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Theory: Fermion degrees of freedom in the interacting boson model.

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Fermion degrees of freedom in the interacting boson model

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1. **Structure of odd-even nuclei in the interacting boson-fermion model**

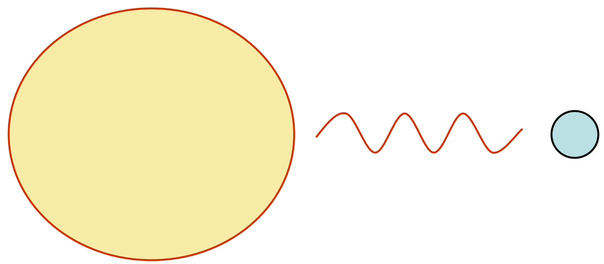
2. **High spin states in the interacting boson and interacting boson-fermion model**

3. **Structure of odd-odd nuclei in the interacting boson-fermion-fermion model**

4. **β decay in the interacting boson-fermion model**

1.

Structure of odd-even nuclei in the interacting boson fermion model



CORE **INTERACTION** **FERMION**

Spherical
Soft
Transitional
Deformed

Laboratory
coordinates

Intrinsic
coordinates

Microscopic
Geometrical
Phenomenological
Algebraic
………..
………..

?

Particle
Hole
Quasiparticle

IBM CORE

Origin : Microscopic

Symmetries : Algebraic

Applications : Phenomenologic

In the same basis applicable for nuclei :

Spherical
Soft
Transitional
Deformed

Coordinates : Laboratory

FERMION

Quasiparticle

INTERACTION

Origin : Microscopic

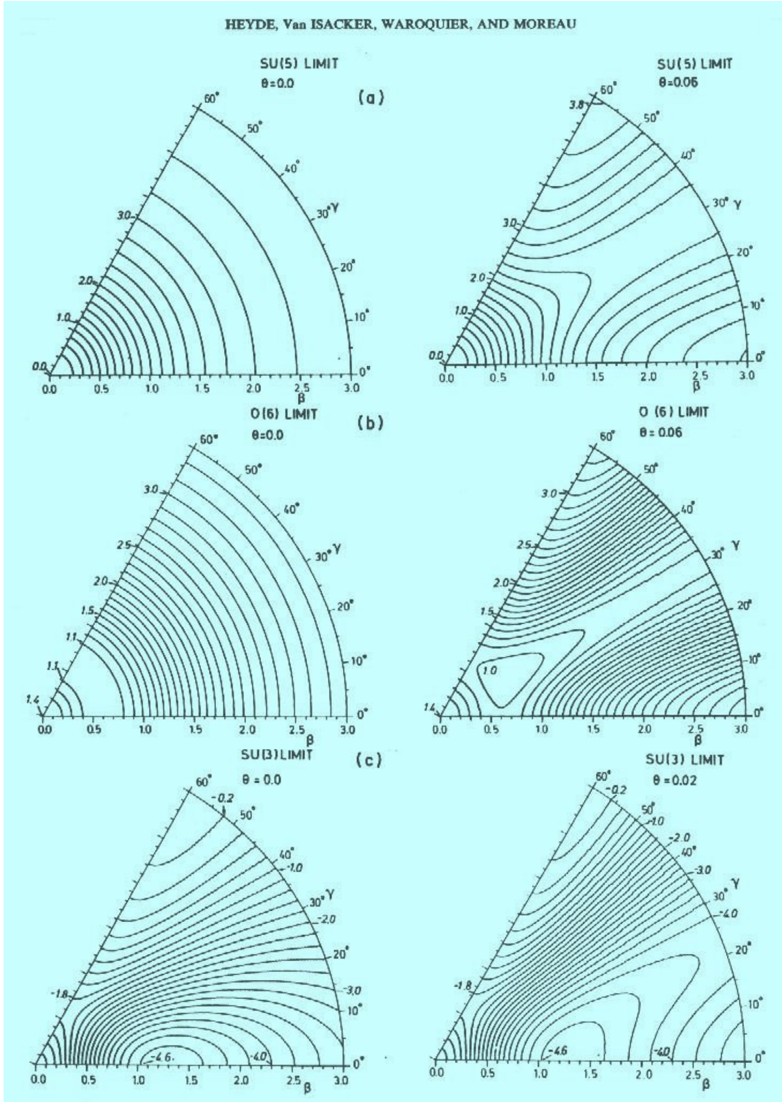
Symmetries : Algebraic and supersymmetry in some cases

Applications : Phenomenologic

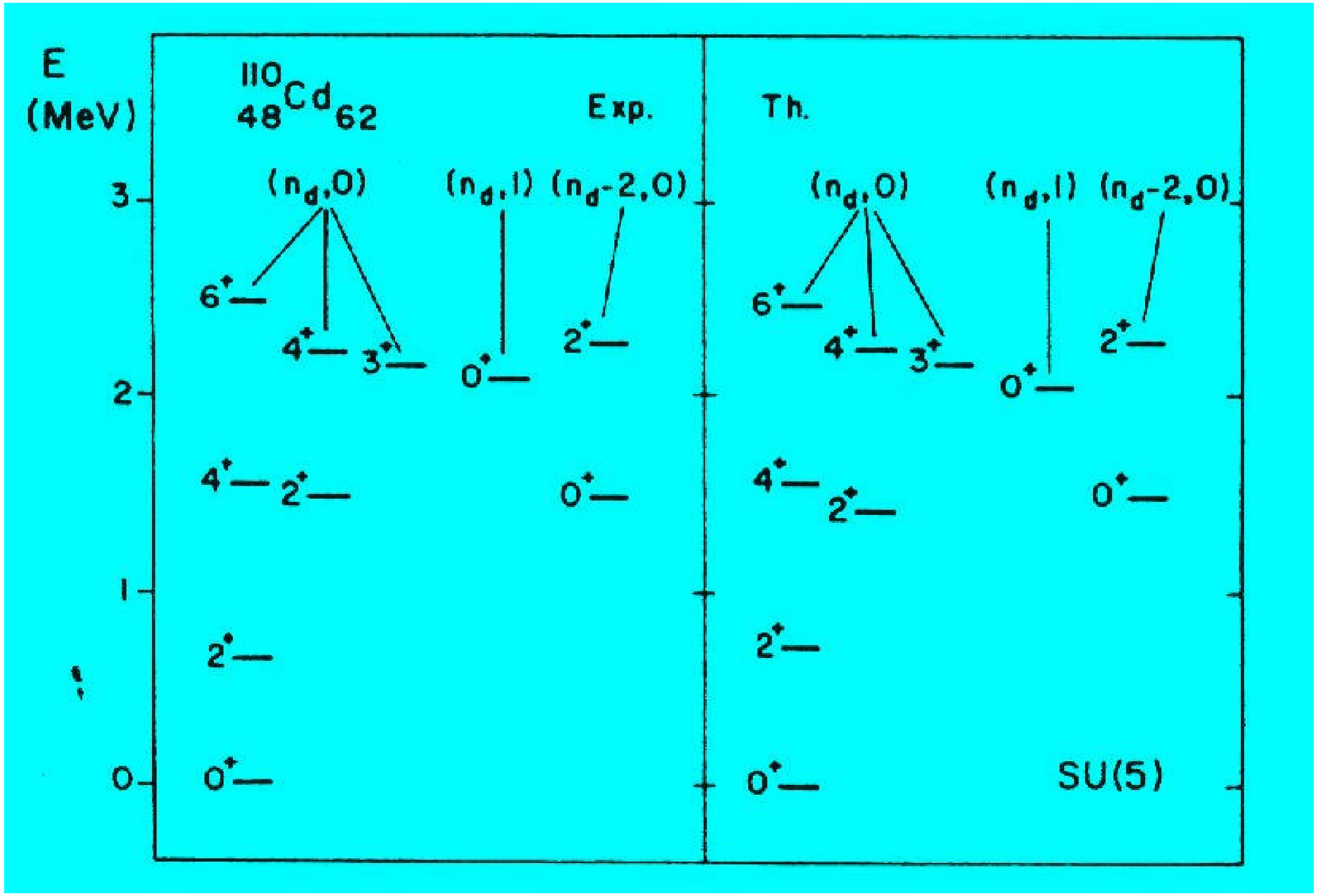
SPHERICAL

γ – SOFT

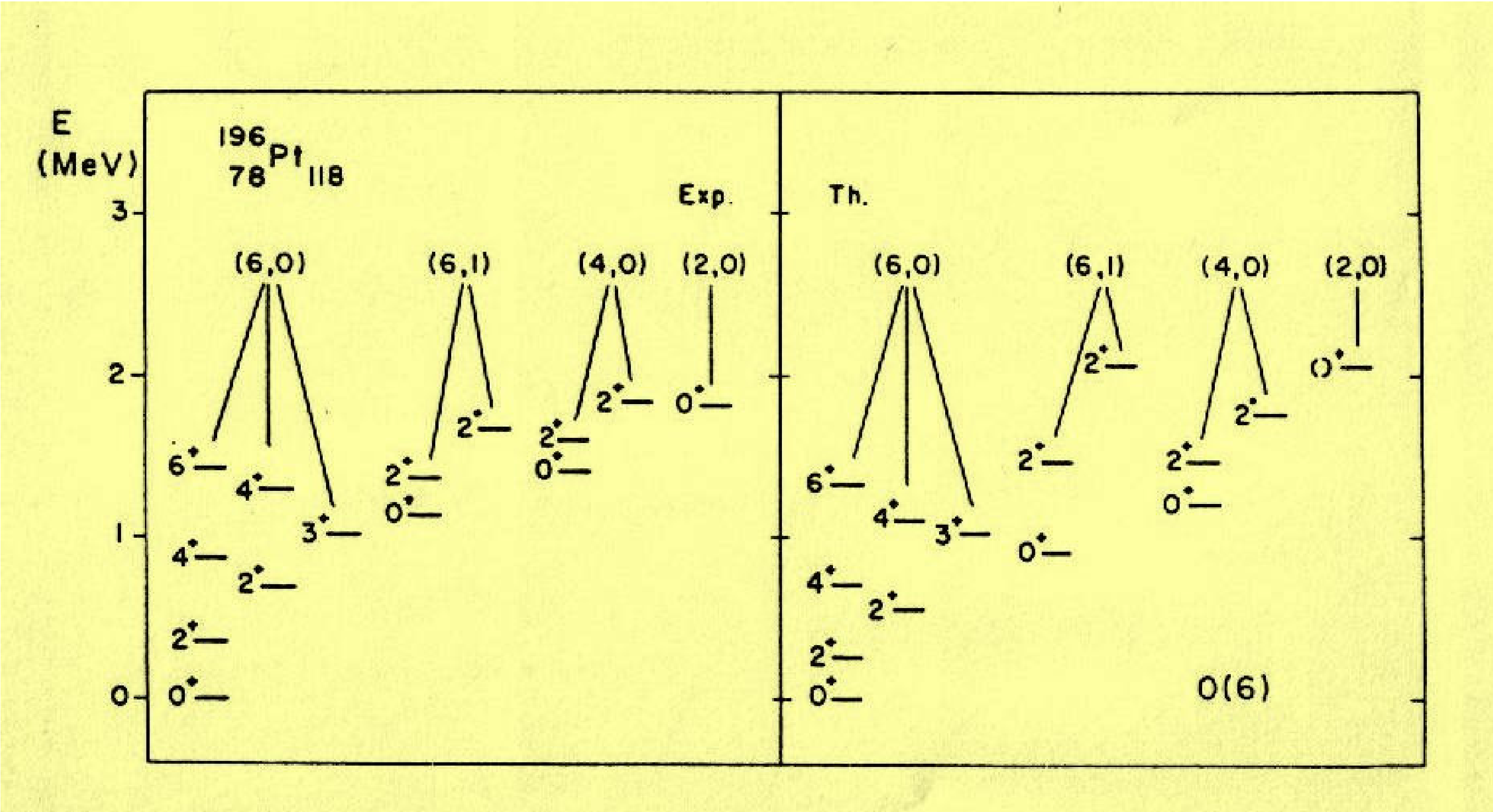
DEFORMED



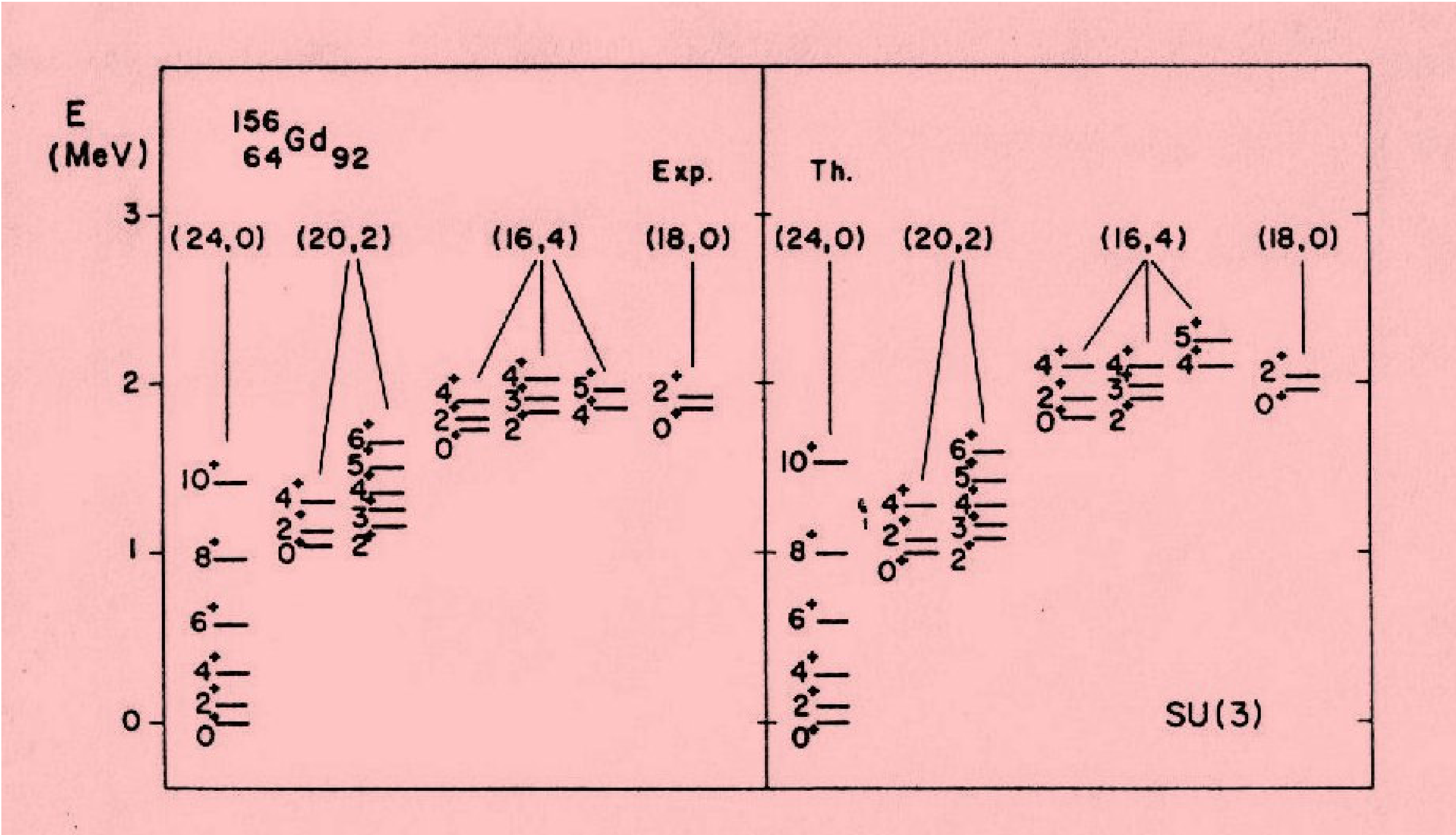
Spherical core



γ - soft core



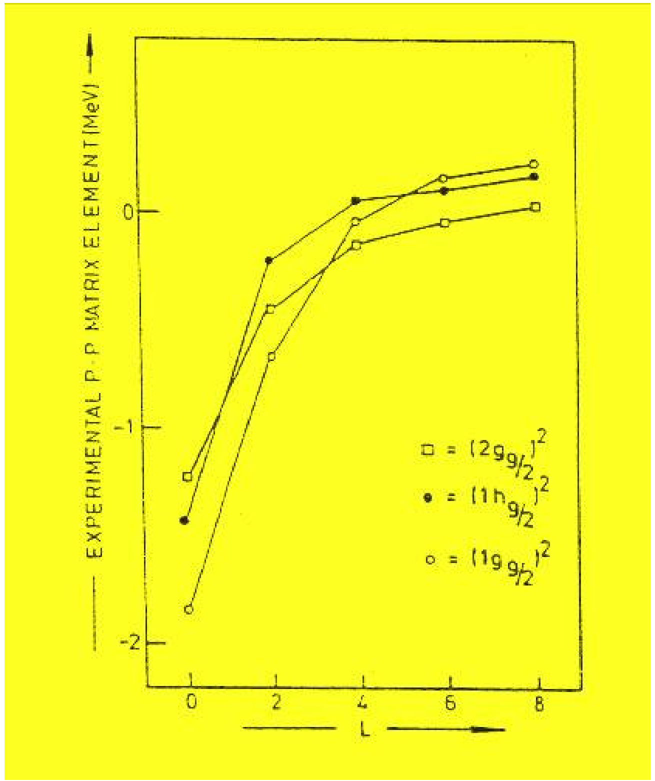
Deformed core



Microscopic origin

Shell model $\longrightarrow$ matrix elements of the effective interaction between identical nucleons are strongly attractive when the two nucleons are in a $J = 0$ state and remain attractive when the two nucleons are in a $J = 2$ state. They become repulsive for $J \geq 4$.

Nucleons tend to form pairs with angular momentum $J = 0$ or $J = 2$



Generalized seniority scheme: generalization of the seniority scheme to several non-degenerate orbits. The number of active nucleons is counted in respect to the nearest closed shell (valence nucleons). Contributions from orbitals outside the valence shell can be neglected since they lie at a too high energy.

A collective $J = 0$ pair is generated by the operator

$$S^\dagger = \sum_j \alpha_j S_j^\dagger$$

$$S_j^\dagger = \frac{1}{2}\sqrt{2j+1}\;(c_j^\dagger \tilde{c}_j)^{(0)}$$

State with generalized seniority $w = 0$ and $n = 2N$ particles

$$|n,\; J = 0,\; w = 0\rangle = (S^\dagger)^N\;|0\rangle$$

An excited 2^+ state is generated by the operator that creates a collective state with $J = 2$ and $w = 2$

$$D^\dagger = \sum_{jj'} \frac{1}{2} \beta_{jj'} \sqrt{1 + \delta_{jj'}}\ (c_j^\dagger \tilde{c}_{j'})^{(2)}$$

State with generalized seniority $w = 2$, $J = 2$ and $n = 2N$ particles

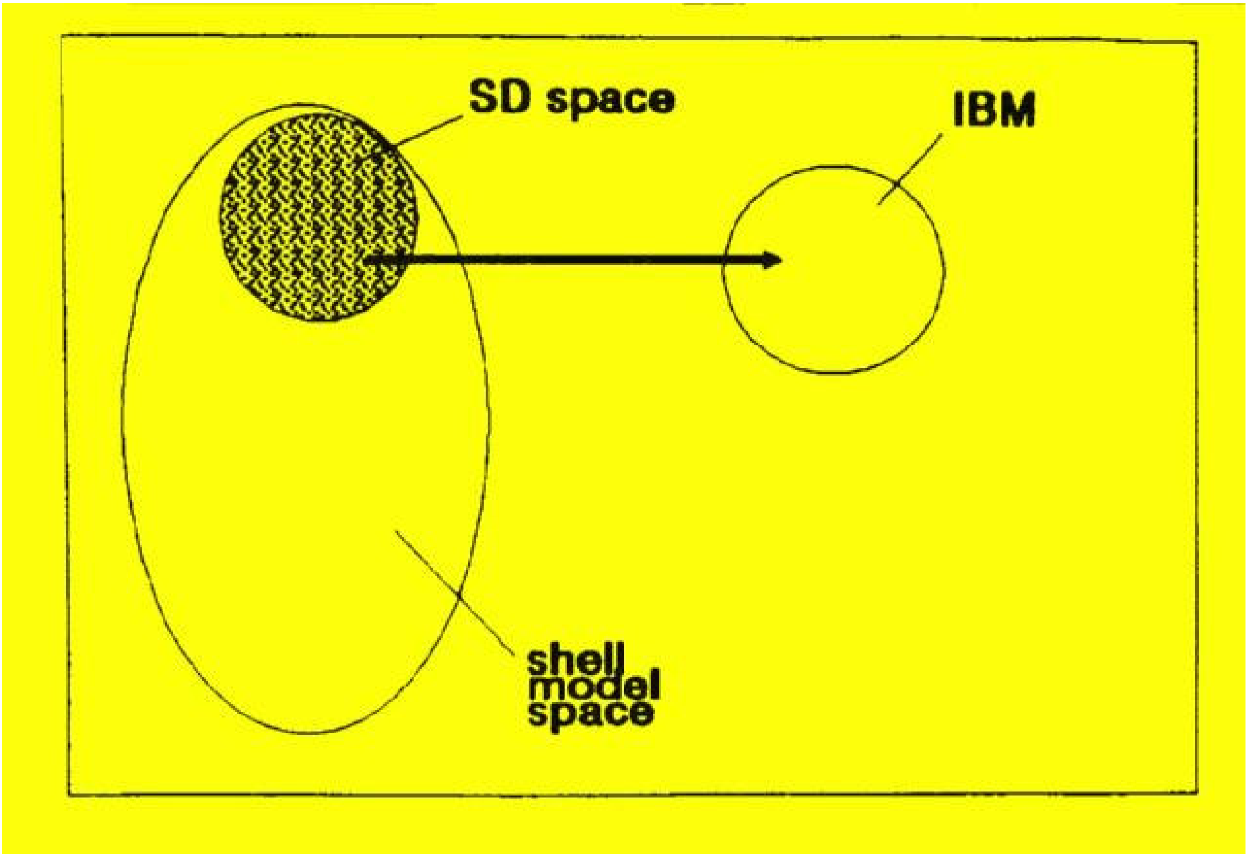
$$|n,\ J = 2,\ w = 2\rangle = D^\dagger (S^\dagger)^{N-1}\ |0\rangle$$

The structure coefficients α_j and $\beta_{jj'}$ can be obtained by diagonalizing the shell model interaction in the space of all $w = 0, 2$ states.

Instead of having to use the full shell model space, it is sufficient to consider the much smaller (S, D) subspace.

- Low-lying collective states can be described very well
- Non collective states can not be described
- The matrix elements of the fermion operators in the (S, D) subspace can be cumbersome
- The space built on S and D fermion pairs is mapped onto a corresponding space built on s and d boson degrees of freedom
- For states containing more than one D fermion pair we have to map the component of the state which is orthogonal to all the states containing fewer D fermion pairs

$$\begin{array}{lcl} |S^N,\ L=0\rangle & \longrightarrow & |s^N,\ L=0\rangle \\ |D\ S^{N-1},\ L=2\rangle & \longrightarrow & |d\ s^{N-1},\ L=2\rangle \\ |D^m\ S^{N-m},\ L\rangle_{\ orth} & \longrightarrow & |d^m\ s^{N-m},\ L\rangle \end{array}$$



- By equating matrix elements in (S, D) and (s, d) spaces the operators in the (s, d) space are obtained

- Since the S and D fermion pairs are always pairs of like nucleons (two protons or two neutrons), one has proton (s_π, d_π) and neutron (s_ν, d_ν) bosons. The model is called IBM-2.

$$H_B = \epsilon_d\,(\hat{n}_{d_\nu} + \hat{n}_{d_\pi}) + \kappa\,(Q^B_\nu \cdot Q^B_\pi) + M_{\nu\pi} + V_{\nu\nu} + V_{\pi\pi}$$

$$\begin{aligned} Q^B_\nu &= d^\dagger_\nu s_\nu + s^\dagger_\nu \tilde{d}_\nu + \chi_\nu\,[d^\dagger_\nu \tilde{d}_\nu]^{(2)} \\ Q^B_\pi &= d^\dagger_\pi s_\pi + s^\dagger_\pi \tilde{d}_\pi + \chi_\pi\,[d^\dagger_\pi \tilde{d}_\pi]^{(2)} \end{aligned}$$

$$\begin{aligned} M_{\nu\pi} &= \frac{1}{2}\xi_2\,((d^\dagger_\nu s^\dagger_\pi - d^\dagger_\pi s^\dagger_\nu) \cdot (\tilde{d}_\nu s_\pi - \tilde{d}_\pi s_\nu)) \\ &\quad - \sum_{K=1,3} \xi_K\,([d^\dagger_\nu d^\dagger_\pi]^{(K)} \cdot [\tilde{d}_\nu \tilde{d}_\pi]^{(K)}) \end{aligned}$$

$$V_{\nu\nu} = \frac{1}{2} \sum_{L=0,2,4} c^\nu_L\,([d^\dagger_\nu d^\dagger_\nu]^{(L)} \cdot [\tilde{d}_\nu \tilde{d}_\nu]^{(L)})$$

$$V_{\pi\pi} = \frac{1}{2} \sum_{L=0,2,4} c^\pi_L\,([d^\dagger_\pi d^\dagger_\pi]^{(L)} \cdot [\tilde{d}_\pi \tilde{d}_\pi]^{(L)})$$

- The major part of the interaction between like particles is contained in the boson energies and a smaller in the $V_{\nu\nu}$ and $V_{\pi\pi}$ terms.
- The $Q^B_\nu \cdot Q^B_\pi$ interaction is the boson image of the neutron-proton quadrupole-quadrupole interaction.
- $M_{\nu\pi}$ (Majorana term) shifts up all states that are not totally symmetric in the neutron-proton degree of freedom. It is a consequence of the truncation of the basis to s and d bosons only.

Introducing the concept of F spin, the IBM-1 Hamiltonian can be obtained by projecting out the part that acts only on the maximal F spin subspace (on states that are totally symmetric in the neutron-proton degree of freedom).

$$
\begin{aligned}
H_B \quad = \quad & \varepsilon\, \hat{N} \;+\; \frac{1}{2}\, v_0 \;\left([d^\dagger d^\dagger]_{(0)}[\tilde{s}\tilde{s}]_{(0)} \;+\; h.c.\right)_{(0)} \\
+ \quad & \frac{1}{\sqrt{2}}\, v_2 \;\left([d^\dagger d^\dagger]_{(2)}[\tilde{d}\tilde{s}]_{(2)} \;+\; h.c.\right)_{(0)} \\
+ \quad & \sum_{L=0,2,4} \frac{1}{2}\, C_L\, \sqrt{2L+1}\;\left([d^\dagger d^\dagger]_{(L)}[\tilde{d}\tilde{d}]_{(L)}\right)_{(0)}
\end{aligned}
$$

LIMITS

$$\begin{array}{llll} U(6) & \supset & U(5) \supset O(5) \supset O(3) \supset O(2) & \textit{vibrational limit} \\ U(6) & \supset & SU(3) \qquad\qquad\quad \supset O(3) \supset O(2) & \textit{rotational limit} \\ U(6) & \supset & O(6) \supset O(5) \supset O(3) \supset O(2) & \gamma - \textit{soft limit} \end{array}$$

The Lie algebra U(6) admits:

- Schwinger boson realization in terms of 6 bosons s, d_μ
- Holstein-Primakoff boson realization in terms of 5 bosons b_μ

$$\begin{array}{ccc} d^\dagger_{2\mu} & \longleftrightarrow & b^\dagger_{2\mu} \\ s^\dagger & \longleftrightarrow & \sqrt{N - \sum_\mu b^\dagger_{2\mu} b_{2\mu}} \end{array}$$

- In the IBFM an odd-nucleon operator $a_j^\dagger$ is introduced in addition to the s and d boson operators.
- The states in the IBFM model space can be related to the shell model basis by using the generalized seniority scheme.
- The odd-nucleon operator $a_j^\dagger$ should not be regarded as a nucleon creation operator (in the shell model sense) but as a generalized seniority raising operator.

$$\begin{aligned} a_j^\dagger \; |s^N\rangle &= \qquad\quad |js^N\rangle \quad \longleftrightarrow \quad |n = 2N+1, J = j, w = 1\rangle \\ (a_j^\dagger d^\dagger)^{(J)} \; |s^{N-1}\rangle &= |(jd)^{(J)} s^{N-1}\rangle \quad \longleftrightarrow \quad |n = 2N+1, J, w = 3\rangle \end{aligned}$$

- The operator $a_j^\dagger$ operating on an N boson state with n_d d -bosons creates a state which corresponds to a shell model state with $n = 2N+1$ and $w = 2n_d + 1$.
- For the shell model single-nucleon operator $c_j^\dagger$
  $c_j^\dagger \; |w = 2\rangle = \alpha|w = 1\rangle + \beta|w = 3\rangle$
- For the odd-nucleon operator $a_j^\dagger$
  $a_j^\dagger \; |w = 2\rangle = |w = 3\rangle$

A microscopic theory for a system that includes both fermionic and bosonic degrees of freedom is complicated.

The dominant interaction in the coupling of the odd-particle to the bosons is the proton-neutron quadrupole interaction → construction of the IBFM image of the shell model quadrupole operator.

There are several methods for obtaining the IBFM image of the shell model quadrupole operator. One of them is to introduce the pseudo particle operator $\check{c}_j^\dagger$ (Scholten).

Condition:

The matrix elements of $\check{c}_j^\dagger$ in the IBFM space are equal to the matrix elements of $c_j^\dagger$ in the shel model space.

For $w \leq 1$ (α_j are the coefficients which enter in the definition of the S pair operator):

$$\hat{n} = \sum_j \alpha_j^2 \sum_m c_{jm}^\dagger c_{jm} = \sum_j \alpha_j^2 \hat{n}_j$$

$$\langle S^N | \, \hat{n} \, | S^N \rangle = 2N$$

Effective degeneracy

$$\Omega_e = \sum_j \alpha_j^2 \Omega_j$$

Here the spherical shell model OCCUPATION PROBABILITIES v_j^2 are introduced
($u_j^2 + v_j^2 = 1$).

$$v_j^2 = n_j/(2j+1)$$
$$n_j = \langle S^N | \, \hat{n}_j \, | S^N \rangle \approx 2N\alpha_j^2 \frac{\Omega_j}{\Omega_e}$$
$$v_j^2 = \alpha_j^2 N/\Omega_e$$

$$\langle S^N j^{'} \parallel c_j^\dagger \parallel S^N \rangle = -\hat{j} u_j \delta_{jj'} = u_j \langle s^N j^{'} \parallel a_j^\dagger \parallel s^N \rangle$$

$$\langle S^N \parallel c_j^\dagger \parallel S^{N-1} j^{'} \rangle = \hat{j} v_j \delta_{jj'} = v_j \langle s^N \parallel (s^\dagger \tilde{a}_j)^{(j)} \parallel s^{N-1} j^{'} \rangle / \sqrt{N}$$

For $w \leq 3$ similar expressions can be obtained. Finally, the IBFM image of the shell model single-nucleon creation operator is

$$\begin{aligned} \check{c}_j^\dagger \quad &= \quad u_j a_j^\dagger - \sum_{j'} \frac{v_j}{\sqrt{N}} \sqrt{\frac{10}{2j+1}} \beta_{j'j} (K_\beta)^{-1} s^\dagger (\tilde{d}\, a_{j'}^\dagger)^{(j)} \\ &+ \quad \frac{v_j}{\sqrt{N}} (s^\dagger \tilde{a}_j)^{(j)} + \sum_{j'} u_j \sqrt{\frac{10}{2j+1}} \beta_{j'j} (K_\beta)^{-1} (d^\dagger \tilde{a}_{j'})^{(j)} \end{aligned}$$

$$K_\beta^2 = \sum_{jj'} \beta_{j'j}^2$$

The coefficients $\beta_{j'j}$ define the microscopic structure of the d-boson.

The matrix elements of the quadrupole operator $\sum_{jj'} Q_{jj'} \ (c_j^\dagger \tilde{c}_{j'})^{(2)}$ in the fermion space are replaced by the matrix elements of the pseudo particle operator $\check{c}_j^\dagger$ acting in the boson space giving the quadrupole operator expressed in terms of boson and odd-particle operators.

$$
\begin{aligned}
Q^{(2)} &= Q_B^{(2)} + Q_F^{(2)} \\
Q_B^{(2)} &= [s^\dagger \tilde{d} + d^\dagger \tilde{s}]^{(2)} + \chi [d^\dagger \tilde{d}]^{(2)} \\
Q_F^{(2)} &= \sum_{jj'} Q_{jj'} \ (u_j u_{j'} - v_j v_{j'}) \ (a_j^\dagger \tilde{a}_{j'})^{(2)} \\
&- \sqrt{\frac{10}{N}} \sum_{jj'j''} Q_{jj'} (u_j v_{j'} + v_j u_{j'}) \beta_{j''j} [(d^\dagger \tilde{a}_{j''})^{(j)} (\tilde{s} a_{j'}^\dagger)^{(j')}]^{(2)} (\hat{j} K_\beta)^{-1}
\end{aligned}
$$

The boson-fermion interaction can be generated by the interaction between like particles or by the proton-neutron quadrupole interaction. The structure of the interactions is identical. The product of $Q_B^{(2)}$ and $Q_F^{(2)}$ contributes to the boson-fermion interaction. By maping the basis from IBM-2 onto IBM-1 and taking terms up to the second order in d-boson operators the standard form of the boson-fermion interaction is obtained.

The IBFM-1 Hamiltonian for an odd-even nucleus

$$H = H_B + H_F + V_{BF}$$

H_B is the boson Hamiltonian of IBM-1 describing a system of N interacting bosons (correlated S and D pairs) that approximate the valence nucleon pairs:

$$\begin{aligned} H_B \quad &= \quad \varepsilon \, \hat{N} \, + \, \frac{1}{2} \, v_0 \, \left([d^\dagger \times d^\dagger]_{(0)} \times [\tilde{s} \times \tilde{s}]_{(0)} \, + \, h.c. \right)_{(0)} \\ &+ \quad \frac{1}{\sqrt{2}} \, v_2 \, \left([d^\dagger \times d^\dagger]_{(2)} \times [\tilde{d} \times \tilde{s}]_{(2)} \, + \, h.c. \right)_{(0)} \\ &+ \quad \sum_{L=0,2,4} \frac{1}{2} \, C_L \, \sqrt{2L+1} \, \left([d^\dagger \times d^\dagger]_{(L)} \times [\tilde{d} \times \tilde{d}]_{(L)} \right)_{(0)} \end{aligned}$$

$$n_s = N - n_d$$

H_F is the fermion Hamiltonian containing quasiparticle energies of odd protons or neutrons. The quasiparticle energies and occupation probabilities contained in the fermion Hamiltonian, and other terms, are obtained in a BCS calculation with some standard set of single fermion energies.

$$H_F = \sum_i \varepsilon_i \, a_i^\dagger \tilde{a}_i$$

V_{BF} is the IBFM-1 boson-fermion interaction containing the dynamical, exchange and monopole term.

- The dynamical interaction V_{DYN} represents the direct component of the quadrupole interaction between the odd particle and the bosons.
- The exchange interaction V_{EXC} is due to the two-particle nature of the bosons, bringing the Pauli exclusion principle into play.
- The monopole interaction V_{MON} can result from a variety of causes, in particular from the blocking of certain degrees of freedom by the odd particle.

$$V_{BF} = V_{DYN} + V_{EXC} + V_{MON}$$

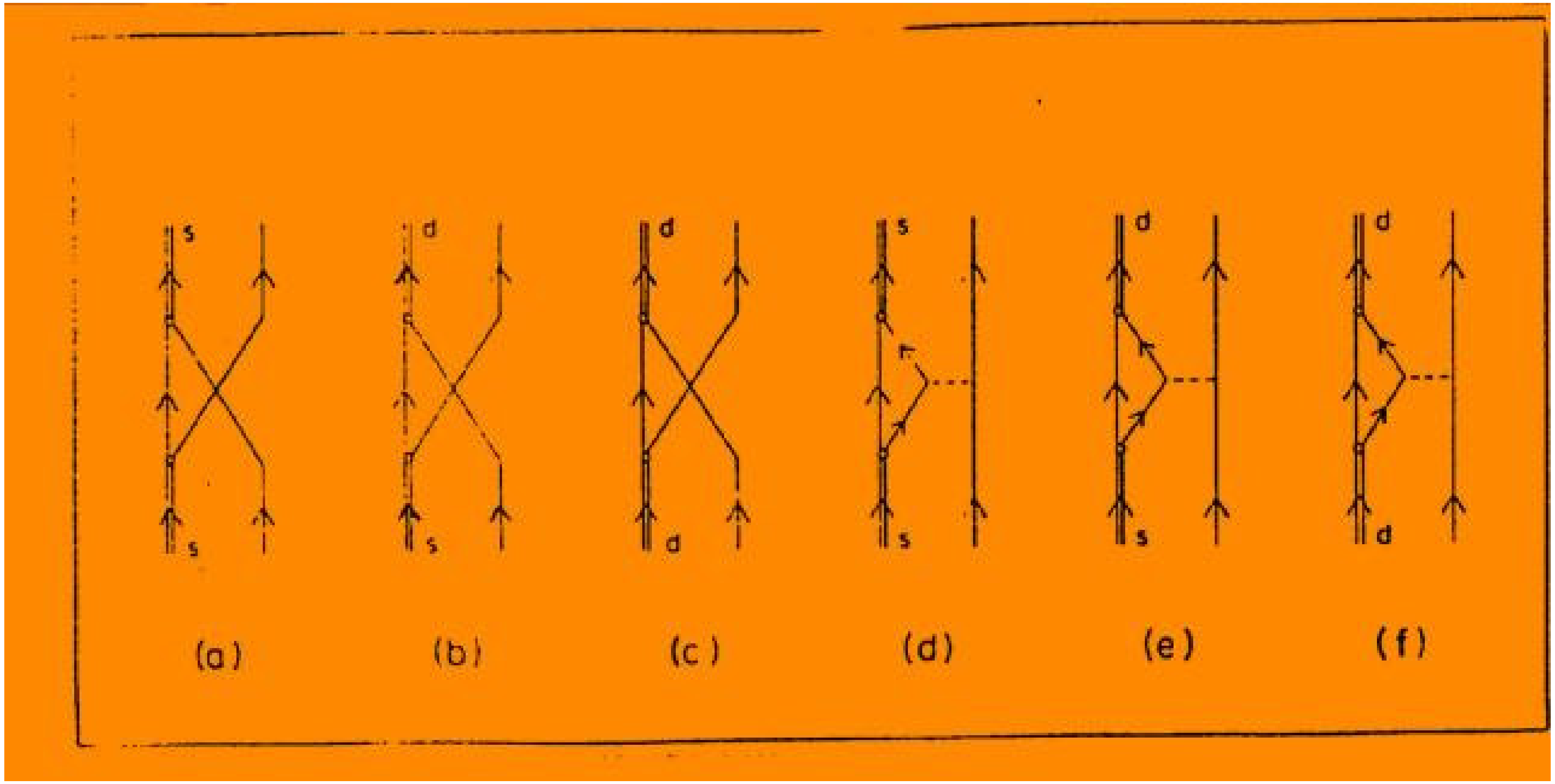
$$V_{DYN} = \Gamma_0 \sum_{j_1 j_2} \sqrt{5}\,(u_{j_1}u_{j_2} - v_{j_1}v_{j_2})\,\langle j_1 \parallel Y_2 \parallel j_2 \rangle\,\left([a^\dagger_{j_1} \times \tilde{a}_{j_2}]^{(2)} \times Q_B^{(2)}\right)^{(0)}$$

$Q_B^{(2)}$ is the standard boson quadrupole operator

$$Q_B^{(2)} = [s^\dagger \times \tilde{d} + d^\dagger \times \tilde{s}]^{(2)} + \chi [d^\dagger \times \tilde{d}]^{(2)}$$

$$V_{EXC} = \Lambda_0 \sum_{j_1 j_2 j_3} (-2) \sqrt{\frac{5}{2j_3+1}}\,(u_{j_1}v_{j_3} + v_{j_1}u_{j_3})\,\,(u_{j_2}v_{j_3} + v_{j_2}u_{j_3})$$
$$\langle j_3 \parallel Y_2 \parallel j_1 \rangle\,\langle j_3 \parallel Y_2 \parallel j_2 \rangle\,\,:\left([a^\dagger_{j_1} \times \tilde{d}\,]_{j_3} \times [\tilde{a}_{j_2} \times d^\dagger\,]_{j_3}\right)^{(0)}:$$

$$V_{MON} = A_0 \sum_j \sqrt{5}\,(2j+1)\,\left([a^\dagger_j \times \tilde{a}_j]^{(0)} \times [d^\dagger \times \tilde{d}]^{(0)}\right)^{(0)}$$



(a), (b), (c) exchange terms
(d), (e), (f) direct terms

The structure coefficients:

- The coefficients v_j are related to the structure coefficients of the fermion S-pair state, which is the microscopic equivalent of the s boson. In practice, they are the occupation probabilities of the single-particle orbits, as follows from a spherical BCS calculation.

- The coefficients $\beta_{j_a j_b} = (u_{j_a} v_{j_b} + v_{j_a} u_{j_b}) \langle j_a \parallel Y_2 \parallel j_b \rangle$ are the structure coefficients of the d boson.

The electromagnetic operators have the form:

$$M(E2) = M_B(E2) + M_F(E2)$$

$$M_B(E2) = \frac{3}{4\pi} R_0^2 \, e^{VIB} \left([s^\dagger \times \tilde{d} + d^\dagger \times \tilde{s}]^{(2)} + \chi [d^\dagger \times \tilde{d}]^{(2)} \right)$$

$$R_0^2 = 0.0144 \; A^{\frac{2}{3}} \qquad barn$$

$$M_F(E2) = \frac{3}{5} R_0^2 \, e_F \, Y_2$$

Common notation:

$$\frac{3}{4\pi} R_0^2 \, e^{VIB} = e_B$$

$$\vec{M}(M1) = \vec{M}_B(M1) + \vec{M}_F(M1)$$

$$\vec{M}_B(M1) = \sqrt{\frac{3}{4\pi}}\,\sqrt{10}\,g_R\,[d^\dagger \times \tilde{d}]^{(1)}$$

$$\vec{M}_F(M1) = \sqrt{\frac{3}{4\pi}}\,[\,g_l\,\vec{l} + g_s\,\vec{s} + g_T\,(Y_2 \times \vec{s}\,)_1\,]$$

Common notations:

$$\sqrt{\frac{3}{4\pi}}\,g_R = g_B$$

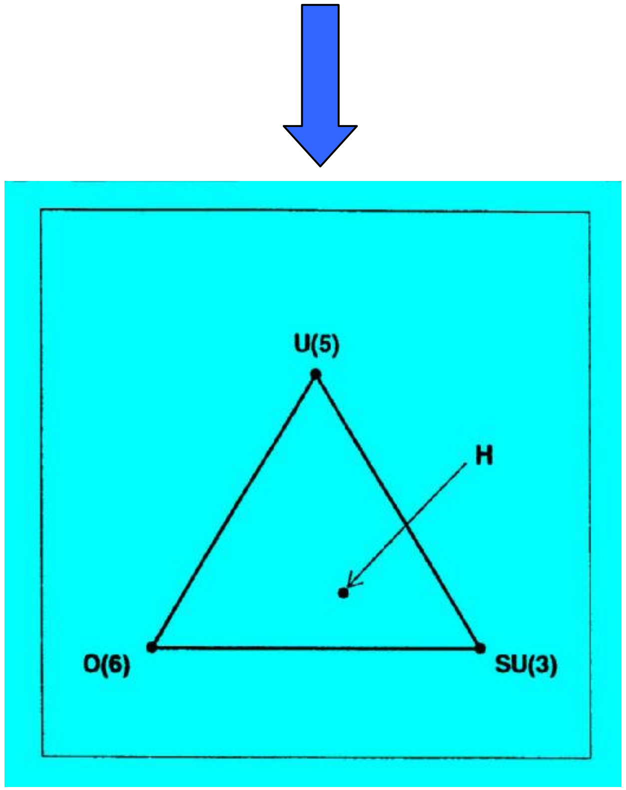
IBFM (and its extensions) provide a consistent description of nuclear structure phenomena in:

- spherical nuclei
- deformed nuclei
- transitional nuclei

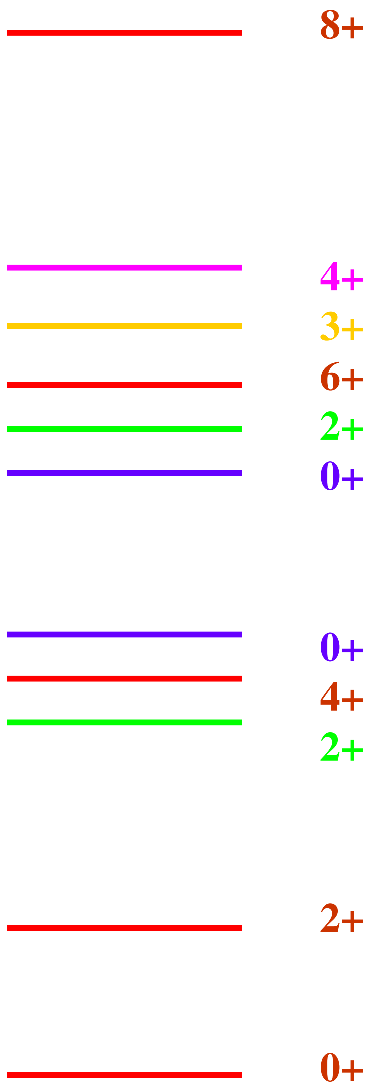
Procedure

1. Boson Hamiltonian

Place the core nucleus in the Casten triangle



Important levels



Important data

excitation energies

γ branchings

B(E2)

$Q(2^+_1)$

$\mu(2^+_1)$

ε

v_0

v_2

C_0

C_2

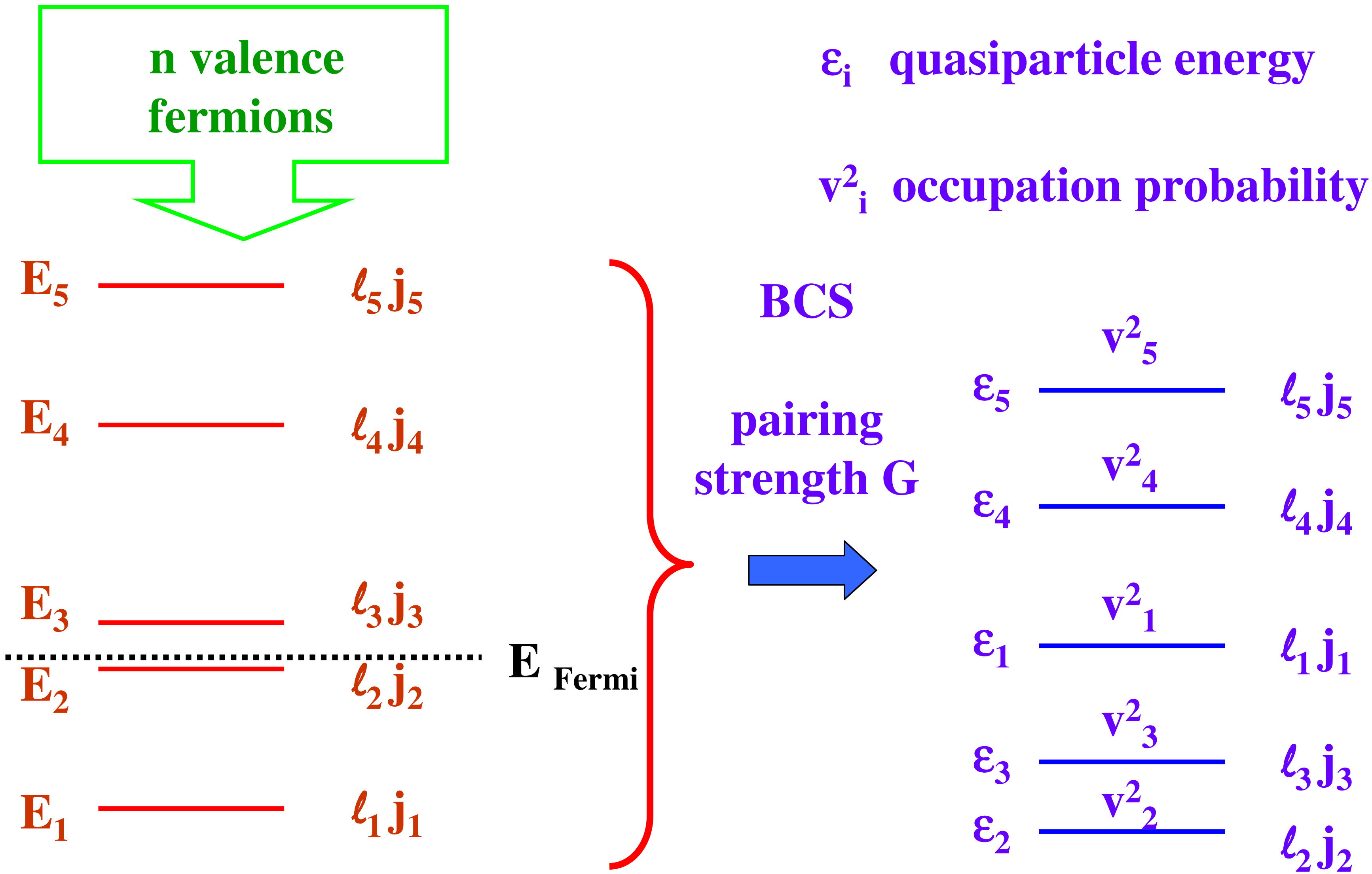
C_4

χ

e_b

g_b

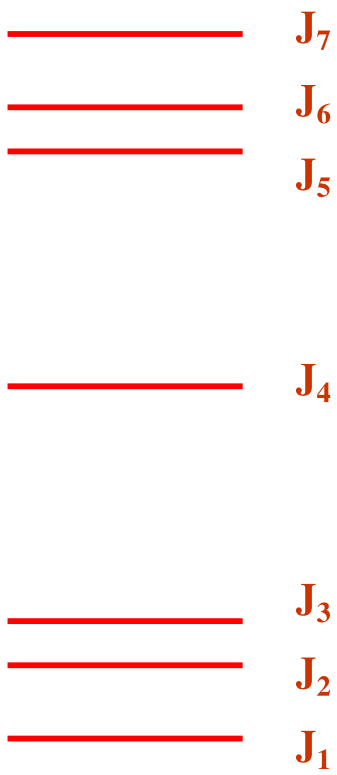
2. Fermion Hamiltonian



3. Boson – Fermion Hamiltonian

Important data

Odd – even nucleus



excitation energies
γ branchings
B(E2)
B(M1)
Q
μ
spectroscopic factors

Γ_0
Λ_0
A_0

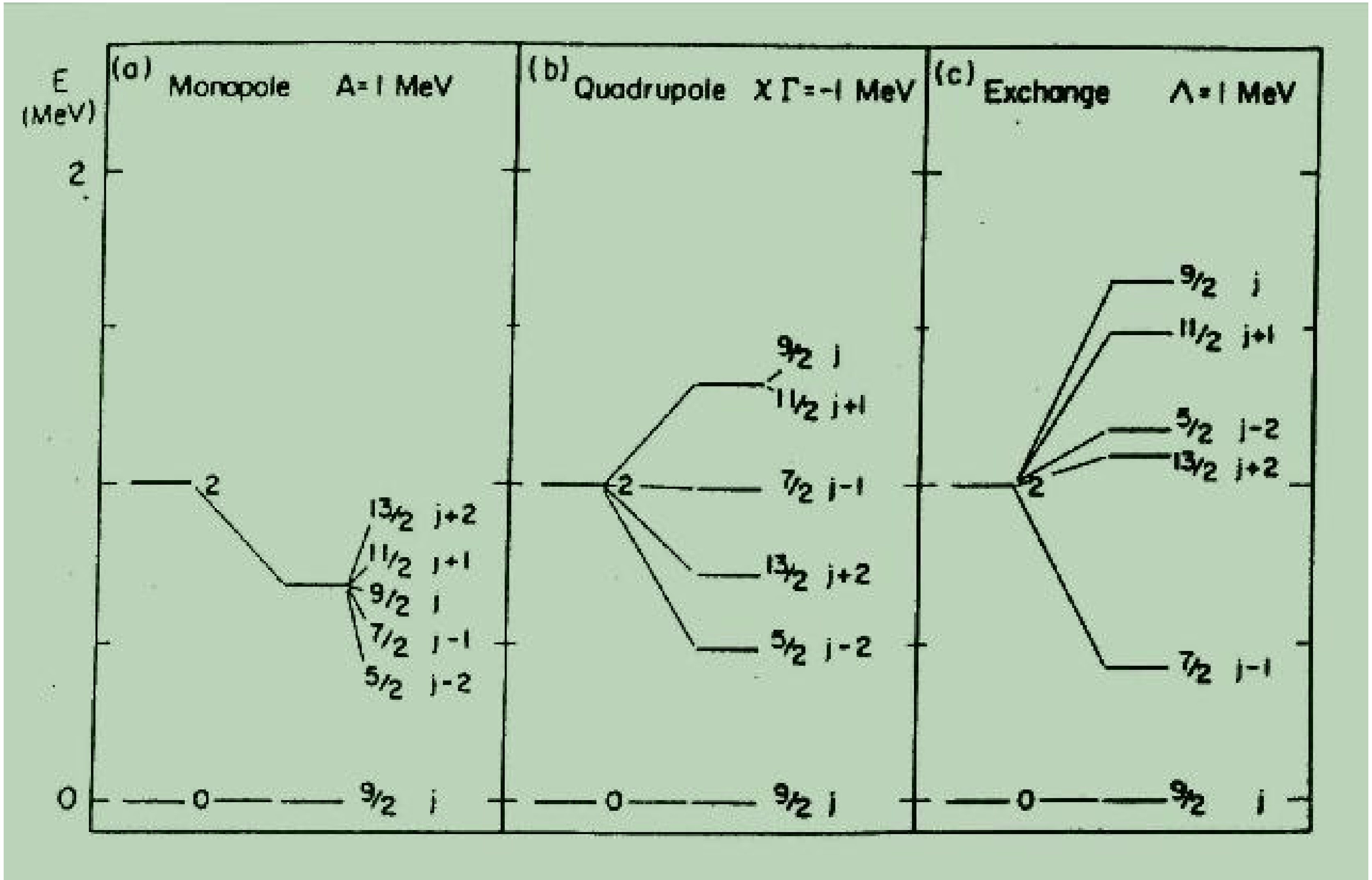
g_s

e_F, g_l **standard**

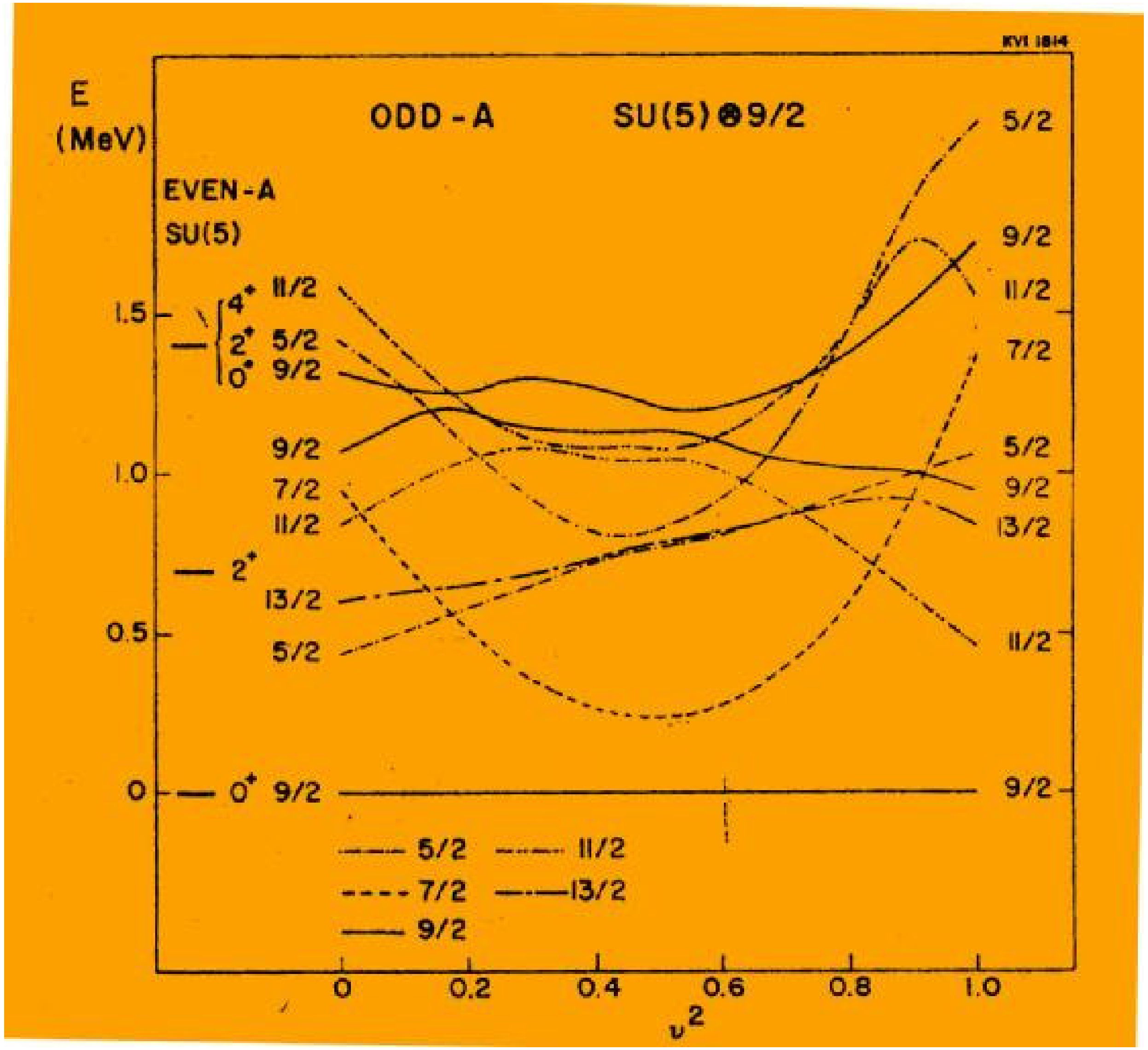
$$H\,\Psi = E\,\Psi$$

$$< \Psi_j \,|\, \mathcal{O} \,|\, \Psi_j >$$

Spherical nuclei


E (MeV)
2
0
(a) Monopole A=1 MeV
2
13/2 j+2
11/2 j+1
9/2 j
7/2 j-1
5/2 j-2
0
9/2 j
(b) Quadrupole χΓ=-1 MeV
9/2 j
11/2 j+1
2
7/2 j-1
13/2 j+2
5/2 j-2
0
9/2 j
(c) Exchange Λ=1 MeV
9/2 j
11/2 j+1
5/2 j-2
13/2 j+2
2
7/2 j-1
0
9/2 j

Scholten



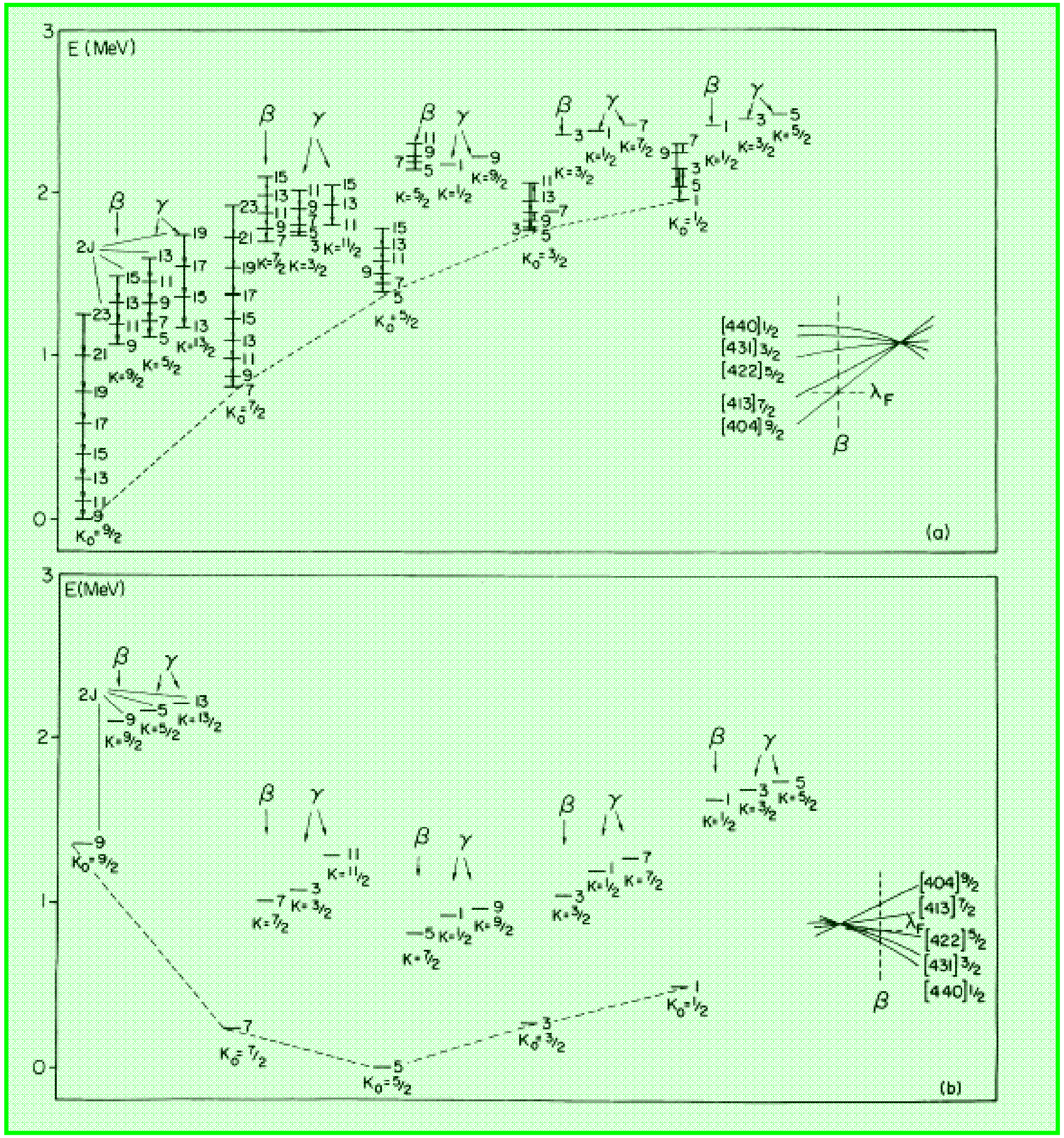
KVI 1814
E
(MeV)
ODD - A
SU(5) ⊗ 9/2
EVEN - A
SU(5)
1.5
1.0
0.5
0
4+
2+
0+
2+
0+
11/2
5/2
9/2
9/2
7/2
11/2
13/2
5/2
9/2
5/2
9/2
11/2
7/2
5/2
9/2
13/2
11/2
9/2
5/2
11/2
7/2
13/2
9/2
0
0.2
0.4
0.6
0.8
1.0
ν^2

Scholten

Deformed nuclei

$j = 9/2$ Iachello, PRL 43

$\Lambda = 0$



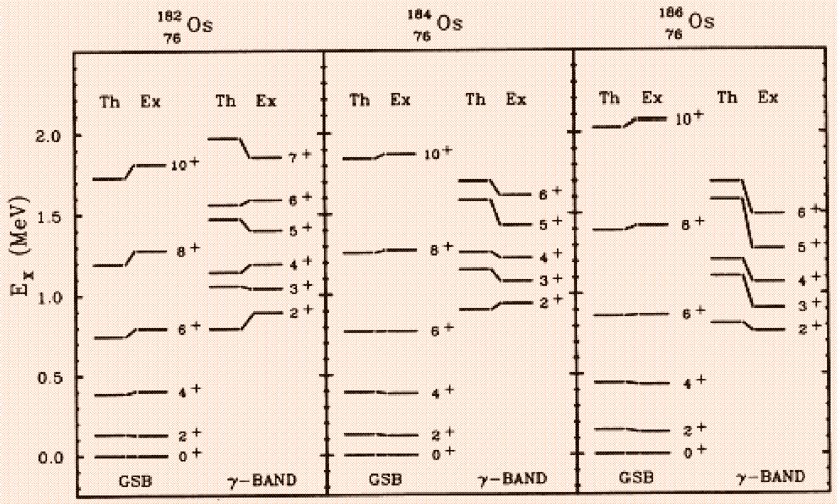
$\Lambda \neq 0$

The levels are arranged into bands denoted by the lowest value of the angular momentum K, contained in the band.

This quantum number is only approximately equivalent to the quantum number K in the Nilsson model.

In the inset, the corresponding situation in the Nilsson model is shown.

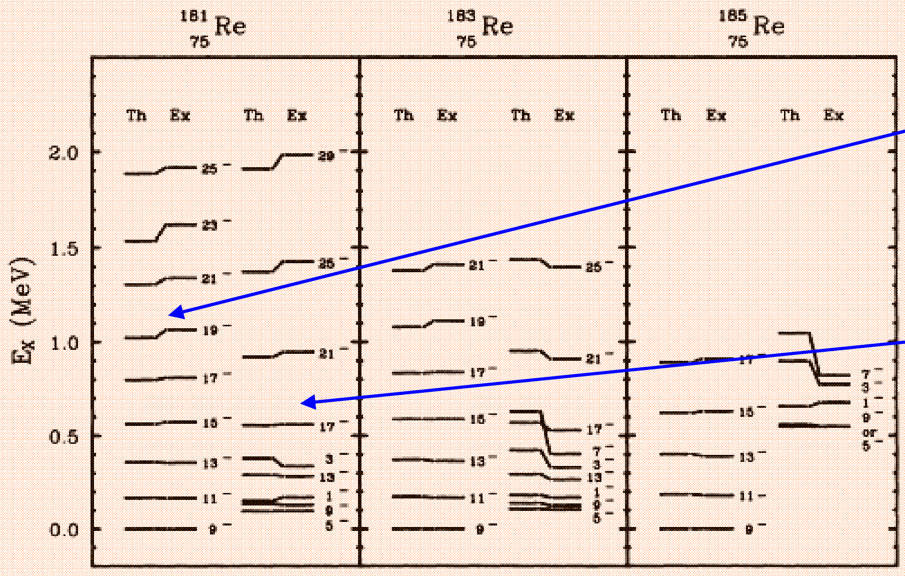
The IBFM generates bands that are analogous to the bands which can be constructed in the Nilsson model. In addition, it generates bands that could be called β and γ bands. While here they arise automatically, in the Nilsson model they must be either placed ad hoc or calculated by use of other methods.



IBA-calculated excitation-energy spectra for the even-even Os core isotopes compared with experimental data.

Prolate

Scholten, PRC 37



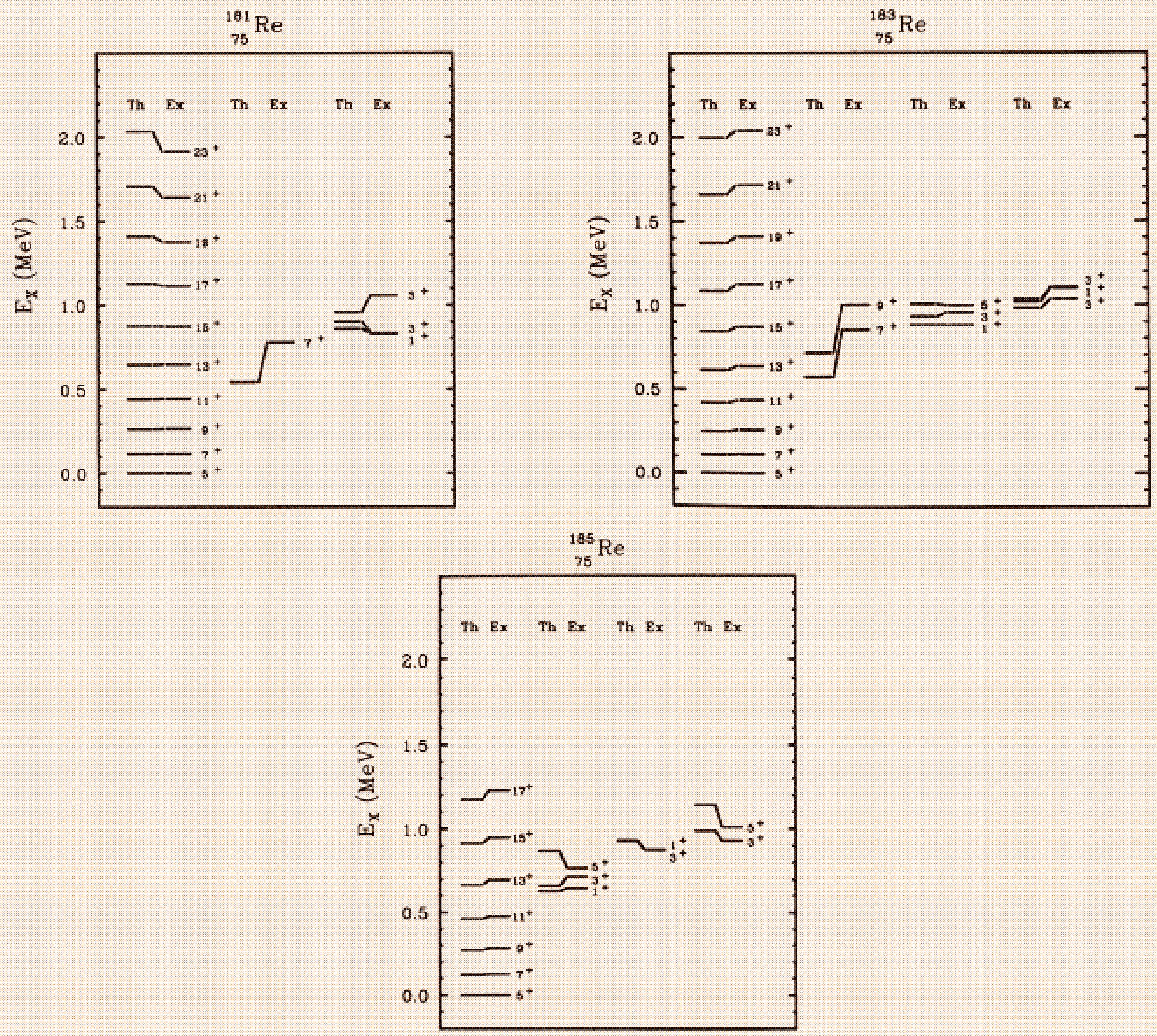
IBFA-calculated excitation energies for negative-parity states in the odd-mass Re isotopes compared with experimental data. States are labeled with 2*J*.

$K^{\pi} = 9/2^{-}$
$(\pi\, h_{11/2})$

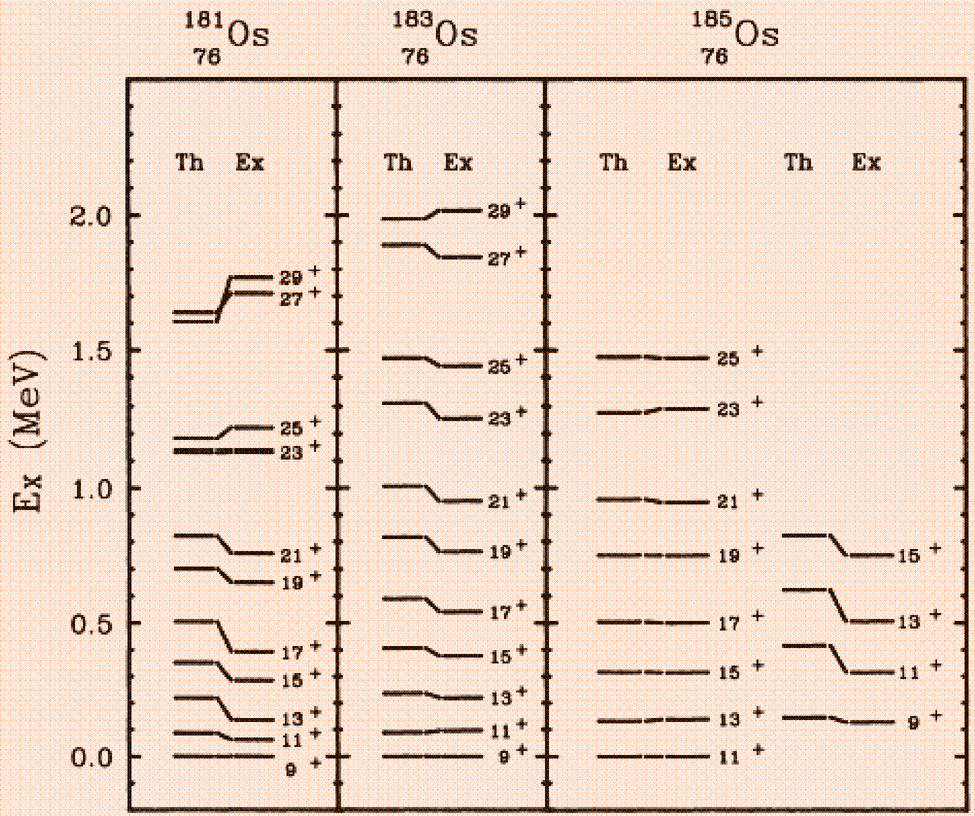
$v^2 = 0.62$

$K^{\pi} = 1/2^{-}$
$(\pi\, h_{9/2})$

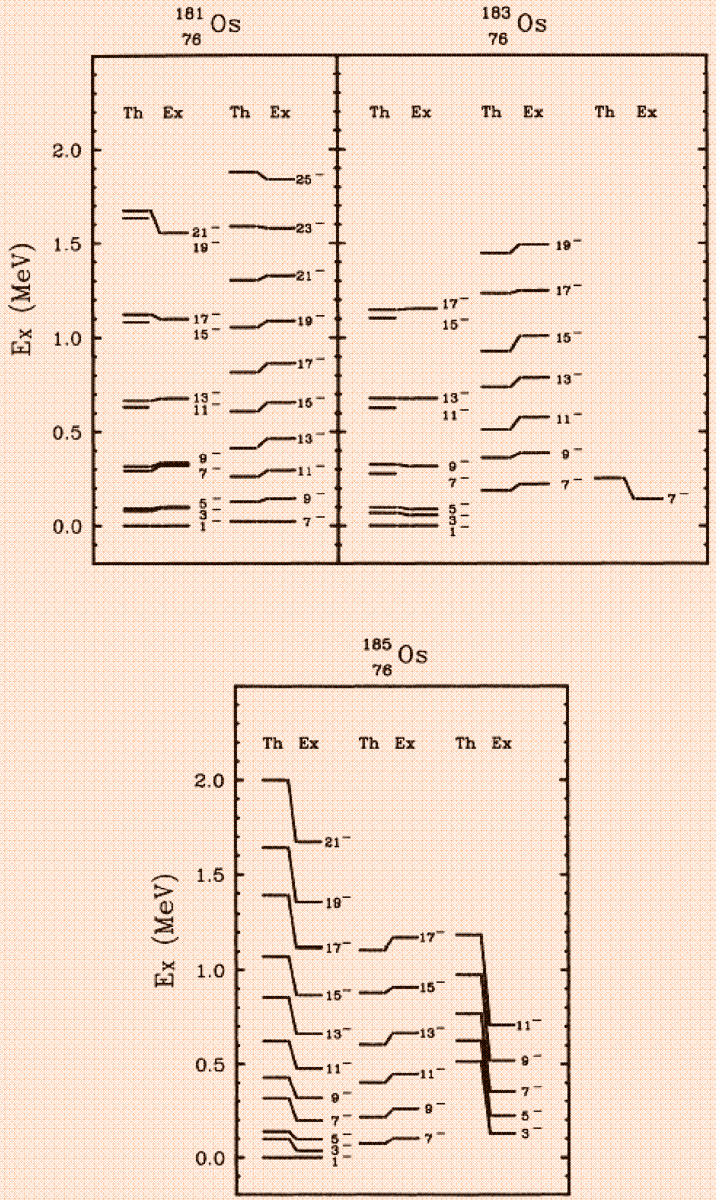
$V^2 = 0.05$



IBFA-calculated excitation energies for positive-parity states in the odd-mass Re isotopes compared with experimental data. States are labeled with $2J$.



IBFA-calculated excitation energies for positive-parity states in the odd-mass Os isotopes compared with experimental data. The states are labeled with $2J$.



IBFA-calculated excitation energies for negative-parity states in the odd-mass Os isotopes compared with experimental data. The states are labeled with 2*J*.

The IBFM states for odd-A Re and Os nuclei are obtained in multi-j calculations.

The choice of the model space has a strong influence on the model parameters. Even if there is a large separation between shells, the mixing due to the strong core-particle quadrupole interaction does not allow for restricting the model space to a single j shell. For example: Levels based on the $g_{9/2}$ particle. Here the $d_{5/2}$ particle from the next major shell has to be included due to the large non-spinflip matrix element $\langle d_{5/2} \parallel Y_2 \parallel g_{9/2} \rangle$. The same situation appears in the case of $h_{11/2}$ ($f_{7/2}$ has to be included in the model space). Restricting the model space requires a renormalization of the interactions. For unique-parity states:

- Strengths of boson-fermion interactions obtained in a single j calculation are effective strengths

- Strengths of boson-fermion interactions obtained in a multi j calculation are real strengths

Intruder deformed bands in odd Ag isotopes

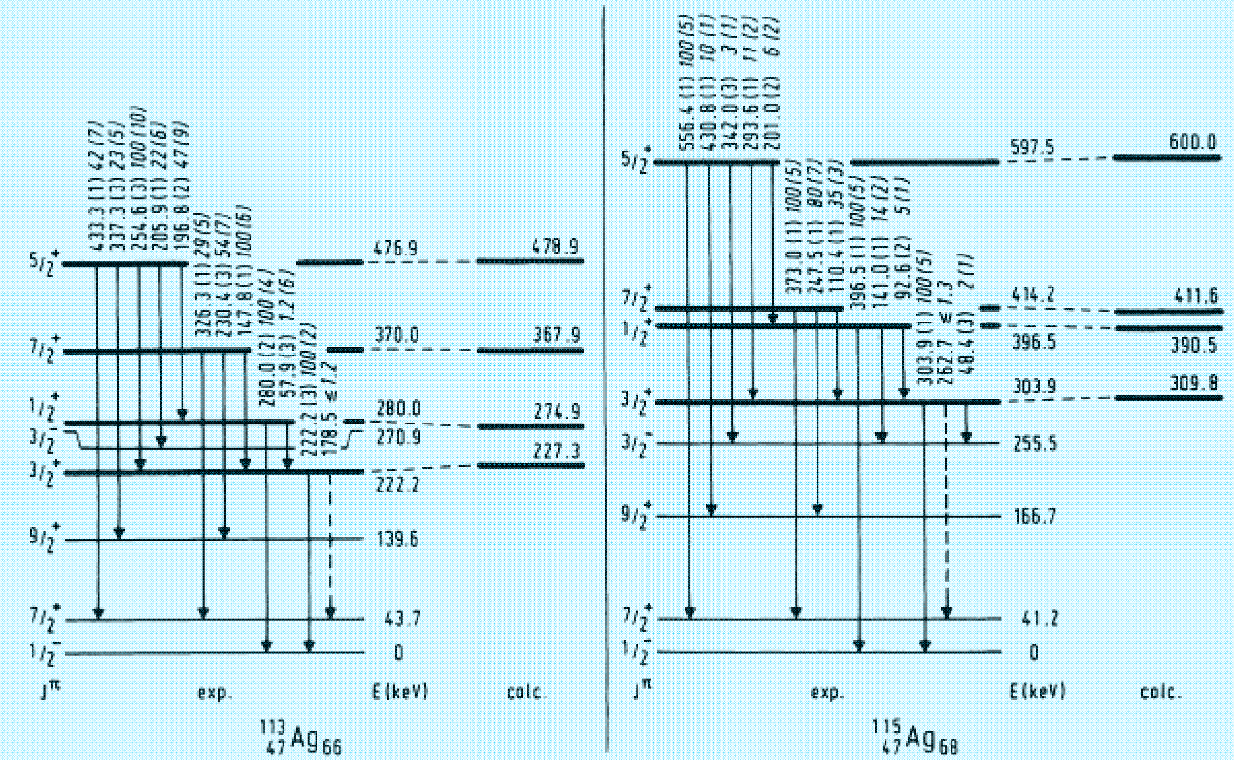


FIG. 2. Partial level schemes of ^{113}Ag and ^{115}Ag. Energies and relative intensities of γ rays are given with their uncertainties in parentheses. The energies of the intruder band members are compared to the values calculated with the rotational formula for $K = \frac{1}{2}$

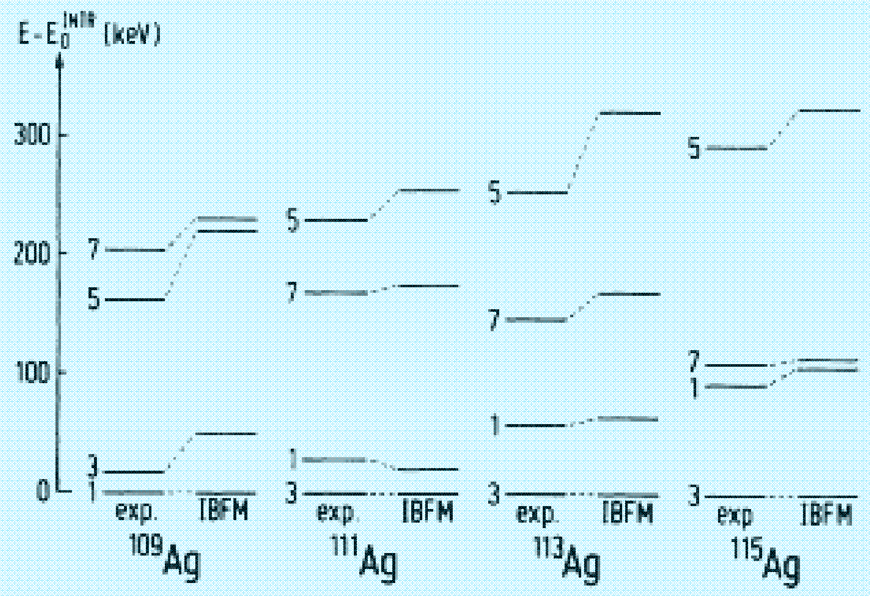
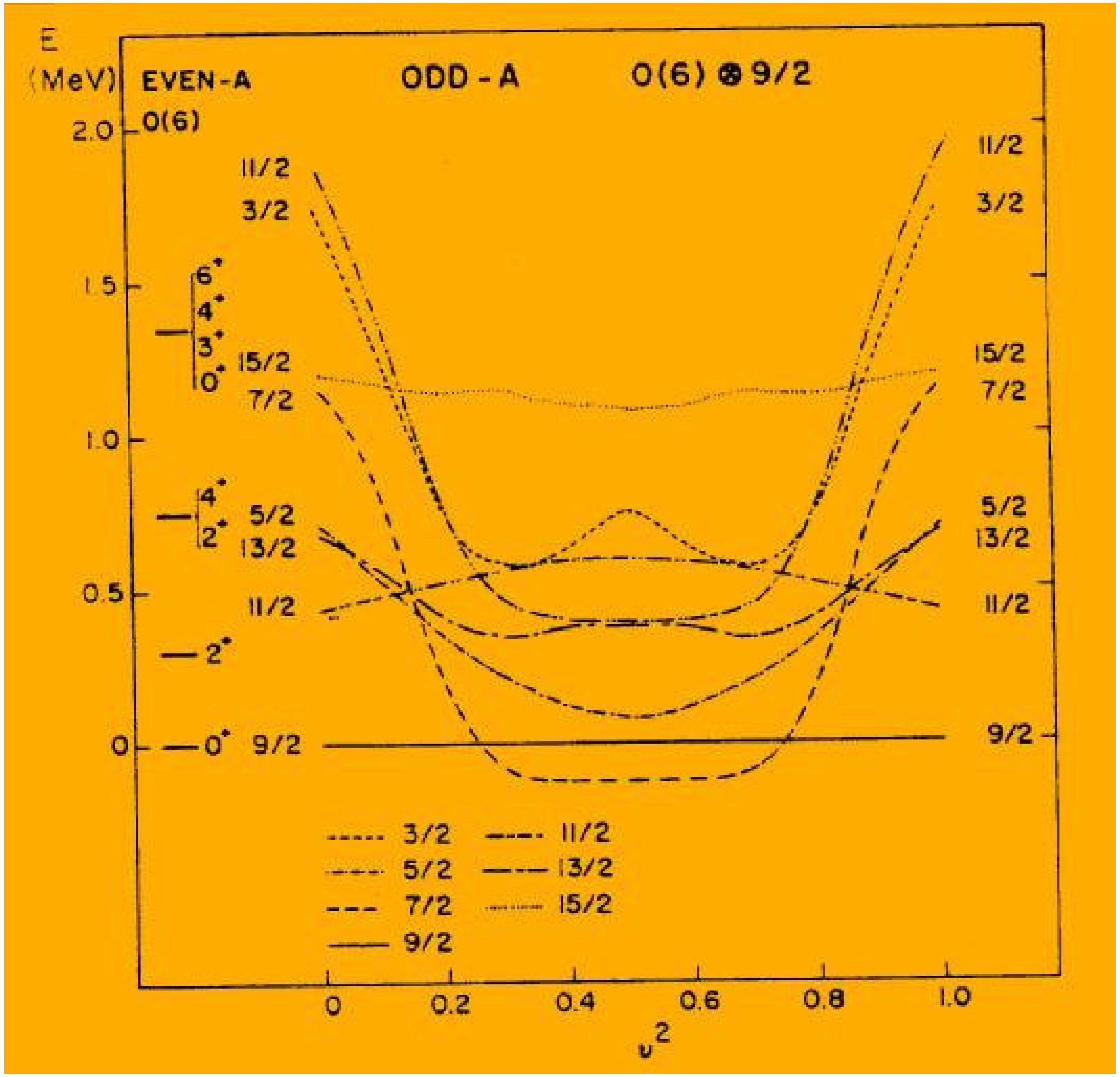


FIG. 3. Intruder positive-parity states of ^{109,111,113,115}Ag in comparison to IBFM calculations. The indicated spin values are twice the actual values.

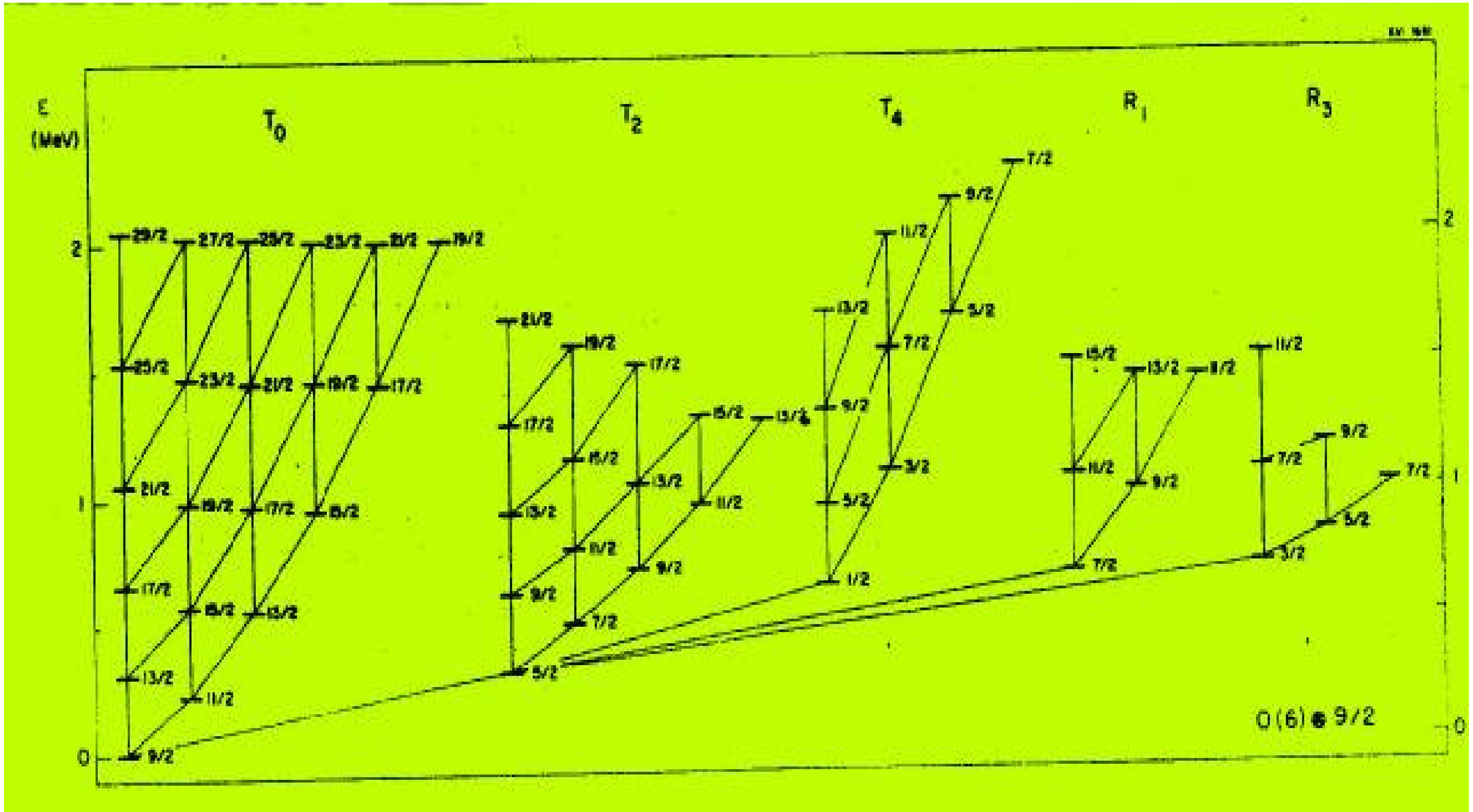
Only the monopole fermion-boson interaction strength is slightly changed from isotope to isotope. All other interaction strengths and occupation probabilities are the same for all isotopes.

O(6) nuclei



E
(MeV)
EVEN-A
ODD-A
O(6) ⊗ 9/2
2.0
O(6)
1.5
1.0
0.5
0
11/2
3/2
6⁺
4⁺
3⁺
0⁺
15/2
7/2
4⁺
2⁺
5/2
13/2
11/2
2⁺
0⁺
9/2
11/2
3/2
15/2
7/2
5/2
13/2
11/2
9/2
3/2
5/2
7/2
9/2
11/2
13/2
15/2
0
0.2
0.4
0.6
0.8
1.0
v^2

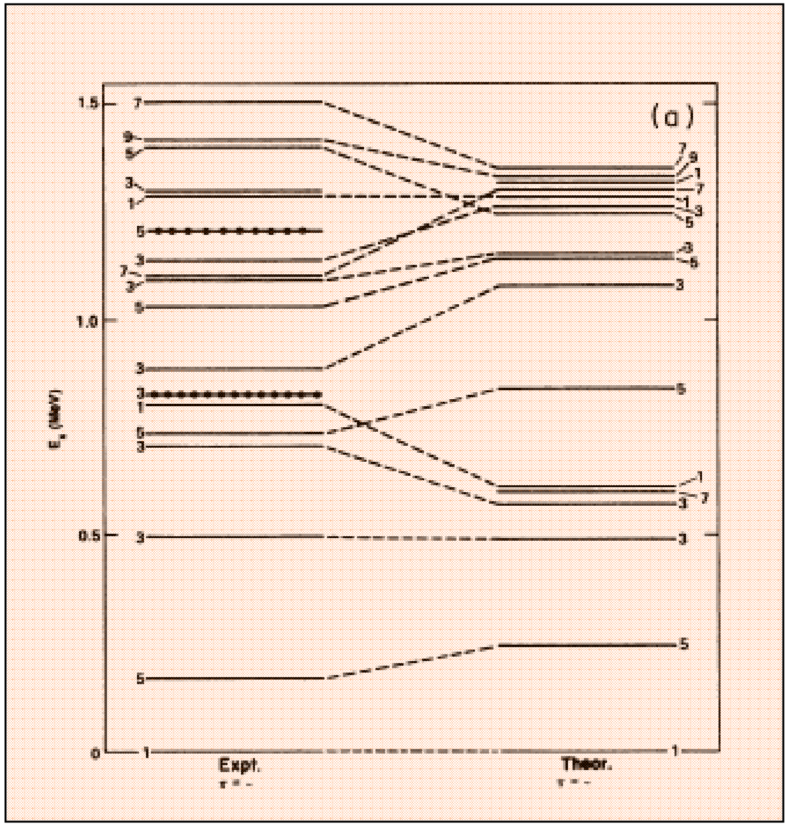
Scholten

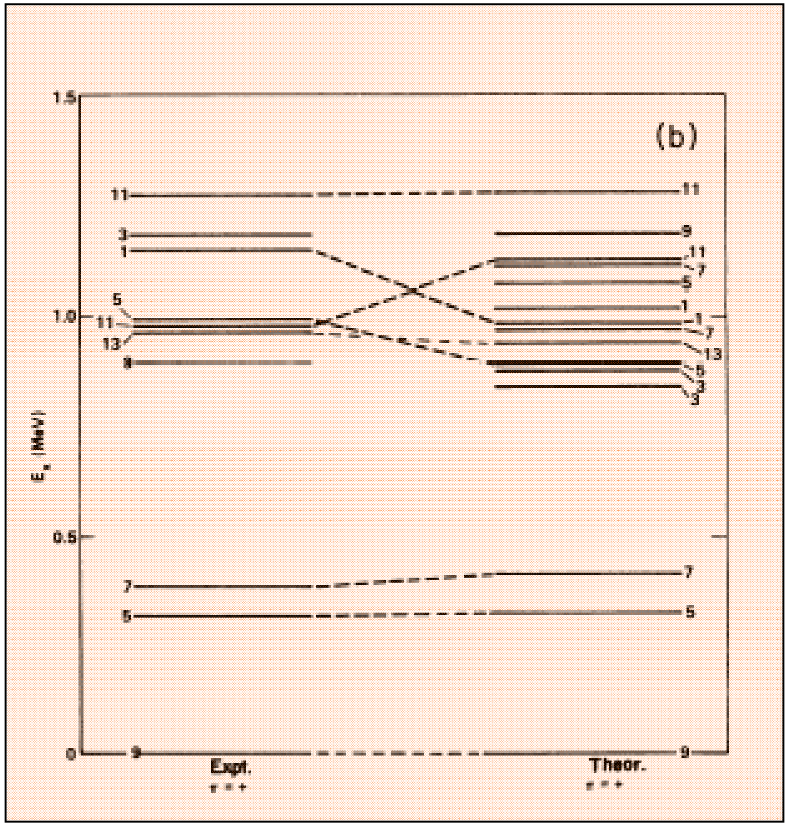


Scholten

Transitional nuclei

^{71}Ge

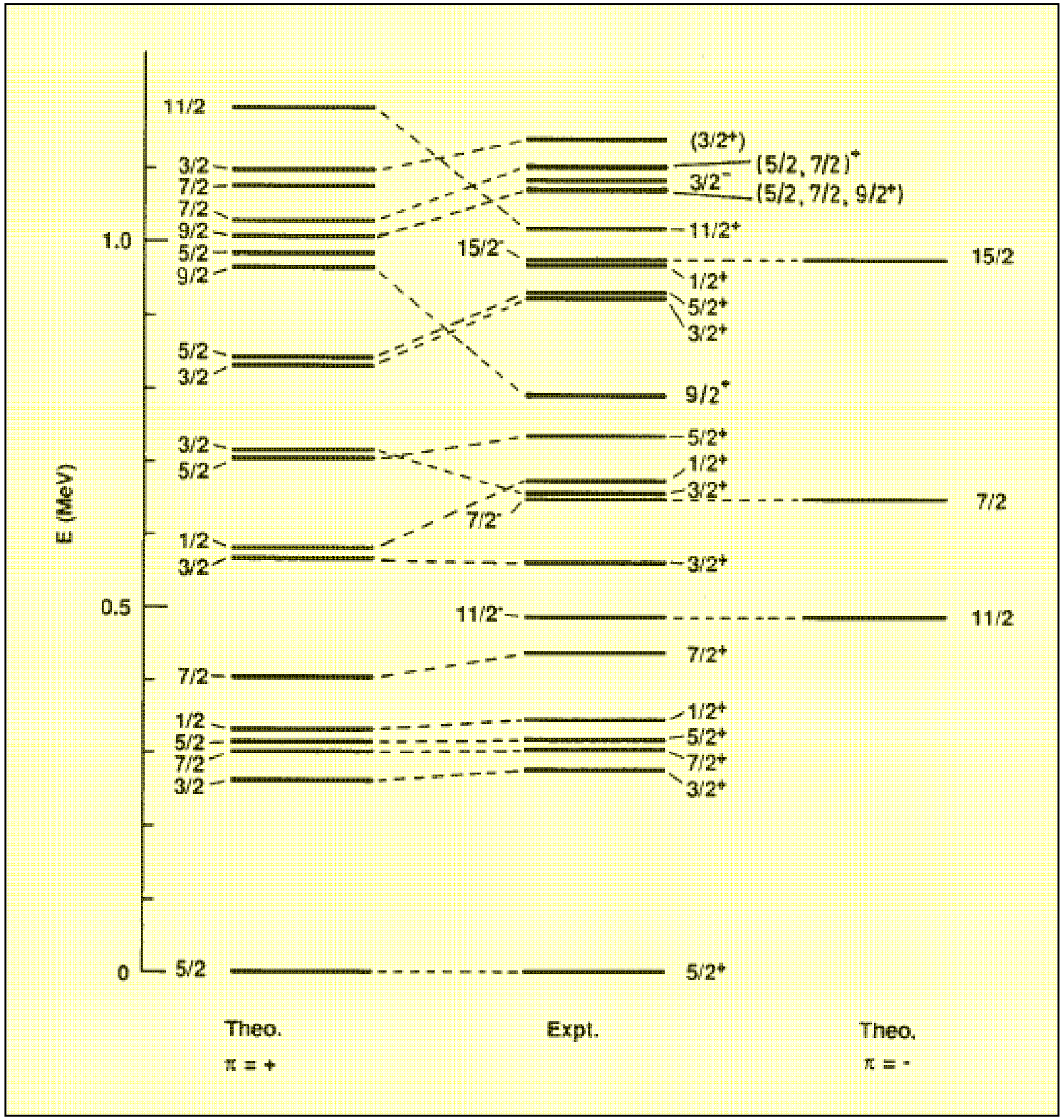




Negative parity levels **Positive parity levels**

Angular momentum x 2

^{105}Pd



J of Level		I_γ	
From	To	Expt.	Theor.
$3/2_1^+$	$5/2_1^+$	100	100
$7/2_1^+$	$3/2_1^+$		0.0005
	$5/2_1^+$	100	100
$5/2_2^+$	$7/2_1^+$		0.0002
	$3/2_1^+$	0.12	0.7
	$5/2_1^+$	100	100
$1/2_1^+$	$5/2_2^+$		0
	$3/2_1^+$	27	93
	$5/2_1^+$	100	100
$7/2_2^+$	$5/2_2^+$		2
	$7/2_1^+$		0.0006
	$3/2_1^+$		0.03
	$5/2_1^+$	100	100
$3/2_2^+$	$7/2_2^+$		0.0005
	$1/2_1^+$	2.4	1
	$5/2_2^+$		2
	$7/2_1^+$		0.005
	$3/2_1^+$		0.4
	$5/2_1^+$	100	100
$3/2_3^+$	$3/2_2^+$	0.8	0.4
	$7/2_2^+$		0.08

Halflives

J	Expt. [a]	Theor.
$3/2_1^+$	0.067 ns	0.03 ns
$5/2_2^+$	0.04 ns	0.1 ns
$7/2_2^+$	3.8 ps	7.7 ps
$1/2_1^+$	0.88 ns	0.22 ns
$3/2_2^+$	1.9 ps	6.3 ps
$1/2_2^+$	>2 ps	1.6 ps

BOSON-FERMION SYMMETRIES
SUPERSYMMETRIES

If the Hamiltonian can be expressed in terms of Casimir invariants of the chain of subgroups, the energy spectrum can be obtained ANALYTICALLY. Other observables (B(E2), B(M1), static moments, spectroscopic factors, ...) can be expressed in analytical form, too.

The symmetry group related to IBM-1 is $U(6)$. The six dimensions are formed by the s boson and five components of d_μ boson. Since the number of bosons is invariant, the group is unitary. There are three chains of subgroups:

$$
\begin{array}{lllll}
U(6) & \supset & U(5) \supset O(5) \supset O(3) \supset O(2) & & \textit{vibrational limit} \\
U(6) & \supset & SU(3) \qquad\qquad \supset O(3) \supset O(2) & & \textit{rotational limit} \\
U(6) & \supset & O(6) \supset O(5) \supset O(3) \supset O(2) & & \gamma - \textit{soft limit}
\end{array}
$$

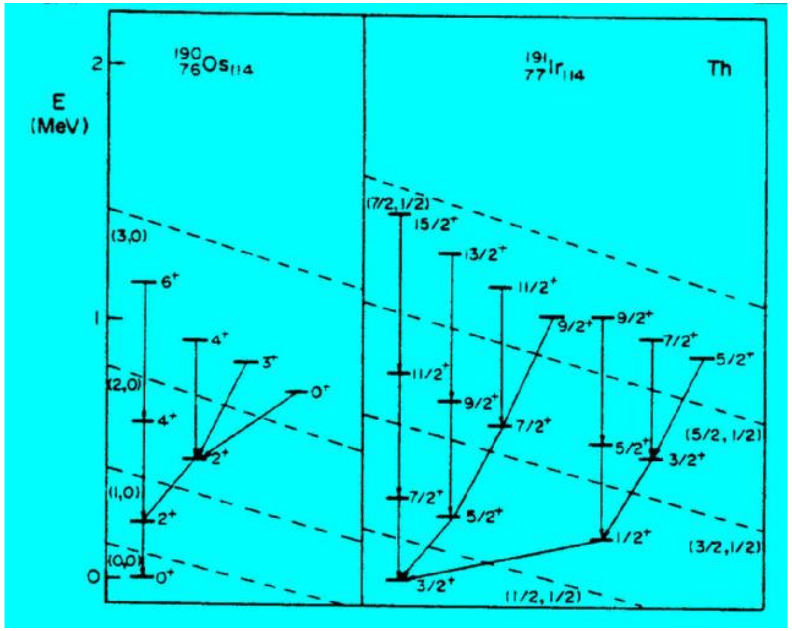
For boson-fermion systems many group chains have been investigated. Example: A $j = 3/2$ particle coupled to an $O(6)$ core ($j = 3/2$ has four different m-states and therefore forms a representation of the $U(4)$ group).

$$U^B(6) \otimes U^F(4) \supset O^B(6) \otimes U^F(4) \supset Spin(6) \supset Spin(5) \supset Spin(3) \supset Spin(2)$$

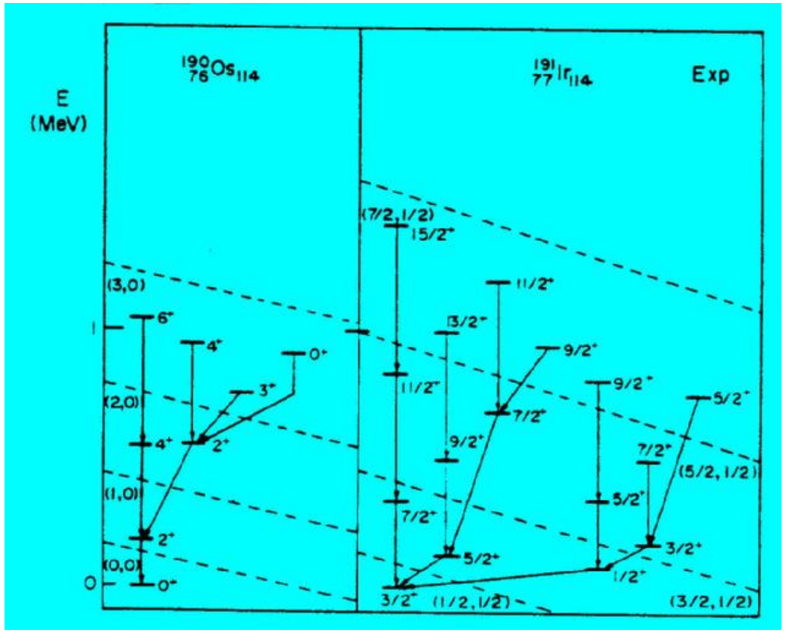
$$E = -\frac{A}{4}[\sigma_1(\sigma_1 + 4) + \sigma_2(\sigma_2 + 2) + \sigma_3^2] + \frac{B}{6}[\tau_1(\tau_1 + 3) + \tau_2(\tau_2 + 1)] + CJ(J + 1) + D\Sigma(\Sigma + 4)$$

$$\begin{array}{rll} U^B(6) & \textit{quantum numbers} & [N] \\ U^F(4) & \textit{quantum numbers} & \{M\} \\ O^B(6) & \textit{quantum numbers} & \Sigma \\ Spin(6) & \textit{quantum numbers} & (\sigma_1, \sigma_2, \sigma_3) \\ Spin(5) & \textit{quantum numbers} & (\tau_1, \tau_2) \\ Spin(3) & \textit{quantum numbers} & J \\ Spin(2) & \textit{quantum numbers} & M_J \end{array}$$

$O^B(6) \otimes U^F(4) \supset Spin(6) \longrightarrow$ Parameters describing the boson system are in a unique relation to the parameters describing the boson-fermion system.



$^{190}_{76}Os_{114}$
$^{191}_{77}Ir_{114}$
Th
E (MeV)
2
1
0
(3,0)
(2,0)
(1,0)
(0,0)
6+
4+
3+
0+
4+
2+
2+
0+
(7/2, 1/2)
15/2+
13/2+
11/2+
9/2+
9/2+
7/2+
5/2+
11/2+
9/2+
7/2+
5/2+
3/2+
7/2+
5/2+
1/2+
3/2+
(5/2, 1/2)
(3/2, 1/2)
(1/2, 1/2)



$^{190}_{76}Os_{114}$
$^{191}_{77}Ir_{114}$
Exp
E (MeV)
1
0
(3,0)
(2,0)
(1,0)
(0,0)
6+
4+
0+
3+
4+
2+
2+
0+
(7/2, 1/2)
15/2+
11/2+
13/2+
9/2+
11/2+
7/2+
9/2+
9/2+
5/2+
7/2+
5/2+
7/2+
3/2+
5/2+
1/2+
3/2+
(5/2, 1/2)
(1/2, 1/2)
(3/2, 1/2)

Problems:

- The symmetry approach to boson-fermion systems is more phenomenological in nature
- It can be applyed only in special cases when one or few fermion configurations are coupled to boson cores in one of the symmetry limits of IBM

Advantages:

- This approach was extended to boson-fermion-fermion systems (odd-odd nuclei)
- The spectra of neighboring even-even, odd-even and odd-odd nuclei can be described with the same set of parameters
- Analytical expressions are available
- Evidence that collective and single-particle degrees of freedom are closely related