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Effects of ground state NN correlations on medium and high energy scattering off nuclei.

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**On the Effects of NN Correlations
on
Medium- and High-Energy Processes off Nuclei**

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1. Exclusive processes off few-body nuclei: ${}^3\text{He}$

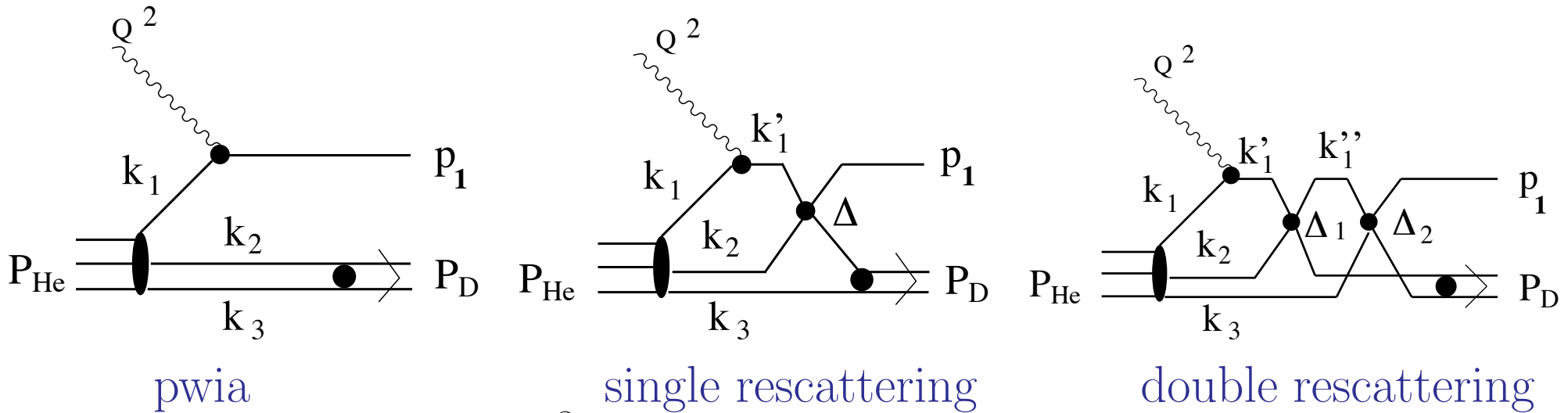
- NN correlations taken into account by realistic, state-of-the-art few-body wave functions obtained with $AV18$ interaction
 \rightarrow *initial state*: $\Psi_{3\text{He}} = \hat{A} e^{i\mathbf{P}\cdot\mathbf{R}} \psi_3(\boldsymbol{\rho}, \mathbf{r})$
 \rightarrow *final state*: $\Phi_f^*(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = \hat{A} S(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) e^{-i\mathbf{p}_1\mathbf{r}_1} e^{-i\mathbf{P}_D\mathbf{R}_D} \Psi_{D/pn}^*(\mathbf{r})$
- Factorization approximation and non-relativistic reduction removed by performing calculations in momentum space; current operator chosen according to the $CC1$ prescription:

$$\begin{aligned} \langle s_f | J_\mu(\mathbf{k}_1, \mathbf{q}) | \lambda \rangle &= \\ &= \bar{u}(\mathbf{k}_1 + \mathbf{q}, s_f) \left[\gamma_\mu \left(F_1(Q^2) + F_2(Q^2) \right) - (2k_1 + q)_\mu F_2(Q^2) \right] u(\mathbf{k}_1, \lambda) \end{aligned}$$

- FSI in $S(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3)$, calculated within generalized eikonal approximation

${}^3\text{He}(e, e'p){}^2\text{H}$: diagrammatic approach to FSI

Δ : momentum transfer in the NN rescattering

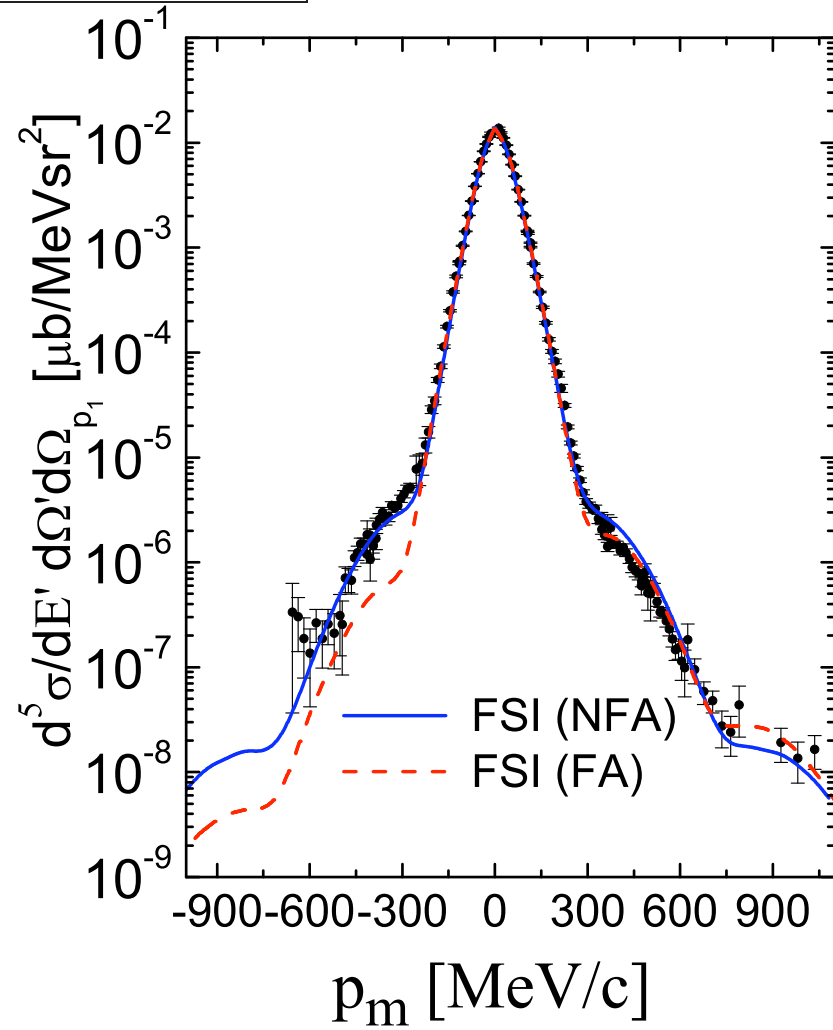
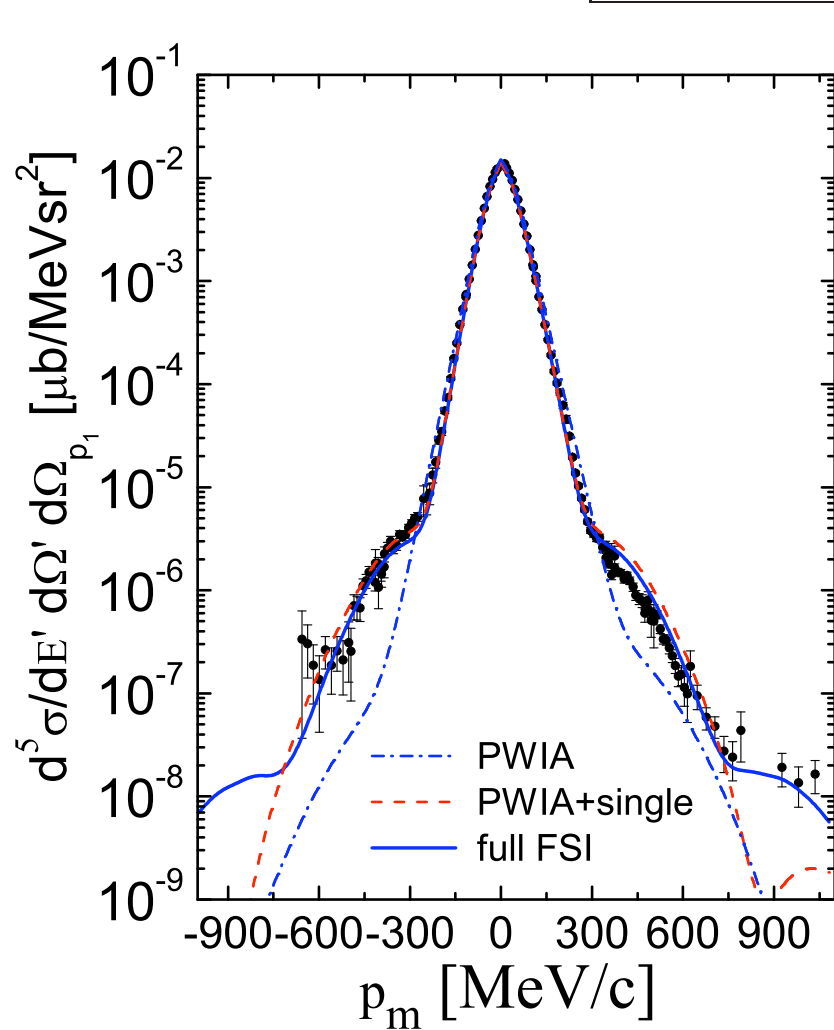


$$S(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = \prod_{j=2}^3 \left[1 - \theta(\mathbf{r}_{j\parallel} - \mathbf{r}_{1\parallel}) \Gamma(\mathbf{r}_{j\perp} - \mathbf{r}_{1\perp}) \right]$$

$$\Gamma(\mathbf{r}_\perp) = \frac{1}{2\pi i K^2} \int d^2\kappa_\perp f_{NN}(\kappa_\perp) e^{-i\kappa_\perp \mathbf{r}_\perp}$$

$$f_{NN}(\Delta_\perp) = K \frac{\sigma^{tot}(i + \alpha)}{4\pi} e^{-b^2 \Delta_\perp^2 / 2}$$

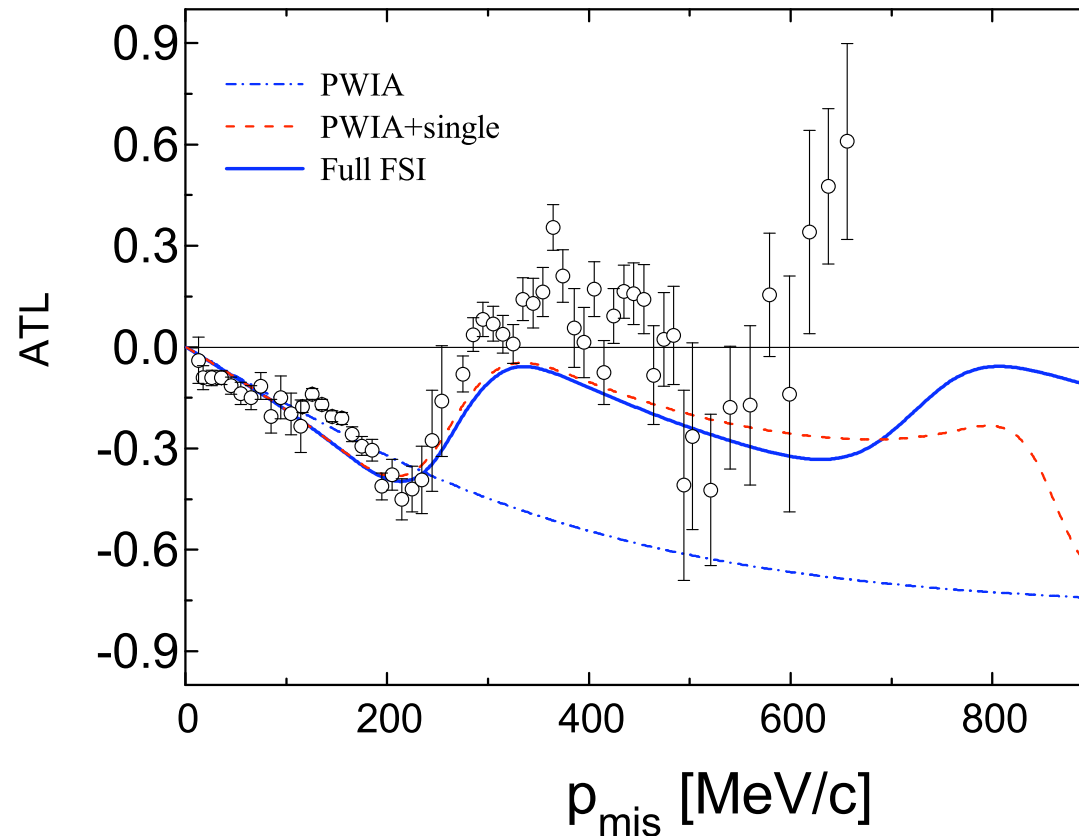
${}^3\text{He}(e, e'p){}^2\text{H}$: results



Data: *M.M. Rvachev et al., PRL94 (2005) 192302*

Calculations: *C.Ciofi degli Atti, L.P. Kaptari, PRL100 (2008) 122301*

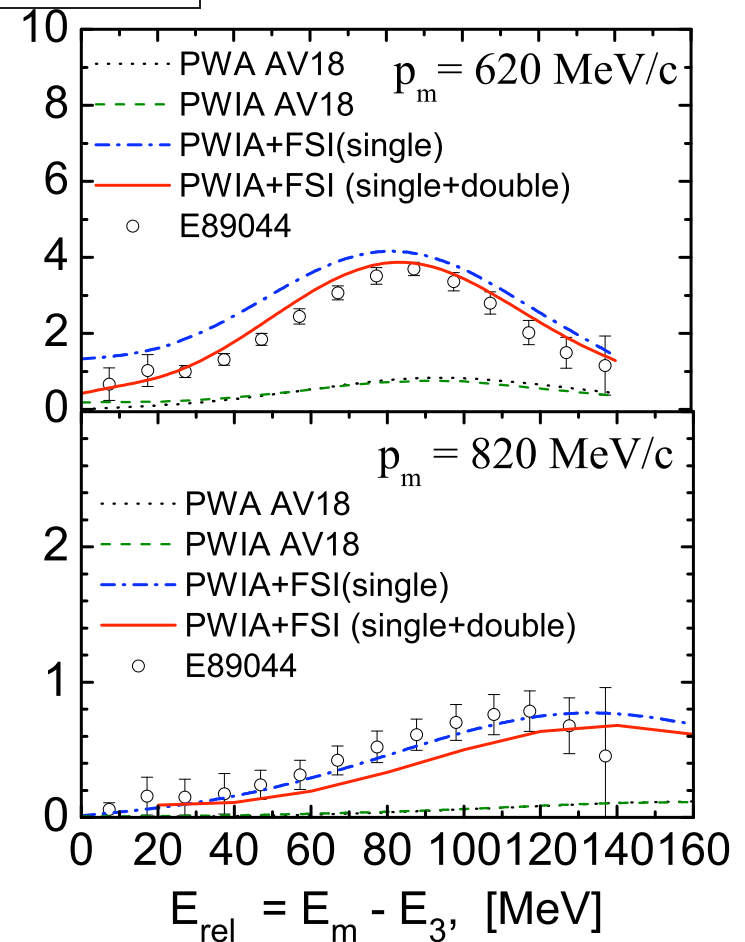
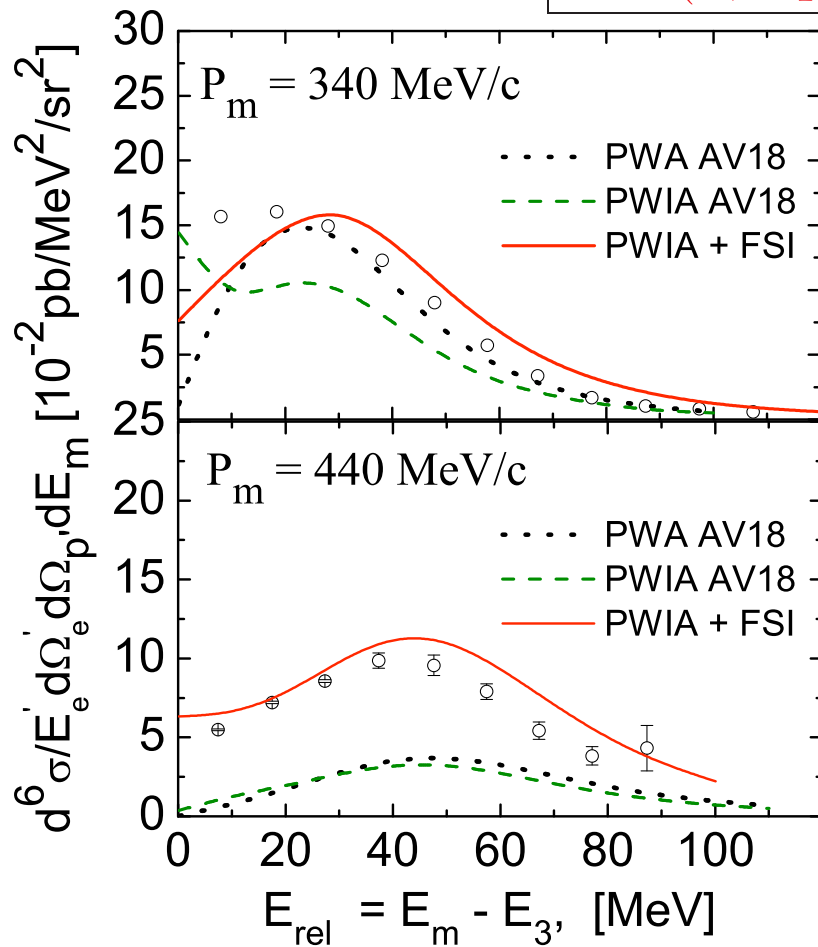
${}^3\text{He}(e, e'p){}^2\text{H}$: results



Data: *M.M. Rvachev et al., PRL94 (2005) 192302*

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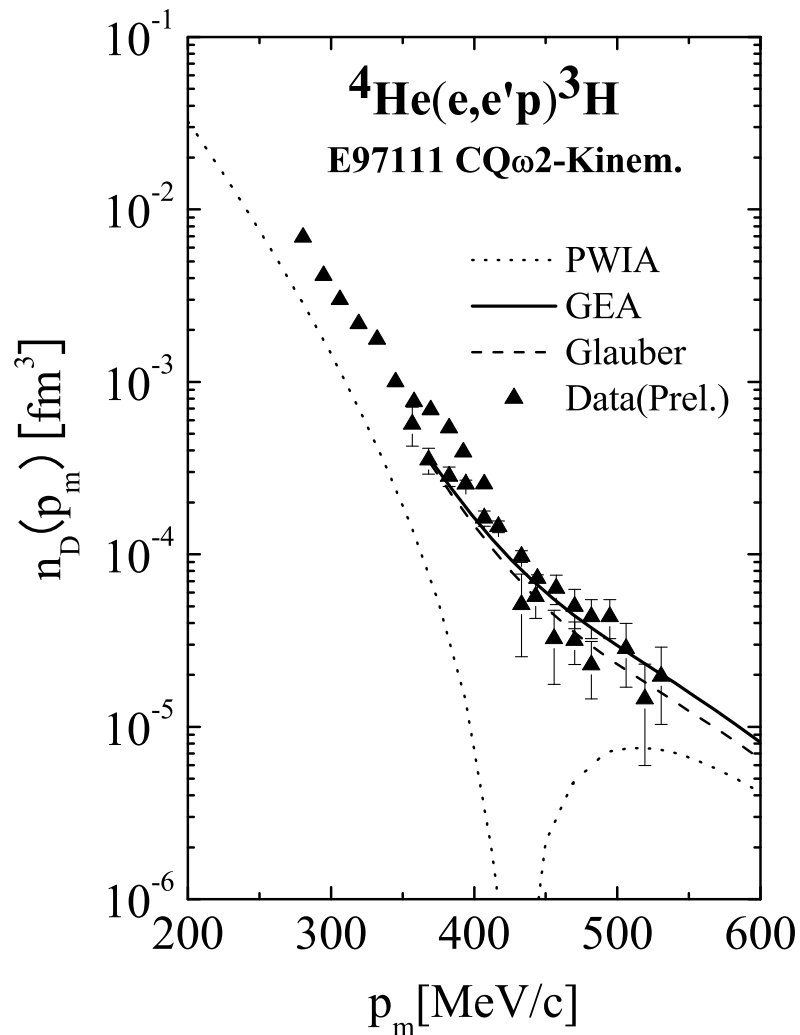
${}^3\text{He}(e, e'p)(pn)$: results



FSI in (n-p) pair- irrelevant; FSI in p-(n-p) system- very large

(preliminary un-factorized calculations show no sensible deviation)

${}^4\text{He}(e, e'p){}^3\text{He}$: results within factorized approximation



$$n_D(\mathbf{p}_m) = \frac{d^5\sigma}{d\Omega' dE' d\Omega_p} (\mathcal{K}\sigma_{ep})^{-1}$$

Data: *B. Reitz et al.*,
EPJ A **S19** (2004) 165

Calculations: *C. Ciofi degli Atti*,
L.P. Kaptari, H. Morita
in "Few Body Systems", *in press*
arXiv:0803.1162

A corresponding *un-factorized* calculation is in progress

2. Spectral Function for Complex Nuclei: TNC Model

$$P_A(|\mathbf{k}|, E) = P_0^A(|\mathbf{k}|, E) + P_1^A(|\mathbf{k}|, E)$$

$$P_0^A(|\mathbf{k}|, E) = \sum_{\alpha < \alpha_F} \tilde{n}_\alpha(|\mathbf{k}|) \delta(E - |\epsilon_\alpha|); \quad \int \tilde{n}_\alpha d\mathbf{k} < 1 \quad (\text{renormalized MF})$$

$$P_1^A(|\mathbf{k}|, E) = \sum_{f \neq \alpha} \left| \int d\mathbf{r} e^{i\mathbf{k} \cdot \mathbf{r}} G_{f0}(\vec{r}) \right|^2 \delta[E - (E_{A-1}^f - E_A)] \quad (\text{correlations})$$

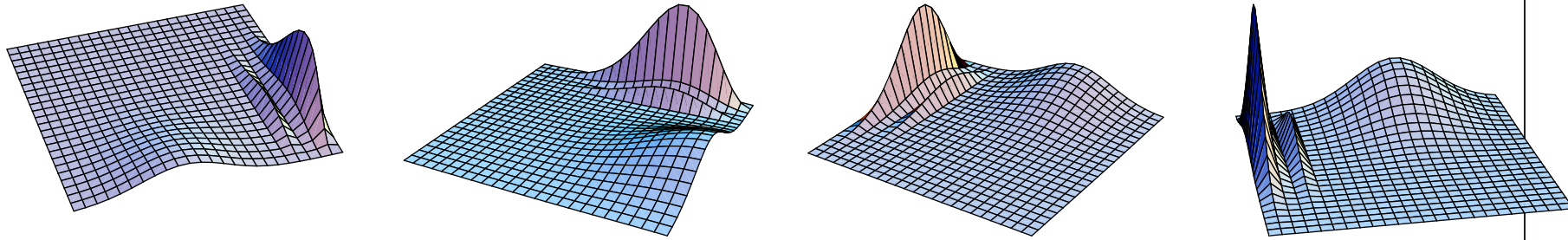
Two-Nucleon Correlation (TNC) Model

(Frankfurt & Strikman; Ciofi & Simula)

$$P_1^A(|\mathbf{k}|, E) = \int d\mathbf{P}_{cm} n_{rel}^A(|\mathbf{k} - \mathbf{P}_{cm}/2|) n_{cm}^A(|\mathbf{P}_{cm}|) \cdot \delta \left[E - E_{thr}^{(2)} - \frac{(A-2)}{2M(A-1)} \cdot \left(\mathbf{k} - \frac{(A-1)\mathbf{P}_{cm}}{(A-2)} \right)^2 \right]$$

TNC Spectral Function - ^{12}C

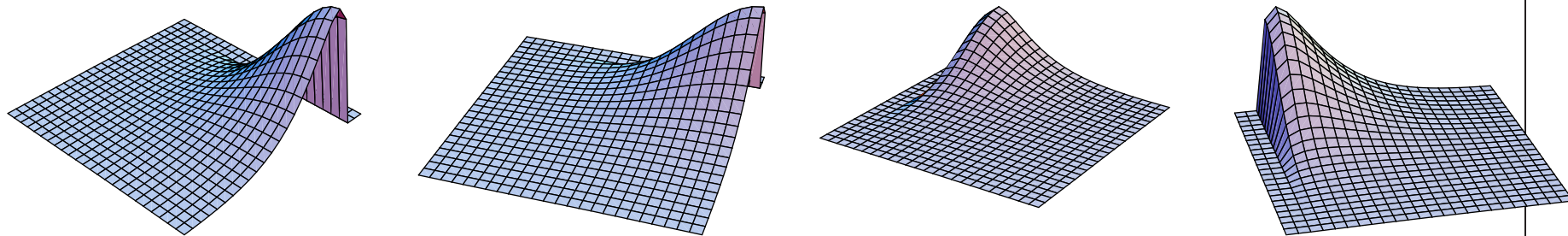
$P_0^p(k, E) :$



$$\int dE dk P_0^p(k, E) = 0.8 Z_p \longrightarrow$$

Shell Model: 80%

$P_1(k, E) :$



$$\int dE dk P_1^p(k, E) = 0.2 Z_p \longrightarrow$$

Correlations: 20%

C. Ciofi degli Atti, S. Simula - PRC 53 (1996)

Many-Body Calculations: Cluster Expansion

- The nuclear many-body problem:

$$\hat{\mathbf{H}} \Psi_n = E_n \Psi_n, \quad \hat{\mathbf{H}} = -\frac{\hbar^2}{2m} \sum_i \hat{\nabla}_i^2 + \frac{1}{2} \sum_{i < j} \hat{v}_{ij}$$

- The ground state energy E_0 is given by:

$$E_0 = -\frac{\hbar^2}{2m} \int d\mathbf{r} \left[\hat{\nabla}^2 \rho^{(1)}(\mathbf{r}, \mathbf{r}') \right]_{\mathbf{r}=\mathbf{r}'} + \sum_n \int d\mathbf{r}_1 d\mathbf{r}_2 \hat{v}^{(n)} \rho_{(n)}^{(2)}(\mathbf{r}_1, \mathbf{r}_2)$$

$$\longrightarrow \rho^{(1)}(\mathbf{r}, \mathbf{r}') = A \int \prod_{j=2}^A d\mathbf{r}_j \Psi_o^\dagger(\mathbf{r}, \mathbf{r}_2, \dots, \mathbf{r}_A) \Psi_o(\mathbf{r}', \mathbf{r}_2, \dots, \mathbf{r}_A)$$

$$\longrightarrow \rho_{(n)}^{(2)}(\mathbf{r}_1, \mathbf{r}_2) = \frac{A(A-1)}{2} \int \prod_{j=3}^A d\mathbf{r}_j \Psi_o^\dagger(\mathbf{r}_1, \dots, \mathbf{r}_A) \hat{O}_{12}^{(n)} \Psi_o(\mathbf{r}_1, \dots, \mathbf{r}_A)$$

- $\rho^{(1)}(\mathbf{r}, \mathbf{r}')$ and $\rho_{(n)}^{(2)}(\mathbf{r}_1, \mathbf{r}_2)$ are *cluster expanded*, with:

$$\Psi_o = \hat{\mathbf{F}} \phi_o$$

where ϕ_o is the *mean-field* wave function and

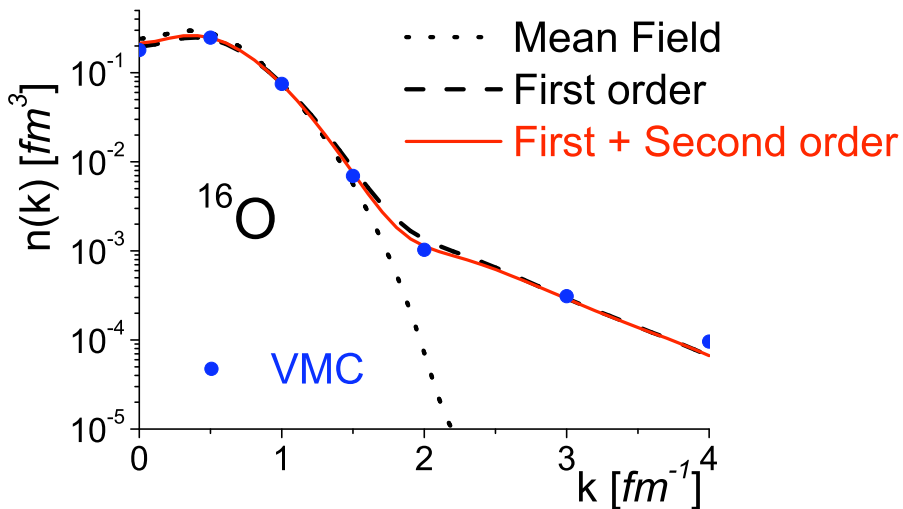
$$\hat{\mathbf{F}} = \hat{S} \prod_{i < j} \hat{f}_{ij} = \hat{S} \prod_{i < j} \sum_n f^{(n)}(r_{ij}) \hat{\mathcal{O}}_{ij}^{(n)}$$

is a *correlation* operator.

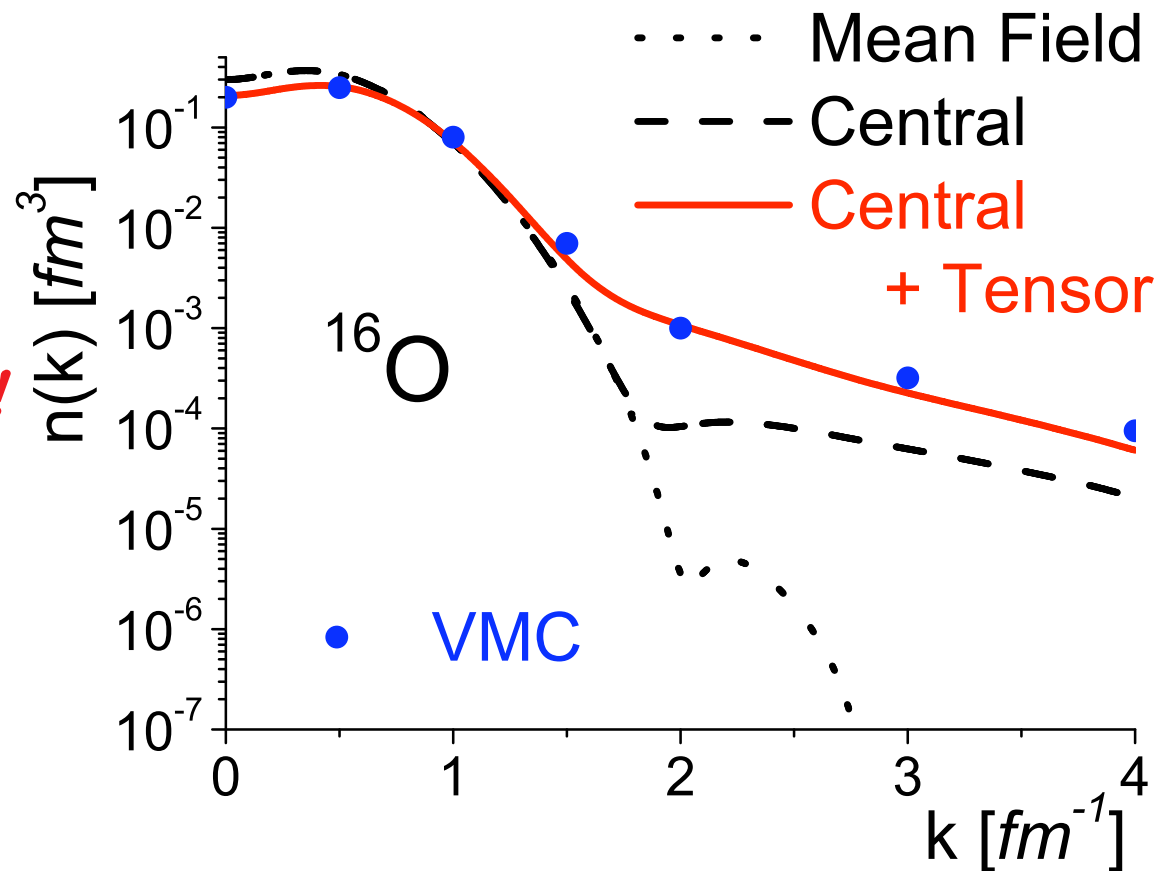
- cluster expansion truncated at *2nd order* in \hat{f}_{ij} ;
- the wave function and correlation functions which minimize the ground-state energy are used to calculate the *expectation value of any operator*
- we can calculate *diagonal* as well as *non-diagonal*, **one-** and **two-body realistically correlated** densities and *Spectral Functions*

(*M. Alvioli, C.Ciofi degli Atti, H.Morita, PRC72 (2005)*)

Momentum Distributions and Tensor Forces

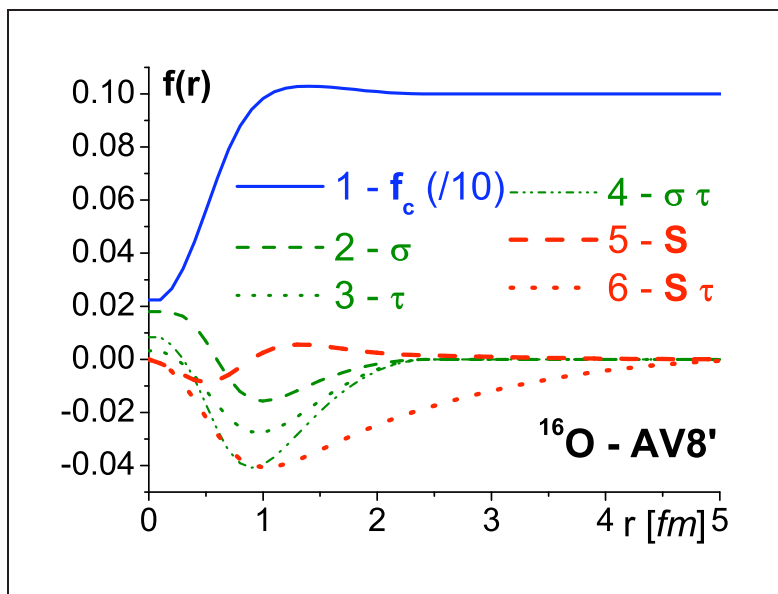


← cluster expansion convergence



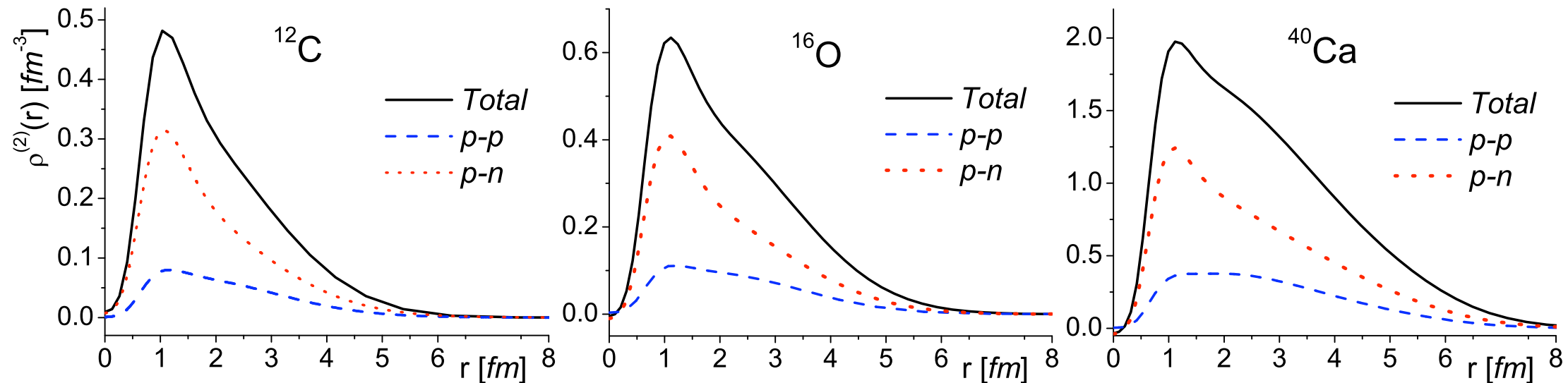
← correlation functions

Tensor ($S + S \otimes \tau$) correlations!
 $k > 1.5 \text{ fm}^{-1} \longrightarrow$



Two-Body Densities: isospin separation

$$\rho^{(2)}(r) = \int d\mathbf{R} \rho^{(2)} \left(\mathbf{R} + \frac{1}{2} \mathbf{r}, \mathbf{R} - \frac{1}{2} \mathbf{r} ; \mathbf{R} + \frac{1}{2} \mathbf{r}, \mathbf{R} - \frac{1}{2} \mathbf{r} \right)$$



- normalization (number of pairs) conserved by the expansion
- isospin separation feasible: $\rho^{(2)} = \rho_{(2)}^{pp} + \rho_{(2)}^{pn} + \rho_{(2)}^{nn}$
- We can build **two-body pp , pn and nn** momentum distributions
(M. Alvioli, C. Ciofi degli Atti, H. Morita, PRL100 (2008))

Two-Body Momentum Distributions

$$\mathbf{k}_{rel} \equiv \mathbf{k} = \frac{1}{2} (\mathbf{k}_1 - \mathbf{k}_2) \quad \mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2 \quad \mathbf{r}' = \mathbf{r}'_1 - \mathbf{r}'_2$$

$$\mathbf{K}_{CM} \equiv \mathbf{K} = \mathbf{k}_1 + \mathbf{k}_2 \quad \mathbf{R} = \frac{1}{2} (\mathbf{r}_1 + \mathbf{r}_2) \quad \mathbf{R}' = \frac{1}{2} (\mathbf{r}'_1 + \mathbf{r}'_2)$$

we have

$$n(\mathbf{k}, \mathbf{K}) = \frac{1}{(2\pi)^6} \int d\mathbf{r} d\mathbf{r}' d\mathbf{R} d\mathbf{R}' e^{-i\mathbf{K}\cdot(\mathbf{R}-\mathbf{R}')} e^{-i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')} \rho^{(2)}(\mathbf{r}, \mathbf{r}'; \mathbf{R}, \mathbf{R}')$$

and

$$n(\mathbf{k}) = \int d\mathbf{K} n(\mathbf{k}, \mathbf{K}) = \frac{1}{(2\pi)^3} \int d\mathbf{r} d\mathbf{r}' d\mathbf{R} e^{-i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')} \rho^{(2)}(\mathbf{r}, \mathbf{r}'; \mathbf{R}, \mathbf{R})$$

$$n(\mathbf{K}) = \int d\mathbf{k} n(\mathbf{k}, \mathbf{K}) = \frac{1}{(2\pi)^3} \int d\mathbf{r} d\mathbf{R} d\mathbf{R}' e^{-i\mathbf{K}\cdot(\mathbf{R}-\mathbf{R}')} \rho^{(2)}(\mathbf{r}, \mathbf{r}; \mathbf{R}, \mathbf{R}')$$

$\mathbf{K}_{CM} = 0$ corresponds to $\mathbf{k}_2 = -\mathbf{k}_1$, *i.e. back-to-back* nucleons

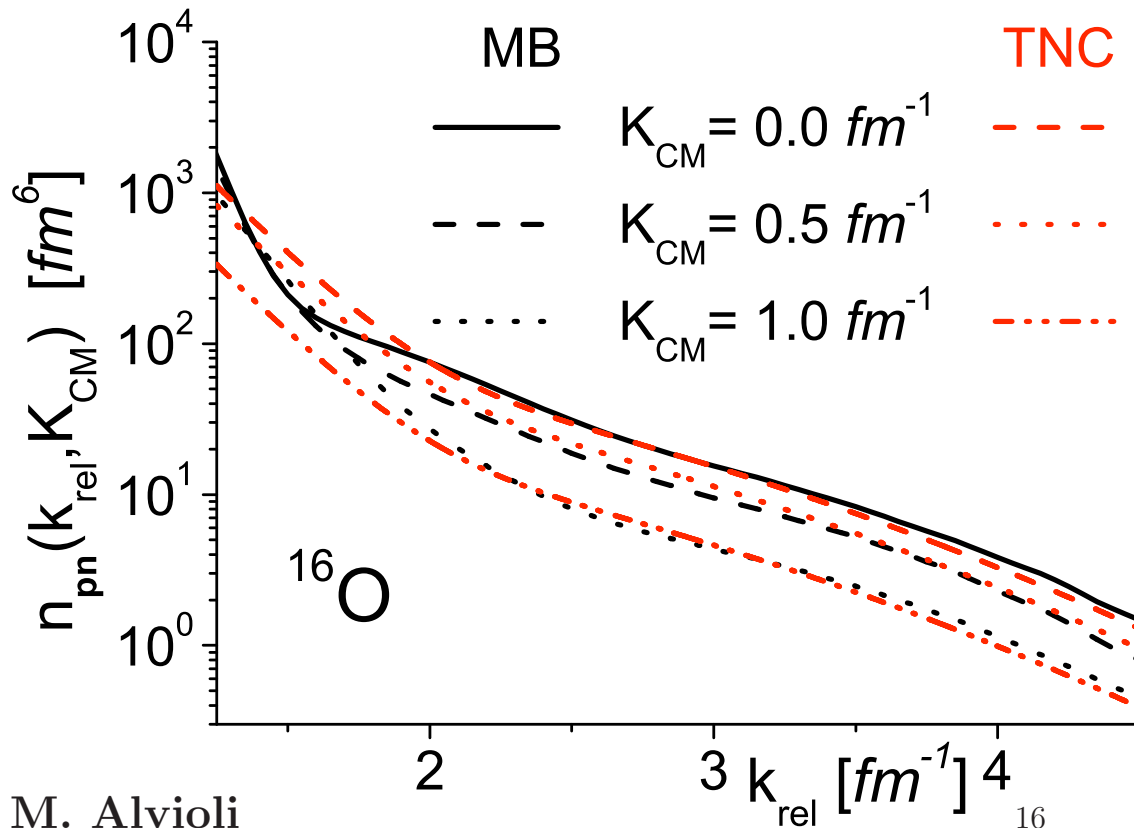
Spectral Function properties at low K_{CM} and high k_{rel}

1-body SF:

$$P_1^A(|\mathbf{k}|, E) = \int d\mathbf{K}_{cm} n_{rel}^A(|\mathbf{k} - \mathbf{K}_{cm}/2|) n_{cm}^A(|\mathbf{K}_{cm}|) \delta \left[E - E_{thr}^{(2)} - \frac{(A-2)}{2M(A-1)} \left(\mathbf{k} - \frac{(A-1)\mathbf{K}_{cm}}{(A-2)} \right)^2 \right]$$

2-body SF:

$$P_2^A(\mathbf{k}, \mathbf{K}_{cm}, E) = n_{rel}^A(|\mathbf{k} - \mathbf{K}_{cm}/2|) n_{cm}^A(|\mathbf{K}_{cm}|) \delta(E - E_{thr}^{(2)})$$

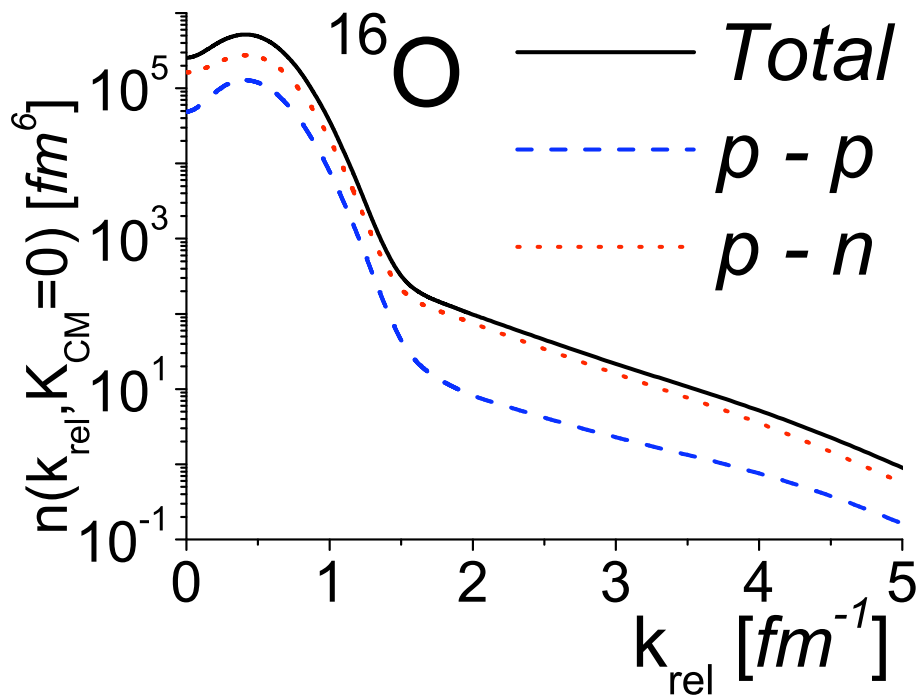


MB: $n_{pn}(\mathbf{k}_{rel}, \mathbf{K}_{cm})$

TNC: Ciofi, Simula
PRC53, (1996)
 $C_A n_{2H}(k_{rel}) n_{cm}(K_{cm})$

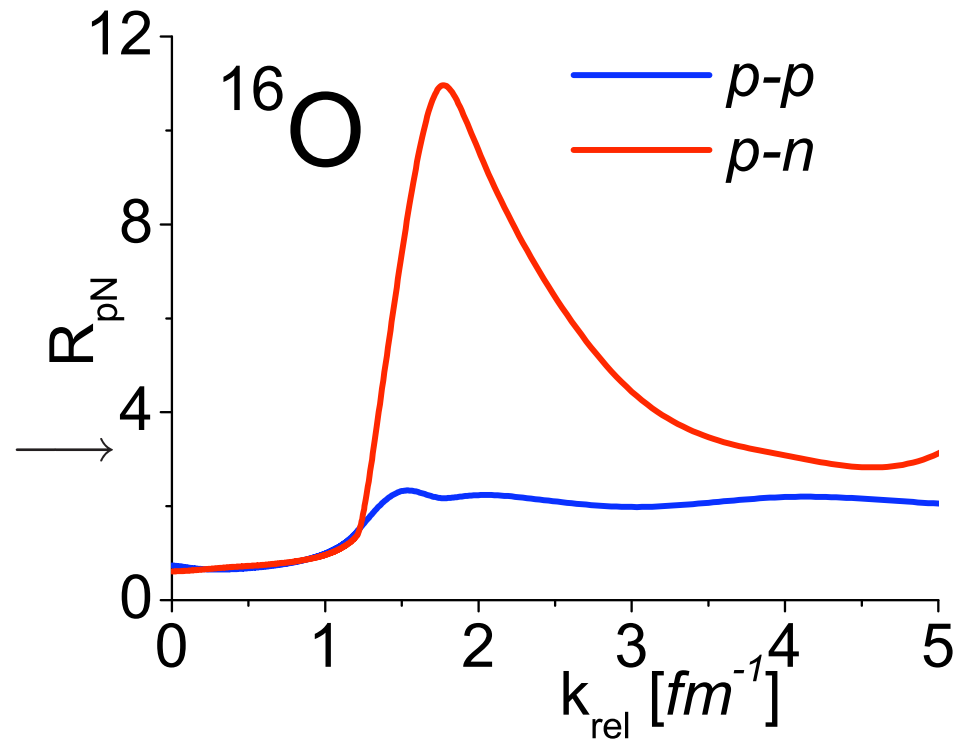
factorization of n_{pn}

Back-to-Back nucleons: pn to pp ratio



effect of tensor forces

$$R_{pN} = n_{pN}(k_{\text{rel}}, 0) / n_{pN}^{\text{central}}(k_{\text{rel}}, 0)$$



Back-to-Back nucleons: pn and pp probabilities

$$P_{pN} = \frac{\int_a^b dk_{rel} k_{rel}^2 n_{pN}(\mathbf{k}_{rel}, 0)}{\int_a^b dk_{rel} k_{rel}^2 (n_{pp}(\mathbf{k}_{rel}, 0) + n_{pn}(\mathbf{k}_{rel}, 0))}; \quad 0 < P_{pN} < 1$$

- integration over the whole k_{rel} range: $(a, b) = [0, \infty]$

A	4	12	16	40
P_{pp} (%)	19.7	30.6	29.5	31.0
P_{pn} (%)	81.3	69.4	70.5	69.0

(*Alvioli, Ciofi degli Atti, Morita PRL100 (2008)*)

- *correlation region*: $(a, b) = [1.5, 3.0] \text{ fm}^{-1}$

A	4	12	16	40
P_{pp} (%)	2.9	13.3	10.8	24.0
P_{pn} (%)	97.1	86.7	89.2	76.0

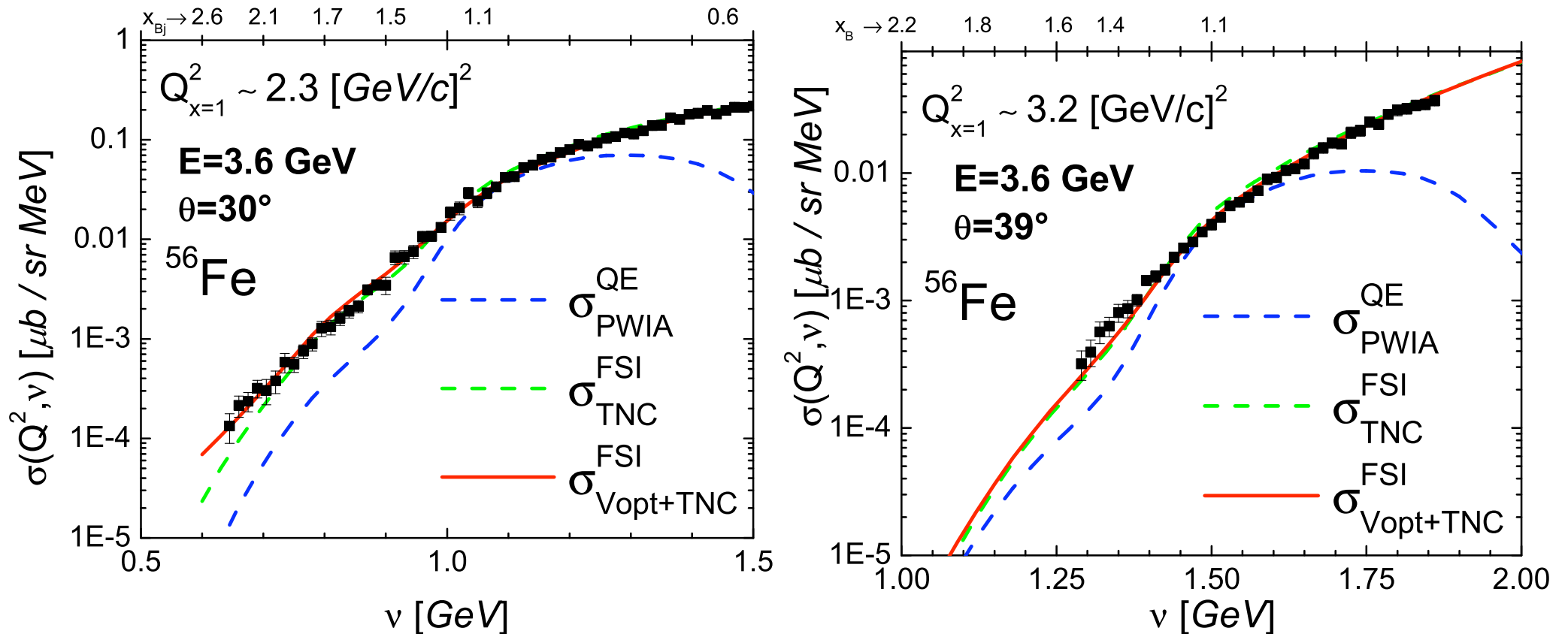
$P_{pN}^{A=4}$ in agreement with *Schiavilla et al., PRL98 (2007) 132501*

(extracted from published figures, AV18: $P_{pp} \simeq 3\%$, $P_{pn} \simeq 97\%$)

$P_{pp} \simeq 10 - 13\%$ consistent with *Shneor et al., PRL99 (2007) 072501*

(extracted from $^{12}\text{C}(e, e'pp)X / ^{12}\text{C}(e, e'p)X$)

3. Effect of NN correlations and FSI in Inclusive Processes

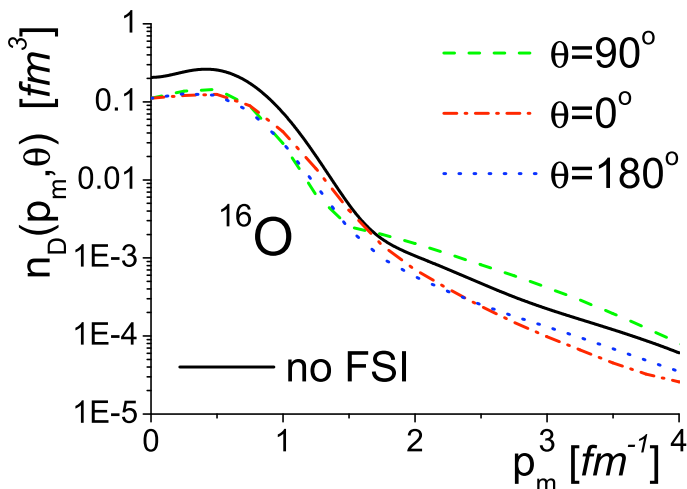
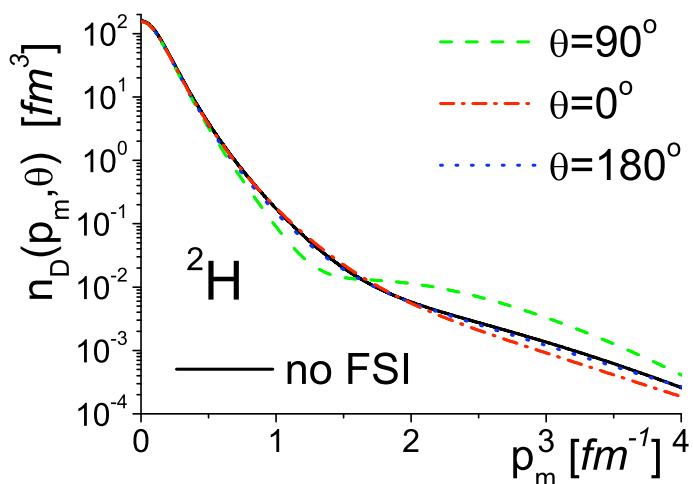


(Preliminary calculations)

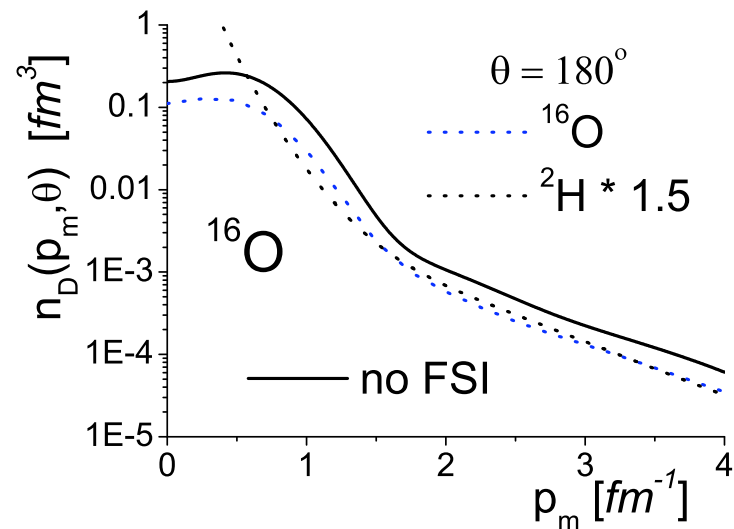
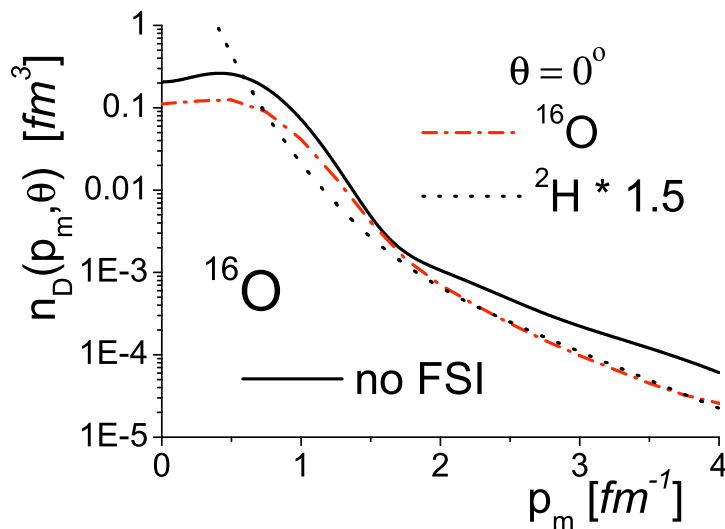
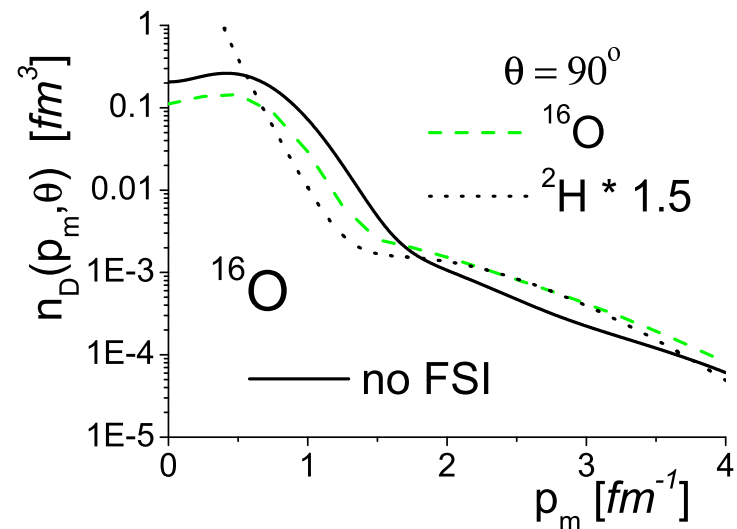
TNC: *OK* for $x_{Bj} \leq 2$: three-body correlations are missing.
 Calculations with many-body wave functions are in progress

Inclusive *Distorted* Momentum Distribution (Glauber)

well known fact: $n_A(k)$ proportional to $n_{2H}(k)$



$$\theta = \mathbf{p}_m \cdot \mathbf{q} / p_m q$$



new result: $n_A^D(k)$ proportional to $n_{2H}^D(k)$!

4. Effects of NN Correlations in High-Energy Processes

- SRC: are they relevant only in dedicated experiments?
- high-energy scattering processes \longrightarrow Glauber multiple scattering
- exact expansion of the many-body WF (*Glauber, Foldy & Walecka*):

$$\begin{aligned}
 |\Psi(\mathbf{r}_1, \dots, \mathbf{r}_A)|^2 &= \prod_{j=1}^A \rho(\mathbf{r}_j) + \sum_{i < j=1}^A \Delta(\mathbf{r}_i, \mathbf{r}_j) \prod_{k \neq (il)}^A \rho_1(\mathbf{r}_k) + \\
 &\quad + \sum_{(i < j) \neq (k < l)} \Delta(\mathbf{r}_i, \mathbf{r}_j) \Delta(\mathbf{r}_k, \mathbf{r}_l) \prod_{m \neq i, j, k, l} \rho_1(\mathbf{r}_m) + \dots \\
 &\simeq \prod_{j=1}^A \rho(\mathbf{r}_j) \longleftarrow \text{usual approximation : is it reliable?}
 \end{aligned}$$

our two-body $\Delta(\mathbf{r}_i, \mathbf{r}_j) = \rho_2(\mathbf{r}_i, \mathbf{r}_j) - \rho_1(\mathbf{r}_i) \rho_1(\mathbf{r}_j)$ provides:

$$\boxed{\int d\mathbf{r}_2 \rho_2(\mathbf{r}_1, \mathbf{r}_2) = \rho_1(\mathbf{r}_1)} \longrightarrow \boxed{\int d\mathbf{r}_2 \Delta(\mathbf{r}_1, \mathbf{r}_2) = 0}$$

Example: total *neutron-Nucleus* cross section at high energies

$$\sigma_{\text{tot}} = \frac{4\pi}{k} \text{Im} [F_{00}(0)] \quad F_{00}(\mathbf{q}) = \frac{ik}{2\pi} \int d^2b_n e^{i\mathbf{q}\cdot\mathbf{b}_n} \left[1 - e^{i\chi_{\text{opt}}(\mathbf{b}_n)} \right]$$

$$e^{i\chi_{\text{opt}}(\mathbf{b}_n)} = \int \prod_{j=1}^A d\mathbf{r}_j \prod_{j=1}^A [1 - \Gamma(\mathbf{b}_n - \mathbf{s}_j)] |\Psi_0(\mathbf{r}_1, \dots, \mathbf{r}_A)|^2 \delta\left(\frac{1}{A} \sum \mathbf{r}_j\right)$$

using the $|\Psi_0|^2$ expansion, with:

$$\Delta(\mathbf{r}_i, \mathbf{r}_j) = \rho_2(\mathbf{r}_i, \mathbf{r}_j) - \rho_1(\mathbf{r}_i) \rho_1(\mathbf{r}_j);$$

one has:

$$\sigma_{\text{tot}} = \sigma_G^{(1)} + \sigma_G^{(2)} + \Delta\sigma_{\text{in}}$$

$$\begin{aligned} \sigma_G^{(1)} + \sigma_G^{(2)} &= \\ &= 2 \int d\mathbf{b} \left\{ 1 - e^{-A \int d\mathbf{r}_1 \rho_1(\mathbf{r}_1) \Gamma(\mathbf{b}-\mathbf{b}_1)} e^{\frac{A^2}{2} \int d\mathbf{r}_1 d\mathbf{r}_2 \Delta(\mathbf{r}_1, \mathbf{r}_2) \Gamma(\mathbf{b}-\mathbf{b}_1) \Gamma(\mathbf{b}-\mathbf{b}_2)} \right\} \end{aligned}$$

$$\text{no correlation case} \longrightarrow \Delta = 0 \longrightarrow \sigma_G^{(2)} = 0$$

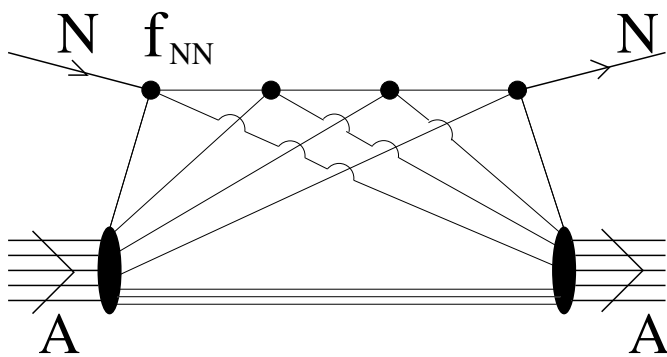
Glauber + Inelastic shadowing

(Diffractive excitation of the projectile)

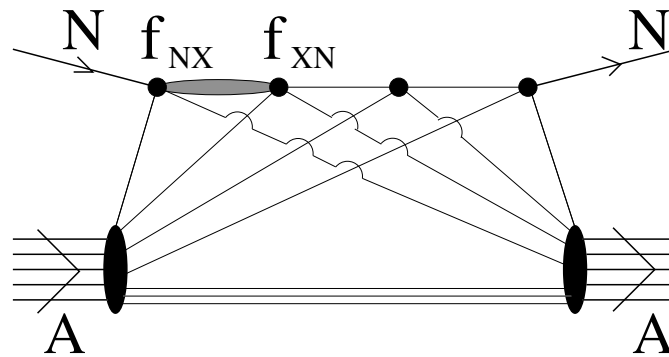
V. N. Gribov, Sov. JETP 29 (1969) 483;

V.A.Karmanov, L.A.Kondratyuk, JETP Lett. 18 (1973) 451;

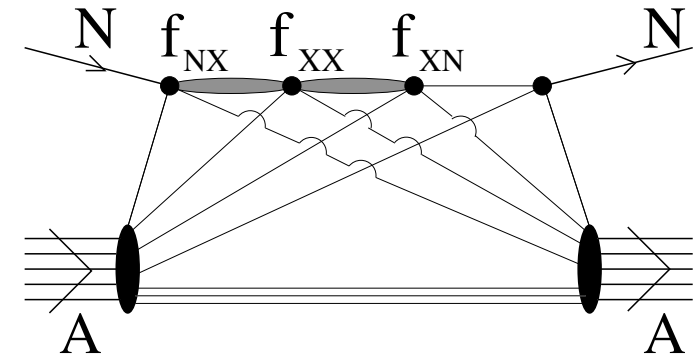
B. K. Jennings, G. A. Miller, Phys. Rev. C49 (1999) 2637



(Glauber)



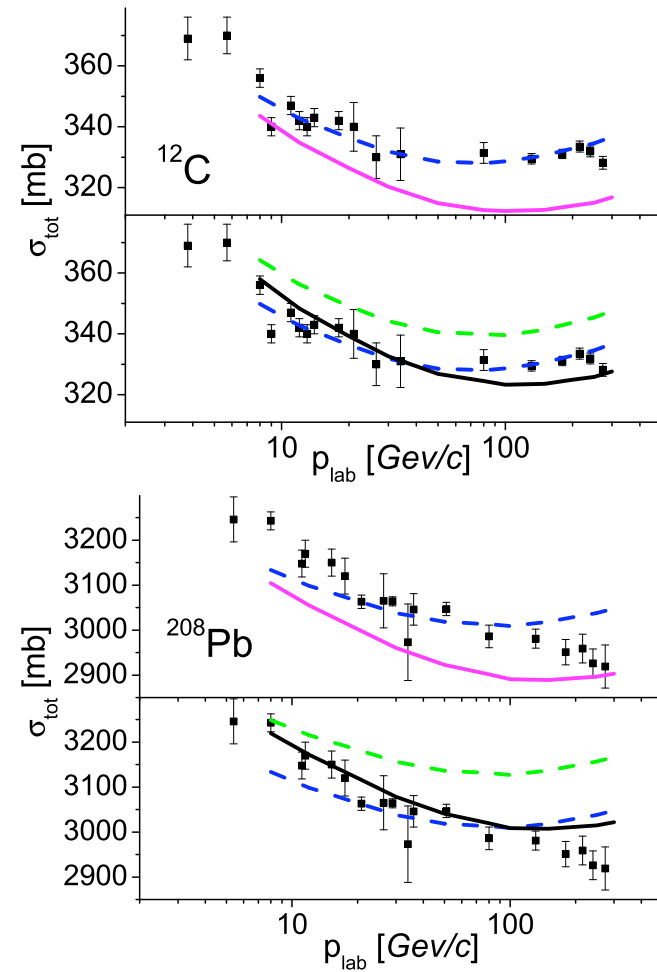
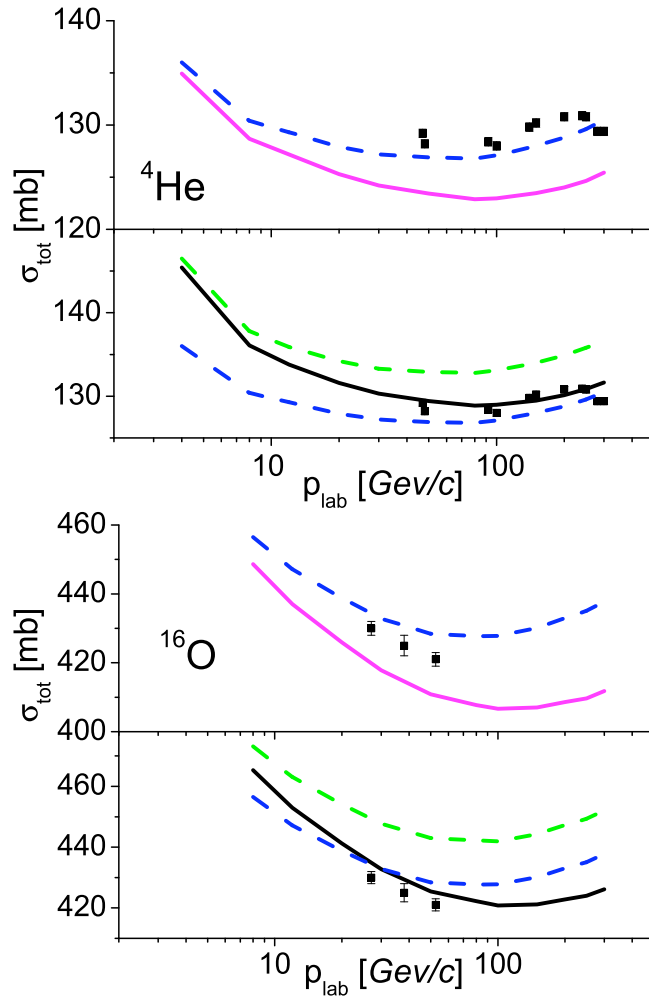
(Inelastic Shadowing)



total neutron-Nucleus cross section:

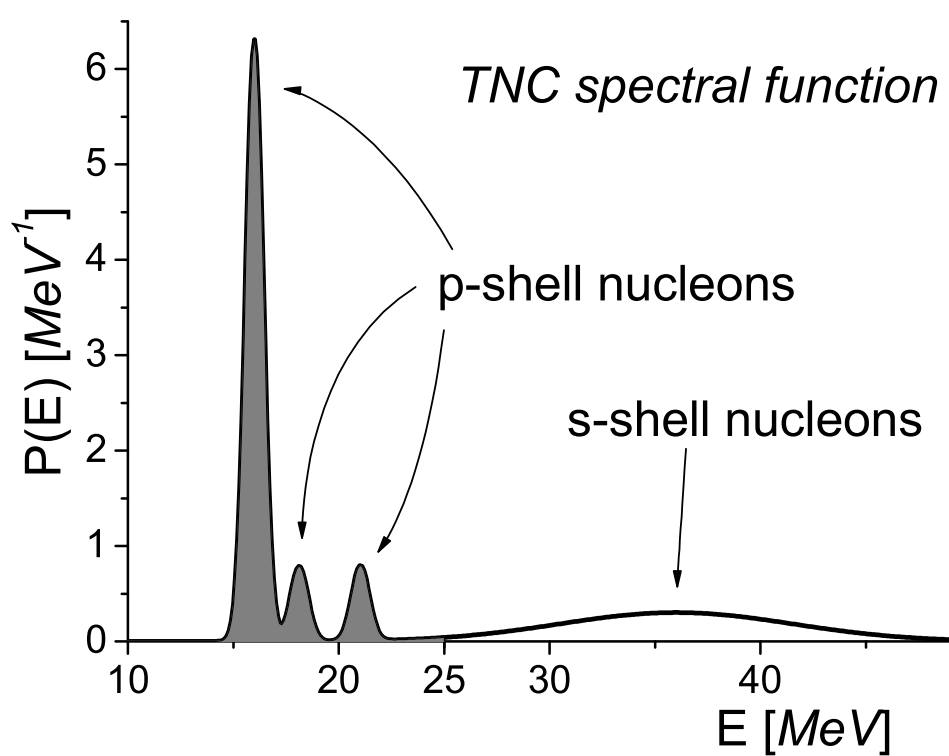
$$\sigma_{tot} = \frac{4\pi}{k} \text{Im} \left[F_{00}^G(0) \right] + \Delta\sigma_{in} = \sigma_G^{(1)} + \sigma_G^{(2)} + \Delta\sigma_{in}$$

RESULTS: $\sigma_{\text{tot}} = \sigma_{\text{G}}^{(1)} + \sigma_{\text{G}}^{(2)} + \Delta\sigma_{\text{in}}$

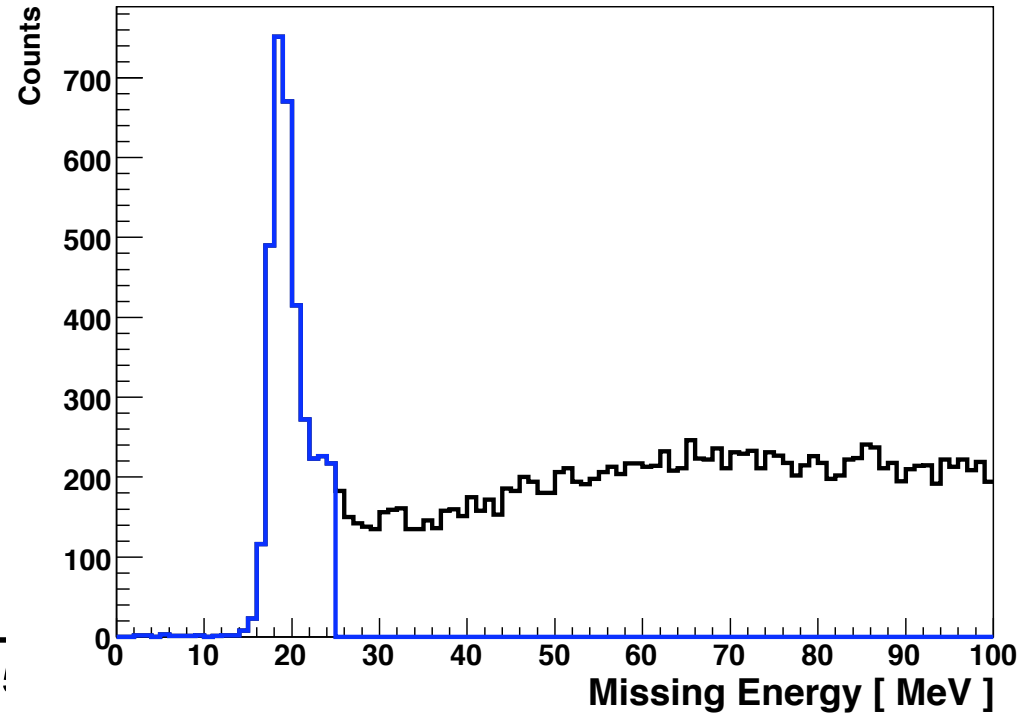


*M. Alvioli, Ciofi degli Atti, I. Marchino, H. Morita and V. Palli,
nucl-th:07053613 - submitted for publication*

The process $^{12}\text{C}(e, e'p)^{11}\text{B}$



↑
TNC spectral function



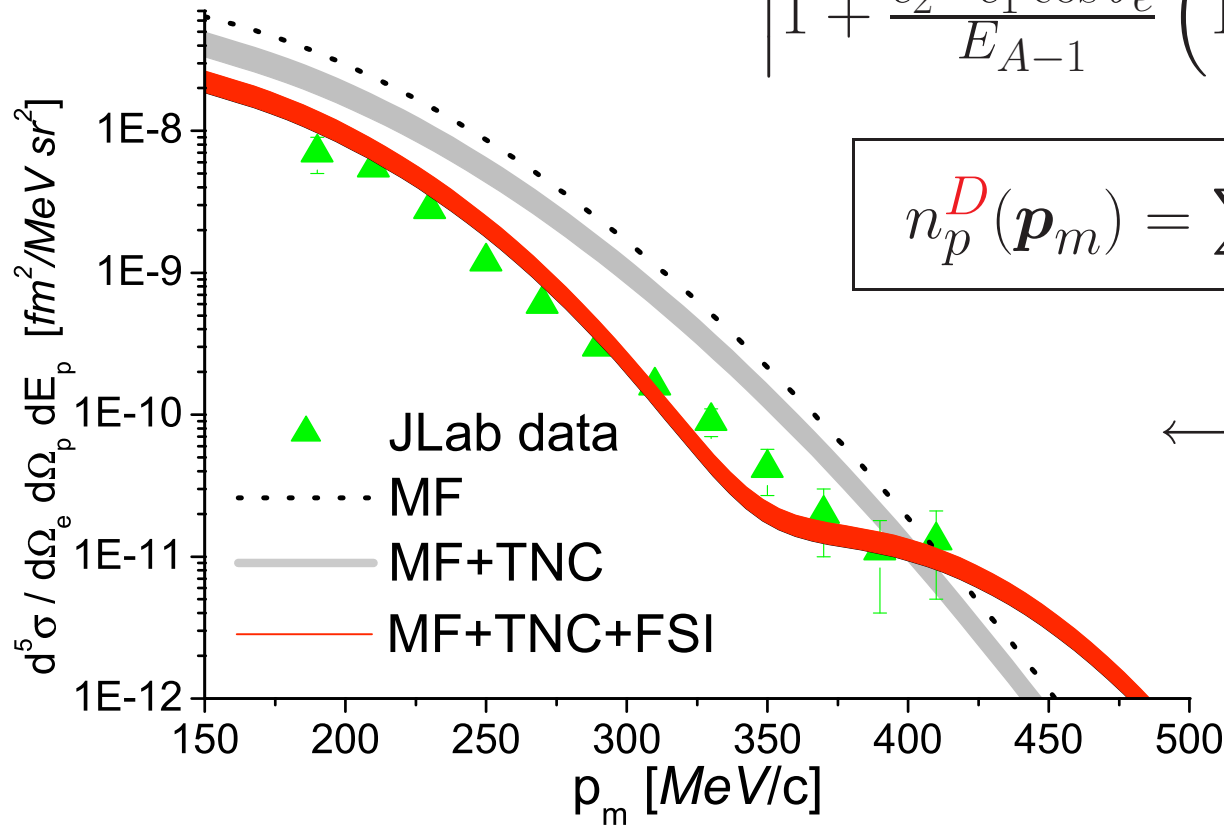
↑
JLab experiment

$$n_p(k) = \int_0^{E_M} dE P_0(k, E), \quad \text{with } E_M = 25 \text{ MeV}$$

$^{12}\text{C}(e, e'p)^{11}\text{B}$ factorized cross section

$$\frac{d^5\sigma}{d\Omega_e d\Omega_p dE_p} = \mathcal{K} \sigma_{ep} Z_p \int_0^{E_M} dE P_0^D(kE) =$$

$$= \frac{E_p p \sigma_{ep} Z_p N_p}{\left| 1 + \frac{\epsilon_2 - \epsilon_1 \cos \theta_e}{E_{A-1}} \left(1 - \frac{p}{q} \cos \theta_{pq} \right) \right|} n_p^D(\mathbf{p}_m)$$



$$n_p^D(\mathbf{p}_m) = \sum_{\alpha} \left| \int d\mathbf{r} e^{i\mathbf{p}_m \cdot \mathbf{r}} \varphi_{\alpha}(\mathbf{r}) G^{A-1}(\mathbf{r}) \right|^2$$

grey and red bands
 ← due to our present uncertainty
 in the calculation of
 p-shell spectroscopic factors
 $0.5 < N_p < 0.7$
 (*Nikhef*: ~ 0.5)

un-factorized calculation feasible

Conclusions

- Exclusive processes off few-body nuclei can be reliably described within generalized Glauber FSI and realistic wave functions
- Cross sections and momentum distributions in inclusive reactions suggest that both ground state high momentum components and FSI in complex nuclei can be described with properly *scaled* 2H corresponding quantities
- Many-body calculations can be reliably performed within a convergent cluster expansion method: any one and two-body quantity can be calculated
- Tensor and Isospin-Tensor correlations appear to be the essential ingredients for a correct description of (one- and two-body) momentum distributions in complex nuclei; at high momentum values TNC model OK
- High-energy processes are also affected by NN correlations and their contribution can be quantitatively be evaluated within the "exact" $|\Psi_o|^2$ expression with two-body correlations

Additional Slides

The Nuclear Many-Body Problem

$$\hat{\mathbf{H}} \Psi_n = E_n \Psi_n, \quad \text{with:} \quad \hat{\mathbf{H}} = -\frac{\hbar^2}{2m} \sum_i \hat{\nabla}_i^2 + \frac{1}{2} \sum_{i<j} \hat{v}_{ij}$$

where

$$\hat{v}_{ij} = \sum_n v^{(n)}(r_{ij}) \hat{\mathcal{O}}_{ij}^{(n)}$$

$$\hat{\mathcal{O}}_{ij}^{(n)} = \left[1, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j, \hat{S}_{ij}, (\mathbf{L} \cdot \mathbf{S})_{ij}, \dots \right] \otimes [1, \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j] .$$

The same operatorial dependence is cast onto Ψ_o :

$$\Psi_o = \hat{\mathbf{F}} \phi_o$$

where ϕ_o is the *mean-field* wave function and

$$\hat{\mathbf{F}} = \hat{S} \prod_{i<j} \hat{f}_{ij} = \hat{S} \prod_{i<j} \sum_n f^{(n)}(r_{ij}) \hat{\mathcal{O}}_{ij}^{(n)}$$

is a *correlation* operator.

• at **first order** of the η -expansion, the **full correlated one-body mixed density matrix** expression is as follows:

$$\rho^{(1)}(\mathbf{r}_1, \mathbf{r}'_1) = \rho_o^{(1)}(\mathbf{r}_1, \mathbf{r}'_1) + \rho_H^{(1)}(\mathbf{r}_1, \mathbf{r}'_1) + \rho_S^{(1)}(\mathbf{r}_1, \mathbf{r}'_1),$$

with

$$\rho_H^{(1)}(\mathbf{r}_1, \mathbf{r}'_1) = \int d\mathbf{r}_2 \left[H_D(r_{12}, r_{1'2}) \rho_o^{(1)}(\mathbf{r}_1, \mathbf{r}'_1) \rho_o(\mathbf{r}_2) - H_E(r_{12}, r_{1'2}) \rho_o^{(1)}(\mathbf{r}_1, \mathbf{r}_2) \rho_o^{(1)}(\mathbf{r}_2, \mathbf{r}'_1) \right]$$

$$\rho_S^{(1)}(\mathbf{r}_1, \mathbf{r}'_1) = - \int d\mathbf{r}_2 d\mathbf{r}_3 \rho_o^{(1)}(\mathbf{r}_1, \mathbf{r}_2) \left[H_D(r_{23}) \rho_o^{(1)}(\mathbf{r}_2, \mathbf{r}'_1) \rho_o(\mathbf{r}_3) - H_E(r_{23}) \rho_o^{(1)}(\mathbf{r}_2, \mathbf{r}_3) \rho_o^{(1)}(\mathbf{r}_3, \mathbf{r}'_1) \right]$$

and the functions H_D and H_E are defined as:

$$H_{D(E)}(r_{ij}, r_{kl}) = \sum_{p,q=1}^6 f^{(p)}(r_{ij}) f^{(q)}(r_{kl}) C_{D(E)}^{(p,q)}(r_{ij}, r_{kl}) - C_{D(E)}^{(1,1)}(r_{ij}, r_{kl})$$

with $C_{D(E)}^{(p,q)}(r_{ij}, r_{kl})$ proper functions arising from spin-isospin traces;

(Alvioli, Ciofi degli Atti, Morita, PRC72 (2005))

• at **first order** of the η –expansion, the **full correlated two-body mixed** density matrix expression is as follows:

$$\rho^{(2)}(\mathbf{r}_1, \mathbf{r}_2; \mathbf{r}'_1, \mathbf{r}'_2) = \rho_{\text{SM}}^{(2)}(\mathbf{r}_1, \mathbf{r}_2; \mathbf{r}'_1, \mathbf{r}'_2) + \rho_{2\text{b}}^{(2)}(\mathbf{r}_1, \mathbf{r}_2; \mathbf{r}'_1, \mathbf{r}'_2) + \rho_{3\text{b}}^{(2)}(\mathbf{r}_1, \mathbf{r}_2; \mathbf{r}'_1, \mathbf{r}'_2) + \rho_{4\text{b}}^{(2)}(\mathbf{r}_1, \mathbf{r}_2; \mathbf{r}'_1, \mathbf{r}'_2)$$

with:

$$\rho_{\text{SM}}^{(2)}(\mathbf{r}_1, \mathbf{r}_2; \mathbf{r}'_1, \mathbf{r}'_2) = C_D \rho_o(\mathbf{r}_1, \mathbf{r}'_1) \rho_o(\mathbf{r}_2, \mathbf{r}'_2) - C_E \rho_o(\mathbf{r}_1, \mathbf{r}'_2) \rho_o(\mathbf{r}_2, \mathbf{r}'_1)$$

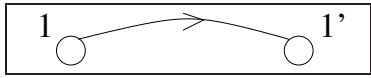
$$\rho_{2\text{b}}^{(2)}(\mathbf{r}_1, \mathbf{r}_2; \mathbf{r}'_1, \mathbf{r}'_2) = \frac{1}{2} \hat{\eta}(r_{12}, r_{1'2'}) \rho_o(\mathbf{r}_1, \mathbf{r}'_1) \rho_o(\mathbf{r}_2, \mathbf{r}'_2) - \frac{1}{2} \hat{\eta}(r_{12}, r_{1'2'}) \rho_o(\mathbf{r}_1, \mathbf{r}'_2) \rho_o(\mathbf{r}_2, \mathbf{r}'_1)$$

$$\begin{aligned} \rho_{3\text{b}}^{(2)}(\mathbf{r}_1, \mathbf{r}_2; \mathbf{r}'_1, \mathbf{r}'_2) = & \int d\mathbf{r}_3 \hat{\eta}(r_{13}, r_{1'3}) [\rho_o(\mathbf{r}_1, \mathbf{r}'_1) \rho_o(\mathbf{r}_2, \mathbf{r}'_2) \rho_o(\mathbf{r}_3, \mathbf{r}_3) + \\ & - \rho_o(\mathbf{r}_1, \mathbf{r}'_1) \rho_o(\mathbf{r}_2, \mathbf{r}_3) \rho_o(\mathbf{r}_3, \mathbf{r}'_2) + \\ & + \rho_o(\mathbf{r}_1, \mathbf{r}_3) \rho_o(\mathbf{r}_2, \mathbf{r}'_1) \rho_o(\mathbf{r}_3, \mathbf{r}'_2) + \\ & - \rho_o(\mathbf{r}_1, \mathbf{r}_3) \rho_o(\mathbf{r}_2, \mathbf{r}'_2) \rho_o(\mathbf{r}_3, \mathbf{r}'_1) + \\ & + \rho_o(\mathbf{r}_1, \mathbf{r}'_2) \rho_o(\mathbf{r}_2, \mathbf{r}_3) \rho_o(\mathbf{r}_3, \mathbf{r}'_1) + \\ & - \rho_o(\mathbf{r}_1, \mathbf{r}'_2) \rho_o(\mathbf{r}_2, \mathbf{r}'_1) \rho_o(\mathbf{r}_3, \mathbf{r}_3)] \end{aligned}$$

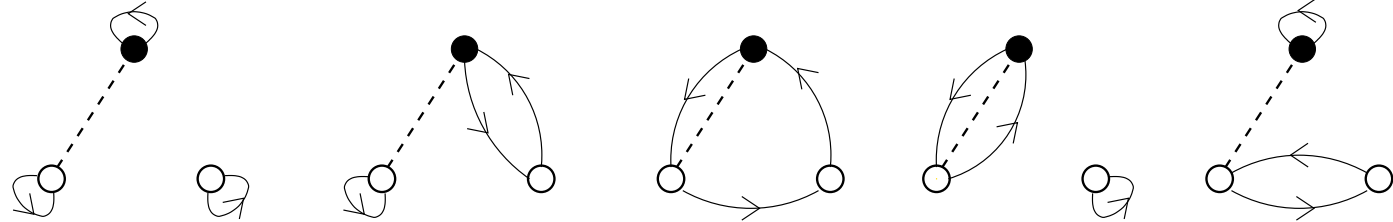
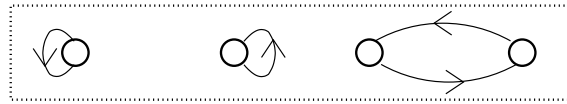
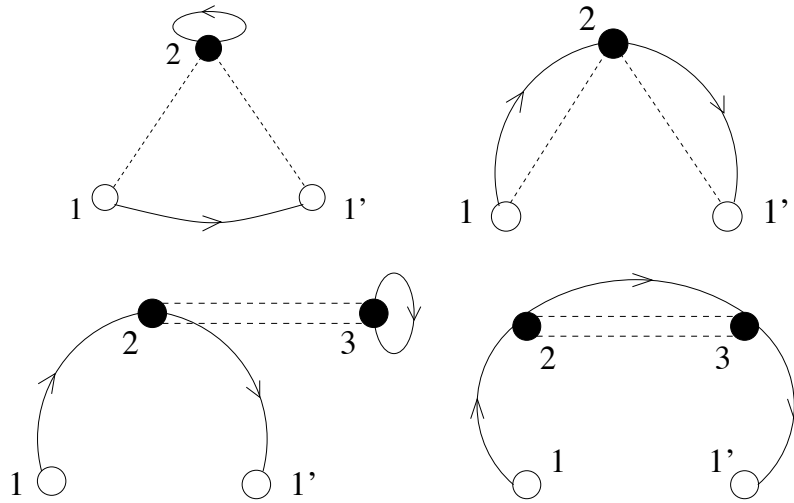
$$\begin{aligned} \rho_{4\text{b}}^{(2)}(\mathbf{r}_1, \mathbf{r}_2; \mathbf{r}'_1, \mathbf{r}'_2) = & \frac{1}{4} \int d\mathbf{r}_3 d\mathbf{r}_4 \hat{\eta}(r_{34}) \cdot \\ & \cdot \sum_{\mathcal{P} \in \mathcal{C}} (-1)^{\mathcal{P}} [\rho_o(\mathbf{r}_1, \mathbf{r}_{\mathcal{P}1'}) \rho_o(\mathbf{r}_2, \mathbf{r}_{\mathcal{P}2'}) \rho_o(\mathbf{r}_3, \mathbf{r}_{\mathcal{P}3}) \rho_o(\mathbf{r}_4, \mathbf{r}_{\mathcal{P}4})] \end{aligned}$$

(*Alvioli, Ciofi degli Atti, Morita, PRC72 (2005)*)

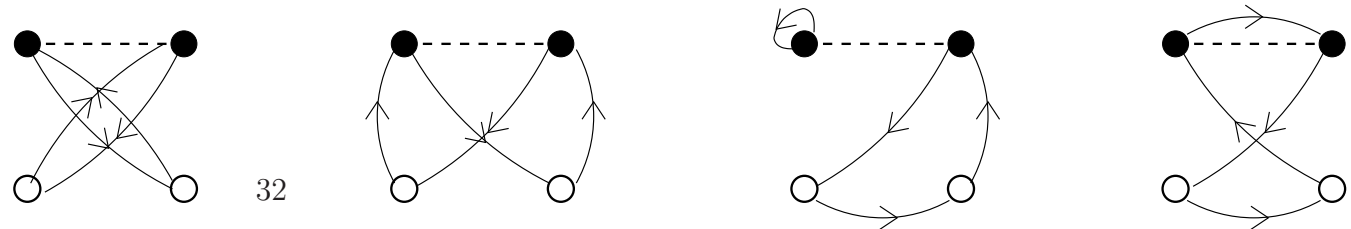
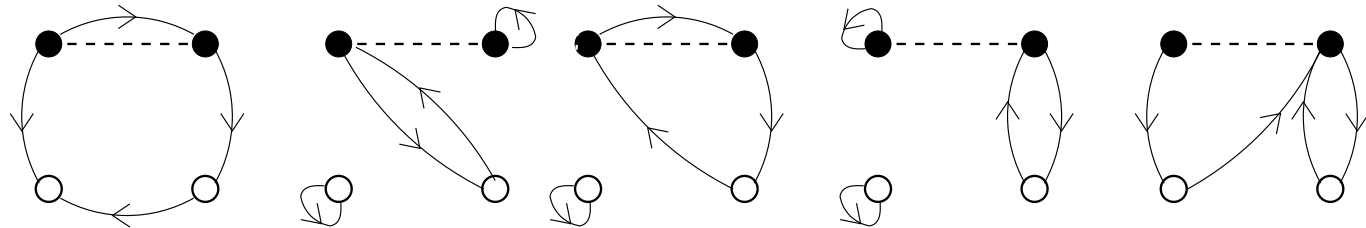
(*Alvioli, Ciofi degli Atti, Morita, PRL100 (2008)*)



one-body, non-diagonal
 $\leftarrow \rho(\mathbf{r}_1, \mathbf{r}'_1)$ diagrams



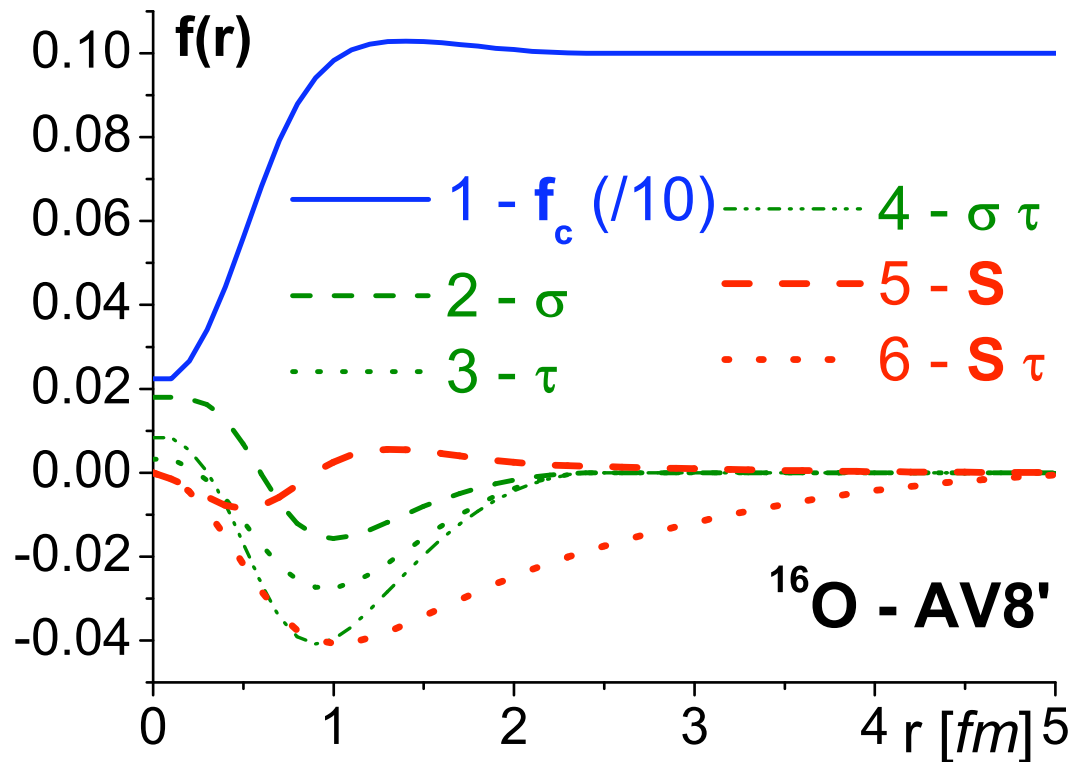
two-body, diagonal
 $\rho(\mathbf{r}_1, \mathbf{r}_2)$ diagrams \longrightarrow



Ground state energy: ^{16}O - Argonne $V8'$

	$\langle V_c \rangle$	$\langle V_\sigma \rangle$	$\langle V_\tau \rangle$	$\langle V_{\sigma\tau} \rangle$	$\langle V_S \rangle$	$\langle V_{S\tau} \rangle$	$\langle \mathbf{V} \rangle$	$\langle \mathbf{T} \rangle$	\mathbf{E}	\mathbf{E}/\mathbf{A} MeV
η -exp	0.19	-35.88	-9.47	-171.32	-0.003	-172.89	-389.40	323.50	-65.90	-4.12
FHNC	0.694	-40.13	-10.61	-180.00	-0.07	-160.32	-390.30	325.18	-65.12	-4.07

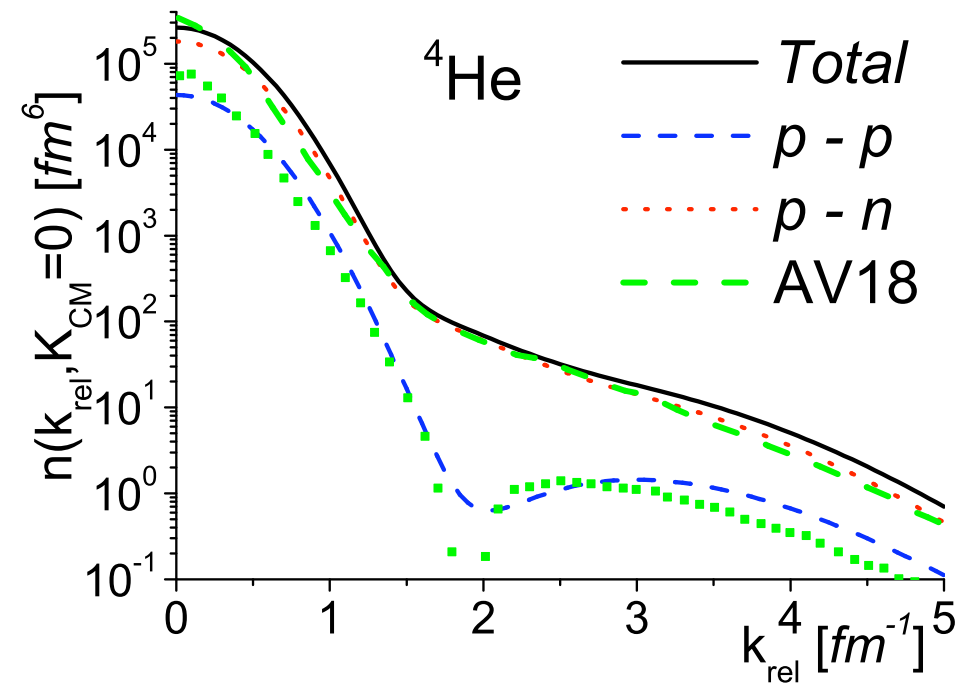
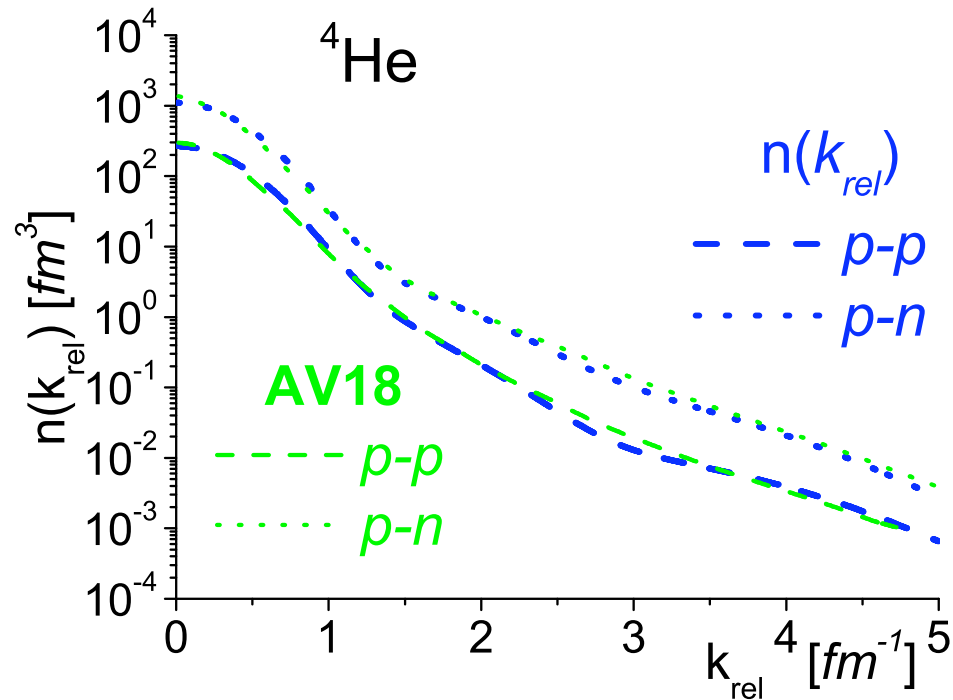
correlation functions: *Central*, *Spin-Isospin*, *Tensor*



${}^4\text{He}$: comparison with VMC

$$n_{pN}(k_{rel}) = \int d\mathbf{K}_{CM} n_{pN}(\mathbf{k}_{rel}, \mathbf{K}_{CM})$$

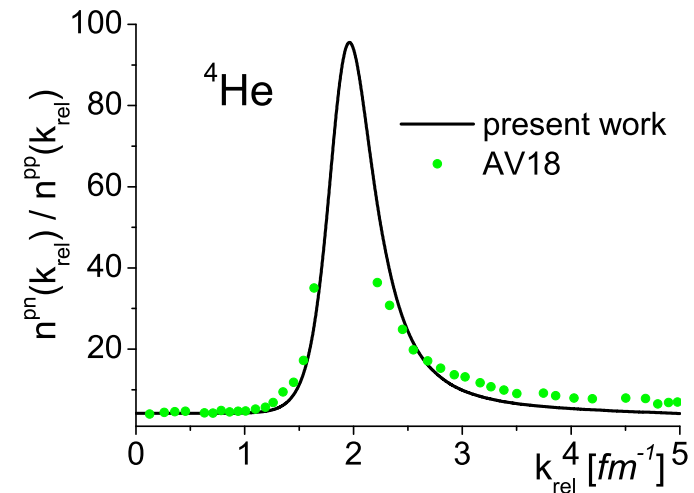
$$n_{NN}(\mathbf{k}_{rel}, \mathbf{K}_{CM} = 0)$$



● good agreement with VMC calculations

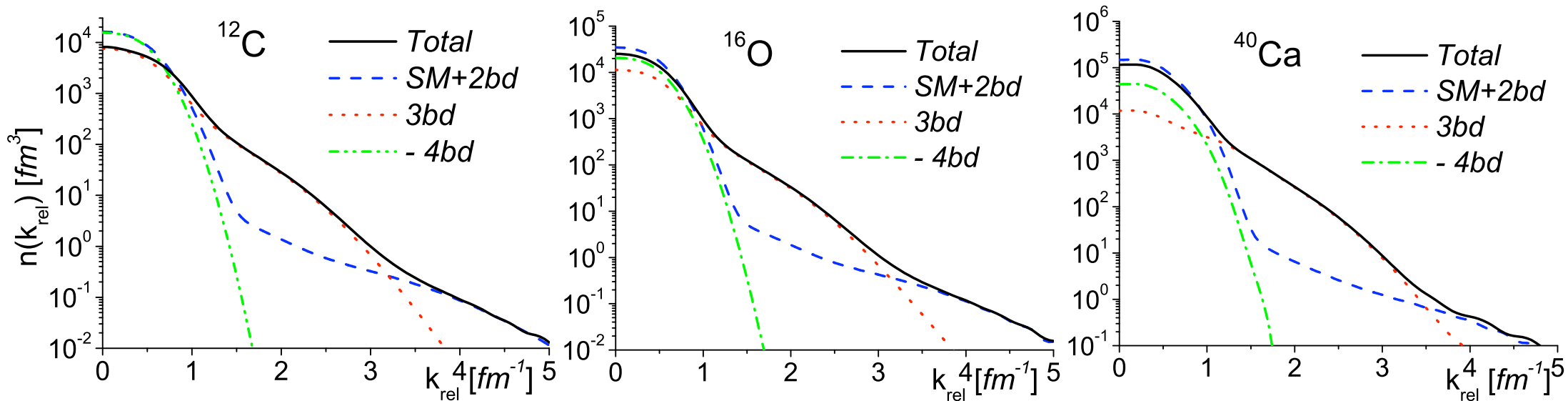
● $n_{pn}(k_{rel}, 0)/n_{pp}(k_{rel}, 0)$ peak location ok →

(AV18: Schiavilla et al. PRL98 (2007))



$n_{NN}(k_{rel})$ for Complex Nuclei

$$n_{NN}(k_{rel}) = \int d\mathbf{K}_{CM} n_{NN}(\mathbf{k}_{rel}, \mathbf{K}_{CM})$$



- normalization (number of pairs) conserved by the expansion
- isospin separation feasible
- closed j-shell nuclei included in the formalism
- *three* and *four-body* diagrams **essential**