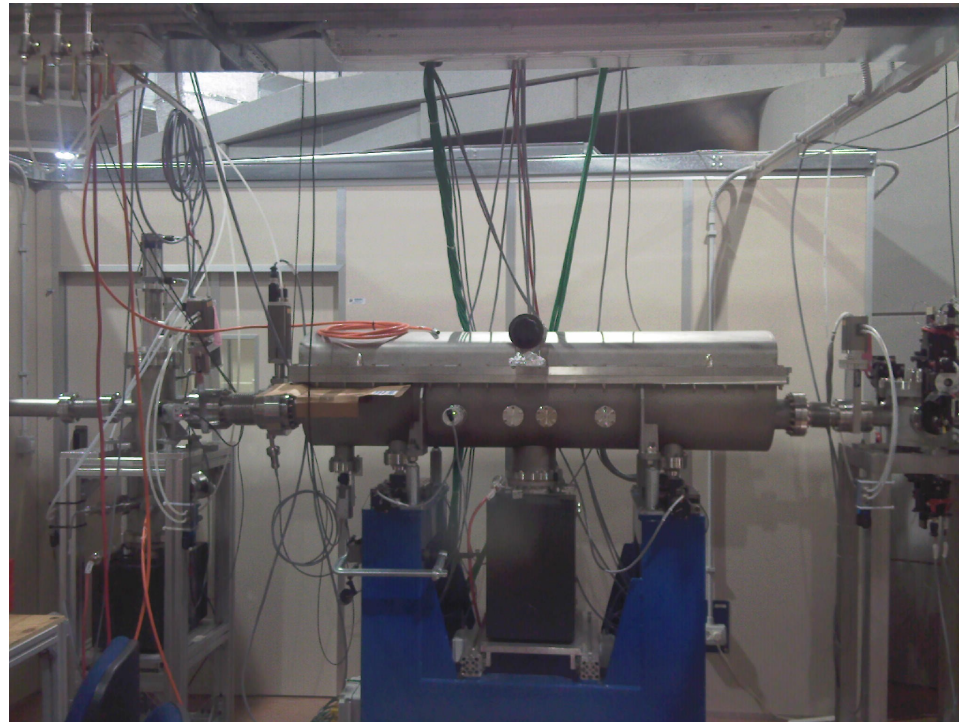


- Choice of wavelength of X-rays



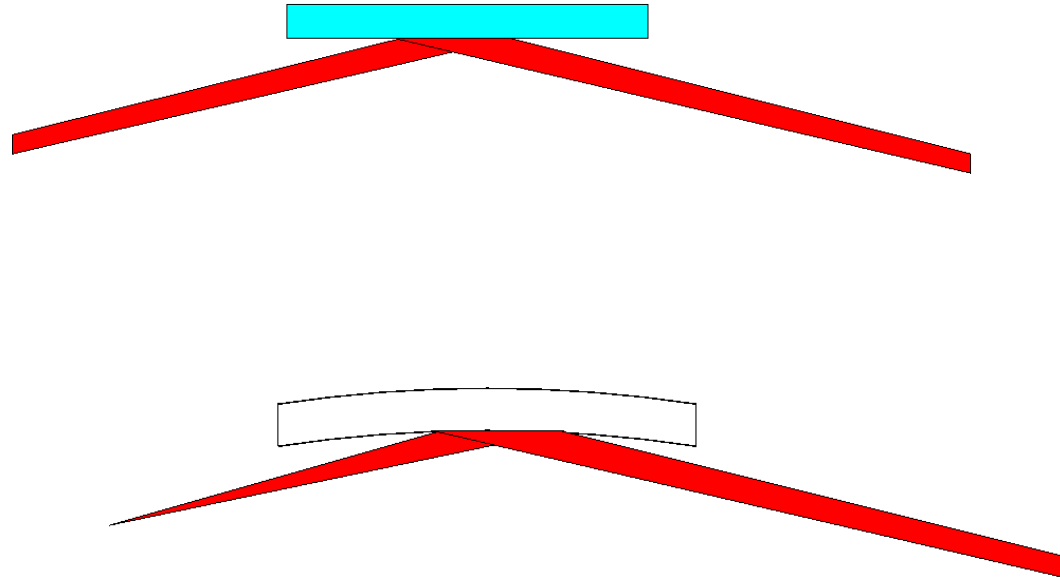
# MCX Experimental setup

## Optics – Mirror 2



# MCX Experimental setup

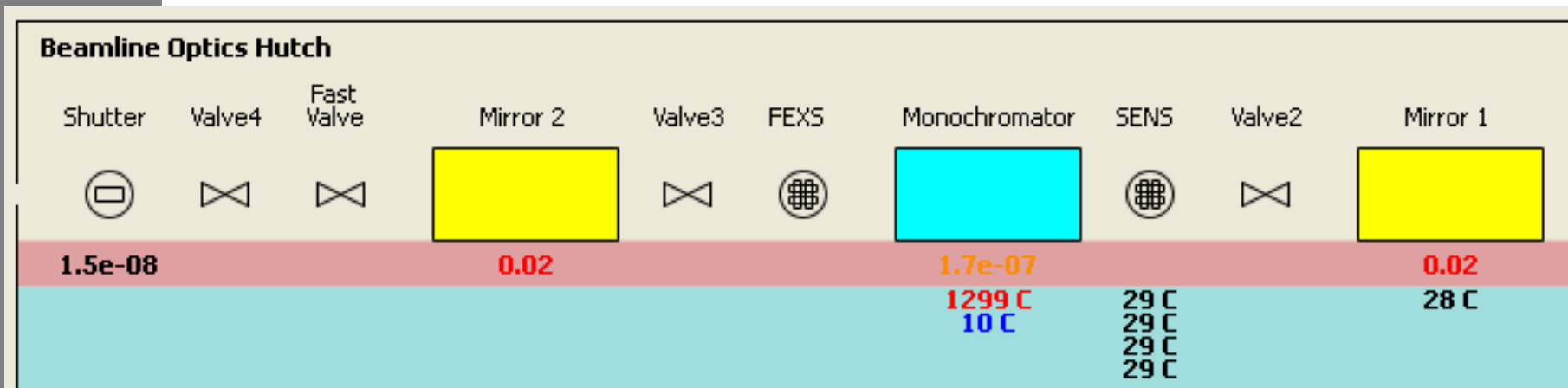
## Optics – Mirror 2



# MCX Experimental setup

## Beamline Optics Hutch

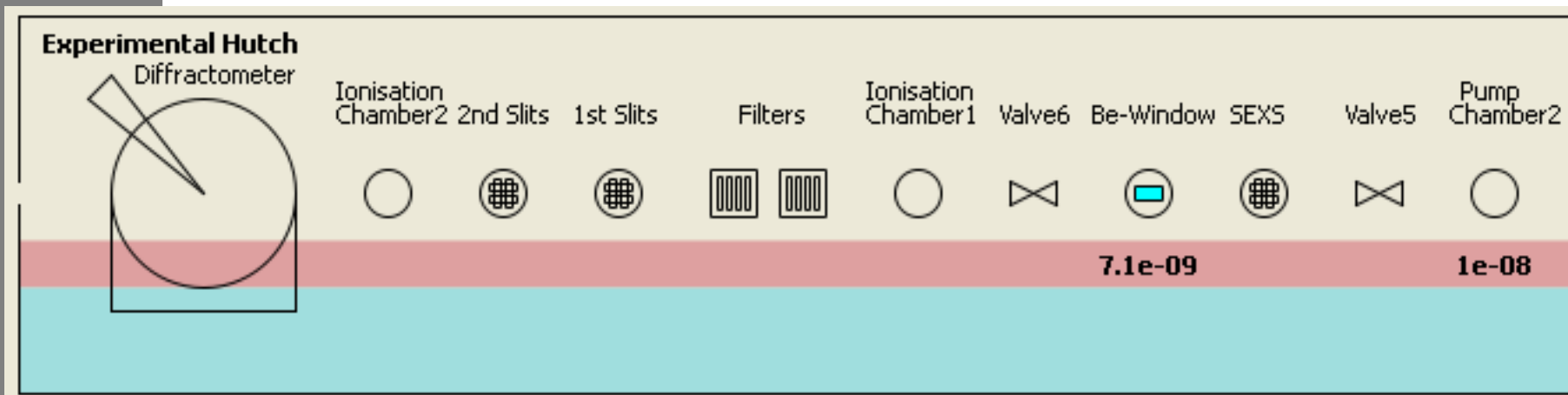
- Mirrors
- Monochromator
- Second entrance/First exit slits
- Shutter
- Several valves



# MCX Experimental setup

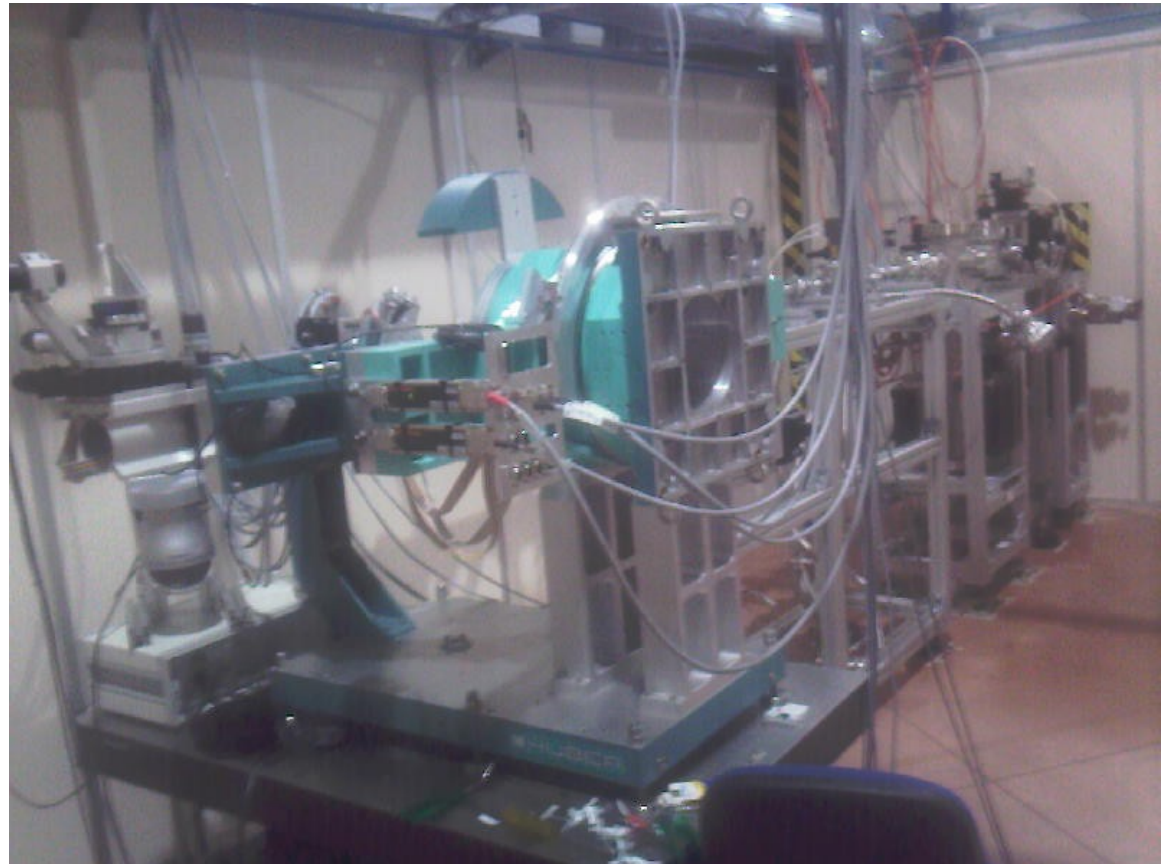
## Experimental Hutch

- Second exit slits
- Be-window
- filters
- slits
- Several valves
- Ionisation chambers



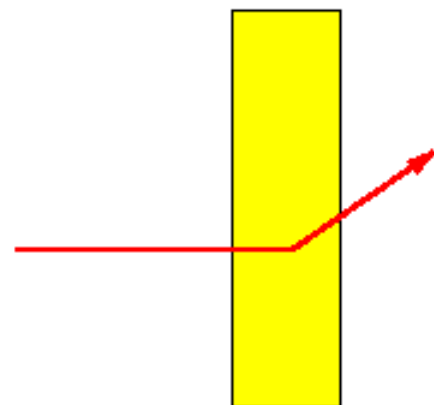
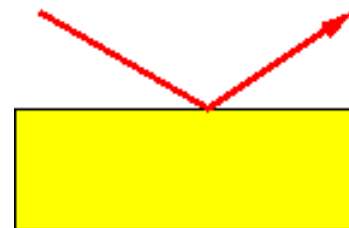
# MCX Experimental setup

## Experimental Hutch

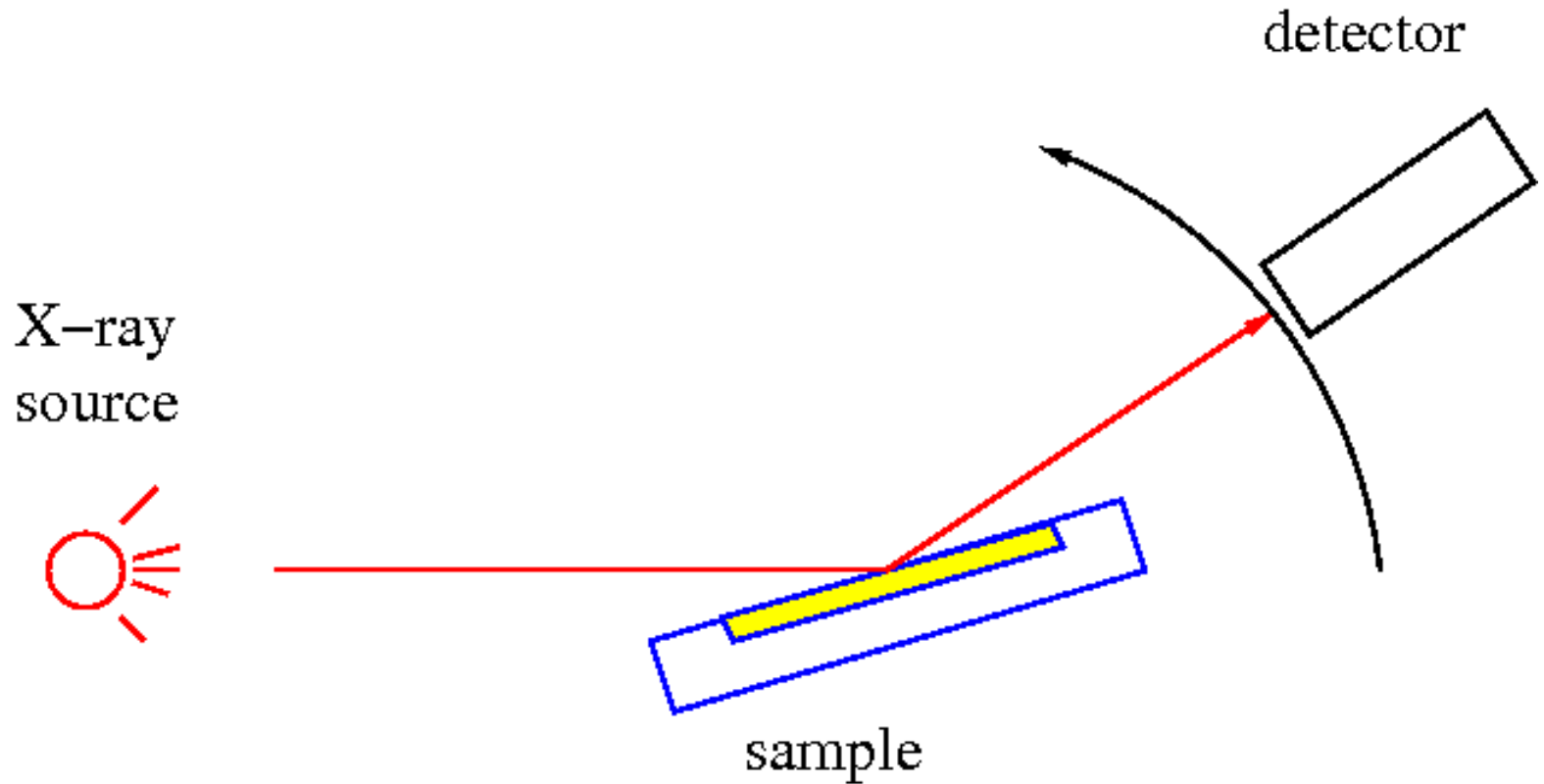


# Diffraction geometry

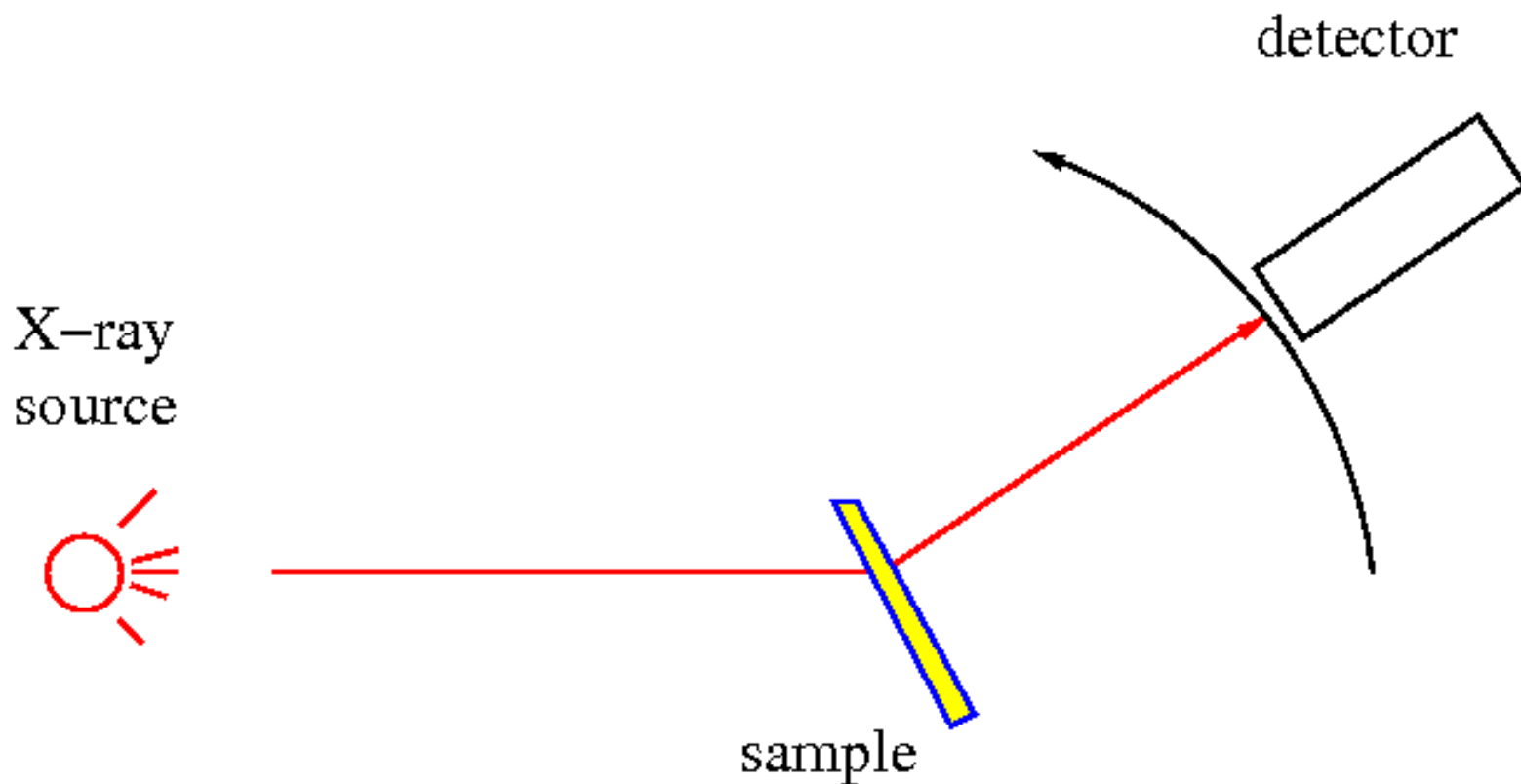
- Reflection geometry
  - Constant diffracting volume
  - Flat sample thick sample
  - Problem of preferred orientation
- Transmission geometry
  - Small sample volume
  - Lower signal background ratio



# Bragg Brentano geometry

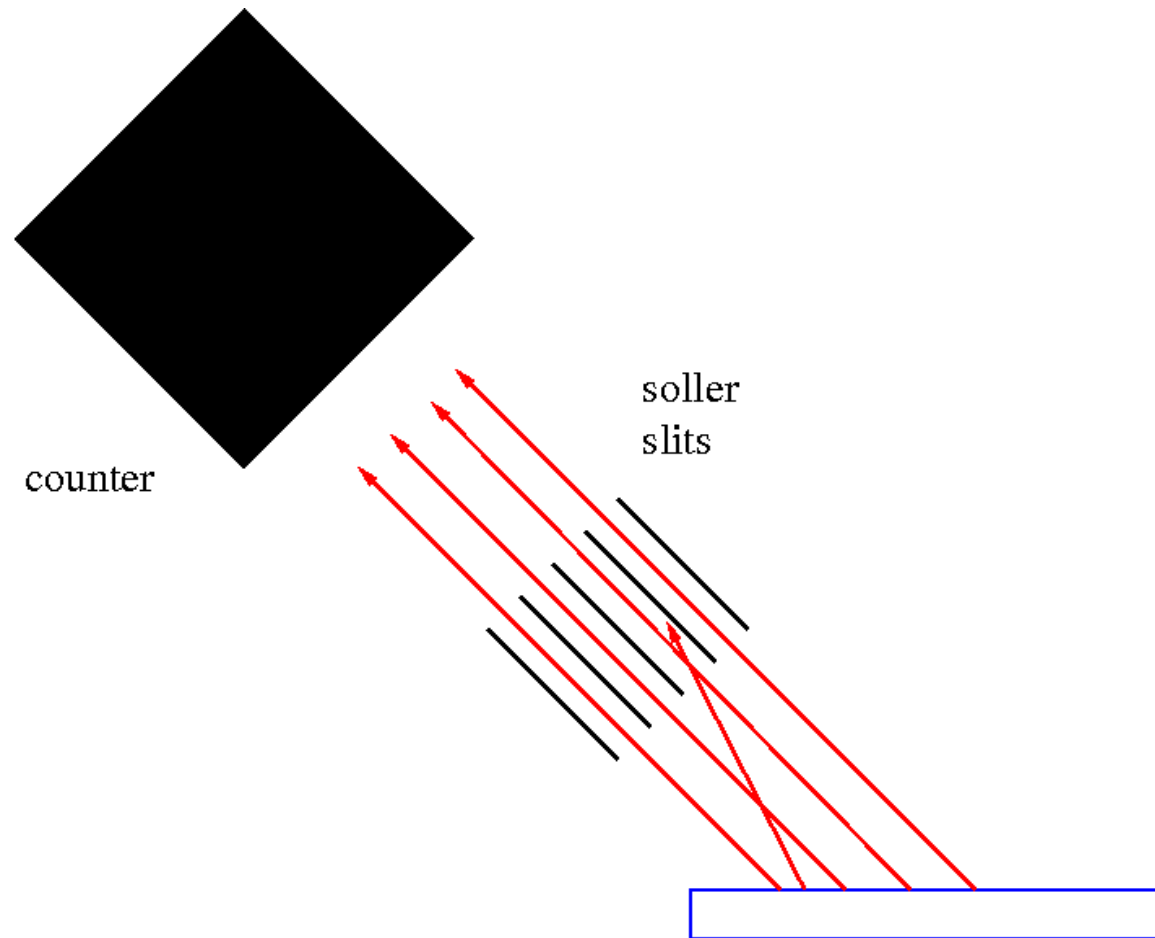


# Debye-Scherrer geometry

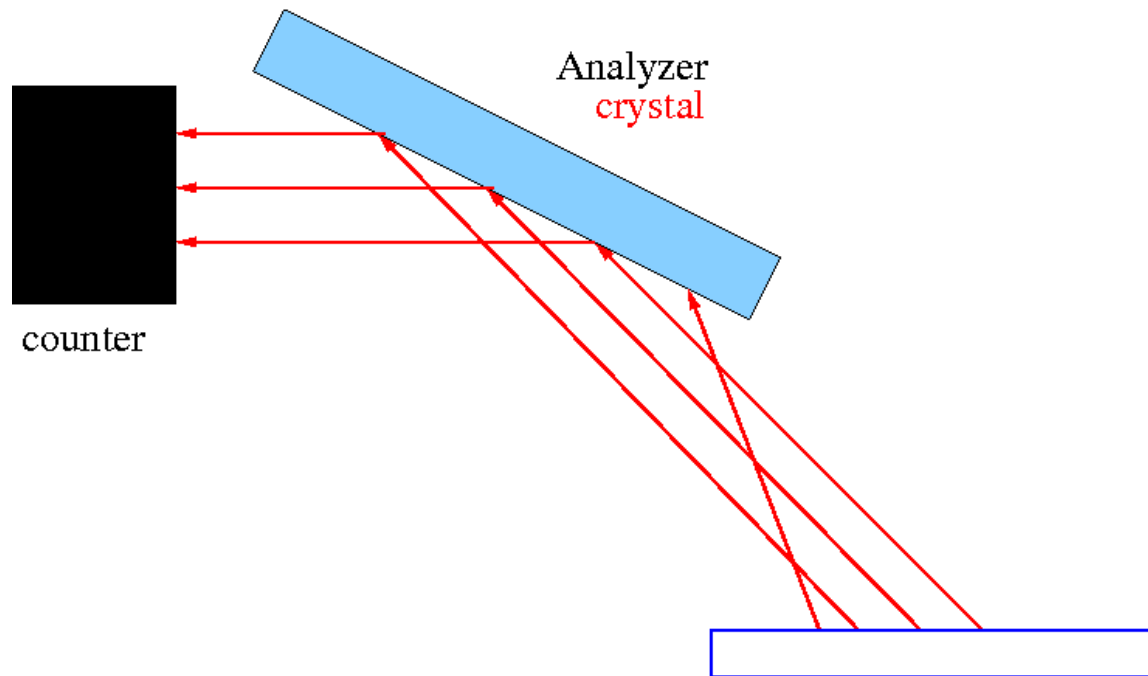




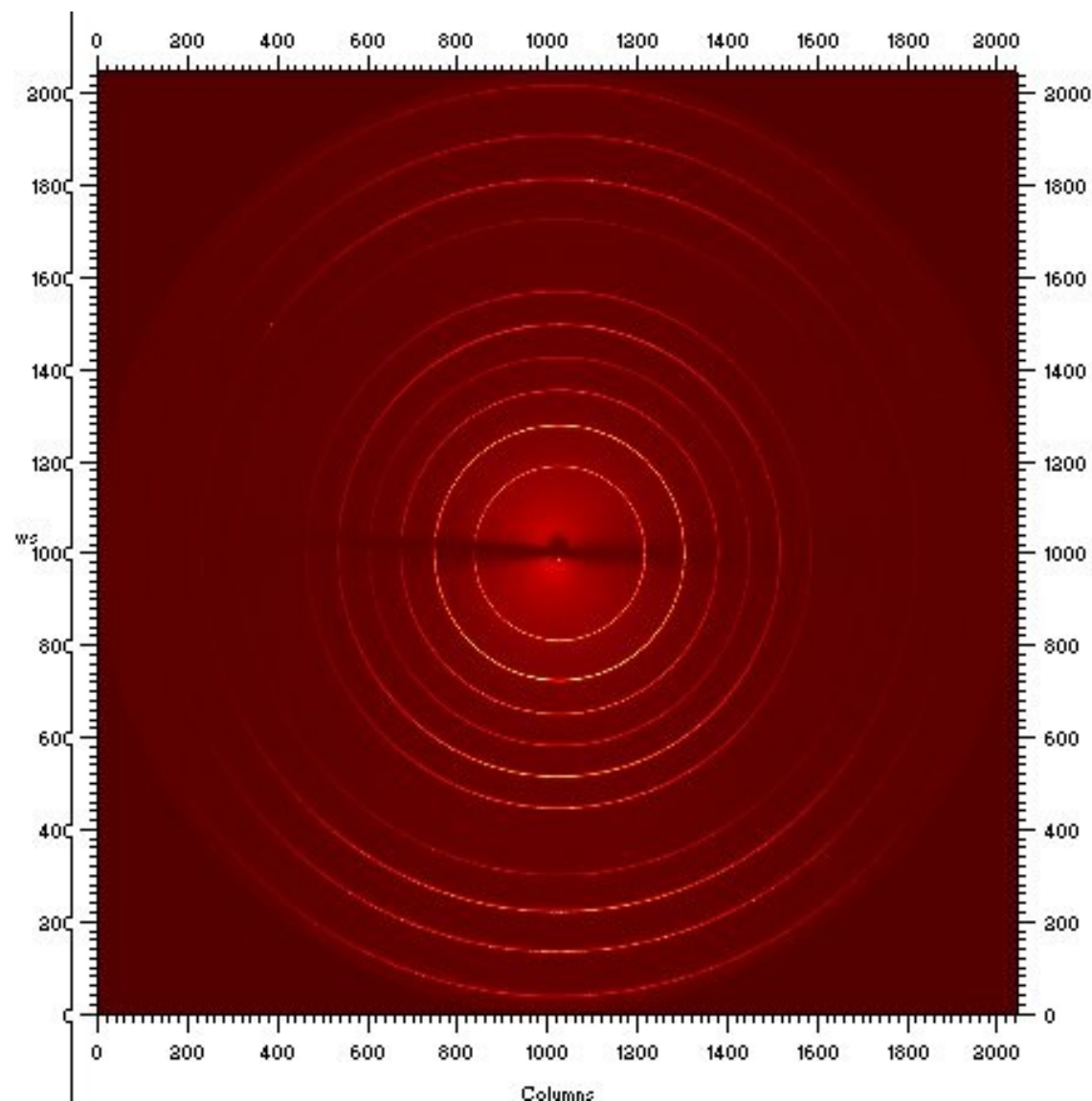
# Detection system



# Detection system



# Using an 2D PSD



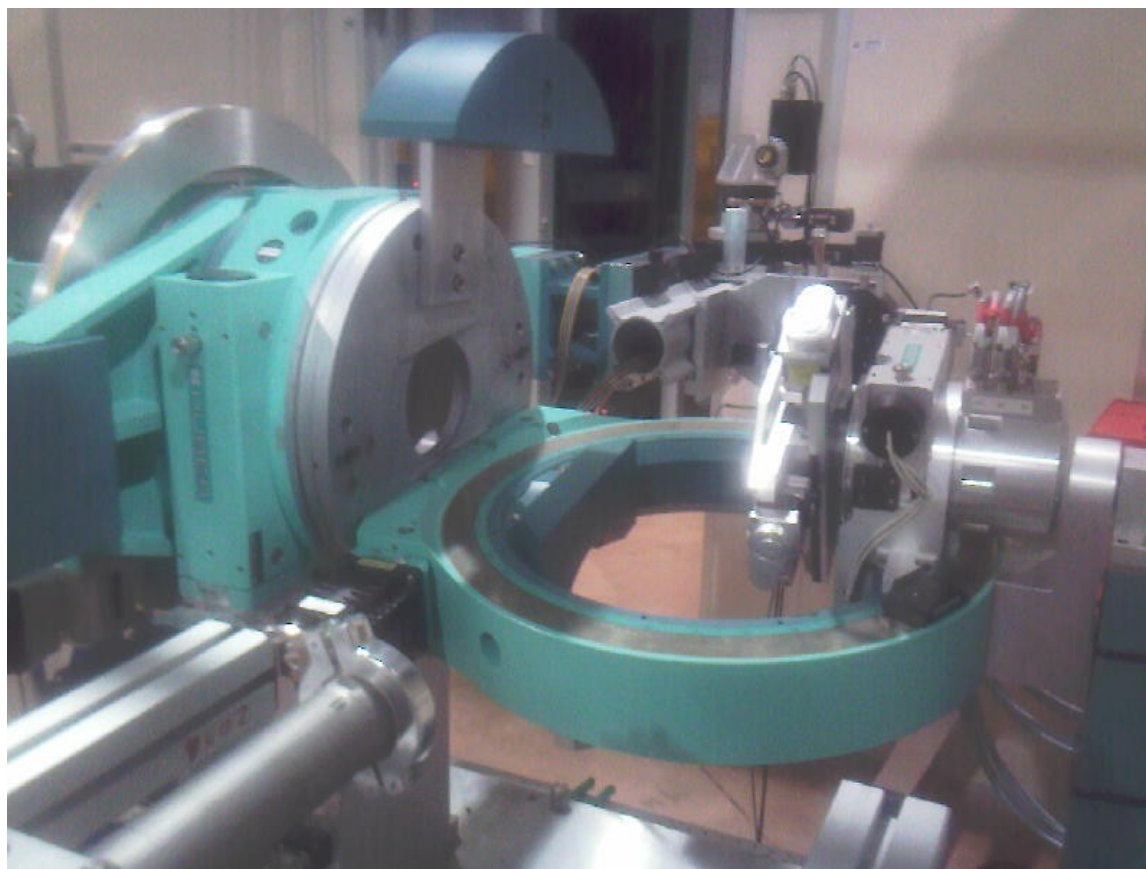


# Multipurpose Diffraction System

- Must be suitable for various techniques
- MDS needs to be flexible
  - Support for different geometries
  - Space for in situ experimental chambers
- High precision control of movements required

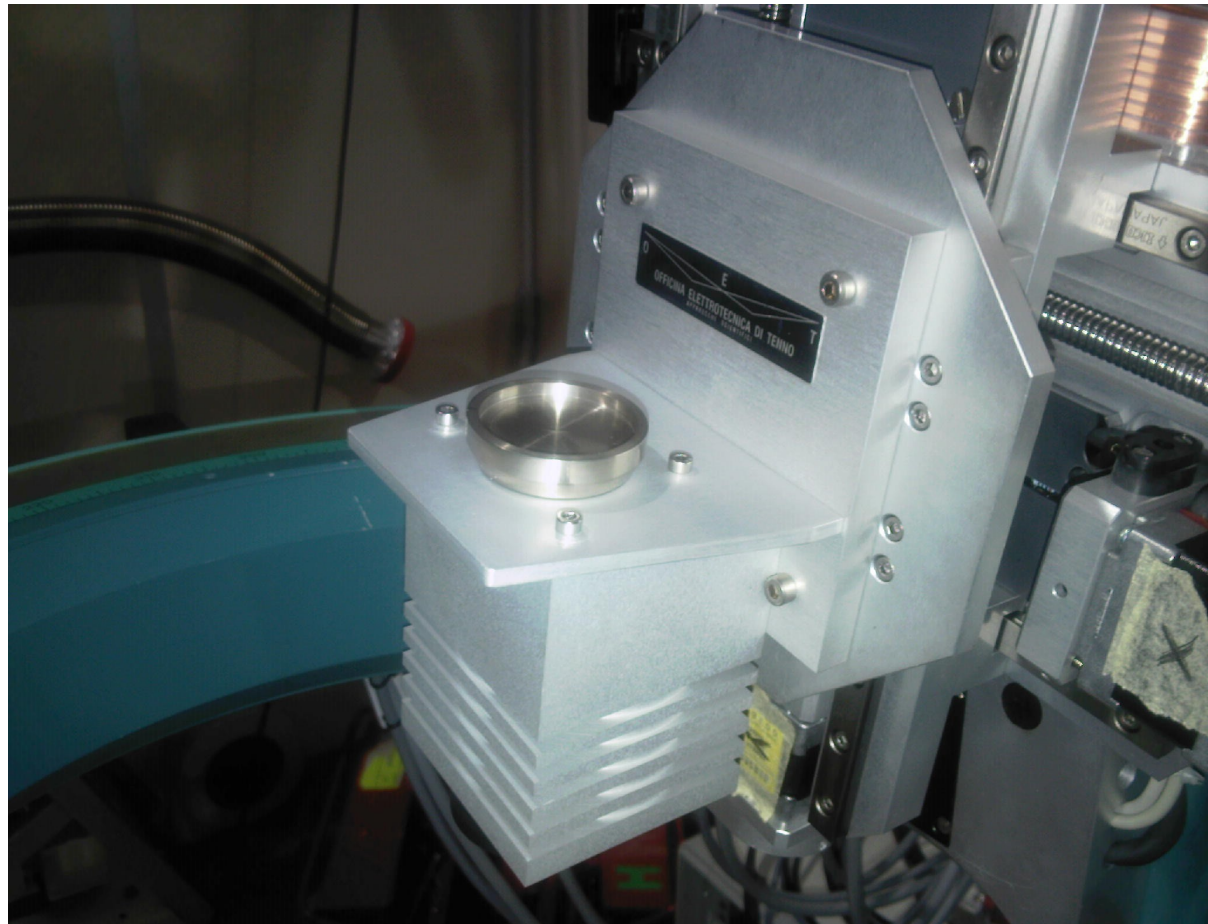
# Experimental setup at MCX

- 4-circle goniometer



# Experimental setup at MCX

- Sample holder



# Experimental setup at MCX

- Detector

