



*The Abdus Salam*  
International Centre for Theoretical Physics



1939-30

**Joint ICTP-IAEA Workshop on Nuclear Structure and Decay Data:  
Theory and Evaluation**

*28 April - 9 May, 2008*

**Experimental techniques to deduce  $J\pi$**

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THE AUSTRALIAN NATIONAL UNIVERSITY

# *Experimental techniques to deduce $J^\pi$*

Joint ICTP-IAEA Workshop on  
Nuclear Structure and Decay Data Theory and Evaluation  
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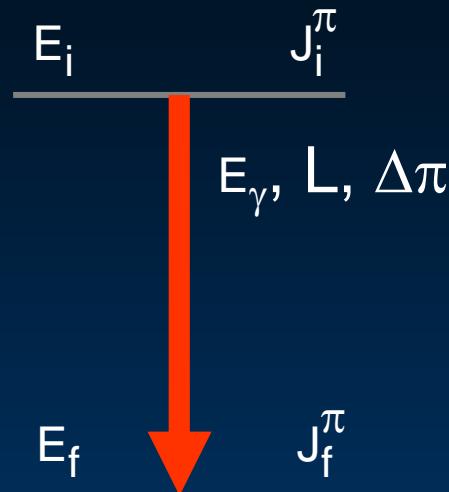
*Tibor Kibédi*

## *Outline:*

### Lecture I: *Experimental techniques to deduce $J^\pi$ from*

- *Internal Conversion Coefficients*
- *Angular distributions and correlations*
- *Directional Correlations from Oriented nuclei (DCO)*

### Lecture II: *New developments in characterizing nuclei far from stability*



## Energetics of $\gamma$ -decay:

$$E_i = E_f + E_\gamma + T_r$$

$$0 = p_R + p_\gamma,$$

where  $T_R = (p_R)^2/2M$ ; usually  $T_R/E_\gamma \sim 10^{-5}$

## Angular momentum and parity selection rules; multipolarities

Multipolarity known

$\Delta J$  may not be unique

unique  $\Delta\pi$

$$|J_i - J_f| \leq L \leq J_i + J_f; L \neq 0$$

$$J_i = J_f$$

$$\Delta\pi = \text{no}; \quad E2, E4, E6$$

$$M1, M3, M5$$

$$E0$$

$$\Delta\pi = \text{yes}; \quad E1, E3, E5$$

$$M2, M4, M6$$

## Mixed multipolarity

$$\delta(\pi`L') = I_\gamma(\pi`L') / I_\gamma(\pi L)$$

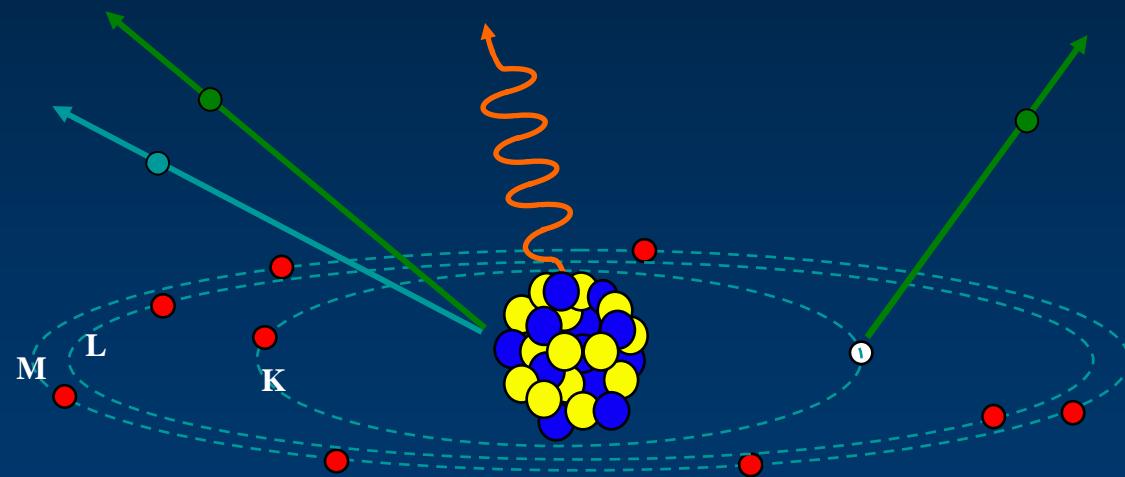
# Electromagnetic decay modes

- Pair conversion coefficients
- E0 transitions:  $\Delta L=0$
- e<sup>-</sup> - e<sup>+</sup> pair**

- Angular distribution with spins oriented
- Angular correlations
- Polarization effects

**$\gamma$ -ray**

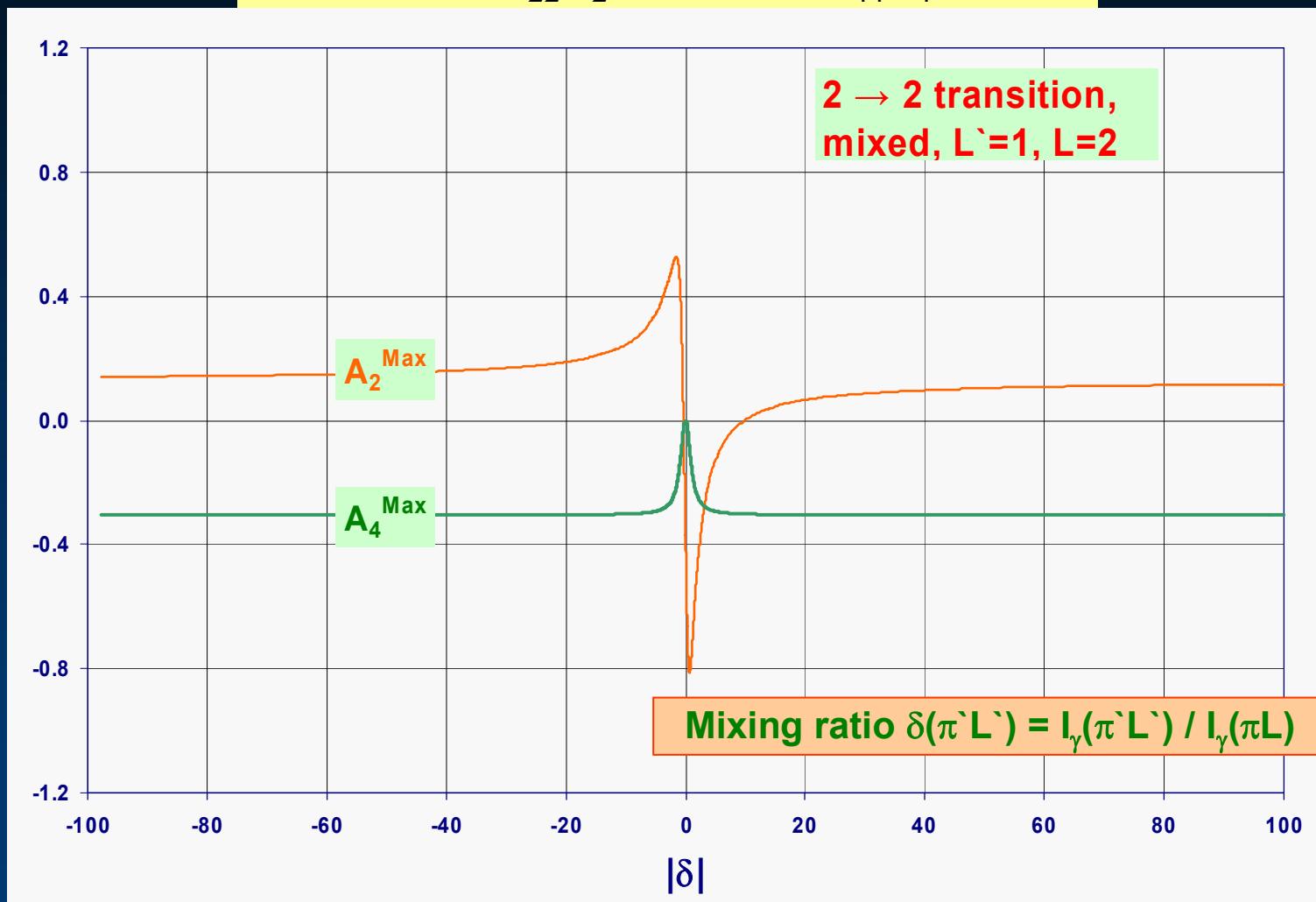
- Electron conversion coefficients
- E0 transitions:  $\Delta L=0$
- electron conversion**



**Higher order effects: for example 2 photon emission is very weak**

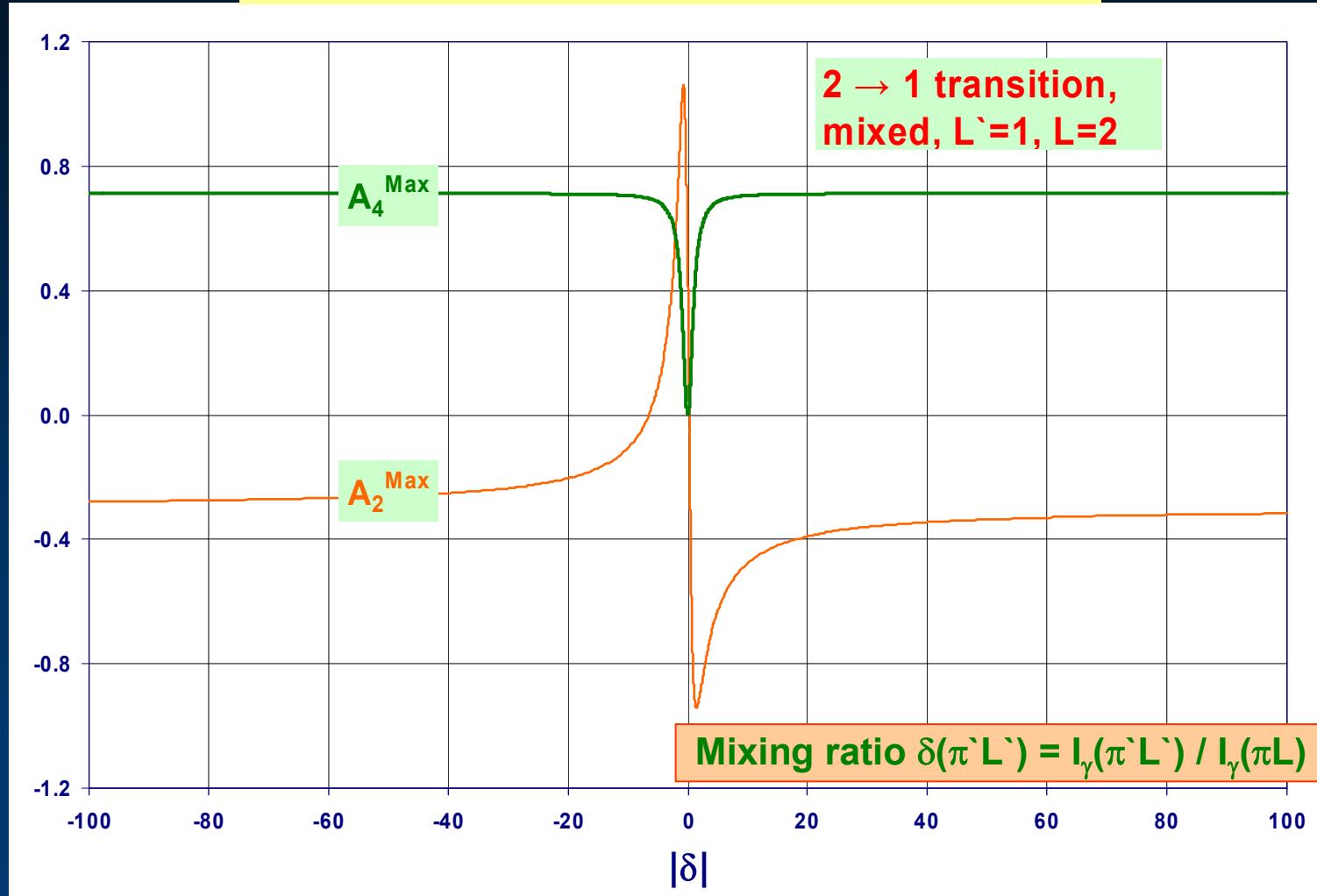
# Angular distributions of Gamma-rays

$$W(\theta) = 1 + A_{22}P_2(\cos \theta) + A_{44}P_4(\cos \theta)$$



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## Attenuation due to relaxation of nuclear orientation

$$0 \leq A_{kk} \leq A_k^{\max}(J_i, J_f, L); k = 2, 4 \dots$$

$$A_k^{\max}(J_i, J_f, L) = \frac{F_k(LLJ_fJ_i) + 2\delta \times F_k(LL+1J_fJ_i) + \delta^2 \times F_k(L+1L+1J_fJ_i)}{1 + \delta^2}$$

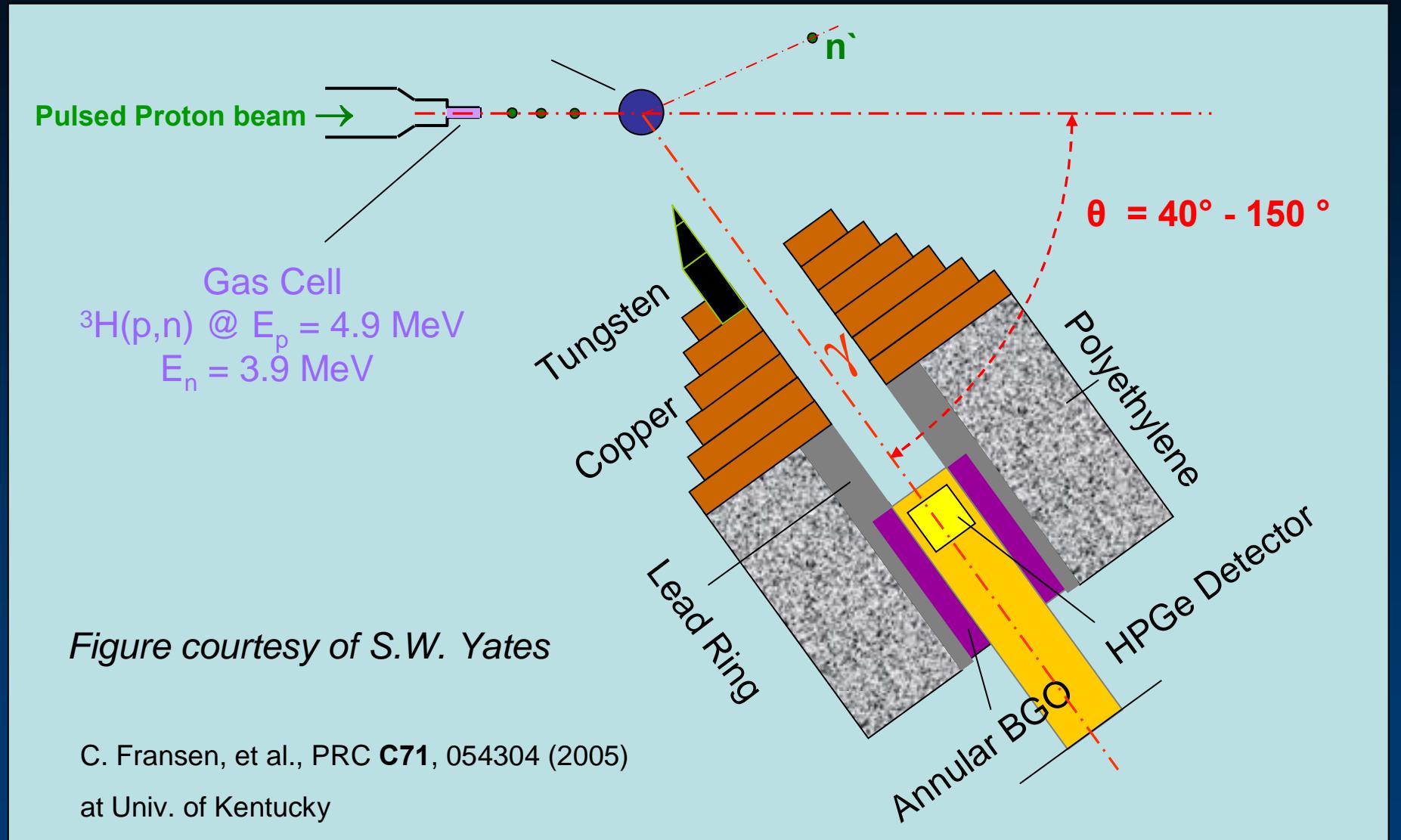
For  $F_k(LL`J_fJ_i)$  see E. Der Mateosian and A.W. Sunyar, ADNDT 13 (1974) 407

$$A_{kk} = B_k(J_i) \times A_k^{\max}(J_i, J_f, L)$$

## Nuclear orientation can be achieved

- ❖ by interaction of external fields (E,B) with the static moments of the nuclei at low temperatures
- ❖ by nuclear reaction

# Angular distributions of Gamma-rays (n,n)'reaction on $^{92}\text{Zr}$



# Angular distributions of Gamma-rays

(n,n')reaction on  $^{92}\text{Zr}$

$^{92}\text{Zr}(n,n')$  reaction

12 angles and

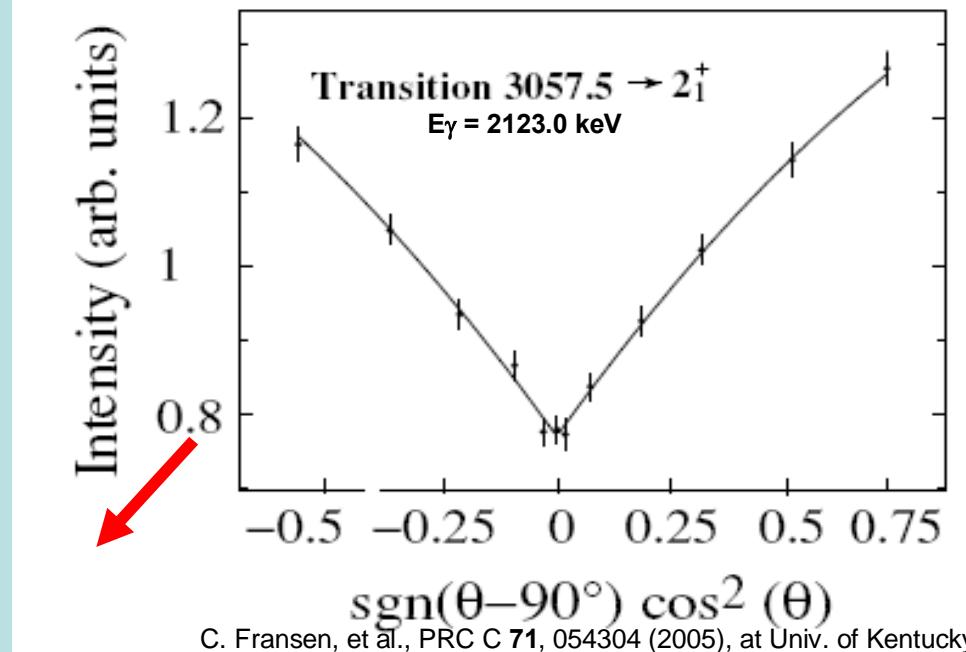
12 hours / angle

$\gamma$ -spectrometer at 1.4 m

Fit to data

$$W(\theta) = A_0 + A_{22}P_2(\cos\theta) + A_{44}P_4(\cos\theta)$$

Deduced     $A_2 = A_{22}/A_0$   
 $A_4 = A_{44}/A_0$



Typical values	<b>A2</b>	<b>A4</b>
$\Delta J=2$ (stretched quadrupole)	+0.3	-0.1
$\Delta J=1$ (stretched dipole)	-0.2	0
$\Delta J=1$ , D+Q	+0.5 to -0.8	>0

# Angular distributions – mixing ratio

Beam defines a symmetry axis

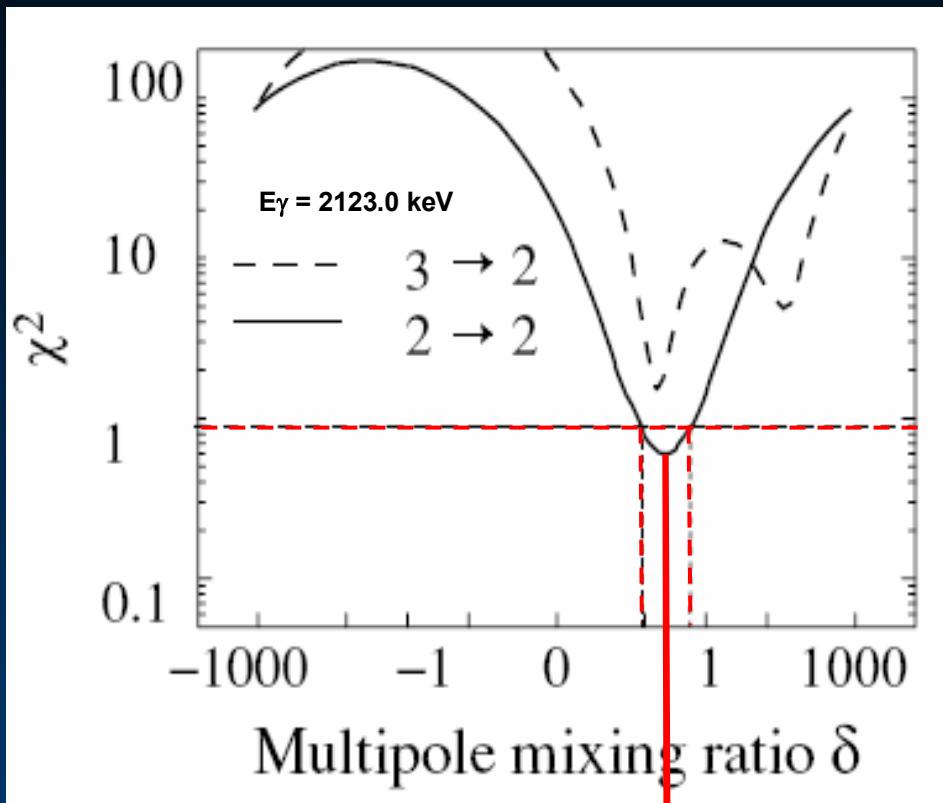
$$W(\theta) = 1 + \sum_{k=2,4} B_k(J_i) \times A_k^{\max}(J_i, J_f, L, \delta)$$

where  $B_k(J)$  is the statistical tensor

$$B_k(J_i) = \sum_{m=-J_i}^{+1} (-1)^{J_i+m} \sqrt{(2k+1)(2J_i+1)} \times$$

$$\begin{pmatrix} J_i & J_i & k \\ -m & m & 0 \end{pmatrix} \times \frac{\text{Exp}(-m^2/2\sigma^2)}{\sum_{m=-1}^{J_i} \text{Exp}(-m^2/2\sigma^2)}$$

Approximation with  
Gaussian distribution



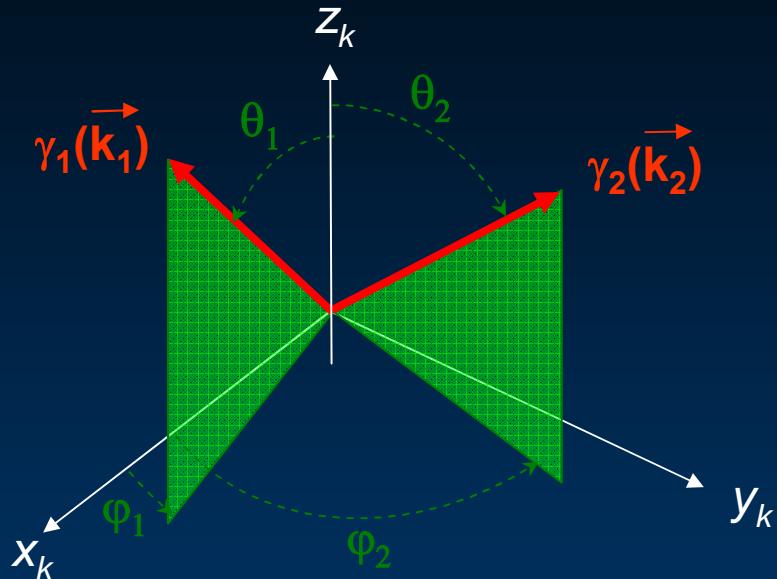
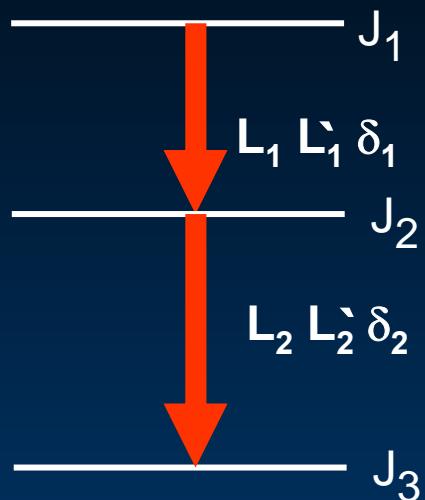
Mixing ratio,  $\delta$  deduced from  $\chi^2$  of

$\frac{W_{\text{exp}}(\theta)}{W_{\text{calc}}(\theta)}$  as a function of  $\delta$

$\delta = 0.69(16) \text{ (D+Q)}$

But no information on Electric or Magnetic character: E1+M2 or M1+E2

## Directional Correlations from Oriented nuclei (DCO)



**For a  $J_1 \rightarrow J_2 \rightarrow J_3$  cascade** (see A.E. Stuchbery, Nucl. Phys. A723 (2003) 69)

$$W(\theta_1, \varphi_1, \theta_2, \varphi_2) = \sum_{k, q, k_1, q_1, k_2, q_2} \rho_{k_1 q_1}(J_1) (-1)^{k_1 + q_1} \sqrt{(2k+1)(2k_1+1)} \begin{pmatrix} k_1 & k & k_2 \\ -q_1 & q & q_2 \end{pmatrix}$$

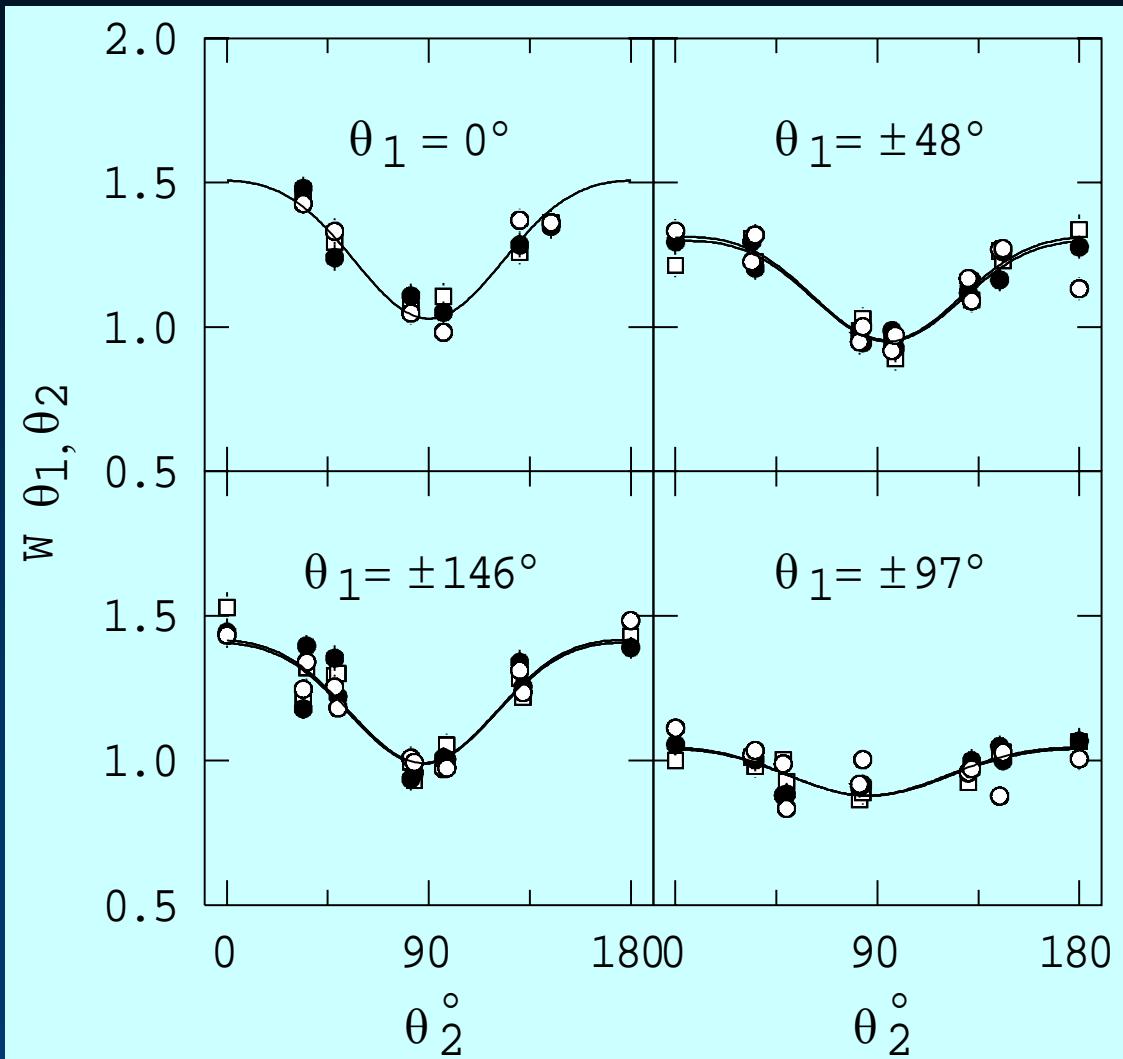
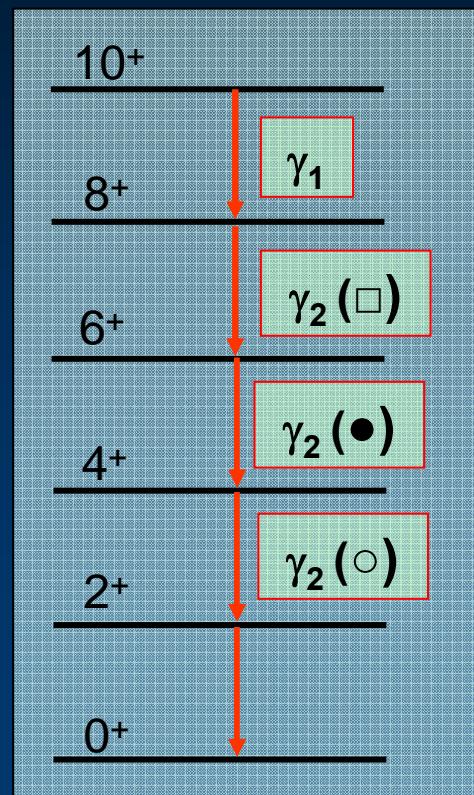
$$\times A_k^{k_2 k_1} (\delta_{\gamma 12} \vec{L} \vec{L} J_2 J_1) Q_k(E_{\gamma 12}) D_{q_0}^{k*}(\varphi_1, \theta_1, 0)$$

$$\times A_{k_2} (\delta_{\gamma 23} \vec{L} \vec{L} J_3 J_2) Q_{k_2}(E_{\gamma 23}) D_{q_2 0}^{k_2 *}(\varphi_2, \theta_2, 0)$$

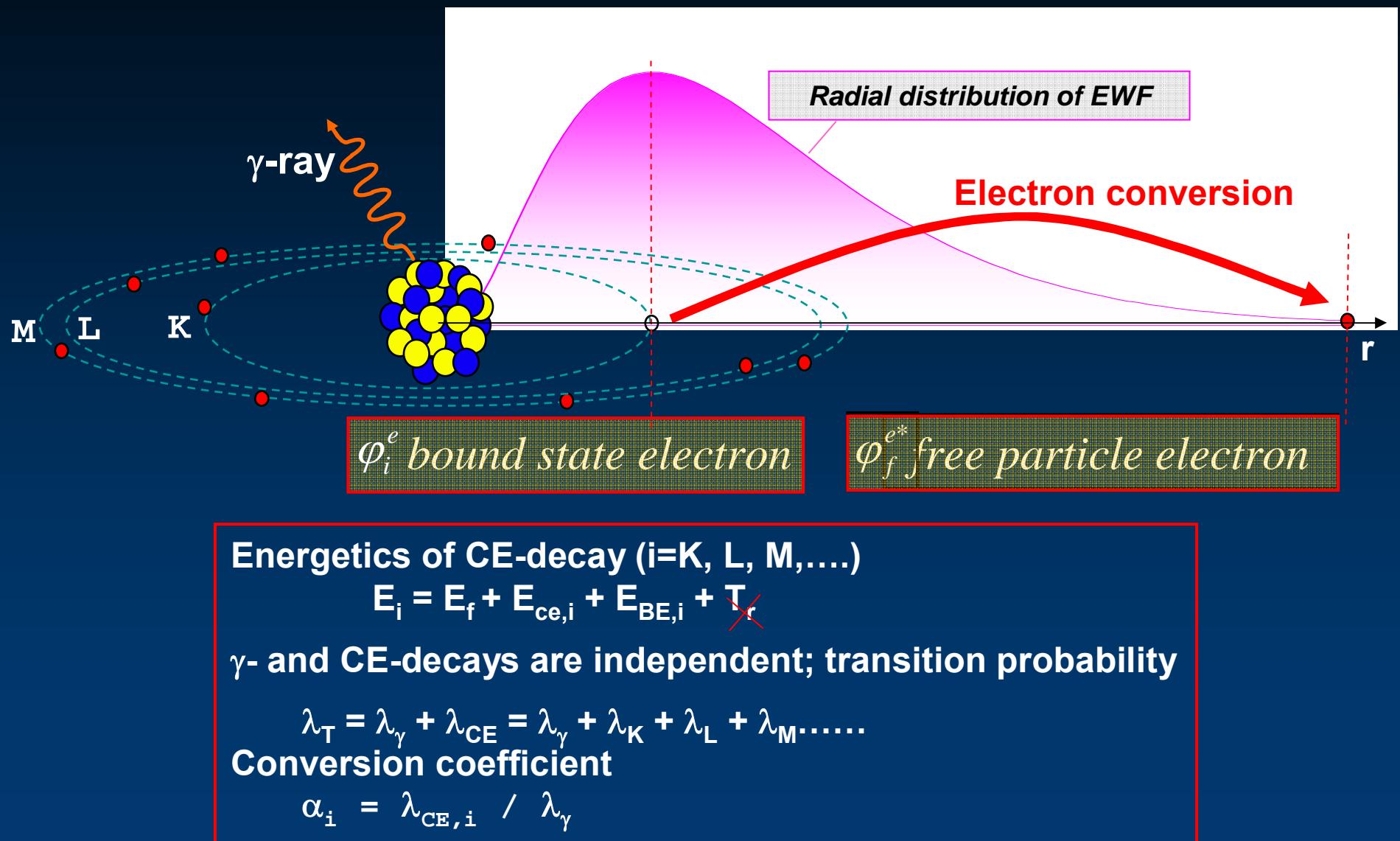
# Directional Correlations from Oriented nuclei (DCO) example

$^{184}\text{Pt}$  from  ${}^{\text{nat}}\text{Gd} + {}^{29}\text{Si}$  @ 145 MeV CAESAR array (ANU)

M.P. Robinson et al., Phys. Lett B530 (2002) 74



## Conversion electrons (CE)



# The physics of conversion coefficients

$$\alpha_K \equiv \frac{\lambda_{e,K}}{\lambda_\gamma}$$

$$\lambda_e = \frac{2\pi}{\hbar} |m_{fi}|^2 \frac{d\rho}{dE} \quad \textit{Fermi's golden rule}$$

*Density of the final electron state  
(continuum)*

$$m_{fi} = \psi_f^{N*} \varphi_f^{e*} F_{\ell,m} \psi_i^N \varphi_i^e$$

**Nuclear**

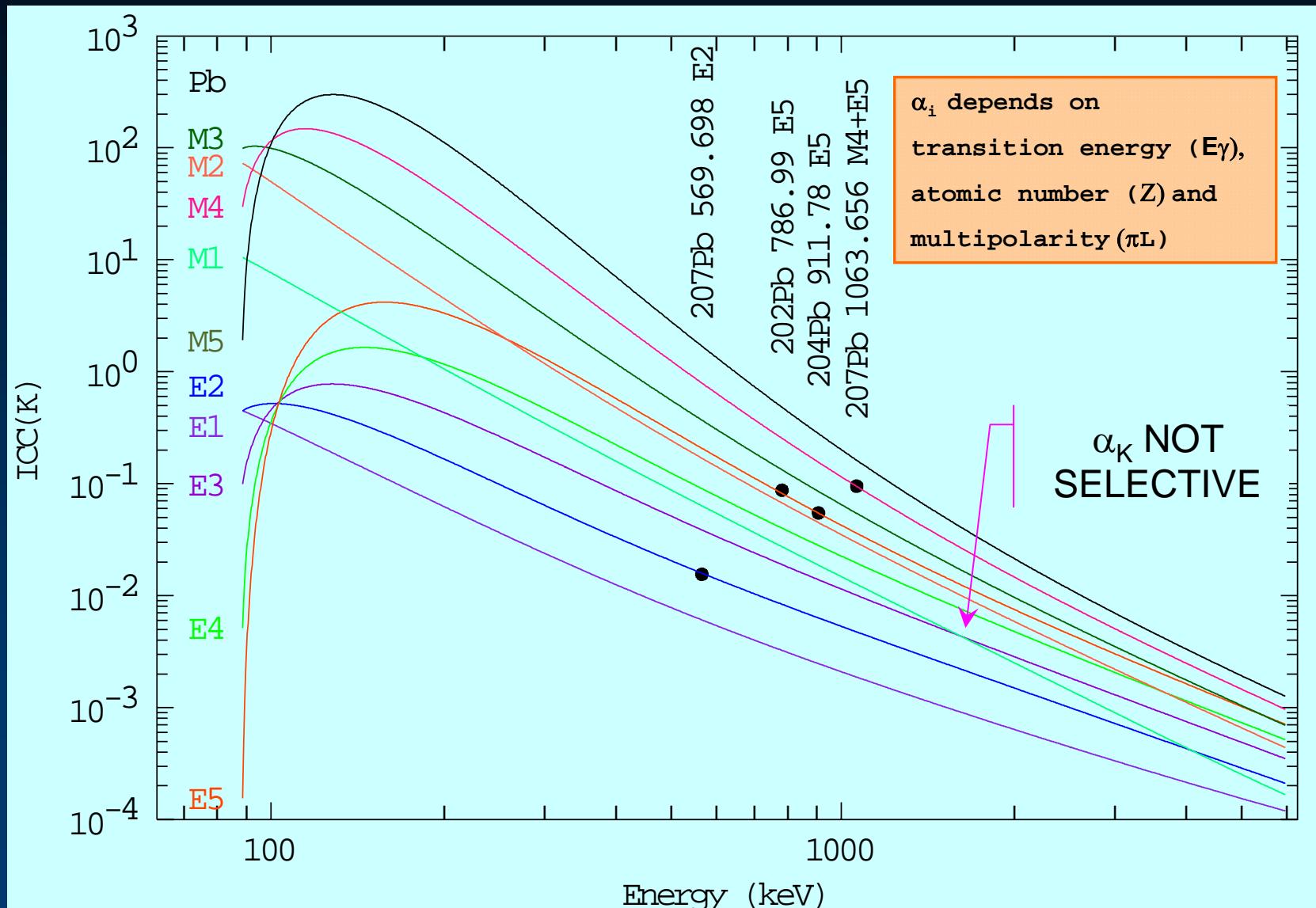
**Electron**

**Multipolar source**

*Same for  $\gamma$  and CE*

$\varphi_i^e$  bound state electron  
 $\varphi_f^{e*}$  free particle electron

## Sensitivity to transition multipolarity



## Mixed multipolarity and E0 transitions

$$\delta^2 = \frac{I_\gamma(E2)}{I_\gamma(M1)}$$

$$\alpha^{M1/E2} = \frac{\alpha_{M1} + \delta^2 \alpha_{E2}}{1 + \delta^2}$$

In some cases the mixing ratio can be deduced

$$\delta^2 = \frac{\alpha_{M1} - \alpha^{\text{exp}}}{\alpha^{\text{exp}} - \alpha_{E2}}$$

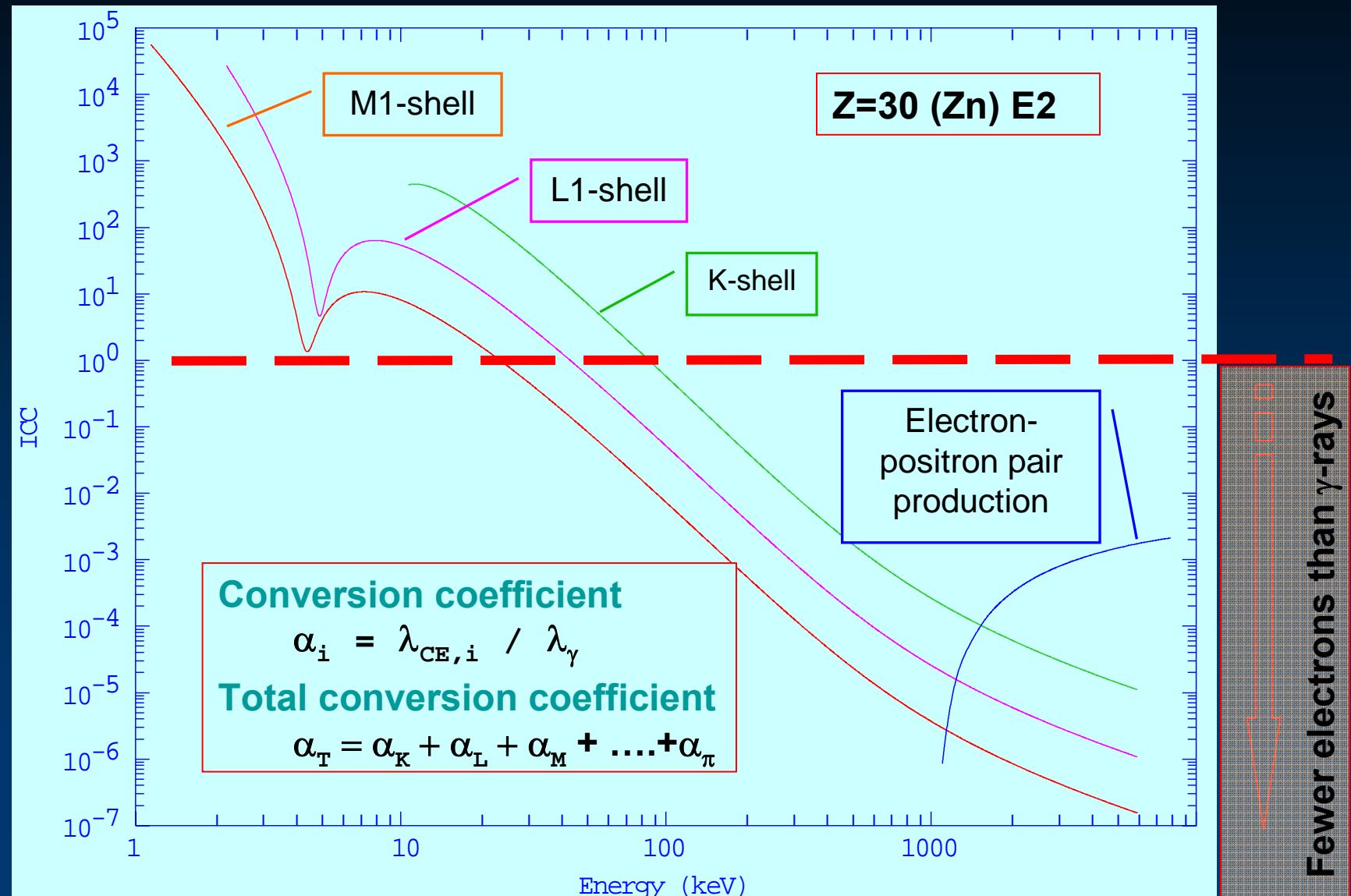
## E0 transitions – pure penetration effect; no $\gamma$ -rays ( $I_g=0$ )

$$\alpha = \frac{I_{CE}}{I_\gamma} = \infty$$

- Pure E0 transition:  $0^+ \rightarrow 0^+$  or  $0^- \rightarrow 0^-$
- $J \rightarrow J$  ( $J \neq 0$ ) transitions can be mixed E0+E2+M1

$$\alpha = \frac{I_{CE}(E0) + I_{CE}(E2) + I_{CE}(M1)}{I_\gamma(E2) + I_\gamma(M1)}$$

## More on conversion coefficients



## Measuring conversion coefficients - methods

- NPG: normalization of relative CE ( $I_{CE,i}$ ) and  $\gamma$  ( $I_\gamma$ ) intensities via intensities of one (or more) transition with known  $\alpha$

$$\alpha_i = \frac{I_{CE,i}}{I_\gamma} \times \left[ \frac{I_\gamma^*}{I_{CE}^*} \times \alpha^* \right]_{KNOWN}$$

- CEL: Coulomb excitation and lifetime measurement

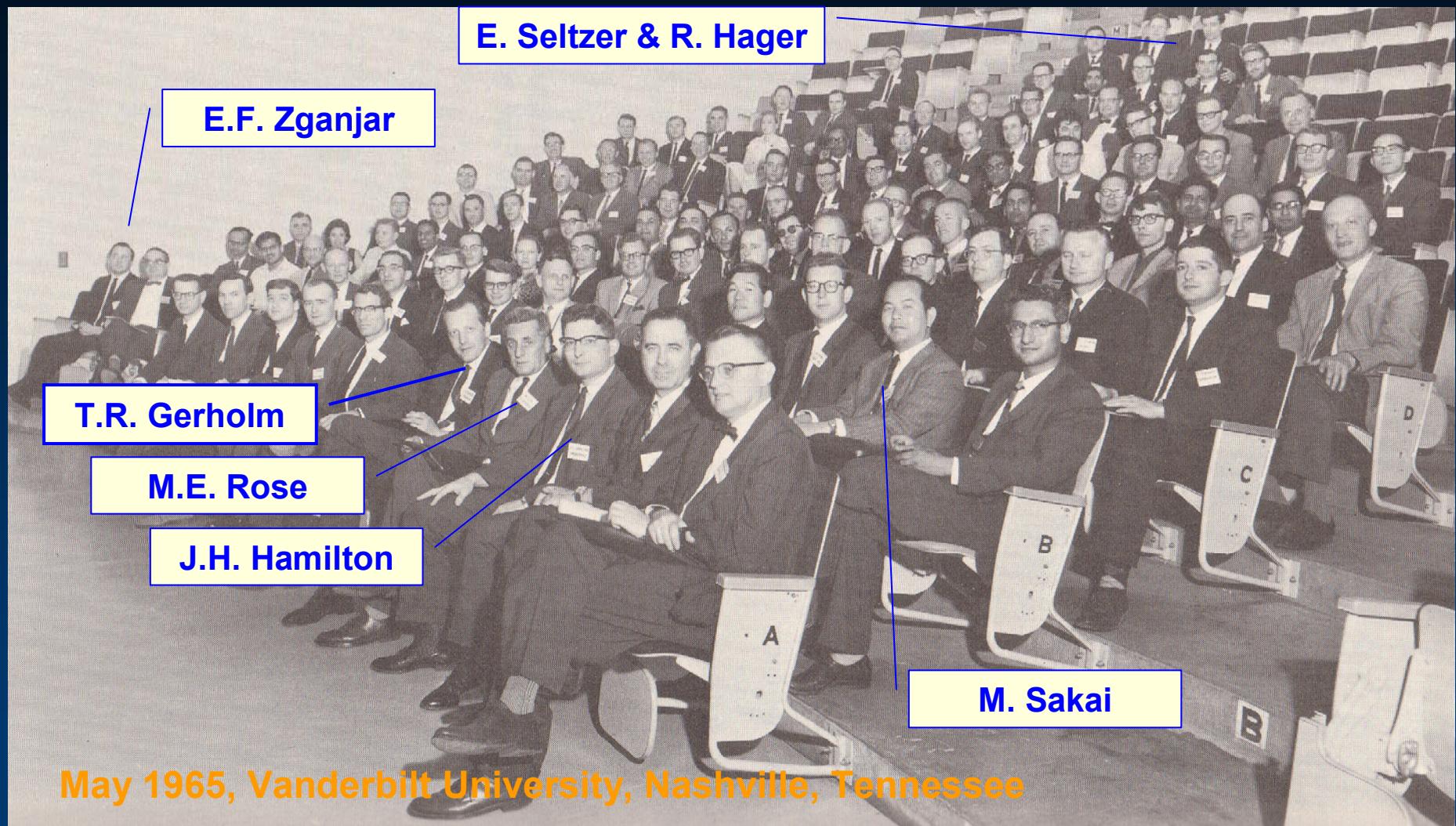
$$\alpha_T = \frac{2.829 \times 10^{11} \times E_\gamma^{-5} (keV)}{B(E2) \uparrow (e^2 b^2) \times T_{1/2} (ns)} - 1$$

- XPG: intensity ratio of K X-rays to  $\gamma$ -rays with K-fluorescent yield,  $\omega_K$

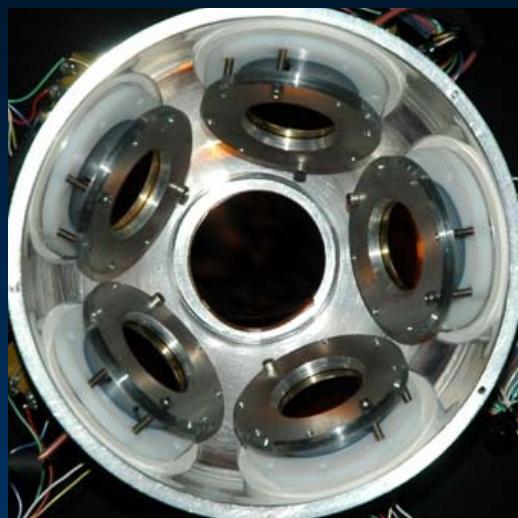
$$\alpha_K = \frac{I_{KX}}{I_\gamma} \times \frac{1}{\omega_K}$$

*And many more, see Hamilton's book*

## Internal Conversion Process – the Pioneers



## Conversion electron spectroscopy with PACES

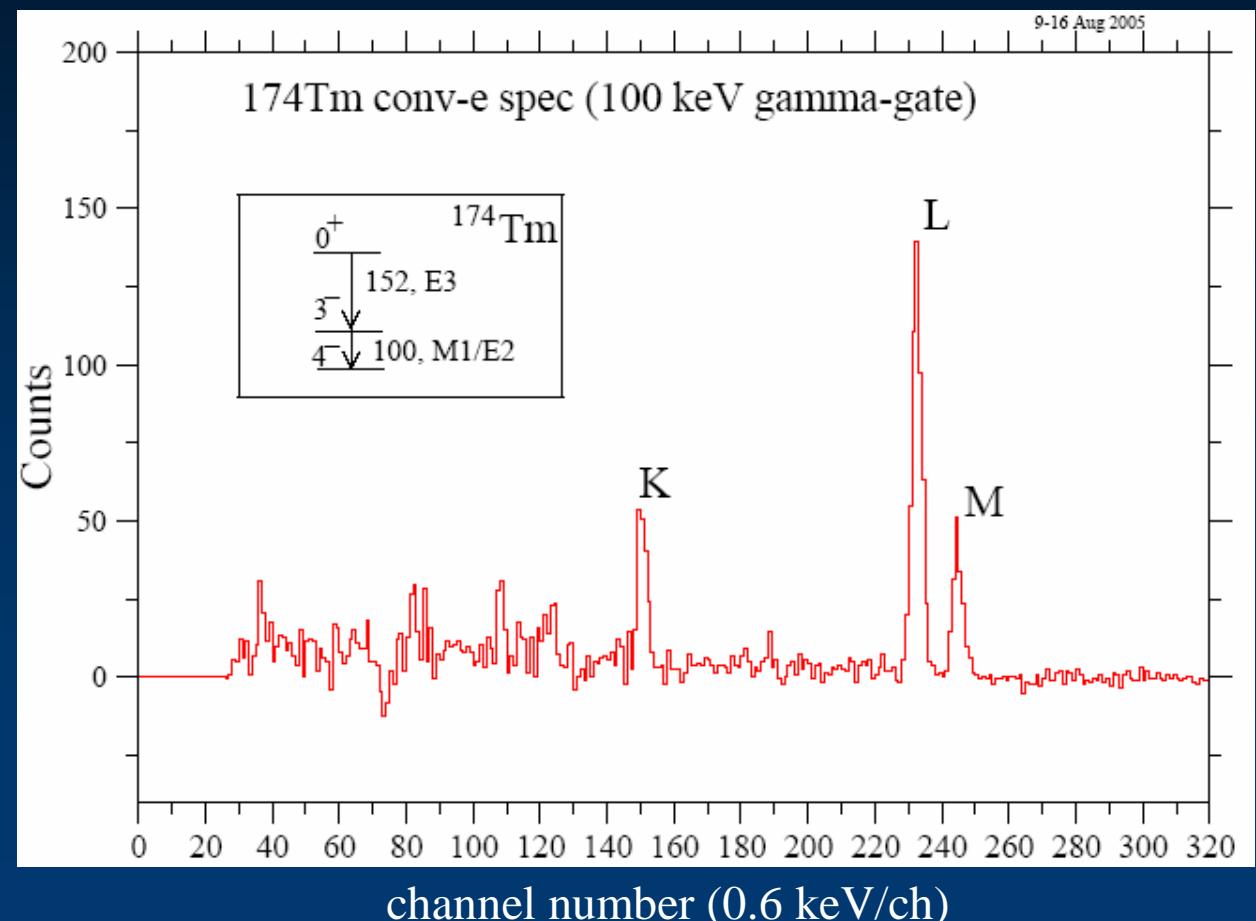


Built by E. Zganjar,  
LSU

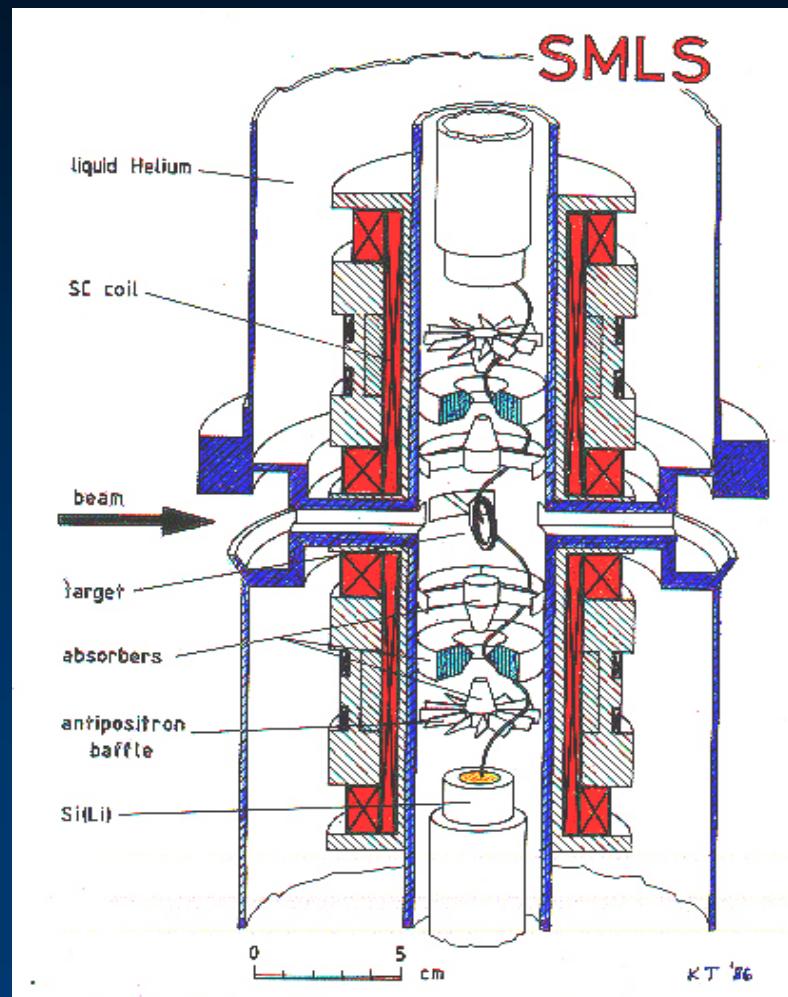
Figure courtesy of  
P.M. Walker (Surrey)  
and P. Garrett (Guelph)

Based on new data, favoured interpretation is that isomer is not a high-K, but  $K^\pi=0^+$

Electrons are vital!



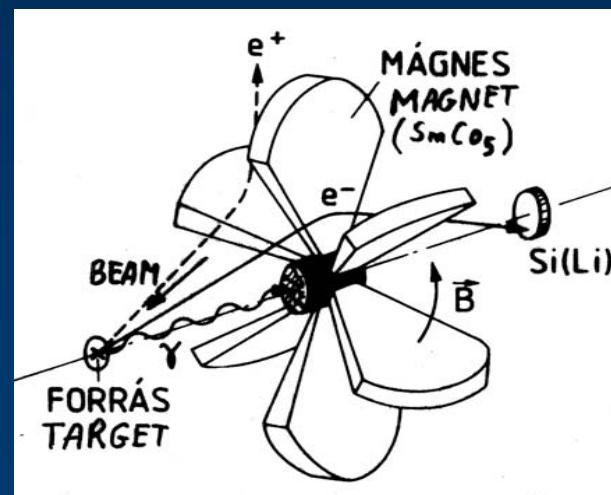
# Magnetic spectrometers



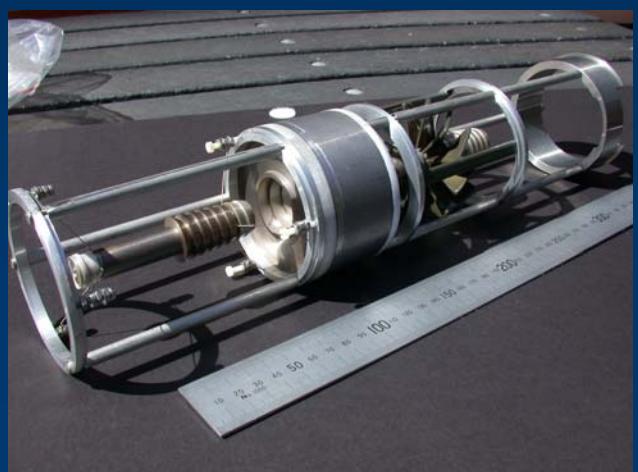
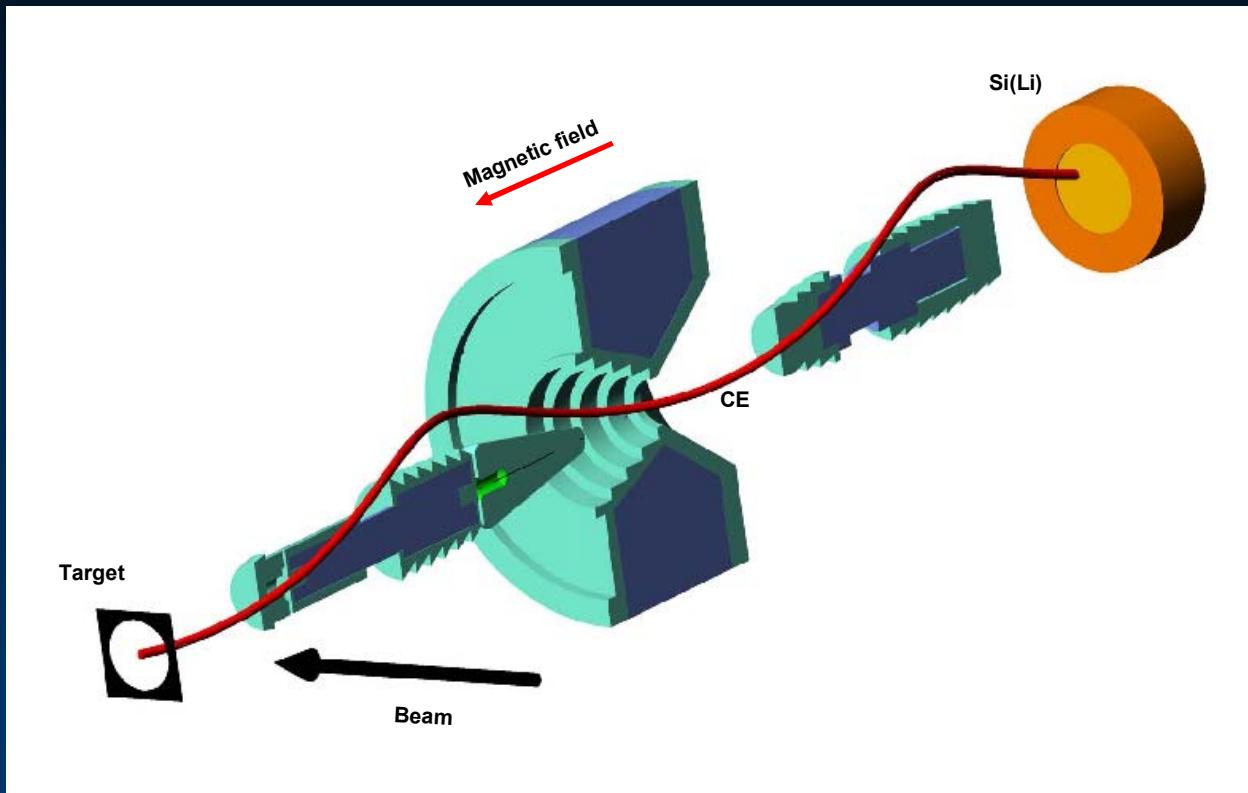
- Superconducting solenoid**
- Broad-range mode – 100 keV up to a few MeV
  - Lens mode – finite transmitted momentum bandwidth ( $\Delta p/p \sim 15\text{-}25\%$ ) – high peak-to-background ratio

- Mini-orange (looks like a peeled orange)**
- transmission > 20%
  - small size and portability, but poorer quality

ATOMKI,  
Debrecen



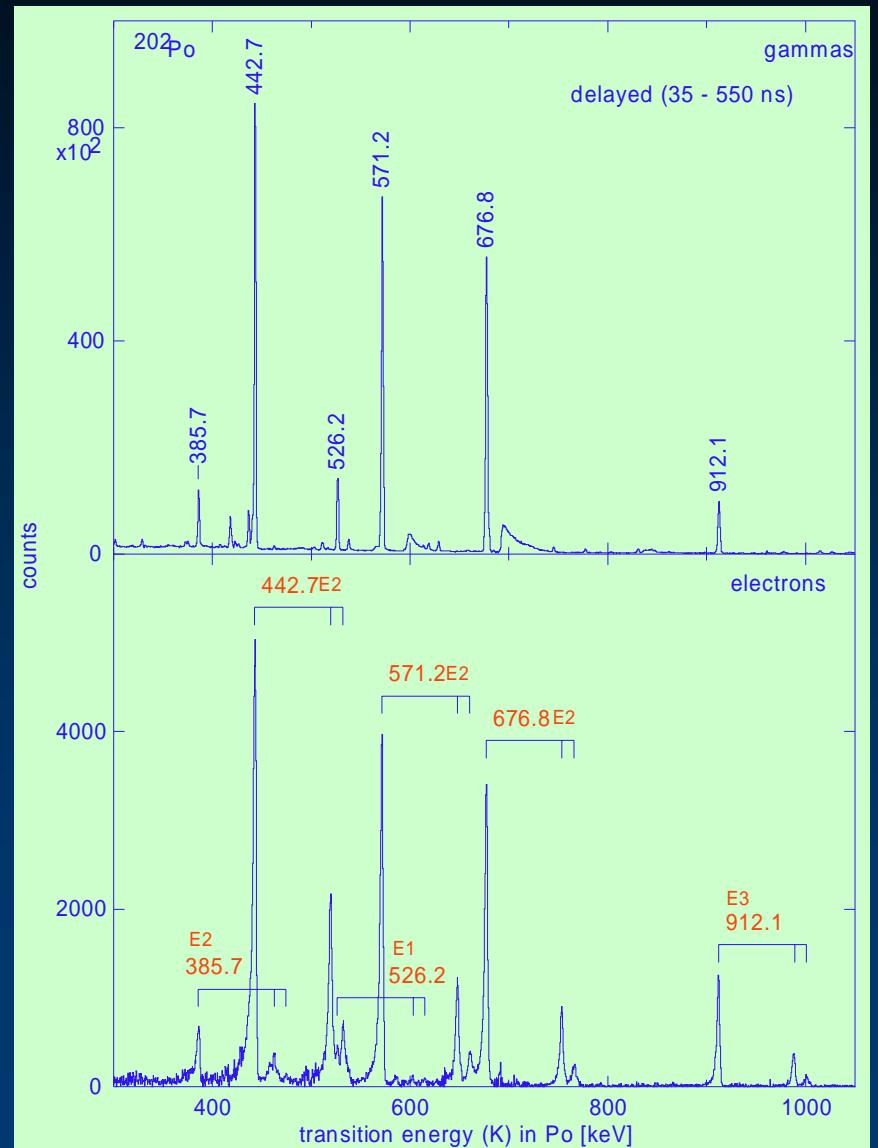
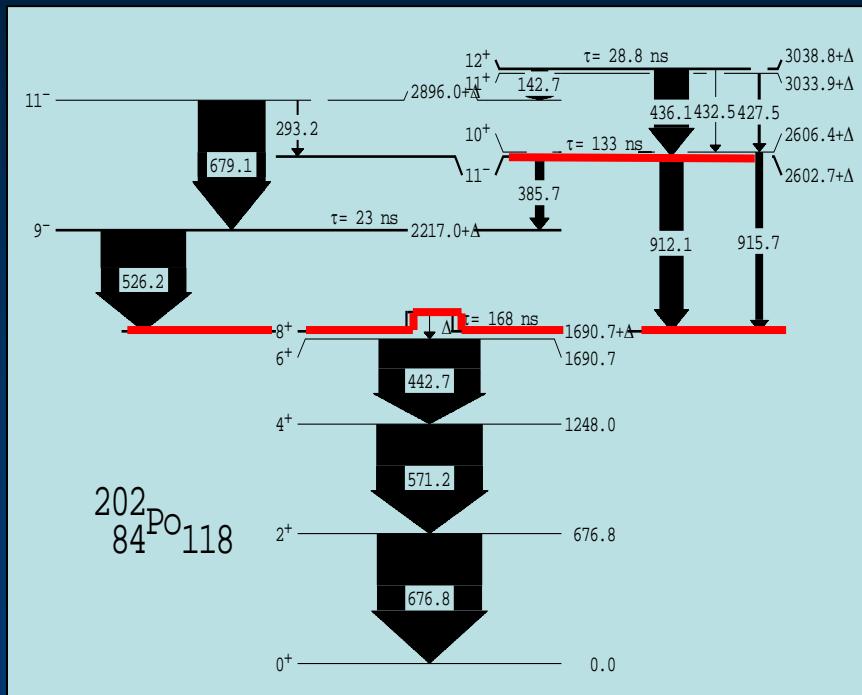
## Super-e Lens (ANU)



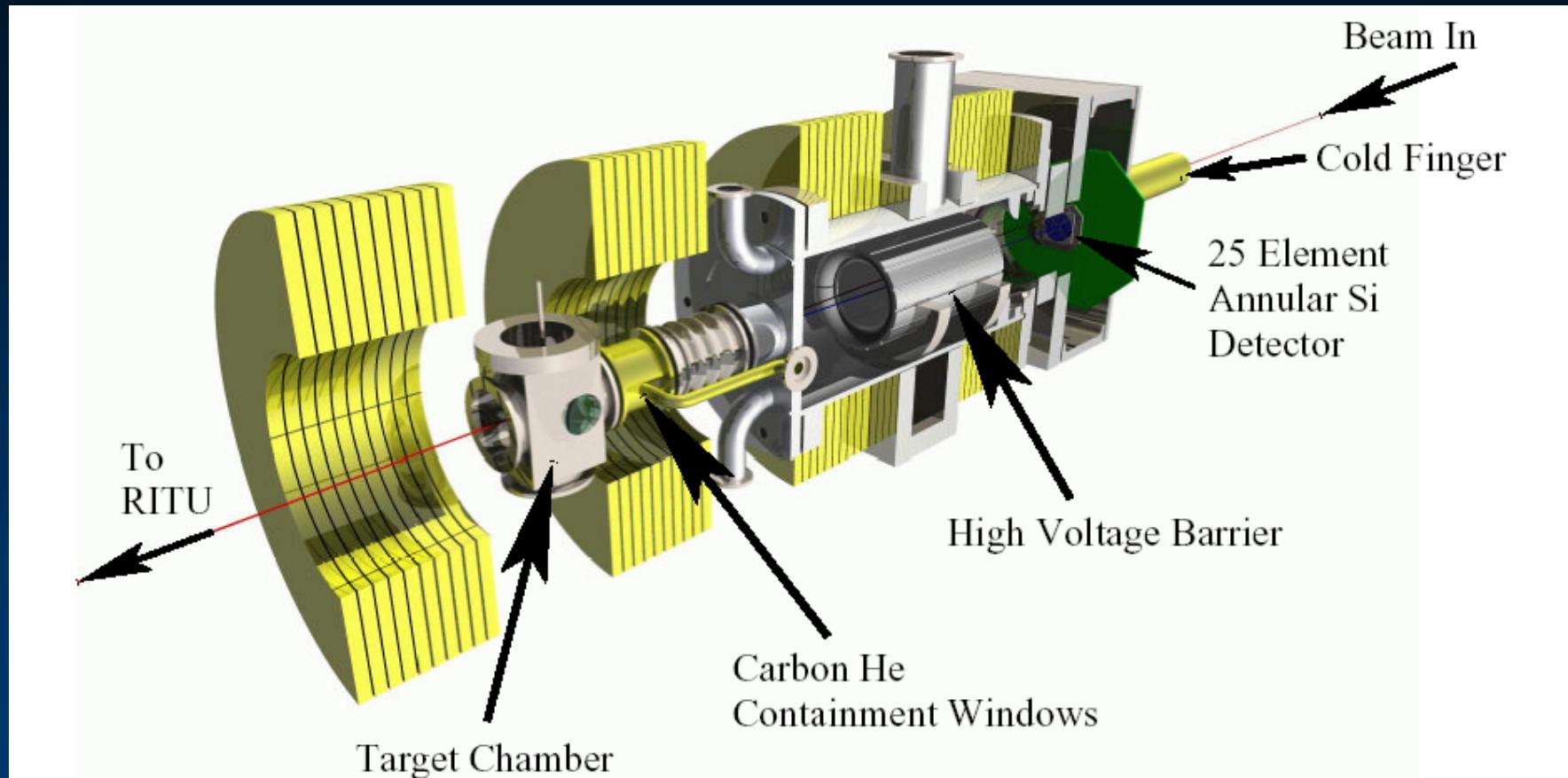
# Super-e Lens (ANU)

$^{194}\text{Pt}(^{12}\text{C},4\text{n})^{202}\text{Po}$  @ 76 MeV

Pulsed beams ( $\sim 1$  ns) with  $1.7 \mu\text{s}$  separation

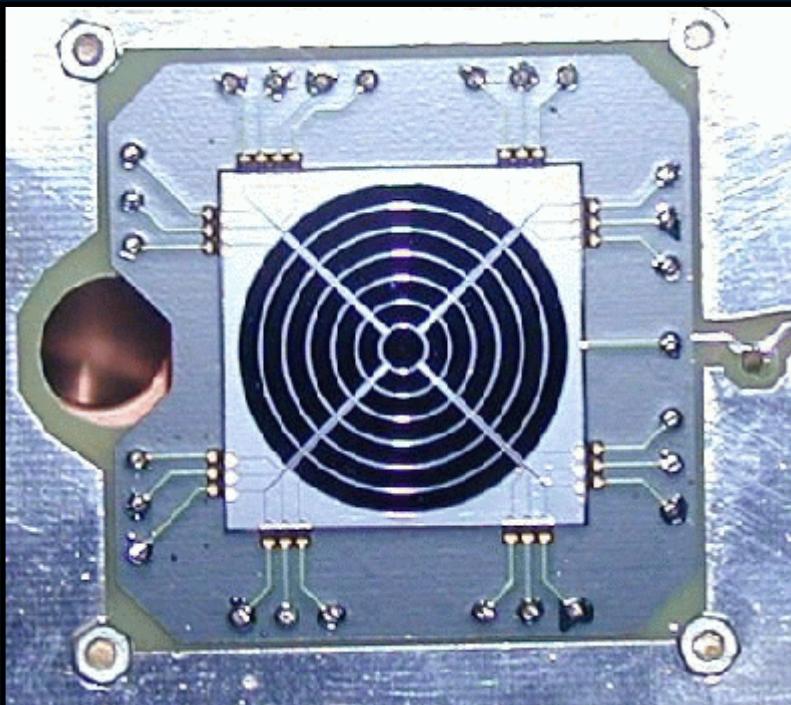


## The SACRED Electron Spectrometer (Liverpool-JYFL)



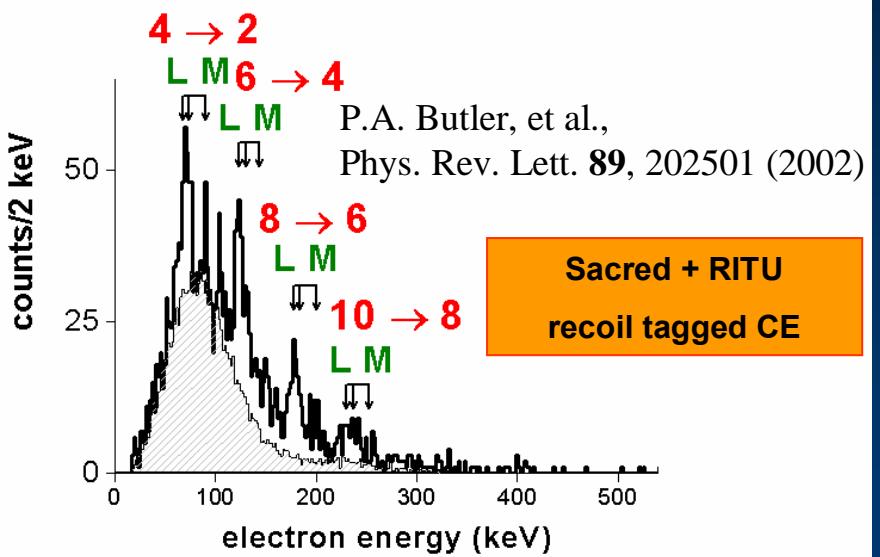
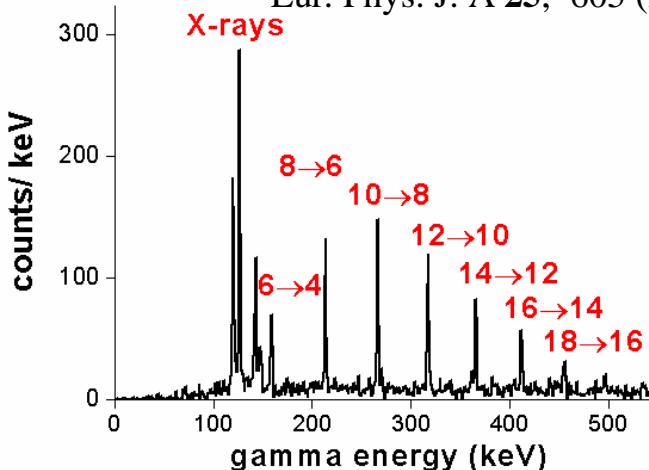
*H. Kankaanpää et al., NIM A534 (2004) 503  
see also P.A. Butler et al., NIM A381 (1996) 433*

## The SACRED array example

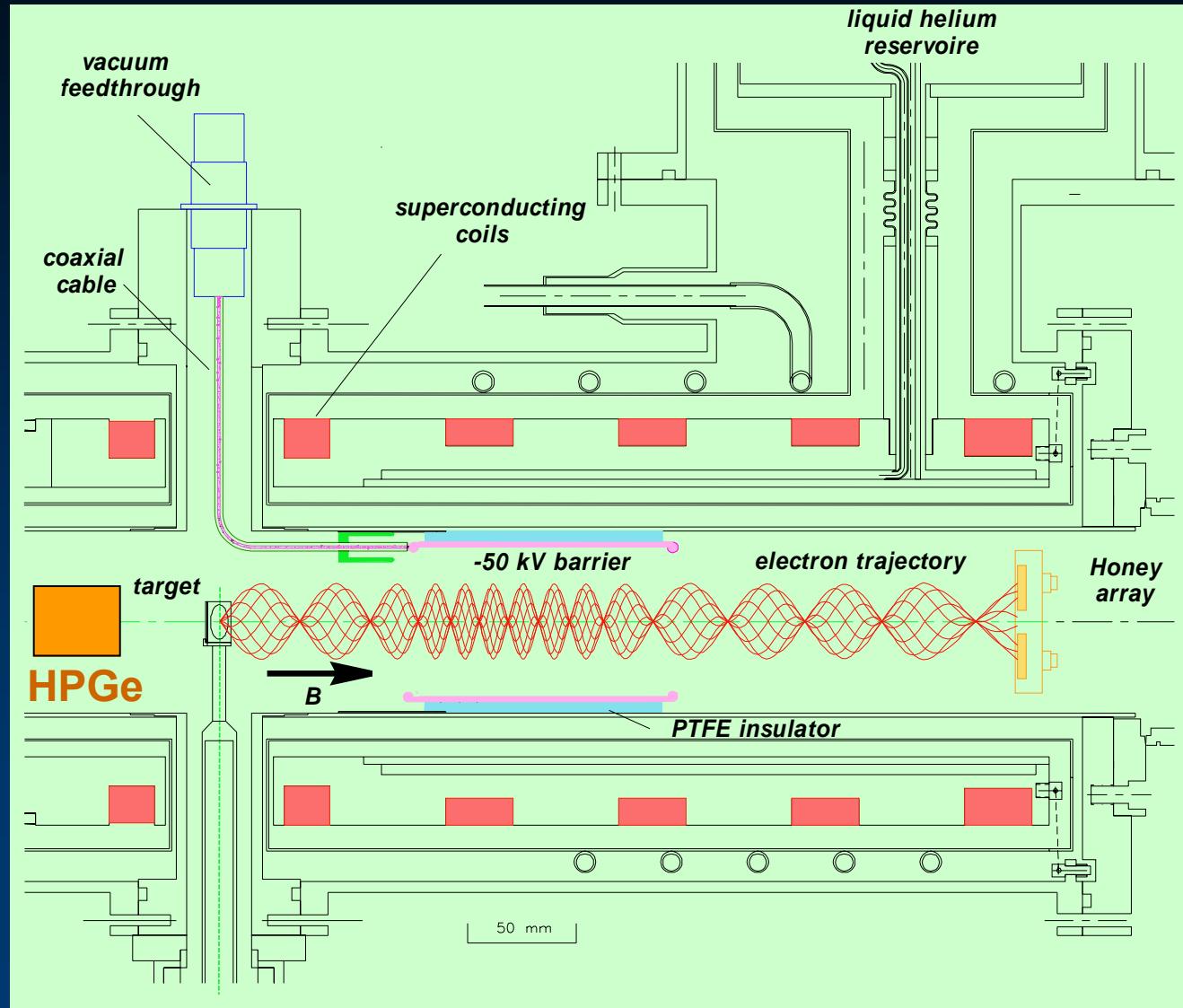


$^{208}\text{Pb}(^{48}\text{Ca},2\text{n})^{254}\text{No}$

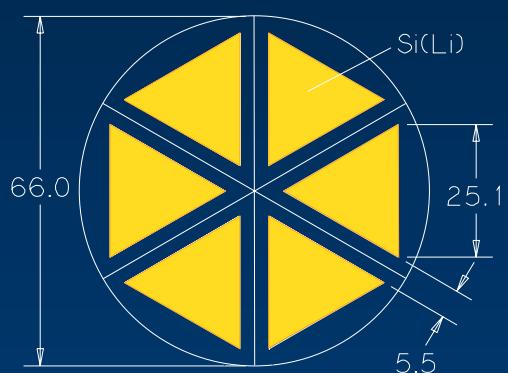
S. Eeckhaudt, et al.,  
 Eur. Phys. J. A **25**, 605 (2005)



# Super-e & Honey (ANU)



Electrons from atomic collisions are the major difficulty in low energy CE spectroscopy using ion induced reactions

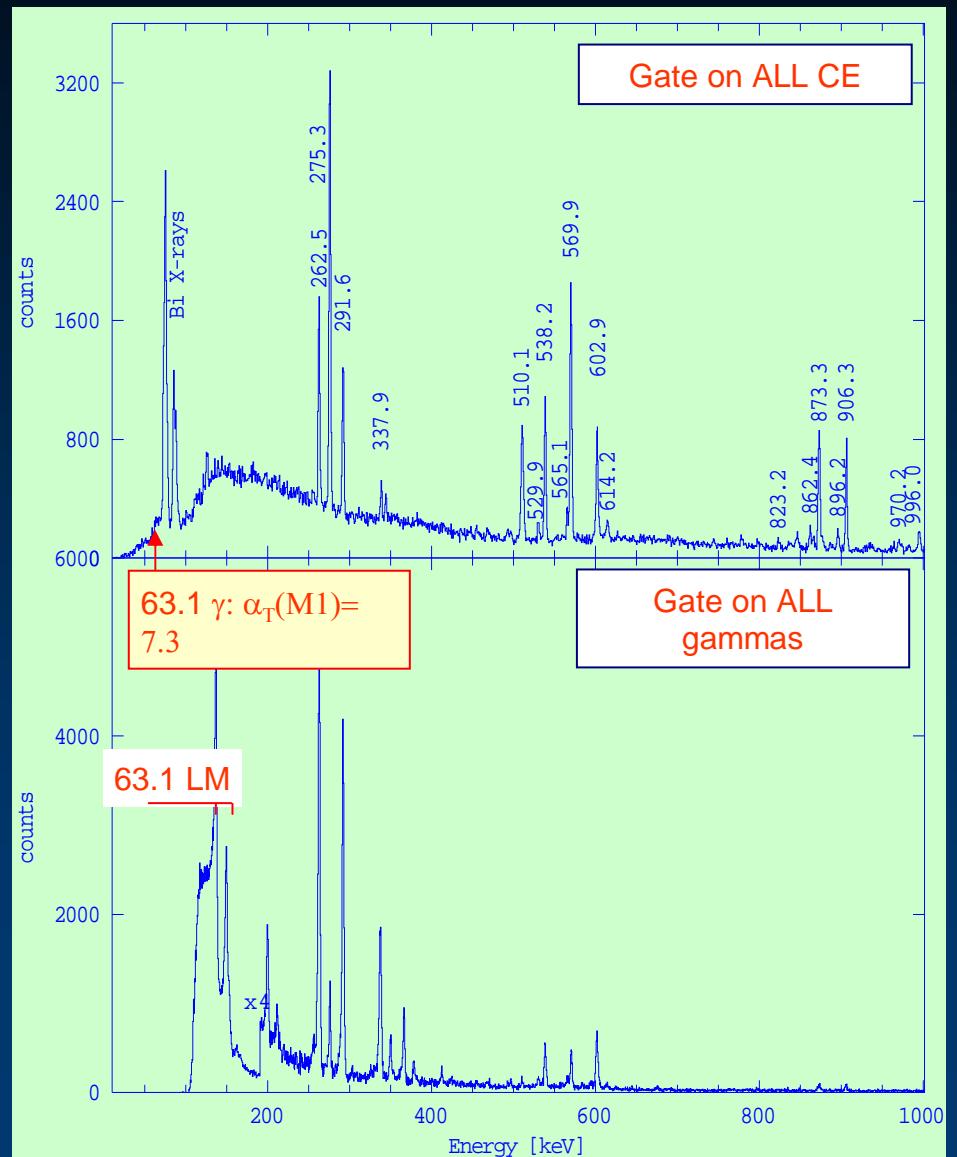


# Super-e Honey (ANU) ee $\gamma$ coincidences

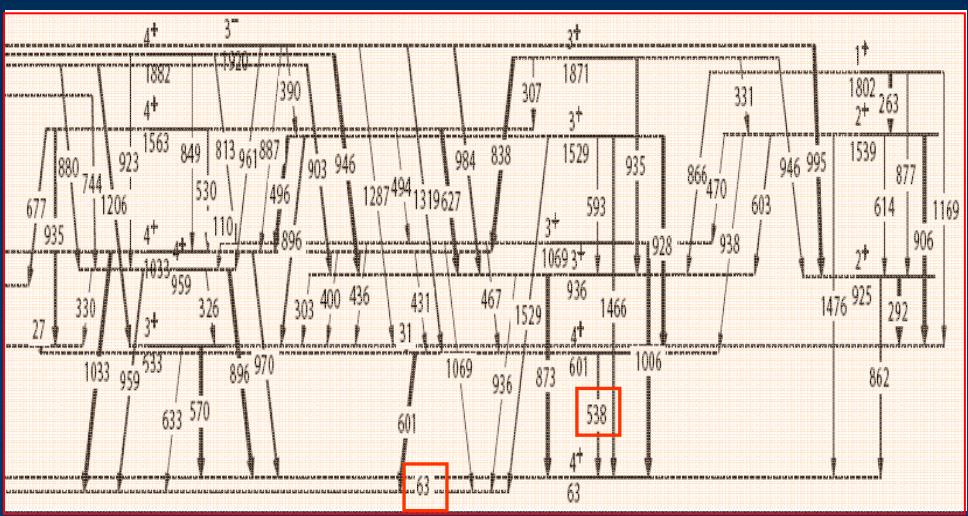
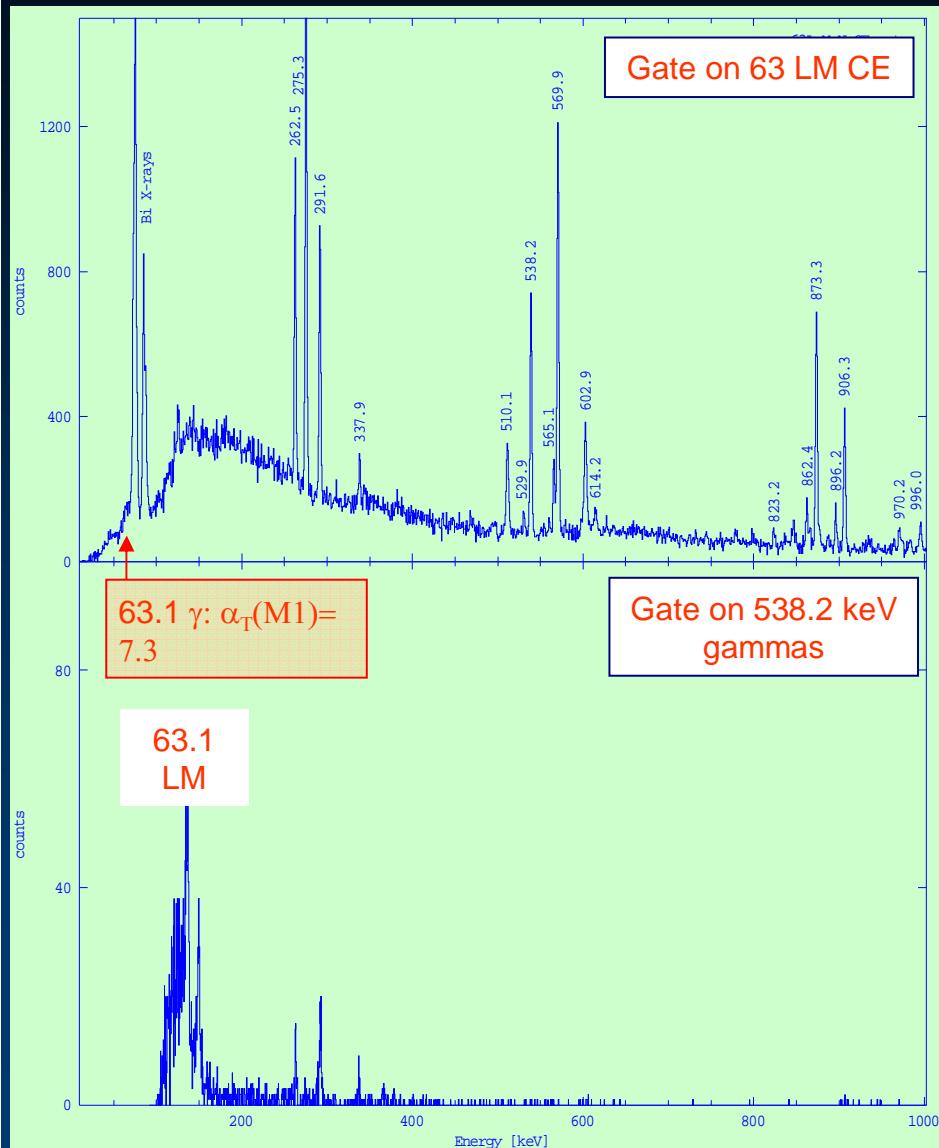
$E_{\gamma}=63.1$  keV transition not  
visible in the gamma spectrum  
 $\alpha_T(M1) = 7.3!$

K-shell conversion not allowed  
 $BE_K=90.5$  keV

$^{208}Pb(p,n)^{208}Bi$  @ 9 MeV  
K.H. Maier et al,  
Phys. Rev. C 76, 064304 (2007)



# Super-e Honey (ANU) eey coincidences

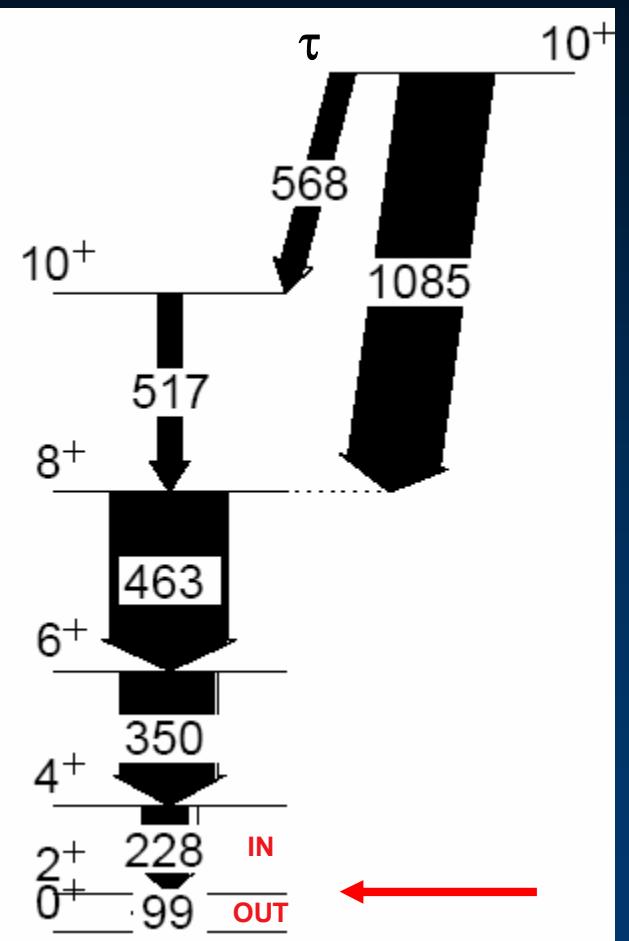
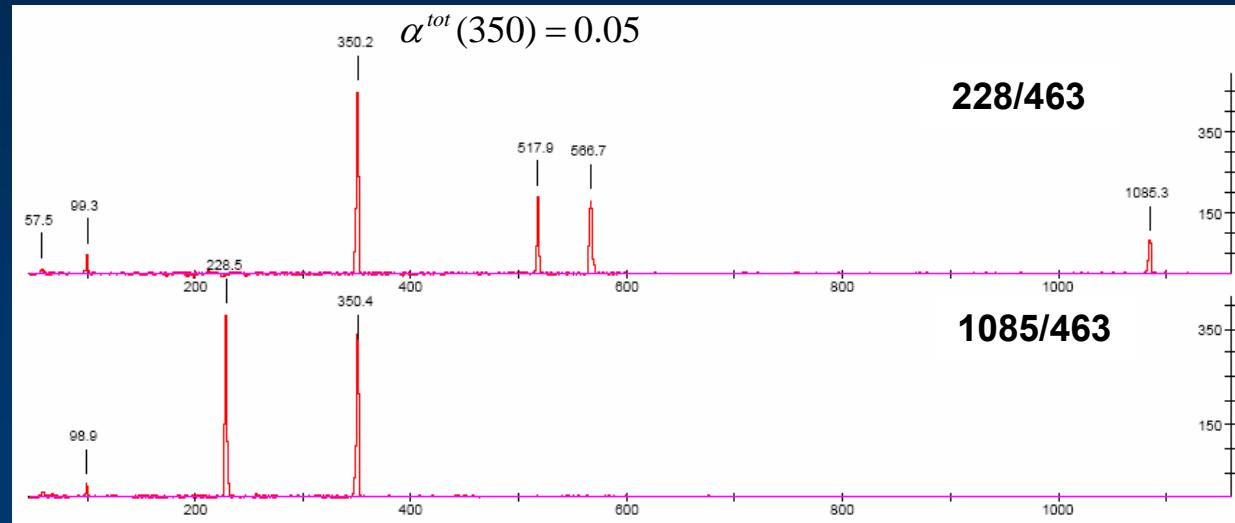


## ICC from total intensity balances –example 1

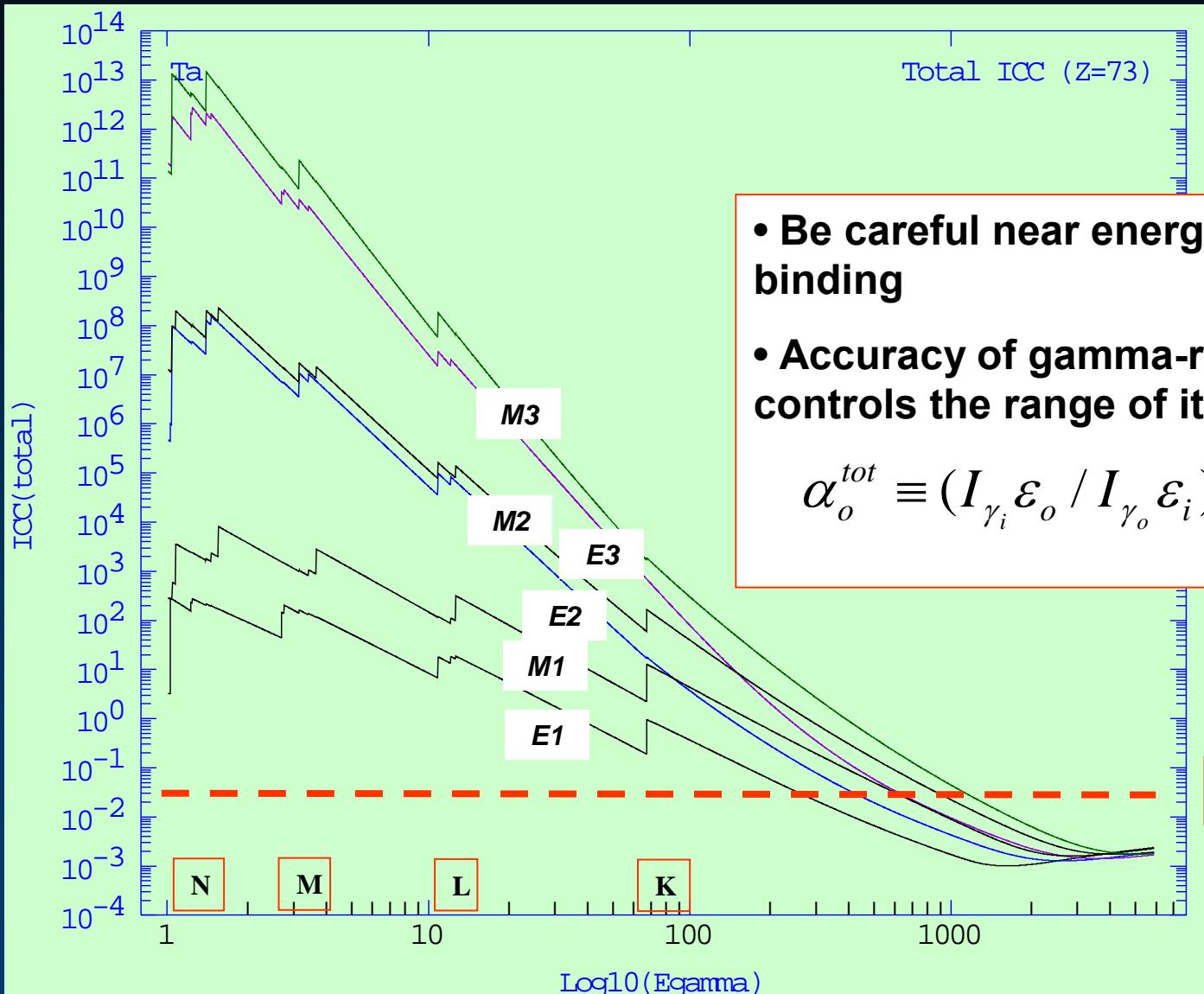
Out-of-beam (or decay) coincidence data

$$I_{\gamma_i}^{tot} = I_{\gamma_i} \times (1 + \alpha_i^{tot}) \equiv I_{\gamma_o}^{tot} = I_{\gamma_o} \times (1 + \alpha_o^{tot})$$

$$\alpha_o^{tot} \equiv (I_{\gamma_i} \varepsilon_o / I_{\gamma_o} \varepsilon_i) \times (1 + \alpha_i^{tot}) - 1$$



## ICC from total intensity balances – when to use



- Be careful near energies close to shell binding
- Accuracy of gamma-ray measurements controls the range of its use

$$\alpha_o^{tot} \equiv (I_{\gamma_i} \varepsilon_o / I_{\gamma_o} \varepsilon_i) \times (1 + \alpha_i^{tot}) - 1$$

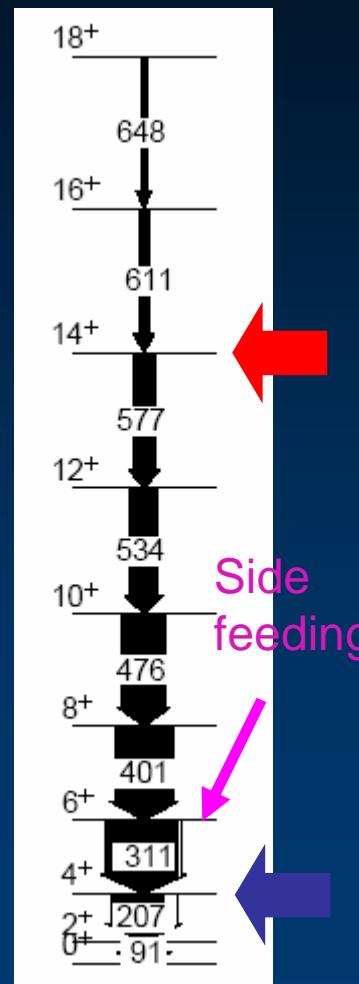
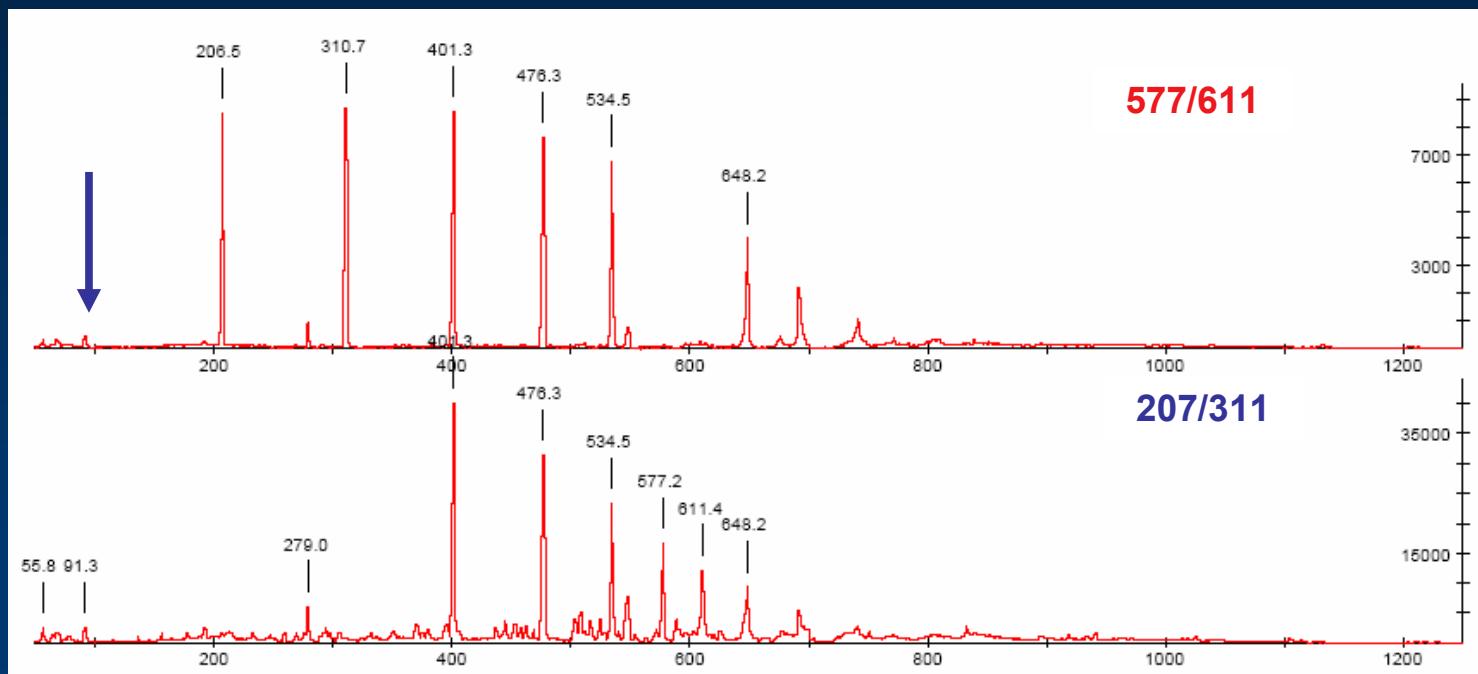
$\delta I_{\gamma} / I_{\gamma} \sim 10\%$

## ICC from total intensity balances –example 2

In-beam: only when gating from “above”

$$I_{\gamma_i}^{tot} = I_{\gamma_i} \times (1 + \alpha_i^{tot}) \equiv I_{\gamma_o}^{tot} = I_{\gamma_o} \times (1 + \alpha_o^{tot})$$

$$I_{\gamma_i}^{tot} = I_{\gamma_o}^{tot} + I_{sf}$$



## Acknowledgements

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**G.D. Dracoulis , G.J. Lane, P. Nieminen, H. Maier (ANU)**  
**F.G. Kondev (ANL)**  
**P.E. Garrett (*University of Guelph and TRIUMF*)**  
**S.W. Yates (*Univ. of Kentucky*)**