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Hadronic light-front wavefunctions from AdS/QCD

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Light-Front Holography:

Hadronic Wavefunctions from AdS/QCD

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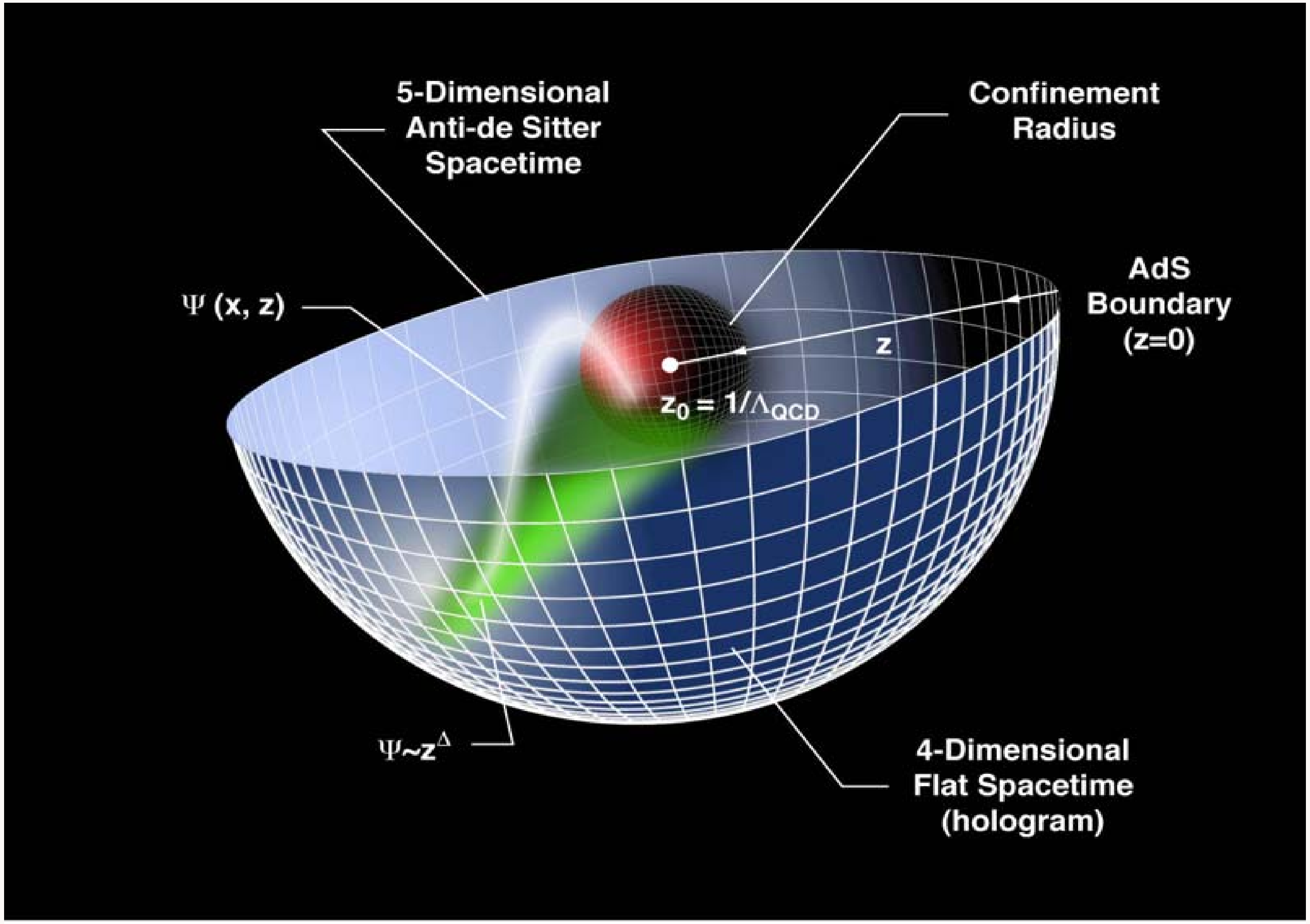
Goal:

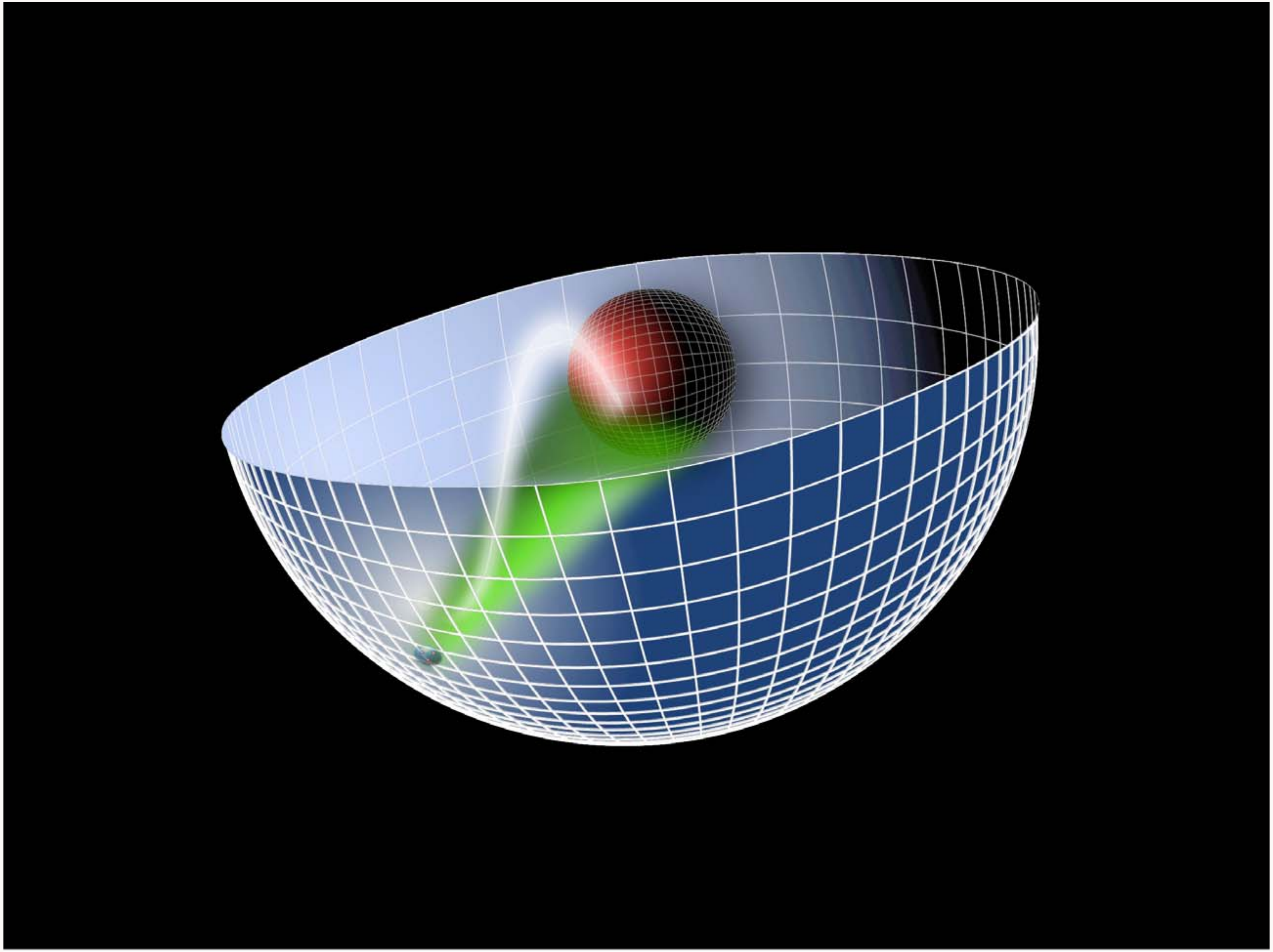
- **Use AdS/CFT to provide an approximate, covariant, and analytic model of hadron structure with confinement at large distances, conformal behavior at short distances**
- **Analogous to the Schrodinger Theory for Atomic Physics**
- *AdS/QCD Light-Front Holography*
- *Hadronic Spectra and Light-Front Wavefunctions*

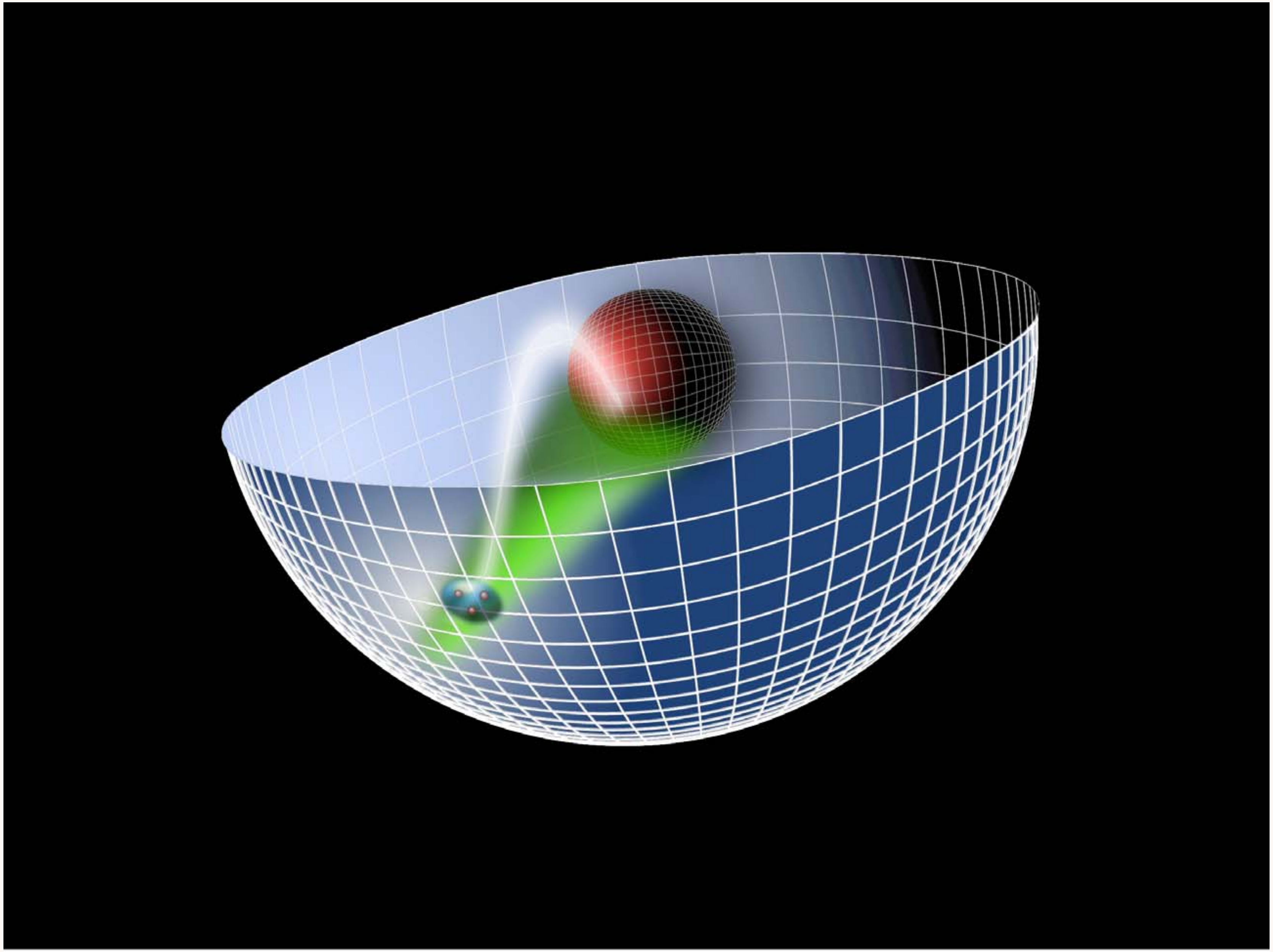
Applications of AdS/CFT to QCD

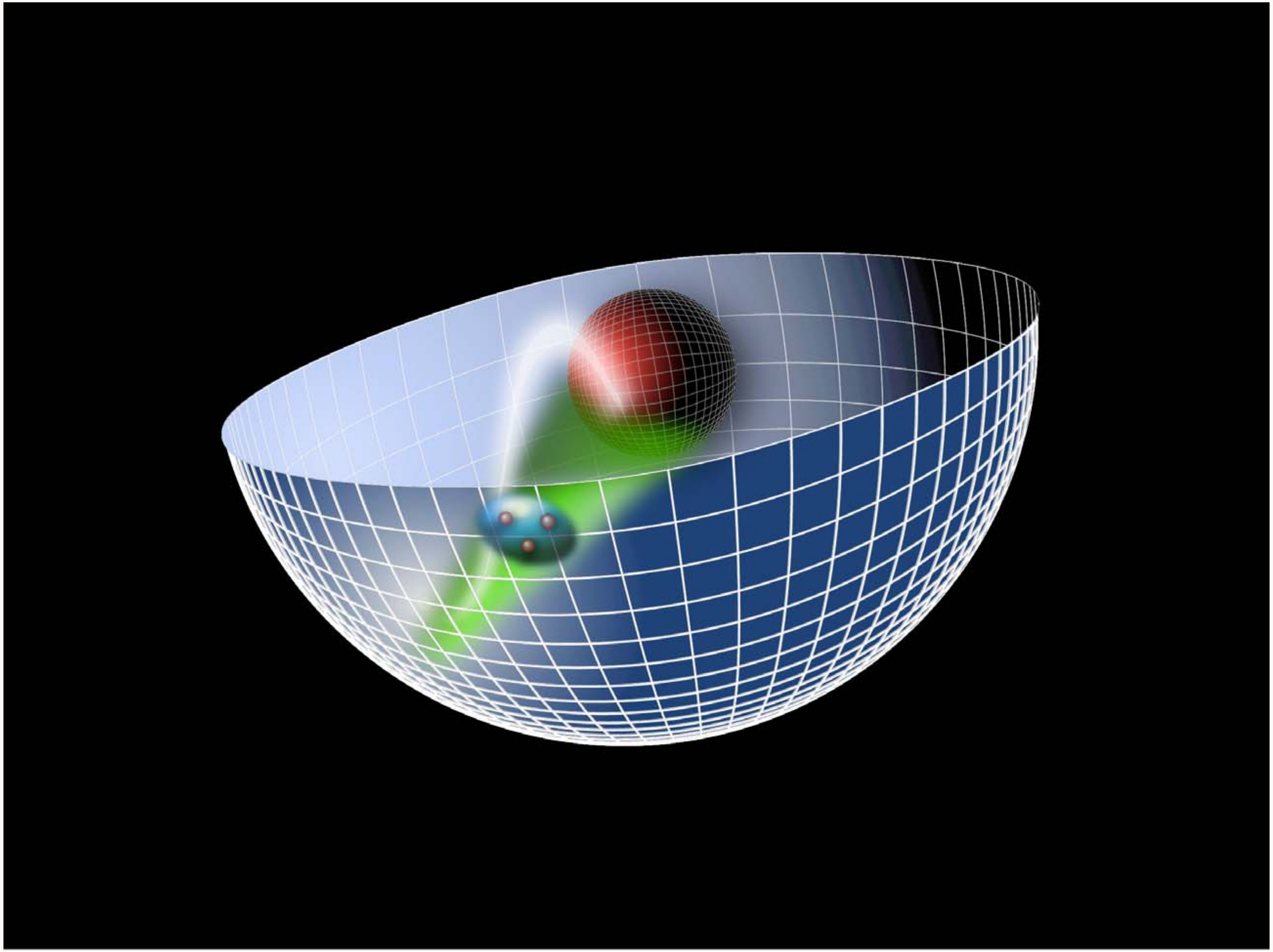


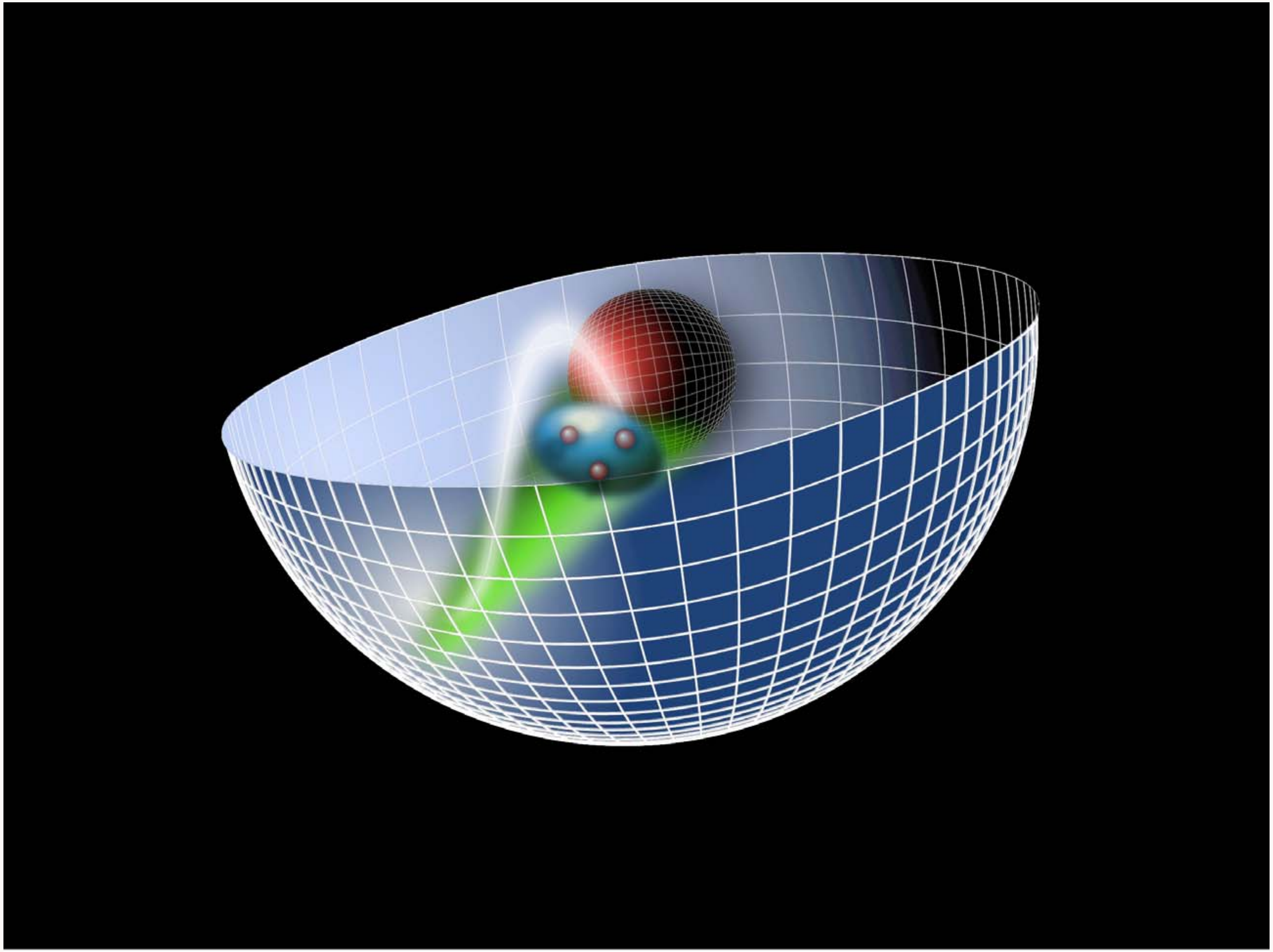
Changes in physical length scale mapped to evolution in the 5th dimension z

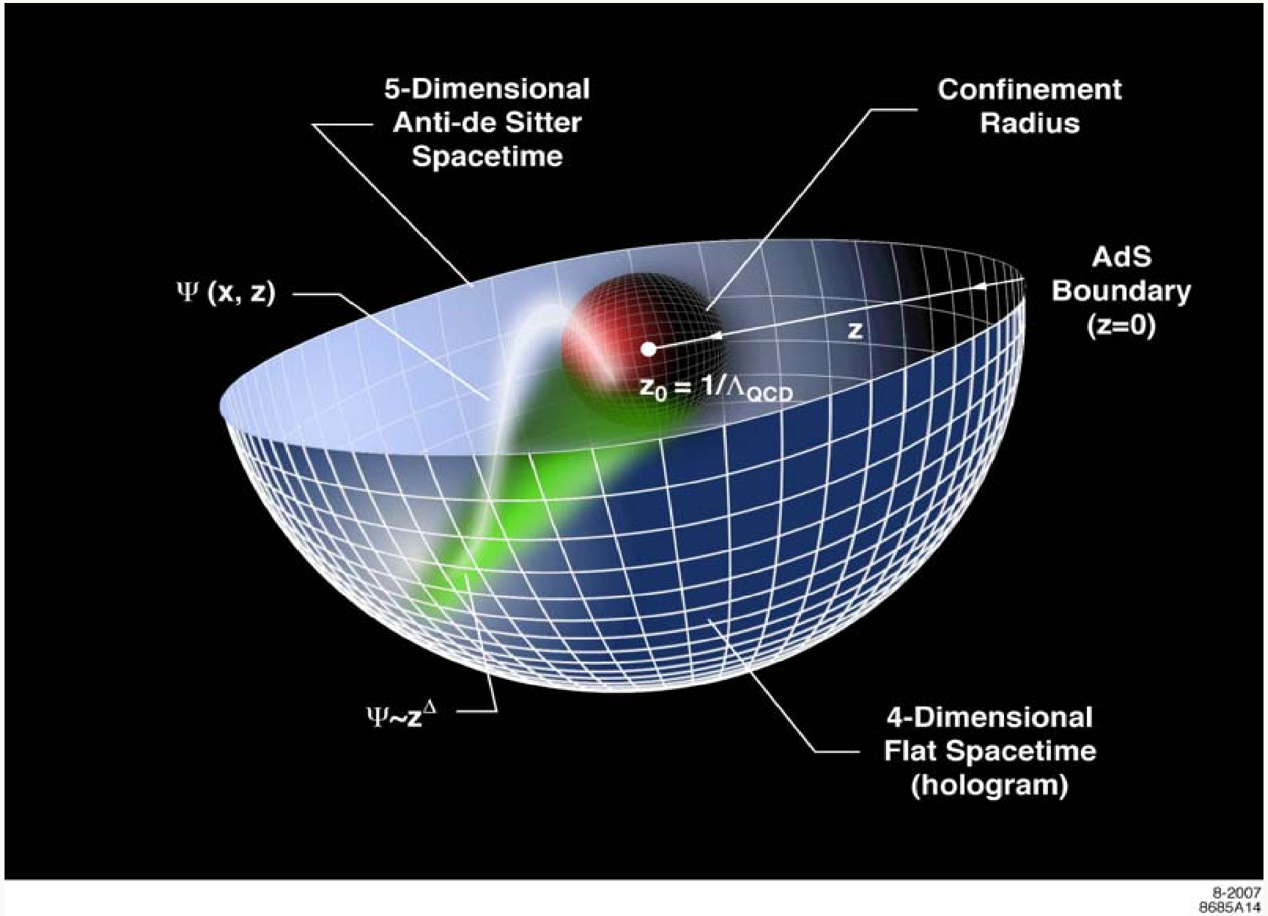
in collaboration with Guy de Teramond











- Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{\rm QCD}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) Polchinski and Strassler (2001).

- Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model) Karch, Katz, Son and Stephanov (2006).

We will consider both holographic models

Conformal Theories are invariant under the Poincare and conformal transformations with

$$\mathbf{M}^{\mu\nu}, \mathbf{P}^{\mu}, \mathbf{D}, \mathbf{K}^{\mu},$$

the generators of SO(4,2)

SO(4,2) has a mathematical representation on AdS5

Scale Transformations

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2),$$

invariant measure

$x^\mu \to \lambda x^\mu, \; z \to \lambda z$, maps scale transformations into the holographic coordinate z.

- AdS mode in z is the extension of the hadron wf into the fifth dimension.

- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.

AdS/CFT: Anti-de Sitter Space / Conformal Field Theory

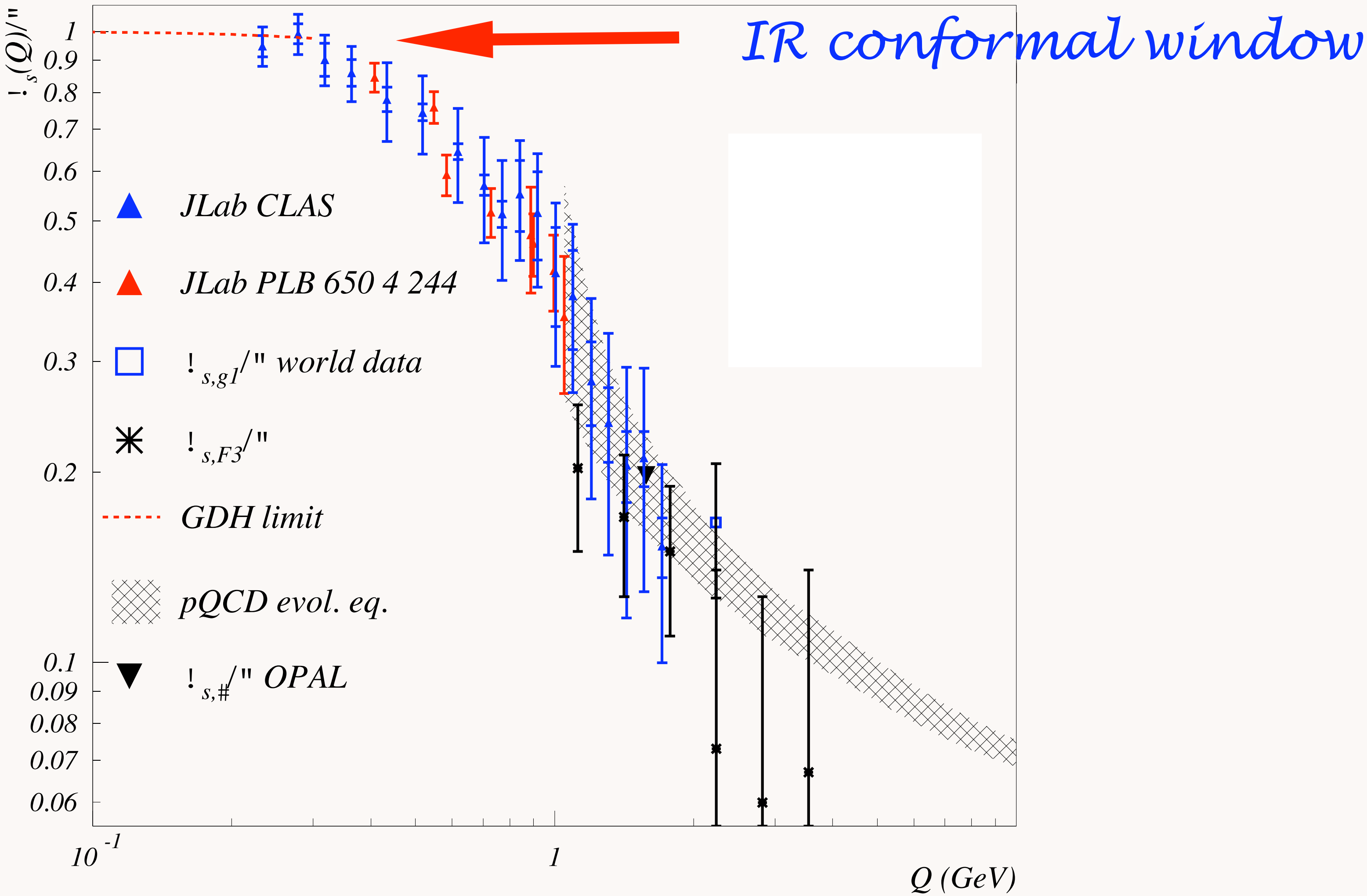
Maldacena:

Map $AdS_5 \times S_5$ to conformal N=4 SUSY

- **QCD is not conformal**; however, it has manifestations of a scale-invariant theory: Bjorken scaling, dimensional counting for hard exclusive processes
- **Conformal window:** $\alpha_s(Q^2) \simeq \text{const at small } Q^2$
- **Use mathematical mapping of the conformal group SO(4,2) to AdS5 space**

Deur, Korsch, et al: Effective Charge from Bjorken Sum Rule

$$\Gamma_{bj}^{p-n}(Q^2) \equiv \frac{g_A}{6}[1 - \frac{\alpha_s^{g_1}(Q^2)}{\pi}]$$

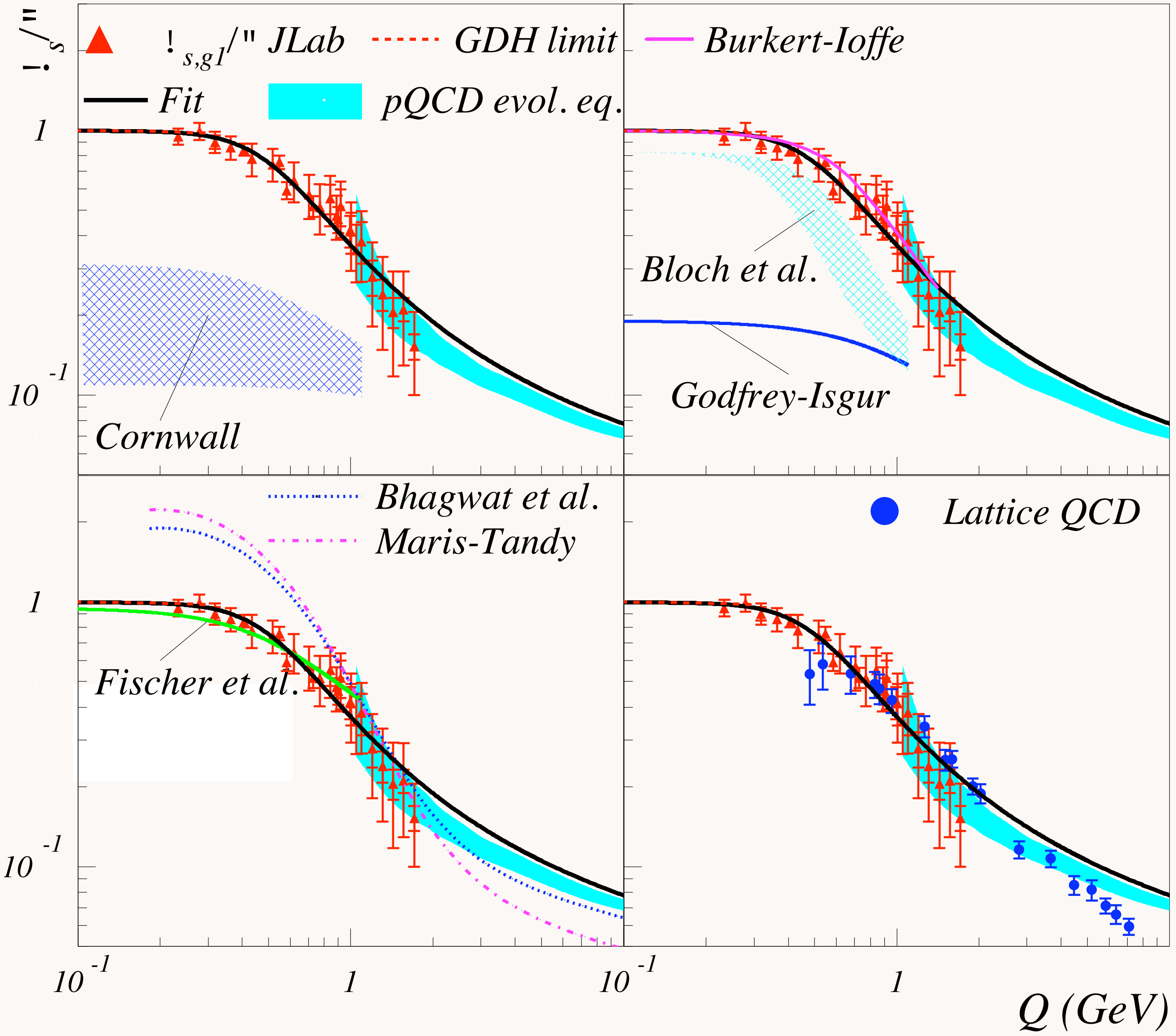


Trieste ICTP
May 12, 2008

AdS/QCD
12

Stan Brodsky
SLAC & IPPP

Deur, Korsch, et al.



IR Conformal Window for QCD?

- *Dyson-Schwinger Analysis:* **QCD Coupling has IR Fixed Point**

- *Evidence from Lattice Gauge Theory*

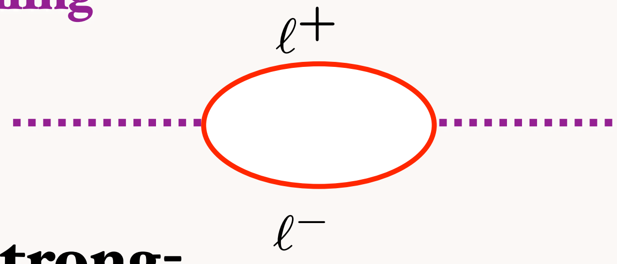
- Define coupling from observable: **indications of IR fixed point for QCD effective charges**

- Confined gluons and quarks have maximum wavelength: **Decoupling of QCD vacuum polarization at small Q²**

Shrock, de Teramond, sjb

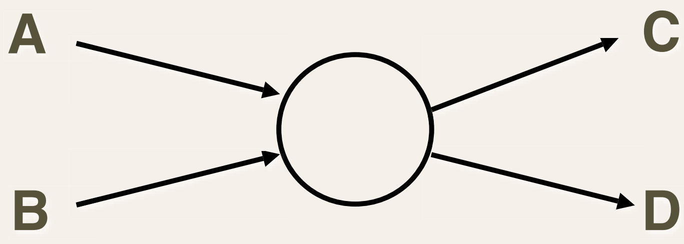
Serber-Uehling

$$\Pi(Q^2) \to \frac{\alpha}{15\pi}\frac{Q^2}{m^2} \qquad Q^2 << 4m^2$$



- **Justifies application of AdS/CFT in strong-coupling conformal window**

Constituent Counting Rules



$$\frac{d\sigma}{dt}(s,t) = \frac{F(\theta_{\rm cm})}{s^{[n_{\rm tot}-2]}} \qquad s = E^2_{\rm cm}$$

$$F_H(Q^2) \sim [\frac{1}{Q^2}]^{n_H - 1}$$

$$n_{tot} = n_A + n_B + n_C + n_D$$

Fixed t/s or $\cos\theta_{cm}$

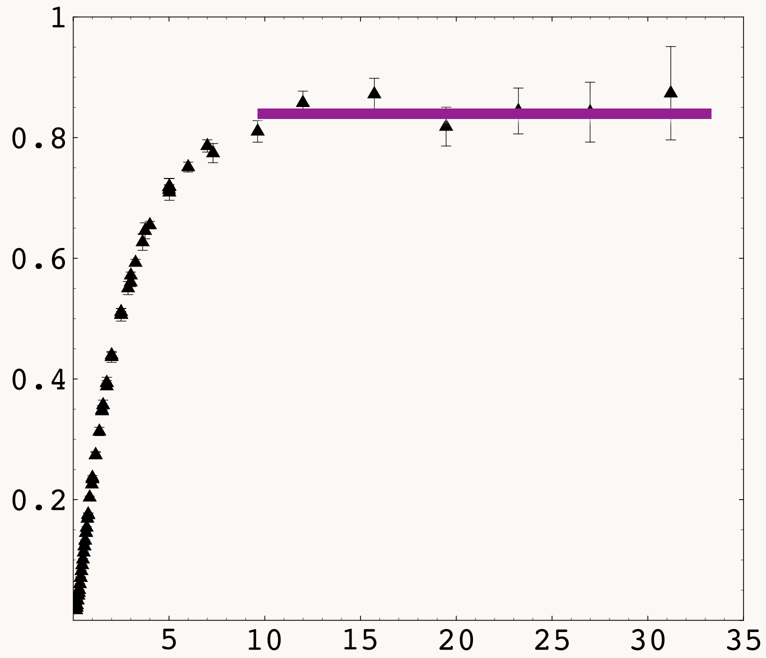
Farrar & sjb; Matveev, Muradyan, Tavkhelidze

Conformal symmetry and PQCD predict leading-twist scaling behavior of fixed-CM angle exclusive amplitudes

Characteristic scale of QCD: 300 MeV

Many new J-PARC, GSI, J-Lab, Belle, Babar tests

$Q^4 F_1^p(Q^2)$ [GeV^4]



Q^2 [GeV^2]

$F_1(Q^2) \sim \left[1/Q^2\right]^{n-1}, \quad n = 3$

From: M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

- Phenomenological success of dimensional scaling laws for exclusive processes

$$d\sigma/dt \sim 1/s^{n-2}, \quad n = n_A + n_B + n_C + n_D,$$

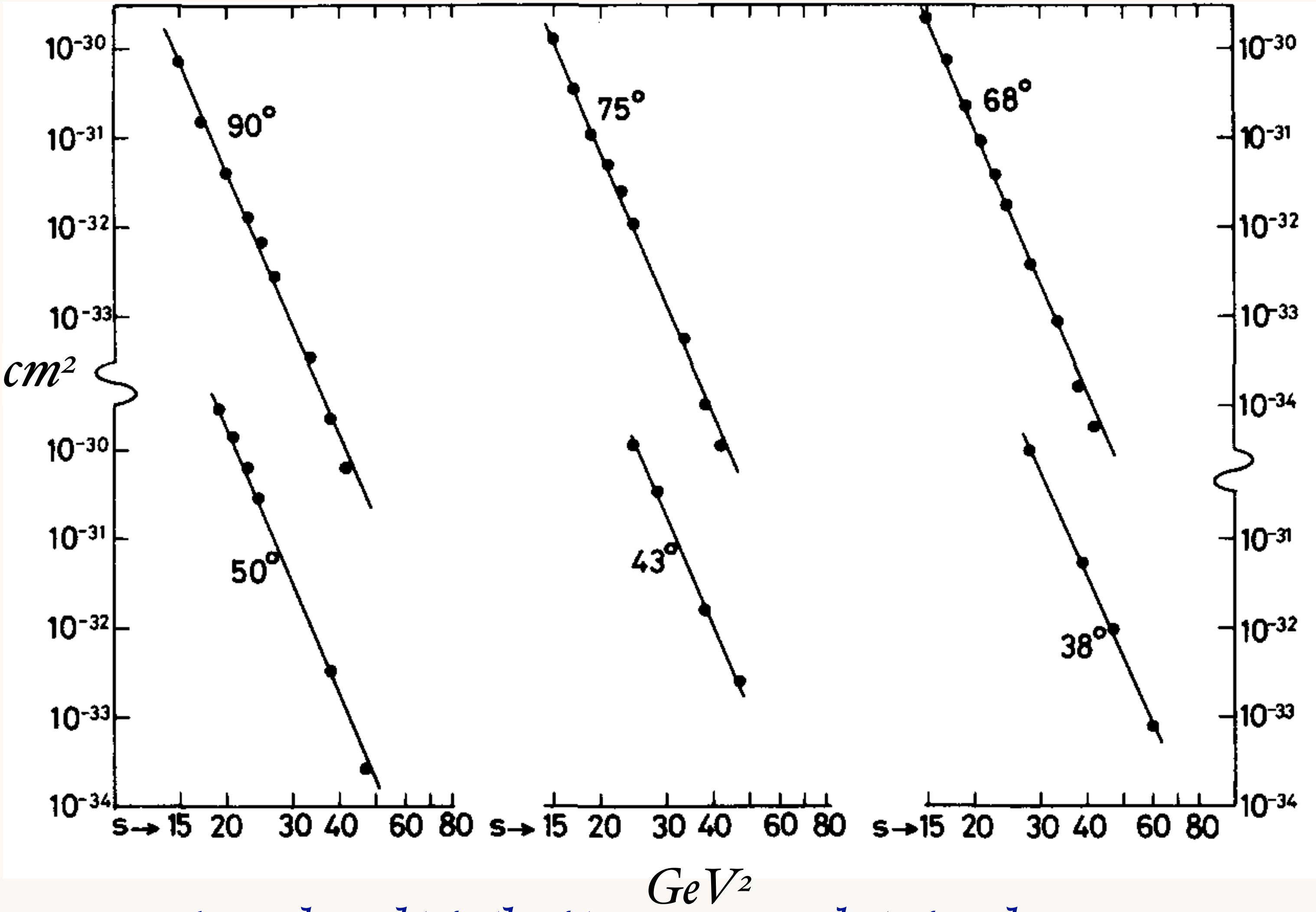
implies QCD is a strongly coupled conformal theory at moderate but not asymptotic energies

Farrar and sjb (1973); Matveev *et al.* (1973).

- Derivation of counting rules for gauge theories with mass gap dual to string theories in warped space (hard behavior instead of soft behavior characteristic of strings) Polchinski and Strassler (2001).

Quark-Counting : $\frac{d\sigma}{dt}(pp \to pp) = \frac{F(\theta_{CM})}{s^{10}}$ $n = 4 \times 3 - 2 = 10$

P.V. LANDSHOFF and J.C. POLKINGHORNE

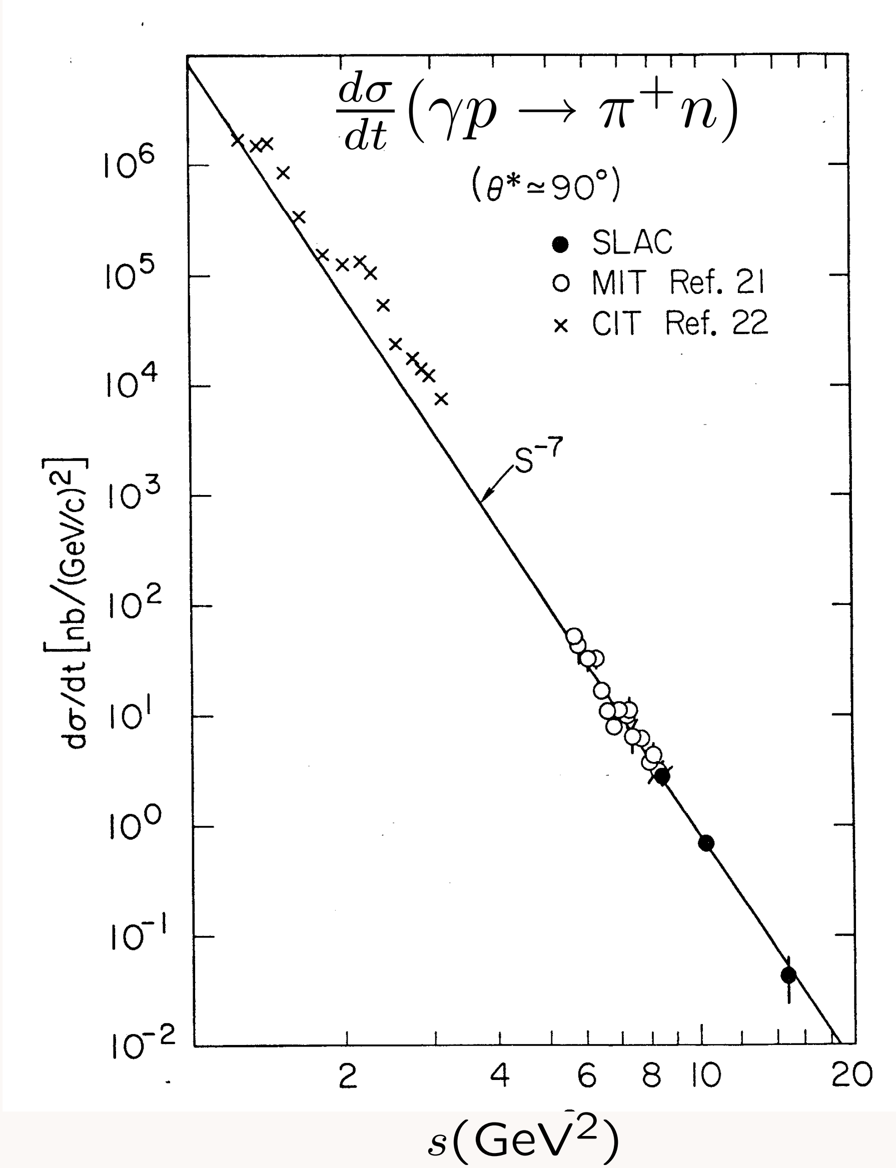


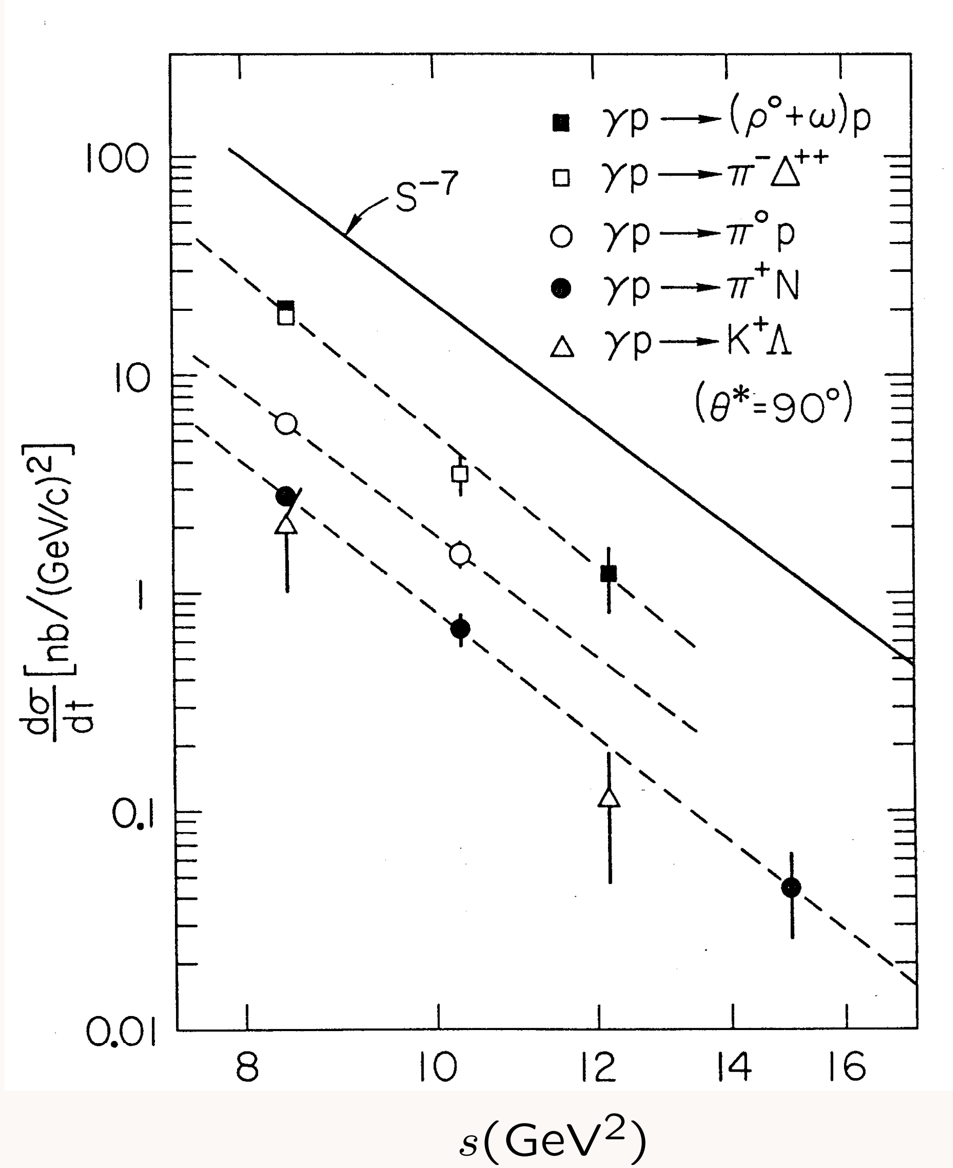
Best Fit

$n = 9.7 \pm 0.5$

Reflects underlying conformal scale-free interactions

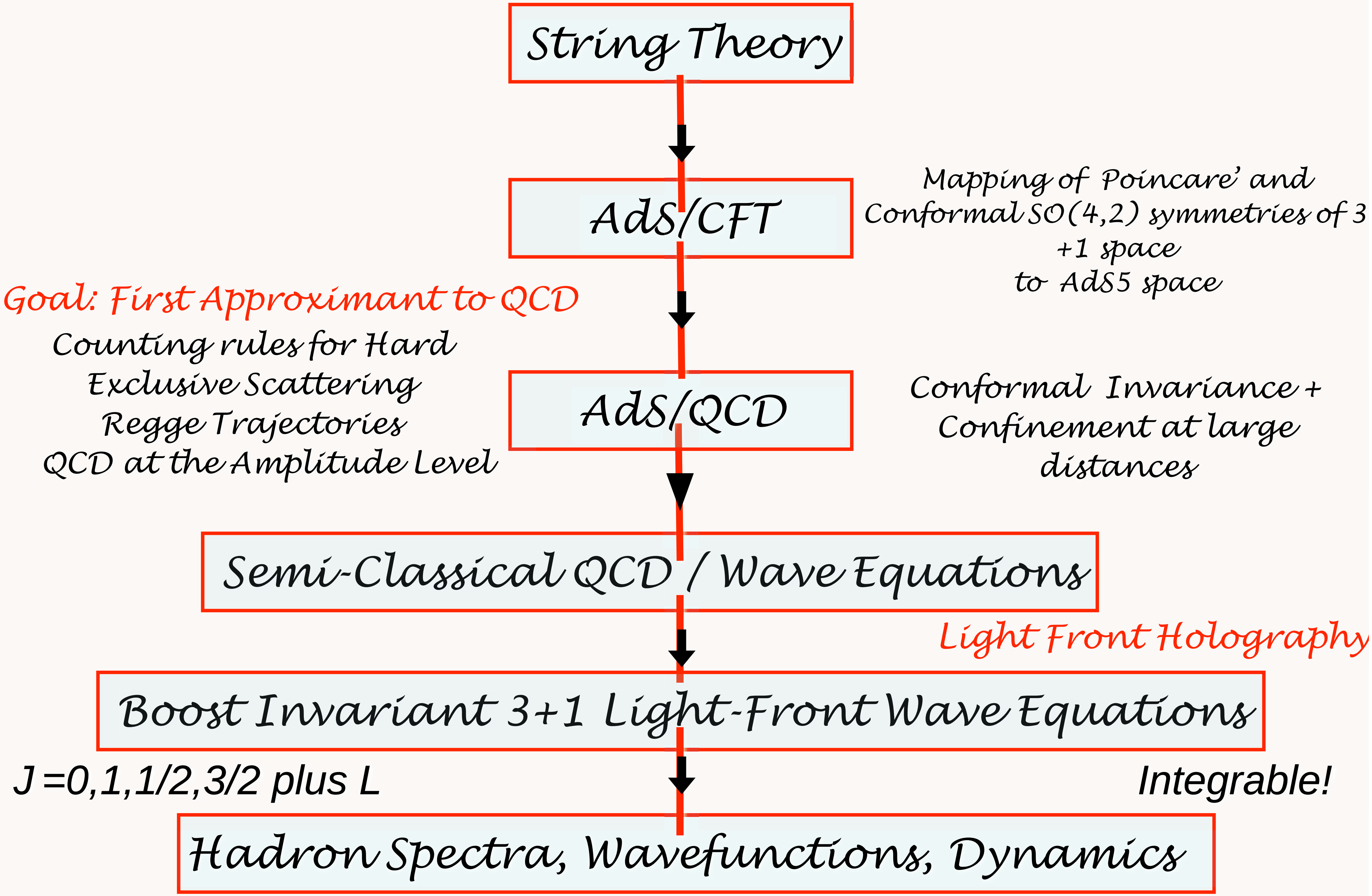
Angular distribution -- quark interchange





Conformal Invariance:

$$\frac{d\sigma}{dt}(\gamma p \to MB) = \frac{F(\theta_{cm})}{s^7}$$

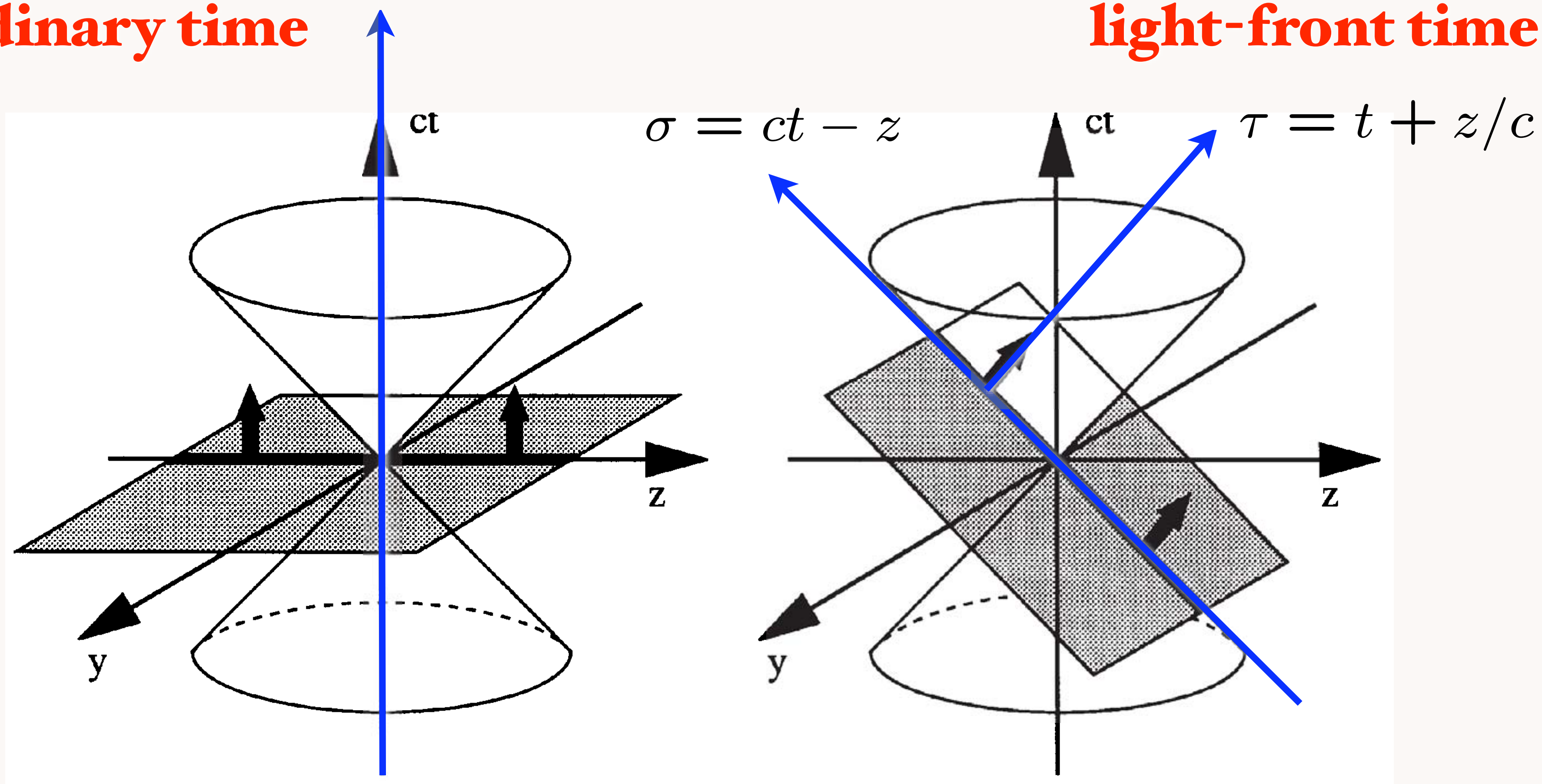

String Theory
AdS/CFT
Mapping of Poincare' and Conformal SO(4,2) symmetries of 3+1 space to AdS5 space
Goal: First Approximant to QCD
Counting rules for Hard Exclusive Scattering
Regge Trajectories
QCD at the Amplitude Level
AdS/QCD
Conformal Invariance + Confinement at large distances
Semi-Classical QCD / Wave Equations
Light Front Holography
Boost Invariant 3+1 Light-Front Wave Equations
J =0,1,1/2,3/2 plus L
Integrable!
Hadron Spectra, Wavefunctions, Dynamics

P.A.M Dirac, Rev. Mod. Phys. 21, 392 (1949)

Dirac's Amazing Idea: The Front Form

Evolve in ordinary time

Evolve in light-front time!



Instant Form **Front Form**

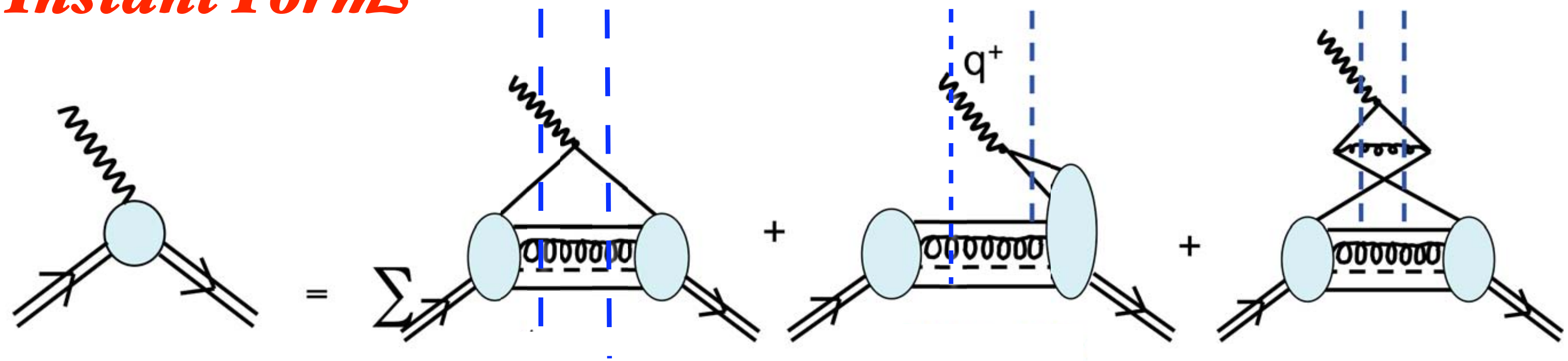
Each element of
flash photograph
illuminated
at same LF time

$$\tau = t + z/c$$

HELEN BRADLEY - PHOTOGRAPHY

Calculation of Form Factors in Equal-Time Theory

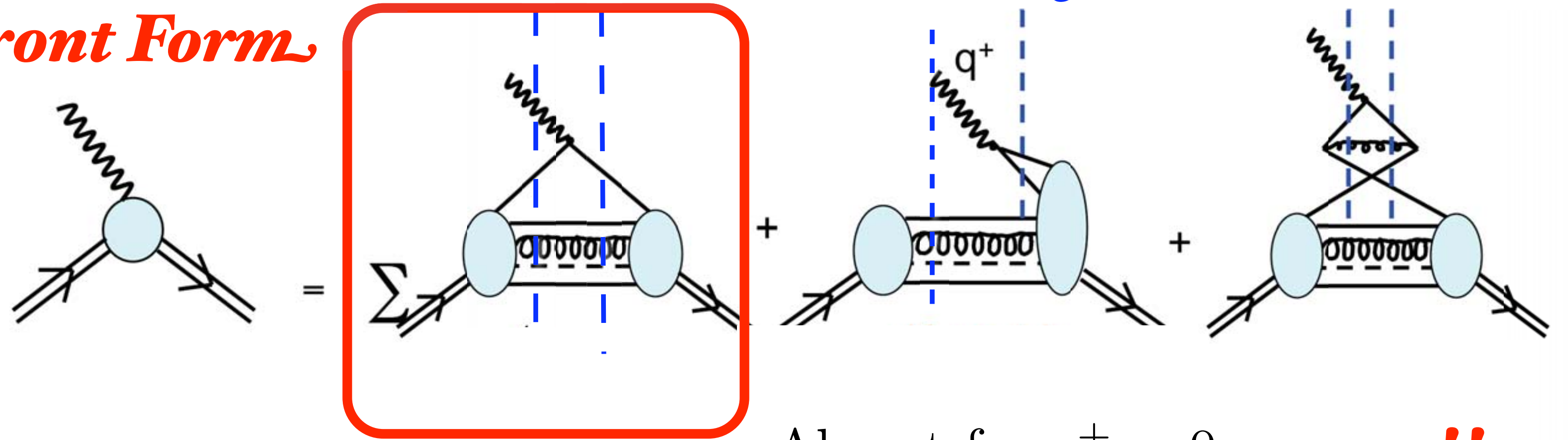
Instant Form



Need vacuum-induced currents

Calculation of Form Factors in Light-Front Theory

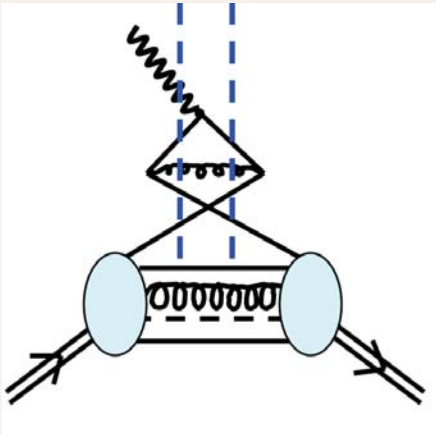
Front Form



Absent for $q^+ = 0$ **zero !!**

Calculation of Hadron Form Factors
Instant Form

- Current matrix elements of hadron include interactions with vacuum-induced currents arising from infinitely-complex vacuum



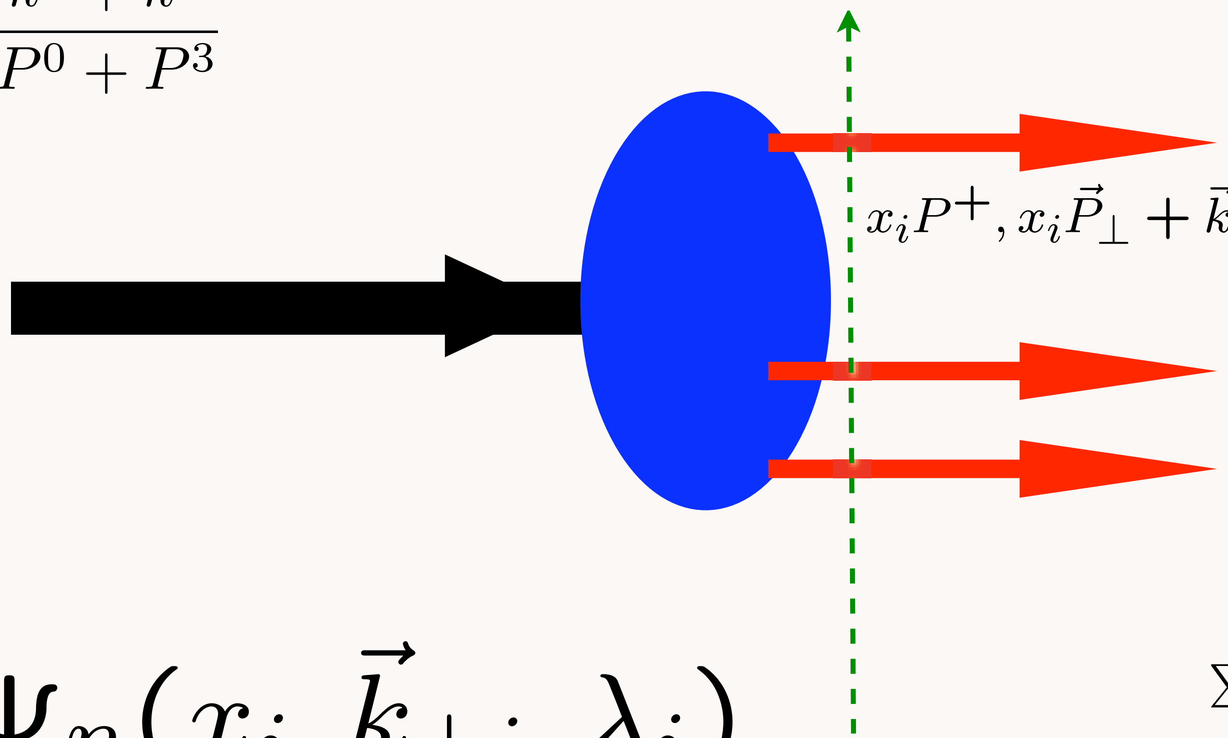
- Pair creation from vacuum occurs at any time before probe acts -- acausal
- Knowledge of hadron wavefunction insufficient to compute current matrix elements
- Requires dynamical boost of hadron wavefunction -- unknown except at weak binding
- Complex vacuum even for QED
- None of these complications occur for quantization at fixed LF time (front form)

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

Fixed $\tau = t + z/c$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

$P^+, \vec{P}_\perp$



$x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}$

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Invariant under boosts! Independent of P^μ

Angular Momentum on the Light-Front

$$J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z .$$

Conserved
LF Fock state by Fock State

$$l_j^z = -i \left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1} \right)$$

n-1 orbital angular momenta

Nonzero Anomalous Moment -->Nonzero orbital angular momentum

$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \times$$

Drell, sjb

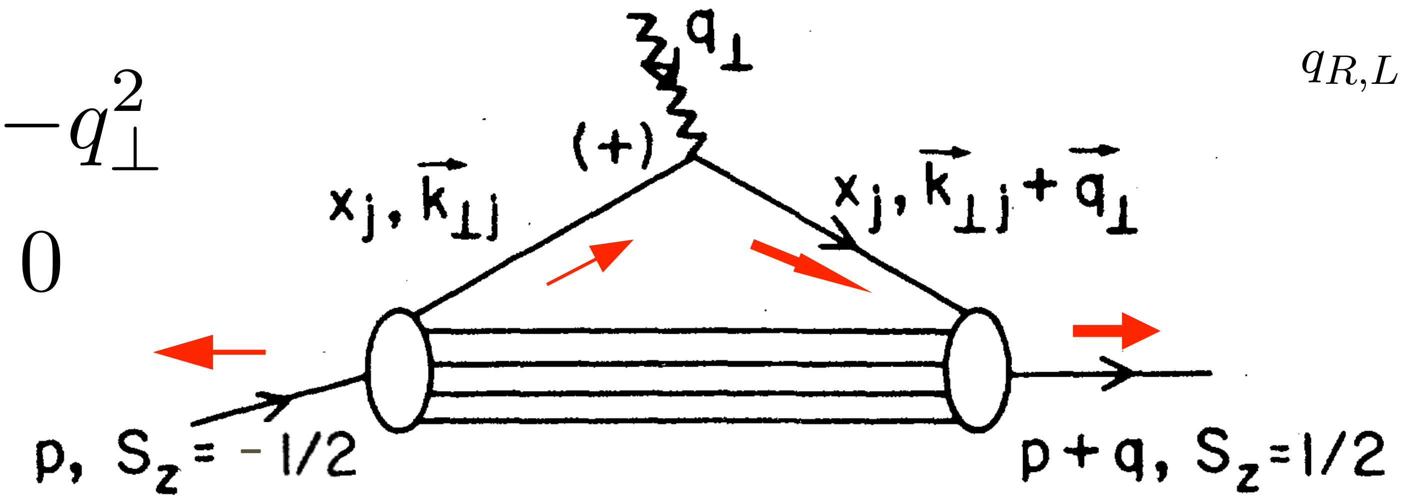
$$\left[-\frac{1}{q^L}\psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R}\psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$

$$q_{R,L} = q^x \pm i q^y$$

$$q^2 = -q_\perp^2$$

$$q^+ = 0$$



Must have $\Delta \ell_z = \pm 1$ to have nonzero $F_2(q^2)$

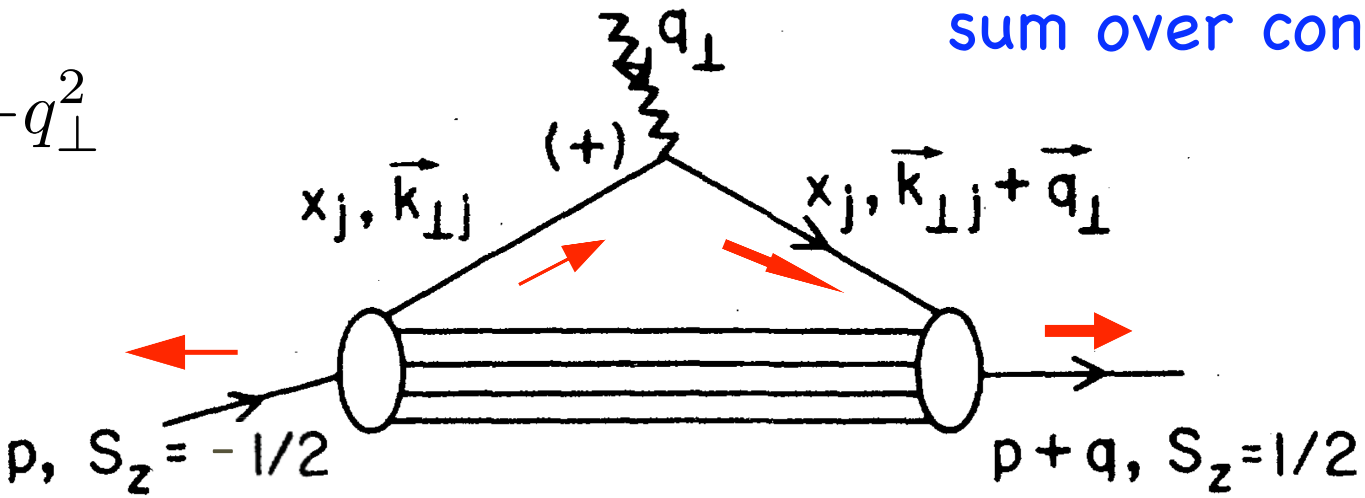
Anomalous gravitomagnetic moment B(0)

Okun, Kobzarev, Teryaev: *B(0) Must vanish because of Equivalence Theorem*

$$q^2 = -q_\perp^2$$

graviton

sum over constituents



Hwang, Schmidt, sjb;
Holstein et al

B(0) = 0

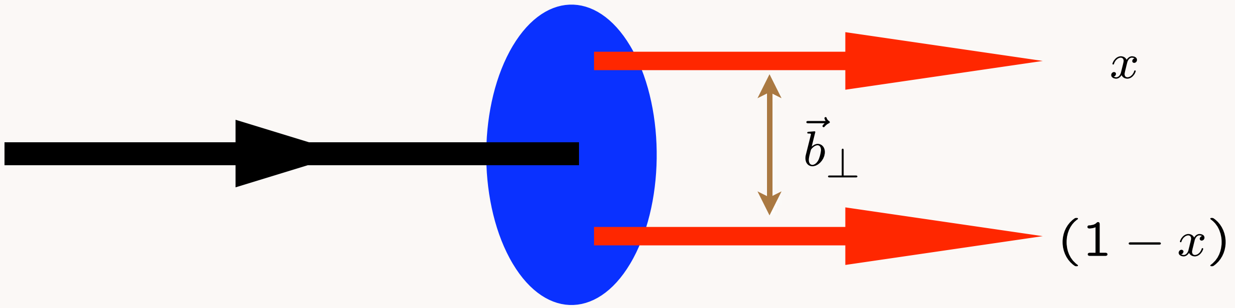
Each Fock State

LF(3+1) $\qquad\qquad$ AdS$_5$

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \longleftrightarrow z$$

$\psi(x, \vec{b}_\perp)$



$$\psi(x, \zeta) = \sqrt{x(1-x)}\,\zeta^{-1/2}\phi(\zeta)$$

Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements: **em and gravitational!**

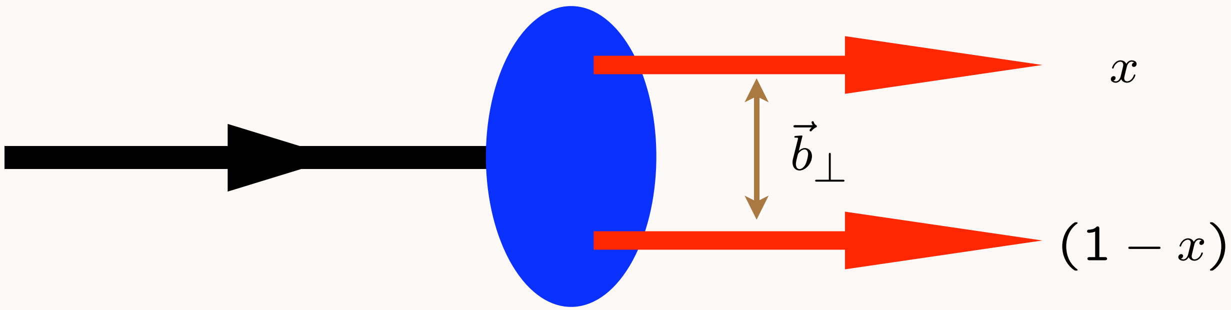
Light-Front Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation — Frame Independent

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$

G. de Teramond, sjb

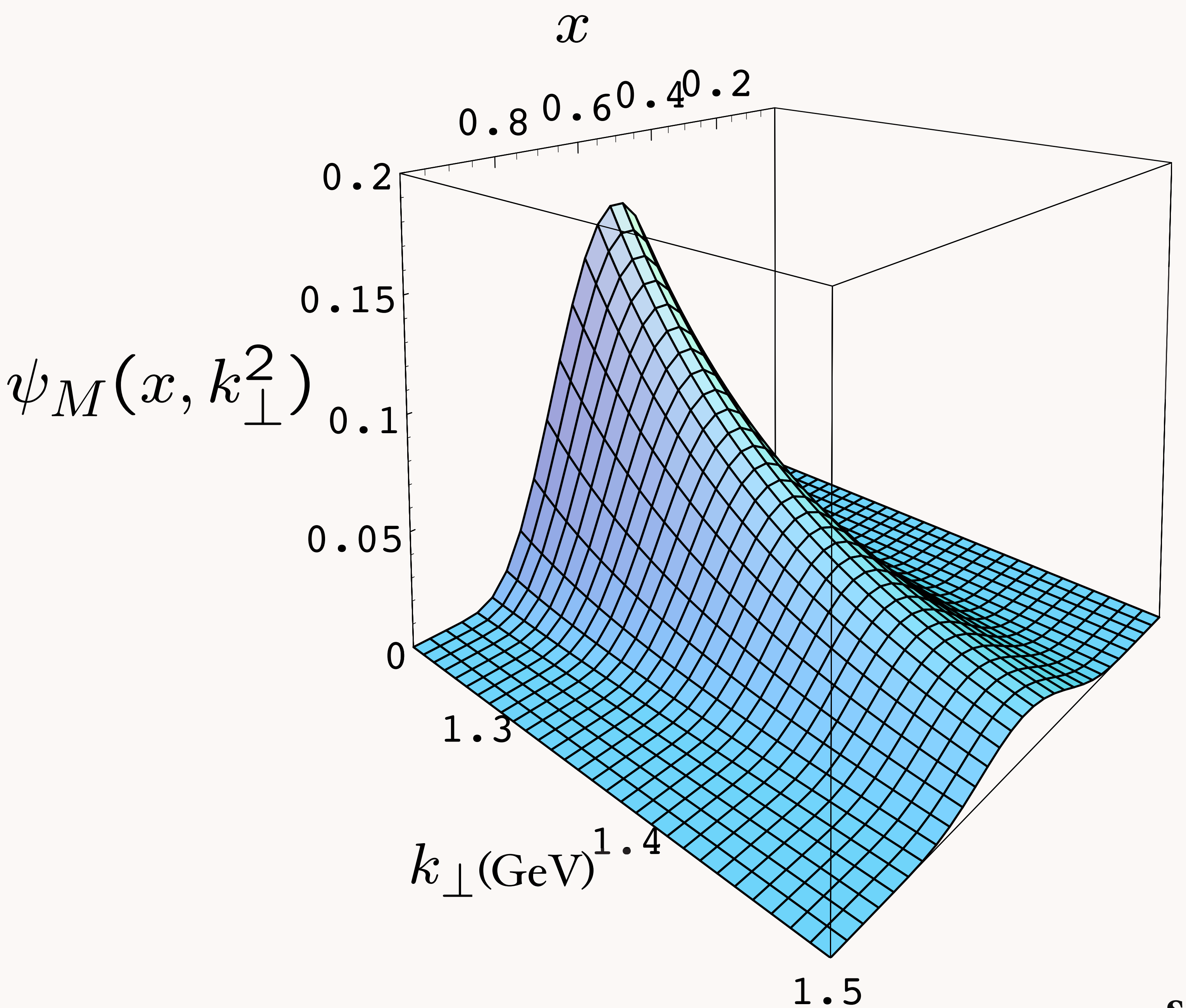


Effective conformal potential:

$$V(\zeta) = -\frac{1-4L^2}{4\zeta^2}.$$

$$+\kappa^4\zeta^2$$ confining potential:

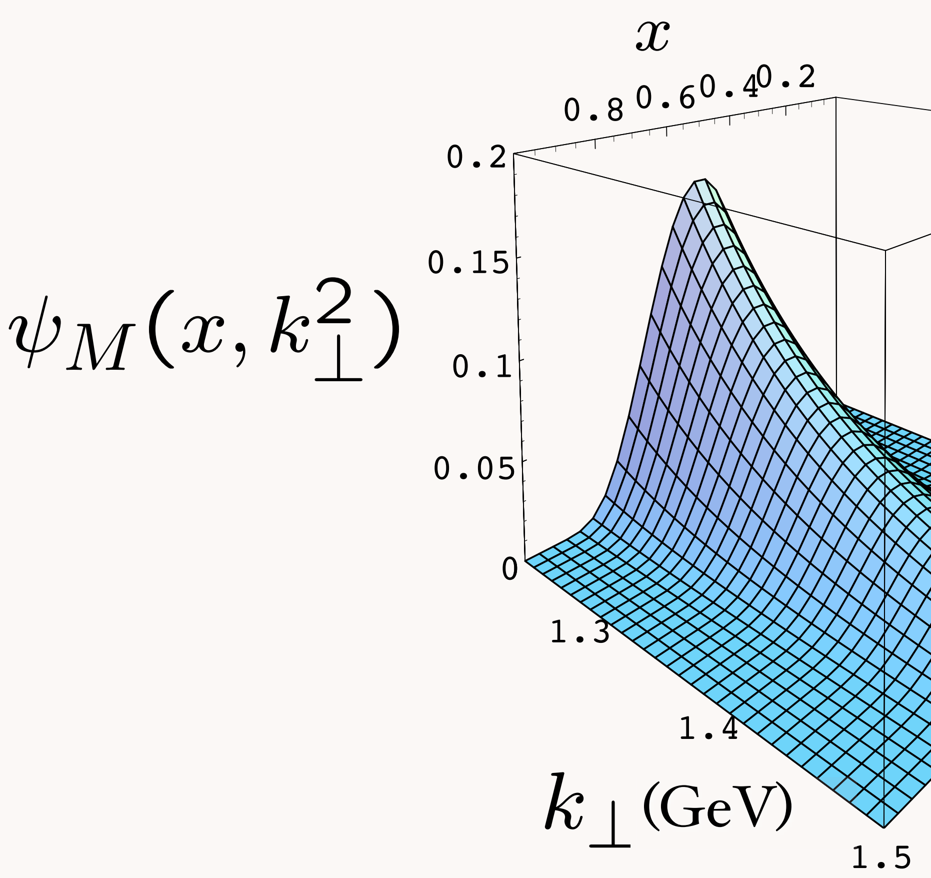
Prediction from AdS/CFT: Meson LFWF



"Soft Wall" model

de Teramond, sjb

Prediction from AdS/CFT: Meson LFWF



de Teramond, sjb

"Soft Wall" model

$\kappa = 0.375$ GeV

massless quarks

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

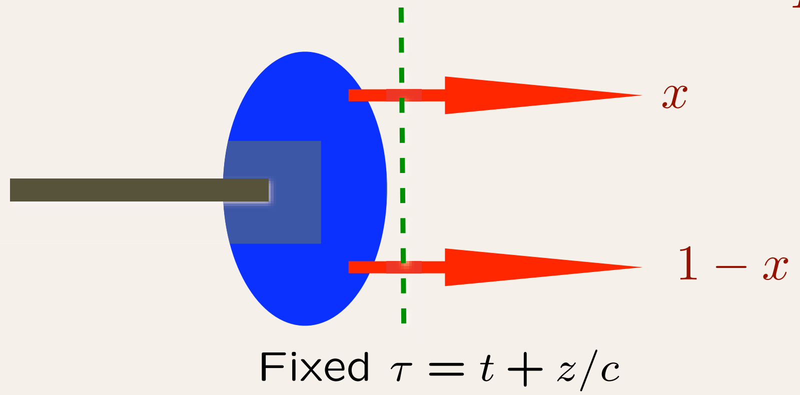
$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$

Hadron Distribution Amplitudes

Lepage, sjb

$$\phi_H(x_i, Q)$$

$$\sum_i x_i = 1$$



$k_\perp^2 < Q^2$

- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for mesons, baryons

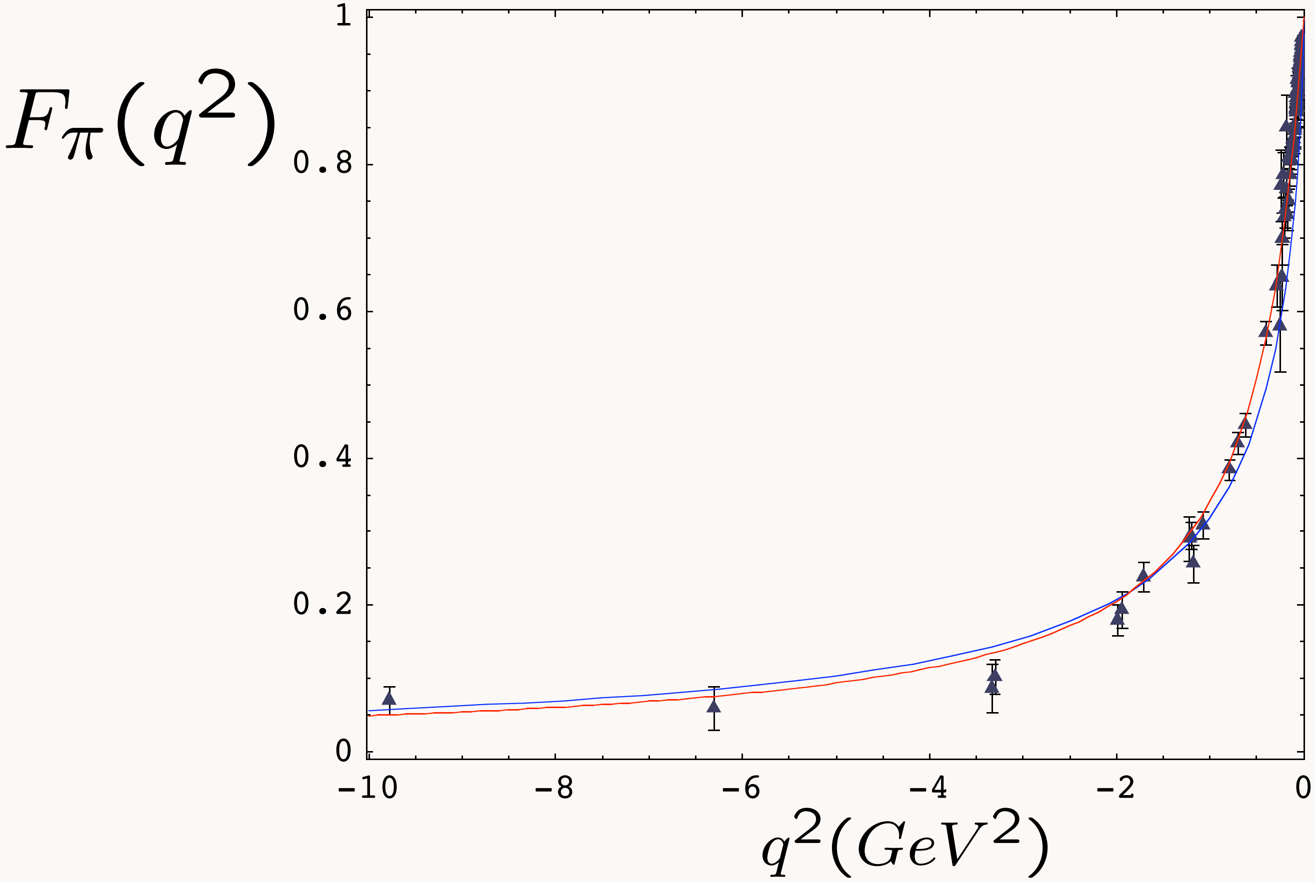
Lepage, sjb
Frishman, Lepage, Sachrajda, sjb
Peskin   Braun
Efremov, Radyushkin   Chernyak etal

- Evolution Equations from PQCD, OPE, Conformal Invariance

- Compute from valence light-front wavefunction in light-cone gauge

$$\phi_M(x, Q) = \int^Q d^2\vec{k}\ \psi_{q\bar{q}}(x, \vec{k}_\perp)$$

Spacelike pion form factor from AdS/CFT



Data Compilation
Baldini, Kloe and Volmer

Soft Wall: Harmonic Oscillator Confinement

Hard Wall: Truncated Space Confinement

One parameter - set by pion decay constant

de Teramond, sjb
See also: Radyushkin

$$|p, S_z> = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)|n; \vec{k}_{\perp_i}, \lambda_i>$$

sum over states with n=3, 4, ...constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

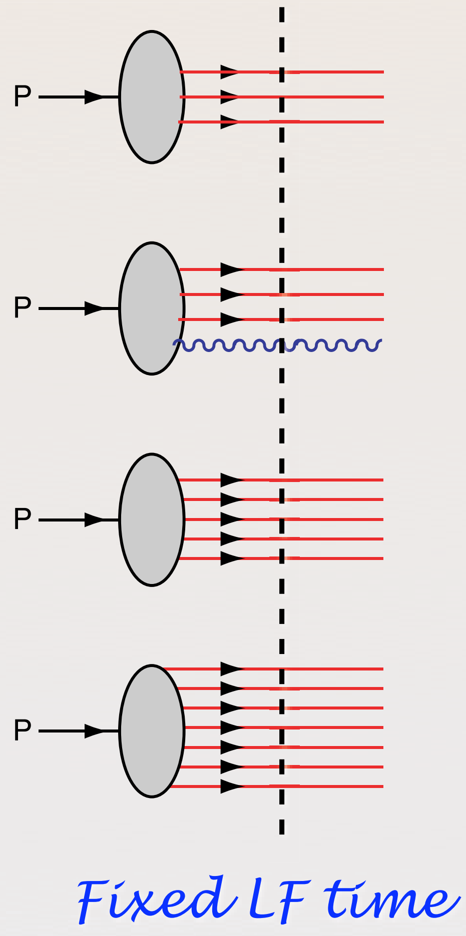
are boost invariant; they are independent of the hadron's energy and momentum P^μ.

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+, \ \sum_i^n x_i = 1, \ \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$



Intrinsic heavy quarks $\bar{u}(x) \neq \bar{d}(x)$

Mueller: BFKL DYNAMICS $\bar{s}(x) \neq s(x)$

Light-Front QCD

Heisenberg Matrix Formulation

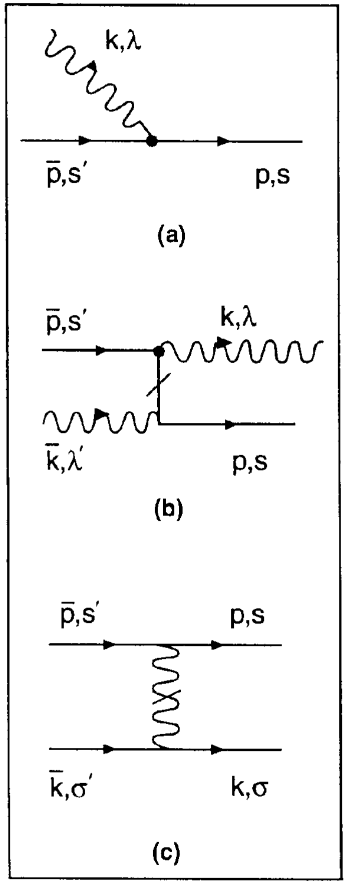
Physical gauge: $A^+ = 0$

$$L^{QCD} \rightarrow H^{QCD}_{LF}$$

$$H^{QCD}_{LF} = \sum_i [\frac{m^2 + k^2_\perp}{x}]_i + H^{int}_{LF}$$

H^{int}_{LF}: Matrix in Fock Space

$$H^{QCD}_{LF} |\Psi_h> = \mathcal{M}^2_h |\Psi_h>$$



Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

DLCQ: Periodic BC in x^-. Discrete k^+; frame-independent truncation

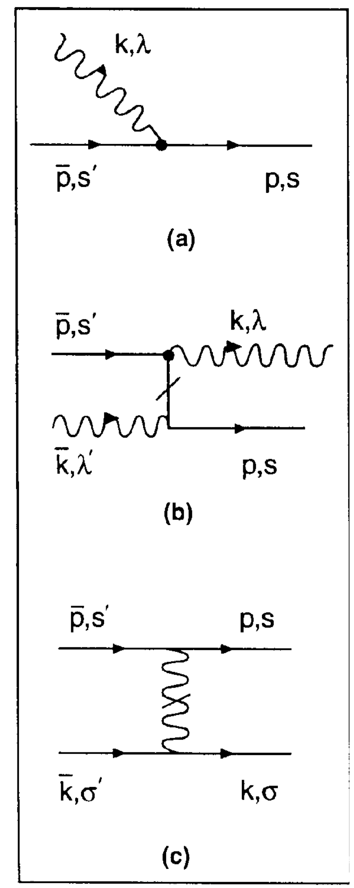
Light-Front QCD

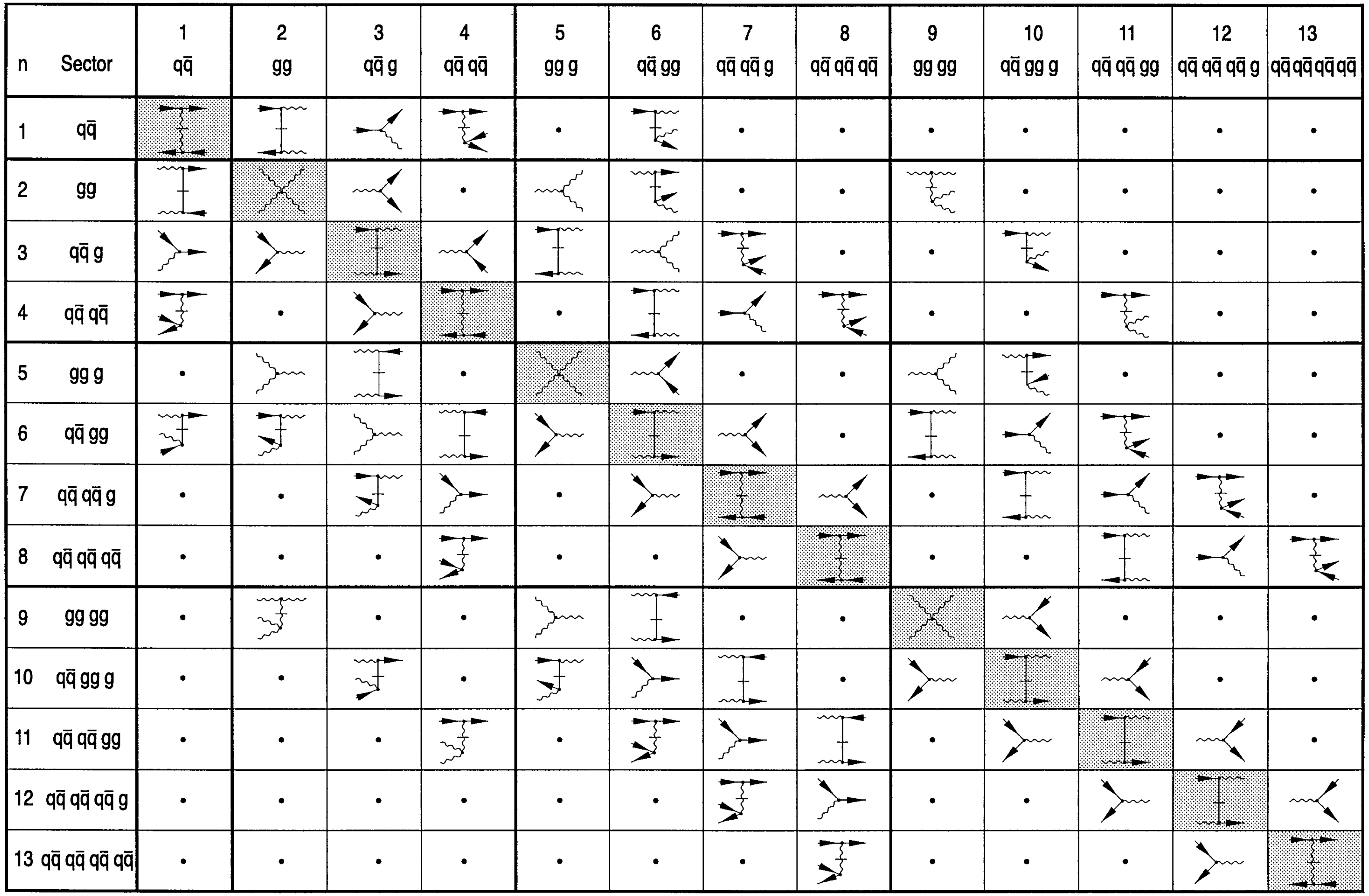
Heisenberg Matrix Formulation

$$H^{QCD}_{LF}|\Psi_h> = \mathcal{M}^2_h|\Psi_h>$$

DLCQ

Discretized Light-Cone Quantization





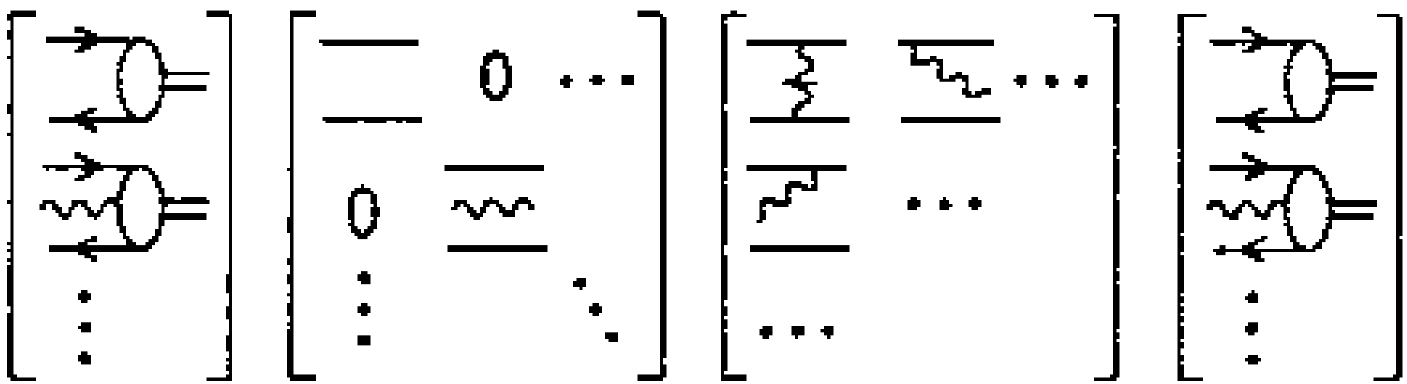
Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

H.C. Pauli & sjb

DLCQ: Frame-independent, No fermion doubling; Minkowski Space

LIGHT-FRONT SCHRODINGER EQUATION

$$\left(M_\pi^2 - \sum_i \frac{\vec{k}^2_{\perp i} + m_i^2}{x_i}\right) \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix} = \begin{bmatrix} \langle q\bar{q}|\, V \,|q\bar{q}\rangle & \langle q\bar{q}|\, V \,|q\bar{q}g\rangle & \cdots \\ \langle q\bar{q}g|\, V \,|q\bar{q}\rangle & \langle q\bar{q}g|\, V \,|q\bar{q}g\rangle & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix}$$



$A^+ = 0$

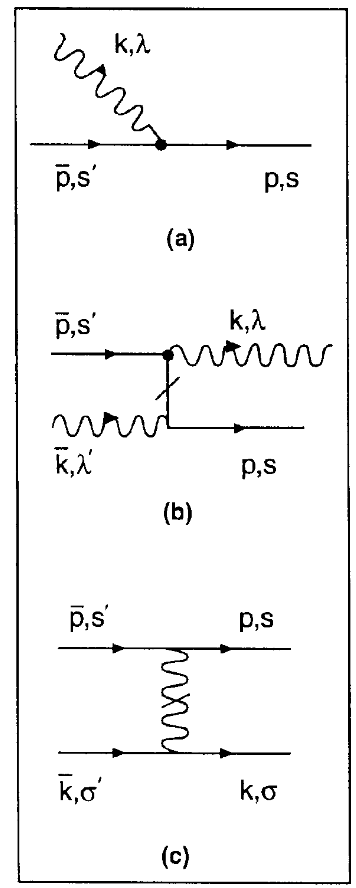
G.P. Lepage, sjb

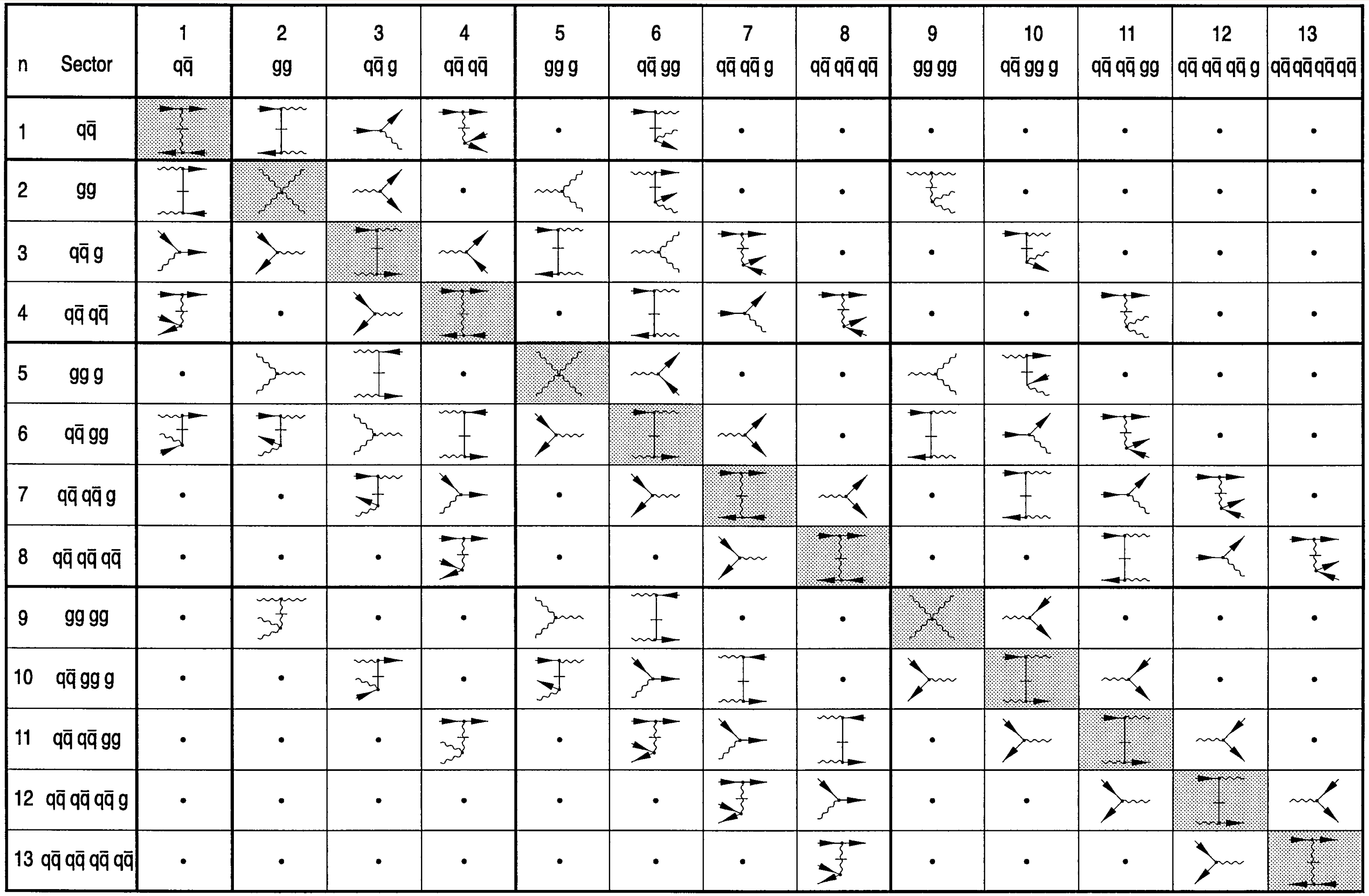
Use AdS/CFT orthonormal LFWFs as a basis for diagonalizing the QCD LF Hamiltonian

- Good initial approximant
- Better than plane wave basis — Pauli, Hornbostel, Hiller, McCartor, sjb
- DLCQ discretization -- highly successful 1+1
- Use independent HO LFWFs, remove CM motion — Vary, Harinandrath, Maris, sjb
- Similar to Shell Model calculations

Light-Front QCD Heisenberg Equation

$$H_{LC}^{QCD} |\Psi_h\rangle = \mathcal{M}_h^2 \, |\Psi_h\rangle$$





Use AdS/QCD basis functions

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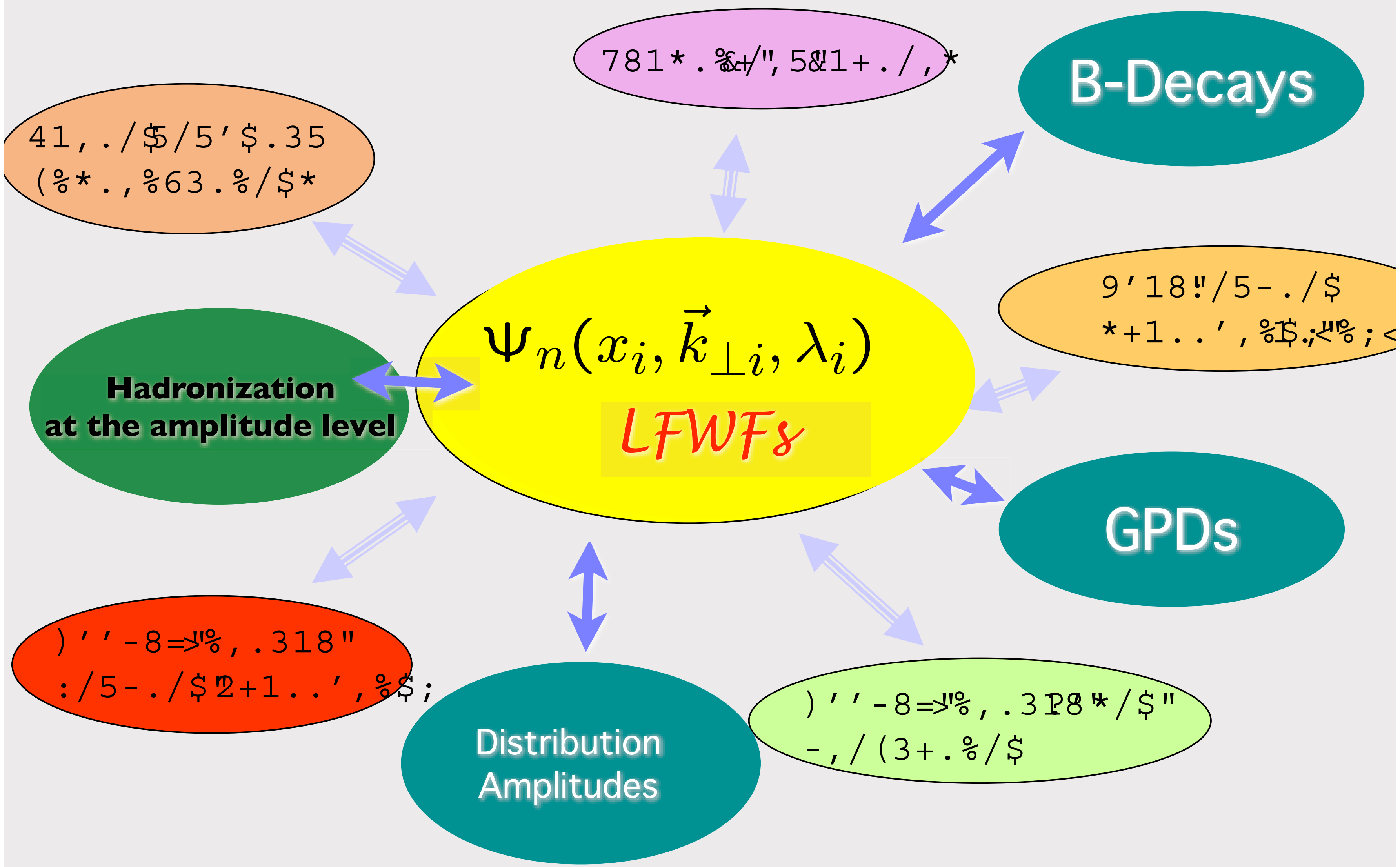

B-Decays
Hadronization
at the amplitude level
$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$
LFWFs
GPDs
Distribution
Amplitudes

Hadron Dynamics at the Amplitude Level

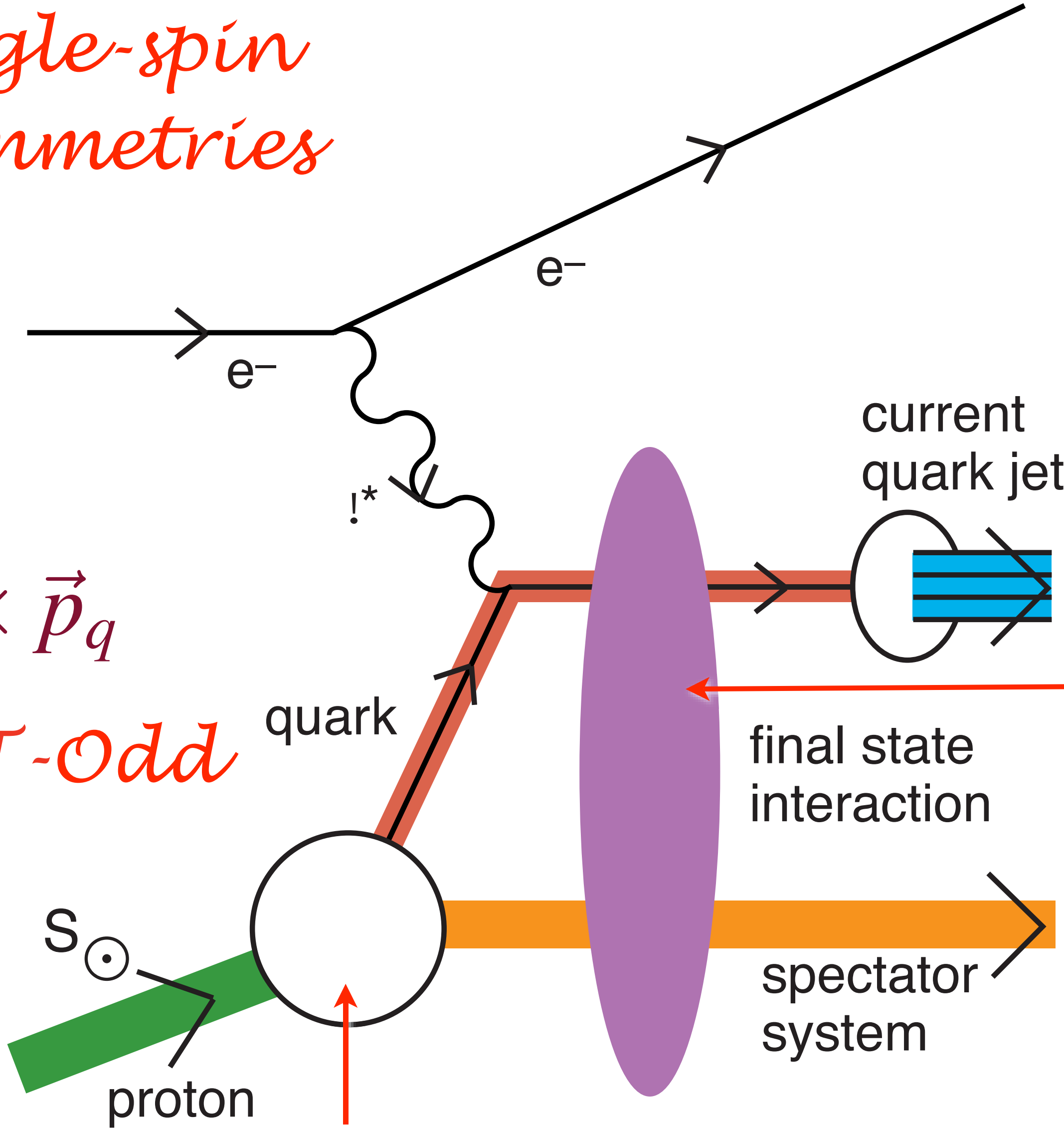
- LFWFS are the universal hadronic amplitudes which underlie structure functions, GPDs, exclusive processes, distribution amplitudes, direct subprocesses, hadronization.
- Relation of spin, momentum, and other distributions to physics of the hadron itself.
- Connections between observables, orbital angular momentum
- Role of FSI and ISIs--Sivers effect

Single-spin asymmetries

Leading-Twist Sivers Effect

$$\mathrm{i}\ \vec{S}_p \cdot \vec{q} \times \vec{p}_q$$

Pseudo-T-Odd



QCD S- and P- Coulomb Phases

Light-Front Wavefunction S and P- Waves

D. S. Hwang, I. A. Schmidt, sjb

- Polchinski & Strassler: AdS/CFT builds in conformal symmetry at short distances; counting rules for form factors and hard exclusive processes; non-perturbative derivation

- Goal: Use AdS/CFT to provide an approximate model of hadron structure with confinement at large distances, conformal behavior at short distances

- de Teramond, sjb: AdS/QCD Holographic Model: Initial "semi-classical" approximation to QCD. Predict light-quark hadron spectroscopy, form factors.

- Karch, Katz, Son, Stephanov: Linear Confinement

- Mapping of AdS amplitudes to 3+ 1 Light-Front equations, wavefunctions

- Use AdS/CFT wavefunctions as expansion basis for diagonalizing H^{LF}_{QCD}; variational methods

AdS/CFT

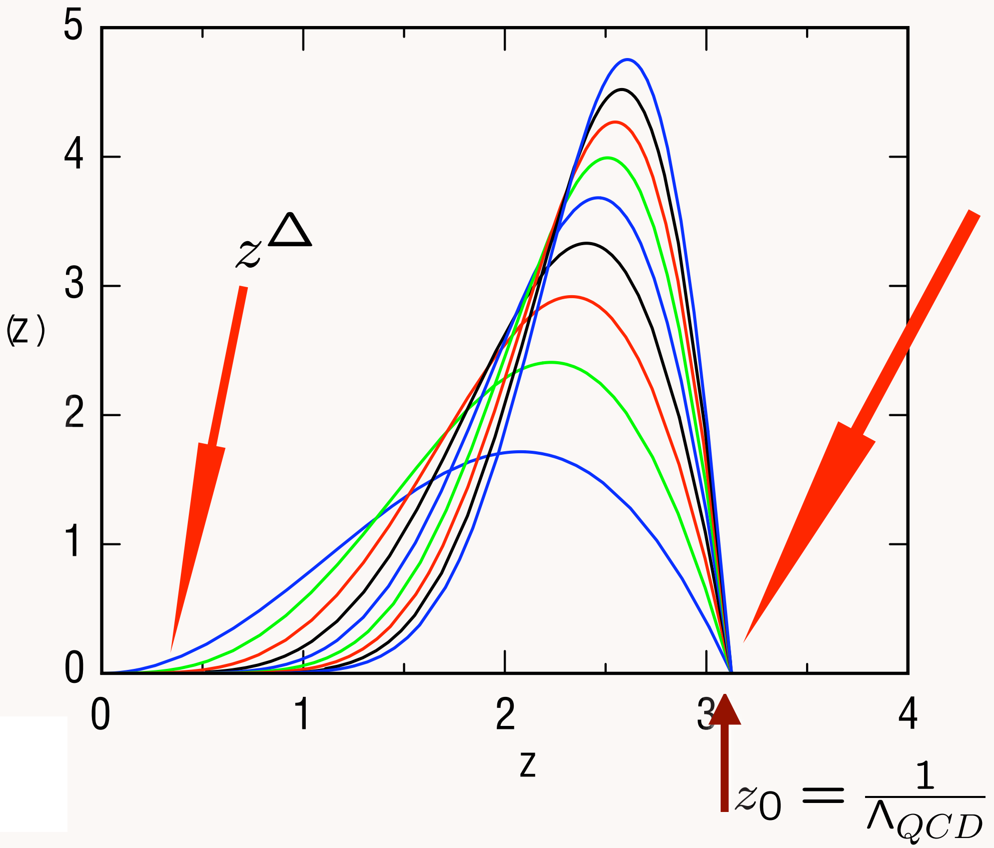
- Use mapping of conformal group SO(4,2) to AdS5
- Scale Transformations represented by wavefunction $\psi(z)$ in 5th dimension $x_\mu^2 \to \lambda^2 x_\mu^2$ $z \to \lambda z$
- Match solutions at small z to conformal dimension of hadron wavefunction at short distances $\psi(z) \sim z^\Delta$ at $z \to 0$
- Hard wall model: Confinement at large distances and conformal symmetry in interior
- Truncated space simulates "bag" boundary conditions

$$0 < z < z_0 \qquad \psi(z_0) = 0 \qquad z_0 = \frac{1}{\Lambda_{QCD}}$$

- Physical AdS modes $\Phi_P(x,z) \sim e^{-iP\cdot x}\,\Phi(z)$ are plane waves along the Poincaré coordinates with four-momentum P^μ and hadronic invariant mass states $P_\mu P^\mu = \mathcal{M}^2$.
- For small-z $\Phi(z) \sim z^\Delta$. The scaling dimension Δ of a normalizable string mode, is the same dimension of the interpolating operator $\mathcal{O}$ which creates a hadron out of the vacuum: $\langle P|\mathcal{O}|0\rangle \neq 0$.

$$\Delta = 2 + L$$

Twist dimension of meson



Confinement in the 5th dimension

de Teramond, sjb

Identify hadron by its interpolating operator at z -- > 0

Bosonic Solutions: Hard Wall Model

- Conformal metric: $ds^2 = g_{\ell m} dx^\ell dx^m$. $x^\ell = (x^\mu, z)$, $g_{\ell m} \to \left(R^2/z^2\right) \eta_{\ell m}$.
- Action for massive scalar modes on AdS$_{d+1}$:

$$S[\Phi] = \frac{1}{2} \int d^{d+1}x \sqrt{g}\, \tfrac{1}{2} \left[g^{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2 \right], \quad \sqrt{g} \to (R/z)^{d+1}.$$

- Equation of motion

$$\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\ell} \left(\sqrt{g}\, g^{\ell m} \frac{\partial}{\partial x^m} \Phi \right) + \mu^2 \Phi = 0.$$

- Factor out dependence along x^μ-coordinates , $\Phi_P(x, z) = e^{-iP\cdot x}\, \Phi(z)$, $P_\mu P^\mu = \mathcal{M}^2$:

$$\left[z^2 \partial_z^2 - (d-1) z\, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi(z) = 0.$$

- Solution: $\Phi(z) \to z^\Delta$ as $z \to 0$,

$$\Phi(z) = C z^{d/2} J_{\Delta - d/2}(z\mathcal{M}) \qquad \Delta = \tfrac{1}{2}\left(d + \sqrt{d^2 + 4\mu^2 R^2}\right).$$

$$\Delta = 2 + L \qquad d = 4 \qquad (\mu R)^2 = L^2 - 4$$

$$\text{Let } \Phi(z) = z^{3/2}\phi(z)$$

AdS Schrodinger Equation for bound state of two scalar constituents:

$$[-\frac{\mathrm{d}^2}{\mathrm{dz}^2} + \mathrm{V(z)}]\phi(\mathrm{z}) = \mathrm{M}^2\phi(\mathrm{z})$$

$$\mathrm{V(z)} = -\frac{1-4\mathrm{L}^2}{4\mathrm{z}^2}$$

Interpret L as orbital angular momentum

Derived from variation of Action in AdS_5

Hard wall model: truncated space

$$\phi(\mathrm{z} = \mathrm{z}_0 = \frac{1}{\Lambda_c}) = 0.$$

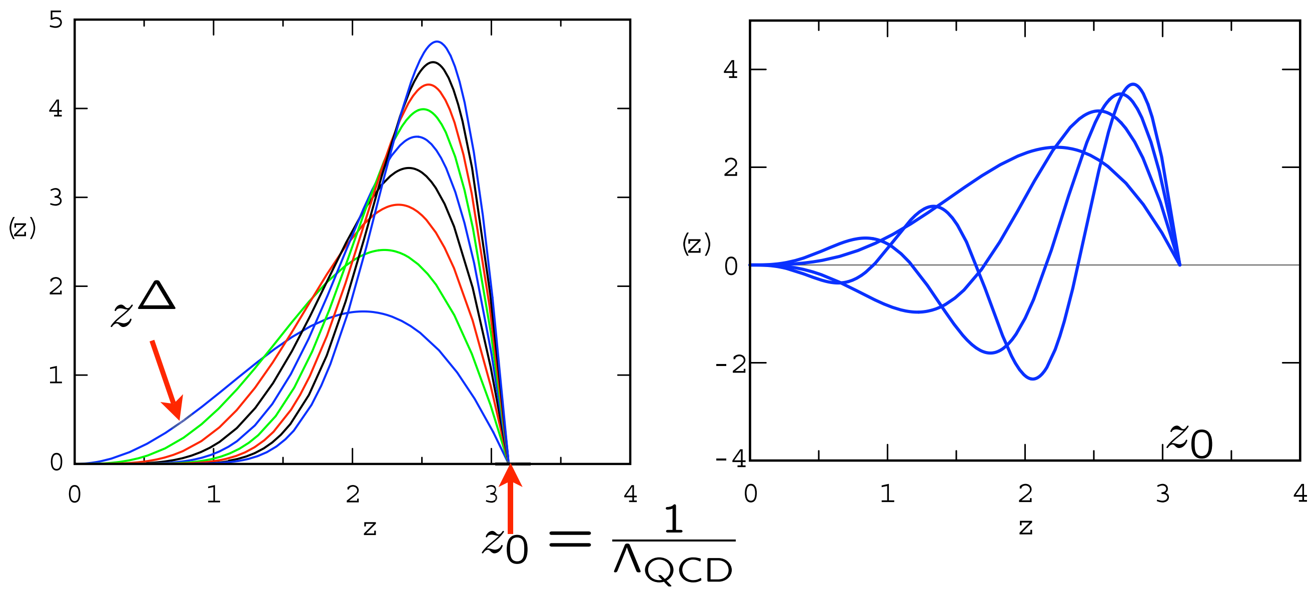
Match fall-off at small z to conformal twist-dimension at short distances

twist

- Pseudoscalar mesons: $\mathcal{O}_{2+L} = \overline{\psi}\gamma_5 D_{\{\ell_1} \dots D_{\ell_m\}}\psi$ ($\Phi_\mu = 0$ gauge). $\Delta = 2 + L$

- 4-d mass spectrum from boundary conditions on the normalizable string modes at $z = z_0$, $\Phi(x, z_o) = 0$, given by the zeros of Bessel functions $\beta_{\alpha,k}$: $\mathcal{M}_{\alpha,k} = \beta_{\alpha,k}\Lambda_{QCD}$

- Normalizable AdS modes $\Phi(z)$



$S = 0$ Meson orbital and radial AdS modes for $\Lambda_{QCD} = 0.32$ GeV.

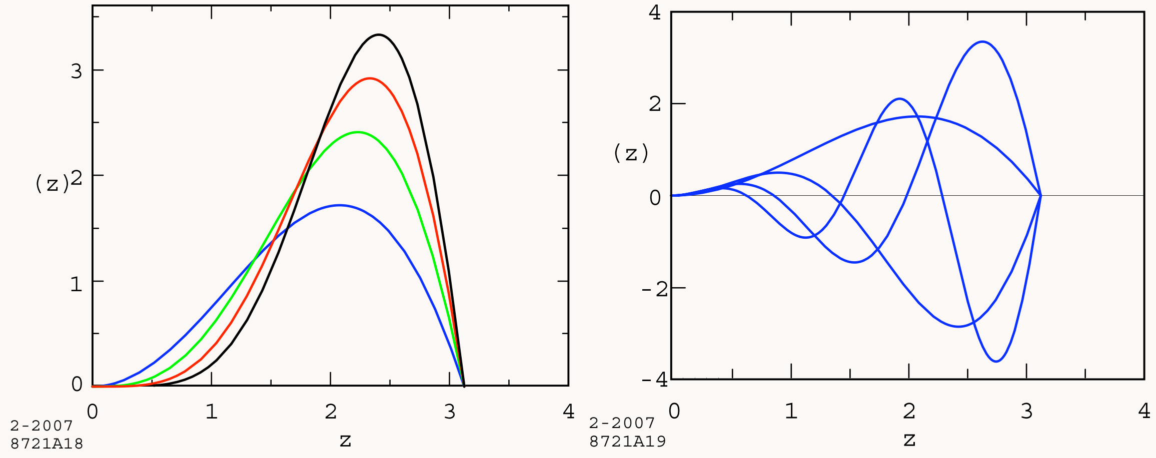


Fig: Orbital and radial AdS modes in the hard wall model for $\Lambda_{\rm QCD}$ = 0.32 GeV .

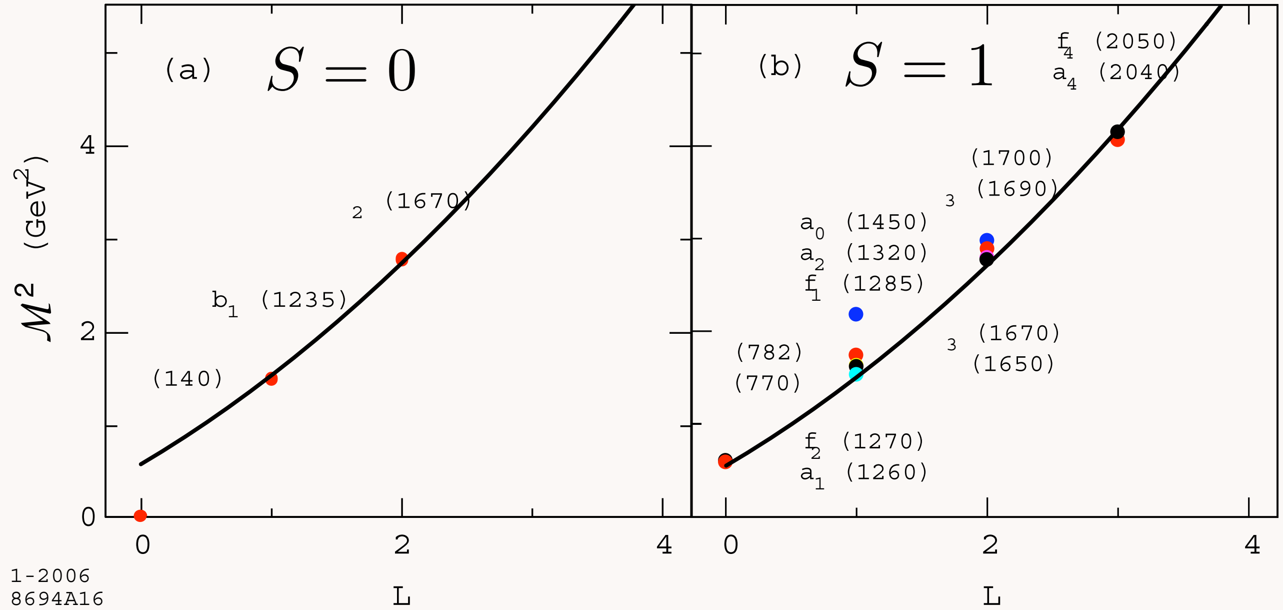


Fig: Light meson and vector meson orbital spectrum $\Lambda_{QCD} = 0.32$ GeV

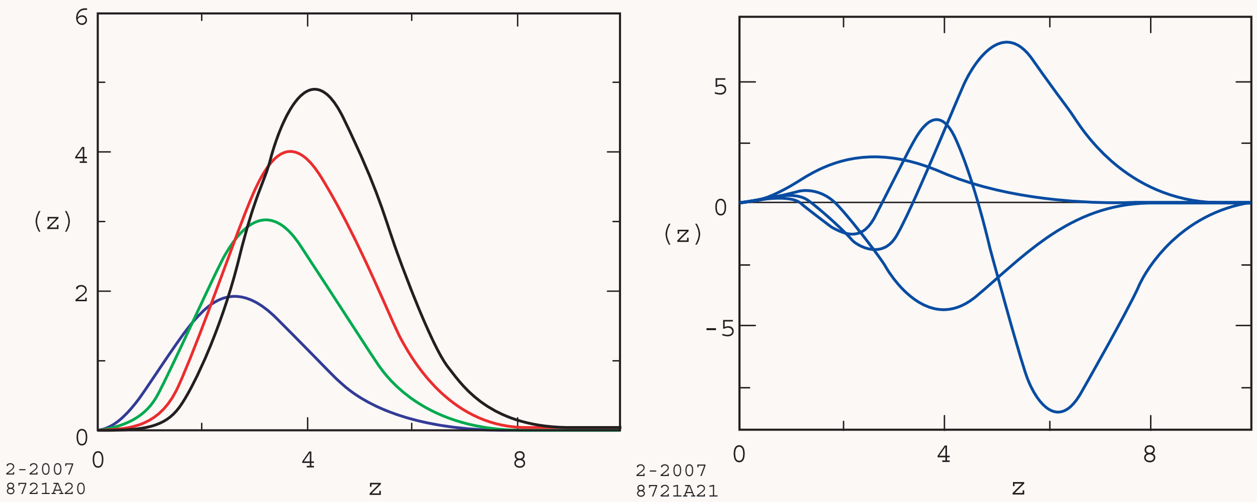
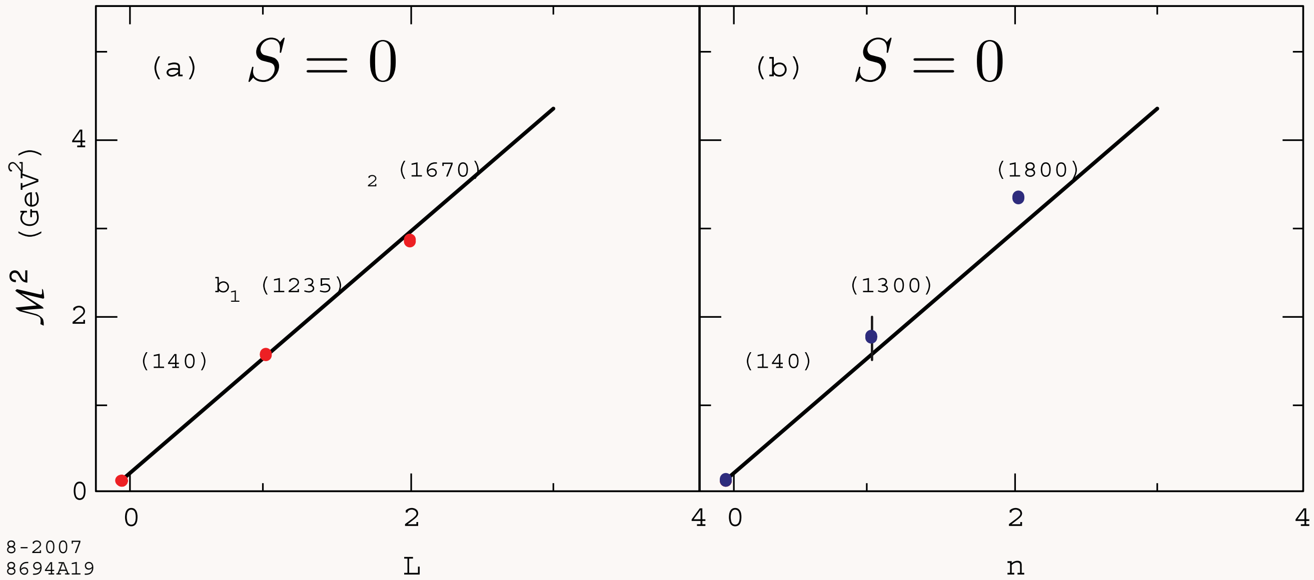


Fig: Orbital and radial AdS modes in the soft wall model for κ = 0.6 GeV .



Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

Higher Spin Bosonic Modes SW

Soft-wall model

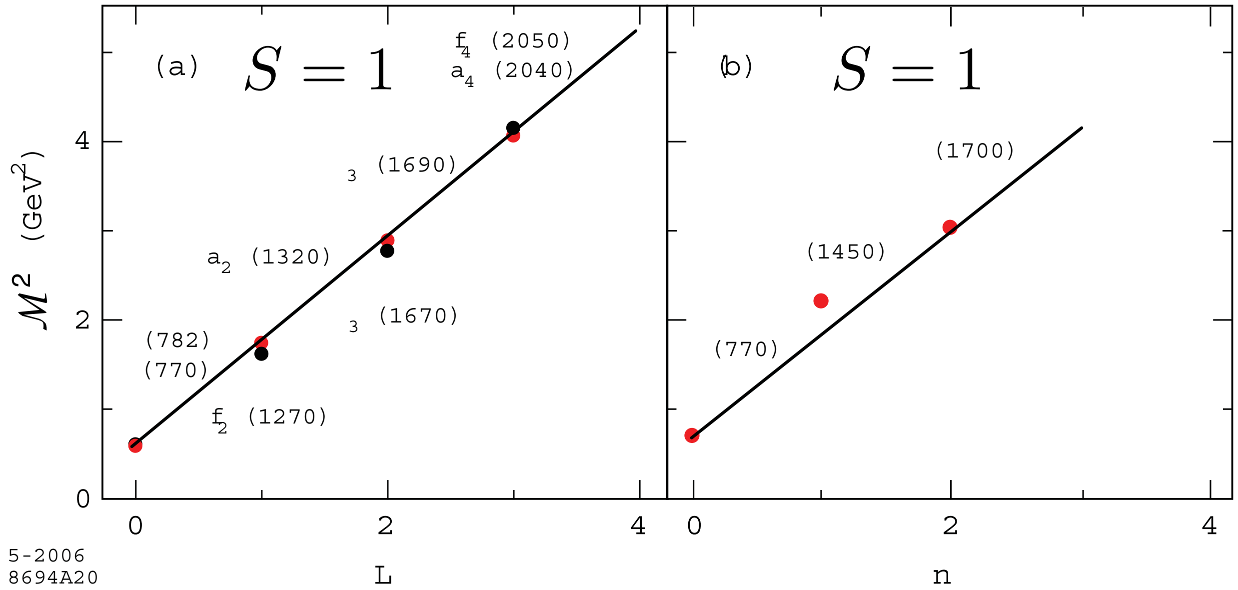
- Effective LF Schrödinger wave equation

$$\left[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} + \kappa^4 z^2 + 2\kappa^2(L+S-1)\right]\phi_S(z) = \mathcal{M}^2\phi_S(z)$$

with eigenvalues $\mathcal{M}^2 = 2\kappa^2(2n + 2L + S)$.

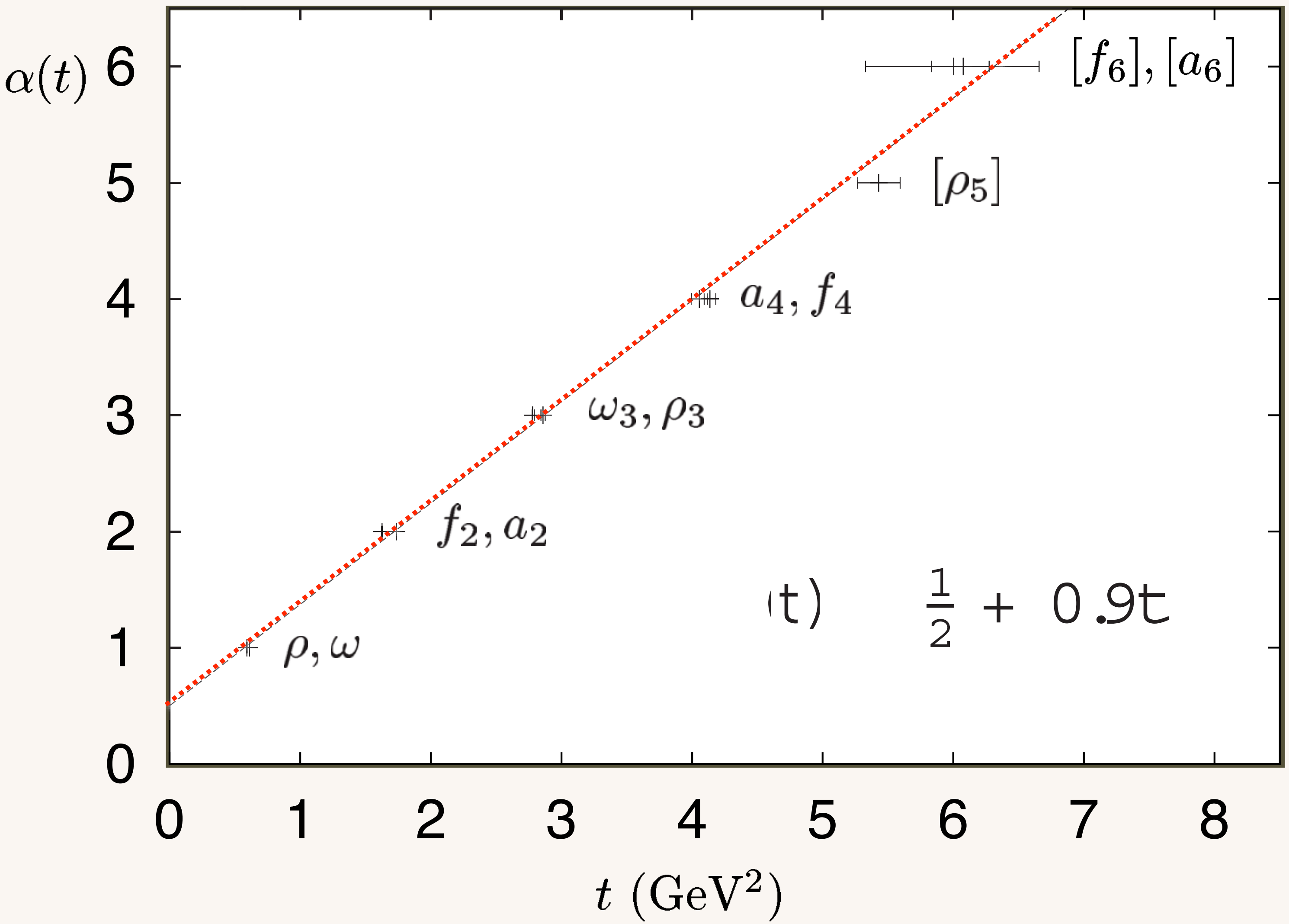
Same slope in n and L

- Compare with Nambu string result (rotating flux tube): $M_n^2(L) = 2\pi\sigma\,(n + L + 1/2)\,.$



Vector mesons orbital (a) and radial (b) spectrum for $\kappa = 0.54$ GeV.

- Glueballs in the bottom-up approach: (HW) Boschi-Filho, Braga and Carrion (2005); (SW) Colangelo, De Facio, Jugeau and Nicotri(2007).



AdS/QCD Soft Wall Model -- Reproduces Linear Regge Trajectories

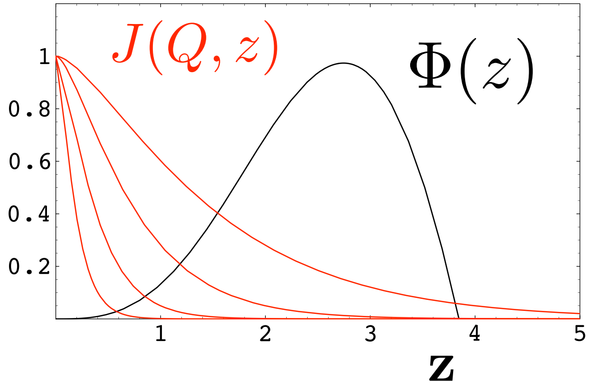
Hadron Form Factors from AdS/CFT

Propagation of external perturbation suppressed inside AdS.

$$J(Q,z) = zQK_1(zQ)$$

$$F(Q^2)_{I\to F} = \int \frac{dz}{z^3}\Phi_F(z)J(Q,z)\Phi_I(z)$$

High Q^2
from
small z ~ 1/Q



Polchinski, Strassler
de Teramond, sjb

Consider a specific AdS mode $\Phi^{(n)}$ dual to an n partonic Fock state $|n\rangle$. At small z, $\Phi^{(n)}$
scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \to \left[\frac{1}{Q^2}\right]^{\tau-1},$$

Dimensional Quark Counting Rule
General result from AdS/CFT

where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^{n}\sigma_i$. The twist is equal to the number of partons, $\tau = n$.

Current Matrix Elements in AdS Space (HW)

- Hadronic matrix element for EM coupling with string mode $\Phi(x^\ell), \; x^\ell = (x^\mu, z)$

$$ig_5 \int d^4x \, dz \, \sqrt{g} \, A^\ell(x,z) \Phi^*_{P'}(x,z) \overleftrightarrow{\partial}_\ell \Phi_P(x,z).$$

- Electromagnetic probe polarized along Minkowski coordinates $(Q^2 = -q^2 > 0)$

$$A(x,z)_\mu = \epsilon_\mu e^{-iQ\cdot x} J(Q,z), \quad A_z = 0.$$

- Propagation of external current inside AdS space described by the AdS wave equation

$$\left[z^2 \partial_z^2 - z \, \partial_z - z^2 Q^2\right] J(Q,z) = 0,$$

subject to boundary conditions $J(Q=0,z) = J(Q, z=0) = 1.$

- Solution

$$J(Q,z) = zQK_1(zQ).$$

- Substitute hadronic modes $\Phi(x,z)$ in the AdS EM matrix element

$$\Phi_P(x,z) = e^{-iP\cdot x} \, \Phi(z), \quad \Phi(z) \to z^\Delta, \quad z \to 0.$$

Current Matrix Elements in AdS Space (SW)

sjb and GdT
Grigoryan and Radyushkin

- Propagation of external current inside AdS space described by the AdS wave equation

$$\left[z^2\partial_z^2 - z\left(1 + 2\kappa^2 z^2\right)\partial_z - Q^2 z^2\right] J_\kappa(Q,z) = 0.$$

- Solution bulk-to-boundary propagator

$$J_\kappa(Q,z) = \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right),$$

where $U(a,b,c)$ is the confluent hypergeometric function

$$\Gamma(a)U(a,b,z) = \int_0^\infty e^{-zt} t^{a-1}(1+t)^{b-a-1} dt.$$

- Form factor in presence of the dilaton background $\varphi = \kappa^2 z^2$

$$F(Q^2) = R^3 \int \frac{dz}{z^3}\, e^{-\kappa^2 z^2}\Phi(z) J_\kappa(Q,z)\Phi(z).$$

- For large $Q^2 \gg 4\kappa^2$

$$J_\kappa(Q,z) \to zQK_1(zQ) = J(Q,z),$$

the external current decouples from the dilaton field.

Space and Time-Like Pion Form Factor

- Hadronic string modes $\Phi_\pi(z) \to z^2$ as $z \to 0$ (twist $\tau = 2$)

$$\Phi_\pi^{HW}(z) = \frac{\sqrt{2}\Lambda_{QCD}}{R^{3/2} J_1(\beta_{0,1})} z^2 J_0\left(z\beta_{0,1}\Lambda_{QCD}\right),$$

$$\Phi_\pi^{SW}(z) = \frac{\sqrt{2}\kappa}{R^{3/2}} z^2.$$

- F_π has analytical solution in the SW model $F_\pi(Q^2) = \frac{4\kappa^2}{4\kappa^2+Q^2}$.

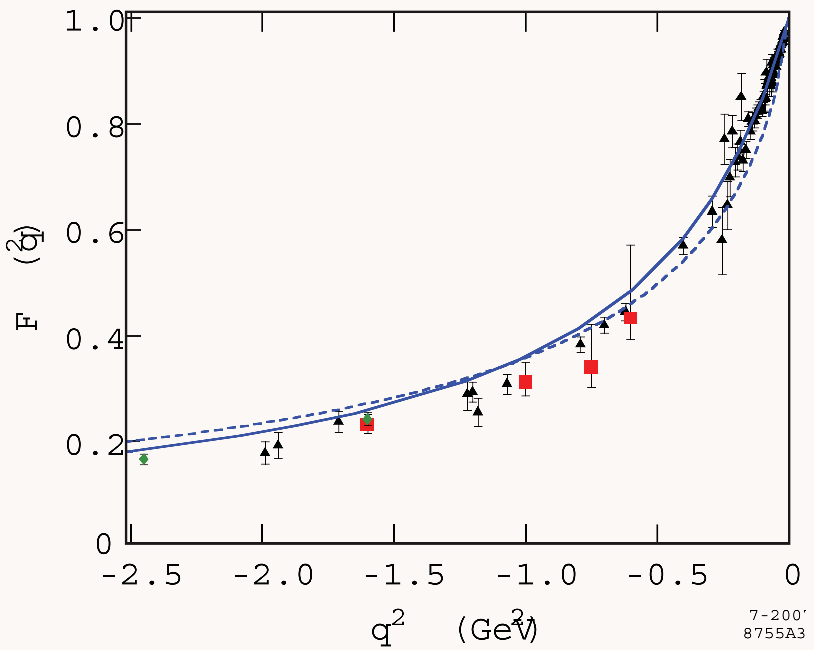


Fig: $F_\pi(q^2)$ for $\kappa = 0.375$ GeV and $\Lambda_{QCD} = 0.22$ GeV. Continuous line: SW, dashed line: HW.

Note: Analytical Form of Hadronic Form Factor for Arbitrary Twist

- Form factor for a string mode with scaling dimension τ, Φ_τ in the SW model

$$F(Q^2) = \Gamma(\tau) \frac{\Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right)}{\Gamma\left(\tau + \frac{Q^2}{4\kappa^2}\right)}.$$

- For $\tau = N$, $\quad \Gamma(N + z) = (N - 1 + z)(N - 2 + z) \ldots (1 + z)\Gamma(1 + z)$.

- Form factor expressed as $N - 1$ product of poles

$$F(Q^2) = \frac{1}{1 + \frac{Q^2}{4\kappa^2}}, \quad N = 2,$$

$$F(Q^2) = \frac{2}{\left(1 + \frac{Q^2}{4\kappa^2}\right)\left(2 + \frac{Q^2}{4\kappa^2}\right)}, \quad N = 3,$$

$$\cdots$$

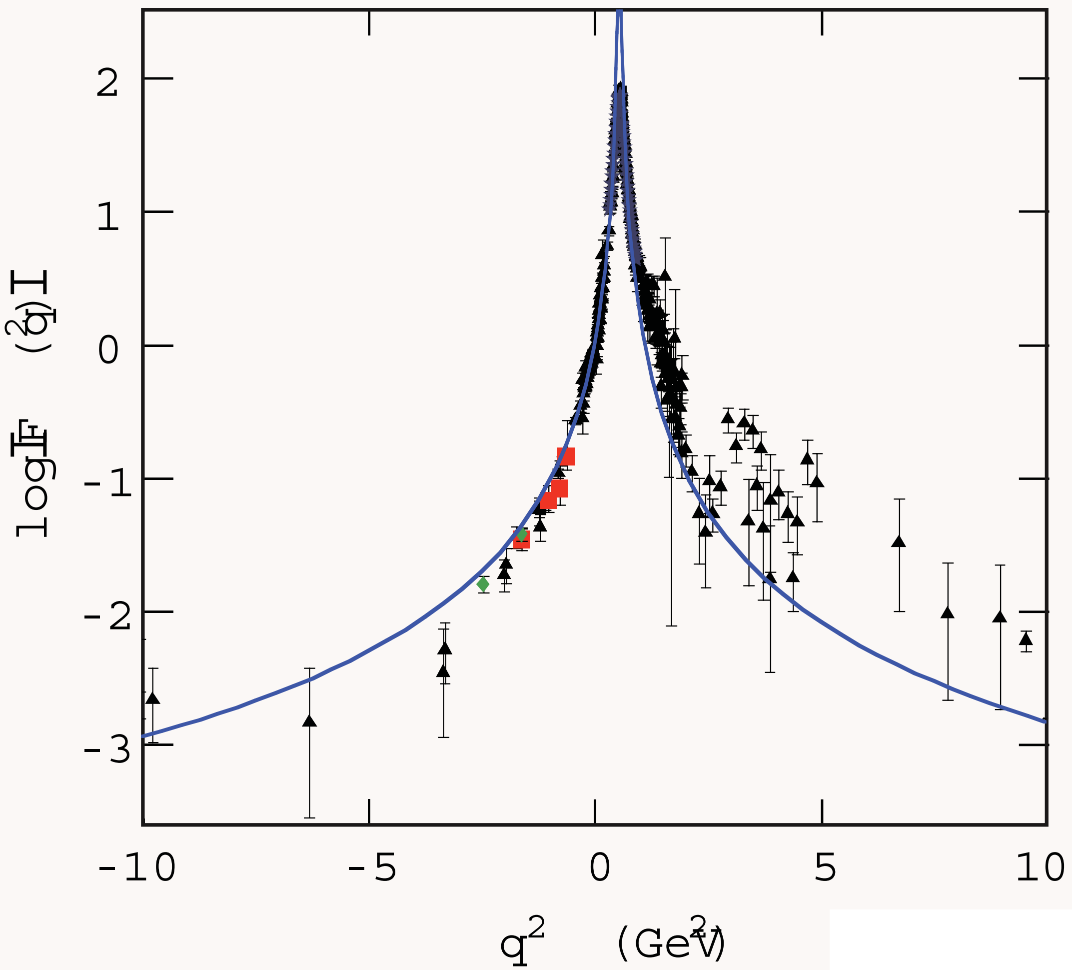
$$F(Q^2) = \frac{(N-1)!}{\left(1 + \frac{Q^2}{4\kappa^2}\right)\left(2 + \frac{Q^2}{4\kappa^2}\right) \cdots \left(N - 1 + \frac{Q^2}{4\kappa^2}\right)}, \quad N.$$

- For large Q^2:

$$F(Q^2) \to (N-1)! \left[\frac{4\kappa^2}{Q^2}\right]^{(N-1)}.$$

- Analytical continuation to time-like region $q^2 \to -q^2$ $\qquad M_\rho = 2\kappa = 750 \text{ MeV}$

- Strongly coupled semiclassical gauge/gravity limit hadrons have zero widths (stable).



Space and time-like pion form factor for $\kappa = 0.375$ GeV in the SW model.

- Vector Mesons: Hong, Yoon and Strassler (2004); Grigoryan and Radyushkin (2007).

Light-Front Representation of Two-Body Meson Form Factor

- Drell-Yan-West form factor

$$F(q^2) = \sum_q e_q \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3}\, \psi^*_{P'}(x, \vec{k}_\perp - x\vec{q}_\perp)\, \psi_P(x, \vec{k}_\perp).$$

- Fourrier transform to impact parameter space $\vec{b}_\perp$

$$\psi(x, \vec{k}_\perp) = \sqrt{4\pi} \int d^2\vec{b}_\perp\, e^{i\vec{b}_\perp \cdot \vec{k}_\perp} \widetilde{\psi}(x, \vec{b}_\perp)$$

- Find ($b = |\vec{b}_\perp|$) :

$$\begin{aligned} F(q^2) &= \int_0^1 dx \int d^2\vec{b}_\perp\, e^{ix\vec{b}_\perp \cdot \vec{q}_\perp} \left|\widetilde{\psi}(x, b)\right|^2 \\ &= 2\pi \int_0^1 dx \int_0^\infty b\, db\, J_0\left(bqx\right)\, \left|\widetilde{\psi}(x, b)\right|^2, \end{aligned}$$

Soper

Holographic Mapping of AdS Modes to QCD LFWFs

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \, \frac{(1-x)}{x} \int \zeta d\zeta J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) \tilde{\rho}(x,\zeta),$$

with $\widetilde{\rho}(x,\zeta)$ QCD effective transverse charge density.

- Transversality variable

$$\zeta = \sqrt{\frac{x}{1-x}} \left| \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j} \right|.$$

- Compare AdS and QCD expressions of FFs for arbitrary Q using identity:

$$\int_0^1 dx J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right) = \zeta Q K_1(\zeta Q),$$

the solution for $J(Q,\zeta) = \zeta Q K_1(\zeta Q)$!

- Electromagnetic form-factor in AdS space:

$$F_{\pi^+}(Q^2) = R^3 \int \frac{dz}{z^3} \, J(Q^2, z) \, |\Phi_{\pi^+}(z)|^2 \, ,$$

where $J(Q^2, z) = zQK_1(zQ)$.

- Use integral representation for $J(Q^2, z)$

$$J(Q^2, z) = \int_0^1 dx \, J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right)$$

- Write the AdS electromagnetic form-factor as

$$F_{\pi^+}(Q^2) = R^3 \int_0^1 dx \int \frac{dz}{z^3} \, J_0\left(zQ\sqrt{\frac{1-x}{x}}\right) |\Phi_{\pi^+}(z)|^2$$

- Compare with electromagnetic form-factor in light-front QCD for arbitrary Q

$$\boxed{\left|\tilde{\psi}_{q\overline{q}/\pi}(x, \zeta)\right|^2 = \frac{R^3}{2\pi} \, x(1-x) \frac{|\Phi_\pi(\zeta)|^2}{\zeta^4}}$$

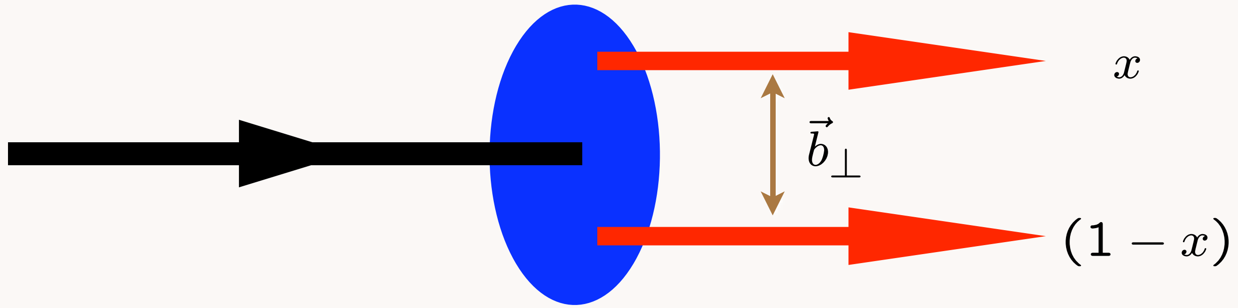
with $\zeta = z, \; 0 \leq \zeta \leq \Lambda_{\rm QCD}$

LF(3+1) $\quad\quad$ AdS$_5$

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \longleftrightarrow z$$

$\psi(x, \vec{b}_\perp)$



$$\psi(x, \zeta) = \sqrt{x(1-x)}\,\zeta^{-1/2}\phi(\zeta)$$

Light-Front Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements

Gravitational Form Factor of Composite Hadrons

- Gravitational FF defined by matrix elements of the energy momentum tensor $\Theta^{++}(x)$

$$\left\langle P' \left| \Theta^{++}(0) \right| P \right\rangle = 2 \left(P^+\right)^2 A(Q^2)$$

- $\Theta^{\mu\nu}$ is computed for each constituent in the hadron from the QCD Lagrangian

$$\mathcal{L}_{\rm QCD} = \overline{\psi} \left(i\gamma^\mu D_\mu - m\right) \psi - \tfrac{1}{4} G^a_{\mu\nu} G^{a\,\mu\nu}$$

- Symmetric and gauge invariant $\Theta^{\mu\nu}$ from variation of $S_{\rm QCD} = \int d^4x \sqrt{g}\, \mathcal{L}_{\rm QCD}$ with respect to four-dim Minkowski metric $g_{\mu\nu}$, $\Theta^{\mu\nu}(x) = -\frac{2}{\sqrt{g}} \frac{\delta S_{\rm QCD}}{\delta g_{\mu\nu}(x)}$:

$$\Theta^{\mu\nu} = \tfrac{1}{2} \overline{\psi} i (\gamma^\mu D^\nu + \gamma^\nu D^\mu)\, \psi - g^{\mu\nu} \overline{\psi} \left(i \not{D} - m\right) \psi - G^{a\,\mu\lambda} G^{a\,\nu}{}_{\lambda} + \tfrac{1}{4} g^{\mu\nu} G^a_{\mu\nu} G^{a\,\mu\nu}$$

- Quark contribution in light front gauge ($A^+ = 0,\ g^{++} = 0$)

$$\Theta^{++}(x) = \frac{i}{2} \sum_f \overline{\psi}^f(x) \gamma^+ \overleftrightarrow{\partial}^+ \psi^f(x)$$

Gravitational Form Factor on the LF

$$A_f(q^2) = \int_0^1 \textcircled{x}\, dx \int d^2\vec{\eta}_\perp e^{i\vec{\eta}_\perp\cdot\vec{q}_\perp} \tilde{\rho}_f(x, \vec{\eta}_\perp),$$

where

$$\tilde{\rho}_f(x, \vec{\eta}_\perp) = \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} e^{-i\vec{\eta}_\perp\cdot\vec{q}_\perp} \rho_f(x, \vec{q}_\perp)$$

$$= \sum_n \prod_{j=1}^{n-1} \int dx_j\, d^2\vec{b}_{\perp j}\, \delta\Big(1 - x - \sum_{j=1}^{n-1} x_j\Big)$$

$$\times \delta^{(2)}\Big(\sum_{j=1}^{n-1} x_j \vec{b}_{\perp j} - \vec{\eta}_\perp\Big) \Big|\tilde{\psi}_n(x_j, \vec{b}_{\perp j})\Big|^2 .$$

Extra factor of x relative to charge form factor

For each quark and

Integrate over angle

$$A_f(q^2) = 2\pi \int_0^1 dx\, (1-x) \int \zeta d\zeta\, J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) \tilde{\rho}_f(x, \zeta)$$

$$\zeta = \sqrt{\frac{x}{1-x}} \Big| \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j} \Big|$$

Gravitational Form Factor in AdS space

- Hadronic gravitational form-factor in AdS space

$$A_\pi(Q^2) = R^3 \int \frac{dz}{z^3} H(Q^2, z) \left|\Phi_\pi(z)\right|^2 ,$$

Abidin & Carlson

where $H(Q^2, z) = \frac{1}{2} Q^2 z^2 K_2(zQ)$

- Use integral representation for $H(Q^2, z)$

$$H(Q^2, z) = 2 \int_0^1 x \, dx \, J_0\left(zQ \sqrt{\frac{1-x}{x}}\right)$$

- Write the AdS gravitational form-factor as

$$A_\pi(Q^2) = 2R^3 \int_0^1 x \, dx \int \frac{dz}{z^3} J_0\left(zQ \sqrt{\frac{1-x}{x}}\right) \left|\Phi_\pi(z)\right|^2$$

- Compare with gravitational form-factor in light-front QCD for arbitrary Q

$$\boxed{\left|\tilde{\psi}_{q\bar{q}/\pi}(x, \zeta)\right|^2 = \frac{R^3}{2\pi} x(1-x) \frac{\left|\Phi_\pi(\zeta)\right|^2}{\zeta^4},}$$

Identical to LF Holography obtained from electromagnetic current

$$H(Q^2, z) = 2\int_0^1 x\,dx\,J_0\left(zQ\sqrt{\frac{1-x}{x}}\right).$$

$$A(Q^2) = 2\,R^3 \int x\,dx \int \frac{dz}{z^3}\,J_0\left(zQ\sqrt{\frac{1-x}{x}}\right)|\Phi(z)|^2.$$ **AdS**

Compare with gravitational form factor from LF

$$A(Q^2) = 2\pi \int_0^1 dx\,(1-x) \int \zeta d\zeta\,J_0\left(\zeta Q\sqrt{\frac{1-x}{x}}\right)\tilde\rho(x,\zeta)$$ **LF**

Holography: identify AdS and LF density for all Q

$$\tilde\rho(x,\zeta) = 2\,\frac{R^3}{2\pi}\,\frac{x}{1-x}\,\frac{|\Phi(\zeta)|^2}{\zeta^4}.$$

with

$$\zeta \equiv z \qquad \zeta = \sqrt{\frac{x}{1-x}}\left|\sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j}\right|$$

Holographic result for LFWF identical for electroweak and gravity couplings! Highly nontrivial consistency test

AdS/QCD can predict

- Momentum fractions for each quark flavor and the gluons $A_f(0) = < x_f >, \sum_f A_f(0) = A(0) = 1$
- Orbital Angular Momentum for each quark flavor and the gluons $B_f(0) = < L^3_f >, \sum_f B_f(0) = B(0) = 0$
- Vanishing Anomalous Gravitomagnetic Moment
- Shape and Asymptotic Behavior of $A_f(Q^2), B_f(Q^2)$

Consider the AdS_5 metric:

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2).$$

ds^2 invariant if $x^\mu \to \lambda x^\mu, \; z \to \lambda z,$

Maps scale transformations to scale changes of the the holographic coordinate z.

We define light-front coordinates $x^\pm = x^0 \pm x^3$.

Then $\eta^{\mu\nu}dx_\mu dx_\nu = {dx_0}^2 - {dx_3}^2 - {dx_\perp}^2 = dx^+dx^- - {dx_\perp}^2$

and

$ds^2 = -\frac{R^2}{z^2}({dx_\perp}^2 + dz^2)$ for $x^+ = 0$.

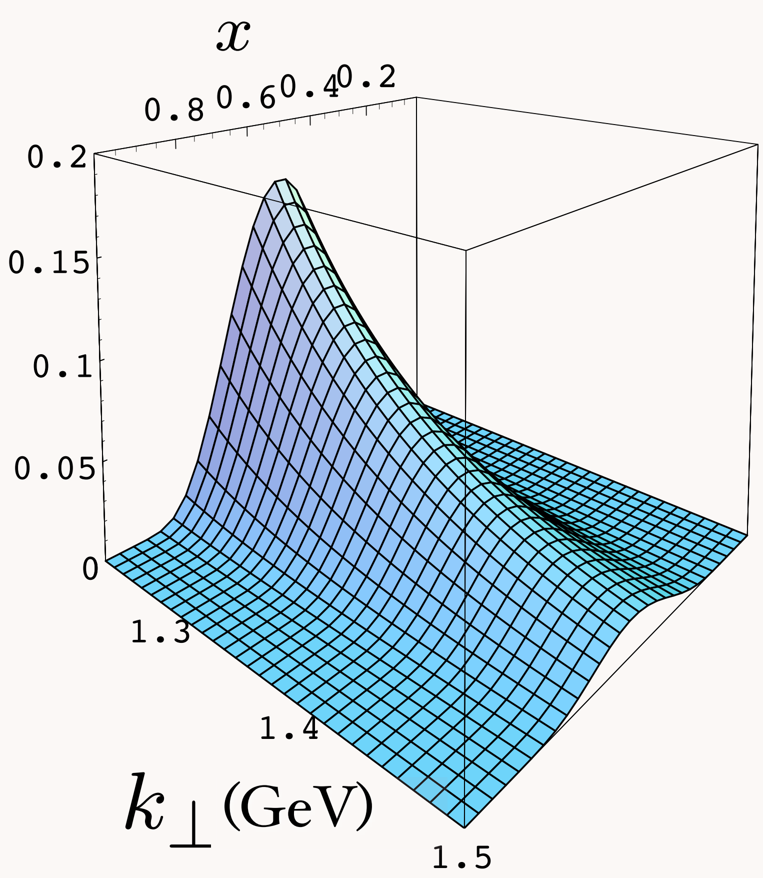
Light-Front AdS_5 Duality

- ds^2 is invariant if ${dx_\perp}^2 \to \lambda^2 {dx_\perp}^2$, and $z \to \lambda z$, at equal LF time.
- Maps scale transformations in transverse LF space to scale changes of the holographic coordinate z.
- Holographic connection of AdS_5 to the light-front.
- The effective wave equation in the two-dim transverse LF plane has the Casimir representation L^2 corresponding to the $SO(2)$ rotation group [The Casimir for $SO(N) \sim S^{N-1}$ is $L(L+N-2)$].

Prediction from AdS/CFT: Meson LFWF

de Teramond, sjb

$\psi_M(x, k_\perp^2)$



"Soft Wall" model

$\kappa = 0.375$ GeV

massless quarks

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}} \qquad \phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$

Example: Pion LFWF

- Two parton LFWF bound state:

$$\widetilde{\psi}^{HW}_{\overline{q}q/\pi}(x,\mathbf{b}_\perp) = \frac{\Lambda_{\rm QCD}\sqrt{x(1-x)}}{\sqrt{\pi}J_{1+L}(\beta_{L,k})} J_L\left(\sqrt{x(1-x)}\,|\mathbf{b}_\perp|\beta_{L,k}\Lambda_{\rm QCD}\right)\theta\left(\mathbf{b}_\perp^2 \le \frac{\Lambda^{-2}_{\rm QCD}}{x(1-x)}\right),$$

$$\widetilde{\psi}^{SW}_{\overline{q}q/\pi}(x,\mathbf{b}_\perp) = \kappa^{L+1}\sqrt{\frac{2n!}{(n+L)!}}\,[x(1-x)]^{\frac{1}{2}+L}|\mathbf{b}_\perp|^L e^{-\frac{1}{2}\kappa^2 x(1-x)\mathbf{b}_\perp^2} L_n^L\left(\kappa^2 x(1-x)\mathbf{b}_\perp^2\right).$$

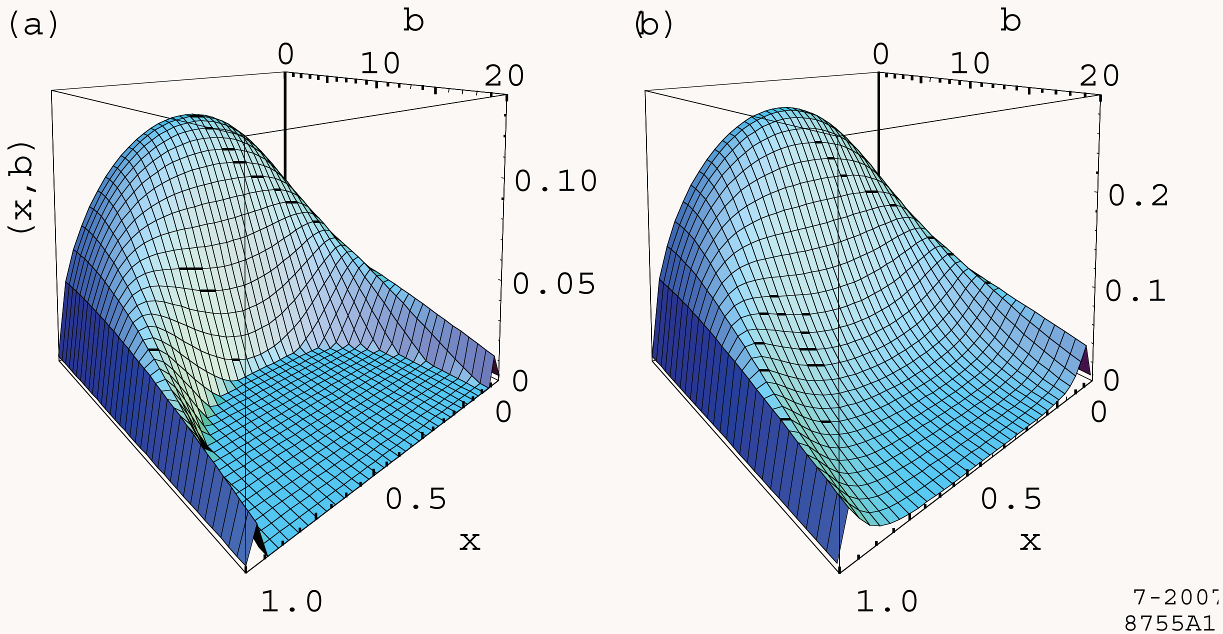


Fig: Ground state pion LFWF in impact space. (a) HW model $\Lambda_{\rm QCD} = 0.32$ GeV, (b) SW model $\kappa = 0.375$ GeV.

Example: Evaluation of QCD Matrix Elements

- Pion decay constant f_π defined by the matrix element of EW current J^+_W:

$$\left\langle 0 \left| \overline{\psi}_u \gamma^+ \tfrac{1}{2}(1-\gamma_5)\psi_d \right| \pi^- \right\rangle = i \frac{P^+ f_\pi}{\sqrt{2}}$$

with

$$\left|\pi^-\right\rangle = |d\overline{u}\rangle = \frac{1}{\sqrt{N_C}} \frac{1}{\sqrt{2}} \sum_{c=1}^{N_C} \left(b^\dagger_{c\, d\downarrow} d^\dagger_{c\, u\uparrow} - b^\dagger_{c\, d\uparrow} d^\dagger_{c\, u\downarrow} \right) |0\rangle .$$

- Find light-front expression (Lepage and Brodsky '80):

$$f_\pi = 2\sqrt{N_C} \int_0^1 dx \int \frac{d^2 \vec{k}_\perp}{16\pi^3}\, \psi_{\overline{q}q/\pi}(x, k_\perp).$$

- Using relation between AdS modes and QCD LFWF in the $\zeta \to 0$ limit

$$f_\pi = \frac{1}{8}\sqrt{\frac{3}{2}}\, R^{3/2} \lim_{\zeta \to 0} \frac{\Phi(\zeta)}{\zeta^2}.$$

- Holographic result ($\Lambda_{\rm QCD} = 0.22$ GeV and $\kappa = 0.375$ GeV from pion FF data): Exp: $f_\pi = 92.4$ MeV

$$f_\pi^{HW} = \frac{\sqrt{3}}{8 J_1(\beta_{0,k})} \Lambda_{\rm QCD} = 91.7 \,\text{MeV}, \;\; f_\pi^{SW} = \frac{\sqrt{3}}{8} \kappa = 81.2 \,\text{MeV},$$

Second Moment of Pion Distribution Amplitude

$$< \xi^2 >= \int_{-1}^{1} d\xi \; \xi^2 \phi(\xi)$$

$$\xi = 1 - 2x$$

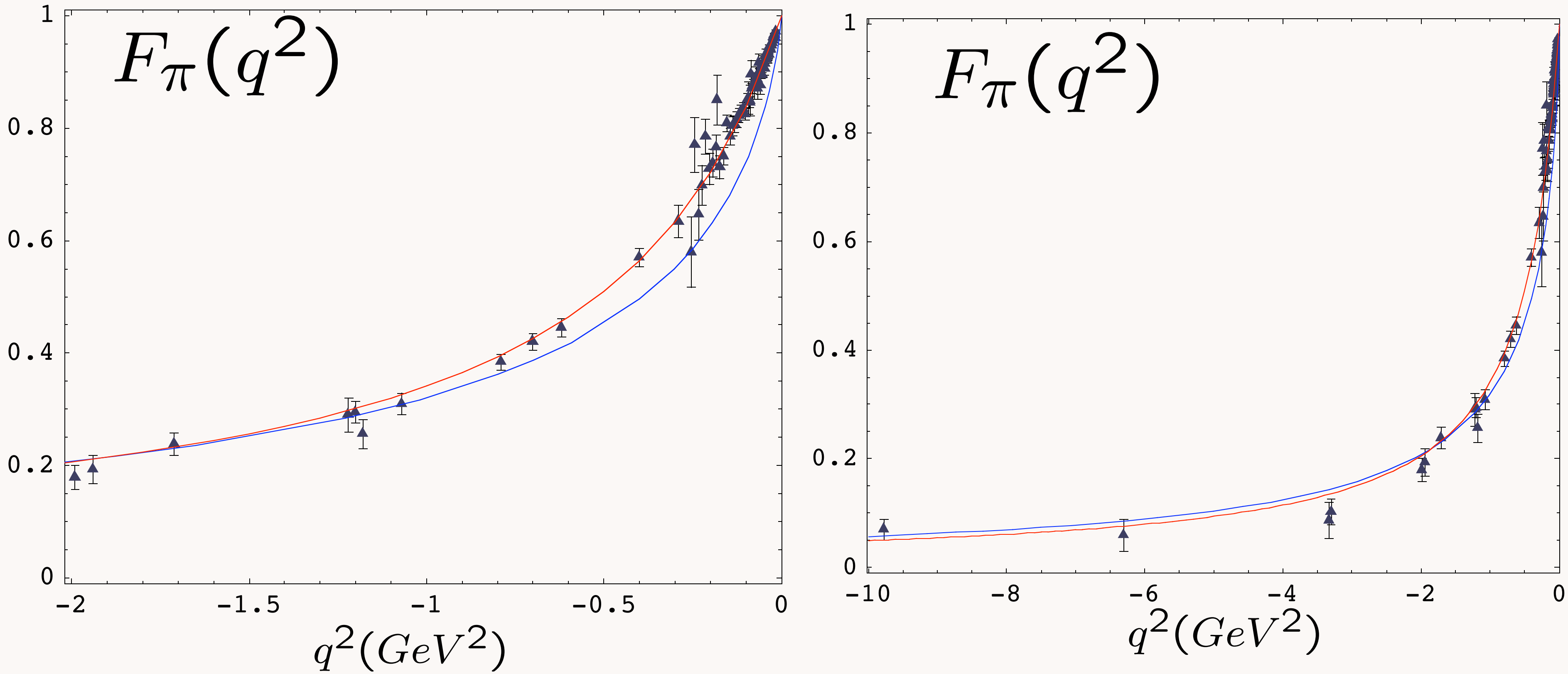
$$< \xi^2 >_\pi = 1/5 = 0.20 \qquad \phi_{asympt} \propto x(1-x)$$

$$< \xi^2 >_\pi = 1/4 = 0.25 \qquad \phi_{AdS/QCD} \propto \sqrt{x(1-x)}$$

Lattice (I) $< \xi^2 >_\pi = 0.28 \pm 0.03$ — Donnellan et al.

Lattice (II) $< \xi^2 >_\pi = 0.269 \pm 0.039$ — Braun et al.

Spacelike pion form factor from AdS/CFT



Data Compilation from Baldini, Kloe and Volmer

SW: Harmonic Oscillator Confinement

HW: Truncated Space Confinement

One parameter - set by pion decay constant

de Teramond, sjb

Note: Contributions to Mesons Form Factors at Large Q in AdS/QCD

- Write form factor in terms of an effective partonic transverse density in impact space $\mathbf{b}_\perp$

$$F_\pi(q^2) = \int_0^1 dx \int db^2 \, \widetilde{\rho}(x, b, Q),$$

with $\widetilde{\rho}(x, b, Q) = \pi J_0 \left[b\, Q(1-x) \right] |\widetilde{\psi}(x, b)|^2$ and $b = |\mathbf{b}_\perp|$.

- Contribution from $\rho(x, b, Q)$ is shifted towards small $|\mathbf{b}_\perp|$ and large $x \to 1$ as Q increases.

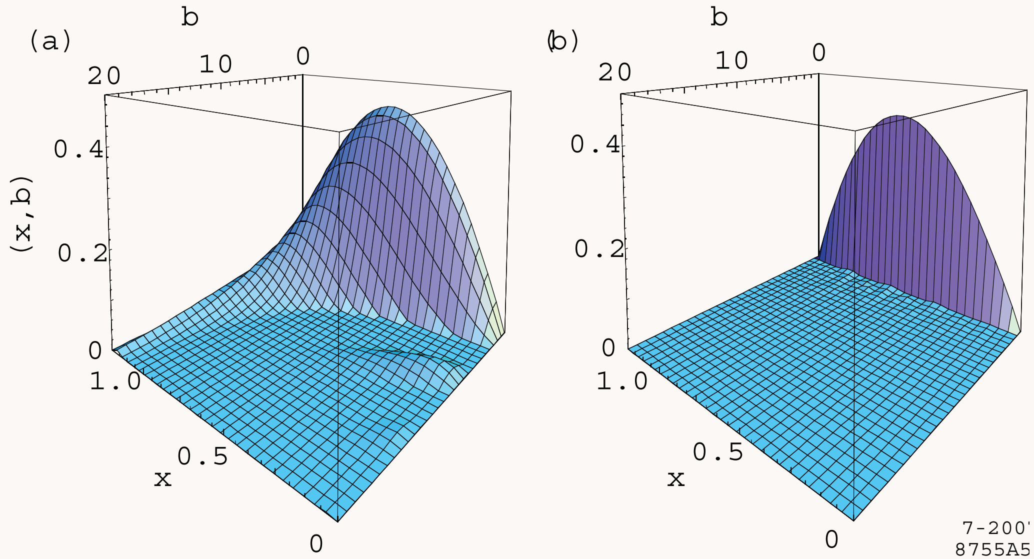


Fig: LF partonic density $\rho(x, b, Q)$: (a) Q = 1 GeV/c, (b) very large Q.

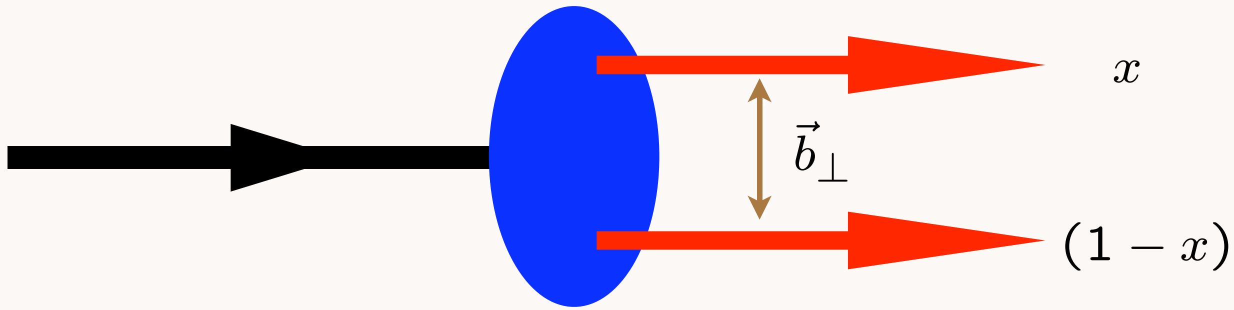
Holography:
Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation   Frame Independent

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

G. de Teramond, sjb

$$\zeta^2 = x(1-x)\mathbf{b}^2_\perp.$$



Effective conformal potential:

$$V(\zeta) = -\frac{1-4L^2}{4\zeta^2}.$$

$$+\kappa^4\zeta^2$$

confining potential:

- Light-front Hamiltonian equation

$$H_{LF}|\phi\rangle = \mathcal{M}^2|\phi\rangle,$$

leads to effective LF Schrödinger wave equation (KKSS)

$$\left[-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4\zeta^2 + 2\kappa^2(L-1)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

with eigenvalues $\mathcal{M}^2 = 4\kappa^2(n+L)$ and eigenfunctions

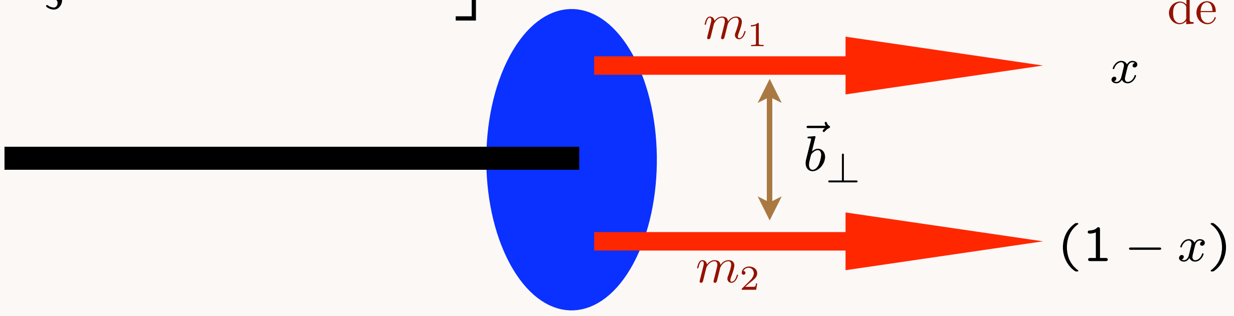
$$\phi_L(\zeta) = \kappa^{1+L}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{1/2+L}e^{-\kappa^2\zeta^2/2}L_n^L\left(\kappa^2\zeta^2\right).$$

- Transverse oscillator in the LF plane with $SO(2)$ rotation subgroup has Casimir L^2 representing rotations for the transverse coordinates $\mathbf{b}_\perp$ in the LF.

- SW model is a remarkable example of integrability to a non-conformal extension of AdS/CFT [Chim and Zamolodchikov (1992) - Potts Model.]

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

de Teramond, sjb



$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$ *Holographic Variable*

$$-\frac{d}{d\zeta^2} \equiv \frac{k_\perp^2}{x(1-x)}$$ *LF Kinetic Energy in momentum space*

Assume LFWF is a dynamical function of the quark-antiquark invariant mass squared

$$-\frac{d}{d\zeta^2} \to -\frac{d}{d\zeta^2} + \frac{m_1^2}{x} + \frac{m_2^2}{1-x} \equiv \frac{k_\perp^2 + m_1^2}{x} + \frac{k_\perp^2 + m_2^2}{1-x}$$

Result: Soft-Wall LFWF for massive constituents

$$\psi(x, \mathbf{k}_\perp) = \frac{4\pi c}{\kappa\sqrt{x(1-x)}} e^{-\frac{1}{2\kappa^2}\left(\frac{\mathbf{k}_\perp^2}{x(1-x)} + \frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right)}$$

LF WF in impact space: soft-wall model with massive quarks

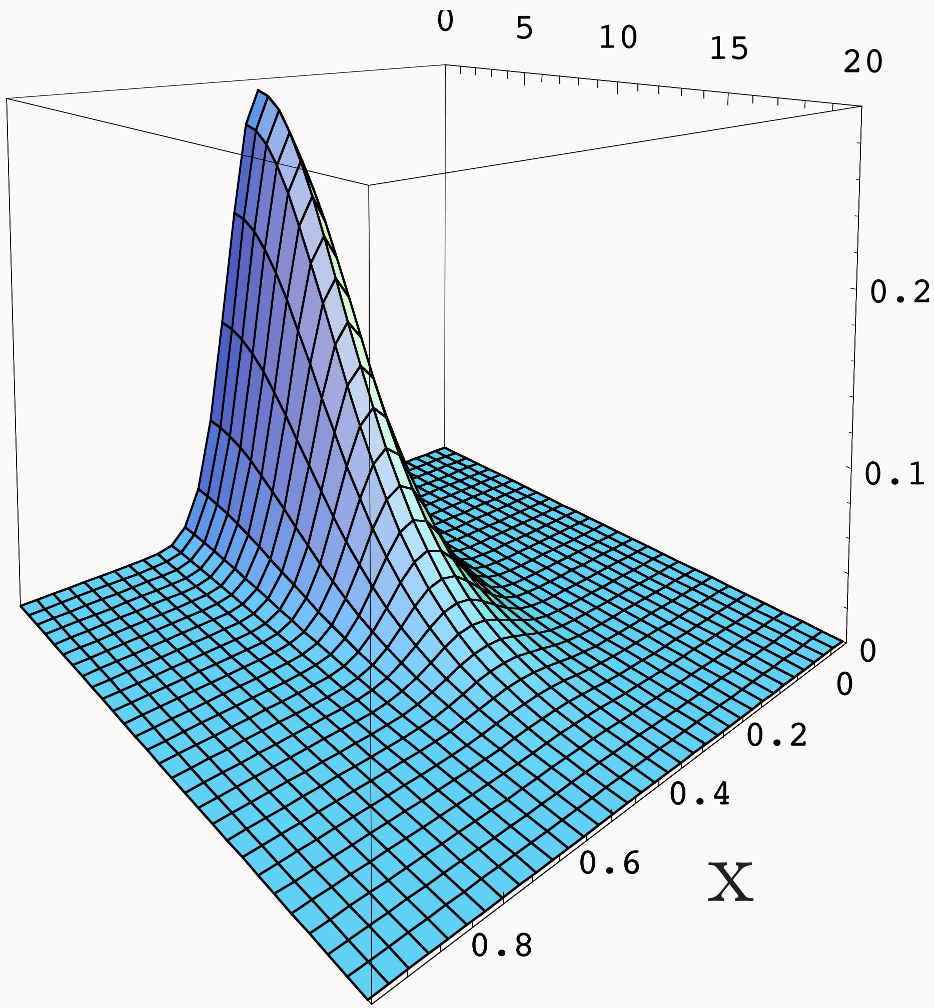
$$\psi(x, \mathbf{b}_\perp) = \frac{c\,\kappa}{\sqrt{\pi}} \sqrt{x(1-x)}\, e^{-\frac{1}{2}\kappa^2 x(1-x)\mathbf{b}_\perp^2 - \frac{1}{2\kappa^2}\left[\frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right]}$$

$$z \to \zeta \to \chi$$

$$\chi^2 = b^2 x(1-x) + \frac{1}{\kappa^4}\left[\frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right]$$

J/ψ

$\psi_{J/\psi}(x,b)$



LFWF peaks at

$$x_i = \frac{m_{\perp i}}{\sum_j^n m_{\perp j}}$$

where

$$m_{\perp i} = \sqrt{m^2 + k_\perp^2}$$

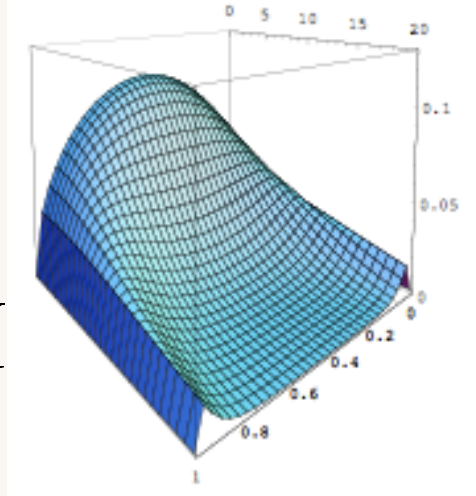
minimum of LF energy denominator

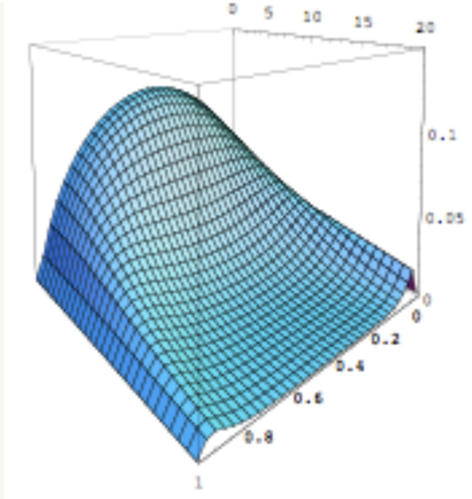
$$\kappa = 0.375 \text{ GeV}$$

$$m_a = m_b = 1.25 \text{ GeV}$$

$|\pi^+ >= |u\bar{d} >$

$m_u = 2\ MeV$
$m_d = 5\ MeV$



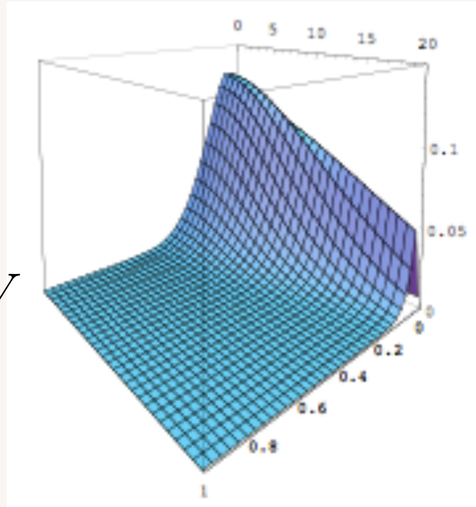


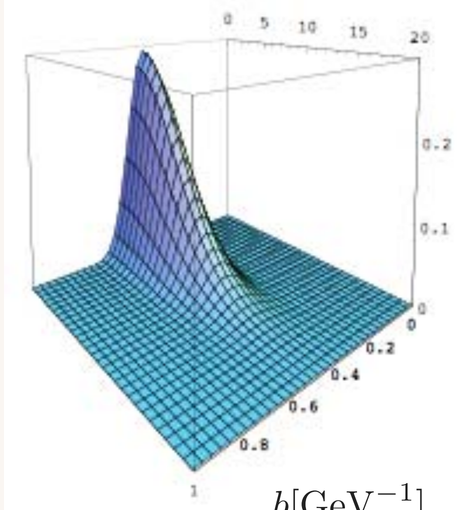
$|K^+ >= |u\bar{s} >$

$m_s = 95\ MeV$

$|D^+ >= |c\bar{d} >$

$m_c = 1.25\ GeV$

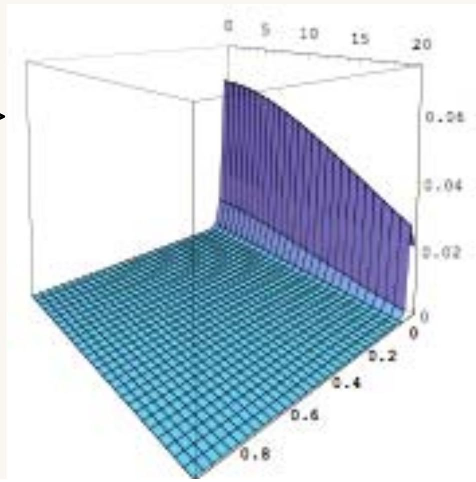


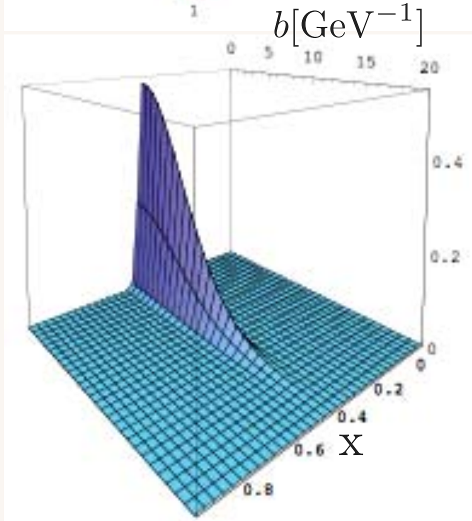


$|\eta_c >= |c\bar{c} >$

$|B^+ >= |u\bar{b} >$

$m_b = 4.2\ GeV$





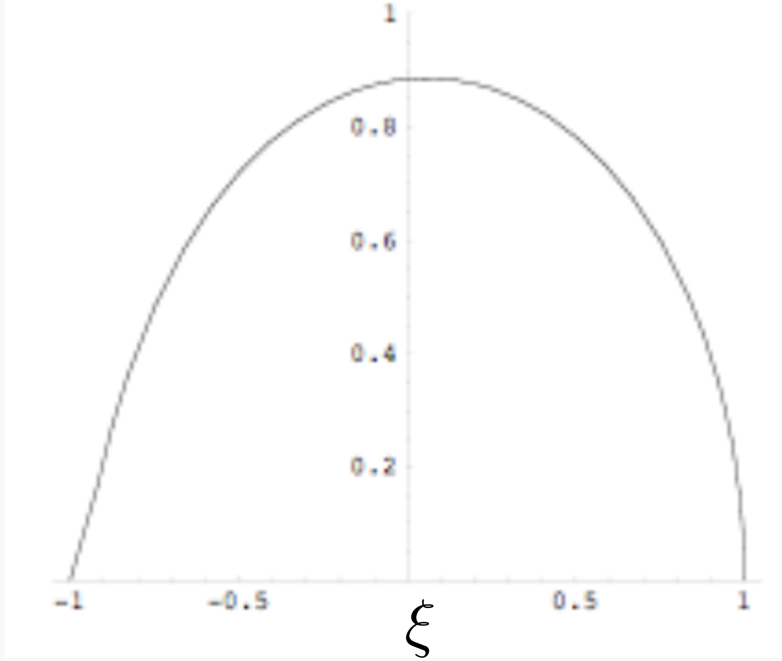
$|\eta_b >= |b\bar{b} >$

$\kappa = 375\ MeV$

First Moment of Kaon Distribution Amplitude

$$< \xi > = \int_{-1}^{1} d\xi \; \xi \; \phi(\xi)$$

$$\xi = 1 - 2x$$



$$< \xi >_K = 0.04 \pm 0.02 \qquad \kappa = 375 \; MeV$$

Range from $m_s = 65 \pm 25$ MeV (PDG)

$< \xi >_K = 0.029 \pm 0.002$ Donnellan et al.

$< \xi >_K = 0.0272 \pm 0.0005$ Braun et al.

AdS/QCD $m_s = 65 \pm 25$ MeV (PDG)

M	$\langle \xi \rangle_M$		$\langle \xi^2 \rangle_M$
π			0.25
K	$0.04 \pm 0.02\,^a$		0.235 ± 0.005^a
D	0.71	AdS/QCD	0.54
η_c			0.02
B	0.96		0.91
η_b			0.002
π			0.28 ± 0.03^b
K	$0.029 \pm 0.002\,^b$		$0.27 \pm 0.02\,^b$
π		Lattice	0.269 ± 0.039^c
K	$0.0272 \pm 0.0005\,^c$		$0.260 \pm 0.006\,^c$

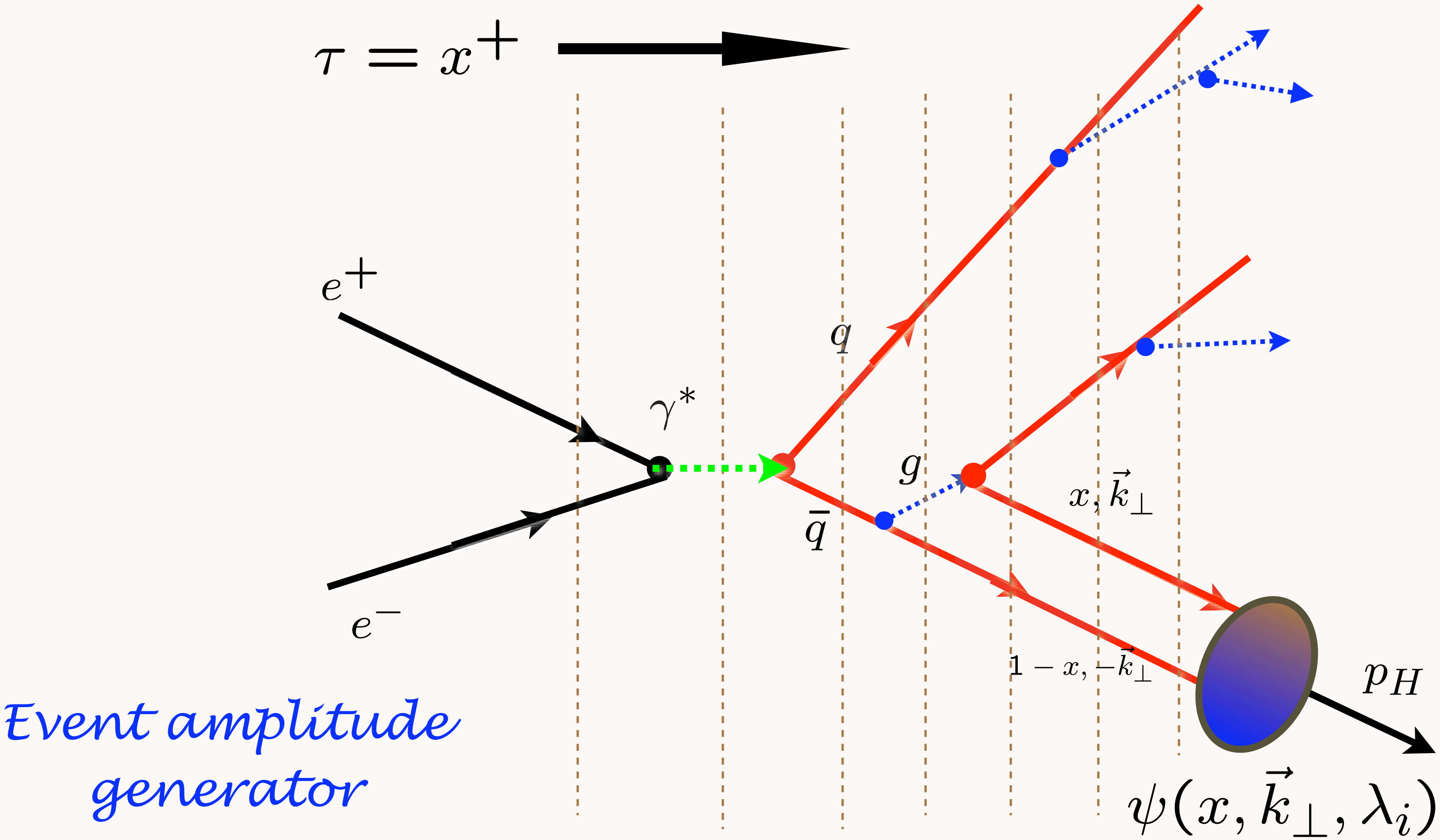
M. A. Donnellan *et al.*, "Lattice Results for Vector Meson Couplings and Parton Distribution Amplitudes," arXiv:0710.0869 [hep-lat].

b: Lattice

V. M. Braun *et al.*, "Moments of pseudoscalar meson distribution amplitudes from the lattice," Phys. Rev. D **74**, 074501 (2006) [arXiv:hep-lat/0606012].

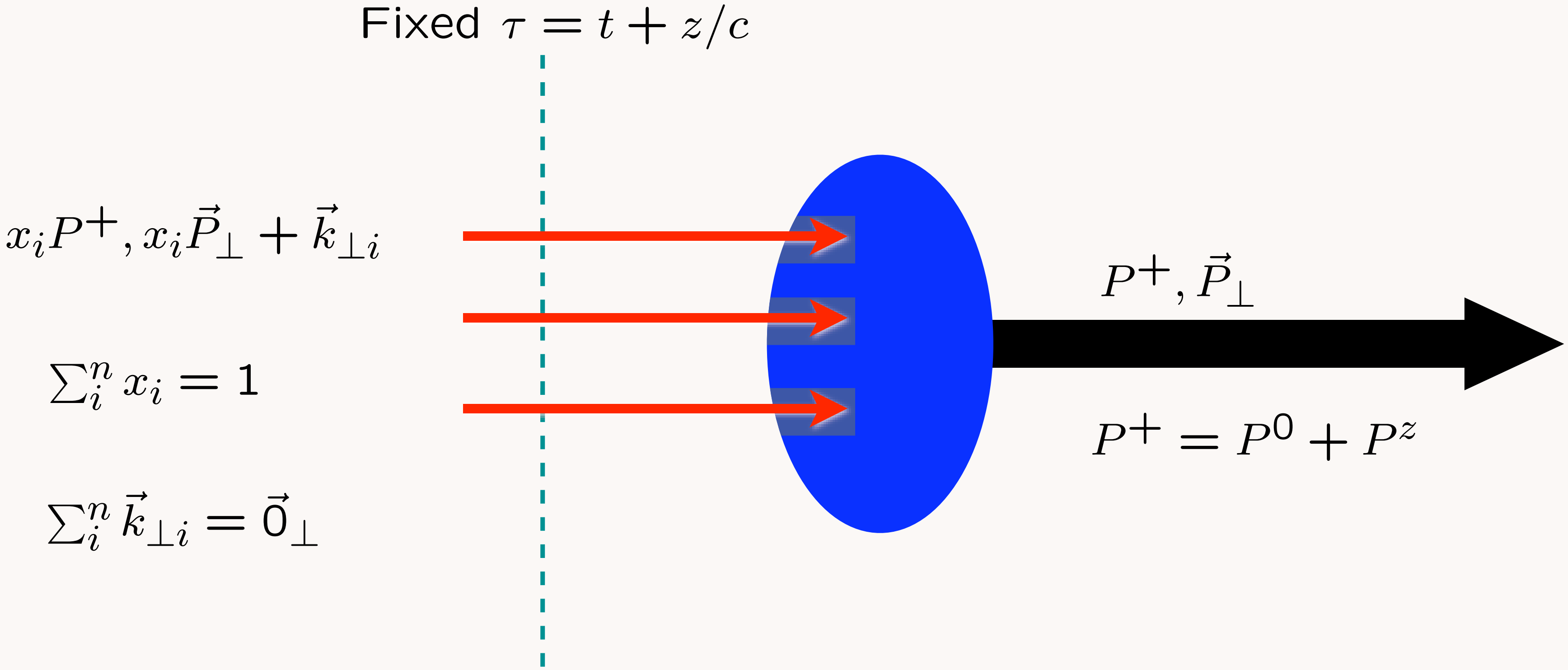
c: Lattice

Hadronization at the Amplitude Level



Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

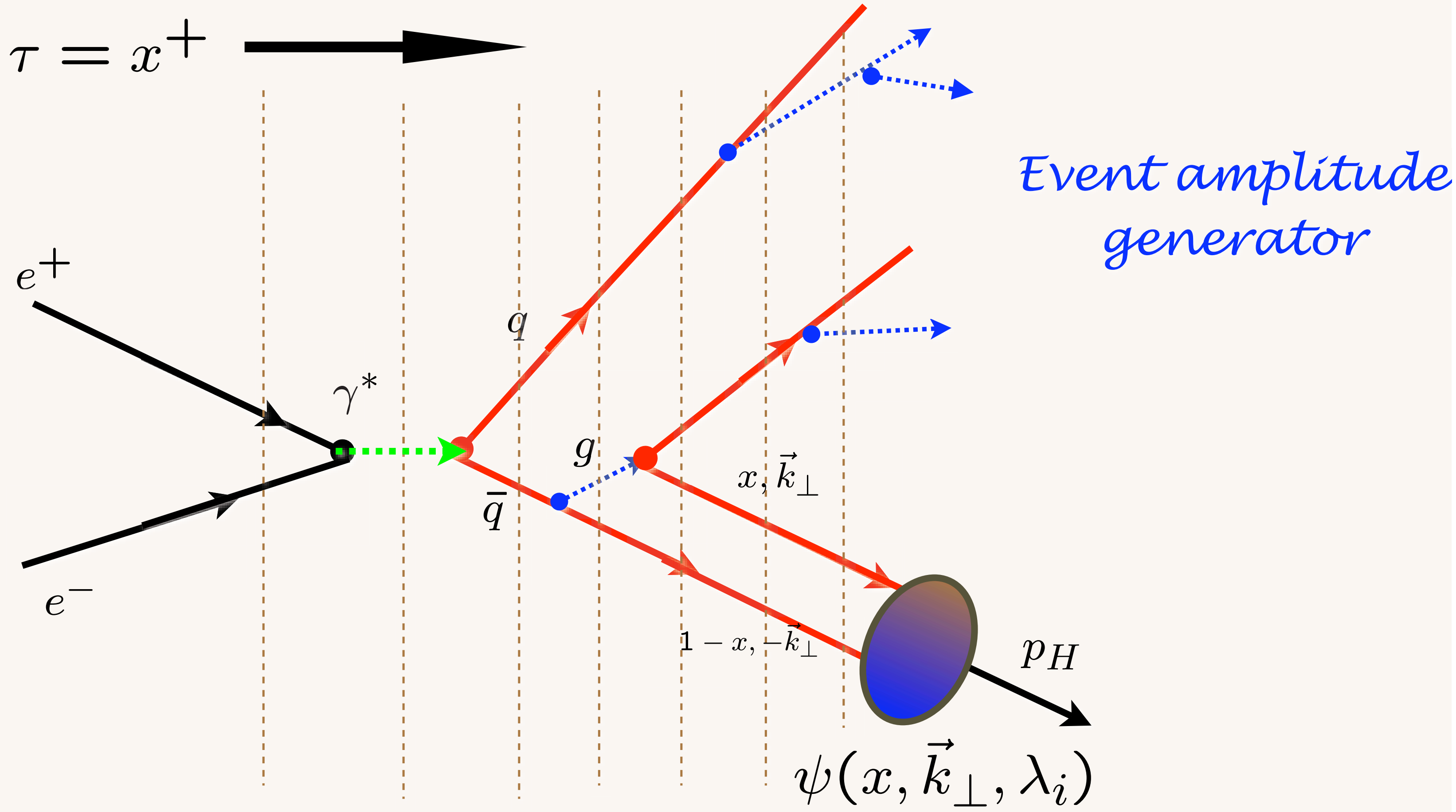
Light-Front Wavefunctions



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Invariant under boosts! Independent of P^μ

Hadronization at the Amplitude Level



AdS/QCD Hard Wall Confinement:

Capture if $\zeta^2 = x(1-x)b_\perp^2 > \frac{1}{\Lambda^2_{QCD}}$

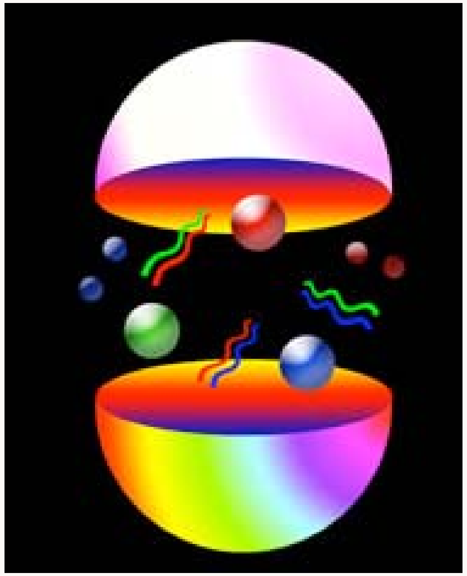
i.e.,

$\mathcal{M}^2 = \frac{k_\perp^2}{x(1-x)} < \Lambda^2_{QCD}$

- Baryons Spectrum in ”bottom-up” holographic QCD

  GdT and Brodsky: hep-th/0409074, hep-th/0501022.

Baryons in Ads/CFT



- Action for massive fermionic modes on AdS$_{d+1}$:

$$S[\overline{\Psi}, \Psi] = \int d^{d+1}x \sqrt{g}\, \overline{\Psi}(x,z) \left(i\Gamma^\ell D_\ell - \mu \right) \Psi(x,z).$$

- Equation of motion: $\left(i\Gamma^\ell D_\ell - \mu \right) \Psi(x,z) = 0$

$$\left[i \left(z\eta^{\ell m}\Gamma_\ell \partial_m + \frac{d}{2}\Gamma_z \right) + \mu R \right] \Psi(x^\ell) = 0.$$

Baryons

Holographic Light-Front Integrable Form and Spectrum

- In the conformal limit fermionic spin-$\frac{1}{2}$ modes $\psi(\zeta)$ and spin-$\frac{3}{2}$ modes $\psi_\mu(\zeta)$ are two-component spinor solutions of the Dirac light-front equation

$$\alpha\Pi(\zeta)\psi(\zeta) = \mathcal{M}\psi(\zeta),$$

where $H_{LF} = \alpha\Pi$ and the operator

$$\Pi_L(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{L+\frac{1}{2}}{\zeta}\gamma_5\right),$$

and its adjoint $\Pi^\dagger_L(\zeta)$ satisfy the commutation relations

$$\left[\Pi_L(\zeta), \Pi^\dagger_L(\zeta)\right] = \frac{2L+1}{\zeta^2}\gamma_5.$$

- Supersymmetric QM between bosonic and fermionic modes in AdS?

- Note: in the Weyl representation ($i\alpha = \gamma_5 \beta$)

$$i\alpha = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}, \qquad \beta = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \qquad \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}.$$

- Baryon: twist-dimension $3 + L \quad (\nu = L + 1)$

$$\mathcal{O}_{3+L} = \psi D_{\{\ell_1} \dots D_{\ell_q} \psi D_{\ell_{q+1}} \dots D_{\ell_m\}} \psi, \qquad L = \sum_{i=1}^{m} \ell_i.$$

- Solution to Dirac eigenvalue equation with UV matching boundary conditions

$$\psi(\zeta) = C\sqrt{\zeta}\left[J_{L+1}(\zeta\mathcal{M})u_+ + J_{L+2}(\zeta\mathcal{M})u_-\right].$$

  Baryonic modes propagating in AdS space have two components: orbital L and $L + 1$.

- Hadronic mass spectrum determined from IR boundary conditions

$$\psi_\pm\left(\zeta = 1/\Lambda_{\rm QCD}\right) = 0,$$

  given by

$$\mathcal{M}^+_{\nu,k} = \beta_{\nu,k}\Lambda_{\rm QCD}, \quad \mathcal{M}^-_{\nu,k} = \beta_{\nu+1,k}\Lambda_{\rm QCD},$$

  with a scale independent mass ratio.

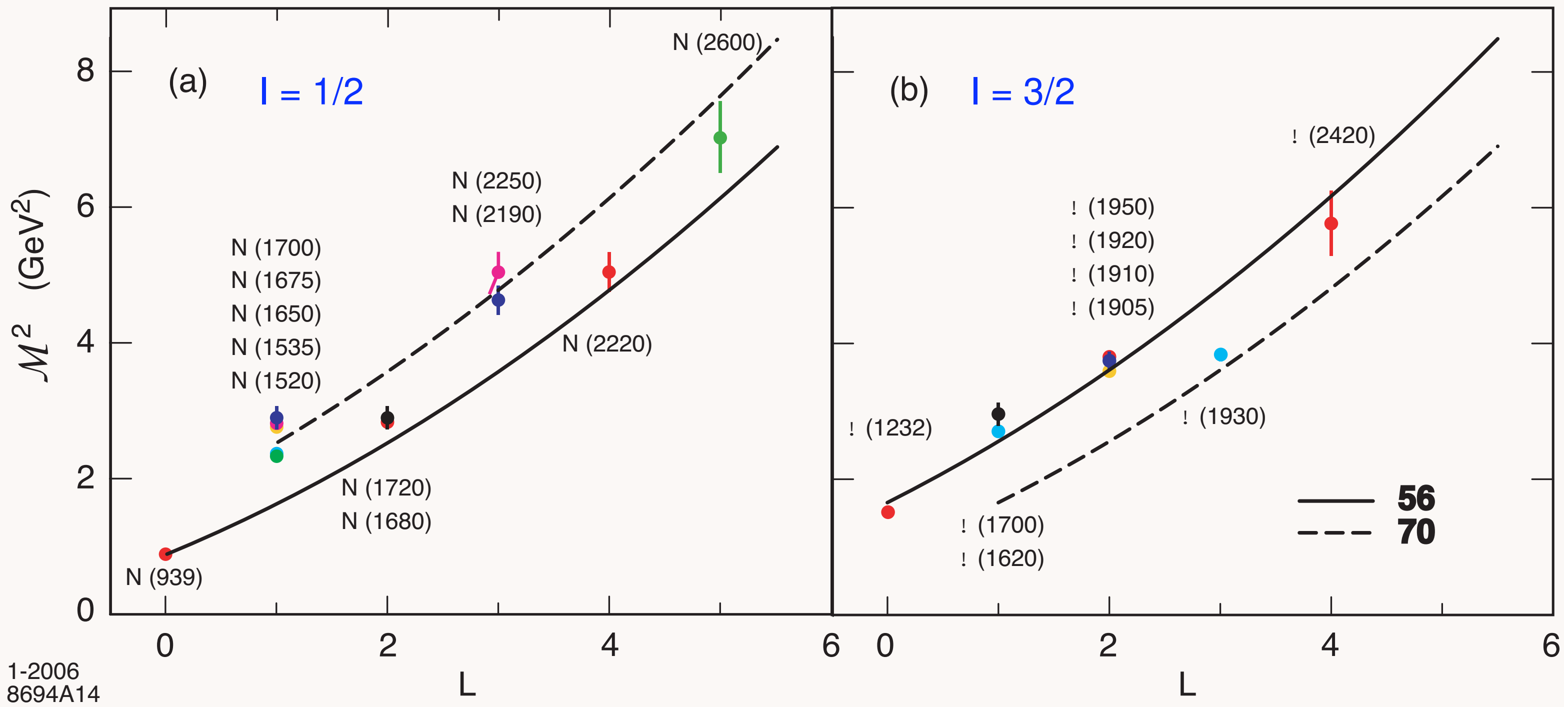


Fig: Light baryon orbital spectrum for Λ_{QCD} = 0.25 GeV in the HW model. The $\mathbf{56}$ trajectory corresponds to L even $P = +$ states, and the $\mathbf{70}$ to L odd $P = -$ states.

$SU(6)$	S	L	Baryon State
56	$\frac{1}{2}$	0	$N\frac{1}{2}^+(939)$
	$\frac{3}{2}$	0	$\Delta\frac{3}{2}^+(1232)$
70	$\frac{1}{2}$	1	$N\frac{1}{2}^-(1535)$ $N\frac{3}{2}^-(1520)$
	$\frac{3}{2}$	1	$N\frac{1}{2}^-(1650)$ $N\frac{3}{2}^-(1700)$ $N\frac{5}{2}^-(1675)$
	$\frac{1}{2}$	1	$\Delta\frac{1}{2}^-(1620)$ $\Delta\frac{3}{2}^-(1700)$
56	$\frac{1}{2}$	2	$N\frac{3}{2}^+(1720)$ $N\frac{5}{2}^+(1680)$
	$\frac{3}{2}$	2	$\Delta\frac{1}{2}^+(1910)$ $\Delta\frac{3}{2}^+(1920)$ $\Delta\frac{5}{2}^+(1905)$ $\Delta\frac{7}{2}^+(1950)$
70	$\frac{1}{2}$	3	$N\frac{5}{2}^-$ $N\frac{7}{2}^-$
	$\frac{3}{2}$	3	$N\frac{3}{2}^-$ $N\frac{5}{2}^-$ $N\frac{7}{2}^-(2190)$ $N\frac{9}{2}^-(2250)$
	$\frac{1}{2}$	3	$\Delta\frac{5}{2}^-(1930)$ $\Delta\frac{7}{2}^-$
56	$\frac{1}{2}$	4	$N\frac{7}{2}^+$ $N\frac{9}{2}^+(2220)$
	$\frac{3}{2}$	4	$\Delta\frac{5}{2}^+$ $\Delta\frac{7}{2}^+$ $\Delta\frac{9}{2}^+$ $\Delta\frac{11}{2}^+(2420)$
70	$\frac{1}{2}$	5	$N\frac{9}{2}^-$ $N\frac{11}{2}^-(2600)$
	$\frac{3}{2}$	5	$N\frac{7}{2}^-$ $N\frac{9}{2}^-$ $N\frac{11}{2}^-$ $N\frac{13}{2}^-$

Non-Conformal Extension of Algebraic Structure (Soft Wall Model)

- We write the Dirac equation

$$(\alpha\Pi(\zeta) - \mathcal{M})\,\psi(\zeta) = 0,$$

in terms of the matrix-valued operator Π

$$\Pi_\nu(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{\nu + \frac{1}{2}}{\zeta}\gamma_5 - \kappa^2\zeta\gamma_5\right),$$

and its adjoint $\Pi^\dagger$, with commutation relations

$$\left[\Pi_\nu(\zeta), \Pi_\nu^\dagger(\zeta)\right] = \left(\frac{2\nu+1}{\zeta^2} - 2\kappa^2\right)\gamma_5.$$

- Solutions to the Dirac equation

$$\begin{aligned}\psi_+(\zeta) &\sim z^{\frac{1}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^\nu(\kappa^2\zeta^2),\\ \psi_-(\zeta) &\sim z^{\frac{3}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^{\nu+1}(\kappa^2\zeta^2).\end{aligned}$$

- Eigenvalues

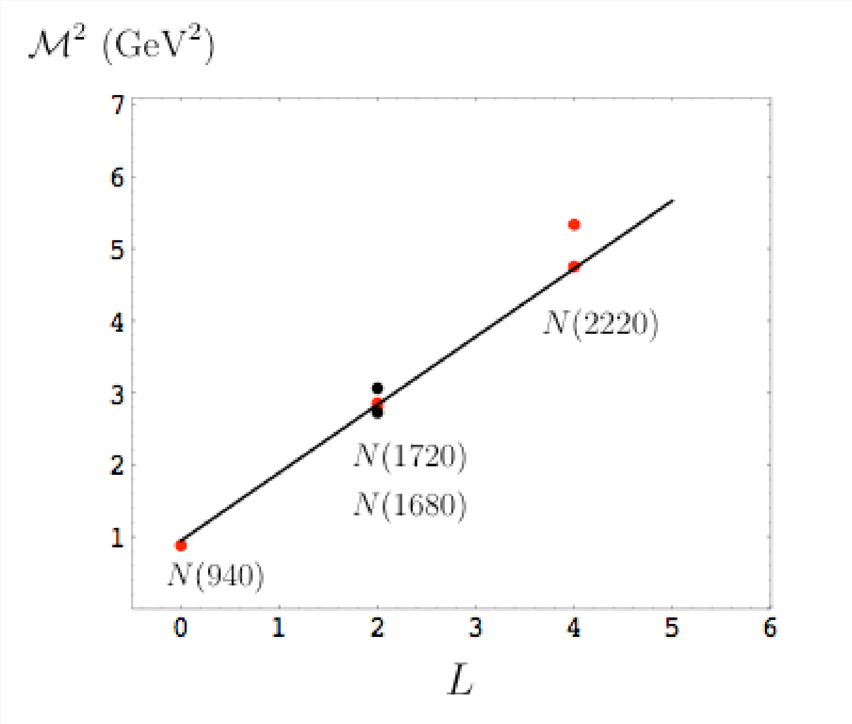
$$\mathcal{M}^2 = 4\kappa^2(n + \nu + 1).$$

- Baryon: twist-dimension $3 + L \quad (\nu = L + 1)$

$$\mathcal{O}_{3+L} = \psi D_{\{\ell_1} \dots D_{\ell_q} \psi D_{\ell_{q+1}} \dots D_{\ell_m\}} \psi, \qquad L = \sum_{i=1}^{m} \ell_i.$$

- Define the zero point energy (identical as in the meson case) $\mathcal{M}^2 \to \mathcal{M}^2 - 4\kappa^2$:

$$\mathcal{M}^2 = 4\kappa^2(n + L + 1).$$



Proton Regge Trajectory $\quad \kappa = 0.49$GeV

Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$F_+(Q^2) = g_+ \int d\zeta\, J(Q,\zeta) |\psi_+(\zeta)|^2,$$
$$F_-(Q^2) = g_- \int d\zeta\, J(Q,\zeta) |\psi_-(\zeta)|^2,$$

where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

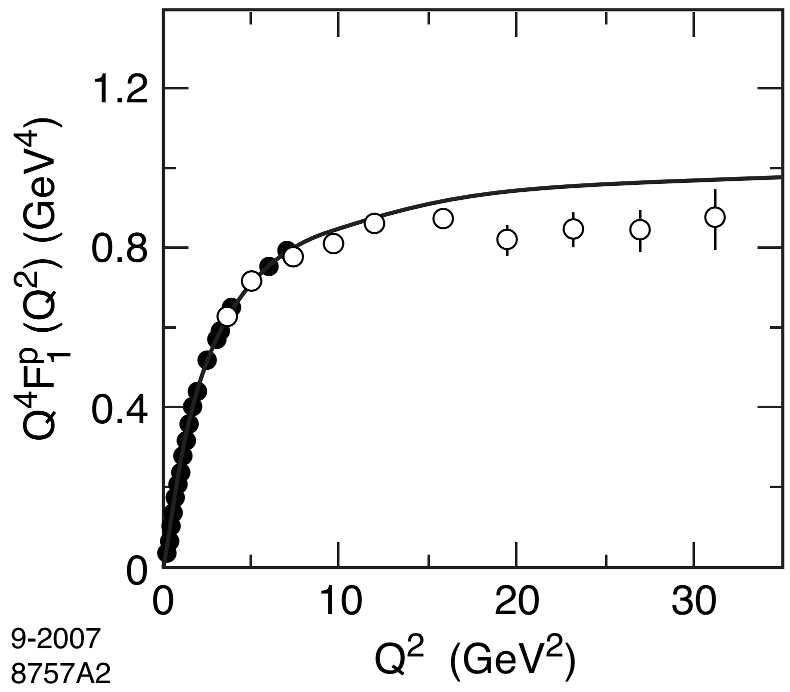
- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

- For $SU(6)$ spin-flavor symmetry

$$F_1^p(Q^2) = \int d\zeta\, J(Q,\zeta) |\psi_+(\zeta)|^2,$$
$$F_1^n(Q^2) = -\frac{1}{3} \int d\zeta\, J(Q,\zeta) \left[|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2 \right],$$

where $F_1^p(0) = 1,\ F_1^n(0) = 0.$

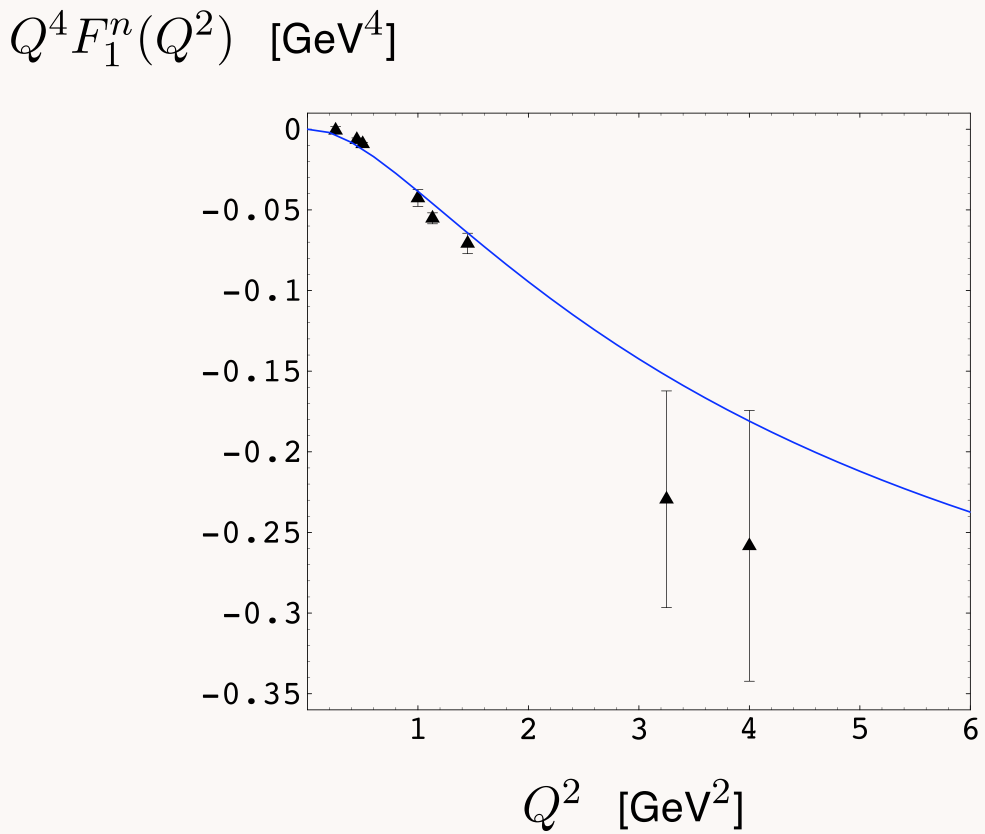
- Scaling behavior for large Q^2: $Q^4 F_1^p(Q^2) \to \text{constant}$ $\boxed{\text{Proton } \tau = 3}$



SW model predictions for $\kappa = 0.424$ GeV. Data analysis from: M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

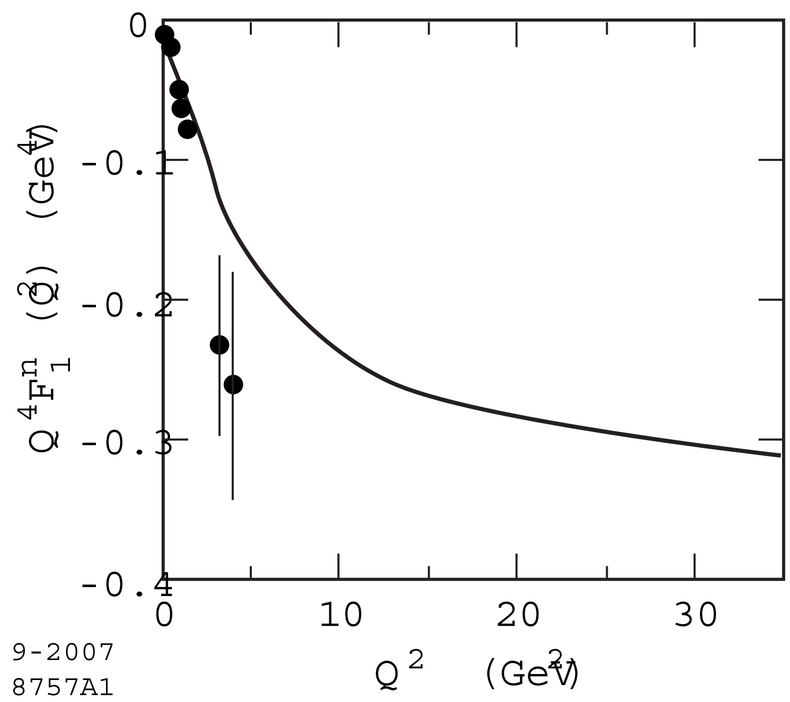
Dirac Neutron Form Factor
(Valence Approximation)

Truncated Space Confinement



Prediction for $Q^4 F_1^n(Q^2)$ for $\Lambda_{\rm QCD} = 0.21$ GeV in the hard wall approximation. Data analysis from Diehl (2005).

- Scaling behavior for large Q^2: $Q^4 F_1^n(Q^2) \to \text{constant}$ $\boxed{\text{Neutron } \tau = 3}$

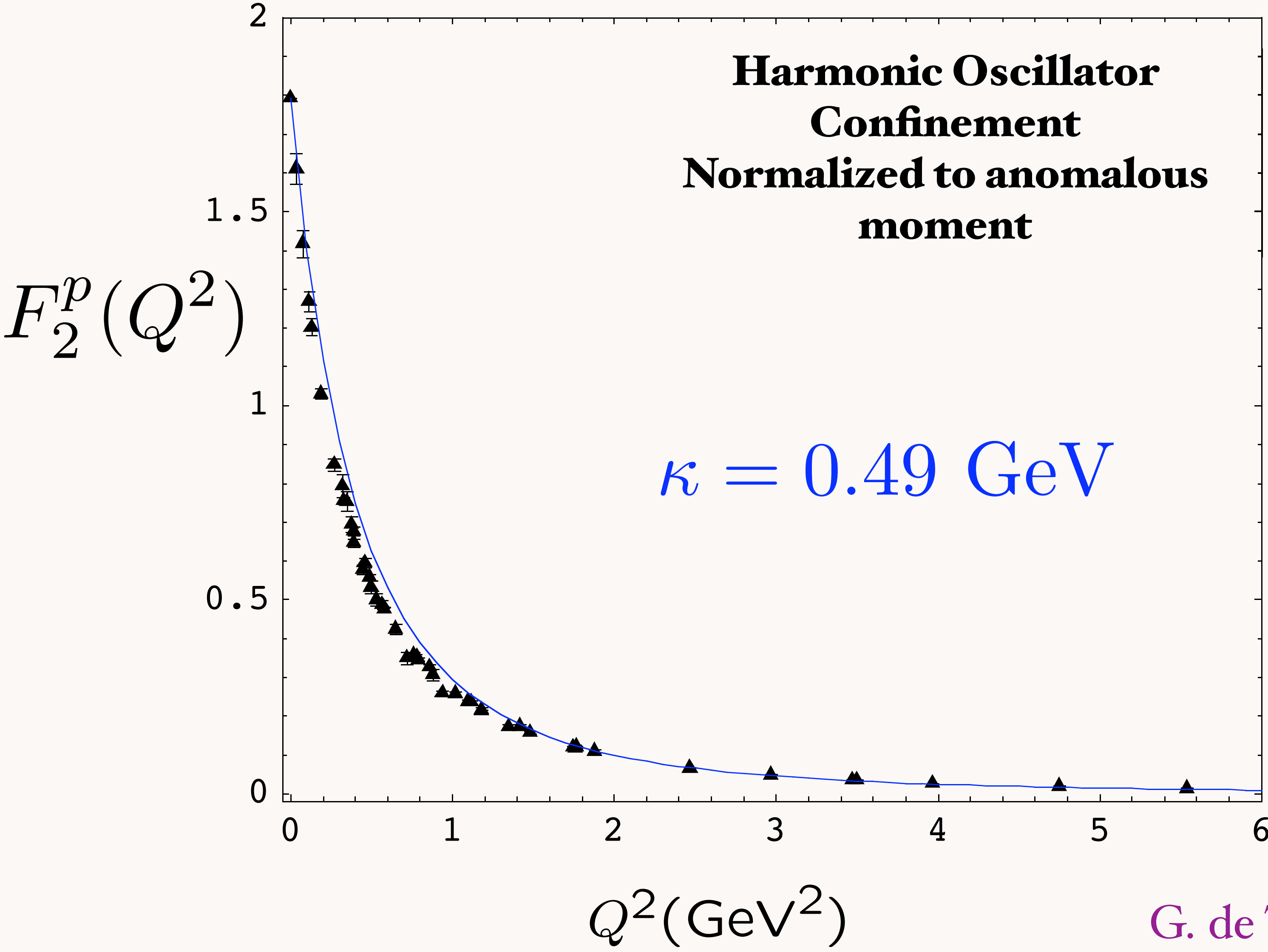


SW model predictions for $\kappa = 0.424$ GeV. Data analysis from M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

Spacelike Pauli Form Factor

Preliminary

From overlap of $L = 1$ and $L = 0$ LFWFs



G. de Teramond, sjb

AdS/CFT and Integrability

- L. Infeld, "On a new treatment of some eigenvalue problems", Phys. Rev. 59, 737 (1941).
- Generate eigenvalues and eigenfunctions using Ladder Operators
- Apply to Covariant Light-Front Radial Dirac and Schrodinger Equations

Algebraic Structure , Integrability and Stability Conditions (HW Model)

- If $L^2 > 0$ the LF Hamiltonian, H_{LF}, can be written as a bilinear form

$$H^L_{LF}(\zeta) = \Pi^\dagger_L(\zeta)\Pi_L(\zeta)$$

in terms of the operator

$$\Pi_L(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{L+\frac{1}{2}}{\zeta}\right),$$

and its adjoint

$$\Pi^\dagger_L(\zeta) = -i\left(\frac{d}{d\zeta} + \frac{L+\frac{1}{2}}{\zeta}\right),$$

with commutation relations

$$\left[\Pi_L(\zeta), \Pi^\dagger_L(\zeta)\right] = \frac{2L+1}{\zeta^2}.$$

- For $L^2 \geq 0$ the Hamiltonian is positive definite

$$\langle\phi\left|H^L_{LF}\right|\phi\rangle = \int d\zeta \left|\Pi_L\phi(z)\right|^2 \geq 0$$

and thus $\mathcal{M}^2 \geq 0$.

Ladder Construction of Orbital States

- Orbital excitations constructed by the L-th application of the raising operator

$$a_L^\dagger = -i\Pi_L$$

on the ground state:

$$a^\dagger|L\rangle = c_L|L+1\rangle.$$

- In the light-front ζ-representation

$$\begin{aligned}\phi_L(\zeta) &= \langle\zeta|L\rangle = C_L\sqrt{\zeta}\,(-\zeta)^L\left(\frac{1}{\zeta}\frac{d}{d\zeta}\right)^L J_0(\zeta\mathcal{M}) \\ &= C_L\sqrt{\zeta}J_L\left(\zeta\mathcal{M}\right).\end{aligned}$$

- The solutions ϕ_L are solutions of the light-front equation $(L = 0, \pm1, \pm2, \cdots)$

$$\left[-\frac{d^2}{d\zeta^2} - \frac{1-L^2}{4\zeta^2}\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta),$$

- Mode spectrum from boundary conditions : $\phi\left(\zeta = 1/\Lambda_{\rm QCD}\right) = 0.$

Non-Conformal Extension of Algebraic Integrability (SW Model)

- Soft-wall model [Karch, Katz, Son and Stephanov (2006)] retain conformal AdS metrics but introduce smooth cutoff which depends on the profile of a dilaton background field $\varphi(z)$.

- Consider the generator (short-distance Coulombic and long-distance linear potential)

$$\Pi_L(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{L+\frac{1}{2}}{\zeta} - \kappa^2\zeta\right),$$

and its adjoint

$$\Pi_L^\dagger(\zeta) = -i\left(\frac{d}{d\zeta} + \frac{L+\frac{1}{2}}{\zeta} + \kappa^2\zeta\right),$$

with commutation relations

$$\left[\Pi_L(\zeta), \Pi_L^\dagger(\zeta)\right] = \frac{2L+1}{\zeta^2} - 2\kappa^2.$$

- The LF Hamiltonian

Integrable !

$$H_{LF} = \Pi_L^\dagger \Pi_L + C$$

is positive definite $\langle\phi|H_{LF}|\phi\rangle \geq 0$ for $L^2 \geq 0$, and $C \geq -4\kappa^2$.

- Orbital and radial excited states are constructed from the ladder operators from the $L = 0$ state.

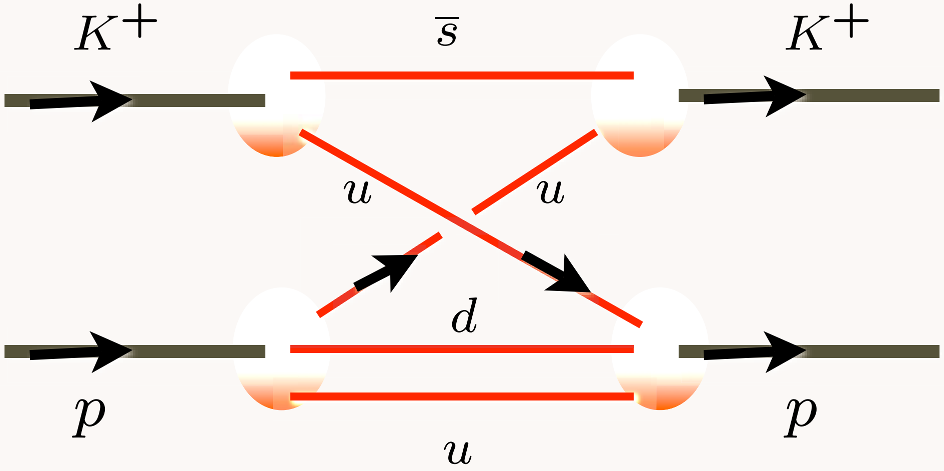
Holographic Connection between LF and AdS/CFT

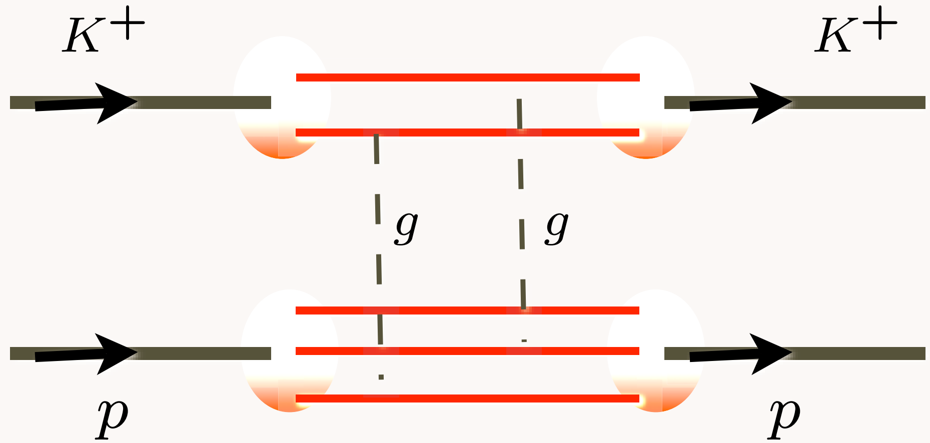
- Predictions for hadronic spectra, light-front wavefunctions, interactions
- Deduce meson and baryon wavefunctions, distribution amplitude, structure function from holographic constraint
- Identification of Orbital Angular Momentum Casimir for SO(2): LF Rotations
- Extension to massive quarks

New Perspectives for QCD from AdS/CFT

- LFWFs: Fundamental frame-independent description of hadrons at amplitude level
- Holographic Model from AdS/CFT : Confinement at large distances and conformal behavior at short distances
- Model for LFWFs, meson and baryon spectra: many applications!
- New basis for diagonalizing Light-Front Hamiltonian
- Physics similar to MIT bag model, but covariant. No problem with support $0 < x < 1$.
- Quark Interchange dominant force at short distances

CIM: Blankenbecler, Gunion, sjb





Quark Interchange
(Spin exchange in atom-atom scattering)

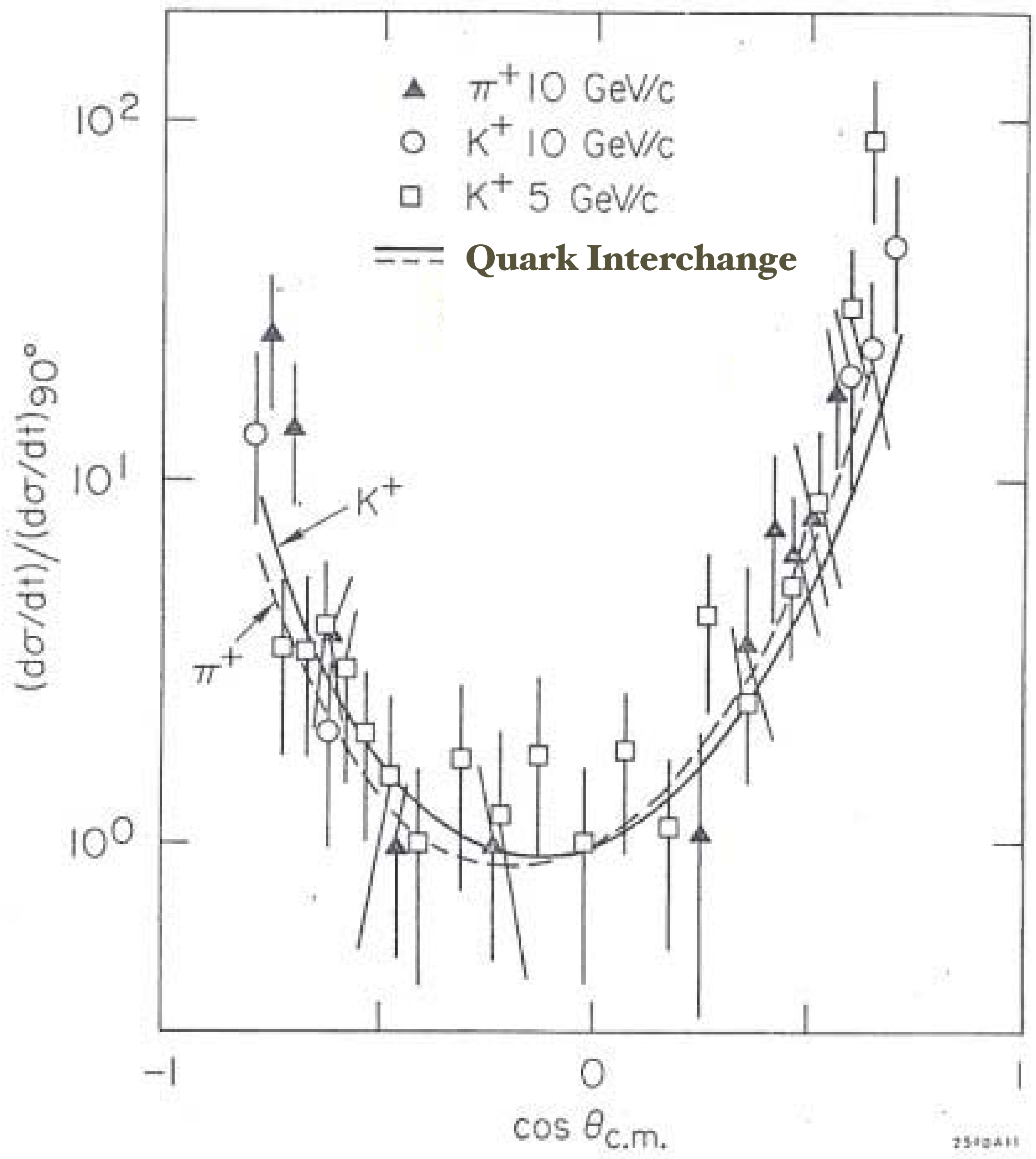
Gluon Exchange
(Van der Waal -- Landshoff)

$$\frac{d\sigma}{dt} = \frac{|M(s,t)|^2}{s^2}$$

$M(t,u)_{\text{interchange}} \propto \frac{1}{ut^2}$

$M(s,t)_{\text{gluonexchange}} \propto sF(t)$

MIT Bag Model (de Tar), large N_C, ('t Hooft), AdS/CFT all predict dominance of quark interchange:



AdS/CFT explains why quark interchange is dominant interaction at high momentum transfer in exclusive reactions

$$M(t,u)_{\text{interchange}} \propto \frac{1}{ut^2}$$

Non-linear Regge behavior:

$$\alpha_R(t) \rightarrow -1$$

Why is quark-interchange dominant over gluon exchange?

Example: $M(K^+p \to K^+p) \propto \frac{1}{ut^2}$

Exchange of common u quark

$M_{QIM} = \int d^2k_\perp dx \ \psi^\dagger_C \psi^\dagger_D \Delta \psi_A \psi_B$

Holographic model (Classical level):

Hadrons enter 5th dimension of AdS_5

Quarks travel freely within cavity as long as separation $z < z_0 = \frac{1}{\Lambda_{QCD}}$

LFWFs obey conformal symmetry producing quark counting rules.

Comparison of Exclusive Reactions at Large t

B. R. Baller,[a] G. C. Blazey,[b] H. Courant, K. J. Heller, S. Heppelmann,[c] M. L. Marshak, E. A. Peterson, M. A. Shupe, and D. S. Wahl[d]
University of Minnesota, Minneapolis, Minnesota 55455

D. S. Barton, G. Bunce, A. S. Carroll, and Y. I. Makdisi
Brookhaven National Laboratory, Upton, New York 11973

and

S. Gushue[e] and J. J. Russell
Southeastern Massachusetts University, North Dartmouth, Massachusetts 02747
(Received 28 October 1987; revised manuscript received 3 February 1988)

Cross sections or upper limits are reported for twelve meson-baryon and two baryon-baryon reactions for an incident momentum of 9.9 GeV/c, near 90° c.m.: $\pi^{\pm}p \to p\pi^{\pm}, p\rho^{\pm}, \pi^{+}\Delta^{\pm}, K^{+}\Sigma^{\pm}, (\Lambda^0/\Sigma^0)K^0$; $K^{\pm}p \to pK^{\pm}$; $p^{\pm}p \to pp^{\pm}$. By studying the flavor dependence of the different reactions, we have been able to isolate the quark-interchange mechanism as dominant over gluon exchange and quark-antiquark annihilation.

$\pi^{\pm}p \to p\pi^{\pm}$,

$K^{\pm}p \to pK^{\pm}$,

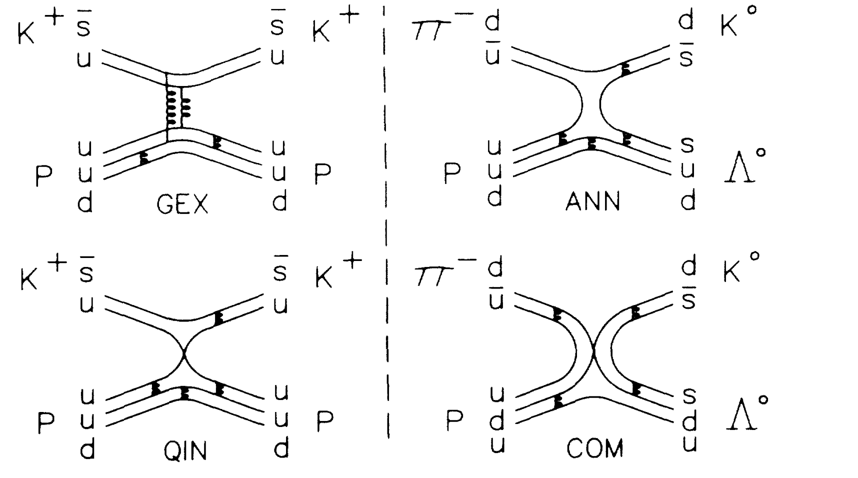
$\pi^{\pm}p \to p\rho^{\pm}$,

$\pi^{\pm}p \to \pi^{+}\Delta^{\pm}$,

$\pi^{\pm}p \to K^{+}\Sigma^{\pm}$,

$\pi^{-}p \to \Lambda^0K^0, \Sigma^0K^0$,

$p^{\pm}p \to pp^{\pm}$.



New Perspectives on QCD Phenomena from AdS/CFT

- AdS/CFT: Duality between string theory in Anti-de Sitter Space and Conformal Field Theory
- New Way to Implement Conformal Symmetry
- Holographic Model: Conformal Symmetry at Short Distances, Confinement at large distances
- Remarkable predictions for hadronic spectra, wavefunctions, interactions
- AdS/CFT provides novel insights into the quark structure of hadrons

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

$$\psi(x, k_\perp)$$

$$x_i = \frac{k_i^+}{P^+}$$

Invariant under boosts. Independent of P^μ

$$\mathrm{H}_{LF}^{QCD}|\psi> = M^2|\psi>$$

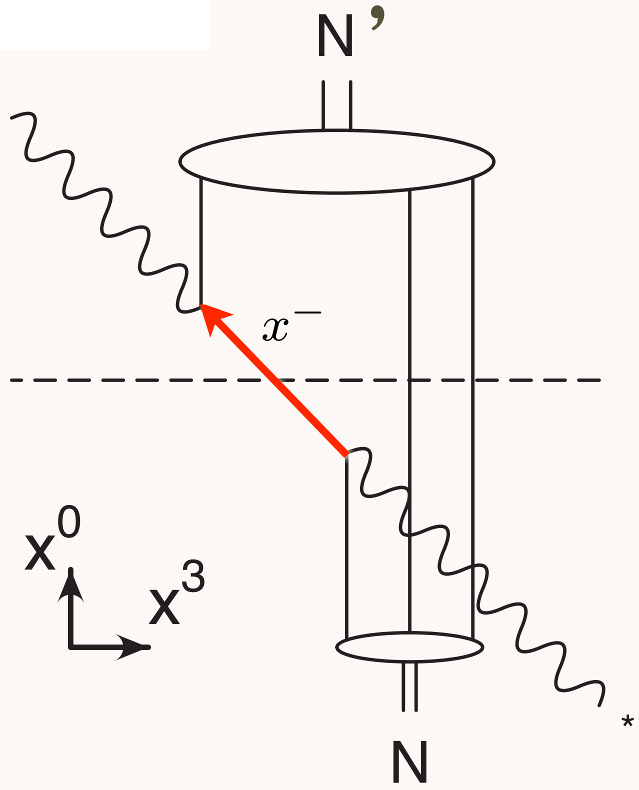
Remarkable new insights from AdS/CFT,
the duality between conformal field theory
and Anti-de Sitter Space

Some Applications of Light-Front Wavefunctions

- Exact formulae for form factors, quark and gluon distributions; vanishing anomalous gravitational moment; edm connection to anm
- Deeply Virtual Compton Scattering, generalized parton distributions, angular momentum sum rules
- Exclusive weak decay amplitudes
- Single spin asymmetries: Role of ISI and FSI
- Factorization theorems, DGLAP, BFKL, ERBL Evolution
- Quark interchange amplitude
- Relation of spin, momentum, and other distributions to physics of the hadron itself.

Space-time picture of DVCS

P. Hoyer



$\sigma = \frac{1}{2} x^- P^+$

$x^+ = \mathbf{x} = 0)$

The position of the struck quark differs by x^- in the two wave functions

Measure x- distribution from DVCS:
Take Fourier transform of skewness, $\xi = \frac{Q^2}{2p.q}$
the longitudinal momentum transfer

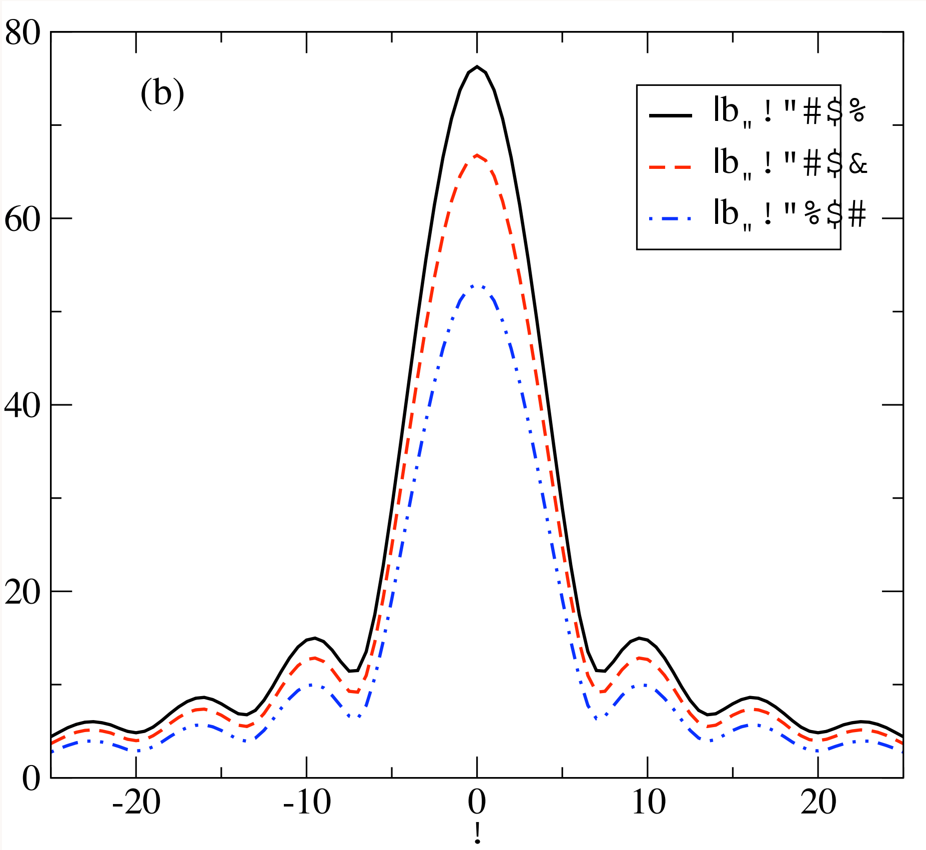
S. J. Brodsky[a], D. Chakrabarti[b], A. Harindranath[c], A. Mukherjee[d], J. P. Vary[e,a,f]

S. J. Brodsky[a], D. Chakrabarti[b], A. Harindranath[c], A. Mukherjee[d], J. P. Vary[e,a,f]

Hadron Optics

$$A(\sigma, \vec{b}_\perp) = \frac{1}{2\pi}\int d\xi e^{i\frac{1}{2}\xi\sigma}\tilde{A}(\xi, \vec{b}_\perp)$$

$$\sigma = \frac{1}{2}x^- P^+ \quad \xi = \frac{Q^2}{2p.q}$$

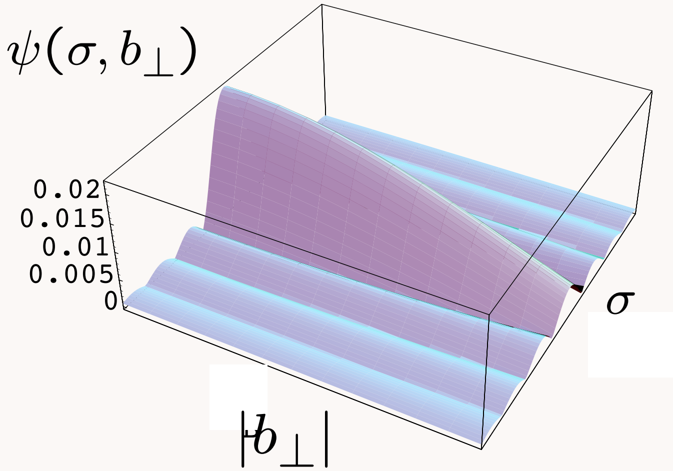


The Fourier Spectrum of the DVCS amplitude in σ space for different fixed values of $|b_\perp|$.

GeV units

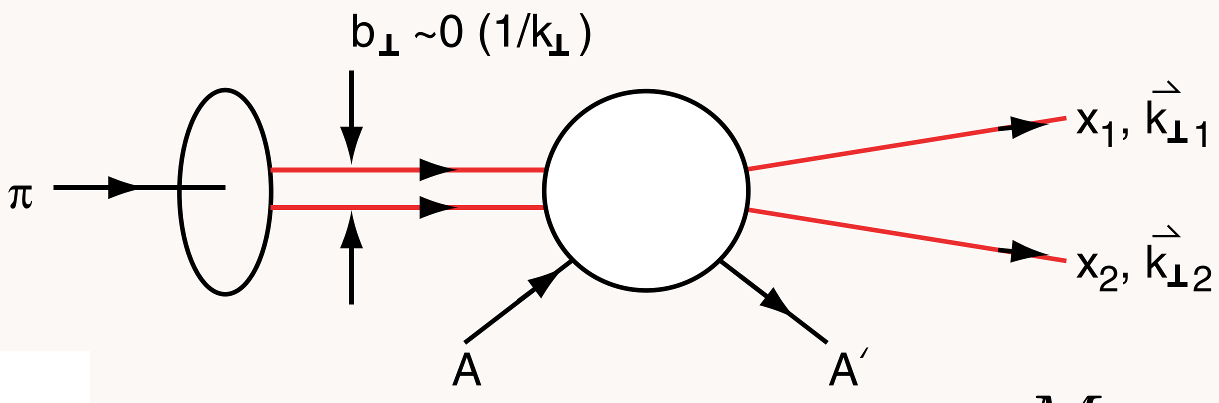
DVCS Amplitude using holographic QCD meson LFWF

$$\Lambda_{QCD} = 0.32$$



Diffractive Dissociation of Pion into Quark Jets

E791 Ashery et al.



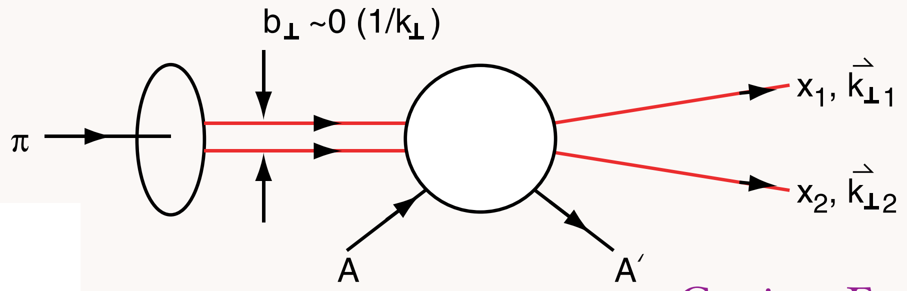
$$M \propto \frac{\partial^2}{\partial^2 k_\perp} \psi_\pi(x, k_\perp)$$

Measure Light-Front Wavefunction of Pion

Minimal momentum transfer to nucleus

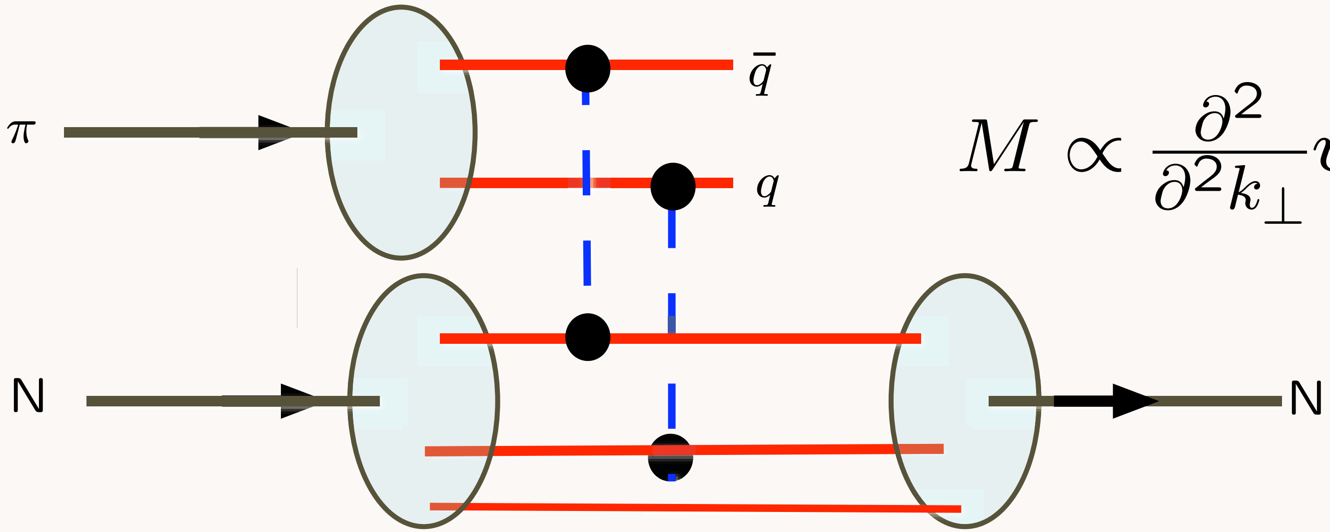
Nucleus left Intact!

E791 FNAL Diffractive DiJet



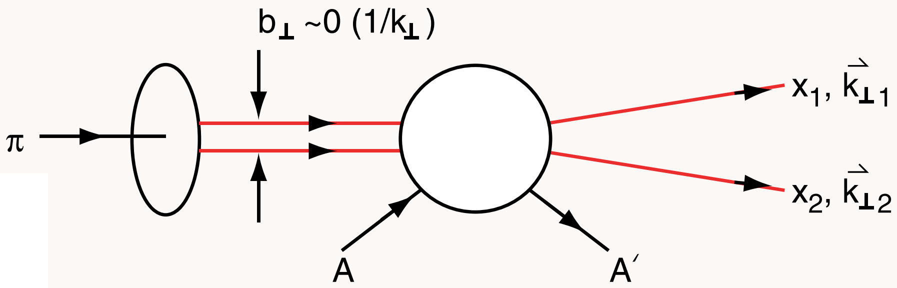
Gunion, Frankfurt, Mueller, Strikman, sjb
Frankfurt, Miller, Strikman

Two-gluon exchange measures the second derivative of the pion light-front wavefunction



$$M \propto \frac{\partial^2}{\partial^2 k_\perp}\psi_\pi(x, k_\perp)$$

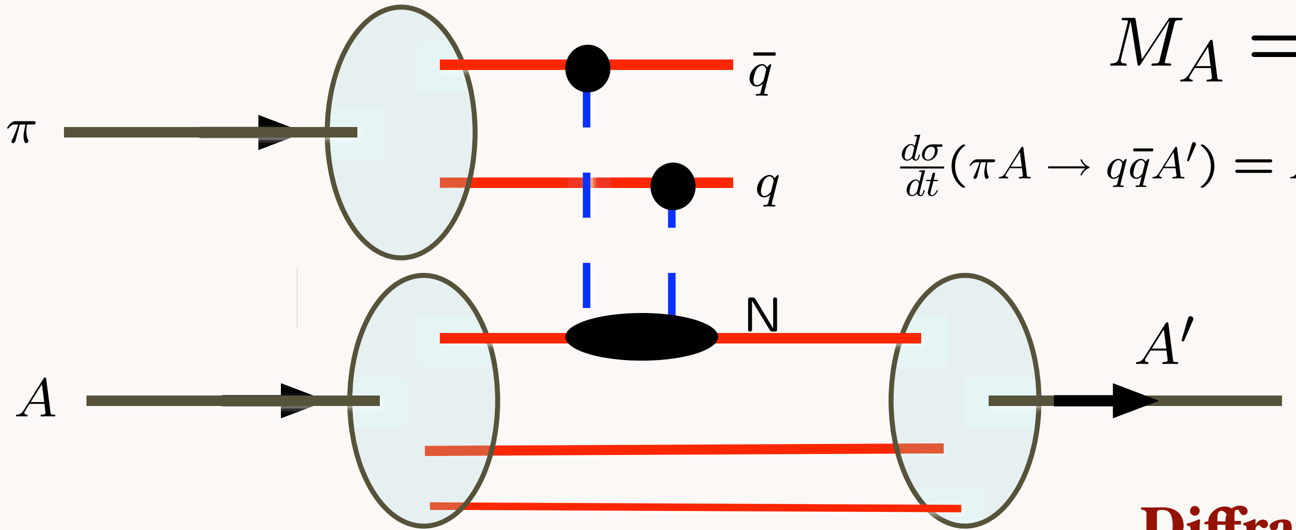
Key Ingredients in E791 Experiment



Brodsky Mueller
Frankfurt Miller Strikman

Small color-dipole moment pion not absorbed; interacts with each nucleon coherently

QCD COLOR Transparency



$$M_A = A\ M_N$$

$$\frac{d\sigma}{dt}(\pi A \to q\bar{q}A') = A^2\ \frac{d\sigma}{dt}(\pi N \to q\bar{q}N')\ F_A^2(t)$$

Target left intact

Diffraction, Rapidity gap

Color Transparency

Bertsch, Gunion, Goldhaber, sjb

A. H. Mueller, sjb

- Fundamental test of gauge theory in hadron physics
- Small color dipole moments interact weakly in nuclei
- Complete coherence at high energies
- Clear Demonstration of CT from Diffractive Di-Jets

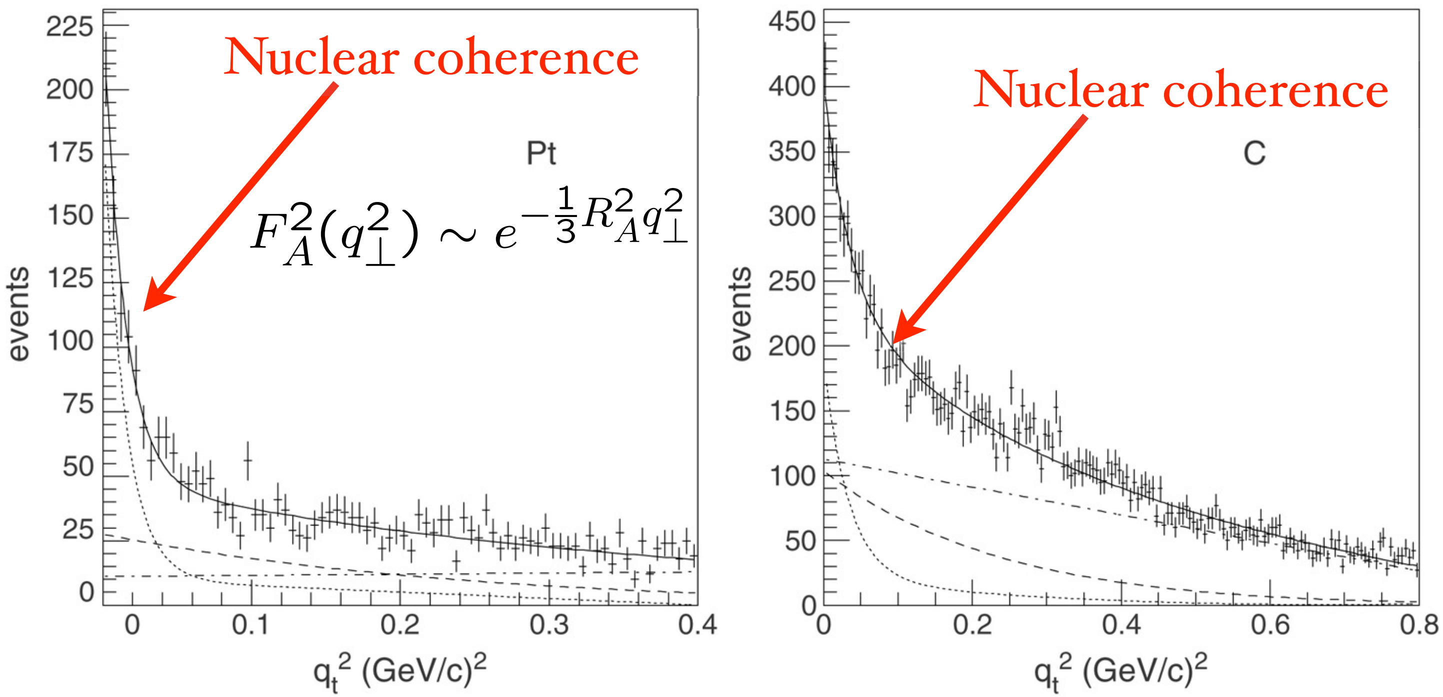
- Fully coherent interactions between pion and nucleons.

- Emerging Di-Jets do not interact with nucleus.

$$\mathcal{M}(\mathcal{A}) = \mathcal{A} \cdot \mathcal{M}(\mathcal{N})$$

$$\frac{d\sigma}{dq_t^2} \propto A^2 \quad q_t^2 \sim 0$$

$$\sigma \propto A^{4/3}$$



Mueller, sjb; Bertsch et al;
Frankfurt, Miller, Strikman

Measure pion LFWF in diffractive dijet production
Confirmation of color transparency

A-Dependence results: A

k_t range (GeV/c)		(CT)
$1.25 < k_t < 1.5$	1.64 + 0.06 -0.12	1.25
$1.5 < k_t < 2.0$	1.52 ± 0.12	1.45
$2.0 < k_t < 2.5$	1.55 ± 0.16	1.60

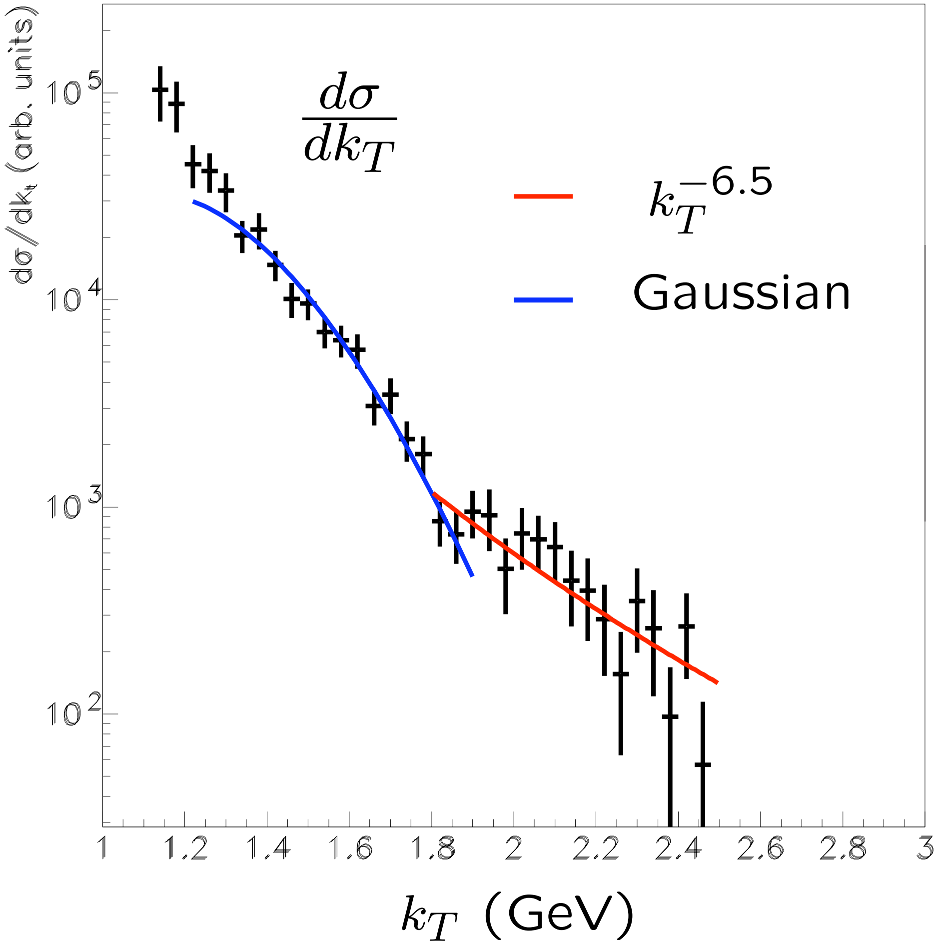
Ashery E791

(Incoh.) = 0.70 ± 0.1

Conventional Glauber Theory Ruled Out !

Factor of 7

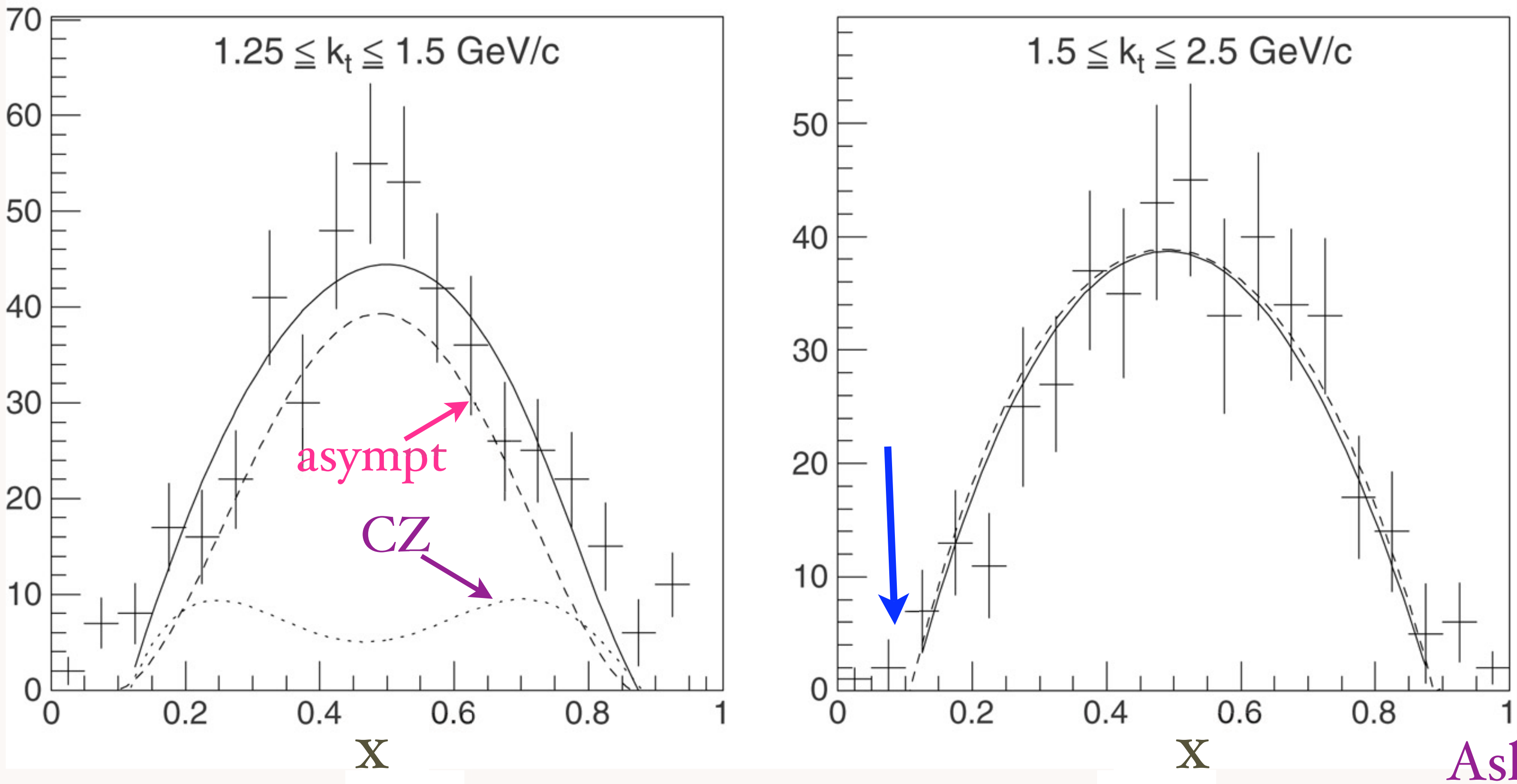
E791 Diffractive Di-Jet transverse momentum distribution



Two Components

High Transverse momentum dependence $k_T^{-6.5}$ consistent with PQCD, ERBL Evolution

Gaussian component similar to AdS/CFT HO LFWF



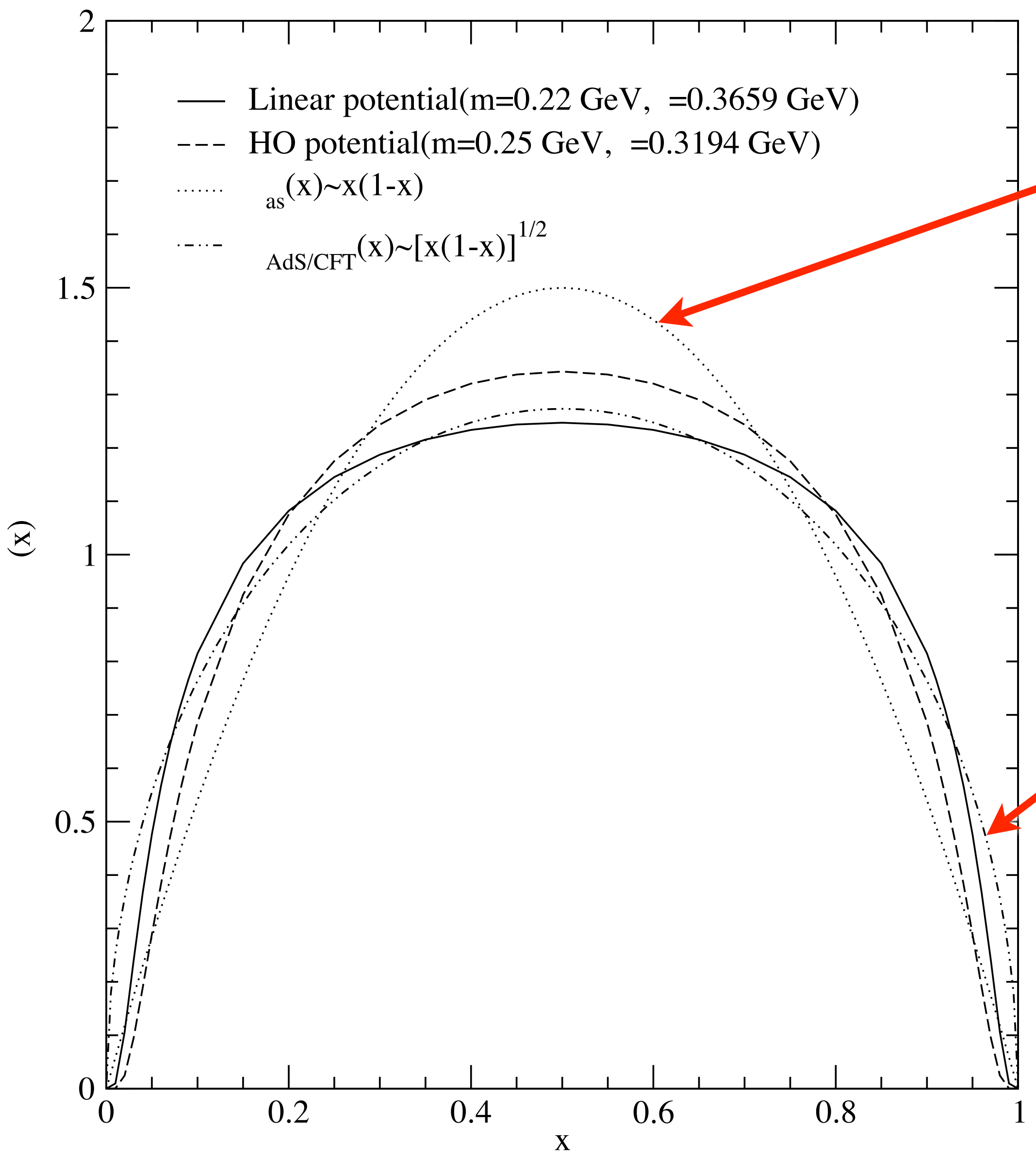
Ashery E791

Narrowing of x distribution at higher jet transverse momentum

x: distribution of diffractive dijets from the platinum target for 1 25 k_t 1 5 GeV/c (left) and for 1 5 k_t 2 5 GeV/c (right). The solid line is a fit to a combination of the asymptotic and CZ distribution amplitudes. The dashed line shows the contribution from the asymptotic function and the dotted line that of the CZ function.

Possibly two components:
Nonperturbative (AdS/CFT) and
Perturbative (ERBL)
Evolution to asymptotic distribution

$$\phi(x) \propto \sqrt{x(1-x)}$$



$$\phi_{asympt} \sim x(1-x)$$

AdS/CFT:

$$\phi(x, Q_0) \propto \sqrt{x(1-x)}$$

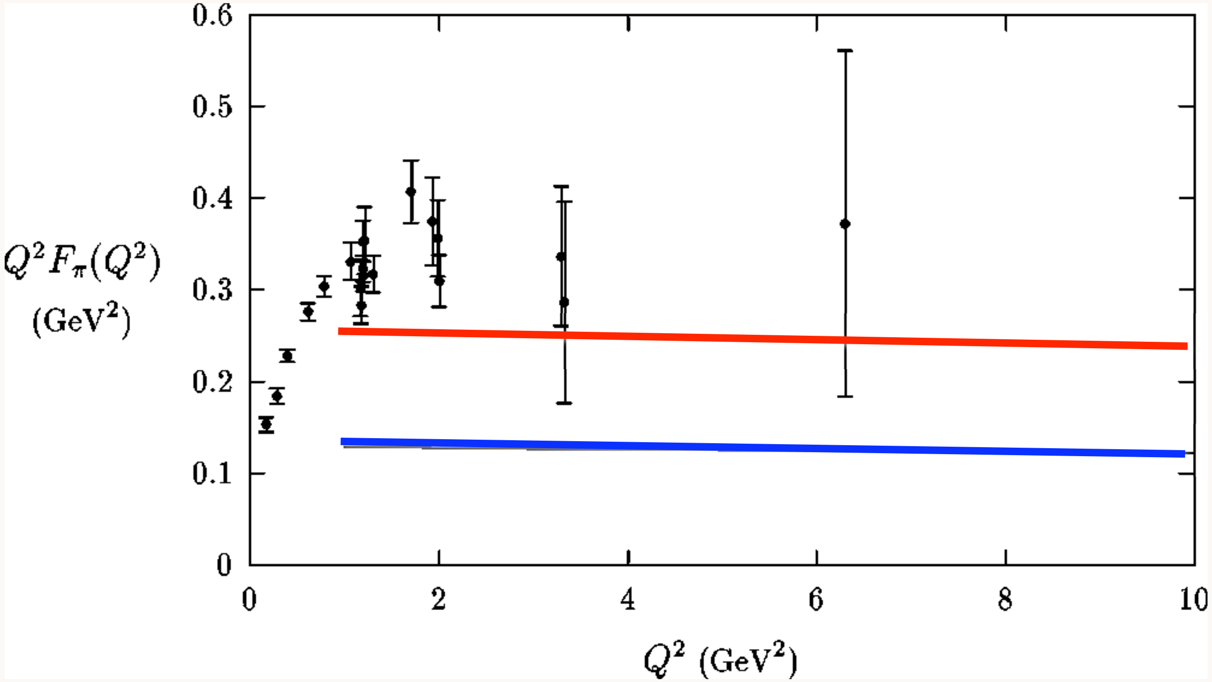
Increases PQCD leading twist predictio
$F_\pi(Q^2)$ by factor 16/9

Lepage, sjb

C. Ji, A. Pang, D. Robertson, sjb

Choi, Ji

$$F_\pi(Q^2) = \int_0^1 dx\,\phi_\pi(x) \int_0^1 dy\,\phi_\pi(y) \frac{16\pi C_F \alpha_V(Q_V)}{(1-x)(1-y)Q^2}$$



$\phi(x, Q_0) \propto \sqrt{x(1-x)}$

$\phi_{asymptotic} \propto x(1-x)$

Normalized to f_π

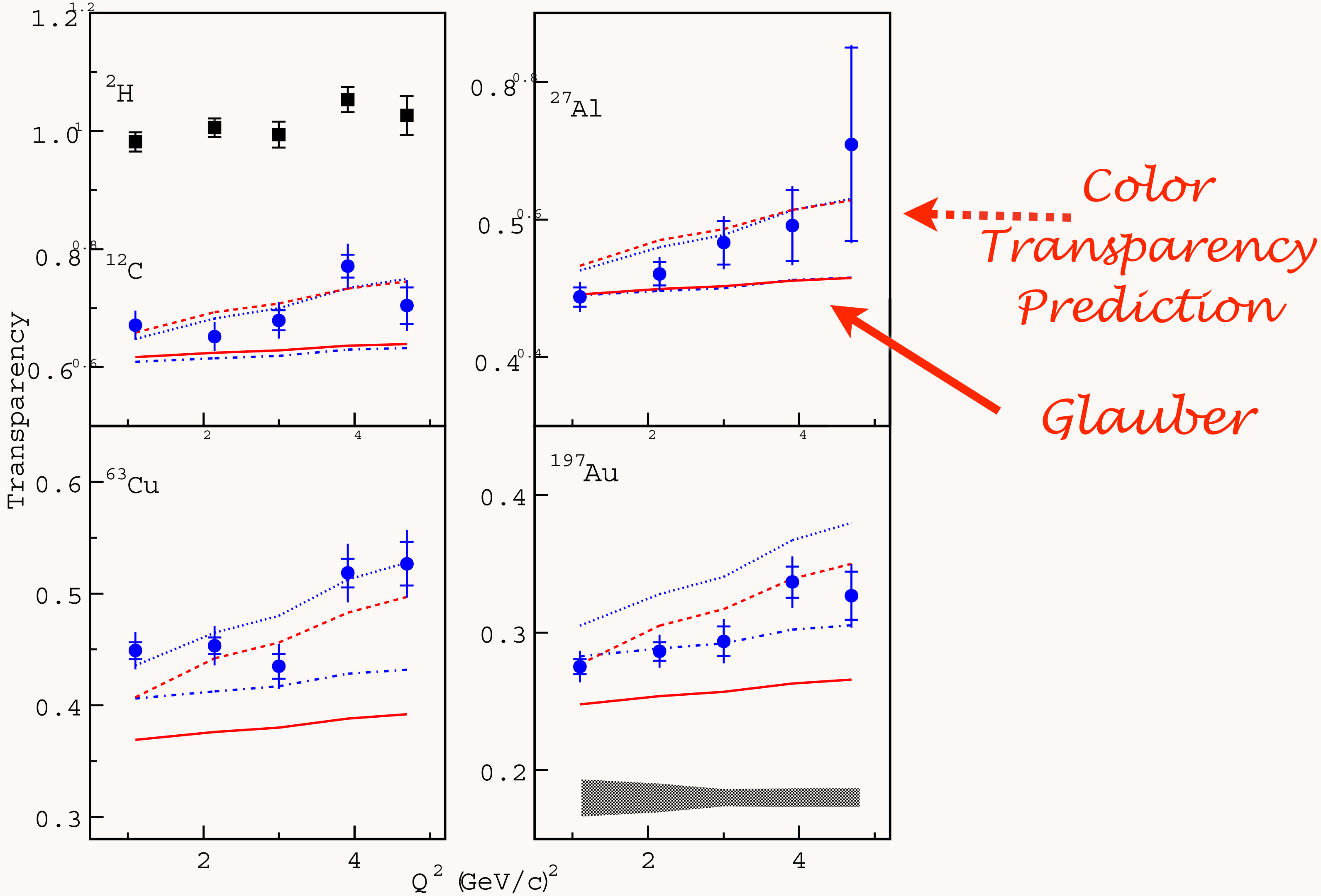
AdS/CFT: Increases PQCD leading twist prediction for $F_\pi(Q^2)$ by factor 16/9

Measurement of Nuclear Transparency for the A e ;e^0 Reaction

$$eA \to e'\pi^+ X$$

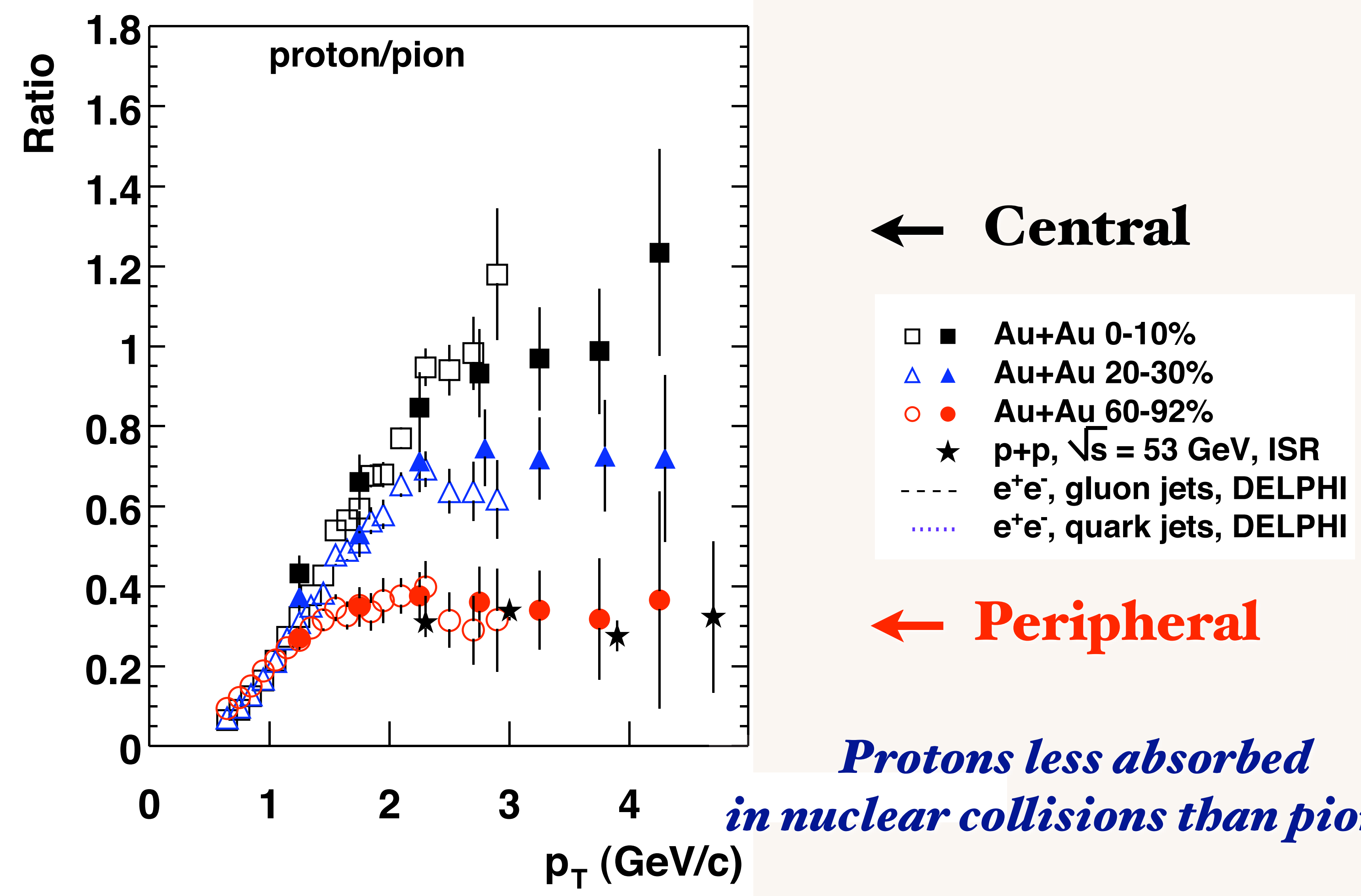
B. Clasie, et al , Jlab

PRL **99,** 242502 (2007)



S. S. Adler *et al*. PHENIX Collaboration *Phys. Rev. Lett.* **91**, 172301 (2003).

Particle ratio changes with centrality!



← **Central**

← **Peripheral**

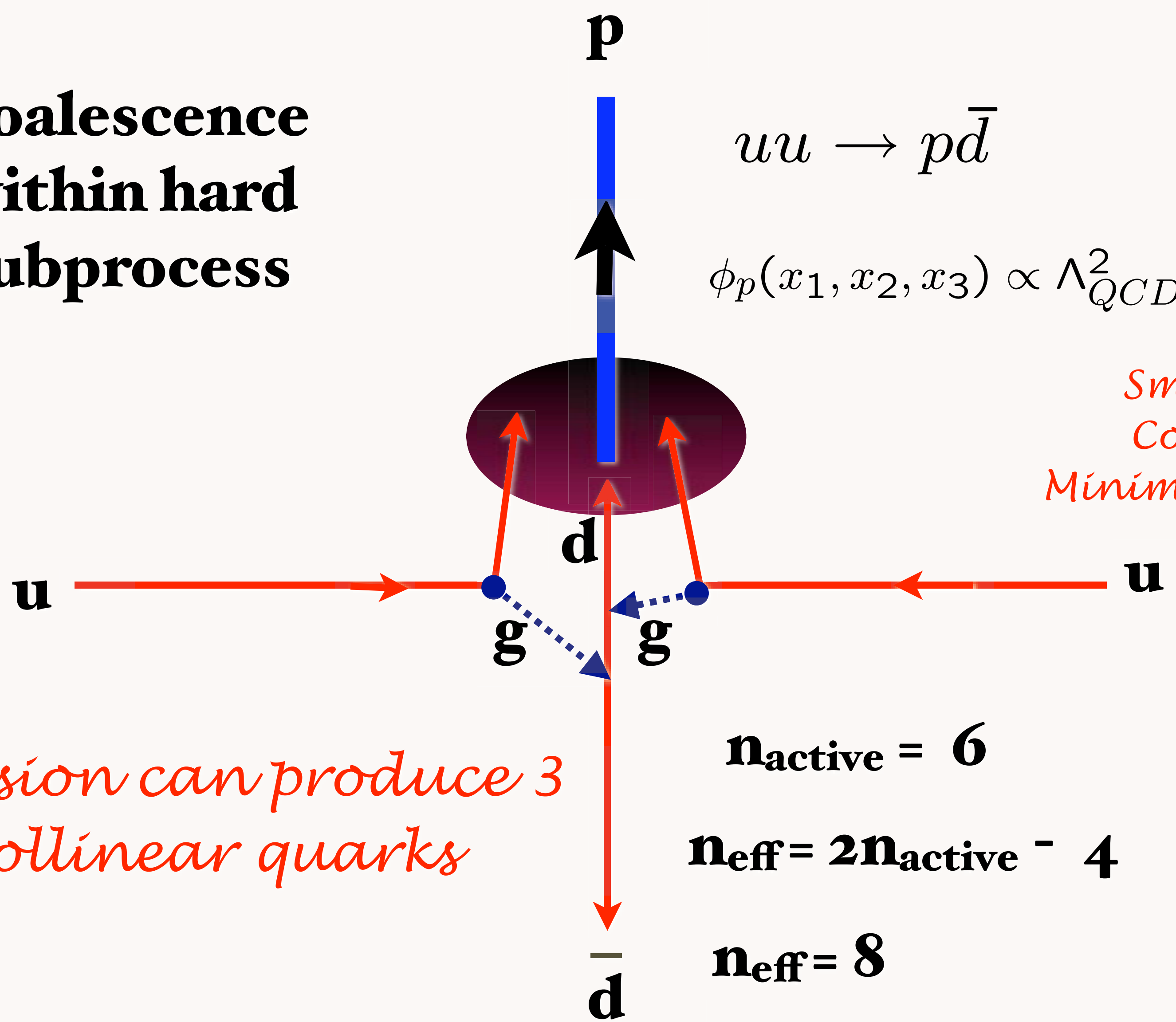
Protons less absorbed in nuclear collisions than pions!

Open (filled) points are for $\pi^{\pm}$ (π^0), respectively.

Baryon can be made directly within hard subprocess

Bjorken
Blankenbecler, Gunion, sjb
Berger, sjb
Hoyer, et al: Semi-Exclusive

Coalescence within hard subprocess



$$uu \to p\bar{d}$$

$$\phi_p(x_1, x_2, x_3) \propto \Lambda^2_{QCD}$$

Small color-singlet
Color Transparent
Minimal same-side energy

Collision can produce 3 collinear quarks

$n_{active} = 6$

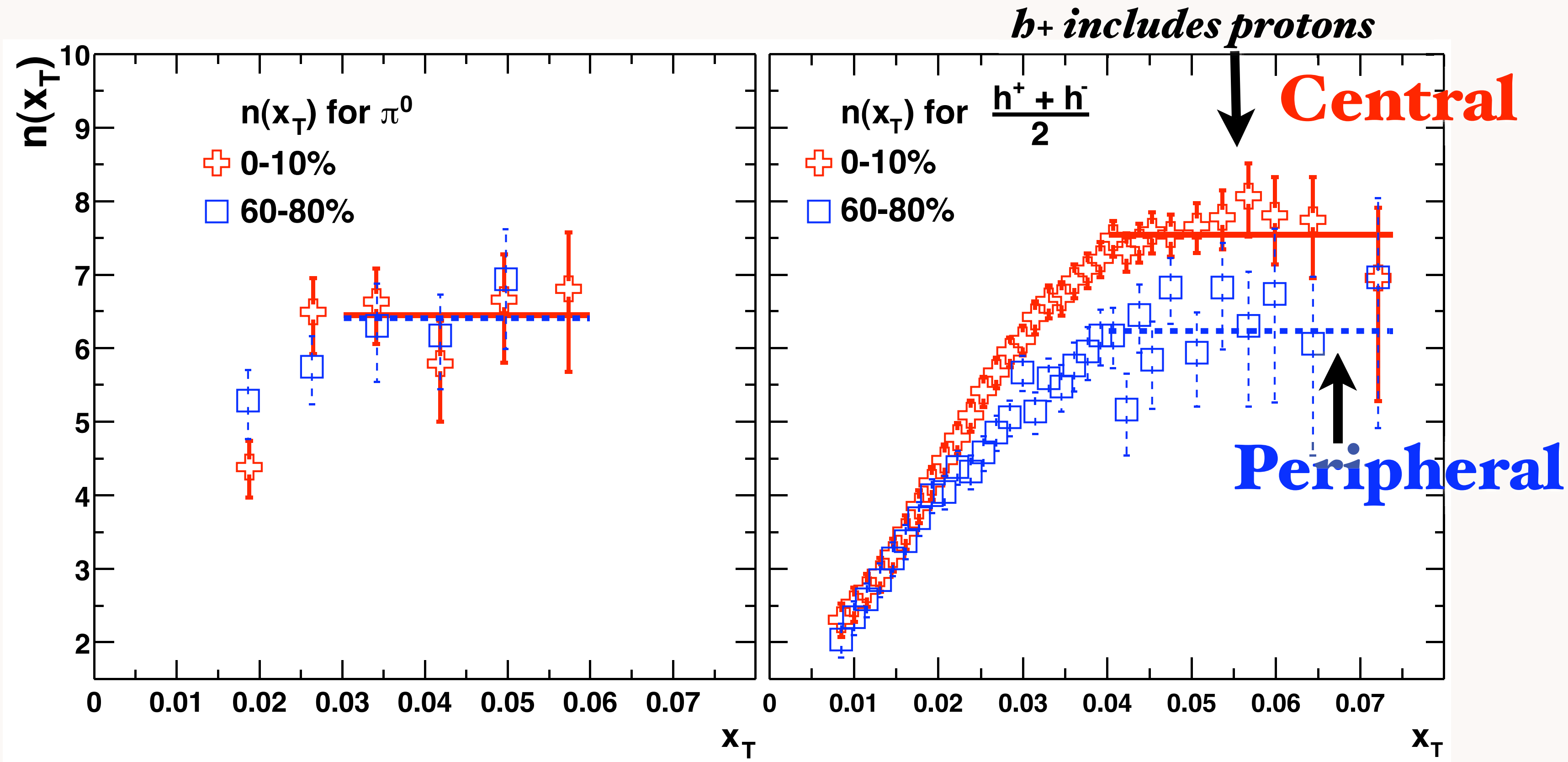
$n_{eff} = 2n_{active} - 4$

$n_{eff} = 8$

$$qq \to B\bar{q}$$

Power-law exponent $n(x_T)$ for π^0 and h spectra in central and peripheral Au+Au collisions at $\sqrt{s_{NN}}$ = 130 and 200 GeV

S. S. Adler, *et al.*, PHENIX Collaboration, *Phys. Rev.* C **69**, 034910 (2004) [`nucl-ex/0308006`].

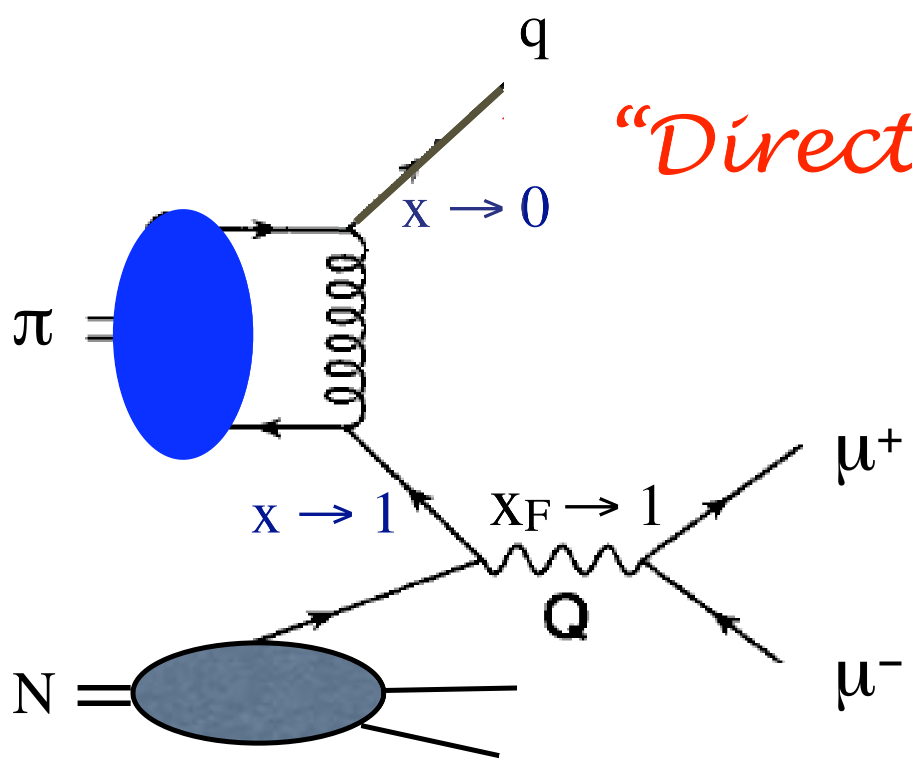


Proton production dominated by color-transparent direct high n_{eff} subprocesses

$\pi N \to \mu^+ \mu^- X$ at high x_F

In the limit where $(1-x_F)Q^2$ is fixed as $Q^2 \to \infty$

Entire pion wf contributes to hard process



"Direct" Subprocess

Virtual photon is longitudinally polarized

Berger and Brodsky, PRL 42 (1979) 940

$\pi^- N \to \mu^+ \mu^- X$ at 80 GeV/c

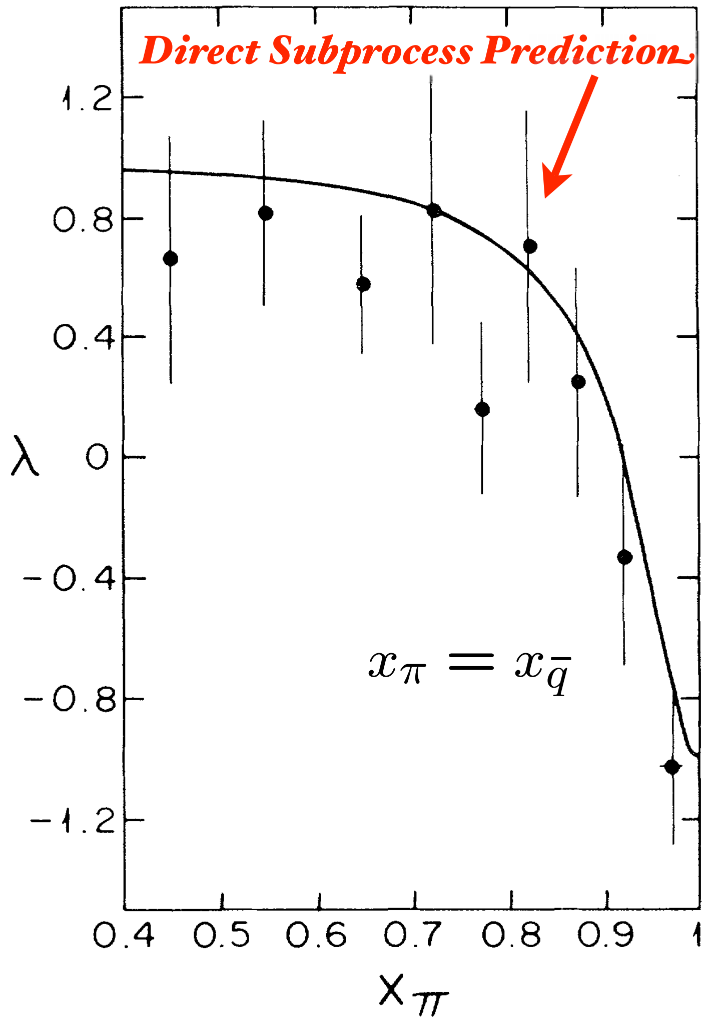
$$\frac{d\sigma}{d\Omega} \propto 1 + \lambda \cos^2\theta + \rho \sin 2\theta \cos\phi + \omega \sin^2\theta \cos 2\phi .$$

$$\frac{d^2\sigma}{dx_\pi d\cos\theta} \propto x_\pi \left[(1-x_\pi)^2 (1+\cos^2\theta) + \frac{4}{9} \frac{\langle k_T^2 \rangle}{M^2} \sin^2\theta \right]$$

$$\langle k_T^2 \rangle = 0.62 \pm 0.16 \ \mathrm{GeV}^2/c^2$$

Dramatic change in angular distribution at large x_F

Example of a higher-twist direct subprocess



Chicago-Princeton Collaboration

Phys.Rev.Lett.55:2649,1985

Hadron Dynamics at the Amplitude Level

- LFWFS are the universal hadronic amplitudes which underlie structure functions, GPDs, exclusive processes, distribution amplitudes, direct subprocesses, hadronization.

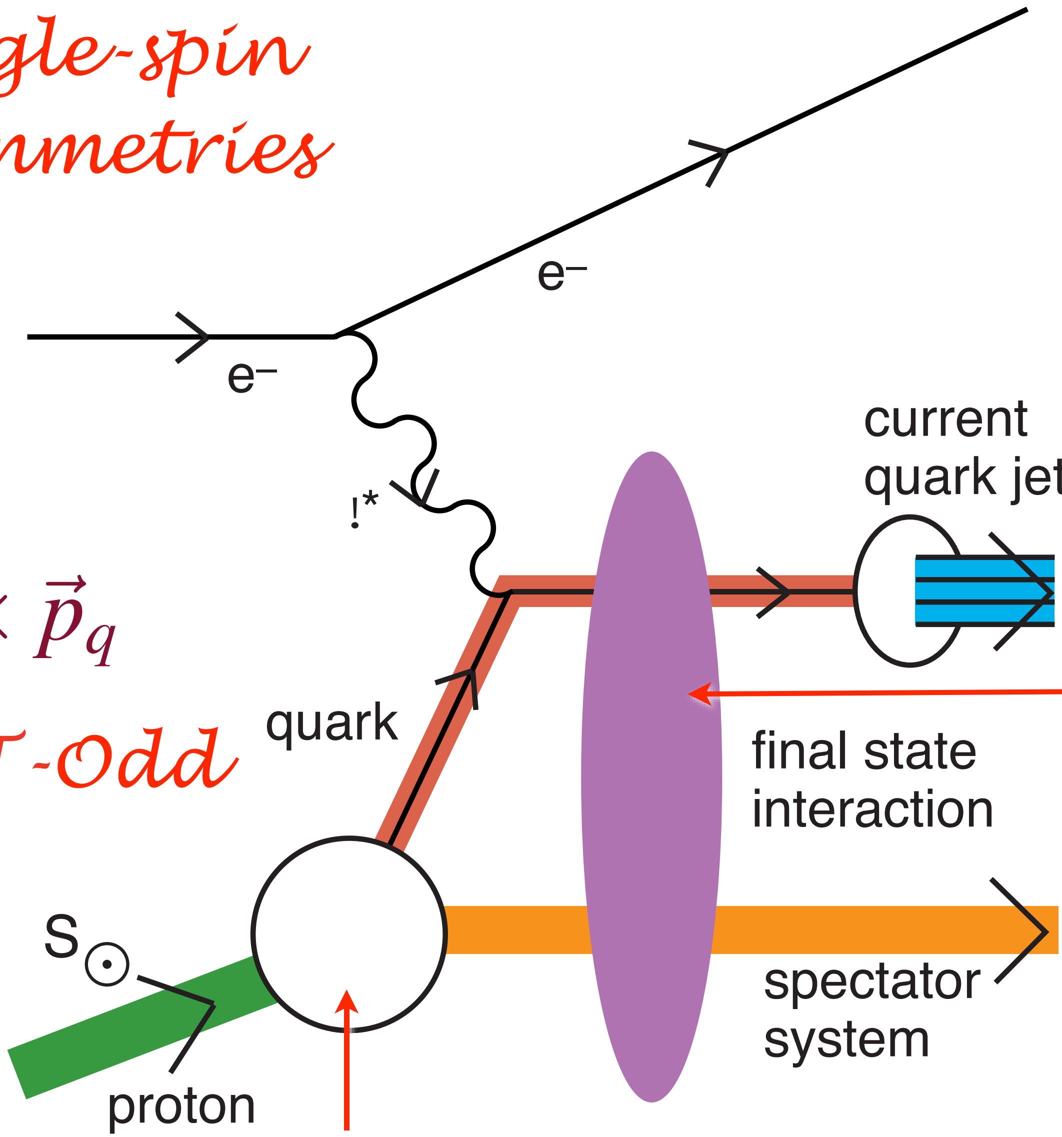
- Relation of spin, momentum, and other distributions to physics of the hadron itself.

- Connections between observables, orbital angular momentum

- Role of FSI and ISIs--Sivers effect

Single-spin asymmetries

Leading-Twist Sivers Effect



$\mathrm{i}\ \vec{S}_p \cdot \vec{q} \times \vec{p}_q$

Pseudo-T-Odd

QCD S- and P- Coulomb Phases

Light-Front Wavefunction S and P- Waves

D. S. Hwang, I. A. Schmidt, sjb

Final-State Interactions Produce T-Odd (Sivers Effect)

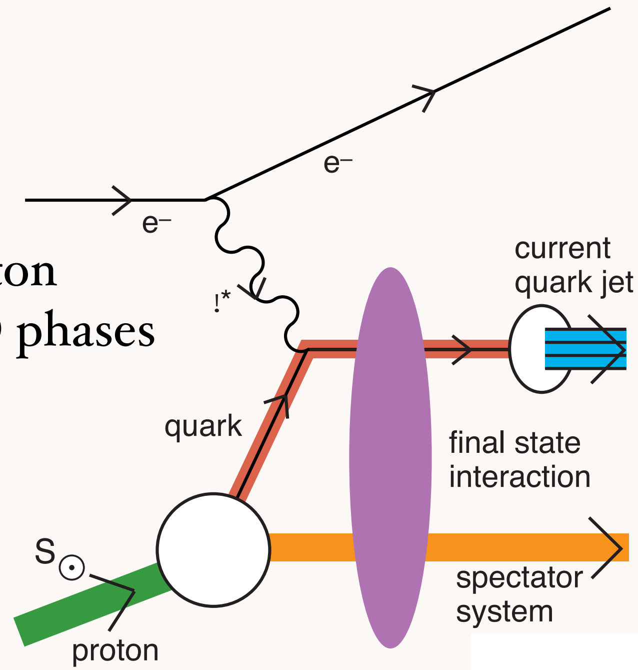
$i\ \vec{S} \cdot \vec{p}_{jet} \times \vec{q}$

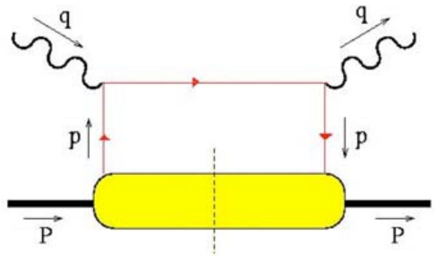
- Bjorken Scaling!
- Arises from Interference of Final-State Coulomb Phases in S and P waves
- Relate to the quark contribution to the target proton anomalous magnetic moment

Hwang, Schmidt. sjb;
Burkardt

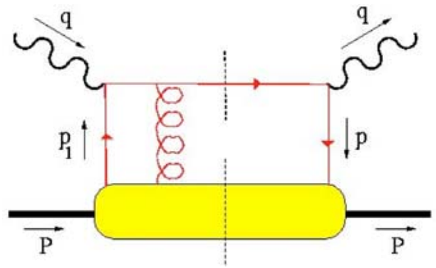
Final-State Interactions Produce Pseudo T-Odd (Sivers Effect)

- Leading-Twist Bjorken Scaling!
- Requires nonzero orbital angular momentum of quark! $\mathbf{i}\, \vec{S} \cdot \vec{p}_{jet} \times \vec{q}$
- Arises from the interference of Final-State QCD Coulomb phases in S- and P- waves; Wilson line effect; gauge independent
- Unexpected QCD Effect -- thought to be zero!
- Relate to the quark contribution to the target proton anomalous magnetic moment and final-state QCD phases
- QCD Coulomb phase at soft scale
- Measure in jet trigger or leading hadron
- Sum of Sivers Functions for all quarks and gluons vanishes. (Zero gravito-anomalous magnetic moment: B(0)= 0)





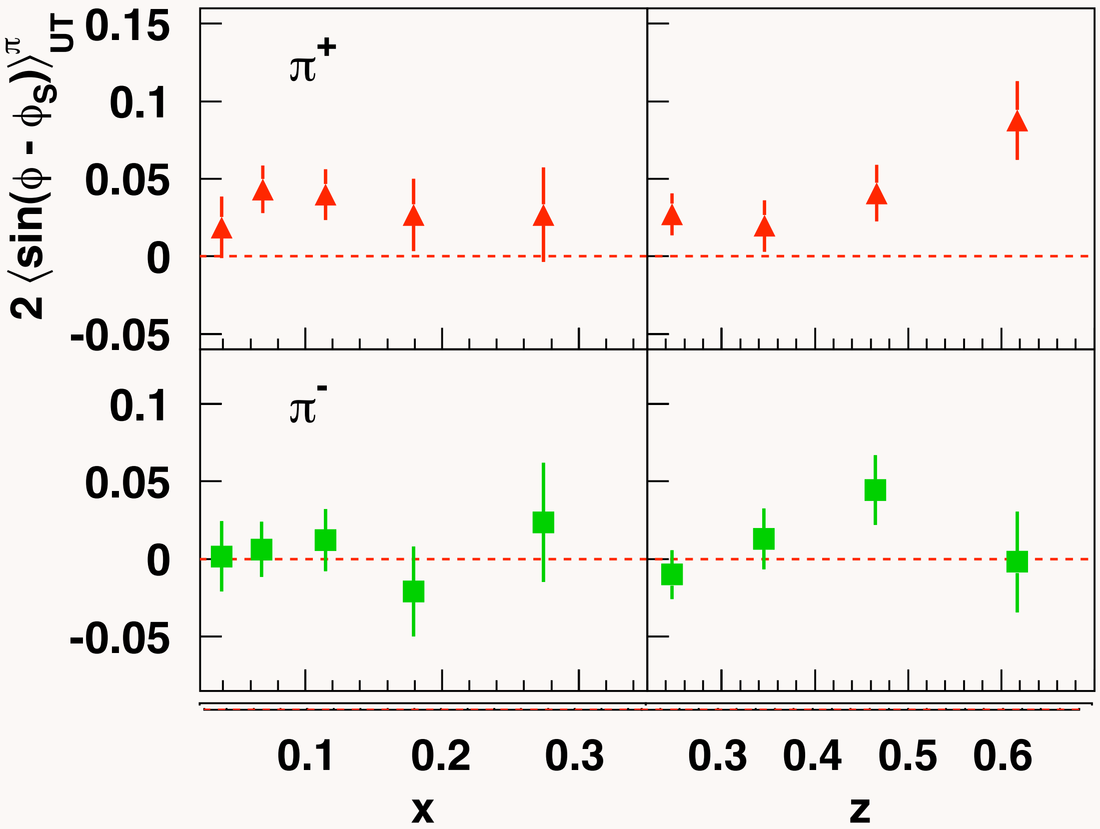
can interfere with



and produce a T-odd effect! (also need $L_z \neq 0$)

Hermes coll A, Airapetian et al., Phys. Rev. Lett. 94 (2005) 012002.

Sivers asymmetry from HERMES

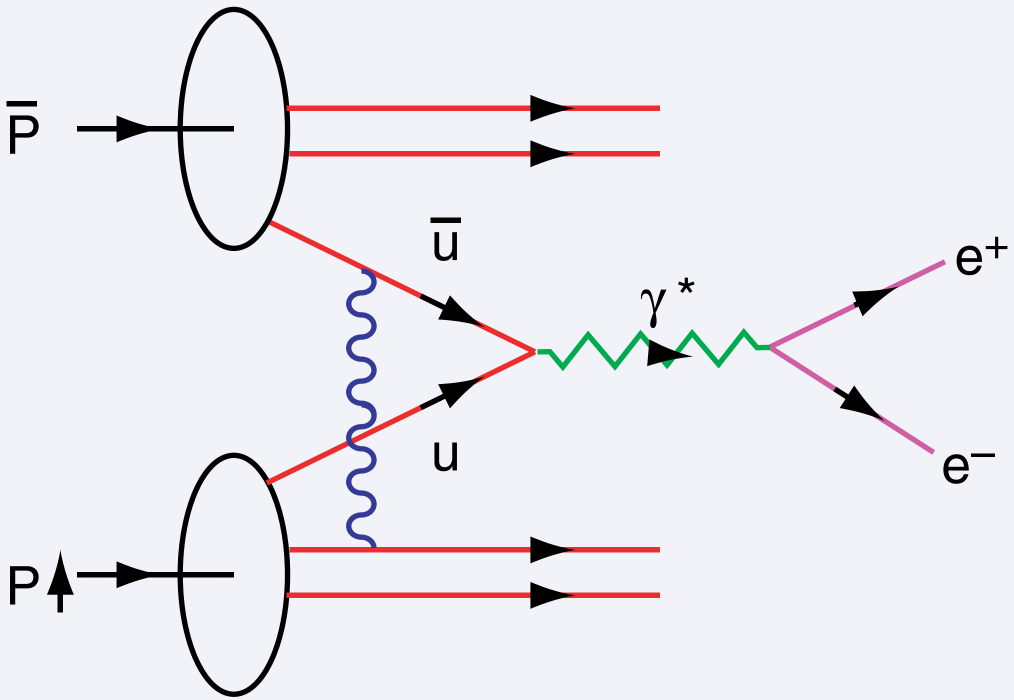


- First evidence for non-zero Sivers function!
- ⇒ presence of non-zero **quark orbital angular momentum**!
- Positive for **π⁺**... Consistent with zero for **π⁻**...

Gamberg: Hermes data compatible with BHS model

Schmidt, Lu: Hermes charge pattern follow quark contributions to anomalous moment

Predict Opposite Sign SSA in DY !



Collins;
Hwang, Schmidt.
sjb

Single Spin Asymmetry In the Drell Yan Process

$\vec{S}_p \cdot \vec{\overline{p}} \times \vec{q}_{\gamma^*}$

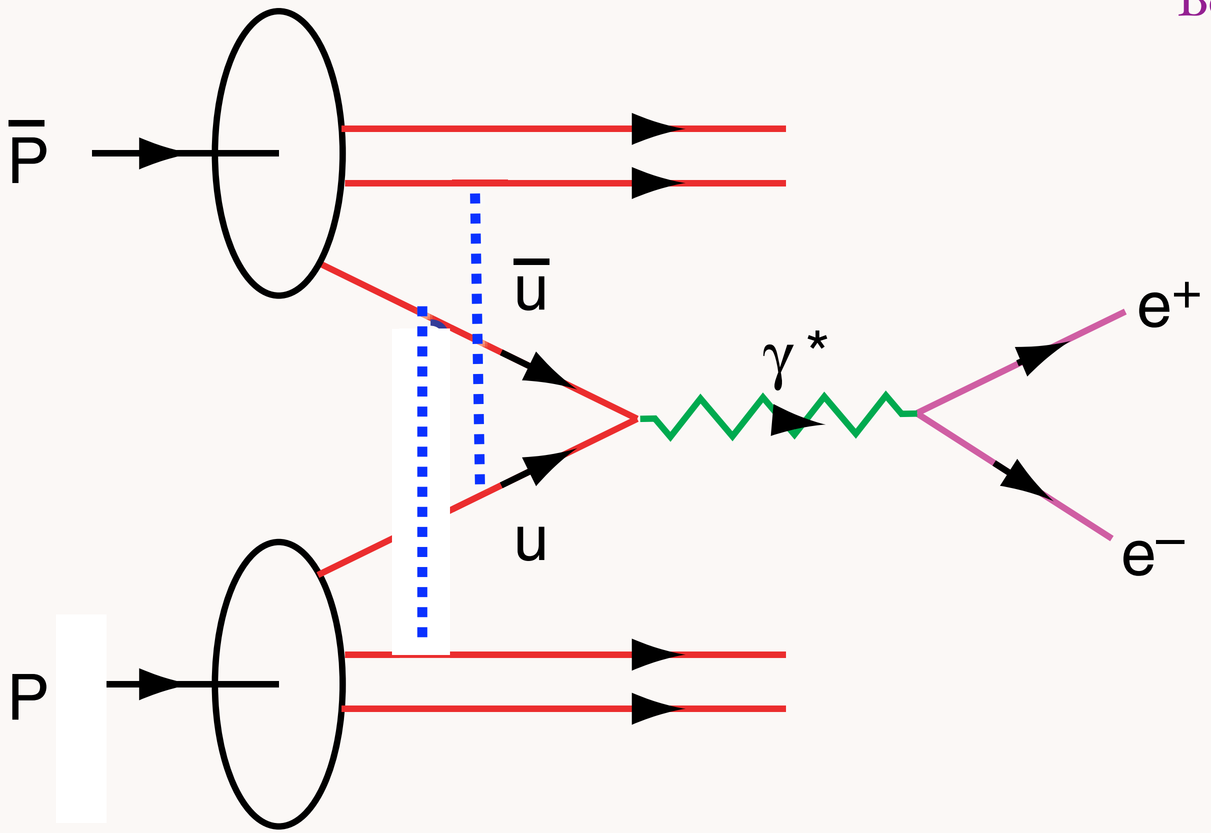
Quarks Interact in the Initial State

Interference of Coulomb Phases for S and P states

Produce Single Spin Asymmetry [Siver's Effect]Proportional to the Proton Anomalous Moment and α_s.

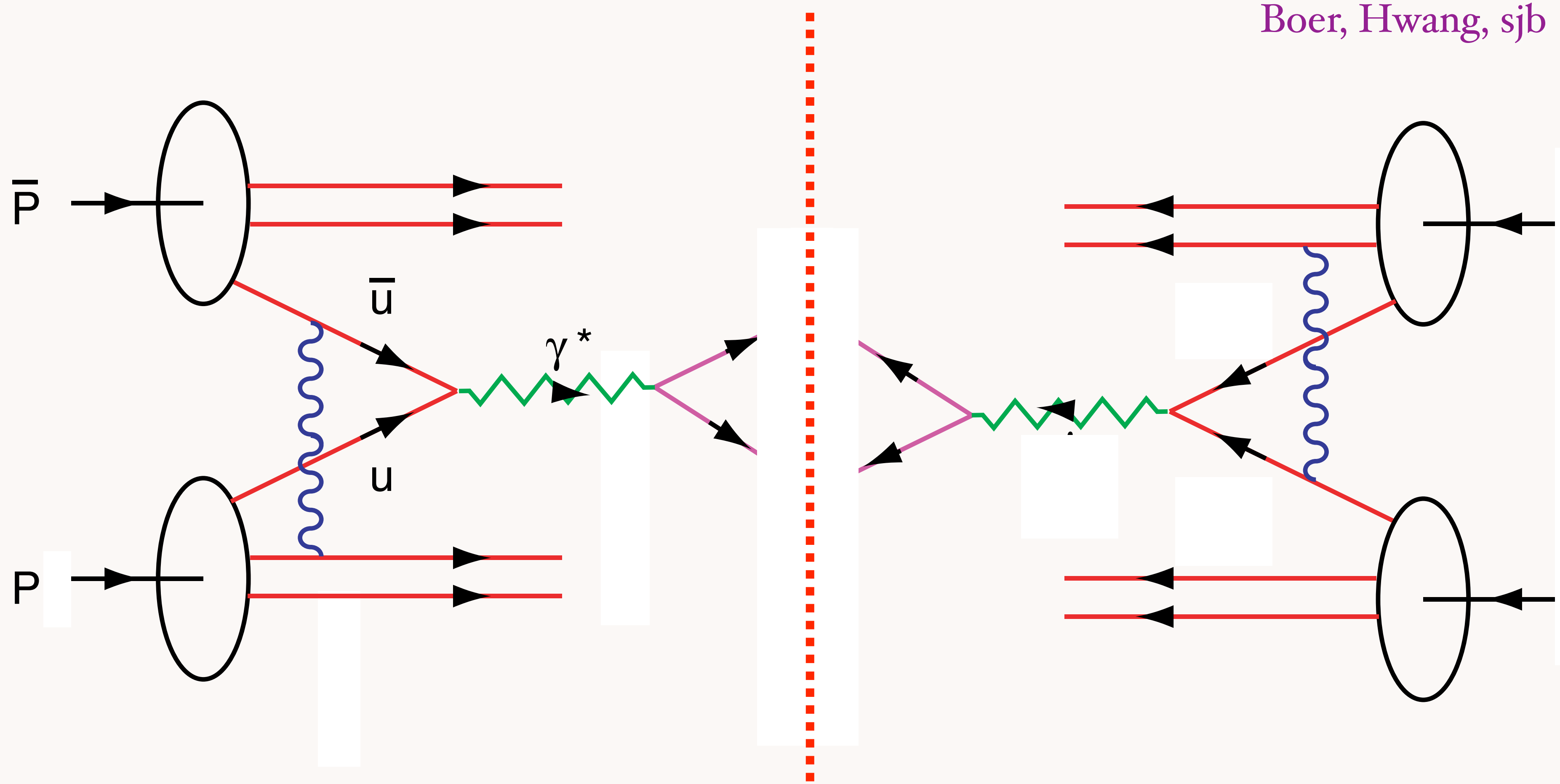
Opposite Sign to DIS! No Factorization

Boer, Hwang, sjb



DY $\cos 2$ **correlation at leading twist from double ISI**

Boer, Hwang, sjb

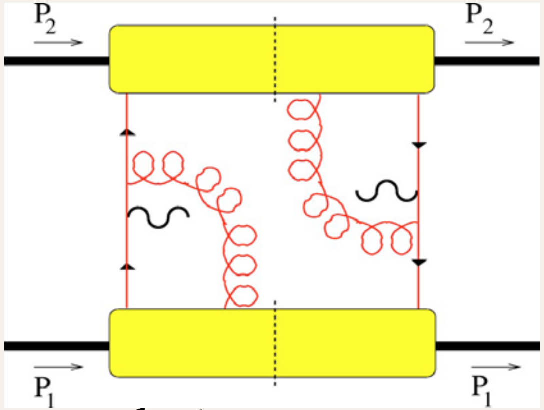


DY cos 2 **correlation at leading twist from double ISI**

Anomalous effect from Double ISI in Massive Lepton Production

Boer, Hwang, sjb

$\cos 2\phi$ correlation



- Leading Twist, valence quark dominated
- Violates Lam-Tung Relation!
- Not obtained from standard PQCD subprocess analysis
- Normalized to the square of the single spin asymmetry in semi-inclusive DIS
- No polarization required
- Challenge to standard picture of PQCD Factorization

Double Initial-State Interactions generate anomalous cos 2

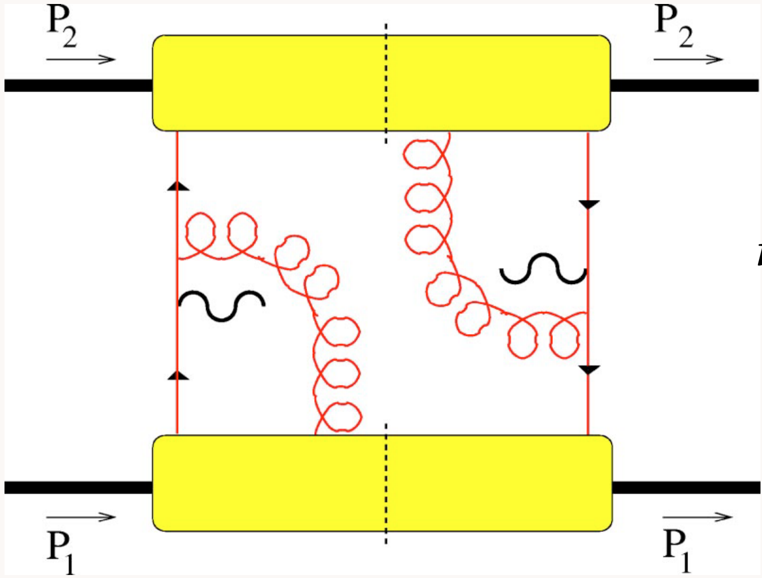
Boer, Hwang, sjb

Drell-Yan planar correlations

1 d / d   1 + cos² + μ sin 2 cos + — / 2 sin² cos2

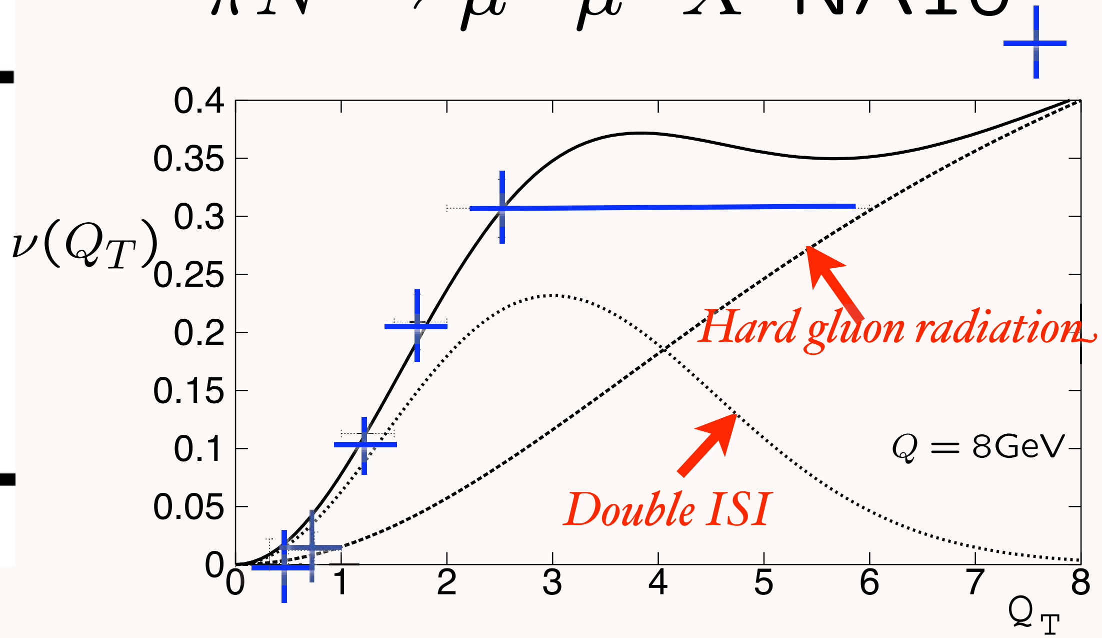
PQCD Factorization (Lam Tung): 1 − − 2 = 0

— / 2   h₁ ( ) h₁ (N)

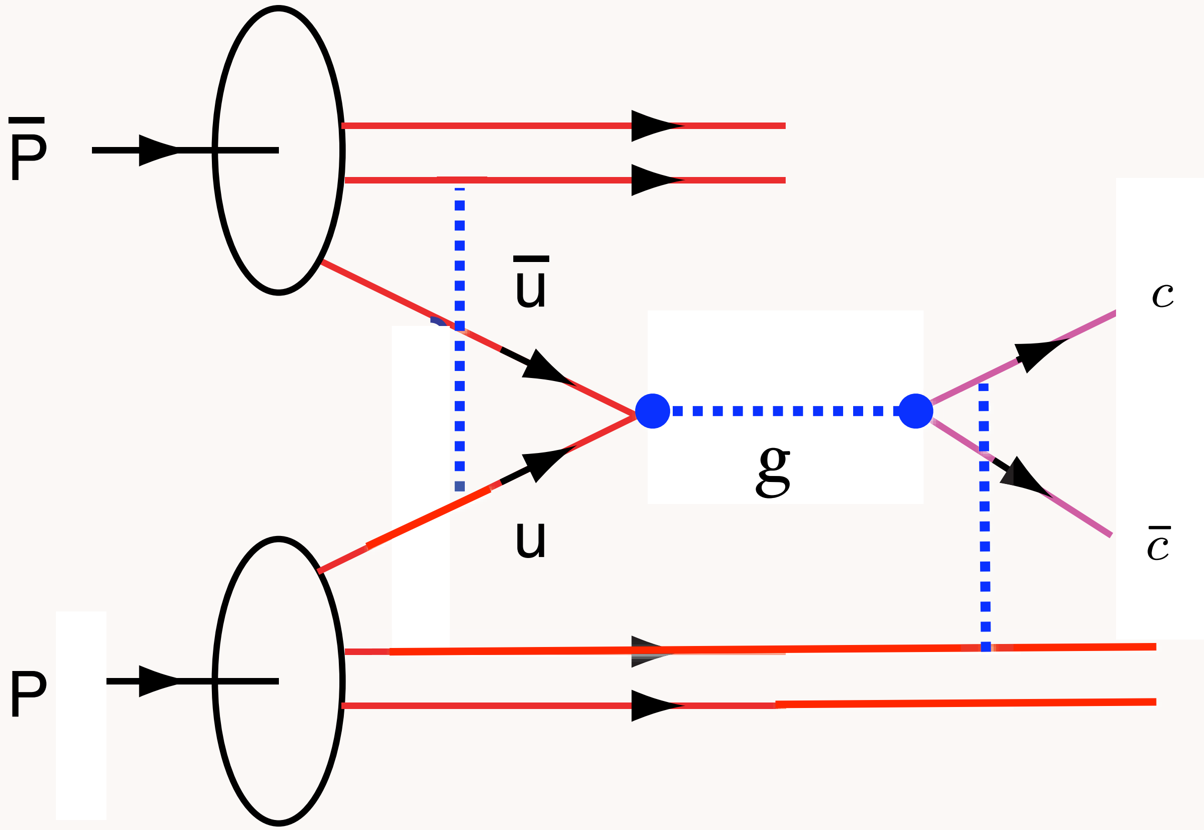


Violates Lam-Tung relation!

$\pi N \to \mu^+\mu^- X$ NA10



Model: Boer,

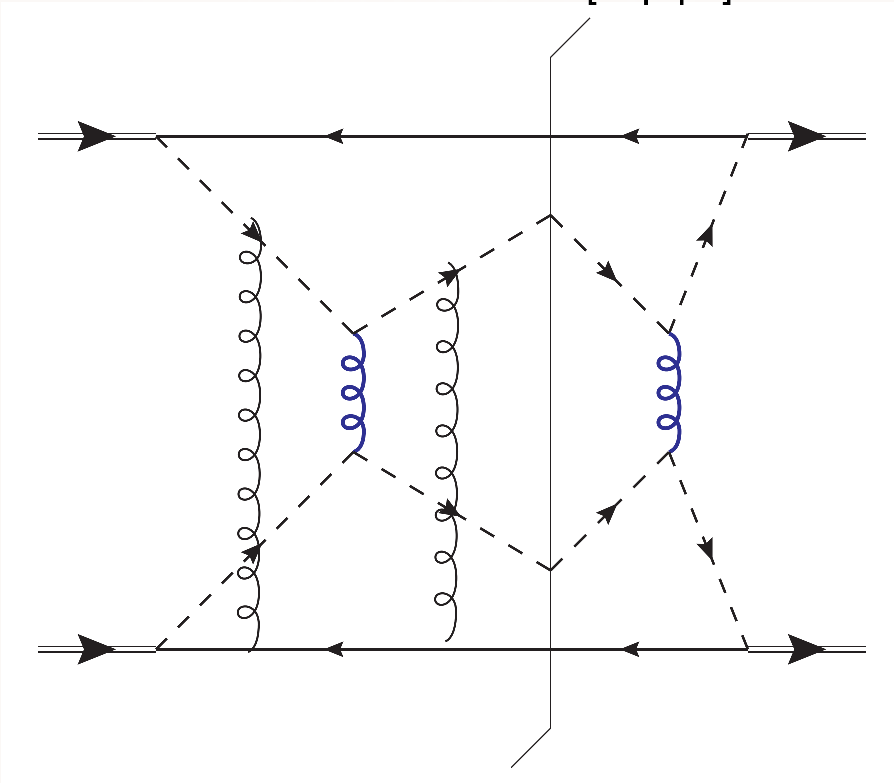


Problem for factorization when both ISI and FSI occur

Factorization is violated in production of high-transverse-momentum particles in hadron-hadron collisions

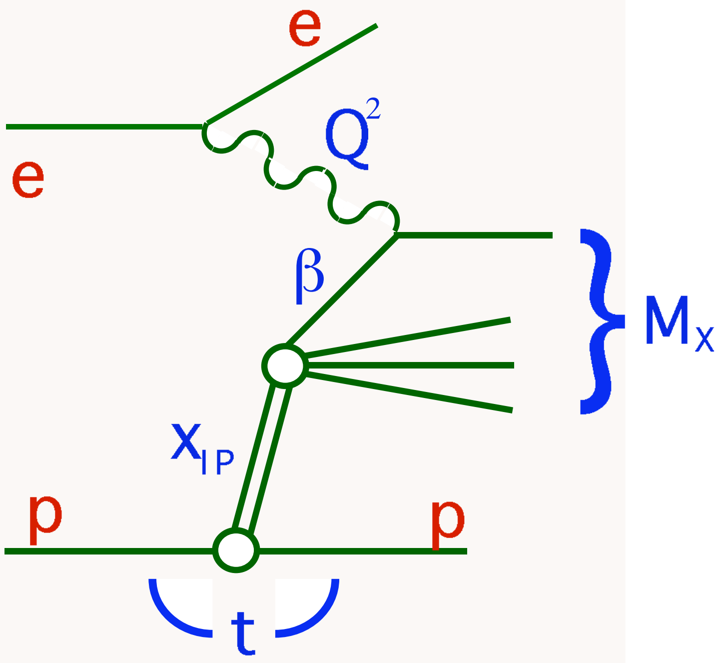
John Collins, Jian-Wei Qiu . ANL-HEP-PR-07-25, May 2007.

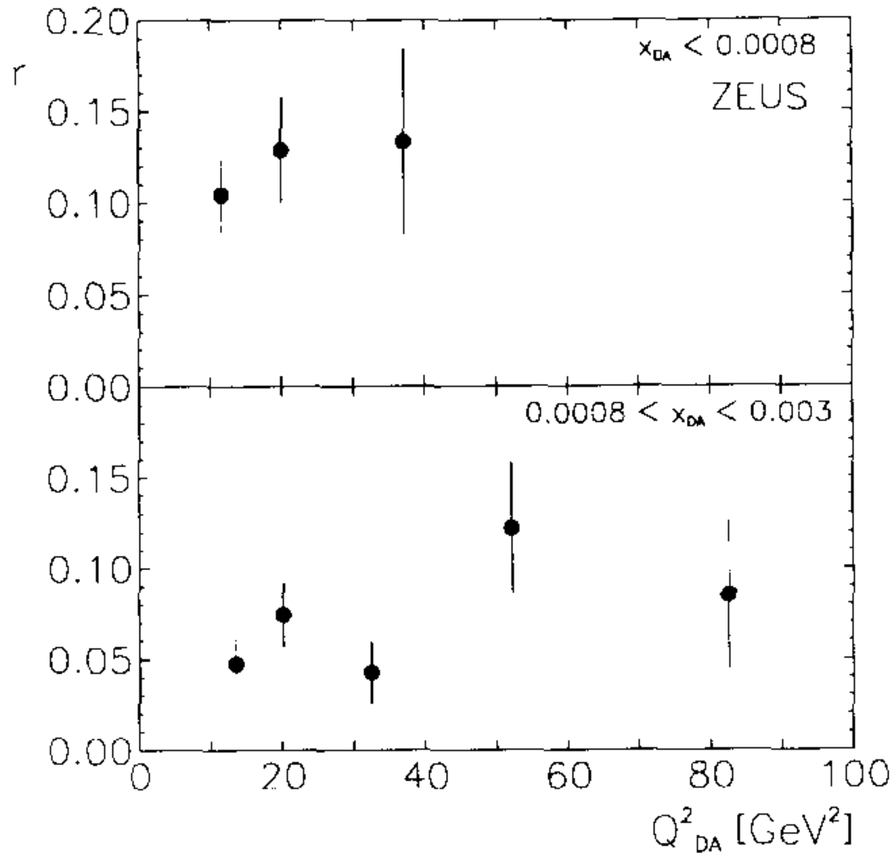
e-Print: **arXiv:0705.2141** [hep-ph]



The exchange of two extra gluons, as in this graph, will tend to give non-factorization in unpolarized cross sections.

Remarkable observation at HERA



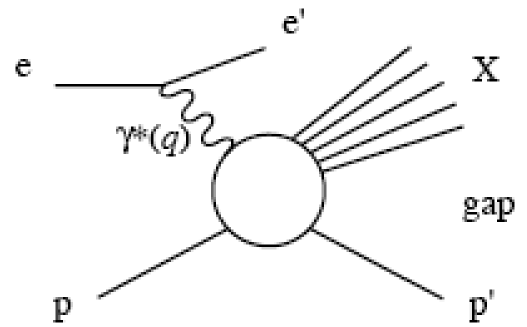


Fraction r of events with a large rapidity gap, $\eta_{\rm max} < 1.5$, as a function of $Q^2_{\rm DA}$ for two ranges of $x_{\rm DA}$. No acceptance corrections have been applied.

10% to 15% of DIS events are diffractive !

M. Derrick et al. [ZEUS Collaboration], Phys. Lett. B 315, 481 (1993).

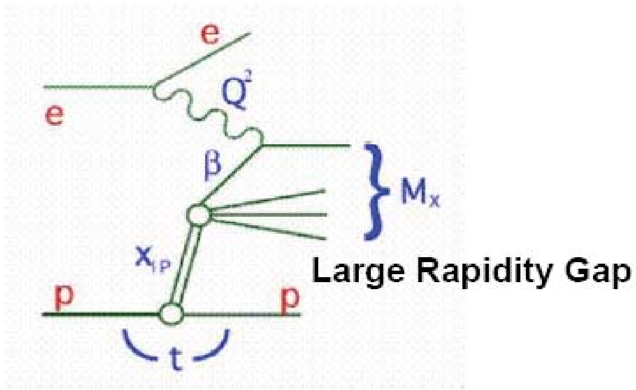
DDIS

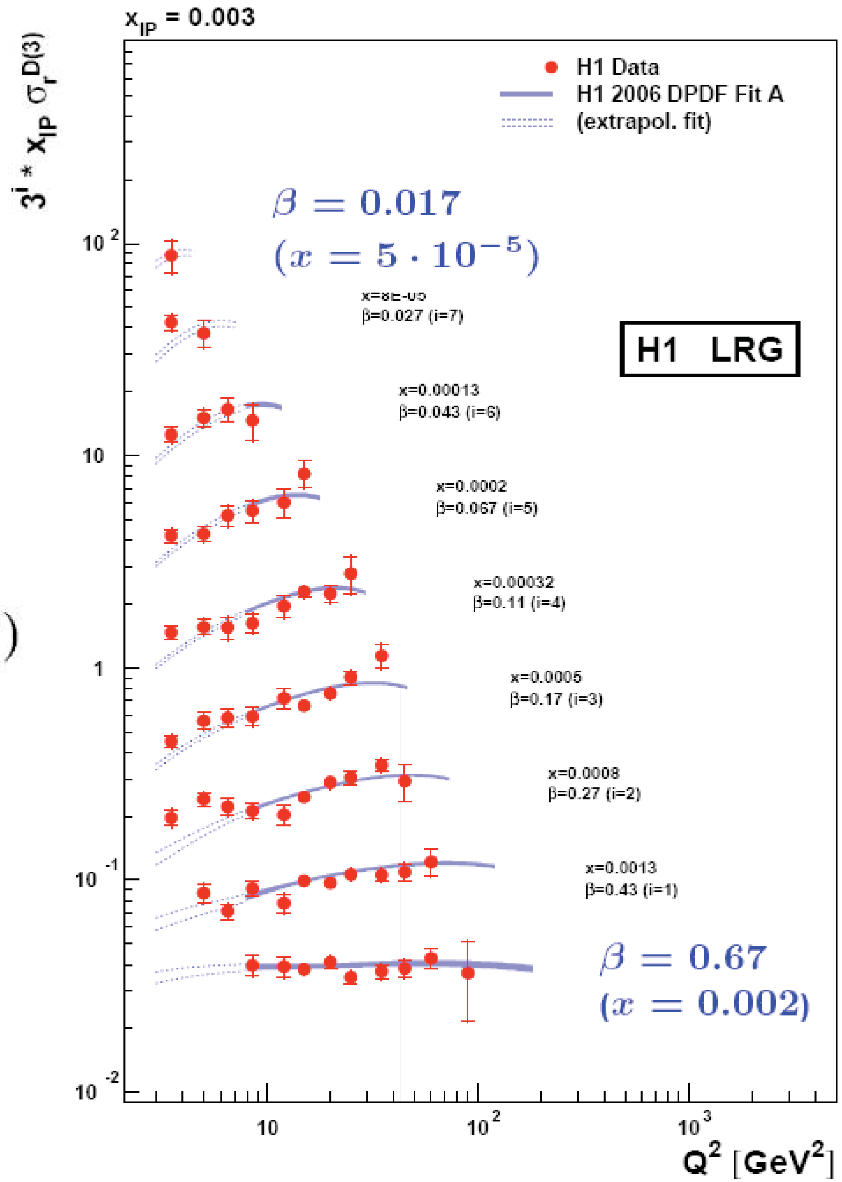


- In a large fraction ($\sim$ 10–15%) of DIS events, the proton escapes intact, keeping a large fraction of its initial momentum
- This leaves a large *rapidity gap* between the proton and the produced particles
- The t-channel exchange must be *color singlet* $\rightarrow$ a pomeron??

Diffractive Deep Inelastic Lepton-Proton Scattering

Diffractive Structure Function F_2^D





Diffractive inclusive cross section

$$\frac{d^3\sigma_{NC}^{diff}}{dx_{I\!P}\, d\beta\, dQ^2} \propto \frac{2\pi\alpha^2}{xQ^4} F_2^{D(3)}(x_{I\!P}, \beta, Q^2)$$

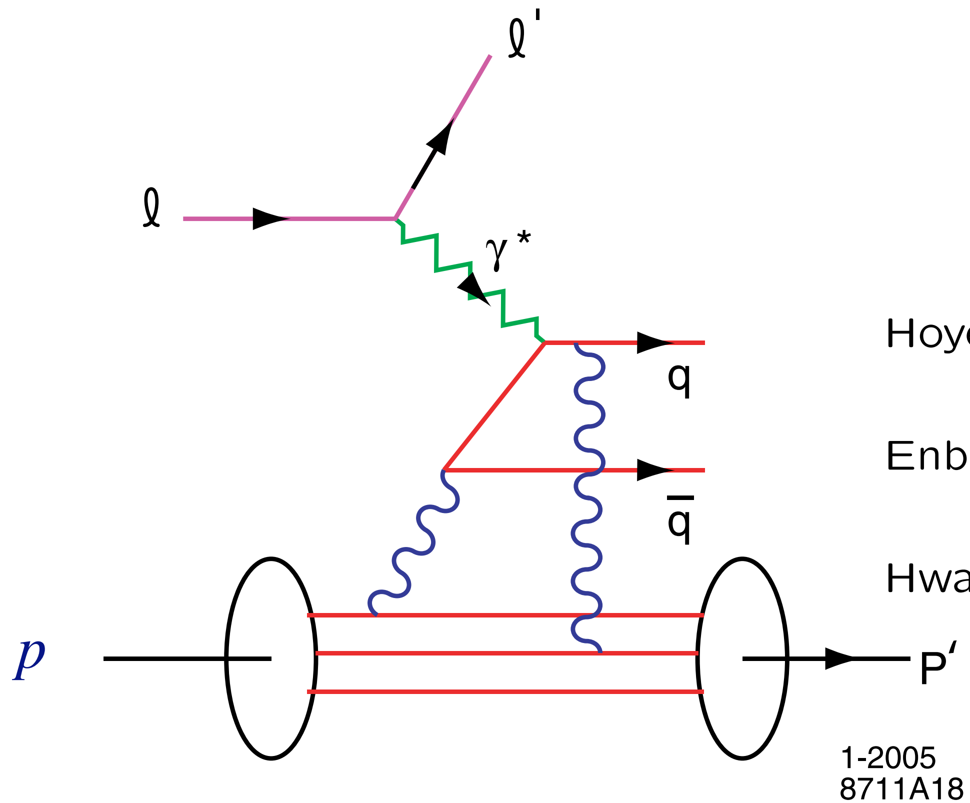
$$F_2^D(x_{I\!P}, \beta, Q^2) = f(x_{I\!P}) \cdot F_2^{I\!P}(\beta, Q^2)$$

extract DPDF and $xg(x)$ from scaling violation

Large kinematic domain $3 < Q^2 < 1600\ \mathrm{GeV}^2$

Precise measurements *sys 5%, stat 5–20 %*

Final-State Interaction Produces Diffractive DIS



Quark Rescattering

Hoyer, Marchal, Peigne, Sannino, SJB (BHM

Enberg, Hoyer, Ingelman, SJB

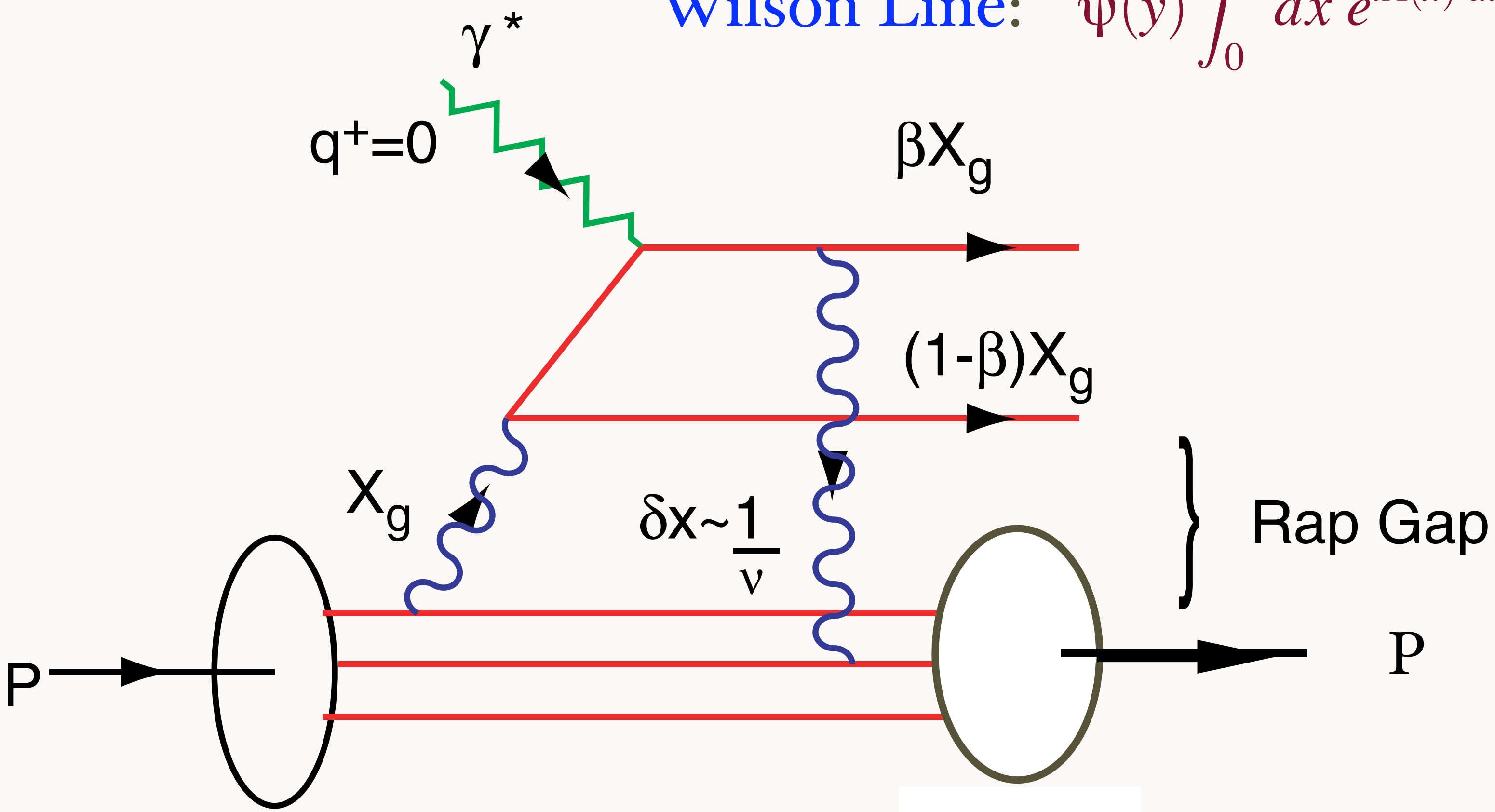
Hwang, Schmidt, SJB

Low-Nussinov model of Pomeron

Hoyer, Marchal, Peigne, Sannino, sjb

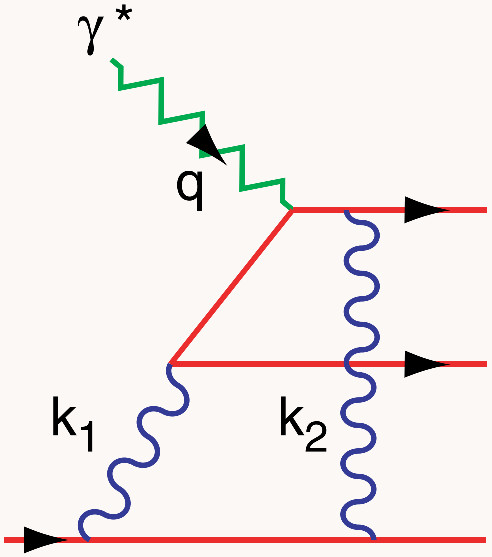
QCD Mechanism for Rapidity Gaps

Wilson Line: $\overline{\psi}(y) \int_0^y dx\, e^{iA(x)\cdot dx}\, \psi(0)$

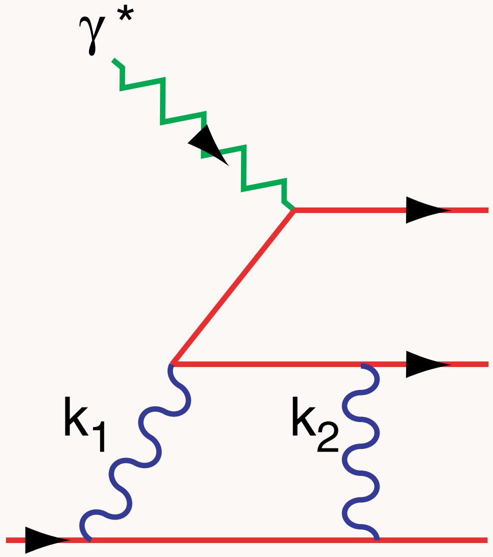


Reproduces lab-frame color dipole approach

Final State Interactions in QCD

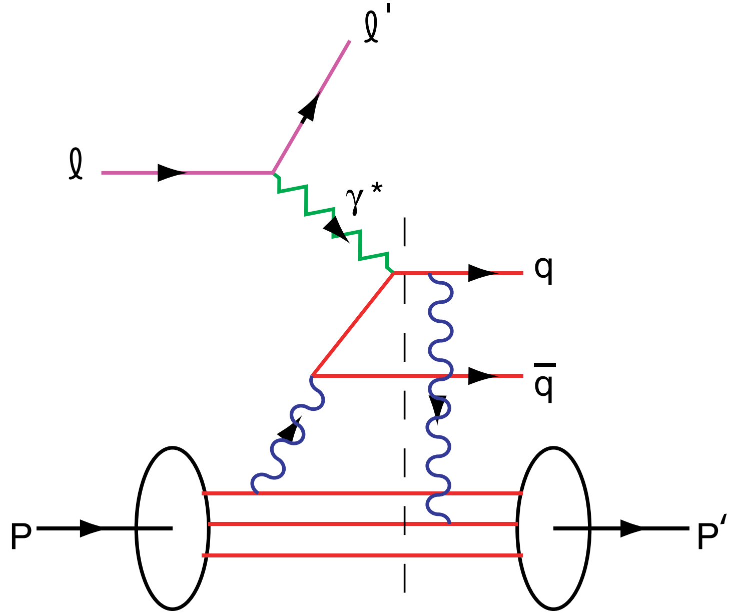


Feynman Gauge



Light-Cone Gauge

Result is Gauge Independent



Integration over on-shell domain produces phase i

Need Imaginary Phase to Generate Pomeron

Need Imaginary Phase to Generate
T-Odd Single-Spin Asymmetry

Physics of FSI not in Wavefunction of Target

Physics of Rescattering

- Sivers Asymmetry and Diffractive DIS: New Insights into Final State Interactions in QCD
- Origin of Hard Pomeron
- Structure Functions not Probability Distributions!
- T-odd SSAs, Shadowing, Antishadowing
- Diffractive dijets/ trijets, doubly diffractive Higgs
- Novel Effects: Color Transparency, Color Opaqueness, Intrinsic Charm, Odderon

Physics of Rescattering

- Diffractive DIS
- Non-Unitary Correction to DIS: Structure functions are not probability distributions
- Nuclear Shadowing, Antishadowing- Not in Target WF
- Single Spin Asymmetries -- opposite sign in DY and DIS
- DY cos 2 distribution at leading twist from double ISI-- not given by PQCD factorization -- breakdown of factorization!
- Wilson Line Effects not 1 even in LCG
- Must correct hard subprocesses for initial and final-state soft gluon attachments
- Corrections to Handbag Approximation in DVCS!

Hoyer, Marchal, Peigne, Sannino, sjb

"Dangling Gluons"

Bodwin, Lepage, sjb
Hoyer, Marchal, Peigne, Sannino, sjb

- Diffractive DIS
- Non-Unitary Correction to DIS: Structure functions are not probability distributions
- Nuclear Shadowing, Antishadowing
- Single Spin Asymmetries -- opposite sign in DY and DIS
- DY $\cos 2$ correlation at leading twist from double ISI-- not given by standard PQCD factorization
- Wilson Line Effects persist even in LCG
- Must correct hard subprocesses for initial and final-state soft gluon attachments -- Ji gauge link, Kovchegov gauge

Light-Front QCD Phenomenology

- Hidden color, Intrinsic glue, sea, Color Transparency
- Near Conformal Behavior of LFWFs at Short Distances; PQCD constraints
- Vanishing anomalous gravitomagnetic moment
- Relation between edm and anomalous magnetic moment
- Cluster Decomposition Theorem for relativistic systems
- OPE: DGLAP, ERBL evolution; invariant mass scheme

$$|p, S_z> = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) | n; \vec{k}_{\perp_i}, \lambda_i >$$

sum over states with n=3, 4, ...constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

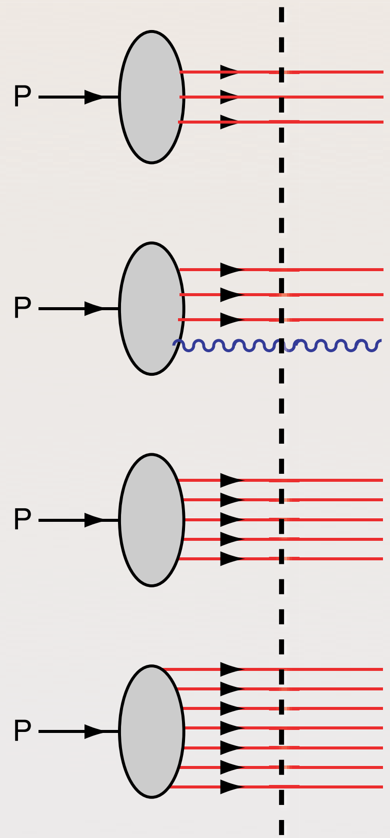
are boost invariant; they are independent of the hadron's energy and momentum P^μ.

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+, \ \sum_i^n x_i = 1, \ \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$



Intrinsic heavy quarks $\bar{u}(x) \neq \bar{d}(x)$

Mueller: BFKL DYNAMICS $\bar{s}(x) \neq s(x)$

Fixed LF time

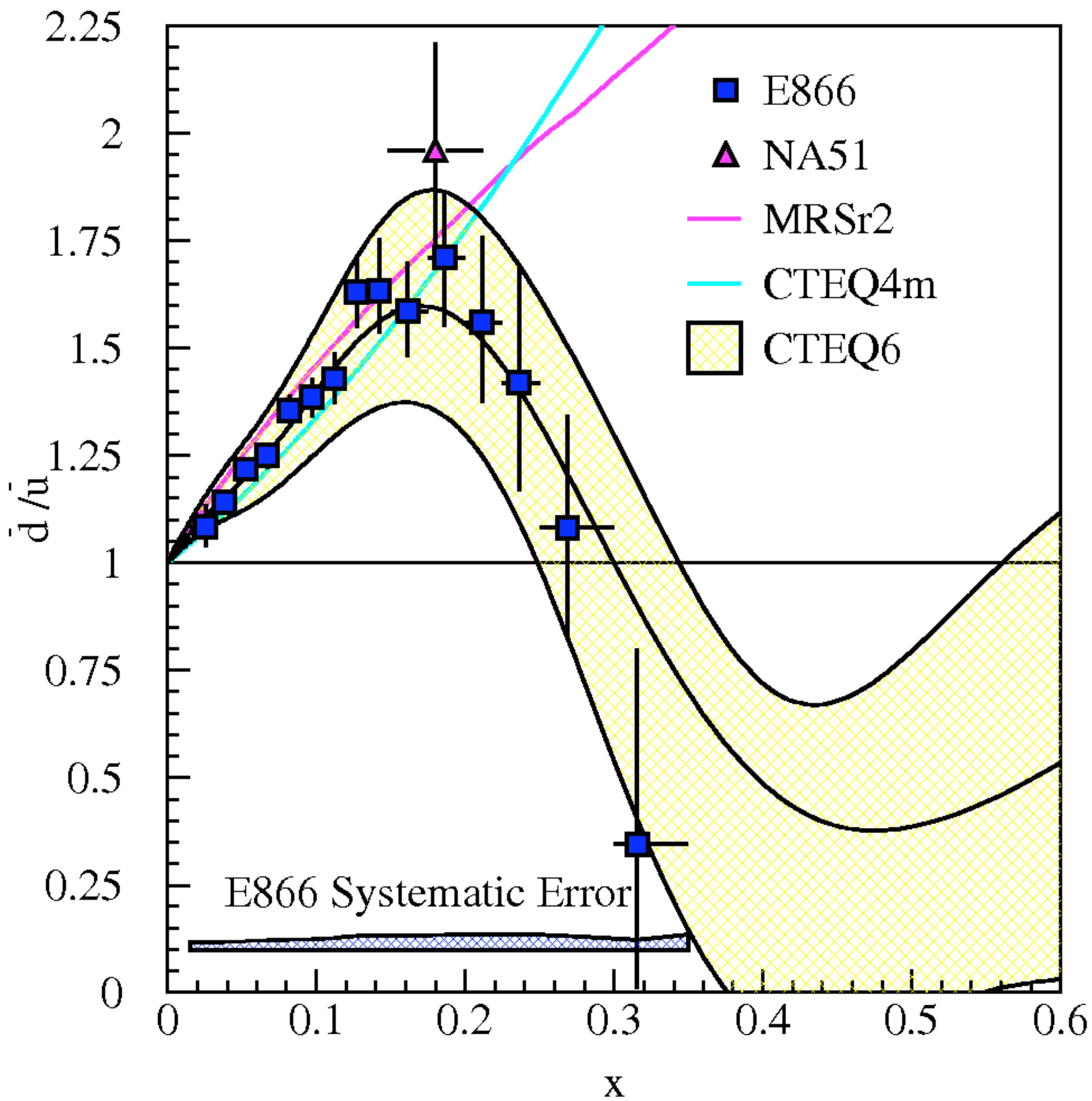
Light Antiquark Flavor Asymmetry

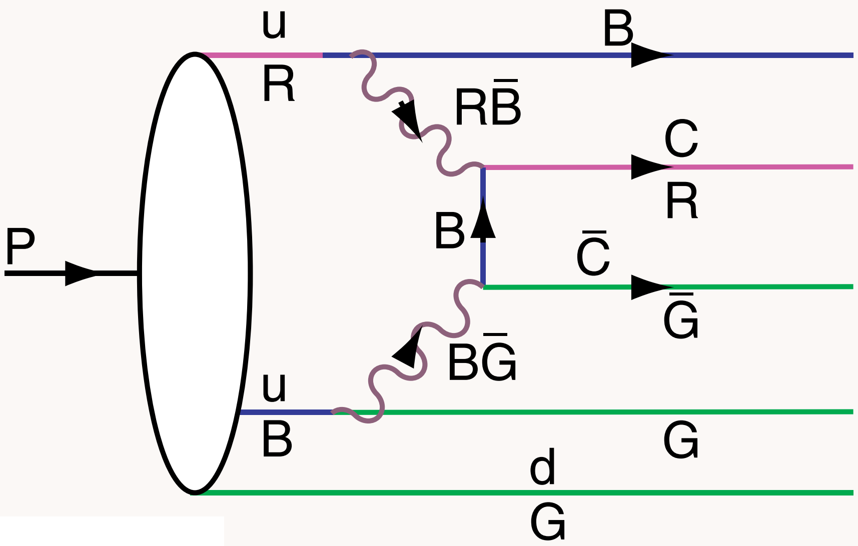
- Naïve Assumption from gluon splitting:

$$\bar{d}(x) = \bar{u}(x)$$

- E866/NuSea (Drell-Yan)

$\bar{d}(x)/\bar{u}(x)$ for $0.015 \leq x \leq 0.35$





$|uudc\bar{c}>$ Fluctuation in Proton

QCD: Probability $\frac{\sim\Lambda^2_{QCD}}{M^2_Q}$

$|e^+e^-\ell^+\ell^->$ Fluctuation in Positronium

QED: Probability $\frac{\sim(m_e\alpha)^4}{M^4_\ell}$

OPE derivation - M.Polyakov et al.

$< p|\frac{G^3_{\mu\nu}}{m^2_Q}|p >$ vs. $< p|\frac{F^4_{\mu\nu}}{m^4_\ell}|p >$

$c\bar{c}$ in Color Octet

Distribution peaks at equal rapidity (velocity) Therefore heavy particles carry the largest momentum fractions

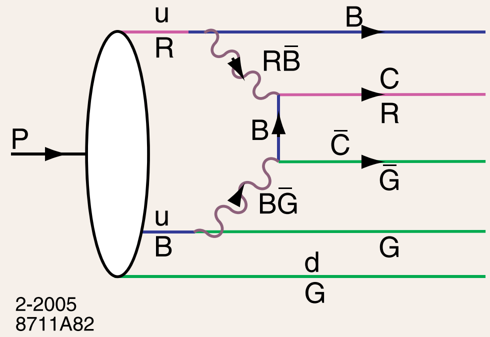
$$\hat{x}_i = \frac{m_{\perp i}}{\sum_j^n m_{\perp j}}$$

High x charm!

Hoyer, Peterson, Sakai, sjb

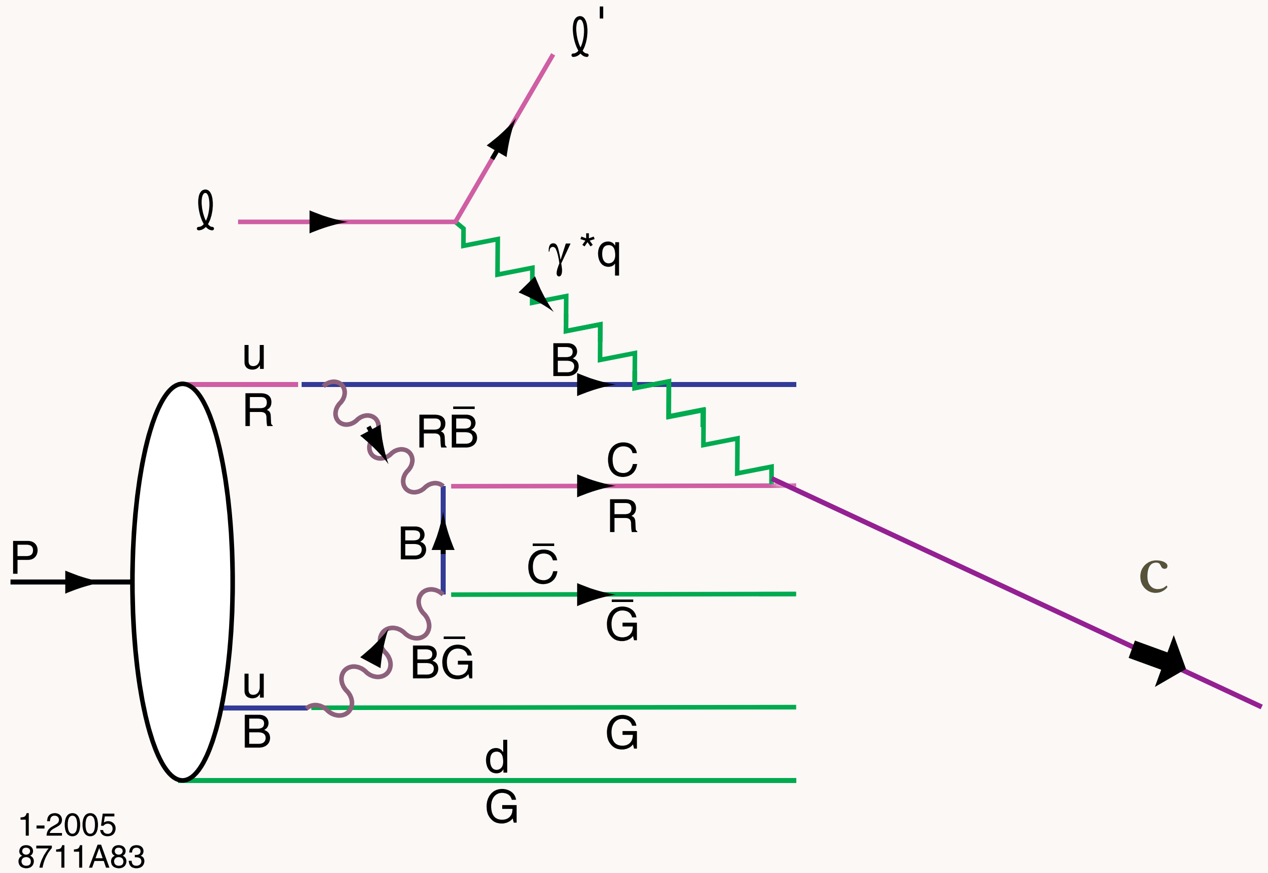
Hoyer, Peterson, Sakai, sjb

Intrinsic Heavy-Quark Fock States



- Rigorous prediction of QCD, OPE
- Color - Octet + Color - Octet Fock State!
- Probability $P_{Q\bar{Q}} \propto \frac{1}{M_Q^2}$ $P_{Q\bar{Q}Q\bar{Q}} \sim \alpha_s^2 P_{Q\bar{Q}}$ $P_{c\bar{c}/p} \simeq 1\%$
- Large Effect at high x
- Greatly increases kinematics of colliders such as Higgs production (Kopeliovich, Schmidt, Soffer, sjb)
- Severely underestimated in conventional parameterizations of heavy quark distributions (Pumplin, Tung)
- Many empirical tests

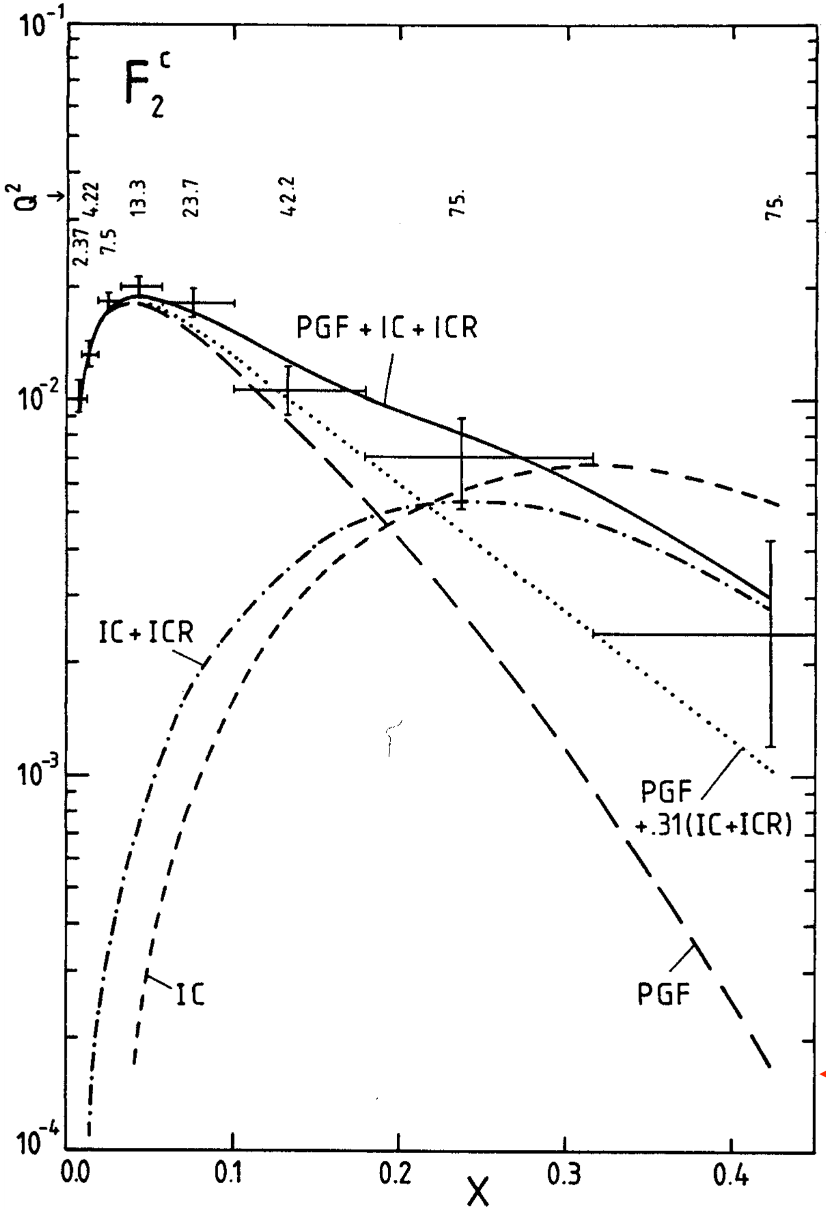
Measure c(x) in Deep Inelastic Lepton-Proton Scattering



Hoyer, Peterson, SJB

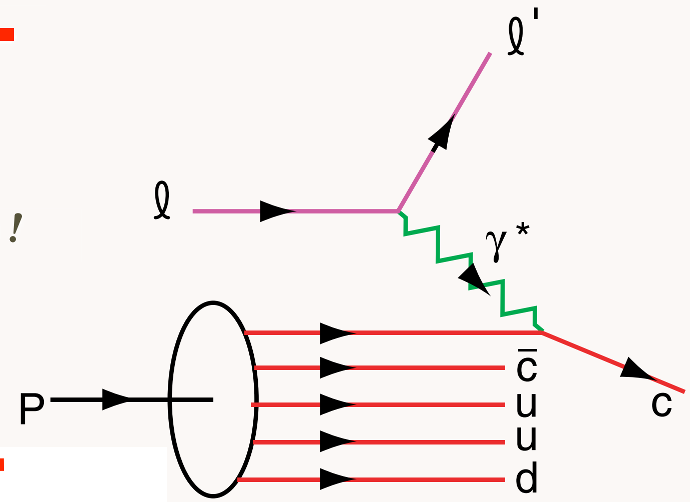
Measurement of Charm Structure Function

J. J. Aubert et al. [European Muon Collaboration], "Production Of Charmed Particles In 250-Gev Mu+ - Iron Interactions," Nucl. Phys. B 213, 31 (1983).



First Evidence for Intrinsic Charm

factor of 30 !



DGLAP / Photon-Gluon Fusion: factor of 30 too small

- EMC data: $c(x, Q^2) > 30 \times$ DGLAP
  $Q^2 = 75$ GeV^2, $x = 0.42$

- High x_F $pp \to J/\psi X$

- High x_F $pp \to J/\psi J/\psi X$

- High x_F $pp \to \Lambda_c X$

- High x_F $pp \to \Lambda_b X$

- High x_F $pp \to \Xi(ccd) X$ (SELEX)

Novel Heavy Flavor Physics

- LFWFS -- remarkable model from AdS/CFT
- AdS/CFT: Hadron Spectra and Dynamics, Counting Rules
- Intrinsic Charm and Bottom: rigorous prediction of QCD
- B decays: Many Novel QCD Effects
- Exclusive Channels: QCD at Amplitude Level
- Test B-analyses in other hard exclusive reactions, such as two-photon reactions
- Initial and Final State QCD Interactions -- Breakdown of QCD Factorization in Heavy Quark Hadroproduction!
- Renormalization scale not arbitrary

Quark and Gluon condensates reside within hadrons, not vacuum

Shrock, sjb

- **Bound-State Dyson-Schwinger Equations**
- **LF vacuum trivial up to $k^+ = 0$ zero modes**
- **Analogous to finite size superconductor**
- **Implications for cosmological constant -- reduction by 55 orders of magnitude!**

String Theory

↓

AdS/CFT

Mapping of Poincare' and Conformal SO(4,2) symmetries of 3+1 space to AdS5 space

↓

Goal: First Approximant to QCD

Counting rules for Hard Exclusive Scattering
Regge Trajectories
QCD at the Amplitude Level

AdS/QCD

Conformal behavior at short distances
+ Confinement at large distance

↓

Semi-Classical QCD / Wave Equations

↓ Holography

Boost Invariant 3+1 Light-Front Wave Equations

J =0,1,1/2,3/2 plus L

↓ Integrable!

Hadron Spectra, Wavefunctions, Dynamics

New Perspectives on QCD Phenomena from AdS/CFT

- AdS/CFT: Duality between string theory in Anti-de Sitter Space and Conformal Field Theory
- New Way to Implement Conformal Symmetry
- Holographic Model: Conformal Symmetry at Short Distances, Confinement at large distances
- Remarkable predictions for hadronic spectra, wavefunctions, interactions
- AdS/CFT provides novel insights into the quark structure of hadrons

Outlook

- Only one scale Λ_{QCD} determines hadronic spectrum (slightly different for mesons and baryons).
- Ratio of Nucleon to Delta trajectories determined by zeroes of Bessel functions.
- String modes dual to baryons extrapolate to three fermion fields at zero separation in the AdS boundary.
- Only dimension $3, \frac{9}{2}$ and 4 states $\overline{q}q$, qqq, and gg appear in the duality at the classical level!
- Non-zero orbital angular momentum and higher Fock-states require introduction of quantum fluctuations.
- Simple description of space and time-like structure of hadronic form factors.
- Dominance of quark-interchange in hard exclusive processes emerges naturally from the classical duality of the holographic model. Modified by gluonic quantum fluctuations.
- Covariant version of the bag model with confinement and conformal symmetry.

Light-Front Holography and AdS/QCD Correspondence.
Stanley J. Brodsky, Guy F. de Teramond . SLAC-PUB-13220, Apr 2008. 14pp.
e-Print: **arXiv:0804.3562** [hep-ph]

Light-Front Dynamics and AdS/QCD Correspondence: Gravitational Form Factors of Composite Hadrons.
Stanley J. Brodsky (SLAC) , Guy F. de Teramond (Ecole Polytechnique, CPHT & Costa Rica U.) . SLAC-PUB-13192, Apr 2008. 12pp. e-Print: **arXiv:0804.0452** [hep-ph]

AdS/CFT and Light-Front QCD.
Stanley J. Brodsky, Guy F. de Teramond . SLAC-PUB-13107, Feb 2008. 38pp.
Invited talk at International School of Subnuclear Physics: 45th Course: Searching for the "Totally Unexpected" in the LHC Era, Erice, Sicily, Italy, 29 Aug - 7 Sep 2007.
e-Print: **arXiv:0802.0514** [hep-ph]

AdS/CFT and Exclusive Processes in QCD.
Stanley J. Brodsky, Guy F. de Teramond . SLAC-PUB-12804, Sep 2007. 29pp. Temporary entry
e-Print: **arXiv:0709.2072** [hep-ph]

Light-Front Dynamics and AdS/QCD Correspondence: The Pion Form Factor in the Space- and Time-Like Regions.
Stanley J. Brodsky (SLAC) , Guy F. de Teramond (Costa Rica U. & SLAC) . SLAC-PUB-12554, SLAC-PUB-12544, Jul 2007. 20pp.
Published in **Phys.Rev.D77:056007,2008**.
e-Print: **arXiv:0707.3859** [hep-ph]

1. **“Light-Front Dynamics and AdS/QCD: The Pion Form Factor in the Space- and Time-Like Regions”**
S. J. Brodsky and G. F. de Teramond
arXiv:0707.3859 [hep-ph]
SLAC-PUB-12554(2007) (Submitted to Phys.Rev.D)

2. **“AdS/CFT and QCD”**
S. J. Brodsky and G. F. de Teramond
arXiv:hep-th/0702205
SLAC-PUB-12361(2007)
Invited talk at 2006 International Workshop on the Origin of Mass and Strong Coupling Gauge Theories (SCGT 06), Nagoya, Japan, 21-24 Nov 2006

3. **“Hadronic spectra and light-front wavefunctions in holographic QCD”**
S. J. Brodsky and G. F. de Teramond
Phys. Rev. Lett. **96**, 201601 (2006) [arXiv:hep-ph/0602252]

4. **“Advances in light-front quantization and new perspectives for QCD from AdS/CFT”**
S. J. Brodsky and G. F. de Teramond
Nucl. Phys. Proc. Suppl. **161**, 34 (2006)
Invited talk at Workshop on Light-Cone QCD and Nonperturbative Hadron Physics 2005 (LC 2005), Cairns, Queensland, Australia, 7-15 Jul 2005

5. **“Hadron spectroscopy and wavefunctions in QCD and the AdS/CFT correspondence”**
S. J. Brodsky and G. F. de Teramond
AIP Conf. Proc. **814**, 108 (2006) [arXiv:hep-ph/0510240]
Invited talk at 11th International Conference on Hadron Spectroscopy (Hadron05), Rio de Janeiro, Brazil, 21-26 Aug 2005

6. **"Applications of AdS/CFT duality to QCD"**
S. J. Brodsky and G. F. de Teramond
Int. J. Mod. Phys. A **21**, 762 (2006) [arXiv:hep-ph/0509269]
Invited talk at International Conference on QCD and Hadronic Physics, Beijing, China, 16-20 Jun 2005

7. **"Nearly conformal QCD and AdS/CFT"**
G. F. de Teramond and S. J. Brodsky
arXiv:hep-ph/0507273
SLAC-PUB-11375(2005)
Presented at 1st Workshop on Quark-Hadron Duality and the Transition to pQCD, Frascati, Rome, Italy, 6-8 Jun 2005

8. **"The hadronic spectrum of a holographic dual of QCD"**
G. F. de Teramond and S. J. Brodsky
Phys. Rev. Lett. **94**, 201601 (2005) [arXiv:hep-th/0501022]

9. **"Baryonic states in QCD from gauge / string duality at large N(c)"**
G. F. de Teramond and S. J. Brodsky
arXiv:hep-th/0409074
SLAC-PUB-10693(2004)
Presented at ECT Workshop on Large Nc QCD 2004, Trento, Italy, 5-9 Jul 2004*

10. **"Light-front hadron dynamics and AdS/CFT correspondence"**
S. J. Brodsky and G. F. de Teramond
Phys. Lett. B **582**, 211 (2004) [arXiv:hep-th/0310227]

A Few References: Bottom-up-Approach

- Derivation of dimensional counting rules of hard exclusive glueball scattering in AdS/CFT:

  Polchinski and Strassler, hep-th/0109174.

- Deep inelastic scattering in AdS/CFT:

  Polchinski and Strassler, hep-th/0209211.

- Unified description of the soft and hard pomeron in AdS/CFT:

  Brower, Polchinski, Strassler and Tan, hep-th/0603115.

- Hadron couplings and form factors in AdS/CFT:

  Hong, Yoon and Strassler, hep-th/0409118.

- Low lying meson spectra, chiral symmetry breaking and hadron couplings in AdS/QCD (Emphasis on axial and vector currents)

  Erlich, Katz, Son and Stephanov, hep-ph/0501128,

  Da Rold and Pomarol, hep-ph/0501218, hep-ph/0510268.

- Gluonium spectrum (top-bottom):

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