Trieste 2008

Note Title 4/29/2008

Macro Entanglement

– Massive & Massless

Vlatko Vedral

v.vedral@quantuminfo.org

Summary

*) Entanglement as Order

*) Peierls Argument

*) Quantum Version

*) Example: XX-model

*) Discussion of Physics

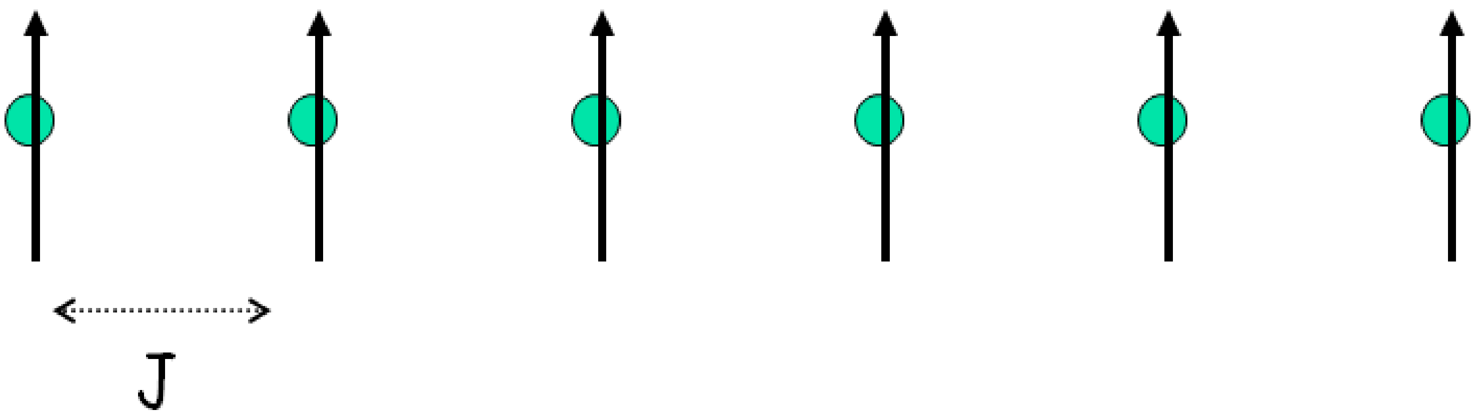
Phase Transitions

Order / Disorder

But, order could mean many different things.

order ≡ magnetisation; long range/short range correlations; persistent current; etc...

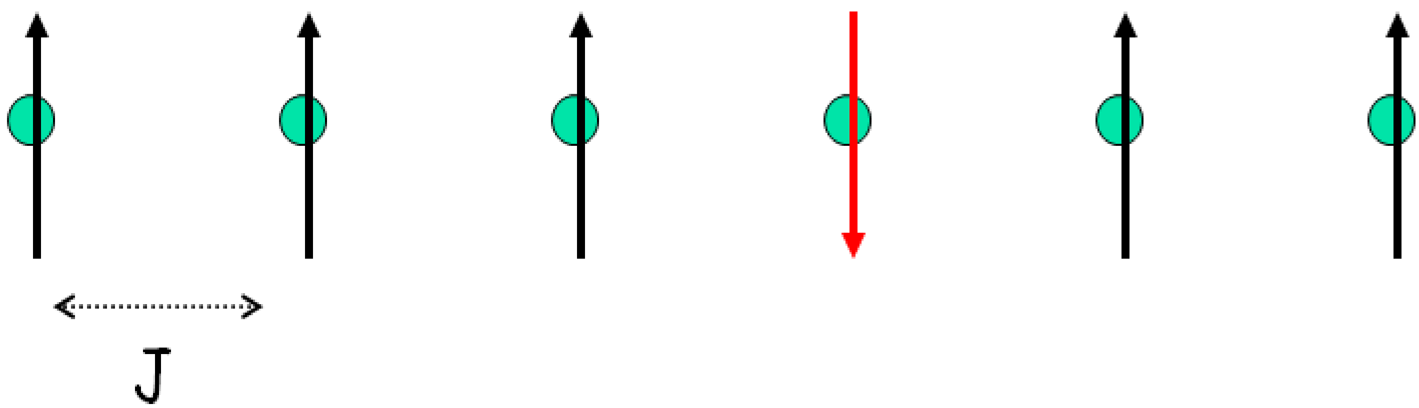
Peierls



$$F = U - TS$$

$dF = 0 \iff$ Equilibrium

1 Dim – No Criticality

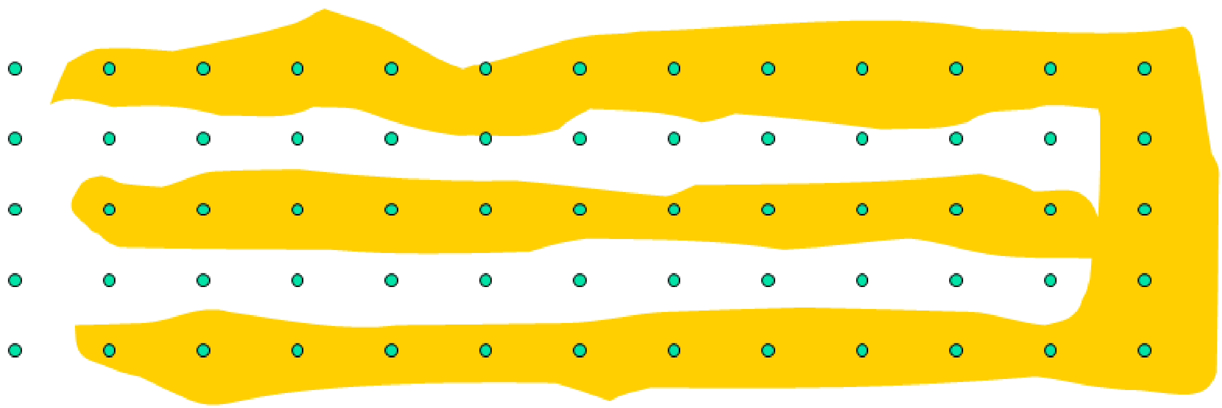


$$\Delta U = 2J$$

$$T\Delta S = k \ln N$$

...no phase transition

2D – Critical



$$\Delta U = 4JN$$

$$T\Delta S = kT \ln 3^N = kTN \ln 3$$

...there is a phase transition $kT_C \ln 3 = 4J$

PTs - Summary

Trade off between energy and entropy. In 1D entropy always wins ⇒ no order.
In 2D can have balance.

Quantum "Peierls"

For quantum criticality, the trade-off is between entropy and entanglement.

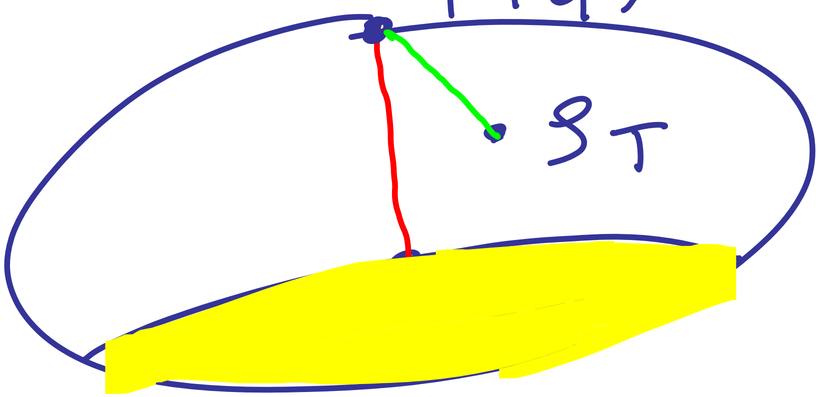
Thm. If

$$S(\rho_T) < E_G$$

$\Rightarrow \rho_T$ is entangled.

Proof

Simple. If ρ_T is closer to $|\Psi_G\rangle$ than the closest separable state $\Longrightarrow$ ρ_T must be entangled



$$S(|\Psi_G\rangle \| \rho_T) < S(|\Psi_G\rangle \| \rho_{sep})$$

$\rotatebox{90}{$\geq$}$ $S(\rho_T)$ $'' \; E(|\Psi_G\rangle)$

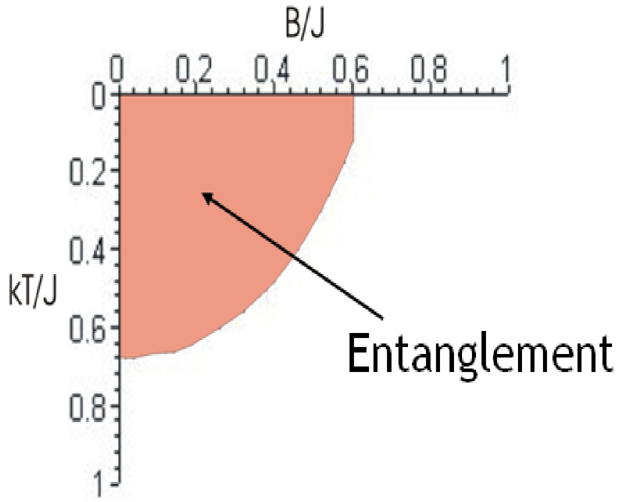
Examples

XX Model

XX Heisenberg Interaction

$$H = J\sum_j \left(S_j^x \cdot S_{j+1}^x + S_j^y \cdot S_{j+1}^y\right) + B\sum_j S_j^z$$

Brukner & Vedral, Int. J.Quant. Info (2005).



Katsura (1962)
Exactly Solvable: *U*, *M*, ...

Cluster States

$$H = -J \sum_{i=1}^{N} \sigma^x_i \sigma^z_{i+1} \sigma^x_{i+2}$$

$$H |\psi_G\rangle = -NJ |\psi_G\rangle$$

Excited states obtained by applying σ^z s to different spins. Each σ^z adds $2J$ of energy.

$$p_G = \left(\frac{1}{e^{-2\beta J}+1}\right)^N$$

$$E_G = \frac{N}{2}$$

$$\Rightarrow \quad \beta_c = \frac{\ln(\sqrt{2}+1)}{2J}$$

SAME AS ONSEGAR!

Physical Issues

* Spins $\uparrow \downarrow \pm \downarrow\uparrow$

* XX model can also be seen as hopping particles.

$$a_n = e^{i\phi_n} \sigma_n^- \quad \Bigg\} \text{J-W}$$
$$a_n^\dagger = e^{-i\phi_n} \sigma_n^+$$

$\Longrightarrow$ Mode Entanglement.

Entangling Mechanism

Spins: Pauli Exclusion

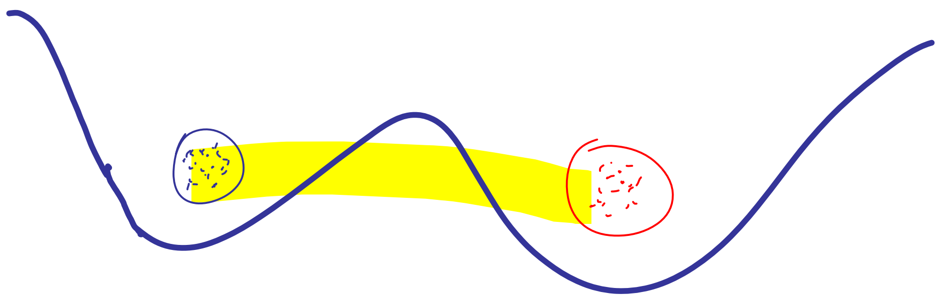
Mode ent: hopping interaction
— tunneling

Modes

- Massless mode entanglement
  e.g. a photon in two output arms of beamsplitter

- Massive particles.
  e.g. electron in a superposition between two quantum dots.

Massive Modes



$$|0\rangle_L |1\rangle_R + |1\rangle_L |0\rangle_R$$

Entangled?

Need $|0\rangle_L \rightarrow |0\rangle_L + |1\rangle_L$

Superselection??

References

$$\underbrace{|0\rangle}_{S}\underbrace{|\alpha\rangle}_{R} \approx (|0\rangle + |1\rangle)|\alpha'\rangle$$

↑ How do we make this?

We do it. We use

$$\rho = \frac{1}{2\pi}\int |\alpha\rangle\langle\alpha|\, d\varphi$$

$$= \sum_{n=0}^{\infty} p_n \underline{|n\rangle\langle n|}$$

mixture

Non locality

If we mix reference,

then superposition =

entanglement = non-locality

J. A. Dunningham + V. V., PRL 2007.

Summary

*) Can think of entanglement as order.

*) There are general criteria for witnessing entanglement

*) But is it real ?

*) If we allow all tools of quantum physics

entanglement = superposition

Literature

1) Amico, Fazio, Osterloh, V.V., Rev. Mod. Phys (2008)

2) M. Terra Cunha, J.A. Dunningham V.V, Proc. Roy. Soc. (2007).

3) Markham, Anders, Miyake, Murao, V.V., EPL (2008).

4) V.V, Nature Insight Issue on Macroscopic Entanglement (2008).

5) J. Hide, W. Son, I. Lawrie, V.V. PRA (2007).