Adiabatic quenches in open quantum critical systems

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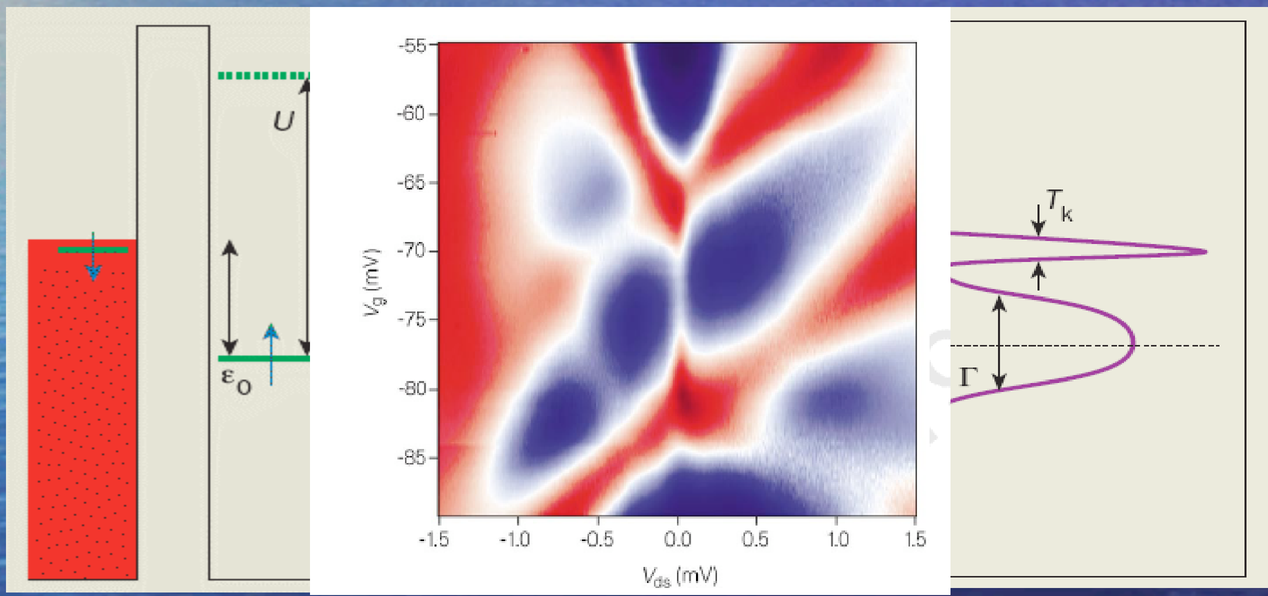
arXiv:0805.0586

Non equilibrium physics in many body systems

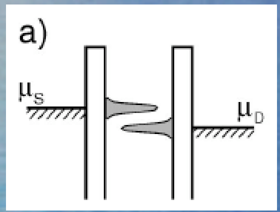
Prototype example: Kondo effect in Quantum Dots

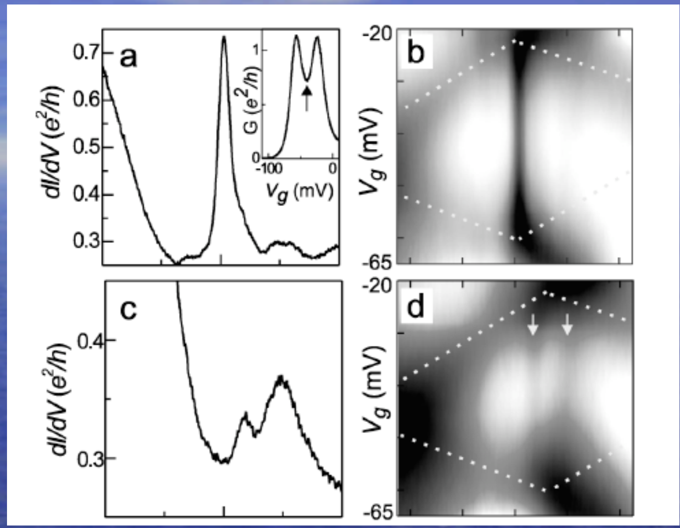


From: L. Kouwenhoven and L. Glazman, Phys. World 14(1), 33 (2001)
D. Goldhaber-Gordon, et al., Nature 391, 156 (1998)

Non equilibrium physics in many body systems

Nonequilibrium splitting of the Kondo resonance

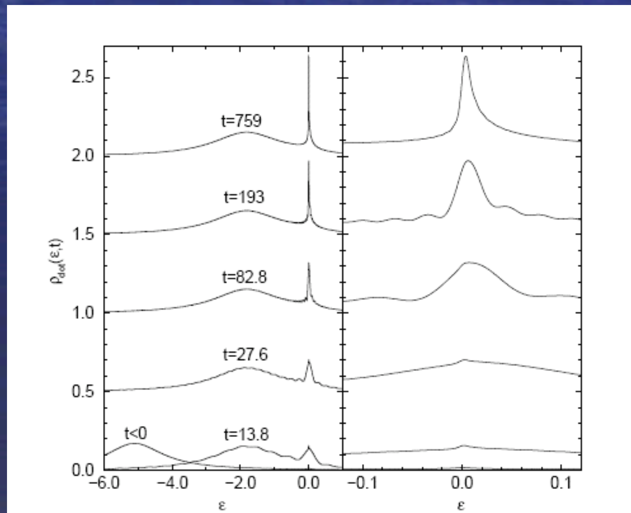




From: De Franceschi, et al, PRL 89, 156801 (2002)

Abrupt quench inside the Kondo valley

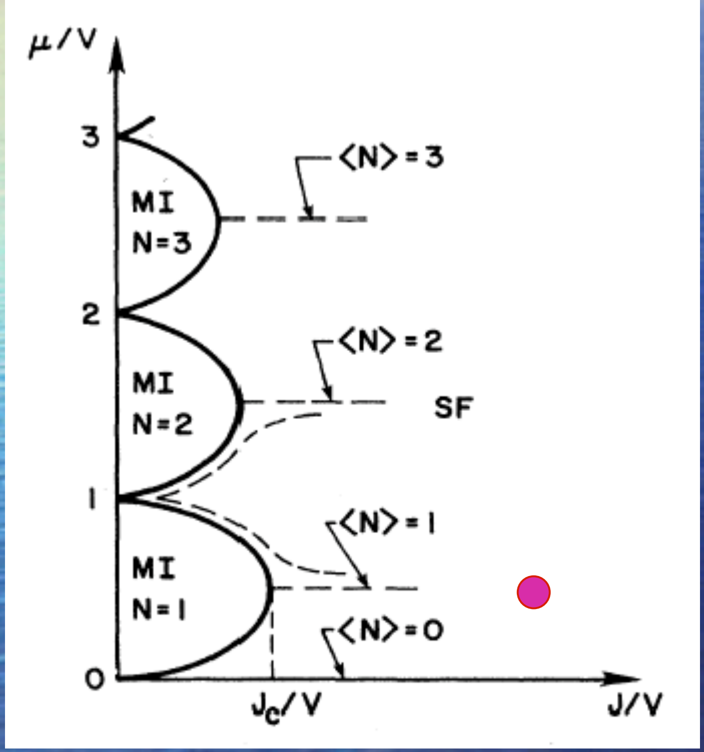
$$\tau \simeq 1/T_K$$

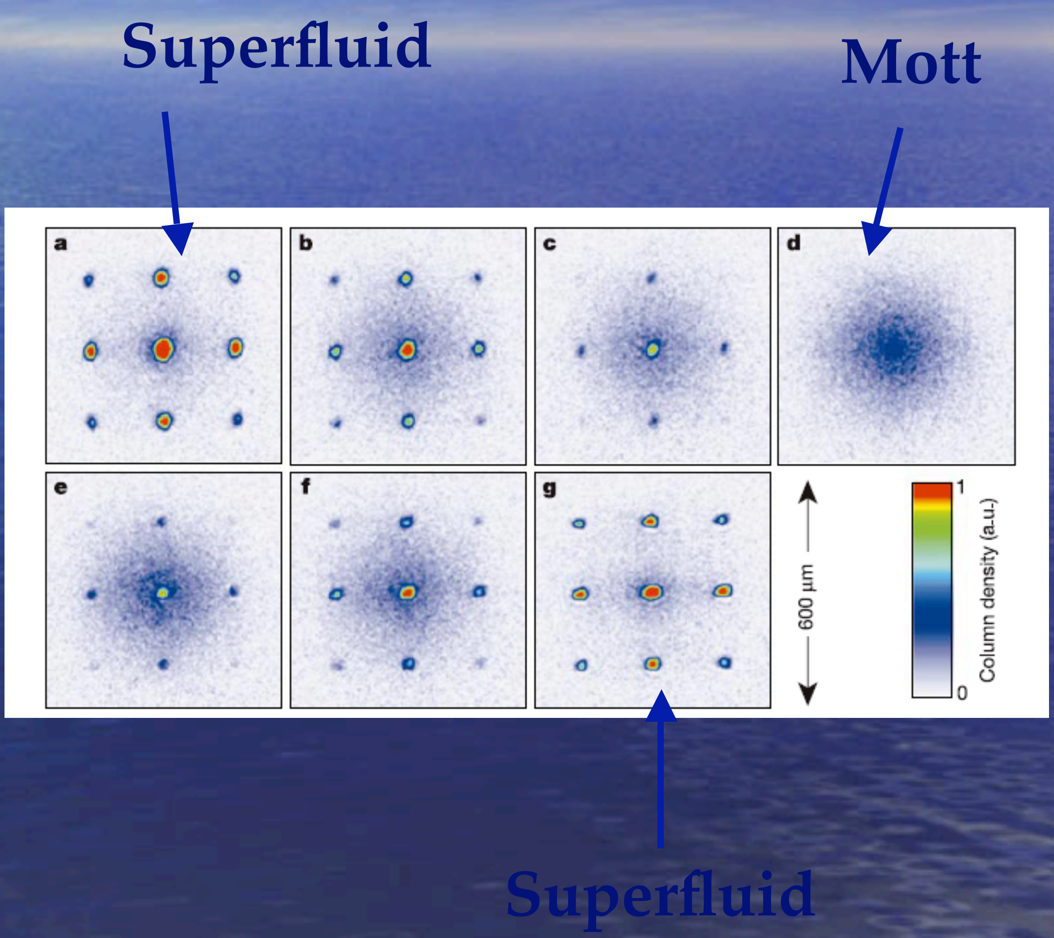


From: Nordlander, et al PRL 83, 808 (1999) .

Non equilibrium physics in many body systems

The nonequilibrium lab: cold atomic gases





From: Fisher et al, Phys Rev B 40, 546 (1989).
See also Jaksch et al, PRL 81, 3108 (1998).

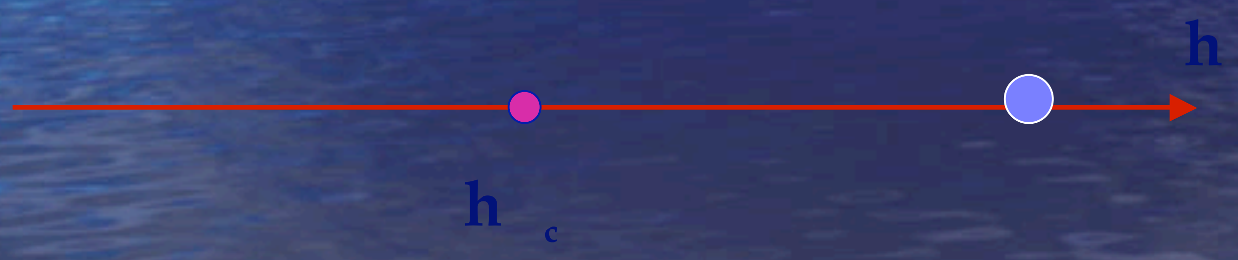
From: Greiner et al, Nature 419, 51 (2002)

Status of the theory

Artificial many body systems
(nanoscience, cold atoms)

nonequilibrium physics

Driven system | **Quantum quenches**



Quantum quenches

Early works by Baruch, McCoy, Dresden, Mazur, Girardeau ('70)

Thermalization vs. Integrability
Nonequilibrium quantum stat. Mech.

Abrupt

Sengupta, Powell, Sachdev ('04)
Calabrese and Cardy ('07)
Rigol et al, ('06)
.
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.
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"Adiabatic"

Zurek, Dorner, Zoller ('05)
Polkovnikov ('05)
Dziarmaga ('05)
Cherng and Levitov ('06)
Gritsev, Polkovnikov ('07)
.
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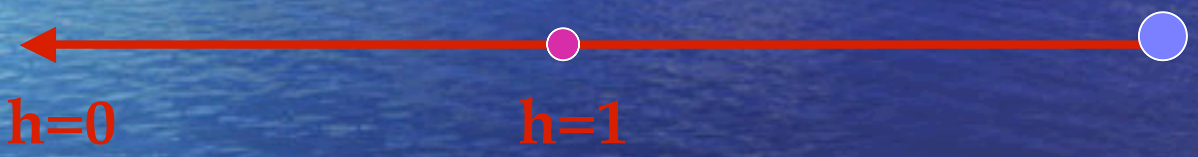
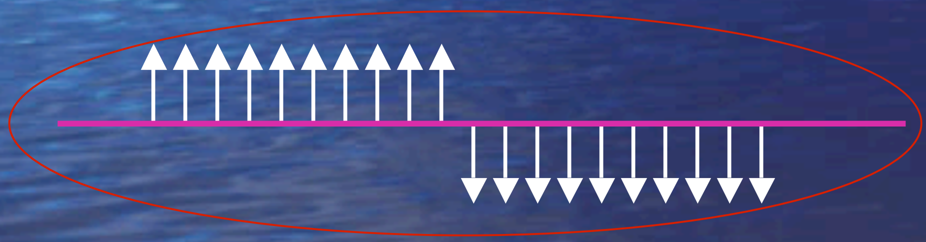
A well posed problem

The quantum Ising model

$$H = -\frac{J}{2}\sum_{j=1}^{N} \sigma_j^x \sigma_{j+1}^x + h\sigma^z$$





How many defects
are generated ??

Adiabatic quantum quenches

Kibble Zurek mechanism

$$h - h_c \ = \ v\, t$$

$$\tau(t) = \frac{1}{\Delta} \approx \frac{1}{|\, h - h_c \,|} = \frac{1}{v\, |\, t\, |}$$

Freezing of dynamics

$$\tau(t_Q) \ = \ t_Q$$

$$t_Q \approx \frac{1}{\sqrt{v}}$$

Adiabatic quantum quenches

Density of defects

$$n_{def} \approx \xi^{-1} \approx v t_Q = \sqrt{v}$$

In general

$$n_{def} \simeq \xi^{-d} \simeq v^{\frac{\nu d}{n u z+1}}$$

Zurek, Dorner, Zoller, Phys. Rev. Lett. 95, 105701 (2005)

Polkovnikov, Phys. Rev. B 72, 161201(R) (2005).

Questions and outline

1)- Defects across a QCP : coherent and universal

Dephasing and dissipation ???

Does universality survive ?

Yes !!

2)- Scaling laws in quantum ising model + bath

3)- Scaling laws for a generic QPT

Defects across a QCP : coherent and universal

Ising model and Landau Zener dynamics

$$H = -\frac{J}{2}\sum_{j=1}^{N} \sigma_j^x \sigma_{j+1}^x + h\sigma^z$$

$$b_i = \sigma_i^x + i\sigma_i^y$$

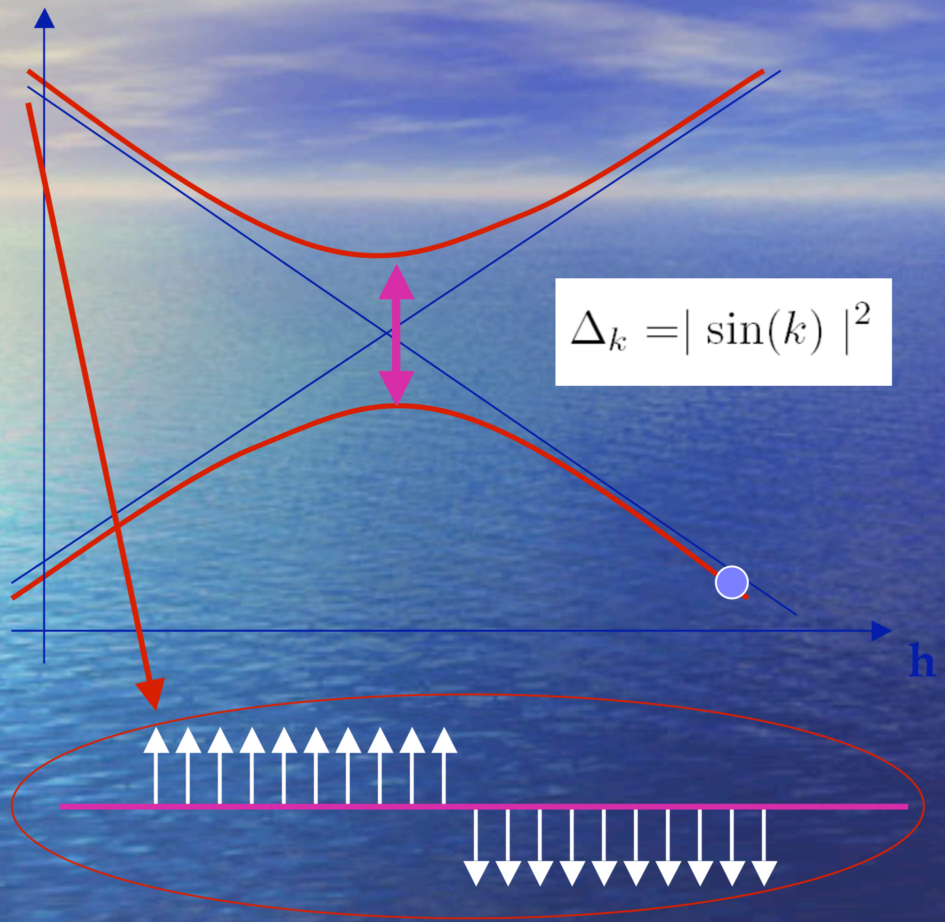
$$\hat{\Psi}_k = \begin{pmatrix} c_k \\ c_{-k}^{\dagger} \end{pmatrix}$$

$$\mathbf{H}_k = J\begin{pmatrix} h(t) - \cos(k) & -i\sin(k) \\ i\sin(k) & -(h(t) - \cos(k)) \end{pmatrix}$$

Jordan-Wigner

$$c_i = e^{i\left[\pi \sum_{k<j} b_k^{\dagger} b_k\right]} b_j$$

Coherent dynamics: Landau Zener problem



$$\Delta_k = |\sin(k)|^2$$

$$P_k = e^{-2\pi \frac{\Delta_k^2}{v}}$$

Cherng and Levitov, PRA 73, 043614 (2006)
Dziarmaga, PRL 95, 245701 (2005)

Density of defects

$$n_{def} = \int dk\ P_k \simeq \int dk\ e^{-2\pi k^2/v} \simeq \sqrt{v}$$

The bath: introducing incoherent effects

Adding the bath

$$H = -\frac{J}{2}\sum_{j}^{N}[\sigma_j^x\sigma_{j+1}^x + (h(t)+X_j)\sigma_j^z] + H_B$$

$$X_j = \sum_\beta \lambda_\beta (b^\dagger_{\beta,j} + b_{\beta,j})$$

$$H_B = \sum_{j,\beta}\omega_\beta b^\dagger_{\beta j} b_{\beta j}$$

$$H = \sum_{k>0}\Psi_k^\dagger \hat{\mathcal{H}}_k \Psi_k + \frac{1}{\sqrt{N}}\sum_{k,q}\Psi_k^\dagger \hat{\tau}^z \Psi_{k+q} X_q + H_B$$

Mixing of all modes

Ohmic $\sum_\beta \lambda_\beta^2 \delta(\omega - \omega_\beta) = 2\alpha\omega \exp(-\omega/\omega_c)$

Relaxation rate to bosons (escape, interaction): universality class does not change

Adding the bath

Dephasing and dissipation: like a qubit

Master equation: weak coupling + Markov

Master equation for density matrix

$$\partial_t \rho = -i[\mathbf{H}, \rho] + \sum_{i=x,y,z} [\mathcal{D}_\mathbf{i} \rho - \mathrm{h.c.}, \sigma_\mathbf{i}].$$

Coherent evolution

Bath

Bath correlators

Kinetic equations

Analogue of density matrix (for 2LS)
or distribution function (for fermions)

$$-i[G_k^<(t,t)]_{i,j} \equiv \langle \Psi^\dagger_{k,j}(t)\Psi_{k,i}(t)\rangle$$

$$\partial_t \hat{G}_k^< + i\left[\hat{\mathcal{H}}_k,\, \hat{G}_k^<\right] = \frac{1}{N}\sum_q \hat{\tau}^z(\hat{1} + i\hat{G}_q^<)\hat{D}_{qk}\hat{G}_k^< \;+\; \hat{\tau}^z\hat{G}_q^<\hat{D}^\dagger_{kq}(\hat{1} + i\hat{G}_k^<) + H.c.$$

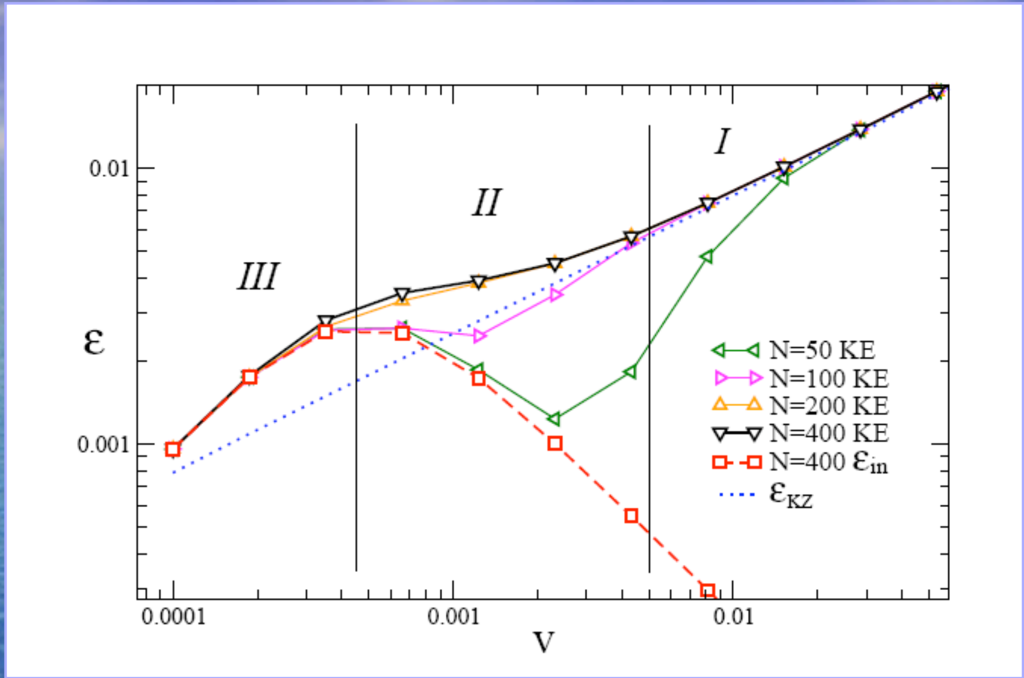
Coherent evolution

Bath

Density of excitations

$$\mathcal{E} = \frac{-i}{2N}\sum_{k>0} \mathrm{Tr}\left((\hat{1} + \hat{\tau}^z)\hat{G}_k^<\right)$$

Some numerics



T=0.1
α=0.001
h_f =0

$$\mathcal{E} \simeq \mathcal{E}_{KZ} + \mathcal{E}_{in}$$

$$\mathcal{E}_{KZ} \simeq \sqrt{v}$$

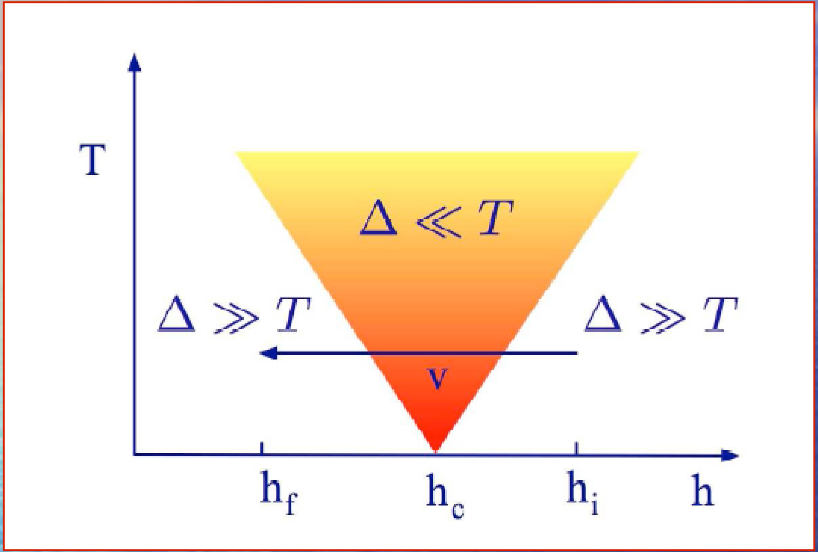
$$\mathcal{E}_{in} = ????$$

Temperature ???

Velocity ???

Universality in the presence of a bath

General understanding



$$h - h_c \; = \; vt$$

$$\Delta \; = \; | \, h - h_c \, |$$

Time to cross the QC region

$$t_{QC} \; \approx \; 2T/v$$

$$P_{fin}(k) \; = \; (1 - e^{-\frac{t_{QC}}{\tau}})P_c(k)$$

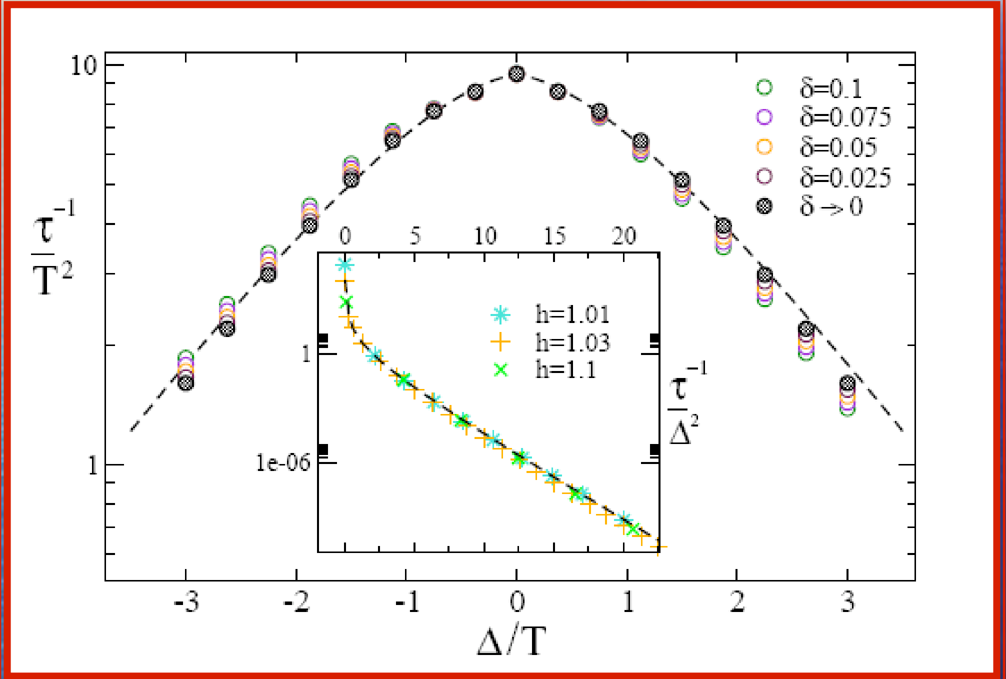
$$\tau^{-1} \approx \alpha T^2$$

$$\mathcal{E}_{in} \approx (1 - e^{-\frac{t_{QC}}{\tau}}) \int dk\, P_c(k) \simeq \quad T\,(1 - e^{-2\frac{T}{v\tau}})$$

Function of k/T

Understanding the behavior of the system

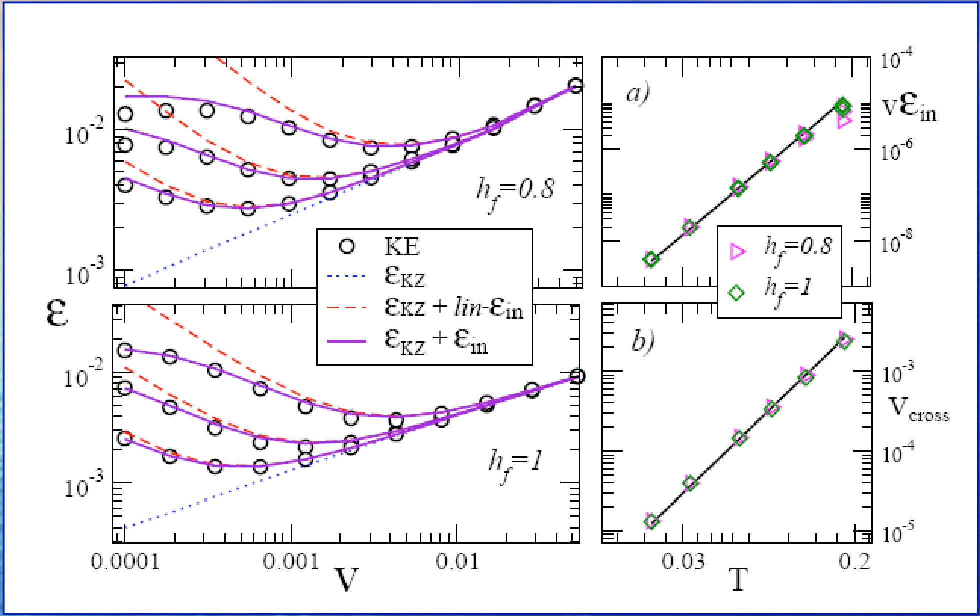
Relaxation times



$$\delta = \sqrt{T^2 + (h - h_c)^2}$$

$$\tau^{-1} \sim \alpha T^2 f(\Delta/T)\ e^{-\Delta/T}$$

Fit of the numerics



$$\mathcal{E}_{in} \simeq \frac{\log 2}{2\pi} T \left(1 - e^{-2T/(\tau v)}\right)$$

$$\mathcal{E}_{in} \propto \alpha v^{-1} T^4$$

$$v_{cross} \propto \alpha^{2/3} T^{8/3}$$

General scaling formulae

$$\mathcal{E}_{in} \propto \alpha v^{-1} T^{\theta+\frac{d\nu+1}{\nu z}}$$

$$v_{cross} \propto \alpha^{\frac{\nu z+1}{\nu(z+d)+1}} T^{\left(1+\frac{(\theta-1)\nu z}{\nu(z+d)+1}\right)\left(1+\frac{1}{\nu z}\right)}$$

Conclusions and Outlook

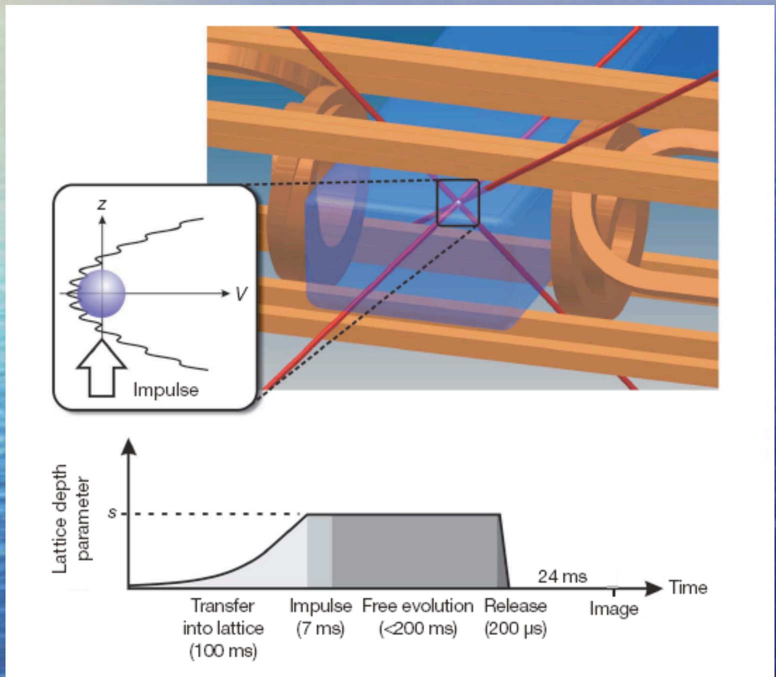
Universal description of incoherent defect production across a QPT.

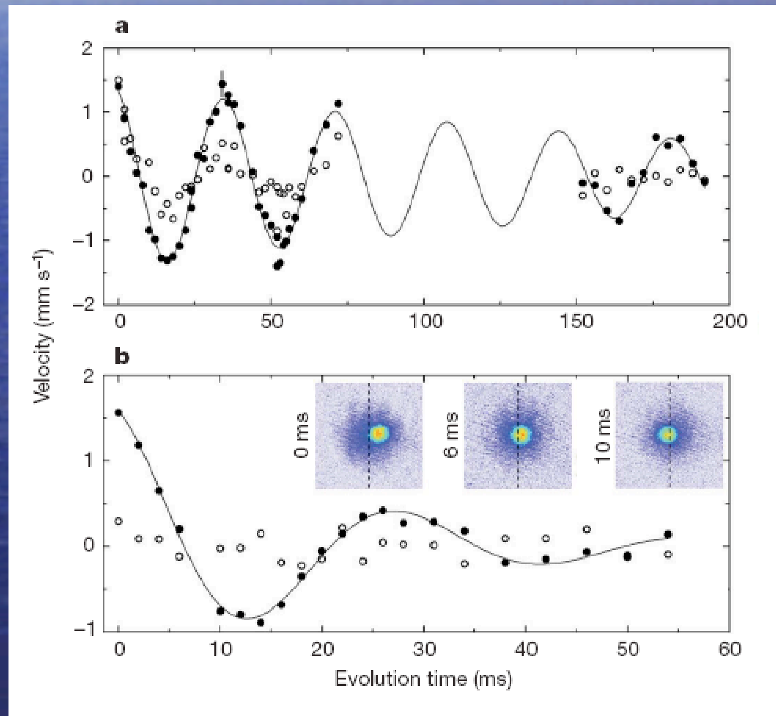
1)- And if the bath does change the universality class ?

2)- What about dephasing ? How does it show up ?

Non equilibrium physics in many body systems

From: MacKay et al., Nature 453, 76 (2008)

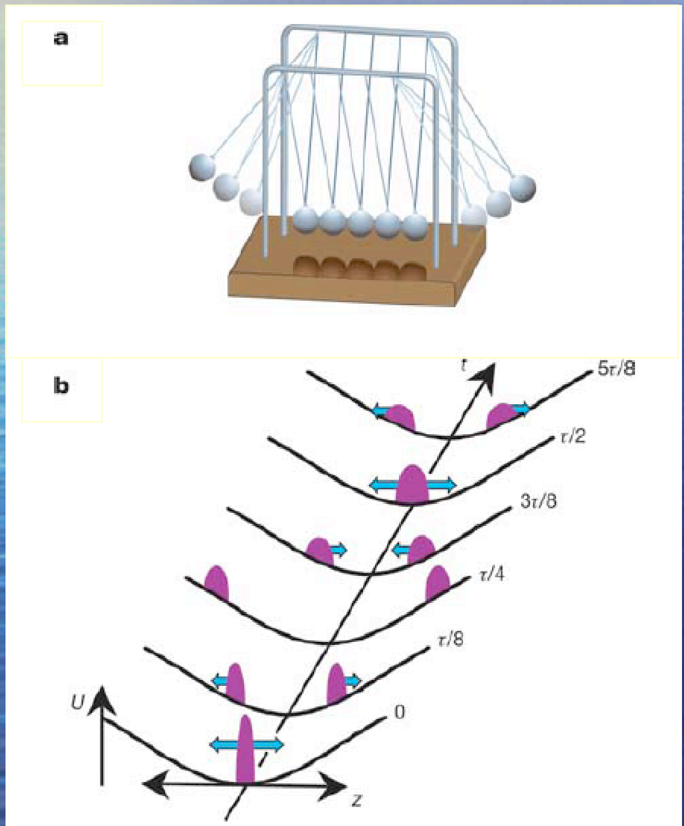


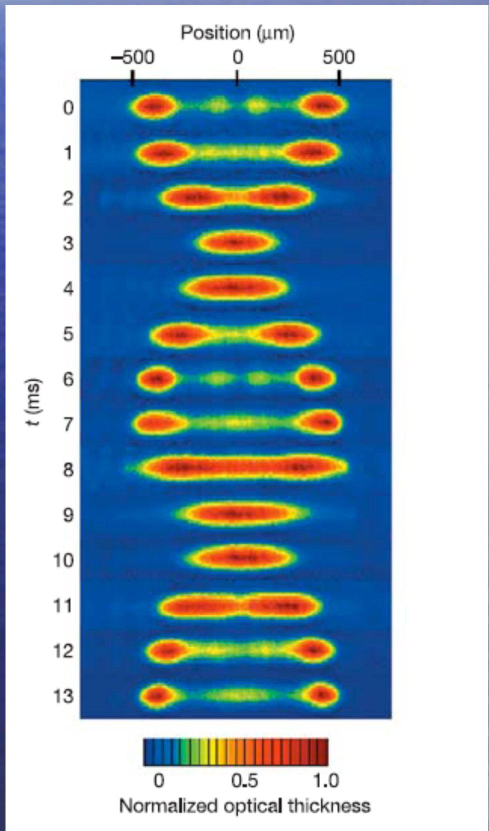


Saturation of damping rate at low T: quantum phase slip !

Non equilibrium physics in many body systems

From: Kinoshita et al., Nature 440, 900 (2006)





40 periods without thermalization: integrability ??